Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.6-Hyperbolic-cosecant/183-6.6.3-Hyperbolic-cosecant-functions

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [175]. This is test number [183].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (175)	0.00 (0)
Mathematica	100.00 (175)	0.00 (0)
Fricas	96.57 (169)	3.43 (6)
Maple	77.71 (136)	22.29 (39)
Maxima	62.29 (109)	37.71 (66)
Giac	61.14 (107)	38.86 (68)
Mupad	52.00 (91)	48.00 (84)
Sympy	1.71 (3)	98.29 (172)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

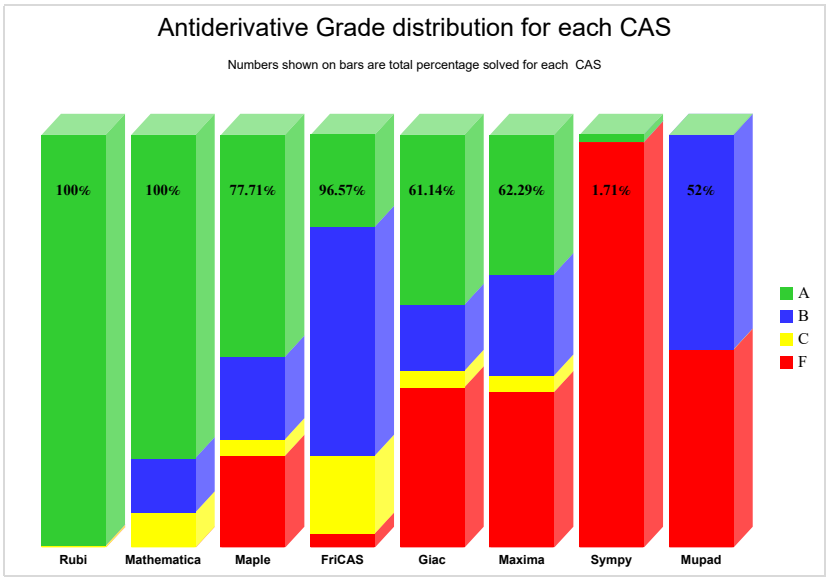
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

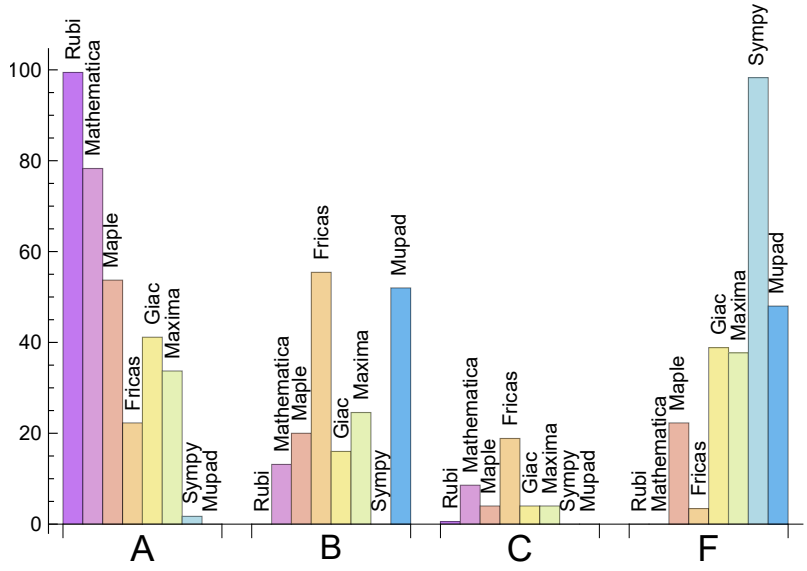
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.429	0.000	0.571	0.000
Mathematica	78.286	13.143	8.571	0.000
Maple	53.714	20.000	4.000	22.286
Giac	41.143	16.000	4.000	38.857
Maxima	33.714	24.571	4.000	37.714
Fricas	22.286	55.429	18.857	3.429
Sympy	1.714	0.000	0.000	98.286
Mupad	0.000	52.000	0.000	48.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of

error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	6	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Maxima	66	96.97	0.00	3.03
Giac	68	58.82	36.76	4.41
Mupad	84	0.00	100.00	0.00
Sympy	172	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.06
Fricas	0.23
Maxima	0.23
Giac	0.29
Sympy	0.30
Mathematica	0.40
Mupad	2.11
Maple	3.75

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	27.00	1.60	22.00	1.42
Rubi	68.71	1.01	62.00	1.00
Mathematica	75.07	1.31	62.00	1.00
Giac	91.97	1.64	62.00	1.46
Maple	102.65	1.78	87.00	1.46
Maxima	105.02	1.76	75.00	1.55
Mupad	151.23	2.36	81.00	2.16
Fricas	415.40	5.06	154.00	3.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

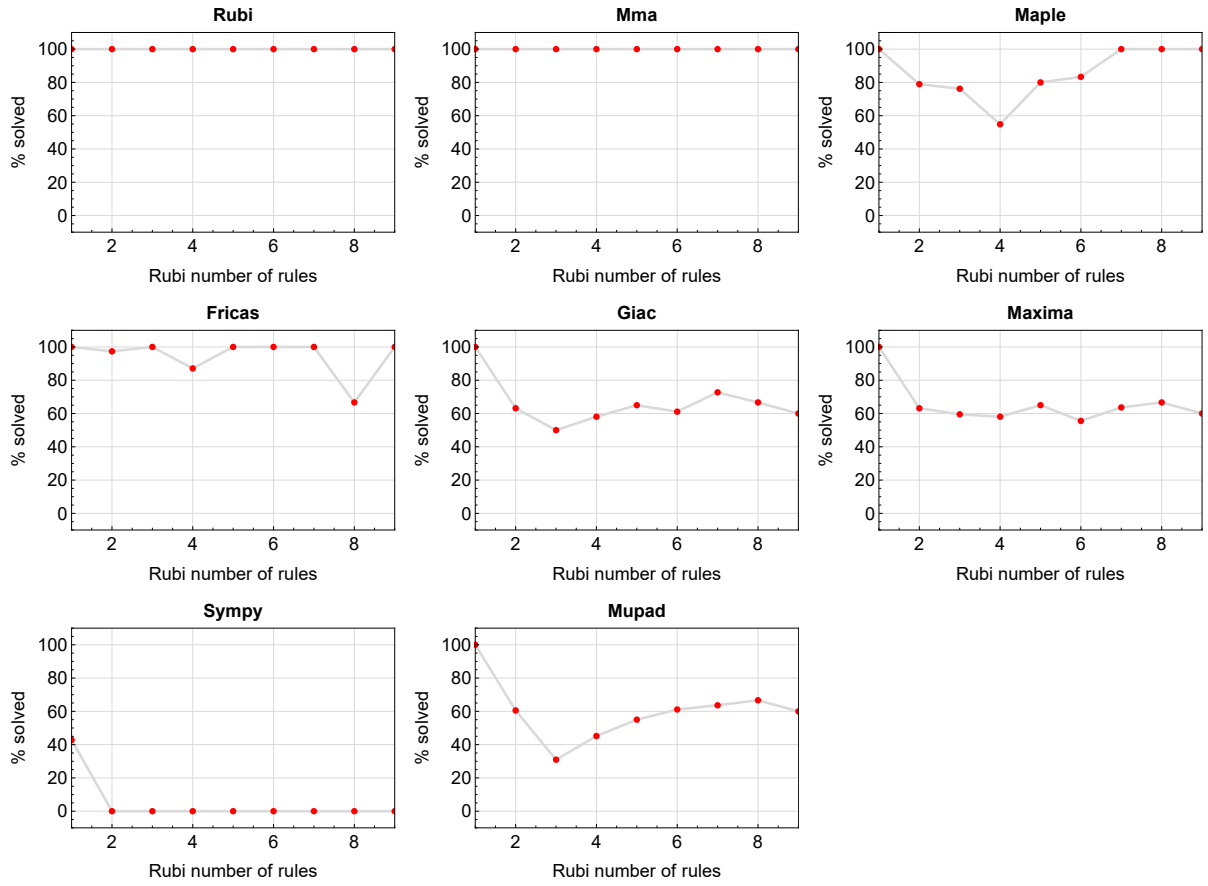


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

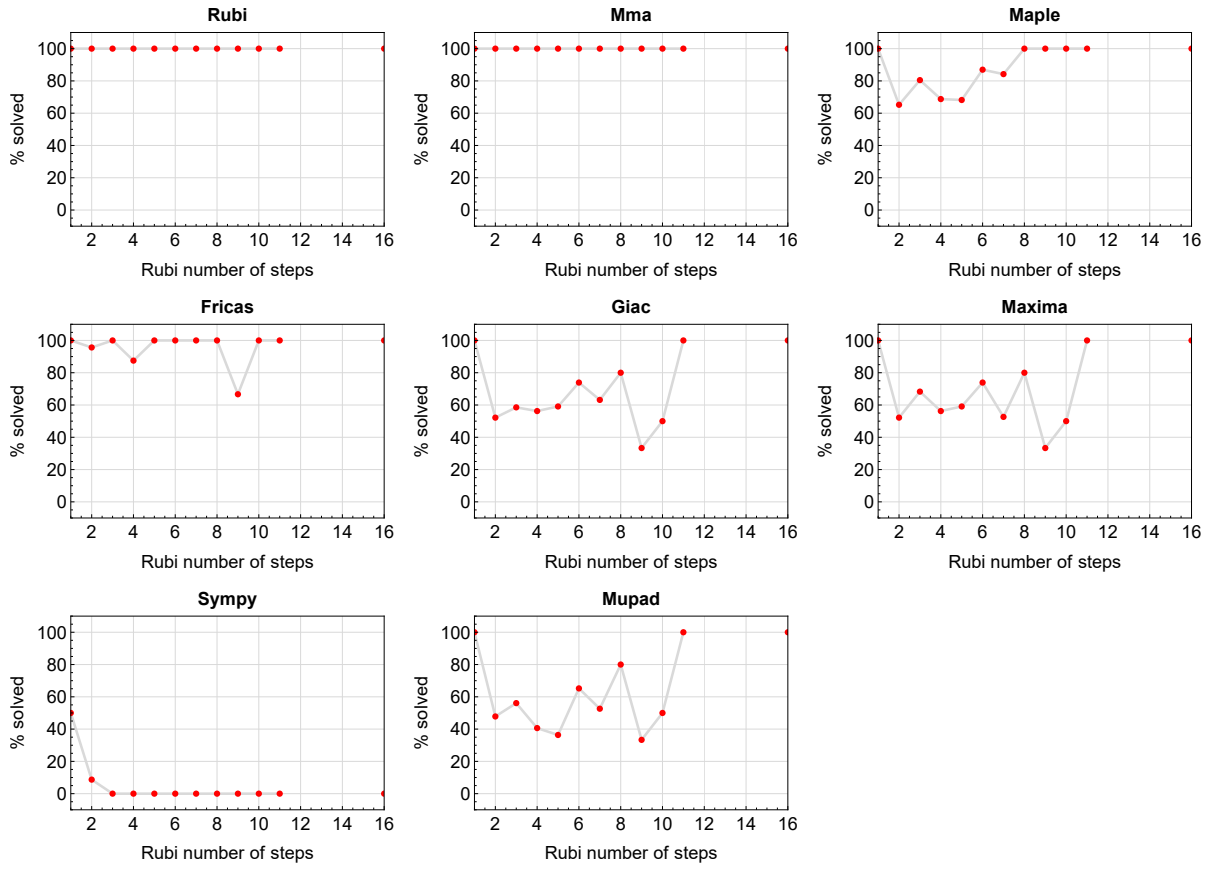


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

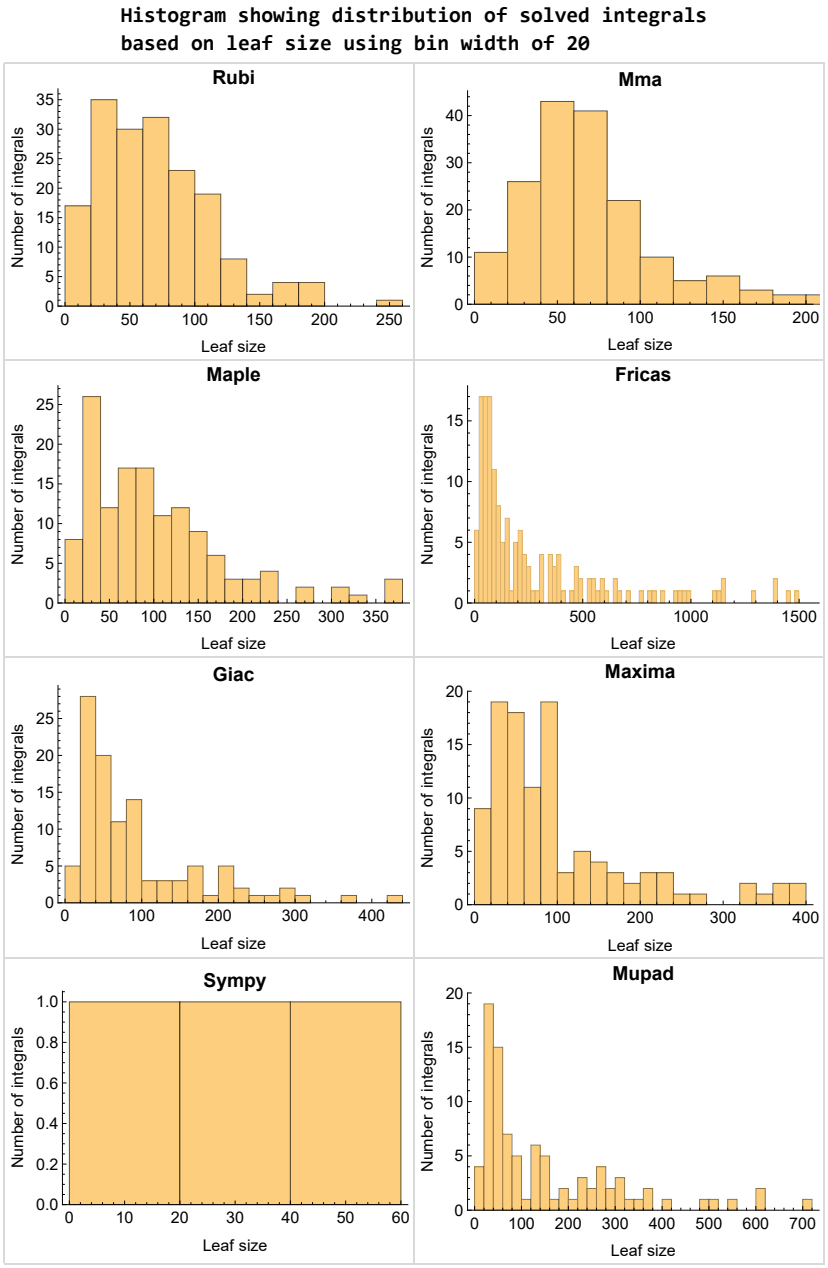


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

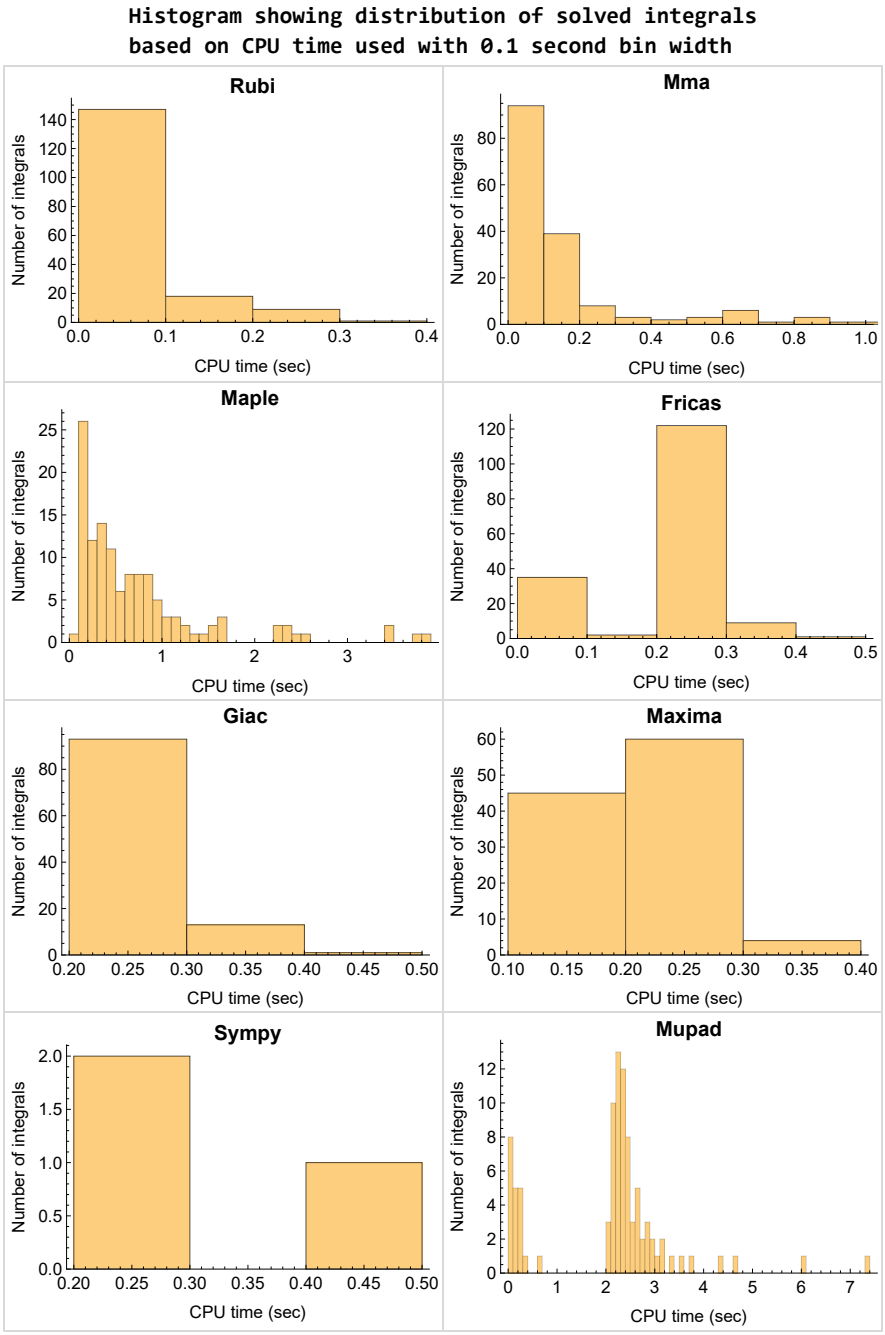


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

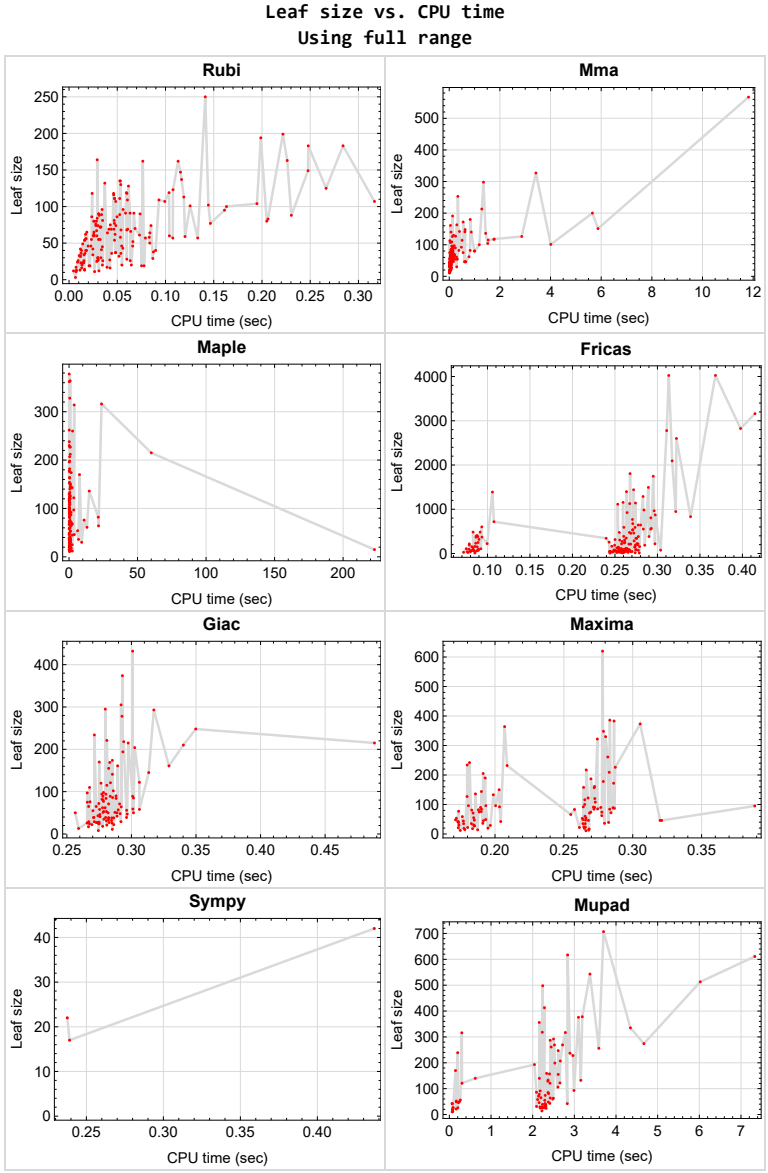


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {164}

Maple {125, 126, 127, 128, 129, 130, 131}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

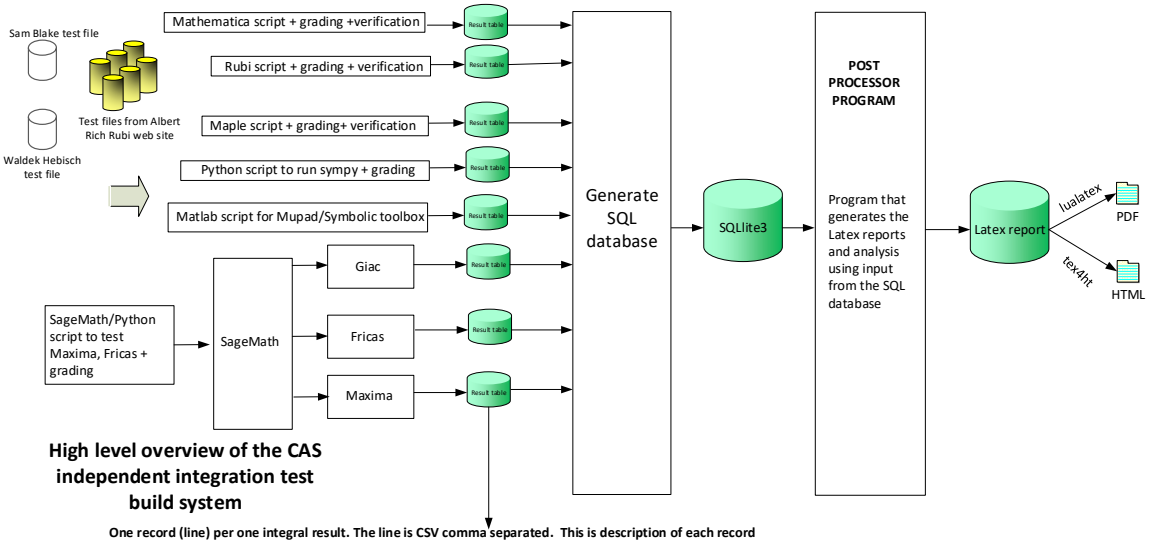
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 26
2.3 Detailed conclusion table specific for Rubi results 62

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 24
Giac 24
Mupad 24
Sympy 25

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade { }

C grade { 160 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 141, 143, 145, 147, 149, 152, 153, 156, 157, 158, 160, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade { 1, 3, 5, 23, 24, 55, 67, 68, 69, 70, 71, 72, 73, 89, 91, 103, 110, 112, 159, 161, 162, 164, 165 }

C grade { 113, 115, 117, 132, 134, 136, 140, 142, 144, 146, 148, 150, 151, 154, 155 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 42, 43, 44, 45, 49, 50, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 85, 87, 88, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 121, 123, 132, 133, 134, 135, 136, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 160, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade { 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 76, 77, 78, 84, 86, 89, 90, 91, 92, 94, 101, 110, 111, 112, 120, 122, 124 }

C grade { 125, 126, 127, 128, 129, 130, 131 }

F normal fail { 13, 14, 15, 16, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 137, 139, 149, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 49, 50, 62, 63, 66, 84, 97, 107, 108, 117, 118, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 151, 152, 153, 154, 155, 161, 162 }

B grade { 1, 2, 3, 4, 5, 6, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 145, 160, 163, 164, 165, 166, 167, 168, 169 }

C grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 36, 37, 38, 39, 40, 41, 170, 171, 172, 173, 174, 175 }

F normal fail { 21, 150, 156, 157, 158, 159 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 87, 93, 95, 97, 98, 105, 106, 107, 114, 115, 116, 117, 118, 119, 123, 126, 127, 128, 129, 130, 131, 133, 145, 162, 165, 166 }

B grade { 3, 4, 5, 6, 42, 43, 44, 68, 69, 70, 76, 83, 85, 86, 88, 89, 92, 94, 96, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 120, 121, 122, 124, 125, 141, 153, 160, 161, 167, 168, 169 }

C grade { 22, 23, 24, 25, 26, 27, 28 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { 90, 91 }

Giac

A grade { 2, 4, 6, 29, 30, 31, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 93, 94, 95, 96, 97, 98, 100, 102, 103, 104, 105, 106, 107, 108, 114, 116, 117, 118, 119, 123, 125, 126, 127, 128, 129, 130, 131, 161, 162, 166, 168 }

B grade { 1, 3, 5, 32, 68, 85, 86, 88, 89, 90, 91, 92, 99, 101, 109, 110, 111, 112, 113, 115, 120, 121, 122, 124, 160, 165, 167, 169 }

C grade { 22, 23, 24, 25, 26, 27, 28 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 133, 135, 156, 157, 158, 159, 164 }

F(-1) timedout fail { 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 163, 170, 171, 172, 173, 174, 175 }

F(-2) exception fail { 134, 136, 148 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 25, 32, 42, 43, 44, 45, 49, 50, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 141, 145, 153, 160, 161, 162, 165, 166, 167, 168, 169 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 76, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

F(-2) exception fail { }

Sympy

A grade { 1, 73, 165 }

B grade { }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	38	15	14	38	17	27	27
N.S.	1	1.00	3.17	1.25	1.17	3.17	1.42	2.25	2.25
time (sec)	N/A	0.005	0.021	0.092	0.187	0.265	0.239	0.273	0.090

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	43	0	18	18
N.S.	1	1.00	1.00	1.09	1.64	3.91	0.00	1.64	1.64
time (sec)	N/A	0.007	0.017	0.763	0.184	0.244	0.000	0.278	0.074

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	75	27	84	387	0	84	86
N.S.	1	1.00	2.21	0.79	2.47	11.38	0.00	2.47	2.53
time (sec)	N/A	0.016	0.019	0.763	0.190	0.260	0.000	0.280	2.091

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	35	23	90	164	0	31	31
N.S.	1	1.00	1.35	0.88	3.46	6.31	0.00	1.19	1.19
time (sec)	N/A	0.009	0.013	0.804	0.189	0.246	0.000	0.281	2.092

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	41	133	1114	0	110	193
N.S.	1	1.00	2.05	0.75	2.42	20.25	0.00	2.00	3.51
time (sec)	N/A	0.031	0.024	0.788	0.199	0.253	0.000	0.268	2.042

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	56	33	205	344	0	42	42
N.S.	1	1.00	1.33	0.79	4.88	8.19	0.00	1.00	1.00
time (sec)	N/A	0.012	0.021	0.755	0.191	0.240	0.000	0.289	0.075

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	61	101	0	194	0	0	0
N.S.	1	1.00	0.76	1.26	0.00	2.42	0.00	0.00	0.00
time (sec)	N/A	0.028	0.101	0.362	0.000	0.078	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	96	0	0	0
N.S.	1	1.00	0.75	2.03	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.028	0.173	0.368	0.000	0.078	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	24	0	0	0
N.S.	1	1.00	0.89	1.61	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.016	0.158	0.308	0.000	0.077	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	150	0	0	0
N.S.	1	1.00	0.93	2.00	0.00	2.78	0.00	0.00	0.00
time (sec)	N/A	0.016	0.031	0.474	0.000	0.088	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	100	0	223	0	0	0
N.S.	1	1.00	0.79	1.25	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.026	0.053	0.393	0.000	0.100	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	67	164	0	370	0	0	0
N.S.	1	1.00	0.84	2.05	0.00	4.62	0.00	0.00	0.00
time (sec)	N/A	0.026	0.094	0.398	0.000	0.086	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	484	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	4.17	0.00	0.00	0.00
time (sec)	N/A	0.046	0.149	0.000	0.000	0.083	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	66	0	0	217	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.031	0.087	0.000	0.000	0.079	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	60	0	0	107	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.029	0.053	0.000	0.000	0.075	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	27	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.017	0.037	0.000	0.000	0.072	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	227	0	154	0	0	0
N.S.	1	1.00	0.93	4.05	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.018	0.047	0.504	0.000	0.082	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	73	0	0	232	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	2.58	0.00	0.00	0.00
time (sec)	N/A	0.031	0.076	0.000	0.000	0.086	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	68	0	0	378	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	4.20	0.00	0.00	0.00
time (sec)	N/A	0.031	0.110	0.000	0.000	0.089	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	80	0	0	483	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.046	0.136	0.000	0.000	0.091	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	51	114	74	117	0	72	0
N.S.	1	1.00	1.28	2.85	1.85	2.92	0.00	1.80	0.00
time (sec)	N/A	0.013	0.154	0.274	0.270	0.263	0.000	0.275	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	51	99	49	67	0	57	0
N.S.	1	1.00	2.12	4.12	2.04	2.79	0.00	2.38	0.00
time (sec)	N/A	0.009	0.040	0.156	0.264	0.256	0.000	0.290	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	30	67	19	15	0	27	0
N.S.	1	1.00	10.00	22.33	6.33	5.00	0.00	9.00	0.00
time (sec)	N/A	0.007	0.009	0.173	0.265	0.258	0.000	0.274	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	11	14	0	25	31
N.S.	1	1.00	1.00	4.46	0.85	1.08	0.00	1.92	2.38
time (sec)	N/A	0.008	0.004	0.146	0.266	0.259	0.000	0.274	2.285

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	118	23	26	0	38	0
N.S.	1	1.00	0.82	3.58	0.70	0.79	0.00	1.15	0.00
time (sec)	N/A	0.011	0.038	0.149	0.266	0.257	0.000	0.282	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	33	178	35	38	0	50	0
N.S.	1	1.00	0.67	3.63	0.71	0.78	0.00	1.02	0.00
time (sec)	N/A	0.014	0.044	0.154	0.264	0.260	0.000	0.289	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	39	238	47	50	0	62	0
N.S.	1	1.00	0.60	3.66	0.72	0.77	0.00	0.95	0.00
time (sec)	N/A	0.018	0.051	0.175	0.266	0.252	0.000	0.280	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	123	92	1128	0	75	0
N.S.	1	1.00	0.78	1.89	1.42	17.35	0.00	1.15	0.00
time (sec)	N/A	0.026	0.120	0.328	0.264	0.267	0.000	0.266	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	103	60	340	0	58	0
N.S.	1	1.00	0.85	2.24	1.30	7.39	0.00	1.26	0.00
time (sec)	N/A	0.020	0.082	0.155	0.265	0.276	0.000	0.275	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	67	24	97	0	29	0
N.S.	1	1.00	1.15	2.58	0.92	3.73	0.00	1.12	0.00
time (sec)	N/A	0.016	0.033	0.169	0.264	0.264	0.000	0.271	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	17	83	0	24	33
N.S.	1	1.00	1.00	4.46	1.31	6.38	0.00	1.85	2.54
time (sec)	N/A	0.011	0.027	0.161	0.268	0.274	0.000	0.275	2.314

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	35	285	0	41	0
N.S.	1	1.00	0.75	3.61	0.97	7.92	0.00	1.14	0.00
time (sec)	N/A	0.017	0.042	0.165	0.266	0.273	0.000	0.276	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	53	590	0	53	0
N.S.	1	1.00	0.65	3.56	0.96	10.73	0.00	0.96	0.00
time (sec)	N/A	0.023	0.053	0.158	0.263	0.274	0.000	0.279	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	71	984	0	65	0
N.S.	1	1.00	0.57	3.54	0.96	13.30	0.00	0.88	0.00
time (sec)	N/A	0.030	0.069	0.148	0.269	0.284	0.000	0.267	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	68	0	0	1389	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	10.29	0.00	0.00	0.00
time (sec)	N/A	0.053	0.139	0.000	0.000	0.106	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	395	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.035	0.106	0.000	0.000	0.087	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	60	42	0	0	60	0	0	0
N.S.	1	1.07	0.75	0.00	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.025	0.036	0.000	0.000	0.081	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	43	0	0	127	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.026	0.070	0.000	0.000	0.083	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	57	0	0	407	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	4.57	0.00	0.00	0.00
time (sec)	N/A	0.034	0.074	0.000	0.000	0.088	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	71	0	0	718	0	0	0
N.S.	1	1.00	0.53	0.00	0.00	5.32	0.00	0.00	0.00
time (sec)	N/A	0.053	0.107	0.000	0.000	0.108	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	59	72	620	2825	0	51	498
N.S.	1	1.00	0.36	0.44	3.78	17.23	0.00	0.31	3.04
time (sec)	N/A	0.029	0.058	0.832	0.278	0.398	0.000	0.278	2.241

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	47	60	322	1493	0	39	356
N.S.	1	1.00	0.40	0.51	2.73	12.65	0.00	0.33	3.02
time (sec)	N/A	0.024	0.044	0.758	0.274	0.289	0.000	0.271	2.158

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	33	46	120	529	0	27	46
N.S.	1	1.00	0.53	0.74	1.94	8.53	0.00	0.44	0.74
time (sec)	N/A	0.017	0.037	0.159	0.272	0.271	0.000	0.275	2.155

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	13	81	0	13	71
N.S.	1	1.00	1.00	1.81	0.81	5.06	0.00	0.81	4.44
time (sec)	N/A	0.013	0.027	0.194	0.268	0.263	0.000	0.259	2.142

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	89	22	253	0	26	0
N.S.	1	1.00	0.67	2.47	0.61	7.03	0.00	0.72	0.00
time (sec)	N/A	0.013	0.043	0.181	0.261	0.262	0.000	0.266	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	46	1141	0	50	0
N.S.	1	1.00	0.44	2.67	0.53	13.27	0.00	0.58	0.00
time (sec)	N/A	0.024	0.058	0.162	0.268	0.274	0.000	0.282	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	72	2600	0	76	0
N.S.	1	1.00	0.42	2.74	0.55	19.70	0.00	0.58	0.00
time (sec)	N/A	0.037	0.103	0.168	0.284	0.322	0.000	0.268	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	27	35	32	0	29	26
N.S.	1	1.00	1.69	0.84	1.09	1.00	0.00	0.91	0.81
time (sec)	N/A	0.012	0.320	0.406	0.183	0.261	0.000	0.292	2.250

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	27	35	32	0	29	26
N.S.	1	1.00	1.69	0.84	1.09	1.00	0.00	0.91	0.81
time (sec)	N/A	0.012	0.255	0.395	0.177	0.261	0.000	0.267	2.286

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	136	0	0	566	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	5.29	0.00	0.00	0.00
time (sec)	N/A	0.099	1.438	0.000	0.000	0.293	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	100	0	0	461	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	6.40	0.00	0.00	0.00
time (sec)	N/A	0.032	1.186	0.000	0.000	0.272	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	383	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	9.58	0.00	0.00	0.00
time (sec)	N/A	0.017	0.989	0.000	0.000	0.278	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	118	0	0	551	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	6.05	0.00	0.00	0.00
time (sec)	N/A	0.067	1.777	0.000	0.000	0.292	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	327	0	0	873	0	0	0
N.S.	1	1.00	2.66	0.00	0.00	7.10	0.00	0.00	0.00
time (sec)	N/A	0.108	3.422	0.000	0.000	0.297	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	383	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	9.58	0.00	0.00	0.00
time (sec)	N/A	0.018	1.007	0.000	0.000	0.290	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	117	0	0	551	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	6.05	0.00	0.00	0.00
time (sec)	N/A	0.063	1.761	0.000	0.000	0.283	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	212	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	9.22	0.00	0.00	0.00
time (sec)	N/A	0.014	0.655	0.000	0.000	0.266	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	212	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	9.22	0.00	0.00	0.00
time (sec)	N/A	0.015	0.630	0.000	0.000	0.269	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	0.00
time (sec)	N/A	0.012	0.630	0.000	0.000	0.277	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	0.00
time (sec)	N/A	0.014	0.602	0.000	0.000	0.297	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	63	65	71	79	0	62	64
N.S.	1	1.00	1.09	1.12	1.22	1.36	0.00	1.07	1.10
time (sec)	N/A	0.053	0.154	1.660	0.184	0.259	0.000	0.278	2.420

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	56	53	59	67	0	50	52
N.S.	1	1.00	1.22	1.15	1.28	1.46	0.00	1.09	1.13
time (sec)	N/A	0.055	0.153	1.068	0.176	0.266	0.000	0.256	0.168

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	46	39	47	55	0	38	38
N.S.	1	1.00	1.28	1.08	1.31	1.53	0.00	1.06	1.06
time (sec)	N/A	0.044	0.160	0.905	0.185	0.258	0.000	0.282	2.306

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	35	25	31	39	0	24	24
N.S.	1	1.00	1.75	1.25	1.55	1.95	0.00	1.20	1.20
time (sec)	N/A	0.035	0.089	0.643	0.184	0.255	0.000	0.268	2.322

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	27	11	12	8	0	8	10
N.S.	1	1.00	1.93	0.79	0.86	0.57	0.00	0.57	0.71
time (sec)	N/A	0.016	0.094	0.282	0.175	0.244	0.000	0.274	0.068

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	46	19	27	31	0	22	26
N.S.	1	1.00	2.71	1.12	1.59	1.82	0.00	1.29	1.53
time (sec)	N/A	0.039	0.130	0.450	0.185	0.247	0.000	0.286	0.204

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	70	35	53	77	0	46	60
N.S.	1	1.00	2.69	1.35	2.04	2.96	0.00	1.77	2.31
time (sec)	N/A	0.063	0.196	0.768	0.172	0.267	0.000	0.276	2.482

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	53	77	120	0	50	63
N.S.	1	1.00	2.43	1.43	2.08	3.24	0.00	1.35	1.70
time (sec)	N/A	0.048	0.241	0.948	0.174	0.255	0.000	0.302	2.496

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	567	92	234	1440	0	169	239
N.S.	1	1.00	5.20	0.84	2.15	13.21	0.00	1.55	2.19
time (sec)	N/A	0.093	11.817	1.480	0.180	0.272	0.000	0.283	0.200

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	151	66	136	769	0	122	170
N.S.	1	1.00	2.01	0.88	1.81	10.25	0.00	1.63	2.27
time (sec)	N/A	0.043	5.874	1.114	0.186	0.271	0.000	0.306	0.147

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	75	37	44	222	0	59	74
N.S.	1	1.00	2.21	1.09	1.29	6.53	0.00	1.74	2.18
time (sec)	N/A	0.025	0.549	0.860	0.177	0.276	0.000	0.301	2.301

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	43	20	19	44	22	32	42
N.S.	1	1.00	2.53	1.18	1.12	2.59	1.29	1.88	2.47
time (sec)	N/A	0.007	0.005	0.133	0.177	0.270	0.238	0.285	0.064

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	82	85	186	0	84	121
N.S.	1	1.00	1.19	1.52	1.57	3.44	0.00	1.56	2.24
time (sec)	N/A	0.047	0.200	0.322	0.275	0.274	0.000	0.288	0.304

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	142	176	187	645	0	161	269
N.S.	1	1.00	1.41	1.74	1.85	6.39	0.00	1.59	2.66
time (sec)	N/A	0.125	0.403	0.344	0.270	0.277	0.000	0.329	2.717

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	213	328	373	2094	0	293	0
N.S.	1	1.00	1.31	2.01	2.29	12.85	0.00	1.80	0.00
time (sec)	N/A	0.226	1.286	0.513	0.305	0.317	0.000	0.317	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	104	198	157	807	0	155	199
N.S.	1	1.00	0.97	1.85	1.47	7.54	0.00	1.45	1.86
time (sec)	N/A	0.317	1.521	0.422	0.272	0.292	0.000	0.282	2.532

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	152	116	456	0	115	157
N.S.	1	1.00	1.02	1.90	1.45	5.70	0.00	1.44	1.96
time (sec)	N/A	0.205	0.825	0.318	0.265	0.269	0.000	0.281	2.404

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	84	238	0	86	129
N.S.	1	1.00	1.07	1.61	1.47	4.18	0.00	1.51	2.26
time (sec)	N/A	0.079	0.287	0.277	0.272	0.261	0.000	0.285	2.364

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	35	54	111	0	56	49
N.S.	1	1.00	1.22	0.95	1.46	3.00	0.00	1.51	1.32
time (sec)	N/A	0.048	0.023	0.103	0.266	0.258	0.000	0.296	0.166

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	67	49	83	156	0	82	287
N.S.	1	1.00	1.34	0.98	1.66	3.12	0.00	1.64	5.74
time (sec)	N/A	0.084	0.214	0.169	0.258	0.279	0.000	0.277	2.421

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	92	73	100	345	0	98	292
N.S.	1	1.00	1.56	1.24	1.69	5.85	0.00	1.66	4.95
time (sec)	N/A	0.120	0.480	0.234	0.265	0.275	0.000	0.278	2.501

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	148	108	158	947	0	141	617
N.S.	1	1.00	1.78	1.30	1.90	11.41	0.00	1.70	7.43
time (sec)	N/A	0.206	0.627	0.302	0.264	0.322	0.000	0.285	2.838

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	116	42	43	0	38	41
N.S.	1	1.00	0.84	3.05	1.11	1.13	0.00	1.00	1.08
time (sec)	N/A	0.087	0.054	0.152	0.173	0.252	0.000	0.285	2.292

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	39	36	0	35	39
N.S.	1	1.00	1.00	0.79	2.05	1.89	0.00	1.84	2.05
time (sec)	N/A	0.075	0.008	222.886	0.174	0.248	0.000	0.279	2.235

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	30	30	31	0	26	29
N.S.	1	1.00	1.00	1.50	1.50	1.55	0.00	1.30	1.45
time (sec)	N/A	0.065	0.042	9.222	0.174	0.252	0.000	0.285	2.213

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	21	28	0	21	12
N.S.	1	1.00	1.00	1.56	1.31	1.75	0.00	1.31	0.75
time (sec)	N/A	0.042	0.011	2.259	0.173	0.252	0.000	0.281	0.084

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	20	30	41	52	0	51	46
N.S.	1	1.00	0.71	1.07	1.46	1.86	0.00	1.82	1.64
time (sec)	N/A	0.060	0.027	1.185	0.173	0.256	0.000	0.287	0.218

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	64	31	81	32	0	27	31
N.S.	1	1.00	3.37	1.63	4.26	1.68	0.00	1.42	1.63
time (sec)	N/A	0.078	0.062	0.606	0.183	0.252	0.000	0.283	2.307

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	32	61	0	142	0	94	122
N.S.	1	1.00	0.80	1.52	0.00	3.55	0.00	2.35	3.05
time (sec)	N/A	0.090	0.043	13.089	0.000	0.254	0.000	0.280	2.655

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	96	93	0	68	0	55	207
N.S.	1	1.00	3.31	3.21	0.00	2.34	0.00	1.90	7.14
time (sec)	N/A	0.087	0.100	0.229	0.000	0.254	0.000	0.270	2.663

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	97	378	242	1398	0	194	228
N.S.	1	1.00	0.95	3.71	2.37	13.71	0.00	1.90	2.24
time (sec)	N/A	0.144	0.198	0.129	0.181	0.264	0.000	0.293	2.968

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	180	215	217	924	0	221	247
N.S.	1	1.00	1.44	1.72	1.74	7.39	0.00	1.77	1.98
time (sec)	N/A	0.266	0.827	60.041	0.266	0.264	0.000	0.281	2.613

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	136	127	476	0	97	121
N.S.	1	1.00	0.98	2.39	2.23	8.35	0.00	1.70	2.12
time (sec)	N/A	0.108	0.028	14.752	0.180	0.255	0.000	0.266	2.415

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	80	122	122	304	0	121	159
N.S.	1	1.00	1.04	1.58	1.58	3.95	0.00	1.57	2.06
time (sec)	N/A	0.147	0.127	3.476	0.268	0.261	0.000	0.284	2.335

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	48	80	0	39	20
N.S.	1	1.00	0.95	1.55	2.40	4.00	0.00	1.95	1.00
time (sec)	N/A	0.055	0.008	0.656	0.171	0.260	0.000	0.271	0.075

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	72	66	57	0	89	93
N.S.	1	1.00	0.88	1.12	1.03	0.89	0.00	1.39	1.45
time (sec)	N/A	0.084	0.048	0.266	0.255	0.259	0.000	0.284	2.991

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	67	81	91	256	0	85	133
N.S.	1	1.00	1.12	1.35	1.52	4.27	0.00	1.42	2.22
time (sec)	N/A	0.104	0.176	0.490	0.271	0.243	0.000	0.302	2.371

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	127	143	161	675	0	218	256
N.S.	1	1.00	1.34	1.51	1.69	7.11	0.00	2.29	2.69
time (sec)	N/A	0.161	0.250	2.237	0.272	0.271	0.000	0.294	3.589

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	114	170	226	1155	0	174	269
N.S.	1	1.00	1.10	1.63	2.17	11.11	0.00	1.67	2.59
time (sec)	N/A	0.195	0.507	7.646	0.287	0.260	0.000	0.285	2.522

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	163	316	348	2778	0	374	513
N.S.	1	1.00	1.09	2.12	2.34	18.64	0.00	2.51	3.44
time (sec)	N/A	0.248	0.299	23.740	0.279	0.311	0.000	0.293	6.022

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	75	97	144	304	0	120	274
N.S.	1	1.00	0.69	0.89	1.32	2.79	0.00	1.10	2.51
time (sec)	N/A	0.061	0.139	3.434	0.190	0.256	0.000	0.276	4.669

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	126	68	96	124	0	58	237
N.S.	1	1.00	2.42	1.31	1.85	2.38	0.00	1.12	4.56
time (sec)	N/A	0.066	0.123	2.424	0.181	0.252	0.000	0.306	2.894

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	71	96	186	0	94	140
N.S.	1	1.00	0.79	0.92	1.25	2.42	0.00	1.22	1.82
time (sec)	N/A	0.047	0.084	1.672	0.200	0.285	0.000	0.277	0.622

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	71	35	42	50	0	34	85
N.S.	1	1.00	1.97	0.97	1.17	1.39	0.00	0.94	2.36
time (sec)	N/A	0.054	0.103	1.218	0.204	0.260	0.000	0.282	2.370

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	45	71	0	53	50
N.S.	1	1.00	0.87	0.78	1.00	1.58	0.00	1.18	1.11
time (sec)	N/A	0.035	0.030	0.804	0.192	0.278	0.000	0.281	0.244

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	15	11	0	11	14
N.S.	1	1.00	1.00	1.15	1.15	0.85	0.00	0.85	1.08
time (sec)	N/A	0.016	0.007	1.027	0.178	0.279	0.000	0.285	2.222

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	22	18	20	16	0	17	21
N.S.	1	1.00	2.00	1.64	1.82	1.45	0.00	1.55	1.91
time (sec)	N/A	0.027	0.071	1.625	0.195	0.274	0.000	0.267	0.164

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	36	40	0	39	27
N.S.	1	1.00	1.00	1.00	3.00	3.33	0.00	3.25	2.25
time (sec)	N/A	0.031	0.013	2.366	0.191	0.269	0.000	0.280	2.219

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	76	46	55	86	0	43	56
N.S.	1	1.00	2.81	1.70	2.04	3.19	0.00	1.59	2.07
time (sec)	N/A	0.042	0.050	3.824	0.184	0.275	0.000	0.274	0.263

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	54	75	100	0	65	81
N.S.	1	1.00	1.00	1.80	2.50	3.33	0.00	2.17	2.70
time (sec)	N/A	0.033	0.012	6.257	0.189	0.264	0.000	0.273	2.383

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	140	76	96	157	0	71	106
N.S.	1	1.00	3.26	1.77	2.23	3.65	0.00	1.65	2.47
time (sec)	N/A	0.056	0.069	10.997	0.193	0.274	0.000	0.291	2.606

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	253	364	383	4025	0	432	611
N.S.	1	1.00	1.30	1.88	1.97	20.75	0.00	2.23	3.15
time (sec)	N/A	0.199	0.340	0.812	0.286	0.368	0.000	0.301	7.331

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	141	207	261	1746	0	215	707
N.S.	1	1.00	0.77	1.13	1.43	9.54	0.00	1.17	3.86
time (sec)	N/A	0.284	0.618	0.468	0.282	0.295	0.000	0.297	3.700

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	191	182	172	965	0	234	335
N.S.	1	1.00	1.69	1.61	1.52	8.54	0.00	2.07	2.96
time (sec)	N/A	0.119	0.133	0.447	0.286	0.296	0.000	0.271	4.346

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	82	95	108	349	0	102	376
N.S.	1	1.00	0.82	0.95	1.08	3.49	0.00	1.02	3.76
time (sec)	N/A	0.163	0.382	0.277	0.273	0.263	0.000	0.286	3.099

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	97	74	75	0	89	132
N.S.	1	1.00	1.03	1.59	1.21	1.23	0.00	1.46	2.16
time (sec)	N/A	0.073	0.040	0.298	0.278	0.304	0.000	0.278	3.156

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	28	27	0	22	25
N.S.	1	1.00	0.58	1.11	1.47	1.42	0.00	1.16	1.32
time (sec)	N/A	0.022	0.007	0.316	0.185	0.265	0.000	0.269	0.088

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	75	84	90	141	0	89	316
N.S.	1	1.00	1.32	1.47	1.58	2.47	0.00	1.56	5.54
time (sec)	N/A	0.133	0.101	0.462	0.272	0.273	0.000	0.301	0.299

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	37	75	82	199	0	80	261
N.S.	1	1.00	1.16	2.34	2.56	6.22	0.00	2.50	8.16
time (sec)	N/A	0.052	0.034	0.670	0.188	0.277	0.000	0.278	2.443

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	172	150	178	831	0	161	378
N.S.	1	1.00	1.95	1.70	2.02	9.44	0.00	1.83	4.30
time (sec)	N/A	0.230	0.549	1.128	0.279	0.339	0.000	0.289	3.191

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	83	174	190	1288	0	170	155
N.S.	1	1.00	1.19	2.49	2.71	18.40	0.00	2.43	2.21
time (sec)	N/A	0.068	0.085	1.597	0.193	0.283	0.000	0.275	2.613

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	298	260	330	3160	0	305	543
N.S.	1	1.00	1.63	1.42	1.80	17.27	0.00	1.67	2.97
time (sec)	N/A	0.248	1.352	2.576	0.280	0.415	0.000	0.292	3.374

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	130	314	364	4024	0	295	317
N.S.	1	1.00	1.09	2.64	3.06	33.82	0.00	2.48	2.66
time (sec)	N/A	0.104	0.171	3.737	0.207	0.313	0.000	0.280	2.784

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	199	199	84	87	386	592	0	90	413
N.S.	1	1.00	0.42	0.44	1.94	2.97	0.00	0.45	2.08
time (sec)	N/A	0.222	0.062	1.368	0.283	0.260	0.000	0.288	2.283

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	147	147	72	65	209	315	0	77	91
N.S.	1	1.00	0.49	0.44	1.42	2.14	0.00	0.52	0.62
time (sec)	N/A	0.116	0.048	1.275	0.283	0.250	0.000	0.278	2.179

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	58	58	56	40	84	121	0	64	78
N.S.	1	1.00	0.97	0.69	1.45	2.09	0.00	1.10	1.34
time (sec)	N/A	0.083	0.034	0.358	0.283	0.253	0.000	0.292	2.156

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	44	29	39	42	0	48	0
N.S.	1	1.00	0.96	0.63	0.85	0.91	0.00	1.04	0.00
time (sec)	N/A	0.066	0.031	0.316	0.282	0.245	0.000	0.291	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	74	48	60	36	66	0	71	0
N.S.	1	1.00	0.65	0.81	0.49	0.89	0.00	0.96	0.00
time (sec)	N/A	0.085	0.036	0.416	0.279	0.250	0.000	0.282	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	162	76	75	62	126	0	204	0
N.S.	1	1.00	0.47	0.46	0.38	0.78	0.00	1.26	0.00
time (sec)	N/A	0.113	0.049	0.719	0.278	0.255	0.000	0.303	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	250	109	88	90	218	0	278	0
N.S.	1	1.00	0.44	0.35	0.36	0.87	0.00	1.11	0.00
time (sec)	N/A	0.141	0.102	0.726	0.277	0.258	0.000	0.293	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	125	0	72	0	0	0
N.S.	1	1.00	0.99	1.54	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.047	0.133	0.822	0.000	0.089	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	39	46	48	0	0	42
N.S.	1	1.00	1.47	1.30	1.53	1.60	0.00	0.00	1.40
time (sec)	N/A	0.030	0.035	0.157	0.320	0.244	0.000	0.000	2.830

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	60	127	0	97	0	0	0
N.S.	1	1.00	0.50	1.07	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.059	0.086	0.653	0.000	0.090	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	74	97	0	92	0	0	0
N.S.	1	1.00	1.07	1.41	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.041	0.098	0.205	0.000	0.266	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	109	0	63	0	0	0
N.S.	1	1.00	0.95	1.82	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.029	0.120	0.560	0.000	0.084	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	75	0	0	86	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.026	0.080	0.000	0.000	0.260	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	90	0	11	0	0	0
N.S.	1	1.00	0.93	1.96	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.023	0.082	0.420	0.000	0.085	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	54	0	0	43	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.034	0.084	0.000	0.000	0.263	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	58	126	0	72	0	0	0
N.S.	1	1.00	0.78	1.70	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.049	0.072	0.679	0.000	0.088	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	33	38	89	37	0	0	58
N.S.	1	1.00	1.32	1.52	3.56	1.48	0.00	0.00	2.32
time (sec)	N/A	0.029	0.029	0.150	0.286	0.260	0.000	0.000	2.118

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	112	0	56	0	0	0
N.S.	1	1.00	0.94	1.75	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.040	0.070	0.569	0.000	0.087	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	121	0	110	0	0	0
N.S.	1	1.00	0.74	0.95	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.062	0.149	0.201	0.000	0.263	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	80	133	0	80	0	0	0
N.S.	1	1.00	0.68	1.13	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.060	0.123	0.571	0.000	0.084	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	44	47	46	56	0	0	42
N.S.	1	1.00	1.47	1.57	1.53	1.87	0.00	0.00	1.40
time (sec)	N/A	0.030	0.036	0.148	0.321	0.257	0.000	0.000	2.296

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	63	140	0	106	0	0	0
N.S.	1	1.00	0.39	0.86	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.076	0.094	0.660	0.000	0.092	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	87	113	0	102	0	0	0
N.S.	1	1.00	0.91	1.18	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.056	0.129	0.200	0.000	0.263	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	65	124	0	71	0	0	0
N.S.	1	1.00	0.76	1.44	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.050	0.089	0.550	0.000	0.082	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	94	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.059	0.117	0.000	0.000	0.261	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	60	152	0	0	0	0	0
N.S.	1	1.00	0.46	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.115	0.646	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	63	130	0	106	0	0	0
N.S.	1	1.00	0.66	1.35	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.034	0.084	0.192	0.000	0.264	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	163	0	60	0	0	0
N.S.	1	1.00	0.81	2.43	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.029	0.098	0.450	0.000	0.080	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	33	0	87	29	0	0	29
N.S.	1	1.00	1.22	0.00	3.22	1.07	0.00	0.00	1.07
time (sec)	N/A	0.030	0.026	0.000	0.287	0.255	0.000	0.000	2.174

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	43	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.044	0.089	0.000	0.000	0.080	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	53	0	0	78	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.047	0.079	0.000	0.000	0.249	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.779	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	126	0	0	0	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	2.863	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	101	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	4.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	200	0	0	0	0	0	0
N.S.	1	1.00	2.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	5.653	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	137	30	82	95	187	0	215	65
N.S.	1	3.26	0.71	1.95	2.26	4.45	0.00	5.12	1.55
time (sec)	N/A	0.117	0.295	21.335	0.389	0.250	0.000	0.488	2.199

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	62	0	76	48	0	38	48
N.S.	1	1.00	2.38	0.00	2.92	1.85	0.00	1.46	1.85
time (sec)	N/A	0.029	0.144	0.000	0.190	0.256	0.000	0.286	2.376

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	65	0	49	49	0	39	36
N.S.	1	1.00	2.50	0.00	1.88	1.88	0.00	1.50	1.38
time (sec)	N/A	0.035	0.070	0.000	0.193	0.248	0.000	0.296	2.313

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	115	0	0	475	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	5.28	0.00	0.00	0.00
time (sec)	N/A	0.074	1.537	0.000	0.000	0.267	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	140	0	0	539	0	0	0
N.S.	1	1.00	2.12	0.00	0.00	8.17	0.00	0.00	0.00
time (sec)	N/A	0.064	0.869	0.000	0.000	0.271	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	54	23	22	65	42	145	43
N.S.	1	1.00	2.70	1.15	1.10	3.25	2.10	7.25	2.15
time (sec)	N/A	0.013	0.044	0.261	0.180	0.256	0.437	0.313	2.411

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	29	71	0	28	25
N.S.	1	1.00	1.00	1.05	1.53	3.74	0.00	1.47	1.32
time (sec)	N/A	0.021	0.062	1.096	0.197	0.250	0.000	0.275	2.180

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	107	45	150	643	0	210	140
N.S.	1	1.00	1.95	0.82	2.73	11.69	0.00	3.82	2.55
time (sec)	N/A	0.032	0.045	2.374	0.203	0.280	0.000	0.340	2.160

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	56	36	92	272	0	47	55
N.S.	1	1.00	1.33	0.86	2.19	6.48	0.00	1.12	1.31
time (sec)	N/A	0.025	0.039	6.915	0.203	0.258	0.000	0.297	2.290

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	161	64	232	1806	0	248	318
N.S.	1	1.00	1.81	0.72	2.61	20.29	0.00	2.79	3.57
time (sec)	N/A	0.052	0.047	21.529	0.209	0.268	0.000	0.350	2.231

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	84	144	0	318	0	0	0
N.S.	1	1.00	0.76	1.30	0.00	2.86	0.00	0.00	0.00
time (sec)	N/A	0.053	0.139	1.589	0.000	0.090	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	159	0	0	0
N.S.	1	1.00	0.75	1.98	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.048	0.077	0.895	0.000	0.085	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	39	0	0	0
N.S.	1	1.00	0.92	1.67	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.032	0.059	0.819	0.000	0.076	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	248	0	0	0
N.S.	1	1.00	0.94	2.03	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.033	0.048	0.958	0.000	0.089	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	86	143	0	370	0	0	0
N.S.	1	1.00	0.77	1.29	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.047	0.087	0.940	0.000	0.093	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	227	0	602	0	0	0
N.S.	1	1.00	0.86	2.05	0.00	5.42	0.00	0.00	0.00
time (sec)	N/A	0.047	0.129	0.930	0.000	0.093	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [114] had the largest ratio of [.6923000000000026]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	3	3	1.00	10	0.300
8	A	3	3	1.00	10	0.300
9	A	2	2	1.00	10	0.200
10	A	2	2	1.00	10	0.200
11	A	3	3	1.00	10	0.300
12	A	3	3	1.00	10	0.300
13	A	4	3	1.00	12	0.250
14	A	3	3	1.00	12	0.250
15	A	3	3	1.00	12	0.250
16	A	2	2	1.00	12	0.167
17	A	2	2	1.00	12	0.167
18	A	3	3	1.00	12	0.250
19	A	3	3	1.00	12	0.250
20	A	4	3	1.00	12	0.250
21	A	2	2	1.00	10	0.200
22	A	4	3	1.00	10	0.300
23	A	3	3	1.00	10	0.300
24	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	2	2	1.00	10	0.200
26	A	3	3	1.00	10	0.300
27	A	4	3	1.00	10	0.300
28	A	5	3	1.00	10	0.300
29	A	5	4	1.00	10	0.400
30	A	4	4	1.00	10	0.400
31	A	3	3	1.00	10	0.300
32	A	2	2	1.00	10	0.200
33	A	3	3	1.00	10	0.300
34	A	4	3	1.00	10	0.300
35	A	5	3	1.00	10	0.300
36	A	7	4	1.00	10	0.400
37	A	5	4	1.00	10	0.400
38	A	4	4	1.07	10	0.400
39	A	4	4	1.00	10	0.400
40	A	5	4	1.00	10	0.400
41	A	7	4	1.00	10	0.400
42	A	3	2	1.00	10	0.200
43	A	3	2	1.00	10	0.200
44	A	3	2	1.00	10	0.200
45	A	3	3	1.00	10	0.300
46	A	3	3	1.00	10	0.300
47	A	5	3	1.00	10	0.300
48	A	7	3	1.00	10	0.300
49	A	2	2	1.00	15	0.133
50	A	2	2	1.00	15	0.133
51	A	5	5	1.00	17	0.294
52	A	4	4	1.00	17	0.235
53	A	2	2	1.00	17	0.118
54	A	5	4	1.00	17	0.235
55	A	6	5	1.00	17	0.294
56	A	2	2	1.00	17	0.118
57	A	5	4	1.00	17	0.235
58	A	2	2	1.00	12	0.167
59	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	2	2	1.00	12	0.167
61	A	2	2	1.00	12	0.167
62	A	7	5	1.00	13	0.385
63	A	6	5	1.00	13	0.385
64	A	5	5	1.00	13	0.385
65	A	4	4	1.00	11	0.364
66	A	1	1	1.00	11	0.091
67	A	3	3	1.00	13	0.231
68	A	4	4	1.00	13	0.308
69	A	6	6	1.00	13	0.462
70	A	6	5	1.00	12	0.417
71	A	5	4	1.00	12	0.333
72	A	4	4	1.00	12	0.333
73	A	2	1	1.00	10	0.100
74	A	4	4	1.00	12	0.333
75	A	6	6	1.00	12	0.500
76	A	7	7	1.00	12	0.583
77	A	8	7	1.00	13	0.538
78	A	7	7	1.00	13	0.538
79	A	6	6	1.00	11	0.546
80	A	4	4	1.00	11	0.364
81	A	6	6	1.00	13	0.462
82	A	7	7	1.00	13	0.538
83	A	8	8	1.00	13	0.615
84	A	7	7	1.00	13	0.538
85	A	6	4	1.00	13	0.308
86	A	5	5	1.00	13	0.385
87	A	4	3	1.00	11	0.273
88	A	6	6	1.00	11	0.546
89	A	6	5	1.00	13	0.385
90	A	7	7	1.00	13	0.538
91	A	7	6	1.00	13	0.462
92	A	5	4	1.00	13	0.308
93	A	7	6	1.00	13	0.462
94	A	5	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	6	1.00	13	0.462
96	A	5	4	1.00	11	0.364
97	A	4	3	1.00	11	0.273
98	A	6	6	1.00	13	0.462
99	A	6	5	1.00	13	0.385
100	A	7	6	1.00	13	0.462
101	A	7	5	1.00	13	0.385
102	A	3	2	1.00	13	0.154
103	A	5	3	1.00	13	0.231
104	A	3	2	1.00	13	0.154
105	A	4	3	1.00	13	0.231
106	A	3	2	1.00	11	0.182
107	A	2	2	1.00	11	0.182
108	A	3	2	1.00	13	0.154
109	A	3	2	1.00	13	0.154
110	A	4	3	1.00	13	0.231
111	A	3	2	1.00	13	0.154
112	A	5	3	1.00	13	0.231
113	A	11	7	1.00	13	0.538
114	A	16	9	1.00	13	0.692
115	A	8	6	1.00	13	0.462
116	A	10	9	1.00	13	0.692
117	A	6	5	1.00	11	0.454
118	A	4	4	1.00	11	0.364
119	A	8	8	1.00	13	0.615
120	A	3	2	1.00	13	0.154
121	A	7	7	1.00	13	0.538
122	A	3	2	1.00	13	0.154
123	A	16	9	1.00	13	0.692
124	A	3	2	1.00	13	0.154
125	A	6	5	1.00	25	0.200
126	A	6	5	1.00	25	0.200
127	A	4	4	1.00	25	0.160
128	A	4	4	1.00	25	0.160
129	A	5	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	6	5	1.00	25	0.200
131	A	6	5	1.00	25	0.200
132	A	6	6	1.00	15	0.400
133	A	3	3	1.00	15	0.200
134	A	9	9	1.00	15	0.600
135	A	6	6	1.00	15	0.400
136	A	5	5	1.00	13	0.385
137	A	6	6	1.00	11	0.546
138	A	3	2	1.00	15	0.133
139	A	5	5	1.00	15	0.333
140	A	7	7	1.00	15	0.467
141	A	3	3	1.00	15	0.200
142	A	5	5	1.00	15	0.333
143	A	8	7	1.00	15	0.467
144	A	7	6	1.00	15	0.400
145	A	3	3	1.00	15	0.200
146	A	10	9	1.00	15	0.600
147	A	7	6	1.00	15	0.400
148	A	6	5	1.00	15	0.333
149	A	7	6	1.00	15	0.400
150	A	9	8	1.00	13	0.615
151	A	7	7	1.00	11	0.636
152	A	4	3	1.00	15	0.200
153	A	3	3	1.00	15	0.200
154	A	5	5	1.00	15	0.333
155	A	6	6	1.00	15	0.400
156	A	4	4	1.00	11	0.364
157	A	4	4	1.00	13	0.308
158	A	4	4	1.00	13	0.308
159	A	4	4	1.00	13	0.308
160	C	9	4	3.26	45	0.089
161	A	3	3	1.00	15	0.200
162	A	4	4	1.00	15	0.267
163	A	3	3	1.00	20	0.150
164	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	1	1.00	15	0.067
166	A	3	2	1.00	17	0.118
167	A	3	2	1.00	17	0.118
168	A	3	1	1.00	17	0.059
169	A	4	2	1.00	17	0.118
170	A	4	3	1.00	19	0.158
171	A	4	3	1.00	19	0.158
172	A	3	2	1.00	19	0.105
173	A	3	2	1.00	19	0.105
174	A	4	3	1.00	19	0.158
175	A	4	3	1.00	19	0.158

CHAPTER 3

LISTING OF INTEGRALS

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3.154 $\int \frac{\text{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$ . . . 812
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3.171 $\int \frac{\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$ . . . 887
3.172 $\int \frac{\sqrt{\text{csch}(a+b\log(cx^n))}}{x}\,dx$ . . . 892
3.173 $\int \frac{1}{x\sqrt{\text{csch}(a+b\log(cx^n))}}\,dx$ . . . 896
3.174 $\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$ . . . 900
3.175 $\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$ . . . 905

3.1 $\int \operatorname{csch}(a+bx)\,dx$

Optimal result 75
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Mathematica [B] (verified) 76
Maple [A] (verified) 76
Fricas [B] (verification not implemented) 76
Sympy [A] (verification not implemented) 77
Maxima [A] (verification not implemented) 77
Giac [B] (verification not implemented) 77
Mupad [B] (verification not implemented) 78

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

```
[In] Integrate[Csch[a + b*x],x]
[Out] -(Log[Cosh[a/2 + (b*x)/2]]/b) + Log[Sinh[a/2 + (b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
default	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
parallelrisch	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
risch	$\frac{\ln\left(e^{bx+a}-1\right)}{b} - \frac{\ln\left(e^{bx+a}+1\right)}{b}$	29

```
[In] int(csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*ln(tanh(1/2*b*x+1/2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)}{b}$$

```
[In] integrate(csch(b*x+a),x, algorithm="fricas")
[Out] -(log(cosh(b*x + a) + sinh(b*x + a) + 1) - log(cosh(b*x + a) + sinh(b*x + a
) - 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}(a+bx)\,dx = \begin{cases} \frac{\log\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ x\operatorname{csch}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(csch(b*x+a),x)
[Out] Piecewise((log(tanh(a/2 + b*x/2))/b, Ne(b, 0)), (x*csch(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \operatorname{csch}(a+bx)\,dx = \frac{\log\left(\tanh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

```
[In] integrate(csch(b*x+a),x, algorithm="maxima")
[Out] log(tanh(1/2*b*x + 1/2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(csch(b*x+a),x, algorithm="giac")
[Out] -(log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{2\,\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/sinh(a + b*x),x)
[Out] -(2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.2 $\int \operatorname{csch}^2(a+bx)\,dx$

Optimal result . . . 79
Rubi [A] (verified) . . . 79
Mathematica [A] (verified) . . . 80
Maple [A] (verified) . . . 80
Fricas [B] (verification not implemented) . . . 81
Sympy [F] . . . 81
Maxima [A] (verification not implemented) . . . 81
Giac [A] (verification not implemented) . . . 81
Mupad [B] (verification not implemented) . . . 82

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852, 8}

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}(\int 1\,dx, x, -i\coth(a+bx))}{b} \\ &= -\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \text{csch}^2(a+bx)\,dx = -\frac{\coth(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]^2,x]
[Out] -(Coth[a + b*x]/b)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\coth(bx+a)}{b}$	12
default	$-\frac{\coth(bx+a)}{b}$	12
risch	$-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	19
parallelrisch	$\frac{-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}$	29

```
[In] int(csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -coth(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(11) = 22$.

Time $= 0.24$ (sec) , antiderivative size $= 43$, normalized size of antiderivative $= 3.91$

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2}{b\cosh(bx+a)^2 + 2\,b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2 - b}$$

```
[In] integrate(csch(b*x+a)^2,x, algorithm="fricas")
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**2,x)
[Out] Integral(csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.18$ (sec) , antiderivative size $= 18$, normalized size of antiderivative $= 1.64$

$$\int \operatorname{csch}^2(a+bx)\,dx = \frac{2}{b(e^{(-2\,bx-2\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2,x, algorithm="maxima")
[Out] 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time $= 0.28$ (sec) , antiderivative size $= 18$, normalized size of antiderivative $= 1.64$

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2,x, algorithm="giac")
[Out] -2/(b*(e^(2*b*x + 2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2}{b\,(\mathrm{e}^{2\,a+2\,b\,x}-1)}$$

```
[In] int(1/sinh(a + b*x)^2,x)
[Out] -2/(b*(exp(2*a + 2*b*x) - 1))
```

3.3 $\int \operatorname{csch}^3(a+bx)\,dx$

Optimal result . . . 83
Rubi [A] (verified) . . . 83
Mathematica [B] (verified) . . . 84
Maple [A] (verified) . . . 84
Fricas [B] (verification not implemented) . . . 85
Sympy [F] . . . 85
Maxima [B] (verification not implemented) . . . 85
Giac [B] (verification not implemented) . . . 86
Mupad [B] (verification not implemented) . . . 86

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3855}

$$\int \operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{1}{2}\int \operatorname{csch}(a+bx)\,dx \\ &= \frac{\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \operatorname{csch}^3(a+bx)\,dx = &-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} \\ &- \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}\end{aligned}$$

```
[In] Integrate[Csch[a + b*x]^3,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b + Log[Cosh[(a + b*x)/2]]/(2*b) - Log[Sinh[(a + b
*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	27
default	$\frac{-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	27
parallelrisch	$\frac{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^2-4\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	43
risch	$-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{bx+a}-1\right)}{2b}+\frac{\ln\left(e^{bx+a}+1\right)}{2b}$	65

```
[In] int(csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(30) = 60$.

Time $= 0.26$ (sec) , antiderivative size $= 387$, normalized size of antiderivative $= 11.38$

$$\int \operatorname{csch}^3(a+bx)\,dx =$$

$$-\frac{2\cosh(bx+a)^3 + 6\cosh(bx+a)\sinh(bx+a)^2 + 2\sinh(bx+a)^3 - (\cosh(bx+a)^4 + 4\cosh(bx+a$$

```
[In] integrate(csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time $= 0.19$ (sec) , antiderivative size $= 84$, normalized size of antiderivative $= 2.47$

$$\int \operatorname{csch}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(-b*x - a) + 1)/b - 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a) +
 e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time $= 0.28$ (sec) , antiderivative size $= 84$, normalized size of antiderivative $= 2.47$

$$\int \operatorname{csch}^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

```
[In] integrate(csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) -
 log(e^(b*x + a) + e^(-b*x - a) + 2) + log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

Mupad [B] (verification not implemented)

Time $= 2.09$ (sec) , antiderivative size $= 86$, normalized size of antiderivative $= 2.53$

$$\int \operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(1/sinh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*
b*x) - 1))
```

3.4 $\int \operatorname{csch}^4(a+bx)\,dx$

Optimal result . . . 87
Rubi [A] (verified) . . . 87
Mathematica [A] (verified) . . . 88
Maple [A] (verified) . . . 88
Fricas [B] (verification not implemented) . . . 88
Sympy [F] . . . 89
Maxima [B] (verification not implemented) . . . 89
Giac [A] (verification not implemented) . . . 89
Mupad [B] (verification not implemented) . . . 90

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \operatorname{csch}^4(a+bx)\,dx = \frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b}$$

```
[Out] coth(b*x+a)/b-1/3*coth(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3852}

$$\int \operatorname{csch}^4(a+bx)\,dx = \frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b}$$

```
[In] Int[Csch[a + b*x]^4,x]
```

```
[Out] Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\operatorname{Subst}\left(\int (1+x^2)\,dx, x, -i\coth(a+bx)\right)}{b} \\ &= \frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \operatorname{csch}^4(a+bx)\,dx = \frac{2\coth(a+bx)}{3b} - \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^4,x]
[Out] (2*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^2}{3}\right)\coth(bx+a)}{b}$	23
default	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^2}{3}\right)\coth(bx+a)}{b}$	23
risch	$-\frac{4\left(3\,\mathrm{e}^{2bx+2a}-1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	32
parallelrisch	$\frac{-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^3+9\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+9\coth\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}$	55

```
[In] int(csch(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.31

$$\int \operatorname{csch}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh(bx+a)^5+5\,b\cosh(bx+a)\sinh(bx+a)^4+b\sinh(bx+a)^5-3\,b\cosh(bx+a)^3+\left(10\,b\cosh(\right.}$$

```
[In] integrate(csch(b*x+a)^4,x, algorithm="fricas")
[Out] -8/3*(cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
*x + a)^2 - 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 9*b*cosh(b*
x + a)^2 + 4*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**4,x)
[Out] Integral(csch(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(24) = 48$.

Time $= 0.19$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 3.46$

$$\int \operatorname{csch}^4(a+bx)\,dx = \frac{4\,e^{(-2\,bx-2\,a)}}{b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)} - \frac{4}{3\,b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^4,x, algorithm="maxima")
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1)) - 4/3/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1))
```

Giac [A] (verification not implemented)

none

Time $= 0.28$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 1.19$

$$\int \operatorname{csch}^4(a+bx)\,dx = -\frac{4\left(3\,e^{(2\,bx+2\,a)} - 1\right)}{3\,b(e^{(2\,bx+2\,a)} - 1)^3}$$

```
[In] integrate(csch(b*x+a)^4,x, algorithm="giac")
[Out] -4/3*(3*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{csch}^4(a+bx)\,dx = -\frac{4\left(3\,\mathrm{e}^{2a+2bx}-1\right)}{3b\left(\mathrm{e}^{2a+2bx}-1\right)^3}$$

```
[In] int(1/sinh(a + b*x)^4,x)
[Out] -(4*(3*exp(2*a + 2*b*x) - 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

3.5 $\int \operatorname{csch}^5(a+bx)\,dx$

Optimal result . . . 91
Rubi [A] (verified) . . . 91
Mathematica [B] (verified) . . . 92
Maple [A] (verified) . . . 92
Fricas [B] (verification not implemented) . . . 93
Sympy [F] . . . 94
Maxima [B] (verification not implemented) . . . 94
Giac [B] (verification not implemented) . . . 95
Mupad [B] (verification not implemented) . . . 95

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \operatorname{csch}^5(a+bx)\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} + \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}$$

```
[Out] -3/8*arctanh(cosh(b*x+a))/b+3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*c
sch(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3855}

$$\int \operatorname{csch}^5(a+bx)\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b} + \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

```
[In] Int[Csch[a + b*x]^5,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) + (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b} - \frac{3}{4}\int \operatorname{csch}^3(a+bx)\,dx \\ &= \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b} + \frac{3}{8}\int \operatorname{csch}(a+bx)\,dx \\ &= -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} + \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time $= 0.02$ (sec) , antiderivative size $= 113$, normalized size of antiderivative $= 2.05$

$$\begin{aligned}\int \operatorname{csch}^5(a+bx)\,dx = &\frac{3\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} \\ &+ \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}\end{aligned}$$

```
[In] Integrate[Csch[a + b*x]^5,x]
```

```
[Out] (3*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (3*Log[Cosh[(
a + b*x)/2]])/(8*b) + (3*Log[Sinh[(a + b*x)/2]])/(8*b) + (3*Sech[(a + b*x)/
2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time $= 0.79$ (sec) , antiderivative size $= 41$, normalized size of antiderivative $= 0.75$

method	result	size
derivativedivides	$\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	41
default	$\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	41
parallelrisch	$\frac{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^4-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^4-8\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+8\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^2+24\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}$	69
risch	$\frac{e^{bx+a}\left(3e^{6bx+6a}-11e^{4bx+4a}-11e^{2bx+2a}+3\right)}{4b\left(e^{2bx+2a}-1\right)^4}-\frac{3\ln\left(e^{bx+a}+1\right)}{8b}+\frac{3\ln\left(e^{bx+a}-1\right)}{8b}$	89

```
[In] int(csch(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((-1/4*csch(b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(49) = 98$.

Time $= 0.25$ (sec) , antiderivative size $= 1114$, normalized size of antiderivative $= 20.25$

$$\int \operatorname{csch}^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a)
^7 + 2*(63*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10*
(21*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(105*cosh(b*x +
 a)^4 - 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 22*cosh(b*x + a)^3 + 2*
(63*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)
^2 - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cos
h(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 3
0*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x
 + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(
b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^
8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*co
sh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 -
30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x
 + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh
(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
```

```
 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x +
a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a)
- 1) + 2*(21*cosh(b*x + a)^6 - 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 + 3)
*sinh(b*x + a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*si
nh(b*x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*si
nh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b
*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x +
 a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*c
osh(b*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x +
 a)^2 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 -
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(49) = 98$.

Time $= 0.20$ (sec) , antiderivative size $= 133$, normalized size of antiderivative $= 2.42$

$$\int \operatorname{csch}^5(a+bx)\,dx = -\frac{3\log\left(e^{(-bx-a)}+1\right)}{8\,b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{8\,b} - \frac{3\,e^{(-bx-a)} - 11\,e^{(-3\,bx-3\,a)} - 11\,e^{(-5\,bx-5\,a)} + 3\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} - 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b - 1/4*(3*e^(-b*x
 - a) - 11*e^(-3*b*x - 3*a) - 11*e^(-5*b*x - 5*a) + 3*e^(-7*b*x - 7*a))/(b*
(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x -
 8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time $= 0.27$ (sec) , antiderivative size $= 110$, normalized size of antiderivative $= 2.00$

$$\int \operatorname{csch}^5(a+bx)\,dx$$

$$= \frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} - 3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

```
[In] integrate(csch(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*(4*(3*(e^(b*x + a) + e^(-b*x - a))^3 - 20*e^(b*x + a) - 20*e^(-b*x - a
))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - 3*log(e^(b*x + a) + e^(-b*x - a
) + 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time $= 2.04$ (sec) , antiderivative size $= 193$, normalized size of antiderivative $= 3.51$

$$\begin{aligned}\int \operatorname{csch}^5(a+bx)\,dx = {} & \frac{3\,\mathrm{e}^{a+bx}}{4\,b\,\left(\mathrm{e}^{2a+2bx}-1\right)} - \frac{\mathrm{e}^{a+bx}}{2\,b\,\left(\mathrm{e}^{4a+4bx}-2\,\mathrm{e}^{2a+2bx}+1\right)} \\ & - \frac{2\,\mathrm{e}^{a+bx}}{b\,\left(3\,\mathrm{e}^{2a+2bx}-3\,\mathrm{e}^{4a+4bx}+\mathrm{e}^{6a+6bx}-1\right)} \\ & - \frac{4\,\mathrm{e}^{3a+3bx}}{b\,\left(6\,\mathrm{e}^{4a+4bx}-4\,\mathrm{e}^{2a+2bx}-4\,\mathrm{e}^{6a+6bx}+\mathrm{e}^{8a+8bx}+1\right)} \\ & - \frac{3\,\operatorname{atan}\left(\frac{\mathrm{e}^{bx}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}\end{aligned}$$

```
[In] int(1/sinh(a + b*x)^5,x)
```

```
[Out] (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1)) - exp(a + b*x)/(2*b*(exp(4*a
+ 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*exp(2*a + 2*b*
x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(3*a + 3*b*x))/(b*
(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8
*b*x) + 1)) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2))
```

3.6 $\int \operatorname{csch}^6(a+bx)\,dx$

Optimal result . . . 96
Rubi [A] (verified) . . . 96
Mathematica [A] (verified) . . . 97
Maple [A] (verified) . . . 97
Fricas [B] (verification not implemented) . . . 97
Sympy [F] . . . 98
Maxima [B] (verification not implemented) . . . 98
Giac [A] (verification not implemented) . . . 99
Mupad [B] (verification not implemented) . . . 99

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{\coth(a+bx)}{b} + \frac{2\coth^3(a+bx)}{3b} - \frac{\coth^5(a+bx)}{5b}$$

```
[Out] -coth(b*x+a)/b+2/3*coth(b*x+a)^3/b-1/5*coth(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3852}

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{\coth^5(a+bx)}{5b} + \frac{2\coth^3(a+bx)}{3b} - \frac{\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^6,x]
```

```
[Out] -(Coth[a + b*x]/b) + (2*Coth[a + b*x]^3)/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\coth(a+bx)\right)}{b} \\ &= -\frac{\coth(a+bx)}{b} + \frac{2\coth^3(a+bx)}{3b} - \frac{\coth^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{8\coth(a+bx)}{15b} + \frac{4\coth(a+bx)\operatorname{csch}^2(a+bx)}{15b} - \frac{\coth(a+bx)\operatorname{csch}^4(a+bx)}{5b}$$

```
[In] Integrate[Csch[a + b*x]^6,x]
```

```
[Out] (-8*Coth[a + b*x])/(15*b) + (4*Coth[a + b*x]*Csch[a + b*x]^2)/(15*b) - (Cot
h[a + b*x]*Csch[a + b*x]^4)/(5*b)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	s
derivativedivides	$\frac{\left(-\frac{8}{15}-\frac{\operatorname{csch}(bx+a)^4}{5}+\frac{4\operatorname{csch}(bx+a)^2}{15}\right)\coth(bx+a)}{b}$	3
default	$\frac{\left(-\frac{8}{15}-\frac{\operatorname{csch}(bx+a)^4}{5}+\frac{4\operatorname{csch}(bx+a)^2}{15}\right)\coth(bx+a)}{b}$	3
risch	$-\frac{16\left(10\,e^{4bx+4a}-5\,e^{2bx+2a}+1\right)}{15b\left(e^{2bx+2a}-1\right)^5}$	4
parallelrisch	$\frac{-3\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^5-3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5+25\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^3+25\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3-150\coth\left(\frac{bx}{2}+\frac{a}{2}\right)-150\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{480b}$	8

```
[In] int(csch(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-8/15-1/5*csch(b*x+a)^4+4/15*csch(b*x+a)^2)*coth(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 344, normalized size of antiderivative = 8.19

$$\int \operatorname{csch}^6(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh(bx+a)^8+8\,b\cosh(bx+a)\sinh(bx+a)^7+b\sinh(bx+a)^8-5\,b\cosh(bx+a)^6+(28\,b\cos$$

```
[In] integrate(csch(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 - 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
b*x + a)^8 - 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 - 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 - 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 -
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 - 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 - 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F]

$$\int \operatorname{csch}^6(a+bx)\,dx = \int \operatorname{csch}^6(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**6,x)
[Out] Integral(csch(a + b*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(38) = 76$.

Time $= 0.19$ (sec) , antiderivative size $= 205$, normalized size of antiderivative $= 4.88$

$$\begin{aligned}
&\int \operatorname{csch}^6(a+bx)\,dx \\
&= -\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)} \\
&\quad + \frac{32\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)} \\
&\quad + \frac{16}{15\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)}
\end{aligned}$$

```
[In] integrate(csch(b*x+a)^6,x, algorithm="maxima")
[Out] -16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 32/3*e^(-4
*b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15/(b*(5*e^(-2*b*x
 - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) +
e^(-10*b*x - 10*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{16\left(10\,e^{(4\,bx+4\,a)} - 5\,e^{(2\,bx+2\,a)} + 1\right)}{15\,b(e^{(2\,bx+2\,a)} - 1)^5}$$

```
[In] integrate(csch(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) - 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1
)^5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{16\left(10\,\mathrm{e}^{4\,a+4\,b\,x} - 5\,\mathrm{e}^{2\,a+2\,b\,x} + 1\right)}{15\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x} - 1\right)^5}$$

```
[In] int(1/sinh(a + b*x)^6,x)
```

```
[Out] -(16*(10*exp(4*a + 4*b*x) - 5*exp(2*a + 2*b*x) + 1))/(15*b*(exp(2*a + 2*b*x
) - 1)^5)
```

3.7 $\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal result . . . 100
Rubi [A] (verified) . . . 100
Mathematica [A] (verified) . . . 101
Maple [A] (verified) . . . 102
Fricas [C] (verification not implemented) . . . 102
Sympy [F] . . . 102
Maxima [F] . . . 103
Giac [F] . . . 103
Mupad [F(-1)] . . . 103

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

```
[Out] -2/3*cosh(b*x+a)*csch(b*x+a)^(3/2)/b-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2
)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2720}

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} + \frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}$$

```
[In] Int[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*x
]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} - \frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx \\ &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} - \frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx \\ &= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} \\ &\quad + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx \\ &= -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\coth(a+bx)+i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}\right)}{3b}\end{aligned}$$

```
[In] Integrate[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Coth[a + b*x] + I*EllipticF[((-2*I)*a + Pi - (2*I)
*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\sinh(bx+a)+2\cosh(bx+a)^2}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)b}$	101

```
[In] int(csch(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.42

$$\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(b}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx}}\right.}{3\left(b\cosh(b}$$

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (sqrt(2)*cosh(b*x + a)^2 +
2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2 - sqrt(2))*
weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx = \int \text{csch}^{\frac{5}{2}}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**(5/2),x)
```

```
[Out] Integral(csch(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = \int \left(\frac{1}{\sinh\left(a+b\,x\right)}\right)^{5/2} dx$$

```
[In] int((1/sinh(a + b*x))^(5/2),x)
[Out] int((1/sinh(a + b*x))^(5/2), x)
```

3.8 $\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal result . . . 104
Rubi [A] (verified) . . . 104
Mathematica [A] (verified) . . . 105
Maple [A] (verified) . . . 106
Fricas [C] (verification not implemented) . . . 106
Sympy [F] . . . 106
Maxima [F] . . . 107
Giac [F] . . . 107
Mupad [F(-1)] . . . 107

Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2*cosh(b*x+a)*csch(b*x+a)^(1/2)/b+2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2719}

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} - \frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|\,2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[In] Int[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*EllipticE[(I*a - Pi/2 + I
*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} + \int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx \\ &= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} + \frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}} \\ &= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.17$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 0.75$

$$\begin{aligned}&\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx \\ &= -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\cosh(a+bx) - E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|\,2\right)\sqrt{i\sinh(a+bx)}\right)}{b}\end{aligned}$$

```
[In] Integrate[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b
*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/b
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

method	result
default	$\frac{2\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}$ … $\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b$

```
[In] int(csch(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}}\left(\cosh(bx+a)+\sinh(bx+a)\right)+\sqrt{2}\text{weierstrassZeta}\right.}{b}$$

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))*(cosh(b*x + a) + sinh(b*x +
a)) + sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x + a)
 + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**(3/2),x)
```

```
[Out] Integral(csch(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \left(\frac{1}{\sinh\left(a+b\,x\right)}\right)^{3/2}\,dx$$

```
[In] int((1/sinh(a + b*x))^(3/2),x)
[Out] int((1/sinh(a + b*x))^(3/2), x)
```

3.9 $\int \sqrt{\operatorname{csch}(a+bx)}\,dx$

Optimal result . . . 108
Rubi [A] (verified) . . . 108
Mathematica [A] (verified) . . . 109
Maple [A] (verified) . . . 109
Fricas [C] (verification not implemented) . . . 110
Sympy [F] . . . 110
Maxima [F] . . . 110
Giac [F] . . . 110
Mupad [F(-1)] . . . 111

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = -\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b
*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2720}

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = -\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b}$$

```
[In] Int[Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sin
h[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx \\ &= -\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \frac{2\operatorname{csch}^{\frac{3}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)(i\sinh(a+bx))^{3/2}}{b}$$

```
[In] Integrate[Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Csch[a + b*x]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[
a + b*x])^(3/2))/b
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{i\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	87

```
[In] int(csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}(a+bx)}\,dx$$

```
[In] integrate(csch(b*x+a)**(1/2),x)
[Out] Integral(sqrt(csch(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}(bx+a)}\,dx$$

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(csch(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}(bx+a)}\,dx$$

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csch(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\frac{1}{\sinh{(a+b\,x)}}}\,dx$$

```
[In] int((1/sinh(a + b*x))^(1/2),x)
[Out] int((1/sinh(a + b*x))^(1/2), x)
```

3.10 $\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal result . . . 112
Rubi [A] (verified) . . . 112
Mathematica [A] (verified) . . . 113
Maple [A] (verified) . . . 113
Fricas [C] (verification not implemented) . . . 114
Sympy [F] . . . 114
Maxima [F] . . . 114
Giac [F] . . . 115
Mupad [F(-1)] . . . 115

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2719}

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|\,2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[In] Int[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}} \\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{2\sqrt{\operatorname{csch}(a+bx)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{b}$$

```
[In] Integrate[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csch[a + b*x]]*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a +
 b*x]])/b
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\right.\right.$ $\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b$
risch	$\frac{\sqrt{2}}{b\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}}-\frac{\left(\frac{2e^{2bx+2a}-2}{\sqrt{e^{bx+a}\left(e^{2bx+2a}-1\right)}}-\frac{\sqrt{e^{bx+a}+1}\,\sqrt{-2e^{bx+a}+2}\,\sqrt{-e^{bx+a}}\left(-2\,\text{EllipticE}\left(\sqrt{e^{bx+a}+1},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{e^{bx+a}+1},\frac{\sqrt{2}}{2}\right.\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.$ $b\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}\left(e^{2bx+2a}-1\right)$

```
[In] int(1/csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.78

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = -\frac{\sqrt{2}\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(bx+a}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)}}}{b}$$

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*(sqrt(2)*cosh(b*x + a) + sqrt
(2)*sinh(b*x + a))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x
 + a) + sinh(b*x + a))))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$

```
[In] integrate(1/csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(csch(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\text{csch}\,(bx+a)}}\,dx$$

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

```
[In] int(1/(1/sinh(a + b*x))^(1/2),x)
[Out] int(1/(1/sinh(a + b*x))^(1/2), x)
```

3.11 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 116
Rubi [A] (verified) . . . 116
Mathematica [A] (verified) . . . 117
Maple [A] (verified) . . . 118
Fricas [C] (verification not implemented) . . . 118
Sympy [F] . . . 118
Maxima [F] . . . 119
Giac [F] . . . 119
Mupad [F(-1)] . . . 119

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

```
[Out] 2/3*cosh(b*x+a)/b/csch(b*x+a)^(1/2)-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} + \frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}$$

```
[In] Int[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x]
]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} - \frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx \\ &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} - \frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx \\ &= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\begin{aligned}&\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx \\ &= \frac{\sqrt{\operatorname{csch}(a+bx)}\left(-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}+\sinh(2(a+bx))\right)}{3b}\end{aligned}$$

```
[In] Integrate[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Csch[a + b*x]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sq
rt[I*Sinh[a + b*x]] + Sinh[2*(a + b*x)]))/(3*b)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(bx+a)^2\sinh(bx+a)}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	100

```
[In] int(1/csch(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)^2
*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.79

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$$

$$= \frac{\sqrt{2}\big(\cosh(bx+a)^4 + 4\cosh(bx+a)^3\sinh(bx+a) + 6\cosh(bx+a)^2\sinh(bx+a)^2 + 4\cosh(bx+a)\sinh$$

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*
x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 4*(sqrt(2)*cosh(b*x + a)^2 + 2*
sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2)*weierstrassP
Inverse(4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh
(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/csch(b*x+a)**(3/2),x)
```

```
[Out] Integral(csch(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(1/sinh(a + b*x))^(3/2),x)
[Out] int(1/(1/sinh(a + b*x))^(3/2), x)
```

3.12 $\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result . . . 120
Rubi [A] (verified) . . . 120
Mathematica [A] (verified) . . . 121
Maple [A] (verified) . . . 122
Fricas [C] (verification not implemented) . . . 122
Sympy [F] . . . 123
Maxima [F] . . . 123
Giac [F] . . . 123
Mupad [F(-1)] . . . 123

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{5b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2/5*cosh(b*x+a)/b/csch(b*x+a)^(3/2)-6/5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[In] Int[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(5*b*Csch[a + b*x]^(3/2)) + (((6*I)/5)*EllipticE[(I*a - P
i/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx \\ &= \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{3\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}} \\ &= \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)}{5b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx \\ &= \frac{2\left(\cosh(a+bx) - 3\operatorname{csch}^2(a+bx)E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|\,2\right)\sqrt{i\sinh(a+bx)}\right)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}\end{aligned}$$

```
[In] Integrate[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*(Cosh[a + b*x] - 3*Csch[a + b*x]^2*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)
/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(5*b*Csch[a + b*x]^(3/2))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

method	result
default	$-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}}{5}$ $\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b$

```
[In] int(1/csch(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.62

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$$

$$= \frac{\sqrt{2}\left(\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + \left(15\cosh(bx+a)^2 + 11\right)\sinh(bx+\right.}{}$$

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x
 + a)^6 + (15*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^4 + 11*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 + 66*cosh(b*x + a)^2 - 13)*sinh(b*x + a)^2 - 13*cosh(b*x + a)^2 + 2*(3*
cosh(b*x + a)^5 + 22*cosh(b*x + a)^3 - 13*cosh(b*x + a))*sinh(b*x + a) + 1)
*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*si
nh(b*x + a) + sinh(b*x + a)^2 - 1)) + 24*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(
2)*cosh(b*x + a)^2*sinh(b*x + a) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2
+ sqrt(2)*sinh(b*x + a)^3)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*s
inh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/csch(b*x+a)**(5/2),x)
[Out] Integral(csch(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(1/sinh(a + b*x))^(5/2),x)
[Out] int(1/(1/sinh(a + b*x))^(5/2), x)
```

3.13 $\int (b\operatorname{csch}(c+dx))^{7/2}\,dx$

Optimal result . . . 124
Rubi [A] (verified) . . . 124
Mathematica [A] (verified) . . . 126
Maple [F] . . . 126
Fricas [C] (verification not implemented) . . . 126
Sympy [F] . . . 127
Maxima [F] . . . 127
Giac [F] . . . 127
Mupad [F(-1)] . . . 127

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d} + \frac{6ib^4E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2/5*b*cosh(d*x+c)*(b*csch(d*x+c))^(5/2)/d+6/5*b^3*cosh(d*x+c)*(b*csch(d*x+
c))^(1/2)/d-6/5*I*b^4*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1
/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch
(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \frac{6ib^4E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}} + \frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d}$$

```
[In] Int[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (6*b^3*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/(5*d) - (2*b*Cosh[c + d*x]*(b*C
sch[c + d*x])^(5/2))/(5*d) + (((6*I)/5)*b^4*EllipticE[(I*c - Pi/2 + I*d*x)/
2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d} - \frac{1}{5}(3b^2)\int (b\operatorname{csch}(c+dx))^{3/2}\,dx \\
&= \frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d} \\
&\quad - \frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d} - \frac{1}{5}(3b^4)\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx \\
&= \frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d} \\
&\quad - \frac{(3b^4)\int \sqrt{i\sinh(c+dx)}\,dx}{5\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}} \\
&= \frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d} \\
&\quad + \frac{6ib^4 E\left(\frac{1}{2}\left(ic - \frac{\pi}{2} + idx\right)\middle|\, 2\right)}{5d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx =$$

$$-\frac{2b^3\sqrt{b\operatorname{csch}(c+dx)}\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3E\left(\tfrac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c}\right.}{5d}$$

```
[In] Integrate[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (-2*b^3*Sqrt[b*Csch[c + d*x]]*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*
x] + 3*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/
(5*d)
```

Maple [F]

$$\int (b\ \operatorname{csch}(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] int((b*csch(d*x+c))^(7/2),x)
```

```
[Out] int((b*csch(d*x+c))^(7/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.17

$$\int (b\operatorname{csch}(c$$

$$+dx))^{7/2}\,dx = \frac{2\left(3\sqrt{2}\left(b^3\cosh(dx+c)^4+4\,b^3\cosh(dx+c)\sinh(dx+c)^3+b^3\sinh(dx+c)^4-2\,b^3\cosh(dx\right.\right.}$$

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(2)*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*
x + c) + sinh(d*x + c))) + sqrt(2)*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d*x
 + c)*sinh(d*x + c)^4 + 3*b^3*sinh(d*x + c)^5 - 8*b^3*cosh(d*x + c)^3 + b^3
*cosh(d*x + c) + 2*(15*b^3*cosh(d*x + c)^2 - 4*b^3)*sinh(d*x + c)^3 + 6*(5*
b^3*cosh(d*x + c)^3 - 4*b^3*cosh(d*x + c))*sinh(d*x + c)^2 + (15*b^3*cosh(d
```

```
*x + c)^4 - 24*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c))*sqrt((b*cosh(d*x +
 c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 - 1)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^
3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \int (b\operatorname{csch}(c+dx))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))**(7/2),x)
[Out] Integral((b*csch(c + d*x))**(7/2), x)
```

Maxima [F]

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \int \left(\frac{b}{\sinh(c+d\,x)}\right)^{7/2} dx$$

```
[In] int((b/sinh(c + d*x))^(7/2),x)
[Out] int((b/sinh(c + d*x))^(7/2), x)
```

3.14 $\int (b\operatorname{csch}(c+dx))^{5/2}\,dx$

Optimal result . . . 128
Rubi [A] (verified) . . . 128
Mathematica [A] (verified) . . . 129
Maple [F] . . . 130
Fricas [C] (verification not implemented) . . . 130
Sympy [F] . . . 130
Maxima [F] . . . 131
Giac [F] . . . 131
Mupad [F(-1)] . . . 131

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} + \frac{2ib^2\sqrt{b\operatorname{csch}(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3d}$$

```
[Out] -2/3*b*cosh(d*x+c)*(b*csch(d*x+c))^(3/2)/d-2/3*I*b^2*(sin(1/2*I*c+1/4*Pi+1/
2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*P
i+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2720}

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} + \frac{2ib^2\sqrt{i\sinh(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3d}$$

```
[In] Int[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(3/2))/(3*d) + (((2*I)/3)*b^2*Sqrt[b*
Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} - \frac{1}{3}b^2\int \sqrt{b\operatorname{csch}(c+dx)}\,dx \\ &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} \\ &\quad -\frac{1}{3}\left(b^2\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx \\ &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} \\ &\quad +\frac{2ib^2\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = -\frac{2b^2\sqrt{b\operatorname{csch}(c+dx)}\left(\coth(c+dx)+i\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)\sqrt{i\sinh(c+dx)}\right)}{3d}$$

```
[In] Integrate[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b^2*Sqrt[b*Csch[c + d*x]]*(Coth[c + d*x] + I*EllipticF[((-2*I)*c + Pi -
 (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(3*d)
```

Maple [F]

$$\int (b \operatorname{csch}(dx+c))^{\frac{5}{2}}\, dx$$

```
[In] int((b*csch(d*x+c))^(5/2),x)
[Out] int((b*csch(d*x+c))^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.47

$$\int (b\operatorname{csch}(c+dx))^{5/2}\, dx =$$

$$-\frac{2\left(\sqrt{2}\left(b^2\cosh(dx+c)^2+2\,b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2-b^2\right)\sqrt{b}\text{weierstrassPInverse}(4$$

$$3\left(d\cosh($$

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -2/3*(sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^
2*sinh(d*x + c)^2 - b^2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) +
sinh(d*x + c)) + sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*
x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c)
)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)))
/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 -
 d)
```

Sympy [F]

$$\int (b\operatorname{csch}(c+dx))^{5/2}\, dx = \int (b\operatorname{csch}(c+dx))^{\frac{5}{2}}\, dx$$

```
[In] integrate((b*csch(d*x+c))**(5/2),x)
[Out] Integral((b*csch(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = \int \left(\frac{b}{\sinh(c+d\,x)}\right)^{5/2} dx$$

```
[In] int((b/sinh(c + d*x))^(5/2),x)
[Out] int((b/sinh(c + d*x))^(5/2), x)
```

3.15 $\int (b\operatorname{csch}(c+dx))^{3/2}\,dx$

Optimal result . . . 132
Rubi [A] (verified) . . . 132
Mathematica [A] (verified) . . . 133
Maple [F] . . . 134
Fricas [C] (verification not implemented) . . . 134
Sympy [F] . . . 134
Maxima [F] . . . 134
Giac [F] . . . 135
Mupad [F(-1)] . . . 135

Optimal result

Integrand size = 12, antiderivative size = 84

$$\int (b\operatorname{csch}(c+dx))^{3/2}\,dx = -\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d} - \frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*b*cosh(d*x+c)*(b*csch(d*x+c))^(1/2)/d+2*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2*I*
d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/
2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$\int (b\operatorname{csch}(c+dx))^{3/2}\,dx = -\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d} - \frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

```
[In] Int[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c -
Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d} + b^2\int\frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx \\ &= -\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d} + \frac{b^2\int\sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}} \\ &= -\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d} - \frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int(b\operatorname{csch}(c+dx))^{3/2}\,dx = -\frac{2b\sqrt{b\operatorname{csch}(c+dx)}\left(\cosh(c+dx) - E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)\sqrt{i\sinh(c+dx)}\right)}{d}$$

```
[In] Integrate[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Csch[c + d*x]]*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*
I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/d
```

Maple [F]

$$\int (b\ \mathrm{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((b*csch(d*x+c))^(3/2),x)
[Out] int((b*csch(d*x+c))^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int (b\mathrm{csch}(c+dx))^{3/2}\,dx =$$

$$-\frac{2\left(\sqrt{2}b^{\frac{3}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\,(dx+c)+\sinh\,(dx+c)))+\sqrt{2}(b\cosh\,(dx+c\right.}{d}$$

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2*(sqrt(2)*b^(3/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*
x + c) + sinh(d*x + c))) + sqrt(2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)))/d
```

Sympy [F]

$$\int (b\mathrm{csch}(c+dx))^{3/2}\,dx = \int (b\,\mathrm{csch}\,(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))**(3/2),x)
[Out] Integral((b*csch(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\mathrm{csch}(c+dx))^{3/2}\,dx = \int (b\,\mathrm{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \int (b\,\text{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \int \left(\frac{b}{\sinh\,(c+d\,x)}\right)^{3/2} dx$$

```
[In] int((b/sinh(c + d*x))^(3/2),x)
[Out] int((b/sinh(c + d*x))^(3/2), x)
```

3.16 $\int \sqrt{b\operatorname{csch}(c+dx)}\,dx$

Optimal result . . . 136
Rubi [A] (verified) . . . 136
Mathematica [A] (verified) . . . 137
Maple [F] . . . 137
Fricas [C] (verification not implemented) . . . 137
Sympy [F] . . . 138
Maxima [F] . . . 138
Giac [F] . . . 138
Mupad [F(-1)] . . . 138

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = -\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2720}

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = -\frac{2i\sqrt{i\sinh(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\operatorname{csch}(c+dx)}}{d}$$

```
[In] Int[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*S
inh[c + d*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int\frac{1}{\sqrt{i\sinh(c+dx)}}\,dx \\ &= -\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int\sqrt{b\operatorname{csch}(c+dx)}\,dx = \frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)\sqrt{i\sinh(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Si
nh[c + d*x]])/d
```

Maple [F]

$$\int\sqrt{b\ \operatorname{csch}\left(dx+c\right)}dx$$

```
[In] int((b*csch(d*x+c))^(1/2),x)
```

```
[Out] int((b*csch(d*x+c))^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int\sqrt{b\operatorname{csch}(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{d}$$

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))/
d
```

Sympy [F]

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = \int \sqrt{b\,\operatorname{csch}(c+dx)}\,dx$$

```
[In] integrate((b*csch(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*csch(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = \int \sqrt{b\,\operatorname{csch}(dx+c)}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = \int \sqrt{b\,\operatorname{csch}(dx+c)}\,dx$$

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = \int \sqrt{\frac{b}{\sinh(c+d\,x)}}\,dx$$

```
[In] int((b/sinh(c + d*x))^(1/2),x)
[Out] int((b/sinh(c + d*x))^(1/2), x)
```

3.17 $\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx$

Optimal result . . . 139
Rubi [A] (verified) . . . 139
Mathematica [A] (verified) . . . 140
Maple [B] (verified) . . . 140
Fricas [C] (verification not implemented) . . . 141
Sympy [F] . . . 141
Maxima [F] . . . 141
Giac [F] . . . 142
Mupad [F(-1)] . . . 142

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*
sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2719}

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}} \\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx = \frac{2iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/(d*Sqrt[b*Csch[c + d*x]
]*Sqrt[I*Sinh[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(79) = 158.

Time = 0.50 (sec) , antiderivative size = 227, normalized size of antiderivative = 4.05

method	result
risch	$\frac{\sqrt{2}}{d\sqrt{\frac{b\,e^{dx+c}}{e^{2dx+2c}-1}}} - \frac{\left(\frac{2b\,e^{2dx+2c}-2b}{b\sqrt{e^{dx+c}\left(b\,e^{2dx+2c}-b\right)}} - \frac{\sqrt{e^{dx+c}+1}\sqrt{-2\,e^{dx+c}+2}\sqrt{-e^{dx+c}}\left(-2\,\text{EllipticE}\left(\sqrt{e^{dx+c}+1},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{e^{dx+c}+1},\frac{\sqrt{2}}{2}\right.\right.\right.}{\sqrt{e^{3dx+3c}b-e^{dx+c}b}}$ $d\sqrt{\frac{b\,e^{dx+c}}{e^{2dx+2c}-1}}\,(e^{2dx+2c}-1)$

```
[In] int(1/(b*csch(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)-1/d*(2*(b*exp(d*x+c)^2-b)
/b/(exp(d*x+c)*(b*exp(d*x+c)^2-b))^(1/2)-(exp(d*x+c)+1)^(1/2)*(-2*exp(d*x+c
)+2)^(1/2)*(-exp(d*x+c))^(1/2)/(exp(d*x+c)^3*b-exp(d*x+c)*b)^(1/2)*(-2*Elli
pticE((exp(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(d*x+c)+1)^(1/2),1/2*
2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)*(b*exp(d*x+c)*(exp
(d*x+c)^2-1))^(1/2)/(exp(d*x+c)^2-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.75

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{b}(\cosh(dx+c)+\sinh(dx+c))\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh(dx+c)+\sinh(dx+c)))}{bd}$$

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*sqrt(b)*(cosh(d*x + c) + sinh(d*x + c))*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt((b*cos
h(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2 - 1)))/(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*csch(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\operatorname{csch}(dx+c)}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{csch}\,(dx+c)}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\sinh(c+d\,x)}}}\,dx$$

```
[In] int(1/(b/sinh(c + d*x))^(1/2),x)
[Out] int(1/(b/sinh(c + d*x))^(1/2), x)
```

3.18 $\int \frac{1}{(b\mathbf{csch}(c+dx))^{3/2}}\,dx$

Optimal result . . . 143
Rubi [A] (verified) . . . 143
Mathematica [A] (verified) . . . 144
Maple [F] . . . 145
Fricas [C] (verification not implemented) . . . 145
Sympy [F] . . . 145
Maxima [F] . . . 146
Giac [F] . . . 146
Mupad [F(-1)] . . . 146

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}} + \frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3b^2d}$$

```
[Out] 2/3*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(1/2)-2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d
*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2
*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}} + \frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3b^2d}$$

```
[In] Int[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c
+ d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}} - \frac{\int \sqrt{b\operatorname{csch}(c+dx)}\,dx}{3b^2} \\ &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}} - \frac{\left(\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2} \\ &= \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}} + \frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3b^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{\operatorname{csch}^2(c+dx)\left(-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)\sqrt{i\sinh(c+dx)}+\sinh(2(c\right.}{3d(b\operatorname{csch}(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (Csch[c + d*x]^2*((-2*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I
*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(3*d*(b*Csch[c + d*x])^(3/2))
```

Maple [F]

$$\int \frac{1}{(b\ \mathrm{csch}\,(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(1/(b*csch(d*x+c))^(3/2),x)
[Out] int(1/(b*csch(d*x+c))^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.58

$$\int \frac{1}{(b\mathrm{csch}(c+dx))^{3/2}}\,dx =$$

$$-\frac{4\sqrt{2}\left(\cosh\left(dx+c\right)^2+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2\right)\sqrt{b}\mathrm{weierstrassPInverse}(4,0,\cosh\left(d$$

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -1/6*(4*sqrt(2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) -
 sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
1)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d
*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{1}{(b\mathrm{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\mathrm{csch}\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))**(3/2),x)
[Out] Integral((b*csch(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(b/sinh(c + d*x))^(3/2),x)
[Out] int(1/(b/sinh(c + d*x))^(3/2), x)
```

3.19 $\int \frac{1}{(b\operatorname{csch}(c+dx))^{5/2}}\,dx$

Optimal result . . . 147
Rubi [A] (verified) . . . 147
Mathematica [A] (verified) . . . 148
Maple [F] . . . 149
Fricas [C] (verification not implemented) . . . 149
Sympy [F] . . . 149
Maxima [F] . . . 150
Giac [F] . . . 150
Mupad [F(-1)] . . . 150

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{5/2}}\,dx = \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5b^2d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2/5*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(3/2)-6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d
*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2
*I*d*x),2^(1/2))/b^2/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{5/2}}\,dx = \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} + \frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

```
[In] Int[(b*Csch[c + d*x])^(-5/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(5*b*d*(b*Csch[c + d*x])^(3/2)) + (((6*I)/5)*EllipticE[(I
*c - Pi/2 + I*d*x)/2, 2])/(b^2*d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]
])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} - \frac{3\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx}{5b^2} \\ &= \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} - \frac{3\int \sqrt{i\sinh(c+dx)}\,dx}{5b^2\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}} \\ &= \frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{5b^2d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{5/2}}\,dx = \frac{-\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)}{\sqrt{i\sinh(c+dx)}} + \sinh(2(c+dx))}{5b^2d\sqrt{b\operatorname{csch}(c+dx)}}$$

```
[In] Integrate[(b*Csch[c + d*x])^(-5/2),x]
```

```
[Out] (((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]]
 + Sinh[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Csch[c + d*x]])
```

Maple [F]

$$\int \frac{1}{(b\ \mathrm{csch}\,(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int(1/(b*csch(d*x+c))^(5/2),x)
[Out] int(1/(b*csch(d*x+c))^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.20

$$\int \frac{1}{(b\mathrm{csch}(c+dx))^{5/2}}\,dx = \frac{24\sqrt{2}(\cosh(dx+c)^3 + 3\cosh(dx+c)^2\sinh(dx+c) + 3\cosh(dx+c)\sinh(dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/20*(24*sqrt(2)*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cos
h(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)*sqrt(b)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(
d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + (15*cosh(d
*x + c)^2 + 11)*sinh(d*x + c)^4 + 11*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3
 + 11*cosh(d*x + c))*sinh(d*x + c)^3 + (15*cosh(d*x + c)^4 + 66*cosh(d*x +
c)^2 - 13)*sinh(d*x + c)^2 - 13*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c)^5 + 22
*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt((b*cosh(d*x +
c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)))/(b^3*d*cosh(d*x + c)^3 + 3*b^3*d*cosh(d*x + c)^2*sinh(
d*x + c) + 3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*d*sinh(d*x + c)^3)
```

Sympy [F]

$$\int \frac{1}{(b\mathrm{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\mathrm{csch}\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))**(5/2),x)
[Out] Integral((b*csch(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(b/sinh(c + d*x))^(5/2),x)
[Out] int(1/(b/sinh(c + d*x))^(5/2), x)
```

3.20 $\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx$

Optimal result . . . 151
Rubi [A] (verified) . . . 151
Mathematica [A] (verified) . . . 153
Maple [F] . . . 153
Fricas [C] (verification not implemented) . . . 153
Sympy [F] . . . 154
Maxima [F] . . . 154
Giac [F] . . . 154
Mupad [F(-1)] . . . 154

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx = \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} - \frac{10i\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{21b^4d}$$

```
[Out] 2/7*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(5/2)-10/21*cosh(d*x+c)/b^3/d/(b*csch(d
*x+c))^(1/2)+10/21*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/
4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*
x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/b^4/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx = -\frac{10i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\operatorname{csch}(c+dx)}}{21b^4d} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} + \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}}$$

```
[In] Int[(b*Csch[c + d*x])^(-7/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(7*b*d*(b*Csch[c + d*x])^(5/2)) - (10*Cosh[c + d*x])/(21*
b^3*d*Sqrt[b*Csch[c + d*x]]) - (((10*I)/21)*Sqrt[b*Csch[c + d*x]]*EllipticF
[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^4*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{5\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx}{7b^2} \\
&= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} + \frac{5\int \sqrt{b\operatorname{csch}(c+dx)}\,dx}{21b^4} \\
&= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} \\
&\quad + \frac{\left(5\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{21b^4} \\
&= \frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}} \\
&\quad - \frac{10i\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{21b^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx = \frac{\sqrt{b\operatorname{csch}(c+dx)}\left(40i\,\text{EllipticF}\left(\tfrac{1}{4}(-2ic+\pi-2idx),2\right)\sqrt{i\sinh(c+dx)}-26\sin\right.}{84b^4d}$$

```
[In] Integrate[(b*Csch[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Csch[c + d*x]]*((40*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*
Sqrt[I*Sinh[c + d*x]] - 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^
4*d)
```

Maple [F]

$$\int \frac{1}{(b\ \operatorname{csch}(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] int(1/(b*csch(d*x+c))^(7/2),x)
```

```
[Out] int(1/(b*csch(d*x+c))^(7/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.09

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx = \frac{80\sqrt{2}\left(\cosh(dx+c)^4+4\cosh(dx+c)^3\sinh(dx+c)+6\cosh(dx+c)^2\sinh(d\right.}{}$$

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/168*(80*sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) +
sqrt(2)*(3*cosh(d*x + c)^8 + 24*cosh(d*x + c)*sinh(d*x + c)^7 + 3*sinh(d*x
+ c)^8 + 2*(42*cosh(d*x + c)^2 - 13)*sinh(d*x + c)^6 - 26*cosh(d*x + c)^6 +
 12*(14*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(7*cosh(d*
x + c)^4 - 13*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(21*cosh(d*x + c)^5 - 65
*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(42*cosh(d*x + c)^6 - 195*cosh(d*x +
c)^4 + 13)*sinh(d*x + c)^2 + 26*cosh(d*x + c)^2 + 4*(6*cosh(d*x + c)^7 - 39
*cosh(d*x + c)^5 + 13*cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt((b*cosh(d*x +
c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)))/(b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^4*d*sinh(d*x + c)^4)
```

Sympy [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))**(7/2),x)
[Out] Integral((b*csch(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int(1/(b/sinh(c + d*x))^(7/2),x)
[Out] int(1/(b/sinh(c + d*x))^(7/2), x)
```

3.21 $\int (b\operatorname{csch}(c+dx))^n\,dx$

Optimal result . . . 155
Rubi [A] (verified) . . . 155
Mathematica [A] (verified) . . . 156
Maple [F] . . . 156
Fricas [F] . . . 157
Sympy [F] . . . 157
Maxima [F] . . . 157
Giac [F] . . . 157
Mupad [F(-1)] . . . 158

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{-1+n}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] b*cosh(d*x+c)*(b*csch(d*x+c))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,-sinh(d*x+c)^2)/d/(1-n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{n-1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Csch[c + d*x])^n,x]
```

```
[Out] (b*Cosh[c + d*x]*(b*Csch[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, -Sinh[c + d*x]^2])/(d*(1 - n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = (b\operatorname{csch}(c+dx))^n \left(\frac{\sinh(c+dx)}{b}\right)^n \int \left(\frac{\sinh(c+dx)}{b}\right)^{-n} dx$$

$$= \frac{\cosh(c+dx)(b\operatorname{csch}(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, -\sinh^2(c+dx)\right) \sinh(c+dx)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int (b\operatorname{csch}(c+dx))^n \, dx =$$

$$-\frac{2^n \left(\frac{e^{c+dx}}{-1+e^{2(c+dx)}}\right)^n \left(-1+e^{2(c+dx)}\right) \operatorname{csch}^{-n}(c+dx)(b\operatorname{csch}(c+dx))^n \operatorname{Hypergeometric2F1}\left(1, 1-\frac{n}{2}, 1+\frac{n}{2}, e^2\right.}{dn}$$

```
[In] Integrate[(b*Csch[c + d*x])^n,x]
```

```
[Out] -((2^n*(E^(c + d*x)/(-1 + E^(2*(c + d*x))))^n*(-1 + E^(2*(c + d*x)))*(b*Csc
h[c + d*x])^n*Hypergeometric2F1[1, 1 - n/2, 1 + n/2, E^(2*(c + d*x))])/(d*n
*Csch[c + d*x]^n))
```

Maple [F]

$$\int (b \operatorname{csch}(dx+c))^n \, dx$$

```
[In] int((b*csch(d*x+c))^n,x)
```

```
[Out] int((b*csch(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\text{csch}(c+dx))^n\,dx = \int (b\,\text{csch}\,(dx+c))^n\,dx$$

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*csch(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\text{csch}(c+dx))^n\,dx = \int (b\,\text{csch}\,(c+dx))^n\,dx$$

```
[In] integrate((b*csch(d*x+c))**n,x)
[Out] Integral((b*csch(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\text{csch}(c+dx))^n\,dx = \int (b\,\text{csch}\,(dx+c))^n\,dx$$

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^n, x)
```

Giac [F]

$$\int (b\text{csch}(c+dx))^n\,dx = \int (b\,\text{csch}\,(dx+c))^n\,dx$$

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*csch(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \int \left(\frac{b}{\sinh(c+d\,x)}\right)^n dx$$

```
[In] int((b/sinh(c + d*x))^n,x)
[Out] int((b/sinh(c + d*x))^n, x)
```

3.22 $\int \left(-\text{csch}^2(x)\right)^{5/2}\,dx$

Optimal result . . . 159
Rubi [A] (verified) . . . 159
Mathematica [A] (verified) . . . 160
Maple [B] (verified) . . . 160
Fricas [C] (verification not implemented) . . . 161
Sympy [F] . . . 161
Maxima [C] (verification not implemented) . . . 161
Giac [C] (verification not implemented) . . . 162
Mupad [F(-1)] . . . 162

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \left(-\text{csch}^2(x)\right)^{5/2}\,dx = \frac{3}{8}\arcsin(\coth(x)) + \frac{3}{8}\coth(x)\sqrt{-\text{csch}^2(x)} + \frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2}$$

```
[Out] 3/8*arcsin(coth(x))+1/4*coth(x)*(-csch(x)^2)^(3/2)+3/8*coth(x)*(-csch(x)^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 201, 222}

$$\int \left(-\text{csch}^2(x)\right)^{5/2}\,dx = \frac{3}{8}\arcsin(\coth(x)) + \frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2} + \frac{3}{8}\coth(x)\sqrt{-\text{csch}^2(x)}$$

```
[In] Int[(-Csch[x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Coth[x]])/8 + (3*Coth[x]*Sqrt[-Csch[x]^2])/8 + (Coth[x]*(-Csch[x]
^2)^(3/2))/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \left(1-x^2\right)^{3/2}\,dx, x, \coth(x)\right)\\ &= \frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2} + \frac{3}{4}\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \coth(x)\right)\\ &= \frac{3}{8}\coth(x)\sqrt{-\text{csch}^2(x)} + \frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \coth(x)\right)\\ &= \frac{3}{8}\arcsin(\coth(x)) + \frac{3}{8}\coth(x)\sqrt{-\text{csch}^2(x)} + \frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(-\text{csch}^2(x)\right)^{5/2}\,dx = \frac{1}{64}\left(-\text{csch}^2(x)\right)^{5/2}\sinh(x)\left(-22\cosh(x)+6\left(\cosh(3x)+4\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\text{si}\right.\right.\right.$$

```
[In] Integrate[(-Csch[x]^2)^(5/2),x]
```

```
[Out] ((-Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*(-Log[Cosh[x/2]
] + Log[Sinh[x/2]])*Sinh[x]^4)))/64
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.85

method	result	size
risch	$\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\left(3e^{6x}-11e^{4x}-11e^{2x}+3\right)}{4(e^{2x}-1)^3} - \frac{3\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}e^{-x}\left(e^{2x}-1\right)\ln(e^x+1)}{8} + \frac{3\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}e^{-x}\left(e^{2x}-1\right)\ln(e^x-1)}{8}$	114

```
[In] int((-csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/4/(exp(2*x)-1)^3*(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)*(3*exp(6*x)-11*exp(4*
x)-11*exp(2*x)+3)-3/8*(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)*exp(-x)*(exp(2*x)-
1)*ln(exp(x)+1)+3/8*(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)*exp(-x)*(exp(2*x)-1)
*ln(exp(x)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.92

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx =$$

$$-\frac{3\left(i\,e^{(8\,x)}-4i\,e^{(6\,x)}+6i\,e^{(4\,x)}-4i\,e^{(2\,x)}+i\right)\log\left(e^{x}+1\right)+3\left(-i\,e^{(8\,x)}+4i\,e^{(6\,x)}-6i\,e^{(4\,x)}+4i\,e^{(2\,x)}-i\right)\text{lo}}{8\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="fricas")
[Out] -1/8*(3*(I*e^(8*x) - 4*I*e^(6*x) + 6*I*e^(4*x) - 4*I*e^(2*x) + I)*log(e^x +
 1) + 3*(-I*e^(8*x) + 4*I*e^(6*x) - 6*I*e^(4*x) + 4*I*e^(2*x) - I)*log(e^x
- 1) - 6*I*e^(7*x) + 22*I*e^(5*x) + 22*I*e^(3*x) - 6*I*e^x)/(e^(8*x) - 4*e^
(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F]

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx = \int \left(-\operatorname{csch}^2(x)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((-csch(x)**2)**(5/2),x)
[Out] Integral((-csch(x)**2)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx = \frac{3i\,e^{(-x)}-11i\,e^{(-3\,x)}-11i\,e^{(-5\,x)}+3i\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}$$
$$+\frac{3}{8}i\,\log\left(e^{(-x)}+1\right)-\frac{3}{8}i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="maxima")
[Out] 1/4*(3*I*e^(-x) - 11*I*e^(-3*x) - 11*I*e^(-5*x) + 3*I*e^(-7*x))/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3/8*I*log(e^(-x) + 1) - 3/8*I*
log(e^(-x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx =$$

$$-\frac{1}{16}\left(\frac{4i\left(3\left(e^{(-x)}+e^{x}\right)^3-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^2-4\right)^2}-3i\log\left(e^{(-x)}+e^{x}+2\right)+3i\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3x)}+e^{x}\right)$$

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(4*I*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2 - 4)
^2 - 3*I*log(e^(-x) + e^x + 2) + 3*I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) +
e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx = \int \left(-\frac{1}{\sinh\left(x\right)^2}\right)^{5/2} dx$$

```
[In] int((-1/sinh(x)^2)^(5/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(5/2), x)
```

3.23 $\int \left(-\text{csch}^2(x)\right)^{3/2}\,dx$

Optimal result . . . 163
Rubi [A] (verified) . . . 163
Mathematica [B] (verified) . . . 164
Maple [B] (verified) . . . 164
Fricas [C] (verification not implemented) . . . 165
Sympy [F] . . . 165
Maxima [C] (verification not implemented) . . . 165
Giac [C] (verification not implemented) . . . 166
Mupad [F(-1)] . . . 166

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \left(-\text{csch}^2(x)\right)^{3/2}\,dx = \frac{1}{2}\arcsin(\coth(x)) + \frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}$$

```
[Out] 1/2*arcsin(coth(x))+1/2*coth(x)*(-csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 201, 222}

$$\int \left(-\text{csch}^2(x)\right)^{3/2}\,dx = \frac{1}{2}\arcsin(\coth(x)) + \frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}$$

```
[In] Int[(-Csch[x]^2)^(3/2),x]
```

```
[Out] ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[-Csch[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \coth(x)\right) \\ &= \frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \coth(x)\right) \\ &= \frac{1}{2}\arcsin(\coth(x)) + \frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.

Time $= 0.04$ (sec) , antiderivative size $= 51$, normalized size of antiderivative $= 2.12$

$$\begin{aligned}\int \left(-\text{csch}^2(x)\right)^{3/2}\,dx &= \frac{1}{8}\sqrt{-\text{csch}^2(x)}\left(\text{csch}^2\left(\frac{x}{2}\right)\right. \\ &\left. - 4\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right) + \text{sech}^2\left(\frac{x}{2}\right)\right)\sinh(x)\end{aligned}$$

```
[In] Integrate[(-Csch[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-Csch[x]^2]*(Csch[x/2]^2 - 4*Log[Cosh[x/2]] + 4*Log[Sinh[x/2]] + Sech
[x/2]^2)*Sinh[x])/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(18) = 36$.

Time $= 0.16$ (sec) , antiderivative size $= 99$, normalized size of antiderivative $= 4.12$

method	result	size
risch	$\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\left(1+e^{2x}\right)}{e^{2x}-1} - \frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\,e^{-x}\left(e^{2x}-1\right)\ln(e^x+1)}{2} + \frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\,e^{-x}\left(e^{2x}-1\right)\ln(e^x-1)}{2}$	99

```
[In] int((-csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(exp(2*x)-1)*(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)*(1+exp(2*x))-1/2*(-1/(exp
(2*x)-1)^2*exp(2*x))^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)+1/2*(-1/(exp(2
*x)-1)^2*exp(2*x))^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2}\,dx = \frac{\left(-i\,e^{(4\,x)} + 2i\,e^{(2\,x)} - i\right)\log\left(e^x+1\right) + \left(i\,e^{(4\,x)} - 2i\,e^{(2\,x)} + i\right)\log\left(e^x-1\right) + 2i\,e^{(3\,x)} -}{2\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)}$$

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*x) + 2*I*e^(2*x) - I)*log(e^x + 1) + (I*e^(4*x) - 2*I*e^(2*x)
 + I)*log(e^x - 1) + 2*I*e^(3*x) + 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F]

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2}\,dx = \int \left(-\operatorname{csch}^2(x)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2}\,dx = \frac{i\,e^{(-x)} + i\,e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{2}i\,\log\left(e^{(-x)}+1\right) - \frac{1}{2}i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (I*e^(-x) + I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*I*log(e^(-x) + 1)
 - 1/2*I*log(e^(-x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2}\,dx = -\frac{1}{4}\left(\frac{4i\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^2-4} - i\log\left(e^{(-x)}+e^{x}+2\right) + i\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3x)}+e^{x}\right)$$

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - I*log(e^(-x) + e^x + 2) +
 I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2}\,dx = \int \left(-\frac{1}{\sinh\left(x\right)^2}\right)^{3/2}\,dx$$

```
[In] int((-1/sinh(x)^2)^(3/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(3/2), x)
```

3.24 $\int \sqrt{-\text{csch}^2(x)}\,dx$

Optimal result . . . 167
Rubi [A] (verified) . . . 167
Mathematica [B] (verified) . . . 168
Maple [B] (verified) . . . 168
Fricas [C] (verification not implemented) . . . 168
Sympy [F] . . . 169
Maxima [C] (verification not implemented) . . . 169
Giac [C] (verification not implemented) . . . 169
Mupad [F(-1)] . . . 169

Optimal result

Integrand size = 10, antiderivative size = 3

$$\int \sqrt{-\text{csch}^2(x)}\,dx = \arcsin(\coth(x))$$

```
[Out] arcsin(coth(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 222}

$$\int \sqrt{-\text{csch}^2(x)}\,dx = \arcsin(\coth(x))$$

```
[In] Int[Sqrt[-Csch[x]^2],x]
```

```
[Out] ArcSin[Coth[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \coth(x)\right)\\ &= \arcsin(\coth(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. $2(3) = 6$.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 10.00

$$\int \sqrt{-\text{csch}^2(x)}\,dx = \sqrt{-\text{csch}^2(x)}\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)$$

```
[In] Integrate[Sqrt[-Csch[x]^2],x]
[Out] Sqrt[-Csch[x]^2]*(-Log[Cosh[x/2]] + Log[Sinh[x/2]])*Sinh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(3) = 6$.

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 22.33

method	result	size
risch	$\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,e^{-x}(e^{2x}-1)\ln(e^x-1) - \sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,e^{-x}(e^{2x}-1)\ln(e^x+1)$	67

```
[In] int((-csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)-(-1/(e
xp(2*x)-1)^2*exp(2*x))^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sqrt{-\text{csch}^2(x)}\,dx = -i\,\log\left(e^x+1\right) + i\,\log\left(e^x-1\right)$$

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="fricas")
[Out] -I*log(e^x + 1) + I*log(e^x - 1)
```

Sympy [F]

$$\int \sqrt{-\text{csch}^2(x)}\,dx = \int \sqrt{-\text{csch}^2\left(x\right)}\,dx$$

```
[In] integrate((-csch(x)**2)**(1/2),x)
[Out] Integral(sqrt(-csch(x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int \sqrt{-\text{csch}^2(x)}\,dx = i\,\log\left(e^{(-x)}+1\right) - i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="maxima")
[Out] I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 9.00

$$\int \sqrt{-\text{csch}^2(x)}\,dx = (i\,\log\left(e^{x}+1\right) - i\,\log\left(|e^{x}-1|\right))\text{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="giac")
[Out] (I*log(e^x + 1) - I*log(abs(e^x - 1)))*sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\text{csch}^2(x)}\,dx = \int \sqrt{-\frac{1}{\sinh\left(x\right)^2}}\,dx$$

```
[In] int((-1/sinh(x)^2)^(1/2),x)
[Out] int((-1/sinh(x)^2)^(1/2), x)
```

3.25 $\int \frac{1}{\sqrt{-\mathbf{csch}^2(x)}}\,dx$

Optimal result . . . 170
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Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

```
[Out] coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

```
[In] Int[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \coth(x)\right)$$

$$= \frac{\coth(x)}{\sqrt{-\text{csch}^2(x)}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\text{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{-\text{csch}^2(x)}}$$

```
[In] Integrate[1/Sqrt[-Csch[x]^2],x]
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(11) = 22.

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}\left(e^{2x}-1\right)} + \frac{1}{2\left(e^{2x}-1\right)\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^2}}}$	58

```
[In] int(1/(-csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-
1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{1}{2}\left(-i\,e^{(2\,x)} - i\right)e^{(-x)}$$

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(-I*e^(2*x) - I)*e^(-x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx$$

```
[In] integrate(1/(-csch(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(-csch(x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{1}{2}i\,e^{(-x)} + \frac{1}{2}i\,e^{x}$$

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*I*e^(-x) + 1/2*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = -\frac{-i\,e^{(-x)} - i\,e^{x}}{2\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(-I*e^(-x) - I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = -\mathrm{e}^{-2\,x}\sqrt{-\frac{1}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^2}}\left(\frac{\mathrm{e}^{4\,x}}{4}-\frac{1}{4}\right)$$

```
[In] int(1/(-1/sinh(x)^2)^(1/2),x)
[Out] -exp(-2*x)*(-1/(exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x)/4 - 1/4)
```

3.26 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx$

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Mathematica [A] (verified) . . . 175
Maple [B] (verified) . . . 175
Fricas [C] (verification not implemented) . . . 176
Sympy [F] . . . 176
Maxima [C] (verification not implemented) . . . 176
Giac [C] (verification not implemented) . . . 177
Mupad [F(-1)] . . . 177

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx = \frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}}$$

```
[Out] 1/3*coth(x)/(-csch(x)^2)^(3/2)+2/3*coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx = \frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}} + \frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}}$$

```
[In] Int[(-Csch[x]^2)^(-3/2),x]
```

```
[Out] Coth[x]/(3*(-Csch[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[-Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx = \frac{9\coth(x) - \cosh(3x)\text{csch}(x)}{12\sqrt{-\text{csch}^2(x)}}$$

```
[In] Integrate[(-Csch[x]^2)^(-3/2),x]
[Out] (9*Coth[x] - Cosh[3*x]*Csch[x])/(12*Sqrt[-Csch[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(25) = 50$.

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.58

method	result	size
risch	$-\frac{e^{4x}}{24(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{3e^{2x}}{8\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}(e^{2x}-1)} + \frac{3}{8(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{e^{-2x}}{24(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}$	118

```
[In] int(1/(-csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*exp(4*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)+3/8/(-1/(exp
(2*x)-1)^2*exp(2*x))^(1/2)/(exp(2*x)-1)*exp(2*x)+3/8/(exp(2*x)-1)/(-1/(exp(
2*x)-1)^2*exp(2*x))^(1/2)-1/24*exp(-2*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*ex
p(2*x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{1}{24}\left(i\,e^{(6\,x)} - 9i\,e^{(4\,x)} - 9i\,e^{(2\,x)} + i\right)e^{(-3\,x)}$$

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/24*(I*e^(6*x) - 9*I*e^(4*x) - 9*I*e^(2*x) + I)*e^(-3*x)
```

Sympy [F]

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-csch(x)**2)**(3/2),x)
[Out] Integral((-csch(x)**2)**(-3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = -\frac{1}{24}i\,e^{(3\,x)} + \frac{3}{8}i\,e^{(-x)} - \frac{1}{24}i\,e^{(-3\,x)} + \frac{3}{8}i\,e^{x}$$

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/24*I*e^(3*x) + 3/8*I*e^(-x) - 1/24*I*e^(-3*x) + 3/8*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\left(9i\,e^{(2\,x)} - i\right)e^{(-3\,x)} - i\,e^{(3\,x)} + 9i\,e^{x}}{24\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/24*((9*I*e^(2*x) - I)*e^(-3*x) - I*e^(3*x) + 9*I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(-1/sinh(x)^2)^(3/2),x)
[Out] int(1/(-1/sinh(x)^2)^(3/2), x)
```

3.27 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{5/2}}\,dx$

Optimal result . . . 178
Rubi [A] (verified) . . . 178
Mathematica [A] (verified) . . . 179
Maple [B] (verified) . . . 180
Fricas [C] (verification not implemented) . . . 180
Sympy [F] . . . 180
Maxima [C] (verification not implemented) . . . 181
Giac [C] (verification not implemented) . . . 181
Mupad [F(-1)] . . . 181

Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{5/2}}\,dx = \frac{\coth(x)}{5\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15\sqrt{-\text{csch}^2(x)}}$$

```
[Out] 1/5*coth(x)/(-csch(x)^2)^(5/2)+4/15*coth(x)/(-csch(x)^2)^(3/2)+8/15*coth(x)
/(-csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{5/2}}\,dx = \frac{8\coth(x)}{15\sqrt{-\text{csch}^2(x)}} + \frac{4\coth(x)}{15\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{\coth(x)}{5\left(-\text{csch}^2(x)\right)^{5/2}}$$

```
[In] Int[(-Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(-Csch[x]^2)^(5/2)) + (4*Coth[x])/(15*(-Csch[x]^2)^(3/2)) + (8*C
oth[x])/(15*Sqrt[-Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{5\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{5\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{8}{15}\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{5\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15\sqrt{-\text{csch}^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.04$ (sec) , antiderivative size $= 33$, normalized size of antiderivative $= 0.67$

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{5/2}}\,dx = \frac{(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\text{csch}(x)}{240\sqrt{-\text{csch}^2(x)}}$$

```
[In] Integrate[(-Csch[x]^2)^(-5/2),x]
```

```
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x])/(240*Sqrt[-Csch[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(37) = 74.

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.63

method	result
risch	$\frac{e^{6x}}{160(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{5e^{4x}}{96(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{5e^{2x}}{16\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}(e^{2x}-1)} + \frac{5}{16(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{5e^{-2x}}{96(e^{2x}-1)\sqrt{-}}$

```
[In] int(1/(-csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160*exp(6*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)-5/96*exp(4*x
)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)+5/16/(-1/(exp(2*x)-1)^2*e
xp(2*x))^(1/2)/(exp(2*x)-1)*exp(2*x)+5/16/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*e
xp(2*x))^(1/2)-5/96*exp(-2*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/
2)+1/160*exp(-4*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{1}{480}\left(-3i\,e^{(10\,x)} + 25i\,e^{(8\,x)} - 150i\,e^{(6\,x)} - 150i\,e^{(4\,x)} + 25i\,e^{(2\,x)} - 3i\right)e^{(-5\,x)}$$

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(10*x) + 25*I*e^(8*x) - 150*I*e^(6*x) - 150*I*e^(4*x) + 25*I*
e^(2*x) - 3*I)*e^(-5*x)
```

Sympy [F]

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-csch(x)**2)**(5/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{1}{160}i\,e^{(5\,x)} - \frac{5}{96}i\,e^{(3\,x)} + \frac{5}{16}i\,e^{(-x)} - \frac{5}{96}i\,e^{(-3\,x)} + \frac{1}{160}i\,e^{(-5\,x)} + \frac{5}{16}i\,e^{x}$$

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*I*e^(5*x) - 5/96*I*e^(3*x) + 5/16*I*e^(-x) - 5/96*I*e^(-3*x) + 1/160*
I*e^(-5*x) + 5/16*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = -\frac{\left(-150i\,e^{(4\,x)} + 25i\,e^{(2\,x)} - 3i\right)e^{(-5\,x)} - 3i\,e^{(5\,x)} + 25i\,e^{(3\,x)} - 150i\,e^{x}}{480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((-150*I*e^(4*x) + 25*I*e^(2*x) - 3*I)*e^(-5*x) - 3*I*e^(5*x) + 25*I
*e^(3*x) - 150*I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(-1/sinh(x)^2)^(5/2),x)
```

```
[Out] int(1/(-1/sinh(x)^2)^(5/2), x)
```

3.28 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx$

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Mathematica [A] (verified) . . . 184
Maple [B] (verified) . . . 184
Fricas [C] (verification not implemented) . . . 184
Sympy [F] . . . 185
Maxima [C] (verification not implemented) . . . 185
Giac [C] (verification not implemented) . . . 185
Mupad [F(-1)] . . . 186

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx = \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}$$

```
[Out] 1/7*coth(x)/(-csch(x)^2)^(7/2)+6/35*coth(x)/(-csch(x)^2)^(5/2)+8/35*coth(x)
/(-csch(x)^2)^(3/2)+16/35*coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx = \frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}} + \frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}}$$

```
[In] Int[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(-Csch[x]^2)^(7/2)) + (6*Coth[x])/(35*(-Csch[x]^2)^(5/2)) + (8*C
oth[x])/(35*(-Csch[x]^2)^(3/2)) + (16*Coth[x])/(35*Sqrt[-Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)^{9/2}}\,dx, x, \coth(x)\right) \\
&= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \coth(x)\right) \\
&= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{24}{35}\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \coth(x)\right) \\
&= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}} \\
&\quad + \frac{16}{35}\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \coth(x)\right) \\
&= \frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}} + \frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx = \frac{(1225\cosh(x) - 245\cosh(3x) + 49\cosh(5x) - 5\cosh(7x))\text{csch}(x)}{2240\sqrt{-\text{csch}^2(x)}}$$

```
[In] Integrate[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] ((1225*Cosh[x] - 245*Cosh[3*x] + 49*Cosh[5*x] - 5*Cosh[7*x])*Csch[x])/(2240
*Sqrt[-Csch[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(49) = 98.

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.66

method	result
risch	$-\frac{e^{8x}}{896(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{7e^{6x}}{640(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{7e^{4x}}{128(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{35e^{2x}}{128\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}(e^{2x}-1)} + \frac{}{128(e^{2x}-}$

```
[In] int(1/(-csch(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/896*exp(8*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)+7/640*exp(6
*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)-7/128*exp(4*x)/(exp(2*x
)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)+35/128/(-1/(exp(2*x)-1)^2*exp(2*x))
^(1/2)/(exp(2*x)-1)*exp(2*x)+35/128/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x
))^(1/2)-7/128*exp(-2*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)+7/
640*exp(-4*x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)-1/896*exp(-6*
x)/(exp(2*x)-1)/(-1/(exp(2*x)-1)^2*exp(2*x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx = \frac{1}{4480}\left(5i\,e^{(14\,x)} - 49i\,e^{(12\,x)} + 245i\,e^{(10\,x)} - 1225i\,e^{(8\,x)} - 1225i\,e^{(6\,x)} + 245i\,e^{(4\,x)} - 49i\right.$$

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*I*e^(14*x) - 49*I*e^(12*x) + 245*I*e^(10*x) - 1225*I*e^(8*x) - 12
25*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x)
```

Sympy [F]

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(-csch(x)**2)**(7/2),x)
[Out] Integral((-csch(x)**2)**(-7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{1}{896}i\,e^{(7\,x)} + \frac{7}{640}i\,e^{(5\,x)} - \frac{7}{128}i\,e^{(3\,x)} + \frac{35}{128}i\,e^{(-x)} - \frac{7}{128}i\,e^{(-3\,x)} + \frac{7}{640}i\,e^{(-5\,x)} - \frac{1}{896}i\,e^{(-7\,x)} + \frac{35}{128}i\,e^{x}$$

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="maxima")
[Out] -1/896*I*e^(7*x) + 7/640*I*e^(5*x) - 7/128*I*e^(3*x) + 35/128*I*e^(-x) - 7/
128*I*e^(-3*x) + 7/640*I*e^(-5*x) - 1/896*I*e^(-7*x) + 35/128*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{\left(-1225i\,e^{(6\,x)} + 245i\,e^{(4\,x)} - 49i\,e^{(2\,x)} + 5i\right)e^{(-7\,x)} + 5i\,e^{(7\,x)} - 49i\,e^{(5\,x)} + 245i\,e^{(3\,x)} - 1225i\,e^{x}}{4480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="giac")
[Out] -1/4480*((-1225*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x) +
5*I*e^(7*x) - 49*I*e^(5*x) + 245*I*e^(3*x) - 1225*I*e^x)/sgn(-e^(3*x) + e^x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{7/2}}\,dx$$

```
[In] int(1/(-1/sinh(x)^2)^(7/2),x)
[Out] int(1/(-1/sinh(x)^2)^(7/2), x)
```

3.29 $\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx = -\frac{3}{8}a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)+\frac{3}{8}a^2\coth(x)\sqrt{a\operatorname{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\operatorname{csch}^2(x)\right)^{3/2}$$

```
[Out] -3/8*a^(5/2)*arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))-1/4*a*coth(x)*(a*
csch(x)^2)^(3/2)+3/8*a^2*coth(x)*(a*csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 212}

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx = -\frac{3}{8}a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)+\frac{3}{8}a^2\coth(x)\sqrt{a\operatorname{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\operatorname{csch}^2(x)\right)^{3/2}$$

```
[In] Int[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/8 + (3*a^2*Coth[x
]*Sqrt[a*Csch[x]^2])/8 - (a*Coth[x]*(a*Csch[x]^2)^(3/2))/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a\text{Subst}\left(\int \left(-a+ax^2\right)^{3/2}\,dx, x, \coth(x)\right)\right) \\
&= -\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{-a+ax^2}\,dx, x, \coth(x)\right) \\
&= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} \\
&\quad - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\,dx, x, \coth(x)\right) \\
&= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2} \\
&\quad - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) \\
&= -\frac{3}{8}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) + \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \left(a\text{csch}^2(x)\right)^{5/2}\,dx = \frac{1}{64}\left(a\text{csch}^2(x)\right)^{5/2}\sinh(x)\left(-22\cosh(x)+6\left(\cosh(3x)+4\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\text{si}\right.\right.\right.$$

```
[In] Integrate[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] ((a*Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*(-Log[Cosh[x/2
]] + Log[Sinh[x/2]])*Sinh[x]^4)))/64
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(49) = 98$.

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.89

method	result	s
risch	$\frac{a^2\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\left(3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3\right)}{4(e^{2x}-1)^3}+\frac{3a^2e^{-x}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\ln(e^x-1)}{8}-\frac{3a^2e^{-x}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\ln(e^x+1)}{8}$	1

```
[In] int((a*csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/(exp(2*x)-1)^3*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp
(4*x)-11*exp(2*x)+3)+3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^
2)^(1/2)*ln(exp(x)-1)-3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 1128, normalized size of antiderivative = 17.35

$$\int \left(a\text{csch}^2(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(6*a^2*cosh(x)^7 - 6*(a^2*e^(2*x) - a^2)*sinh(x)^7 - 22*a^2*cosh(x)^5
- 42*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^6 + 2*(63*a^2*cosh(x)^2 -
11*a^2 - (63*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^5 - 22*a^2*cosh(x)^3
+ 10*(21*a^2*cosh(x)^3 - 11*a^2*cosh(x) - (21*a^2*cosh(x)^3 - 11*a^2*cosh(x
))*e^(2*x))*sinh(x)^4 + 2*(105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2 -
 (105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 + 6*a^
```

```
2*cosh(x) + 2*(63*a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) - (63*
a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 - 2*
(3*a^2*cosh(x)^7 - 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(
2*x) + 3*(a^2*cosh(x)^8 - (a^2*e^(2*x) - a^2)*sinh(x)^8 - 4*a^2*cosh(x)^6 -
 8*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 - a^2
 - (7*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*
cosh(x)^3 - 3*a^2*cosh(x) - (7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2*x))*sinh
(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2 - (35*a^2*cosh(x)^4
 - 30*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2
*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x) - (7*a^2*cosh(x)^5 - 10*a^2*c
osh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2 - (7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9
*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^8 - 4*a^2*cos
h(x)^6 + 6*a^2*cosh(x)^4 - 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^
7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^7 - 3*a^
2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x))*log((cosh(x)
 + sinh(x) - 1)/(cosh(x) + sinh(x) + 1)) + 2*(21*a^2*cosh(x)^6 - 55*a^2*cos
h(x)^4 - 33*a^2*cosh(x)^2 + 3*a^2 - (21*a^2*cosh(x)^6 - 55*a^2*cosh(x)^4 -
33*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1
))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x*s
inh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 - 30
*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh(x
)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (co
sh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx = \int \left(a\,\operatorname{csch}^2\left(x\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*csch(x)**2)**(5/2),x)
[Out] Integral((a*csch(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.42

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx = \frac{3}{8}\,a^{\frac{5}{2}}\log\left(e^{(-x)}+1\right) - \frac{3}{8}\,a^{\frac{5}{2}}\log\left(e^{(-x)}-1\right) + \frac{3\,a^{\frac{5}{2}}e^{(-x)} - 11\,a^{\frac{5}{2}}e^{(-3\,x)} - 11\,a^{\frac{5}{2}}e^{(-5\,x)} + 3\,a^{\frac{5}{2}}e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)}$$

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^(5/2)*log(e^(-x) + 1) - 3/8*a^(5/2)*log(e^(-x) - 1) + 1/4*(3*a^(5/2)*
e^(-x) - 11*a^(5/2)*e^(-3*x) - 11*a^(5/2)*e^(-5*x) + 3*a^(5/2)*e^(-7*x))/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx = \frac{1}{16}\,a^{\frac{5}{2}}\left(\frac{4\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3\,\log\left(e^{(-x)}+e^{x}+2\right)+3\,\log\left(e^{(-x)}-e^{x}\right)\right.$$

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*a^(5/2)*(4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2
 - 4)^2 - 3*log(e^(-x) + e^x + 2) + 3*log(e^(-x) + e^x - 2))*sgn(e^(3*x) -
e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2}\,dx = \int \left(\frac{a}{\sinh\left(x\right)^2}\right)^{5/2}\,dx$$

```
[In] int((a/sinh(x)^2)^(5/2),x)
```

```
[Out] int((a/sinh(x)^2)^(5/2), x)
```

3.30 $\int \left(a\operatorname{csch}^2(x)\right)^{3/2}\,dx$

Optimal result . . . 192
Rubi [A] (verified) . . . 192
Mathematica [A] (verified) . . . 193
Maple [B] (verified) . . . 194
Fricas [B] (verification not implemented) . . . 194
Sympy [F] . . . 195
Maxima [A] (verification not implemented) . . . 195
Giac [A] (verification not implemented) . . . 195
Mupad [F(-1)] . . . 196

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \left(a\operatorname{csch}^2(x)\right)^{3/2}\,dx = \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

```
[Out] 1/2*a^(3/2)*arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))-1/2*a*coth(x)*(a*c
sch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 212}

$$\int \left(a\operatorname{csch}^2(x)\right)^{3/2}\,dx = \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

```
[In] Int[(a*Csch[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/2 - (a*Coth[x]*Sqrt[
a*Csch[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a\text{Subst}\left(\int \sqrt{-a+ax^2}\,dx, x, \coth(x)\right)\right) \\
&= -\frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\,dx, x, \coth(x)\right) \\
&= -\frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) \\
&= \frac{1}{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\text{csch}^2(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx = \\
&-\frac{1}{2}a\sqrt{a\text{csch}^2(x)}\left(\coth(x)\text{csch}(x) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)
\end{aligned}$$

```
[In] Integrate[(a*Csch[x]^2)^(3/2),x]
```

```
[Out] -1/2*(a*Sqrt[a*Csch[x]^2]*(Coth[x]*Csch[x] - Log[Cosh[x/2]] + Log[Sinh[x/2]
])*Sinh[x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time $= 0.16$ (sec) , antiderivative size $= 103$, normalized size of antiderivative $= 2.24$

method	result	size
risch	$-\frac{a\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\,(1+e^{2x})}{e^{2x}-1}+\frac{a\,e^{-x}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\,\ln(e^{x}+1)}{2}-\frac{a\,e^{-x}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\,\ln(e^{x}-1)}{2}$	103

```
[In] int((a*csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(1+exp(2*x))+1/2*a*exp(-x
)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)-1/2*a*exp(-x)
*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(34) = 68$.

Time $= 0.28$ (sec) , antiderivative size $= 340$, normalized size of antiderivative $= 7.39$

$$\int \left(a\operatorname{csch}^2(x)\right)^{3/2}\,dx = \frac{\left(2\,a\cosh(x)^3 - 2\left(ae^{(2\,x)} - a\right)\sinh(x)^3 - 6\left(a\cosh(x)\,e^{(2\,x)} - a\cosh(x)\right)\sinh(x)^2 +}{}$$

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cosh(x)^3 - 2*(a*e^(2*x) - a)*sinh(x)^3 - 6*(a*cosh(x)*e^(2*x) - a
*cosh(x))*sinh(x)^2 + 2*a*cosh(x) - 2*(a*cosh(x)^3 + a*cosh(x))*e^(2*x) - (
a*cosh(x)^4 - (a*e^(2*x) - a)*sinh(x)^4 - 4*(a*cosh(x)*e^(2*x) - a*cosh(x))
*sinh(x)^3 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)^2 - a)*e^(2*x)
 - a)*sinh(x)^2 - (a*cosh(x)^4 - 2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cosh(x)^
3 - a*cosh(x) - (a*cosh(x)^3 - a*cosh(x))*e^(2*x))*sinh(x) + a)*log((cosh(x
) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(4*cosh
(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(
cosh(x)^3 - cosh(x))*e^x*sinh(x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx = \int \left(a\,\text{csch}^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*csch(x)**2)**(3/2),x)
[Out] Integral((a*csch(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx = -\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}+1\right) + \frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}-1\right) - \frac{a^{\frac{3}{2}}e^{(-x)} + a^{\frac{3}{2}}e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1}$$

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*a^(3/2)*log(e^(-x) + 1) + 1/2*a^(3/2)*log(e^(-x) - 1) - (a^(3/2)*e^(-x
) + a^(3/2)*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx = \\ -\frac{1}{4}\,a^{\frac{3}{2}}\left(\frac{4\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^2-4} - \log\left(e^{(-x)}+e^{x}+2\right) + \log\left(e^{(-x)}+e^{x}-2\right)\right)\text{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/4*a^(3/2)*(4*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - log(e^(-x) + e^x +
2) + log(e^(-x) + e^x - 2))*sgn(e^(3*x) - e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\operatorname{csch}^2(x)\right)^{3/2}\,dx = \int \left(\frac{a}{\sinh\left(x\right)^2}\right)^{3/2}\,dx$$

```
[In] int((a/sinh(x)^2)^(3/2),x)
[Out] int((a/sinh(x)^2)^(3/2), x)
```

3.31 $\int \sqrt{a\operatorname{csch}^2(x)}\,dx$

Optimal result . . . 197
Rubi [A] (verified) . . . 197
Mathematica [A] (verified) . . . 198
Maple [B] (verified) . . . 198
Fricas [B] (verification not implemented) . . . 199
Sympy [F] . . . 199
Maxima [A] (verification not implemented) . . . 199
Giac [A] (verification not implemented) . . . 200
Mupad [F(-1)] . . . 200

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)$$

```
[Out] -arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))*a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 223, 212}

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)$$

```
[In] Int[Sqrt[a*Csch[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(a\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\,dx, x, \coth(x)\right)\right) \\ &= -\left(a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)\right) \\ &= -\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \sqrt{a\text{csch}^2(x)}\,dx = \sqrt{a\text{csch}^2(x)}\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)$$

```
[In] Integrate[Sqrt[a*Csch[x]^2],x]
```

```
[Out] Sqrt[a*Csch[x]^2]*(-Log[Cosh[x/2]] + Log[Sinh[x/2]])*Sinh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(20) = 40.

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

method	result	size
risch	$-\mathrm{e}^{-x}(\mathrm{e}^{2x}-1)\sqrt{\frac{a\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2}}\,\ln(\mathrm{e}^{x}+1)+\mathrm{e}^{-x}(\mathrm{e}^{2x}-1)\sqrt{\frac{a\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2}}\,\ln(\mathrm{e}^{x}-1)$	67

```
[In] int((a*csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)+exp(-x
)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.

Time $= 0.26$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 3.73$

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx$$

$$= \left[\sqrt{\frac{a}{e^{(4\,x)} - 2\,e^{(2\,x)} + 1}}\left(e^{(2\,x)} - 1\right)\log\left(\frac{\cosh(x) + \sinh(x) - 1}{\cosh(x) + \sinh(x) + 1}\right), 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{e^{(4\,x)} - 2\,e^{(2\,x)} + 1}}\left(e^{(}\right.\right.\right.$$
$$\left.\left.\left.a\cosh(x)\,e^{x} + a e^{x}\,\mathrm{s}\right.\right.\right.$$

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*(e^(2*x) - 1)*log((cosh(x) + sinh(x) - 1
)/(cosh(x) + sinh(x) + 1)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(a/(e^(4*x) - 2*
e^(2*x) + 1))*(e^(2*x) - 1)*e^x/(a*cosh(x)*e^x + a*e^x*sinh(x)))]
```

Sympy [F]

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = \int \sqrt{a\,\operatorname{csch}^2(x)}\,dx$$

```
[In] integrate((a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.26$ (sec) , antiderivative size $= 24$, normalized size of antiderivative $= 0.92$

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = \sqrt{a}\log\left(e^{(-x)} + 1\right) - \sqrt{a}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*log(e^(-x) + 1) - sqrt(a)*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = -\sqrt{a}(\log\left(e^x+1\right)-\log\left(|e^x-1|\right))\operatorname{sgn}\left(e^{(3\,x)}-e^x\right)$$

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="giac")
[Out] -sqrt(a)*(log(e^x + 1) - log(abs(e^x - 1)))*sgn(e^(3*x) - e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = \int \sqrt{\frac{a}{\sinh\left(x\right)^2}}\,dx$$

```
[In] int((a/sinh(x)^2)^(1/2),x)
[Out] int((a/sinh(x)^2)^(1/2), x)
```

3.32 $\int \frac{1}{\sqrt{a\mathbf{csch}^2(x)}}\,dx$

Optimal result . . . 201
Rubi [A] (verified) . . . 201
Mathematica [A] (verified) . . . 202
Maple [B] (verified) . . . 202
Fricas [B] (verification not implemented) . . . 203
Sympy [F] . . . 203
Maxima [A] (verification not implemented) . . . 203
Giac [B] (verification not implemented) . . . 204
Mupad [B] (verification not implemented) . . . 204

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] coth(x)/(a*csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

```
[In] Int[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(a\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)\right) \\ &= \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}$$

```
[In] Integrate[1/Sqrt[a*Csch[x]^2],x]
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(11) = 22.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\left(e^{2x}-1\right)}+\frac{1}{2\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}}$	58

```
[In] int(1/(a*csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(11) = 22$.

Time $= 0.27$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 6.38$

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx$$
$$= \frac{\left(\left(e^{(2\,x)}-1\right)\sinh(x)^2-\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2\,x)}+2\left(\cosh(x)\,e^{(2\,x)}-\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{}{e^{(4}}}}{2\left(a\cosh(x)\,e^x+ae^x\sinh(x)\right)}$$

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) - 1)*sinh(x)^2 - cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) - cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = \int \frac{1}{\sqrt{a\,\operatorname{csch}^2(x)}}\,dx$$

```
[In] integrate(1/(a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.27$ (sec) , antiderivative size $= 17$, normalized size of antiderivative $= 1.31$

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = -\frac{e^{(-x)}}{2\sqrt{a}} - \frac{e^x}{2\sqrt{a}}$$

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) - 1/2*e^x/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time $= 0.28$ (sec) , antiderivative size $= 24$, normalized size of antiderivative $= 1.85$

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = \frac{e^{(-x)} + e^{x}}{2\sqrt{a}\operatorname{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(e^(-x) + e^x)/(sqrt(a)*sgn(e^(3*x) - e^x))
```

Mupad [B] (verification not implemented)

Time $= 2.31$ (sec) , antiderivative size $= 33$, normalized size of antiderivative $= 2.54$

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = -\frac{\left(\frac{\mathrm{e}^{-2\,x}}{2} - \frac{\mathrm{e}^{2\,x}}{2}\right)\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{-x}}{2} - \frac{\mathrm{e}^{x}}{2}\right)^2}}}{2\sqrt{a}}$$

```
[In] int(1/(a/sinh(x)^2)^(1/2),x)
[Out] -((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 - exp(x)/2)^2)^(1/2))/(2*a^(1/2)
)
```

3.33 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx$

Optimal result . . . 205
Rubi [A] (verified) . . . 205
Mathematica [A] (verified) . . . 206
Maple [B] (verified) . . . 206
Fricas [B] (verification not implemented) . . . 207
Sympy [F] . . . 207
Maxima [A] (verification not implemented) . . . 208
Giac [A] (verification not implemented) . . . 208
Mupad [F(-1)] . . . 208

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] 1/3*coth(x)/(a*csch(x)^2)^(3/2)-2/3*coth(x)/a/(a*csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

```
[In] Int[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] Coth[x]/(3*(a*Csch[x]^2)^(3/2)) - (2*Coth[x])/(3*a*Sqrt[a*Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(a\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)\right) \\ &= \frac{\coth(x)}{3\left(a\text{csch}^2(x)\right)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{3\left(a\text{csch}^2(x)\right)^{3/2}} - \frac{2\coth(x)}{3a\sqrt{a\text{csch}^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{3/2}}\,dx = \frac{(-9\cosh(x)+\cosh(3x))\text{csch}^3(x)}{12\left(a\text{csch}^2(x)\right)^{3/2}}$$

```
[In] Integrate[(a*Csch[x]^2)^(-3/2),x]
[Out] ((-9*Cosh[x] + Cosh[3*x])*Csch[x]^3)/(12*(a*Csch[x]^2)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(28) = 56.

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.61

method	result	size
risch	$\frac{e^{4x}}{24a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{3\,e^{2x}}{8a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}} - \frac{3}{8\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}\,(e^{2x}-1)a} + \frac{e^{-2x}}{24a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^2}}}$	130

```
[In] int(1/(a*csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a*exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/a*exp(2*
x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/(a*exp(2*x)/(exp(2*x)
-1)^2)^(1/2)/(exp(2*x)-1)/a+1/24/a*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(
2*x)-1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(28) = 56$.

Time $= 0.27$ (sec) , antiderivative size $= 285$, normalized size of antiderivative $= 7.92$

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\left(\left(e^{(2\,x)}-1\right)\sinh(x)^6 - \cosh(x)^6 + 6\left(\cosh(x)\,e^{(2\,x)} - \cosh(x)\right)\sinh(x)^5 - 3\left(5\,\cos\right.}{}$$

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) - 1)*sinh(x)^6 - cosh(x)^6 + 6*(cosh(x)*e^(2*x) - cosh(x))*s
inh(x)^5 - 3*(5*cosh(x)^2 - (5*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^4 + 9*co
sh(x)^4 - 4*(5*cosh(x)^3 - (5*cosh(x)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*s
inh(x)^3 - 3*(5*cosh(x)^4 - 18*cosh(x)^2 - (5*cosh(x)^4 - 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^6 - 9*cosh(x)^4 - 9*cosh(x
)^2 + 1)*e^(2*x) - 6*(cosh(x)^5 - 6*cosh(x)^3 - (cosh(x)^5 - 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^2(x)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = -\frac{e^{(3\,x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}} - \frac{e^{(-3\,x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*e^(3*x)/a^(3/2) + 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^
x/a^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = -\frac{\left(9\,e^{(2\,x)} - 1\right)e^{(-3\,x)} - e^{(3\,x)} + 9\,e^{x}}{24\,a^{\frac{3}{2}}\operatorname{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) - 1)*e^(-3*x) - e^(3*x) + 9*e^x)/(a^(3/2)*sgn(e^(3*x) - e
^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a/sinh(x)^2)^(3/2), x)
```

3.34 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx$

Optimal result . . . 209
Rubi [A] (verified) . . . 209
Mathematica [A] (verified) . . . 210
Maple [B] (verified) . . . 211
Fricas [B] (verification not implemented) . . . 211
Sympy [F] . . . 212
Maxima [A] (verification not implemented) . . . 212
Giac [A] (verification not implemented) . . . 212
Mupad [F(-1)] . . . 213

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] 1/5*coth(x)/(a*csch(x)^2)^(5/2)-4/15*coth(x)/a/(a*csch(x)^2)^(3/2)+8/15*cot
h(x)/a^2/(a*csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}} - \frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}$$

```
[In] Int[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(a*Csch[x]^2)^(5/2)) - (4*Coth[x])/(15*a*(a*Csch[x]^2)^(3/2)) +
(8*Coth[x])/(15*a^2*Sqrt[a*Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx, x, \coth(x)\right)\right) \\
&= \frac{\coth(x)}{5\left(a\text{csch}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right) \\
&= \frac{\coth(x)}{5\left(a\text{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\text{csch}^2(x)\right)^{3/2}} - \frac{8\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)}{15a} \\
&= \frac{\coth(x)}{5\left(a\text{csch}^2(x)\right)^{5/2}} - \frac{4\coth(x)}{15a\left(a\text{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15a^2\sqrt{a\text{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx = \frac{(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\sqrt{a\text{csch}^2(x)}\sinh(x)}{240a^3}$$

```
[In] Integrate[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Sqrt[a*Csch[x]^2]*Sinh[x])/(240
*a^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(43) = 86$.

Time $= 0.16$ (sec) , antiderivative size $= 196$, normalized size of antiderivative $= 3.56$

method	result
risch	$\frac{e^{6x}}{160a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{5\,e^{4x}}{96a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} + \frac{5\,e^{2x}}{16a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} + \frac{5}{16\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)a^2} - \frac{5}{96a^2(e^{2x}-$

```
[In] int(1/(a*csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/a^2*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96/a^2*
exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/a^2*exp(2*x)/(
exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)/(exp(2*x)-1)/a^2-5/96/a^2*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp
(2*x)-1)^2)^(1/2)+1/160/a^2*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(43) = 86$.

Time $= 0.27$ (sec) , antiderivative size $= 590$, normalized size of antiderivative $= 10.73$

$$\int \frac{1}{\left(a\mathrm{csch}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) - 1)*sinh(x)^10 - 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) - co
sh(x))*sinh(x)^9 - 5*(27*cosh(x)^2 - (27*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x
)^8 + 25*cosh(x)^8 - 40*(9*cosh(x)^3 - (9*cosh(x)^3 - 5*cosh(x))*e^(2*x) -
5*cosh(x))*sinh(x)^7 - 10*(63*cosh(x)^4 - 70*cosh(x)^2 - (63*cosh(x)^4 - 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 - 150*cosh(x)^6 - 4*(189*cosh(x)^5
 - 350*cosh(x)^3 - (189*cosh(x)^5 - 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 - 10*(63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 -
 (63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^
4 - 150*cosh(x)^4 - 40*(9*cosh(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 - (9*cosh
(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sin
h(x)^3 - 5*(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 -
(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 + 25*cosh(x)^2 + (3*cosh(x)^10 - 25*cosh(x)^8 + 150*cosh(x)^
6 + 150*cosh(x)^4 - 25*cosh(x)^2 + 3)*e^(2*x) - 10*(3*cosh(x)^9 - 20*cosh(x
)^7 + 90*cosh(x)^5 + 60*cosh(x)^3 - (3*cosh(x)^9 - 20*cosh(x)^7 + 90*cosh(x
)^5 + 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
```

```
^(4*x) - 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^2(x)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = -\frac{e^{(5\,x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{(3\,x)}}{96\,a^{\frac{5}{2}}} - \frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}} + \frac{5\,e^{(-3\,x)}}{96\,a^{\frac{5}{2}}} - \frac{e^{(-5\,x)}}{160\,a^{\frac{5}{2}}} - \frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) + 5/96*
e^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) - 5/16*e^x/a^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{\left(150\,e^{(4\,x)} - 25\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + 3\,e^{(5\,x)} - 25\,e^{(3\,x)} + 150\,e^{x}}{480\,a^{\frac{5}{2}}\operatorname{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*((150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 3*e^(5*x) - 25*e^(3*x) + 1
50*e^x)/(a^(5/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^2)^(5/2),x)
[Out] int(1/(a/sinh(x)^2)^(5/2), x)
```

3.35 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{16\coth(x)}{35a^3\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] 1/7*coth(x)/(a*csch(x)^2)^(7/2)-6/35*coth(x)/a/(a*csch(x)^2)^(5/2)+8/35*cot
h(x)/a^2/(a*csch(x)^2)^(3/2)-16/35*coth(x)/a^3/(a*csch(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{16\coth(x)}{35a^3\sqrt{a\operatorname{csch}^2(x)}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}}$$

```
[In] Int[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(a*Csch[x]^2)^(7/2)) - (6*Coth[x])/(35*a*(a*Csch[x]^2)^(5/2)) +
(8*Coth[x])/(35*a^2*(a*Csch[x]^2)^(3/2)) - (16*Coth[x])/(35*a^3*Sqrt[a*Csch
[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{9/2}}\,dx, x, \coth(x)\right)\right) \\
&= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx, x, \coth(x)\right) \\
&= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}} - \frac{24\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)}{35a} \\
&= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\text{csch}^2(x)\right)^{3/2}} \\
&\quad + \frac{16\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)}{35a^2} \\
&= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\text{csch}^2(x)\right)^{3/2}} - \frac{16\coth(x)}{35a^3\sqrt{a\text{csch}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \frac{(-1225\cosh(x)+245\cosh(3x)-49\cosh(5x)+5\cosh(7x))\sqrt{a\operatorname{csch}^2(x)}\sinh(x)}{2240a^4}$$

```
[In] Integrate[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] ((-1225*Cosh[x] + 245*Cosh[3*x] - 49*Cosh[5*x] + 5*Cosh[7*x])*Sqrt[a*Csch[x
]^2]*Sinh[x])/(2240*a^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(58) = 116.

Time = 0.15 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

method	result
risch	$\frac{e^{8x}}{896a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{7e^{6x}}{640a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} + \frac{7e^{4x}}{128a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{35e^{2x}}{128a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{}{128\sqrt{\frac{}{(e^{2}}}}$

```
[In] int(1/(a*csch(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/a^3*exp(8*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/640/a^3
*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/128/a^3*exp(4*x)
/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/a^3*exp(2*x)/(exp(2*
x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/(a*exp(2*x)/(exp(2*x)-1)^2)^
(1/2)/(exp(2*x)-1)/a^3+7/128/a^3*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*
x)-1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)
^(1/2)+1/896/a^3*exp(-6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 984, normalized size of antiderivative = 13.30

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) - 1)*sinh(x)^14 - 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) - c
osh(x))*sinh(x)^13 - 7*(65*cosh(x)^2 - (65*cosh(x)^2 - 7)*e^(2*x) - 7)*sinh
```

```
(x)^12 + 49*cosh(x)^12 - 28*(65*cosh(x)^3 - (65*cosh(x)^3 - 21*cosh(x))*e^(
2*x) - 21*cosh(x))*sinh(x)^11 - 7*(715*cosh(x)^4 - 462*cosh(x)^2 - (715*cos
h(x)^4 - 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 - 245*cosh(x)^10 - 70
*(143*cosh(x)^5 - 154*cosh(x)^3 - (143*cosh(x)^5 - 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 - 35*(429*cosh(x)^6 - 693*cosh(x)^4 + 3
15*cosh(x)^2 - (429*cosh(x)^6 - 693*cosh(x)^4 + 315*cosh(x)^2 - 35)*e^(2*x)
 - 35)*sinh(x)^8 + 1225*cosh(x)^8 - 8*(2145*cosh(x)^7 - 4851*cosh(x)^5 + 36
75*cosh(x)^3 - (2145*cosh(x)^7 - 4851*cosh(x)^5 + 3675*cosh(x)^3 - 1225*cos
h(x))*e^(2*x) - 1225*cosh(x))*sinh(x)^7 - 7*(2145*cosh(x)^8 - 6468*cosh(x)^
6 + 7350*cosh(x)^4 - 4900*cosh(x)^2 - (2145*cosh(x)^8 - 6468*cosh(x)^6 + 73
50*cosh(x)^4 - 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 + 1225*cosh(x
)^6 - 14*(715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3
 - (715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 - 35*(143*cosh(x)^10 - 693*cosh(x)
^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 - (143*cosh(x)^10 - 69
3*cosh(x)^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 + 7)*e^(2*x)
 + 7)*sinh(x)^4 - 245*cosh(x)^4 - 140*(13*cosh(x)^11 - 77*cosh(x)^9 + 210*co
sh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 - (13*cosh(x)^11 - 77*cosh(x)^9 + 2
10*cosh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 + 7*cosh(x))*e^(2*x) + 7*cosh(
x))*sinh(x)^3 - 7*(65*cosh(x)^12 - 462*cosh(x)^10 + 1575*cosh(x)^8 - 4900*c
osh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - (65*cosh(x)^12 - 462*cosh(x)^10
 + 1575*cosh(x)^8 - 4900*cosh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 + 49*cosh(x)^2 + (5*cosh(x)^14 - 49*cosh(x)^12 + 245*c
osh(x)^10 - 1225*cosh(x)^8 - 1225*cosh(x)^6 + 245*cosh(x)^4 - 49*cosh(x)^2
 + 5)*e^(2*x) - 14*(5*cosh(x)^13 - 42*cosh(x)^11 + 175*cosh(x)^9 - 700*cosh(
x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - (5*cosh(x)^13 - 42*cosh(x)^11 + 175*c
osh(x)^9 - 700*cosh(x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^2\left(x\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**2)**(7/2),x)
[Out] Integral((a*csch(x)**2)**(-7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{e^{(7\,x)}}{896\,a^{\frac{7}{2}}} + \frac{7\,e^{(5\,x)}}{640\,a^{\frac{7}{2}}} - \frac{7\,e^{(3\,x)}}{128\,a^{\frac{7}{2}}} + \frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}} - \frac{7\,e^{(-3\,x)}}{128\,a^{\frac{7}{2}}} + \frac{7\,e^{(-5\,x)}}{640\,a^{\frac{7}{2}}} - \frac{e^{(-7\,x)}}{896\,a^{\frac{7}{2}}} + \frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) - 7/128*e^(3*x)/a^(7/2) + 35
/128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) + 7/640*e^(-5*x)/a^(7/2) - 1/8
96*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{\left(1225\,e^{(6\,x)} - 245\,e^{(4\,x)} + 49\,e^{(2\,x)} - 5\right)e^{(-7\,x)} - 5\,e^{(7\,x)} + 49\,e^{(5\,x)} - 245\,e^{(3\,x)} + 1225\,e^{x}}{4480\,a^{\frac{7}{2}}\operatorname{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x) - 5*e^(7*x)
 + 49*e^(5*x) - 245*e^(3*x) + 1225*e^x)/(a^(7/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{7/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^2)^(7/2),x)
```

```
[Out] int(1/(a/sinh(x)^2)^(7/2), x)
```

3.36 $\int \left(a\operatorname{csch}^3(x)\right)^{5/2} dx$

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Optimal result

Integrand size = 10, antiderivative size = 135

$$\int \left(a\operatorname{csch}^3(x)\right)^{5/2} dx = -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) - \frac{154ia^2\sqrt{a\operatorname{csch}^3(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{195\sqrt{i\sinh(x)}}$$

```
[Out] -154/585*a^2*coth(x)*(a*csch(x)^3)^(1/2)+22/117*a^2*coth(x)*csch(x)^2*(a*cs
ch(x)^3)^(1/2)-2/13*a^2*coth(x)*csch(x)^4*(a*csch(x)^3)^(1/2)+154/195*a^2*c
osh(x)*sinh(x)*(a*csch(x)^3)^(1/2)-154/195*I*a^2*(sin(1/4*Pi+1/2*I*x)^2)^(1
/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2*(a
*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$\int \left(a\operatorname{csch}^3(x)\right)^{5/2} dx = -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{154}{195}a^2\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{154ia^2\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{csch}^3(x)}}{195\sqrt{i\sinh(x)}}$$

```
[In] Int[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-154*a^2*Coth[x]*Sqrt[a*Csch[x]^3])/585 + (22*a^2*Coth[x]*Csch[x]^2*Sqrt[a
*Csch[x]^3])/117 - (2*a^2*Coth[x]*Csch[x]^4*Sqrt[a*Csch[x]^3])/13 + (154*a^
2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x])/195 - (((154*I)/195)*a^2*Sqrt[a*Csch[x
]^3]*EllipticE[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{15/2}\,dx}{(i\operatorname{csch}(x))^{3/2}}$$

$$= -\frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{\left(11a^2\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{11/2}\,dx}{13(i\operatorname{csch}(x))^{3/2}}$$

$$= \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{\left(77a^2\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{7/2}\,dx}{117(i\operatorname{csch}(x))^{3/2}}$$

$$
\begin{aligned}
&= -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{\left(77a^2\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{3/2}\,dx}{195(i\operatorname{csch}(x))^{3/2}} \\
&= -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) \\
&\quad + \frac{\left(77a^2\sqrt{a\operatorname{csch}^3(x)}\right)\int \frac{1}{\sqrt{i\operatorname{csch}(x)}}\,dx}{195(i\operatorname{csch}(x))^{3/2}} \\
&= -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) \\
&\quad - \frac{\left(77a^2\sqrt{a\operatorname{csch}^3(x)}\sinh^2(x)\right)\int \sqrt{i\sinh(x)}\,dx}{195\sqrt{i\sinh(x)}} \\
&= -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) \\
&\quad - \frac{154ia^2\sqrt{a\operatorname{csch}^3(x)}E\left(\frac{\pi}{4} - \frac{ix}{2}\middle|\, 2\right)\sinh^2(x)}{195\sqrt{i\sinh(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$
\begin{aligned}
&\int \left(a\operatorname{csch}^3(x)\right)^{5/2}\,dx = \\
&-\frac{2}{585}a^2\sqrt{a\operatorname{csch}^3(x)}\left(-231\cosh(x) + \coth(x)\operatorname{csch}(x)\left(77 - 55\operatorname{csch}^2(x) + 45\operatorname{csch}^4(x)\right)\right. \\
&\left. + 231E\left(\frac{1}{4}(\pi - 2ix)\middle|\, 2\right)\sqrt{i\sinh(x)}\right)\sinh(x)
\end{aligned}
$$

```
[In] Integrate[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a*Csch[x]^3]*(-231*Cosh[x] + Coth[x]*Csch[x]*(77 - 55*Csch[x]^
2 + 45*Csch[x]^4) + 231*EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sin
h[x])/585
```

Maple [F]

$$\int \left(a \operatorname{csch}(x)^3\right)^{\frac{5}{2}} dx$$

```
[In] int((a*csch(x)^3)^(5/2),x)
[Out] int((a*csch(x)^3)^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 1389, normalized size of antiderivative = 10.29

$$\int \left(a\operatorname{csch}^3(x)\right)^{5/2} dx = \text{Too large to display}$$

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="fricas")
[Out] 2/585*(231*sqrt(2)*(a^2*cosh(x)^12 + 12*a^2*cosh(x)*sinh(x)^11 + a^2*sinh(x
)^12 - 6*a^2*cosh(x)^10 + 6*(11*a^2*cosh(x)^2 - a^2)*sinh(x)^10 + 15*a^2*co
sh(x)^8 + 20*(11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^9 + 15*(33*a^2*cosh
(x)^4 - 18*a^2*cosh(x)^2 + a^2)*sinh(x)^8 - 20*a^2*cosh(x)^6 + 24*(33*a^2*c
osh(x)^5 - 30*a^2*cosh(x)^3 + 5*a^2*cosh(x))*sinh(x)^7 + 4*(231*a^2*cosh(x)
^6 - 315*a^2*cosh(x)^4 + 105*a^2*cosh(x)^2 - 5*a^2)*sinh(x)^6 + 15*a^2*cosh
(x)^4 + 24*(33*a^2*cosh(x)^7 - 63*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 - 5*a^2*
cosh(x))*sinh(x)^5 + 15*(33*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 70*a^2*cosh(
x)^4 - 20*a^2*cosh(x)^2 + a^2)*sinh(x)^4 - 6*a^2*cosh(x)^2 + 20*(11*a^2*cos
h(x)^9 - 36*a^2*cosh(x)^7 + 42*a^2*cosh(x)^5 - 20*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 6*(11*a^2*cosh(x)^10 - 45*a^2*cosh(x)^8 + 70*a^2*cosh(x)^
6 - 50*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 12*(a^2*co
sh(x)^11 - 5*a^2*cosh(x)^9 + 10*a^2*cosh(x)^7 - 10*a^2*cosh(x)^5 + 5*a^2*co
sh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a)*weierstrassZeta(4, 0, weierstrassPI
nverse(4, 0, cosh(x) + sinh(x))) + sqrt(2)*(231*a^2*cosh(x)^13 + 3003*a^2*c
osh(x)*sinh(x)^12 + 231*a^2*sinh(x)^13 - 1540*a^2*cosh(x)^11 + 154*(117*a^2
*cosh(x)^2 - 10*a^2)*sinh(x)^11 + 4367*a^2*cosh(x)^9 + 1694*(39*a^2*cosh(x)
^3 - 10*a^2*cosh(x))*sinh(x)^10 + 11*(15015*a^2*cosh(x)^4 - 7700*a^2*cosh(x
)^2 + 397*a^2)*sinh(x)^9 - 6808*a^2*cosh(x)^7 + 33*(9009*a^2*cosh(x)^5 - 77
00*a^2*cosh(x)^3 + 1191*a^2*cosh(x))*sinh(x)^8 + 4*(99099*a^2*cosh(x)^6 - 1
27050*a^2*cosh(x)^4 + 39303*a^2*cosh(x)^2 - 1702*a^2)*sinh(x)^7 + 1277*a^2*
cosh(x)^5 + 28*(14157*a^2*cosh(x)^7 - 25410*a^2*cosh(x)^5 + 13101*a^2*cosh(
x)^3 - 1702*a^2*cosh(x))*sinh(x)^6 + (297297*a^2*cosh(x)^8 - 711480*a^2*cos
h(x)^6 + 550242*a^2*cosh(x)^4 - 142968*a^2*cosh(x)^2 + 1277*a^2)*sinh(x)^5
- 484*a^2*cosh(x)^3 + (165165*a^2*cosh(x)^9 - 508200*a^2*cosh(x)^7 + 550242
*a^2*cosh(x)^5 - 238280*a^2*cosh(x)^3 + 6385*a^2*cosh(x))*sinh(x)^4 + 2*(33
033*a^2*cosh(x)^10 - 127050*a^2*cosh(x)^8 + 183414*a^2*cosh(x)^6 - 119140*a
```

```
^2*cosh(x)^4 + 6385*a^2*cosh(x)^2 - 242*a^2)*sinh(x)^3 + 77*a^2*cosh(x) + 2
*(9009*a^2*cosh(x)^11 - 42350*a^2*cosh(x)^9 + 78606*a^2*cosh(x)^7 - 71484*a
^2*cosh(x)^5 + 6385*a^2*cosh(x)^3 - 726*a^2*cosh(x))*sinh(x)^2 + (3003*a^2*
cosh(x)^12 - 16940*a^2*cosh(x)^10 + 39303*a^2*cosh(x)^8 - 47656*a^2*cosh(x)
^6 + 6385*a^2*cosh(x)^4 - 1452*a^2*cosh(x)^2 + 77*a^2)*sinh(x))*sqrt((a*cos
h(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(cosh(x
)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10
 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^
4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh
(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(
x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*c
osh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh
(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*
cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh
(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sin
h(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cos
h(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(a\operatorname{csch}^{3}(x)\right)^{5/2}\,dx = \int \left(a\,\operatorname{csch}^{3}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*csch(x)**3)**(5/2),x)
[Out] Integral((a*csch(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\operatorname{csch}^{3}(x)\right)^{5/2}\,dx = \int \left(a\,\operatorname{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\text{csch}^3(x)\right)^{5/2}\,dx = \int \left(a\,\text{csch}\,(x)^3\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{csch}^3(x)\right)^{5/2}\,dx = \int \left(\frac{a}{\sinh\,(x)^3}\right)^{5/2} dx$$

```
[In] int((a/sinh(x)^3)^(5/2),x)
[Out] int((a/sinh(x)^3)^(5/2), x)
```

3.37 $\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx$

Optimal result . . . 225
Rubi [A] (verified) . . . 225
Mathematica [A] (verified) . . . 227
Maple [F] . . . 227
Fricas [C] (verification not implemented) . . . 227
Sympy [F] . . . 228
Maxima [F] . . . 228
Giac [F] . . . 228
Mupad [F(-1)] . . . 228

Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{10}{21}ia\sqrt{a\operatorname{csch}^3(x)}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sinh(x)$$

```
[Out] 10/21*a*cosh(x)*(a*csch(x)^3)^(1/2)-2/7*a*coth(x)*csch(x)*(a*csch(x)^3)^(1/
2)+10/21*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(co
s(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)*(a*csch(x)^3)^(1/2)*(I*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2720}

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{10}{21}ia\sqrt{i\sinh(x)}\sinh(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{a\operatorname{csch}^3(x)}$$

```
[In] Int[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (10*a*Cosh[x]*Sqrt[a*Csch[x]^3])/21 - (2*a*Coth[x]*Csch[x]*Sqrt[a*Csch[x]^3
])/7 + ((10*I)/21)*a*Sqrt[a*Csch[x]^3]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*
Sinh[x]]*Sinh[x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(ia\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{9/2}\,dx}{(i\operatorname{csch}(x))^{3/2}} \\
&= -\frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{\left(5ia\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{5/2}\,dx}{7(i\operatorname{csch}(x))^{3/2}} \\
&= \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad + \frac{\left(5ia\sqrt{a\operatorname{csch}^3(x)}\right)\int \sqrt{i\operatorname{csch}(x)}\,dx}{21(i\operatorname{csch}(x))^{3/2}} \\
&= \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad + \frac{1}{21}\left(5a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}\sinh(x)\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx \\
&= \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} \\
&\quad + \frac{10}{21}ia\sqrt{a\operatorname{csch}^3(x)}\,\text{EllipticF}\left(\frac{\pi}{4} - \frac{ix}{2}, 2\right)\sqrt{i\sinh(x)}\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \left(a\text{csch}^3(x)\right)^{3/2}\,dx = -\frac{2}{21}a\sqrt{a\text{csch}^3(x)}\left(\coth(x)\left(-5+3\text{csch}^2(x)\right)\right.$$
$$\left.-5i\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

```
[In] Integrate[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a*Csch[x]^3]*(Coth[x]*(-5 + 3*Csch[x]^2) - (5*I)*EllipticF[(Pi -
 (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/21
```

Maple [F]

$$\int \left(a\,\text{csch}\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] int((a*csch(x)^3)^(3/2),x)
```

```
[Out] int((a*csch(x)^3)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.88

$$\int \left(a\text{csch}^3(x)\right)^{3/2}\,dx = \frac{2\left(5\sqrt{2}\left(a\cosh\left(x\right)^6+6\,a\cosh\left(x\right)\sinh\left(x\right)^5+a\sinh\left(x\right)^6-3\,a\cosh\left(x\right)^4+3\left(5\,a\cosh\right.\right.\right.}{}$$

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*sqrt(2)*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*co
sh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))
*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^
2 + 6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a)*weiers
trassPInverse(4, 0, cosh(x) + sinh(x)) + sqrt(2)*(5*a*cosh(x)^6 + 30*a*cosh
(x)*sinh(x)^5 + 5*a*sinh(x)^6 - 17*a*cosh(x)^4 + (75*a*cosh(x)^2 - 17*a)*si
nh(x)^4 + 4*(25*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x)^3 - 17*a*cosh(x)^2 + (7
5*a*cosh(x)^4 - 102*a*cosh(x)^2 - 17*a)*sinh(x)^2 + 2*(15*a*cosh(x)^5 - 34*
a*cosh(x)^3 - 17*a*cosh(x))*sinh(x) + 5*a)*sqrt((a*cosh(x) + a*sinh(x))/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(
x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 +
 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [F]

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \int \left(a\operatorname{csch}^3(x)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*csch(x)**3)**(3/2),x)
[Out] Integral((a*csch(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \int \left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \int \left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \int \left(\frac{a}{\sinh(x)^3}\right)^{3/2}\,dx$$

```
[In] int((a/sinh(x)^3)^(3/2),x)
[Out] int((a/sinh(x)^3)^(3/2), x)
```

3.38 $\int \sqrt{a\operatorname{csch}^3(x)}\,dx$

Optimal result . . . 229
Rubi [A] (verified) . . . 229
Mathematica [A] (verified) . . . 230
Maple [F] . . . 231
Fricas [C] (verification not implemented) . . . 231
Sympy [F] . . . 231
Maxima [F] . . . 232
Giac [F] . . . 232
Mupad [F(-1)] . . . 232

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = -2i\sqrt{a\operatorname{csch}^3(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)(i\sinh(x))^{3/2} - 2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x)$$

```
[Out] -2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi
+1/2*I*x),2^(1/2))*(I*sinh(x))^(3/2)*(a*csch(x)^3)^(1/2)-2*cosh(x)*sinh(x)*
(a*csch(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = -2\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{csch}^3(x)}}{\sqrt{i\sinh(x)}}$$

```
[In] Int[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x] + ((2*I)*Sqrt[a*Csch[x]^3]*EllipticE[P
i/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a\operatorname{csch}^3(x)}\int (i\operatorname{csch}(x))^{3/2}\,dx}{(i\operatorname{csch}(x))^{3/2}} \\
&= -2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) - \frac{\sqrt{a\operatorname{csch}^3(x)}\int \frac{1}{\sqrt{i\operatorname{csch}(x)}}\,dx}{(i\operatorname{csch}(x))^{3/2}} \\
&= -2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) + \frac{\left(\sqrt{a\operatorname{csch}^3(x)}\sinh^2(x)\right)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}} \\
&= -2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) + \frac{2i\sqrt{a\operatorname{csch}^3(x)}E\left(\frac{\pi}{4} - \frac{ix}{2}\middle|\, 2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = -2\sqrt{a\operatorname{csch}^3(x)}\left(\cosh(x) - E\left(\frac{1}{4}(\pi - 2ix)\middle|\, 2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

```
[In] Integrate[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Sqrt[a*Csch[x]^3]*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[
x]])*Sinh[x]
```

Maple [F]

$$\int \sqrt{a \operatorname{csch}(x)^3}\, dx$$

```
[In] int((a*csch(x)^3)^(1/2),x)
[Out] int((a*csch(x)^3)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\begin{aligned}
&\int \sqrt{a\operatorname{csch}^3(x)}\, dx \\
&= -2\sqrt{2}\sqrt{\frac{a\cosh(x) + a\sinh(x)}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}}(\cosh(x) + \sinh(x)) \\
&\quad - 2\sqrt{2}\sqrt{a}\,\text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cosh(x) + \sinh(x)))
\end{aligned}$$

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(2)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1))*(cosh(x) + sinh(x)) - 2*sqrt(2)*sqrt(a)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cosh(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{a\operatorname{csch}^3(x)}\, dx = \int \sqrt{a \operatorname{csch}^3(x)}\, dx$$

```
[In] integrate((a*csch(x)**3)**(1/2),x)
[Out] Integral(sqrt(a*csch(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = \int \sqrt{a\,\operatorname{csch}(x)^3}\,dx$$

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*csch(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = \int \sqrt{a\,\operatorname{csch}(x)^3}\,dx$$

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*csch(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = \int \sqrt{\frac{a}{\sinh(x)^3}}\,dx$$

```
[In] int((a/sinh(x)^3)^(1/2),x)
[Out] int((a/sinh(x)^3)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx$

Optimal result . . . 233
Rubi [A] (verified) . . . 233
Mathematica [A] (verified) . . . 234
Maple [F] . . . 235
Fricas [C] (verification not implemented) . . . 235
Sympy [F] . . . 235
Maxima [F] . . . 236
Giac [F] . . . 236
Mupad [F(-1)] . . . 236

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} - \frac{2i\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\operatorname{csch}^3(x)}}$$

```
[Out] 2/3*coth(x)/(a*csch(x)^3)^(1/2)-2/3*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(1/
2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))^(
1/2)/(a*csch(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2720}

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} - \frac{2i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{3\sqrt{a\operatorname{csch}^3(x)}}$$

```
[In] Int[1/Sqrt[a*Csch[x]^3],x]
```

```
[Out] (2*Coth[x])/(3*Sqrt[a*Csch[x]^3]) - (((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (
I/2)*x, 2]*Sqrt[I*Sinh[x]])/Sqrt[a*Csch[x]^3]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(i\operatorname{csch}(x))^{3/2}\int \frac{1}{(i\operatorname{csch}(x))^{3/2}}\,dx}{\sqrt{a\operatorname{csch}^3(x)}} \\ &= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} + \frac{(i\operatorname{csch}(x))^{3/2}\int \sqrt{i\operatorname{csch}(x)}\,dx}{3\sqrt{a\operatorname{csch}^3(x)}} \\ &= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{a\operatorname{csch}^3(x)}} \\ &= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}} - \frac{2i\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\operatorname{csch}^3(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \frac{2\left(\coth(x) + \frac{\operatorname{csch}(x)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)}{\sqrt{i\sinh(x)}}\right)}{3\sqrt{a\operatorname{csch}^3(x)}}$$

```
[In] Integrate[1/Sqrt[a*Csch[x]^3],x]
```

```
[Out] (2*(Coth[x] + (Csch[x]*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(3
*Sqrt[a*Csch[x]^3])
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,\mathrm{csch}\,(x)^3}}dx$$

```
[In] int(1/(a*csch(x)^3)^(1/2),x)
[Out] int(1/(a*csch(x)^3)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{a\mathrm{csch}^3(x)}}\,dx =$$
$$-\frac{4\sqrt{2}\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)\sqrt{a}\mathrm{weierstrassPInverse}(4,0,\cosh\left(x\right)+\sinh\left(x\right))-\sqrt{2}}{6\left(a\cosh\left(x\right)\right.}$$

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="fricas")
[Out] -1/6*(4*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a)*weierst
rassPInverse(4, 0, cosh(x) + sinh(x)) - sqrt(2)*(cosh(x)^4 + 4*cosh(x)^3*si
nh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*sqrt((
a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\mathrm{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\mathrm{csch}^3\left(x\right)}}\,dx$$

```
[In] integrate(1/(a*csch(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*csch(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\operatorname{csch}\,(x)^3}}\,dx$$

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\operatorname{csch}\,(x)^3}}\,dx$$

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\sinh(x)^3}}}\,dx$$

```
[In] int(1/(a/sinh(x)^3)^(1/2),x)
[Out] int(1/(a/sinh(x)^3)^(1/2), x)
```

3.40 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx$

Optimal result . . . 237
Rubi [A] (verified) . . . 237
Mathematica [A] (verified) . . . 239
Maple [F] . . . 239
Fricas [C] (verification not implemented) . . . 239
Sympy [F] . . . 240
Maxima [F] . . . 240
Giac [F] . . . 240
Mupad [F(-1)] . . . 241

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}$$

```
[Out] -14/45*cosh(x)/a/(a*csch(x)^3)^(1/2)+2/9*cosh(x)*sinh(x)^2/a/(a*csch(x)^3)^
(1/2)+14/15*I*csch(x)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ell
ipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))/a/(a*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2719}

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\sinh^2(x)\cosh(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{i\sinh(x)}\sqrt{a\operatorname{csch}^3(x)}}$$

```
[In] Int[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-14*Cosh[x])/(45*a*Sqrt[a*Csch[x]^3]) + (((14*I)/15)*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2])/(a*Sqrt[a*Csch[x]^3]*Sqrt[I*Sinh[x]]) + (2*Cosh[x]*Sinh[x]
^2)/(9*a*Sqrt[a*Csch[x]^3])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{9/2}}\,dx}{a\sqrt{a\operatorname{csch}^3(x)}} \\
&= \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{5/2}}\,dx}{9a\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{\sqrt{i\operatorname{csch}(x)}}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} + \frac{(7\operatorname{csch}(x))\int \sqrt{i\sinh(x)}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} \\
&= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4} - \frac{ix}{2}\middle|\, 2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = \frac{-33\cosh(x)+5\cosh(3x)+84\operatorname{csch}^2(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}}{90a\sqrt{a\operatorname{csch}^3(x)}}$$

```
[In] Integrate[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-33*Cosh[x] + 5*Cosh[3*x] + 84*Csch[x]^2*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]])/(90*a*Sqrt[a*Csch[x]^3])
```

Maple [F]

$$\int \frac{1}{\left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a*csch(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.57

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx =$$

$$-\frac{672\sqrt{2}\left(\cosh(x)^5+5\cosh(x)^4\sinh(x)+10\cosh(x)^3\sinh(x)^2+10\cosh(x)^2\sinh(x)^3+5\cosh(x)\sinh\right.}{}$$

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/720*(672*sqrt(2)*(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)
^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)*sqrt(a)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(x))) - sqrt(2)*(
5*cosh(x)^10 + 50*cosh(x)*sinh(x)^9 + 5*sinh(x)^10 + (225*cosh(x)^2 - 43)*s
inh(x)^8 - 43*cosh(x)^8 + 8*(75*cosh(x)^3 - 43*cosh(x))*sinh(x)^7 + 2*(525*
cosh(x)^4 - 602*cosh(x)^2 - 149)*sinh(x)^6 - 298*cosh(x)^6 + 4*(315*cosh(x)
^5 - 602*cosh(x)^3 - 447*cosh(x))*sinh(x)^5 + 2*(525*cosh(x)^6 - 1505*cosh(
x)^4 - 2235*cosh(x)^2 + 187)*sinh(x)^4 + 374*cosh(x)^4 + 8*(75*cosh(x)^7 -
301*cosh(x)^5 - 745*cosh(x)^3 + 187*cosh(x))*sinh(x)^3 + (225*cosh(x)^8 - 1
204*cosh(x)^6 - 4470*cosh(x)^4 + 2244*cosh(x)^2 - 43)*sinh(x)^2 - 43*cosh(x
```

```
)^2 + 2*(25*cosh(x)^9 - 172*cosh(x)^7 - 894*cosh(x)^5 + 748*cosh(x)^3 - 43*
cosh(x))*sinh(x) + 5)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)))/(a^2*cosh(x)^5 + 5*a^2*cosh(x)^4*sinh(x) + 10*a^2
*cosh(x)^3*sinh(x)^2 + 10*a^2*cosh(x)^2*sinh(x)^3 + 5*a^2*cosh(x)*sinh(x)^4
 + a^2*sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^{3}(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^{3}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**3)**(3/2),x)
[Out] Integral((a*csch(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\operatorname{csch}^{3}(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\operatorname{csch}^{3}(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^3}\right)^{3/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^3)^(3/2),x)
[Out] int(1/(a/sinh(x)^3)^(3/2), x)
```

3.41 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 135

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{26i\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}$$

```
[Out] -26/77*coth(x)/a^2/(a*csch(x)^3)^(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*csch(x
)^3)^(1/2)-26/165*cosh(x)*sinh(x)^3/a^2/(a*csch(x)^3)^(1/2)+2/15*cosh(x)*si
nh(x)^5/a^2/(a*csch(x)^3)^(1/2)+26/77*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(
1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))
^(1/2)/a^2/(a*csch(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2720}

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\sinh^5(x)\cosh(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\sinh^3(x)\cosh(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{26i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}$$

```
[In] Int[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (-26*Coth[x])/(77*a^2*Sqrt[a*Csch[x]^3]) + (((26*I)/77)*Csch[x]^2*EllipticF
[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]])/(a^2*Sqrt[a*Csch[x]^3]) + (78*Cosh[x]*
Sinh[x])/(385*a^2*Sqrt[a*Csch[x]^3]) - (26*Cosh[x]*Sinh[x]^3)/(165*a^2*Sqrt
[a*Csch[x]^3]) + (2*Cosh[x]*Sinh[x]^5)/(15*a^2*Sqrt[a*Csch[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(i\operatorname{csch}(x))^{3/2}\int \frac{1}{(i\operatorname{csch}(x))^{15/2}}\,dx}{a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(13(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{11/2}}\,dx}{15a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(39(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{7/2}}\,dx}{55a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&\quad + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(39(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{3/2}}\,dx}{77a^2\sqrt{a\operatorname{csch}^3(x)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&\quad + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(13(i\operatorname{csch}(x))^{3/2}\right)\int\sqrt{i\operatorname{csch}(x)}\,dx}{77a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&\quad + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{\left(13\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\right)\int\frac{1}{\sqrt{i\sinh(x)}}\,dx}{77a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{26i\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&\quad + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$
\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a\operatorname{csch}^3(x)}\sinh(x)\left(24960i\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)\sqrt{i\sinh(x)} - 19122\sinh(2x) + \right.}{73920a^3}
$$

```
[In] Integrate[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^3]*Sinh[x]*((24960*I)*EllipticF[(Pi - (2*I)*x)/4, 2]*Sqrt[I
*Sinh[x]] - 19122*Sinh[2*x] + 4406*Sinh[4*x] - 826*Sinh[6*x] + 77*Sinh[8*x]
))/(73920*a^3)
```

Maple [F]

$$
\int \frac{1}{\left(a\,\operatorname{csch}\left(x\right)^3\right)^{\frac{5}{2}}}\,dx
$$

```
[In] int(1/(a*csch(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 718, normalized size of antiderivative = 5.32

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/147840*(49920*sqrt(2)*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sin
h(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8)*sqrt(a)*
weierstrassPInverse(4, 0, cosh(x) + sinh(x)) + sqrt(2)*(77*cosh(x)^16 + 123
2*cosh(x)*sinh(x)^15 + 77*sinh(x)^16 + 14*(660*cosh(x)^2 - 59)*sinh(x)^14 -
 826*cosh(x)^14 + 196*(220*cosh(x)^3 - 59*cosh(x))*sinh(x)^13 + 2*(70070*co
sh(x)^4 - 37583*cosh(x)^2 + 2203)*sinh(x)^12 + 4406*cosh(x)^12 + 8*(42042*c
osh(x)^5 - 37583*cosh(x)^3 + 6609*cosh(x))*sinh(x)^11 + 2*(308308*cosh(x)^6
 - 413413*cosh(x)^4 + 145398*cosh(x)^2 - 9561)*sinh(x)^10 - 19122*cosh(x)^1
0 + 4*(220220*cosh(x)^7 - 413413*cosh(x)^5 + 242330*cosh(x)^3 - 47805*cosh(
x))*sinh(x)^9 + 6*(165165*cosh(x)^8 - 413413*cosh(x)^6 + 363495*cosh(x)^4 -
 143415*cosh(x)^2)*sinh(x)^8 + 16*(55055*cosh(x)^9 - 177177*cosh(x)^7 + 218
097*cosh(x)^5 - 143415*cosh(x)^3)*sinh(x)^7 + 2*(308308*cosh(x)^10 - 124023
9*cosh(x)^8 + 2035572*cosh(x)^6 - 2007810*cosh(x)^4 + 9561)*sinh(x)^6 + 191
22*cosh(x)^6 + 4*(84084*cosh(x)^11 - 413413*cosh(x)^9 + 872388*cosh(x)^7 -
1204686*cosh(x)^5 + 28683*cosh(x))*sinh(x)^5 + 2*(70070*cosh(x)^12 - 413413
*cosh(x)^10 + 1090485*cosh(x)^8 - 2007810*cosh(x)^6 + 143415*cosh(x)^2 - 22
03)*sinh(x)^4 - 4406*cosh(x)^4 + 8*(5390*cosh(x)^13 - 37583*cosh(x)^11 + 12
1165*cosh(x)^9 - 286830*cosh(x)^7 + 47805*cosh(x)^3 - 2203*cosh(x))*sinh(x)
^3 + 2*(4620*cosh(x)^14 - 37583*cosh(x)^12 + 145398*cosh(x)^10 - 430245*cos
h(x)^8 + 143415*cosh(x)^4 - 13218*cosh(x)^2 + 413)*sinh(x)^2 + 826*cosh(x)^
2 + 4*(308*cosh(x)^15 - 2891*cosh(x)^13 + 13218*cosh(x)^11 - 47805*cosh(x)^
9 + 28683*cosh(x)^5 - 4406*cosh(x)^3 + 413*cosh(x))*sinh(x) - 77)*sqrt((a*c
osh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(a^3*
cosh(x)^8 + 8*a^3*cosh(x)^7*sinh(x) + 28*a^3*cosh(x)^6*sinh(x)^2 + 56*a^3*c
osh(x)^5*sinh(x)^3 + 70*a^3*cosh(x)^4*sinh(x)^4 + 56*a^3*cosh(x)^3*sinh(x)^
5 + 28*a^3*cosh(x)^2*sinh(x)^6 + 8*a^3*cosh(x)*sinh(x)^7 + a^3*sinh(x)^8)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^3\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**3)**(5/2),x)
[Out] Integral((a*csch(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^3}\right)^{5/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^3)^(5/2),x)
[Out] int(1/(a/sinh(x)^3)^(5/2), x)
```

3.42 $\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 164

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx &= 2a^3\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}\\ &- 3a^3\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{20}{7}a^3\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)}\\ &- \frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\operatorname{csch}^4(x)}\\ &- \frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\operatorname{csch}^4(x)} - a^3\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)\end{aligned}$$

```
[Out] 2*a^3*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-3*a^3*cosh(x)^2*coth(x)^3*(a*cs
ch(x)^4)^(1/2)+20/7*a^3*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-5/3*a^3*cos
h(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)+6/11*a^3*cosh(x)^2*coth(x)^9*(a*csch(x
)^4)^(1/2)-1/13*a^3*cosh(x)^2*coth(x)^11*(a*csch(x)^4)^(1/2)-a^3*cosh(x)*si
nh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx &= -\frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\operatorname{csch}^4(x)}\\ &+ \frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)}\\ &+ \frac{20}{7}a^3\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)} - 3a^3\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}\\ &+ 2a^3\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - a^3\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\end{aligned}$$

```
[In] Int[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] 2*a^3*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4] - 3*a^3*Cosh[x]^2*Coth[x]^3*Sqrt[
a*Csch[x]^4] + (20*a^3*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (5*a^3*Co
sh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/3 + (6*a^3*Cosh[x]^2*Coth[x]^9*Sqrt[a*
Csch[x]^4])/11 - (a^3*Cosh[x]^2*Coth[x]^11*Sqrt[a*Csch[x]^4])/13 - a^3*Cosh
[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \left(a^3\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\int \operatorname{csch}^{14}(x)\,dx$$

$$= -\left(\left(ia^3\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)\,dx, x, -i\coth(x)\right)\right)$$

$$= 2a^3\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - 3a^3\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{20}{7}a^3\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\operatorname{csch}^4(x)} - a^3\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \left(a\text{csch}^4(x)\right)^{7/2}\,dx =$$

$$-\frac{a^3\cosh(x)\sqrt{a\text{csch}^4(x)}\left(1024-512\text{csch}^2(x)+384\text{csch}^4(x)-320\text{csch}^6(x)+280\text{csch}^8(x)-252\text{csch}^{10}(x)+\right.}{3003}$$

```
[In] Integrate[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] -1/3003*(a^3*Cosh[x]*Sqrt[a*Csch[x]^4]*(1024 - 512*Csch[x]^2 + 384*Csch[x]^
4 - 320*Csch[x]^6 + 280*Csch[x]^8 - 252*Csch[x]^10 + 231*Csch[x]^12)*Sinh[x
])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{2048a^3\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^4}}\left(1716\,\mathrm{e}^{12x}-1287\,\mathrm{e}^{10x}+715\,\mathrm{e}^{8x}-286\,\mathrm{e}^{6x}+78\,\mathrm{e}^{4x}-13\,\mathrm{e}^{2x}+1\right)}{3003\left(\mathrm{e}^{2x}-1\right)^{11}}$	72

```
[In] int((a*csch(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(exp(2*x)-1)^11*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*
(1716*exp(12*x)-1287*exp(10*x)+715*exp(8*x)-286*exp(6*x)+78*exp(4*x)-13*exp
(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2825 vs. 2(142) = 284.

Time = 0.40 (sec) , antiderivative size = 2825, normalized size of antiderivative = 17.23

$$\int \left(a\text{csch}^4(x)\right)^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) -
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) - 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 - a^3 + (88*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(88*a^3*cosh(x)^2 - a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (88*a^3*co
```

```
sh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 - 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 - 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(11
88*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 - 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) - 2*(1188*a^3*cosh(x)^5 - 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3 + (5544*a^3*c
osh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(5544*a^
3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^
3*cosh(x))*e^(4*x) - 2*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 - 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
- 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) - 2*(32670*a^3*cosh(x)^8 - 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) - 2*(3630*a^3
*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 - 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3
 + (8712*a^3*cosh(x)^10 - 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(8712*a^3*cosh(x)^10 - 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 - a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) - 2*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x) + (792*a^3*cosh(x)^11 - 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a
^3*cosh(x))*e^(4*x) - 2*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 - 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 - 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 -
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 - 483*co
sh(x)^4 + 63*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 - 483*
cosh(x)^5 + 105*cosh(x)^3 - 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 - 12236*cosh(x)^6 + 3990*cosh(x)^4 - 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 - 15732*cosh(x)^7 + 7182*cosh(x)^5 - 1140*cosh
```

```
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 - 66861*cosh
(x)^8 + 40698*cosh(x)^6 - 9690*cosh(x)^4 + 765*cosh(x)^2 - 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 - 81719*cosh(x)^9 + 63954*cosh(x)^7 - 21318*c
osh(x)^5 + 2805*cosh(x)^3 - 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 - 490314*cosh(x)^10 + 479655*cosh(x)^8 - 213180*cosh(x)^6 + 42075*c
osh(x)^4 - 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
 - 4056234*cosh(x)^11 + 4849845*cosh(x)^9 - 2771340*cosh(x)^7 + 765765*cosh(
x)^5 - 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
(x)^14 - 676039*cosh(x)^12 + 969969*cosh(x)^10 - 692835*cosh(x)^8 + 255255*
cosh(x)^6 - 45045*cosh(x)^4 + 3003*cosh(x)^2 - 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 - 156009*cosh(x)^13 + 264537*cosh(x)^11 - 230945*cosh(x)
^9 + 109395*cosh(x)^7 - 27027*cosh(x)^5 + 3003*cosh(x)^3 - 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 - 178296*cosh(x)^14 + 352716*cosh(x)
^12 - 369512*cosh(x)^10 + 218790*cosh(x)^8 - 72072*cosh(x)^6 + 12012*cosh(x
)^4 - 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 - 59432
*cosh(x)^15 + 135660*cosh(x)^13 - 167960*cosh(x)^11 + 121550*cosh(x)^9 - 51
480*cosh(x)^7 + 12012*cosh(x)^5 - 1320*cosh(x)^3 + 45*cosh(x))*e^(2*x)*sinh
(x)^9 + 143*(10925*cosh(x)^18 - 66861*cosh(x)^16 + 174420*cosh(x)^14 - 2519
40*cosh(x)^12 + 218790*cosh(x)^10 - 115830*cosh(x)^8 + 36036*cosh(x)^6 - 59
40*cosh(x)^4 + 405*cosh(x)^2 - 5)*e^(2*x)*sinh(x)^8 + 1144*(575*cosh(x)^19
 - 3933*cosh(x)^17 + 11628*cosh(x)^15 - 19380*cosh(x)^13 + 19890*cosh(x)^11
 - 12870*cosh(x)^9 + 5148*cosh(x)^7 - 1188*cosh(x)^5 + 135*cosh(x)^3 - 5*cos
h(x))*e^(2*x)*sinh(x)^7 + 286*(805*cosh(x)^20 - 6118*cosh(x)^18 + 20349*cos
h(x)^16 - 38760*cosh(x)^14 + 46410*cosh(x)^12 - 36036*cosh(x)^10 + 18018*co
sh(x)^8 - 5544*cosh(x)^6 + 945*cosh(x)^4 - 70*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 572*(115*cosh(x)^21 - 966*cosh(x)^19 + 3591*cosh(x)^17 - 7752*cosh(x)
^15 + 10710*cosh(x)^13 - 9828*cosh(x)^11 + 6006*cosh(x)^9 - 2376*cosh(x)^7
 + 567*cosh(x)^5 - 70*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^5 + 26*(575*cos
h(x)^22 - 5313*cosh(x)^20 + 21945*cosh(x)^18 - 53295*cosh(x)^16 + 84150*cos
h(x)^14 - 90090*cosh(x)^12 + 66066*cosh(x)^10 - 32670*cosh(x)^8 + 10395*cos
h(x)^6 - 1925*cosh(x)^4 + 165*cosh(x)^2 - 3)*e^(2*x)*sinh(x)^4 + 104*(25*co
sh(x)^23 - 253*cosh(x)^21 + 1155*cosh(x)^19 - 3135*cosh(x)^17 + 5610*cosh(x
)^15 - 6930*cosh(x)^13 + 6006*cosh(x)^11 - 3630*cosh(x)^9 + 1485*cosh(x)^7
 - 385*cosh(x)^5 + 55*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 13*(25*cosh
(x)^24 - 276*cosh(x)^22 + 1386*cosh(x)^20 - 4180*cosh(x)^18 + 8415*cosh(x)^
16 - 11880*cosh(x)^14 + 12012*cosh(x)^12 - 8712*cosh(x)^10 + 4455*cosh(x)^8
 - 1540*cosh(x)^6 + 330*cosh(x)^4 - 36*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 2
6*(cosh(x)^25 - 12*cosh(x)^23 + 66*cosh(x)^21 - 220*cosh(x)^19 + 495*cosh(x
)^17 - 792*cosh(x)^15 + 924*cosh(x)^13 - 792*cosh(x)^11 + 495*cosh(x)^9 - 2
20*cosh(x)^7 + 66*cosh(x)^5 - 12*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (co
sh(x)^26 - 13*cosh(x)^24 + 78*cosh(x)^22 - 286*cosh(x)^20 + 715*cosh(x)^18
 - 1287*cosh(x)^16 + 1716*cosh(x)^14 - 1716*cosh(x)^12 + 1287*cosh(x)^10 - 7
15*cosh(x)^8 + 286*cosh(x)^6 - 78*cosh(x)^4 + 13*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx = \int \left(a\,\operatorname{csch}^4(x)\right)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a*csch(x)**4)**(7/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(142) = 284$.

Time = 0.28 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.78

$$\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] -2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*
e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^(-16*x
) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(-26*x)
 - 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x)
- 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^
(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(
-26*x) - 1) - 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(
-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1
287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x)
 + e^(-26*x) - 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) - 78*e^(-4*x) +
286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14
*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^
(-24*x) + e^(-26*x) - 1) - 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) - 78*e^(-4
*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*
e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) -
 13*e^(-24*x) + e^(-26*x) - 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) - 78
*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
 1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-2
2*x) - 13*e^(-24*x) + e^(-26*x) - 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) - 78*
e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22
*x) - 13*e^(-24*x) + e^(-26*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

$$\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx = -\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}-1287\,e^{(10\,x)}+715\,e^{(8\,x)}-286\,e^{(6\,x)}+78\,e^{(4\,x)}-13\,e^{(2\,x)}+1\right)}{3003\left(e^{(2\,x)}-1\right)^{13}}$$

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) - 1287*e^(10*x) + 715*e^(8*x) - 286*e^(6*
x) + 78*e^(4*x) - 13*e^(2*x) + 1)/(e^(2*x) - 1)^13
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.04

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{7/2}\,dx = &-\frac{2048\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{7\left(\mathrm{e}^{2\,x}-1\right)^7\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\\ &-\frac{1536\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{\left(\mathrm{e}^{2\,x}-1\right)^8\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\\ &-\frac{10240\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{3\left(\mathrm{e}^{2\,x}-1\right)^9\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\\ &-\frac{4096\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{\left(\mathrm{e}^{2\,x}-1\right)^{10}\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\\ &-\frac{30720\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{11\left(\mathrm{e}^{2\,x}-1\right)^{11}\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\\ &-\frac{1024\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{\left(\mathrm{e}^{2\,x}-1\right)^{12}\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\\ &-\frac{2048\,a^3\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{13\left(\mathrm{e}^{2\,x}-1\right)^{13}\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}\end{aligned}$$

```
[In] int((a/sinh(x)^4)^(7/2),x)
```

```
[Out] - (2048*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*exp(4*x) + exp
(6*x))) - (1536*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(
2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x)
 + exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) -
 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^9*(exp(2*x) - 2
*exp(4*x) + exp(6*x))) - (4096*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*ex
p(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^10*(exp(2
*x) - 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/
2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(2*x) - 1
)^11*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*a^3*(a/(exp(-x)/2 - exp(x)
/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2
*x) - 1)^12*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(exp(-x)/2 -
 exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/
(13*(exp(2*x) - 1)^13*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

3.43 $\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 118

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx &= \frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}\\ &-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)}\\ &-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)}-a^2\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)\end{aligned}$$

```
[Out] 4/3*a^2*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-6/5*a^2*cosh(x)^2*coth(x)^3*(
a*csch(x)^4)^(1/2)+4/7*a^2*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-1/9*a^2*
cosh(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)-a^2*cosh(x)*sinh(x)*(a*csch(x)^4)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx &= -\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)}\\ &+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}\\ &+\frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-a^2\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\end{aligned}$$

```
[In] Int[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] (4*a^2*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (6*a^2*Cosh[x]^2*Coth[x]^3*
Sqrt[a*Csch[x]^4])/5 + (4*a^2*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (a
^2*Cosh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/9 - a^2*Cosh[x]*Sqrt[a*Csch[x]^4]
*Sinh[x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\int \operatorname{csch}^{10}(x)\,dx \\
&= -\left(\left(ia^2\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\coth(x)\right)\right) \\
&= \frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} \\
&\quad + \frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)} \\
&\quad - \frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)} - a^2\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\begin{aligned}
\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx &= -\frac{1}{315}a^2\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\big(128 \\
&\quad - 64\operatorname{csch}^2(x) + 48\operatorname{csch}^4(x) - 40\operatorname{csch}^6(x) + 35\operatorname{csch}^8(x)\big)\sinh(x)
\end{aligned}$$

```
[In] Integrate[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] -1/315*(a^2*Cosh[x]*Sqrt[a*Csch[x]^4]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 -
40*Csch[x]^6 + 35*Csch[x]^8)*Sinh[x])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256a^2\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^4}}\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315(\mathrm{e}^{2x}-1)^7}$	60

```
[In] int((a*csch(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -256/315*a^2*exp(-2*x)/(exp(2*x)-1)^7*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(12
6*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1493 vs. $2(100) = 200$.

Time = 0.29 (sec) , antiderivative size = 1493, normalized size of antiderivative = 12.65

$$\int \left(a\mathrm{csch}^4(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) - 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 - 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) - 2*a^2*cosh(x)*e^(2*x)
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 - a^2 + (42*a^2*cosh(x)^2
- a^2)*e^(4*x) - 2*(42*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 - a^2*cosh(x) + (14*a^2*cosh(x)^3 - a^2*cosh
(x))*e^(4*x) - 2*(14*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 - 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) - 2*(245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 - 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) - 2*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 -
a^2 + (392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^(4
*x) - 2*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(126*a^2*cosh(x)^8 - 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x) + (56*a^2*cosh(x)
^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(56*a^2*
cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 - 1)*e^(2
```

```
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 - 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 - 10*c
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 - 195*cosh(x)^4
 + 39*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 - 273*cosh(x)^5
 + 91*cosh(x)^3 - 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 - 4004
*cosh(x)^6 + 2002*cosh(x)^4 - 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 - 25740*cosh(x)^7 + 18018*cosh(x)^5 - 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 - 6435*cosh(x)^8 + 6006*cosh
(x)^6 - 2310*cosh(x)^4 + 315*cosh(x)^2 - 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 - 715*cosh(x)^9 + 858*cosh(x)^7 - 462*cosh(x)^5 + 105*cosh(x)^3 -
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 - 858*cosh(x)^10 + 1287*c
osh(x)^8 - 924*cosh(x)^6 + 315*cosh(x)^4 - 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 - 78*cosh(x)^11 + 143*cosh(x)^9 - 132*cosh(x)^7 +
63*cosh(x)^5 - 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 - 455*cosh(x)^12 + 1001*cosh(x)^10 - 1155*cosh(x)^8 + 735*cosh(x)^6 - 245
*cosh(x)^4 + 35*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 - 105*
cosh(x)^13 + 273*cosh(x)^11 - 385*cosh(x)^9 + 315*cosh(x)^7 - 147*cosh(x)^5
 + 35*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 - 120*cos
h(x)^14 + 364*cosh(x)^12 - 616*cosh(x)^10 + 630*cosh(x)^8 - 392*cosh(x)^6 +
 140*cosh(x)^4 - 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 - 8*c
osh(x)^15 + 28*cosh(x)^13 - 56*cosh(x)^11 + 70*cosh(x)^9 - 56*cosh(x)^7 + 2
8*cosh(x)^5 - 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 - 9*cosh
(x)^16 + 36*cosh(x)^14 - 84*cosh(x)^12 + 126*cosh(x)^10 - 126*cosh(x)^8 + 8
4*cosh(x)^6 - 36*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx = \int \left(a\,\operatorname{csch}^4\left(x\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(100) = 200$.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.73

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx =$$

$$-\frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18}\right.}$$

$$+\frac{1024\,a^{\frac{5}{2}}e^{(-4\,x)}}{35\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18\,x}\right.}$$

$$-\frac{1024\,a^{\frac{5}{2}}e^{(-6\,x)}}{15\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18}\right.}$$

$$+\frac{512\,a^{\frac{5}{2}}e^{(-8\,x)}}{5\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18\,x)}\right.}$$

$$+\frac{256\,a^{\frac{5}{2}}}{315\left(9\,e^{(-2\,x)}-36\,e^{(-4\,x)}+84\,e^{(-6\,x)}-126\,e^{(-8\,x)}+126\,e^{(-10\,x)}-84\,e^{(-12\,x)}+36\,e^{(-14\,x)}-9\,e^{(-16\,x)}+e^{(-18}\right.}$$

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-
8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) -
 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) +
 e^(-18*x) - 1) - 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e
^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^
(-16*x) + e^(-18*x) - 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1) + 256/315*a^(5/2)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx = -\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}-84\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\left(e^{(2\,x)}-1\right)^9}$$

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(e
^(2*x) - 1)^9
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.02

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx = -\frac{128\,a^2\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{5\,\left(\mathrm{e}^{2\,x}-1\right)^5\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}$$
$$-\frac{256\,a^2\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{3\,\left(\mathrm{e}^{2\,x}-1\right)^6\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}$$
$$-\frac{768\,a^2\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{7\,\left(\mathrm{e}^{2\,x}-1\right)^7\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}$$
$$-\frac{64\,a^2\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{\left(\mathrm{e}^{2\,x}-1\right)^8\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}$$
$$-\frac{128\,a^2\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{9\,\left(\mathrm{e}^{2\,x}-1\right)^9\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}$$

```
[In] int((a/sinh(x)^4)^(5/2),x)
```

```
[Out] - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*
exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) - 1)^5*(exp(2*x) - 2*exp(4*x) + exp(
6*x))) - (256*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*
x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x)
 + exp(6*x))) - (768*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4
*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*e
xp(4*x) + exp(6*x))) - (64*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*
x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) -
 2*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*e
xp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) - 1)^9*(exp
(2*x) - 2*exp(4*x) + exp(6*x)))
```

3.44 $\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} - a\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)$$

```
[Out] 2/3*a*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-1/5*a*cosh(x)^2*coth(x)^3*(a*cs
ch(x)^4)^(1/2)-a*cosh(x)*sinh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = -\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - a\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^4(x)}$$

```
[In] Int[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] (2*a*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (a*Cosh[x]^2*Coth[x]^3*Sqrt[a
*Csch[x]^4])/5 - a*Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\int \operatorname{csch}^6(x)\,dx \\ &= -\left(\left(ia\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\coth(x)\right)\right) \\ &= \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} - \frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} \\ &\quad - a\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.53

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = -\frac{1}{15}a\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\left(8-4\operatorname{csch}^2(x)+3\operatorname{csch}^4(x)\right)\sinh(x)$$

```
[In] Integrate[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] -1/15*(a*Cosh[x]*Sqrt[a*Csch[x]^4]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{16a\,\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^4}}\left(10\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+1\right)}{15(\mathrm{e}^{2x}-1)^3}$	46

```
[In] int((a*csch(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*a*exp(-2*x)/(exp(2*x)-1)^3*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(10*exp
(4*x)-5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(52) = 104$.

Time $= 0.27$ (sec) , antiderivative size $= 529$, normalized size of antiderivative $= 8.53$

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx =$$

$$-\frac{16\left(10\,a\cosh\left(x\right)^4+10\right.}{15\left(10\,\cosh\left(x\right)e^{(2\,x)}\sinh\left(x\right)^9+e^{(2\,x)}\sinh\left(x\right)^{10}+5\left(9\,\cosh\left(x\right)^2-1\right)e^{(2\,x)}\sinh\left(x\right)^8+40\left(3\,\cosh\left(x\right)^3-\mathrm{co}\right.}$$

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 - 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 - a)*e^(4*x) - 2*(12*a*cosh(x)^2 -
 a)*e^(2*x) - a)*sinh(x)^2 + (10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(4*x) -
 2*(10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 - a*cos
h(x) + (4*a*cosh(x)^3 - a*cosh(x))*e^(4*x) - 2*(4*a*cosh(x)^3 - a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 - 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 - 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 - 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 - 35*cos
h(x)^4 + 15*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 - 7*cosh(x)^
5 + 5*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 - 28*cosh(x)^
6 + 30*cosh(x)^4 - 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 - 4*
cosh(x)^7 + 6*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 - 5*cosh(x)^8 + 10*cosh(x)^6 - 10*cosh(x)^4 + 5*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = \int \left(a\,\operatorname{csch}^4\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = -\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)} + \frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)} + \frac{16\,a^{\frac{3}{2}}}{15\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}$$

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^
(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) + 16/15*a^(3/2)/(5*e^(-2*x) - 10*e^(-4
*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.44

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = -\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)}-5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}-1\right)^5}$$

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \left(a\operatorname{csch}^4(x)\right)^{3/2}\,dx = -\frac{4\,a\,\mathrm{e}^{-2\,x}\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\left(\mathrm{e}^{2\,x}-1\right)^3}$$

```
[In] int((a/sinh(x)^4)^(3/2),x)
```

```
[Out] -(4*a*exp(-2*x)*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x
) + 1))/(15*(exp(2*x) - 1)^3)
```

3.45 $\int \sqrt{a\operatorname{csch}^4(x)}\,dx$

Optimal result . . . 265
Rubi [A] (verified) . . . 265
Mathematica [A] (verified) . . . 266
Maple [A] (verified) . . . 266
Fricas [B] (verification not implemented) . . . 267
Sympy [F] . . . 267
Maxima [A] (verification not implemented) . . . 267
Giac [A] (verification not implemented) . . . 268
Mupad [B] (verification not implemented) . . . 268

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)$$

```
[Out] -cosh(x)*sinh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 3852, 8}

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = \sinh(x)(-\cosh(x))\sqrt{a\operatorname{csch}^4(x)}$$

```
[In] Int[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\int \operatorname{csch}^2(x)\,dx \\ &= -\left(\left(i\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)\right)\operatorname{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\right) \\ &= -\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.03$ (sec) , antiderivative size $= 16$, normalized size of antiderivative $= 1.00$

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)$$

```
[In] Integrate[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Maple [A] (verified)

Time $= 0.19$ (sec) , antiderivative size $= 29$, normalized size of antiderivative $= 1.81$

method	result	size
risch	$-2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}\,\mathrm{e}^{-2x}(\mathrm{e}^{2x}-1)$	29

```
[In] int((a*csch(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time $= 0.26$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 5.06$

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\frac{2\sqrt{\frac{a}{e^{(8x)}-4e^{(6x)}+6e^{(4x)}-4e^{(2x)}+1}}\left(e^{(4x)}-2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}}$$

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*(e^(4*x) - 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 - 1)*e^(2*x))
```

Sympy [F]

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = \int \sqrt{a\,\operatorname{csch}^4(x)}\,dx$$

```
[In] integrate((a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.27$ (sec) , antiderivative size $= 13$, normalized size of antiderivative $= 0.81$

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = \frac{2\sqrt{a}}{e^{(-2x)}-1}$$

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)/(e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\frac{2\sqrt{a}}{e^{(2x)}-1}$$

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(a)/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\frac{\sqrt{a}\,\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}\left(3\,\mathrm{e}^{4x}-2\,\mathrm{e}^{2x}-2\,\mathrm{e}^{6x}+\frac{\mathrm{e}^{8x}}{2}+\frac{1}{2}\right)}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{2x}-2\,\mathrm{e}^{4x}+\mathrm{e}^{6x}\right)}$$

```
[In] int((a/sinh(x)^4)^(1/2),x)
[Out] -(a^(1/2)*(1/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(3*exp(4*x) - 2*exp(2*x) - 2*e
xp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) - 1)*(exp(2*x) - 2*exp(4*x) + exp(6
*x)))
```

3.46 $\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx$

Optimal result . . . 269
Rubi [A] (verified) . . . 269
Mathematica [A] (verified) . . . 270
Maple [B] (verified) . . . 270
Fricas [B] (verification not implemented) . . . 271
Sympy [F] . . . 271
Maxima [A] (verification not implemented) . . . 271
Giac [A] (verification not implemented) . . . 272
Mupad [F(-1)] . . . 272

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}} - \frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[Out] 1/2*coth(x)/(a*csch(x)^4)^(1/2)-1/2*x*csch(x)^2/(a*csch(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}} - \frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[In] Int[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] Coth[x]/(2*Sqrt[a*Csch[x]^4]) - (x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\operatorname{csch}^2(x)\int \sinh^2(x)\,dx}{\sqrt{a\operatorname{csch}^4(x)}} \\ &= \frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}} - \frac{\operatorname{csch}^2(x)\int 1\,dx}{2\sqrt{a\operatorname{csch}^4(x)}} \\ &= \frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}} - \frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \frac{\coth(x) - x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[In] Integrate[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] (Coth[x] - x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(28) = 56.

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

method	result	size
risch	$-\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}\left(e^{2x}-1\right)^2}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}\left(e^{2x}-1\right)^2}-\frac{1}{8(e^{2x}-1)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}$	89

```
[In] int(1/(a*csch(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(2*x)*x+1/8/(a*exp
(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(4*x)-1/8/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(28) = 56$.

Time $= 0.26$ (sec) , antiderivative size $= 253$, normalized size of antiderivative $= 7.03$

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx$$

$$= \frac{\left(\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\sinh\left(x\right)^4 + \cosh\left(x\right)^4 + 4\left(\cosh\left(x\right)e^{(4\,x)} - 2\,\cosh\left(x\right)e^{(2\,x)} + \cosh\left(x\right)\right)\sinh\left(x\right)^3 - 4\,x\right.}{}$$

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) -
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 - 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 - 2*x)*e^(4*x) - 2*(3*cosh(x)^2 - 2*x)*e^(2*x) - 2*x)*sinh(x)^
2 + (cosh(x)^4 - 4*x*cosh(x)^2 - 1)*e^(4*x) - 2*(cosh(x)^4 - 4*x*cosh(x)^2
- 1)*e^(2*x) + 4*(cosh(x)^3 - 2*x*cosh(x) + (cosh(x)^3 - 2*x*cosh(x))*e^(4*
x) - 2*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*
e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \int \frac{1}{\sqrt{a\,\operatorname{csch}^4\left(x\right)}}\,dx$$

```
[In] integrate(1/(a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.26$ (sec) , antiderivative size $= 22$, normalized size of antiderivative $= 0.61$

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = -\frac{\left(e^{(-4\,x)} - 1\right)e^{(2\,x)}}{8\,\sqrt{a}} - \frac{x}{2\,\sqrt{a}}$$

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(e^(-4*x) - 1)*e^(2*x)/sqrt(a) - 1/2*x/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \frac{\left(2\,e^{(2\,x)}-1\right)e^{(-2\,x)}-4\,x+e^{(2\,x)}}{8\,\sqrt{a}}$$

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="giac")
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\sinh(x)^4}}}\,dx$$

```
[In] int(1/(a/sinh(x)^4)^(1/2),x)
[Out] int(1/(a/sinh(x)^4)^(1/2), x)
```

3.47 $\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx$

Optimal result . . . 273
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Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \frac{5\coth(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5x\operatorname{csch}^2(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\operatorname{csch}^4(x)}}$$

```
[Out] 5/16*coth(x)/a/(a*csch(x)^4)^(1/2)-5/16*x*csch(x)^2/a/(a*csch(x)^4)^(1/2)-5
/24*cosh(x)*sinh(x)/a/(a*csch(x)^4)^(1/2)+1/6*cosh(x)*sinh(x)^3/a/(a*csch(x
)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = -\frac{5x\operatorname{csch}^2(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} + \frac{5\coth(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\sinh^3(x)\cosh(x)}{6a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\operatorname{csch}^4(x)}}$$

```
[In] Int[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (5*Coth[x])/(16*a*Sqrt[a*Csch[x]^4]) - (5*x*Csch[x]^2)/(16*a*Sqrt[a*Csch[x]
^4]) - (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^3)/(
6*a*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{csch}^2(x)\int \sinh^6(x)\,dx}{a\sqrt{a\operatorname{csch}^4(x)}} \\
&= \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\operatorname{csch}^4(x)}} - \frac{\left(5\operatorname{csch}^2(x)\right)\int \sinh^4(x)\,dx}{6a\sqrt{a\operatorname{csch}^4(x)}} \\
&= -\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\left(5\operatorname{csch}^2(x)\right)\int \sinh^2(x)\,dx}{8a\sqrt{a\operatorname{csch}^4(x)}} \\
&= \frac{5\coth(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\operatorname{csch}^4(x)}} - \frac{\left(5\operatorname{csch}^2(x)\right)\int 1\,dx}{16a\sqrt{a\operatorname{csch}^4(x)}} \\
&= \frac{5\coth(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5x\operatorname{csch}^2(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\operatorname{csch}^4(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{csch}^6(x)(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))}{192\left(a\operatorname{csch}^4(x)\right)^{3/2}}$$

```
[In] Integrate[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (Csch[x]^6*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Csch[x
]^4)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(70) = 140.

Time = 0.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

method	result
risch	$-\frac{5\,e^{2x}x}{16a\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}+\frac{e^{8x}}{384a\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}-\frac{3\,e^{6x}}{128a\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}+\frac{15\,e^{4x}}{128a\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}}-\frac{}{128\sqrt{}}$

```
[In] int(1/(a*csch(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/16/a*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/384/a
*exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-3/128/a*exp(6*x)
/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/128/a*exp(4*x)/(exp(2*
x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/128/(a*exp(4*x)/(exp(2*x)-1)^4
)^(1/2)/(exp(2*x)-1)^2/a+3/128/a*exp(-2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^
4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.27

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 - 3)*e^(4*x) - 2*(22*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^10 - 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 - 9*cosh(x))*e^(4*x) - 2*(22*cosh(x
)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 - 9*cosh
(x)^2 + (11*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(4*x) - 2*(11*cosh(x)^4 - 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + (11*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(11*cosh(x)
^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 - 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 - 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 -
315*cosh(x)^4 + 210*cosh(x)^2 - 20*x)*e^(4*x) - 2*(154*cosh(x)^6 - 315*cosh
(x)^4 + 210*cosh(x)^2 - 20*x)*e^(2*x) - 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
- 63*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x) + (22*cosh(x)^7 - 63*cosh(x)^5
 + 70*cosh(x)^3 - 20*x*cosh(x))*e^(4*x) - 2*(22*cosh(x)^7 - 63*cosh(x)^5 +
70*cosh(x)^3 - 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 - 42*cos
```

```
h(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 + (11*cosh(x)^8 - 42*cosh(x)^6 + 70*
cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(4*x) - 2*(11*cosh(x)^8 - 42*cosh(x)^6 +
70*cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 + (11*cos
h(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) - 2*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 +
420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 - 135*cosh(
x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + 3)*e^(4*x) - 2*(22*
cosh(x)^10 - 135*cosh(x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2
 + 3)*e^(2*x) + 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^12 - 9*cosh(x)^10 + 4
5*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(4*x) - 2
*(cosh(x)^12 - 9*cosh(x)^10 + 45*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4
 + 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(x)^7
 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(
x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x))*e^(4*x) - 2*(2*cosh(x)^1
1 - 15*cosh(x)^9 + 60*cosh(x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x
))*e^(2*x) + 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x
) - 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**4)**(3/2),x)
[Out] Integral((a*csch(x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = -\frac{\left(9\,e^{(-2\,x)} - 45\,e^{(-4\,x)} + 45\,e^{(-8\,x)} - 9\,e^{(-10\,x)} + e^{(-12\,x)} - 1\right)e^{(6\,x)}}{384\,a^{\frac{3}{2}}} - \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*x) - 45*e^(-4*x) + 45*e^(-8*x) - 9*e^(-10*x) + e^(-12*x) -
1)*e^(6*x)/a^(3/2) - 5/16*x/a^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \frac{\left(110\,e^{(6\,x)} - 45\,e^{(4\,x)} + 9\,e^{(2\,x)} - 1\right)e^{(-6\,x)} - 120\,x + e^{(6\,x)} - 9\,e^{(4\,x)} + 45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))/a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^4}\right)^{3/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^4)^(3/2),x)
```

```
[Out] int(1/(a/sinh(x)^4)^(3/2), x)
```

3.48 $\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx$

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Mupad [F(-1)] . . . 284

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[Out] 63/256*coth(x)/a^2/(a*csch(x)^4)^(1/2)-63/256*x*csch(x)^2/a^2/(a*csch(x)^4)
^(1/2)-21/128*cosh(x)*sinh(x)/a^2/(a*csch(x)^4)^(1/2)+21/160*cosh(x)*sinh(x
)^3/a^2/(a*csch(x)^4)^(1/2)-9/80*cosh(x)*sinh(x)^5/a^2/(a*csch(x)^4)^(1/2)+
1/10*cosh(x)*sinh(x)^7/a^2/(a*csch(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = -\frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\sinh^7(x)\cosh(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\sinh^5(x)\cosh(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{21\sinh^3(x)\cosh(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{21\sinh(x)\cosh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[In] Int[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (63*Coth[x])/(256*a^2*Sqrt[a*Csch[x]^4]) - (63*x*Csch[x]^2)/(256*a^2*Sqrt[a
*Csch[x]^4]) - (21*Cosh[x]*Sinh[x])/(128*a^2*Sqrt[a*Csch[x]^4]) + (21*Cosh[
```

```
x]*Sinh[x]^3)/(160*a^2*Sqrt[a*Csch[x]^4]) - (9*Cosh[x]*Sinh[x]^5)/(80*a^2*S
qrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^7)/(10*a^2*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{csch}^2(x)\int \sinh^{10}(x)\,dx}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$= \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{\left(9\operatorname{csch}^2(x)\right)\int \sinh^8(x)\,dx}{10a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$= -\frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\left(63\operatorname{csch}^2(x)\right)\int \sinh^6(x)\,dx}{80a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$= \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{\left(21\operatorname{csch}^2(x)\right)\int \sinh^4(x)\,dx}{32a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$= -\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\left(63\operatorname{csch}^2(x)\right)\int \sinh^2(x)\,dx}{128a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$= \frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{\left(63\operatorname{csch}^2(x)\right)\int 1\,dx}{256a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$= \frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \frac{\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25\sin$$

```
[In] Integrate[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^4]*Sinh[x]^2*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x] + 15
0*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(108) = 216.

Time = 0.17 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.74

method	result
risch	$-\frac{63\,e^{2x}x}{256a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}} + \frac{e^{12x}}{10240a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}} - \frac{5\,e^{10x}}{4096a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}} + \frac{15\,e^{8x}}{2048a^2\left(e^{2x}-1\right)^2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^4}}} -$

```
[In] int(1/(a*csch(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -63/256/a^2*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/1
0240/a^2*exp(12*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-5/4096/
a^2*exp(10*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/2048/a^2*
exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/512/a^2*exp(6*
x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+105/1024/a^2*exp(4*x)/(
exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-105/1024/(a*exp(4*x)/(exp(2
*x)-1)^4)^(1/2)/(exp(2*x)-1)^2/a^2+15/512/a^2*exp(-2*x)/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4
*x)/(exp(2*x)-1)^4)^(1/2)+5/4096/a^2*exp(-6*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(
exp(2*x)-1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(108) = 216$.

Time = 0.32 (sec) , antiderivative size = 2600, normalized size of antiderivative = 19.70

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 - 5)*e^(4*x) - 2*(76*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^18 - 25
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 - 15*cosh(x))*e^(4*x) - 2*(7
6*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 - 255*cosh(x)^2 + (646*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(4*x) - 2*(646
*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
+ 48*(646*cosh(x)^5 - 425*cosh(x)^3 + (646*cosh(x)^5 - 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) - 2*(646*cosh(x)^5 - 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(4*x) - 2*(129
2*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^14
 - 600*cosh(x)^14 + 120*(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4
 - 910*cosh(x)^2 + (4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4 - 910*
cosh(x)^2 + 35)*e^(4*x) - 2*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)
^4 - 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + (4199*co
sh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) - 2*(4199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 - 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x
)^6 - 300300*cosh(x)^4 + 69300*cosh(x)^2 - 2520*x)*e^(4*x) - 2*(184756*cosh
(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^4 + 69300*cos
h(x)^2 - 2520*x)*e^(2*x) - 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 - 6077
5*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 2520*x*
cosh(x) + (16796*cosh(x)^11 - 60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cos
h(x)^5 + 23100*cosh(x)^3 - 2520*x*cosh(x))*e^(4*x) - 2*(16796*cosh(x)^11 -
60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 - 36465*cosh(x)^10 +
64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 - 36465*cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 346
```

```
50*cosh(x)^4 - 7560*x*cosh(x)^2 - 70)*e^(4*x) - 2*(8398*cosh(x)^12 - 36465*
cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*cosh(x)^5 - 2
520*x*cosh(x)^3 + (646*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580
*cosh(x)^7 + 6930*cosh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*(6
46*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*co
sh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 - 7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8
+ 32340*cosh(x)^6 - 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 -
7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17
640*x*cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(4*x) - 2*(1292*cosh(x)^14 - 7735*c
osh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17640*x*
cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^6 + 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 +
 69300*cosh(x)^7 - 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 -
8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7 - 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(4*x) - 2*(1292*cosh(x)^1
5 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7
- 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(2*x) + 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*
cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cos
h(x)^2 + (323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*cosh(x
)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cosh(x)^2
 - 5)*e^(4*x) - 2*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 200
20*cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
- 170*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040
*x*cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 + (19*cosh(x)^17 - 170*cosh(x)
^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*cosh(x)^7
- 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) - 2*(19*cosh(x)^17 - 1
70*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*
cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))
*sinh(x)^3 + 5*(76*cosh(x)^18 - 765*cosh(x)^16 + 3600*cosh(x)^14 - 10920*co
sh(x)^12 + 27720*cosh(x)^10 - 45360*x*cosh(x)^8 - 11760*cosh(x)^6 + 1800*co
sh(x)^4 - 180*cosh(x)^2 + (76*cosh(x)^18 - 765*cosh(x)^16 + 3600*cosh(x)^14
 - 10920*cosh(x)^12 + 27720*cosh(x)^10 - 45360*x*cosh(x)^8 - 11760*cosh(x)^
6 + 1800*cosh(x)^4 - 180*cosh(x)^2 + 5)*e^(4*x) - 2*(76*cosh(x)^18 - 765*co
sh(x)^16 + 3600*cosh(x)^14 - 10920*cosh(x)^12 + 27720*cosh(x)^10 - 45360*x*
cosh(x)^8 - 11760*cosh(x)^6 + 1800*cosh(x)^4 - 180*cosh(x)^2 + 5)*e^(2*x) +
 5)*sinh(x)^2 + 25*cosh(x)^2 + (2*cosh(x)^20 - 25*cosh(x)^18 + 150*cosh(x)^
16 - 600*cosh(x)^14 + 2100*cosh(x)^12 - 5040*x*cosh(x)^10 - 2100*cosh(x)^8
+ 600*cosh(x)^6 - 150*cosh(x)^4 + 25*cosh(x)^2 - 2)*e^(4*x) - 2*(2*cosh(x)^
20 - 25*cosh(x)^18 + 150*cosh(x)^16 - 600*cosh(x)^14 + 2100*cosh(x)^12 - 50
40*x*cosh(x)^10 - 2100*cosh(x)^8 + 600*cosh(x)^6 - 150*cosh(x)^4 + 25*cosh(
```

```
x)^2 - 2)*e^(2*x) + 10*(4*cosh(x)^19 - 45*cosh(x)^17 + 240*cosh(x)^15 - 840
*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 - 1680*cosh(x)^7 + 360*cos
h(x)^5 - 60*cosh(x)^3 + (4*cosh(x)^19 - 45*cosh(x)^17 + 240*cosh(x)^15 - 84
0*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 - 1680*cosh(x)^7 + 360*co
sh(x)^5 - 60*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(4*cosh(x)^19 - 45*cosh(x)^
17 + 240*cosh(x)^15 - 840*cosh(x)^13 + 2520*cosh(x)^11 - 5040*x*cosh(x)^9 -
 1680*cosh(x)^7 + 360*cosh(x)^5 - 60*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cos
h(x))*sinh(x) - 2)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
)*e^(2*x)/(a^3*cosh(x)^10*e^(2*x) + 10*a^3*cosh(x)^9*e^(2*x)*sinh(x) + 45*a
^3*cosh(x)^8*e^(2*x)*sinh(x)^2 + 120*a^3*cosh(x)^7*e^(2*x)*sinh(x)^3 + 210*
a^3*cosh(x)^6*e^(2*x)*sinh(x)^4 + 252*a^3*cosh(x)^5*e^(2*x)*sinh(x)^5 + 210
*a^3*cosh(x)^4*e^(2*x)*sinh(x)^6 + 120*a^3*cosh(x)^3*e^(2*x)*sinh(x)^7 + 45
*a^3*cosh(x)^2*e^(2*x)*sinh(x)^8 + 10*a^3*cosh(x)*e^(2*x)*sinh(x)^9 + a^3*e
^(2*x)*sinh(x)^10)
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\operatorname{csch}^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx =$$

$$-\frac{\left(25\,e^{(-2\,x)} - 150\,e^{(-4\,x)} + 600\,e^{(-6\,x)} - 2100\,e^{(-8\,x)} + 2100\,e^{(-12\,x)} - 600\,e^{(-14\,x)} + 150\,e^{(-16\,x)} - 25\,e^{(-18\,x)} -\right.}{20480\,a^{\frac{5}{2}}}$$

$$-\frac{63\,x}{256\,a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20480*(25*e^(-2*x) - 150*e^(-4*x) + 600*e^(-6*x) - 2100*e^(-8*x) + 2100*
e^(-12*x) - 600*e^(-14*x) + 150*e^(-16*x) - 25*e^(-18*x) + 2*e^(-20*x) - 2)
*e^(10*x)/a^(5/2) - 63/256*x/a^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \frac{\left(5754\,e^{(10\,x)} - 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} - 150\,e^{(4\,x)} + 25\,e^{(2\,x)} - 2\right)e^{(-10\,x)} - 5040\,x + 2\,e^{(}}{20480\,a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*((5754*e^(10*x) - 2100*e^(8*x) + 600*e^(6*x) - 150*e^(4*x) + 25*e^(
2*x) - 2)*e^(-10*x) - 5040*x + 2*e^(10*x) - 25*e^(8*x) + 150*e^(6*x) - 600*
e^(4*x) + 2100*e^(2*x))/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^4}\right)^{5/2}}\,dx$$

```
[In] int(1/(a/sinh(x)^4)^(5/2),x)
```

```
[Out] int(1/(a/sinh(x)^4)^(5/2), x)
```

3.49 $\int \frac{1}{a+ia\mathbf{csch}(a+bx)}\,dx$

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Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{a+ia\operatorname{csch}(a+bx)}\,dx = \frac{x}{a} - \frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}$$

```
[Out] x/a-coth(b*x+a)/b/(a+I*a*csch(b*x+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3862, 8}

$$\int \frac{1}{a+ia\operatorname{csch}(a+bx)}\,dx = \frac{x}{a} - \frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}$$

```
[In] Int[(a + I*a*Csch[a + b*x])^(-1),x]

[Out] x/a - Coth[a + b*x]/(b*(a + I*a*Csch[a + b*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))} + \frac{\int a\,dx}{a^2} \\ &= \frac{x}{a} - \frac{\coth(a+bx)}{b(a+ia\operatorname{csch}(a+bx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+ia\operatorname{csch}(a+bx)}\,dx = \frac{1}{b} + \frac{x}{a} - \frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right) - i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}$$

```
[In] Integrate[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] - I*Sinh[(a +
b*x)/2]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{x}{a} + \frac{2i}{ba\left(e^{bx+a}+i\right)}$	27
parallelrisch	$\frac{ixb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(bx-2i)}{ba\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)}$	44
derivativedivides	$\frac{\ln\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i}-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{ab}$	54
default	$\frac{\ln\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i}-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{ab}$	54

```
[In] int(1/(a+I*a*csch(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] x/a+2*I/b/a/(exp(b*x+a)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)}\,dx = \frac{bxe^{(bx+a)} + i\,bx + 2i}{abe^{(bx+a)} + i\,ab}$$

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="fricas")
[Out] (b*x*e^(b*x + a) + I*b*x + 2*I)/(a*b*e^(b*x + a) + I*a*b)
```

Sympy [F]

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)}\,dx = -\frac{i \int \frac{1}{\operatorname{csch}(a+bx) - i}\,dx}{a}$$

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x)
[Out] -I*Integral(1/(csch(a + b*x) - I), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)}\,dx = \frac{bx + a}{ab} + \frac{2i}{(ae^{(-bx-a)} - i\,a)b}$$

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="maxima")
[Out] (b*x + a)/(a*b) + 2*I/((a*e^(-b*x - a) - I*a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)}\,dx = \frac{\frac{bx+a}{a} + \frac{2i}{a(e^{(bx+a)}+i)}}{b}$$

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="giac")
[Out] ((b*x + a)/a + 2*I/(a*(e^(b*x + a) + I)))/b
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{a + ia\operatorname{csch}(a + bx)}\,dx = \frac{x}{a} + \frac{2\mathrm{i}}{a\,b\,\left(\mathrm{e}^{a+b\,x} + 1\mathrm{i}\right)}$$

```
[In] int(1/(a + (a*1i)/sinh(a + b*x)),x)
[Out] x/a + 2i/(a*b*(exp(a + b*x) + 1i))
```

3.50 $\int \frac{1}{a-ia\mathbf{csch}(a+bx)}\,dx$

Optimal result . . . 289
Rubi [A] (verified) . . . 289
Mathematica [A] (verified) . . . 290
Maple [A] (verified) . . . 290
Fricas [A] (verification not implemented) . . . 291
Sympy [F] . . . 291
Maxima [A] (verification not implemented) . . . 291
Giac [A] (verification not implemented) . . . 291
Mupad [B] (verification not implemented) . . . 292

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{a-ia\operatorname{csch}(a+bx)}\,dx = \frac{x}{a} - \frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}$$

```
[Out] x/a-coth(b*x+a)/b/(a-I*a*csch(b*x+a))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3862, 8}

$$\int \frac{1}{a-ia\operatorname{csch}(a+bx)}\,dx = \frac{x}{a} - \frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}$$

```
[In] Int[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a - I*a*Csch[a + b*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))} + \frac{\int a\,dx}{a^2} \\ &= \frac{x}{a} - \frac{\coth(a+bx)}{b(a-ia\operatorname{csch}(a+bx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{a-ia\operatorname{csch}(a+bx)}\,dx = \frac{1}{b} + \frac{x}{a} - \frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right) + i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}$$

```
[In] Integrate[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] + I*Sinh[(a +
b*x)/2]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{x}{a} - \frac{2i}{ba\left(e^{bx+a}-i\right)}$	27
parallelrisch	$\frac{(-bx-2i)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+ixb}{ba\left(-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)}$	47
derivativedivides	$\frac{-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i}}{ab}$	54
default	$\frac{-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i}}{ab}$	54

```
[In] int(1/(a-I*a*csch(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] x/a-2*I/b/a/(exp(b*x+a)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - i\text{acsch}(a + bx)}\,dx = \frac{bxe^{(bx+a)} - i\,bx - 2i}{abe^{(bx+a)} - i\,ab}$$

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="fricas")
[Out] (b*x*e^(b*x + a) - I*b*x - 2*I)/(a*b*e^(b*x + a) - I*a*b)
```

Sympy [F]

$$\int \frac{1}{a - i\text{acsch}(a + bx)}\,dx = \frac{i\int \frac{1}{\text{csch}\,(a+bx)+i}\,dx}{a}$$

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x)
[Out] I*Integral(1/(csch(a + b*x) + I), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{a - i\text{acsch}(a + bx)}\,dx = \frac{bx + a}{ab} - \frac{2i}{(ae^{(-bx-a)} + i\,a)b}$$

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="maxima")
[Out] (b*x + a)/(a*b) - 2*I/((a*e^(-b*x - a) + I*a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1}{a - i\text{acsch}(a + bx)}\,dx = \frac{\frac{bx+a}{a} - \frac{2i}{a\left(e^{(bx+a)}-i\right)}}{b}$$

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="giac")
[Out] ((b*x + a)/a - 2*I/(a*(e^(b*x + a) - I)))/b
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{a - ia\operatorname{csch}(a + bx)}\,dx = \frac{x}{a} - \frac{2\mathrm{i}}{a\,b\,\left(\mathrm{e}^{a+b\,x} - \mathrm{i}\right)}$$

```
[In] int(1/(a - (a*1i)/sinh(a + b*x)),x)
[Out] x/a - 2i/(a*b*(exp(a + b*x) - 1i))
```

3.51 $\int (a + ia\mathbf{csch}(c + dx))^{5/2}\, dx$

Optimal result . . . 293
Rubi [A] (verified) . . . 293
Mathematica [A] (verified) . . . 295
Maple [F] . . . 295
Fricas [B] (verification not implemented) . . . 296
Sympy [F] . . . 296
Maxima [F] . . . 297
Giac [F] . . . 297
Mupad [F(-1)] . . . 297

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\, dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} + \frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}} + \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d+14/3*a^3
*coth(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)+2/3*a^2*coth(d*x+c)*(a+I*a*csch(d*
x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3860, 4000, 3859, 209, 3877}

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\, dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} + \frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}} + \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}$$

```
[In] Int[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (14*a^3*Coth[c + d*x])/(3*d*Sqrt[a + I*a*Csch[c + d*x]]) + (2*a^2*Coth[c
+ d*x]*Sqrt[a + I*a*Csch[c + d*x]])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2a^2 \coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d} \\ &\quad + \frac{1}{3}(2a)\int \sqrt{a+ia\operatorname{csch}(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}ia\operatorname{csch}(c+dx)\right)\,dx \\ &= \frac{2a^2 \coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d} \\ &\quad + \frac{1}{3}(7ia^2)\int \operatorname{csch}(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}\,dx + a^2\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx\end{aligned}$$

$$= \frac{14a^3 \coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}} + \frac{2a^2 \coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d} - \frac{(2ia^3)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$

$$= \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} + \frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}} + \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.27

$$\int (a+ia\operatorname{csch}(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+ia\operatorname{csch}(c+dx)}\left(-7i+\coth(c+dx)+\frac{3(-1)^{3/4}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{i+\operatorname{csch}(c+dx)}\right)\coth(c+dx)}{(-i+\operatorname{csch}(c+dx))\sqrt{i+\operatorname{csch}(c+dx)}}\right.}{3d}$$

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[a + I*a*Csch[c + d*x]]*(-7*I + Coth[c + d*x] + (3*(-1)^(3/4)*Ar
cTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x])/((-I + Csch[c + d*
x])*Sqrt[I + Csch[c + d*x]]) + (14*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2])))/(3*d)
```

Maple [F]

$$\int (a+ia\ \operatorname{csch}(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] int((a+I*a*csch(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(87) = 174$.

Time $= 0.29$ (sec) , antiderivative size $= 566$, normalized size of antiderivative $= 5.29$

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\,dx = \frac{3\sqrt{\frac{a^5}{d^2}}\left(de^{(2\,dx+2\,c)} - d\right)\log\left(\frac{2\left(a^3e^{(dx+c)} + ia^3 + \sqrt{\frac{a^5}{d^2}}\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-dx-c)}}{d}\right) - 3\sqrt{\frac{a^5}{d^2}}\left(de^{(2}\right.}{}$$

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(a^3*e^(d*x + c) + I*a^3
 + sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(
-d*x - c)/d) - 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(a^3*e^(d*x +
c) + I*a^3 - sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c)
- 1)))*e^(-d*x - c)/d) + 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(sqr
t(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d) + (a^3*e^(3*d*
x + 3*c) - 2*I*a^3*e^(2*d*x + 2*c) - a^3*e^(d*x + c) + 2*I*a^3)*sqrt(a/(e^(
2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c
) - d)*log(-2*(sqrt(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a
*d) - (a^3*e^(3*d*x + 3*c) - 2*I*a^3*e^(2*d*x + 2*c) - a^3*e^(d*x + c) + 2*
I*a^3)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + 8*(4*a^2*e^(3*d
*x + 3*c) - 3*I*a^2*e^(2*d*x + 2*c) - 3*a^2*e^(d*x + c) + 4*I*a^2)*sqrt(a/(
e^(2*d*x + 2*c) - 1)))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\,dx = \int \left(ia\operatorname{csch}\left(c + dx\right) + a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+I*a*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(5/2), x)
```

Maxima [F]

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\, dx = \int (i\, a \operatorname{csch}(dx + c) + a)^{\frac{5}{2}}\, dx$$

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\, dx = \int (i\, a \operatorname{csch}(dx + c) + a)^{\frac{5}{2}}\, dx$$

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + ia\operatorname{csch}(c + dx))^{5/2}\, dx = \int \left(a + \frac{a\,1\mathrm{i}}{\sinh(c + d\,x)}\right)^{5/2} dx$$

```
[In] int((a + (a*1i)/sinh(c + d*x))^(5/2),x)
[Out] int((a + (a*1i)/sinh(c + d*x))^(5/2), x)
```

3.52 $\int (a + ia\,\mathbf{csch}(c + dx))^{3/2}\,dx$

Optimal result . . . 298
Rubi [A] (verified) . . . 298
Mathematica [A] (verified) . . . 300
Maple [F] . . . 300
Fricas [B] (verification not implemented) . . . 300
Sympy [F] . . . 301
Maxima [F] . . . 301
Giac [F] . . . 301
Mupad [F(-1)] . . . 301

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int (a + ia\,\mathrm{csch}(c + dx))^{3/2}\,dx = \frac{2a^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d} + \frac{2a^2\coth(c+dx)}{d\sqrt{a + ia\,\mathrm{csch}(c+dx)}}$$

```
[Out] 2*a^(3/2)*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d+2*a^2*co
th(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3860, 21, 3859, 209}

$$\int (a + ia\,\mathrm{csch}(c + dx))^{3/2}\,dx = \frac{2a^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d} + \frac{2a^2\coth(c+dx)}{d\sqrt{a + ia\,\mathrm{csch}(c+dx)}}$$

```
[In] Int[(a + I*a*Csch[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (2*a^2*Coth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}} + (2a)\int \frac{\frac{a}{2}+\frac{1}{2}ia\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx \\
&= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}} + a\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx \\
&= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}} - \frac{(2ia^2)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} \\
&= \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} + \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx = -\frac{2ia\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}\left(-\sqrt[4]{-1}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{i+\operatorname{csch}(c+dx)}\right)+\sqrt{i+\operatorname{csch}(c+dx)}\right)}{d(-i+\operatorname{csch}(c+dx))\sqrt{i+\operatorname{csch}(c+dx)}}$$

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Coth[c + d*x]*Sqrt[a + I*a*Csch[c + d*x]]*(-((-1)^(1/4)*ArcTanh[(
-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]) + Sqrt[I + Csch[c + d*x]]))/(d*(-I + Cs
ch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Maple [F]

$$\int (a+ia\ \operatorname{csch}(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((a+I*a*csch(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 461, normalized size of antiderivative = 6.40

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx = \frac{\sqrt{\frac{a^3}{d^2}}d\log\left(\frac{2\left(a^2e^{(dx+c)}+i\,a^2+\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-dx-c)}}{d}\right)-\sqrt{\frac{a^3}{d^2}}d\log\left(\frac{2\left(a^2e^{(dx+c)}+i\,a^2-(\right.\right.}{}$$

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^3/d^2)*d*log(2*(a^2*e^(d*x + c) + I*a^2 + (d*e^(2*d*x + 2*c) -
d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) - sqrt(a^3/
d^2)*d*log(2*(a^2*e^(d*x + c) + I*a^2 - (d*e^(2*d*x + 2*c) - d)*sqrt(a^3/d^
2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) + sqrt(a^3/d^2)*d*log(2*(
(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^2) + (a^2*e^(3
*d*x + 3*c) - 2*I*a^2*e^(2*d*x + 2*c) - a^2*e^(d*x + c) + 2*I*a^2)*sqrt(a/(
```

```
e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - sqrt(a^3/d^2)*d*log(-2*((a*d*e
^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^2) - (a^2*e^(3*d*x +
 3*c) - 2*I*a^2*e^(2*d*x + 2*c) - a^2*e^(d*x + c) + 2*I*a^2)*sqrt(a/(e^(2*d
*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + 4*(a*e^(d*x + c) - I*a)*sqrt(a/(e^(2
*d*x + 2*c) - 1)))/d
```

Sympy [F]

$$\int (a + ia\text{csch}(c + dx))^{3/2}\, dx = \int (ia\,\text{csch}\,(c + dx) + a)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+I*a*csch(d*x+c))**(3/2),x)
[Out] Integral((I*a*csch(c + d*x) + a)**(3/2), x)
```

Maxima [F]

$$\int (a + ia\text{csch}(c + dx))^{3/2}\, dx = \int (i\,a\,\text{csch}\,(dx + c) + a)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a + ia\text{csch}(c + dx))^{3/2}\, dx = \int (i\,a\,\text{csch}\,(dx + c) + a)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + ia\text{csch}(c + dx))^{3/2}\, dx = \int \left(a + \frac{a\,1\text{i}}{\sinh\,(c + d\,x)}\right)^{3/2} dx$$

```
[In] int((a + (a*1i)/sinh(c + d*x))^(3/2),x)
[Out] int((a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

3.53 $\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx$

Optimal result . . . 302
Rubi [A] (verified) . . . 302
Mathematica [A] (verified) . . . 303
Maple [F] . . . 303
Fricas [B] (verification not implemented) . . . 303
Sympy [F] . . . 304
Maxima [F] . . . 305
Giac [F] . . . 305
Mupad [F(-1)] . . . 305

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3859, 209}

$$\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$

```
[In] Int[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2ia)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \sqrt{a+ia\text{csch}(c+dx)}\,dx$$

$$= \frac{2(-1)^{3/4}\text{arctanh}\left((-1)^{3/4}\sqrt{i+\text{csch}(c+dx)}\right)\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{d(-i+\text{csch}(c+dx))\sqrt{i+\text{csch}(c+dx)}}$$

```
[In] Integrate[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x]*Sqr
t[a + I*a*Csch[c + d*x]])/(d*(-I + Csch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Maple [F]

$$\int \sqrt{a+ia\ \text{csch}\left(dx+c\right)}dx$$

```
[In] int((a+I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 383, normalized size of antiderivative = 9.58

$$\int \sqrt{a+i a \operatorname{csch}(c+dx)}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}+ae^{(dx+c)}+i\,a\right)e^{(-dx-c)}}{d}\right)$$

$$-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}-ae^{(dx+c)}-i\,a\right)e^{(-dx-c)}}{d}\right)$$

$$+\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)}-2i\,ae^{(2\,dx+2\,c)}-ae^{(dx+c)}+2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}+\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}-\right.\right.\right.}{d}\right)$$

$$-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)}-2i\,ae^{(2\,dx+2\,c)}-ae^{(dx+c)}+2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}-\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}-\right.\right.\right.}{d}\right)$$

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)
)*sqrt(a/d^2) + a*e^(d*x + c) + I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*log(
-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) - a*e
^(d*x + c) - I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*
c) - 2*I*a*e^(2*d*x + 2*c) - a*e^(d*x + c) + 2*I*a)*sqrt(a/(e^(2*d*x + 2*c)
 - 1)) + (a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(
-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*c) - 2*I*a*e^(2*d
*x + 2*c) - a*e^(d*x + c) + 2*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)) - (a*d*e^(
2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]

$$\int \sqrt{a+i a \operatorname{csch}(c+dx)}\,dx = \int \sqrt{ia\operatorname{csch}(c+dx)+a}\,dx$$

```
[In] integrate((a+I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx = \int \sqrt{i\,a\,\operatorname{csch}\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx = \int \sqrt{i\,a\,\operatorname{csch}\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx = \int \sqrt{a+\frac{a\,1\mathrm{i}}{\sinh\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int((a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.54 $\int \frac{1}{\sqrt{a+ia\mathbf{csch}(c+dx)}}\,dx$

Optimal result . . . 306
Rubi [A] (verified) . . . 306
Mathematica [A] (verified) . . . 307
Maple [F] . . . 308
Fricas [B] (verification not implemented) . . . 308
Sympy [F] . . . 309
Maxima [F] . . . 309
Giac [F] . . . 309
Mupad [F(-1)] . . . 310

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{1}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d/a^(1/2)-arctanh(
1/2*coth(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3861, 3859, 209, 3880}

$$\int \frac{1}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(i\int \frac{\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx\right) + \frac{\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx}{a}$$

$$= -\frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d} + \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx$$

$$= \frac{\sqrt{a}\left(2\arctan\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{a}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\right)\coth(c+dx)}{d\sqrt{ia(i+\operatorname{csch}(c+dx))}\sqrt{a+ia\operatorname{csch}(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*ArcTan[
Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d*Sqrt[I*
a*(I + Csch[c + d*x])]*Sqrt[a + I*a*Csch[c + d*x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+ia\ \mathrm{csch}\left(dx+c\right)}}dx$$

```
[In] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(72) = 144$.

Time = 0.29 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.05

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+ia\mathrm{csch}(c+dx)}}\,dx = \\
&-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(2\left(\sqrt{2}\left(ade^{(2\,dx+2\,c)}-ad\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{1}{ad^2}}+ae^{(dx+c)}-i\,a\right)e^{(-dx-c)}\right) \\
&+\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-2\left(\sqrt{2}\left(ade^{(2\,dx+2\,c)}-ad\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{1}{ad^2}}-ae^{(dx+c)}+i\,a\right)e^{(-dx-c)}\right) \\
&+\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{1}{ad^2}}+e^{(dx+c)}+i\right)e^{(-dx-c)}}{d}\right) \\
&-\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{1}{ad^2}}-e^{(dx+c)}-i\right)e^{(-dx-c)}}{d}\right) \\
&+\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}-2\,ad\right)\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\left(e^{(3\,dx+3\,c)}-2i\,e^{(2\,dx+2\,c)}-e^{(dx+c}\right.\right.\right. \\
&-\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}-2\,ad\right)\sqrt{\frac{1}{ad^2}}-\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\left(e^{(3\,dx+3\,c)}-2i\,e^{(2\,dx+2\,c)}-e^{(dx}\right.\right.\right.
\end{aligned}$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(1/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sqr
t(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + a*e^(d*x + c) - I*a)*e^(-d*x -
 c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a
*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - a*e^(d*x + c) + I*a)*e^
(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^
```

```
(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + e^(d*x + c) + I)*e^(-d*x - c)/d) - 1/
2*sqrt(1/(a*d^2))*log(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) -
 1))*sqrt(1/(a*d^2)) - e^(d*x + c) - I)*e^(-d*x - c)/d) + 1/2*sqrt(1/(a*d^2
))*log(2*((a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2))
 + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e
^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(a*d^2))*log(-2*((a*d*e
^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2)) - sqrt(a/(e^(2*
d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d*x + c) + 2*I
))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{ia\,\text{csch}\,(c+dx)+a}}\,dx$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\,\text{csch}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\,\text{csch}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}}}\,dx$$

```
[In] int(1/(a + (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int(1/(a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.55 $\int \frac{1}{(a+ia\,\mathbf{csch}(c+dx))^{3/2}}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d-1/2*coth
(d*x+c)/d/(a+I*a*csch(d*x+c))^(3/2)-5/4*arctanh(1/2*coth(d*x+c)*a^(1/2)*2^(
1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3862, 4005, 3859, 209, 3880}

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(a^(3/2)*d
) - (5*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - Coth[c + d*x]/(2*d*(a + I*a*Csch[c + d*x])^(3/2
))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}ia\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}} + \frac{\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx}{a^2} - \frac{(5i)\int \frac{\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx}{4a}$$

$$= -\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}} - \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{ad}$$
$$+ \frac{(5i)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{2ad}$$
$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 327 vs. $2(123) = 246$.

Time = 3.42 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{\left(-2\sqrt{a} - 8\arctan\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{a}}\right)\right)\sqrt{ia(i+\operatorname{csch}(c+dx))} + 5\sqrt{2}\,\text{arcta}$$

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] ((-2*Sqrt[a] - 8*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I
+ Csch[c + d*x])] + 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]
*Sqrt[a])]*Sqrt[I*a*(I + Csch[c + d*x])] + I*Csch[c + d*x]*(2*Sqrt[a] - 8*A
rcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I + Csch[c + d*x])]
+ 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])]*Sqrt[I*
a*(I + Csch[c + d*x])]))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))/(4*a^(3
/2)*d*(I + Csch[c + d*x])*Sqrt[a + I*a*Csch[c + d*x]]*(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2]))
```

Maple [F]

$$\int \frac{1}{(a+ia\ \operatorname{csch}(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(96) = 192$.

Time = 0.30 (sec) , antiderivative size = 873, normalized size of antiderivative = 7.10

$$\int \frac{1}{(a + ia\text{csch}(c + dx))^{3/2}}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(5*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*s
qrt(1/(a^3*d^2))*log(2*(2*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(a/
(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) + a*e^(d*x + c) - I*a)*e^(-d*x - c
)) - 5*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sq
rt(1/(a^3*d^2))*log(-2*(2*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(a/
(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) - a*e^(d*x + c) + I*a)*e^(-d*x - c
)) - 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^3*
d^2))*log(2*((a*d*e^(2*d*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt
(1/(a^3*d^2)) + e^(d*x + c) + I)*e^(-d*x - c)/(a*d)) + 2*(a^2*d*e^(2*d*x +
2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^3*d^2))*log(-2*((a*d*e^(2*d
*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) - e^(d*x +
 c) - I)*e^(-d*x - c)/(a*d)) - 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x
+ c) - a^2*d)*sqrt(1/(a^3*d^2))*log(2*((a^2*d*e^(2*d*x + 2*c) - I*a^2*d*e^(
d*x + c) - 2*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3
*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/(a
*d)) + 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^
3*d^2))*log(-2*((a^2*d*e^(2*d*x + 2*c) - I*a^2*d*e^(d*x + c) - 2*a^2*d)*sqr
t(1/(a^3*d^2)) - sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*
d*x + 2*c) - e^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/(a*d)) + 2*sqrt(a/(e^(2*d
*x + 2*c) - 1))*(e^(3*d*x + 3*c) - I*e^(2*d*x + 2*c) - e^(d*x + c) + I))/(a
^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)
```

Sympy [F]

$$\int \frac{1}{(a + ia\text{csch}(c + dx))^{3/2}}\, dx = \int \frac{1}{(ia\,\text{csch}\,(c + dx) + a)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\,a\,\operatorname{csch}\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\,a\,\operatorname{csch}\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + (a*1i)/sinh(c + d*x))^(3/2),x)
[Out] int(1/(a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

3.56 $\int \sqrt{a - ia\operatorname{csch}(c+dx)}\,dx$

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Mupad [F(-1)] . . . 319

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sqrt{a - ia\operatorname{csch}(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a-I*a*csch(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3859, 209}

$$\int \sqrt{a - ia\operatorname{csch}(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}$$

```
[In] Int[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2ia)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \sqrt{a-ia\text{csch}(c+dx)}\,dx$$

$$= -\frac{2(-1)^{3/4}\arctan\left((-1)^{3/4}\sqrt{-i+\text{csch}(c+dx)}\right)\coth(c+dx)\sqrt{a-ia\text{csch}(c+dx)}}{d\sqrt{-i+\text{csch}(c+dx)}(i+\text{csch}(c+dx))}$$

```
[In] Integrate[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (-2*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[-I + Csch[c + d*x]]]*Coth[c + d*x]*Sq
rt[a - I*a*Csch[c + d*x]])/(d*Sqrt[-I + Csch[c + d*x]]*(I + Csch[c + d*x]))
```

Maple [F]

$$\int \sqrt{a-ia\ \text{csch}\left(dx+c\right)}dx$$

```
[In] int((a-I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a-I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(32) = 64$.

Time = 0.29 (sec) , antiderivative size = 383, normalized size of antiderivative = 9.58

$$\int \sqrt{a - ia\operatorname{csch}(c+dx)}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}+ae^{(dx+c)}-i\,a\right)e^{(-dx-c)}}{d}\right)$$
$$-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}-ae^{(dx+c)}+i\,a\right)e^{(-dx-c)}}{d}\right)$$
$$+\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)}+2i\,ae^{(2\,dx+2\,c)}-ae^{(dx+c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}+\left(ade^{(2\,dx+2\,c)}+i\,ade^{(dx+c)}-\right.\right.\right.}{d}\right)$$
$$-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)}+2i\,ae^{(2\,dx+2\,c)}-ae^{(dx+c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}-\left(ade^{(2\,dx+2\,c)}+i\,ade^{(dx+c)}-\right.\right.\right.}{d}\right)$$

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)
)*sqrt(a/d^2) + a*e^(d*x + c) - I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*log(
-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) - a*e
^(d*x + c) + I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*
c) + 2*I*a*e^(2*d*x + 2*c) - a*e^(d*x + c) - 2*I*a)*sqrt(a/(e^(2*d*x + 2*c)
 - 1)) + (a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(
-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*c) + 2*I*a*e^(2*d
*x + 2*c) - a*e^(d*x + c) - 2*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)) - (a*d*e^(
2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]

$$\int \sqrt{a - ia\operatorname{csch}(c+dx)}\,dx = \int \sqrt{-ia\operatorname{csch}(c+dx)+a}\,dx$$

```
[In] integrate((a-I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a - ia\operatorname{csch}(c + dx)}\, dx = \int \sqrt{-i\, a \operatorname{csch}(dx + c) + a}\, dx$$

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a - ia\operatorname{csch}(c + dx)}\, dx = \int \sqrt{-i\, a \operatorname{csch}(dx + c) + a}\, dx$$

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - ia\operatorname{csch}(c + dx)}\, dx = \int \sqrt{a - \frac{a\,1\mathrm{i}}{\sinh(c + d\,x)}}\, dx$$

```
[In] int((a - (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int((a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.57 $\int \frac{1}{\sqrt{a-ia\,\mathbf{csch}(c+dx)}}\,dx$

Optimal result . . . 320
Rubi [A] (verified) . . . 320
Mathematica [A] (verified) . . . 321
Maple [F] . . . 322
Fricas [B] (verification not implemented) . . . 322
Sympy [F] . . . 323
Maxima [F] . . . 323
Giac [F] . . . 323
Mupad [F(-1)] . . . 324

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{1}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a-I*a*csch(d*x+c))^(1/2))/d/a^(1/2)-arctanh(
1/2*coth(d*x+c)*a^(1/2)*2^(1/2)/(a-I*a*csch(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3861, 3859, 209, 3880}

$$\int \frac{1}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a - I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int \frac{\operatorname{csch}(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\,dx + \frac{\int \sqrt{a-ia\operatorname{csch}(c+dx)}\,dx}{a} \\ &= \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d} - \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d} \\ &= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\begin{aligned}&\int \frac{1}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\,dx \\ &= \frac{\sqrt{a}\left(2\arctan\left(\frac{\sqrt{-ia(-i+\operatorname{csch}(c+dx))}}{\sqrt{a}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{-ia(-i+\operatorname{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\right)\coth(c+dx)}{d\sqrt{a(-1-i\operatorname{csch}(c+dx))}\sqrt{a-ia\operatorname{csch}(c+dx)}}\end{aligned}$$

```
[In] Integrate[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*Arc
Tan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d
*Sqrt[a*(-1 - I*Csch[c + d*x])]*Sqrt[a - I*a*Csch[c + d*x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a - ia\ \mathrm{csch}\left(dx + c\right)}}dx$$

```
[In] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.05

$$\begin{aligned}
&\int \frac{1}{\sqrt{a - ia\mathrm{csch}(c + dx)}}\,dx = \\
&\quad -\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(2\left(\sqrt{2}\left(ade^{(2\,dx+2\,c)} - ad\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)} - 1}}\sqrt{\frac{1}{ad^2}} + ae^{(dx+c)} + i\,a\right)e^{(-dx-c)}\right) \\
&\quad +\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-2\left(\sqrt{2}\left(ade^{(2\,dx+2\,c)} - ad\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)} - 1}}\sqrt{\frac{1}{ad^2}} - ae^{(dx+c)} - i\,a\right)e^{(-dx-c)}\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{1}{ad^2}} + e^{(dx+c)} - i\right)e^{(-dx-c)}}{d}\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{1}{ad^2}} - e^{(dx+c)} + i\right)e^{(-dx-c)}}{d}\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(ade^{(2\,dx+2\,c)} + i\,ade^{(dx+c)} - 2\,ad\right)\sqrt{\frac{1}{ad^2}} + \sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\left(e^{(3\,dx+3\,c)} + 2i\,e^{(2\,dx+2\,c)} - e^{(dx+c}\right.\right.\right.}{d}\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(ade^{(2\,dx+2\,c)} + i\,ade^{(dx+c)} - 2\,ad\right)\sqrt{\frac{1}{ad^2}} - \sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\left(e^{(3\,dx+3\,c)} + 2i\,e^{(2\,dx+2\,c)} - e^{(dx}\right.\right.\right.}{d}\right)
\end{aligned}$$

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(1/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sqr
t(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + a*e^(d*x + c) + I*a)*e^(-d*x -
 c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a
*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - a*e^(d*x + c) - I*a)*e^
(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^
```

```
(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + e^(d*x + c) - I)*e^(-d*x - c)/d) - 1/
2*sqrt(1/(a*d^2))*log(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) -
 1))*sqrt(1/(a*d^2)) - e^(d*x + c) + I)*e^(-d*x - c)/d) + 1/2*sqrt(1/(a*d^2
))*log(2*((a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2))
 + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) - e
^(d*x + c) - 2*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(a*d^2))*log(-2*((a*d*e
^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2)) - sqrt(a/(e^(2*
d*x + 2*c) - 1))*(e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) - e^(d*x + c) - 2*I
))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-ia\,\text{csch}\,(c + dx) + a}}\,dx$$

```
[In] integrate(1/(a-I*a*csch(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(-I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-i\,a\,\text{csch}\,(dx + c) + a}}\,dx$$

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-i\,a\,\text{csch}\,(dx + c) + a}}\,dx$$

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - ia\operatorname{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a - \frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}}}\,dx$$

```
[In] int(1/(a - (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int(1/(a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.58 $\int \sqrt{3+3i\operatorname{csch}(x)}\,dx$

Optimal result . . . 325
Rubi [A] (verified) . . . 325
Mathematica [A] (verified) . . . 326
Maple [F] . . . 326
Fricas [B] (verification not implemented) . . . 326
Sympy [F] . . . 327
Maxima [F] . . . 327
Giac [F] . . . 328
Mupad [F(-1)] . . . 328

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{3+3i\operatorname{csch}(x)}\,dx = 2\sqrt{3}\operatorname{arctanh}\left(\frac{\coth(x)}{\sqrt{1+i\operatorname{csch}(x)}}\right)$$

```
[Out] 2*arctanh(coth(x)/(1+I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 209}

$$\int \sqrt{3+3i\operatorname{csch}(x)}\,dx = 2\sqrt{3}\operatorname{arctanh}\left(\frac{\coth(x)}{\sqrt{1+i\operatorname{csch}(x)}}\right)$$

```
[In] Int[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 + I*Csch[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{3+3i\text{csch}(x)}}\right)\right)\\ &= 2\sqrt{3}\text{arctanh}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\sqrt{-1+i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1+i\text{csch}(x)}\sqrt{1+i\text{csch}(x)}}$$

```
[In] Integrate[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqrt
[1 + I*Csch[x]])
```

Maple [F]

$$\int \sqrt{3+3i\,\text{csch}\,(x)}dx$$

```
[In] int((3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((3+3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 9.22

$$\begin{aligned}
&\int \sqrt{3+3i\operatorname{csch}(x)}\,dx \\
&= \frac{1}{2}\sqrt{3}\log\left(2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+3\,e^{x}+3i\right)e^{(-x)}\right) \\
&\quad -\frac{1}{2}\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3\,e^{x}-3i\right)e^{(-x)}\right) \\
&\quad +\frac{1}{2}\sqrt{3}\log\left(6\left(\sqrt{3}e^{(2x)}-i\sqrt{3}e^{x}+\frac{\sqrt{3}\left(e^{(3x)}-2i\,e^{(2x)}-e^{x}+2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right) \\
&\quad -\frac{1}{2}\sqrt{3}\log\left(-6\left(\sqrt{3}e^{(2x)}-i\sqrt{3}e^{x}-\frac{\sqrt{3}\left(e^{(3x)}-2i\,e^{(2x)}-e^{x}+2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)
\end{aligned}$$

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
3*e^x + 3*I)*e^(-x)) - 1/2*sqrt(3)*log(-2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(
3))/sqrt(e^(2*x) - 1) - 3*e^x - 3*I)*e^(-x)) + 1/2*sqrt(3)*log(6*(sqrt(3)*e
^(2*x) - I*sqrt(3)*e^x + sqrt(3)*(e^(3*x) - 2*I*e^(2*x) - e^x + 2*I)/sqrt(e
^(2*x) - 1) - 2*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-6*(sqrt(3)*e^(2*x) -
I*sqrt(3)*e^x - sqrt(3)*(e^(3*x) - 2*I*e^(2*x) - e^x + 2*I)/sqrt(e^(2*x) -
1) - 2*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{3+3i\operatorname{csch}(x)}\,dx = \sqrt{3}\int \sqrt{i\,\operatorname{csch}(x)+1}\,dx$$

```
[In] integrate((3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{3+3i\operatorname{csch}(x)}\,dx = \int \sqrt{3i\,\operatorname{csch}(x)+3}\,dx$$

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

Giac [F]

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \int \sqrt{3i\ \text{csch}\,(x)+3}\,dx$$

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \int \sqrt{3+\frac{3\text{i}}{\sinh\,(x)}}\,dx$$

```
[In] int((3i/sinh(x) + 3)^(1/2),x)
[Out] int((3i/sinh(x) + 3)^(1/2), x)
```

3.59 $\int \sqrt{3 - 3i\operatorname{csch}(x)}\, dx$

Optimal result . . . 329
Rubi [A] (verified) . . . 329
Mathematica [A] (verified) . . . 330
Maple [F] . . . 330
Fricas [B] (verification not implemented) . . . 330
Sympy [F] . . . 331
Maxima [F] . . . 331
Giac [F] . . . 332
Mupad [F(-1)] . . . 332

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{3 - 3i\operatorname{csch}(x)}\, dx = 2\sqrt{3}\operatorname{arctanh}\left(\frac{\coth(x)}{\sqrt{1 - i\operatorname{csch}(x)}}\right)$$

```
[Out] 2*arctanh(coth(x)/(1-I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 209}

$$\int \sqrt{3 - 3i\operatorname{csch}(x)}\, dx = 2\sqrt{3}\operatorname{arctanh}\left(\frac{\coth(x)}{\sqrt{1 - i\operatorname{csch}(x)}}\right)$$

```
[In] Int[Sqrt[3 - (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 - I*Csch[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = 6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{3-3i\text{csch}(x)}}\right)$$

$$= 2\sqrt{3}\text{arctanh}\left(\frac{\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{3-3i\text{csch}(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\sqrt{-1-i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1-i\text{csch}(x)}\sqrt{1-i\text{csch}(x)}}$$

```
[In] Integrate[Sqrt[3 - (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqrt
[1 - I*Csch[x]])
```

Maple [F]

$$\int \sqrt{3-3i\,\text{csch}\,(x)}dx$$

```
[In] int((3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((3-3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 9.22

$$\begin{aligned}
&\int \sqrt{3-3i\operatorname{csch}(x)}\,dx \\
&= \frac{1}{2}\sqrt{3}\log\left(2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+3\,e^{x}-3i\right)e^{(-x)}\right) \\
&\quad -\frac{1}{2}\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3\,e^{x}+3i\right)e^{(-x)}\right) \\
&\quad +\frac{1}{2}\sqrt{3}\log\left(6\left(\sqrt{3}e^{(2x)}+i\sqrt{3}e^{x}+\frac{\sqrt{3}\left(e^{(3x)}+2i\,e^{(2x)}-e^{x}-2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right) \\
&\quad -\frac{1}{2}\sqrt{3}\log\left(-6\left(\sqrt{3}e^{(2x)}+i\sqrt{3}e^{x}-\frac{\sqrt{3}\left(e^{(3x)}+2i\,e^{(2x)}-e^{x}-2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)
\end{aligned}$$

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
3*e^x - 3*I)*e^(-x)) - 1/2*sqrt(3)*log(-2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(
3))/sqrt(e^(2*x) - 1) - 3*e^x + 3*I)*e^(-x)) + 1/2*sqrt(3)*log(6*(sqrt(3)*e
^(2*x) + I*sqrt(3)*e^x + sqrt(3)*(e^(3*x) + 2*I*e^(2*x) - e^x - 2*I)/sqrt(e
^(2*x) - 1) - 2*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-6*(sqrt(3)*e^(2*x) +
I*sqrt(3)*e^x - sqrt(3)*(e^(3*x) + 2*I*e^(2*x) - e^x - 2*I)/sqrt(e^(2*x) -
1) - 2*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{3-3i\operatorname{csch}(x)}\,dx = \sqrt{3}\int \sqrt{-i\,\operatorname{csch}(x)+1}\,dx$$

```
[In] integrate((3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{3-3i\operatorname{csch}(x)}\,dx = \int \sqrt{-3i\,\operatorname{csch}(x)+3}\,dx$$

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

Giac [F]

$$\int \sqrt{3 - 3i\text{csch}(x)}\, dx = \int \sqrt{-3i\ \text{csch}\,(x) + 3}\, dx$$

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 3i\text{csch}(x)}\, dx = \int \sqrt{3 - \frac{3\text{i}}{\sinh\,(x)}}\, dx$$

```
[In] int((3 - 3i/sinh(x))^(1/2),x)
[Out] int((3 - 3i/sinh(x))^(1/2), x)
```

3.60 $\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx$

Optimal result . . . 333
Rubi [A] (verified) . . . 333
Mathematica [A] (verified) . . . 334
Maple [F] . . . 334
Fricas [B] (verification not implemented) . . . 334
Sympy [F] . . . 335
Maxima [F] . . . 335
Giac [F] . . . 336
Mupad [F(-1)] . . . 336

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx = -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)$$

```
[Out] -2*arctan(coth(x)/(-1+I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 213}

$$\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx = -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}}\right)$$

```
[In] Int[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 + I*Csch[x]]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(6i\text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{-3+3i\text{csch}(x)}}\right)\right) \\ &= -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1+i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = -\frac{2\sqrt{3}\text{arctanh}\left(\sqrt{1+i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1+i\text{csch}(x)}\sqrt{1+i\text{csch}(x)}}$$

```
[In] Integrate[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqr
t[1 + I*Csch[x]])
```

Maple [F]

$$\int \sqrt{-3+3i\,\text{csch}\,(x)}dx$$

```
[In] int((-3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3+3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 9.48

$$\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx = -\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(i\sqrt{3}e^{(2x)}-i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3i\,e^{x}-3\right)e^{(-x)}\right)$$
$$+\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(-i\sqrt{3}e^{(2x)}+i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3i\,e^{x}-3\right)e^{(-x)}\right)$$
$$-\frac{1}{2}i\sqrt{3}\log\left(-6\left(i\sqrt{3}e^{(2x)}-\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-i\,e^{(3x)}+2\,e^{(2x)}+i\,e^{x}-2\right)}{\sqrt{e^{(2x)}-1}}-2i\sqrt{3}\right)e^{(-2x)}\right)$$
$$+\frac{1}{2}i\sqrt{3}\log\left(-6\left(-i\sqrt{3}e^{(2x)}+\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-i\,e^{(3x)}+2\,e^{(2x)}+i\,e^{x}-2\right)}{\sqrt{e^{(2x)}-1}}+2i\sqrt{3}\right)e^{(-2x)}\right)$$

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(3)*log(-2*(sqrt(3)*(I*sqrt(3)*e^(2*x) - I*sqrt(3))/sqrt(e^(2*x)
 - 1) - 3*I*e^x - 3)*e^(-x)) + 1/2*I*sqrt(3)*log(-2*(sqrt(3)*(-I*sqrt(3)*e^
(2*x) + I*sqrt(3))/sqrt(e^(2*x) - 1) - 3*I*e^x - 3)*e^(-x)) - 1/2*I*sqrt(3)
*log(-6*(I*sqrt(3)*e^(2*x) - sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) + 2*e^(2*x)
+ I*e^x - 2)/sqrt(e^(2*x) - 1) - 2*I*sqrt(3))*e^(-2*x)) + 1/2*I*sqrt(3)*log
(-6*(-I*sqrt(3)*e^(2*x) + sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) + 2*e^(2*x) + I
*e^x - 2)/sqrt(e^(2*x) - 1) + 2*I*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx = \sqrt{3}\int \sqrt{i\operatorname{csch}(x)-1}\,dx$$

```
[In] integrate((-3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) - 1), x)
```

Maxima [F]

$$\int \sqrt{-3+3i\operatorname{csch}(x)}\,dx = \int \sqrt{3i\,\operatorname{csch}(x)-3}\,dx$$

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

Giac [F]

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \int \sqrt{3i\ \text{csch}\,(x)-3}\,dx$$

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \int \sqrt{-3+\frac{3\text{i}}{\sinh\,(x)}}\,dx$$

```
[In] int((3i/sinh(x) - 3)^(1/2),x)
[Out] int((3i/sinh(x) - 3)^(1/2), x)
```

3.61 $\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx$

Optimal result . . . 337
Rubi [A] (verified) . . . 337
Mathematica [A] (verified) . . . 338
Maple [F] . . . 338
Fricas [B] (verification not implemented) . . . 338
Sympy [F] . . . 339
Maxima [F] . . . 339
Giac [F] . . . 339
Mupad [F(-1)] . . . 340

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1-i\operatorname{csch}(x)}}\right)$$

```
[Out] -2*arctan(coth(x)/(-1-I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 213}

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1-i\operatorname{csch}(x)}}\right)$$

```
[In] Int[Sqrt[-3 - (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 - I*Csch[x]]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 6i\text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{-3-3i\operatorname{csch}(x)}}\right)\\ &= -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1-i\operatorname{csch}(x)}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = -\frac{2\sqrt{3}\operatorname{arctanh}\left(\sqrt{1-i\operatorname{csch}(x)}\right)\coth(x)}{\sqrt{-1-i\operatorname{csch}(x)}\sqrt{1-i\operatorname{csch}(x)}}$$

```
[In] Integrate[Sqrt[-3 - (3*I)*Csch[x]],x]
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqr
t[1 - I*Csch[x]])
```

Maple [F]

$$\int \sqrt{-3-3i\,\operatorname{csch}(x)}dx$$

```
[In] int((-3-3*I*csch(x))^(1/2),x)
[Out] int((-3-3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(17) = 34.

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 9.48

$$\begin{aligned}\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx &= -\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(i\sqrt{3}e^{(2x)}-i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3i\,e^{x}+3\right)e^{(-x)}\right)\\ &+\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(-i\sqrt{3}e^{(2x)}+i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3i\,e^{x}+3\right)e^{(-x)}\right)\\ &-\frac{1}{2}i\sqrt{3}\log\left(-6\left(i\sqrt{3}e^{(2x)}+\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-i\,e^{(3x)}-2\,e^{(2x)}+i\,e^{x}+2\right)}{\sqrt{e^{(2x)}-1}}-2i\sqrt{3}\right)e^{(-2x)}\right)\\ &+\frac{1}{2}i\sqrt{3}\log\left(-6\left(-i\sqrt{3}e^{(2x)}-\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-i\,e^{(3x)}-2\,e^{(2x)}+i\,e^{x}+2\right)}{\sqrt{e^{(2x)}-1}}+2i\sqrt{3}\right)e^{(-2x)}\right)\end{aligned}$$

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(3)*log(-2*(sqrt(3)*(I*sqrt(3)*e^(2*x) - I*sqrt(3))/sqrt(e^(2*x)
 - 1) - 3*I*e^x + 3)*e^(-x)) + 1/2*I*sqrt(3)*log(-2*(sqrt(3)*(-I*sqrt(3)*e^
(2*x) + I*sqrt(3))/sqrt(e^(2*x) - 1) - 3*I*e^x + 3)*e^(-x)) - 1/2*I*sqrt(3)
*log(-6*(I*sqrt(3)*e^(2*x) + sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) - 2*e^(2*x)
+ I*e^x + 2)/sqrt(e^(2*x) - 1) - 2*I*sqrt(3))*e^(-2*x)) + 1/2*I*sqrt(3)*log
(-6*(-I*sqrt(3)*e^(2*x) - sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) - 2*e^(2*x) + I
*e^x + 2)/sqrt(e^(2*x) - 1) + 2*I*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \sqrt{3}\int \sqrt{-i\,\operatorname{csch}(x)-1}\,dx$$

```
[In] integrate((-3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) - 1), x)
```

Maxima [F]

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \int \sqrt{-3i\ \operatorname{csch}(x)-3}\,dx$$

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

Giac [F]

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \int \sqrt{-3i\ \operatorname{csch}(x)-3}\,dx$$

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \int \sqrt{-3-\frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

```
[In] int((- 3i/sinh(x) - 3)^(1/2),x)
[Out] int((- 3i/sinh(x) - 3)^(1/2), x)
```

3.62 $\int \frac{\sinh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 341
Rubi [A] (verified) . . . 341
Mathematica [A] (verified) . . . 343
Maple [A] (verified) . . . 343
Fricas [A] (verification not implemented) . . . 343
Sympy [F] . . . 344
Maxima [A] (verification not implemented) . . . 344
Giac [A] (verification not implemented) . . . 344
Mupad [B] (verification not implemented) . . . 345

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15ix}{8} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)}$$

```
[Out] -15/8*I*x-4*cosh(x)+4/3*cosh(x)^3+15/8*I*cosh(x)*sinh(x)-5/4*I*cosh(x)*sinh
(x)^3-cosh(x)*sinh(x)^3/(I+csch(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2715, 8, 2713}

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15ix}{8} + \frac{4\cosh^3(x)}{3} - 4\cosh(x) - \frac{5}{4}i\sinh^3(x)\cosh(x) + \frac{15}{8}i\sinh(x)\cosh(x) - \frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[In] Int[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-15*I)/8)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 + ((15*I)/8)*Cosh[x]*Sinh[x] -
((5*I)/4)*Cosh[x]*Sinh[x]^3 - (Cosh[x]*Sinh[x]^3)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \int (-5i + 4\operatorname{csch}(x))\sinh^4(x)\,dx \\
&= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} - 5i\int \sinh^4(x)\,dx + 4\int \sinh^3(x)\,dx \\
&= -\frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} \\
&\quad + \frac{15}{4}i\int \sinh^2(x)\,dx - 4\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) \\
&= -4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) \\
&\quad - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} - \frac{15}{8}i\int 1\,dx \\
&= -\frac{15ix}{8} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) \\
&\quad - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{96}\left(-180ix - 168\cosh(x) + 8\cosh(3x) + \frac{192\sinh\left(\frac{x}{2}\right)}{-i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)} + 48i\sinh(2x) - 3i\sinh(4x)\right)$$

```
[In] Integrate[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-180*I)*x - 168*Cosh[x] + 8*Cosh[3*x] + (192*Sinh[x/2])/((-I)*Cosh[x/2] +
 Sinh[x/2]) + (48*I)*Sinh[2*x] - (3*I)*Sinh[4*x])/96
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{15ix}{8} - \frac{ie^{4x}}{64} + \frac{e^{3x}}{24} + \frac{ie^{2x}}{4} - \frac{7e^{x}}{8} - \frac{7e^{-x}}{8} - \frac{ie^{-2x}}{4} + \frac{e^{-3x}}{24} + \frac{ie^{-4x}}{64} - \frac{2}{e^{x}-i}$
parallelrisch	$\frac{432+360(i-i\cosh(x)-\sinh(x))\ln(1-\coth(x)+\operatorname{csch}(x))+360(-i+i\cosh(x)+\sinh(x))\ln(\coth(x)-\operatorname{csch}(x)+1)-80i\sinh(2x)-4}{192i\sinh(x)-192\cos}$
default	$-\frac{i}{4(\tanh(\frac{x}{2})-1)^4} + \frac{15i\ln(\tanh(\frac{x}{2})-1)}{8} + \frac{\frac{3}{2}+\frac{7i}{8}}{\tanh(\frac{x}{2})-1} + \frac{-\frac{1}{2}+\frac{5i}{8}}{(\tanh(\frac{x}{2})-1)^2} + \frac{-\frac{1}{3}-\frac{i}{2}}{(\tanh(\frac{x}{2})-1)^3} + \frac{2i}{\tanh(\frac{x}{2})-i} - \frac{15i\ln($

```
[In] int(sinh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -15/8*I*x-1/64*I*exp(x)^4+1/24*exp(x)^3+1/4*I*exp(x)^2-7/8*exp(x)-7/8/exp(x
)-1/4*I/exp(x)^2+1/24/exp(x)^3+1/64*I/exp(x)^4-2/(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{24\,(15i\,x-7i)e^{(5\,x)} + 24\,(15\,x+23)e^{(4\,x)} + 3i\,e^{(9\,x)} - 5\,e^{(8\,x)} - 40i\,e^{(7\,x)} + 120\,e^{(6\,x)} - 120i\,e^{(3\,x)} + 40\,e^{(}}{192\,(e^{(5\,x)} - i\,e^{(4\,x)})}$$

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/192*(24*(15*I*x - 7*I)*e^(5*x) + 24*(15*x + 23)*e^(4*x) + 3*I*e^(9*x) -
5*e^(8*x) - 40*I*e^(7*x) + 120*e^(6*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e
^x - 3)/(e^(5*x) - I*e^(4*x))
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(sinh(x)**4/(I+csch(x)),x)
[Out] Integral(sinh(x)**4/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15}{8}i\,x - \frac{-5i\,e^{(-x)} + 40\,e^{(-2\,x)} + 120i\,e^{(-3\,x)} + 552\,e^{(-4\,x)} - 3}{16\,(12i\,e^{(-4\,x)} + 12\,e^{(-5\,x)})} - \frac{7}{8}\,e^{(-x)} - \frac{1}{4}i\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{64}i\,e^{(-4\,x)}$$

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="maxima")
[Out] -15/8*I*x - 1/16*(-5*I*e^(-x) + 40*e^(-2*x) + 120*I*e^(-3*x) + 552*e^(-4*x)
 - 3)/(12*I*e^(-4*x) + 12*e^(-5*x)) - 7/8*e^(-x) - 1/4*I*e^(-2*x) + 1/24*e^
(-3*x) + 1/64*I*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15}{8}i\,x - \frac{\left(552\,e^{(4\,x)} - 120i\,e^{(3\,x)} + 40\,e^{(2\,x)} + 5i\,e^{x} - 3\right)e^{(-4\,x)}}{192\,(e^{x} - i)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{4}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x}$$

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="giac")
[Out] -15/8*I*x - 1/192*(552*e^(4*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e^x - 3)*
e^(-4*x)/(e^x - I) - 1/64*I*e^(4*x) + 1/24*e^(3*x) + 1/4*I*e^(2*x) - 7/8*e^
x
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-3x}}{24} - \frac{7\,\mathrm{e}^{-x}}{8} - \frac{\mathrm{e}^{-2x}\,1\mathrm{i}}{4} + \frac{\mathrm{e}^{2x}\,1\mathrm{i}}{4} - \frac{x\,15\mathrm{i}}{8} + \frac{\mathrm{e}^{3x}}{24} + \frac{\mathrm{e}^{-4x}\,1\mathrm{i}}{64} - \frac{\mathrm{e}^{4x}\,1\mathrm{i}}{64} - \frac{7\,\mathrm{e}^{x}}{8} - \frac{2}{\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(sinh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] (exp(2*x)*1i)/4 - (7*exp(-x))/8 - (exp(-2*x)*1i)/4 - (x*15i)/8 + exp(-3*x)/
24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 - (7*exp(x))/8 - 2/
(exp(x) - 1i)
```

3.63 $\int \frac{\sinh^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 346
Rubi [A] (verified) . . . 346
Mathematica [A] (verified) . . . 347
Maple [A] (verified) . . . 348
Fricas [A] (verification not implemented) . . . 348
Sympy [F] . . . 349
Maxima [A] (verification not implemented) . . . 349
Giac [A] (verification not implemented) . . . 349
Mupad [B] (verification not implemented) . . . 350

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3x}{2} + 4i\cosh(x) - \frac{4}{3}i\cosh^3(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}$$

```
[Out] -3/2*x+4*I*cosh(x)-4/3*I*cosh(x)^3+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I
+csch(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2713, 2715, 8}

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3x}{2} - \frac{4}{3}i\cosh^3(x) + 4i\cosh(x) + \frac{3}{2}\sinh(x)\cosh(x) - \frac{\sinh^2(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[In] Int[Sinh[x]^3/(I + Csch[x]),x]
```

```
[Out] (-3*x)/2 + (4*I)*Cosh[x] - ((4*I)/3)*Cosh[x]^3 + (3*Cosh[x]*Sinh[x])/2 - (C
osh[x]*Sinh[x]^2)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} + \int (-4i + 3\operatorname{csch}(x))\sinh^3(x)\,dx \\
&= -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} - 4i\int \sinh^3(x)\,dx + 3\int \sinh^2(x)\,dx \\
&= \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)} + 4i\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) - \frac{3\int 1\,dx}{2} \\
&= -\frac{3x}{2} + 4i\cosh(x) - \frac{4}{3}i\cosh^3(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\begin{aligned}
\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{12}\Bigg(& 21i\cosh(x) - i\cosh(3x) \\
& + 3\left(-6x + \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)} + \sinh(2x)\right)\Bigg)
\end{aligned}$$

```
[In] Integrate[Sinh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((21*I)*Cosh[x] - I*Cosh[3*x] + 3*(-6*x + (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh
[x/2]) + Sinh[2*x]))/12
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{3x}{2}-\frac{i\mathrm{e}^{3x}}{24}+\frac{\mathrm{e}^{2x}}{8}+\frac{7i\mathrm{e}^{x}}{8}+\frac{7i\mathrm{e}^{-x}}{8}-\frac{\mathrm{e}^{-2x}}{8}-\frac{i\mathrm{e}^{-3x}}{24}+\frac{2i}{\mathrm{e}^{x}-i}$
default	$\frac{i}{3(\tanh(\frac{x}{2})-1)^3}+\frac{\frac{1}{2}+\frac{i}{2}}{(\tanh(\frac{x}{2})-1)^2}+\frac{\frac{1}{2}-\frac{3i}{2}}{\tanh(\frac{x}{2})-1}+\frac{3\ln(\tanh(\frac{x}{2})-1)}{2}-\frac{i}{3(\tanh(\frac{x}{2})+1)^3}+\frac{\frac{1}{2}+\frac{3i}{2}}{\tanh(\frac{x}{2})+1}+\frac{-\frac{1}{2}+\frac{i}{2}}{(\tanh(\frac{x}{2})+$
parallelrisch	$\frac{(36i\sinh(x)-36\cosh(x)+36)\ln(1-\coth(x)+\operatorname{csch}(x))+(-36i\sinh(x)+36\cosh(x)-36)\ln(\coth(x)-\operatorname{csch}(x)+1)+23i\cosh(x)-20i}{24i\sinh(x)-24\cosh(x)+24}$

```
[In] int(sinh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x-1/24*I*exp(x)^3+1/8*exp(x)^2+7/8*I*exp(x)+7/8*I/exp(x)-1/8/exp(x)^2-
1/24*I/exp(x)^3+2*I/(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3\,(12\,x-7)e^{(4\,x)}+3\,(-12i\,x-23i)e^{(3\,x)}+i\,e^{(7\,x)}-2\,e^{(6\,x)}-18i\,e^{(5\,x)}-18\,e^{(2\,x)}-2i\,e^{x}+1}{24\,(e^{(4\,x)}-i\,e^{(3\,x)})}$$

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(12*x - 7)*e^(4*x) + 3*(-12*I*x - 23*I)*e^(3*x) + I*e^(7*x) - 2*e^
(6*x) - 18*I*e^(5*x) - 18*e^(2*x) - 2*I*e^x + 1)/(e^(4*x) - I*e^(3*x))
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh^3(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(sinh(x)**3/(I+csch(x)),x)
[Out] Integral(sinh(x)**3/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3}{2}x + \frac{2i\,e^{(-x)} - 18\,e^{(-2x)} + 69i\,e^{(-3x)} + 1}{8\left(3i\,e^{(-3x)} + 3\,e^{(-4x)}\right)} + \frac{7}{8}i\,e^{(-x)} - \frac{1}{8}\,e^{(-2x)} - \frac{1}{24}i\,e^{(-3x)}$$

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="maxima")
[Out] -3/2*x + 1/8*(2*I*e^(-x) - 18*e^(-2*x) + 69*I*e^(-3*x) + 1)/(3*I*e^(-3*x) +
 3*e^(-4*x)) + 7/8*I*e^(-x) - 1/8*e^(-2*x) - 1/24*I*e^(-3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3}{2}x - \frac{\left(-69i\,e^{(3x)} - 18\,e^{(2x)} - 2i\,e^{x} + 1\right)e^{(-3x)}}{24\left(e^{x} - i\right)} - \frac{1}{24}i\,e^{(3x)} + \frac{1}{8}\,e^{(2x)} + \frac{7}{8}i\,e^{x}$$

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="giac")
[Out] -3/2*x - 1/24*(-69*I*e^(3*x) - 18*e^(2*x) - 2*I*e^x + 1)*e^(-3*x)/(e^x - I)
 - 1/24*I*e^(3*x) + 1/8*e^(2*x) + 7/8*I*e^x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{2\,x}}{8}+\frac{\mathrm{e}^{-x}\,7\mathrm{i}}{8}-\frac{\mathrm{e}^{-2\,x}}{8}-\frac{3\,x}{2}-\frac{\mathrm{e}^{-3\,x}\,1\mathrm{i}}{24}-\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{24}+\frac{\mathrm{e}^{x}\,7\mathrm{i}}{8}+\frac{2\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(sinh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*7i)/8 - (3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - (exp(-3*x)*1i)/24 - (
exp(3*x)*1i)/24 + (exp(x)*7i)/8 + 2i/(exp(x) - 1i)
```

3.64 $\int \frac{\sinh^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 351
Rubi [A] (verified) . . . 351
Mathematica [A] (verified) . . . 352
Maple [A] (verified) . . . 353
Fricas [B] (verification not implemented) . . . 353
Sympy [F] . . . 353
Maxima [A] (verification not implemented) . . . 354
Giac [A] (verification not implemented) . . . 354
Mupad [B] (verification not implemented) . . . 354

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}$$

```
[Out] 3/2*I*x+2*cosh(x)-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)/(I+csch(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2715, 8, 2718}

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\sinh(x)\cosh(x) - \frac{\sinh(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[In] Int[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] ((3*I)/2)*x + 2*Cosh[x] - ((3*I)/2)*Cosh[x]*Sinh[x] - (Cosh[x]*Sinh[x])/(I
+ Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} + \int (-3i + 2\operatorname{csch}(x))\sinh^2(x)\,dx \\
&= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} - 3i\int \sinh^2(x)\,dx + 2\int \sinh(x)\,dx \\
&= 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)} + \frac{3}{2}i\int 1\,dx \\
&= \frac{3ix}{2} + 2\cosh(x) - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.16$ (sec) , antiderivative size $= 46$, normalized size of antiderivative $= 1.28$

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \cosh(x) + \frac{1}{4}i\left(6x - \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)} - \sinh(2x)\right)$$

```
[In] Integrate[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] Cosh[x] + (I/4)*(6*x - (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2]) - Sinh[2*x])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result
risch	$\frac{3ix}{2}-\frac{ie^{2x}}{8}+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{ie^{-2x}}{8}+\frac{2}{e^{x}-i}$
default	$-\frac{3i\ln(\tanh(\frac{x}{2})-1)}{2}-\frac{i}{2(\tanh(\frac{x}{2})-1)^{2}}+\frac{-1-\frac{i}{2}}{\tanh(\frac{x}{2})-1}-\frac{2i}{\tanh(\frac{x}{2})-i}+\frac{i}{2(\tanh(\frac{x}{2})+1)^{2}}+\frac{3i\ln(\tanh(\frac{x}{2})+1)}{2}+\tan$
parallelrisch	$\frac{(12i\cosh(x)-12i+12\sinh(x))\ln(1-\coth(x)+\operatorname{csch}(x))+(-12i\cosh(x)+12i-12\sinh(x))\ln(\coth(x)-\operatorname{csch}(x)+1)+5i\sinh(x)+2}{8i\sinh(x)-8\cosh(x)+8}$

```
[In] int(sinh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I*x-1/8*I*exp(x)^2+1/2*exp(x)+1/2/exp(x)+1/8*I/exp(x)^2+2/(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx=-\frac{4\,(-3i\,x+i)e^{(3\,x)}-4\,(3\,x+5)e^{(2\,x)}+i\,e^{(5\,x)}-3\,e^{(4\,x)}+3i\,e^{x}-1}{8\,(e^{(3\,x)}-i\,e^{(2\,x)})}$$

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-3*I*x + I)*e^(3*x) - 4*(3*x + 5)*e^(2*x) + I*e^(5*x) - 3*e^(4*x)
+ 3*I*e^x - 1)/(e^(3*x) - I*e^(2*x))
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\sinh^2(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(sinh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)**2/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3}{2}i\,x + \frac{3i\,e^{(-x)} + 20\,e^{(-2\,x)} + 1}{4\left(2i\,e^{(-2\,x)} + 2\,e^{(-3\,x)}\right)} + \frac{1}{2}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)}$$

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x + 1/4*(3*I*e^(-x) + 20*e^(-2*x) + 1)/(2*I*e^(-2*x) + 2*e^(-3*x)) +
1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3}{2}i\,x + \frac{\left(20\,e^{(2\,x)} - 3i\,e^{x} + 1\right)e^{(-2\,x)}}{8\left(e^{x} - i\right)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x + 1/8*(20*e^(2*x) - 3*I*e^x + 1)*e^(-2*x)/(e^x - I) - 1/8*I*e^(2*x)
 + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{x\,3\mathrm{i}}{2} + \frac{\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{x}}{2} + \frac{2}{\mathrm{e}^{x} - \mathrm{i}}$$

```
[In] int(sinh(x)^2/(1/sinh(x) + 1i),x)
```

```
[Out] (x*3i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2 + 2/(e
xp(x) - 1i)
```

3.65 $\int \frac{\sinh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 355
Rubi [A] (verified) . . . 355
Mathematica [A] (verified) . . . 356
Maple [A] (verified) . . . 356
Fricas [B] (verification not implemented) . . . 357
Sympy [F] . . . 357
Maxima [A] (verification not implemented) . . . 357
Giac [A] (verification not implemented) . . . 358
Mupad [B] (verification not implemented) . . . 358

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x - 2i\cosh(x) - \frac{\cosh(x)}{i+\operatorname{csch}(x)}$$

```
[Out] x-2*I*cosh(x)-cosh(x)/(I+csch(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3904, 3872, 2718, 8}

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x - 2i\cosh(x) - \frac{\cosh(x)}{\operatorname{csch}(x)+i}$$

```
[In] Int[Sinh[x]/(I + Csch[x]),x]
```

```
[Out] x - (2*I)*Cosh[x] - Cosh[x]/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cosh(x)}{i+\operatorname{csch}(x)} + \int (-2i + \operatorname{csch}(x))\sinh(x)\,dx \\ &= -\frac{\cosh(x)}{i+\operatorname{csch}(x)} - 2i\int \sinh(x)\,dx + \int 1\,dx \\ &= x - 2i\cosh(x) - \frac{\cosh(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.09$ (sec) , antiderivative size $= 35$, normalized size of antiderivative $= 1.75$

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x - i\cosh(x) - \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}$$

```
[In] Integrate[Sinh[x]/(I + Csch[x]),x]
[Out] x - I*Cosh[x] - (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A] (verified)

Time $= 0.64$ (sec) , antiderivative size $= 25$, normalized size of antiderivative $= 1.25$

method	result	size
risch	$x - \frac{ie^{x}}{2} - \frac{ie^{-x}}{2} - \frac{2i}{e^{x}-i}$	25
parallelrisch	$\frac{i\cosh(2x)+\sinh(2x)+(2ix+6)\sinh(x)-2x\cosh(x)-i+2x}{2i\sinh(x)-2\cosh(x)+2}$	46
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)-i} - \frac{i}{\tanh\left(\frac{x}{2}\right)+1} + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{i}{\tanh\left(\frac{x}{2}\right)-1} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	51

```
[In] int(sinh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/2*I*exp(x)-1/2*I/exp(x)-2*I/(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time $= 0.26$ (sec) , antiderivative size $= 39$, normalized size of antiderivative $= 1.95$

$$\int \frac{\sinh(x)}{i + \operatorname{csch}(x)}\,dx = \frac{(2\,x-1)e^{(2\,x)} + (-2i\,x - 5i)e^{x} - i\,e^{(3\,x)} - 1}{2\,(e^{(2\,x)} - i\,e^{x})}$$

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*x - 1)*e^(2*x) + (-2*I*x - 5*I)*e^x - I*e^(3*x) - 1)/(e^(2*x) - I*e
^x)
```

Sympy [F]

$$\int \frac{\sinh(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\sinh(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(sinh(x)/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.18$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 1.55$

$$\int \frac{\sinh(x)}{i + \operatorname{csch}(x)}\,dx = x - \frac{5i\,e^{(-x)} - 1}{2\,(i\,e^{(-x)} + e^{(-2\,x)})} - \frac{1}{2}i\,e^{(-x)}$$

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] x - 1/2*(5*I*e^(-x) - 1)/(I*e^(-x) + e^(-2*x)) - 1/2*I*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x + \frac{(-5i\,e^{x}-1)e^{(-x)}}{2\,(e^{x}-i)} - \frac{1}{2}i\,e^{x}$$

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="giac")
[Out] x + 1/2*(-5*I*e^x - 1)*e^(-x)/(e^x - I) - 1/2*I*e^x
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x - \frac{\mathrm{e}^{-x}\,1\mathrm{i}}{2} - \frac{\mathrm{e}^{x}\,1\mathrm{i}}{2} - \frac{2\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(sinh(x)/(1/sinh(x) + 1i),x)
[Out] x - (exp(-x)*1i)/2 - (exp(x)*1i)/2 - 2i/(exp(x) - 1i)
```

3.66 $\int \frac{\mathbf{csch}(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 359
Rubi [A] (verified) . . . 359
Mathematica [A] (verified) . . . 360
Maple [A] (verified) . . . 360
Fricas [A] (verification not implemented) . . . 360
Sympy [F] . . . 361
Maxima [A] (verification not implemented) . . . 361
Giac [A] (verification not implemented) . . . 361
Mupad [B] (verification not implemented) . . . 361

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

```
[Out] I*coth(x)/(I+csch(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3879}

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

```
[In] Int[Csch[x]/(I + Csch[x]),x]
```

```
[Out] (I*Coth[x])/(I + Csch[x])
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

```
[In] Integrate[Csch[x]/(I + Csch[x]),x]
[Out] (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{2i}{e^x-i}$	11
default	$\frac{2}{\tanh\left(\frac{x}{2}\right)-i}$	12
parallelrisch	$-\frac{2}{-\tanh\left(\frac{x}{2}\right)+i}$	14

```
[In] int(csch(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] 2*I/(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2i}{e^x-i}$$

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="fricas")
[Out] 2*I/(e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(csch(x)/(I+csch(x)),x)
[Out] Integral(csch(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{2}{i\,e^{(-x)}-1}$$

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="maxima")
[Out] -2/(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2i}{e^{x}-i}$$

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="giac")
[Out] 2*I/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(1/(sinh(x)*(1/sinh(x) + 1i)),x)
[Out] 2i/(exp(x) - 1i)
```

3.67 $\int \frac{\mathbf{csch}^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 362
Rubi [A] (verified) . . . 362
Mathematica [B] (verified) . . . 363
Maple [A] (verified) . . . 363
Fricas [B] (verification not implemented) . . . 364
Sympy [F] . . . 364
Maxima [A] (verification not implemented) . . . 364
Giac [A] (verification not implemented) . . . 364
Mupad [B] (verification not implemented) . . . 365

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) + \frac{\coth(x)}{i+\operatorname{csch}(x)}$$

```
[Out] -arctanh(cosh(x))+coth(x)/(I+csch(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3874, 3855, 3879}

$$\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) + \frac{\coth(x)}{\operatorname{csch}(x)+i}$$

```
[In] Int[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + Coth[x]/(I + Csch[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i \int \frac{\operatorname{csch}(x)}{i + \operatorname{csch}(x)}\, dx\right) + \int \operatorname{csch}(x)\, dx \\ &= -\operatorname{arctanh}(\cosh(x)) + \frac{\coth(x)}{i + \operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time $= 0.13$ (sec) , antiderivative size $= 46$, normalized size of antiderivative $= 2.71$

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\, dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{2i \sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i \sinh\left(\frac{x}{2}\right)}$$

```
[In] Integrate[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] -Log[Cosh[x/2]] + Log[Sinh[x/2]] - ((2*I)*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/
2])
```

Maple [A] (verified)

Time $= 0.45$ (sec) , antiderivative size $= 19$, normalized size of antiderivative $= 1.12$

method	result	size
default	$-\frac{2i}{\tanh\left(\frac{x}{2}\right)-i} + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$	19
risch	$\frac{2}{e^x - i} + \ln\left(e^x - 1\right) - \ln\left(e^x + 1\right)$	23
parallelrisch	$\frac{2i+\ln\left(\tanh\left(\frac{x}{2}\right)\right)\left(-\tanh\left(\frac{x}{2}\right)+i\right)}{-\tanh\left(\frac{x}{2}\right)+i}$	31

```
[In] int(csch(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(tanh(1/2*x)-I)+ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time $= 0.25$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 1.82$

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{(e^x - i)\log(e^x + 1) - (e^x - i)\log(e^x - 1) - 2}{e^x - i}$$

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="fricas")
[Out] -((e^x - I)*log(e^x + 1) - (e^x - I)*log(e^x - 1) - 2)/(e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(csch(x)**2/(I+csch(x)),x)
[Out] Integral(csch(x)**2/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.18$ (sec) , antiderivative size $= 27$, normalized size of antiderivative $= 1.59$

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{2}{e^{(-x)} + i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="maxima")
[Out] 2/(e^(-x) + I) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time $= 0.29$ (sec) , antiderivative size $= 22$, normalized size of antiderivative $= 1.29$

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{2}{e^x - i} - \log\left(e^x + 1\right) + \log\left(|e^x - 1|\right)$$

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] 2/(e^x - I) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx = \ln\left(2-2\,\mathrm{e}^x\right)-\ln\left(-2\,\mathrm{e}^x-2\right)+\frac{2}{\mathrm{e}^x-\mathrm{i}}$$

```
[In] int(1/(sinh(x)^2*(1/sinh(x) + 1i)),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2/(exp(x) - 1i)
```

3.68 $\int \frac{\text{csch}^3(x)}{i+\text{csch}(x)}\,dx$

Optimal result . . . 366
Rubi [A] (verified) . . . 366
Mathematica [B] (verified) . . . 367
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Sympy [F] . . . 368
Maxima [B] (verification not implemented) . . . 369
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Mupad [B] (verification not implemented) . . . 369

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\text{csch}^3(x)}{i+\text{csch}(x)}\,dx = i\text{arctanh}(\cosh(x)) - \coth(x) - \frac{i\coth(x)}{i+\text{csch}(x)}$$

```
[Out] I*arctanh(cosh(x))-coth(x)-I*coth(x)/(I+csch(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3875, 3874, 3855, 3879}

$$\int \frac{\text{csch}^3(x)}{i+\text{csch}(x)}\,dx = i\text{arctanh}(\cosh(x)) - \coth(x) - \frac{i\coth(x)}{\text{csch}(x)+i}$$

```
[In] Int[Csch[x]^3/(I + Csch[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] - Coth[x] - (I*Coth[x])/(I + Csch[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
```

```
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\coth(x) - i\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx \\ &= -\coth(x) - i\int \operatorname{csch}(x)\,dx - \int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx \\ &= i\operatorname{arctanh}(\cosh(x)) - \coth(x) - \frac{i\coth(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx = &-\frac{1}{2}\coth\left(\frac{x}{2}\right) + i\log\left(\cosh\left(\frac{x}{2}\right)\right) - i\log\left(\sinh\left(\frac{x}{2}\right)\right) \\ &- \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)} - \frac{1}{2}\tanh\left(\frac{x}{2}\right)\end{aligned}$$

```
[In] Integrate[Csch[x]^3/(I + Csch[x]),x]
```

```
[Out] -1/2*Coth[x/2] + I*Log[Cosh[x/2]] - I*Log[Sinh[x/2]] - (2*Sinh[x/2])/(Cosh[
x/2] + I*Sinh[x/2]) - Tanh[x/2]/2
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)-i}$	35
risch	$-\frac{2i\left(\mathrm{e}^{2x}-2-i\mathrm{e}^{x}\right)}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}-i\right)}+i\ln\left(\mathrm{e}^{x}+1\right)-i\ln\left(\mathrm{e}^{x}-1\right)$	47
parallelrisch	$\frac{2i\tanh\left(\frac{x}{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)-i\coth\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)^{2}+2\ln\left(\tanh\left(\frac{x}{2}\right)\right)+6}{-2\tanh\left(\frac{x}{2}\right)+2i}$	47

```
[In] int(csch(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-I*ln(tanh(1/2*x))-2/(tanh(1/2*x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.96

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx$$
$$=\frac{\left(i\,e^{(3\,x)}+e^{(2\,x)}-i\,e^{x}-1\right)\log\left(e^{x}+1\right)+\left(-i\,e^{(3\,x)}-e^{(2\,x)}+i\,e^{x}+1\right)\log\left(e^{x}-1\right)-2i\,e^{(2\,x)}-2\,e^{x}+4i}{e^{(3\,x)}-i\,e^{(2\,x)}-e^{x}+i}$$

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^(3*x) + e^(2*x) - I*e^x - 1)*log(e^x + 1) + (-I*e^(3*x) - e^(2*x) + I
*e^x + 1)*log(e^x - 1) - 2*I*e^(2*x) - 2*e^x + 4*I)/(e^(3*x) - I*e^(2*x) -
e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{csch}^3(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(csch(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.

Time $= 0.17$ (sec) , antiderivative size $= 53$, normalized size of antiderivative $= 2.04$

$$\int \frac{\operatorname{csch}^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{2\left(e^{(-x)} - i\,e^{(-2\,x)} + 2i\right)}{e^{(-x)} - i\,e^{(-2\,x)} - e^{(-3\,x)} + i} + i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2*(e^(-x) - I*e^(-2*x) + 2*I)/(e^(-x) - I*e^(-2*x) - e^(-3*x) + I) + I*log
(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time $= 0.28$ (sec) , antiderivative size $= 46$, normalized size of antiderivative $= 1.77$

$$\int \frac{\operatorname{csch}^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{2\left(e^{(2\,x)} - i\,e^{x} - 2\right)}{i\,e^{(3\,x)} + e^{(2\,x)} - i\,e^{x} - 1} + i\,\log\left(e^{x} + 1\right) - i\,\log\left(\left|e^{x} - 1\right|\right)$$

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) - I*e^x - 2)/(I*e^(3*x) + e^(2*x) - I*e^x - 1) + I*log(e^x + 1)
- I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time $= 2.48$ (sec) , antiderivative size $= 60$, normalized size of antiderivative $= 2.31$

$$\int \frac{\operatorname{csch}^3(x)}{i + \operatorname{csch}(x)}\,dx = -\ln\left(\mathrm{e}^{x}\,2\mathrm{i} - 2\mathrm{i}\right)\,1\mathrm{i} + \ln\left(\mathrm{e}^{x}\,2\mathrm{i} + 2\mathrm{i}\right)\,1\mathrm{i} + \frac{\mathrm{e}^{2\,x}\,2\mathrm{i} + 2\,\mathrm{e}^{x} - 4\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i} - \mathrm{e}^{3\,x} + \mathrm{e}^{x} - \mathrm{i}}$$

```
[In] int(1/(sinh(x)^3*(1/sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i + (exp(2*x)*2i + 2*exp(x) -
 4i)/(exp(2*x)*1i - exp(3*x) + exp(x) - 1i)
```

3.69 $\int \frac{\mathbf{csch}^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 370
Rubi [A] (verified) . . . 370
Mathematica [B] (verified) . . . 372
Maple [A] (verified) . . . 372
Fricas [B] (verification not implemented) . . . 373
Sympy [F] . . . 373
Maxima [B] (verification not implemented) . . . 373
Giac [A] (verification not implemented) . . . 374
Mupad [B] (verification not implemented) . . . 374

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3}{2}\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}$$

```
[Out] 3/2*arctanh(cosh(x))+2*I*coth(x)-3/2*coth(x)*csch(x)+coth(x)*csch(x)^2/(I+c
sch(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3903, 3872, 3852, 8, 3853, 3855}

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3}{2}\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{\operatorname{csch}(x)+i} - \frac{3}{2}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 + (2*I)*Coth[x] - (3*Coth[x]*Csch[x])/2 + (Coth[x]*C
sch[x]^2)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - \int (2i - 3\operatorname{csch}(x))\operatorname{csch}^2(x)\,dx \\
&= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - 2i\int \operatorname{csch}^2(x)\,dx + 3\int \operatorname{csch}^3(x)\,dx \\
&= -\frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - \frac{3}{2}\int \operatorname{csch}(x)\,dx - 2\text{Subst}\Big(\int 1\,dx, x, -i\coth(x)\Big) \\
&= \frac{3}{2}\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 90 vs. $2(37) = 74$.

Time $= 0.24$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 2.43$

$$\int \frac{\operatorname{csch}^4(x)}{i + \operatorname{csch}(x)}\,dx = \frac{1}{8}\left(4i\coth\left(\frac{x}{2}\right) - \operatorname{csch}^2\left(\frac{x}{2}\right) + 12\log\left(\cosh\left(\frac{x}{2}\right)\right) - 12\log\left(\sinh\left(\frac{x}{2}\right)\right) - \operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{16\sinh\left(\frac{x}{2}\right)}{-i\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)} + 4i\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] ((4*I)*Coth[x/2] - Csch[x/2]^2 + 12*Log[Cosh[x/2]] - 12*Log[Sinh[x/2]] - Se
ch[x/2]^2 + (16*Sinh[x/2])/((-I)*Cosh[x/2] + Sinh[x/2]) + (4*I)*Tanh[x/2])/
8
```

Maple [A] (verified)

Time $= 0.95$ (sec) , antiderivative size $= 53$, normalized size of antiderivative $= 1.43$

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2} + \frac{\tanh\left(\frac{x}{2}\right)^2}{8} - \frac{1}{8\tanh\left(\frac{x}{2}\right)^2} + \frac{i}{2\tanh\left(\frac{x}{2}\right)} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} + \frac{2i}{\tanh\left(\frac{x}{2}\right)-i}$	53
risch	$-\frac{-5\,e^{2x}-3ie^{3x}+3\,e^{4x}+4+ie^{x}}{\left(e^{2x}-1\right)^2\left(e^{x}-i\right)} - \frac{3\ln\left(e^{x}-1\right)}{2} + \frac{3\ln\left(e^{x}+1\right)}{2}$	59
parallelrisch	$\frac{\left(-12i+12\tanh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right) - i\coth\left(\frac{x}{2}\right)^2 - 3i\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^3 - 24i - 3\coth\left(\frac{x}{2}\right)}{-8\tanh\left(\frac{x}{2}\right)+8i}$	63

```
[In] int(csch(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-1/8/tanh(1/2*x)^2+1/2*I/tanh(1/2*x)-3/2
*ln(tanh(1/2*x))+2*I/(tanh(1/2*x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.24

$$\int \frac{\operatorname{csch}^4(x)}{i + \operatorname{csch}(x)}\,dx$$
$$= \frac{3\left(e^{(5\,x)} - i\,e^{(4\,x)} - 2\,e^{(3\,x)} + 2i\,e^{(2\,x)} + e^{x} - i\right)\log\left(e^{x} + 1\right) - 3\left(e^{(5\,x)} - i\,e^{(4\,x)} - 2\,e^{(3\,x)} + 2i\,e^{(2\,x)} + e^{x} - i\right)}{2\left(e^{(5\,x)} - i\,e^{(4\,x)} - 2\,e^{(3\,x)} + 2i\,e^{(2\,x)} + e^{x} - i\right)}$$

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*(e^(5*x) - I*e^(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)*log(e^x +
1) - 3*(e^(5*x) - I*e^(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)*log(e^x -
1) - 6*e^(4*x) + 6*I*e^(3*x) + 10*e^(2*x) - 2*I*e^x - 8)/(e^(5*x) - I*e^(4*
x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(csch(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(29) = 58$.

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.08

$$\int \frac{\operatorname{csch}^4(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{-i\,e^{(-x)} - 5\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 4}{e^{(-x)} - 2i\,e^{(-2\,x)} - 2\,e^{(-3\,x)} + i\,e^{(-4\,x)} + e^{(-5\,x)} + i} + \frac{3}{2}\log\left(e^{(-x)} + 1\right) - \frac{3}{2}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -(-I*e^(-x) - 5*e^(-2*x) + 3*I*e^(-3*x) + 3*e^(-4*x) + 4)/(e^(-x) - 2*I*e^(
-2*x) - 2*e^(-3*x) + I*e^(-4*x) + e^(-5*x) + I) + 3/2*log(e^(-x) + 1) - 3/2
*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{e^{(3\,x)}-2i\,e^{(2\,x)}+e^x+2i}{\left(e^{(2\,x)}-1\right)^2}-\frac{2i}{i\,e^x+1}+\frac{3}{2}\log\left(e^x+1\right)-\frac{3}{2}\log\left(\left|e^x-1\right|\right)$$

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -(e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 2*I/(I*e^x + 1) + 3/
2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3\,\ln\left(3\,\mathrm{e}^x+3\right)}{2}-\frac{3\,\ln\left(3\,\mathrm{e}^x-3\right)}{2}-\frac{\mathrm{e}^x}{\mathrm{e}^{2\,x}-1}-\frac{2\,\mathrm{e}^x}{\left(\mathrm{e}^{2\,x}-1\right)^2}-\frac{2}{\mathrm{e}^x-\mathrm{i}}+\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(1/(sinh(x)^4*(1/sinh(x) + 1i)),x)
```

```
[Out] (3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1) -
 (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) - 1i) + 2i/(exp(2*x) - 1)
```

3.70 $\int (a + b\text{csch}(c + dx))^4\, dx$

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Optimal result

Integrand size = 12, antiderivative size = 109

$$\int (a + b\text{csch}(c + dx))^4\, dx = a^4 x - \frac{2ab(2a^2 - b^2)\,\text{arctanh}(\cosh(c + dx))}{d} - \frac{b^2(17a^2 - 2b^2)\coth(c + dx)}{3d} - \frac{4ab^3\coth(c + dx)\text{csch}(c + dx)}{3d} - \frac{b^2\coth(c + dx)(a + b\text{csch}(c + dx))^2}{3d}$$

```
[Out] a^4*x-2*a*b*(2*a^2-b^2)*arctanh(cosh(d*x+c))/d-1/3*b^2*(17*a^2-2*b^2)*coth(
d*x+c)/d-4/3*a*b^3*coth(d*x+c)*csch(d*x+c)/d-1/3*b^2*coth(d*x+c)*(a+b*csch(
d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3867, 4133, 3855, 3852, 8}

$$\int (a + b\text{csch}(c + dx))^4\, dx = a^4 x - \frac{2ab(2a^2 - b^2)\,\text{arctanh}(\cosh(c + dx))}{d} - \frac{b^2(17a^2 - 2b^2)\coth(c + dx)}{3d} - \frac{4ab^3\coth(c + dx)\text{csch}(c + dx)}{3d} - \frac{b^2\coth(c + dx)(a + b\text{csch}(c + dx))^2}{3d}$$

```
[In] Int[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] a^4*x - (2*a*b*(2*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (b^2*(17*a^2 - 2*b
^2)*Coth[c + d*x])/(3*d) - (4*a*b^3*Coth[c + d*x]*Csch[c + d*x])/(3*d) - (b
^2*Coth[c + d*x]*(a + b*Csch[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^2 \coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d} \\
&\quad + \frac{1}{3}\int (a+b\operatorname{csch}(c+dx))\left(3a^3+b\left(9a^2-2b^2\right)\operatorname{csch}(c+dx)+8ab^2\operatorname{csch}^2(c+dx)\right)\,dx \\
&= -\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d} - \frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d} \\
&\quad + \frac{1}{6}\int \left(6a^4+12ab\left(2a^2-b^2\right)\operatorname{csch}(c+dx)+2b^2\left(17a^2-2b^2\right)\operatorname{csch}^2(c+dx)\right)\,dx \\
&= a^4x - \frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d} - \frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d} \\
&\quad + \frac{1}{3}\left(b^2\left(17a^2-2b^2\right)\right)\int \operatorname{csch}^2(c+dx)\,dx + \left(2ab\left(2a^2-b^2\right)\right)\int \operatorname{csch}(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= a^4 x - \frac{2ab(2a^2-b^2)\operatorname{arctanh}(\cosh(c+dx))}{d} \\
&\quad - \frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d} - \frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d} \\
&\quad - \frac{(ib^2(17a^2-2b^2))\operatorname{Subst}(\int 1\,dx, x, -i\coth(c+dx))}{3d} \\
&= a^4 x - \frac{2ab(2a^2-b^2)\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b^2(17a^2-2b^2)\coth(c+dx)}{3d} \\
&\quad - \frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d} - \frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 567 vs. $2(109) = 218$.

Time = 11.82 (sec) , antiderivative size = 567, normalized size of antiderivative = 5.20

$$
\begin{aligned}
\int (a+b\operatorname{csch}(c+dx))^4\,dx &= \frac{a^4(c+dx)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4} \\
&+ \frac{\left(-9a^2b^2\cosh\left(\frac{1}{2}(c+dx)\right)+b^4\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{3d(b+a\sinh(c+dx))^4} \\
&- \frac{ab^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{2d(b+a\sinh(c+dx))^4} \\
&- \frac{b^4\coth\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{24d(b+a\sinh(c+dx))^4} \\
&+ \frac{2(-2a^3b+ab^3)(a+b\operatorname{csch}(c+dx))^4\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4} \\
&- \frac{2(-2a^3b+ab^3)(a+b\operatorname{csch}(c+dx))^4\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4} \\
&- \frac{ab^3(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\sinh^4(c+dx)}{2d(b+a\sinh(c+dx))^4} \\
&+ \frac{(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-9a^2b^2\sinh\left(\frac{1}{2}(c+dx)\right)+b^4\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{3d(b+a\sinh(c+dx))^4} \\
&+ \frac{b^4(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\sinh^4(c+dx)\tanh\left(\frac{1}{2}(c+dx)\right)}{24d(b+a\sinh(c+dx))^4}
\end{aligned}
$$

```
[In] Integrate[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] (a^4*(c + d*x)*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c +
d*x])^4) + ((-9*a^2*b^2*Cosh[(c + d*x)/2] + b^4*Cosh[(c + d*x)/2])*Csch[(c
+ d*x)/2]*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x
])^4) - (a*b^3*Csch[(c + d*x)/2]^2*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)
/(2*d*(b + a*Sinh[c + d*x])^4) - (b^4*Coth[(c + d*x)/2]*Csch[(c + d*x)/2]^2
```

```
*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(24*d*(b + a*Sinh[c + d*x])^4) +
(2*(-2*a^3*b + a*b^3)*(a + b*Csch[c + d*x])^4*Log[Cosh[(c + d*x)/2]]*Sinh[c
 + d*x]^4)/(d*(b + a*Sinh[c + d*x])^4) - (2*(-2*a^3*b + a*b^3)*(a + b*Csch[
c + d*x])^4*Log[Sinh[(c + d*x)/2]]*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c + d*x]
)^4) - (a*b^3*(a + b*Csch[c + d*x])^4*Sech[(c + d*x)/2]^2*Sinh[c + d*x]^4)/
(2*d*(b + a*Sinh[c + d*x])^4) + ((a + b*Csch[c + d*x])^4*Sech[(c + d*x)/2]*
(-9*a^2*b^2*Sinh[(c + d*x)/2] + b^4*Sinh[(c + d*x)/2])*Sinh[c + d*x]^4)/(3*
d*(b + a*Sinh[c + d*x])^4) + (b^4*(a + b*Csch[c + d*x])^4*Sech[(c + d*x)/2]
^2*Sinh[c + d*x]^4*Tanh[(c + d*x)/2])/(24*d*(b + a*Sinh[c + d*x])^4)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^4(dx+c)-8a^3b\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-6a^2b^2\coth(dx+c)+4a\,b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^4\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c}{3}\right.}{d}$
default	$\frac{a^4(dx+c)-8a^3b\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-6a^2b^2\coth(dx+c)+4a\,b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^4\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c}{3}\right.}{d}$
parts	$x\,a^4+\frac{b^4\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{d}+\frac{4a\,b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}-\frac{6a^2b^2\coth(dx+c)}{d}+$
parallelrisch	$\frac{48\left(2a^3b-a\,b^3\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^3b^4-12\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\,b^3+9\left(-8a^2b^2+b^4\right)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{24d}$
risch	$x\,a^4-\frac{4b^2\left(3ab\,\mathrm{e}^{5dx+5c}+9a^2\mathrm{e}^{4dx+4c}-18a^2\mathrm{e}^{2dx+2c}+3\,\mathrm{e}^{2dx+2c}b^2-3\,\mathrm{e}^{dx+c}ab+9a^2-b^2\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3}-\frac{4a^3b\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}+\frac{2a\,b^3}{}$

```
[In] int((a+b*csch(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)-8*a^3*b*arctanh(exp(d*x+c))-6*a^2*b^2*coth(d*x+c)+4*a*b^3*
(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^4*(2/3-1/3*csch(d*x+c)
^2)*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(103) = 206$.

Time = 0.27 (sec) , antiderivative size = 1440, normalized size of antiderivative = 13.21

$$\int (a+b\operatorname{csch}(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 - 12*a*b^3*cosh(
d*x + c)^5 - 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) - 2*a*b^3)*sinh(d*x + c
```

```
)^5 + 12*a*b^3*cosh(d*x + c) - 9*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 - 3*a^4*d*x - 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 + 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 -
10*a*b^3*cosh(d*x + c)^2 - 3*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 - 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x + 24*a^2*b^2 - 4*b^
4 - 18*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((2*a^3*b
 - a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)
^5 + (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^
4 - 3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^4 - 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b
- a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)
^2 + 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b
- a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x +
 c)^5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x +
c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 6*((2*a^3*b - a
*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 +
 (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 -
3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 -
 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 +
 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b - a*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x + c)^
5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(3*a^4*d*x*cosh(d
*x + c)^5 - 10*a*b^3*cosh(d*x + c)^4 + 2*a*b^3 - 6*(a^4*d*x + 4*a^2*b^2)*co
sh(d*x + c)^3 + (3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c
)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
 c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
) - d)
```

Sympy [F]

$$\int (a + b\text{csch}(c + dx))^4 \, dx = \int (a + b \, \text{csch} \, (c + dx))^4 \, dx$$

```
[In] integrate((a+b*csch(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(103) = 206$.

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int (a + b\operatorname{csch}(c + dx))^4\,dx$$

$$= a^4 x + 2ab^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3dx-3c)}\right)}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right)$$
$$+ \frac{4}{3}b^4\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right)$$
$$+ \frac{4a^3 b\log\left(\tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d} + \frac{12a^2b^2}{d(e^{(-2dx-2c)} - 1)}$$

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*a*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 4/3*b^4*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1))) + 4*a^3*b*log(tanh(1/2*d*x + 1/2*c))/d + 12*a^2*
b^2/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int (a + b\operatorname{csch}(c + dx))^4\,dx$$

$$= \frac{3(dx+c)a^4 - 6(2a^3b - ab^3)\log\left(e^{(dx+c)}+1\right) + 6(2a^3b - ab^3)\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{4\left(3ab^3e^{(5dx+5c)} + 9a^2b^2e^{(4d}\right.}{3d}$$

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 - 6*(2*a^3*b - a*b^3)*log(e^(d*x + c) + 1) + 6*(2*a^3*
b - a*b^3)*log(abs(e^(d*x + c) - 1)) - 4*(3*a*b^3*e^(5*d*x + 5*c) + 9*a^2*b
^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) - 3
*a*b^3*e^(d*x + c) + 9*a^2*b^2 - b^4)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.19

$$\int (a+b\operatorname{csch}(c+dx))^4\,dx = a^4\,x - \frac{\frac{12\,a^2\,b^2}{d}+\frac{4\,a\,b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x}-1} - \frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,e^{c+d\,x}}{d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1} - \frac{8\,b^4}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)} + \frac{4\,\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\left(a\,b^3\,\sqrt{-d^2}-2\,a^3\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{4\,a^6\,b^2-4\,a^4\,b^4+a^2\,b^6}}\right)\,\sqrt{4\,a^6\,b^2-4\,a^4\,b^4+a^2\,b^6}}{\sqrt{-d^2}}$$

```
[In] int((a + b/sinh(c + d*x))^4,x)
```

```
[Out] a^4*x - ((12*a^2*b^2)/d + (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1)
- ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1) - (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(-d^2)^(1/2) - 2*a^3*b*(-
d^2)^(1/2)))/(d*(a^2*b^6 - 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 - 4*a^4*
b^4 + 4*a^6*b^2)^(1/2))/(-d^2)^(1/2)
```

3.71 $\int (a + b\text{csch}(c + dx))^3\, dx$

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Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (a + b\text{csch}(c + dx))^3\, dx = a^3 x - \frac{b(6a^2 - b^2)\,\text{arctanh}(\cosh(c + dx))}{2d} - \frac{5ab^2 \coth(c + dx)}{2d} - \frac{b^2 \coth(c + dx)(a + b\text{csch}(c + dx))}{2d}$$

```
[Out] a^3*x-1/2*b*(6*a^2-b^2)*arctanh(cosh(d*x+c))/d-5/2*a*b^2*coth(d*x+c)/d-1/2*
b^2*coth(d*x+c)*(a+b*csch(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3867, 3855, 3852, 8}

$$\int (a + b\text{csch}(c + dx))^3\, dx = a^3 x - \frac{b(6a^2 - b^2)\,\text{arctanh}(\cosh(c + dx))}{2d} - \frac{5ab^2 \coth(c + dx)}{2d} - \frac{b^2 \coth(c + dx)(a + b\text{csch}(c + dx))}{2d}$$

```
[In] Int[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] a^3*x - (b*(6*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/(2*d) - (5*a*b^2*Coth[c +
d*x])/(2*d) - (b^2*Coth[c + d*x]*(a + b*Csch[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^2 \coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d} \\
&\quad + \frac{1}{2}\int \left(2a^3 + b\left(6a^2 - b^2\right)\operatorname{csch}(c+dx) + 5ab^2\operatorname{csch}^2(c+dx)\right)\,dx \\
&= a^3 x - \frac{b^2 \coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d} \\
&\quad + \frac{1}{2}\left(5ab^2\right)\int \operatorname{csch}^2(c+dx)\,dx + \frac{1}{2}\left(b\left(6a^2 - b^2\right)\right)\int \operatorname{csch}(c+dx)\,dx \\
&= a^3 x - \frac{b(6a^2 - b^2)\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{b^2 \coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d} \\
&\quad - \frac{(5iab^2)\operatorname{Subst}(\int 1\,dx, x, -i\coth(c+dx))}{2d} \\
&= a^3 x - \frac{b(6a^2 - b^2)\operatorname{arctanh}(\cosh(c+dx))}{2d} \\
&\quad - \frac{5ab^2 \coth(c+dx)}{2d} - \frac{b^2 \coth(c+dx)(a+b\operatorname{csch}(c+dx))}{2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. $2(75) = 150$.

Time = 5.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int (a + b\operatorname{csch}(c + dx))^3\,dx =$$

$$-\frac{-8a^3c - 8a^3dx + 12ab^2\coth\left(\frac{1}{2}(c+dx)\right) + b^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 24a^2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) - 4b^3\log$$

```
[In] Integrate[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] -1/8*(-8*a^3*c - 8*a^3*d*x + 12*a*b^2*Coth[(c + d*x)/2] + b^3*Csch[(c + d*x
)/2]^2 + 24*a^2*b*Log[Cosh[(c + d*x)/2]] - 4*b^3*Log[Cosh[(c + d*x)/2]] - 2
4*a^2*b*Log[Sinh[(c + d*x)/2]] + 4*b^3*Log[Sinh[(c + d*x)/2]] + b^3*Sech[(c
 + d*x)/2]^2 + 12*a*b^2*Tanh[(c + d*x)/2])/d
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^3(dx+c)-6a^2b\,\operatorname{arctanh}\left(e^{dx+c}\right)-3\coth(dx+c)a\,b^2+b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a^3(dx+c)-6a^2b\,\operatorname{arctanh}\left(e^{dx+c}\right)-3\coth(dx+c)a\,b^2+b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}-\frac{3a\,b^2\coth(dx+c)}{d}+\frac{3a^2b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$
parallelrisch	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+8a^3dx+24\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-12\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a\,b^2-12}{8d}$
risch	$a^3x-\frac{b^2\left(e^{3dx+3c}b+6\,e^{2dx+2c}a+e^{dx+c}b-6a\right)}{d\left(e^{2dx+2c}-1\right)^2}-\frac{3b\ln\left(e^{dx+c}+1\right)a^2}{d}+\frac{b^3\ln\left(e^{dx+c}+1\right)}{2d}+\frac{3b\ln\left(e^{dx+c}-1\right)a^2}{d}-\frac{b^3}{}$

```
[In] int((a+b*csch(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))-3*coth(d*x+c)*a*b^2+b^3*(-1/2*
csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(69) = 138$.

Time $= 0.27$ (sec) , antiderivative size $= 769$, normalized size of antiderivative $= 10.25$

$$\int (a + b\text{csch}(c + dx))^3 \, dx$$

$$= \frac{2\,a^3 dx \cosh(dx+c)^4 + 2\,a^3 dx \sinh(dx+c)^4 - 2\,b^3 \cosh(dx+c)^3 + 2\,a^3 dx - 2\,b^3 \cosh(dx+c) + 2\,(4\,a^3 d$$

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*d*x*cosh(d*x + c)^4 + 2*a^3*d*x*sinh(d*x + c)^4 - 2*b^3*cosh(d*x
 + c)^3 + 2*a^3*d*x - 2*b^3*cosh(d*x + c) + 2*(4*a^3*d*x*cosh(d*x + c) - b^
3)*sinh(d*x + c)^3 + 12*a*b^2 - 4*(a^3*d*x + 3*a*b^2)*cosh(d*x + c)^2 + 2*(
6*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x - 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh
(d*x + c)^2 - ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2
*(6*a^2*b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2
*b - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x + c)*si
nh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2*(6*a^2*
b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*
(4*a^3*d*x*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c)^2 - b^3 - 4*(a^3*d*x + 3*a
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh
(d*x + c) + d)
```

Sympy [F]

$$\int (a + b\text{csch}(c + dx))^3 \, dx = \int (a + b\,\text{csch}\,(c + dx))^3 \, dx$$

```
[In] integrate((a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.81

$$\int (a + b\operatorname{csch}(c + dx))^3 \, dx$$
$$= a^3 x + \frac{1}{2} b^3 \left(\frac{\log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)} \right)$$
$$+ \frac{3\,a^2 b \log\left(\tanh\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d} + \frac{6\,ab^2}{d(e^{(-2\,dx-2\,c)} - 1)}$$

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/2*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 3*a^2*b*log(tanh(1/2*d*x + 1/2*c))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) -
 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int (a + b\operatorname{csch}(c + dx))^3 \, dx$$
$$= \frac{2\,(dx + c)a^3 - (6\,a^2 b - b^3)\log\left(e^{(dx+c)} + 1\right) + (6\,a^2 b - b^3)\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{2\left(b^3 e^{(3\,dx+3\,c)} + 6\,ab^2 e^{(2\,dx+2\,c)} + b^3 e^{(}\right.}{\left(e^{(2\,dx+2\,c)} - 1\right)^2}}{2\,d}$$

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 - (6*a^2*b - b^3)*log(e^(d*x + c) + 1) + (6*a^2*b - b^
3)*log(abs(e^(d*x + c) - 1)) - 2*(b^3*e^(3*d*x + 3*c) + 6*a*b^2*e^(2*d*x +
2*c) + b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.27

$$\int (a + b\operatorname{csch}(c + dx))^3\,dx = a^3\,x - \frac{\frac{6\,a\,b^2}{d} + \frac{b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x} - 1} + \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\left(b^3\,\sqrt{-d^2} - 6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{36\,a^4\,b^2 - 12\,a^2\,b^4 + b^6}}\right)\,\sqrt{36\,a^4\,b^2 - 12\,a^2\,b^4 + b^6}}{\sqrt{-d^2}} - \frac{2\,b^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x} - 2\,e^{2\,c+2\,d\,x} + 1\right)}$$

```
[In] int((a + b/sinh(c + d*x))^3,x)
```

```
[Out] a^3*x - ((6*a*b^2)/d + (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1) + (atan
((exp(d*x)*exp(c)*(b^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(b^6 - 12*a
^2*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 - 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(-d^2)^
(1/2) - (2*b^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)
)
```

3.72 $\int (a + b\text{csch}(c + dx))^2\, dx$

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Optimal result

Integrand size = 12, antiderivative size = 34

$$\int (a + b\text{csch}(c + dx))^2\, dx = a^2 x - \frac{2ab\text{arctanh}(\cosh(c + dx))}{d} - \frac{b^2 \coth(c + dx)}{d}$$

```
[Out] a^2*x-2*a*b*arctanh(cosh(d*x+c))/d-b^2*coth(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3858, 3855, 3852, 8}

$$\int (a + b\text{csch}(c + dx))^2\, dx = a^2 x - \frac{2ab\text{arctanh}(\cosh(c + dx))}{d} - \frac{b^2 \coth(c + dx)}{d}$$

```
[In] Int[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] a^2*x - (2*a*b*ArcTanh[Cosh[c + d*x]])/d - (b^2*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a^2x + (2ab)\int \operatorname{csch}(c+dx)\,dx + b^2\int \operatorname{csch}^2(c+dx)\,dx \\ &= a^2x - \frac{2ab\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{(ib^2)\operatorname{Subst}(\int 1\,dx, x, -i\coth(c+dx))}{d} \\ &= a^2x - \frac{2ab\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b^2\coth(c+dx)}{d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx = -\frac{b^2\coth\left(\frac{1}{2}(c+dx)\right) - 2a\left(ac+adx-2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right) + b^2\tanh\left(\right.}{2d}$$

```
[In] Integrate[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] -1/2*(b^2*Coth[(c + d*x)/2] - 2*a*(a*c + a*d*x - 2*b*Log[Cosh[(c + d*x)/2]]
 + 2*b*Log[Sinh[(c + d*x)/2]]) + b^2*Tanh[(c + d*x)/2])/d
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{a^2(dx+c)-4ab\ \operatorname{arctanh}\left(e^{dx+c}\right)-\coth(dx+c)b^2}{d}$	37
default	$\frac{a^2(dx+c)-4ab\ \operatorname{arctanh}\left(e^{dx+c}\right)-\coth(dx+c)b^2}{d}$	37
parts	$a^2x-\frac{b^2\coth(dx+c)}{d}+\frac{2ab\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	38
parallelrisch	$\frac{2a^2dx+4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b^2-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{2d}$	56
risch	$a^2x-\frac{2b^2}{d\left(e^{2dx+2c}-1\right)}-\frac{2ab\ln\left(e^{dx+c}+1\right)}{d}+\frac{2ab\ln\left(e^{dx+c}-1\right)}{d}$	60

```
[In] int((a+b*csch(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c)-4*a*b*arctanh(exp(d*x+c))-coth(d*x+c)*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(34) = 68$.

Time $= 0.28$ (sec) , antiderivative size $= 222$, normalized size of antiderivative $= 6.53$

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx$$

$$= \frac{a^2dx\cosh(dx+c)^2+2\,a^2dx\cosh(dx+c)\sinh(dx+c)+a^2dx\sinh(dx+c)^2-a^2dx-2\,b^2-2\,(ab\cosh(d$$

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 - a^2*d*x - 2*b^2 - 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) +
 a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(d*cosh
(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F]

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx = \int (a+b\operatorname{csch}(c+dx))^2\,dx$$

```
[In] integrate((a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx = a^2x + \frac{2\,ab\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d} + \frac{2\,b^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*log(tanh(1/2*d*x + 1/2*c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) - 1
))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\begin{aligned}&\int (a+b\operatorname{csch}(c+dx))^2\,dx \\ &= \frac{(dx+c)a^2 - 2\,ab\log\left(e^{(dx+c)}+1\right) + 2\,ab\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\,b^2}{e^{(2\,dx+2\,c)}-1}}{d}\end{aligned}$$

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 - 2*a*b*log(e^(d*x + c) + 1) + 2*a*b*log(abs(e^(d*x + c) - 1
)) - 2*b^2/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx = a^2\,x - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}$$

```
[In] int((a + b/sinh(c + d*x))^2,x)
```

```
[Out] a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) - (4*atan((a*b*exp(d*x)*exp(c)*(
-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1/2)
```

3.73 $\int (a + b\mathbf{csch}(c + dx))\, dx$

Optimal result . . . 392
Rubi [A] (verified) . . . 392
Mathematica [B] (verified) . . . 393
Maple [A] (verified) . . . 393
Fricas [B] (verification not implemented) . . . 393
Sympy [A] (verification not implemented) . . . 394
Maxima [A] (verification not implemented) . . . 394
Giac [A] (verification not implemented) . . . 394
Mupad [B] (verification not implemented) . . . 395

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int (a + b\operatorname{csch}(c + dx))\, dx = ax - \frac{b\operatorname{arctanh}(\cosh(c + dx))}{d}$$

```
[Out] a*x-b*arctanh(cosh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3855}

$$\int (a + b\operatorname{csch}(c + dx))\, dx = ax - \frac{b\operatorname{arctanh}(\cosh(c + dx))}{d}$$

```
[In] Int[a + b*Csch[c + d*x],x]
```

```
[Out] a*x - (b*ArcTanh[Cosh[c + d*x]])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \operatorname{csch}(c + dx)\, dx \\ &= ax - \frac{b\operatorname{arctanh}(\cosh(c + dx))}{d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int (a + b\operatorname{csch}(c + dx))\, dx = ax - \frac{b \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{b \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

```
[In] Integrate[a + b*Csch[c + d*x],x]
[Out] a*x - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result	size
default	$ax + \frac{b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d}$	20
parallelrisch	$ax + \frac{b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d}$	20
parts	$ax + \frac{b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d}$	20
derivativedivides	$\frac{a(dx+c) + b \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d}$	25
risch	$ax + \frac{b \ln\left(e^{dx+c} - 1\right)}{d} - \frac{b \ln\left(e^{dx+c} + 1\right)}{d}$	34

```
[In] int(a+b*csch(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b/d*ln(tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int (a + b\operatorname{csch}(c + dx))\, dx$$
$$= \frac{adx - b \log\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right) + 1\right) + b \log\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right) - 1\right)}{d}$$

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="fricas")
[Out] (a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) + s
inh(d*x + c) - 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int (a + b\text{csch}(c + dx))\, dx = ax + b\left(\begin{cases} \frac{\log\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} & \text{for } d \neq 0 \\ x\,\text{csch}\left(c\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*csch(d*x+c),x)
[Out] a*x + b*Piecewise((log(tanh(c/2 + d*x/2))/d, Ne(d, 0)), (x*csch(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (a + b\text{csch}(c + dx))\, dx = ax + \frac{b\log\left(\tanh\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)\right)}{d}$$

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="maxima")
[Out] a*x + b*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int (a + b\text{csch}(c + dx))\, dx = ax - \frac{b\left(\log\left(e^{(dx+c)} + 1\right) - \log\left(\left|e^{(dx+c)} - 1\right|\right)\right)}{d}$$

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="giac")
[Out] a*x - b*(log(e^(d*x + c) + 1) - log(abs(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int (a + b\operatorname{csch}(c + dx))\,dx = a\,x - \frac{2\operatorname{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

```
[In] int(a + b/sinh(c + d*x),x)
```

```
[Out] a*x - (2*atan((b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2)
)/(-d^2)^(1/2)
```

3.74 $\int \frac{1}{a+b\mathbf{csch}(c+dx)}\,dx$

Optimal result . . . 396
Rubi [A] (verified) . . . 396
Mathematica [A] (verified) . . . 397
Maple [A] (verified) . . . 398
Fricas [B] (verification not implemented) . . . 398
Sympy [F] . . . 399
Maxima [A] (verification not implemented) . . . 399
Giac [A] (verification not implemented) . . . 399
Mupad [B] (verification not implemented) . . . 400

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \frac{x}{a} + \frac{2b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

```
[Out] x/a+2*b*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a/d/(a^2+b^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3868, 2739, 632, 210}

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \frac{2b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} + \frac{x}{a}$$

```
[In] Int[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] x/a + (2*b*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2
+ b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a \sinh(c+dx)}{b}}\,dx}{a} \\ &= \frac{x}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad} \\ &= \frac{x}{a} - \frac{(4i)\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad} \\ &= \frac{x}{a} + \frac{2b\text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{1}{a+b\text{csch}(c+dx)}\,dx = \frac{\frac{c}{d}+x-\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}}{a}$$

```
[In] Integrate[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] (c/d + x - (2*b*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-
a^2 - b^2]*d))/a
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\ \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	82
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\ \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	82
risch	$\frac{x}{a}+\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,da}-\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,da}$	124

```
[In] int(1/(a+b*csch(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(1+tanh(1/2*d*x+1/2*c))-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2*b/a/(a^2
+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(51) = 102.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.44

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)dx+\sqrt{a^2+b^2}b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+a^2+2\,b^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)+2\sqrt{a^2+}}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh(dx}\right)}{(a^3+ab^2)d}$$

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(d*
x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b)*
sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b))/
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x
+ c) + b)*sinh(d*x + c) - a)))/((a^3 + a*b^2)*d)
```

Sympy [F]

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \int \frac{1}{a+b\,\operatorname{csch}\left(c+dx\right)}\,dx$$

```
[In] integrate(1/(a+b*csch(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*csch(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = -\frac{b\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ad} + \frac{dx+c}{ad}$$

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c) - b + sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + (d*x + c)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = -\frac{\frac{b\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a} - \frac{dx+c}{a}}{d}$$

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*x + c)
+ 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) - (d*x + c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.24

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \frac{x}{a} - \frac{b\,\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} - \frac{2\,b\,\left(a-b\,e^{c+d\,x}\right)}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}} + \frac{b\,\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} + \frac{2\,b\,\left(a-b\,e^{c+d\,x}\right)}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

```
[In] int(1/(a + b/sinh(c + d*x)),x)
```

```
[Out] x/a - (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a - b*exp(c + d*x)))/(a^2*(a^2
+ b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2)) + (b*log((2*b*exp(c + d*x))/a^2 + (
2*b*(a - b*exp(c + d*x)))/(a^2*(a^2 + b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2))
```

3.75 $\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = \frac{x}{a^2} + \frac{2b(2a^2+b^2)\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}$$

```
[Out] x/a^2+2*b*(2*a^2+b^2)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^
2/(a^2+b^2)^(3/2)/d-b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4004, 3916, 2739, 632, 210}

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = \frac{2b(2a^2+b^2)\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}} - \frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\operatorname{csch}(c+dx))} + \frac{x}{a^2}$$

```
[In] Int[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] x/a^2 + (2*b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2
]])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b
*Csch[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2 \coth(c+dx)}{a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))} - \frac{\int \frac{-a^2-b^2+ab\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{a\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2 \coth(c+dx)}{a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))} - \frac{(b(2a^2+b^2)) \int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$
\begin{aligned}
&= \frac{x}{a^2} - \frac{b^2 \coth(c+dx)}{a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))} - \frac{\left(2a^2+b^2\right) \int \frac{1}{1+\frac{a \sinh(c+dx)}{b}}\, dx}{a^2\left(a^2+b^2\right)} \\
&= \frac{x}{a^2} - \frac{b^2 \coth(c+dx)}{a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))} \\
&\quad + \frac{\left(2i\left(2a^2+b^2\right)\right) \operatorname{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\, dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^2\left(a^2+b^2\right) d} \\
&= \frac{x}{a^2} - \frac{b^2 \coth(c+dx)}{a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))} \\
&\quad - \frac{\left(4i\left(2a^2+b^2\right)\right) \operatorname{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\, dx, x, -\frac{2ia}{b}+2\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^2\left(a^2+b^2\right) d} \\
&= \frac{x}{a^2} + \frac{2b\left(2a^2+b^2\right) \operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2} d} - \frac{b^2 \coth(c+dx)}{a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$
\begin{aligned}
&\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\, dx \\
&= \frac{\operatorname{csch}(c+dx)\left(-\frac{ab^2 \coth(c+dx)}{a^2+b^2} + (c+dx)(a+b\operatorname{csch}(c+dx)) + \frac{2b\left(2a^2+b^2\right) \arctan\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)(a+b\operatorname{csch}(c+dx)}{\left(-a^2-b^2\right)^{3/2}}\right.}{a^2 d(a+b\operatorname{csch}(c+dx))^2}
\end{aligned}
$$

```
[In] Integrate[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] (Csch[c + d*x]*(-((a*b^2*Coth[c + d*x])/(a^2 + b^2)) + (c + d*x)*(a + b*Csc
h[c + d*x]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2
 - b^2]]*(a + b*Csch[c + d*x]))/(-a^2 - b^2)^(3/2))*(b + a*Sinh[c + d*x]))/
(a^2*d*(a + b*Csch[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{2b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}{2}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right)}{a^2}}{d}$
default	$\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{2b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}{2}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right)}{a^2}}{d}$
risch	$\frac{x}{a^2}-\frac{2b^2\left(-\mathrm{e}^{dx+c}b+a\right)}{d\,a^2(a^2+b^2)\left(\mathrm{e}^{2dx+2c}a+2\,\mathrm{e}^{dx+c}b-a\right)}+\frac{2b\ln\left(\mathrm{e}^{dx+c}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}+\frac{b^3\ln\left(\mathrm{e}^{dx+c}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b}{a\left(a\right.}\right.}{(a^2+b^2)^{\frac{3}{2}}d}$

```
[In] int(1/(a+b*csch(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(1+tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-2/a^2*
b*((1/2*a^2/(a^2+b^2)*tanh(1/2*d*x+1/2*c)+1/2*b*a/(a^2+b^2))/(-1/2*tanh(1/2
*d*x+1/2*c)^2*b+tanh(1/2*d*x+1/2*c)*a+1/2*b)-2*(2*a^2+b^2)/(2*a^2+2*b^2)/(a
^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(98) = 196.

Time = 0.28 (sec) , antiderivative size = 645, normalized size of antiderivative = 6.39

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = -\frac{2\,a^3b^2+2\,ab^4-(a^5+2\,a^3b^2+ab^4)dx\cosh(dx+c)^2-(a^5+2\,a^3b^2+ab^4)dx\sinh(dx+c)^2+(a^5+2\,a^3b$$

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^
5 + 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d*x
+ (2*a^3*b + a*b^3 - (2*a^3*b + a*b^3)*cosh(d*x + c)^2 - (2*a^3*b + a*b^3)*
sinh(d*x + c)^2 - 2*(2*a^2*b^2 + b^4)*cosh(d*x + c) - 2*(2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*co
sh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2
*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x +
```

```
c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cos
h(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(a^2*b^3 + b^5
 + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) - 2*(a^2*b^3 + b^5 + (a^5 +
 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*sinh
(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^5*b^
2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x
 + c) - (a^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*co
sh(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx = \int \frac{1}{\left(a+b\,\text{csch}\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx = &-\frac{(2\,a^2b+b^3)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}d}\\ &-\frac{2\left(b^3e^{(-dx-c)}+ab^2\right)}{(a^5+a^3b^2+2\left(a^4b+a^2b^3\right)e^{(-dx-c)}-(a^5+a^3b^2)e^{(-2\,dx-2\,c)})d}\\ &+\frac{dx+c}{a^2d}\end{aligned}$$

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2*b + b^3)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c)
 - b + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(b^3*e^(-d
*x - c) + a*b^2)/((a^5 + a^3*b^2 + 2*(a^4*b + a^2*b^3)*e^(-d*x - c) - (a^5
+ a^3*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx$$
$$= -\frac{\frac{(2\,a^2b+b^3)\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}} - \frac{2\left(b^3e^{(dx+c)}-ab^2\right)}{(a^4+a^2b^2)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}-a\right)} - \frac{dx+c}{a^2}}{d}$$

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((2*a^2*b + b^3)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2))
 - 2*(b^3*e^(d*x + c) - a*b^2)/((a^4 + a^2*b^2)*(a*e^(2*d*x + 2*c) + 2*b*e^
(d*x + c) - a)) - (d*x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{\frac{2\,b^2}{d\,(a^3+a\,b^2)} - \frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{a\,d\,(a^3+a\,b^2)}}{2\,b\,\mathrm{e}^{c+d\,x} - a + a\,\mathrm{e}^{2\,c+2\,d\,x}}$$
$$- \frac{b\,\ln\left(\frac{2\,\mathrm{e}^{c+d\,x}\,(2\,a^2\,b+b^3)}{a^3\,(a^2+b^2)} - \frac{2\,b\,(2\,a^2+b^2)\,(a-b\,\mathrm{e}^{c+d\,x})}{a^3\,(a^2+b^2)^{3/2}}\right)\,(2\,a^2+b^2)}{a^2\,d\,(a^2+b^2)^{3/2}}$$
$$+ \frac{b\,\ln\left(\frac{2\,\mathrm{e}^{c+d\,x}\,(2\,a^2\,b+b^3)}{a^3\,(a^2+b^2)} + \frac{2\,b\,(2\,a^2+b^2)\,(a-b\,\mathrm{e}^{c+d\,x})}{a^3\,(a^2+b^2)^{3/2}}\right)\,(2\,a^2+b^2)}{a^2\,d\,(a^2+b^2)^{3/2}}$$

```
[In] int(1/(a + b/sinh(c + d*x))^2,x)
```

```
[Out] x/a^2 - ((2*b^2)/(d*(a*b^2 + a^3)) - (2*b^3*exp(c + d*x))/(a*d*(a*b^2 + a^3
)))/(2*b*exp(c + d*x) - a + a*exp(2*c + 2*d*x)) - (b*log((2*exp(c + d*x)*(2
*a^2*b + b^3))/(a^3*(a^2 + b^2)) - (2*b*(2*a^2 + b^2)*(a - b*exp(c + d*x)))
/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b^2)^(3/2)) + (b*log
((2*exp(c + d*x)*(2*a^2*b + b^3))/(a^3*(a^2 + b^2)) + (2*b*(2*a^2 + b^2)*(a
 - b*exp(c + d*x)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b
^2)^(3/2))
```

3.76 $\int \frac{1}{(a+b\mathbf{csch}(c+dx))^3}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 163

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{b(6a^4+5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{5/2}d} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))}$$

```
[Out] x/a^3+b*(6*a^4+5*a^2*b^2+2*b^4)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)
^(1/2))/a^3/(a^2+b^2)^(5/2)/d-1/2*b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d
*x+c))^2-1/2*b^2*(5*a^2+2*b^2)*coth(d*x+c)/a^2/(a^2+b^2)^2/d/(a+b*csch(d*x+
c))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3870, 4145, 4004, 3916, 2739, 632, 210}

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2d\left(a^2+b^2\right)^2(a+b\operatorname{csch}(c+dx))} - \frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\operatorname{csch}(c+dx))^2} + \frac{b(6a^4+5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{5/2}}$$

```
[In] Int[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] x/a^3 + (b*(6*a^4 + 5*a^2*b^2 + 2*b^4)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(5/2)*d) - (b^2*Coth[c + d*x])/(2*a*(a^2 +
 b^2)*d*(a + b*Csch[c + d*x])^2) - (b^2*(5*a^2 + 2*b^2)*Coth[c + d*x])/(2*a
^2*(a^2 + b^2)^2*d*(a + b*Csch[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b^2 \coth(c+dx)}{2a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))^2} - \frac{\int \frac{-2\left(a^2+b^2\right)+2ab\operatorname{csch}(c+dx)-b^2\operatorname{csch}^2(c+dx)}{(a+b\operatorname{csch}(c+dx))^2}\,dx}{2a\left(a^2+b^2\right)} \\
&= -\frac{b^2 \coth(c+dx)}{2a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))} \\
&\quad + \frac{\int \frac{2\left(a^2+b^2\right)^2-ab\left(4a^2+b^2\right)\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{2a^2\left(a^2+b^2\right)^2} \\
&= \frac{x}{a^3} - \frac{b^2 \coth(c+dx)}{2a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))} \\
&\quad - \frac{\left(b\left(6a^4+5a^2b^2+2b^4\right)\right)\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}\,dx}{2a^3\left(a^2+b^2\right)^2} \\
&= \frac{x}{a^3} - \frac{b^2 \coth(c+dx)}{2a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))^2} \\
&\quad - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))} - \frac{\left(6a^4+5a^2b^2+2b^4\right)\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{2a^3\left(a^2+b^2\right)^2} \\
&= \frac{x}{a^3} - \frac{b^2 \coth(c+dx)}{2a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))} \\
&\quad + \frac{\left(i\left(6a^4+5a^2b^2+2b^4\right)\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^3\left(a^2+b^2\right)^2 d} \\
&= \frac{x}{a^3} - \frac{b^2 \coth(c+dx)}{2a\left(a^2+b^2\right) d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))} \\
&\quad - \frac{\left(2i\left(6a^4+5a^2b^2+2b^4\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, -\frac{2ia}{b}+2\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^3\left(a^2+b^2\right)^2 d}
\end{aligned}
$$

$$= \frac{x}{a^3} + \frac{b(6a^4+5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{5/2}d} - \frac{b^2\coth(c+dx)}{2a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\left(a^2+b^2\right)^2 d(a+b\operatorname{csch}(c+dx))}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx$$

$$= \frac{\operatorname{csch}^2(c+dx)(b+a\sinh(c+dx))\left(\frac{ab^3\coth(c+dx)}{a^2+b^2} - \frac{3ab^2\left(2a^2+b^2\right)\coth(c+dx)(b+a\sinh(c+dx))}{\left(a^2+b^2\right)^2} + 2(c+dx)\operatorname{csch}(c+d\right.}{2a^3d(a+b\operatorname{csch}(c+d$$

```
[In] Integrate[(a + b*Csch[c + d*x])^(-3),x]

[Out] (Csch[c + d*x]^2*(b + a*Sinh[c + d*x])*((a*b^3*Coth[c + d*x])/(a^2 + b^2) -
 (3*a*b^2*(2*a^2 + b^2)*Coth[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2)^2
+ 2*(c + d*x)*Csch[c + d*x]*(b + a*Sinh[c + d*x])^2 - (2*b*(6*a^4 + 5*a^2*b
^2 + 2*b^4)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]*Csch[c + d*x
]*(b + a*Sinh[c + d*x])^2)/(-a^2 - b^2)^(5/2)))/(2*a^3*d*(a + b*Csch[c + d*
x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(154) = 308.

Time = 0.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.01

method	result
derivativedivides	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b\left(-\frac{b\,a^2\left(4a^2+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{4a\left(10a^4-a^2b^2-2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^4+16a^2b^2+8b^4}+\frac{4a^2}{\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{d}$
default	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b\left(-\frac{b\,a^2\left(4a^2+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{4a\left(10a^4-a^2b^2-2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^4+16a^2b^2+8b^4}+\frac{4a^2}{\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{d}$
risch	$\frac{x}{a^3}-\frac{b^2\left(-7a^3b\,\mathrm{e}^{3dx+3c}-4a\,b^3\mathrm{e}^{3dx+3c}+6a^4\mathrm{e}^{2dx+2c}-9a^2b^2\mathrm{e}^{2dx+2c}-6b^4\mathrm{e}^{2dx+2c}+17a^3b\,\mathrm{e}^{dx+c}+8\,\mathrm{e}^{dx+c}a\,b^3-6a^4-3a^2\right.}{a^3(a^2+b^2)^2d\left(\mathrm{e}^{2dx+2c}a+2\,\mathrm{e}^{dx+c}b-a\right)^2}$

```
[In] int(1/(a+b*csch(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(1+tanh(1/2*d*x+1/2*c))-2/a^3
*b*(4*(-1/8*b*a^2*(4*a^2+b^2)/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3+1/8
*a*(10*a^4-a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2+1/8*a^2
*b*(16*a^2+7*b^2)/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)+1/8*a*b^2*(5*a^2+
2*b^2)/(a^4+2*a^2*b^2+b^4))/(-tanh(1/2*d*x+1/2*c)^2*b+2*tanh(1/2*d*x+1/2*c)
*a+b)^2-2*(6*a^4+5*a^2*b^2+2*b^4)/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*a
rctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(156) = 312$.

Time $= 0.32$ (sec) , antiderivative size $= 2094$, normalized size of antiderivative $= 12.85$

$$\int \frac{1}{(a+b\mathrm{csch}(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^6*b^2 + 18*a^4*b^4 + 6*a^2*b^6 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 +
 a^2*b^6)*d*x*cosh(d*x + c)^4 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d
*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5
*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 + 11*a^3*b^5
+ 4*a*b^7 + 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c) + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*sinh(d*x + c)^3 + 2*(a^8 + 3*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6
- 6*b^8 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x +
 c)^2 - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8 - 6*(a^8 + 3*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 -
```

```
 5*a^2*b^6 - 2*b^8)*d*x - 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b +
3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a^6
*b + 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c
)^4 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 + 5*
a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6 + (
6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^6*
b - 7*a^4*b^3 - 8*a^2*b^5 - 4*b^7)*cosh(d*x + c)^2 - 2*(6*a^6*b - 7*a^4*b^3
 - 8*a^2*b^5 - 4*b^7 - 3*(6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2
- 6*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(6
*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) - 4*(6*a^5*b^2 + 5*a^3*b^4 +
2*a*b^6 - (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 - 3*(6*a^5*b^2
+ 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b - 7*a^4*b^3 - 8*a^2*b^5 -
 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*cosh(d*x + c
)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh
(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sin
h(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
 + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(17*a^5*b^3 + 25*a^3*b^5
 + 8*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c) -
 2*(17*a^5*b^3 + 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^
2*b^6)*d*x*cosh(d*x + c)^3 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x
- 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8
 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*si
nh(d*x + c))/((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c)^4 +
(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b + 3*
a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c)^3 - 2*(a^11 + a^9*b^2 - 3*a^
7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 + 3*a^9*b^2 + 3
*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c) + (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4
*b^7)*d)*sinh(d*x + c)^3 - 4*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c) + 2*(3*(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c
)^2 + 6*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c) - (a^11
+ a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 +
 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d + 4*((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^
5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c)^2 - (a^11 + a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cos
h(d*x + c) - (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{csch}\,(c+dx))^3}\,dx$$

```
[In] integrate(1/(a+b*csch(d*x+c))**3,x)
[Out] Integral((a + b*csch(c + d*x))**(-3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(156) = 312$.

Time $= 0.31$ (sec) , antiderivative size $= 373$, normalized size of antiderivative $= 2.29$

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = -\frac{(6\,a^4b+5\,a^2b^3+2\,b^5)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{2\,(a^7+2\,a^5b^2+a^3b^4)\sqrt{a^2+b^2}d} - \frac{6\,a^4b^2+3\,a^2b^4+(17\,a^3b^3+8\,ab^5)e^{(-dx-c)}-3\,(2\,a^4b^2-3\,a^2b^4-2\,b^6)e^{(-2\,dx-2}}{(a^9+2\,a^7b^2+a^5b^4+4\,(a^8b+2\,a^6b^3+a^4b^5)e^{(-dx-c)}-2\,(a^9-3\,a^5b^4-2\,a^3b^6)e^{(-2\,dx-2\,c)}-4\,(a^8b+2\,a^6} + \frac{dx+c}{a^3d}$$

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(6*a^4*b + 5*a^2*b^3 + 2*b^5)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2
))/(a*e^(-d*x - c) - b + sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b^2 + a^3*b^4)*sqr
t(a^2 + b^2)*d) - (6*a^4*b^2 + 3*a^2*b^4 + (17*a^3*b^3 + 8*a*b^5)*e^(-d*x -
 c) - 3*(2*a^4*b^2 - 3*a^2*b^4 - 2*b^6)*e^(-2*d*x - 2*c) - (7*a^3*b^3 + 4*a
*b^5)*e^(-3*d*x - 3*c))/((a^9 + 2*a^7*b^2 + a^5*b^4 + 4*(a^8*b + 2*a^6*b^3
 + a^4*b^5)*e^(-d*x - c) - 2*(a^9 - 3*a^5*b^4 - 2*a^3*b^6)*e^(-2*d*x - 2*c)
 - 4*(a^8*b + 2*a^6*b^3 + a^4*b^5)*e^(-3*d*x - 3*c) + (a^9 + 2*a^7*b^2 + a^5
*b^4)*e^(-4*d*x - 4*c))*d) + (d*x + c)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time $= 0.32$ (sec) , antiderivative size $= 293$, normalized size of antiderivative $= 1.80$

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = -\frac{\frac{(6\,a^4b+5\,a^2b^3+2\,b^5)\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{(a^7+2\,a^5b^2+a^3b^4)\sqrt{a^2+b^2}} - \frac{2\,(7\,a^3b^3e^{(3\,dx+3\,c)}+4\,ab^5e^{(3\,dx+3\,c)}-6\,a^4b^2e^{(2\,dx+2\,c)}+9\,a^2b^4e^{(2\,dx+2\,c)}+6\,b^6e^{(2\,c}}{(a^7+2\,a^5b^2+a^3b^4)(ae^{(2\,dx+2\,c)}+2\,be^{(}}}{2\,d}$$

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((6*a^4*b + 5*a^2*b^3 + 2*b^5)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b
^2 + a^3*b^4)*sqrt(a^2 + b^2)) - 2*(7*a^3*b^3*e^(3*d*x + 3*c) + 4*a*b^5*e^(
3*d*x + 3*c) - 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x + 2*c) + 6*b^
6*e^(2*d*x + 2*c) - 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x + c) + 6*a^4*b^
2 + 3*a^2*b^4)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c) + 2*b*e^(d*x
 + c) - a)^2) - 2*(d*x + c)/a^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{b}{\sinh(c+dx)}\right)^3}\,dx$$

```
[In] int(1/(a + b/sinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b/sinh(c + d*x))^3, x)
```

3.77 $\int \frac{\sinh^3(x)}{a+b\mathbf{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{2b^4\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a}$$

```
[Out] 1/2*b*(a^2-2*b^2)*x/a^4-1/3*(2*a^2-3*b^2)*cosh(x)/a^3-1/2*b*cosh(x)*sinh(x)
/a^2+1/3*cosh(x)*sinh(x)^2/a-2*b^4*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2
))/a^4/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\sinh(x)\cosh(x)}{2a^2} - \frac{2b^4\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{bx(a^2-2b^2)}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} + \frac{\sinh^2(x)\cosh(x)}{3a}$$

```
[In] Int[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (b*(a^2 - 2*b^2)*x)/(2*a^4) - (2*b^4*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - ((2*a^2 - 3*b^2)*Cosh[x])/(3*a^3) - (b*Cosh[x
]*Sinh[x])/(2*a^2) + (Cosh[x]*Sinh[x]^2)/(3*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
```

```
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{i\int \frac{\left(-3ib-2ia\operatorname{csch}(x)-2ib\operatorname{csch}^2(x)\right)\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{\int \frac{\left(-2(2a^2-3b^2)-ab\operatorname{csch}(x)+3b^2\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{6a^2}$$

$$= -\frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{i\int \frac{-3ib(a^2-2b^2)-3iab^2\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{6a^3}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^4\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{a^4}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^3\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^4}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^4}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{(4b^3)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a^4}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{2b^4\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{(-9a^3+12ab^2)\cosh(x)+a^3\cosh(3x)+3b\left(2a^2x-4b^2x+\frac{8b^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-a^2\sinh(2x)\right)}{12a^4}$$

```
[In] Integrate[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] + a^3*Cosh[3*x] + 3*b*(2*a^2*x - 4*b^2*x + (8*
b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - a^2*Sinh
[2*x]))/(12*a^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(93) = 186.

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.85

method	result
default	$-\frac{2b^4\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{a-b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{a^2+ab-2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{b\left(a^2-2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^4}$
risch	$\frac{bx}{2a^2}-\frac{b^3x}{a^4}+\frac{\mathrm{e}^{3x}}{24a}-\frac{b\mathrm{e}^{2x}}{8a^2}-\frac{3\mathrm{e}^{x}}{8a}+\frac{\mathrm{e}^{x}b^2}{2a^3}-\frac{3\mathrm{e}^{-x}}{8a}+\frac{\mathrm{e}^{-x}b^2}{2a^3}+\frac{b\mathrm{e}^{-2x}}{8a^2}+\frac{\mathrm{e}^{-3x}}{24a}+\frac{b^4\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^4}-\frac{b^4\ln($

```
[In] int(sinh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^4/a^4/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/
2))+1/3/a/(tanh(1/2*x)+1)^3-1/2*(a-b)/a^2/(tanh(1/2*x)+1)^2-1/2*(a^2+a*b-2*
b^2)/a^3/(tanh(1/2*x)+1)+1/2*b*(a^2-2*b^2)/a^4*ln(tanh(1/2*x)+1)-1/3/a/(tan
h(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(-a^2+a*b+2*b^2)/a^3/(tan
h(1/2*x)-1)-1/2*b*(a^2-2*b^2)/a^4*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(95) = 190$.

Time $= 0.29$ (sec) , antiderivative size $= 807$, normalized size of antiderivative $= 7.54$

$$\int \frac{\sinh^3(x)}{a + b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^5 + a^3*b^2)*cosh(x)^6 + (a^5 + a^3*b^2)*sinh(x)^6 - 3*(a^4*b + a^
2*b^3)*cosh(x)^5 - 3*(a^4*b + a^2*b^3 - 2*(a^5 + a^3*b^2)*cosh(x))*sinh(x)^
5 + a^5 + a^3*b^2 + 12*(a^4*b - a^2*b^3 - 2*b^5)*x*cosh(x)^3 - 3*(3*a^5 - a
^3*b^2 - 4*a*b^4)*cosh(x)^4 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b
^2)*cosh(x)^2 + 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 + a^3*b
^2)*cosh(x)^3 - 15*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b - a^2*b^3 - 2*b^5
)*x - 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 - a^3*b^2
 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b^2)*co
sh(x)^4 + 10*(a^4*b + a^2*b^3)*cosh(x)^3 - 12*(a^4*b - a^2*b^3 - 2*b^5)*x*c
osh(x) + 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 24*(b^4*cosh(
x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2
*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sin
h(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sin
h(x) - a)) + 3*(a^4*b + a^2*b^3)*cosh(x) + 3*(2*(a^5 + a^3*b^2)*cosh(x)^5 +
 a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 12*(a^4*b - a^2*b^3 - 2*
b^5)*x*cosh(x)^2 - 4*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 - a^3
*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x)^3 + 3*(a^6 + a^4
*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 + a^4*
b^2)*sinh(x)^3)
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\sinh^3(x)}{a + b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(sinh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b^4 \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^4} - \frac{\left(3\,abe^{(-x)} - a^2 + 3\,(3\,a^2-4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} + \frac{3\,abe^{(-2\,x)} + a^2e^{(-3\,x)} - 3\,(3\,a^2-4\,b^2)e^{(-x)}}{24\,a^3} + \frac{(a^2b-2\,b^3)x}{2\,a^4}$$

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^4) - 1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*
x))*e^(3*x)/a^3 + 1/24*(3*a*b*e^(-2*x) + a^2*e^(-3*x) - 3*(3*a^2 - 4*b^2)*e
^(-x))/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b^4 \log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} + \frac{a^2e^{(3\,x)} - 3\,abe^{(2\,x)} - 9\,a^2e^x + 12\,b^2e^x}{24\,a^3} + \frac{(a^2b-2\,b^3)x}{2\,a^4} + \frac{\left(3\,a^2be^x + a^3 - 3\,(3\,a^3-4\,ab^2)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/24*(a^2*e^(3*x) - 3*a*b*e^(2*x) - 9*a
^2*e^x + 12*b^2*e^x)/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4 + 1/24*(3*a^2*b*e^x +
a^3 - 3*(3*a^3 - 4*a*b^2)*e^(2*x))*e^(-3*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-3x}}{24a} + \frac{\mathrm{e}^{3x}}{24a} + \frac{x\,(a^2 b - 2b^3)}{2a^4} - \frac{\mathrm{e}^{x}\,(3a^2 - 4b^2)}{8a^3} + \frac{b\,\mathrm{e}^{-2x}}{8a^2} - \frac{b\,\mathrm{e}^{2x}}{8a^2} - \frac{\mathrm{e}^{-x}\,(3a^2 - 4b^2)}{8a^3} - \frac{b^4\,\ln\left(-\frac{2b^4\,\mathrm{e}^{x}}{a^5} - \frac{2b^4\,(a - b\,\mathrm{e}^{x})}{a^5\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{b^4\,\ln\left(\frac{2b^4\,(a - b\,\mathrm{e}^{x})}{a^5\sqrt{a^2+b^2}} - \frac{2b^4\,\mathrm{e}^{x}}{a^5}\right)}{a^4\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) + exp(3*x)/(24*a) + (x*(a^2*b - 2*b^3))/(2*a^4) - (exp(x)*
(3*a^2 - 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (
exp(-x)*(3*a^2 - 4*b^2))/(8*a^3) - (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^4*(
a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2))))/(a^4*(a^2 + b^2)^(1/2)) + (b^4*log
((2*b^4*(a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2)) - (2*b^4*exp(x))/a^5))/(a^4
*(a^2 + b^2)^(1/2))
```

3.78 $\int \frac{\sinh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

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Rubi [A] (verified) . . . 422
Mathematica [A] (verified) . . . 424
Maple [B] (verified) . . . 425
Fricas [B] (verification not implemented) . . . 425
Sympy [F] . . . 426
Maxima [A] (verification not implemented) . . . 426
Giac [A] (verification not implemented) . . . 426
Mupad [B] (verification not implemented) . . . 427

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(a^2-2b^2)\,x}{2a^3} + \frac{2b^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] -1/2*(a^2-2*b^2)*x/a^3-b*cosh(x)/a^2+1/2*cosh(x)*sinh(x)/a+2*b^3*arctanh((a
-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\cosh(x)}{a^2} + \frac{2b^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{x(a^2-2b^2)}{2a^3} + \frac{\sinh(x)\cosh(x)}{2a}$$

```
[In] Int[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -1/2*((a^2 - 2*b^2)*x)/a^3 + (2*b^3*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^
2]])/(a^3*Sqrt[a^2 + b^2]) - (b*Cosh[x])/a^2 + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(x)\sinh(x)}{2a} - \frac{i\int \frac{\left(-2ib-ia\operatorname{csch}(x)-ib\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{2a} \\
&= -\frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{-a^2+2b^2-ab\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{2a^2} \\
&= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{a^3} \\
&= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^3} \\
&= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3} \\
&= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} \\
&\quad + \frac{(4b^2)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a^3} \\
&= -\frac{(a^2-2b^2)\,x}{2a^3} + \frac{2b^3\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{-2a^2x+4b^2x-\frac{8b^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\cosh(x) + a^2\sinh(2x)}{4a^3}$$

```
[In] Integrate[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(70) = 140$.

Time $= 0.32$ (sec) , antiderivative size $= 152$, normalized size of antiderivative $= 1.90$

method	result
default	$-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{-a+2b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{\left(-a^2+2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^3} + \frac{2b^3\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}$
risch	$-\frac{x}{2a} + \frac{x\,b^2}{a^3} + \frac{\mathrm{e}^{2x}}{8a} - \frac{b\,\mathrm{e}^{x}}{2a^2} - \frac{b\,\mathrm{e}^{-x}}{2a^2} - \frac{\mathrm{e}^{-2x}}{8a} + \frac{b^3\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{b^3\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^3}$

```
[In] int(sinh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2-1/2*(-a+2*b)/a^2/(tanh(1/2*x)+1)+1/2/a^3*(-a^2+2*b
^2)*ln(tanh(1/2*x)+1)+2*b^3/a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*
x)+2*a)/(a^2+b^2)^(1/2))+1/2/a/(tanh(1/2*x)-1)^2+1/2/a^3*(a^2-2*b^2)*ln(tan
h(1/2*x)-1)-1/2*(-a-2*b)/a^2/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(72) = 144$.

Time $= 0.27$ (sec) , antiderivative size $= 456$, normalized size of antiderivative $= 5.70$

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{\left(a^4+a^2b^2\right)\cosh(x)^4 + \left(a^4+a^2b^2\right)\sinh(x)^4 - a^4 - a^2b^2 - 4\left(a^4-a^2b^2-2\,b^4\right)x\cosh(x)^2 - 4\left(a^3b+ab^3\right)($$

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^4 + a^2*b^2)*cosh(x)^4 + (a^4 + a^2*b^2)*sinh(x)^4 - a^4 - a^2*b^2
- 4*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 + a^2*b^2)*c
osh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/
(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x)^3
+ 2*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^5 + a^3*b^2)*cosh(x)*sinh(x) + (a^5 +
a^3*b^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\sinh^2(x)}{a+b\,\text{csch}\,(x)}\,dx$$

```
[In] integrate(sinh(x)**2/(a+b*csch(x)),x)
[Out] Integral(sinh(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{b^3 \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3} - \frac{\left(4\,be^{(-x)}-a\right)e^{(2\,x)}}{8\,a^2} - \frac{4\,be^{(-x)}+ae^{(-2\,x)}}{8\,a^2} - \frac{(a^2-2\,b^2)x}{2\,a^3}$$

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
[Out] -b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3) - 1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x)
 + a*e^(-2*x))/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{b^3 \log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{ae^{(2\,x)}-4\,be^{x}}{8\,a^2} - \frac{(a^2-2\,b^2)x}{2\,a^3} - \frac{(4\,abe^{x}+a^2)e^{(-2\,x)}}{8\,a^3}$$

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="giac")
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a
^2 - 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x + a^2)*e^(-2*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.96

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{e^{2x}}{8a} - \frac{e^{-2x}}{8a} - \frac{b\,e^{x}}{2a^2} - \frac{b\,e^{-x}}{2a^2} - \frac{x\,(a^2-2b^2)}{2a^3} - \frac{b^3\,\ln\left(\frac{2b^3\,e^{x}}{a^4} - \frac{2b^3\,(a-b\,e^{x})}{a^4\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{b^3\,\ln\left(\frac{2b^3\,e^{x}}{a^4} + \frac{2b^3\,(a-b\,e^{x})}{a^4\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2)
 - (x*(a^2 - 2*b^2))/(2*a^3) - (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a - b*
exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2 + b^2)^(1/2)) + (b^3*log((2*b^
3*exp(x))/a^4 + (2*b^3*(a - b*exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2
+ b^2)^(1/2))
```

3.79 $\int \frac{\sinh(x)}{a+b\mathbf{csch}(x)}\,dx$

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Mathematica [A] (verified) . . . 430
Maple [A] (verified) . . . 430
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Sympy [F] . . . 431
Maxima [A] (verification not implemented) . . . 431
Giac [A] (verification not implemented) . . . 432
Mupad [B] (verification not implemented) . . . 432

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{bx}{a^2} - \frac{2b^2\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} + \frac{\cosh(x)}{a}$$

```
[Out] -b*x/a^2+cosh(x)/a-2*b^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3938, 12, 3868, 2739, 632, 212}

$$\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{2b^2\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} - \frac{bx}{a^2} + \frac{\cosh(x)}{a}$$

```
[In] Int[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*x)/a^2) - (2*b^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2*Sqrt[a^2 + b^2]) + Cosh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(x)}{a} - \frac{\int \frac{b}{a+b\operatorname{csch}(x)}\,dx}{a} \\
&= \frac{\cosh(x)}{a} - \frac{b\int \frac{1}{a+b\operatorname{csch}(x)}\,dx}{a} \\
&= -\frac{bx}{a^2} + \frac{\cosh(x)}{a} + \frac{b\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^2} \\
&= -\frac{bx}{a^2} + \frac{\cosh(x)}{a} + \frac{(2b)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}
\end{aligned}$$

$$= -\frac{bx}{a^2} + \frac{\cosh(x)}{a} - \frac{(4b)\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b} - 2\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

$$= -\frac{bx}{a^2} - \frac{2b^2 \text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} + \frac{\cosh(x)}{a}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sinh(x)}{a+b\text{csch}(x)}\,dx = \frac{b\left(-x + \frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right) + a\cosh(x)}{a^2}$$

```
[In] Integrate[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] (b*(-x + (2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + a*Cosh[x])/a^2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^2} - \frac{2b^2\,\text{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} - \frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^2}$	92
risch	$-\frac{bx}{a^2} + \frac{e^x}{2a} + \frac{e^{-x}}{2a} + \frac{b^2\ln\left(e^x + \frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^2} - \frac{b^2\ln\left(e^x + \frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^2}$	132

```
[In] int(sinh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(tanh(1/2*x)+1)-1/a^2*b*ln(tanh(1/2*x)+1)-2*b^2/a^2/(a^2+b^2)^(1/2)*arc
tanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))-1/a/(tanh(1/2*x)-1)+1/a^2*
b*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(53) = 106$.

Time $= 0.26$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 4.18$

$$\int \frac{\sinh(x)}{a + b\operatorname{csch}(x)}\,dx$$

$$= \frac{a^3 + ab^2 - 2\left(a^2b + b^3\right)x\cosh\left(x\right) + \left(a^3 + ab^2\right)\cosh\left(x\right)^2 + \left(a^3 + ab^2\right)\sinh\left(x\right)^2 + 2\left(b^2\cosh\left(x\right) + b^2\sinh\left(x\right)\right.}{2}$$

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3 + a*b^2 - 2*(a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*cosh(x)^2 + (a
^3 + a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 + b^2)*log((
a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x
)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 2*((a^2
*b + b^3)*x - (a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + a^2*b^2)*cosh(x) + (a
^4 + a^2*b^2)*sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\sinh\left(x\right)}{a + b\,\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(sinh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.27$ (sec) , antiderivative size $= 84$, normalized size of antiderivative $= 1.47$

$$\int \frac{\sinh(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{b^2\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2 + b^2}a^2} - \frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b^2 \log\left(\frac{\left|2\,a e^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a e^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} - \frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.26

$$\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-x}}{2\,a} + \frac{\mathrm{e}^x}{2\,a} - \frac{b\,x}{a^2} - \frac{b^2\,\ln\left(-\frac{2\,b^2\,\mathrm{e}^x}{a^3} - \frac{2\,b^2\,(a-b\,\mathrm{e}^x)}{a^3\,\sqrt{a^2+b^2}}\right)}{a^2\,\sqrt{a^2+b^2}} + \frac{b^2\,\ln\left(\frac{2\,b^2\,(a-b\,\mathrm{e}^x)}{a^3\,\sqrt{a^2+b^2}} - \frac{2\,b^2\,\mathrm{e}^x}{a^3}\right)}{a^2\,\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)/(a + b/sinh(x)),x)
```

```
[Out] exp(-x)/(2*a) + exp(x)/(2*a) - (b*x)/a^2 - (b^2*log(- (2*b^2*exp(x))/a^3 -
(2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2))))/(a^2*(a^2 + b^2)^(1/2)) +
(b^2*log((2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2)) - (2*b^2*exp(x))/a^
3))/(a^2*(a^2 + b^2)^(1/2))
```

3.80 $\int \frac{\mathbf{csch}(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 433
Rubi [A] (verified) . . . 433
Mathematica [A] (verified) . . . 434
Maple [A] (verified) . . . 435
Fricas [B] (verification not implemented) . . . 435
Sympy [F] . . . 435
Maxima [A] (verification not implemented) . . . 436
Giac [A] (verification not implemented) . . . 436
Mupad [B] (verification not implemented) . . . 436

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3916, 2739, 632, 212}

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] Int[Csch[x]/(a + b*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b} \\ &= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b} \\ &= -\frac{4\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b} \\ &= -\frac{2\text{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}\,dx = \frac{2\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

```
[In] Integrate[Csch[x]/(a + b*Csch[x]),x]
[Out] (2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	35
risch	$\frac{\ln\left(e^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}}-\frac{\ln\left(e^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}}$	97

```
[In] int(csch(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
[Out] -2/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)-2\sqrt{a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="fricas")
[Out] log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*c
osh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))/sqrt
(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)/(a+b*csch(x)),x)
[Out] Integral(csch(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/sqrt
(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log\left(\left|\frac{2\,ae^{x}+2\,b-2\,\sqrt{a^2+b^2}}{2\,ae^{x}+2\,b+2\,\sqrt{a^2+b^2}}\right|\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 +
 b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2\operatorname{atan}\left(\frac{b}{\sqrt{-a^2-b^2}}+\frac{a\,e^{x}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

```
[In] int(1/(sinh(x)*(a + b/sinh(x))),x)
```

```
[Out] (2*atan(b/(- a^2 - b^2)^(1/2) + (a*exp(x))/(- a^2 - b^2)^(1/2)))/(- a^2 - b
^2)^(1/2)
```

3.81 $\int \frac{\mathbf{csch}^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 437
Rubi [A] (verified) . . . 437
Mathematica [A] (verified) . . . 439
Maple [A] (verified) . . . 439
Fricas [B] (verification not implemented) . . . 439
Sympy [F] . . . 440
Maxima [A] (verification not implemented) . . . 440
Giac [A] (verification not implemented) . . . 440
Mupad [B] (verification not implemented) . . . 441

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{2a\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] -arctanh(cosh(x))/b+2*a*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3874, 3855, 3916, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2a\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}} - \frac{\operatorname{arctanh}(\cosh(x))}{b}$$

```
[In] Int[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (2*a*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b
*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \operatorname{csch}(x)\,dx}{b} - \frac{a\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b} \\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{b} - \frac{a\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^2} \\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{b} - \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{(4a)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{2a\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{-\frac{2a\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)}{b}$$

```
[In] Integrate[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] ((-2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - Log[C
osh[x/2]] + Log[Sinh[x/2]])/b
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2a\ \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$	49
risch	$\frac{\ln(e^x-1)}{b} + \frac{a\ln\left(e^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b} - \frac{a\ln\left(e^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b} - \frac{\ln(e^x+1)}{b}$	124

```
[In] int(csch(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))+1
/b*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2ab\cosh(x)+a^2+2b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2b\cosh(x)+2\left(a\cosh(x)+b\right)\sinh(x)-a}\right) - (a^2}{a^2b+b^3}$$

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a
*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)
```

```
*sinh(x) - a)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^2*b + b^3)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{\log\left(e^{(-x)}+1\right)}{b} + \frac{\log\left(e^{(-x)}-1\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b) - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a\log\left(\frac{\left|2ae^{x}+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^{x}+2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b} - \frac{\log\left(e^{x}+1\right)}{b} + \frac{\log\left(\left|e^{x}-1\right|\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) - log(e^x + 1)/b + log(abs(e^x - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\ln\left(32\,b-32\,b\,\mathrm{e}^x\right)}{b} - \frac{\ln\left(32\,b+32\,b\,\mathrm{e}^x\right)}{b}$$
$$+\frac{a\,\ln\left(128\,b^5\,\mathrm{e}^x-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^x+160\,a^2\,b^3\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{a^2+b^2}+3\right.}{a^2\,b+b^3}$$
$$-\frac{a\,\ln\left(64\,a\,b^4+64\,a^3\,b^2-128\,b^5\,\mathrm{e}^x-128\,b^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}-32\,a^4\,b\,\mathrm{e}^x-160\,a^2\,b^3\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{a^2+b^2}+3\right.}{a^2\,b+b^3}$$

```
[In] int(1/(sinh(x)^2*(a + b/sinh(x))),x)
```

```
[Out] log(32*b - 32*b*exp(x))/b - log(32*b + 32*b*exp(x))/b + (a*log(128*b^5*exp(
x) - 64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*ex
p(x) + 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^
2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^2*b +
 b^3) - (a*log(64*a*b^4 + 64*a^3*b^2 - 128*b^5*exp(x) - 128*b^4*exp(x)*(a^2
 + b^2)^(1/2) - 32*a^4*b*exp(x) - 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)
^(1/2) + 32*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/(a^2*b + b^3)
```

3.82 $\int \frac{\text{csch}^3(x)}{a+b\text{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\text{csch}^3(x)}{a+b\text{csch}(x)}\,dx = \frac{a\text{arctanh}(\cosh(x))}{b^2} - \frac{2a^2\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\coth(x)}{b}$$

```
[Out] a*arctanh(cosh(x))/b^2-coth(x)/b-2*a^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^
(1/2))/b^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3875, 3874, 3855, 3916, 2739, 632, 212}

$$\int \frac{\text{csch}^3(x)}{a+b\text{csch}(x)}\,dx = -\frac{2a^2\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{a\text{arctanh}(\cosh(x))}{b^2} - \frac{\coth(x)}{b}$$

```
[In] Int[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (2*a^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(b^2*Sqrt[a^2 + b^2]) - Coth[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(x)}{b} - \frac{a\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx}{b} \\ &= -\frac{\coth(x)}{b} - \frac{a\int \operatorname{csch}(x)\,dx}{b^2} + \frac{a^2\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^2} \\ &= \frac{a\operatorname{arctanh}(\cosh(x))}{b^2} - \frac{\coth(x)}{b} + \frac{a^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^3}\end{aligned}$$

$$= \frac{a\operatorname{arctanh}(\cosh(x))}{b^2} - \frac{\coth(x)}{b} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3}$$

$$= \frac{a\operatorname{arctanh}(\cosh(x))}{b^2} - \frac{\coth(x)}{b} - \frac{(4a^2)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b} - 2\tanh\left(\frac{x}{2}\right)\right)}{b^3}$$

$$= \frac{a\operatorname{arctanh}(\cosh(x))}{b^2} - \frac{2a^2\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\coth(x)}{b}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\left(-b\cosh(x) + a\left(\frac{2a\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \log\left(\cosh\left(\frac{x}{2}\right)\right) - \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)\right)}{2b^2}$$

```
[In] Integrate[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x/2]*Sech[x/2]*(-(b*Cosh[x]) + a*((2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt[
-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[Cosh[x/2]] - Log[Sinh[x/2]])*Sinh[x]))
/(2*b^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2b} - \frac{2a^2\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{1}{2b\tanh\left(\frac{x}{2}\right)} - \frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}$	73
risch	$-\frac{2}{b(e^{2x}-1)} + \frac{a^2\ln\left(e^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b^2} - \frac{a^2\ln\left(e^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b^2} + \frac{a\ln(e^x+1)}{b^2} - \frac{a\ln(e^x-1)}{b^2}$	143

```
[In] int(csch(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*tanh(1/2*x)-2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+
2*a)/(a^2+b^2)^(1/2))-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(55) = 110$.

Time $= 0.27$ (sec) , antiderivative size $= 345$, normalized size of antiderivative $= 5.85$

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

$$= \frac{2\,a^2 b + 2\,b^3 - \left(a^2\cosh(x)^2 + 2\,a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 - a^2\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2}{a\,c}\right.}{}$$

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^2*b + 2*b^3 - (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 -
 a^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x)
+ a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
 b)*sinh(x) - a)) + (a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2
)*cosh(x)*sinh(x) - (a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2)*cosh(x)*sinh(x) - (
a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 + b^4 - (a^2*b
^2 + b^4)*cosh(x)^2 - 2*(a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^2*b^2 + b^4)*s
inh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.27$ (sec) , antiderivative size $= 100$, normalized size of antiderivative $= 1.69$

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{a^2\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2} + \frac{a\log\left(e^{(-x)}+1\right)}{b^2} - \frac{a\log\left(e^{(-x)}-1\right)}{b^2} + \frac{2}{be^{(-2x)}-b}$$

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^2) + a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(
b*e^(-2*x) - b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a^2 \log\left(\frac{\left|2\,ae^x+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} + \frac{a\log\left(e^x+1\right)}{b^2} - \frac{a\log\left(\left|e^x-1\right|\right)}{b^2} - \frac{2}{b(e^{(2\,x)}-1)}$$

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1)
)/b^2 - 2/(b*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.95

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2}{b-b\,\mathrm{e}^{2\,x}} - \frac{a\,\ln\left(32\,\mathrm{e}^x-32\right)}{b^2} + \frac{a\,\ln\left(32\,\mathrm{e}^x+32\right)}{b^2} + \frac{a^2\,\ln\left(32\,a^4\,\mathrm{e}^x-64\,a\,b^3-64\,a^3\,b-32\,a^3\,\sqrt{a^2+b^2}+128\,b^4\,\mathrm{e}^x+128\,b^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x-64\,a\,b\right.}{a^2\,b^2+b^4} - \frac{a^2\,\ln\left(32\,a^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+32\,a^4\,\mathrm{e}^x+128\,b^4\,\mathrm{e}^x-128\,b^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x+64\,a\,b\right.}{a^2\,b^2+b^4}$$

```
[In] int(1/(sinh(x)^3*(a + b/sinh(x))),x)
```

```
[Out] 2/(b - b*exp(2*x)) - (a*log(32*exp(x) - 32))/b^2 + (a*log(32*exp(x) + 32))/
b^2 + (a^2*log(32*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*a^3*(a^2 + b^2)^(1/
2) + 128*b^4*exp(x) + 128*b^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x)
 - 64*a*b^2*(a^2 + b^2)^(1/2) + 96*a^2*b*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b
^2)^(1/2))/(b^4 + a^2*b^2) - (a^2*log(32*a^3*(a^2 + b^2)^(1/2) - 64*a*b^3 -
 64*a^3*b + 32*a^4*exp(x) + 128*b^4*exp(x) - 128*b^3*exp(x)*(a^2 + b^2)^(1/
2) + 160*a^2*b^2*exp(x) + 64*a*b^2*(a^2 + b^2)^(1/2) - 96*a^2*b*exp(x)*(a^2
 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^4 + a^2*b^2)
```

3.83 $\int \frac{\mathbf{csch}^4(x)}{a+b\mathbf{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{2a^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[Out] -1/2*(2*a^2-b^2)*arctanh(cosh(x))/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b+2
*a^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3936, 4167, 4083, 3855, 3916, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{2a^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[In] Int[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/2*((2*a^2 - b^2)*ArcTanh[Cosh[x]])/b^3 + (2*a^3*ArcTanh[(a - b*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]) + (a*Coth[x])/b^2 - (Coth[x]*Csch
[x])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\int \frac{\operatorname{csch}(x)\left(a+b\operatorname{csch}(x)+2a\operatorname{csch}^2(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b}$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{i\int \frac{\operatorname{csch}(x)\left(iab-i\left(2a^2-b^2\right)\operatorname{csch}(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b^2}$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^3} + \frac{\left(2a^2-b^2\right)\int \operatorname{csch}(x)\,dx}{2b^3}$$

$$= -\frac{\left(2a^2-b^2\right)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^4}$$

$$= -\frac{\left(2a^2-b^2\right)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{(2a^3)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^4}$$

$$= -\frac{\left(2a^2-b^2\right)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{(4a^3)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^4}$$

$$= -\frac{\left(2a^2-b^2\right)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{2a^3\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.78

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\frac{16a^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{x}{2}\right) + b^2\operatorname{csch}^2\left(\frac{x}{2}\right) + 8a^2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 4b^2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 8a^2\log\left(\sinh\left(\frac{x}{2}\right)\right) + 4b^2\log\left(\sinh\left(\frac{x}{2}\right)\right) + b^2\operatorname{sech}^2\left(\frac{x}{2}\right) - 4ab\tanh\left(\frac{x}{2}\right)}{8b^3}$$

```
[In] Integrate[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/8*((16*a^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- 4*a*b*Coth[x/2] + b^2*Csch[x/2]^2 + 8*a^2*Log[Cosh[x/2]] - 4*b^2*Log[Cosh
[x/2]] - 8*a^2*Log[Sinh[x/2]] + 4*b^2*Log[Sinh[x/2]] + b^2*Sech[x/2]^2 - 4*
a*b*Tanh[x/2])/b^3
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)^2 b}{2}+2a\tanh\left(\frac{x}{2}\right) + \frac{2a^3\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} - \frac{1}{8b\tanh\left(\frac{x}{2}\right)^2} + \frac{(4a^2-2b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^3} + \frac{a}{2b^2\tanh\left(\frac{x}{2}\right)}$
risch	$\frac{-e^{3x}b+2a\,e^{2x}-e^{x}b-2a}{(e^{2x}-1)^2b^2} + \frac{a^3\ln\left(e^{x}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b^3} - \frac{a^3\ln\left(e^{x}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b^3} - \frac{\ln(e^{x}+1)a^2}{b^3} + \frac{\ln(e^{x}+1)}{2b} + \frac{\ln(e^{x}-1}{b^3}$

```
[In] int(csch(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/2*tanh(1/2*x)^2*b+2*a*tanh(1/2*x))+2/b^3*a^3/(a^2+b^2)^(1/2)*arc
tanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))-1/8/b/tanh(1/2*x)^2+1/4/b^
3*(4*a^2-2*b^2)*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(75) = 150$.

Time = 0.32 (sec) , antiderivative size = 947, normalized size of antiderivative = 11.41

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x)^3 + 2*(a^2*b^2 + b^4)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^2*b^2
+ b^4)*cosh(x))*sinh(x)^2 - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^
3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 +
 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 2*(a^2*b^2 + b^4)*c
osh(x) + ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cos
h(x)*sinh(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4
- 2*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4
 + a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)
^3 - (2*a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) -
 ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cosh(x)*sin
h(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4 - 2*(2*a
^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4 + a^2*b
^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)^3 - (2*
a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*
b^2 + b^4 + 3*(a^2*b^2 + b^4)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^2*b^3 + b^5 + (a^2*b^3 + b^5)*cosh(x)^4 + 4*(a^2*b^3 + b^5)*cosh(x)*s
inh(x)^3 + (a^2*b^3 + b^5)*sinh(x)^4 - 2*(a^2*b^3 + b^5)*cosh(x)^2 - 2*(a^2
*b^3 + b^5 - 3*(a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 + b^5)*co
sh(x)^3 - (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)**4/(a+b*csch(x)),x)
[Out] Integral(csch(x)**4/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(75) = 150$.

Time $= 0.26$ (sec) , antiderivative size $= 158$, normalized size of antiderivative $= 1.90$

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^3} + \frac{be^{(-x)}+2\,ae^{(-2\,x)}+be^{(-3\,x)}-2\,a}{2\,b^2e^{(-2\,x)}-b^2e^{(-4\,x)}-b^2} - \frac{(2\,a^2-b^2)\log\left(e^{(-x)}+1\right)}{2\,b^3} + \frac{(2\,a^2-b^2)\log\left(e^{(-x)}-1\right)}{2\,b^3}$$

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^3) + (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^
2*e^(-2*x) - b^2*e^(-4*x) - b^2) - 1/2*(2*a^2 - b^2)*log(e^(-x) + 1)/b^3 +
1/2*(2*a^2 - b^2)*log(e^(-x) - 1)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^3 \log\left(\frac{\left|2\,ae^x+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^3} - \frac{(2\,a^2-b^2)\log\left(e^x+1\right)}{2\,b^3} + \frac{(2\,a^2-b^2)\log\left(|e^x-1|\right)}{2\,b^3} - \frac{be^{(3\,x)}-2\,ae^{(2\,x)}+be^x+2\,a}{b^2\left(e^{(2\,x)}-1\right)^2}$$

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 1/2*(2*a^2 - b^2)*log(e^x + 1)/b^3 + 1
/2*(2*a^2 - b^2)*log(abs(e^x - 1))/b^3 - (b*e^(3*x) - 2*a*e^(2*x) + b*e^x +
 2*a)/(b^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.43

$$\begin{aligned}
&\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx \\
&= \frac{\mathrm{e}^x}{b-b\,\mathrm{e}^{2\,x}} - \frac{2\,\mathrm{e}^x}{b-2\,b\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{4\,x}} - \frac{\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2-24\,a^4\,\mathrm{e}^x-4\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,b} \\
&\quad + \frac{\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2+24\,a^4\,\mathrm{e}^x+4\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,b} + \frac{2\,a}{b^2\,\mathrm{e}^{2\,x}-b^2} \\
&\quad + \frac{a^2\,\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2-24\,a^4\,\mathrm{e}^x-4\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{b^3} \\
&\quad - \frac{a^2\,\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2+24\,a^4\,\mathrm{e}^x+4\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{b^3} \\
&\quad - \frac{a^3\,\ln\left(16\,a\,b^5-24\,a^5\,\sqrt{a^2+b^2}-48\,a^5\,b-32\,a^3\,b^3+24\,a^6\,\mathrm{e}^x-32\,b^6\,\mathrm{e}^x-40\,a^3\,b^2\,\sqrt{a^2+b^2}-32\,b^5\,\mathrm{e}^x\,\sqrt{a^2+b^2}+\right.}{a^2\,b^3} \\
&\quad + \frac{a^3\,\ln\left(24\,a^5\,\sqrt{a^2+b^2}+16\,a\,b^5-48\,a^5\,b-32\,a^3\,b^3+24\,a^6\,\mathrm{e}^x-32\,b^6\,\mathrm{e}^x+40\,a^3\,b^2\,\sqrt{a^2+b^2}+32\,b^5\,\mathrm{e}^x\,\sqrt{a^2+b^2}+\right.}{a^2\,b^3}
\end{aligned}$$

```
[In] int(1/(sinh(x)^4*(a + b/sinh(x))),x)
```

```
[Out] exp(x)/(b - b*exp(2*x)) - (2*exp(x))/(b - 2*b*exp(2*x) + b*exp(4*x)) - log(
24*a^4 + 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp
(x))/(2*b) + log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp(x)
 - 20*a^2*b^2*exp(x))/(2*b) + (2*a)/(b^2*exp(2*x) - b^2) + (a^2*log(24*a^4
+ 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp(x)))/b
^3 - (a^2*log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp(x) -
20*a^2*b^2*exp(x)))/b^3 - (a^3*log(16*a*b^5 - 24*a^5*(a^2 + b^2)^(1/2) - 48
*a^5*b - 32*a^3*b^3 + 24*a^6*exp(x) - 32*b^6*exp(x) - 40*a^3*b^2*(a^2 + b^2
)^(1/2) - 32*b^5*exp(x)*(a^2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2
*exp(x) + 16*a*b^4*(a^2 + b^2)^(1/2) + 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) +
72*a^2*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3) + (
a^3*log(24*a^5*(a^2 + b^2)^(1/2) + 16*a*b^5 - 48*a^5*b - 32*a^3*b^3 + 24*a^
6*exp(x) - 32*b^6*exp(x) + 40*a^3*b^2*(a^2 + b^2)^(1/2) + 32*b^5*exp(x)*(a^
2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2*exp(x) - 16*a*b^4*(a^2 + b
^2)^(1/2) - 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^2*b^3*exp(x)*(a^2 + b^
2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3)
```

3.84 $\int \frac{\cosh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 454
Rubi [A] (verified) . . . 454
Mathematica [A] (verified) . . . 456
Maple [B] (verified) . . . 456
Fricas [A] (verification not implemented) . . . 456
Sympy [F] . . . 457
Maxima [A] (verification not implemented) . . . 457
Giac [A] (verification not implemented) . . . 457
Mupad [B] (verification not implemented) . . . 458

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{ix}{8} + \frac{\cosh^3(x)}{3} + \frac{1}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x)$$

```
[Out] 1/8*I*x+1/3*cosh(x)^3+1/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3957, 2918, 2645, 30, 2648, 2715, 8}

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{ix}{8} + \frac{\cosh^3(x)}{3} - \frac{1}{4}i\sinh(x)\cosh^3(x) + \frac{1}{8}i\sinh(x)\cosh(x)$$

```
[In] Int[Cosh[x]^4/(I + Csch[x]),x]
[Out] (I/8)*x + Cosh[x]^3/3 + (I/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i \int \frac{\cosh^4(x)\sinh(x)}{i - \sinh(x)}\,dx \\
&= -\left(i \int \cosh^2(x)\sinh^2(x)\,dx\right) + \int \cosh^2(x)\sinh(x)\,dx \\
&= -\frac{1}{4} i \cosh^3(x)\sinh(x) + \frac{1}{4} i \int \cosh^2(x)\,dx + \text{Subst}\left(\int x^2\,dx, x, \cosh(x)\right) \\
&= \frac{\cosh^3(x)}{3} + \frac{1}{8} i \cosh(x)\sinh(x) - \frac{1}{4} i \cosh^3(x)\sinh(x) + \frac{1}{8} i \int 1\,dx
\end{aligned}
$$

$$= \frac{ix}{8} + \frac{\cosh^3(x)}{3} + \frac{1}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^4(x)}{i + \text{csch}(x)}\,dx = \frac{ix}{8} + \frac{\cosh(x)}{4} + \frac{1}{12}\cosh(3x) - \frac{1}{32}i\sinh(4x)$$

```
[In] Integrate[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]/4 + Cosh[3*x]/12 - (I/32)*Sinh[4*x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(27) = 54.

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.05

$$-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8} - \frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} + \frac{-\frac{1}{3}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} + \frac{-\frac{1}{2}-\frac{3i}{8}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{-\frac{1}{2}-\frac{i}{8}}{\tanh\left(\frac{x}{2}\right)-1} + \frac{i}{4\left(\tanh\left(\frac{x}{2}\right)+\right.}$$

```
[In] int(cosh(x)^4/(I+csch(x)),x)
```

```
[Out] -1/8*I*ln(tanh(1/2*x)-1)-1/4*I/(tanh(1/2*x)-1)^4-(1/3+1/2*I)/(tanh(1/2*x)-1
)^3-(1/2+3/8*I)/(tanh(1/2*x)-1)^2-(1/2+1/8*I)/(tanh(1/2*x)-1)+1/4*I/(tanh(1
/2*x)+1)^4+1/8*I*ln(tanh(1/2*x)+1)+(1/3-1/2*I)/(tanh(1/2*x)+1)^3+(1/2-1/8*I
)/(tanh(1/2*x)+1)+(-1/2+3/8*I)/(tanh(1/2*x)+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^4(x)}{i + \text{csch}(x)}\,dx = \frac{1}{192}\left(24i\,x e^{(4x)} - 3i\,e^{(8x)} + 8\,e^{(7x)} + 24\,e^{(5x)} + 24\,e^{(3x)} + 8\,e^{x} + 3i\right)e^{(-4x)}$$

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(24*I*x*e^(4*x) - 3*I*e^(8*x) + 8*e^(7*x) + 24*e^(5*x) + 24*e^(3*x) +
 8*e^x + 3*I)*e^(-4*x)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\cosh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(cosh(x)**4/(I+csch(x)),x)
[Out] Integral(cosh(x)**4/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{192}\left(8\,e^{(-x)} + 24\,e^{(-3\,x)} - 3i\right)e^{(4\,x)} + \frac{1}{8}i\,x + \frac{1}{8}\,e^{(-x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{64}i\,e^{(-4\,x)}$$

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="maxima")
[Out] 1/192*(8*e^(-x) + 24*e^(-3*x) - 3*I)*e^(4*x) + 1/8*I*x + 1/8*e^(-x) + 1/24*
e^(-3*x) + 1/64*I*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{192}\left(24\,e^{(3\,x)} + 8\,e^{x} + 3i\right)e^{(-4\,x)} + \frac{1}{8}i\,x - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{8}\,e^{x}$$

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="giac")
[Out] 1/192*(24*e^(3*x) + 8*e^x + 3*I)*e^(-4*x) + 1/8*I*x - 1/64*I*e^(4*x) + 1/24
*e^(3*x) + 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} + \frac{\mathrm{e}^{x}}{8} + \frac{x\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{-4\,x}\,1\mathrm{i}}{64} - \frac{\mathrm{e}^{4\,x}\,1\mathrm{i}}{64}$$

```
[In] int(cosh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] (x*1i)/8 + exp(-x)/8 + exp(-3*x)/24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (ex
p(4*x)*1i)/64 + exp(x)/8
```

3.85 $\int \frac{\cosh^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 459
Rubi [A] (verified) . . . 459
Mathematica [A] (verified) . . . 460
Maple [A] (verified) . . . 461
Fricas [B] (verification not implemented) . . . 461
Sympy [F] . . . 461
Maxima [B] (verification not implemented) . . . 462
Giac [B] (verification not implemented) . . . 462
Mupad [B] (verification not implemented) . . . 462

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\sinh^2(x)}{2} - \frac{1}{3}i\sinh^3(x)$$

```
[Out] 1/2*sinh(x)^2-1/3*I*sinh(x)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3957, 2914, 2644, 30}

$$\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\sinh^2(x)}{2} - \frac{1}{3}i\sinh^3(x)$$

```
[In] Int[Cosh[x]^3/(I + Csch[x]),x]
```

```
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\cosh^3(x)\sinh(x)}{i - \sinh(x)}\,dx \\
&= -\left(i \int \cosh(x)\sinh^2(x)\,dx\right) + \int \cosh(x)\sinh(x)\,dx \\
&= -\text{Subst}(\int x\,dx, x, i\sinh(x)) + \text{Subst}\left(\int x^2\,dx, x, i\sinh(x)\right) \\
&= \frac{\sinh^2(x)}{2} - \frac{1}{3}i\sinh^3(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{i + \text{csch}(x)}\,dx = \frac{\sinh^2(x)}{2} - \frac{1}{3}i\sinh^3(x)$$

```
[In] Integrate[Cosh[x]^3/(I + Csch[x]),x]
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Maple [A] (verified)

Time = 222.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{i}{3\,\mathrm{csch}(x)^3}+\frac{1}{2\,\mathrm{csch}(x)^2}$	15
default	$-\frac{i}{3\,\mathrm{csch}(x)^3}+\frac{1}{2\,\mathrm{csch}(x)^2}$	15
risch	$-\frac{ie^{3x}}{24}+\frac{e^{2x}}{8}+\frac{ie^{x}}{8}-\frac{ie^{-x}}{8}+\frac{e^{-2x}}{8}+\frac{ie^{-3x}}{24}$	40

```
[In] int(cosh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -1/3*I/csch(x)^3+1/2/csch(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^3(x)}{i+\mathrm{csch}(x)}\,dx = \frac{1}{24}\left(-i\,e^{(6\,x)}+3\,e^{(5\,x)}+3i\,e^{(4\,x)}-3i\,e^{(2\,x)}+3\,e^{x}+i\right)e^{(-3\,x)}$$

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="fricas")
[Out] 1/24*(-I*e^(6*x) + 3*e^(5*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + 3*e^x + I)*e^(-3
*x)
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{i+\mathrm{csch}(x)}\,dx = \int \frac{\cosh^3(x)}{\mathrm{csch}(x)+i}\,dx$$

```
[In] integrate(cosh(x)**3/(I+csch(x)),x)
[Out] Integral(cosh(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{1}{24}\left(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - i\right)e^{(3\,x)} - \frac{1}{8}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)} + \frac{1}{24}i\,e^{(-3\,x)}$$

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 3*I*e^(-2*x) - I)*e^(3*x) - 1/8*I*e^(-x) + 1/8*e^(-2*x) +
1/24*I*e^(-3*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\cosh^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{1}{24}\left(3i\,e^{(2\,x)} - 3\,e^{x} - i\right)e^{(-3\,x)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{8}\,e^{(2\,x)} + \frac{1}{8}i\,e^{x}$$

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(3*I*e^(2*x) - 3*e^x - I)*e^(-3*x) - 1/24*I*e^(3*x) + 1/8*e^(2*x) + 1
/8*I*e^x
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-2\,x}}{8} - \frac{\mathrm{e}^{-x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{2\,x}}{8} + \frac{\mathrm{e}^{-3\,x}\,1\mathrm{i}}{24} - \frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{24} + \frac{\mathrm{e}^{x}\,1\mathrm{i}}{8}$$

```
[In] int(cosh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] exp(-2*x)/8 - (exp(-x)*1i)/8 + exp(2*x)/8 + (exp(-3*x)*1i)/24 - (exp(3*x)*1
i)/24 + (exp(x)*1i)/8
```

3.86 $\int \frac{\cosh^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 463
Rubi [A] (verified) . . . 463
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Mupad [B] (verification not implemented) . . . 466

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{ix}{2} + \cosh(x) - \frac{1}{2}i\cosh(x)\sinh(x)$$

```
[Out] 1/2*I*x+cosh(x)-1/2*I*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2918, 2718, 2715, 8}

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{ix}{2} + \cosh(x) - \frac{1}{2}i\sinh(x)\cosh(x)$$

```
[In] Int[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/2)*Cosh[x]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i \int \frac{\cosh^2(x)\sinh(x)}{i - \sinh(x)}\,dx \\ &= -\left(i \int \sinh^2(x)\,dx\right) + \int \sinh(x)\,dx \\ &= \cosh(x) - \frac{1}{2} i \cosh(x)\sinh(x) + \frac{1}{2} i \int 1\,dx \\ &= \frac{ix}{2} + \cosh(x) - \frac{1}{2} i \cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.04$ (sec) , antiderivative size $= 20$, normalized size of antiderivative $= 1.00$

$$\int \frac{\cosh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{ix}{2} + \cosh(x) - \frac{1}{4} i \sinh(2x)$$

```
[In] Integrate[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/4)*Sinh[2*x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 9.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

method	result	size
risch	$\frac{ix}{2} - \frac{ie^{2x}}{8} + \frac{e^{x}}{2} + \frac{e^{-x}}{2} + \frac{ie^{-2x}}{8}$	30
default	$-\frac{i\ln(\tanh(\frac{x}{2})-1)}{2} - \frac{i}{2(\tanh(\frac{x}{2})-1)^2} + \frac{-1-\frac{i}{2}}{\tanh(\frac{x}{2})-1} + \frac{i}{2(\tanh(\frac{x}{2})+1)^2} + \frac{i\ln(\tanh(\frac{x}{2})+1)}{2} + \frac{1-\frac{i}{2}}{\tanh(\frac{x}{2})+1}$	68

```
[In] int(cosh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*x-1/8*I*exp(x)^2+1/2*exp(x)+1/2/exp(x)+1/8*I/exp(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{8}\left(4i\,x e^{(2\,x)} - i\,e^{(4\,x)} + 4\,e^{(3\,x)} + 4\,e^{x} + i\right)e^{(-2\,x)}$$

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="fricas")
[Out] 1/8*(4*I*x*e^(2*x) - I*e^(4*x) + 4*e^(3*x) + 4*e^x + I)*e^(-2*x)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\cosh^2(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(cosh(x)**2/(I+csch(x)),x)
[Out] Integral(cosh(x)**2/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{1}{8}\left(4\,e^{(-x)} - i\right)e^{(2\,x)} + \frac{1}{2}i\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)}$$

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="maxima")
[Out] 1/8*(4*e^(-x) - I)*e^(2*x) + 1/2*I*x + 1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{1}{8}\left(4\,e^{x} + i\right)e^{(-2\,x)} + \frac{1}{2}i\,x - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] 1/8*(4*e^x + I)*e^(-2*x) + 1/2*I*x - 1/8*I*e^(2*x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{x}}{2} + \frac{x\,1\mathrm{i}}{2} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}$$

```
[In] int(cosh(x)^2/(1/sinh(x) + 1i),x)
[Out] (x*1i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2
```

3.87 $\int \frac{\cosh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 467
Rubi [A] (verified) . . . 467
Mathematica [A] (verified) . . . 468
Maple [A] (verified) . . . 468
Fricas [B] (verification not implemented) . . . 469
Sympy [F] . . . 469
Maxima [A] (verification not implemented) . . . 469
Giac [A] (verification not implemented) . . . 470
Mupad [B] (verification not implemented) . . . 470

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx = \log(i-\sinh(x)) - i\sinh(x)$$

```
[Out] ln(I-sinh(x))-I*sinh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3957, 2912, 45}

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx = \log(-\sinh(x)+i) - i\sinh(x)$$

```
[In] Int[Cosh[x]/(I + Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
```

```
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i \int \frac{\cosh(x)\sinh(x)}{i-\sinh(x)}\,dx \\ &= i\text{Subst}\left(\int \frac{x}{i+x}\,dx, x, -\sinh(x)\right) \\ &= i\text{Subst}\left(\int \left(1-\frac{i}{i+x}\right)\,dx, x, -\sinh(x)\right) \\ &= \log(i-\sinh(x)) - i\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.01$ (sec) , antiderivative size $= 16$, normalized size of antiderivative $= 1.00$

$$\int \frac{\cosh(x)}{i+\text{csch}(x)}\,dx = \log(i-\sinh(x)) - i\sinh(x)$$

```
[In] Integrate[Cosh[x]/(I + Csch[x]),x]
[Out] Log[I - Sinh[x]] - I*Sinh[x]
```

Maple [A] (verified)

Time $= 2.26$ (sec) , antiderivative size $= 25$, normalized size of antiderivative $= 1.56$

method	result	size
risch	$-x - \frac{i\mathrm{e}^{x}}{2} + \frac{i\mathrm{e}^{-x}}{2} + 2\ln\left(\mathrm{e}^{x} - i\right)$	25
derivativedivides	$-\ln\left(\text{csch}\left(x\right)\right) - \frac{i}{\text{csch}(x)} + \frac{\ln\left(1+\text{csch}(x)^2\right)}{2} - i\arctan\left(\text{csch}\left(x\right)\right)$	29
default	$-\ln\left(\text{csch}\left(x\right)\right) - \frac{i}{\text{csch}(x)} + \frac{\ln\left(1+\text{csch}(x)^2\right)}{2} - i\arctan\left(\text{csch}\left(x\right)\right)$	29

```
[In] int(cosh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x-1/2*I*exp(x)+1/2*I/exp(x)+2*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time $= 0.25$ (sec) , antiderivative size $= 28$, normalized size of antiderivative $= 1.75$

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{1}{2}\left(2\,xe^{x} - 4\,e^{x}\log\left(e^{x} - i\right) + i\,e^{(2\,x)} - i\right)e^{(-x)}$$

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="fricas")
[Out] -1/2*(2*x*e^x - 4*e^x*log(e^x - I) + I*e^(2*x) - I)*e^(-x)
```

Sympy [F]

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\cosh\left(x\right)}{\operatorname{csch}\left(x\right) + i}\,dx$$

```
[In] integrate(cosh(x)/(I+csch(x)),x)
[Out] Integral(cosh(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.17$ (sec) , antiderivative size $= 21$, normalized size of antiderivative $= 1.31$

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\,dx = x + \frac{1}{2}i\,e^{(-x)} - \frac{1}{2}i\,e^{x} + 2\,\log\left(e^{(-x)} + i\right)$$

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="maxima")
[Out] x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\,dx = -x + \frac{1}{2} i\, e^{(-x)} - \frac{1}{2} i\, e^{x} + 2\,\log\left(e^{x} - i\right)$$

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="giac")
[Out] -x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\,dx = \ln\left(\sinh(x) - \mathrm{i}\right) - \sinh(x)\ 1\mathrm{i}$$

```
[In] int(cosh(x)/(1/sinh(x) + 1i),x)
[Out] log(sinh(x) - 1i) - sinh(x)*1i
```

3.88 $\int \frac{\mathbf{sech}(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 471
Rubi [A] (verified) . . . 471
Mathematica [A] (verified) . . . 473
Maple [A] (verified) . . . 473
Fricas [B] (verification not implemented) . . . 473
Sympy [F] . . . 474
Maxima [B] (verification not implemented) . . . 474
Giac [B] (verification not implemented) . . . 474
Mupad [B] (verification not implemented) . . . 475

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{2}i\arctan(\sinh(x)) - \frac{\operatorname{sech}^2(x)}{2} + \frac{1}{2}i\operatorname{sech}(x)\tanh(x)$$

```
[Out] -1/2*I*arctan(sinh(x))-1/2*sech(x)^2+1/2*I*sech(x)*tanh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3957, 2785, 2686, 30, 2691, 3855}

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{2}i\arctan(\sinh(x)) - \frac{1}{2}\operatorname{sech}^2(x) + \frac{1}{2}i\tanh(x)\operatorname{sech}(x)$$

```
[In] Int[Sech[x]/(I + Csch[x]),x]
```

```
[Out] (-1/2*I)*ArcTan[Sinh[x]] - Sech[x]^2/2 + (I/2)*Sech[x]*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\tanh(x)}{i - \sinh(x)}\, dx \\
&= -\left(i \int \operatorname{sech}(x) \tanh^2(x)\, dx \right) + \int \operatorname{sech}^2(x) \tanh(x)\, dx \\
&= \frac{1}{2} i \operatorname{sech}(x) \tanh(x) - \frac{1}{2} i \int \operatorname{sech}(x)\, dx - \operatorname{Subst}\left(\int x\, dx, x, \operatorname{sech}(x) \right) \\
&= -\frac{1}{2} i \arctan(\sinh(x)) - \frac{\operatorname{sech}^2(x)}{2} + \frac{1}{2} i \operatorname{sech}(x) \tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{2}i\left(\arctan(\sinh(x)) + \frac{1}{i-\sinh(x)}\right)$$

```
[In] Integrate[Sech[x]/(I + Csch[x]),x]
[Out] (-1/2*I)*(ArcTan[Sinh[x]] + (I - Sinh[x])^(-1))
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{ie^x}{(e^x-i)^2} - \frac{\ln(e^x-i)}{2} + \frac{\ln(e^x+i)}{2}$	30
default	$\frac{\ln(\tanh(\frac{x}{2})+i)}{2} - \frac{i}{\tanh(\frac{x}{2})-i} + \frac{1}{(\tanh(\frac{x}{2})-i)^2} - \frac{\ln(\tanh(\frac{x}{2})-i)}{2}$	43

```
[In] int(sech(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] I*exp(x)/(exp(x)-I)^2-1/2*ln(exp(x)-I)+1/2*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\left(e^{(2\,x)} - 2i\,e^x - 1\right)\log\left(e^x + i\right) - \left(e^{(2\,x)} - 2i\,e^x - 1\right)\log\left(e^x - i\right) + 2i\,e^x}{2\left(e^{(2\,x)} - 2i\,e^x - 1\right)}$$

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="fricas")
[Out] 1/2*((e^(2*x) - 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) - 2*I*e^x - 1)*log(e^x
 - I) + 2*I*e^x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(sech(x)/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time $= 0.17$ (sec) , antiderivative size $= 41$, normalized size of antiderivative $= 1.46$

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{2i\,e^{(-x)}}{4i\,e^{(-x)}+2\,e^{(-2\,x)}-2} - \frac{1}{2}\log\left(e^{(-x)}+i\right) + \frac{1}{2}\log\left(e^{(-x)}-i\right)$$

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2*I*e^(-x)/(4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(18) = 36$.

Time $= 0.29$ (sec) , antiderivative size $= 51$, normalized size of antiderivative $= 1.82$

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{e^{(-x)}-e^{x}-2i}{4\left(e^{(-x)}-e^{x}+2i\right)} + \frac{1}{4}\log\left(-e^{(-x)}+e^{x}+2i\right) - \frac{1}{4}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(e^(-x) - e^x - 2*I)/(e^(-x) - e^x + 2*I) + 1/4*log(-e^(-x) + e^x + 2*I
) - 1/4*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\ln\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2} - \frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2} + \frac{1}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} + \frac{1\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(1/(cosh(x)*(1/sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 + 1/(exp(x)*2i - exp(2*x) + 1)
+ 1i/(exp(x) - 1i)
```

3.89 $\int \frac{\mathbf{sech}^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 476
Rubi [A] (verified) . . . 476
Mathematica [B] (verified) . . . 477
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Fricas [B] (verification not implemented) . . . 478
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Maxima [B] (verification not implemented) . . . 479
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Mupad [B] (verification not implemented) . . . 479

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{3}\operatorname{sech}^3(x) - \frac{1}{3}i\tanh^3(x)$$

```
[Out] -1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2918, 2686, 30, 2687}

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{3}\operatorname{sech}^3(x) - \frac{1}{3}i\tanh^3(x)$$

```
[In] Int[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] -1/3*Sech[x]^3 - (I/3)*Tanh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\operatorname{sech}(x)\tanh(x)}{i - \sinh(x)}\,dx \\
&= -\left(i \int \operatorname{sech}^2(x)\tanh^2(x)\,dx \right) + \int \operatorname{sech}^3(x)\tanh(x)\,dx \\
&= -\operatorname{Subst}\left(\int x^2\,dx, x, \operatorname{sech}(x) \right) + \operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(x) \right) \\
&= -\frac{1}{3}\operatorname{sech}^3(x) - \frac{1}{3} i \tanh^3(x)
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 64 vs. $2(19) = 38$.

Time $= 0.06$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 3.37$

$$\int \frac{\operatorname{sech}^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{-3 + \cosh(x) + \cosh(2x) - 2i\sinh(x) + i\cosh(x)\sinh(x)}{6\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] (-3 + Cosh[x] + Cosh[2*x] - (2*I)*Sinh[x] + I*Cosh[x]*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

Time $= 0.61$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 1.63$

method	result	size
risch	$\frac{2i(-2ie^x+3e^{2x}-1)}{3(e^x+i)(e^x-i)^3}$	31
default	$-\frac{i}{2(\tanh(\frac{x}{2})+i)}+\frac{i}{2\tanh(\frac{x}{2})-2i}-\frac{2i}{3(\tanh(\frac{x}{2})-i)^3}-\frac{1}{(\tanh(\frac{x}{2})-i)^2}$	49

```
[In] int(sech(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(-2*I*exp(x)+3*exp(2*x)-1)/(exp(x)+I)/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time $= 0.25$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 1.68$

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{2\left(-3i\,e^{(2\,x)}-2\,e^x+i\right)}{3\left(e^{(4\,x)}-2i\,e^{(3\,x)}-2i\,e^x-1\right)}$$

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(-3*I*e^(2*x) - 2*e^x + I)/(e^(4*x) - 2*I*e^(3*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(sech(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time $= 0.18$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 4.26$

$$\int \frac{\operatorname{sech}^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{8\,e^{(-x)}}{12i\,e^{(-x)} + 12i\,e^{(-3\,x)} + 6\,e^{(-4\,x)} - 6} - \frac{12i\,e^{(-2\,x)}}{12i\,e^{(-x)} + 12i\,e^{(-3\,x)} + 6\,e^{(-4\,x)} - 6} + \frac{4i}{12i\,e^{(-x)} + 12i\,e^{(-3\,x)} + 6\,e^{(-4\,x)} - 6}$$

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 8*e^(-x)/(12*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) - 12*I*e^(-2*x)/(12
*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) + 4*I/(12*I*e^(-x) + 12*I*e^(-3
*x) + 6*e^(-4*x) - 6)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time $= 0.28$ (sec) , antiderivative size $= 27$, normalized size of antiderivative $= 1.42$

$$\int \frac{\operatorname{sech}^2(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{i}{2\,(i\,e^x - 1)} + \frac{3\,e^{(2\,x)} - 1}{6\,(e^x - i)^3}$$

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/2*I/(I*e^x - 1) + 1/6*(3*e^(2*x) - 1)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time $= 2.31$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 1.63$

$$\int \frac{\operatorname{sech}^2(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{\frac{2}{3} - 2\,\mathrm{e}^{2\,x} + \frac{\mathrm{e}^x\,4\mathrm{i}}{3}}{(\mathrm{e}^x + 1\mathrm{i})\,(1 + \mathrm{e}^x\,1\mathrm{i})^3}$$

```
[In] int(1/(cosh(x)^2*(1/sinh(x) + 1i)),x)
```

```
[Out] -((exp(x)*4i)/3 - 2*exp(2*x) + 2/3)/((exp(x) + 1i)*(exp(x)*1i + 1)^3)
```

3.90 $\int \frac{\mathbf{sech}^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 480
Rubi [A] (verified) . . . 480
Mathematica [A] (verified) . . . 482
Maple [B] (verified) . . . 482
Fricas [B] (verification not implemented) . . . 483
Sympy [F] . . . 483
Maxima [F(-2)] . . . 483
Giac [B] (verification not implemented) . . . 484
Mupad [B] (verification not implemented) . . . 484

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{8}i\arctan(\sinh(x)) - \frac{\operatorname{sech}^4(x)}{4} - \frac{1}{8}i\operatorname{sech}(x)\tanh(x) + \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)$$

```
[Out] -1/8*I*arctan(sinh(x))-1/4*sech(x)^4-1/8*I*sech(x)*tanh(x)+1/4*I*sech(x)^3*
tanh(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{8}i\arctan(\sinh(x)) - \frac{1}{4}\operatorname{sech}^4(x) + \frac{1}{4}i\tanh(x)\operatorname{sech}^3(x) - \frac{1}{8}i\tanh(x)\operatorname{sech}(x)$$

```
[In] Int[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] (-1/8*I)*ArcTan[Sinh[x]] - Sech[x]^4/4 - (I/8)*Sech[x]*Tanh[x] + (I/4)*Sech
[x]^3*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i \int \frac{\operatorname{sech}^2(x)\tanh(x)}{i - \sinh(x)}\,dx \\ &= -\left(i \int \operatorname{sech}^3(x)\tanh^2(x)\,dx\right) + \int \operatorname{sech}^4(x)\tanh(x)\,dx\end{aligned}$$

$$= \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x) - \frac{1}{4}i\int \operatorname{sech}^3(x)\,dx - \operatorname{Subst}\left(\int x^3\,dx, x, \operatorname{sech}(x)\right)$$

$$= -\frac{1}{4}\operatorname{sech}^4(x) - \frac{1}{8}i\operatorname{sech}(x)\tanh(x) + \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x) - \frac{1}{8}i\int \operatorname{sech}(x)\,dx$$

$$= -\frac{1}{8}i\arctan(\sinh(x)) - \frac{\operatorname{sech}^4(x)}{4} - \frac{1}{8}i\operatorname{sech}(x)\tanh(x) + \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{sech}^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{1}{8}\left(-i\arctan(\sinh(x)) + \frac{1}{(-i+\sinh(x))^2} - \frac{i}{i+\sinh(x)}\right)$$

```
[In] Integrate[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-I)*ArcTan[Sinh[x]] + (-I + Sinh[x])^(-2) - I/(I + Sinh[x]))/8
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 13.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{i\mathrm{e}^x\left(-2i\mathrm{e}^{3x}+\mathrm{e}^{4x}+2i\mathrm{e}^x-10\,\mathrm{e}^{2x}+1\right)}{4\left(\mathrm{e}^x-i\right)^4\left(\mathrm{e}^x+i\right)^2} - \frac{\ln(\mathrm{e}^x-i)}{8} + \frac{\ln(\mathrm{e}^x+i)}{8}$
default	$\frac{i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^3} - \frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)} - \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^4} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{8} + \frac{i}{4\tanh\left(\frac{x}{2}\right)+4i} + \frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}$

```
[In] int(sech(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*exp(x)*(-2*I*exp(x)^3+exp(x)^4+2*I*exp(x)-10*exp(x)^2+1)/(exp(x)-I)^
4/(exp(x)+I)^2-1/8*ln(exp(x)-I)+1/8*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.55

$$\int \frac{\operatorname{sech}^3(x)}{i + \operatorname{csch}(x)}\,dx$$
$$= \frac{\left(e^{(6\,x)} - 2i\,e^{(5\,x)} + e^{(4\,x)} - 4i\,e^{(3\,x)} - e^{(2\,x)} - 2i\,e^{x} - 1\right)\log\left(e^{x} + i\right) - \left(e^{(6\,x)} - 2i\,e^{(5\,x)} + e^{(4\,x)} - 4i\,e^{(3\,x)} - e\right.}{8\left(e^{(6\,x)} - 2i\,e^{(5\,x)} + e^{(4\,x)} - 4i\,e^{(3\,x)} - e^{(2\,x)}\right.}$$

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I*e^x - 1
)*log(e^x + I) - (e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) -
 2*I*e^x - 1)*log(e^x - I) - 2*I*e^(5*x) - 4*e^(4*x) + 20*I*e^(3*x) + 4*e^(
2*x) - 2*I*e^x)/(e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) -
2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(sech(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(csch(x) + I), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{i + \operatorname{csch}(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(26) = 52$.

Time $= 0.28$ (sec) , antiderivative size $= 94$, normalized size of antiderivative $= 2.35$

$$\int \frac{\operatorname{sech}^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{-i\,e^{(-x)} + i\,e^{x} - 6}{16\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)} + \frac{3\left(e^{(-x)} - e^{x}\right)^2 + 12i\,e^{(-x)} - 12i\,e^{x} + 4}{32\left(e^{(-x)} - e^{x} + 2i\right)^2} + \frac{1}{16}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{16}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/16*(-I*e^(-x) + I*e^x - 6)/(-I*e^(-x) + I*e^x - 2) + 1/32*(3*(e^(-x) - e
^x)^2 + 12*I*e^(-x) - 12*I*e^x + 4)/(e^(-x) - e^x + 2*I)^2 + 1/16*log(-e^(-
x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time $= 2.65$ (sec) , antiderivative size $= 122$, normalized size of antiderivative $= 3.05$

$$\int \frac{\operatorname{sech}^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{\ln\left(-\frac{1}{4} + \frac{e^x\,1i}{4}\right)}{8} - \frac{\ln\left(\frac{1}{4} + \frac{e^x\,1i}{4}\right)}{8} - \frac{1i}{e^{2x}\,3i - e^{3x} + 3\,e^{x} - i} - \frac{1}{4\left(e^{2x} - 1 + e^{x}\,2i\right)} - \frac{1}{2\left(e^{4x} - 6\,e^{2x} + 1 - e^{3x}\,4i + e^{x}\,4i\right)} - \frac{1}{2\left(1 - e^{2x} + e^{x}\,2i\right)} - \frac{1i}{4\left(e^{x} + 1i\right)}$$

```
[In] int(1/(cosh(x)^3*(1/sinh(x) + 1i)),x)
```

```
[Out] log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 1i/(exp(2*x)*3i -
 exp(3*x) + 3*exp(x) - 1i) - 1/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(2*(exp(4
*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) - 1/(2*(exp(x)*2i - exp(2*
x) + 1)) - 1i/(4*(exp(x) + 1i))
```

3.91 $\int \frac{\mathbf{sech}^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 485
Rubi [A] (verified) . . . 485
Mathematica [B] (verified) . . . 487
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Giac [B] (verification not implemented) . . . 488
Mupad [B] (verification not implemented) . . . 488

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{5}\operatorname{sech}^5(x) - \frac{1}{3}i\tanh^3(x) + \frac{1}{5}i\tanh^5(x)$$

```
[Out] -1/5*sech(x)^5-1/3*I*tanh(x)^3+1/5*I*tanh(x)^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2918, 2686, 30, 2687, 14}

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{5}i\tanh^5(x) - \frac{1}{3}i\tanh^3(x) - \frac{1}{5}\operatorname{sech}^5(x)$$

```
[In] Int[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] -1/5*Sech[x]^5 - (I/3)*Tanh[x]^3 + (I/5)*Tanh[x]^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i \int \frac{\operatorname{sech}^3(x)\tanh(x)}{i - \sinh(x)}\,dx \\
&= -\left(i \int \operatorname{sech}^4(x)\tanh^2(x)\,dx\right) + \int \operatorname{sech}^5(x)\tanh(x)\,dx \\
&= -\operatorname{Subst}\left(\int x^4\,dx, x, \operatorname{sech}(x)\right) + \operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, i\tanh(x)\right) \\
&= -\frac{1}{5}\operatorname{sech}^5(x) + \operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, i\tanh(x)\right) \\
&= -\frac{1}{5}\operatorname{sech}^5(x) - \frac{1}{3}i\tanh^3(x) + \frac{1}{5}i\tanh^5(x)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 96 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.31

$$\int \frac{\operatorname{sech}^4(x)}{i + \operatorname{csch}(x)}\,dx$$
$$= \frac{-240 + 54\cosh(x) + 32\cosh(2x) + 18\cosh(3x) + 16\cosh(4x) - 96i\sinh(x) + 18i\sinh(2x) - 32i\sinh(3x}{960\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^5}$$

```
[In] Integrate[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] (-240 + 54*Cosh[x] + 32*Cosh[2*x] + 18*Cosh[3*x] + 16*Cosh[4*x] - (96*I)*Si
nh[x] + (18*I)*Sinh[2*x] - (32*I)*Sinh[3*x] + (9*I)*Sinh[4*x])/(960*(Cosh[x
/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$-\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right) - i\right)^3} + \frac{3i}{8\left(\tanh\left(\frac{x}{2}\right) - i\right)} + \frac{2i}{5\left(\tanh\left(\frac{x}{2}\right) - i\right)^5} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right) - i\right)^4} - \frac{1}{\left(\tanh\left(\frac{x}{2}\right) - i\right)^2} + \frac{i}{6\left(\tanh\left(\frac{x}{2}\right)\right.}$$

```
[In] int(sech(x)^4/(I+csch(x)),x)
```

```
[Out] -4/3*I/(tanh(1/2*x)-I)^3+3/8*I/(tanh(1/2*x)-I)+2/5*I/(tanh(1/2*x)-I)^5+1/(t
anh(1/2*x)-I)^4-1/(tanh(1/2*x)-I)^2+1/6*I/(tanh(1/2*x)+I)^3-3/8*I/(tanh(1/2
*x)+I)-1/4/(tanh(1/2*x)+I)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\operatorname{sech}^4(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{4\left(-15i\,e^{(4x)} - 6\,e^{(3x)} + 2i\,e^{(2x)} - 2\,e^x + i\right)}{15\left(e^{(8x)} - 2i\,e^{(7x)} + 2\,e^{(6x)} - 6i\,e^{(5x)} - 6i\,e^{(3x)} - 2\,e^{(2x)} - 2i\,e^x - 1\right)}$$

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(-15*I*e^(4*x) - 6*e^(3*x) + 2*I*e^(2*x) - 2*e^x + I)/(e^(8*x) - 2*I*
e^(7*x) + 2*e^(6*x) - 6*I*e^(5*x) - 6*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(sech(x)**4/(I+csch(x)),x)
[Out] Integral(sech(x)**4/(csch(x) + I), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time $= 0.27$ (sec) , antiderivative size $= 55$, normalized size of antiderivative $= 1.90$

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{-3i\,e^{(2x)}+12\,e^x+5i}{24\,(i\,e^x-1)^3} + \frac{15\,e^{(4x)}-60i\,e^{(3x)}-10\,e^{(2x)}+20i\,e^x+7}{120\,(e^x-i)^5}$$

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="giac")
[Out] -1/24*(-3*I*e^(2*x) + 12*e^x + 5*I)/(I*e^x - 1)^3 + 1/120*(15*e^(4*x) - 60*
I*e^(3*x) - 10*e^(2*x) + 20*I*e^x + 7)/(e^x - I)^5
```

Mupad [B] (verification not implemented)

Time $= 2.66$ (sec) , antiderivative size $= 207$, normalized size of antiderivative $= 7.14$

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = &-\frac{1\mathrm{i}}{4\,(\mathrm{e}^{2x}-1+\mathrm{e}^x\,2\mathrm{i})} + \frac{1}{20\,(\mathrm{e}^x-\mathrm{i})} - \frac{1}{8\,(\mathrm{e}^x+1\mathrm{i})}\\ &+ \frac{\frac{\mathrm{e}^{3x}}{40}-\frac{\mathrm{e}^{2x}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^x}{8}+\frac{1}{40}\mathrm{i}}{\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1-\mathrm{e}^{3x}\,4\mathrm{i}+\mathrm{e}^x\,4\mathrm{i}} - \frac{\frac{\mathrm{e}^{2x}}{40}+\frac{1}{24}-\frac{\mathrm{e}^x\,1\mathrm{i}}{20}}{\mathrm{e}^{2x}\,3\mathrm{i}-\mathrm{e}^{3x}+3\,\mathrm{e}^x-\mathrm{i}}\\ &- \frac{1}{6\,(\mathrm{e}^{2x}\,3\mathrm{i}+\mathrm{e}^{3x}-3\,\mathrm{e}^x-\mathrm{i})} + \frac{\frac{\mathrm{e}^{2x}}{4}+\frac{\mathrm{e}^{4x}}{40}+\frac{1}{40}-\frac{\mathrm{e}^{3x}\,1\mathrm{i}}{10}+\frac{\mathrm{e}^x\,1\mathrm{i}}{10}}{\mathrm{e}^{2x}\,10\mathrm{i}-10\,\mathrm{e}^{3x}-\mathrm{e}^{4x}\,5\mathrm{i}+\mathrm{e}^{5x}+5\,\mathrm{e}^x-\mathrm{i}}\end{aligned}$$

```
[In] int(1/(cosh(x)^4*(1/sinh(x) + 1i)),x)
```

```
[Out] 1/(20*(exp(x) - 1i)) - 1i/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(8*(exp(x) + 1
i)) + (exp(3*x)/40 - (exp(2*x)*3i)/40 + exp(x)/8 + 1i/40)/(exp(4*x) - exp(3
*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) - (exp(2*x)/40 - (exp(x)*1i)/20 + 1/24
)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x) -
 3*exp(x) - 1i)) + (exp(2*x)/4 - (exp(3*x)*1i)/10 + exp(4*x)/40 + (exp(x)*1
i)/10 + 1/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) + 5*exp(
x) - 1i)
```

3.92 $\int \frac{\cosh^5(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 490
Rubi [A] (verified) . . . 490
Mathematica [A] (verified) . . . 492
Maple [B] (verified) . . . 492
Fricas [B] (verification not implemented) . . . 492
Sympy [F] . . . 493
Maxima [B] (verification not implemented) . . . 494
Giac [B] (verification not implemented) . . . 494
Mupad [B] (verification not implemented) . . . 495

Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b(a^2+b^2)^2\log(b+a\sinh(x))}{a^6} + \frac{(a^2+b^2)^2\sinh(x)}{a^5} - \frac{b(2a^2+b^2)\sinh^2(x)}{2a^4} + \frac{(2a^2+b^2)\sinh^3(x)}{3a^3} - \frac{b\sinh^4(x)}{4a^2} + \frac{\sinh^5(x)}{5a}$$

```
[Out] -b*(a^2+b^2)^2*ln(b+a*sinh(x))/a^6+(a^2+b^2)^2*sinh(x)/a^5-1/2*b*(2*a^2+b^2
)*sinh(x)^2/a^4+1/3*(2*a^2+b^2)*sinh(x)^3/a^3-1/4*b*sinh(x)^4/a^2+1/5*sinh(
x)^5/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3957, 2916, 12, 786}

$$\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\sinh^4(x)}{4a^2} - \frac{b(a^2+b^2)^2\log(a\sinh(x)+b)}{a^6} + \frac{(a^2+b^2)^2\sinh(x)}{a^5} - \frac{b(2a^2+b^2)\sinh^2(x)}{2a^4} + \frac{(2a^2+b^2)\sinh^3(x)}{3a^3} + \frac{\sinh^5(x)}{5a}$$

```
[In] Int[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/a^6) + ((a^2 + b^2)^2*Sinh[x])/a^5 -
 (b*(2*a^2 + b^2)*Sinh[x]^2)/(2*a^4) + ((2*a^2 + b^2)*Sinh[x]^3)/(3*a^3) -
(b*Sinh[x]^4)/(4*a^2) + Sinh[x]^5/(5*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\cosh^5(x)\sinh(x)}{ib + ia\sinh(x)}\,dx \\
&= -\frac{i\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^5} \\
&= -\frac{i\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^6} \\
&= \\
&\quad -\frac{i\text{Subst}\left(\int \left(\left(a^2+b^2\right)^2 - \frac{b\left(a^2+b^2\right)^2}{b-ix} + ib\left(2a^2+b^2\right)x - \left(2a^2+b^2\right)x^2 - ibx^3 + x^4\right)\,dx, x, ia\sinh(x)\right.}{a^6} \\
&= -\frac{b\left(a^2+b^2\right)^2\log(b+a\sinh(x))}{a^6} + \frac{\left(a^2+b^2\right)^2\sinh(x)}{a^5} \\
&\quad - \frac{b\left(2a^2+b^2\right)\sinh^2(x)}{2a^4} + \frac{\left(2a^2+b^2\right)\sinh^3(x)}{3a^3} - \frac{b\sinh^4(x)}{4a^2} + \frac{\sinh^5(x)}{5a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{-60b(a^2+b^2)^2\log(b+a\sinh(x)) + 60a(a^2+b^2)^2\sinh(x) - 30a^2b(2a^2+b^2)\sinh^2(x) + 20a^3(2a^2+b^2)\sinh$$
$$60a^6$$

```
[In] Integrate[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^2 + b^2)^2*Log[b + a*Sinh[x]] + 60*a*(a^2 + b^2)^2*Sinh[x] - 30*a
^2*b*(2*a^2 + b^2)*Sinh[x]^2 + 20*a^3*(2*a^2 + b^2)*Sinh[x]^3 - 15*a^4*b*Si
nh[x]^4 + 12*a^5*Sinh[x]^5)/(60*a^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(94) = 188.

Time = 0.13 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.71

$$-\frac{1}{5a\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\frac{2a+b}{4a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{11a^2+6ab+4b^2}{12a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{7a^3+9a^2b+4a\,b^2+4b^3}{8a^4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{8a^4+7a^3b+}{8a^5\left(t}$$

```
[In] int(cosh(x)^5/(a+b*csch(x)),x)
```

```
[Out] -1/5/a/(tanh(1/2*x)-1)^5-1/4*(2*a+b)/a^2/(tanh(1/2*x)-1)^4-1/12*(11*a^2+6*a
*b+4*b^2)/a^3/(tanh(1/2*x)-1)^3-1/8*(7*a^3+9*a^2*b+4*a*b^2+4*b^3)/a^4/(tanh
(1/2*x)-1)^2-1/8*(8*a^4+7*a^3*b+16*a^2*b^2+4*a*b^3+8*b^4)/a^5/(tanh(1/2*x)-
1)+b*(a^4+2*a^2*b^2+b^4)/a^6*ln(tanh(1/2*x)-1)-2*b/a^6*(1/2*a^4+a^2*b^2+1/2
*b^4)*ln(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)-1/5/a/(tanh(1/2*x)+1)^5-1/4*(-2
*a+b)/a^2/(tanh(1/2*x)+1)^4-1/12*(11*a^2-6*a*b+4*b^2)/a^3/(tanh(1/2*x)+1)^3
-1/8*(-7*a^3+9*a^2*b-4*a*b^2+4*b^3)/a^4/(tanh(1/2*x)+1)^2-1/8*(8*a^4-7*a^3*
b+16*a^2*b^2-4*a*b^3+8*b^4)/a^5/(tanh(1/2*x)+1)+b*(a^4+2*a^2*b^2+b^4)/a^6*l
n(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. 2(94) = 188.

Time = 0.26 (sec) , antiderivative size = 1398, normalized size of antiderivative = 13.71

$$\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*a^5*cosh(x)^10 + 6*a^5*sinh(x)^10 - 15*a^4*b*cosh(x)^9 + 15*(4*a^5
*cosh(x) - a^4*b)*sinh(x)^9 + 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^8 + 5*(54*a^5*
cosh(x)^2 - 27*a^4*b*cosh(x) + 10*a^5 + 8*a^3*b^2)*sinh(x)^8 - 60*(3*a^4*b
+ 2*a^2*b^3)*cosh(x)^7 + 20*(36*a^5*cosh(x)^3 - 27*a^4*b*cosh(x)^2 - 9*a^4*
b - 6*a^2*b^3 + 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x)^7 + 960*(a^4*b + 2*a
^2*b^3 + b^5)*x*cosh(x)^5 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^6 + 2
0*(63*a^5*cosh(x)^4 - 63*a^4*b*cosh(x)^3 + 15*a^5 + 42*a^3*b^2 + 24*a*b^4 +
 14*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 - 21*(3*a^4*b + 2*a^2*b^3)*cosh(x))*sinh(
x)^6 - 15*a^4*b*cosh(x) + 2*(756*a^5*cosh(x)^5 - 945*a^4*b*cosh(x)^4 + 280*
(5*a^5 + 4*a^3*b^2)*cosh(x)^3 - 630*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 + 480*(
a^4*b + 2*a^2*b^3 + b^5)*x + 180*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x))*si
nh(x)^5 - 6*a^5 - 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 + 10*(126*a^5
*cosh(x)^6 - 189*a^4*b*cosh(x)^5 - 30*a^5 - 84*a^3*b^2 - 48*a*b^4 + 70*(5*a
^5 + 4*a^3*b^2)*cosh(x)^4 - 210*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 480*(a^4*
b + 2*a^2*b^3 + b^5)*x*cosh(x) + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^
2)*sinh(x)^4 - 60*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 20*(36*a^5*cosh(x)^7 -
63*a^4*b*cosh(x)^6 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^5 - 9*a^4*b - 6*a^2*b^3
 - 105*(3*a^4*b + 2*a^2*b^3)*cosh(x)^4 + 480*(a^4*b + 2*a^2*b^3 + b^5)*x*co
sh(x)^2 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 12*(5*a^5 + 14*a^3*
b^2 + 8*a*b^4)*cosh(x))*sinh(x)^3 - 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 + 10*(
27*a^5*cosh(x)^8 - 54*a^4*b*cosh(x)^7 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^6 -
126*(3*a^4*b + 2*a^2*b^3)*cosh(x)^5 - 5*a^5 - 4*a^3*b^2 + 960*(a^4*b + 2*a^
2*b^3 + b^5)*x*cosh(x)^3 + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 - 36
*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^2 - 18*(3*a^4*b + 2*a^2*b^3)*cosh(x
))*sinh(x)^2 - 960*((a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^5 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4*sinh(x) + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^3*sinh(
x)^2 + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2*sinh(x)^3 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)*sinh(x)^4 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^5)*log(2*(
a*sinh(x) + b)/(cosh(x) - sinh(x))) + 5*(12*a^5*cosh(x)^9 - 27*a^4*b*cosh(x
)^8 + 16*(5*a^5 + 4*a^3*b^2)*cosh(x)^7 - 84*(3*a^4*b + 2*a^2*b^3)*cosh(x)^6
 + 960*(a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^4 + 72*(5*a^5 + 14*a^3*b^2 + 8*a
*b^4)*cosh(x)^5 - 3*a^4*b - 48*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 3
6*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 - 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x))
/(a^6*cosh(x)^5 + 5*a^6*cosh(x)^4*sinh(x) + 10*a^6*cosh(x)^3*sinh(x)^2 + 10
*a^6*cosh(x)^2*sinh(x)^3 + 5*a^6*cosh(x)*sinh(x)^4 + a^6*sinh(x)^5)
```

Sympy [F]

$$\int \frac{\cosh^5(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\cosh^5(x)}{a + b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(cosh(x)**5/(a+b*csch(x)),x)
[Out] Integral(cosh(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(94) = 188$.

Time = 0.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.37

$$\int \frac{\cosh^5(x)}{a + b\operatorname{csch}(x)}\,dx =$$
$$-\frac{\left(15\,a^3be^{(-x)} - 6\,a^4 - 10\,(5\,a^4 + 4\,a^2b^2)e^{(-2\,x)} + 60\,(3\,a^3b + 2\,ab^3)e^{(-3\,x)} - 60\,(5\,a^4 + 14\,a^2b^2 + 8\,b^4)e^{(-4\,x)}\right.}{960\,a^5}$$
$$-\frac{15\,a^3be^{(-4\,x)} + 6\,a^4e^{(-5\,x)} + 60\,(5\,a^4 + 14\,a^2b^2 + 8\,b^4)e^{(-x)} + 60\,(3\,a^3b + 2\,ab^3)e^{(-2\,x)} + 10\,(5\,a^4 + 4\,a^2b^2)e^{}}{960\,a^5}$$
$$-\frac{(a^4b + 2\,a^2b^3 + b^5)x}{a^6} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^6}$$

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a^3*b*e^(-x) - 6*a^4 - 10*(5*a^4 + 4*a^2*b^2)*e^(-2*x) + 60*(3*a
^3*b + 2*a*b^3)*e^(-3*x) - 60*(5*a^4 + 14*a^2*b^2 + 8*b^4)*e^(-4*x))*e^(5*x
)/a^5 - 1/960*(15*a^3*b*e^(-4*x) + 6*a^4*e^(-5*x) + 60*(5*a^4 + 14*a^2*b^2
+ 8*b^4)*e^(-x) + 60*(3*a^3*b + 2*a*b^3)*e^(-2*x) + 10*(5*a^4 + 4*a^2*b^2)*
e^(-3*x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*x/a^6 - (a^4*b + 2*a^2*b^3 + b^5)
*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(94) = 188$.

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.90

$$\int \frac{\cosh^5(x)}{a + b\operatorname{csch}(x)}\,dx =$$
$$-\frac{6\,a^4\left(e^{(-x)} - e^{x}\right)^5 + 15\,a^3b\left(e^{(-x)} - e^{x}\right)^4 + 80\,a^4\left(e^{(-x)} - e^{x}\right)^3 + 40\,a^2b^2\left(e^{(-x)} - e^{x}\right)^3 + 240\,a^3b\left(e^{(-x)} - e^{x}\right.}{960\,a^5}$$
$$-\frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^6}$$

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/960*(6*a^4*(e^(-x) - e^x)^5 + 15*a^3*b*(e^(-x) - e^x)^4 + 80*a^4*(e^(-x)
 - e^x)^3 + 40*a^2*b^2*(e^(-x) - e^x)^3 + 240*a^3*b*(e^(-x) - e^x)^2 + 120*
a*b^3*(e^(-x) - e^x)^2 + 480*a^4*(e^(-x) - e^x) + 960*a^2*b^2*(e^(-x) - e^x
) + 480*b^4*(e^(-x) - e^x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(-a*(e^(
-x) - e^x) + 2*b))/a^6
```

Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx = {} & \frac{\mathrm{e}^{5x}}{160\,a} - \frac{\mathrm{e}^{-5x}}{160\,a} - \frac{\mathrm{e}^{-2x}\,(3\,a^2\,b+2\,b^3)}{16\,a^4} - \frac{\mathrm{e}^{2x}\,(3\,a^2\,b+2\,b^3)}{16\,a^4} \\ & + \frac{\mathrm{e}^{x}\,(5\,a^4+14\,a^2\,b^2+8\,b^4)}{16\,a^5} - \frac{b\,\mathrm{e}^{-4x}}{64\,a^2} - \frac{b\,\mathrm{e}^{4x}}{64\,a^2} \\ & - \frac{\ln\left(2\,b\,\mathrm{e}^{x}-a+a\,\mathrm{e}^{2x}\right)\,(a^4\,b+2\,a^2\,b^3+b^5)}{a^6} \\ & - \frac{\mathrm{e}^{-x}\,(5\,a^4+14\,a^2\,b^2+8\,b^4)}{16\,a^5} - \frac{\mathrm{e}^{-3x}\,(5\,a^2+4\,b^2)}{96\,a^3} \\ & + \frac{\mathrm{e}^{3x}\,(5\,a^2+4\,b^2)}{96\,a^3} + \frac{b\,x\,(a^2+b^2)^2}{a^6}\end{aligned}$$

```
[In] int(cosh(x)^5/(a + b/sinh(x)),x)
```

```
[Out] exp(5*x)/(160*a) - exp(-5*x)/(160*a) - (exp(-2*x)*(3*a^2*b + 2*b^3))/(16*a^
4) - (exp(2*x)*(3*a^2*b + 2*b^3))/(16*a^4) + (exp(x)*(5*a^4 + 8*b^4 + 14*a^
2*b^2))/(16*a^5) - (b*exp(-4*x))/(64*a^2) - (b*exp(4*x))/(64*a^2) - (log(2*
b*exp(x) - a + a*exp(2*x))*(a^4*b + b^5 + 2*a^2*b^3))/a^6 - (exp(-x)*(5*a^4
 + 8*b^4 + 14*a^2*b^2))/(16*a^5) - (exp(-3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (
exp(3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (b*x*(a^2 + b^2)^2)/a^6
```

3.93 $\int \frac{\cosh^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 496
Rubi [A] (verified) . . . 496
Mathematica [A] (verified) . . . 498
Maple [A] (verified) . . . 499
Fricas [B] (verification not implemented) . . . 499
Sympy [F] . . . 500
Maxima [A] (verification not implemented) . . . 500
Giac [A] (verification not implemented) . . . 501
Mupad [B] (verification not implemented) . . . 501

Optimal result

Integrand size = 13, antiderivative size = 125

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{(3a^4+12a^2b^2+8b^4)\,x}{8a^5} + \frac{2b(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{8a^4}$$

```
[Out] 1/8*(3*a^4+12*a^2*b^2+8*b^4)*x/a^5+2*b*(a^2+b^2)^(3/2)*arctanh((a-b*tanh(1/
2*x))/(a^2+b^2)^(1/2))/a^5-1/12*cosh(x)^3*(4*b-3*a*sinh(x))/a^2-1/8*cosh(x)
*(8*b*(a^2+b^2)-a*(3*a^2+4*b^2)*sinh(x))/a^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2944, 2814, 2739, 632, 210}

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} + \frac{2b(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{8a^4} + \frac{x(3a^4+12a^2b^2+8b^4)}{8a^5}$$

```
[In] Int[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*a^4 + 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b*(a^2 + b^2)^(3/2)*ArcTanh[(
a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - (Cosh[x]^3*(4*b - 3*a*Sinh[x]))/(1
2*a^2) - (Cosh[x]*(8*b*(a^2 + b^2) - a*(3*a^2 + 4*b^2)*Sinh[x]))/(8*a^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = i \int \frac{\cosh^4(x)\sinh(x)}{ib + ia\sinh(x)}\,dx$$

$$= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} + \frac{\int \frac{\cosh^2(x)\left(-iab+i\left(3a^2+4b^2\right)\sinh(x)\right)}{ib+ia\sinh(x)}\,dx}{4a^2}$$

$$= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4} + \frac{\int \frac{-iab\left(5a^2+4b^2\right)+i\left(3a^4+12a^2b^2+8b^4\right)\sinh(x)}{ib+ia\sinh(x)}\,dx}{8a^4}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4} - \frac{\left(ib\left(a^2+b^2\right)^2\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^5}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4} - \frac{\left(2ib\left(a^2+b^2\right)^2\right)\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^5}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4} + \frac{\left(4ib\left(a^2+b^2\right)^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, 2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{a^5}$$

$$= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5} + \frac{2b\left(a^2+b^2\right)^{3/2}\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx$$

$$= \frac{-24ab\left(5a^2+4b^2\right)\cosh(x) - 8a^3b\cosh(3x) + 3\left(12a^4x+48a^2b^2x+32b^4x+64a^2b\sqrt{-a^2-b^2}\arctan\left(\frac{a-b\tan}{\sqrt{-a}}\right.\right.}{96a^5}$$

```
[In] Integrate[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (-24*a*b*(5*a^2 + 4*b^2)*Cosh[x] - 8*a^3*b*Cosh[3*x] + 3*(12*a^4*x + 48*a^2
*b^2*x + 32*b^4*x + 64*a^2*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt
```

```
[-a^2 - b^2]] + 64*b^3*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2
- b^2]] + 8*a^2*(a^2 + b^2)*Sinh[2*x] + a^4*Sinh[4*x]))/(96*a^5)
```

Maple [A] (verified)

Time = 60.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.72

method	result
risch	$\frac{3x}{8a}+\frac{3x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{\mathrm{e}^{4x}}{64a}-\frac{b\,\mathrm{e}^{3x}}{24a^2}+\frac{\mathrm{e}^{2x}}{8a}+\frac{\mathrm{e}^{2x}b^2}{8a^3}-\frac{5b\,\mathrm{e}^{x}}{8a^2}-\frac{b^3\mathrm{e}^{x}}{2a^4}-\frac{5b\,\mathrm{e}^{-x}}{8a^2}-\frac{b^3\mathrm{e}^{-x}}{2a^4}-\frac{\mathrm{e}^{-2x}}{8a}-\frac{\mathrm{e}^{-2x}b^2}{8a^3}-\frac{b\,\mathrm{e}^{-3x}}{24a^2}$
default	$\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{-3a-2b}{6a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{-7a^2-4ab-4b^2}{8a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{\left(-3a^4-12a^2b^2-8b^4\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a^5}-\frac{-5a^3-12a^2b-4a\,b^2}{8a^4\left(\tanh\left(\frac{x}{2}\right)-1\right)}$

```
[In] int(cosh(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x/a+3/2*x/a^3*b^2+x/a^5*b^4+1/64/a*exp(x)^4-1/24/a^2*b*exp(x)^3+1/8/a*e
xp(x)^2+1/8/a^3*exp(x)^2*b^2-5/8*b/a^2*exp(x)-1/2*b^3/a^4*exp(x)-5/8*b/a^2/
exp(x)-1/2*b^3/a^4/exp(x)-1/8/a/exp(x)^2-1/8/a^3/exp(x)^2*b^2-1/24/a^2*b/ex
p(x)^3-1/64/a/exp(x)^4+(a^2+b^2)^(3/2)*b/a^5*ln(exp(x)+(b+(a^2+b^2)^(1/2))/
a)-(a^2+b^2)^(3/2)*b/a^5*ln(exp(x)-(-b+(a^2+b^2)^(1/2))/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(114) = 228$.

Time = 0.26 (sec) , antiderivative size = 924, normalized size of antiderivative = 7.39

$$\int \frac{\cosh^4(x)}{a+b\mathrm{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cos
h(x) - a^3*b)*sinh(x)^7 + 24*(a^4 + a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^
2 - 14*a^3*b*cosh(x) + 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 + 12*a^2*b^
2 + 8*b^4)*x*cosh(x)^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(
x)^3 - 7*a^3*b*cosh(x)^2 - 5*a^3*b - 4*a*b^3 + 6*(a^4 + a^2*b^2)*cosh(x))*s
inh(x)^5 - 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 + 1
80*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x - 60*(5*a^
3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^
3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b - 12*a*b^3 + 60*(a^
4 + a^2*b^2)*cosh(x)^3 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x) - 30*(5*
a^3*b + 4*a*b^3)*cosh(x)^2)*sinh(x)^3 - 24*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(
7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 + 30*(a^4 + a^2*b^2)*cosh(x)^4 - 2*a^4
 - 2*a^2*b^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 - 20*(5*a^3*b +
```

```
4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a^2*b
 + b^3)*cosh(x)^4 + 4*(a^2*b + b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b + b^3)*cos
h(x)^2*sinh(x)^2 + 4*(a^2*b + b^3)*cosh(x)*sinh(x)^3 + (a^2*b + b^3)*sinh(x
)^4)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a
^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) +
 a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
b)*sinh(x) - a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 + 18*(a^4 + a^2*b
^2)*cosh(x)^5 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 - 15*(5*a^3*b +
 4*a*b^3)*cosh(x)^4 - a^3*b - 9*(5*a^3*b + 4*a*b^3)*cosh(x)^2 - 6*(a^4 + a^
2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*c
osh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\cosh^4(x)}{a+b\,\text{csch}\,(x)}\,dx$$

```
[In] integrate(cosh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx = &-\frac{\left(8\,a^2be^{(-x)} - 3\,a^3 - 24\,(a^3+ab^2)e^{(-2\,x)} + 24\,(5\,a^2b+4\,b^3)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,a^4}\\ &-\frac{8\,a^2be^{(-3\,x)} + 3\,a^3e^{(-4\,x)} + 24\,(5\,a^2b+4\,b^3)e^{(-x)} + 24\,(a^3+ab^2)e^{(-2\,x)}}{192\,a^4}\\ &+\frac{(3\,a^4+12\,a^2b^2+8\,b^4)x}{8\,a^5} - \frac{(a^4b+2\,a^2b^3+b^5)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^5}\end{aligned}$$

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a^2*b*e^(-x) - 3*a^3 - 24*(a^3 + a*b^2)*e^(-2*x) + 24*(5*a^2*b +
4*b^3)*e^(-3*x))*e^(4*x)/a^4 - 1/192*(8*a^2*b*e^(-3*x) + 3*a^3*e^(-4*x) + 2
4*(5*a^2*b + 4*b^3)*e^(-x) + 24*(a^3 + a*b^2)*e^(-2*x))/a^4 + 1/8*(3*a^4 +
12*a^2*b^2 + 8*b^4)*x/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log((a*e^(-x) - b - s
qrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} + 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} - 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} + \frac{(3\,a^4 + 12\,a^2b^2 + 8\,b^4)x}{8\,a^5} - \frac{\left(8\,a^3be^x + 3\,a^4 + 24\,(5\,a^3b + 4\,ab^3)e^{(3\,x)} + 24\,(a^4 + a^2b^2)e^{(2\,x)}\right)e^{(-4\,x)}}{192\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(\frac{\left|2\,ae^x + 2\,b - 2\sqrt{a^2+b^2}\right|}{\left|2\,ae^x + 2\,b + 2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^5}$$

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
- 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x/a^5
- 1/192*(8*a^3*b*e^x + 3*a^4 + 24*(5*a^3*b + 4*a*b^3)*e^(3*x) + 24*(a^4 + a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(2*a*e^x +
 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2
 + b^2)*a^5)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.98

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{4\,x}}{64\,a} - \frac{\mathrm{e}^{-4\,x}}{64\,a} + \frac{x\,(3\,a^4 + 12\,a^2\,b^2 + 8\,b^4)}{8\,a^5} - \frac{\mathrm{e}^{-2\,x}\,(a^2+b^2)}{8\,a^3} + \frac{\mathrm{e}^{2\,x}\,(a^2+b^2)}{8\,a^3} - \frac{\mathrm{e}^{-x}\,(5\,a^2\,b + 4\,b^3)}{8\,a^4} - \frac{b\,\mathrm{e}^{-3\,x}}{24\,a^2} - \frac{b\,\mathrm{e}^{3\,x}}{24\,a^2} - \frac{\mathrm{e}^x\,(5\,a^2\,b + 4\,b^3)}{8\,a^4} - \frac{b\,\ln\left(\frac{2\,b\,\mathrm{e}^x\,{(a^2+b^2)}^2}{a^6} - \frac{2\,b\,(a - b\,\mathrm{e}^x)\,{(a^2+b^2)}^{3/2}}{a^6}\right)\,{(a^2+b^2)}^{3/2}}{a^5} + \frac{b\,\ln\left(\frac{2\,b\,(a - b\,\mathrm{e}^x)\,{(a^2+b^2)}^{3/2}}{a^6} + \frac{2\,b\,\mathrm{e}^x\,{(a^2+b^2)}^2}{a^6}\right)\,{(a^2+b^2)}^{3/2}}{a^5}$$

```
[In] int(cosh(x)^4/(a + b/sinh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 12*a^2*b^2))/(8*a^
5) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) - (ex
p(-x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(2
```

```
4*a^2) - (exp(x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*log((2*b*exp(x)*(a^2 + b^2
)^2)/a^6 - (2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6)*(a^2 + b^2)^(3/2))/a
^5 + (b*log((2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6 + (2*b*exp(x)*(a^2 +
 b^2)^2)/a^6)*(a^2 + b^2)^(3/2))/a^5
```

3.94 $\int \frac{\cosh^3(x)}{a+b\mathbf{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b(a^2+b^2)\log(b+a\sinh(x))}{a^4}+\frac{(a^2+b^2)\sinh(x)}{a^3}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

```
[Out] -b*(a^2+b^2)*ln(b+a*sinh(x))/a^4+(a^2+b^2)*sinh(x)/a^3-1/2*b*sinh(x)^2/a^2+
1/3*sinh(x)^3/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3957, 2916, 12, 786}

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\sinh^2(x)}{2a^2}-\frac{b(a^2+b^2)\log(a\sinh(x)+b)}{a^4}+\frac{(a^2+b^2)\sinh(x)}{a^3}+\frac{\sinh^3(x)}{3a}$$

```
[In] Int[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)*Log[b + a*Sinh[x]])/a^4) + ((a^2 + b^2)*Sinh[x])/a^3 - (b*
Sinh[x]^2)/(2*a^2) + Sinh[x]^3/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\cosh^3(x)\sinh(x)}{ib + ia\sinh(x)}\,dx \\
&= -\frac{i\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^3} \\
&= -\frac{i\text{Subst}\left(\int \frac{x(a^2-x^2)}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^4} \\
&= -\frac{i\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right) - \frac{b(a^2+b^2)}{b-ix} + ibx - x^2\right)\,dx, x, ia\sinh(x)\right)}{a^4} \\
&= -\frac{b(a^2+b^2)\log(b+a\sinh(x))}{a^4} + \frac{(a^2+b^2)\sinh(x)}{a^3} - \frac{b\sinh^2(x)}{2a^2} + \frac{\sinh^3(x)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \frac{\cosh^3(x)}{a + b\text{csch}(x)}\,dx \\
&= \frac{-6b(a^2+b^2)\log(b+a\sinh(x)) + 6a(a^2+b^2)\sinh(x) - 3a^2b\sinh^2(x) + 2a^3\sinh^3(x)}{6a^4}
\end{aligned}$$

```
[In] Integrate[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-6*b*(a^2 + b^2)*Log[b + a*Sinh[x]] + 6*a*(a^2 + b^2)*Sinh[x] - 3*a^2*b*Si
nh[x]^2 + 2*a^3*Sinh[x]^3)/(6*a^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time $= 14.75$ (sec) , antiderivative size $= 136$, normalized size of antiderivative $= 2.39$

method	result
risch	$\frac{bx}{a^2}+\frac{b^3x}{a^4}+\frac{e^{3x}}{24a}-\frac{be^{2x}}{8a^2}+\frac{3e^x}{8a}+\frac{e^xb^2}{2a^3}-\frac{3e^{-x}}{8a}-\frac{e^{-x}b^2}{2a^3}-\frac{be^{-2x}}{8a^2}-\frac{e^{-3x}}{24a}-\frac{b\ln\left(e^{2x}+\frac{2be^x}{a}-1\right)}{a^2}-\frac{b^3\ln\left(e^{2x}+\frac{2be^x}{a}$
default	$\frac{2b\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\ln\left(-\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^4}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-a+b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{2a^2-ab+2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(a^2+b^2)b\,l$

```
[In] int(cosh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b*x/a^2+1/a^4*b^3*x+1/24/a*exp(3*x)-1/8/a^2*b*exp(2*x)+3/8/a*exp(x)+1/2/a^3
*exp(x)*b^2-3/8/a*exp(-x)-1/2/a^3*exp(-x)*b^2-1/8/a^2*b*exp(-2*x)-1/24/a*ex
p(-3*x)-1/a^2*b*ln(exp(2*x)+2*b/a*exp(x)-1)-1/a^4*b^3*ln(exp(2*x)+2*b/a*exp
(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(53) = 106$.

Time $= 0.26$ (sec) , antiderivative size $= 476$, normalized size of antiderivative $= 8.35$

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{a^3\cosh(x)^6+a^3\sinh(x)^6-3a^2b\cosh(x)^5+3(2a^3\cosh(x)-a^2b)\sinh(x)^5+24(a^2b+b^3)x\cosh(x)^3+}{}$$

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 + 24*(a^2*b + b^3)*x*cosh(x)^3 + 3*(3*a^3 + 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) + 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 + 12*(a^2*b +
b^3)*x + 6*(3*a^3 + 4*a*b^2)*cosh(x))*sinh(x)^3 - a^3 - 3*(3*a^3 + 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 - 4*a*b^2 + 24
*(a^2*b + b^3)*x*cosh(x) + 6*(3*a^3 + 4*a*b^2)*cosh(x)^2)*sinh(x)^2 - 24*((
a^2*b + b^3)*cosh(x)^3 + 3*(a^2*b + b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b + b^3
)*cosh(x)*sinh(x)^2 + (a^2*b + b^3)*sinh(x)^3)*log(2*(a*sinh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 + 24*(a^2*b + b^3)*
x*cosh(x)^2 + 4*(3*a^3 + 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 + 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\cosh^3(x)}{a+b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(cosh(x)**3/(a+b*csch(x)),x)
[Out] Integral(cosh(x)**3/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx = &-\frac{\left(3\,abe^{(-x)} - a^2 - 3\,(3\,a^2+4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3}\\ &-\frac{3\,abe^{(-2\,x)} + a^2e^{(-3\,x)} + 3\,(3\,a^2+4\,b^2)e^{(-x)}}{24\,a^3}\\ &-\frac{(a^2b+b^3)x}{a^4} - \frac{(a^2b+b^3)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^4}\end{aligned}$$

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
[Out] -1/24*(3*a*b*e^(-x) - a^2 - 3*(3*a^2 + 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) + a^2*e^(-3*x) + 3*(3*a^2 + 4*b^2)*e^(-x))/a^3 - (a^2*b + b
^3)*x/a^4 - (a^2*b + b^3)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

$$\begin{aligned}&\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx\\ &= -\frac{a^2\left(e^{(-x)} - e^x\right)^3 + 3\,ab\left(e^{(-x)} - e^x\right)^2 + 12\,a^2\left(e^{(-x)} - e^x\right) + 12\,b^2\left(e^{(-x)} - e^x\right)}{24\,a^3}\\ &\quad - \frac{(a^2b+b^3)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^4}\end{aligned}$$

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="giac")
[Out] -1/24*(a^2*(e^(-x) - e^x)^3 + 3*a*b*(e^(-x) - e^x)^2 + 12*a^2*(e^(-x) - e^x
) + 12*b^2*(e^(-x) - e^x))/a^3 - (a^2*b + b^3)*log(abs(-a*(e^(-x) - e^x) +
2*b))/a^4
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{e^{3x}}{24a} - \frac{e^{-3x}}{24a} + \frac{x\,(a^2 b + b^3)}{a^4} + \frac{e^{x}\,(3a^2 + 4b^2)}{8a^3} - \frac{b\,e^{-2x}}{8a^2} - \frac{b\,e^{2x}}{8a^2} - \frac{\ln\left(2b\,e^{x} - a + a\,e^{2x}\right)\,(a^2 b + b^3)}{a^4} - \frac{e^{-x}\,(3a^2 + 4b^2)}{8a^3}$$

```
[In] int(cosh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) + (x*(a^2*b + b^3))/a^4 + (exp(x)*(3*a^2
 + 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (log(2*
b*exp(x) - a + a*exp(2*x))*(a^2*b + b^3))/a^4 - (exp(-x)*(3*a^2 + 4*b^2))/(
8*a^3)
```

3.95 $\int \frac{\cosh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 508
Rubi [A] (verified) . . . 508
Mathematica [A] (verified) . . . 510
Maple [A] (verified) . . . 510
Fricas [B] (verification not implemented) . . . 511
Sympy [F] . . . 511
Maxima [A] (verification not implemented) . . . 512
Giac [A] (verification not implemented) . . . 512
Mupad [B] (verification not implemented) . . . 512

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{(a^2+2b^2)\,x}{2a^3} + \frac{2b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3} - \frac{\cosh(x)(2b-a\sinh(x))}{2a^2}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3-1/2*cosh(x)*(2*b-a*sinh(x))/a^2+2*b*arctanh((a-b*tanh
(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2944, 2814, 2739, 632, 210}

$$\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\cosh(x)(2b-a\sinh(x))}{2a^2} + \frac{2b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3} + \frac{x(a^2+2b^2)}{2a^3}$$

```
[In] Int[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/
Sqrt[a^2 + b^2]])/a^3 - (Cosh[x]*(2*b - a*Sinh[x]))/(2*a^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = i \int \frac{\cosh^2(x)\sinh(x)}{ib + ia\sinh(x)}\,dx$$

$$
\begin{aligned}
&= -\frac{\cosh(x)(2b-a\sinh(x))}{2a^2} + \frac{\int \frac{-iab+i\left(a^2+2b^2\right)\sinh(x)}{ib+ia\sinh(x)}\,dx}{2a^2} \\
&= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{\cosh(x)(2b-a\sinh(x))}{2a^2} - \frac{(ib(a^2+b^2))\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^3} \\
&= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{\cosh(x)(2b-a\sinh(x))}{2a^2} - \frac{(2ib(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3} \\
&= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{\cosh(x)(2b-a\sinh(x))}{2a^2} \\
&\quad + \frac{(4ib(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{a^3} \\
&= \frac{\left(a^2+2b^2\right)x}{2a^3} + \frac{2b\sqrt{a^2+b^2}\,\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3} - \frac{\cosh(x)(2b-a\sinh(x))}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int \frac{\cosh^2(x)}{a+b\,\mathrm{csch}(x)}\,dx \\
&= \frac{2a^2x+4b^2x+8b\sqrt{-a^2-b^2}\,\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right) - 4ab\cosh(x) + a^2\sinh(2x)}{4a^3}
\end{aligned}
$$

```
[In] Integrate[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + 8*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^
2 - b^2]] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

method	result
risch	$\frac{x}{2a} + \frac{x\,b^2}{a^3} + \frac{\mathrm{e}^{2x}}{8a} - \frac{b\,\mathrm{e}^{x}}{2a^2} - \frac{b\,\mathrm{e}^{-x}}{2a^2} - \frac{\mathrm{e}^{-2x}}{8a} + \frac{\sqrt{a^2+b^2}\,b\ln\left(\mathrm{e}^{x}+\frac{b+\sqrt{a^2+b^2}}{a}\right)}{a^3} - \frac{\sqrt{a^2+b^2}\,b\ln\left(\mathrm{e}^{x}-\frac{-b+\sqrt{a^2+b^2}}{a}\right)}{a^3}$
default	$\frac{2b\sqrt{a^2+b^2}\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a^3} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{-a+2b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{\left(a^2+2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^3} + \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right.}$

```
[In] int(cosh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/a+x/a^3*b^2+1/8/a*exp(x)^2-1/2*b/a^2*exp(x)-1/2*b/a^2/exp(x)-1/8/a/ex
p(x)^2+(a^2+b^2)^(1/2)*b/a^3*ln(exp(x)+(b+(a^2+b^2)^(1/2))/a)-(a^2+b^2)^(1/
2)*b/a^3*ln(exp(x)-(-b+(a^2+b^2)^(1/2))/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(68) = 136$.

Time $= 0.26$ (sec) , antiderivative size $= 304$, normalized size of antiderivative $= 3.95$

$$\int \frac{\cosh^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

$$= \frac{a^2\cosh(x)^4 + a^2\sinh(x)^4 - 4\,ab\cosh(x)^3 + 4\,(a^2 + 2\,b^2)x\cosh(x)^2 + 4\,(a^2\cosh(x) - ab)\sinh(x)^3 - 4\,a}{}$$

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 + 4*(a^2 + 2*b^2)*x*co
sh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*a^2*cosh(x
)^2 - 6*a*b*cosh(x) + 2*(a^2 + 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*c
osh(x)*sinh(x) + b*sinh(x)^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh
(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqr
t(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*
cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a*b*
cosh(x)^2 + 2*(a^2 + 2*b^2)*x*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\cosh^2(x)}{a + b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(cosh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\left(4\,be^{(-x)}-a\right)e^{(2\,x)}}{8\,a^2} - \frac{4\,be^{(-x)}+ae^{(-2\,x)}}{8\,a^2} + \frac{(a^2+2\,b^2)x}{2\,a^3} - \frac{(a^2b+b^3)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3}$$

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x) + a*e^(-2*x))/a^2 + 1/2
*(a^2 + 2*b^2)*x/a^3 - (a^2*b + b^3)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(
a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{ae^{(2\,x)}-4\,be^{x}}{8\,a^2} + \frac{(a^2+2\,b^2)x}{2\,a^3} - \frac{(4\,abe^{x}+a^2)e^{(-2\,x)}}{8\,a^3} - \frac{(a^2b+b^3)\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}$$

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x +
a^2)*e^(-2*x)/a^3 - (a^2*b + b^3)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2)
)/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{2\,x}}{8\,a} - \frac{\mathrm{e}^{-2\,x}}{8\,a} - \frac{b\,\mathrm{e}^{x}}{2\,a^2} - \frac{b\,\mathrm{e}^{-x}}{2\,a^2} + \frac{x\,(a^2+2\,b^2)}{2\,a^3} - \frac{b\,\ln\left(\frac{2\,b\,\mathrm{e}^{x}\,(a^2+b^2)}{a^4} - \frac{2\,b\,(a-b\,\mathrm{e}^{x})\,\sqrt{a^2+b^2}}{a^4}\right)\,\sqrt{a^2+b^2}}{a^3} + \frac{b\,\ln\left(\frac{2\,b\,(a-b\,\mathrm{e}^{x})\,\sqrt{a^2+b^2}}{a^4} + \frac{2\,b\,\mathrm{e}^{x}\,(a^2+b^2)}{a^4}\right)\,\sqrt{a^2+b^2}}{a^3}$$

```
[In] int(cosh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2)
 + (x*(a^2 + 2*b^2))/(2*a^3) - (b*log((2*b*exp(x)*(a^2 + b^2))/a^4 - (2*b*(
a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4)*(a^2 + b^2)^(1/2))/a^3 + (b*log((2*b*
(a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4 + (2*b*exp(x)*(a^2 + b^2))/a^4)*(a^2
+ b^2)^(1/2))/a^3
```

3.96 $\int \frac{\cosh(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 514
Rubi [A] (verified) . . . 514
Mathematica [A] (verified) . . . 515
Maple [A] (verified) . . . 516
Fricas [B] (verification not implemented) . . . 516
Sympy [F] . . . 516
Maxima [B] (verification not implemented) . . . 517
Giac [A] (verification not implemented) . . . 517
Mupad [B] (verification not implemented) . . . 517

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\cosh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\log(b+a\sinh(x))}{a^2}+\frac{\sinh(x)}{a}$$

```
[Out] -b*ln(b+a*sinh(x))/a^2+sinh(x)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\cosh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\sinh(x)}{a}-\frac{b\log(a\sinh(x)+b)}{a^2}$$

```
[In] Int[Cosh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*Log[b + a*Sinh[x]])/a^2) + Sinh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\cosh(x)\sinh(x)}{ib + ia\sinh(x)}\,dx \\
&= -\frac{i\text{Subst}\left(\int \frac{x}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a} \\
&= -\frac{i\text{Subst}\left(\int \frac{x}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^2} \\
&= -\frac{i\text{Subst}\left(\int \left(1 - \frac{b}{b-ix}\right)\,dx, x, ia\sinh(x)\right)}{a^2} \\
&= -\frac{b\log(b + a\sinh(x))}{a^2} + \frac{\sinh(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.01$ (sec) , antiderivative size $= 19$, normalized size of antiderivative $= 0.95$

$$\int \frac{\cosh(x)}{a + b\text{csch}(x)}\,dx = \frac{-b\log(b + a\sinh(x)) + a\sinh(x)}{a^2}$$

```
[In] Integrate[Cosh[x]/(a + b*Csch[x]),x]
```

```
[Out] (-(b*Log[b + a*Sinh[x]]) + a*Sinh[x])/a^2
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{1}{a\ \mathrm{csch}(x)} + \frac{b\ln(\mathrm{csch}(x))}{a^2} - \frac{b\ln(a+b\ \mathrm{csch}(x))}{a^2}$	31
default	$\frac{1}{a\ \mathrm{csch}(x)} + \frac{b\ln(\mathrm{csch}(x))}{a^2} - \frac{b\ln(a+b\ \mathrm{csch}(x))}{a^2}$	31
risch	$\frac{bx}{a^2} + \frac{e^x}{2a} - \frac{e^{-x}}{2a} - \frac{b\ln\left(e^{2x}+\frac{2be^x}{a}-1\right)}{a^2}$	45

```
[In] int(cosh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
[Out] 1/a/csch(x)+1/a^2*b*ln(csch(x))-1/a^2*b*ln(a+b*csch(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.00

$$\int \frac{\cosh(x)}{a+b\mathrm{csch}(x)}\,dx$$

$$= \frac{2\,bx\cosh(x) + a\cosh(x)^2 + a\sinh(x)^2 - 2\,(b\cosh(x)+b\sinh(x))\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right) + 2\,(bx+a\cosh(x)}{2\,(a^2\cosh(x)+a^2\sinh(x))}$$

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="fricas")
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) -
a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F]

$$\int \frac{\cosh(x)}{a+b\mathrm{csch}(x)}\,dx = \int \frac{\cosh(x)}{a+b\,\mathrm{csch}(x)}\,dx$$

```
[In] integrate(cosh(x)/(a+b*csch(x)),x)
[Out] Integral(cosh(x)/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time $= 0.17$ (sec) , antiderivative size $= 48$, normalized size of antiderivative $= 2.40$

$$\int \frac{\cosh(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{b\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^2}$$

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 - 1/2*e^(-x)/a + 1/2*e^x/a - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
^2
```

Giac [A] (verification not implemented)

none

Time $= 0.27$ (sec) , antiderivative size $= 39$, normalized size of antiderivative $= 1.95$

$$\int \frac{\cosh(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{e^{(-x)} - e^{x}}{2\,a} - \frac{b\log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^2}$$

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/a - b*log(abs(-a*(e^(-x) - e^x) + 2*b))/a^2
```

Mupad [B] (verification not implemented)

Time $= 0.08$ (sec) , antiderivative size $= 20$, normalized size of antiderivative $= 1.00$

$$\int \frac{\cosh(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\sinh(x)}{a} - \frac{b\,\ln\left(b + a\sinh(x)\right)}{a^2}$$

```
[In] int(cosh(x)/(a + b/sinh(x)),x)
```

```
[Out] sinh(x)/a - (b*log(b + a*sinh(x)))/a^2
```

3.97 $\int \frac{\mathbf{sech}(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 518
Rubi [A] (verified) . . . 518
Mathematica [A] (verified) . . . 519
Maple [A] (verified) . . . 520
Fricas [A] (verification not implemented) . . . 520
Sympy [F] . . . 520
Maxima [A] (verification not implemented) . . . 521
Giac [A] (verification not implemented) . . . 521
Mupad [B] (verification not implemented) . . . 521

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log(i-\sinh(x))}{2(ia+b)} - \frac{\log(i+\sinh(x))}{2(ia-b)} - \frac{b\log(b+a\sinh(x))}{a^2+b^2}$$

```
[Out] 1/2*ln(I-sinh(x))/(I*a+b)-1/2*ln(I+sinh(x))/(I*a-b)-b*ln(b+a*sinh(x))/(a^2+
b^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3957, 2800, 815}

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\log(a\sinh(x)+b)}{a^2+b^2} + \frac{\log(-\sinh(x)+i)}{2(b+ia)} - \frac{\log(\sinh(x)+i)}{2(-b+ia)}$$

```
[In] Int[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]]/(2*(I*a + b)) - Log[I + Sinh[x]]/(2*(I*a - b)) - (b*Log[b
+ a*Sinh[x]])/(a^2 + b^2)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\tanh(x)}{ib + ia\sinh(x)}\,dx \\
&= -\left(i\text{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)}\,dx, x, ia\sinh(x)\right)\right) \\
&= \\
&\quad -\left(i\text{Subst}\left(\int \left(\frac{1}{2(a+ib)(a-x)} - \frac{b}{\left(a^2+b^2\right)(b-ix)} + \frac{1}{2(a-ib)(a+x)}\right)\,dx, x, ia\sinh(x)\right)\right) \\
&= \frac{\log(i-\sinh(x))}{2(ia+b)} - \frac{\log(i+\sinh(x))}{2(ia-b)} - \frac{b\log(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\begin{aligned}
&\int \frac{\text{sech}(x)}{a + b\text{csch}(x)}\,dx \\
&= \frac{(-ia+b)\log(i-\sinh(x)) + (ia+b)\log(i+\sinh(x)) - 2b\log(b+a\sinh(x))}{2\left(a^2+b^2\right)}
\end{aligned}$$

```
[In] Integrate[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] (((-I)*a + b)*Log[I - Sinh[x]] + (I*a + b)*Log[I + Sinh[x]] - 2*b*Log[b + a
*Sinh[x]])/(2*(a^2 + b^2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+4a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^2+2b^2}-\frac{2b\ln\left(-\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{2a^2+2b^2}$	72
risch	$\frac{i\ln(e^x+i)a}{a^2+b^2}+\frac{\ln(e^x+i)b}{a^2+b^2}-\frac{i\ln(e^x-i)a}{a^2+b^2}+\frac{\ln(e^x-i)b}{a^2+b^2}-\frac{b\ln\left(e^{2x}+\frac{2b\,e^x}{a}-1\right)}{a^2+b^2}$	101

```
[In] int(sech(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(2*a^2+2*b^2)*(1/2*b*ln(1+tanh(1/2*x)^2)+a*arctan(tanh(1/2*x)))-2*b/(2*a^
2+2*b^2)*ln(-tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b\log\left(\frac{2\left(a\sinh(x)+b\right)}{\cosh(x)-\sinh(x)}\right)+b\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) - b*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x)
)) + b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(sech(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2} - \frac{b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^2+b^2} + \frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^2
+ b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{ab\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^3+ab^2} + \frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)} + \frac{b\log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) + 1/2*b*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{b+a\,1\mathrm{i}} - \frac{b\,\ln\left(a^3\,\mathrm{e}^{2\,x}-4\,a\,b^2-a^3+8\,b^3\,\mathrm{e}^{x}+2\,a^2\,b\,\mathrm{e}^{x}+4\,a\,b^2\,\mathrm{e}^{2\,x}\right)}{a^2+b^2} + \frac{\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

```
[In] int(1/(cosh(x)*(a + b/sinh(x))),x)
```

```
[Out] log(exp(x)*1i + 1)/(a*1i + b) + (log(exp(x) + 1i)*1i)/(a + b*1i) - (b*log(a
^3*exp(2*x) - 4*a*b^2 - a^3 + 8*b^3*exp(x) + 2*a^2*b*exp(x) + 4*a*b^2*exp(2
*x)))/(a^2 + b^2)
```

3.98 $\int \frac{\mathbf{sech}^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 522
Rubi [A] (verified) . . . 522
Mathematica [A] (verified) . . . 524
Maple [A] (verified) . . . 524
Fricas [B] (verification not implemented) . . . 525
Sympy [F] . . . 525
Maxima [A] (verification not implemented) . . . 525
Giac [A] (verification not implemented) . . . 526
Mupad [B] (verification not implemented) . . . 526

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2ab\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}} - \frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

```
[Out] 2*a*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-sech(x)*(b
-a*sinh(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2945, 12, 2739, 632, 210}

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2ab\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}} - \frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

```
[In] Int[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (Sec
h[x]*(b - a*Sinh[x]))/(a^2 + b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\operatorname{sech}(x)\tanh(x)}{ib + ia\sinh(x)}\,dx \\
&= -\frac{\operatorname{sech}(x)(b - a\sinh(x))}{a^2 + b^2} - \frac{i \int \frac{ab}{ib + ia\sinh(x)}\,dx}{a^2 + b^2} \\
&= -\frac{\operatorname{sech}(x)(b - a\sinh(x))}{a^2 + b^2} - \frac{(iab) \int \frac{1}{ib + ia\sinh(x)}\,dx}{a^2 + b^2} \\
&= -\frac{\operatorname{sech}(x)(b - a\sinh(x))}{a^2 + b^2} - \frac{(2iab)\operatorname{Subst}\left(\int \frac{1}{ib + 2iax - ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2 + b^2}
\end{aligned}$$

$$= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2} + \frac{(4iab)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$= \frac{2ab\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}} - \frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{-b\operatorname{sech}(x) + a\left(-\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \tanh(x)\right)}{a^2+b^2}$$

```
[In] Integrate[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((-2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2] + Tanh[x]))/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{4ab\ \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{(2a^2+2b^2)\sqrt{a^2+b^2}} - \frac{2\left(-a\tanh\left(\frac{x}{2}\right)+b\right)}{(a^2+b^2)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$	81
risch	$-\frac{2(\mathrm{e}^x b+a)}{(1+\mathrm{e}^{2x})(a^2+b^2)} + \frac{ba\ln\left(\mathrm{e}^x+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{ba\ln\left(\mathrm{e}^x+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b-a^4-2a^2b^2-b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	142

```
[In] int(sech(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2
+b^2)^(1/2))-2/(a^2+b^2)*(-a*tanh(1/2*x)+b)/(1+tanh(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(56) = 112$.

Time $= 0.24$ (sec) , antiderivative size $= 256$, normalized size of antiderivative $= 4.27$

$$\int \frac{\operatorname{sech}^2(x)}{a + b\operatorname{csch}(x)}\,dx =$$
$$-\frac{2\,a^3 + 2\,ab^2 - \left(ab\cosh\left(x\right)^2 + 2\,ab\cosh\left(x\right)\sinh\left(x\right) + ab\sinh\left(x\right)^2 + ab\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)}{a^4 + 2\,a^2b^2 + b^4 + \left(a^4 + 2\,a^2b^2 + b^4\right)\cosh\left(x\right)^2 + 2\,(a}\right.$$

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
+ a*b)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) - a)) + 2*(a^2*b + b^3)*cosh(x) + 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^2\left(x\right)}{a + b\,\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.27$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 1.52$

$$\int \frac{\operatorname{sech}^2(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{ab\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}} - \frac{2\left(be^{(-x)} - a\right)}{a^2 + b^2 + (a^2+b^2)e^{(-2\,x)}}$$

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{ab\log\left(\frac{\left|2\,ae^x+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}} - \frac{2\left(be^x+a\right)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x + a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a\,b\,\ln\left(\frac{2\,b\,(a-b\,\mathrm{e}^x)}{(a^2+b^2)^{3/2}}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}\right)}{\left(a^2+b^2\right)^{3/2}} - \frac{a\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}-\frac{2\,b\,(a-b\,\mathrm{e}^x)}{(a^2+b^2)^{3/2}}\right)}{\left(a^2+b^2\right)^{3/2}} - \frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1}$$

```
[In] int(1/(cosh(x)^2*(a + b/sinh(x))),x)
```

```
[Out] (a*b*log((2*b*(a - b*exp(x)))/(a^2 + b^2)^(3/2) + (2*b*exp(x))/(a^2 + b^2))
)/(a^2 + b^2)^(3/2) - (a*b*log((2*b*exp(x))/(a^2 + b^2) - (2*b*(a - b*exp(x
)))/(a^2 + b^2)^(3/2)))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) + (2*b*exp(x
))/(a^2 + b^2))/(exp(2*x) + 1)
```

3.99 $\int \frac{\mathbf{sech}^3(x)}{a+b\mathbf{csch}(x)}\,dx$

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Mupad [B] (verification not implemented) . . . 532

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{ia\log(i-\sinh(x))}{4(a-ib)^2} + \frac{ia\log(i+\sinh(x))}{4(a+ib)^2} - \frac{a^2b\log(b+a\sinh(x))}{(a^2+b^2)^2} - \frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2(a^2+b^2)}$$

```
[Out] -1/4*I*a*ln(I-sinh(x))/(a-I*b)^2+1/4*I*a*ln(I+sinh(x))/(a+I*b)^2-a^2*b*ln(b
+a*sinh(x))/(a^2+b^2)^2-1/2*sech(x)^2*(b-a*sinh(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2916, 12, 837, 815}

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^2b\log(a\sinh(x)+b)}{(a^2+b^2)^2} - \frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2(a^2+b^2)} - \frac{ia\log(-\sinh(x)+i)}{4(a-ib)^2} + \frac{ia\log(\sinh(x)+i)}{4(a+ib)^2}$$

```
[In] Int[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-1/4*I)*a*Log[I - Sinh[x]])/(a - I*b)^2 + ((I/4)*a*Log[I + Sinh[x]])/(a +
 I*b)^2 - (a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^2 - (Sech[x]^2*(b - a*Sinh
[x]))/(2*(a^2 + b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\operatorname{sech}^2(x)\tanh(x)}{ib + ia\sinh(x)}\,dx \\
&= -\left((ia^3)\,\text{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x) \right) \right) \\
&= -\left((ia^2)\,\text{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x) \right) \right) \\
&= -\frac{\operatorname{sech}^2(x)(b - a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\text{Subst}\left(\int \frac{-ia^2b+a^2x}{(ib+x)(a^2-x^2)}\,dx, x, ia\sinh(x) \right)}{2\left(a^2+b^2\right)}
\end{aligned}$$

$$= -\frac{\operatorname{sech}^2(x)(b - a\sinh(x))}{2\left(a^2 + b^2\right)}$$
$$- \frac{i\text{Subst}\left(\int \left(\frac{a(a-ib)}{2(a+ib)(a-x)} - \frac{2a^2b}{(a^2+b^2)(b-ix)} + \frac{a(a+ib)}{2(a-ib)(a+x)}\right) dx, x, ia\sinh(x)\right)}{2\left(a^2 + b^2\right)}$$
$$= -\frac{ia\log(i - \sinh(x))}{4(a - ib)^2} + \frac{ia\log(i + \sinh(x))}{4(a + ib)^2} - \frac{a^2b\log(b + a\sinh(x))}{\left(a^2 + b^2\right)^2} - \frac{\operatorname{sech}^2(x)(b - a\sinh(x))}{2\left(a^2 + b^2\right)}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\, dx =$$
$$-\frac{i\operatorname{csch}(x)(b + a\sinh(x))\left(\frac{a\log(i-\sinh(x))}{(a-ib)^2} - \frac{a\log(i+\sinh(x))}{(a+ib)^2} - \frac{4ia^2b\log(b+a\sinh(x))}{(a^2+b^2)^2} + \frac{1}{(ia+b)(i-\sinh(x))} + \frac{i}{(a+ib)(i+\sinh}{4(a + b\operatorname{csch}(x))}$$

```
[In] Integrate[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-1/4*I)*Csch[x]*(b + a*Sinh[x])*((a*Log[I - Sinh[x]])/(a - I*b)^2 - (a*Lo
g[I + Sinh[x]])/(a + I*b)^2 - ((4*I)*a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^
2 + 1/((I*a + b)*(I - Sinh[x])) + I/((a + I*b)*(I + Sinh[x])))/(a + b*Csch
[x])
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.51

method	result
default	$\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(a^2b+b^3\right)\tanh\left(\frac{x}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+a\left(ab\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+\left(a^2-b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{\left(a^2+b^2\right)^2} - \frac{c}{}$
risch	$\frac{e^x\left(a\,e^{2x}-2\,e^xb-a\right)}{\left(1+e^{2x}\right)^2\left(a^2+b^2\right)} - \frac{ia^3\ln\left(e^x-i\right)}{2\left(a^4+2a^2b^2+b^4\right)} + \frac{ia\ln\left(e^x-i\right)b^2}{2a^4+4a^2b^2+2b^4} + \frac{a^2\ln\left(e^x-i\right)b}{a^4+2a^2b^2+b^4} + \frac{ia^3\ln\left(e^x+i\right)}{2a^4+4a^2b^2+2b^4} - \frac{ia\ln\left(e^x+i\right)b^2}{2\left(a^4+2a^2b^2+b^4\right)} + \frac{a^2}{a^4+}$

```
[In] int(sech(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*x)^3+(a^2*b+b^3)*tanh(1/2*x)^
2+(1/2*a^3+1/2*a*b^2)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+1/2*a*(a*b*ln(1+tanh
(1/2*x)^2)+(a^2-b^2)*arctan(tanh(1/2*x))))-a^2*b/(a^2+b^2)^2*ln(-tanh(1/2*x
)^2*b+2*a*tanh(1/2*x)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(78) = 156$.

Time $= 0.27$ (sec) , antiderivative size $= 675$, normalized size of antiderivative $= 7.11$

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

$$= \frac{(a^3 + ab^2)\cosh(x)^3 + (a^3 + ab^2)\sinh(x)^3 - 2\,(a^2b + b^3)\cosh(x)^2 - (2\,a^2b + 2\,b^3 - 3\,(a^3 + ab^2)\cosh(x))\,\mathrm{si}}{}$$

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 - 2*(a^2*b + b^3)*cosh(x
)^2 - (2*a^2*b + 2*b^3 - 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 - a*b^2
)*cosh(x)^4 + 4*(a^3 - a*b^2)*cosh(x)*sinh(x)^3 + (a^3 - a*b^2)*sinh(x)^4 +
 a^3 - a*b^2 + 2*(a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))
*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(x) - (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3
 + a*b^2)*cosh(x)^2 + 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^
2*b^2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*
cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)
*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)
*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(sech(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = &-\frac{a^2 b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^4+2\,a^2b^2+b^4}\\ &+\frac{a^2 b\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{(a^3-ab^2)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}\\ &+\frac{ae^{(-x)}-2\,be^{(-2\,x)}-ae^{(-3\,x)}}{a^2+b^2+2\,(a^2+b^2)e^{(-2\,x)}+(a^2+b^2)e^{(-4\,x)}}\end{aligned}$$

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^4 + 2*a^2*b^2 + b^4) + a^2*b*lo
g(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 - a*b^2)*arctan(e^(-x))/(a^4
 + 2*a^2*b^2 + b^4) + (a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.29

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = &-\frac{a^3 b\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^5+2\,a^3b^2+ab^4}+\frac{a^2 b\log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)}\\ &+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^3-ab^2)}{4\,(a^4+2\,a^2b^2+b^4)}\\ &-\frac{a^2 b\left(e^{(-x)}-e^{x}\right)^2+2\,a^3\left(e^{(-x)}-e^{x}\right)+2\,ab^2\left(e^{(-x)}-e^{x}\right)+8\,a^2b+4\,b^3}{2\,(a^4+2\,a^2b^2+b^4)\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}\end{aligned}$$

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) + 1/2*a^
2*b*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*(a^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2
*b*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) + 2*a*b^2*(e^(-x) - e^x) + 8*a^2
*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.69

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{2\,b}{a^2+b^2}-\frac{2\,a\,\mathrm{e}^x}{a^2+b^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} - \frac{\frac{2\,(a^2\,b+b^3)}{(a^2+b^2)^2}-\frac{\mathrm{e}^x\,(a^3+a\,b^2)}{(a^2+b^2)^2}}{\mathrm{e}^{2\,x}+1} + \frac{a\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)}{2\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}$$
$$-\frac{a^2\,b\,\ln\left(a^6\,\mathrm{e}^{2\,x}-a^6-a^2\,b^4-14\,a^4\,b^2+a^2\,b^4\,\mathrm{e}^{2\,x}+14\,a^4\,b^2\,\mathrm{e}^{2\,x}+2\,a\,b^5\,\mathrm{e}^x+2\,a^5\,b\,\mathrm{e}^x+28\,a^3\,b^3\,\mathrm{e}^x\right)}{a^4+2\,a^2\,b^2+b^4}$$
$$+\frac{a\,\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

```
[In] int(1/(cosh(x)^3*(a + b/sinh(x))),x)
```

```
[Out] ((2*b)/(a^2 + b^2) - (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) + exp(4*x) + 1)
- ((2*(a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/
(exp(2*x) + 1) + (a*log(exp(x)*1i + 1)*1i)/(2*(a*b*2i - a^2 + b^2)) + (a*lo
g(exp(x) + 1i))/(2*(2*a*b - a^2*1i + b^2*1i)) - (a^2*b*log(a^6*exp(2*x) - a
^6 - a^2*b^4 - 14*a^4*b^2 + a^2*b^4*exp(2*x) + 14*a^4*b^2*exp(2*x) + 2*a*b^
5*exp(x) + 2*a^5*b*exp(x) + 28*a^3*b^3*exp(x)))/(a^4 + b^4 + 2*a^2*b^2)
```

3.100 $\int \frac{\mathbf{sech}^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 533
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Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2a^3b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

```
[Out] 2*a^3*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-1/3*sech
(x)^3*(b-a*sinh(x))/(a^2+b^2)-1/3*sech(x)*(3*a^2*b-a*(2*a^2-b^2)*sinh(x))/(
a^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2945, 12, 2739, 632, 210}

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{2a^3b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[In] Int[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (2*a^3*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (S
ech[x]^3*(b - a*Sinh[x]))/(3*(a^2 + b^2)) - (Sech[x]*(3*a^2*b - a*(2*a^2 -
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = i \int \frac{\operatorname{sech}^3(x) \tanh(x)}{ib + ia \sinh(x)}\, dx$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} + \frac{\int \frac{\operatorname{sech}^2(x)(-iab+2ia^2\sinh(x))}{ib+ia\sinh(x)}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\int -\frac{3ia^3b}{ib+ia\sinh(x)}\,dx}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} - \frac{(ia^3b)\int \frac{1}{ib+ia\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} - \frac{(2ia^3b)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{(4ia^3b)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)^2}$$

$$= \frac{2a^3b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\frac{6a^3b\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3a^2b\operatorname{sech}(x) + b\left(a^2+b^2\right)\operatorname{sech}^3(x) + \left(-2a^3+ab^2\right)\tanh(x) - a\left(a^2+b^2\right)\operatorname{sech}^2(x}{3\left(a^2+b^2\right)^2}$$

```
[In] Integrate[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/3*((6*a^3*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 + 3*a^2*b*Sech[x] + b*(a^2 + b^2)*Sech[x]^3 + (-2*a^3 + a*b^2)*Tanh[x] - a
*(a^2 + b^2)*Sech[x]^2*Tanh[x])/(a^2 + b^2)^2
```

Maple [A] (verified)

Time = 7.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.63

method	result
default	$\frac{4a^3b\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{(2a^4+4a^2b^2+2b^4)\sqrt{a^2+b^2}}-\frac{2\left(-a^3\tanh\left(\frac{x}{2}\right)^5+(2a^2b+b^3)\tanh\left(\frac{x}{2}\right)^4+\left(-\frac{2}{3}a^3+\frac{4}{3}ab^2\right)\tanh\left(\frac{x}{2}\right)^3+2a^2b\tanh\left(\frac{x}{2}\right)^2-a^3\tanh\left(\frac{x}{2}\right)\right.}{(a^4+2a^2b^2+b^4)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}$
risch	$-\frac{2\left(3a^2b\,e^{5x}-3ab^2e^{4x}+10a^2b\,e^{3x}+4b^3e^{3x}+6a^3e^{2x}+3a^2b\,e^x+2a^3-ab^2\right)}{3(1+e^{2x})^3(a^2+b^2)^2}+\frac{ba^3\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}b+a^6+3a^4b^2+3a^2b^4+b^6}{a\left(a^2+b^2\right)^{\frac{5}{2}}}\right)}{(a^2+b^2)^{\frac{5}{2}}}-\frac{ba^3}{}$

```
[In] int(sech(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*a^3*b/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*
x)+2*a)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(-a^3*tanh(1/2*x)^5+(2*a^2*b
+b^3)*tanh(1/2*x)^4+(-2/3*a^3+4/3*a*b^2)*tanh(1/2*x)^3+2*a^2*b*tanh(1/2*x)^
2-a^3*tanh(1/2*x)+4/3*a^2*b+1/3*b^3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. 2(97) = 194.

Time = 0.26 (sec) , antiderivative size = 1155, normalized size of antiderivative = 11.11

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 4*a^5
 + 2*a^3*b^2 - 2*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4
 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5)
*cosh(x)^3 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5 + 15*(a^4*b + a^2*b^3)*cosh(x)^
2 - 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 +
 12*(a^5 + a^3*b^2 + 5*(a^4*b + a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 + a*b^4)*co
sh(x)^2 + (5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(
x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 + 3*a^3*b*cosh(x)^4 + 3*
a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^4 + a^3*b + 4*(5*a^
3*b*cosh(x)^3 + 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 + 6*a^3*b
*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 + 2*a^3*b*cosh(x)^3 + a^
3*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 +
2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) - a)) + 6*(a^4*b + a^2*b^3)*cosh(x) + 6*(a^4*b
+ a^2*b^3 + 5*(a^4*b + a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 +
```

```
 2*(5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 + a^3*b^2)*cosh(x))*sin
h(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
inh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sin
h(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\text{sech}^4(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\text{sech}^4(x)}{a + b\,\text{csch}(x)}\,dx$$

```
[In] integrate(sech(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(97) = 194$.

Time $= 0.29$ (sec) , antiderivative size $= 226$, normalized size of antiderivative $= 2.17$

$$\int \frac{\text{sech}^4(x)}{a + b\text{csch}(x)}\,dx = -\frac{a^3 b \log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{(a^4 + 2a^2b^2 + b^4)\sqrt{a^2+b^2}} - \frac{2\left(3a^2be^{(-x)} - 6a^3e^{(-2x)} + 3ab^2e^{(-4x)} + 3a^2be^{(-5x)} - 2a^3 + ab^2 + 2(5a^2b + 2b^3)e^{(-3x)}\right)}{3(a^4 + 2a^2b^2 + b^4 + 3(a^4 + 2a^2b^2 + b^4)e^{(-2x)} + 3(a^4 + 2a^2b^2 + b^4)e^{(-4x)} + (a^4 + 2a^2b^2 + b^4)e^{(-6x)})}$$

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^3*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)
))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(-x) - 6*a^3*
e^(-2*x) + 3*a*b^2*e^(-4*x) + 3*a^2*b*e^(-5*x) - 2*a^3 + a*b^2 + 2*(5*a^2*b
 + 2*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-
2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x
))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= -\frac{a^3 b \log\left(\frac{\left|2\,a e^x+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,a e^x+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}$$
$$-\frac{2\left(3\,a^2 b e^{(5\,x)}-3\,a b^2 e^{(4\,x)}+10\,a^2 b e^{(3\,x)}+4\,b^3 e^{(3\,x)}+6\,a^3 e^{(2\,x)}+3\,a^2 b e^x+2\,a^3-a b^2\right)}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqr
t(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(
5*x) - 3*a*b^2*e^(4*x) + 10*a^2*b*e^(3*x) + 4*b^3*e^(3*x) + 6*a^3*e^(2*x) +
 3*a^2*b*e^x + 2*a^3 - a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{2\,a\,b^2}{(a^2+b^2)^2}-\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2+b^2)^2}}{\mathrm{e}^{2\,x}+1}-\frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2}+\frac{8\,\mathrm{e}^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{\frac{8\,a}{3\,(a^2+b^2)}+\frac{8\,b\,\mathrm{e}^x}{3\,(a^2+b^2)}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1}$$
$$-\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2+b^2)^2}-\frac{2\,a^2\,b\,(a-b\,\mathrm{e}^x)}{(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}}+\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,(a-b\,\mathrm{e}^x)}{(a^2+b^2)^{5/2}}+\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2+b^2)^2}\right)}{(a^2+b^2)^{5/2}}$$

```
[In] int(1/(cosh(x)^4*(a + b/sinh(x))),x)
```

```
[Out] ((2*a*b^2)/(a^2 + b^2)^2 - (2*a^2*b*exp(x))/(a^2 + b^2)^2)/(exp(2*x) + 1) -
 ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 + (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)
^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3
*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (a^3*b*log((2*a^2
*b*exp(x))/(a^2 + b^2)^2 - (2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2)))/(a^
2 + b^2)^(5/2) + (a^3*b*log((2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2) + (2
*a^2*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

3.101 $\int \frac{\mathbf{sech}^5(x)}{a+b\mathbf{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a(3ia+b)\log(i-\sinh(x))}{16(a-ib)^3} + \frac{a(3a+ib)\log(i+\sinh(x))}{16(ia-b)^3} - \frac{a^4 b\log(b+a\sinh(x))}{(a^2+b^2)^3} - \frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4(a^2+b^2)} - \frac{\operatorname{sech}^2(x)\left(4a^2b-a(3a^2-b^2)\sinh(x)\right)}{8(a^2+b^2)^2}$$

```
[Out] -1/16*a*(3*I*a+b)*ln(I-sinh(x))/(a-I*b)^3+1/16*a*(3*a+I*b)*ln(I+sinh(x))/(I
*a-b)^3-a^4*b*ln(b+a*sinh(x))/(a^2+b^2)^3-1/4*sech(x)^4*(b-a*sinh(x))/(a^2+
b^2)-1/8*sech(x)^2*(4*a^2*b-a*(3*a^2-b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2916, 12, 837, 815}

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4(a^2+b^2)} - \frac{\operatorname{sech}^2(x)\left(4a^2b-a(3a^2-b^2)\sinh(x)\right)}{8(a^2+b^2)^2} - \frac{a^4 b\log(a\sinh(x)+b)}{(a^2+b^2)^3} - \frac{a(b+3ia)\log(-\sinh(x)+i)}{16(a-ib)^3} + \frac{a(3a+ib)\log(\sinh(x)+i)}{16(-b+ia)^3}$$

```
[In] Int[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -1/16*(a*((3*I)*a + b)*Log[I - Sinh[x]])/(a - I*b)^3 + (a*(3*a + I*b)*Log[I
 + Sinh[x]])/(16*(I*a - b)^3) - (a^4*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^3 -
(Sech[x]^4*(b - a*Sinh[x]))/(4*(a^2 + b^2)) - (Sech[x]^2*(4*a^2*b - a*(3*a^
2 - b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i \int \frac{\operatorname{sech}^4(x)\tanh(x)}{ib + ia\sinh(x)}\,dx \\ &= -\left((ia^5)\,\text{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^3}\,dx, x, ia\sinh(x) \right) \right)\end{aligned}$$

$$= -\left((ia^4)\,\text{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^3}\,dx, x, ia\sinh(x) \right) \right)$$

$$= -\frac{\text{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{-ia^2b+3a^2x}{(ib+x)(a^2-x^2)^2}\,dx, x, ia\sinh(x) \right)}{4\left(a^2+b^2\right)}$$

$$= -\frac{\text{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\text{sech}^2(x)\left(4a^2b-a(3a^2-b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}$$
$$- \frac{i\,\text{Subst}\left(\int \frac{-ia^2b(5a^2+b^2)+a^2(3a^2-b^2)x}{(ib+x)(a^2-x^2)}\,dx, x, ia\sinh(x) \right)}{8\left(a^2+b^2\right)^2}$$

$$= -\frac{\text{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\text{sech}^2(x)\left(4a^2b-a(3a^2-b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}$$
$$- \frac{i\,\text{Subst}\left(\int \left(\frac{a(a-ib)^2(3a+ib)}{2(a+ib)(a-x)} - \frac{8a^4b}{(a^2+b^2)(b-ix)} + \frac{a(3a-ib)(a+ib)^2}{2(a-ib)(a+x)} \right)\,dx, x, ia\sinh(x) \right)}{8\left(a^2+b^2\right)^2}$$

$$= -\frac{a(3ia+b)\log(i-\sinh(x))}{16(a-ib)^3} + \frac{a(3a+ib)\log(i+\sinh(x))}{16(ia-b)^3} - \frac{a^4b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^3}$$
$$- \frac{\text{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)} - \frac{\text{sech}^2(x)\left(4a^2b-a(3a^2-b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{\text{sech}^5(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{a(ia-b)^3(3a-ib)\log(i-\sinh(x)) - a(3a+ib)(ia+b)^3\log(i+\sinh(x)) - 16a^4b\log(b+a\sinh(x)) - 8a}{1}$$

```
[In] Integrate[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (a*(I*a - b)^3*(3*a - I*b)*Log[I - Sinh[x]] - a*(3*a + I*b)*(I*a + b)^3*Log
[I + Sinh[x]] - 16*a^4*b*Log[b + a*Sinh[x]] - 8*a^2*b*(a^2 + b^2)*Sech[x]^2
 - 4*b*(a^2 + b^2)^2*Sech[x]^4 + 2*a*(3*a^4 + 2*a^2*b^2 - b^4)*Sech[x]*Tanh
[x] + 4*a*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(16*(a^2 + b^2)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(135) = 270$.

Time = 23.74 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.12

method	result
default	$\frac{2\left(\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\tanh\left(\frac{x}{2}\right)^7+\left(2a^4b+3a^2b^3+b^5\right)\tanh\left(\frac{x}{2}\right)^6+\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}a\,b^4\right)\tanh\left(\frac{x}{2}\right)^5+\left(2a^4b+2a^2b^3\right)\tanh\left(\frac{x}{2}\right)^4+\left(-\frac{3}{8}a^5-\frac{5}{4}a^3b\right.}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^4}$ $\frac{}{(a^4+2a^2$
risch	$\frac{(3a^3e^{6x}-a\,b^2e^{6x}-8a^2b\,e^{5x}+11a^3e^{4x}+7a\,b^2e^{4x}-32a^2b\,e^{3x}-16b^3e^{3x}-11a^3e^{2x}-7a\,b^2e^{2x}-8a^2b\,e^x-3a^3+a\,b^2)e^x}{4(a^4+2a^2b^2+b^4)(1+e^{2x})^4}-\frac{3ia^5\ln(e^x-}{8(a^6+3a^4b^2+3a^2}$

```
[In] int(sech(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-5/8*a^5-3/4*a^3*b^2-1/8*a*b^4)*tanh(1/2
*x)^7+(2*a^4*b+3*a^2*b^3+b^5)*tanh(1/2*x)^6+(3/8*a^5+5/4*a^3*b^2+7/8*a*b^4)
*tanh(1/2*x)^5+(2*a^4*b+2*a^2*b^3)*tanh(1/2*x)^4+(-3/8*a^5-5/4*a^3*b^2-7/8*
a*b^4)*tanh(1/2*x)^3+(2*a^4*b+3*a^2*b^3+b^5)*tanh(1/2*x)^2+(5/8*a^5+3/4*a^3
*b^2+1/8*a*b^4)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^4+1/8*a*(4*a^3*b*ln(1+tanh(1
/2*x)^2)+(3*a^4-6*a^2*b^2-b^4)*arctan(tanh(1/2*x))))-a^4*b/(a^4+2*a^2*b^2+b
^4)/(a^2+b^2)*ln(-tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2778 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 2778, normalized size of antiderivative = 18.64

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^7 + (3*a^5 + 2*a^3*b^2 - a*b^4)*si
nh(x)^7 - 8*(a^4*b + a^2*b^3)*cosh(x)^6 - (8*a^4*b + 8*a^2*b^3 - 7*(3*a^5 +
 2*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*co
sh(x)^5 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4 + 21*(3*a^5 + 2*a^3*b^2 - a*b^4)*c
osh(x)^2 - 48*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^5 - 16*(2*a^4*b + 3*a^2*b^
3 + b^5)*cosh(x)^4 - (32*a^4*b + 48*a^2*b^3 + 16*b^5 - 35*(3*a^5 + 2*a^3*b^
2 - a*b^4)*cosh(x)^3 + 120*(a^4*b + a^2*b^3)*cosh(x)^2 - 5*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*cosh(x
)^3 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4 - 35*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(
x)^4 + 160*(a^4*b + a^2*b^3)*cosh(x)^3 - 10*(11*a^5 + 18*a^3*b^2 + 7*a*b^4)
*cosh(x)^2 + 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 - 8*(a^4*b +
 a^2*b^3)*cosh(x)^2 + (21*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^5 - 8*a^4*b -
```

```
 8*a^2*b^3 - 120*(a^4*b + a^2*b^3)*cosh(x)^4 + 10*(11*a^5 + 18*a^3*b^2 + 7*
a*b^4)*cosh(x)^3 - 96*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 3*(11*a^5 + 1
8*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 - 6*a^3*b^2 - a*b^4)*cosh
(x)^8 + 8*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 - 6*a^3*b^
2 - a*b^4)*sinh(x)^8 + 4*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 4*(3*a^5 -
 6*a^3*b^2 - a*b^4 + 7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^6 + 8
*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^5 + 3*a^5 - 6*a^3*b^2 - a*b^4 + 6*(3*a^5 - 6*a^3*b^2 - a*b^
4)*cosh(x)^4 + 2*(9*a^5 - 18*a^3*b^2 - 3*a*b^4 + 35*(3*a^5 - 6*a^3*b^2 - a*
b^4)*cosh(x)^4 + 30*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7
*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 10*(3*a^5 - 6*a^3*b^2 - a*b^4)*cos
h(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 - 6*a^
3*b^2 - a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 3*a
^5 - 6*a^3*b^2 - a*b^4 + 15*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^4 + 9*(3*a^
5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 6*a^3*b^2 - a*b^4
)*cosh(x)^7 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(3*a^5 - 6*a^3*b^
2 - a*b^4)*cosh(x)^3 + (3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan
(cosh(x) + sinh(x)) - (3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x) - 4*(a^4*b*cosh(x
)^8 + 8*a^4*b*cosh(x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a
^4*b*cosh(x)^4 + 4*a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^
6 + 8*(7*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b
*cosh(x)^4 + 30*a^4*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5
 + 10*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 +
 15*a^4*b*cosh(x)^4 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(
x)^7 + 3*a^4*b*cosh(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(
2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 4*(a^4*b*cosh(x)^8 + 8*a^4*b*cosh(
x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a^4*b*cosh(x)^4 + 4*
a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^6 + 8*(7*a^4*b*cosh
(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b*cosh(x)^4 + 30*a^4
*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5 + 10*a^4*b*cosh(x)
^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 + 15*a^4*b*cosh(x)^4
 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(x)^7 + 3*a^4*b*cosh
(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
- sinh(x))) + (7*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^6 - 48*(a^4*b + a^2*b^
3)*cosh(x)^5 - 3*a^5 - 2*a^3*b^2 + a*b^4 + 5*(11*a^5 + 18*a^3*b^2 + 7*a*b^4
)*cosh(x)^4 - 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x)^2 - 16*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^8 + 4*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 + 6*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2 + 9*a^2*b
^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 30*(a^6 + 3
```

```
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)
^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^
2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 + 3*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(sech(x)**5/(a+b*csch(x)),x)
[Out] Integral(sech(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(130) = 260$.

Time $= 0.28$ (sec) , antiderivative size $= 348$, normalized size of antiderivative $= 2.34$

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^4 b \log\left(-2\,b e^{(-x)} + a e^{(-2\,x)} - a\right)}{a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6} + \frac{a^4 b \log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6} - \frac{\left(3\,a^5 - 6\,a^3 b^2 - a b^4\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6\right)} - \frac{8\,a^2 b e^{(-2\,x)} + 8\,a^2 b e^{(-6\,x)} - (3\,a^3 - a b^2) e^{(-x)} - (11\,a^3 + 7\,a b^2) e^{(-3\,x)} + 16\,(2\,a^2 b + b^3) e^{(-4\,x)} + (11\,a^3 + 7\,a$$

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="maxima")
[Out] -a^4*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + a^4*b*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5 - 6*a^3*b^2 - a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 1/4*(8*a^2*b*e^(-2*x) + 8*a^2*b*e^(-6*x) - (3*a^3 - a*b^2)*e^(-x) - (11*a^
3 + 7*a*b^2)*e^(-3*x) + 16*(2*a^2*b + b^3)*e^(-4*x) + (11*a^3 + 7*a*b^2)*e^
(-5*x) + (3*a^3 - a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b
^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(130) = 260$.

Time $= 0.29$ (sec) , antiderivative size $= 374$, normalized size of antiderivative $= 2.51$

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^5 b \log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6} + \frac{a^4 b \log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$
$$+ \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)\left(3\,a^5-6\,a^3b^2-ab^4\right)}{16\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$
$$- \frac{3\,a^4 b\left(e^{(-x)}-e^{x}\right)^4 + 3\,a^5\left(e^{(-x)}-e^{x}\right)^3 + 2\,a^3b^2\left(e^{(-x)}-e^{x}\right)^3 - ab^4\left(e^{(-x)}-e^{x}\right)^3 + 32\,a^4 b\left(e^{(-x)}-e^{x}\right)^2 +}{4\left(a^6+3\,a^4b^2+3\,a^2b^4}$$

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^5*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6) + 1/2*a^4*b*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 - 6*a^3*b^2 - a*
b^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b*(e^(-x) - e^x)^4 +
3*a^5*(e^(-x) - e^x)^3 + 2*a^3*b^2*(e^(-x) - e^x)^3 - a*b^4*(e^(-x) - e^x)^
3 + 32*a^4*b*(e^(-x) - e^x)^2 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
 - e^x) + 24*a^3*b^2*(e^(-x) - e^x) + 4*a*b^4*(e^(-x) - e^x) + 96*a^4*b + 6
4*a^2*b^3 + 16*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2
+ 4)^2)
```

Mupad [B] (verification not implemented)

Time $= 6.02$ (sec) , antiderivative size $= 513$, normalized size of antiderivative $= 3.44$

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{8\left(a^2b+b^3\right)}{\left(a^2+b^2\right)^2} - \frac{6\,\mathrm{e}^{x}\left(a^3+a\,b^2\right)}{\left(a^2+b^2\right)^2}}{3\,\mathrm{e}^{2x}+3\,\mathrm{e}^{4x}+\mathrm{e}^{6x}+1} - \frac{\frac{2\left(a^2b+2\,b^3\right)}{\left(a^2+b^2\right)^2} - \frac{\mathrm{e}^{x}\left(a^3+5\,a\,b^2\right)}{2\left(a^2+b^2\right)^2}}{2\,\mathrm{e}^{2x}+\mathrm{e}^{4x}+1}$$
$$- \frac{\frac{4\,b}{a^2+b^2} - \frac{4\,a\,\mathrm{e}^{x}}{a^2+b^2}}{4\,\mathrm{e}^{2x}+6\,\mathrm{e}^{4x}+4\,\mathrm{e}^{6x}+\mathrm{e}^{8x}+1} - \frac{\frac{2\left(a^4b+a^2b^3\right)}{\left(a^2+b^2\right)^3} - \frac{\mathrm{e}^{x}\left(3\,a^5+2\,a^3b^2-a\,b^4\right)}{4\left(a^2+b^2\right)^3}}{\mathrm{e}^{2x}+1}$$
$$+ \frac{\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\left(a\,b-a^2\,3\mathrm{i}\right)}{8\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)\left(-3\,a^2+a\,b\,1\mathrm{i}\right)}{8\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$- \frac{a^4\,b\,\ln\left(9\,a^{10}\,\mathrm{e}^{2x}-9\,a^{10}-a^2\,b^8-12\,a^4\,b^6-30\,a^6\,b^4-220\,a^8\,b^2+a^2\,b^8\,\mathrm{e}^{2x}+12\,a^4\,b^6\,\mathrm{e}^{2x}+30\,a^6\,b^4\,\mathrm{e}^{2x}\right.}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}$$

```
[In] int(1/(cosh(x)^5*(a + b/sinh(x))),x)
```

```
[Out] ((8*(a^2*b + b^3))/(a^2 + b^2)^2 - (6*exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/
(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((2*(a^2*b + 2*b^3))/(a^2 + b^2)
```

```
^2 - (exp(x)*(5*a*b^2 + a^3))/(2*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1
) - ((4*b)/(a^2 + b^2) - (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) - ((2*(a^4*b + a^2*b^3))/(a^2 + b^2)^3 - (exp
(x)*(3*a^5 - a*b^4 + 2*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(e
xp(x) + 1i)*(a*b - a^2*3i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(
exp(x)*1i + 1)*(a*b*1i - 3*a^2))/(8*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) -
(a^4*b*log(9*a^10*exp(2*x) - 9*a^10 - a^2*b^8 - 12*a^4*b^6 - 30*a^6*b^4 - 2
20*a^8*b^2 + a^2*b^8*exp(2*x) + 12*a^4*b^6*exp(2*x) + 30*a^6*b^4*exp(2*x) +
 220*a^8*b^2*exp(2*x) + 2*a*b^9*exp(x) + 18*a^9*b*exp(x) + 24*a^3*b^7*exp(x
) + 60*a^5*b^5*exp(x) + 440*a^7*b^3*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)
```

3.102 $\int \frac{\tanh^5(x)}{i+\mathbf{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{21}{32}i\log(i-\sinh(x)) - \frac{11}{32}i\log(i+\sinh(x)) + \frac{i}{32(1-i\sinh(x))^2} - \frac{i}{4(1-i\sinh(x))} - \frac{i}{24(1+i\sinh(x))^3} + \frac{9i}{32(1+i\sinh(x))^2} - \frac{15i}{16(1+i\sinh(x))}$$

```
[Out] -21/32*I*ln(I-sinh(x))-11/32*I*ln(I+sinh(x))+1/32*I/(1-I*sinh(x))^2-1/4*I/(
1-I*sinh(x))-1/24*I/(1+I*sinh(x))^3+9/32*I/(1+I*sinh(x))^2-15/16*I/(1+I*sin
h(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 90}

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{i}{4(1-i\sinh(x))} - \frac{15i}{16(1+i\sinh(x))} + \frac{i}{32(1-i\sinh(x))^2} + \frac{9i}{32(1+i\sinh(x))^2} - \frac{i}{24(1+i\sinh(x))^3} - \frac{21}{32}i\log(-\sinh(x)+i) - \frac{11}{32}i\log(\sinh(x)+i)$$

```
[In] Int[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-21*I)/32)*Log[I - Sinh[x]] - ((11*I)/32)*Log[I + Sinh[x]] + (I/32)/(1 -
I*Sinh[x])^2 - (I/4)/(1 - I*Sinh[x]) - (I/24)/(1 + I*Sinh[x])^3 + ((9*I)/32
)/(1 + I*Sinh[x])^2 - ((15*I)/16)/(1 + I*Sinh[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^6}{(i-ix)^3(i+ix)^4}\,dx, x, i\sinh(x)\right) \\
&= \text{Subst}\left(\int \left(-\frac{i}{16(-1+x)^3} - \frac{i}{4(-1+x)^2} - \frac{11i}{32(-1+x)} + \frac{i}{8(1+x)^4} - \frac{9i}{16(1+x)^3}\right.\right. \\
&\qquad \left.\left. + \frac{15i}{16(1+x)^2} - \frac{21i}{32(1+x)}\right)\,dx, x, i\sinh(x)\right) \\
&= -\frac{21}{32}i\log(i-\sinh(x)) - \frac{11}{32}i\log(i+\sinh(x)) + \frac{i}{32(1-i\sinh(x))^2} \\
&\quad - \frac{i}{4(1-i\sinh(x))} - \frac{i}{24(1+i\sinh(x))^3} + \frac{9i}{32(1+i\sinh(x))^2} - \frac{15i}{16(1+i\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time $= 0.14$ (sec) , antiderivative size $= 75$, normalized size of antiderivative $= 0.69$

$$\begin{aligned}
\int \frac{\tanh^5(x)}{i+\text{csch}(x)}\,dx = \frac{1}{96}\Bigg(&-63i\log(i-\sinh(x)) - 33i\log(i+\sinh(x)) \\
&- \frac{2\left(44+29i\sinh(x)+79\sinh^2(x)+39i\sinh^3(x)+33\sinh^4(x)\right)}{(-i+\sinh(x))^3(i+\sinh(x))^2}\Bigg)
\end{aligned}$$

```
[In] Integrate[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-63*I)*Log[I - Sinh[x]] - (33*I)*Log[I + Sinh[x]] - (2*(44 + (29*I)*Sinh[
x] + 79*Sinh[x]^2 + (39*I)*Sinh[x]^3 + 33*Sinh[x]^4))/((-I + Sinh[x])^3*(I
+ Sinh[x])^2))/96
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

method	result
risch	$ix-\frac{-78ie^{2x}+184e^{3x}+2ie^{4x}+270e^{5x}+33e^{x}-2ie^{6x}+184e^{7x}+78ie^{8x}+33e^{9x}}{24(e^{x}-i)^{6}(e^{x}+i)^{4}}-\frac{11i\ln(e^{x}+i)}{16}-\frac{21i\ln(e^{x}-i)}{16}$
default	$-\frac{21i\ln(\tanh(\frac{x}{2})-i)}{16}+\frac{3i}{8(\tanh(\frac{x}{2})-i)^{2}}+\frac{i}{3(\tanh(\frac{x}{2})-i)^{6}}-\frac{3i}{8(\tanh(\frac{x}{2})-i)^{4}}+\frac{1}{(\tanh(\frac{x}{2})-i)^{5}}+\frac{11}{12(\tanh(\frac{x}{2})-i)^{3}}+\frac{\tan}{}$

```
[In] int(tanh(x)^5/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*x-1/24*(-78*I*exp(x)^2+184*exp(x)^3+2*I*exp(x)^4+270*exp(x)^5+33*exp(x)-2
*I*exp(x)^6+184*exp(x)^7+78*I*exp(x)^8+33*exp(x)^9)/(exp(x)-I)^6/(exp(x)+I)
^4-11/16*I*ln(exp(x)+I)-21/16*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.79

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx$$
$$= \frac{48i\,xe^{(10\,x)}+6\,(16\,x-11)e^{(9\,x)}-12\,(-12i\,x+13i)e^{(8\,x)}+16\,(24\,x-23)e^{(7\,x)}-4\,(-24i\,x-i)e^{(6\,x)}+36\,(}{}$$

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*I*x*e^(10*x) + 6*(16*x - 11)*e^(9*x) - 12*(-12*I*x + 13*I)*e^(8*x)
 + 16*(24*x - 23)*e^(7*x) - 4*(-24*I*x - I)*e^(6*x) + 36*(16*x - 15)*e^(5*x
) - 4*(24*I*x + I)*e^(4*x) + 16*(24*x - 23)*e^(3*x) - 12*(12*I*x - 13*I)*e^
(2*x) + 6*(16*x - 11)*e^x - 33*(I*e^(10*x) + 2*e^(9*x) + 3*I*e^(8*x) + 8*e^
(7*x) + 2*I*e^(6*x) + 12*e^(5*x) - 2*I*e^(4*x) + 8*e^(3*x) - 3*I*e^(2*x) + 
2*e^x - I)*log(e^x + I) - 63*(I*e^(10*x) + 2*e^(9*x) + 3*I*e^(8*x) + 8*e^(7
*x) + 2*I*e^(6*x) + 12*e^(5*x) - 2*I*e^(4*x) + 8*e^(3*x) - 3*I*e^(2*x) + 2*
e^x - I)*log(e^x - I) - 48*I*x)/(e^(10*x) - 2*I*e^(9*x) + 3*e^(8*x) - 8*I*e
^(7*x) + 2*e^(6*x) - 12*I*e^(5*x) - 2*e^(4*x) - 8*I*e^(3*x) - 3*e^(2*x) - 2
*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh^5(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(tanh(x)**5/(I+csch(x)),x)
[Out] Integral(tanh(x)**5/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(67) = 134$.

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx &= -i\,x \\ &+ \frac{33\,e^{(-x)} + 78i\,e^{(-2\,x)} + 184\,e^{(-3\,x)} - 2i\,e^{(-4\,x)} + 270\,e^{(-5\,x)} + 2i\,e^{(-6\,x)} + 184\,e^{(-7\,x)} - 78i\,e^{(-8\,x)}}{48i\,e^{(-x)} - 72\,e^{(-2\,x)} + 192i\,e^{(-3\,x)} - 48\,e^{(-4\,x)} + 288i\,e^{(-5\,x)} + 48\,e^{(-6\,x)} + 192i\,e^{(-7\,x)} + 72\,e^{(-8\,x)} + 48i\,e^{(} \\ &- \frac{11}{16}i\,\log\left(e^{(-x)} - i\right) - \frac{21}{16}i\,\log\left(i\,e^{(-x)} - 1\right)\end{aligned}$$

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x + (33*e^(-x) + 78*I*e^(-2*x) + 184*e^(-3*x) - 2*I*e^(-4*x) + 270*e^(-5
*x) + 2*I*e^(-6*x) + 184*e^(-7*x) - 78*I*e^(-8*x) + 33*e^(-9*x))/(48*I*e^(-
x) - 72*e^(-2*x) + 192*I*e^(-3*x) - 48*e^(-4*x) + 288*I*e^(-5*x) + 48*e^(-6
*x) + 192*I*e^(-7*x) + 72*e^(-8*x) + 48*I*e^(-9*x) + 24*e^(-10*x) - 24) - 1
1/16*I*log(e^(-x) - I) - 21/16*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\begin{aligned}&\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx \\ &= -\frac{33i\left(e^{(-x)} - e^{x}\right)^2 + 100\,e^{(-x)} - 100\,e^{x} - 76i}{64\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)^2} \\ &\quad - \frac{-231i\left(e^{(-x)} - e^{x}\right)^3 + 1026\left(e^{(-x)} - e^{x}\right)^2 + 1548i\,e^{(-x)} - 1548i\,e^{x} - 776}{192\left(e^{(-x)} - e^{x} + 2i\right)^3} \\ &\quad - \frac{11}{32}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{21}{32}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)\end{aligned}$$

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/64*(33*I*(e^(-x) - e^x)^2 + 100*e^(-x) - 100*e^x - 76*I)/(-I*e^(-x) + I*
e^x - 2)^2 - 1/192*(-231*I*(e^(-x) - e^x)^3 + 1026*(e^(-x) - e^x)^2 + 1548*
I*e^(-x) - 1548*I*e^x - 776)/(e^(-x) - e^x + 2*I)^3 - 11/32*I*log(-e^(-x) +
 e^x + 2*I) - 21/32*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.51

$$\begin{aligned}\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = {} & x\,1\mathrm{i} - \ln\left(\left(\frac{5\,\mathrm{e}^x}{8}-\frac{5}{8}\mathrm{i}\right)\left(\frac{5\,\mathrm{e}^x}{8}+\frac{5}{8}\mathrm{i}\right)\right)\,1\mathrm{i} + \frac{5\,\operatorname{atan}(\mathrm{e}^x)}{8} \\ & + \frac{1\mathrm{i}}{3\,\left(15\,\mathrm{e}^{4\,x}-15\,\mathrm{e}^{2\,x}-\mathrm{e}^{6\,x}+1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^x\,6\mathrm{i}\right)} \\ & - \frac{1}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^x-\mathrm{i}} \\ & - \frac{31}{12\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^x-\mathrm{i}\right)} - \frac{5\mathrm{i}}{8\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}\right)} \\ & + \frac{17\mathrm{i}}{8\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^x\,4\mathrm{i}\right)} \\ & + \frac{1\mathrm{i}}{8\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)} + \frac{3\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}} \\ & - \frac{15}{8\,\left(\mathrm{e}^x-\mathrm{i}\right)} + \frac{1}{2\,\left(\mathrm{e}^x+1\mathrm{i}\right)} - \frac{1}{4\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}\right)}\end{aligned}$$

```
[In] int(tanh(x)^5/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(((5*exp(x))/8 - 5i/8)*((5*exp(x))/8 + 5i/8))*1i + (5*atan(exp(x)
))/8 + 1i/(3*(15*exp(4*x) - exp(3*x)*20i - 15*exp(2*x) + exp(5*x)*6i - exp(
6*x) + exp(x)*6i + 1)) - 1/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(
5*x) + 5*exp(x) - 1i) - 31/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) -
5i/(8*(exp(2*x) + exp(x)*2i - 1)) + 17i/(8*(exp(4*x) - exp(3*x)*4i - 6*exp(
2*x) + exp(x)*4i + 1)) + 1i/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x
)*4i + 1)) + 3i/(exp(x)*2i - exp(2*x) + 1) - 15/(8*(exp(x) - 1i)) + 1/(2*(e
xp(x) + 1i)) - 1/(4*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

3.103 $\int \frac{\tanh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 552
Rubi [A] (verified) . . . 552
Mathematica [B] (verified) . . . 553
Maple [A] (verified) . . . 554
Fricas [B] (verification not implemented) . . . 554
Sympy [F] . . . 555
Maxima [B] (verification not implemented) . . . 555
Giac [A] (verification not implemented) . . . 555
Mupad [B] (verification not implemented) . . . 556

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -ix + \frac{1}{15}(15i - 8\operatorname{csch}(x))\tanh(x) + \frac{1}{15}(5i - 4\operatorname{csch}(x))\tanh^3(x) + \frac{1}{5}(i - \operatorname{csch}(x))\tanh^5(x)$$

```
[Out] -I*x+1/15*(15*I-8*csch(x))*tanh(x)+1/15*(5*I-4*csch(x))*tanh(x)^3+1/5*(I-cs
ch(x))*tanh(x)^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3967, 8}

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -ix + \frac{1}{5}\tanh^5(x)(-\operatorname{csch}(x) + i) + \frac{1}{15}\tanh^3(x)(-4\operatorname{csch}(x) + 5i) + \frac{1}{15}\tanh(x)(-8\operatorname{csch}(x) + 15i)$$

```
[In] Int[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((15*I - 8*Csch[x])*Tanh[x])/15 + ((5*I - 4*Csch[x])*Tanh[x]^3)/15
 + ((I - Csch[x])*Tanh[x]^5)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (-i + \operatorname{csch}(x)) \tanh^6(x)\, dx \\
&= \frac{1}{5}(i - \operatorname{csch}(x)) \tanh^5(x) - \frac{1}{5}\int (5i - 4\operatorname{csch}(x)) \tanh^4(x)\, dx \\
&= \frac{1}{15}(5i - 4\operatorname{csch}(x)) \tanh^3(x) + \frac{1}{5}(i - \operatorname{csch}(x)) \tanh^5(x) + \frac{1}{15}\int (-15i + 8\operatorname{csch}(x)) \tanh^2(x)\, dx \\
&= \frac{1}{15}(15i - 8\operatorname{csch}(x)) \tanh(x) + \frac{1}{15}(5i - 4\operatorname{csch}(x)) \tanh^3(x) \\
&\quad + \frac{1}{5}(i - \operatorname{csch}(x)) \tanh^5(x) - \frac{1}{15}\int 15i\, dx \\
&= -ix + \frac{1}{15}(15i - 8\operatorname{csch}(x)) \tanh(x) + \frac{1}{15}(5i - 4\operatorname{csch}(x)) \tanh^3(x) + \frac{1}{5}(i - \operatorname{csch}(x)) \tanh^5(x)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 126 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.42

$$
\begin{aligned}
&\int \frac{\tanh^4(x)}{i + \operatorname{csch}(x)}\, dx \\
&= \frac{-200 + 6(89 - 120ix)\cosh(x) - 128\cosh(2x) + 178\cosh(3x) - 240ix\cosh(3x) - 184\cosh(4x) + 64i\sin}{960\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\right.\right.}
\end{aligned}
$$

```
[In] Integrate[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-200 + 6*(89 - (120*I)*x)*Cosh[x] - 128*Cosh[2*x] + 178*Cosh[3*x] - (240*I
)*x*Cosh[3*x] - 184*Cosh[4*x] + (64*I)*Sinh[x] + (178*I)*Sinh[2*x] + 240*x*
Sinh[2*x] + (128*I)*Sinh[3*x] + (89*I)*Sinh[4*x] + 120*x*Sinh[4*x])/(960*(C
osh[x/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

method	result
risch	$-ix-\frac{2(-31ie^{2x}+73\,e^{3x}+31\,e^{x}-25ie^{4x}+65\,e^{5x}-23i+15ie^{6x}+15\,e^{7x})}{15(e^{x}+i)^{3}(e^{x}-i)^{5}}$
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{11i}{8(\tanh(\frac{x}{2})-i)}+\frac{2i}{5(\tanh(\frac{x}{2})-i)^{5}}+\frac{1}{(\tanh(\frac{x}{2})-i)^{4}}+\frac{1}{(\tanh(\frac{x}{2})-i)^{2}}+\frac{5i}{8(\tanh(\frac{x}{2})+i)}+\frac{}{6(\tanh}$

```
[In] int(tanh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x-2/15*(-31*I*exp(x)^2+73*exp(x)^3+31*exp(x)-25*I*exp(x)^4+65*exp(x)^5-2
3*I+15*I*exp(x)^6+15*exp(x)^7)/(exp(x)+I)^3/(exp(x)-I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.38

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx$$
$$=\frac{-15i\,xe^{(8\,x)}-30\,(x+1)e^{(7\,x)}-30\,(i\,x+i)e^{(6\,x)}-10\,(9\,x+13)e^{(5\,x)}-2\,(45\,x+73)e^{(3\,x)}-2\,(-15i\,x-31i}{15\,(e^{(8\,x)}-2i\,e^{(7\,x)}+2\,e^{(6\,x)}-6i\,e^{(5\,x)}-6i\,e^{(3\,x)}-2\,e^{(2\,x)}-2i\,e^{}}$$

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/15*(-15*I*x*e^(8*x) - 30*(x + 1)*e^(7*x) - 30*(I*x + I)*e^(6*x) - 10*(9*x
 + 13)*e^(5*x) - 2*(45*x + 73)*e^(3*x) - 2*(-15*I*x - 31*I)*e^(2*x) - 2*(15
*x + 31)*e^x + 15*I*x + 50*I*e^(4*x) + 46*I)/(e^(8*x) - 2*I*e^(7*x) + 2*e^(
6*x) - 6*I*e^(5*x) - 6*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(tanh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \frac{2\left(31\,e^{(-x)} + 31i\,e^{(-2\,x)} + 73\,e^{(-3\,x)} + 25i\,e^{(-4\,x)} + 65\,e^{(-5\,x)} - 15i\,e^{(-6\,x)} + 15\,e^{(-7\,x)} + 23i\right)}{30i\,e^{(-x)} - 30\,e^{(-2\,x)} + 90i\,e^{(-3\,x)} + 90i\,e^{(-5\,x)} + 30\,e^{(-6\,x)} + 30i\,e^{(-7\,x)} + 15\,e^{(-8\,x)} - 15}$$

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*(31*e^(-x) + 31*I*e^(-2*x) + 73*e^(-3*x) + 25*I*e^(-4*x) + 65*e^(-
5*x) - 15*I*e^(-6*x) + 15*e^(-7*x) + 23*I)/(30*I*e^(-x) - 30*e^(-2*x) + 90*
I*e^(-3*x) + 90*I*e^(-5*x) + 30*e^(-6*x) + 30*I*e^(-7*x) + 15*e^(-8*x) - 15
)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \frac{21i\,e^{(2\,x)} - 36\,e^{x} - 19i}{24\,(i\,e^{x} - 1)^3} - \frac{115\,e^{(4\,x)} - 380i\,e^{(3\,x)} - 530\,e^{(2\,x)} + 340i\,e^{x} + 91}{40\,(e^{x} - i)^5}$$

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -I*x - 1/24*(21*I*e^(2*x) - 36*e^x - 19*I)/(I*e^x - 1)^3 - 1/40*(115*e^(4*x
) - 380*I*e^(3*x) - 530*e^(2*x) + 340*I*e^x + 91)/(e^x - I)^5
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.56

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -x\,1\mathrm{i} - \frac{1\mathrm{i}}{4\,(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i})} + \frac{\frac{23\,\mathrm{e}^{x}}{40}-\frac{3}{8}\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{23}{40\,(\mathrm{e}^{x}-\mathrm{i})} + \frac{7}{8\,(\mathrm{e}^{x}+1\mathrm{i})} + \frac{\frac{\mathrm{e}^{2\,x}\,9\mathrm{i}}{8}-\frac{23\,\mathrm{e}^{3\,x}}{40}+\frac{9\,\mathrm{e}^{x}}{8}-\frac{3}{8}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}} - \frac{\frac{3}{8}-\frac{23\,\mathrm{e}^{2\,x}}{40}+\frac{\mathrm{e}^{x}\,3\mathrm{i}}{4}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}} - \frac{1}{6\,(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i})} - \frac{\frac{23\,\mathrm{e}^{4\,x}}{40}-\frac{9\,\mathrm{e}^{2\,x}}{4}+\frac{23}{40}-\frac{\mathrm{e}^{3\,x}\,3\mathrm{i}}{2}+\frac{\mathrm{e}^{x}\,3\mathrm{i}}{2}}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(tanh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] ((23*exp(x))/40 - 3i/8)/(exp(x)*2i - exp(2*x) + 1) - 1i/(4*(exp(2*x) + exp(
x)*2i - 1)) - x*1i - 23/(40*(exp(x) - 1i)) + 7/(8*(exp(x) + 1i)) + ((exp(2*
x)*9i)/8 - (23*exp(3*x))/40 + (9*exp(x))/8 - 3i/8)/(exp(4*x) - exp(3*x)*4i
- 6*exp(2*x) + exp(x)*4i + 1) - ((exp(x)*3i)/4 - (23*exp(2*x))/40 + 3/8)/(e
xp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x) - 3*e
xp(x) - 1i)) - ((23*exp(4*x))/40 - (exp(3*x)*3i)/2 - (9*exp(2*x))/4 + (exp(
x)*3i)/2 + 23/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) + 5*
exp(x) - 1i)
```

3.104 $\int \frac{\tanh^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 557
Rubi [A] (verified) . . . 557
Mathematica [A] (verified) . . . 558
Maple [A] (verified) . . . 559
Fricas [B] (verification not implemented) . . . 559
Sympy [F] . . . 559
Maxima [B] (verification not implemented) . . . 560
Giac [A] (verification not implemented) . . . 560
Mupad [B] (verification not implemented) . . . 560

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{11}{16}i\log(i-\sinh(x)) - \frac{5}{16}i\log(i+\sinh(x)) - \frac{i}{8(1-i\sinh(x))} + \frac{i}{8(1+i\sinh(x))^2} - \frac{3i}{4(1+i\sinh(x))}$$

```
[Out] -11/16*I*ln(I-sinh(x))-5/16*I*ln(I+sinh(x))-1/8*I/(1-I*sinh(x))+1/8*I/(1+I*
sinh(x))^2-3/4*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 90}

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{i}{8(1-i\sinh(x))} - \frac{3i}{4(1+i\sinh(x))} + \frac{i}{8(1+i\sinh(x))^2} - \frac{11}{16}i\log(-\sinh(x)+i) - \frac{5}{16}i\log(\sinh(x)+i)$$

```
[In] Int[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)/16)*Log[I - Sinh[x]] - ((5*I)/16)*Log[I + Sinh[x]] - (I/8)/(1 - I*
Sinh[x]) + (I/8)/(1 + I*Sinh[x])^2 - ((3*I)/4)/(1 + I*Sinh[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x^4}{(i-ix)^2(i+ix)^3}\,dx, x, i\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{8(-1+x)^2} - \frac{5i}{16(-1+x)} - \frac{i}{4(1+x)^3} + \frac{3i}{4(1+x)^2}\right.\right.\\ &\qquad\left.\left. - \frac{11i}{16(1+x)}\right)\,dx, x, i\sinh(x)\right)\\ &= -\frac{11}{16}i\log(i-\sinh(x)) - \frac{5}{16}i\log(i+\sinh(x))\\ &\quad - \frac{i}{8(1-i\sinh(x))} + \frac{i}{8(1+i\sinh(x))^2} - \frac{3i}{4(1+i\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(x)}{i+\text{csch}(x)}\,dx = \frac{1}{16}\left(-11i\log(i-\sinh(x)) - 5i\log(i+\sinh(x)) - \frac{2\left(6+3i\sinh(x)+5\sinh^2(x)\right)}{(-i+\sinh(x))^2(i+\sinh(x))}\right)$$

```
[In] Integrate[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)*Log[I - Sinh[x]] - (5*I)*Log[I + Sinh[x]] - (2*(6 + (3*I)*Sinh[x]
+ 5*Sinh[x]^2))/((-I + Sinh[x])^2*(I + Sinh[x])))/16
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

method	result
risch	$ix-\frac{5\,\mathrm{e}^{x}-6i\mathrm{e}^{2x}+14\,\mathrm{e}^{3x}+6i\mathrm{e}^{4x}+5\,\mathrm{e}^{5x}}{4(\mathrm{e}^{x}+i)^{2}(\mathrm{e}^{x}-i)^{4}}-\frac{5i\ln(\mathrm{e}^{x}+i)}{8}-\frac{11i\ln(\mathrm{e}^{x}-i)}{8}$
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{5i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{11i\ln(\tan$

```
[In] int(tanh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*x-1/4*(5*exp(x)-6*I*exp(x)^2+14*exp(x)^3+6*I*exp(x)^4+5*exp(x)^5)/(exp(x)
+I)^2/(exp(x)-I)^4-5/8*I*ln(exp(x)+I)-11/8*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx$$
$$= \frac{8i\,xe^{(6\,x)}+2\,(8\,x-5)e^{(5\,x)}-4\,(-2i\,x+3i)e^{(4\,x)}+4\,(8\,x-7)e^{(3\,x)}-4\,(2i\,x-3i)e^{(2\,x)}+2\,(8\,x-5)e^{x}-5}{8\,(e^{(6\,x)}}$$

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*I*x*e^(6*x) + 2*(8*x - 5)*e^(5*x) - 4*(-2*I*x + 3*I)*e^(4*x) + 4*(8*
x - 7)*e^(3*x) - 4*(2*I*x - 3*I)*e^(2*x) + 2*(8*x - 5)*e^x - 5*(I*e^(6*x) +
 2*e^(5*x) + I*e^(4*x) + 4*e^(3*x) - I*e^(2*x) + 2*e^x - I)*log(e^x + I) -
11*(I*e^(6*x) + 2*e^(5*x) + I*e^(4*x) + 4*e^(3*x) - I*e^(2*x) + 2*e^x - I)*
log(e^x - I) - 8*I*x)/(e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2
*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh^3(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(tanh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^3(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{5\,e^{(-x)} + 6i\,e^{(-2\,x)} + 14\,e^{(-3\,x)} - 6i\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{8i\,e^{(-x)} - 4\,e^{(-2\,x)} + 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} + 8i\,e^{(-5\,x)} + 4\,e^{(-6\,x)} - 4} - \frac{5}{8}i\,\log\left(e^{(-x)} - i\right) - \frac{11}{8}i\,\log\left(i\,e^{(-x)} - 1\right)$$

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (5*e^(-x) + 6*I*e^(-2*x) + 14*e^(-3*x) - 6*I*e^(-4*x) + 5*e^(-5*x))/
(8*I*e^(-x) - 4*e^(-2*x) + 16*I*e^(-3*x) + 4*e^(-4*x) + 8*I*e^(-5*x) + 4*e^
(-6*x) - 4) - 5/8*I*log(e^(-x) - I) - 11/8*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{\tanh^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{5\,e^{(-x)} - 5\,e^{x} - 6i}{16\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)} + \frac{33i\left(e^{(-x)} - e^{x}\right)^2 - 84\,e^{(-x)} + 84\,e^{x} - 52i}{32\left(e^{(-x)} - e^{x} + 2i\right)^2} - \frac{5}{16}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{11}{16}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/16*(5*e^(-x) - 5*e^x - 6*I)/(-I*e^(-x) + I*e^x - 2) + 1/32*(33*I*(e^(-x)
- e^x)^2 - 84*e^(-x) + 84*e^x - 52*I)/(e^(-x) - e^x + 2*I)^2 - 5/16*I*log(-
e^(-x) + e^x + 2*I) - 11/16*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

$$\int \frac{\tanh^3(x)}{i + \operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\left(\frac{3\,\mathrm{e}^{x}}{4} - \frac{3}{4}\mathrm{i}\right)\left(\frac{3\,\mathrm{e}^{x}}{4} + \frac{3}{4}\mathrm{i}\right)\right)\,1\mathrm{i} + \frac{3\,\operatorname{atan}(\mathrm{e}^{x})}{4} - \frac{1}{\mathrm{e}^{2\,x}\,3\mathrm{i} - \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} - \mathrm{i}} - \frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}\right)} + \frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 - \mathrm{e}^{3\,x}\,4\mathrm{i} + \mathrm{e}^{x}\,4\mathrm{i}\right)} + \frac{2\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{3}{2\,\left(\mathrm{e}^{x} - \mathrm{i}\right)} + \frac{1}{4\,\left(\mathrm{e}^{x} + 1\mathrm{i}\right)}$$

```
[In] int(tanh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(((3*exp(x))/4 - 3i/4)*((3*exp(x))/4 + 3i/4))*1i + (3*atan(exp(x)
))/4 - 1/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1i/(4*(exp(2*x) + exp(x
)*2i - 1)) + 1i/(2*(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) +
 2i/(exp(x)*2i - exp(2*x) + 1) - 3/(2*(exp(x) - 1i)) + 1/(4*(exp(x) + 1i))
```

3.105 $\int \frac{\tanh^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 562
Rubi [A] (verified) . . . 562
Mathematica [A] (verified) . . . 563
Maple [A] (verified) . . . 563
Fricas [B] (verification not implemented) . . . 564
Sympy [F] . . . 564
Maxima [A] (verification not implemented) . . . 564
Giac [A] (verification not implemented) . . . 565
Mupad [B] (verification not implemented) . . . 565

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx = -ix + \frac{1}{3}(3i-2\operatorname{csch}(x))\tanh(x) + \frac{1}{3}(i-\operatorname{csch}(x))\tanh^3(x)$$

```
[Out] -I*x+1/3*(3*I-2*csch(x))*tanh(x)+1/3*(I-csch(x))*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3967, 8}

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx = -ix + \frac{1}{3}\tanh^3(x)(-\operatorname{csch}(x)+i) + \frac{1}{3}\tanh(x)(-2\operatorname{csch}(x)+3i)$$

```
[In] Int[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((3*I - 2*Csch[x])*Tanh[x])/3 + ((I - Csch[x])*Tanh[x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
```

```
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (-i + \operatorname{csch}(x)) \tanh^4(x)\, dx \\ &= \frac{1}{3}(i - \operatorname{csch}(x)) \tanh^3(x) - \frac{1}{3}\int (3i - 2\operatorname{csch}(x)) \tanh^2(x)\, dx \\ &= \frac{1}{3}(3i - 2\operatorname{csch}(x)) \tanh(x) + \frac{1}{3}(i - \operatorname{csch}(x)) \tanh^3(x) + \frac{1}{3}\int -3i\, dx \\ &= -ix + \frac{1}{3}(3i - 2\operatorname{csch}(x)) \tanh(x) + \frac{1}{3}(i - \operatorname{csch}(x)) \tanh^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \frac{\tanh^2(x)}{i + \operatorname{csch}(x)}\, dx = \frac{-4\cosh(2x) + 2i\sinh(x) + (5i + 6x)\cosh(x)(-i + \sinh(x))}{6\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-4*Cosh[2*x] + (2*I)*Sinh[x] + (5*I + 6*x)*Cosh[x]*(-I + Sinh[x]))/(6*(Cos
h[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result
risch	$-ix - \frac{2\left(-4i+5\,\mathrm{e}^{x}+3\,\mathrm{e}^{3x}\right)}{3\left(\mathrm{e}^{x}+i\right)\left(\mathrm{e}^{x}-i\right)^{3}}$
default	$\frac{3i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)} + \frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}} - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{i}{2\tanh\left(\frac{x}{2}\right)+2i} + i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right.$

```
[In] int(tanh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x-2/3*(-4*I+5*exp(x)+3*exp(3*x))/(exp(x)+I)/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time $= 0.26$ (sec) , antiderivative size $= 50$, normalized size of antiderivative $= 1.39$

$$\int \frac{\tanh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{-3i\,x e^{(4\,x)} - 6\,(x+1)e^{(3\,x)} - 2\,(3\,x+5)e^{x} + 3i\,x + 8i}{3\,(e^{(4\,x)} - 2i\,e^{(3\,x)} - 2i\,e^{x} - 1)}$$

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*x*e^(4*x) - 6*(x + 1)*e^(3*x) - 2*(3*x + 5)*e^x + 3*I*x + 8*I)/(e
^(4*x) - 2*I*e^(3*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\tanh^2(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(tanh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.20$ (sec) , antiderivative size $= 42$, normalized size of antiderivative $= 1.17$

$$\int \frac{\tanh^2(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x - \frac{2\left(5\,e^{(-x)} + 3\,e^{(-3\,x)} + 4i\right)}{6i\,e^{(-x)} + 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3}$$

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*(5*e^(-x) + 3*e^(-3*x) + 4*I)/(6*I*e^(-x) + 6*I*e^(-3*x) + 3*e^(-4
*x) - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x + \frac{i}{2\,(i\,e^x-1)} - \frac{15\,e^{(2\,x)} - 24 i\,e^x - 13}{6\,(e^x-i)^3}$$

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -I*x + 1/2*I/(I*e^x - 1) - 1/6*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx = -x\,1\mathrm{i} + \frac{\frac{5\,\mathrm{e}^x}{6} - \frac{1}{2}\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^x\,2\mathrm{i}} - \frac{5}{6\,\left(\mathrm{e}^x - \mathrm{i}\right)} + \frac{1}{2\,\left(\mathrm{e}^x + 1\mathrm{i}\right)} - \frac{\frac{5}{6} - \frac{5\,\mathrm{e}^{2\,x}}{6} + \mathrm{e}^x\,1\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i} - \mathrm{e}^{3\,x} + 3\,\mathrm{e}^x - \mathrm{i}}$$

```
[In] int(tanh(x)^2/(1/sinh(x) + 1i),x)
```

```
[Out] ((5*exp(x))/6 - 1i/2)/(exp(x)*2i - exp(2*x) + 1) - x*1i - 5/(6*(exp(x) - 1i
)) + 1/(2*(exp(x) + 1i)) - (exp(x)*1i - (5*exp(2*x))/6 + 5/6)/(exp(2*x)*3i
- exp(3*x) + 3*exp(x) - 1i)
```

3.106 $\int \frac{\tanh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 566
Rubi [A] (verified) . . . 566
Mathematica [A] (verified) . . . 567
Maple [A] (verified) . . . 567
Fricas [B] (verification not implemented) . . . 568
Sympy [F] . . . 568
Maxima [A] (verification not implemented) . . . 568
Giac [A] (verification not implemented) . . . 569
Mupad [B] (verification not implemented) . . . 569

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3}{4}i\log(i-\sinh(x)) - \frac{1}{4}i\log(i+\sinh(x)) - \frac{i}{2(1+i\sinh(x))}$$

```
[Out] -3/4*I*ln(I-sinh(x))-1/4*I*ln(I+sinh(x))-1/2*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3964, 90}

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{i}{2(1+i\sinh(x))} - \frac{3}{4}i\log(-\sinh(x)+i) - \frac{1}{4}i\log(\sinh(x)+i)$$

```
[In] Int[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)/4)*Log[I - Sinh[x]] - (I/4)*Log[I + Sinh[x]] - (I/2)/(1 + I*Sinh[x]
)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x^2}{(i-ix)(i+ix)^2}\,dx, x, i\sinh(x)\right) \\ &= \text{Subst}\left(\int \left(-\frac{i}{4(-1+x)} + \frac{i}{2(1+x)^2} - \frac{3i}{4(1+x)}\right)\,dx, x, i\sinh(x)\right) \\ &= -\frac{3}{4}i\log(i-\sinh(x)) - \frac{1}{4}i\log(i+\sinh(x)) - \frac{i}{2(1+i\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{\tanh(x)}{i+\text{csch}(x)}\,dx = \frac{1}{4}\left(-3i\log(i-\sinh(x)) - i\log(i+\sinh(x)) - \frac{2}{-i+\sinh(x)}\right)$$

```
[In] Integrate[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)*Log[I - Sinh[x]] - I*Log[I + Sinh[x]] - 2/(-I + Sinh[x]))/4
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result
risch	$ix - \frac{e^x}{(e^x-i)^2} - \frac{i\ln(e^x+i)}{2} - \frac{3i\ln(e^x-i)}{2}$
default	$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2} + \frac{i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^2} + \frac{1}{\tanh\left(\frac{x}{2}\right)-i} - \frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2} + i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right)-$

```
[In] int(tanh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*x-1/(exp(x)-I)^2*exp(x)-1/2*I*ln(exp(x)+I)-3/2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{\tanh(x)}{i + \operatorname{csch}(x)}\,dx$$
$$= \frac{2i\,x e^{(2\,x)} + 2\,(2\,x-1)e^{x} + \left(-i\,e^{(2\,x)} - 2\,e^{x} + i\right)\log\left(e^{x}+i\right) - 3\left(i\,e^{(2\,x)} + 2\,e^{x} - i\right)\log\left(e^{x}-i\right) - 2i\,x}{2\left(e^{(2\,x)} - 2i\,e^{x} - 1\right)}$$

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*I*x*e^(2*x) + 2*(2*x - 1)*e^x + (-I*e^(2*x) - 2*e^x + I)*log(e^x + I
) - 3*(I*e^(2*x) + 2*e^x - I)*log(e^x - I) - 2*I*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\tanh\left(x\right)}{\operatorname{csch}\left(x\right) + i}\,dx$$

```
[In] integrate(tanh(x)/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{e^{(-x)}}{2i\,e^{(-x)} + e^{(-2\,x)} - 1} - \frac{1}{2}i\,\log\left(i\,e^{(-x)} + 1\right) - \frac{3}{2}i\,\log\left(i\,e^{(-x)} - 1\right)$$

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) - 1/2*I*log(I*e^(-x) + 1) - 3/2*I
*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{4\,(e^{(-x)} - e^{x} + 2i)} - \frac{1}{4}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{4}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/4*I*log(-e^(-x) + e
^x + 2*I) - 3/4*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = x\,1\mathrm{i} + \operatorname{atan}(\mathrm{e}^{x}) - \ln\left((\mathrm{e}^{x} - \mathrm{i})\,(\mathrm{e}^{x} + 1\mathrm{i})\right)\,1\mathrm{i} + \frac{1\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{1}{\mathrm{e}^{x} - \mathrm{i}}$$

```
[In] int(tanh(x)/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i + atan(exp(x)) - log((exp(x) - 1i)*(exp(x) + 1i))*1i + 1i/(exp(x)*2i -
 exp(2*x) + 1) - 1/(exp(x) - 1i)
```

3.107 $\int \frac{\coth(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 570
Rubi [A] (verified) . . . 570
Mathematica [A] (verified) . . . 571
Maple [A] (verified) . . . 571
Fricas [A] (verification not implemented) . . . 572
Sympy [F] . . . 572
Maxima [A] (verification not implemented) . . . 572
Giac [A] (verification not implemented) . . . 572
Mupad [B] (verification not implemented) . . . 573

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = -i\log(i-\sinh(x))$$

```
[Out] -I*ln(I-sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3964, 31}

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = -i\log(-\sinh(x)+i)$$

```
[In] Int[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
```

[n]

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{i+ix}\,dx, x, i\sinh(x)\right)\\ &= -i\log(i-\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{i+\text{csch}(x)}\,dx = -i\log(i-\sinh(x))$$

```
[In] Integrate[Coth[x]/(I + Csch[x]),x]
[Out] (-I)*Log[I - Sinh[x]]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
risch	$ix-2i\ln\left(\mathrm{e}^{x}-i\right)$	15
derivativedivides	$i\ln\left(\text{csch}\left(x\right)\right)-\frac{i\ln\left(1+\text{csch}(x)^2\right)}{2}-\arctan\left(\text{csch}\left(x\right)\right)$	23
default	$i\ln\left(\text{csch}\left(x\right)\right)-\frac{i\ln\left(1+\text{csch}(x)^2\right)}{2}-\arctan\left(\text{csch}\left(x\right)\right)$	23

```
[In] int(coth(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] I*x-2*I*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = i\,x - 2i\,\log\left(e^{x} - i\right)$$

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="fricas")
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [F]

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

```
[In] integrate(coth(x)/(I+csch(x)),x)
[Out] Integral(coth(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - 2i\,\log\left(i\,e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x - 2*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = i\,x - 2i\,\log\left(e^{x} - i\right)$$

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="giac")
[Out] I*x - 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

```
[In] int(coth(x)/(1/sinh(x) + 1i),x)
[Out] x*1i - log(exp(x) - 1i)*2i
```

3.108 $\int \frac{\coth^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 574
Rubi [A] (verified) . . . 574
Mathematica [A] (verified) . . . 575
Maple [A] (verified) . . . 575
Fricas [A] (verification not implemented) . . . 576
Sympy [F] . . . 576
Maxima [B] (verification not implemented) . . . 576
Giac [A] (verification not implemented) . . . 576
Mupad [B] (verification not implemented) . . . 577

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -ix - \operatorname{arctanh}(\cosh(x))$$

```
[Out] -I*x-arctanh(cosh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3973, 3855}

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) - ix$$

```
[In] Int[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (-i + \operatorname{csch}(x))\, dx \\ &= -ix + \int \operatorname{csch}(x)\, dx \\ &= -ix - \operatorname{arctanh}(\cosh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(x)}{i + \operatorname{csch}(x)}\, dx = -ix - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Coth[x]^2/(I + Csch[x]),x]
[Out] (-I)*x - Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

method	result	size
risch	$-ix + \ln\left(\mathrm{e}^{x} - 1\right) - \ln\left(\mathrm{e}^{x} + 1\right)$	18
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right) - i\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)$	27

```
[In] int(coth(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -I*x+ln(exp(x)-1)-ln(exp(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \log\left(e^x+1\right) + \log\left(e^x-1\right)$$

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="fricas")
[Out] -I*x - log(e^x + 1) + log(e^x - 1)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^2(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(coth(x)**2/(I+csch(x)),x)
[Out] Integral(coth(x)**2/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \log\left(e^x+1\right) + \log\left(\left|e^x-1\right|\right)$$

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] -I*x - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{\coth^2(x)}{i + \operatorname{csch}(x)}\,dx = \ln\left(2 - 2\,\mathrm{e}^{x}\right) - \ln\left(-2\,\mathrm{e}^{x} - 2\right) - x\,1\mathrm{i}$$

```
[In] int(coth(x)^2/(1/sinh(x) + 1i),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) - x*1i
```

3.109 $\int \frac{\coth^3(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 578
Rubi [A] (verified) . . . 578
Mathematica [A] (verified) . . . 579
Maple [A] (verified) . . . 579
Fricas [B] (verification not implemented) . . . 580
Sympy [F] . . . 580
Maxima [B] (verification not implemented) . . . 580
Giac [B] (verification not implemented) . . . 581
Mupad [B] (verification not implemented) . . . 581

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = -\operatorname{csch}(x) - i\log(\sinh(x))$$

```
[Out] -csch(x)-I*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 45}

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = -\operatorname{csch}(x) - i\log(\sinh(x))$$

```
[In] Int[Coth[x]^3/(I + Csch[x]),x]
```

```
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{i-ix}{x^2}\,dx, x, i\sinh(x)\right) \\ &= \text{Subst}\left(\int \left(\frac{i}{x^2}-\frac{i}{x}\right)\,dx, x, i\sinh(x)\right) \\ &= -\text{csch}(x) - i\log(\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{i+\text{csch}(x)}\,dx = -\text{csch}(x) - i\log(\sinh(x))$$

```
[In] Integrate[Coth[x]^3/(I + Csch[x]),x]
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\text{csch}(x) + i\ln(\text{csch}(x))$	12
default	$-\text{csch}(x) + i\ln(\text{csch}(x))$	12
risch	$ix - \frac{2e^x}{e^{2x}-1} - i\ln(e^{2x}-1)$	28

```
[In] int(coth(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -csch(x)+I*ln(csch(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(10) = 20$.

Time $= 0.27$ (sec) , antiderivative size $= 40$, normalized size of antiderivative $= 3.33$

$$\int \frac{\coth^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{i\,x e^{(2\,x)} + \left(-i\,e^{(2\,x)} + i\right)\log\left(e^{(2\,x)} - 1\right) - i\,x - 2\,e^{x}}{e^{(2\,x)} - 1}$$

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) + (-I*e^(2*x) + I)*log(e^(2*x) - 1) - I*x - 2*e^x)/(e^(2*x) -
1)
```

Sympy [F]

$$\int \frac{\coth^3(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\coth^3(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(coth(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time $= 0.19$ (sec) , antiderivative size $= 36$, normalized size of antiderivative $= 3.00$

$$\int \frac{\coth^3(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{2\,e^{(-x)}}{e^{(-2\,x)} - 1} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 2*e^(-x)/(e^(-2*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time $= 0.28$ (sec) , antiderivative size $= 39$, normalized size of antiderivative $= 3.25$

$$\int \frac{\coth^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{-i\,e^{(-x)} + i\,e^x - 2}{e^{(-x)} - e^x} - i\,\log\left(\left|-e^{(-x)} + e^x\right|\right)$$

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="giac")
[Out] -(-I*e^(-x) + I*e^x - 2)/(e^(-x) - e^x) - I*log(abs(-e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time $= 2.22$ (sec) , antiderivative size $= 27$, normalized size of antiderivative $= 2.25$

$$\int \frac{\coth^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{2\,\mathrm{e}^x}{\mathrm{e}^{2\,x} - 1} + x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - 1\right)\,1\mathrm{i}$$

```
[In] int(coth(x)^3/(1/sinh(x) + 1i),x)
[Out] x*1i - log(exp(2*x) - 1)*1i - (2*exp(x))/(exp(2*x) - 1)
```

3.110 $\int \frac{\coth^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 582
Rubi [A] (verified) . . . 582
Mathematica [B] (verified) . . . 583
Maple [B] (verified) . . . 584
Fricas [B] (verification not implemented) . . . 584
Sympy [F] . . . 584
Maxima [B] (verification not implemented) . . . 585
Giac [B] (verification not implemented) . . . 585
Mupad [B] (verification not implemented) . . . 585

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx = -ix - \frac{1}{2}\operatorname{arctanh}(\cosh(x)) + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x))$$

```
[Out] -I*x-1/2*arctanh(cosh(x))+1/2*coth(x)*(2*I-csch(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{2}\operatorname{arctanh}(\cosh(x)) - ix + \frac{1}{2}\coth(x)(-\operatorname{csch}(x)+2i)$$

```
[In] Int[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]/2 + (Coth[x]*(2*I - Csch[x]))/2
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
```

```
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \coth^2(x)(-i + \operatorname{csch}(x))\,dx \\
&= \frac{1}{2}\coth(x)(2i - \operatorname{csch}(x)) + \frac{1}{2}\int (-2i + \operatorname{csch}(x))\,dx \\
&= -ix + \frac{1}{2}\coth(x)(2i - \operatorname{csch}(x)) + \frac{1}{2}\int \operatorname{csch}(x)\,dx \\
&= -ix - \frac{1}{2}\operatorname{arctanh}(\cosh(x)) + \frac{1}{2}\coth(x)(2i - \operatorname{csch}(x))
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 76 vs. $2(27) = 54$.

Time $= 0.05$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 2.81$

$$\begin{aligned}
\int \frac{\coth^4(x)}{i + \operatorname{csch}(x)}\,dx = -ix + \frac{1}{2}i\coth\left(\frac{x}{2}\right) - \frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) \\
+ \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{2}i\tanh\left(\frac{x}{2}\right)
\end{aligned}$$

```
[In] Integrate[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + (I/2)*Coth[x/2] - Csch[x/2]^2/8 - Log[Cosh[x/2]]/2 + Log[Sinh[x/2]
]/2 - Sech[x/2]^2/8 + (I/2)*Tanh[x/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

Time = 3.82 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
risch	$-ix - \frac{-2ie^{2x}+e^{3x}+2i+e^{x}}{(e^{2x}-1)^2} + \frac{\ln(e^x-1)}{2} - \frac{\ln(e^x+1)}{2}$
default	$\frac{i\tanh(\frac{x}{2})}{2} + \frac{\tanh(\frac{x}{2})^2}{8} - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{1}{8\tanh(\frac{x}{2})^2} + \frac{i}{2\tanh(\frac{x}{2})} + \frac{\ln(\tanh(\frac{x}{2}))}{2}$

```
[In] int(coth(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x-(-2*I*exp(x)^2+exp(x)^3+2*I+exp(x))/(exp(x)^2-1)^2+1/2*ln(exp(x)-1)-1/
2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.19

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx$$
$$= \frac{-2i\,xe^{(4\,x)} - 4\,(-i\,x-i)e^{(2\,x)} - \left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\log\left(e^x+1\right) + \left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\log\left(e^x-1\right) - 2i\,x}{2\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)}$$

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x*e^(4*x) - 4*(-I*x - I)*e^(2*x) - (e^(4*x) - 2*e^(2*x) + 1)*log(
e^x + 1) + (e^(4*x) - 2*e^(2*x) + 1)*log(e^x - 1) - 2*I*x - 2*e^(3*x) - 2*e
^x - 4*I)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F]

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^4(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(coth(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time $= 0.18$ (sec) , antiderivative size $= 55$, normalized size of antiderivative $= 2.04$

$$\int \frac{\coth^4(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{e^{(-x)} + 2i\,e^{(-2\,x)} + e^{(-3\,x)} - 2i}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (e^(-x) + 2*I*e^(-2*x) + e^(-3*x) - 2*I)/(2*e^(-2*x) - e^(-4*x) - 1)
 - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.

Time $= 0.27$ (sec) , antiderivative size $= 43$, normalized size of antiderivative $= 1.59$

$$\int \frac{\coth^4(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x - \frac{e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + 2i}{\left(e^{(2\,x)} - 1\right)^2} - \frac{1}{2}\log\left(e^{x} + 1\right) + \frac{1}{2}\log\left(\left|e^{x} - 1\right|\right)$$

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -I*x - (e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 1/2*log(e^x +
1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time $= 0.26$ (sec) , antiderivative size $= 56$, normalized size of antiderivative $= 2.07$

$$\int \frac{\coth^4(x)}{i + \operatorname{csch}(x)}\,dx = \frac{\ln\left(1 - \mathrm{e}^{x}\right)}{2} - \frac{\ln\left(-\mathrm{e}^{x} - 1\right)}{2} - \frac{\mathrm{e}^{x} - 2\mathrm{i}}{\mathrm{e}^{2\,x} - 1} - \frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - x\,1\mathrm{i}$$

```
[In] int(coth(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - x*1i - (exp(x) - 2i)/(exp(2*x) -
1) - (2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

3.111 $\int \frac{\coth^5(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 586
Rubi [A] (verified) . . . 586
Mathematica [A] (verified) . . . 587
Maple [B] (verified) . . . 587
Fricas [B] (verification not implemented) . . . 588
Sympy [F] . . . 588
Maxima [B] (verification not implemented) . . . 588
Giac [B] (verification not implemented) . . . 589
Mupad [B] (verification not implemented) . . . 589

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx = -\operatorname{csch}(x) + \frac{1}{2}i\operatorname{csch}^2(x) - \frac{\operatorname{csch}^3(x)}{3} - i\log(\sinh(x))$$

```
[Out] -csch(x)+1/2*I*csch(x)^2-1/3*csch(x)^3-I*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 76}

$$\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{3}\operatorname{csch}^3(x) + \frac{1}{2}i\operatorname{csch}^2(x) - \operatorname{csch}(x) - i\log(\sinh(x))$$

```
[In] Int[Coth[x]^5/(I + Csch[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(i-ix)^2(i+ix)}{x^4}\,dx, x, i\sinh(x)\right) \\ &= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{i}{x^3}+\frac{i}{x^2}-\frac{i}{x}\right)\,dx, x, i\sinh(x)\right) \\ &= -\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}-i\log(\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\coth^5(x)}{i+\text{csch}(x)}\,dx = -\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}-i\log(\sinh(x))$$

```
[In] Integrate[Coth[x]^5/(I + Csch[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 6.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

method	result
risch	$ix-\frac{2\,\mathrm{e}^{x}\left(-3i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}+3i\mathrm{e}^{x}-2\,\mathrm{e}^{2x}+3\right)}{3\left(\mathrm{e}^{2x}-1\right)^3}-i\ln\left(\mathrm{e}^{2x}-1\right)$
default	$\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)^3}{24}+\frac{i\tanh\left(\frac{x}{2}\right)^2}{8}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^3}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{i}{8\tanh\left(\frac{x}{2}\right)^2}-\frac{3}{8\tanh\left(\frac{x}{2}\right)}+i\ln\left(\tanh\right.$

```
[In] int(coth(x)^5/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*x-2/3*exp(x)*(-3*I*exp(3*x)+3*exp(4*x)+3*I*exp(x)-2*exp(2*x)+3)/(exp(2*x)
-1)^3-I*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(22) = 44$.

Time $= 0.26$ (sec) , antiderivative size $= 100$, normalized size of antiderivative $= 3.33$

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx$$
$$= \frac{3i\,x e^{(6\,x)} - 3\,(3i\,x - 2i)e^{(4\,x)} - 3\,(-3i\,x + 2i)e^{(2\,x)} - 3\,\left(i\,e^{(6\,x)} - 3i\,e^{(4\,x)} + 3i\,e^{(2\,x)} - i\right)\log\left(e^{(2\,x)} - 1\right) - 3i\,x}{3\,\left(e^{(6\,x)} - 3\,e^{(4\,x)} + 3\,e^{(2\,x)} - 1\right)}$$

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*x*e^(6*x) - 3*(3*I*x - 2*I)*e^(4*x) - 3*(-3*I*x + 2*I)*e^(2*x) - 3
*(I*e^(6*x) - 3*I*e^(4*x) + 3*I*e^(2*x) - I)*log(e^(2*x) - 1) - 3*I*x - 6*e
^(5*x) + 4*e^(3*x) - 6*e^x)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [F]

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\coth^5(x)}{\operatorname{csch}(x) + i}\,dx$$

```
[In] integrate(coth(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time $= 0.19$ (sec) , antiderivative size $= 75$, normalized size of antiderivative $= 2.50$

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{2\,\left(3\,e^{(-x)} - 3i\,e^{(-2\,x)} - 2\,e^{(-3\,x)} + 3i\,e^{(-4\,x)} + 3\,e^{(-5\,x)}\right)}{3\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 2/3*(3*e^(-x) - 3*I*e^(-2*x) - 2*e^(-3*x) + 3*I*e^(-4*x) + 3*e^(-5*x
))/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(
-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time $= 0.27$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 2.17$

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = \frac{11i\left(e^{(-x)} - e^{x}\right)^3 + 12\left(e^{(-x)} - e^{x}\right)^2 + 12i\,e^{(-x)} - 12i\,e^{x} + 16}{6\left(e^{(-x)} - e^{x}\right)^3} - i\,\log\left(\left|-e^{(-x)} + e^{x}\right|\right)$$

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/6*(11*I*(e^(-x) - e^x)^3 + 12*(e^(-x) - e^x)^2 + 12*I*e^(-x) - 12*I*e^x +
 16)/(e^(-x) - e^x)^3 - I*log(abs(-e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time $= 2.38$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 2.70$

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2x} - 1\right)\,1\mathrm{i} - \frac{8\,\mathrm{e}^{x}}{3\,\left(3\,\mathrm{e}^{2x} - 3\,\mathrm{e}^{4x} + \mathrm{e}^{6x} - 1\right)} - \frac{\frac{8\,\mathrm{e}^{x}}{3} - 2\mathrm{i}}{\mathrm{e}^{4x} - 2\,\mathrm{e}^{2x} + 1} - \frac{2\,\mathrm{e}^{x} - 2\mathrm{i}}{\mathrm{e}^{2x} - 1}$$

```
[In] int(coth(x)^5/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(exp(2*x) - 1)*1i - (8*exp(x))/(3*(3*exp(2*x) - 3*exp(4*x) + exp(
6*x) - 1)) - ((8*exp(x))/3 - 2i)/(exp(4*x) - 2*exp(2*x) + 1) - (2*exp(x) -
2i)/(exp(2*x) - 1)
```

3.112 $\int \frac{\coth^6(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result . . . 590
Rubi [A] (verified) . . . 590
Mathematica [B] (verified) . . . 591
Maple [B] (verified) . . . 592
Fricas [B] (verification not implemented) . . . 592
Sympy [F] . . . 593
Maxima [B] (verification not implemented) . . . 593
Giac [B] (verification not implemented) . . . 593
Mupad [B] (verification not implemented) . . . 594

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = -ix - \frac{3}{8}\operatorname{arctanh}(\cosh(x)) + \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x))$$

```
[Out] -I*x-3/8*arctanh(cosh(x))+1/12*coth(x)^3*(4*I-3*csch(x))+1/8*coth(x)*(8*I-3
*csch(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3}{8}\operatorname{arctanh}(\cosh(x)) - ix + \frac{1}{12}\coth^3(x)(-3\operatorname{csch}(x)+4i) + \frac{1}{8}\coth(x)(-3\operatorname{csch}(x)+8i)$$

```
[In] Int[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x - (3*ArcTanh[Cosh[x]])/8 + (Coth[x]^3*(4*I - 3*Csch[x]))/12 + (Coth[
x]*(8*I - 3*Csch[x]))/8
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \coth^4(x)(-i+\operatorname{csch}(x))\,dx \\
&= \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{4}\int \coth^2(x)(-4i+3\operatorname{csch}(x))\,dx \\
&= \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x)) + \frac{1}{8}\int (-8i+3\operatorname{csch}(x))\,dx \\
&= -ix + \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x)) + \frac{3}{8}\int \operatorname{csch}(x)\,dx \\
&= -ix - \frac{3}{8}\operatorname{arctanh}(\cosh(x)) + \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x))
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 140 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.26

$$\begin{aligned}
\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = -ix &+ \frac{2}{3}i\coth\left(\frac{x}{2}\right) - \frac{5}{32}\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{24}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right) \\
&- \frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{5}{32}\operatorname{sech}^2\left(\frac{x}{2}\right) \\
&+ \frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right) + \frac{2}{3}i\tanh\left(\frac{x}{2}\right) - \frac{1}{24}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)
\end{aligned}$$

```
[In] Integrate[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((2*I)/3)*Coth[x/2] - (5*Csch[x/2]^2)/32 + (I/24)*Coth[x/2]*Csch[x
/2]^2 - Csch[x/2]^4/64 - (3*Log[Cosh[x/2]])/8 + (3*Log[Sinh[x/2]])/8 - (5*S
ech[x/2]^2)/32 + Sech[x/2]^4/64 + ((2*I)/3)*Tanh[x/2] - (I/24)*Sech[x/2]^2*
Tanh[x/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 11.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

method	result
risch	$-ix - \frac{-48ie^{6x}+15\,e^{7x}+96ie^{4x}+9\,e^{5x}-80ie^{2x}+9\,e^{3x}+32i+15\,e^{x}}{12(e^{2x}-1)^4} - \frac{3\ln(e^x+1)}{8} + \frac{3\ln(e^x-1)}{8}$
default	$\frac{5i\tanh(\frac{x}{2})}{8} + \frac{\tanh(\frac{x}{2})^4}{64} + \frac{i\tanh(\frac{x}{2})^3}{24} + \frac{\tanh(\frac{x}{2})^2}{8} - \frac{1}{64\tanh(\frac{x}{2})^4} + \frac{5i}{8\tanh(\frac{x}{2})} + \frac{i}{24\tanh(\frac{x}{2})^3} - \frac{1}{8\tanh(\frac{x}{2})^2} + \frac{3\ln(\tan$

```
[In] int(coth(x)^6/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x-1/12*(-48*I*exp(x)^6+15*exp(x)^7+96*I*exp(x)^4+9*exp(x)^5-80*I*exp(x)^
2+9*exp(x)^3+32*I+15*exp(x))/(exp(x)^2-1)^4-3/8*ln(exp(x)+1)+3/8*ln(exp(x)-
1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.65

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx$$
$$= \frac{-24i\,xe^{(8\,x)} - 96\,(-i\,x-i)e^{(6\,x)} - 48\,(3i\,x+4i)e^{(4\,x)} - 32\,(-3i\,x-5i)e^{(2\,x)} - 9\,\left(e^{(8\,x)} - 4\,e^{(6\,x)} + 6\,e^{(4\,x)} -\right.}{24\,(e^{(8\,x)}$$

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-24*I*x*e^(8*x) - 96*(-I*x - I)*e^(6*x) - 48*(3*I*x + 4*I)*e^(4*x) -
32*(-3*I*x - 5*I)*e^(2*x) - 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x)
+ 1)*log(e^x + 1) + 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)*log
(e^x - 1) - 24*I*x - 30*e^(7*x) - 18*e^(5*x) - 18*e^(3*x) - 30*e^x - 64*I)/
(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F]

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^6(x)}{\operatorname{csch}(x)+i}\,dx$$

```
[In] integrate(coth(x)**6/(I+csch(x)),x)
[Out] Integral(coth(x)**6/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(31) = 62$.

Time $= 0.19$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 2.23$

$$\begin{aligned}
&\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx \\
&\quad = -i\,x \\
&\qquad + \frac{15\,e^{(-x)} + 80i\,e^{(-2\,x)} + 9\,e^{(-3\,x)} - 96i\,e^{(-4\,x)} + 9\,e^{(-5\,x)} + 48i\,e^{(-6\,x)} + 15\,e^{(-7\,x)} - 32i}{12\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)} \\
&\qquad - \frac{3}{8}\log\left(e^{(-x)}+1\right) + \frac{3}{8}\log\left(e^{(-x)}-1\right)
\end{aligned}$$

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x + 1/12*(15*e^(-x) + 80*I*e^(-2*x) + 9*e^(-3*x) - 96*I*e^(-4*x) + 9*e^(
-5*x) + 48*I*e^(-6*x) + 15*e^(-7*x) - 32*I)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^
(-6*x) - e^(-8*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time $= 0.29$ (sec) , antiderivative size $= 71$, normalized size of antiderivative $= 1.65$

$$\begin{aligned}
&\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx \\
&\quad = -i\,x - \frac{15\,e^{(7\,x)} - 48i\,e^{(6\,x)} + 9\,e^{(5\,x)} + 96i\,e^{(4\,x)} + 9\,e^{(3\,x)} - 80i\,e^{(2\,x)} + 15\,e^{x} + 32i}{12\left(e^{(2\,x)}-1\right)^4} \\
&\qquad - \frac{3}{8}\log\left(e^{x}+1\right) + \frac{3}{8}\log\left(\left|e^{x}-1\right|\right)
\end{aligned}$$

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="giac")
[Out] -I*x - 1/12*(15*e^(7*x) - 48*I*e^(6*x) + 9*e^(5*x) + 96*I*e^(4*x) + 9*e^(3*
x) - 80*I*e^(2*x) + 15*e^x + 32*I)/(e^(2*x) - 1)^4 - 3/8*log(e^x + 1) + 3/8
*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3\,\ln\left(\frac{3}{4}-\frac{3\,e^x}{4}\right)}{8} - x\,1\mathrm{i} - \frac{3\,\ln\left(\frac{3\,e^x}{4}+\frac{3}{4}\right)}{8} - \frac{5\,e^x}{4\,(e^{2x}-1)} - \frac{9\,e^x}{2\,(e^{2x}-1)^2} - \frac{6\,e^x}{(e^{2x}-1)^3} - \frac{4\,e^x}{(e^{2x}-1)^4} + \frac{4\mathrm{i}}{e^{2x}-1} + \frac{4\mathrm{i}}{(e^{2x}-1)^2} + \frac{8\mathrm{i}}{3\,(e^{2x}-1)^3}$$

```
[In] int(coth(x)^6/(1/sinh(x) + 1i),x)
```

```
[Out] (3*log(3/4 - (3*exp(x))/4))/8 - x*1i - (3*log((3*exp(x))/4 + 3/4))/8 - (5*e
xp(x))/(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(e
xp(2*x) - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 4i/(exp(2*x) - 1) + 4i/(exp(
2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

3.113 $\int \frac{\tanh^5(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 595
Rubi [A] (verified) . . . 595
Mathematica [C] (verified) . . . 598
Maple [A] (verified) . . . 599
Fricas [B] (verification not implemented) . . . 599
Sympy [F] . . . 602
Maxima [B] (verification not implemented) . . . 602
Giac [B] (verification not implemented) . . . 603
Mupad [B] (verification not implemented) . . . 603

Optimal result

Integrand size = 13, antiderivative size = 194

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = &-\frac{b^5\arctan(\sinh(x))}{(a^2+b^2)^3}-\frac{b^3\arctan(\sinh(x))}{2(a^2+b^2)^2}\\&-\frac{3b\arctan(\sinh(x))}{8(a^2+b^2)}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^3}+\frac{\log(\sinh(x))}{a}\\&-\frac{a(a^4+3a^2b^2+3b^4)\log(\tanh(x))}{(a^2+b^2)^3}+\frac{3b\operatorname{sech}(x)\tanh(x)}{8(a^2+b^2)}\\&-\frac{(a(a^2+2b^2)-b^3\operatorname{csch}(x))\tanh^2(x)}{2(a^2+b^2)^2}-\frac{(a-b\operatorname{csch}(x))\tanh^4(x)}{4(a^2+b^2)}\end{aligned}$$

```
[Out] -b^5*arctan(sinh(x))/(a^2+b^2)^3-1/2*b^3*arctan(sinh(x))/(a^2+b^2)^2-3/8*b*
arctan(sinh(x))/(a^2+b^2)+b^6*ln(a+b*csch(x))/a/(a^2+b^2)^3+ln(sinh(x))/a-a
*(a^4+3*a^2*b^2+3*b^4)*ln(tanh(x))/(a^2+b^2)^3+3/8*b*sech(x)*tanh(x)/(a^2+b
^2)-1/2*(a*(a^2+2*b^2)-b^3*csch(x))*tanh(x)^2/(a^2+b^2)^2-1/4*(a-b*csch(x))
*tanh(x)^4/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used

$= \{3970, 908, 653, 205, 209, 649, 266\}$

$$\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{3b\arctan(\sinh(x))}{8\left(a^2+b^2\right)} - \frac{b^5\arctan(\sinh(x))}{\left(a^2+b^2\right)^3} - \frac{b^3\arctan(\sinh(x))}{2\left(a^2+b^2\right)^2}$$
$$- \frac{\tanh^4(x)(a-b\operatorname{csch}(x))}{4\left(a^2+b^2\right)} + \frac{3b\tanh(x)\operatorname{sech}(x)}{8\left(a^2+b^2\right)}$$
$$+ \frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3} - \frac{\tanh^2(x)\left(a(a^2+2b^2)-b^3\operatorname{csch}(x)\right)}{2\left(a^2+b^2\right)^2}$$
$$- \frac{a(a^4+3a^2b^2+3b^4)\log(\tanh(x))}{\left(a^2+b^2\right)^3} + \frac{\log(\sinh(x))}{a}$$

```
[In] Int[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b^5*ArcTan[Sinh[x]])/(a^2 + b^2)^3) - (b^3*ArcTan[Sinh[x]])/(2*(a^2 + b^
2)^2) - (3*b*ArcTan[Sinh[x]])/(8*(a^2 + b^2)) + (b^6*Log[a + b*Csch[x]])/(a
*(a^2 + b^2)^3) + Log[Sinh[x]]/a - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Tanh[x]
])/(a^2 + b^2)^3 + (3*b*Sech[x]*Tanh[x])/(8*(a^2 + b^2)) - ((a*(a^2 + 2*b^2
) - b^3*Csch[x])*Tanh[x]^2)/(2*(a^2 + b^2)^2) - ((a - b*Csch[x])*Tanh[x]^4)
/(4*(a^2 + b^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^6 \text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^3}\,dx, x, b\text{csch}(x)\right)$$

$$= b^6 \text{Subst}\left(\int \left(-\frac{1}{ab^6 x} + \frac{1}{a\left(a^2+b^2\right)^3(a+x)} + \frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^3} + \frac{b^4+a\left(a^2+2b^2\right)x}{b^4\left(a^2+b^2\right)^2\left(b^2+x^2\right)^2} + \frac{b^6+a\left(a^4+3a^2b^2+3b^4\right)x}{b^6\left(a^2+b^2\right)^3\left(b^2+x^2\right)}\right)\,dx, x, b\text{csch}(x)\right)$$

$$= \frac{b^6 \log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^3} + \frac{\log(\sinh(x))}{a} + \frac{\text{Subst}\left(\int \frac{b^6+a\left(a^4+3a^2b^2+3b^4\right)x}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{\left(a^2+b^2\right)^3} + \frac{b^2 \text{Subst}\left(\int \frac{b^4+a\left(a^2+2b^2\right)x}{\left(b^2+x^2\right)^2}\,dx, x, b\text{csch}(x)\right)}{\left(a^2+b^2\right)^2} + \frac{b^4 \text{Subst}\left(\int \frac{b^2+ax}{\left(b^2+x^2\right)^3}\,dx, x, b\text{csch}(x)\right)}{a^2+b^2}$$

$$= \frac{b^6 \log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^3} + \frac{\log(\sinh(x))}{a} - \frac{\left(a\left(a^2+2b^2\right)-b^3\text{csch}(x)\right)\tanh^2(x)}{2\left(a^2+b^2\right)^2} - \frac{(a-b\text{csch}(x))\tanh^4(x)}{4\left(a^2+b^2\right)} + \frac{b^6 \text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{\left(a^2+b^2\right)^3} + \frac{b^4 \text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{2\left(a^2+b^2\right)^2} + \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{\left(b^2+x^2\right)^2}\,dx, x, b\text{csch}(x)\right)}{4\left(a^2+b^2\right)} + \frac{\left(a\left(a^4+3a^2b^2+3b^4\right)\right)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{\left(a^2+b^2\right)^3}$$

$$
\begin{aligned}
&= -\frac{b^5 \arctan(\sinh(x))}{\left(a^2+b^2\right)^3} - \frac{b^3 \arctan(\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{b^6 \log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3} \\
&\quad + \frac{\log(\sinh(x))}{a} - \frac{a(a^4+3a^2b^2+3b^4)\log(\tanh(x))}{\left(a^2+b^2\right)^3} \\
&\quad + \frac{3b\operatorname{sech}(x)\tanh(x)}{8\left(a^2+b^2\right)} - \frac{(a(a^2+2b^2)-b^3\operatorname{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)^2} \\
&\quad - \frac{(a-b\operatorname{csch}(x))\tanh^4(x)}{4\left(a^2+b^2\right)} + \frac{(3b^2)\operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{8\left(a^2+b^2\right)} \\
&= -\frac{b^5 \arctan(\sinh(x))}{\left(a^2+b^2\right)^3} - \frac{b^3 \arctan(\sinh(x))}{2\left(a^2+b^2\right)^2} \\
&\quad - \frac{3b \arctan(\sinh(x))}{8\left(a^2+b^2\right)} + \frac{b^6 \log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3} + \frac{\log(\sinh(x))}{a} \\
&\quad - \frac{a(a^4+3a^2b^2+3b^4)\log(\tanh(x))}{\left(a^2+b^2\right)^3} + \frac{3b\operatorname{sech}(x)\tanh(x)}{8\left(a^2+b^2\right)} \\
&\quad - \frac{(a(a^2+2b^2)-b^3\operatorname{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)^2} - \frac{(a-b\operatorname{csch}(x))\tanh^4(x)}{4\left(a^2+b^2\right)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.30

$$
\begin{aligned}
&\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx \\
&= \frac{ab(5a^4+14a^2b^2+9b^4)\arctan(\sinh(x)) + 4a(a^5+ia^4b+3a^3b^2+3ia^2b^3+3ab^4+3ib^5)\log(i-\sinh(x)) + 4}{}
\end{aligned}
$$

```
[In] Integrate[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*ArcTan[Sinh[x]] + 4*a*(a^5 + I*a^4*b + 3*
a^3*b^2 + (3*I)*a^2*b^3 + 3*a*b^4 + (3*I)*b^5)*Log[I - Sinh[x]] + 4*a*(a^5
 - I*a^4*b + 3*a^3*b^2 - (3*I)*a^2*b^3 + 3*a*b^4 - (3*I)*b^5)*Log[I + Sinh[x
]] + 8*b^6*Log[b + a*Sinh[x]] + 4*a^2*(2*a^4 + 5*a^2*b^2 + 3*b^4)*Sech[x]^2
 - 2*a^2*(a^2 + b^2)^2*Sech[x]^4 + a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*Sech[x]
*Tanh[x] - 2*a*b*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(8*a*(a^2 + b^2)^3)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.88

method	result
default	$\frac{2\left(\left(-\frac{3}{8}a^4b-\frac{5}{4}a^2b^3-\frac{7}{8}b^5\right)\tanh\left(\frac{x}{2}\right)^7+\left(-a^5-3a^3b^2-2a\,b^4\right)\tanh\left(\frac{x}{2}\right)^6+\left(-\frac{13}{4}a^2b^3-\frac{15}{8}b^5-\frac{11}{8}a^4b\right)\tanh\left(\frac{x}{2}\right)^5+\left(-4a^5-10a^3b^2-6a\,b^4\right)\tanh\left(\frac{x}{2}\right)$ $\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^4$
risch	$\frac{x}{a}-\frac{2x\,a^5}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6x\,a^3b^2}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6xa\,b^4}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2x\,b^6}{a(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{(5a^2b\,e^{6x}+9b^3e^{6x}$

```
[In] int(tanh(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-3/8*a^4*b-5/4*a^2*b^3-7/8*b^5)*tanh(1/2
*x)^7+(-a^5-3*a^3*b^2-2*a*b^4)*tanh(1/2*x)^6+(-13/4*a^2*b^3-15/8*b^5-11/8*a
^4*b)*tanh(1/2*x)^5+(-4*a^5-10*a^3*b^2-6*a*b^4)*tanh(1/2*x)^4+(13/4*a^2*b^3
+15/8*b^5+11/8*a^4*b)*tanh(1/2*x)^3+(-a^5-3*a^3*b^2-2*a*b^4)*tanh(1/2*x)^2+
(3/8*a^4*b+5/4*a^2*b^3+7/8*b^5)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^4+1/16*(8*a^
5+24*a^3*b^2+24*a*b^4)*ln(1+tanh(1/2*x)^2)+1/8*(-3*a^4*b-10*a^2*b^3-15*b^5)
*arctan(tanh(1/2*x)))-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+b^6/(a^4+
2*a^2*b^2+b^4)/(a^2+b^2)/a*ln(-tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4025 vs. $2(185) = 370$.

Time = 0.37 (sec) , antiderivative size = 4025, normalized size of antiderivative = 20.75

$$\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^8 + 4*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*sinh(x)^8 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)
^7 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*x*cosh(x))*sinh(x)^7 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^6 - (16*a^6 + 40*a^4*b^2 + 24*a^2*b^4 -
112*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 16*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*x + 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^6
 + (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^5 + (3*a^5*b + 2*a^3*b^3 - a*b^5 +
 224*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 21*(5*a^5*b + 14*a^3
*b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^5 - 8*(2*a^6 + 6*a^4*b^2 + 4
*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 - (16*a^6 + 4
8*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^
```

```
4 + 35*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 120*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 - 24*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))
*sinh(x)^4 - (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^3 + (224*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 3*a^5*b - 2*a^3*b^3 + a*b^5 - 35*(5*a^5*
b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 - 160*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 -
2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 10*(3*a^5*b + 2*a^3*b^
3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*
b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + (112*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 - 16*a^6 - 40*a^4*b^2 - 24*a^2*b^4 - 2
1*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 - 120*(2*a^6 + 5*a^4*b^2 + 3*a
^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 10*(3*a^5*b +
 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + 16*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*x - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))*sinh(x)^2 + 4*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + ((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^8 + 8*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (3*a^5*b + 10
*a^3*b^3 + 15*a*b^5)*sinh(x)^8 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^6 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 7*(3*a^5*b + 10*a^3*b^3 + 15*a*b
^5)*cosh(x)^2)*sinh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 8*(7*(3*a^5*b
+ 10*a^3*b^3 + 15*a*b^5)*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*co
sh(x))*sinh(x)^5 + 6*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4 + 2*(9*a^5
*b + 30*a^3*b^3 + 45*a*b^5 + 35*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4
 + 30*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5*
b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 10*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)
*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x)^3 + 4*(3*
a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2 + 4*(7*(3*a^5*b + 10*a^3*b^3 + 15*
a*b^5)*cosh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 15*(3*a^5*b + 10*a^3*b
^3 + 15*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*s
inh(x)^2 + 8*((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^7 + 3*(3*a^5*b + 10
*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x
)^3 + (3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) +
sinh(x)) + (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x) - 4*(b^6*cosh(x)^8 + 8*
b^6*cosh(x)*sinh(x)^7 + b^6*sinh(x)^8 + 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 +
 4*b^6*cosh(x)^2 + 4*(7*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cos
h(x)^3 + 3*b^6*cosh(x))*sinh(x)^5 + 2*(35*b^6*cosh(x)^4 + 30*b^6*cosh(x)^2
+ 3*b^6)*sinh(x)^4 + 8*(7*b^6*cosh(x)^5 + 10*b^6*cosh(x)^3 + 3*b^6*cosh(x))
*sinh(x)^3 + 4*(7*b^6*cosh(x)^6 + 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 + b^6)
*sinh(x)^2 + 8*(b^6*cosh(x)^7 + 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 + b^6*cos
h(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) - 4*((a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)*sinh(
x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 7*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^2)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 8*(7*(a^6 + 3*
```

```
a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*s
inh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2
 + 9*a^2*b^4 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 30*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)
*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)
^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*
a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7
+ 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^
7 - 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^6 + 5*a^5*b + 14*a^3*b^3 + 9
*a*b^5 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*x)*cosh(x)^5 + 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^4 - 32*(2*a^
6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x
)^3 - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^2 - 16*(2*a^6 + 5*a^4*b^2 + 3
*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/((a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^8 + 8*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*cosh(x)*sinh(x)^7 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sinh(x
)^8 + a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
+ a*b^6)*cosh(x)^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 7*(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7 + 3*a^5*b^2 + 3
*a^3*b^4 + a*b^6)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(
x))*sinh(x)^5 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(3*a^
7 + 9*a^5*b^2 + 9*a^3*b^4 + 3*a*b^6 + 35*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x)^4 + 30*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)
^4 + 8*(7*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 + 10*(a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cosh(x))*sinh(x)^3 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 +
 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 7*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cosh(x)^6 + 15*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 9*(
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^7 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cosh(x)^5 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 + (a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\tanh^5(x)}{a+b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(tanh(x)**5/(a+b*csch(x)),x)
[Out] Integral(tanh(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(185) = 370$.

Time $= 0.29$ (sec) , antiderivative size $= 383$, normalized size of antiderivative $= 1.97$

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{b^6\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6}\\ &+\frac{\left(3\,a^4b+10\,a^2b^3+15\,b^5\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\left(a^5+3\,a^3b^2+3\,ab^4\right)\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}\\ &+\frac{(5\,a^2b+9\,b^3)e^{(-x)}+8\,(2\,a^3+3\,ab^2)e^{(-2\,x)}-(3\,a^2b-b^3)e^{(-3\,x)}+16\,(a^3+2\,ab^2)e^{(-4\,x)}+(3\,a^2b-b^3)e^{(-5}}{4\,(a^4+2\,a^2b^2+b^4+4\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+4\,(a^4+2\,a^2b^2+}\\ &+\frac{x}{a}\end{aligned}$$

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
[Out] b^6*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 + 1/4*(3*a^4*b + 10*a^2*b^3 + 15*b^5)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6) + (a^5 + 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) + 1/4*((5*a^2*b + 9*b^3)*e^(-x) + 8*(2*a^3 + 3*a*b^
2)*e^(-2*x) - (3*a^2*b - b^3)*e^(-3*x) + 16*(a^3 + 2*a*b^2)*e^(-4*x) + (3*a
^2*b - b^3)*e^(-5*x) + 8*(2*a^3 + 3*a*b^2)*e^(-6*x) - (5*a^2*b + 9*b^3)*e^(
-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4
 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 +
2*a^2*b^2 + b^4)*e^(-8*x)) + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(185) = 370$.

Time $= 0.30$ (sec) , antiderivative size $= 432$, normalized size of antiderivative $= 2.23$

$$\int \frac{\tanh^5(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{b^6 \log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^7 + 3\,a^5 b^2 + 3\,a^3 b^4 + a b^6}$$
$$- \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^4 b + 10\,a^2 b^3 + 15\,b^5\right)}{16\left(a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6\right)}$$
$$+ \frac{\left(a^5 + 3\,a^3 b^2 + 3\,a b^4\right)\log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)}{2\left(a^6 + 3\,a^4 b^2 + 3\,a^2 b^4 + b^6\right)}$$
$$- \frac{3\,a^5\left(e^{(-x)} - e^{x}\right)^4 + 9\,a^3 b^2\left(e^{(-x)} - e^{x}\right)^4 + 9\,a b^4\left(e^{(-x)} - e^{x}\right)^4 + 5\,a^4 b\left(e^{(-x)} - e^{x}\right)^3 + 14\,a^2 b^3\left(e^{(-x)} - e^{x}\right)^3}$$

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 - 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^4*b + 10*a^2*b^3 + 1
5*b^5)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^5 + 3*a^3*b^2 + 3*a*b^4
)*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5*(e^(-x) - e^x)^4 + 9*a^3*b^2*(e^(-x) - e^x)^4 + 9*a*b^4*(e^(-x) - e^x)^4
+ 5*a^4*b*(e^(-x) - e^x)^3 + 14*a^2*b^3*(e^(-x) - e^x)^3 + 9*b^5*(e^(-x) -
e^x)^3 + 8*a^5*(e^(-x) - e^x)^2 + 32*a^3*b^2*(e^(-x) - e^x)^2 + 48*a*b^4*(e
^(-x) - e^x)^2 + 12*a^4*b*(e^(-x) - e^x) + 40*a^2*b^3*(e^(-x) - e^x) + 28*b
^5*(e^(-x) - e^x) + 16*a^3*b^2 + 64*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*((e^(-x) - e^x)^2 + 4)^2)
```

Mupad [B] (verification not implemented)

Time $= 7.33$ (sec) , antiderivative size $= 611$, normalized size of antiderivative $= 3.15$

$$\int \frac{\tanh^5(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\frac{6\,\mathrm{e}^{x}\left(a^2 b + b^3\right)}{\left(a^2 + b^2\right)^2} + \frac{8\left(a^4 + a^2 b^2\right)}{a\left(a^2 + b^2\right)^2}}{3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1}$$
$$- \frac{\frac{4\,a}{a^2 + b^2} + \frac{4\,b\,\mathrm{e}^{x}}{a^2 + b^2}}{4\,\mathrm{e}^{2\,x} + 6\,\mathrm{e}^{4\,x} + 4\,\mathrm{e}^{6\,x} + \mathrm{e}^{8\,x} + 1} - \frac{x}{a} - \frac{\frac{\mathrm{e}^{x}\left(9\,a^2 b + 13\,b^3\right)}{2\left(a^2 + b^2\right)^2} + \frac{2\left(4\,a^4 + 5\,a^2 b^2\right)}{a\left(a^2 + b^2\right)^2}}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1}$$
$$+ \frac{\frac{\mathrm{e}^{x}\left(5\,a^4 b + 14\,a^2 b^3 + 9\,b^5\right)}{4\left(a^2 + b^2\right)^3} + \frac{2\left(2\,a^6 + 5\,a^4 b^2 + 3\,a^2 b^4\right)}{a\left(a^2 + b^2\right)^3}}{\mathrm{e}^{2\,x} + 1} + \frac{\ln\left(1 + \mathrm{e}^{x}\,1\mathrm{i}\right)\left(-8\,a^2 + a\,b\,21\mathrm{i} + 15\,b^2\right)}{8\left(-a^3 + a^2\,b\,3\mathrm{i} + 3\,a\,b^2 - b^3\,1\mathrm{i}\right)}$$
$$+ \frac{b^6\,\ln\left(64\,a^{13}\,\mathrm{e}^{2\,x} - 64\,a\,b^{12} - 64\,a^{13} + 159\,a^3\,b^{10} - 492\,a^5\,b^8 - 1214\,a^7\,b^6 - 1020\,a^9\,b^4 - 393\,a^{11}\,b^2 + 128\right.}{}$$
$$+ \frac{\ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\left(-a^2\,8\mathrm{i} + 21\,a\,b + b^2\,15\mathrm{i}\right)}{8\left(-a^3\,1\mathrm{i} + 3\,a^2\,b + a\,b^2\,3\mathrm{i} - b^3\right)}$$

```
[In] int(tanh(x)^5/(a + b/sinh(x)),x)
```

```
[Out] ((6*exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (8*(a^4 + a^2*b^2))/(a*(a^2 + b^2
)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*a)/(a^2 + b^2) + (4*b*
exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
- x/a - ((exp(x)*(9*a^2*b + 13*b^3))/(2*(a^2 + b^2)^2) + (2*(4*a^4 + 5*a^2*
b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((exp(x)*(5*a^4*b +
9*b^5 + 14*a^2*b^3))/(4*(a^2 + b^2)^3) + (2*(2*a^6 + 3*a^2*b^4 + 5*a^4*b^2)
)/(a*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(exp(x)*1i + 1)*(a*b*21i - 8*a^2
+ 15*b^2))/(8*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) + (b^6*log(64*a^13*exp(2
*x) - 64*a*b^12 - 64*a^13 + 159*a^3*b^10 - 492*a^5*b^8 - 1214*a^7*b^6 - 102
0*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) - 159*a^3*b^10*exp(2*x) + 492*a^
5*b^8*exp(2*x) + 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) + 393*a^11*b
^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) - 318*a^2*b^11*exp(x)
+ 984*a^4*b^9*exp(x) + 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(x) + 786*a^10
*b^3*exp(x)))/(a*b^6 + a^7 + 3*a^3*b^4 + 3*a^5*b^2) + (log(exp(x) + 1i)*(21
*a*b - a^2*8i + b^2*15i))/(8*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3))
```

3.114 $\int \frac{\tanh^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 605
Rubi [A] (verified) . . . 605
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Optimal result

Integrand size = 13, antiderivative size = 183

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{2b^5\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}} + \frac{b^3\operatorname{sech}(x)}{\left(a^2+b^2\right)^2} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}$$

```
[Out] a*b^2*x/(a^2+b^2)^2+b^4*x/a/(a^2+b^2)^2+a*x/(a^2+b^2)+2*b^5*arctanh((a-b*ta
nh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(5/2)+b^3*sech(x)/(a^2+b^2)^2+b*sec
h(x)/(a^2+b^2)-1/3*b*sech(x)^3/(a^2+b^2)-a*b^2*tanh(x)/(a^2+b^2)^2-a*tanh(x
)/(a^2+b^2)-1/3*a*tanh(x)^3/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3983, 2981, 2686, 3554, 8, 2814, 2739, 632, 210}

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2b^5\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}} + \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} + \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{b^3\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}$$

```
[In] Int[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (b^4*x)/(a*(a^2 + b^2)^2) + (a*x)/(a^2 + b^2) + (
2*b^5*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5/2)) + (
b^3*Sech[x])/(a^2 + b^2)^2 + (b*Sech[x])/(a^2 + b^2) - (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a*b^2*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x])/(a^2 + b^2) - (a*Ta
nh[x]^3)/(3*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
```

```
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = i \int \frac{\sinh(x)\tanh^4(x)}{ib + ia\sinh(x)}\,dx$$

$$= \frac{a\int \tanh^4(x)\,dx}{a^2+b^2} - \frac{b\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a^2+b^2} + \frac{(ib^2)\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} + \frac{(ab^2)\int \tanh^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{b^3\int \operatorname{sech}(x)\tanh(x)\,dx}{\left(a^2+b^2\right)^2}$$
$$+ \frac{(ib^4)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\int \tanh^2(x)\,dx}{a^2+b^2} - \frac{b\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \operatorname{sech}(x)\right)}{a^2+b^2}$$

$$= \frac{b^4 x}{a\left(a^2+b^2\right)^2} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}$$
$$+ \frac{(ab^2)\int 1\,dx}{\left(a^2+b^2\right)^2} + \frac{b^3\text{Subst}(\int 1\,dx, x, \operatorname{sech}(x))}{\left(a^2+b^2\right)^2} - \frac{(ib^5)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a\left(a^2+b^2\right)^2} + \frac{a\int 1\,dx}{a^2+b^2}$$

$$= \frac{ab^2 x}{\left(a^2+b^2\right)^2} + \frac{b^4 x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}$$
$$+ \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh(x)}{a^2+b^2}$$
$$- \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} - \frac{(2ib^5)\,\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2+b^2\right)^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\operatorname{sech}(x)}{\left(a^2+b^2\right)^2} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2}$$
$$- \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} + \frac{(4ib^5)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia - 2ib\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2+b^2\right)^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b^4x}{a\left(a^2+b^2\right)^2} + \frac{ax}{a^2+b^2} + \frac{2b^5\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{5/2}} + \frac{b^3\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}$$
$$+ \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{1}{3}\left(\frac{3\left(x - \frac{2b^5\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{5/2}}\right)}{a} + \frac{3b(a^2+2b^2)\operatorname{sech}(x)}{\left(a^2+b^2\right)^2} - \frac{b\operatorname{sech}^3(x)}{a^2+b^2}\right.$$
$$\left. - \frac{a(4a^2+7b^2)\tanh(x)}{\left(a^2+b^2\right)^2} + \frac{a\operatorname{sech}^2(x)\tanh(x)}{a^2+b^2}\right)$$

```
[In] Integrate[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*(x - (2*b^5*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(5
/2)))/a + (3*b*(a^2 + 2*b^2)*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x]^3)/(a^2 +
b^2) - (a*(4*a^2 + 7*b^2)*Tanh[x])/(a^2 + b^2)^2 + (a*Sech[x]^2*Tanh[x])/(a
^2 + b^2))/3
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{a}+\frac{\ln(\tanh(\frac{x}{2})+1)}{a}+\frac{2(-a^3-2ab^2)\tanh(\frac{x}{2})^5+2b^3\tanh(\frac{x}{2})^4+2(-\frac{10}{3}a^3-\frac{16}{3}ab^2)\tanh(\frac{x}{2})^3+2(2a^2b+4b^3)\tanh(\frac{x}{2})^2+(-a^3-2ab^2)\tanh(\frac{x}{2})+\frac{2a^2b}{3}+\frac{5b^3}{3}}{(a^4+2a^2b^2+b^4)(1+\tanh(\frac{x}{2})^2)^3}+\frac{b^5\ln\left(\tanh(\frac{x}{2})+\frac{a}{b}-\frac{\sqrt{a^2+b^2}}{b}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{b^5\ln\left(\tanh(\frac{x}{2})+\frac{a}{b}+\frac{\sqrt{a^2+b^2}}{b}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}$
risch	$\frac{x}{a}+\frac{2a^2be^{5x}+4b^3e^{5x}+4a^3e^{4x}+6ab^2e^{4x}+\frac{4a^2be^{3x}}{3}+\frac{16b^3e^{3x}}{3}+4a^3e^{2x}+8ab^2e^{2x}+2a^2be^x+4b^3e^x+\frac{8a^3}{3}+\frac{14ab^2}{3}}{(a^4+2a^2b^2+b^4)(1+e^{2x})^3}+\frac{b^5\ln\left(e^x+\frac{(a^2+$

```
[In] int(tanh(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)+2/(a^4+2*a^2*b^2+b^4)*((-a^3-2
*a*b^2)*tanh(1/2*x)^5+b^3*tanh(1/2*x)^4+(-10/3*a^3-16/3*a*b^2)*tanh(1/2*x)^
3+(2*a^2*b+4*b^3)*tanh(1/2*x)^2+(-a^3-2*a*b^2)*tanh(1/2*x)+2/3*a^2*b+5/3*b^
3)/(1+tanh(1/2*x)^2)^3+2/a*b^5/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(
1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1746 vs. 2(175) = 350.

Time = 0.30 (sec) , antiderivative size = 1746, normalized size of antiderivative = 9.54

$$\int \frac{\tanh^4(x)}{a+b\mathrm{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 + 3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*x*sinh(x)^6 + 8*a^6 + 22*a^4*b^2 + 14*a^2*b^4 + 6*(a^5*
b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5 + 3*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x))*sinh(x)^5 + 3*(4*a^6 + 10*a^4*b
^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 3*(4*
a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cos
h(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 10*(a^5*b + 3*a^3*b^3 +
2*a*b^5)*cosh(x))*sinh(x)^4 + 4*(a^5*b + 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 + 4
*(a^5*b + 5*a^3*b^3 + 4*a*b^5 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*co
sh(x)^3 + 15*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 + 10*a^4*b^
2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3
 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*x)*cosh(x)^2 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*x*cosh(x)^4 + 20*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 +
 6*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
```

```
x)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 4*(a^5*b + 5*a^3*b
^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 + 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)
^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cos
h(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3
*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2
*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 +
 a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x
) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 3*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*x + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 + a^5*b + 3*a^3*b^3 + 2*a*b^5 + 5*(
a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 + 2*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4
 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 2*(a^5*b + 5*a^3*b^3
 + 4*a*b^5)*cosh(x)^2 + (4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^6 + 6*(a^7 + 3*a^5*b^2
 + 3*a^3*b^4 + a*b^6)*cosh(x)*sinh(x)^5 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*sinh(x)^6 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 3*(a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x)^2)*sinh(x)^4 + 4*(5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3
 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^3 + 3*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 6*(a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^5 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 +
 (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\tanh^4(x)}{a+b\,\text{csch}\,(x)}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.43

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b^5 \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{a^2+b^2}}$$
$$-\frac{2\left(4\,a^3+7\,ab^2-3\,(a^2b+2\,b^3)e^{(-x)}+6\,(a^3+2\,ab^2)e^{(-2\,x)}-2\,(a^2b+4\,b^3)e^{(-3\,x)}+3\,(2\,a^3+3\,ab^2)e^{(-4\,x)}\right.}{3\,(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+}$$
$$+\frac{x}{a}$$

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) - 2/3*(4*a^3 + 7*a*b^2 - 3*(a^
2*b + 2*b^3)*e^(-x) + 6*(a^3 + 2*a*b^2)*e^(-2*x) - 2*(a^2*b + 4*b^3)*e^(-3*
x) + 3*(2*a^3 + 3*a*b^2)*e^(-4*x) - 3*(a^2*b + 2*b^3)*e^(-5*x))/(a^4 + 2*a^
2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4
)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x)) + x/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b^5 \log\left(\frac{\left|2\,ae^x+2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\sqrt{a^2+b^2}\right|}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{a^2+b^2}}+\frac{x}{a}$$
$$+\frac{2\left(3\,a^2be^{(5\,x)}+6\,b^3e^{(5\,x)}+6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}+2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,a^2b\right.}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) + x/a + 2/3*(3*a^2
*b*e^(5*x) + 6*b^3*e^(5*x) + 6*a^3*e^(4*x) + 9*a*b^2*e^(4*x) + 2*a^2*b*e^(3
*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*a^2*b*e^x + 6*b^
3*e^x + 4*a^3 + 7*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.86

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{x}{a} + \frac{\frac{8a}{3\left(a^2+b^2\right)} + \frac{8be^x}{3\left(a^2+b^2\right)}}{3e^{2x}+3e^{4x}+e^{6x}+1} - \frac{\frac{8e^x\left(a^2b+b^3\right)}{3\left(a^2+b^2\right)^2} + \frac{4\left(a^4+a^2b^2\right)}{a\left(a^2+b^2\right)^2}}{2e^{2x}+e^{4x}+1} + \frac{\frac{2e^x\left(a^2b+2b^3\right)}{\left(a^2+b^2\right)^2} + \frac{2\left(2a^4+3a^2b^2\right)}{a\left(a^2+b^2\right)^2}}{e^{2x}+1}$$
$$+ \frac{2\operatorname{atan}\left(\left(e^x\left(\frac{2b^5}{a^3\sqrt{b^{10}}\left(a^2+b^2\right)^2\left(a^5+2a^3b^2+ab^4\right)} + \frac{2\left(2a^3b^3\sqrt{b^{10}}+ab^5\sqrt{b^{10}}+a^5b\sqrt{b^{10}}\right)}{a^2b^4\sqrt{-a^2\left(a^2+b^2\right)^5}\left(a^5+2a^3b^2+ab^4\right)\sqrt{-a^{12}-5a^{10}b^2-10a^8b^4-10a^6b^6-5a^4b}}\right.\right.\right.}{}$$

```
[In] int(tanh(x)^4/(a + b/sinh(x)),x)
```

```
[Out] x/a + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) +
3*exp(4*x) + exp(6*x) + 1) - ((8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2) +
(4*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((2*ex
p(x)*(a^2*b + 2*b^3))/(a^2 + b^2)^2 + (2*(2*a^4 + 3*a^2*b^2))/(a*(a^2 + b^2
)^2))/(exp(2*x) + 1) + (2*atan((exp(x)*((2*b^5)/(a^3*(b^10)^(1/2)*(a^2 + b^
2)^2*(a*b^4 + a^5 + 2*a^3*b^2)) + (2*(2*a^3*b^3*(b^10)^(1/2) + a*b^5*(b^10)
^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 +
a^5 + 2*a^3*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
 5*a^10*b^2)^(1/2))) - (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) + 2*a^4*
b^2*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 + a^5 + 2*a^3
*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2
)^(1/2)))*((a^6*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
5*a^10*b^2)^(1/2))/2 + (a^2*b^4*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6
 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2))/2 + a^4*b^2*(- a^12 - a^2*b^10 - 5*a^4*b
^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(- a^12 -
a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

3.115 $\int \frac{\tanh^3(x)}{a+b\mathbf{csch}(x)}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b^3\arctan(\sinh(x))}{(a^2+b^2)^2} - \frac{b\arctan(\sinh(x))}{2(a^2+b^2)} + \frac{b^4\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^2} + \frac{\log(\sinh(x))}{a} - \frac{a(a^2+2b^2)\log(\tanh(x))}{(a^2+b^2)^2} - \frac{(a-b\operatorname{csch}(x))\tanh^2(x)}{2(a^2+b^2)}$$

```
[Out] -b^3*arctan(sinh(x))/(a^2+b^2)^2-1/2*b*arctan(sinh(x))/(a^2+b^2)+b^4*ln(a+b
*csch(x))/a/(a^2+b^2)^2+ln(sinh(x))/a-a*(a^2+2*b^2)*ln(tanh(x))/(a^2+b^2)^2
-1/2*(a-b*csch(x))*tanh(x)^2/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3970, 908, 653, 209, 649, 266}

$$\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\arctan(\sinh(x))}{2(a^2+b^2)} - \frac{b^3\arctan(\sinh(x))}{(a^2+b^2)^2} - \frac{a(a^2+2b^2)\log(\tanh(x))}{(a^2+b^2)^2} - \frac{\tanh^2(x)(a-b\operatorname{csch}(x))}{2(a^2+b^2)} + \frac{b^4\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^2} + \frac{\log(\sinh(x))}{a}$$

```
[In] Int[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b^3*ArcTan[Sinh[x]])/(a^2 + b^2)^2) - (b*ArcTan[Sinh[x]])/(2*(a^2 + b^2)
) + (b^4*Log[a + b*Csch[x]])/(a*(a^2 + b^2)^2) + Log[Sinh[x]]/a - (a*(a^2 +
 2*b^2)*Log[Tanh[x]])/(a^2 + b^2)^2 - ((a - b*Csch[x])*Tanh[x]^2)/(2*(a^2 +
 b^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(b^4 \text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\,\text{csch}(x)\right)\right)$$

$$= -\left(b^4 \text{Subst}\left(\int \left(\frac{1}{ab^4 x} - \frac{1}{a\left(a^2+b^2\right)^2(a+x)} + \frac{-b^2-ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^2} + \frac{-b^4-a\left(a^2+2b^2\right)x}{b^4\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx\right.\right.$$

$$= \frac{b^4 \log(a + b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^2} + \frac{\log(\sinh(x))}{a} - \frac{\operatorname{Subst}\left(\int \frac{-b^4-a\left(a^2+2b^2\right)x}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{\left(a^2+b^2\right)^2}$$
$$- \frac{b^2 \operatorname{Subst}\left(\int \frac{-b^2-ax}{\left(b^2+x^2\right)^2}\,dx, x, b\operatorname{csch}(x)\right)}{a^2+b^2}$$
$$= \frac{b^4 \log(a + b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^2} + \frac{\log(\sinh(x))}{a} - \frac{(a - b\operatorname{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)}$$
$$+ \frac{b^4 \operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{\left(a^2+b^2\right)^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{2\left(a^2+b^2\right)}$$
$$+ \frac{\left(a\left(a^2+2b^2\right)\right) \operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{\left(a^2+b^2\right)^2}$$
$$= -\frac{b^3 \arctan(\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b \arctan(\sinh(x))}{2\left(a^2+b^2\right)} + \frac{b^4 \log(a + b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^2}$$
$$+ \frac{\log(\sinh(x))}{a} - \frac{a\left(a^2+2b^2\right)\log(\tanh(x))}{\left(a^2+b^2\right)^2} - \frac{(a - b\operatorname{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.69

$$\int \frac{\tanh^3(x)}{a + b\operatorname{csch}(x)}\,dx$$
$$= \frac{ab\left(a^2+b^2\right)\arctan(\sinh(x)) + a^4\log(i - \sinh(x)) + ia^3b\log(i - \sinh(x)) + 2a^2b^2\log(i - \sinh(x)) + 2iab^3}{}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^4*Log[I - Sinh[x]] + I*a^3*b*Log[I - S
inh[x]] + 2*a^2*b^2*Log[I - Sinh[x]] + (2*I)*a*b^3*Log[I - Sinh[x]] + a^4*L
og[I + Sinh[x]] - I*a^3*b*Log[I + Sinh[x]] + 2*a^2*b^2*Log[I + Sinh[x]] - (
2*I)*a*b^3*Log[I + Sinh[x]] + 2*b^4*Log[b + a*Sinh[x]] + a^2*(a^2 + b^2)*Se
ch[x]^2 + a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*a*(a^2 + b^2)^2)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.61

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})+1)}{a}-\frac{\ln(\tanh(\frac{x}{2})-1)}{a}+\frac{b^4\ln\left(-\tanh(\frac{x}{2})^2b+2a\tanh(\frac{x}{2})+b\right)}{a(a^2+b^2)^2}+\frac{2\left(\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh(\frac{x}{2})^3+\left(-a^3-ab^2\right)\tanh(\frac{x}{2})^2$
risch	$\frac{x}{a}-\frac{2xa^3}{a^4+2a^2b^2+b^4}-\frac{4xab^2}{a^4+2a^2b^2+b^4}-\frac{2xb^4}{a(a^4+2a^2b^2+b^4)}+\frac{e^x(be^{2x}+2ae^x-b)}{(1+e^{2x})^2(a^2+b^2)}+\frac{i\ln(e^x-i)a^2b}{2a^4+4a^2b^2+2b^4}+\frac{3i\ln(e^x-i)b^3}{2(a^4+2a^2b^2+b^4)}+\frac{1}{a^4}$

```
[In] int(tanh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+b^4/a/(a^2+b^2)^2*ln(-tanh(1/2
*x)^2*b+2*a*tanh(1/2*x)+b)+2/(a^2+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*x)
^3+(-a^3-a*b^2)*tanh(1/2*x)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*x))/(1+tanh(1/2*
x)^2)^2+1/4*(2*a^3+4*a*b^2)*ln(1+tanh(1/2*x)^2)+1/2*(-a^2*b-3*b^3)*arctan(t
anh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(110) = 220$.

Time = 0.30 (sec) , antiderivative size = 965, normalized size of antiderivative = 8.54

$$\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*x*sinh(x)^4
 - (a^3*b + a*b^3)*cosh(x)^3 - (a^3*b + a*b^3 - 4*(a^4 + 2*a^2*b^2 + b^4)*x
*cosh(x))*sinh(x)^3 - 2*(a^4 + a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x)
^2 - (2*a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 2*(a^4 +
2*a^2*b^2 + b^4)*x + 3*(a^3*b + a*b^3)*cosh(x))*sinh(x)^2 + (a^4 + 2*a^2*b^
2 + b^4)*x + ((a^3*b + 3*a*b^3)*cosh(x)^4 + 4*(a^3*b + 3*a*b^3)*cosh(x)*sin
h(x)^3 + (a^3*b + 3*a*b^3)*sinh(x)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3
)*cosh(x)^2 + 2*(a^3*b + 3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(x)^2)*sinh(x)^2
 + 4*((a^3*b + 3*a*b^3)*cosh(x)^3 + (a^3*b + 3*a*b^3)*cosh(x))*sinh(x))*arc
tan(cosh(x) + sinh(x)) + (a^3*b + a*b^3)*cosh(x) - (b^4*cosh(x)^4 + 4*b^4*c
osh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*log(2*(a*sin
h(x) + b)/(cosh(x) - sinh(x))) - ((a^4 + 2*a^2*b^2)*cosh(x)^4 + 4*(a^4 + 2*
a^2*b^2)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2)*sinh(x)^4 + a^4 + 2*a^2*b^2
+ 2*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*
cosh(x)^2)*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*a^2*b^2)*c
```

```
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (4*(a^4 + 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 + a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 - 4*(a^4 +
a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2
+ a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^3 + (a^5 + 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\tanh^3(x)}{a+b\,\text{csch}(x)}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.52

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx = {} & \frac{b^4\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^5+2\,a^3b^2+ab^4} \\ & + \frac{\left(a^2b+3\,b^3\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4} \\ & + \frac{be^{(-x)}+2\,ae^{(-2\,x)}-be^{(-3\,x)}}{a^2+b^2+2\left(a^2+b^2\right)e^{(-2\,x)}+\left(a^2+b^2\right)e^{(-4\,x)}} + \frac{x}{a}\end{aligned}$$

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^5 + 2*a^3*b^2 + a*b^4) + (a^2*b +
3*b^3)*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 2*a*b^2)*log(e^(-2*x
) + 1)/(a^4 + 2*a^2*b^2 + b^4) + (b*e^(-x) + 2*a*e^(-2*x) - b*e^(-3*x))/(a^
2 + b^2 + 2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x)) + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(110) = 220$.

Time $= 0.27$ (sec) , antiderivative size $= 234$, normalized size of antiderivative $= 2.07$

$$\begin{aligned}
&\int \frac{\tanh^3(x)}{a + b\operatorname{csch}(x)}\,dx \\
&= \frac{b^4 \log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^5 + 2\,a^3 b^2 + a b^4} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^2 b + 3\,b^3\right)}{4\left(a^4 + 2\,a^2 b^2 + b^4\right)} \\
&\quad + \frac{\left(a^3 + 2\,a b^2\right)\log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2 b^2 + b^4\right)} \\
&\quad - \frac{a^3\left(e^{(-x)} - e^{x}\right)^2 + 2\,a b^2\left(e^{(-x)} - e^{x}\right)^2 + 2\,a^2 b\left(e^{(-x)} - e^{x}\right) + 2\,b^3\left(e^{(-x)} - e^{x}\right) + 4\,a b^2}{2\left(a^4 + 2\,a^2 b^2 + b^4\right)\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)}
\end{aligned}$$

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) - 1/4*(pi +
 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4
) + 1/2*(a^3 + 2*a*b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) -
 1/2*(a^3*(e^(-x) - e^x)^2 + 2*a*b^2*(e^(-x) - e^x)^2 + 2*a^2*b*(e^(-x) - e
^x) + 2*b^3*(e^(-x) - e^x) + 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e
^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time $= 4.35$ (sec) , antiderivative size $= 335$, normalized size of antiderivative $= 2.96$

$$\begin{aligned}
&\int \frac{\tanh^3(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\frac{\mathrm{e}^{x}\left(a^2 b + b^3\right)}{\left(a^2 + b^2\right)^2} + \frac{2\left(a^4 + a^2 b^2\right)}{a\left(a^2 + b^2\right)^2}}{\mathrm{e}^{2\,x} + 1} - \frac{\frac{2\,a}{a^2 + b^2} + \frac{2\,b\,\mathrm{e}^{x}}{a^2 + b^2}}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} - \frac{x}{a} \\
&\quad + \frac{b^4 \ln\left(4\,a^9\,\mathrm{e}^{2\,x} - 4\,a\,b^8 - 4\,a^9 + 7\,a^3 b^6 - 14\,a^5 b^4 - 17\,a^7 b^2 + 8\,b^9\,\mathrm{e}^{x} - 7\,a^3 b^6\,\mathrm{e}^{2\,x} + 14\,a^5 b^4\,\mathrm{e}^{2\,x} + 17\,a^7 b^2
}{a^5 + 2\,a^3 b^2 + a\,b^4} \\
&\quad + \frac{\ln\left(1 + \mathrm{e}^{x}\,1\mathrm{i}\right)\left(3\,b + a\,2\mathrm{i}\right)}{2\left(a^2\,1\mathrm{i} + 2\,a\,b - b^2\,1\mathrm{i}\right)} + \frac{\ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\left(2\,a + b\,3\mathrm{i}\right)}{2\left(a^2 + a\,b\,2\mathrm{i} - b^2\right)}
\end{aligned}$$

```
[In] int(tanh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] ((exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (2*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^
2))/(exp(2*x) + 1) - ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(2*exp(
2*x) + exp(4*x) + 1) - x/a + (b^4*log(4*a^9*exp(2*x) - 4*a*b^8 - 4*a^9 + 7*
a^3*b^6 - 14*a^5*b^4 - 17*a^7*b^2 + 8*b^9*exp(x) - 7*a^3*b^6*exp(2*x) + 14*
a^5*b^4*exp(2*x) + 17*a^7*b^2*exp(2*x) + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x)
- 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x) + 34*a^6*b^3*exp(x)))/(a*b^4 + a^5
+ 2*a^3*b^2) + (log(exp(x)*1i + 1)*(a*2i + 3*b))/(2*(2*a*b + a^2*1i - b^2*1
i)) + (log(exp(x) + 1i)*(2*a + b*3i))/(2*(a*b*2i + a^2 - b^2))
```

3.116 $\int \frac{\tanh^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 619
Rubi [A] (verified) . . . 619
Mathematica [A] (verified) . . . 621
Maple [A] (verified) . . . 622
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Sympy [F] . . . 623
Maxima [A] (verification not implemented) . . . 623
Giac [A] (verification not implemented) . . . 623
Mupad [B] (verification not implemented) . . . 624

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{ax}{a^2+b^2} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{2b^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)+b^2*x/a/(a^2+b^2)+2*b^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(
1/2))/a/(a^2+b^2)^(3/2)+b*sech(x)/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3983, 2981, 2686, 8, 3554, 2814, 2739, 632, 210}

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2b^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}} + \frac{b^2x}{a\left(a^2+b^2\right)} + \frac{ax}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} + \frac{b\operatorname{sech}(x)}{a^2+b^2}$$

```
[In] Int[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b^2*x)/(a*(a^2 + b^2)) + (2*b^3*ArcTanh[(a - b*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)) + (b*Sech[x])/(a^2 + b^2) - (a*
Tanh[x])/(a^2 + b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i \int \frac{\sinh(x)\tanh^2(x)}{ib + ia\sinh(x)}\,dx \\
&= \frac{a\int \tanh^2(x)\,dx}{a^2+b^2} - \frac{b\int \operatorname{sech}(x)\tanh(x)\,dx}{a^2+b^2} + \frac{(ib^2)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2} \\
&= \frac{b^2 x}{a\left(a^2+b^2\right)} - \frac{a\tanh(x)}{a^2+b^2} + \frac{a\int 1\,dx}{a^2+b^2} + \frac{b\text{Subst}(\int 1\,dx, x, \operatorname{sech}(x))}{a^2+b^2} - \frac{(ib^3)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a\left(a^2+b^2\right)} \\
&= \frac{ax}{a^2+b^2} + \frac{b^2 x}{a\left(a^2+b^2\right)} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{(2ib^3)\,\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2+b^2\right)} \\
&= \frac{ax}{a^2+b^2} + \frac{b^2 x}{a\left(a^2+b^2\right)} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} \\
&\quad + \frac{(4ib^3)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, 2ia - 2ib\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2+b^2\right)} \\
&= \frac{ax}{a^2+b^2} + \frac{b^2 x}{a\left(a^2+b^2\right)} + \frac{2b^3 \operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x + \frac{2b^3 \arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}}{a} + \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (x + (2*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2))
/a + (b*Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{a}+\frac{\ln(\tanh(\frac{x}{2})+1)}{a}+\frac{-2a\tanh(\frac{x}{2})+2b}{(a^2+b^2)\left(1+\tanh(\frac{x}{2})^2\right)}+\frac{2b^3\operatorname{arctanh}\left(\frac{-2b\tanh(\frac{x}{2})+2a}{2\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{\frac{3}{2}}}$	95
risch	$\frac{x}{a}+\frac{2e^x b+2a}{(1+e^{2x})(a^2+b^2)}+\frac{b^3\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a}-\frac{b^3\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b-a^4-2a^2b^2-b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a}$	155

```
[In] int(tanh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)+2/(a^2+b^2)*(-a*tanh(1/2*x)+b)
/(1+tanh(1/2*x)^2)+2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*
a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.49

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{2\,a^4+2\,a^2b^2+(a^4+2\,a^2b^2+b^4)x\cosh(x)^2+(a^4+2\,a^2b^2+b^4)x\sinh(x)^2+\left(b^3\cosh(x)^2+2\,b^3\cosh(x)\,\mathrm{si}\right.}{\mathrm{c}}$$

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^4 + 2*a^2*b^2 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 + (a^4 + 2*a^2*b^2
 + b^4)*x*sinh(x)^2 + (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^
2 + b^3)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) - a)) + (a^4 + 2*a^2*b^2 + b^4)*x + 2*(a^3*b + a*b^3)*cosh(x
) + 2*(a^3*b + a*b^3 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 + 2
*a^3*b^2 + a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*cosh(x)*sinh(x) + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\tanh^2(x)}{a+b\,\text{csch}(x)}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*csch(x)),x)
[Out] Integral(tanh(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{b^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}} + \frac{2\left(be^{(-x)}-a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}} + \frac{x}{a}$$

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
[Out] -b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2
)*e^(-2*x)) + x/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{b^3\log\left(\frac{\left|2ae^x+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^x+2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}} + \frac{x}{a} + \frac{2\left(be^x+a\right)}{(a^2+b^2)(e^{(2x)}+1)}$$

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="giac")
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + x/a + 2*(b*e^x + a)/((a^2 +
b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.76

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a} + \frac{\frac{2\,a}{a^2+b^2} + \frac{2\,b\,e^x}{a^2+b^2}}{e^{2\,x}+1} + \frac{2\,\operatorname{atan}\left(\left(\frac{a^4\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}{2} + \frac{a^2\,b^2\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}{2}\right)\left(e^x\left(\frac{2\,b^3}{a^3\,(a^3+a\,b^2)\,\sqrt{b^6\,(a^2+b^2)}} + \frac{}{a^2\,b^2\,\sqrt{-a^2\,(a^2}\right.\right.\right.}{\sqrt{-a^8-3\,a^6\,b^2-3\,a^4}}$$

```
[In] int(tanh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] x/a + ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) + (2*at
an(((a^4*(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2 + (a^2*b^2*(- a
^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2)*(exp(x)*((2*b^3)/(a^3*(a*b^
2 + a^3)*(b^6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*(b^6)^(1/2) + a^3*b*(b^6)^(1/
2)))/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6 - 3
*a^4*b^4 - 3*a^6*b^2)^(1/2))) - (2*(a^4*(b^6)^(1/2) + a^2*b^2*(b^6)^(1/2)))
/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6 - 3*a^4
*b^4 - 3*a^6*b^2)^(1/2))))*(b^6)^(1/2))/(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^
6*b^2)^(1/2)
```

3.117 $\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result . . . 625
Rubi [A] (verified) . . . 625
Mathematica [C] (verified) . . . 627
Maple [A] (verified) . . . 627
Fricas [A] (verification not implemented) . . . 627
Sympy [F] . . . 628
Maxima [A] (verification not implemented) . . . 628
Giac [A] (verification not implemented) . . . 628
Mupad [B] (verification not implemented) . . . 629

Optimal result

Integrand size = 11, antiderivative size = 61

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\arctan(\sinh(x))}{a^2+b^2} + \frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)} + \frac{\log(\sinh(x))}{a} - \frac{a\log(\tanh(x))}{a^2+b^2}$$

```
[Out] -b*arctan(sinh(x))/(a^2+b^2)+b^2*ln(a+b*csch(x))/a/(a^2+b^2)+ln(sinh(x))/a-
a*ln(tanh(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3970, 908, 649, 209, 266}

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\arctan(\sinh(x))}{a^2+b^2} - \frac{a\log(\tanh(x))}{a^2+b^2} + \frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)} + \frac{\log(\sinh(x))}{a}$$

```
[In] Int[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/(a^2 + b^2)) + (b^2*Log[a + b*Csch[x]])/(a*(a^2 + b^2
)) + Log[Sinh[x]]/a - (a*Log[Tanh[x]])/(a^2 + b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)}\,dx, x, b\operatorname{csch}(x)\right) \\
&= b^2 \text{Subst}\left(\int \left(-\frac{1}{ab^2x}+\frac{1}{a\left(a^2+b^2\right)(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)\,dx, x, b\operatorname{csch}(x)\right) \\
&= \frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{b^2+ax}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{a^2+b^2} \\
&= \frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a} \\
&\quad +\frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{a^2+b^2}+\frac{b^2\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{a^2+b^2} \\
&= -\frac{b\arctan(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}-\frac{a\log(\tanh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a(a+ib)\log(i-\sinh(x))+a(a-ib)\log(i+\sinh(x))+2b^2\log(b+a\sinh(x))}{2a\left(a^2+b^2\right)}$$

```
[In] Integrate[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] (a*(a + I*b)*Log[I - Sinh[x]] + a*(a - I*b)*Log[I + Sinh[x]] + 2*b^2*Log[b
+ a*Sinh[x]])/(2*a*(a^2 + b^2))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

method	result	siz
default	$\frac{4a\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)-8b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^2+4b^2}+\frac{b^2\ln\left(-\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a\left(a^2+b^2\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	97
risch	$\frac{x}{a}-\frac{2ax}{a^2+b^2}-\frac{2b^2x}{a\left(a^2+b^2\right)}+\frac{i\ln\left(\mathrm{e}^x-i\right)b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x-i\right)a}{a^2+b^2}-\frac{i\ln\left(\mathrm{e}^x+i\right)b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x+i\right)a}{a^2+b^2}+\frac{b^2\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^x}{a}-1\right)}{a\left(a^2+b^2\right)}$	14

```
[In] int(tanh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/(4*a^2+4*b^2)*(1/2*a*ln(1+tanh(1/2*x)^2)-b*arctan(tanh(1/2*x)))+b^2/a/(a^
2+b^2)*ln(-tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-1/a*ln(tanh(1/2*x)-1)-1/a*ln(
tanh(1/2*x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{2\,ab\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b^2\log\left(\frac{2\left(a\sinh(x)+b\right)}{\cosh(x)-\sinh(x)}\right)-a^2\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\left(a^2+b^2\right)x}{a^3+ab^2}$$

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*arctan(cosh(x) + sinh(x)) - b^2*log(2*(a*sinh(x) + b)/(cosh(x) - si
nh(x))) - a^2*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^2 + b^2)*x)/(a^3 + a*
b^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(tanh(x)/(a+b*csch(x)),x)
[Out] Integral(tanh(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b^2\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^3+ab^2} + \frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2} + \frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2} + \frac{x}{a}$$

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="maxima")
[Out] b^2*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^3 + a*b^2) + 2*b*arctan(e^(-x))/(a
^2 + b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2) + x/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b^2\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^3+ab^2} - \frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\left(a^2+b^2\right)} + \frac{a\log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="giac")
[Out] b^2*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) - 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{a-b\,1\mathrm{i}} - \frac{x}{a}$$
$$+ \frac{b^2\,\ln\left(a^5\,\mathrm{e}^{2\,x} - a\,b^4 - a^5 + a^3\,b^2 + 2\,b^5\,\mathrm{e}^{x} - a^3\,b^2\,\mathrm{e}^{2\,x} + 2\,a^4\,b\,\mathrm{e}^{x} + a\,b^4\,\mathrm{e}^{2\,x} - 2\,a^2\,b^3\,\mathrm{e}^{x}\right)}{a^3 + a\,b^2}$$
$$+ \frac{\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,1\mathrm{i}}{-b+a\,1\mathrm{i}}$$

```
[In] int(tanh(x)/(a + b/sinh(x)),x)
```

```
[Out] log(exp(x)*1i + 1)/(a - b*1i) + (log(exp(x) + 1i)*1i)/(a*1i - b) - x/a + (b
^2*log(a^5*exp(2*x) - a*b^4 - a^5 + a^3*b^2 + 2*b^5*exp(x) - a^3*b^2*exp(2*
x) + 2*a^4*b*exp(x) + a*b^4*exp(2*x) - 2*a^2*b^3*exp(x)))/(a*b^2 + a^3)
```

3.118 $\int \frac{\coth(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 630
Rubi [A] (verified) . . . 630
Mathematica [A] (verified) . . . 631
Maple [A] (verified) . . . 631
Fricas [A] (verification not implemented) . . . 632
Sympy [F] . . . 632
Maxima [A] (verification not implemented) . . . 632
Giac [A] (verification not implemented) . . . 633
Mupad [B] (verification not implemented) . . . 633

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log(a+b\operatorname{csch}(x))}{a} + \frac{\log(\sinh(x))}{a}$$

```
[Out] ln(a+b*csch(x))/a+ln(sinh(x))/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3970, 36, 29, 31}

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log(a+b\operatorname{csch}(x))}{a} + \frac{\log(\sinh(x))}{a}$$

```
[In] Int[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[a + b*Csch[x]]/a + Log[Sinh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\text{csch}(x)\right) \\ &= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\text{csch}(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\text{csch}(x)\right)}{a} \\ &= \frac{\log(a+b\text{csch}(x))}{a} + \frac{\log(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx = \frac{\log(b+a\sinh(x))}{a}$$

```
[In] Integrate[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[b + a*Sinh[x]]/a
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\frac{\ln(\text{csch}(x))}{a} + \frac{\ln(a+b\ \text{csch}(x))}{a}$	21
default	$-\frac{\ln(\text{csch}(x))}{a} + \frac{\ln(a+b\ \text{csch}(x))}{a}$	21
risch	$-\frac{x}{a} + \frac{\ln\left(e^{2x} + \frac{2b\,e^{x}}{a} - 1\right)}{a}$	27

```
[In] int(coth(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(csch(x))+ln(a+b*csch(x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{x-\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [F]

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\coth\left(x\right)}{a+b\,\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(coth(x)/(a+b*csch(x)),x)
[Out] Integral(coth(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a}+\frac{\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="maxima")
[Out] x/a + log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="giac")
[Out] log(abs(-a*(e^(-x) - e^x) + 2*b))/a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{x-\ln\left(2\,b\,\mathrm{e}^{x}-a+a\,\mathrm{e}^{2\,x}\right)}{a}$$

```
[In] int(coth(x)/(a + b/sinh(x)),x)
[Out] -(x - log(2*b*exp(x) - a + a*exp(2*x)))/a
```

3.119 $\int \frac{\coth^2(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 634
Rubi [A] (verified) . . . 634
Mathematica [A] (verified) . . . 636
Maple [A] (verified) . . . 636
Fricas [B] (verification not implemented) . . . 637
Sympy [F] . . . 637
Maxima [A] (verification not implemented) . . . 638
Giac [A] (verification not implemented) . . . 638
Mupad [B] (verification not implemented) . . . 638

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}$$

```
[Out] x/a-arctanh(cosh(x))/b+2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^
2)^(1/2)/a/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3979, 4136, 3855, 4004, 3916, 2739, 632, 212}

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab} + \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b}$$

```
[In] Int[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] x/a - ArcTanh[Cosh[x]]/b + (2*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4136

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{-1-\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx \\
&= \frac{i\int \frac{-ib+ia\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b} + \frac{\int \operatorname{csch}(x)\,dx}{b} \\
&= \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} - \frac{(a^2+b^2)\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{ab} \\
&= \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} - \left(\frac{1}{a}+\frac{a}{b^2}\right)\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx \\
&= \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} - \left(2\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} + \left(4\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a} - \frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{ab}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{bx+2\sqrt{-a^2-b^2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-a\log\left(\cosh\left(\frac{x}{2}\right)\right)+a\log\left(\sinh\left(\frac{x}{2}\right)\right)}{ab}$$

```
[In] Integrate[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (b*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]] - a*Lo
g[Cosh[x/2]] + a*Log[Sinh[x/2]])/(a*b)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\ln(\tanh(\frac{x}{2})+1)}{a}+\frac{\ln(\tanh(\frac{x}{2}))}{b}-\frac{\ln(\tanh(\frac{x}{2})-1)}{a}+\frac{(2a^2+2b^2)\operatorname{arctanh}\left(\frac{-2b\tanh(\frac{x}{2})+2a}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}$	84
risch	$\frac{x}{a}+\frac{\sqrt{a^2+b^2}\ln\left(e^x+\frac{b+\sqrt{a^2+b^2}}{a}\right)}{ba}-\frac{\sqrt{a^2+b^2}\ln\left(e^x-\frac{-b+\sqrt{a^2+b^2}}{a}\right)}{ba}+\frac{\ln(e^x-1)}{b}-\frac{\ln(e^x+1)}{b}$	100

```
[In] int(coth(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)+1/b*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)+(2*a^2+2*b^
2)/a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(53)=106$.

Time $=0.27$ (sec) , antiderivative size $=141$, normalized size of antiderivative $=2.47$

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$=\frac{bx-a\log(\cosh(x)+\sinh(x)+1)+a\log(\cosh(x)+\sinh(x)-1)+\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2}{a}\right.}{ab}$$

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(cosh(x) + sinh(x) + 1) + a*log(cosh(x) + sinh(x) - 1) + sqrt(a
^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2
+ 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x)
+ b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x)
- a)))/(a*b)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx=\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a} - \frac{\log\left(e^{(-x)}+1\right)}{b} + \frac{\log\left(e^{(-x)}-1\right)}{b} - \frac{\sqrt{a^2+b^2}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{ab}$$

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b - sqrt(a^2 + b^2)*log((a*e^(-x)
 - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a} - \frac{\log\left(e^{x}+1\right)}{b} + \frac{\log\left(\left|e^{x}-1\right|\right)}{b} - \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{ab}$$

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - log(e^x + 1)/b + log(abs(e^x - 1))/b - sqrt(a^2 + b^2)*log(abs(2*a*e^
x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 5.54

$$\begin{aligned}
&\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx \\
&= \frac{x}{a} + \frac{\ln\left(32\,a^2\,b+32\,b^3-32\,b^3\,\mathrm{e}^{x}-32\,a^2\,b\,\mathrm{e}^{x}\right)}{b} - \frac{\ln\left(32\,a^2\,b+32\,b^3+32\,b^3\,\mathrm{e}^{x}+32\,a^2\,b\,\mathrm{e}^{x}\right)}{b} \\
&\quad + \frac{\ln\left(128\,b^5\,\mathrm{e}^{x}-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,\mathrm{e}^{x}\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^{x}+160\,a^2\,b^3\,\mathrm{e}^{x}+64\,a\,b^3\,\sqrt{a^2+b^2}+32\,a^3\right.}{a\,b} \\
&\quad - \frac{\ln\left(128\,b^5\,\mathrm{e}^{x}-64\,a^3\,b^2-64\,a\,b^4+128\,b^4\,\mathrm{e}^{x}\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^{x}+160\,a^2\,b^3\,\mathrm{e}^{x}-64\,a\,b^3\,\sqrt{a^2+b^2}-32\,a^3\right.}{a\,b}
\end{aligned}$$

```
[In] int(coth(x)^2/(a + b/sinh(x)),x)
```

```
[Out] x/a + log(32*a^2*b + 32*b^3 - 32*b^3*exp(x) - 32*a^2*b*exp(x))/b - log(32*a
^2*b + 32*b^3 + 32*b^3*exp(x) + 32*a^2*b*exp(x))/b + (log(128*b^5*exp(x) -
```

```
64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*exp(x)
+ 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^2)^(1
/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b) - (log(
128*b^5*exp(x) - 64*a^3*b^2 - 64*a*b^4 + 128*b^4*exp(x)*(a^2 + b^2)^(1/2) +
 32*a^4*b*exp(x) + 160*a^2*b^3*exp(x) - 64*a*b^3*(a^2 + b^2)^(1/2) - 32*a^3
*b*(a^2 + b^2)^(1/2) + 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/
2))/(a*b)
```

3.120 $\int \frac{\coth^3(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 640
Rubi [A] (verified) . . . 640
Mathematica [A] (verified) . . . 641
Maple [B] (verified) . . . 641
Fricas [B] (verification not implemented) . . . 642
Sympy [F] . . . 642
Maxima [B] (verification not implemented) . . . 642
Giac [B] (verification not implemented) . . . 643
Mupad [B] (verification not implemented) . . . 643

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{\coth^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\operatorname{csch}(x)}{b} + \left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\operatorname{csch}(x)) + \frac{\log(\sinh(x))}{a}$$

```
[Out] -csch(x)/b+(1/a+a/b^2)*ln(a+b*csch(x))+ln(sinh(x))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\int \frac{\coth^3(x)}{a+b\operatorname{csch}(x)}\,dx = \left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\operatorname{csch}(x)) + \frac{\log(\sinh(x))}{a} - \frac{\operatorname{csch}(x)}{b}$$

```
[In] Int[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -(Csch[x]/b) + (a^(-1) + a/b^2)*Log[a + b*Csch[x]] + Log[Sinh[x]]/a
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)}\,dx, x, b\operatorname{csch}(x)\right)}{b^2} \\ &= \frac{\text{Subst}\left(\int \left(-1-\frac{b^2}{ax}+\frac{a^2+b^2}{a(a+x)}\right)\,dx, x, b\operatorname{csch}(x)\right)}{b^2} \\ &= -\frac{\operatorname{csch}(x)}{b}+\left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\operatorname{csch}(x))+\frac{\log(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{\coth^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{-ab\operatorname{csch}(x)-a^2\log(\sinh(x))+(a^2+b^2)\log(b+a\sinh(x))}{ab^2}$$

```
[In] Integrate[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-(a*b*Csch[x]) - a^2*Log[Sinh[x]] + (a^2 + b^2)*Log[b + a*Sinh[x]])/(a*b^2
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.

Time = 0.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

method	result
risch	$-\frac{x}{a}-\frac{2\,e^{x}}{b(e^{2x}-1)}+\frac{a\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}-1\right)}{b^2}+\frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}-1\right)}{a}-\frac{a\ln\left(e^{2x}-1\right)}{b^2}$
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2b}+\frac{\left(2a^2+2b^2\right)\ln\left(-\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{2a\,b^2}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+\right.}{a}$

```
[In] int(coth(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x/a-2*exp(x)/b/(exp(2*x)-1)+a/b^2*ln(exp(2*x)+2*b/a*exp(x)-1)+1/a*ln(exp(2
*x)+2*b/a*exp(x)-1)-a/b^2*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(32) = 64$.

Time $= 0.28$ (sec) , antiderivative size $= 199$, normalized size of antiderivative $= 6.22$

$$\int \frac{\coth^3(x)}{a + b\operatorname{csch}(x)}\,dx =$$

$$-\frac{b^2 x\cosh(x)^2 + b^2 x\sinh(x)^2 - b^2 x + 2ab\cosh(x) - \left((a^2+b^2)\cosh(x)^2 + 2(a^2+b^2)\cosh(x)\sinh(x) + \right.}{a}$$

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 - b^2*x + 2*a*b*cosh(x) - ((a^2 + b^2)*
cosh(x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2 - a^2 - b
^2)*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) + a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 - a*b^2)
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\coth^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(32) = 64$.

Time $= 0.19$ (sec) , antiderivative size $= 82$, normalized size of antiderivative $= 2.56$

$$\int \frac{\coth^3(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{x}{a} + \frac{2e^{(-x)}}{be^{(-2x)} - b} - \frac{a\log\left(e^{(-x)} + 1\right)}{b^2} - \frac{a\log\left(e^{(-x)} - 1\right)}{b^2} + \frac{(a^2+b^2)\log\left(-2be^{(-x)} + ae^{(-2x)} - a\right)}{ab^2}$$

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) - b) - a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)
/b^2 + (a^2 + b^2)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(32) = 64$.

Time $= 0.28$ (sec) , antiderivative size $= 80$, normalized size of antiderivative $= 2.50$

$$\int \frac{\coth^3(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{a\log\left(\left|-e^{(-x)} + e^x\right|\right)}{b^2} + \frac{(a^2+b^2)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{ab^2} + \frac{a\left(e^{(-x)} - e^x\right) + 2\,b}{b^2\left(e^{(-x)} - e^x\right)}$$

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(-e^(-x) + e^x))/b^2 + (a^2 + b^2)*log(abs(-a*(e^(-x) - e^x) + 2*
b))/(a*b^2) + (a*(e^(-x) - e^x) + 2*b)/(b^2*(e^(-x) - e^x))
```

Mupad [B] (verification not implemented)

Time $= 2.44$ (sec) , antiderivative size $= 261$, normalized size of antiderivative $= 8.16$

$$\begin{aligned}\int \frac{\coth^3(x)}{a + b\operatorname{csch}(x)}\,dx &= \frac{2\,\mathrm{e}^x}{b - b\,\mathrm{e}^{2\,x}} - \frac{x}{a} \\ &+ \frac{\ln\left(16\,a^5\,\mathrm{e}^{2\,x} - 4\,a\,b^4 - 16\,a^5 - 16\,a^3\,b^2 + 8\,b^5\,\mathrm{e}^x + 16\,a^3\,b^2\,\mathrm{e}^{2\,x} + 32\,a^4\,b\,\mathrm{e}^x + 4\,a\,b^4\,\mathrm{e}^{2\,x} + 32\,a^2\,b^3\,\mathrm{e}^x\right)}{a} \\ &+ \frac{a\,\ln\left(16\,a^5\,\mathrm{e}^{2\,x} - 4\,a\,b^4 - 16\,a^5 - 16\,a^3\,b^2 + 8\,b^5\,\mathrm{e}^x + 16\,a^3\,b^2\,\mathrm{e}^{2\,x} + 32\,a^4\,b\,\mathrm{e}^x + 4\,a\,b^4\,\mathrm{e}^{2\,x} + 32\,a^2\,b^3\,\mathrm{e}^x\right)}{b^2} \\ &- \frac{a\,\ln\left(16\,a^6\,\mathrm{e}^{2\,x} + 4\,b^6\,\mathrm{e}^{2\,x} - 16\,a^6 - 4\,b^6 - 20\,a^2\,b^4 - 32\,a^4\,b^2 + 20\,a^2\,b^4\,\mathrm{e}^{2\,x} + 32\,a^4\,b^2\,\mathrm{e}^{2\,x}\right)}{b^2}\end{aligned}$$

```
[In] int(coth(x)^3/(a + b/sinh(x)),x)
```

```
[Out] (2*exp(x))/(b - b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) - 4*a*b^4 - 16*a^5
- 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) + 32*a^2*b^3*exp(x))/a + (a*log(16*a^5*exp(2*x) - 4*a*b^4 - 16*
a^5 - 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4
*a*b^4*exp(2*x) + 32*a^2*b^3*exp(x)))/b^2 - (a*log(16*a^6*exp(2*x) + 4*b^6*
exp(2*x) - 16*a^6 - 4*b^6 - 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(2*x) +
 32*a^4*b^2*exp(2*x)))/b^2
```

3.121 $\int \frac{\coth^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 644
Rubi [A] (verified) . . . 644
Mathematica [A] (verified) . . . 646
Maple [A] (verified) . . . 647
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Maxima [B] (verification not implemented) . . . 648
Giac [B] (verification not implemented) . . . 648
Mupad [B] (verification not implemented) . . . 649

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a} - \frac{(2a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[Out] x/a-1/2*(2*a^2+3*b^2)*arctanh(cosh(x))/b^3+2*(a^2+b^2)^(3/2)*arctanh((a-b*t
anh(1/2*x))/(a^2+b^2)^(1/2))/a/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3983, 2972, 3136, 2739, 632, 210, 3855}

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(2a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3} + \frac{a\coth(x)}{b^2} + \frac{x}{a} - \frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[In] Int[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] x/a - ((2*a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*(a^2 + b^2)^(3/2)*Arc
Tanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^3) + (a*Coth[x])/b^2 - (Coth[
x]*Csch[x])/(2*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = i \int \frac{\cosh(x)\coth^3(x)}{ib + ia\sinh(x)}\,dx$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{i\int \frac{\operatorname{csch}(x)\left(-2a^2-3b^2+ab\sinh(x)-2b^2\sinh^2(x)\right)}{ib+ia\sinh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\left(i(a^2+b^2)^2\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^3} + \frac{(2a^2+3b^2)\int \operatorname{csch}(x)\,dx}{2b^3}$$

$$= \frac{x}{a} - \frac{(2a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\left(2i(a^2+b^2)^2\right)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{ab^3}$$

$$= \frac{x}{a} - \frac{(2a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{\left(4i(a^2+b^2)^2\right)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{ab^3}$$

$$= \frac{x}{a} - \frac{(2a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} + \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{\operatorname{csch}(x)(b+a\sinh(x))\left(8b^3x - 16(-a^2-b^2)^{3/2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right) + 4a^2b\coth\left(\frac{x}{2}\right) - ab^2\operatorname{csch}^2\left(\frac{x}{2}\right) - 4a(2\right.}{8ab^3(a+b\operatorname{csch}(x))}$$

```
[In] Integrate[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(8*b^3*x - 16*(-a^2 - b^2)^(3/2)*ArcTan[(a - b*Tan
h[x/2])/Sqrt[-a^2 - b^2]] + 4*a^2*b*Coth[x/2] - a*b^2*Csch[x/2]^2 - 4*a*(2*
a^2 + 3*b^2)*Log[Cosh[x/2]] + 4*a*(2*a^2 + 3*b^2)*Log[Sinh[x/2]] - a*b^2*Se
ch[x/2]^2 + 4*a^2*b*Tanh[x/2]))/(8*a*b^3*(a + b*Csch[x]))
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^2 b}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\left(8a^4+16a^2b^2+8b^4\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{4a\,b^3\sqrt{a^2+b^2}}-\frac{1}{8b\tanh\left(\frac{x}{2}\right)^2}+\frac{\left(4a^2+6b^2\right)\ln$
risch	$\frac{x}{a}+\frac{-\mathrm{e}^{3x}b+2a\,\mathrm{e}^{2x}-\mathrm{e}^{x}b-2a}{\left(\mathrm{e}^{2x}-1\right)^2b^2}-\frac{\ln\left(\mathrm{e}^{x}+1\right)a^2}{b^3}-\frac{3\ln\left(\mathrm{e}^{x}+1\right)}{2b}+\frac{\ln\left(\mathrm{e}^{x}-1\right)a^2}{b^3}+\frac{3\ln\left(\mathrm{e}^{x}-1\right)}{2b}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}\ln\left(\mathrm{e}^{x}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}+a^2b}{\left(a^2+b^2\right)a}\right.}{b^3a}$

```
[In] int(coth(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/2*tanh(1/2*x)^2*b+2*a*tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)+1/4*(8*
a^4+16*a^2*b^2+8*b^4)/a/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2
*a)/(a^2+b^2)^(1/2))-1/8/b/tanh(1/2*x)^2+1/4/b^3*(4*a^2+6*b^2)*ln(tanh(1/2*
x))+1/2*a/b^2/tanh(1/2*x)+1/a*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(80) = 160$.

Time = 0.34 (sec) , antiderivative size = 831, normalized size of antiderivative = 9.44

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x*cosh(x)^4 + 2*b^3*x*sinh(x)^4 - 2*a*b^2*cosh(x)^3 + 2*b^3*x -
2*a*b^2*cosh(x) + 2*(4*b^3*x*cosh(x) - a*b^2)*sinh(x)^3 - 4*a^2*b - 4*(b^3*
x - a^2*b)*cosh(x)^2 + 2*(6*b^3*x*cosh(x)^2 - 2*b^3*x - 3*a*b^2*cosh(x) + 2
*a^2*b)*sinh(x)^2 + 2*((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*co
sh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^
2 + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a
^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cos
h(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - ((2*a^3 + 3*a*b^2)*cosh(x)^4 + 4*(
2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)^4 + 2*a^3 +
3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 - 3*(2*a^3 + 3
*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3 - (2*a^3 + 3*
a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((2*a^3 + 3*a*b^2)*co
sh(x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)
^4 + 2*a^3 + 3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 -
```

```
 3*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3
- (2*a^3 + 3*a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*b^3
*x*cosh(x)^3 - 3*a*b^2*cosh(x)^2 - a*b^2 - 4*(b^3*x - a^2*b)*cosh(x))*sinh(
x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 - 2*a*b^
3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 - a*b^3)*sinh(x)^2 + 4*(a*b^3*co
sh(x)^3 - a*b^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\coth^4(x)}{a+b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(coth(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(80) = 160$.

Time $= 0.28$ (sec) , antiderivative size $= 178$, normalized size of antiderivative $= 2.02$

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b e^{(-x)} + 2\,a e^{(-2\,x)} + b e^{(-3\,x)} - 2\,a}{2\,b^2 e^{(-2\,x)} - b^2 e^{(-4\,x)} - b^2} + \frac{x}{a} - \frac{(2\,a^2+3\,b^2)\log\left(e^{(-x)}+1\right)}{2\,b^3} + \frac{(2\,a^2+3\,b^2)\log\left(e^{(-x)}-1\right)}{2\,b^3} - \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{a e^{(-x)}-b-\sqrt{a^2+b^2}}{a e^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ab^3}$$

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x)
 - b^2) + x/a - 1/2*(2*a^2 + 3*b^2)*log(e^(-x) + 1)/b^3 + 1/2*(2*a^2 + 3*b^
2)*log(e^(-x) - 1)/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log((a*e^(-x) - b - sqrt(a
^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.83

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x}{a} - \frac{(2\,a^2+3\,b^2)\log\left(e^x+1\right)}{2\,b^3} + \frac{(2\,a^2+3\,b^2)\log\left(\left|e^x-1\right|\right)}{2\,b^3} - \frac{(a^4+2\,a^2b^2+b^4)\log\left(\left|\frac{2\,ae^x+2\,b-2\sqrt{a^2+b^2}}{2\,ae^x+2\,b+2\sqrt{a^2+b^2}}\right|\right)}{\sqrt{a^2+b^2}ab^3} - \frac{be^{(3\,x)}-2\,ae^{(2\,x)}+be^x+2\,a}{b^2\left(e^{(2\,x)}-1\right)^2}$$

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2*(2*a^2 + 3*b^2)*log(e^x + 1)/b^3 + 1/2*(2*a^2 + 3*b^2)*log(abs(e^
x - 1))/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3) - (b*
e^(3*x) - 2*a*e^(2*x) + b*e^x + 2*a)/(b^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.30

$$\int \frac{\coth^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{2\,a}{b^2}-\frac{\mathrm{e}^x}{b}}{\mathrm{e}^{2\,x}-1} + \frac{x}{a} + \frac{\ln\left(\mathrm{e}^x-1\right)\,\left(2\,a^2+3\,b^2\right)}{2\,b^3} - \frac{\ln\left(\mathrm{e}^x+1\right)\,\left(2\,a^2+3\,b^2\right)}{2\,b^3} - \frac{2\,\mathrm{e}^x}{b\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} + \frac{\ln\left(a^3\sqrt{\left(a^2+b^2\right)^3}-2\,a^5\,b-2\,a\,b^5-4\,a^3\,b^3+a^6\,\mathrm{e}^x+4\,b^6\,\mathrm{e}^x+2\,a\,b^2\sqrt{\left(a^2+b^2\right)^3}-4\,b^3\,\mathrm{e}^x\sqrt{\left(a^2+b^2\right)^3}\right.}{a\,b^3} - \frac{\ln\left(a^6\,\mathrm{e}^x-2\,a^5\,b-a^3\sqrt{\left(a^2+b^2\right)^3}-4\,a^3\,b^3-2\,a\,b^5+4\,b^6\,\mathrm{e}^x-2\,a\,b^2\sqrt{\left(a^2+b^2\right)^3}+4\,b^3\,\mathrm{e}^x\sqrt{\left(a^2+b^2\right)^3}\right.}{a\,b^3}$$

```
[In] int(coth(x)^4/(a + b/sinh(x)),x)
```

```
[Out] ((2*a)/b^2 - exp(x)/b)/(exp(2*x) - 1) + x/a + (log(exp(x) - 1)*(2*a^2 + 3*b
^2))/(2*b^3) - (log(exp(x) + 1)*(2*a^2 + 3*b^2))/(2*b^3) - (2*exp(x))/(b*(e
xp(4*x) - 2*exp(2*x) + 1)) + (log(a^3*((a^2 + b^2)^3)^(1/2) - 2*a^5*b - 2*a
*b^5 - 4*a^3*b^3 + a^6*exp(x) + 4*b^6*exp(x) + 2*a*b^2*((a^2 + b^2)^3)^(1/2
) - 4*b^3*exp(x)*((a^2 + b^2)^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x
) - 3*a^2*b*exp(x)*((a^2 + b^2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3) -
(log(a^6*exp(x) - 2*a^5*b - a^3*((a^2 + b^2)^3)^(1/2) - 4*a^3*b^3 - 2*a*b^5
 + 4*b^6*exp(x) - 2*a*b^2*((a^2 + b^2)^3)^(1/2) + 4*b^3*exp(x)*((a^2 + b^2)
^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x) + 3*a^2*b*exp(x)*((a^2 + b^
2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3)
```

3.122 $\int \frac{\coth^5(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 650
Rubi [A] (verified) . . . 650
Mathematica [A] (verified) . . . 651
Maple [B] (verified) . . . 652
Fricas [B] (verification not implemented) . . . 652
Sympy [F] . . . 653
Maxima [B] (verification not implemented) . . . 653
Giac [B] (verification not implemented) . . . 654
Mupad [B] (verification not implemented) . . . 654

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3} + \frac{a\operatorname{csch}^2(x)}{2b^2} - \frac{\operatorname{csch}^3(x)}{3b} + \frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4} + \frac{\log(\sinh(x))}{a}$$

```
[Out] -(a^2+2*b^2)*csch(x)/b^3+1/2*a*csch(x)^2/b^2-1/3*csch(x)^3/b+(a^2+b^2)^2*ln
(a+b*csch(x))/a/b^4+ln(sinh(x))/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4} - \frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3} + \frac{a\operatorname{csch}^2(x)}{2b^2} + \frac{\log(\sinh(x))}{a} - \frac{\operatorname{csch}^3(x)}{3b}$$

```
[In] Int[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -(((a^2 + 2*b^2)*Csch[x])/b^3) + (a*Csch[x]^2)/(2*b^2) - Csch[x]^3/(3*b) +
((a^2 + b^2)^2*Log[a + b*Csch[x]])/(a*b^4) + Log[Sinh[x]]/a
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^2}{x(a+x)}\,dx, x, b\text{csch}(x)\right)}{b^4} \\ &= -\frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{\left(a^2+b^2\right)^2}{a(a+x)}\right)\,dx, x, b\text{csch}(x)\right)}{b^4} \\ &= -\frac{\left(a^2+2b^2\right)\text{csch}(x)}{b^3}+\frac{a\text{csch}^2(x)}{2b^2}-\frac{\text{csch}^3(x)}{3b}+\frac{\left(a^2+b^2\right)^2\log(a+b\text{csch}(x))}{ab^4}+\frac{\log(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{-6ab\left(a^2+2b^2\right)\text{csch}(x)+3a^2b^2\text{csch}^2(x)-2ab^3\text{csch}^3(x)-6a^2\left(a^2+2b^2\right)\log(\sinh(x))+6\left(a^2+b^2\right)^2\log(b+}{6ab^4}$$

```
[In] Integrate[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-6*a*b*(a^2 + 2*b^2)*Csch[x] + 3*a^2*b^2*Csch[x]^2 - 2*a*b^3*Csch[x]^3 - 6
*a^2*(a^2 + 2*b^2)*Log[Sinh[x]] + 6*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/(6*a*
b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(66) = 132$.

Time $= 1.60$ (sec) , antiderivative size $= 174$, normalized size of antiderivative $= 2.49$

method	result
risch	$-\frac{x}{a} - \frac{2\,\mathrm{e}^{x}\left(3a^2\mathrm{e}^{4x}+6b^2\mathrm{e}^{4x}-3ab\,\mathrm{e}^{3x}-6a^2\mathrm{e}^{2x}-8b^2\mathrm{e}^{2x}+3b\,\mathrm{e}^{x}a+3a^2+6b^2\right)}{3b^3\left(\mathrm{e}^{2x}-1\right)^3} - \frac{a^3\ln\left(\mathrm{e}^{2x}-1\right)}{b^4} - \frac{2a\ln\left(\mathrm{e}^{2x}-1\right)}{b^2} + \frac{a^3\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-\right.}{b^4}$
default	$\frac{\tanh\left(\frac{x}{2}\right)^3b^2}{3}+\tanh\left(\frac{x}{2}\right)^2ab+4a^2\tanh\left(\frac{x}{2}\right)+7b^2\tanh\left(\frac{x}{2}\right)}{8b^3} + \frac{\left(8a^4+16a^2b^2+8b^4\right)\ln\left(-\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{8b^4a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$

```
[In] int(coth(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x/a-2/3*exp(x)*(3*a^2*exp(4*x)+6*b^2*exp(4*x)-3*a*b*exp(3*x)-6*a^2*exp(2*x
)-8*b^2*exp(2*x)+3*b*exp(x)*a+3*a^2+6*b^2)/b^3/(exp(2*x)-1)^3-1/b^4*a^3*ln(
exp(2*x)-1)-2*a/b^2*ln(exp(2*x)-1)+a^3/b^4*ln(exp(2*x)+2*b/a*exp(x)-1)+2*a/
b^2*ln(exp(2*x)+2*b/a*exp(x)-1)+1/a*ln(exp(2*x)+2*b/a*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(66) = 132$.

Time $= 0.28$ (sec) , antiderivative size $= 1288$, normalized size of antiderivative $= 18.40$

$$\int \frac{\coth^5(x)}{a + b\mathrm{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b + 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b + 2*a*b^3)*sinh(x)^5 - 3*b^4*x - 3*(3*b^4*x +
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 - 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b + 2*a*b^3)*cosh(x))*sinh(x)^4 - 4*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 - 3*a^3*b - 4*a*b^3 + 15*(a^3*b + 2*a*b^3)*cosh(x)^2 - 3*
(3*b^4*x + 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x + 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x + 2*a^2*b^2 + 20*(a^3*b + 2*a*b^3)*cosh
(x)^3 - 6*(3*b^4*x + 2*a^2*b^2)*cosh(x)^2 - 4*(3*a^3*b + 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b + 2*a*b^3)*cosh(x) - 3*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2 + b^4)
*sinh(x)^6 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 + b^4
 - 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 - a^4 - 2*a^2*b^2 - b^4 +
 4*(5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 + 2*a^2*b^2 + b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2 +
```

```
 b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*sinh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 + 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 + 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2)*sinh(x)^6 - 3*(a^4 + 2*a^2*b^
2)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 - 5*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 - a^4 - 2*a^2*b^2 + 4*(5*(a^4 + 2*a^2*b^2)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^
2)*cosh(x)^4 + a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 + 2*a^2*b^2)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b + 2*a*b^3)*cosh(x)^4 + a^3*b + 2*a*b^3 - 2*(3*b^4*x +
2*a^2*b^2)*cosh(x)^3 - 2*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (3*b^4*x + 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 - 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 - a*b^4 + 3*(5*a*b^4*cosh
(x)^2 - a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 - 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 - 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 - 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\coth^5(x)}{a+b\,\text{csch}(x)}\,dx$$

```
[In] integrate(coth(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(66) = 132$.

Time = 0.19 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}\,dx =$$
$$-\frac{2\left(3\,abe^{(-2\,x)} - 3\,abe^{(-4\,x)} - 3\left(a^2+2\,b^2\right)e^{(-x)} + 2\left(3\,a^2+4\,b^2\right)e^{(-3\,x)} - 3\left(a^2+2\,b^2\right)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)} - 3\,b^3e^{(-4\,x)} + b^3e^{(-6\,x)} - b^3\right)}$$
$$+\frac{x}{a} - \frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-x)}+1\right)}{b^4} - \frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-x)}-1\right)}{b^4}$$
$$+\frac{\left(a^4+2\,a^2b^2+b^4\right)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{ab^4}$$

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) - 3*(a^2 + 2*b^2)*e^(-x) + 2*(3*a^2 +
 4*b^2)*e^(-3*x) - 3*(a^2 + 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-4*
x) + b^3*e^(-6*x) - b^3) + x/a - (a^3 + 2*a*b^2)*log(e^(-x) + 1)/b^4 - (a^3
 + 2*a*b^2)*log(e^(-x) - 1)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*b*e^(-x) +
 a*e^(-2*x) - a)/(a*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(66) = 132$.

Time $= 0.28$ (sec) , antiderivative size $= 170$, normalized size of antiderivative $= 2.43$

$$\int \frac{\coth^5(x)}{a + b\operatorname{csch}(x)}\,dx$$
$$= -\frac{(a^3 + 2\,ab^2)\log\left(\left|-e^{(-x)} + e^{x}\right|\right)}{b^4} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{ab^4}$$
$$+ \frac{11\,a^3\left(e^{(-x)} - e^{x}\right)^3 + 22\,ab^2\left(e^{(-x)} - e^{x}\right)^3 + 12\,a^2b\left(e^{(-x)} - e^{x}\right)^2 + 24\,b^3\left(e^{(-x)} - e^{x}\right)^2 + 12\,ab^2\left(e^{(-x)} - e^{x}\right)}{6\,b^4\left(e^{(-x)} - e^{x}\right)^3}$$

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(-e^(-x) + e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(
abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) - e^x)^3 + 22*a
*b^2*(e^(-x) - e^x)^3 + 12*a^2*b*(e^(-x) - e^x)^2 + 24*b^3*(e^(-x) - e^x)^2
 + 12*a*b^2*(e^(-x) - e^x) + 16*b^3)/(b^4*(e^(-x) - e^x)^3)
```

Mupad [B] (verification not implemented)

Time $= 2.61$ (sec) , antiderivative size $= 155$, normalized size of antiderivative $= 2.21$

$$\int \frac{\coth^5(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\frac{2\,a}{b^2} - \frac{2\,\mathrm{e}^{x}\,(a^2+2\,b^2)}{b^3}}{\mathrm{e}^{2\,x} - 1} - \frac{x}{a} + \frac{\frac{2\,a}{b^2} - \frac{8\,\mathrm{e}^{x}}{3\,b}}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{8\,\mathrm{e}^{x}}{3\,b\,\left(3\,\mathrm{e}^{2\,x} - 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} - 1\right)}$$
$$- \frac{\ln\left(\mathrm{e}^{2\,x} - 1\right)\,\left(a^3 + 2\,a\,b^2\right)}{b^4} + \frac{\ln\left(2\,b\,\mathrm{e}^{x} - a + a\,\mathrm{e}^{2\,x}\right)\,\left(a^4 + 2\,a^2\,b^2 + b^4\right)}{a\,b^4}$$

```
[In] int(coth(x)^5/(a + b/sinh(x)),x)
```

```
[Out] ((2*a)/b^2 - (2*exp(x)*(a^2 + 2*b^2))/b^3)/(exp(2*x) - 1) - x/a + ((2*a)/b^
2 - (8*exp(x))/(3*b))/(exp(4*x) - 2*exp(2*x) + 1) - (8*exp(x))/(3*b*(3*exp(
2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (log(exp(2*x) - 1)*(2*a*b^2 + a^3))/b^
4 + (log(2*b*exp(x) - a + a*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))/(a*b^4)
```

3.123 $\int \frac{\coth^6(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 655
Rubi [A] (verified) . . . 656
Mathematica [A] (verified) . . . 659
Maple [A] (verified) . . . 659
Fricas [B] (verification not implemented) . . . 660
Sympy [F] . . . 662
Maxima [A] (verification not implemented) . . . 662
Giac [A] (verification not implemented) . . . 663
Mupad [B] (verification not implemented) . . . 663

Optimal result

Integrand size = 13, antiderivative size = 183

$$\begin{aligned}\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx = {} & \frac{x}{a}-\frac{3\operatorname{arctanh}(\cosh(x))}{8b}+\frac{(a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3}\\ & -\frac{(a^4+3a^2b^2+3b^4)\operatorname{arctanh}(\cosh(x))}{b^5}\\ & +\frac{2(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}-\frac{a\coth(x)}{b^2}\\ & +\frac{a(a^2+3b^2)\coth(x)}{b^4}+\frac{a\coth^3(x)}{3b^2}+\frac{3\coth(x)\operatorname{csch}(x)}{8b}\\ & -\frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2b^3}-\frac{\coth(x)\operatorname{csch}^3(x)}{4b}\end{aligned}$$

```
[Out] x/a-3/8*arctanh(cosh(x))/b+1/2*(a^2+3*b^2)*arctanh(cosh(x))/b^3-(a^4+3*a^2*
b^2+3*b^4)*arctanh(cosh(x))/b^5+2*(a^2+b^2)^(5/2)*arctanh((a-b*tanh(1/2*x))
/(a^2+b^2)^(1/2))/a/b^5-a*coth(x)/b^2+a*(a^2+3*b^2)*coth(x)/b^4+1/3*a*coth(
x)^3/b^2+3/8*coth(x)*csch(x)/b-1/2*(a^2+3*b^2)*coth(x)*csch(x)/b^3-1/4*coth
(x)*csch(x)^3/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3983, 2976, 3855, 3852, 8, 3853, 2739, 632, 210}

$$\begin{aligned}\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx = {} & \frac{2(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5} + \frac{(a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} \\ & + \frac{a(a^2+3b^2)\coth(x)}{b^4} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2b^3} \\ & - \frac{(a^4+3a^2b^2+3b^4)\operatorname{arctanh}(\cosh(x))}{b^5} + \frac{a\coth^3(x)}{3b^2} - \frac{a\coth(x)}{b^2} \\ & + \frac{x}{a} - \frac{3\operatorname{arctanh}(\cosh(x))}{8b} - \frac{\coth(x)\operatorname{csch}^3(x)}{4b} + \frac{3\coth(x)\operatorname{csch}(x)}{8b}\end{aligned}$$

```
[In] Int[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cosh[x]])/(8*b) + ((a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3)
 - ((a^4 + 3*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/b^5 + (2*(a^2 + b^2)^(5/2)*
ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^5) - (a*Coth[x])/b^2 + (a*
(a^2 + 3*b^2)*Coth[x])/b^4 + (a*Coth[x]^3)/(3*b^2) + (3*Coth[x]*Csch[x])/(8
*b) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*b^3) - (Coth[x]*Csch[x]^3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = i \int \frac{\cosh(x)\coth^5(x)}{ib + ia\sinh(x)}\,dx$$

$$= -\int \left(-\frac{1}{a} - \frac{(a^4 + 3a^2b^2 + 3b^4)\operatorname{csch}(x)}{b^5} - \frac{a(-a^2 - 3b^2)\operatorname{csch}^2(x)}{b^4} \right.$$

$$\left. - \frac{(a^2 + 3b^2)\operatorname{csch}^3(x)}{b^3} + \frac{a\operatorname{csch}^4(x)}{b^2} - \frac{\operatorname{csch}^5(x)}{b} + \frac{i(a^2 + b^2)^3}{ab^5(ib + ia\sinh(x))} \right) dx$$

$$
\begin{aligned}
&= \frac{x}{a} - \frac{a\int \operatorname{csch}^4(x)\,dx}{b^2} + \frac{\int \operatorname{csch}^5(x)\,dx}{b} \\
&\quad - \frac{\left(i(a^2+b^2)^3\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^5} - \frac{(a(a^2+3b^2))\int \operatorname{csch}^2(x)\,dx}{b^4} \\
&\quad + \frac{(a^2+3b^2)\int \operatorname{csch}^3(x)\,dx}{b^3} + \frac{(a^4+3a^2b^2+3b^4)\int \operatorname{csch}(x)\,dx}{b^5} \\
&= \frac{x}{a} - \frac{(a^4+3a^2b^2+3b^4)\operatorname{arctanh}(\cosh(x))}{b^5} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2b^3} \\
&\quad - \frac{\coth(x)\operatorname{csch}^3(x)}{4b} - \frac{(ia)\operatorname{Subst}\left(\int (1+x^2)\ dx, x, -i\coth(x)\right)}{b^2} \\
&\quad - \frac{3\int \operatorname{csch}^3(x)\,dx}{4b} - \frac{\left(2i(a^2+b^2)^3\right)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{ab^5} \\
&\quad + \frac{(ia(a^2+3b^2))\operatorname{Subst}(\int 1\,dx, x, -i\coth(x))}{b^4} - \frac{(a^2+3b^2)\int \operatorname{csch}(x)\,dx}{2b^3} \\
&= \frac{x}{a} + \frac{(a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} - \frac{(a^4+3a^2b^2+3b^4)\operatorname{arctanh}(\cosh(x))}{b^5} \\
&\quad - \frac{a\coth(x)}{b^2} + \frac{a(a^2+3b^2)\coth(x)}{b^4} + \frac{a\coth^3(x)}{3b^2} + \frac{3\coth(x)\operatorname{csch}(x)}{8b} \\
&\quad - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2b^3} - \frac{\coth(x)\operatorname{csch}^3(x)}{4b} + \frac{3\int \operatorname{csch}(x)\,dx}{8b} \\
&\quad + \frac{\left(4i(a^2+b^2)^3\right)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{ab^5} \\
&= \frac{x}{a} - \frac{3\operatorname{arctanh}(\cosh(x))}{8b} + \frac{(a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3} \\
&\quad - \frac{(a^4+3a^2b^2+3b^4)\operatorname{arctanh}(\cosh(x))}{b^5} + \frac{2(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5} \\
&\quad - \frac{a\coth(x)}{b^2} + \frac{a(a^2+3b^2)\coth(x)}{b^4} + \frac{a\coth^3(x)}{3b^2} \\
&\quad + \frac{3\coth(x)\operatorname{csch}(x)}{8b} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2b^3} - \frac{\coth(x)\operatorname{csch}^3(x)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.63

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{\operatorname{csch}(x)(b+a\sinh(x))\left(192b^5x+384(-a^2-b^2)^{5/2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+32a^2b(3a^2+7b^2)\coth\left(\frac{x}{2}\right)-6a\right.}{}$$

```
[In] Integrate[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(192*b^5*x + 384*(-a^2 - b^2)^(5/2)*ArcTan[(a - b*
Tanh[x/2])/Sqrt[-a^2 - b^2]] + 32*a^2*b*(3*a^2 + 7*b^2)*Coth[x/2] - 6*a*b^2
*(4*a^2 + 9*b^2)*Csch[x/2]^2 - 3*a*b^4*Csch[x/2]^4 - 24*a*(8*a^4 + 20*a^2*b
^2 + 15*b^4)*Log[Cosh[x/2]] + 24*a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*Log[Sinh[x
/2]] - 6*a*b^2*(4*a^2 + 9*b^2)*Sech[x/2]^2 + 3*a*b^4*Sech[x/2]^4 - 64*a^2*b
^3*Csch[x]^3*Sinh[x/2]^4 + 4*a^2*b^3*Csch[x/2]^4*Sinh[x] + 32*a^2*b*(3*a^2
+ 7*b^2)*Tanh[x/2]))/(192*a*b^5*(a + b*Csch[x]))
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.42

method	result
default	$\frac{\frac{b^3\tanh\left(\frac{x}{2}\right)^4}{4}+\frac{2a\tanh\left(\frac{x}{2}\right)^3b^2}{3}+2a^2b\tanh\left(\frac{x}{2}\right)^2+4b^3\tanh\left(\frac{x}{2}\right)^2+8a^3\tanh\left(\frac{x}{2}\right)+18ab^2\tanh\left(\frac{x}{2}\right)}{16b^4}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right.}{a}$
risch	$\frac{x}{a}+\frac{-12a^2b\mathrm{e}^{7x}-27b^3\mathrm{e}^{7x}+24a^3\mathrm{e}^{6x}+72ab^2\mathrm{e}^{6x}+12a^2b\mathrm{e}^{5x}+3b^3\mathrm{e}^{5x}-72a^3\mathrm{e}^{4x}-168ab^2\mathrm{e}^{4x}+12a^2b\mathrm{e}^{3x}+3b^3\mathrm{e}^{3x}+72a^3\mathrm{e}^{2x}+152ab^2\mathrm{e}^{2}}{12b^4\left(\mathrm{e}^{2x}-1\right)^4}$

```
[In] int(coth(x)^6/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b^4*(1/4*b^3*tanh(1/2*x)^4+2/3*a*tanh(1/2*x)^3*b^2+2*a^2*b*tanh(1/2*x)
^2+4*b^3*tanh(1/2*x)^2+8*a^3*tanh(1/2*x)+18*a*b^2*tanh(1/2*x))+1/a*ln(tanh(
1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+1/16*(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/
a/b^5/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))-1
/64/b/tanh(1/2*x)^4-1/32*(4*a^2+8*b^2)/b^3/tanh(1/2*x)^2+1/16/b^5*(16*a^4+4
0*a^2*b^2+30*b^4)*ln(tanh(1/2*x))+1/24*a/b^2/tanh(1/2*x)^3+1/8*a*(4*a^2+9*b
^2)/b^4/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3160 vs. $2(167) = 334$.

Time = 0.41 (sec) , antiderivative size = 3160, normalized size of antiderivative = 17.27

$$\int \frac{\coth^6(x)}{a + b\text{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^5*x*cosh(x)^8 + 24*b^5*x*sinh(x)^8 - 6*(4*a^3*b^2 + 9*a*b^4)*cos
h(x)^7 + 6*(32*b^5*x*cosh(x) - 4*a^3*b^2 - 9*a*b^4)*sinh(x)^7 - 48*(2*b^5*x
 - a^4*b - 3*a^2*b^3)*cosh(x)^6 + 6*(112*b^5*x*cosh(x)^2 - 16*b^5*x + 8*a^4
*b + 24*a^2*b^3 - 7*(4*a^3*b^2 + 9*a*b^4)*cosh(x))*sinh(x)^6 + 24*b^5*x + 6
*(4*a^3*b^2 + a*b^4)*cosh(x)^5 + 6*(224*b^5*x*cosh(x)^3 + 4*a^3*b^2 + a*b^4
 - 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^2 - 48*(2*b^5*x - a^4*b - 3*a^2*b^3)*co
sh(x))*sinh(x)^5 - 48*a^4*b - 112*a^2*b^3 + 48*(3*b^5*x - 3*a^4*b - 7*a^2*b
^3)*cosh(x)^4 + 6*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b - 56*a^2*b^3 -
 35*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^3 - 120*(2*b^5*x - a^4*b - 3*a^2*b^3)*cos
h(x)^2 + 5*(4*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 6*(4*a^3*b^2 + a*b^4)*c
osh(x)^3 + 6*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 + a*b^4 - 35*(4*a^3*b^2 + 9*a
*b^4)*cosh(x)^4 - 160*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^3 + 10*(4*a^3*b
^2 + a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x))*sinh(x)
^3 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cosh(x)^2 + 2*(336*b^5*x*cosh(x)^6
 - 48*b^5*x - 63*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^5 + 72*a^4*b + 152*a^2*b^3 -
 360*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^4 + 30*(4*a^3*b^2 + a*b^4)*cosh(
x)^3 + 144*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 + a*b^4
)*cosh(x))*sinh(x)^2 + 24*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 + 2*a
^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^8 - 4*(a^
4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 4*(a^4 + 2*a^2*b^2 + b^4 - 7*(a^4 + 2*a^2*
b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 -
3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 + 2*a^2*b^2 + b^4)*co
sh(x)^4 + 2*(35*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 + 6*a^2*b^2 + 3*b
^4 - 30*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^
4 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 10*(a^4 + 2*a^2*b^2 + b^4)*cos
h(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 4*(a^4 + 2*a^2*b^2
+ b^4)*cosh(x)^2 + 4*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 15*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^4 - a^4 - 2*a^2*b^2 - b^4 + 9*(a^4 + 2*a^2*b^2 + b^4)*c
osh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^7 - 3*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 + 2*a^2*
b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(
x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt
(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*c
osh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 6*(4*a^3*b^2 + 9*a*b^4)*cosh(x)
- 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^2 + 15
```

```
*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 - 4*(
8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4
 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 2
0*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)
)*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 + 15*a*
b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a^3*b^2
 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh
(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sin
h(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 + 1) + 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^
2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8
 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a
^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*c
osh(x))*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 +
 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2
)*sinh(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) + 2*(96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^6 -
144*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^5 - 12*a^3*b^2 - 27*a*b^4 + 15*(4
*a^3*b^2 + a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^3
+ 9*(4*a^3*b^2 + a*b^4)*cosh(x)^2 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cos
h(x))*sinh(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)
^8 - 4*a*b^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 - 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5
*cosh(x)^2 - a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 - 3*a*b^5*cosh
(x))*sinh(x)^5 + 2*(35*a*b^5*cosh(x)^4 - 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh
(x)^4 + 8*(7*a*b^5*cosh(x)^5 - 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x
)^3 + 4*(7*a*b^5*cosh(x)^6 - 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 - a*b^5
)*sinh(x)^2 + 8*(a*b^5*cosh(x)^7 - 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 -
a*b^5*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\coth^6(x)}{a+b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(coth(x)**6/(a+b*csch(x)),x)
[Out] Integral(coth(x)**6/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.80

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx =$$
$$-\frac{24\,a^3+56\,ab^2-3\,(4\,a^2b+9\,b^3)e^{(-x)}-8\,(9\,a^3+19\,ab^2)e^{(-2\,x)}+3\,(4\,a^2b+b^3)e^{(-3\,x)}+24\,(3\,a^3+7\,ab^2)e^{(}}{12\,(4\,b^4e^{(-2\,x)}-6\,b^4e^{(-4\,x)}+4\,b^4e^{(-6\,x)}-b^4}$$
$$+\frac{x}{a}-\frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log\left(e^{(-x)}+1\right)}{8\,b^5}+\frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log\left(e^{(-x)}-1\right)}{8\,b^5}$$
$$-\frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ab^5}$$

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="maxima")
[Out] -1/12*(24*a^3 + 56*a*b^2 - 3*(4*a^2*b + 9*b^3)*e^(-x) - 8*(9*a^3 + 19*a*b^2
)*e^(-2*x) + 3*(4*a^2*b + b^3)*e^(-3*x) + 24*(3*a^3 + 7*a*b^2)*e^(-4*x) + 3
*(4*a^2*b + b^3)*e^(-5*x) - 24*(a^3 + 3*a*b^2)*e^(-6*x) - 3*(4*a^2*b + 9*b^
3)*e^(-7*x))/(4*b^4*e^(-2*x) - 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) - b^4*e^(-8*
x) - b^4) + x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) + 1)/b^5 + 1
/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) - 1)/b^5 - (a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.67

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{x}{a} - \frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log\left(e^x+1\right)}{8\,b^5} + \frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log\left(\left|e^x-1\right|\right)}{8\,b^5}$$
$$- \frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ab^5}$$
$$- \frac{12\,a^2be^{(7\,x)}+27\,b^3e^{(7\,x)}-24\,a^3e^{(6\,x)}-72\,ab^2e^{(6\,x)}-12\,a^2be^{(5\,x)}-3\,b^3e^{(5\,x)}+72\,a^3e^{(4\,x)}+168\,ab^2e^{(4\,x)}-}{12\,b^4(e^{(2\,x)}-1)^4}$$

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^x + 1)/b^5 + 1/8*(8*a^4 + 20*
a^2*b^2 + 15*b^4)*log(abs(e^x - 1))/b^5 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5) - 1/12*(12*a^2*b*e^(7*x) + 27*b^3*e^(7*x
) - 24*a^3*e^(6*x) - 72*a*b^2*e^(6*x) - 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) +
72*a^3*e^(4*x) + 168*a*b^2*e^(4*x) - 12*a^2*b*e^(3*x) - 3*b^3*e^(3*x) - 72*
a^3*e^(2*x) - 152*a*b^2*e^(2*x) + 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 + 56*a
*b^2)/(b^4*(e^(2*x) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.97

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{8\,a}{3\,b^2}-\frac{6\,e^x}{b}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} - \frac{\frac{e^x\,(4\,a^2+9\,b^2)}{4\,b^3}-\frac{2\,(a^4+3\,a^2\,b^2)}{a\,b^4}}{e^{2\,x}-1}$$
$$+ \frac{\frac{4\,a}{b^2}-\frac{e^x\,(4\,a^2+13\,b^2)}{2\,b^3}}{e^{4\,x}-2\,e^{2\,x}+1} + \frac{x}{a} + \frac{\ln\left(e^x-1\right)\,(8\,a^4+20\,a^2\,b^2+15\,b^4)}{8\,b^5}$$
$$- \frac{\ln\left(e^x+1\right)\,(8\,a^4+20\,a^2\,b^2+15\,b^4)}{8\,b^5} - \frac{4\,e^x}{b\,(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1)}$$
$$+ \frac{\ln\left(a^3\,\sqrt{(a^2+b^2)^5}-2\,a^7\,b-2\,a\,b^7-6\,a^3\,b^5-6\,a^5\,b^3+a^8\,e^x+4\,b^8\,e^x+2\,a\,b^2\,\sqrt{(a^2+b^2)^5}-4\,b^3\,e^x\,\sqrt{}\right.}{a\,b^5}$$
$$- \frac{\ln\left(a^8\,e^x-2\,a^7\,b-a^3\,\sqrt{(a^2+b^2)^5}-6\,a^3\,b^5-6\,a^5\,b^3-2\,a\,b^7+4\,b^8\,e^x-2\,a\,b^2\,\sqrt{(a^2+b^2)^5}+4\,b^3\,e^x\,\sqrt{}\right.}{a\,b^5}$$

```
[In] int(coth(x)^6/(a + b/sinh(x)),x)
```

```
[Out] ((8*a)/(3*b^2) - (6*exp(x))/b)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - (
(exp(x)*(4*a^2 + 9*b^2))/(4*b^3) - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x)
 - 1) + ((4*a)/b^2 - (exp(x)*(4*a^2 + 13*b^2))/(2*b^3))/(exp(4*x) - 2*exp(2
*x) + 1) + x/a + (log(exp(x) - 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) -
(log(exp(x) + 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) - (4*exp(x))/(b*(6*
exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) + (log(a^3*((a^2 + b^2)
^5)^(1/2) - 2*a^7*b - 2*a*b^7 - 6*a^3*b^5 - 6*a^5*b^3 + a^8*exp(x) + 4*b^8*
exp(x) + 2*a*b^2*((a^2 + b^2)^5)^(1/2) - 4*b^3*exp(x)*((a^2 + b^2)^5)^(1/2)
 + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) - 3*a^2*b*exp(x
)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5) - (log(a^8*exp(x) -
 2*a^7*b - a^3*((a^2 + b^2)^5)^(1/2) - 6*a^3*b^5 - 6*a^5*b^3 - 2*a*b^7 + 4*
b^8*exp(x) - 2*a*b^2*((a^2 + b^2)^5)^(1/2) + 4*b^3*exp(x)*((a^2 + b^2)^5)^(
1/2) + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) + 3*a^2*b*e
xp(x)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5)
```

3.124 $\int \frac{\coth^7(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result . . . 665
Rubi [A] (verified) . . . 665
Mathematica [A] (verified) . . . 666
Maple [B] (verified) . . . 667
Fricas [B] (verification not implemented) . . . 667
Sympy [F] . . . 670
Maxima [B] (verification not implemented) . . . 670
Giac [B] (verification not implemented) . . . 671
Mupad [B] (verification not implemented) . . . 671

Optimal result

Integrand size = 13, antiderivative size = 119

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5} + \frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4} - \frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3} + \frac{a\operatorname{csch}^4(x)}{4b^2} - \frac{\operatorname{csch}^5(x)}{5b} + \frac{(a^2+b^2)^3\log(a+b\operatorname{csch}(x))}{ab^6} + \frac{\log(\sinh(x))}{a}$$

```
[Out] -(a^4+3*a^2*b^2+3*b^4)*csch(x)/b^5+1/2*a*(a^2+3*b^2)*csch(x)^2/b^4-1/3*(a^2
+3*b^2)*csch(x)^3/b^3+1/4*a*csch(x)^4/b^2-1/5*csch(x)^5/b+(a^2+b^2)^3*ln(a+
b*csch(x))/a/b^6+ln(sinh(x))/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{(a^2+b^2)^3\log(a+b\operatorname{csch}(x))}{ab^6} + \frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4} - \frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3} - \frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5} + \frac{a\operatorname{csch}^4(x)}{4b^2} + \frac{\log(\sinh(x))}{a} - \frac{\operatorname{csch}^5(x)}{5b}$$

```
[In] Int[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] -(((a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x])/b^5) + (a*(a^2 + 3*b^2)*Csch[x]^2)/(2
*b^4) - ((a^2 + 3*b^2)*Csch[x]^3)/(3*b^3) + (a*Csch[x]^4)/(4*b^2) - Csch[x]
^5/(5*b) + ((a^2 + b^2)^3*Log[a + b*Csch[x]])/(a*b^6) + Log[Sinh[x]]/a
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^3}{x(a+x)}\,dx, x, b\text{csch}(x)\right)}{b^6}$$

$$= \frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2\left(a^2+b^2\right)}{a^4}\right) - \frac{b^6}{ax} + a\left(a^2+3b^2\right)x - \left(a^2+3b^2\right)x^2 + ax^3 - x^4 + \frac{\left(a^2+b^2\right)^3}{a(a+x)}\right)\,dx, x, b\right.}{b^6}$$

$$= -\frac{\left(a^4+3a^2b^2+3b^4\right)\text{csch}(x)}{b^5} + \frac{a\left(a^2+3b^2\right)\text{csch}^2(x)}{2b^4} - \frac{\left(a^2+3b^2\right)\text{csch}^3(x)}{3b^3}$$
$$+ \frac{a\text{csch}^4(x)}{4b^2} - \frac{\text{csch}^5(x)}{5b} + \frac{\left(a^2+b^2\right)^3\log(a+b\text{csch}(x))}{ab^6} + \frac{\log(\sinh(x))}{a}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

$$\int \frac{\coth^7(x)}{a+b\text{csch}(x)}\,dx$$

$$= \frac{-60b\left(a^4+3a^2b^2+3b^4\right)\text{csch}(x) + 30ab^2\left(a^2+3b^2\right)\text{csch}^2(x) - 20b^3\left(a^2+3b^2\right)\text{csch}^3(x) + 15ab^4\text{csch}^4(x) - 12}{60b^6}$$

```
[In] Integrate[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x] + 30*a*b^2*(a^2 + 3*b^2)*Csch[x]^2
 - 20*b^3*(a^2 + 3*b^2)*Csch[x]^3 + 15*a*b^4*Csch[x]^4 - 12*b^5*Csch[x]^5 -
 60*a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + (60*(a^2 + b^2)^3*Log[b + a*
Sinh[x]])/a)/(60*b^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(111) = 222.

Time = 3.74 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.64

method	result
default	$\frac{\frac{b^4\tanh\left(\frac{x}{2}\right)^5}{5}+\frac{a\tanh\left(\frac{x}{2}\right)^4b^3}{2}+\frac{4a^2b^2\tanh\left(\frac{x}{2}\right)^3}{3}+3\tanh\left(\frac{x}{2}\right)^3b^4+4a^3b\tanh\left(\frac{x}{2}\right)^2+10b^3\tanh\left(\frac{x}{2}\right)^2a+16a^4\tanh\left(\frac{x}{2}\right)+44a^2b^2\tanh\left(\frac{x}{2}\right)+3}{32b^5}$
risch	$-\frac{x}{a}-\frac{2\mathrm{e}^x\left(15a^4\mathrm{e}^{8x}+45a^2b^2\mathrm{e}^{8x}+45b^4\mathrm{e}^{8x}-15a^3b\,\mathrm{e}^{7x}-45a\,b^3\mathrm{e}^{7x}-60a^4\mathrm{e}^{6x}-160a^2b^2\mathrm{e}^{6x}-120b^4\mathrm{e}^{6x}+45a^3b\,\mathrm{e}^{5x}+105a\,b^3\mathrm{e}^{5x}+90a^4\mathrm{e}\right.}{15b^5}$

```
[In] int(coth(x)^7/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b^5*(1/5*b^4*tanh(1/2*x)^5+1/2*a*tanh(1/2*x)^4*b^3+4/3*a^2*b^2*tanh(1/
2*x)^3+3*tanh(1/2*x)^3*b^4+4*a^3*b*tanh(1/2*x)^2+10*b^3*tanh(1/2*x)^2*a+16*
a^4*tanh(1/2*x)+44*a^2*b^2*tanh(1/2*x)+38*tanh(1/2*x)*b^4)+1/32/a/b^6*(32*a
^6+96*a^4*b^2+96*a^2*b^4+32*b^6)*ln(-tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-1/a
*ln(tanh(1/2*x)+1)-1/160/b/tanh(1/2*x)^5-1/96/b^3*(4*a^2+9*b^2)/tanh(1/2*x)
^3-1/32*(16*a^4+44*a^2*b^2+38*b^4)/b^5/tanh(1/2*x)+1/64*a/b^2/tanh(1/2*x)^4
+1/16*a/b^4*(2*a^2+5*b^2)/tanh(1/2*x)^2-1/b^6*a*(a^4+3*a^2*b^2+3*b^4)*ln(ta
nh(1/2*x))-1/a*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4024 vs. 2(111) = 222.

Time = 0.31 (sec) , antiderivative size = 4024, normalized size of antiderivative = 33.82

$$\int \frac{\coth^7(x)}{a+b\mathrm{csch}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 + 30*(a^5*b + 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) + a^5*b + 3*a^3*b^3 + 3*a*b^5)*sin
h(x)^9 - 15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 - 5*b^6*x - 2*a^4*b^2 - 6*a^2*b^4 + 18*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 + 27*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 - 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x))*sinh(x)^7
- 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 + 21*a^2*b^4 + 252*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 - 42*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 + 4*(45*a^5*b + 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 + 45*a^5*b + 115*a^3*b^3 +
```

```
99*a*b^5 + 945*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 210*(5*b^6*x + 2*a
^4*b^2 + 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)
^2 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^5 - 30*(5*b^6*x
 + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 - 15*b^6*x - 9
*a^4*b^2 - 21*a^2*b^4 + 378*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 - 105*(
5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b + 8*a^3*b^3 + 6*a
*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^2 + 2*(45*a^
5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b + 8*a^3*b^3
 + 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 + 63*(a^5*b + 3*a^3*b^3 + 3*a
*b^5)*cosh(x)^6 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 - 21*(5*b^6*x + 2*a^4*b^2 +
 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(
5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 + (45*a^5*b + 115*a^3*b^3 + 99*a
*b^5)*cosh(x)^2 - 3*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^3 +
15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 + 2
16*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x - 84*(5*b^6*x + 2*a^4
*b^2 + 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 + 18*a^2*b^4 - 168*(3*a^5*b + 8*a^3
*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4
 + 8*(45*a^5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 - 36*(5*b^6*x + 3*a^4*b^2
 + 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(
x)^2 + 30*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x) - 15*((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*
sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^10 - 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 -
 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*cosh(x))*sinh(x)^7 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 10
*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 -
 a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 + 4*(63*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 15*(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*cosh(x)^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(
x)^4 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 - 7*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 5*(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 5*(9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cos
h(x)^4 - 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 10*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*cosh(x)^7 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 - 4*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 15*
```

```
((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 - 5*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 - 9*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*(63*(a^6 + 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 +
3*a^2*b^4)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 - a^6
 - 3*a^4*b^2 - 3*a^2*b^4 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 15*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4)*cosh(x)^7 - 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 5*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x
))*sinh(x)^3 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 5*(9*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 +
 a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 -
 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 + 3*a^4*b
^2 + 3*a^2*b^4)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7 + 6*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 10*(15*b^6*x*cosh(x)^9 + 27*(a^5*b + 3*a^3*b^3 + 3*a*b^
5)*cosh(x)^8 - 12*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^7 - 28*(3*a^5*b
 + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^6 + 3*a^5*b + 9*a^3*b^3 + 9*a*b^5 + 18*(5*b
^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^5 + 2*(45*a^5*b + 115*a^3*b^3 + 99*a*
b^5)*cosh(x)^4 - 12*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 - 12*(3*a^5
*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*c
osh(x))*sinh(x))/(a*b^6*cosh(x)^10 + 10*a*b^6*cosh(x)*sinh(x)^9 + a*b^6*sin
h(x)^10 - 5*a*b^6*cosh(x)^8 + 10*a*b^6*cosh(x)^6 - 10*a*b^6*cosh(x)^4 + 5*a
*b^6*cosh(x)^2 + 5*(9*a*b^6*cosh(x)^2 - a*b^6)*sinh(x)^8 + 40*(3*a*b^6*cosh
(x)^3 - a*b^6*cosh(x))*sinh(x)^7 - a*b^6 + 10*(21*a*b^6*cosh(x)^4 - 14*a*b^
6*cosh(x)^2 + a*b^6)*sinh(x)^6 + 4*(63*a*b^6*cosh(x)^5 - 70*a*b^6*cosh(x)^3
 + 15*a*b^6*cosh(x))*sinh(x)^5 + 10*(21*a*b^6*cosh(x)^6 - 35*a*b^6*cosh(x)^
4 + 15*a*b^6*cosh(x)^2 - a*b^6)*sinh(x)^4 + 40*(3*a*b^6*cosh(x)^7 - 7*a*b^6
*cosh(x)^5 + 5*a*b^6*cosh(x)^3 - a*b^6*cosh(x))*sinh(x)^3 + 5*(9*a*b^6*cosh
(x)^8 - 28*a*b^6*cosh(x)^6 + 30*a*b^6*cosh(x)^4 - 12*a*b^6*cosh(x)^2 + a*b^
6)*sinh(x)^2 + 10*(a*b^6*cosh(x)^9 - 4*a*b^6*cosh(x)^7 + 6*a*b^6*cosh(x)^5
- 4*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\coth^7(x)}{a+b\,\operatorname{csch}(x)}\,dx$$

```
[In] integrate(coth(x)**7/(a+b*csch(x)),x)
[Out] Integral(coth(x)**7/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(111) = 222$.

Time $= 0.21$ (sec) , antiderivative size $= 364$, normalized size of antiderivative $= 3.06$

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{2\left(15\left(a^4+3\,a^2b^2+3\,b^4\right)e^{(-x)} - 15\left(a^3b+3\,ab^3\right)e^{(-2\,x)} - 20\left(3\,a^4+8\,a^2b^2+6\,b^4\right)e^{(-3\,x)} + 15\left(3\,a^3b+7\,ab^3\right)e^{(-4\,x)} + 2\left(45\,a^4+115\,a^2b^2+99\,b^4\right)e^{(-5\,x)} - 15\left(3\,a^3b+7\,ab^3\right)e^{(-6\,x)} - 20\left(3\,a^4+8\,a^2b^2+6\,b^4\right)e^{(-7\,x)} + 15\left(a^3b+3\,ab^3\right)e^{(-8\,x)} + 15\left(a^4+3\,a^2b^2+3\,b^4\right)e^{(-9\,x)}\right)}{15\left(5\,b^5e^{(-}\right.}$$
$$+ \frac{x}{a} - \frac{\left(a^5+3\,a^3b^2+3\,ab^4\right)\log\left(e^{(-x)}+1\right)}{b^6} - \frac{\left(a^5+3\,a^3b^2+3\,ab^4\right)\log\left(e^{(-x)}-1\right)}{b^6}$$
$$+ \frac{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{ab^6}$$

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="maxima")
[Out] 2/15*(15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b + 3*a*b^3)*e^(-2*x) -
 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-3*x) + 15*(3*a^3*b + 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 + 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b + 7*a*b^3)*e^(-6
*x) - 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-7*x) + 15*(a^3*b + 3*a*b^3)*e^(-8*
x) + 15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) - 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) - 5*b^5*e^(-8*x) + b^5*e^(-10*x) - b^5) + x/a - (a^5 +
 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + 1)/b^6 - (a^5 + 3*a^3*b^2 + 3*a*b^4)*log
(e^(-x) - 1)/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*b*e^(-x) + a*
e^(-2*x) - a)/(a*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(111) = 222$.

Time $= 0.28$ (sec) , antiderivative size $= 295$, normalized size of antiderivative $= 2.48$

$$\int \frac{\coth^7(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{(a^5 + 3\,a^3b^2 + 3\,ab^4)\log\left(\left|-e^{(-x)} + e^x\right|\right)}{b^6} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{ab^6} + \frac{137\,a^5\left(e^{(-x)} - e^x\right)^5 + 411\,a^3b^2\left(e^{(-x)} - e^x\right)^5 + 411\,ab^4\left(e^{(-x)} - e^x\right)^5 + 120\,a^4b\left(e^{(-x)} - e^x\right)^4 + 360\,a^2b^3\left(e$$

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^5 + 3*a^3*b^2 + 3*a*b^4)*log(abs(-e^(-x) + e^x))/b^6 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^6) + 1/60*(137*a
^5*(e^(-x) - e^x)^5 + 411*a^3*b^2*(e^(-x) - e^x)^5 + 411*a*b^4*(e^(-x) - e^
x)^5 + 120*a^4*b*(e^(-x) - e^x)^4 + 360*a^2*b^3*(e^(-x) - e^x)^4 + 360*b^5*
(e^(-x) - e^x)^4 + 120*a^3*b^2*(e^(-x) - e^x)^3 + 360*a*b^4*(e^(-x) - e^x)^
3 + 160*a^2*b^3*(e^(-x) - e^x)^2 + 480*b^5*(e^(-x) - e^x)^2 + 240*a*b^4*(e^
(-x) - e^x) + 384*b^5)/(b^6*(e^(-x) - e^x)^5)
```

Mupad [B] (verification not implemented)

Time $= 2.78$ (sec) , antiderivative size $= 317$, normalized size of antiderivative $= 2.66$

$$\int \frac{\coth^7(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\frac{4\,a}{b^2} - \frac{64\,\mathrm{e}^x}{5\,b}}{6\,\mathrm{e}^{4\,x} - 4\,\mathrm{e}^{2\,x} - 4\,\mathrm{e}^{6\,x} + \mathrm{e}^{8\,x} + 1} + \frac{\frac{8\,a}{b^2} - \frac{8\,\mathrm{e}^x\,(5\,a^2 + 27\,b^2)}{15\,b^3}}{3\,\mathrm{e}^{2\,x} - 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} - 1} - \frac{\frac{8\,\mathrm{e}^x\,(a^2 + 3\,b^2)}{3\,b^3} - \frac{2\,(a^4 + 5\,a^2\,b^2)}{a\,b^4}}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{x}{a} - \frac{\frac{2\,\mathrm{e}^x\,(a^4 + 3\,a^2\,b^2 + 3\,b^4)}{b^5} - \frac{2\,(a^4 + 3\,a^2\,b^2)}{a\,b^4}}{\mathrm{e}^{2\,x} - 1} - \frac{32\,\mathrm{e}^x}{5\,b\,\left(5\,\mathrm{e}^{2\,x} - 10\,\mathrm{e}^{4\,x} + 10\,\mathrm{e}^{6\,x} - 5\,\mathrm{e}^{8\,x} + \mathrm{e}^{10\,x} - 1\right)} - \frac{\ln\left(\mathrm{e}^{2\,x} - 1\right)\,\left(a^5 + 3\,a^3\,b^2 + 3\,a\,b^4\right)}{b^6} + \frac{\ln\left(2\,b\,\mathrm{e}^x - a + a\,\mathrm{e}^{2\,x}\right)\,\left(a^6 + 3\,a^4\,b^2 + 3\,a^2\,b^4 + b^6\right)}{a\,b^6}$$

```
[In] int(coth(x)^7/(a + b/sinh(x)),x)
```

```
[Out] ((4*a)/b^2 - (64*exp(x))/(5*b))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp
(8*x) + 1) + ((8*a)/b^2 - (8*exp(x)*(5*a^2 + 27*b^2))/(15*b^3))/(3*exp(2*x)
 - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(x)*(a^2 + 3*b^2))/(3*b^3) - (2*(a^4
 + 5*a^2*b^2))/(a*b^4))/(exp(4*x) - 2*exp(2*x) + 1) - x/a - ((2*exp(x)*(a^4
```

```
 + 3*b^4 + 3*a^2*b^2))/b^5 - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x) - 1)
- (32*exp(x))/(5*b*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + e
xp(10*x) - 1)) - (log(exp(2*x) - 1)*(3*a*b^4 + a^5 + 3*a^3*b^2))/b^6 + (log
(2*b*exp(x) - a + a*exp(2*x))*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(a*b^6)
```

3.125 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx$

Optimal result . . . 673
Rubi [A] (verified) . . . 673
Mathematica [A] (verified) . . . 675
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Fricas [B] (verification not implemented) . . . 676
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Giac [A] (verification not implemented) . . . 677
Mupad [B] (verification not implemented) . . . 678

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx = -\frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6} + \frac{192\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5} - \frac{48\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{64\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}$$

```
[Out] -32/3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^6+
192/5*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^5-
48*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^4+64/
3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx = \frac{64\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{48\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{192\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{5bc\left(1-e^{2c(a+bx)}\right)^5} - \frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^6}$$

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b
*x)))^6) + (192*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^
(2*c*(a + b*x)))^5) - (48*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c
*(1 - E^(2*c*(a + b*x)))^4) + (64*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*
x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^7(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{128x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(128\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^3}{(-1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{1}{(-1+x)^4}\right)dx, x, e^{2c}\right.}{bc}$$

$$= -\frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6} + \frac{192\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5}$$
$$- \frac{48\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{64\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx =$$
$$-\frac{16\left(-1+6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\sqrt{\operatorname{csch}^2(c(a+bx))}\sinh(c(a+bx))}{15bc\left(-1+e^{2c(a+bx)}\right)^6}$$

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sqrt[Csch[c*(a + b*x)]^2]*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b
*x)))^6)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.44

method	result	siz
default	$-\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(\frac{\coth(c(bx+a))^6}{6}+\frac{\coth(c(bx+a))^5}{5}-\frac{\coth(c(bx+a))^4}{2}-\frac{2\coth(c(bx+a))^3}{3}+\frac{\coth(c(bx+a))^2}{2}+\coth(c(bx+a))\right)}{cb}$	87
risch	$-\frac{16\left(20\,e^{6c(bx+a)}-15\,e^{4c(bx+a)}+6\,e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{-c(bx+a)}}{15bc\left(e^{2c(bx+a)}-1\right)^5}$	91

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -csgn(csch(c*(b*x+a)))/c/b*(1/6*coth(c*(b*x+a))^6+1/5*coth(c*(b*x+a))^5-1/2
*coth(c*(b*x+a))^4-2/3*coth(c*(b*x+a))^3+1/2*coth(c*(b*x+a))^2+coth(c*(b*x+
a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(173) = 346$.

Time $= 0.26$ (sec) , antiderivative size $= 592$, normalized size of antiderivative $= 2.97$

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9-6\,bc\cosh\left(bcx+ac\right)^7+\right.}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx = e^{ac}\int \left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{7}{2}} e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(7/2)*exp(b*c*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.

Time $= 0.28$ (sec) , antiderivative size $= 386$, normalized size of antiderivative $= 1.94$

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx =$$

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

$$+\frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

$$-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

$$+\frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time $= 0.29$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 0.45$

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}-15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}-1\right)}{15\,bc(e^{(2\,bcx+2\,ac)}-1)^6\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) - 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6*sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.08

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac + bcx)^{7/2}\,dx = \frac{32\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2e^{2ac+2bcx}+1\right)}{3bc\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^3} + \frac{24\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2e^{2ac+2bcx}+1\right)}{bc\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^4} + \frac{96\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2e^{2ac+2bcx}+1\right)}{5bc\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^5} + \frac{16\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2e^{2ac+2bcx}+1\right)}{3bc\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^6}$$

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (32*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*
b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c
 + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^3) + (24*(1/(exp(a*c + b*c*x)/2 - ex
p(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x)
 + 1))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 - 1)^4) + (96*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp
(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(5*b*c*(exp(a*c + b*c*x) -
 exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^5) + (16*(1/(exp(a*c + b*
c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c
 + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a
*c + 2*b*c*x) - 1)^6)
```

3.126 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx$

Optimal result . . . 679
Rubi [A] (verified) . . . 679
Mathematica [A] (verified) . . . 681
Maple [C] (warning: unable to verify) . . . 681
Fricas [B] (verification not implemented) . . . 682
Sympy [F] . . . 682
Maxima [A] (verification not implemented) . . . 682
Giac [A] (verification not implemented) . . . 683
Mupad [B] (verification not implemented) . . . 683

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = -\frac{4\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{8\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[Out] -4*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^4+32/
3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^3-8*si
nh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = -\frac{8\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{4\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)
))^4) + (32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3) - (8*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 -
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^5(ac+bcx)\,dx \\
&= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(16\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

$$= \frac{\left(16\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int\left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= -\frac{4\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}$$
$$- \frac{8\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx =$$
$$-\frac{4\left(1-4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\sqrt{\operatorname{csch}^2(c(a+bx))}\sinh(c(a+bx))}{3bc\left(-1+e^{2c(a+bx)}\right)^4}$$

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sqrt[Csch[c*(a + b*x)]^
2]*Sinh[c*(a + b*x)])/(3*b*c*(-1 + E^(2*c*(a + b*x)))^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(\frac{\coth(c(bx+a))^4}{4}+\frac{\coth(c(bx+a))^3}{3}-\frac{\coth(c(bx+a))^2}{2}-\coth(c(bx+a))\right)}{cb}$	65
risch	$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}-4\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\,\mathrm{e}^{-c(bx+a)}}{3bc\left(\mathrm{e}^{2c(bx+a)}-1\right)^3}$	80

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -csgn(csch(c*(b*x+a)))/c/b*(1/4*coth(c*(b*x+a))^4+1/3*coth(c*(b*x+a))^3-1/2
*coth(c*(b*x+a))^2-coth(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(130) = 260$.

Time $= 0.25$ (sec) , antiderivative size $= 315$, normalized size of antiderivative $= 2.14$

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^6+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^5+bc\sinh\left(bcx+ac\right)^6-4\,bc\cosh\left(bcx+ac\right)^4+\right.}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = e^{ac}\int \left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{5}{2}} e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(5/2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

none

Time $= 0.28$ (sec) , antiderivative size $= 209$, normalized size of antiderivative $= 1.42$

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}$$

$$+\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}$$

$$-\frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)} - 4\,e^{(2\,bcx+2\,ac)} + 1\right)}{3\,bc(e^{(2\,bcx+2\,ac)}-1)^4\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) - 1)^4*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = -\frac{2\,\mathrm{e}^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)^2}}\left(6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x} - 4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x} + 1\right)}{3\,b\,c\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)^3}$$

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1
/2)*(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*
c + 2*b*c*x) - 1)^3)
```

3.127 $\int e^{c(a+bx)}\mathbf{csch}^2(ac+bcx)^{3/2}\,dx$

Optimal result . . . 684
Rubi [A] (verified) . . . 684
Mathematica [A] (verified) . . . 685
Maple [C] (warning: unable to verify) . . . 686
Fricas [B] (verification not implemented) . . . 686
Sympy [F] . . . 686
Maxima [A] (verification not implemented) . . . 687
Giac [A] (verification not implemented) . . . 687
Mupad [B] (verification not implemented) . . . 687

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{2e^{4c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[Out] -2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*
c*(b*x+a)))^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 270}

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1
- E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx \\
&= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(8\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= -\frac{2e^{4c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{2e^{4c(a+bx)}\operatorname{csch}^2(c(a+bx))^{3/2}\sinh^3(c(a+bx))}{bc\left(-1+e^{2c(a+bx)}\right)^2}$$

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*(Csch[c*(a + b*x)]^2)^(3/2)*Sinh[c*(a + b*x)]^3)/(b*c
*(-1 + E^(2*c*(a + b*x)))^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(-\frac{\coth(c(bx+a))^2}{2}-\coth(c(bx+a))\right)}{cb}$	40
risch	$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\,\mathrm{e}^{-c(bx+a)}}{bc\left(\mathrm{e}^{2c(bx+a)}-1\right)}$	69

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] csgn(csch(c*(b*x+a)))/c/b*(-1/2*coth(c*(b*x+a))^2-coth(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(52) = 104.

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.09

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx =$$

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3\left(bc\right.}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = e^{ac}\int \left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{3}{2}} e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(3/2),x)
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(3/2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)} + \frac{2}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}-1\right)}{bc(e^{(2\,bcx+2\,ac)}-1)^2\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{\mathrm{e}^{-a\,c-b\,c\,x}\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)^2}}}{b\,c\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}$$

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1))
```

3.128 $\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx$

Optimal result . . . 688
Rubi [A] (verified) . . . 688
Mathematica [A] (verified) . . . 689
Maple [C] (warning: unable to verify) . . . 690
Fricas [A] (verification not implemented) . . . 690
Sympy [F] . . . 690
Maxima [A] (verification not implemented) . . . 691
Giac [A] (verification not implemented) . . . 691
Mupad [F(-1)] . . . 691

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}$$

```
[Out] ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 266}

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (Sqrt[Csch[a*c + b*c*x]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b
*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx \\
&= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(2\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\sqrt{\operatorname{csch}^2(c(a+bx))}\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (Sqrt[Csch[c*(a + b*x)]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b
*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
default	$\operatorname{csgn}\left(\operatorname{csch}\left(c(bx+a)\right)\right)\left(x+\frac{\ln(\sinh(c(bx+a)))}{cb}\right)$	29
risch	$\frac{\ln\left(e^{2bcx}-e^{-2ac}\right)\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{-c(bx+a)}}{bc}$	68

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] csgn(csch(c*(b*x+a)))*(x+1/c/b*ln(sinh(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = e^{ac}\int \sqrt{\operatorname{csch}^2\left(ac+bcx\right)}e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(sqrt(csch(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] log(abs(e^(2*b*c*x + 2*a*c) - 1))/(b*c*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*
c)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \int \mathrm{e}^{c\,(a+b\,x)}\,\sqrt{\frac{1}{\sinh\left(a\,c+b\,c\,x\right)^2}}\,dx$$

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

3.129 $\int \frac{e^{c(a+bx)}}{\sqrt{\text{csch}^2(ac+bcx)}}\,dx$

Optimal result . . . 692
Rubi [A] (verified) . . . 692
Mathematica [A] (verified) . . . 694
Maple [C] (warning: unable to verify) . . . 694
Fricas [A] (verification not implemented) . . . 694
Sympy [F] . . . 695
Maxima [A] (verification not implemented) . . . 695
Giac [A] (verification not implemented) . . . 695
Mupad [F(-1)] . . . 696

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{e^{c(a+bx)}}{\sqrt{\text{csch}^2(ac+bcx)}}\,dx = \frac{e^{2c(a+bx)}\text{csch}(ac+bcx)}{4bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{x\text{csch}(ac+bcx)}{2\sqrt{\text{csch}^2(ac+bcx)}}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-1/2*x*cs
ch(b*c*x+a*c)/(csch(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 14}

$$\int \frac{e^{c(a+bx)}}{\sqrt{\text{csch}^2(ac+bcx)}}\,dx = \frac{e^{2c(a+bx)}\text{csch}(ac+bcx)}{4bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{x\text{csch}(ac+bcx)}{2\sqrt{\text{csch}^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(4*b*c*Sqrt[Csch[a*c + b*c*x]^2]) - (
x*Csch[a*c + b*c*x])/(2*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = \frac{\left(e^{2c(a+bx)}-2bcx\right)\operatorname{csch}(c(a+bx))}{4bc\sqrt{\operatorname{csch}^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)])/(4*b*c*Sqrt[Csch[c*(a + b
*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(\frac{\sinh(bcx+ac)\cosh(bcx+ac)}{2}-\frac{bcx}{2}-\frac{ac}{2}+\frac{\cosh(bcx+ac)^2}{2}\right)}{cb}$	60
risch	$-\frac{x\,\mathrm{e}^{c(bx+a)}}{2\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}+\frac{\mathrm{e}^{3c(bx+a)}}{4bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}$	106

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(csch(c*(b*x+a)))/c/b*(1/2*sinh(b*c*x+a*c)*cosh(b*c*x+a*c)-1/2*b*c*x-1/
2*a*c+1/2*cosh(b*c*x+a*c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = -\frac{(2\,bcx-1)\cosh(bcx+ac)-(2\,bcx+1)\sinh(bcx+ac)}{4\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(csch(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = -\frac{bcx+ac}{2\,bc} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = -\frac{1}{2}\,x\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) + \frac{e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = \int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{\sinh(a\,c+b\,c\,x)^2}}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

3.130 $\int \frac{e^{c(a+bx)}}{\text{csch}^2(ac+bcx)^{3/2}}\,dx$

Optimal result . . . 697
Rubi [A] (verified) . . . 697
Mathematica [A] (verified) . . . 699
Maple [C] (warning: unable to verify) . . . 699
Fricas [A] (verification not implemented) . . . 700
Sympy [F] . . . 700
Maxima [A] (verification not implemented) . . . 700
Giac [A] (verification not implemented) . . . 701
Mupad [F(-1)] . . . 701

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{e^{c(a+bx)}}{\text{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{e^{-2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{3e^{2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\text{csch}(ac+bcx)}{32bc\sqrt{\text{csch}^2(ac+bcx)}} + \frac{3x\text{csch}(ac+bcx)}{8\sqrt{\text{csch}^2(ac+bcx)}}$$

```
[Out] 1/16*csch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-3/16*ex
p(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c*(
b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+3/8*x*csch(b*c*x+a*c)
/(csch(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\int \frac{e^{c(a+bx)}}{\text{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{e^{-2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{3e^{2c(a+bx)}\text{csch}(ac+bcx)}{16bc\sqrt{\text{csch}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\text{csch}(ac+bcx)}{32bc\sqrt{\text{csch}^2(ac+bcx)}} + \frac{3x\text{csch}(ac+bcx)}{8\sqrt{\text{csch}^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (3
*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(16*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
```

```
(E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(3*x*Csch[a*c + b*c*x])/(8*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&\quad + \frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{3x\operatorname{csch}(ac+bcx)}{8\sqrt{\operatorname{csch}^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$
\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(e^{-2c(a+bx)} - 3e^{2c(a+bx)} + \frac{1}{2}e^{4c(a+bx)} + 6bcx\right)\operatorname{csch}^3(c(a+bx))}{16bc\operatorname{csch}^2(c(a+bx))^{3/2}}
$$

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3)/(16*b*c*(Csch[c*(a + b*x)]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
default	$\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(\left(\frac{\sinh(bcx+ac)^3}{4}-\frac{3\sinh(bcx+ac)}{8}\right)\cosh(bcx+ac)+\frac{3bcx}{8}+\frac{3ac}{8}+\frac{\sinh(bcx+ac)^4}{4}\right)}{cb}$
risch	$\frac{3x\,\mathrm{e}^{c(bx+a)}}{8\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}+\frac{\mathrm{e}^{5c(bx+a)}}{32bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}-\frac{3\,\mathrm{e}^{3c(bx+a)}}{16bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}+\frac{}{16bc(\mathrm{e}^{2c}$

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(csch(c*(b*x+a)))/c/b*((1/4*sinh(b*c*x+a*c)^3-3/8*sinh(b*c*x+a*c))*cosh
(b*c*x+a*c)+3/8*b*c*x+3/8*a*c+1/4*sinh(b*c*x+a*c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{3\cosh(bcx+ac)^3 + 9\cosh(bcx+ac)\sinh(bcx+ac)^2 - \sinh(bcx+ac)^3 + 6(2b$$
$$32(bc\cosh(bcx+ac) - b$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{csch}^2(ac+bcx)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(e^{(6\,bcx+6\,ac)} - 6\,e^{(4\,bcx+4\,ac)} + 2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc} + \frac{3\,(bcx+ac)}{8\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(12\,bcxe^{(-ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right.\right.\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2
*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*
c) - e^(-b*c*x - a*c)))*e^(-2*b*c*x - 3*a*c) + (e^(4*b*c*x + 9*a*c)*sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 7*a*c)*sgn(e^(b*c*x + a*c
) - e^(-b*c*x - a*c)))*e^(-6*a*c))*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\left(\frac{1}{\sinh(a\,c+b\,c\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2), x)
```

3.131 $\int \frac{e^{c(a+bx)}}{\text{csch}^2(ac+bcx)^{5/2}}\,dx$

Optimal result . . . 702
Rubi [A] (verified) . . . 702
Mathematica [A] (verified) . . . 704
Maple [C] (warning: unable to verify) . . . 705
Fricas [A] (verification not implemented) . . . 705
Sympy [F] . . . 705
Maxima [A] (verification not implemented) . . . 706
Giac [A] (verification not implemented) . . . 706
Mupad [F(-1)] . . . 707

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{e^{c(a+bx)}}{\text{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{e^{-4c(a+bx)}\text{csch}(ac+bcx)}{128bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\text{csch}(ac+bcx)}{64bc\sqrt{\text{csch}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\text{csch}(ac+bcx)}{32bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{5e^{4c(a+bx)}\text{csch}(ac+bcx)}{128bc\sqrt{\text{csch}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\text{csch}(ac+bcx)}{192bc\sqrt{\text{csch}^2(ac+bcx)}} - \frac{5x\text{csch}(ac+bcx)}{16\sqrt{\text{csch}^2(ac+bcx)}}$$

```
[Out] 1/128*csch(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-5/64*c
sch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*c*
(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/128*exp(4*c*(b*x+a
))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))*csc
h(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/16*x*csch(b*c*x+a*c)/(csch(b*c
*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{6852, 2320, 12, 272, 45\}$

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\operatorname{csch}(ac+bcx)}{192bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5x\operatorname{csch}(ac+bcx)}{16\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (
5*Csch[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) +
 (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2])
 - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(128*b*c*Sqrt[Csch[a*c + b*c*x]^
2]) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x])/(192*b*c*Sqrt[Csch[a*c + b*c*x]
^2]) - (5*x*Csch[a*c + b*c*x])/(16*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right)\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} \\
&\quad - \frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\operatorname{csch}(ac+bcx)}{192bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5x\operatorname{csch}(ac+bcx)}{16\sqrt{\operatorname{csch}^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(\frac{1}{128}e^{-4c(a+bx)} - \frac{5}{64}e^{-2c(a+bx)} + \frac{5}{32}e^{2c(a+bx)} - \frac{5}{128}e^{4c(a+bx)} + \frac{1}{192}e^{6c(a+bx)} - \frac{5bcx}{16}\right)\operatorname{csch}}{bc\operatorname{csch}^2(c(a+bx))^{5/2}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(128*E^(4*c*(a + b*x))) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b*x
)))/32 - (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 - (5*b*c*x)/16)*
Csch[c*(a + b*x)]^5)/(b*c*(Csch[c*(a + b*x)]^2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.35

method	result
default	$\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(\left(\frac{\sinh(bcx+ac)^5}{6}-\frac{5\sinh(bcx+ac)^3}{24}+\frac{5\sinh(bcx+ac)}{16}\right)\cosh(bcx+ac)-\frac{5bcx}{16}-\frac{5ac}{16}+\frac{\sinh(bcx+ac)^6}{6}\right)}{cb}$
risch	$-\frac{5x\,e^{c(bx+a)}}{16\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{e^{7c(bx+a)}}{192bc\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}}-\frac{5\,e^{5c(bx+a)}}{128bc\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}}+\frac{}{32}$

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(csch(c*(b*x+a)))/c/b*((1/6*sinh(b*c*x+a*c)^5-5/24*sinh(b*c*x+a*c)^3+5/
16*sinh(b*c*x+a*c))*cosh(b*c*x+a*c)-5/16*b*c*x-5/16*a*c+1/6*sinh(b*c*x+a*c)
^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{5\,\cosh(bcx+ac)^5 + 25\,\cosh(bcx+ac)\sinh(bcx+ac)^4 - \sinh(bcx+ac)^5 - 5\,(}{}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{csch}^2(ac+bcx)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.36

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(2\,e^{(10\,bcx+10\,ac)} - 15\,e^{(8\,bcx+8\,ac)} + 60\,e^{(6\,bcx+6\,ac)} - 30\,e^{(2\,bcx+2\,ac)} + 3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc} - \frac{5\,(bcx+ac)}{16\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.11

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = -\frac{\left(120\,bcxe^{(-ac)}\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 3\left(30\,e^{(4\,bcx+4\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 10\,e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(b}\right.\right.\right.$$

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*
e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))*e^(-4*b*c*x - 5*a*c) - (2*e^(6*b*c*x + 20*a*c)*sgn(e^(b*c
*x + a*c) - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) + 60*e^(2*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)))*e^(-15*a*c))*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\left(\frac{1}{\sinh(a\,c+b\,c\,x)^2}\right)^{5/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2),x)
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2), x)
```

3.132 $\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result . . . 708
Rubi [A] (verified) . . . 708
Mathematica [C] (verified) . . . 710
Maple [A] (verified) . . . 710
Fricas [A] (verification not implemented) . . . 711
Sympy [F] . . . 711
Maxima [F] . . . 711
Giac [F] . . . 712
Mupad [F(-1)] . . . 712

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -2/21*x^2/c^4/csch(2*ln(c*x))^(1/2)+1/7*x^6/csch(2*ln(c*x))^(1/2)+2/21*Elli
pticF(1/c/x,I)/c^7/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 283, 331, 227}

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{21c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[In] Int[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (-2*x^2)/(21*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^6/(7*Sqrt[Csch[2*Log[c*x]]]) +
 (2*EllipticF[ArcCsc[c*x], -1])/(21*c^7*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2
*Log[c*x]]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^5}{\sqrt{\text{csch}(2\log(x))}}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^6\,dx, x, cx\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= \frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}} + \frac{2\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= -\frac{2x^2}{21c^4\sqrt{\text{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= -\frac{2x^2}{21c^4\sqrt{\text{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt{\text{csch}(2\log(cx))}}\,dx = \frac{x^2\left(-(1-c^4x^4)^{3/2} + \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, c^4x^4\right)\right)}{7c^4\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*(-(1 - c^4*x^4)^(3/2) + Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4]))/(
7*c^4*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

method	result	size
risch	$\frac{x^2\left(3c^4x^4-2\right)\sqrt{2}}{42c^4\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\ \text{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\,x}{21c^4\sqrt{-c^2}\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	125

```
[In] int(x^5/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*x^2*(3*c^4*x^4-2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/21/c^4/(-c
^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^
2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\sqrt{2}(3\,c^{10}x^8 - 5\,c^6x^4 + 2\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 2\,\sqrt{2}\sqrt{c^4}F(\arcsin\left(\frac{1}{cx}\right)\,|\,-1)}{42\,c^8}$$

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/42*(sqrt(2)*(3*c^10*x^8 - 5*c^6*x^4 + 2*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1))
+ 2*sqrt(2)*sqrt(c^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^8
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x**5/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\ \log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

```
[In] int(x^5/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x^5/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.133 $\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result . . . 713
Rubi [A] (verified) . . . 713
Mathematica [A] (verified) . . . 714
Maple [A] (verified) . . . 714
Fricas [A] (verification not implemented) . . . 715
Sympy [F] . . . 715
Maxima [A] (verification not implemented) . . . 715
Giac [F] . . . 716
Mupad [B] (verification not implemented) . . . 716

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/6*(c^4-1/x^4)*x^5/c^4/csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5671, 5669, 270}

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x^5\left(c^4-\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[In] Int[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] ((c^4 - x^(-4))*x^5)/(6*c^4*Sqrt[Csch[2*Log[c*x]]])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{\sqrt{\text{csch}(2\log(x))}}\,dx, x, cx\right)}{c^5} \\ &= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^5\,dx, x, cx\right)}{c^6\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\ &= \frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\text{csch}(2\log(cx))}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x^4}{\sqrt{\text{csch}(2\log(cx))}}\,dx = \frac{\left(-1+c^4x^4\right)^2\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}{6c^6x}$$

```
[In] Integrate[x^4/Sqrt[Csch[2*Log[c*x]]],x]
[Out] ((-1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(6*c^6*x)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{\sqrt{2}\,x\left(c^4x^4-1\right)}{12\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^4}$	39

```
[In] int(x^4/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^4*x^4-1)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\sqrt{2}(c^8x^8 - 2\,c^4x^4 + 1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{12\,c^6x}$$

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*(c^8*x^8 - 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c^6*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x**4/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(csch(2*log(c*x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\left(\sqrt{2}c^4x^4 - \sqrt{2}\right)\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{cx-1}}{12\,c^5}$$

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*c^4*x^4 - sqrt(2))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x -
 1)/c^5
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(csch(2*log(c*x))), x)
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\left(c^4\,x^4-1\right)^2\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{12\,c^6\,x}$$

```
[In] int(x^4/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] ((c^4*x^4 - 1)^2*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(12*c^6*x)
```

3.134 $\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result . . . 717
Rubi [A] (verified) . . . 717
Mathematica [C] (verified) . . . 720
Maple [A] (verified) . . . 720
Fricas [A] (verification not implemented) . . . 721
Sympy [F] . . . 721
Maxima [F] . . . 721
Giac [F(-2)] . . . 722
Mupad [F(-1)] . . . 722

Optimal result

Integrand size = 15, antiderivative size = 119

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2E(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] -2/5/c^4/csch(2*ln(c*x))^(1/2)+1/5*x^4/csch(2*ln(c*x))^(1/2)-2/5*EllipticE(
1/c/x,I)/c^5/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)+2/5*EllipticF(1/c/
x,I)/c^5/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5671, 5669, 342, 283, 331, 313, 227, 1195, 435}

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2E(\csc^{-1}(cx)|-1)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[In] Int[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] -2/(5*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^4/(5*Sqrt[Csch[2*Log[c*x]]]) - (2*Ell
ipticE[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x
]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[C
sch[2*Log[c*x]]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{\sqrt{\text{csch}(2\log(x))}}\,dx, x, cx\right)}{c^4} \\
&= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^4\,dx, x, cx\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\
&= \frac{x^4}{5\sqrt{\text{csch}(2\log(cx))}} + \frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\
&= -\frac{2}{5c^4\sqrt{\text{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\text{csch}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\
&= -\frac{2}{5c^4\sqrt{\text{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\text{csch}(2\log(cx))}} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}
\end{aligned}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$+ \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$- \frac{2E(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^4x^4\right)}{3\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^4*Hypergeometric2F1[-1/2, 3/4, 7/4, c^4*x^4])/(3*Sqrt[2 - 2*c^4*x^4]*Sqr
t[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{\sqrt{2}\,x^4}{10\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{-c^2},i\right)-\text{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}\,x}{5\sqrt{-c^2}\,(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	127

```
[In] int(x^3/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*2^(1/2)*x^4/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/5/(-c^2)^(1/2)*(c^2*x^2+1)^(
1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^2*(EllipticF(x*(-c^2)^(1/2),I)-Ellipt
icE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$
$$= \frac{\sqrt{2}(c^{10}x^8 - 3\,c^6x^4 + 2\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 2\sqrt{c^4}(\sqrt{2}x^2E(\arcsin\left(\frac{1}{cx}\right)\mid -1) - \sqrt{2}x^2F(\arcsin\left(\frac{1}{cx}\right)\mid -1))}{10\,c^8x^2}$$

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(2)*(c^10*x^8 - 3*c^6*x^4 + 2*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) -
2*sqrt(c^4)*(sqrt(2)*x^2*elliptic_e(arcsin(1/(c*x)), -1) - sqrt(2)*x^2*elli
ptic_f(arcsin(1/(c*x)), -1)))/(c^8*x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x**3/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\ \log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to c
onvert to real %%{poly1[1.000000000000000000000000000000,0.000000000000000
000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

```
[In] int(x^3/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^3/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.135 $\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result . . . 723
Rubi [A] (verified) . . . 723
Mathematica [A] (verified) . . . 725
Maple [A] (verified) . . . 725
Fricas [A] (verification not implemented) . . . 726
Sympy [F] . . . 726
Maxima [F] . . . 726
Giac [F] . . . 727
Mupad [F(-1)] . . . 727

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/4*x^3/csch(2*ln(c*x))^(1/2)-1/4*arctanh((1-1/c^4/x^4)^(1/2))/c^4/x/(1-1/c
^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 272, 43, 65, 212}

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[In] Int[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Csch[2*Log[c*x]]]) - ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{\text{csch}(2\log(x))}}\,dx, x, cx\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^3\,dx, x, cx\right)}{c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\text{csch}(2\log(cx))}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\text{csch}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\text{csch}(2\log(cx))}} - \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{\text{csch}(2\log(cx))}}\,dx = \frac{x\left(c^2x^2\sqrt{1-c^4x^4} + \arcsin\left(c^2x^2\right)\right)}{4c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 - c^4*x^4] + ArcSin[c^2*x^2]))/(4*c^2*Sqrt[2 - 2*c^4*x^4
]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

method	result	size
risch	$\frac{\sqrt{2}\,x^3}{8\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{8\sqrt{c^4}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\sqrt{c^4x^4-1}}$	97

```
[In] int(x^2/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*x^3/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)/(c^4*x^4-1
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$
$$= \frac{2\sqrt{2}(c^5x^5 - cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + \sqrt{2}\log\left(2\,c^4x^4 - 2\,(c^5x^5 - cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 1\right)}{16\,c^3}$$

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + sqrt(2)*log(2
*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) - 1))/c^3
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x**2/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

```
[In] int(x^2/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x^2/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.136 $\int \frac{x}{\sqrt{\text{csch}(2\log(cx))}}\,dx$

Optimal result . . . 728
Rubi [A] (verified) . . . 728
Mathematica [C] (verified) . . . 730
Maple [A] (verified) . . . 730
Fricas [A] (verification not implemented) . . . 730
Sympy [F] . . . 731
Maxima [F] . . . 731
Giac [F(-2)] . . . 731
Mupad [F(-1)] . . . 731

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{x}{\sqrt{\text{csch}(2\log(cx))}}\,dx = \frac{x^2}{3\sqrt{\text{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

```
[Out] 1/3*x^2/csch(2*ln(c*x))^(1/2)+2/3*EllipticF(1/c/x,I)/c^3/x/(1-1/c^4/x^4)^(1
/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5671, 5669, 342, 283, 227}

$$\int \frac{x}{\sqrt{\text{csch}(2\log(cx))}}\,dx = \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{3c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}} + \frac{x^2}{3\sqrt{\text{csch}(2\log(cx))}}$$

```
[In] Int[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Csch[2*Log[c*x]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(3*c^3*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{\text{csch}(2\log(x))}}\,dx, x, cx\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x^2\,dx, x, cx\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\
&= \frac{x^2}{3\sqrt{\text{csch}(2\log(cx))}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}
\end{aligned}$$

$$= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, c^4x^4\right)}{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4])/(Sqrt[2 - 2*c^4*x^4]*Sqrt[
(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{\sqrt{2}\,x^2}{6\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\text{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\,x}{3\sqrt{-c^2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	109

```
[In] int(x/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*x^2/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/3/(-c^2)^(1/2)*(c^2*x^2+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c
^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\sqrt{2}(c^6x^4-c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 2\sqrt{2}\sqrt{c^4}F(\arcsin\left(\frac{1}{cx}\right)\,|\,-1)}{6\,c^4}$$

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(c^6*x^4 - c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + 2*sqrt(2)*sqrt(c
^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^4
```

Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(x/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{csch}\left(2\ \log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to c
onvert to real %%{poly1[1.0000000000000000000000000000000,0.000000000000000
000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\frac{1}{\sinh(2\ \ln(c\,x))}}}\,dx$$

```
[In] int(x/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result . . . 732
Rubi [A] (verified) . . . 732
Mathematica [A] (verified) . . . 734
Maple [F] . . . 734
Fricas [A] (verification not implemented) . . . 734
Sympy [F] . . . 735
Maxima [F] . . . 735
Giac [F(-1)] . . . 735
Mupad [F(-1)] . . . 735

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\csc^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/2*x/csch(2*ln(c*x))^(1/2)+1/2*arccsc(c^2*x^2)/c^2/x/(1-1/c^4/x^4)^(1/2)/c
sch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5665, 5663, 342, 281, 283, 222}

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\csc^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[In] Int[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Csch[2*Log[c*x]]]) + ArcCsc[c^2*x^2]/(2*c^2*Sqrt[1 - 1/(c^4*x^4)]
*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5663

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Csch[d*(a
+ b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(2\log(x))}}\,dx, x, cx\right)}{c} \\ &= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}x\,dx, x, cx\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}} \\ &= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= \frac{x}{2\sqrt{\text{csch}(2\log(cx))}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

$$= \frac{x}{2\sqrt{\text{csch}(2\log(cx))}} + \frac{\csc^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\text{csch}(2\log(cx))}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\text{csch}(2\log(cx))}}\,dx = \frac{x\left(\sqrt{-1+c^4x^4} - \arctan\left(\sqrt{-1+c^4x^4}\right)\right)}{2\sqrt{2}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\sqrt{-1+c^4x^4}}$$

```
[In] Integrate[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(Sqrt[-1 + c^4*x^4] - ArcTan[Sqrt[-1 + c^4*x^4]]))/(2*Sqrt[2]*Sqrt[(c^2*
x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Maple [F]

$$\int \frac{1}{\sqrt{\text{csch}\left(2\ln\left(cx\right)\right)}}dx$$

```
[In] int(1/csch(2*ln(c*x))^(1/2),x)
```

```
[Out] int(1/csch(2*ln(c*x))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{\text{csch}(2\log(cx))}}\,dx = -\frac{\sqrt{2}cx\arctan\left(\frac{\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right) - \sqrt{2}\left(c^4x^4-1\right)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{4\,c^2x}$$

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) -
 sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^2*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(1/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(1/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}\left(2\ \log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \text{Timed out}$$

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\sinh(2\ \ln(c\,x))}}}\,dx$$

```
[In] int(1/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(1/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.138 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx$

Optimal result . . . 736
Rubi [A] (verified) . . . 736
Mathematica [A] (verified) . . . 737
Maple [A] (verified) . . . 737
Fricas [A] (verification not implemented) . . . 738
Sympy [F] . . . 738
Maxima [F] . . . 738
Giac [F(-1)] . . . 738
Mupad [F(-1)] . . . 739

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = i\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\frac{\pi}{4} - i\log(cx), 2\right)\sqrt{i\sinh(2\log(cx))}$$

```
[Out] I*(sin(1/4*Pi+I*ln(c*x))^2)^(1/2)/sin(1/4*Pi+I*ln(c*x))*EllipticF(cos(1/4*P
i+I*ln(c*x)),2^(1/2))*csch(2*ln(c*x))^(1/2)*(I*sinh(2*ln(c*x)))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3856, 2720}

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = i\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\frac{\pi}{4} - i\log(cx), 2\right)$$

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] I*Sqrt[Csch[2*Log[c*x]]]*EllipticF[Pi/4 - I*Log[c*x], 2]*Sqrt[I*Sinh[2*Log[
c*x]]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \sqrt{\text{csch}(2x)}\,dx, x, \log(cx)\right) \\ &= \left(\sqrt{\text{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(2x)}}\,dx, x, \log(cx)\right) \\ &= i\sqrt{\text{csch}(2\log(cx))}\,\text{EllipticF}\left(\frac{\pi}{4} - i\log(cx), 2\right)\sqrt{i\sinh(2\log(cx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\text{csch}(2\log(cx))}}{x}\,dx = \text{csch}^{\frac{3}{2}}(2\log(cx))\,\text{EllipticF}\left(\frac{\pi}{4} - i\log(cx), 2\right)(i\sinh(2\log(cx)))^{3/2}$$

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] Csch[2*Log[c*x]]^(3/2)*EllipticF[Pi/4 - I*Log[c*x], 2]*(I*Sinh[2*Log[c*x]])
^(3/2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

method	result	size
derivativedivides	$\frac{i\sqrt{-i(\sinh(2\ln(cx))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(2\ln(cx))+i)}\,\sqrt{i\sinh(2\ln(cx))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(2\ln(cx))+i)},\frac{\sqrt{2}}{2}\right)}{2\cosh(2\ln(cx))\sqrt{\sinh(2\ln(cx))}}$	90
default	$\frac{i\sqrt{-i(\sinh(2\ln(cx))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(2\ln(cx))+i)}\,\sqrt{i\sinh(2\ln(cx))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(2\ln(cx))+i)},\frac{\sqrt{2}}{2}\right)}{2\cosh(2\ln(cx))\sqrt{\sinh(2\ln(cx))}}$	90

```
[In] int(csch(2*ln(c*x))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(-I*(sinh(2*ln(c*x))+I))^(1/2)*2^(1/2)*(-I*(-sinh(2*ln(c*x))+I))^(1/2
)*(I*sinh(2*ln(c*x)))^(1/2)*EllipticF((-I*(sinh(2*ln(c*x))+I))^(1/2),1/2*2^
(1/2))/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = -i\sqrt{2}F(\arcsin(cx)\mid -1)$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
[Out] -I*sqrt(2)*elliptic_f(arcsin(c*x), -1)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x,x)
[Out] Integral(sqrt(csch(2*log(c*x)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(csch(2*log(c*x)))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x, x)
```

3.139 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$

Optimal result . . . 740
Rubi [A] (verified) . . . 740
Mathematica [A] (verified) . . . 742
Maple [F] . . . 742
Fricas [A] (verification not implemented) . . . 742
Sympy [F] . . . 742
Maxima [F] . . . 743
Giac [F(-1)] . . . 743
Mupad [F(-1)] . . . 743

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = -\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] -1/2*c^2*x*arccsc(c^2*x^2)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 281, 222}

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = -\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] -1/2*(c^2*Sqrt[1 - 1/(c^4*x^4)]*x*ArcCsc[c^2*x^2]*Sqrt[Csch[2*Log[c*x]]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^2}\,dx, x, cx\right) \\
&= \left(c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^3}\,dx, x, cx\right) \\
&= -\left(\left(c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right) \\
&= -\left(\frac{1}{2}\left(c^2\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right) \\
&= -\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \frac{\sqrt{-1+c^4x^4}\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}\arctan\left(\sqrt{-1+c^4x^4}\right)}{x}$$

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] (Sqrt[-1 + c^4*x^4]*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)]*ArcTan[Sqrt[-1 + c^4*x
^4]])/x
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{csch}\left(2\ln\left(cx\right)\right)}}{x^2}dx$$

```
[In] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

```
[Out] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \frac{1}{2}\sqrt{2}c\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*c*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^2}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(csch(2*log(c*x)))/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^2}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^2,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^2, x)
```

3.140 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$

Optimal result . . . 744
Rubi [A] (verified) . . . 744
Mathematica [C] (verified) . . . 746
Maple [A] (verified) . . . 747
Fricas [A] (verification not implemented) . . . 747
Sympy [F] . . . 747
Maxima [F] . . . 748
Giac [F(-1)] . . . 748
Mupad [F(-1)] . . . 748

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = -c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,E\left(\csc^{-1}(cx)\middle|-1\right) + c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

```
[Out] -c^3*x*EllipticE(1/c/x,I)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)+c^3*x*E
llipticF(1/c/x,I)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5671, 5669, 342, 313, 227, 1195, 435}

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = c^3x\sqrt{1-\frac{1}{c^4x^4}}\,\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)\sqrt{\operatorname{csch}(2\log(cx))} - c^3x\sqrt{1-\frac{1}{c^4x^4}}\,E\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticE[ArcCsc[c*x],
 -1]) + c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc
[c*x], -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = c^2 \text{Subst}\left(\int \frac{\sqrt{\text{csch}(2\log(x))}}{x^3}\, dx, x, cx \right)$$

$$= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^4}\,dx, x, cx\right)$$

$$= -\left(\left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)$$
$$- \left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)$$

$$= c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\csc^{-1}(cx), -1\right)$$
$$- \left(c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{1}{cx}\right)$$

$$= -c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}E\left(\csc^{-1}(cx)\middle| -1\right)$$
$$+ c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\csc^{-1}(cx), -1\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = -\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, c^4x^4\right)}{x^2}$$

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/
4, 1/2, 3/4, c^4*x^4])/x^2)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

method	result	size
risch	$\frac{(c^4x^4-1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2} - \frac{c^2\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{-c^2},i\right)-\text{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{\sqrt{-c^2}\,x}$	126

```
[In] int(csch(2*ln(c*x))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c^4*x^4-1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)-c^2/(-c^2)^(1/2)*(c^2*x
^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(EllipticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c
^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$$
$$= \frac{i\sqrt{2}c^4x^2E(\arcsin(cx)\mid -1) - i\sqrt{2}c^4x^2F(\arcsin(cx)\mid -1) + \sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*c^4*x^2*elliptic_e(arcsin(c*x), -1) - I*sqrt(2)*c^4*x^2*elliptic
_f(arcsin(c*x), -1) + sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/x^
2
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^3}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(csch(2*log(c*x)))/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^3}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^3,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^3, x)
```

3.141 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx$

Optimal result . . . 749
Rubi [A] (verified) . . . 749
Mathematica [A] (verified) . . . 750
Maple [A] (verified) . . . 750
Fricas [A] (verification not implemented) . . . 751
Sympy [F] . . . 751
Maxima [B] (verification not implemented) . . . 751
Giac [F(-1)] . . . 752
Mupad [B] (verification not implemented) . . . 752

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{1}{2}\left(c^4 - \frac{1}{x^4}\right) x\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] 1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5671, 5669, 267}

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{1}{2}x\left(c^4 - \frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] ((c^4 - x^(-4))*x*Sqrt[Csch[2*Log[c*x]]])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= c^3 \text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^4}\,dx, x, cx\right) \\ &= \left(c^4\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^5}\,dx, x, cx\right) \\ &= \frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\sqrt{\operatorname{csch}(2\log(cx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{c^2}{2x\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}$$

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
[Out] c^2/(2*x*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{(c^4x^4-1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2x^3}$	38

```
[In] int(csch(2*ln(c*x))^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/2*(c^4*x^4-1)/x^3*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{\sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2\,x^3}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/2*sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**4,x)
[Out] Integral(sqrt(csch(2*log(c*x)))/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx$$
$$= \frac{1}{2}c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{cx}+1}\sqrt{-\frac{1}{cx}+1}\sqrt{\frac{1}{c^2x^2}+1}} - \frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{cx}+1}\sqrt{-\frac{1}{cx}+1}\sqrt{\frac{1}{c^2x^2}+1}}\right)$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/2*c^3*(sqrt(2)/(sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2) + 1
)) - sqrt(2)/(c^4*x^4*sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2)
 + 1)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2} - \frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2\,x^3}$$

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^4,x)
```

```
[Out] (c^4*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/2 - ((2*c^2*x^2)/(c^4*x^4 - 1))^(
1/2)/(2*x^3)
```

3.142 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx$

Optimal result . . . 753
Rubi [A] (verified) . . . 753
Mathematica [C] (verified) . . . 755
Maple [A] (verified) . . . 755
Fricas [A] (verification not implemented) . . . 756
Sympy [F] . . . 756
Maxima [F] . . . 756
Giac [F(-1)] . . . 756
Mupad [F(-1)] . . . 757

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \frac{1}{3}\left(c^4 - \frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

```
[Out] 1/3*(c^4-1/x^4)*csch(2*ln(c*x))^(1/2)-1/3*c^5*x*EllipticF(1/c/x,I)*(1-1/c^4
/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 327, 227}

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \frac{1}{3}\left(c^4 - \frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}c^5x\sqrt{1-\frac{1}{c^4x^4}}\,\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] ((c^4 - x^(-4))*Sqrt[Csch[2*Log[c*x]]])/3 - (c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sq
rt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc[c*x], -1])/3
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = c^4 \text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^5}\,dx, x, cx\right)$$

$$= \left(c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}x^6}\,dx, x, cx\right)$$

$$= -\left(\left(c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= \frac{1}{3}\left(c^4 - \frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}\left(c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)$$

$$= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\operatorname{EllipticF}\left(\csc^{-1}(cx), -1\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = -\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\operatorname{Hypergeometric2F1}\left(-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, c^4x^4\right)}{3x^4}$$

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*(Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[
-3/4, 1/2, 1/4, c^4*x^4])/x^4
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

method	result	size
risch	$\frac{(c^4x^4-1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{3x^4} + \frac{c^4\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\operatorname{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{3\sqrt{-c^2}\,x}$	112

```
[In] int(csch(2*ln(c*x))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c^4*x^4-1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)+1/3*c^4/(-c^2)^(1/2
)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*
(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \frac{-i\sqrt{2}c^4x^4F(\arcsin(cx)\mid -1) + \sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{3\,x^4}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*c^4*x^4*elliptic_f(arcsin(c*x), -1) + sqrt(2)*(c^4*x^4 - 1)
*sqrt(c^2*x^2/(c^4*x^4 - 1)))/x^4
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^5}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^5}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^5}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^5,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^5, x)
```

3.143 $\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 758
Rubi [A] (verified) . . . 758
Mathematica [A] (verified) . . . 761
Maple [A] (verified) . . . 761
Fricas [A] (verification not implemented) . . . 761
Sympy [F] . . . 762
Maxima [F] . . . 762
Giac [F(-1)] . . . 762
Mupad [F(-1)] . . . 762

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/32*x/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-1/16*x^5/(c^4-1/x^4)/csch(2*ln
(c*x))^(3/2)+1/12*x^9/csch(2*ln(c*x))^(3/2)+1/32*arctanh((1-1/c^4/x^4)^(1/2
))/c^12/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5671, 5669, 272, 43, 44, 65, 212}

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - x^5/(16*(c^4 - x^(-4))*C
sch[2*Log[c*x]]^(3/2)) + x^9/(12*Csch[2*Log[c*x]]^(3/2)) + ArcTanh[Sqrt[1 -
 1/(c^4*x^4)]]/(32*c^12*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^8}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^9}{12\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{x^9}{12\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{c^4x^4}\right)}{64c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{x^9}{12\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{x^9}{12\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1-c^4x^4}(3-14c^4x^4+8c^8x^8)-3cx\arcsin\left(c^2x^2\right)}{192c^9\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(3 - 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSin[c^2*
x^2])/(192*c^9*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{x^3(8c^8x^8-14c^4x^4+3)\sqrt{2}}{384c^6\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{128c^6\sqrt{c^4}\,\sqrt{c^4x^4-1}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	121

```
[In] int(x^8/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*x^3*(8*c^8*x^8-14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{2\sqrt{2}(8\,c^{13}x^{13}-22\,c^9x^9+17\,c^5x^5-3\,cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+3\sqrt{2}\log\left(2\,c^4x^4+2\,(c^5x^5-cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{768\,c^9}$$

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 - 22*c^9*x^9 + 17*c^5*x^5 - 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 - 1)) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^
2/(c^4*x^4 - 1)) - 1))/c^9
```

Sympy [F]

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**8/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(x**8/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{csch}\left(2\ \log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\left(\frac{1}{\sinh(2\ \ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^8/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^8/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.144 $\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 763
Rubi [A] (verified) . . . 763
Mathematica [C] (verified) . . . 765
Maple [A] (verified) . . . 766
Fricas [A] (verification not implemented) . . . 766
Sympy [F] . . . 766
Maxima [F] . . . 767
Giac [F(-1)] . . . 767
Mupad [F(-1)] . . . 767

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 4/77/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-6/77*x^4/(c^4-1/x^4)/csch(2*ln(c
*x))^(3/2)+1/11*x^8/csch(2*ln(c*x))^(3/2)-4/77*EllipticF(1/c/x,I)/c^11/(1-1
/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 283, 331, 227}

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{77c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - (6*x^4)/(77*(c^4 - x^(-4
))*Csch[2*Log[c*x]]^(3/2)) + x^8/(11*Csch[2*Log[c*x]]^(3/2)) - (4*EllipticF
```

```
[ArcCsc[c*x], -1])/(77*c^11*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3
/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^7}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^8}{11\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{x^8}{11\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{x^8}{11\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x^2\left(\left(1-c^4x^4\right)^{5/2} - \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}, \frac{5}{4}, c^4x^4\right)\right)}{22c^6\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x^2*((1 - c^4*x^4)^(5/2) - Hypergeometric2F1[-3/2, 1/4, 5/4, c^4*x^4]))/(2
2*c^6*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{x^2(7c^8x^8-13c^4x^4+4)\sqrt{2}}{308c^6\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\,\text{EllipticF}\left(x\sqrt{-c^2},i\right)\sqrt{2}\,x}{77c^6\sqrt{-c^2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	133

```
[In] int(x^7/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/308*x^2*(7*c^8*x^8-13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/77/c^6/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*Elli
pticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{\sqrt{2}(7\,c^{14}x^{12}-20\,c^{10}x^8+17\,c^6x^4-4\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-4\,\sqrt{2}\sqrt{c^4}F(\arcsin\left(\frac{1}{cx}\right)\,|\,-1)}{308\,c^{10}}$$

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/308*(sqrt(2)*(7*c^14*x^12 - 20*c^10*x^8 + 17*c^6*x^4 - 4*c^2)*sqrt(c^2*x^
2/(c^4*x^4 - 1)) - 4*sqrt(2)*sqrt(c^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^1
0
```

Sympy [F]

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**7/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**7/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^7/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^7/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.145 $\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 768
Rubi [A] (verified) . . . 768
Mathematica [A] (verified) . . . 769
Maple [A] (verified) . . . 769
Fricas [B] (verification not implemented) . . . 770
Sympy [F] . . . 770
Maxima [A] (verification not implemented) . . . 770
Giac [F(-1)] . . . 771
Mupad [B] (verification not implemented) . . . 771

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(c^4-\frac{1}{x^4}\right)x^7}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/10*(c^4-1/x^4)*x^7/c^4/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5671, 5669, 270}

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x^7\left(c^4-\frac{1}{x^4}\right)}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 - x^(-4))*x^7)/(10*c^4*Csch[2*Log[c*x]]^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7} \\ &= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\ &= \frac{\left(c^4-\frac{1}{x^4}\right)x^7}{10c^4\text{csch}^{\frac{3}{2}}(2\log(cx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(-1+c^4x^4\right)^3\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}{20c^8x}$$

```
[In] Integrate[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((-1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(20*c^8*x)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

method	result	size
risch	$\frac{\sqrt{2}x\left(c^8x^8-2c^4x^4+1\right)}{40c^6\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	47

```
[In] int(x^6/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^8*x^8-2*c^4*x^4+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{2}(c^{12}x^{12} - 3\,c^8x^8 + 3\,c^4x^4 - 1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{40\,c^8x}$$

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(2)*(c^12*x^12 - 3*c^8*x^8 + 3*c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4
- 1))/(c^8*x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**6/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**6/csch(2*log(c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(\sqrt{2}c^4x^4 - \sqrt{2}\right)(c^2x^2+1)^{\frac{3}{2}}(cx+1)^{\frac{3}{2}}(cx-1)^{\frac{3}{2}}}{40\,c^7}$$

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/40*(sqrt(2)*c^4*x^4 - sqrt(2))*(c^2*x^2 + 1)^(3/2)*(c*x + 1)^(3/2)*(c*x -
 1)^(3/2)/c^7
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(c^4x^4-1\right)^3\sqrt{\frac{2c^2x^2}{c^4x^4-1}}}{40c^8x}$$

```
[In] int(x^6/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] ((c^4*x^4 - 1)^3*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(40*c^8*x)
```

3.146 $\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 772
Rubi [A] (verified) . . . 772
Mathematica [C] (verified) . . . 775
Maple [A] (verified) . . . 775
Fricas [A] (verification not implemented) . . . 776
Sympy [F] . . . 776
Maxima [F] . . . 776
Giac [F(-1)] . . . 777
Mupad [F(-1)] . . . 777

Optimal result

Integrand size = 15, antiderivative size = 162

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4E(\csc^{-1}(cx)|-1)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 4/15/c^4/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)-2/15*x^2/(c^4-1/x^4)/csch(2*
ln(c*x))^(3/2)+1/9*x^6/csch(2*ln(c*x))^(3/2)+4/15*EllipticE(1/c/x,I)/c^9/(1
-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)-4/15*EllipticF(1/c/x,I)/c^9/(1-
1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5671, 5669, 342, 283, 331, 313, 227, 1195, 435}

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{15c^4x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4E(\csc^{-1}(cx)|-1)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(15*c^4*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) - (2*x^2)/(15*(c^4 - x
^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^6/(9*Csch[2*Log[c*x]]^(3/2)) + (4*Ellipt
icE[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2)) - (4*EllipticF[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^6}{9\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{2\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{x^6}{9\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$- \frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4\operatorname{Subst}\left(\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$- \frac{4\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$+ \frac{4E(\csc^{-1}(cx)|-1)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.39

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{x^4\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)}{6c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/6*(x^4*Hypergeometric2F1[-3/2, 3/4, 7/4, c^4*x^4])/(c^2*Sqrt[2 - 2*c^4*x
^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{x^4\left(5c^4x^4-11\right)\sqrt{2}}{180c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}} + \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}\,x}{15\sqrt{-c^2}\left(c^4x^4-1\right)c^4\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	140

```
[In] int(x^5/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*x^4*(5*c^4*x^4-11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/15/(-c^2
)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^4*(EllipticF(x*(
```

```
-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{\sqrt{2}(5\,c^{14}x^{12} - 16\,c^{10}x^8 + 23\,c^6x^4 - 12\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 12\sqrt{c^4}(\sqrt{2}x^2E(\arcsin\left(\frac{1}{cx}\right)\,|-1) - \sqrt{2}x^2F(\arcsin\left(\frac{1}{cx}\right)$$
$$180\,c^{10}x^2$$

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/180*(sqrt(2)*(5*c^14*x^12 - 16*c^10*x^8 + 23*c^6*x^4 - 12*c^2)*sqrt(c^2*x
^2/(c^4*x^4 - 1)) + 12*sqrt(c^4)*(sqrt(2)*x^2*elliptic_e(arcsin(1/(c*x)), -
1) - sqrt(2)*x^2*elliptic_f(arcsin(1/(c*x)), -1)))/(c^10*x^2)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**5/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**5/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^5/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^5/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.147 $\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 778
Rubi [A] (verified) . . . 778
Mathematica [A] (verified) . . . 780
Maple [A] (verified) . . . 780
Fricas [A] (verification not implemented) . . . 781
Sympy [F] . . . 781
Maxima [F] . . . 781
Giac [F(-1)] . . . 782
Mupad [F(-1)] . . . 782

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/16*x/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/8*x^5/csch(2*ln(c*x))^(3/2)+3/1
6*arctanh((1-1/c^4/x^4)^(1/2))/c^8/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^
(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 272, 43, 65, 212}

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (-3*x)/(16*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^5/(8*Csch[2*Log[c*x]]
^(3/2)) + (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(16*c^8*(1 - 1/(c^4*x^4))^(3/2
)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^7\, dx, x, cx\right)}{c^8 \left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^3}\, dx, x, \frac{1}{c^4x^4}\right)}{4c^8 \left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{8\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\, dx, x, \frac{1}{c^4x^4}\right)}{16c^8 \left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\, dx, x, \frac{1}{c^4x^4}\right)}{32c^8 \left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8 \left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8 \left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\, dx = \frac{c^3x^3\sqrt{1-c^4x^4}(-5+2c^4x^4) - 3cx\arcsin\left(c^2x^2\right)}{32c^5\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(-5 + 2*c^4*x^4) - 3*c*x*ArcSin[c^2*x^2])/(32*c^
5*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{x^3\left(2c^4x^4-5\right)\sqrt{2}}{64c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}} + \frac{3\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{64\sqrt{c^4}\,c^2\sqrt{c^4x^4-1}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	113

```
[In] int(x^4/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*x^3*(2*c^4*x^4-5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4-1)^(1
/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{2\sqrt{2}(2\,c^9x^9 - 7\,c^5x^5 + 5\,cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 3\sqrt{2}\log\left(2\,c^4x^4 + 2\,(c^5x^5 - cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 1\right)}{128\,c^5}$$

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 - 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)
) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1))
 - 1))/c^5
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**4/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**4/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^4/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^4/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.148 $\int \frac{x^3}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 783
Rubi [A] (verified) . . . 783
Mathematica [C] (verified) . . . 785
Maple [A] (verified) . . . 785
Fricas [A] (verification not implemented) . . . 786
Sympy [F] . . . 786
Maxima [F] . . . 786
Giac [F(-2)] . . . 786
Mupad [F(-1)] . . . 787

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{x^3}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -2/7/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/7*x^4/csch(2*ln(c*x))^(3/2)-4/7*El
lipticF(1/c/x,I)/c^7/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 283, 227}

$$\int \frac{x^3}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{7c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -2/(7*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^4/(7*Csch[2*Log[c*x]]^(3/2
)) - (4*EllipticF[ArcCsc[c*x], -1])/(7*c^7*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch
[2*Log[c*x]]^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4} \\ &= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))} \\ &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}\end{aligned}$$

$$= \frac{x^4}{7\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{1-c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}, \frac{5}{4}, c^4x^4\right)}{2\sqrt{2}c^4}$$

```
[In] Integrate[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 - c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-3/2, 1
/4, 5/4, c^4*x^4])/(2*Sqrt[2]*c^4)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{x^2(c^4x^4-3)\sqrt{2}}{28c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}} + \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\,\text{EllipticF}\left(x\sqrt{-c^2}, i\right)\sqrt{2}\,x}{7\sqrt{-c^2}\,(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	124

```
[In] int(x^3/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^2*(c^4*x^4-3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/7/(-c^2)^(1/
2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2
),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{2}(c^{10}x^8 - 4\,c^6x^4 + 3\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 4\,\sqrt{2}\sqrt{c^4}F(\arcsin\left(\frac{1}{cx}\right)\,|\,-1)}{28\,c^6}$$

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
[Out] 1/28*(sqrt(2)*(c^10*x^8 - 4*c^6*x^4 + 3*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) -
4*sqrt(2)*sqrt(c^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^6
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**3/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(x**3/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to c
onvert to real %%{poly1[1.0000000000000000000000000000000,0.000000000000000
000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\left(\frac{1}{\sinh(2\,\ln(cx))}\right)^{3/2}}\,dx$$

```
[In] int(x^3/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^3/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.149 $\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 788
Rubi [A] (verified) . . . 788
Mathematica [A] (verified) . . . 790
Maple [F] . . . 790
Fricas [A] (verification not implemented) . . . 791
Sympy [F] . . . 791
Maxima [F] . . . 791
Giac [F(-1)] . . . 791
Mupad [F(-1)] . . . 792

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -1/2/(c^4-1/x^4)/x/csch(2*ln(c*x))^(3/2)+1/6*x^3/csch(2*ln(c*x))^(3/2)-1/2*
arccsc(c^2*x^2)/c^6/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 281, 283, 222}

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{1}{2x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/2*1/((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + x^3/(6*Csch[2*Log[c*x]]^
(3/2)) - ArcCsc[c^2*x^2]/(2*c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]
]^(3/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^3}{6\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x\left(\left(-4+c^4x^4\right)\sqrt{-1+c^4x^4} + 3\arctan\left(\sqrt{-1+c^4x^4}\right)\right)}{12\sqrt{2}c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\sqrt{-1+c^4x^4}}$$

```
[In] Integrate[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x*((-4 + c^4*x^4)*Sqrt[-1 + c^4*x^4] + 3*ArcTan[Sqrt[-1 + c^4*x^4]]))/(12*
Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Maple [F]

$$\int \frac{x^2}{\text{csch}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\sqrt{2}cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right) + \sqrt{2}(c^8x^8 - 5\,c^4x^4 + 4)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{24\,c^4x}$$

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
 + sqrt(2)*(c^8*x^8 - 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**2/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^2/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^2/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.150 $\int \frac{x}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 793
Rubi [A] (verified) . . . 793
Mathematica [C] (verified) . . . 796
Maple [A] (verified) . . . 796
Fricas [F] . . . 797
Sympy [F] . . . 797
Maxima [F] . . . 797
Giac [F(-1)] . . . 797
Mupad [F(-1)] . . . 798

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{x}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E(\csc^{-1}(cx)|-1)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -6/5/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)+1/5*x^2/csch(2*ln(c*x))^(3/2)-12
/5*EllipticE(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)+12/
5*EllipticF(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5671, 5669, 342, 283, 313, 227, 1195, 435}

$$\int \frac{x}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{6}{5x^2\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E(\csc^{-1}(cx)|-1)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -6/(5*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + x^2/(5*Csch[2*Log[c*x]]^
(3/2)) - (12*EllipticE[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2)) + (12*EllipticF[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c
^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^2}{5\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{12\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\text{Subst}\left(\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&\quad + \frac{12\,\text{EllipticF}\left(\csc^{-1}(cx), -1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E(\csc^{-1}(cx)|-1)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.46

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},c^4x^4\right)}{2c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/4, 3/4, c^4*x^4]/(2*c^2*Sqrt[2 - 2*c^4*x^4]*Sqrt
[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17

method	result	size
risch	$\frac{(c^8x^8+4c^4x^4-5)\sqrt{2}}{20(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{3\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{-c^2},i\right)-\text{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}x}{5\sqrt{-c^2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	152

```
[In] int(x/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(c^8*x^8+4*c^4*x^4-5)/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/5/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*(Elli
pticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^
4*x^4-1))^(1/2)
```

Fricas [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
[Out] integral(x/csch(2*log(c*x))^(3/2), x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(x/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.151 $\int \frac{1}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result . . . 799
Rubi [A] (verified) . . . 799
Mathematica [C] (verified) . . . 801
Maple [A] (verified) . . . 802
Fricas [A] (verification not implemented) . . . 802
Sympy [F] . . . 802
Maxima [F] . . . 803
Giac [F(-1)] . . . 803
Mupad [F(-1)] . . . 803

Optimal result

Integrand size = 11, antiderivative size = 96

$$\int \frac{1}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/4/(c^4-1/x^4)/x^3/csch(2*ln(c*x))^(3/2)+1/4*x/csch(2*ln(c*x))^(3/2)-3/4*a
rctanh((1-1/c^4/x^4)^(1/2))/c^4/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/
2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5665, 5663, 272, 43, 52, 65, 212}

$$\int \frac{1}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3}{4x^3\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] 3/(4*(c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2)) + x/(4*Csch[2*Log[c*x]]^(3/
2)) - (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(4*c^4*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5663

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Csch[d*(a
+ b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x}{4\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\text{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{1}{\text{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, c^4x^4\right)}{4c^2x\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

```
[In] Integrate[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/2, 1/2, c^4*x^4]/(4*c^2*x*Sqrt[2 - 2*c^4*x^4]*Sq
rt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.35

method	result	size
risch	$\frac{(c^8x^8+c^4x^4-2)\sqrt{2}}{16x(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}} - \frac{3c^2\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{16\sqrt{c^4}\sqrt{c^4x^4-1}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	130

```
[In] int(1/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(c^8*x^8+c^4*x^4-2)/x/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{3\sqrt{2}c^3x^3\log\left(2\,c^4x^4 - 2\,(c^5x^5 - cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 1\right) + 2\sqrt{2}(c^8x^8 + c^4x^4 - 2)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{32\,c^4x^3}$$

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(2*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4
*x^4 - 1)) - 1) + 2*sqrt(2)*(c^8*x^8 + c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4 -
 1)))/(c^4*x^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(csch(2*log(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(csch(2*log(c*x))^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(1/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(1/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.152 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal result . . . 804
Rubi [A] (verified) . . . 804
Mathematica [A] (verified) . . . 805
Maple [A] (verified) . . . 806
Fricas [A] (verification not implemented) . . . 806
Sympy [F] . . . 806
Maxima [F] . . . 807
Giac [F(-1)] . . . 807
Mupad [F(-1)] . . . 807

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))} + \frac{iE\left(\frac{\pi}{4} - i\log(cx)\middle|2\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}$$

```
[Out] -cosh(2*ln(c*x))*csch(2*ln(c*x))^(1/2)+I*(sin(1/4*Pi+I*ln(c*x))^2)^(1/2)/si
n(1/4*Pi+I*ln(c*x))*EllipticE(cos(1/4*Pi+I*ln(c*x)),2^(1/2))/csch(2*ln(c*x)
)^(1/2)/(I*sinh(2*ln(c*x)))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3853, 3856, 2719}

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))} + \frac{iE\left(\frac{\pi}{4} - i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] -(Cosh[2*Log[c*x]]*Sqrt[Csch[2*Log[c*x]]]) + (I*EllipticE[Pi/4 - I*Log[c*x]
, 2])/(Sqrt[Csch[2*Log[c*x]]]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \text{csch}^{\frac{3}{2}}(2x)\,dx, x, \log(cx)\right) \\
&= -\cosh(2\log(cx))\sqrt{\text{csch}(2\log(cx))} + \text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(2x)}}\,dx, x, \log(cx)\right) \\
&= -\cosh(2\log(cx))\sqrt{\text{csch}(2\log(cx))} + \frac{\text{Subst}\left(\int \sqrt{i\sinh(2x)}\,dx, x, \log(cx)\right)}{\sqrt{\text{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}} \\
&= -\cosh(2\log(cx))\sqrt{\text{csch}(2\log(cx))} + \frac{iE\left(\frac{\pi}{4} - i\log(cx)\middle|\, 2\right)}{\sqrt{\text{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{\text{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \sqrt{\text{csch}(2\log(cx))}\Big(-\cosh(2\log(cx)) + E\Big(\frac{\pi}{4} - i\log(cx)\Big|\, 2\Big)\sqrt{i\sinh(2\log(cx))}\Big)$$

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] Sqrt[Csch[2*Log[c*x]]]*(-Cosh[2*Log[c*x]] + EllipticE[Pi/4 - I*Log[c*x], 2]
*Sqrt[I*Sinh[2*Log[c*x]]])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(2\ln(cx))}\,\sqrt{2}\,\sqrt{i\sinh(2\ln(cx))+1}\,\sqrt{i\sinh(2\ln(cx))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(2\ln(cx))},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(2\ln(cx)}$ $2\cosh(2\ln(cx))\sqrt{\sinh(2\ln}$
default	$\frac{2\sqrt{1-i\sinh(2\ln(cx))}\,\sqrt{2}\,\sqrt{i\sinh(2\ln(cx))+1}\,\sqrt{i\sinh(2\ln(cx))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(2\ln(cx))},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(2\ln(cx)}$ $2\cosh(2\ln(cx))\sqrt{\sinh(2\ln}$

```
[In] int(csch(2*ln(c*x))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*(1-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(I*sinh(2*ln(c*x))+1)^(1/2)*(I*s
inh(2*ln(c*x)))^(1/2)*EllipticE((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-(1
-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(I*sinh(2*ln(c*x))+1)^(1/2)*(I*sinh(2*ln(
c*x)))^(1/2)*EllipticF((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-2*cosh(2*ln
(c*x))^2)/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = -\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2x^2 - i\sqrt{2}c^2E(\arcsin(cx)\,|\,-1) + i\sqrt{2}c^2F(\arcsin(cx)\,|\,-1)$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x^2 - I*sqrt(2)*c^2*elliptic_e(arc
sin(c*x), -1) + I*sqrt(2)*c^2*elliptic_f(arcsin(c*x), -1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(csch(2*log(c*x))^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x,x)
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x, x)
```

3.153 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal result . . . 808
Rubi [A] (verified) . . . 808
Mathematica [A] (verified) . . . 809
Maple [F] . . . 809
Fricas [A] (verification not implemented) . . . 810
Sympy [F] . . . 810
Maxima [B] (verification not implemented) . . . 810
Giac [F(-1)] . . . 811
Mupad [B] (verification not implemented) . . . 811

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x^3*csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5671, 5669, 267}

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\frac{1}{2}x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] -1/2*((c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right) \\ &= \left(c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right) \\ &= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\sqrt{2}c^2x\sqrt{\frac{c^2x^2}{-1+c^4x^4}}$$

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^2,x]
[Out] -(Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [F]

$$\int \frac{\operatorname{csch}\left(2\ln(cx)\right)^{\frac{3}{2}}}{x^2}dx$$

```
[In] int(csch(2*ln(c*x))^(3/2)/x^2,x)
[Out] int(csch(2*ln(c*x))^(3/2)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2x$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^2}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**2,x)
[Out] Integral(csch(2*log(c*x))**(3/2)/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(23) = 46.

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$
$$= -c\left(\frac{\sqrt{2}}{\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}}{c^4x^4\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
[Out] -c*(sqrt(2)/((1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2) + 1)^(3/
2)) - sqrt(2)/(c^4*x^4*(1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2
) + 1)^(3/2)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}$$

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^2,x)
[Out] -c^2*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2)
```

3.154 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal result . . . 812
Rubi [A] (verified) . . . 812
Mathematica [C] (verified) . . . 814
Maple [F] . . . 814
Fricas [A] (verification not implemented) . . . 814
Sympy [F] . . . 815
Maxima [F] . . . 815
Giac [F(-1)] . . . 815
Mupad [F(-1)] . . . 815

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

```
[Out] -1/2*(c^4-1/x^4)*x^2*csch(2*ln(c*x))^(3/2)+1/2*c^5*(1-1/c^4/x^4)^(3/2)*x^3*
csch(2*ln(c*x))^(3/2)*EllipticF(1/c/x,I)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 294, 227}

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \frac{1}{2}c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{EllipticF}\left(\csc^{-1}(cx),-1\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] -1/2*((c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + (c^5*(1 - 1/(c^4*x^4))^(
3/2)*x^3*Csch[2*Log[c*x]]^(3/2)*EllipticF[ArcCsc[c*x], -1])/2
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^2 \text{Subst}\left(\int \frac{\text{csch}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right) \\
&= \left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^6}\,dx, x, cx\right) \\
&= -\left(\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{(1-x^4)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)
\end{aligned}$$

$$= -\frac{1}{2}\left(c^4 - \frac{1}{x^4}\right) x^2 \operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}\left(c^5\left(1 - \frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)$$

$$= -\frac{1}{2}\left(c^4 - \frac{1}{x^4}\right) x^2 \operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^5\left(1 - \frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))\,\operatorname{EllipticF}\left(\csc^{-1}(cx), -1\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = -\sqrt{2}c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\left(1 + \sqrt{1-c^4x^4}\,\operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, c^4x^4\right)\right)$$

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^3,x]
[Out] -(Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*(1 + Sqrt[1 - c^4*x^4]*Hyperge
ometric2F1[1/4, 1/2, 5/4, c^4*x^4]))
```

Maple [F]

$$\int \frac{\operatorname{csch}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^3}dx$$

```
[In] int(csch(2*ln(c*x))^(3/2)/x^3,x)
[Out] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = i\sqrt{2}c^2F(\arcsin\left(cx\right)\,|\,-1) - \sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
[Out] I*sqrt(2)*c^2*elliptic_f(arcsin(c*x), -1) - sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 -
 1))*c^2
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^3}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**3,x)
[Out] Integral(csch(2*log(c*x))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(csch(2*log(c*x))^(3/2)/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\left(\frac{1}{\sinh(2\,\ln(cx))}\right)^{3/2}}{x^3}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^3,x)
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x^3, x)
```

3.155 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal result . . . 816
Rubi [A] (verified) . . . 816
Mathematica [C] (verified) . . . 818
Maple [F] . . . 818
Fricas [A] (verification not implemented) . . . 819
Sympy [F] . . . 819
Maxima [F] . . . 819
Giac [F(-1)] . . . 819
Mupad [F(-1)] . . . 820

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(3/2)+1/2*c^6*(1-1/c^4/x^4)^(3/2)*x^3*ar
ccsc(c^2*x^2)*csch(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 281, 294, 222}

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -1/2*((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + (c^6*(1 - 1/(c^4*x^4))^(3/
2)*x^3*ArcCsc[c^2*x^2]*Csch[2*Log[c*x]]^(3/2))/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = c^3 \text{Subst}\left(\int \frac{\text{csch}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)$$

$$= \left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{csch}^{\frac{3}{2}}(2\log(cx))\right) \text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2} x^7}\,dx, x, cx\right)$$

$$= -\left(\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\operatorname{Subst}\left(\int\frac{x^5}{(1-x^4)^{3/2}}\,dx,x,\frac{1}{cx}\right)\right)$$

$$= -\left(\frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\operatorname{Subst}\left(\int\frac{x^2}{(1-x^2)^{3/2}}\,dx,x,\frac{1}{c^2x^2}\right)\right)$$

$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$
$$+\frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,\frac{1}{c^2x^2}\right)$$

$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))+\frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int\frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = -\frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-c^4x^4\right)}{x}$$

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -((Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/
2, 1 - c^4*x^4])/x)
```

Maple [F]

$$\int\frac{\operatorname{csch}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^4}dx$$

```
[In] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = -\frac{\sqrt{2}c^3 x \arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right) + \sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2}{x}$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*c^3*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) + s
qrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2)/x
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^4}\,dx$$

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**4,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^4,x)
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x^4, x)
```

3.156 $\int \operatorname{csch}(a + b \log(cx^n))\, dx$

Optimal result . . . 821
Rubi [A] (verified) . . . 821
Mathematica [A] (verified) . . . 822
Maple [F] . . . 823
Fricas [F] . . . 823
Sympy [F] . . . 823
Maxima [F] . . . 823
Giac [F] . . . 824
Mupad [F(-1)] . . . 824

Optimal result

Integrand size = 11, antiderivative size = 62

$$\int \operatorname{csch}(a + b \log(cx^n))\, dx = -\frac{2e^a x (cx^n)^b \operatorname{Hypergeometric2F1}\left(1, \frac{b+\frac{1}{n}}{2b}, \frac{1}{2}\left(3 + \frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1 + bn}$$

```
[Out] -2*exp(a)*x*(c*x^n)^b*hypergeom([1, 1/2*(b+1/n)/b],[3/2+1/2/b/n],exp(2*a)*(
c*x^n)^(2*b))/(b*n+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5665, 5667, 269, 371}

$$\int \operatorname{csch}(a + b \log(cx^n))\, dx = -\frac{2e^a x (cx^n)^b \operatorname{Hypergeometric2F1}\left(1, \frac{b+\frac{1}{n}}{2b}, \frac{1}{2}\left(3 + \frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{bn + 1}$$

```
[In] Int[Csch[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/
2, E^(2*a)*(c*x^n)^(2*b)])/(1 + b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}(a+b\log(x))\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1-e^{-2a}x^{-2b}}\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{-e^{-2a}+x^{2b}}\,dx, x, cx^n\right)}{n} \\ &= -\frac{2e^a x(cx^n)^b \operatorname{Hypergeometric2F1}\left(1, \frac{b+\frac{1}{n}}{2b}, \frac{1}{2}\left(3+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \operatorname{csch}(a+b\log(cx^n))\,dx \\ &= -\frac{2e^a x(cx^n)^b \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1+\frac{1}{bn}\right), \frac{1}{2}\left(3+\frac{1}{bn}\right), e^{2(a+b\log(cx^n))}\right)}{1+bn}\end{aligned}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
E^(2*(a + b*Log[c*x^n]))])/(1 + b*n)
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

```
[In] int(csch(a+b*ln(c*x^n)),x)
[Out] int(csch(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \operatorname{csch}(a+b\log\left(cx^{n}\right))\;dx=\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)\;dx$$

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(csch(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \operatorname{csch}(a+b\log\left(cx^{n}\right))\;dx=\int \operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)dx$$

```
[In] integrate(csch(a+b*ln(c*x**n)),x)
[Out] Integral(csch(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \operatorname{csch}(a+b\log\left(cx^{n}\right))\;dx=\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)\;dx$$

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \operatorname{csch}(a + b \log(cx^n))\ dx = \int \operatorname{csch}(b \log(cx^n) + a)\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}(a + b \log(cx^n))\ dx = \int \frac{1}{\sinh(a + b \ln(c\, x^n))}\, dx$$

```
[In] int(1/sinh(a + b*log(c*x^n)),x)
[Out] int(1/sinh(a + b*log(c*x^n)), x)
```

3.157 $\int \operatorname{csch}^2(a + b\log(cx^n))\,dx$

Optimal result . . . 825
Rubi [A] (verified) . . . 825
Mathematica [A] (verified) . . . 826
Maple [F] . . . 827
Fricas [F] . . . 827
Sympy [F] . . . 827
Maxima [F] . . . 827
Giac [F] . . . 828
Mupad [F(-1)] . . . 828

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \operatorname{csch}^2(a + b\log(cx^n))\,dx = \frac{4e^{2a}x(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2+\frac{1}{bn}\right), \frac{1}{2}\left(4+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

```
[Out] 4*exp(2*a)*x*(c*x^n)^(2*b)*hypergeom([2, 1+1/2/b/n],[2+1/2/b/n],exp(2*a)*(c
*x^n)^(2*b))/(2*b*n+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5665, 5667, 269, 371}

$$\int \operatorname{csch}^2(a + b\log(cx^n))\,dx = \frac{4e^{2a}x(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2+\frac{1}{bn}\right), \frac{1}{2}\left(4+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 2*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^2(a+b\log(x))\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^2}\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^2}\,dx, x, cx^n\right)}{n} \\ &= \frac{4e^{2a}x(cx^n)^{2b}\operatorname{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2+\frac{1}{bn}\right), \frac{1}{2}\left(4+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$\begin{aligned}&\int \operatorname{csch}^2(a+b\log(cx^n))\,dx \\ &= \frac{x\left(-\coth(a+b\log(cx^n)) - \frac{e^{2a}(cx^n)^{2b}\operatorname{Hypergeometric2F1}\left(1,1+\frac{1}{2bn},2+\frac{1}{2bn},e^{2(a+b\log(cx^n))}\right)}{1+2bn} - \operatorname{Hypergeometric2F1}\left(1,\right.\right.}{bn}\end{aligned}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-Coth[a + b*Log[c*x^n]] - (E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1
 + 1/(2*b*n), 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))])/(1 + 2*b*n) - Hyper
geometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))]))/(b*n)
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

```
[In] int(csch(a+b*ln(c*x^n))^2,x)
[Out] int(csch(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \operatorname{csch}^{2}(a+b\log\left(cx^{n}\right))\ dx=\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{2}\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(csch(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int \operatorname{csch}^{2}(a+b\log\left(cx^{n}\right))\ dx=\int \operatorname{csch}^{2}\left(a+b\log\left(cx^{n}\right)\right)dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**2,x)
[Out] Integral(csch(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \operatorname{csch}^{2}(a+b\log\left(cx^{n}\right))\ dx=\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{2}\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 4*integrate(1/4/(b*c^b*n*
e^(b*log(x^n) + a) + b*n), x) + 4*integrate(1/4/(b*c^b*n*e^(b*log(x^n) + a)
 - b*n), x)
```

Giac [F]

$$\int \operatorname{csch}^2(a + b\log(cx^n))\, dx = \int \operatorname{csch}(b\log(cx^n) + a)^2\, dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(csch(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(a + b\log(cx^n))\, dx = \int \frac{1}{\sinh(a + b\,\ln(c\,x^n))^2}\, dx$$

```
[In] int(1/sinh(a + b*log(c*x^n))^2,x)
[Out] int(1/sinh(a + b*log(c*x^n))^2, x)
```

3.158 $\int \operatorname{csch}^3(a + b\log(cx^n))\,dx$

Optimal result . . . 829
Rubi [A] (verified) . . . 829
Mathematica [A] (verified) . . . 830
Maple [F] . . . 831
Fricas [F] . . . 831
Sympy [F] . . . 831
Maxima [F] . . . 831
Giac [F] . . . 832
Mupad [F(-1)] . . . 832

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \operatorname{csch}^3(a + b\log(cx^n))\,dx = -\frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3, \frac{3b+\frac{1}{n}}{2b}, \frac{1}{2}\left(5+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

```
[Out] -8*exp(3*a)*x*(c*x^n)^(3*b)*hypergeom([3, 1/2*(3*b+1/n)/b],[5/2+1/2/b/n],ex
p(2*a)*(c*x^n)^(2*b))/(3*b*n+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5665, 5667, 269, 371}

$$\int \operatorname{csch}^3(a + b\log(cx^n))\,dx = -\frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3, \frac{3b+\frac{1}{n}}{2b}, \frac{1}{2}\left(5+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
 1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^3(a+b\log(x))\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^3}\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^3}\,dx, x, cx^n\right)}{n} \\ &= -\frac{8e^{3a}x(cx^n)^{3b}\operatorname{Hypergeometric2F1}\left(3, \frac{3b+\frac{1}{n}}{2b}, \frac{1}{2}\left(5+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+3bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 4.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\begin{aligned}&\int \operatorname{csch}^3(a+b\log(cx^n))\,dx \\ &= \frac{-4x(1+bn\coth(a+b\log(cx^n)))\operatorname{csch}(a+b\log(cx^n)) + 8e^a(-1+bn)x(cx^n)^b\operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}(1\right.}{8b^2n^2}\end{aligned}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-4*x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]] + 8*E^a*(-1 +
 b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, E^
(2*(a + b*Log[c*x^n]))])/(8*b^2*n^2)
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

```
[In] int(csch(a+b*ln(c*x^n))^3,x)
[Out] int(csch(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \operatorname{csch}^{3}(a+b\log\left(cx^{n}\right))\ dx=\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{3}\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="fricas")
[Out] integral(csch(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int \operatorname{csch}^{3}(a+b\log\left(cx^{n}\right))\ dx=\int \operatorname{csch}^{3}\left(a+b\log\left(cx^{n}\right)\right)dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**3,x)
[Out] Integral(csch(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \operatorname{csch}^{3}(a+b\log\left(cx^{n}\right))\ dx=\int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{3}\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] -8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) + b^2*n^2),
 x) - 8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) - b^2*
n^2), x) - ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c
^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b*log(x^n) + 4*a) - 2*b^2*c
^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Giac [F]

$$\int \operatorname{csch}^3(a + b\log(cx^n))\, dx = \int \operatorname{csch}(b\log(cx^n) + a)^3\, dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] integrate(csch(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(a + b\log(cx^n))\, dx = \int \frac{1}{\sinh(a + b\,\ln(c\,x^n))^3}\, dx$$

```
[In] int(1/sinh(a + b*log(c*x^n))^3,x)
[Out] int(1/sinh(a + b*log(c*x^n))^3, x)
```

3.159 $\int \operatorname{csch}^4(a + b\log(cx^n))\,dx$

Optimal result . . . 833
Rubi [A] (verified) . . . 833
Mathematica [B] (verified) . . . 834
Maple [F] . . . 835
Fricas [F] . . . 835
Sympy [F] . . . 835
Maxima [F] . . . 835
Giac [F] . . . 836
Mupad [F(-1)] . . . 836

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \operatorname{csch}^4(a + b\log(cx^n))\,dx = \frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4+\frac{1}{bn}\right), \frac{1}{2}\left(6+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+4bn}$$

```
[Out] 16*exp(4*a)*x*(c*x^n)^(4*b)*hypergeom([4, 2+1/2/b/n],[3+1/2/b/n],exp(2*a)*(
c*x^n)^(2*b))/(4*b*n+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5665, 5667, 269, 371}

$$\int \operatorname{csch}^4(a + b\log(cx^n))\,dx = \frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4+\frac{1}{bn}\right), \frac{1}{2}\left(6+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 4*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^4(a+b\log(x))\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^4}\,dx, x, cx^n\right)}{n} \\ &= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^4}\,dx, x, cx^n\right)}{n} \\ &= \frac{16e^{4a}x(cx^n)^{4b}\operatorname{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4+\frac{1}{bn}\right), \frac{1}{2}\left(6+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+4bn}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 200 vs. $2(68) = 136$.

Time $= 5.65$ (sec) , antiderivative size $= 200$, normalized size of antiderivative $= 2.94$

$$\begin{aligned}&\int \operatorname{csch}^4(a+b\log(cx^n))\,dx \\ &= \frac{x\left(4e^{2a}(-1+2bn)(cx^n)^{2b}\operatorname{Hypergeometric2F1}\left(1, 1+\frac{1}{2bn}, 2+\frac{1}{2bn}, e^{2(a+b\log(cx^n))}\right)+4(-1+4b^2n^2)\operatorname{Hyperge}\right.}\end{aligned}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(4*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n)
, 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + 4*(-1 + 4*b^2*n^2)*Hypergeomet
ric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + Csch[a + b*
Log[c*x^n]]^3*((1 - 12*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (-1 + 4*b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 4*b*n*Sinh[a + b*Log[c*x^n]])))/(24*b^3*n^3)
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}dx$$

```
[In] int(csch(a+b*ln(c*x^n))^4,x)
[Out] int(csch(a+b*ln(c*x^n))^4,x)
```

Fricas [F]

$$\int \operatorname{csch}^{4}(a+b\log\left(cx^{n}\right))\ dx = \int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{4}\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="fricas")
[Out] integral(csch(b*log(c*x^n) + a)^4, x)
```

Sympy [F]

$$\int \operatorname{csch}^{4}(a+b\log\left(cx^{n}\right))\ dx = \int \operatorname{csch}^{4}\left(a+b\log\left(cx^{n}\right)\right)dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**4,x)
[Out] Integral(csch(a + b*log(c*x**n))**4, x)
```

Maxima [F]

$$\int \operatorname{csch}^{4}(a+b\log\left(cx^{n}\right))\ dx = \int \operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{4}\ dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="maxima")
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) + b^3*n^3
), x) - 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) -
 b^3*n^3), x) - 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) + 2
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) - 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) - b^3*n^
3)
```

Giac [F]

$$\int \operatorname{csch}^4(a + b\log(cx^n))\, dx = \int \operatorname{csch}(b\log(cx^n) + a)^4\, dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="giac")
[Out] integrate(csch(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(a + b\log(cx^n))\, dx = \int \frac{1}{\sinh(a + b\,\ln(c\,x^n))^4}\, dx$$

```
[In] int(1/sinh(a + b*log(c*x^n))^4,x)
[Out] int(1/sinh(a + b*log(c*x^n))^4, x)
```

3.160 $\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}$

Optimal result . . . 837
Rubi [C] (verified) . . . 837
Mathematica [A] (verified) . . . 839
Maple [A] (verified) . . . 839
Fricas [B] (verification not implemented) . . . 840
Sympy [F] . . . 840
Maxima [B] (verification not implemented) . . . 840
Giac [B] (verification not implemented) . . . 841
Mupad [B] (verification not implemented) . . . 841

Optimal result

Integrand size = 45, antiderivative size = 42

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=-x\operatorname{csch}(a+b\log\left(cx^n\right))-bnx\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}(a+b\log\left(cx^n\right))$$

```
[Out] -x*csch(a+b*ln(c*x^n))-b*n*x*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.26, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {5665, 5667, 269, 371}

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=2e^a x(1-bn)\left(cx^n\right)^b\operatorname{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),e^{2a}\left(cx^n\right)^{2b}\right)$$
$$-\frac{16e^{3a}b^2n^2x\left(cx^n\right)^{3b}\operatorname{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

```
[In] Int[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Log[c*x^
n]]^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)] - (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*Hyp
ergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, E^(2*a)*(c*x^n)^(2
*b)])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(2b^2n^2\right)\int \operatorname{csch}^3(a+b\log\left(cx^n\right))\,dx + \left(-1+b^2n^2\right)\int \operatorname{csch}(a+b\log\left(cx^n\right))\,dx \\
&= \left(2b^2nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^3(a+b\log(x))\,dx, x, cx^n\right) \\
&\quad + \frac{\left(\left(-1+b^2n^2\right)x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}(a+b\log(x))\,dx, x, cx^n\right)}{n} \\
&= \left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{\left(1-e^{-2a}x^{-2b}\right)^3}\,dx, x, cx^n\right) \\
&\quad + \frac{\left(2e^{-a}\left(-1+b^2n^2\right)x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1-e^{-2a}x^{-2b}}\,dx, x, cx^n\right)}{n} \\
&= \left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{\left(-e^{-2a}+x^{2b}\right)^3}\,dx, x, cx^n\right) \\
&\quad + \frac{\left(2e^{-a}\left(-1+b^2n^2\right)x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{-e^{-2a}+x^{2b}}\,dx, x, cx^n\right)}{n}
\end{aligned}$$

$$= 2e^a(1-bn)x(cx^n)^b \operatorname{Hypergeometric2F1}\left(1, \frac{b+\frac{1}{n}}{2b}, \frac{1}{2}\left(3+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right) - \frac{16b^2e^{3a}n^2x(cx^n)^{3b} \operatorname{Hypergeometric2F1}\left(3, \frac{3b+\frac{1}{n}}{2b}, \frac{1}{2}\left(5+\frac{1}{bn}\right), e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log(cx^n))\right)+2b^2n^2\operatorname{csch}^3(a+b\log(cx^n))\right)\,dx = -x(1+bn\coth(a+b\log(cx^n)))\operatorname{csch}(a+b\log(cx^n))$$

```
[In] Integrate[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] -(x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]])
```

Maple [A] (verified)

Time = 21.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

method	result
parallelrisch	$\frac{x\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)^4bn+2\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)^3-bn-2\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)\right)}{4\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)^2}$
risch	$-\frac{2c^b(x^n)^bx\left(nb(x^n)^{2b}c^{2b}\mathrm{e}^{3a}\mathrm{e}^{\frac{3ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{-\frac{3ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}{2}}\mathrm{e}^{-\frac{3ib\pi\operatorname{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{\frac{3ib\pi\operatorname{csgn}(ic\,x^n)}{2}}\right.$

```
[In] int(-(-b^2*n^2+1)*csch(a+b*ln(c*x^n))+2*b^2*n^2*csch(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(tanh(1/2*a+b*ln((c*x^n)^(1/2)))^4*b*n+2*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^3-b*n-2*tanh(1/2*a+b*ln((c*x^n)^(1/2))))/tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.45

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx =$$

$$-\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\left(bn+1\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(bn+1\right)x-\left(bn-1\right)x\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="fricas")
```

```
[Out] -2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b
*n*log(x) + b*log(c) + a)^2 + (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)
^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 +
sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a)^2 -
1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$= \int \left(2b^2n^2\operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)+b^2n^2-1\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)dx$$

```
[In] integrate(-(-b**2*n**2+1)*csch(a+b*ln(c*x**n))+2*b**2*n**2*csch(a+b*ln(c*x*
*n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*csch(a + b*log(c*x**n))**2 + b**2*n**2 - 1)*csch(a +
b*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$= -\frac{2\left(\left(bc^{3\,b}n+c^{3\,b}\right)xe^{(3\,b\log(x^n)+3\,a)}+\left(bc^bn-c^b\right)xe^{(b\log(x^n)+a)}\right)}{c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}-2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1}$$

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="maxima")
```

```
[Out] -2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c^b)*x*e^
(b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) - 2*c^(2*b)*e^(2*b*log(x^
n) + 2*a) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(42) = 84$.

Time $= 0.49$ (sec) , antiderivative size $= 215$, normalized size of antiderivative $= 5.12$

$$\begin{aligned}
&\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx \\
&= -\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1} \\
&\quad -\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{b}xx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}
\end{aligned}$$

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="giac")
```

```
[Out] -2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x
^(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a)
- 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) - 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*b)
*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*e^a
/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time $= 2.20$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 1.55$

$$\begin{aligned}
&\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx \\
&= -\frac{2\,x\,\mathrm{e}^{a}\left(c\,x^{n}\right)^{b}\left(b\,n+\mathrm{e}^{2\,a}\left(c\,x^{n}\right)^{2\,b}+b\,n\,\mathrm{e}^{2\,a}\left(c\,x^{n}\right)^{2\,b}-1\right)}{\left(\mathrm{e}^{2\,a}\left(c\,x^{n}\right)^{2\,b}-1\right)^{2}}
\end{aligned}$$

```
[In] int((b^2*n^2 - 1)/sinh(a + b*log(c*x^n)) + (2*b^2*n^2)/sinh(a + b*log(c*x^n
))^3,x)
```

```
[Out] -(2*x*exp(a)*(c*x^n)^b*(b*n + exp(2*a)*(c*x^n)^(2*b) + b*n*exp(2*a)*(c*x^n)
^(2*b) - 1))/(exp(2*a)*(c*x^n)^(2*b) - 1)^2
```

3.161 $\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal result . . . 842
Rubi [A] (verified) . . . 842
Mathematica [B] (verified) . . . 843
Maple [F] . . . 843
Fricas [A] (verification not implemented) . . . 844
Sympy [F] . . . 844
Maxima [B] (verification not implemented) . . . 844
Giac [A] (verification not implemented) . . . 844
Mupad [B] (verification not implemented) . . . 845

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

```
[Out] -2*c^6/exp(a)/(c^4-1/exp(2*a)/x^2)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5665, 5667, 267}

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2e^{-a}c^6}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

```
[In] Int[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6)/(E^a*(c^4 - 1/(E^(2*a)*x^2))^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int x\text{csch}^3(a+2\log(x))\,dx, x, c\sqrt{x}\right)}{c^2} \\ &= \frac{(16e^{-3a})\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3 x^5}\,dx, x, c\sqrt{x}\right)}{c^2} \\ &= -\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.

Time $= 0.14$ (sec) , antiderivative size $= 62$, normalized size of antiderivative $= 2.38$

$$\begin{aligned}&\int \text{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx \\ &= \frac{2(\cosh(a)-\sinh(a))\left(-2c^4x^2+\cosh^2(a)-2\cosh(a)\sinh(a)+\sinh^2(a)\right)}{c^2\left((-1+c^4x^2)\cosh(a)+(1+c^4x^2)\sinh(a)\right)^2}\end{aligned}$$

```
[In] Integrate[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (2*(Cosh[a] - Sinh[a])*(-2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((-1 + c^4*x^2)*Cosh[a] + (1 + c^4*x^2)*Sinh[a])^2)
```

Maple [F]

$$\int \text{csch}\left(a+2\ln\left(c\sqrt{x}\right)\right)^3\,dx$$

```
[In] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)}{c^{10}x^4e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)/(c^10*x^4*e^(5*a) - 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

Sympy [F]

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = \int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

```
[In] integrate(csch(a+2*ln(c*x**(1/2)))**3,x)
[Out] Integral(csch(a + 2*log(c*sqrt(x)))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}-\frac{1}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a) - 1/(c^8*
x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}-1\right)^2c^2}$$

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)*e^(-a)/((c^4*x^2*e^(2*a) - 1)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=\frac{\frac{2\,\mathrm{e}^{-a}}{c^2}-4\,c^2\,x^2\,\mathrm{e}^{a}}{\mathrm{e}^{4\,a}\,c^8\,x^4-2\,\mathrm{e}^{2\,a}\,c^4\,x^2+1}$$

```
[In] int(1/sinh(a + 2*log(c*x^(1/2)))^3,x)
```

```
[Out] ((2*exp(-a))/c^2 - 4*c^2*x^2*exp(a))/(c^8*x^4*exp(4*a) - 2*c^4*x^2*exp(2*a)
 + 1)
```

3.162 $\int \text{csch}^3\left(a + 2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal result . . . 846
Rubi [A] (verified) . . . 846
Mathematica [B] (verified) . . . 847
Maple [F] . . . 848
Fricas [A] (verification not implemented) . . . 848
Sympy [F] . . . 848
Maxima [A] (verification not implemented) . . . 848
Giac [A] (verification not implemented) . . . 849
Mupad [B] (verification not implemented) . . . 849

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \text{csch}^3\left(a + 2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \frac{2c^2e^{-3a}}{\left(e^{-2a} - \frac{c^4}{x^2}\right)^2}$$

```
[Out] 2*c^2/exp(3*a)/(exp(-2*a)-c^4/x^2)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5665, 5667, 269, 267}

$$\int \text{csch}^3\left(a + 2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \frac{2e^{-3a}c^2}{\left(e^{-2a} - \frac{c^4}{x^2}\right)^2}$$

```
[In] Int[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) - c^4/x^2)^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(2c^2\right) \text{Subst}\left(\int \frac{\operatorname{csch}^3(a+2\log(x))}{x^3}\,dx, x, \frac{c}{\sqrt{x}} \right) \right) \\
&= -\left(\left(16c^2e^{-3a}\right) \text{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3 x^9}\,dx, x, \frac{c}{\sqrt{x}} \right) \right) \\
&= -\left(\left(16c^2e^{-3a}\right) \text{Subst}\left(\int \frac{x^3}{\left(-e^{-2a}+x^4\right)^3}\,dx, x, \frac{c}{\sqrt{x}} \right) \right) \\
&= \frac{2c^2e^{-3a}}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\begin{aligned}
&\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx \\
&= -\frac{2c^6\left(\left(c^4-2x^2\right)\cosh(a)+\left(c^4+2x^2\right)\sinh(a)\right)\left(\cosh(2a)+\sinh(2a)\right)}{\left(\left(-c^4+x^2\right)\cosh(a)-\left(c^4+x^2\right)\sinh(a)\right)^2}
\end{aligned}$$

```
[In] Integrate[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6*((c^4 - 2*x^2)*Cosh[a] + (c^4 + 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((-c^4 + x^2)*Cosh[a] - (c^4 + x^2)*Sinh[a])^2
```

Maple [F]

$$\int \operatorname{csch}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

```
[In] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = -\frac{2\left(c^{10}e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}\right)}{c^8e^{(4\,a)}-2\,c^4x^2e^{(2\,a)}+x^4}$$

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

Sympy [F]

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = \int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx$$

```
[In] integrate(csch(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(csch(a + 2*log(c/sqrt(x)))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = -\frac{2\left(c^{10}e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}\right)}{c^8e^{(4\,a)}-2\,c^4x^2e^{(2\,a)}+x^4}$$

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2\left(c^{10}e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}\right)}{\left(c^4e^{(2\,a)}-x^2\right)^2}$$

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) - x^2)^2
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \frac{2\,c^2\,x^4\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8-2\,\mathrm{e}^{2\,a}\,c^4\,x^2+x^4}$$

```
[In] int(1/sinh(a + 2*log(c/x^(1/2)))^3,x)
[Out] (2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 - 2*c^4*x^2*exp(2*a))
```

3.163 $\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result . . . 850
Rubi [A] (verified) . . . 850
Mathematica [A] (verified) . . . 851
Maple [F] . . . 852
Fricas [B] (verification not implemented) . . . 852
Sympy [F] . . . 853
Maxima [F] . . . 853
Giac [F(-1)] . . . 853
Mupad [F(-1)] . . . 853

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = -\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] -1/2*exp(2*a)*(2-p)*x*(1-(c*x^n)^(2/n/(2-p))/exp(2*a))*csch(a-ln(c*x^n)/n/(
2-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5665, 5669, 267}

$$\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = -\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[In] Int[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] -1/2*(E^(2*a)*(2 - p)*x*(1 - (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Csch[a - Log[
c*x^n]/(n*(2 - p))]^p)/((1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\text{csch}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\text{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1-e^{-2a}x^{-\frac{2}{n(-2+p)}}\right)^p\text{csch}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)\,dx, x, cx^n\right)}{n}$$

$$= -\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\text{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \text{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$$

$$= \frac{2^{-1+p}(-2+p)x\left(\frac{e^a(cx^n)^{\frac{1}{n(-2+p)}}}{-1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p\left(1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}\left(-1+\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

```
[In] Integrate[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^(2*a)*(c*x^n
)^(2/(n*(-2 + p)))))^p*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 - 1/(
E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

Maple [F]

$$\int \operatorname{csch}\left(a+\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

```
[In] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
[Out] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(76) = 152$.

Time $= 0.27$ (sec) , antiderivative size $= 475$, normalized size of antiderivative $= 5.28$

$$\int \operatorname{csch}^{p}\left(a+\frac{\log\left(cx^{n}\right)}{n(-2+p)}\right)\,dx = -\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+2\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{an}{}\right.\right.\right.\right.}$$

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
[Out] -((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p
- 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log
(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p -
2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh
((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(p*log(2*
(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*
n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(
c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a
*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh((a*n*p - 2*a*n + n*log(x
) + log(c))/(n*p - 2*n)))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n
)))
```

Sympy [F]

$$\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)\,dx$$

```
[In] integrate(csch(a+ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(csch(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \int \operatorname{csch}\left(a+\frac{\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F(-1)]

Timed out.

$$\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \text{Timed out}$$

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \int \left(\frac{1}{\sinh\left(a+\frac{\ln\left(c\,x^n\right)}{n\,(p-2)}\right)}\right)^p\,dx$$

```
[In] int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

3.164 $\int \text{csch}^p\left(a - \frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result . . . 854
Rubi [A] (verified) . . . 854
Mathematica [B] (warning: unable to verify) . . . 855
Maple [F] . . . 856
Fricas [B] (verification not implemented) . . . 856
Sympy [F] . . . 857
Maxima [F] . . . 857
Giac [F] . . . 857
Mupad [F(-1)] . . . 857

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \text{csch}^p\left(a - \frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{(2-p)x\left(1 - e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\text{csch}^p\left(a + \frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1-1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*csch(a+ln(c*x^n)/n/(2-p))^
p/(1-p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5665, 5669, 270}

$$\int \text{csch}^p\left(a - \frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{(2-p)x\left(1 - e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\text{csch}^p\left(a + \frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[In] Int[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 - 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Csch[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\text{csch}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\text{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-e^{-2}\right.\right.}{n}$$

$$= \frac{(2-p)x\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\text{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 140 vs. $2(66) = 132$.

Time = 0.87 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.12

$$\int \text{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx$$

$$= \frac{2^{-1+p}e^{-\frac{2ap}{-2+p}}(-2+p)x\left(e^{\frac{2ap}{-2+p}}-e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}\right)\left(-\frac{e^{\frac{a(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{-e^{\frac{2ap}{-2+p}}+e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p}{-1+p}$$

```
[In] Integrate[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*(E^((2*a*p)/(-2 + p)) - E^((4*a)/(-2 + p))*(c*x^n)^(
2/(n*(-2 + p))))*(-((E^((a*(2 + p))/(-2 + p))*(c*x^n)^(1/(n*(-2 + p))))/(-E
^((2*a*p)/(-2 + p)) + E^((4*a)/(-2 + p))*(c*x^n)^(2/(n*(-2 + p))))))^p)/(E^
((2*a*p)/(-2 + p))*(-1 + p))
```

Maple [F]

$$\int \operatorname{csch}\left(a-\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

```
[In] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
[Out] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(55) = 110$.

Time $= 0.27$ (sec) , antiderivative size $= 539$, normalized size of antiderivative $= 8.17$

$$\int \operatorname{csch}^{p}\left(a-\frac{\log\left(cx^{n}\right)}{n(-2+p)}\right)\,dx = -\frac{(p-2)x\cosh\left(p\log\left(-\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right.}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^{2}+2\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\right.}\right.\right.$$

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
[Out] -((p - 2)*x*cosh(p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*
n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n -
n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(
n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)
))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(
p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c)
)/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sin
h(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)))*sinh(-(a*n*p -
 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/((p - 1)*cosh(-(a*n*p - 2*a*n - n
*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) -
log(c))/(n*p - 2*n)))
```

Sympy [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)\,dx$$

```
[In] integrate(csch(a-ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(csch(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(a-\frac{\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate((-csch(-a + log(c*x^n)/(n*(p - 2))))^p, x)
```

Giac [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(a-\frac{\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] integrate(csch(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\sinh\left(a-\frac{\ln\left(cx^n\right)}{n\left(p-2\right)}\right)}\right)^p\,dx$$

```
[In] int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

3.165 $\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 858
Rubi [A] (verified) . . . 858
Mathematica [B] (verified) . . . 859
Maple [A] (verified) . . . 859
Fricas [B] (verification not implemented) . . . 859
Sympy [A] (verification not implemented) . . . 860
Maxima [A] (verification not implemented) . . . 860
Giac [B] (verification not implemented) . . . 860
Mupad [B] (verification not implemented) . . . 861

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = -\frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{bn}$$

```
[Out] -arctanh(cosh(a+b*ln(c*x^n)))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3855}

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = -\frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{bn}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{\operatorname{csch}(a + b\log(cx^n))}{x}\,dx = -\frac{\log\left(\cosh\left(\frac{a}{2} + \frac{1}{2}b\log(cx^n)\right)\right)}{bn} + \frac{\log\left(\sinh\left(\frac{a}{2} + \frac{1}{2}b\log(cx^n)\right)\right)}{bn}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(Log[Cosh[a/2 + (b*Log[c*x^n])/2]]/(b*n)) + Log[Sinh[a/2 + (b*Log[c*x^n])/
2]]/(b*n)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}{nb}$
default	$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}{nb}$
parallelrisch	$\frac{\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)\right)}{nb}$
risch	$\frac{\ln\left(c^b(x^n)^b\mathrm{e}^a\mathrm{e}^{\frac{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{-\frac{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{-\frac{ib\pi\,\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{\frac{ib\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}{2}}-1\right.$ / bn

```
[In] int(csch(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*ln(tanh(1/2*a+1/2*b*ln(c*x^n)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{csch}(a + b\log(cx^n))}{x}\,dx =$$
$$-\frac{\log\left(\cosh\left(bn\log(x) + b\log(c) + a\right) + \sinh\left(bn\log(x) + b\log(c) + a\right) + 1\right) - \log\left(\cosh\left(bn\log(x) + b\lo\right.}{bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -(log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1
) - log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) -
 1))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = -\begin{cases} -\log(x)\operatorname{csch}(a) & \text{for } b=0 \\ -\log(x)\operatorname{csch}(a+b\log(c)) & \text{for } n=0 \\ -\frac{\log\left(\tanh\left(\frac{a}{2}+\frac{b\log(cx^n)}{2}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(csch(a+b*ln(c*x**n))/x,x)
```

```
[Out] -Piecewise((-log(x)*csch(a), Eq(b, 0)), (-log(x)*csch(a + b*log(c)), Eq(n,
0)), (-log(tanh(a/2 + b*log(c*x**n)/2))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(\tanh\left(\frac{1}{2}\,b\log(cx^n)+\frac{1}{2}\,a\right)\right)}{bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*b*log(c*x^n) + 1/2*a))/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(20) = 40.

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 7.25

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx =$$

$$-c^b\left(\frac{c^b e^{(-a)}\log\left(\sqrt{2\,x^{bn}|c|^b\cos\left(-\frac{1}{2}\,\pi b\operatorname{sgn}(c)+\frac{1}{2}\,\pi b\right)e^a + x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{2\,b}n} - \frac{c^b e^{(-a)}\log\left(\sqrt{-2\,x^{bn}|c|}\right.}{}\right.$$

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] -c^b*(c^b*e^(-a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*
b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(2*b)*n) - c^b*e^(-a)*lo
g(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a + x^(2*b*n)
*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(2*b)*n))*e^a
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{csch}(a + b\log(cx^n))}{x}\,dx = -\frac{2\operatorname{atan}\left(\frac{\mathrm{e}^{-a}\sqrt{-b^2n^2}}{bn(cx^n)^b}\right)}{\sqrt{-b^2n^2}}$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))),x)
[Out] -(2*atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(-b^2*n^2)^(1/2)
```

3.166 $\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 862
Rubi [A] (verified) . . . 862
Mathematica [A] (verified) . . . 863
Maple [A] (verified) . . . 863
Fricas [B] (verification not implemented) . . . 864
Sympy [F] . . . 864
Maxima [A] (verification not implemented) . . . 864
Giac [A] (verification not implemented) . . . 865
Mupad [B] (verification not implemented) . . . 865

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx = -\frac{\coth(a+b\log(cx^n))}{bn}$$

```
[Out] -coth(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3852, 8}

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx = -\frac{\coth(a+b\log(cx^n))}{bn}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{csch}^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= -\frac{i\text{Subst}\left(\int 1\,dx, x, -i\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\ &= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(a+b\log\left(cx^n\right))}{x}\,dx = -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2/x,x]
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\coth(a+b\ln(c\,x^n))}{bn}$
default	$-\frac{\coth(a+b\ln(c\,x^n))}{bn}$
parallelrisch	$\frac{-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)}{2bn}$
risch	$-\frac{2}{bn\left((x^n)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}\mathrm{e}^{-ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}\mathrm{e}^{-ib\pi\,\mathrm{csgn}(ic\,x^n)^3}\mathrm{e}^{ib\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}-\right.$

```
[In] int(csch(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
[Out] -coth(a+b*ln(c*x^n))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.74

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx =$$

$$-\frac{2}{bn\cosh(bn\log(x) + b\log(c) + a)^2 + 2\,bn\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) + a)}$$

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 - b*n)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx = -\frac{2}{bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx = -\frac{2}{(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1)bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx = \frac{2}{b\,n - b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^2),x)
[Out] 2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b))
```

3.167 $\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 866
Rubi [A] (verified) . . . 866
Mathematica [A] (verified) . . . 867
Maple [A] (verified) . . . 867
Fricas [B] (verification not implemented) . . . 868
Sympy [F] . . . 869
Maxima [B] (verification not implemented) . . . 869
Giac [B] (verification not implemented) . . . 869
Mupad [B] (verification not implemented) . . . 870

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{2bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}$$

```
[Out] 1/2*arctanh(cosh(a+b*ln(c*x^n)))/b/n-1/2*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*
x^n))/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3853, 3855}

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{2bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTanh[Cosh[a + b*Log[c*x^n]]]/(2*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a +
b*Log[c*x^n]])/(2*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \text{csch}^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)\text{csch}(a+b\log\left(cx^n\right))}{2bn} - \frac{\text{Subst}\left(\int \text{csch}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{2n} \\ &= \frac{\text{arctanh}(\cosh\left(a+b\log\left(cx^n\right)\right))}{2bn} - \frac{\coth\left(a+b\log\left(cx^n\right)\right)\text{csch}(a+b\log\left(cx^n\right))}{2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\begin{aligned}\int \frac{\text{csch}^3(a+b\log\left(cx^n\right))}{x}\,dx = &-\frac{\text{csch}^2\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{8bn} + \frac{\log\left(\cosh\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)\right)}{2bn} \\ &- \frac{\log\left(\sinh\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)\right)}{2bn} - \frac{\text{sech}^2\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{8bn}\end{aligned}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/8*Csch[(a + b*Log[c*x^n])/2]^2/(b*n) + Log[Cosh[(a + b*Log[c*x^n])/2]]/(
2*b*n) - Log[Sinh[(a + b*Log[c*x^n])/2]]/(2*b*n) - Sech[(a + b*Log[c*x^n])/
2]^2/(8*b*n)
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{\operatorname{csch}(a+b\ln(c\,x^n))\coth(a+b\ln(c\,x^n))}{2}+\operatorname{arctanh}\left(e^{a+b\ln(c\,x^n)}\right)}{nb}$
default	$\frac{-\frac{\operatorname{csch}(a+b\ln(c\,x^n))\coth(a+b\ln(c\,x^n))}{2}+\operatorname{arctanh}\left(e^{a+b\ln(c\,x^n)}\right)}{nb}$
parallelrisch	$\frac{-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2-4\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)\right)}{8bn}$
risch	$-\frac{c^b(x^n)^b\left((x^n)^{2b}c^{2b}e^{3a}e^{\frac{3ib\pi\ \operatorname{csgn}(ix^n)\ \operatorname{csgn}(ic\,x^n)^2}{2}}e^{-\frac{3ib\pi\ \operatorname{csgn}(ix^n)\ \operatorname{csgn}(ic\,x^n)\ \operatorname{csgn}(ic)}{2}}e^{-\frac{3ib\pi\ \operatorname{csgn}(ic\,x^n)^3}{2}}e^{\frac{3ib\pi\ \operatorname{csgn}(ic\,x^n}{2}}\right.}{bn\left((x^n)^{2b}c^{2b}e^{2a}e^{ib\pi\ \operatorname{csgn}(ix^n)\ \operatorname{csgn}(ic\,x^n)^2}e^{-ib\pi\ \operatorname{csgn}(ix^n)\ \operatorname{csg}}\right.}$

```
[In] int(csch(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/2*csch(a+b*ln(c*x^n))*coth(a+b*ln(c*x^n))+arctanh(exp(a+b*ln(c*x^
n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 643, normalized size of antiderivative = 11.69

$$\int \frac{\operatorname{csch}^3(a + b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sinh(b*n*log(x) + b*log(c) + a)^3
- (cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*
cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 -
2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3
- cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log
(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + (
cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cos
h(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*c
osh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(co
sh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(3
*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a) + 2
*cosh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 +
4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b
*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a
```

```
)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b
*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(51) = 102$.

Time $= 0.20$ (sec) , antiderivative size $= 150$, normalized size of antiderivative $= 2.73$

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = -\frac{c^{3b}e^{(3b\log(x^n)+3a)} + c^b e^{(b\log(x^n)+a)}}{bc^{4b}ne^{(4b\log(x^n)+4a)} - 2bc^{2b}ne^{(2b\log(x^n)+2a)} + bn} + \frac{\log\left(\frac{\left(c^b e^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^b}\right)}{2bn} - \frac{\log\left(\frac{\left(c^b e^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^b}\right)}{2bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -(c^(3*b)*e^(3*b*log(x^n) + 3*a) + c^b*e^(b*log(x^n) + a))/(b*c^(4*b)*n*e^(
4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/2*log
((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) - 1/2*log((c^b*e^(b*log(x^n
) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(51) = 102$.

Time $= 0.34$ (sec) , antiderivative size $= 210$, normalized size of antiderivative $= 3.82$

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx$$

$$= \frac{1}{2}c^{3b}\left(\frac{c^b e^{(-3a)}\log\left(\sqrt{2\,x^{bn}|c|^b\cos\left(-\frac{1}{2}\pi b\operatorname{sgn}(c)+\frac{1}{2}\pi b\right)e^a + x^{2bn}|c|^{2b}e^{(2a)}+1}\right)}{bc^{4b}n} - \frac{c^b e^{(-3a)}\log\left(\sqrt{-2\,x}\right.\right.$$

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/2*c^(3*b)*(c^b*e^(-3*a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c)
+ 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(4*b)*n) - c^b*
e^(-3*a)*log(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a
+ x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(4*b)*n) - 2*(c^(2*b)*x^(3*b*n)
*e^(2*a) + x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*c^(2*b)*n
))*e^(3*a)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.55

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{atan}\left(\frac{e^{-a}\sqrt{-b^2n^2}}{bn(cx^n)^b}\right)}{\sqrt{-b^2n^2}} + \frac{e^{-a}}{(cx^n)^b\left(bn - \frac{bne^{-2a}}{(cx^n)^{2b}}\right)} - \frac{2e^{-a}}{(cx^n)^b\left(bn - \frac{2bne^{-2a}}{(cx^n)^{2b}} + \frac{bne^{-4a}}{(cx^n)^{4b}}\right)}$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^3),x)
```

```
[Out] atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(-b^2*n^2)^(1/2) + exp(-a)
/((c*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (2*exp(-a))/((c*x^n)^b
*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*x^n)^(4*b)))
```

3.168 $\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 871
Rubi [A] (verified) . . . 871
Mathematica [A] (verified) . . . 872
Maple [A] (verified) . . . 872
Fricas [B] (verification not implemented) . . . 873
Sympy [F] . . . 873
Maxima [B] (verification not implemented) . . . 873
Giac [A] (verification not implemented) . . . 874
Mupad [B] (verification not implemented) . . . 874

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx = \frac{\coth(a+b\log(cx^n))}{bn} - \frac{\coth^3(a+b\log(cx^n))}{3bn}$$

```
[Out] coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3852}

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx = \frac{\coth(a+b\log(cx^n))}{bn} - \frac{\coth^3(a+b\log(cx^n))}{3bn}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Coth[a + b*Log[c*x^n]]/(b*n) - Coth[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{csch}^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= \frac{i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\ &= \frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^4(a+b\log\left(cx^n\right))}{x}\,dx = \frac{2\coth\left(a+b\log\left(cx^n\right)\right)}{3bn} - \frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^2(a+b\log\left(cx^n\right))}{3bn}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (2*Coth[a + b*Log[c*x^n]])/(3*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log
[c*x^n]]^2)/(3*b*n)
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}\left(a+b\ln\left(c\,x^n\right)\right)^2}{3}\right)\coth\left(a+b\ln\left(c\,x^n\right)\right)}{nb}$
default	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}\left(a+b\ln\left(c\,x^n\right)\right)^2}{3}\right)\coth\left(a+b\ln\left(c\,x^n\right)\right)}{nb}$
parallelrisch	$\frac{-\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3+9\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+9\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)}{24bn}$
risch	$-\frac{4\left(3(x^n)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\ \mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)^2\mathrm{e}^{-ib\pi\ \mathrm{csgn}\left(ix^n\right)\mathrm{csgn}\left(ic\,x^n\right)\mathrm{csgn}(ic)}\mathrm{e}^{-ib\pi\,\mathrm{csgn}\left(ic\,x^n\right)^3\mathrm{e}^{ib\pi\,\mathrm{csgn}\left(ic\,x^n\right)^2\mathrm{csgn}(ic)}}$

```
[In] int(csch(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(2/3-1/3*csch(a+b*ln(c*x^n))^2)*coth(a+b*ln(c*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(40) = 80$.

Time $= 0.26$ (sec) , antiderivative size $= 272$, normalized size of antiderivative $= 6.48$

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx =$$

$$-\frac{}{3\left(bn\cosh\left(bn\log(x) + b\log(c) + a\right)^5 + 5\,bn\cosh\left(bn\log(x) + b\log(c) + a\right)\sinh\left(bn\log(x) + b\log(c)\right.}$$

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*n*log(x) + b*log(c) + a) + 2*sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 - 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 2*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 - 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 4*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^4\left(a + b\log(cx^n)\right)}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(40) = 80$.

Time $= 0.20$ (sec) , antiderivative size $= 92$, normalized size of antiderivative $= 2.19$

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx$$
$$= -\frac{4\left(3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} - 1\right)}{3\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn\right)}$$

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) - b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^3 bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) - 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^3*b
*n)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx = \frac{4\,\mathrm{e}^{4\,a}\left(c\,x^n\right)^{4\,b}\left(\mathrm{e}^{2\,a}\left(c\,x^n\right)^{2\,b}-3\right)}{3\,b\,n\left(\mathrm{e}^{2\,a}\left(c\,x^n\right)^{2\,b}-1\right)^3}$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^4),x)
```

```
[Out] (4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) - 3))/(3*b*n*(exp(2*a)*(c
*x^n)^(2*b) - 1)^3)
```

3.169 $\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 875
Rubi [A] (verified) . . . 875
Mathematica [A] (verified) . . . 876
Maple [A] (verified) . . . 877
Fricas [B] (verification not implemented) . . . 877
Sympy [F] . . . 879
Maxima [B] (verification not implemented) . . . 879
Giac [B] (verification not implemented) . . . 880
Mupad [B] (verification not implemented) . . . 880

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{8bn} + \frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}$$

```
[Out] -3/8*arctanh(cosh(a+b*ln(c*x^n)))/b/n+3/8*coth(a+b*ln(c*x^n))*csch(a+b*ln(c
*x^n))/b/n-1/4*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3853, 3855}

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{8bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn} + \frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Coth[a + b*Log[c*x^n]]*Cs
ch[a + b*Log[c*x^n]])/(8*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^
n]]^3)/(4*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^5(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{4bn} - \frac{3\operatorname{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{4n} \\
&= \frac{3\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{8bn} \\
&\quad - \frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{4bn} \\
&\quad + \frac{3\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{8n} \\
&= -\frac{3\operatorname{arctanh}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn} + \frac{3\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}{8bn} \\
&\quad - \frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^3\left(a+b\log\left(cx^n\right)\right)}{4bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

$$\begin{aligned}
\int \frac{\operatorname{csch}^5(a+b\log\left(cx^n\right))}{x}\,dx &= \frac{3\operatorname{csch}^2\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{32bn} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{64bn} \\
&\quad - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)\right)}{8bn} \\
&\quad + \frac{3\log\left(\sinh\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)\right)}{8bn} \\
&\quad + \frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{32bn} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{64bn}
\end{aligned}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*Csch[(a + b*Log[c*x^n])/2]^2)/(32*b*n) - Csch[(a + b*Log[c*x^n])/2]^4/(6
4*b*n) - (3*Log[Cosh[(a + b*Log[c*x^n])/2]])/(8*b*n) + (3*Log[Sinh[(a + b*L
og[c*x^n])/2]])/(8*b*n) + (3*Sech[(a + b*Log[c*x^n])/2]^2)/(32*b*n) + Sech[
(a + b*Log[c*x^n])/2]^4/(64*b*n)
```

Maple [A] (verified)

Time = 21.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(-\frac{\operatorname{csch}(a+b\ln(cx^n))^3}{4}+\frac{3\operatorname{csch}(a+b\ln(cx^n))}{8}\right)\coth(a+b\ln(cx^n))-\frac{3\operatorname{arctanh}\left(e^{a+b\ln(cx^n)}\right)}{4}}{nb}$
default	$\frac{\left(-\frac{\operatorname{csch}(a+b\ln(cx^n))^3}{4}+\frac{3\operatorname{csch}(a+b\ln(cx^n))}{8}\right)\coth(a+b\ln(cx^n))-\frac{3\operatorname{arctanh}\left(e^{a+b\ln(cx^n)}\right)}{4}}{nb}$
parallelrisch	$\frac{-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)^4+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)^4-8\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)^2+24\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{cx^n}\right)\right)\right)+8\coth\left(\frac{a}{2}+\right.}{64bn}$
risch	Expression too large to display

```
[In] int(csch(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*((-1/4*csch(a+b*ln(c*x^n))^3+3/8*csch(a+b*ln(c*x^n)))*coth(a+b*ln(c*x
^n))-3/4*arctanh(exp(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 1806, normalized size of antiderivative = 20.29

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*n*log(x) + b*log(c) + a)^7 + 42*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*sinh(b*n*log(x) + b*log(c) + a)^7
+ 2*(63*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c)
+ a)^5 - 22*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*(21*cosh(b*n*log(x) + b*
log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log
(c) + a)^4 + 2*(105*cosh(b*n*log(x) + b*log(c) + a)^4 - 110*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 22*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + 2*(63*cosh(b*n*log(x) + b*log(c) + a)^5 - 110*cosh
(b*n*log(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log
```

```
(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*
n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cos
h(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*c
osh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6
*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5
 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^
6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*log(x) + b*log(c)
 + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
+ a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log(x) + b*log(c) +
 a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + 3*(cosh(b*n*log(x) + b*log(c)
+ a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^
7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(
c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*l
og(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) +
 b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x)
 + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x)
+ b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x
) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log
(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*l
og(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*lo
g(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(21*cosh(b*n
*log(x) + b*log(c) + a)^6 - 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(
b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(
b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*
cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sin
h(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 +
4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c
) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(
x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(
x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*b*n
*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)
^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b
*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*
log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*cosh(b*n*log(x
```

```
) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**5/x,x)
[Out] Integral(csch(a + b*log(c*x**n))**5/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(83) = 166$.

Time $= 0.21$ (sec) , antiderivative size $= 232$, normalized size of antiderivative $= 2.61$

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{3\,c^{7\,b}e^{(7\,b\log(x^n)+7\,a)} - 11\,c^{5\,b}e^{(5\,b\log(x^n)+5\,a)} - 11\,c^{3\,b}e^{(3\,b\log(x^n)+3\,a)} + 3\,c^{b}e^{(b\log(x^n)+a)}}{4\,(bc^{8\,b}ne^{(8\,b\log(x^n)+8\,a)} - 4\,bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} + 6\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} - 4\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} + bn)}$$
$$- \frac{3\log\left(\frac{\left(c^{b}e^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^{b}}\right)}{8\,bn} + \frac{3\log\left(\frac{\left(c^{b}e^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^{b}}\right)}{8\,bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
[Out] 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) - 11*c^(5*b)*e^(5*b*log(x^n) + 5*a) -
 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) + 3*c^b*e^(b*log(x^n) + a))/(b*c^(8*b)*n
*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b
)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) -
3/8*log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + 3/8*log((c^b*e^(b*
log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(83) = 166.

Time = 0.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.79

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = -\frac{1}{8}c^{5b}\left(\frac{3\,c^b e^{(-5a)}\log\left(\sqrt{2\,x^{bn}|c|^b\cos\left(-\frac{1}{2}\,\pi b\operatorname{sgn}(c)+\frac{1}{2}\,\pi b\right)e^a+x^{2bn}|c|^{2b}e^{(2a)}+1}\right)}{bc^{6b}n} - \frac{3\,c^b e^{(-5a)}\log\left(\sqrt{}\right.}{}\right.$$

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] -1/8*c^(5*b)*(3*c^b*e^(-5*a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(
c) + 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(6*b)*n) - 3
*c^b*e^(-5*a)*log(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)
*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(6*b)*n) - 2*(3*c^(6*b)*x^
(7*b*n)*e^(6*a) - 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x^(3*b*n)*e^(2*
a) + 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*c^(4*b)*n))*e
^(5*a)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.57

$$\begin{aligned}\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = {} & \frac{2\,\mathrm{e}^{-a}}{(cx^n)^b\left(bn-\frac{3bn\mathrm{e}^{-2a}}{(cx^n)^{2b}}+\frac{3bn\mathrm{e}^{-4a}}{(cx^n)^{4b}}-\frac{bn\mathrm{e}^{-6a}}{(cx^n)^{6b}}\right)} \\ & -\frac{3\operatorname{atan}\left(\frac{\mathrm{e}^{-a}\sqrt{-b^2n^2}}{bn(cx^n)^b}\right)}{4\sqrt{-b^2n^2}} - \frac{3\,\mathrm{e}^{-a}}{4(cx^n)^b\left(bn-\frac{bn\mathrm{e}^{-2a}}{(cx^n)^{2b}}\right)} \\ & -\frac{4\,\mathrm{e}^{-3a}}{(cx^n)^{3b}\left(bn-\frac{4bn\mathrm{e}^{-2a}}{(cx^n)^{2b}}+\frac{6bn\mathrm{e}^{-4a}}{(cx^n)^{4b}}-\frac{4bn\mathrm{e}^{-6a}}{(cx^n)^{6b}}+\frac{bn\mathrm{e}^{-8a}}{(cx^n)^{8b}}\right)} \\ & -\frac{\mathrm{e}^{-a}}{2(cx^n)^b\left(bn-\frac{2bn\mathrm{e}^{-2a}}{(cx^n)^{2b}}+\frac{bn\mathrm{e}^{-4a}}{(cx^n)^{4b}}\right)}\end{aligned}$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^5),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n - (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp(
-4*a))/(c*x^n)^(4*b) - (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)*(
-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(-b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(c
*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (4*exp(-3*a))/((c*x^n)^(3*
```

```
b)*(b*n - (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4*b)
 - (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) - exp(
-a)/(2*(c*x^n)^b*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(
c*x^n)^(4*b)))
```

3.170 $\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 882
Rubi [A] (verified) . . . 882
Mathematica [A] (verified) . . . 884
Maple [A] (verified) . . . 884
Fricas [C] (verification not implemented) . . . 884
Sympy [F] . . . 885
Maxima [F] . . . 885
Giac [F(-1)] . . . 885
Mupad [F(-1)] . . . 886

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn}$$

```
[Out] -2/3*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(3/2)/b/n-2/3*I*(sin(1/2*I*a+1
/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csch(a+b*ln(c*x^n))^(
1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2720}

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3bn}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + (((2*I)/
3)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn} - \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{3n} \\
&= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn} \\
&\quad - \frac{\left(\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{3n} \\
&= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn} \\
&\quad + \frac{2i\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right), 2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\coth(a+b\log(cx^n)) + i\,\text{EllipticF}\left(\tfrac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)\sqrt{i\sinh(a}\right.}{3bn}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Coth[a + b*Log[c*x^n]] + I*EllipticF[((-2
*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(3*
b*n)
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{3n\sinh(a+b\ln(cx^n))^{\frac{3}{2}}\cosh(a+b\ln(cx^n))b}$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{3n\sinh(a+b\ln(cx^n))^{\frac{3}{2}}\cosh(a+b\ln(cx^n))b}$

```
[In] int(csch(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.86

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\left(\sqrt{2}(\cosh(bn\log(x)+b\log(c)+a)^2 + 2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+\right.}$$

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
^2 + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
+ a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1))
 + (sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) +
 b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
b*log(c) + a)^2 - sqrt(2))*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*lo
g(c) + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log\left(cx^n\right))}{x}\,dx = \int \frac{\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**(5/2)/x,x)
[Out] Integral(csch(a + b*log(c*x**n))**(5/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log\left(cx^n\right))}{x}\,dx = \int \frac{\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
[Out] integrate(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log\left(cx^n\right))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

```
[In] int((1/sinh(a + b*log(c*x^n)))^(5/2)/x,x)
[Out] int((1/sinh(a + b*log(c*x^n)))^(5/2)/x, x)
```

3.171 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result . . . 887
Rubi [A] (verified) . . . 887
Mathematica [A] (verified) . . . 889
Maple [A] (verified) . . . 889
Fricas [C] (verification not implemented) . . . 889
Sympy [F] . . . 890
Maxima [F] . . . 890
Giac [F(-1)] . . . 890
Mupad [F(-1)] . . . 891

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] -2*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(1/2)/b/n+2*I*(sin(1/2*I*a+1/4*P
i+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n))^(
1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2719}

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} - \frac{2iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

```
[In] Int[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Sqrt[Csch[a + b*Log[c*x^n]]])/(b*n) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c*x^n]
]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \text{csch}^{\frac{3}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn} + \frac{\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn} - \frac{2iE\left(\frac{1}{2}\left(ia - \frac{\pi}{2} + ib\log\left(cx^n\right)\right)\middle|\, 2\right)}{bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\cosh(a+b\log(cx^n)) - E\left(\tfrac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log}\right.}{bn}$$

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I
)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{n\cosh(a+}$
default	$\frac{2\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{n\cosh(a+}$

```
[In] int(csch(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^
n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)}{\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)^2-1}}(\cosh\right.}$$

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log
(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
 1))*(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) +
sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x) + b
*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}\left(b\log(cx^n) + a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

```
[In] int((1/sinh(a + b*log(c*x^n)))^(3/2)/x,x)
[Out] int((1/sinh(a + b*log(c*x^n)))^(3/2)/x, x)
```

3.172 $\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}\,dx$

Optimal result . . . 892
Rubi [A] (verified) . . . 892
Mathematica [A] (verified) . . . 893
Maple [A] (verified) . . . 893
Fricas [C] (verification not implemented) . . . 894
Sympy [F] . . . 894
Maxima [F] . . . 894
Giac [F(-1)] . . . 895
Mupad [F(-1)] . . . 895

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{2i\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right),2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csc
h(a+b*ln(c*x^n))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2720}

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right),2\right)}{bn}$$

```
[In] Int[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n]
)/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{\text{csch}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= \frac{\left(\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= -\frac{2i\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right),2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int \frac{\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx \\ &= \frac{2\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\text{EllipticF}\left(\frac{1}{4}\left(-2ia+\pi-2ib\log\left(cx^n\right)\right),2\right)\left(i\sinh\left(a+b\log\left(cx^n\right)\right)\right)^{3/2}}{bn}\end{aligned}$$

```
[In] Integrate[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Csch[a + b*Log[c*x^n]]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^
n])/4, 2]*(I*Sinh[a + b*Log[c*x^n]])^(3/2))/(b*n)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{i\sqrt{-i(\sinh(a+b\ln(cx^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(cx^n))+i)}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(cx^n}\right.$ $\frac{}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b}$
default	$\frac{i\sqrt{-i(\sinh(a+b\ln(cx^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(cx^n))+i)}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(cx^n}\right.$ $\frac{}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b}$

```
[In] int(csch(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+I)
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$
$$= \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(
b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$

```
[In] integrate(csch(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \text{Timed out}$$

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(a+b\,\ln(c\,x^n))}}}{x}\,dx$$

```
[In] int((1/sinh(a + b*log(c*x^n)))^(1/2)/x,x)
[Out] int((1/sinh(a + b*log(c*x^n)))^(1/2)/x, x)
```

3.173 $\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx$

Optimal result . . . 896
Rubi [A] (verified) . . . 896
Mathematica [A] (verified) . . . 897
Maple [A] (verified) . . . 897
Fricas [C] (verification not implemented) . . . 898
Sympy [F] . . . 898
Maxima [F] . . . 899
Giac [F(-1)] . . . 899
Mupad [F(-1)] . . . 899

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|\,2\right)}{bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n
/csch(a+b*ln(c*x^n))^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2719}

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|\,2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[In] Int[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b
*Log[c*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n} \\ &= \frac{\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}} \\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|\,2\right)}{bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int \frac{1}{x\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx \\ &= \frac{2\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+b\log\left(cx^n\right))\right)\middle|\,2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn}\end{aligned}$$

```
[In] Integrate[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.03

method	result
derivativedivides	$\frac{\sqrt{-i(\sinh(a+b\ln(cx^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(cx^n))+i)}\,\sqrt{i\sinh(a+b\ln(cx^n))}\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c$ $\frac{}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b}$
default	$\frac{\sqrt{-i(\sinh(a+b\ln(cx^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(cx^n))+i)}\,\sqrt{i\sinh(a+b\ln(cx^n))}\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c$ $\frac{}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b}$

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\right.}{}$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
)/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)) + 2
*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x) +
 b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n*log(x) +
b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(csch(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\frac{1}{\sinh(a+b\,\ln(c\,x^n))}}}\,dx$$

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)),x)
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)), x)
```

3.174 $\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result . . . 900
Rubi [A] (verified) . . . 900
Mathematica [A] (verified) . . . 902
Maple [A] (verified) . . . 902
Fricas [C] (verification not implemented) . . . 902
Sympy [F] . . . 903
Maxima [F] . . . 903
Giac [F(-1)] . . . 903
Mupad [F(-1)] . . . 904

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}} + \frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn}$$

```
[Out] 2/3*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(1/2)-2/3*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csch(a+b*ln(c*x^n))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}} + \frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3bn}$$

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Csch[a + b*Log[c*x^n]]]) + (((2*I)/3
)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2
]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{3}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}} - \frac{\text{Subst}\left(\int \sqrt{\text{csch}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{3n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}} - \frac{\left(\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{3n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}} + \frac{2i\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right),2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}+\sinh(2($$
$$3bn$$

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Csch[a + b*Log[c*x^n]]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Lo
g[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]] + Sinh[2*(a + b*Log[c*x^n])]
))/(3*b*n)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(a+b\ln$ $n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(a+b\ln$ $n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b$

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.33

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{\sqrt{2}\big(\cosh(bn\log(x)+b\log(c)+a)^4+4\cosh(bn\log(x)+b\log(c)+a)^3\sinh(bn\log(x)+b\log(c)+a)+$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log
(c) + a)^3*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(b*n*log(x) + b*log(c) +
 a)^2*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 - 1)
*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/(
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)) - 4*(s
qrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a)^2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sin
h(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b
*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*si
nh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F]

$$\int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}(a + b \log(cx^n))} \, dx = \int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}(a + b \log(cx^n))} \, dx$$

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}(a + b \log(cx^n))} \, dx = \int \frac{1}{x \operatorname{csch}(b \log(cx^n) + a)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}(a + b \log(cx^n))} \, dx = \text{Timed out}$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)),x)
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)), x)
```

3.175 $\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result . . . 905
Rubi [A] (verified) . . . 905
Mathematica [A] (verified) . . . 906
Maple [A] (verified) . . . 907
Fricas [C] (verification not implemented) . . . 907
Sympy [F] . . . 908
Maxima [F] . . . 908
Giac [F(-1)] . . . 908
Mupad [F(-1)] . . . 909

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\cosh(a+b\log(cx^n))}{5bn\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)}{5bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] 2/5*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(3/2)-6/5*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n)
)^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\cosh(a+b\log(cx^n))}{5bn\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(5*b*n*Csch[a + b*Log[c*x^n]]^(3/2)) + (((6*I)/5
)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c
*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{5n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{3\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{5n\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{5bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx$$

$$= \frac{2\Big(\cosh\left(a+b\log\left(cx^n\right)\right) - 3\text{csch}^2\left(a+b\log\left(cx^n\right)\right)E\left(\frac{1}{4}\left(-2ia+\pi-2ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right.}}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Cosh[a + b*Log[c*x^n]] - 3*Csch[a + b*Log[c*x^n]]^2*EllipticE[((-2*I)*a
 + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(5*b*n*C
sch[a + b*Log[c*x^n]]^(3/2))
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.05

method	result
derivativedivides	$-\frac{6\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sin}}{n\cosh(a+b}$
default	$-\frac{6\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sin}}{n\cosh(a+b}$

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*
ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 602, normalized size of antiderivative = 5.42

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^6 + 6*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^5 + sinh(b*n*log(x) + b*log(c) +
a)^6 + (15*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(
c) + a)^4 + 11*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*(5*cosh(b*n*log(x) + b
*log(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*lo
g(c) + a)^3 + (15*cosh(b*n*log(x) + b*log(c) + a)^4 + 66*cosh(b*n*log(x) +
b*log(c) + a)^2 - 13)*sinh(b*n*log(x) + b*log(c) + a)^2 - 13*cosh(b*n*log(x
) + b*log(c) + a)^2 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 22*cosh(b*n*
log(x) + b*log(c) + a)^3 - 13*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log
(x) + b*log(c) + a) + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*l
og(x) + b*log(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(
```

```
x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*lo
g(c) + a)^2 - 1)) + 24*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*sqrt(
2)*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*sq
rt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
qrt(2)*sinh(b*n*log(x) + b*log(c) + a)^3)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
 + a))))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b
*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*b*n*cosh(b*n*log(x) + b*
log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n*sinh(b*n*log(x) + b*log
(c) + a)^3)
```

Sympy [F]

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\,\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(5/2),x)
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\,\operatorname{csch}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)),x)
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 911

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'^')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])   #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
         #is checked before calling the grading function that is passed.
         #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```