Computer Algebra Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.7-Miscellaneous/185-6.7.1-Hyperbolic-functions

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1059]. This is test number [185].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1059)	0.00 (0)
Mathematica	99.62 (1055)	0.38 (4)
Fricas	93.67 (992)	6.33 (67)
Maple	88.67 (939)	11.33 (120)
Giac	76.68 (812)	23.32 (247)
Maxima	71.95 (762)	28.05 (297)
Mupad	69.88 (740)	30.12 (319)
Sympy	32.48 (344)	67.52 (715)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

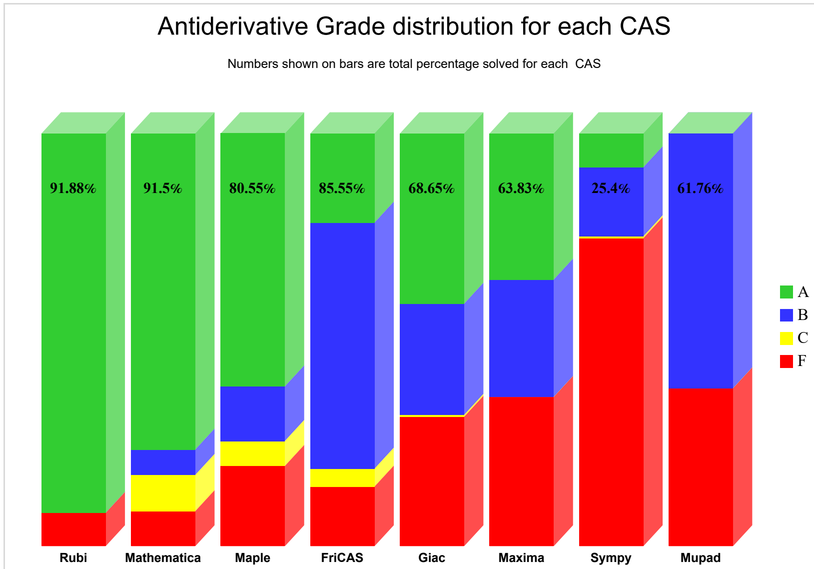
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

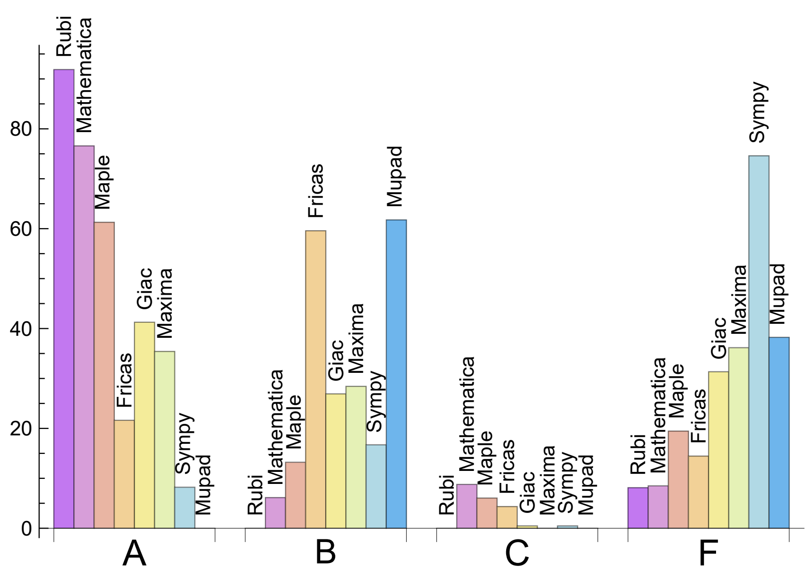
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.879	0.000	0.000	8.121
Mathematica	76.582	6.138	8.782	8.499
Maple	61.284	13.220	6.043	19.452
Giac	41.265	26.912	0.472	31.350
Maxima	35.411	28.423	0.000	36.166
Fricas	21.624	59.585	4.344	14.448
Sympy	8.215	16.714	0.472	74.599
Mupad	0.000	61.756	0.000	38.244

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	0.00	100.00	0.00
Fricas	67	49.25	0.00	50.75
Maple	120	99.17	0.83	0.00
Giac	247	91.09	1.21	7.69
Maxima	297	61.62	0.00	38.38
Mupad	319	0.00	100.00	0.00
Sympy	715	75.94	23.92	0.14

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.09
Maxima	0.25
Fricas	0.27
Giac	0.48
Mupad	1.74
Mathematica	2.89
Sympy	8.02
Maple	8.40

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	69.25	1.00	50.00	1.00
Giac	89.39	1.73	54.00	1.44
Maxima	99.11	2.55	70.00	1.64
Mupad	99.59	2.14	51.00	1.46
Maple	157.66	1.59	51.00	1.11
Mathematica	163.02	2.06	52.00	1.00
Sympy	324.97	5.05	59.50	1.89
Fricas	591.30	6.27	164.00	3.35

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

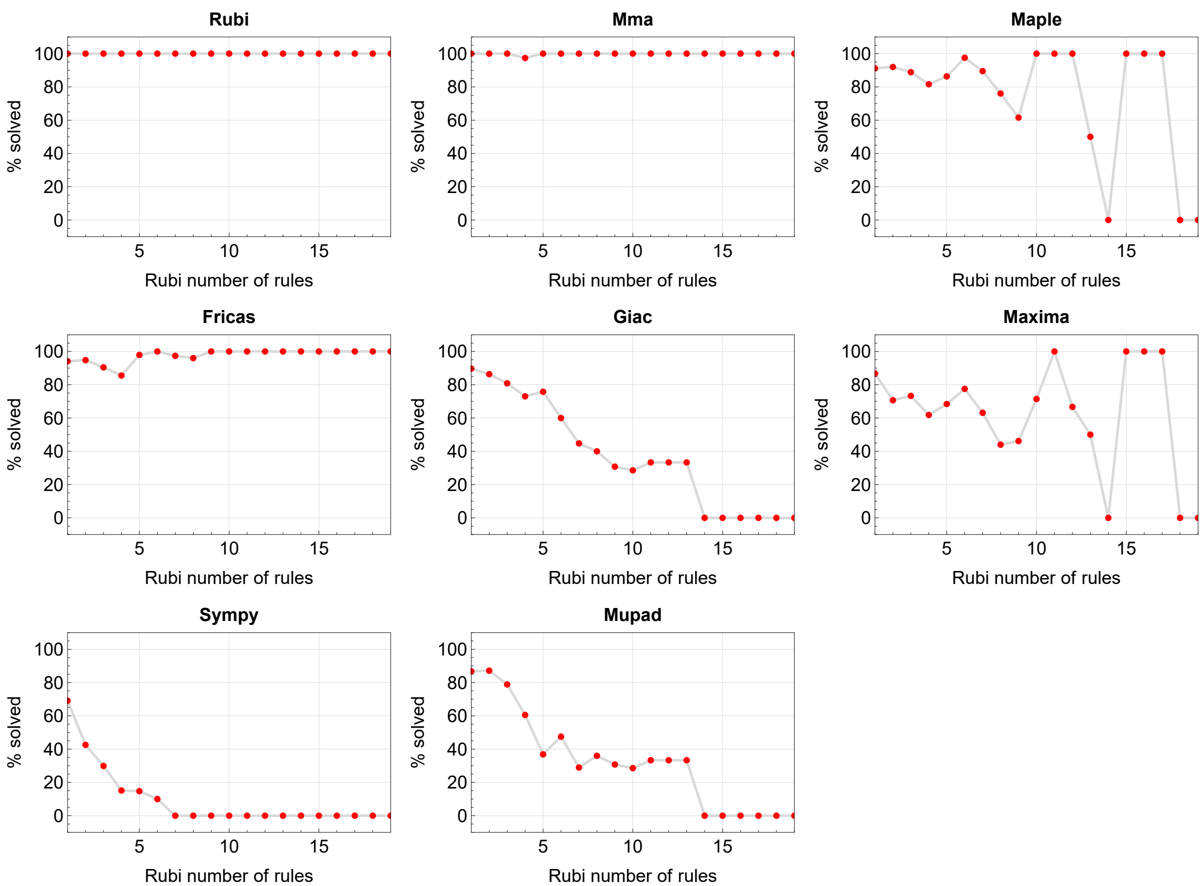


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

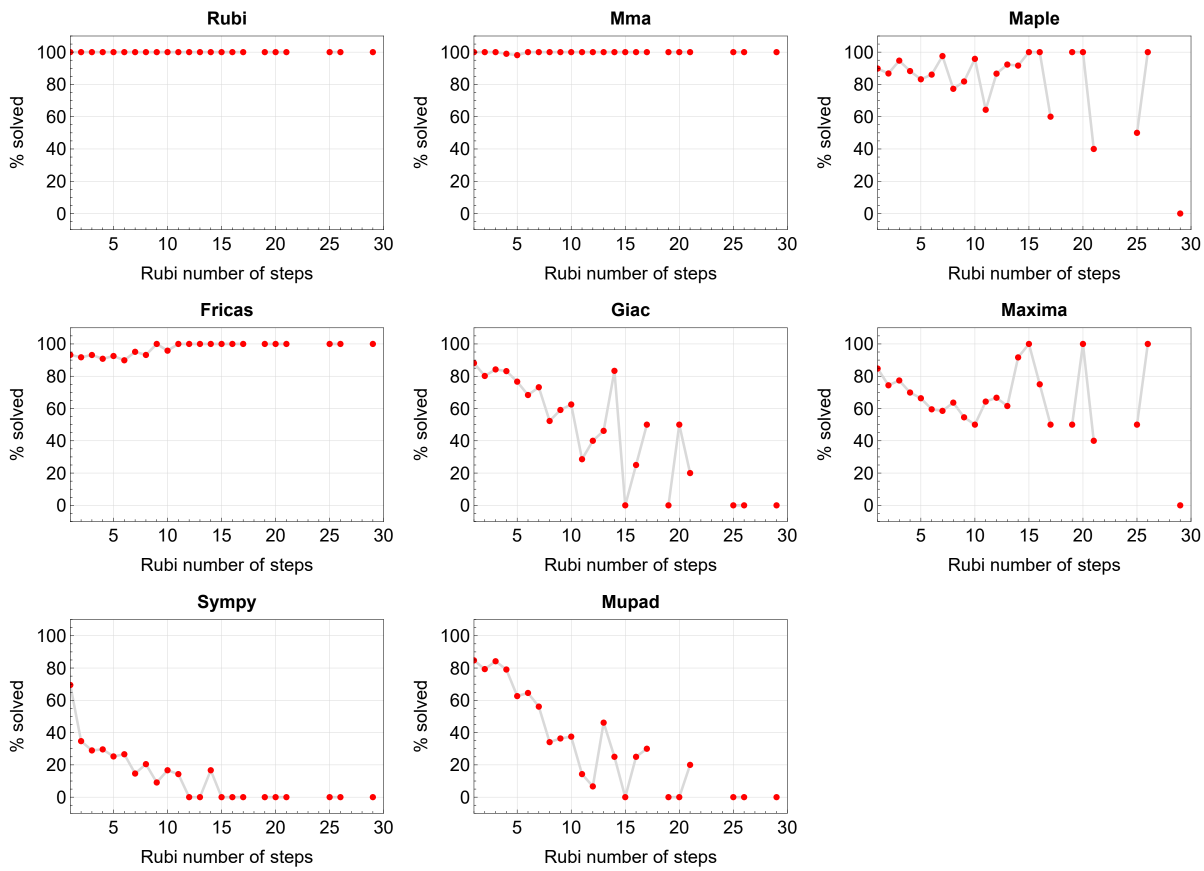


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

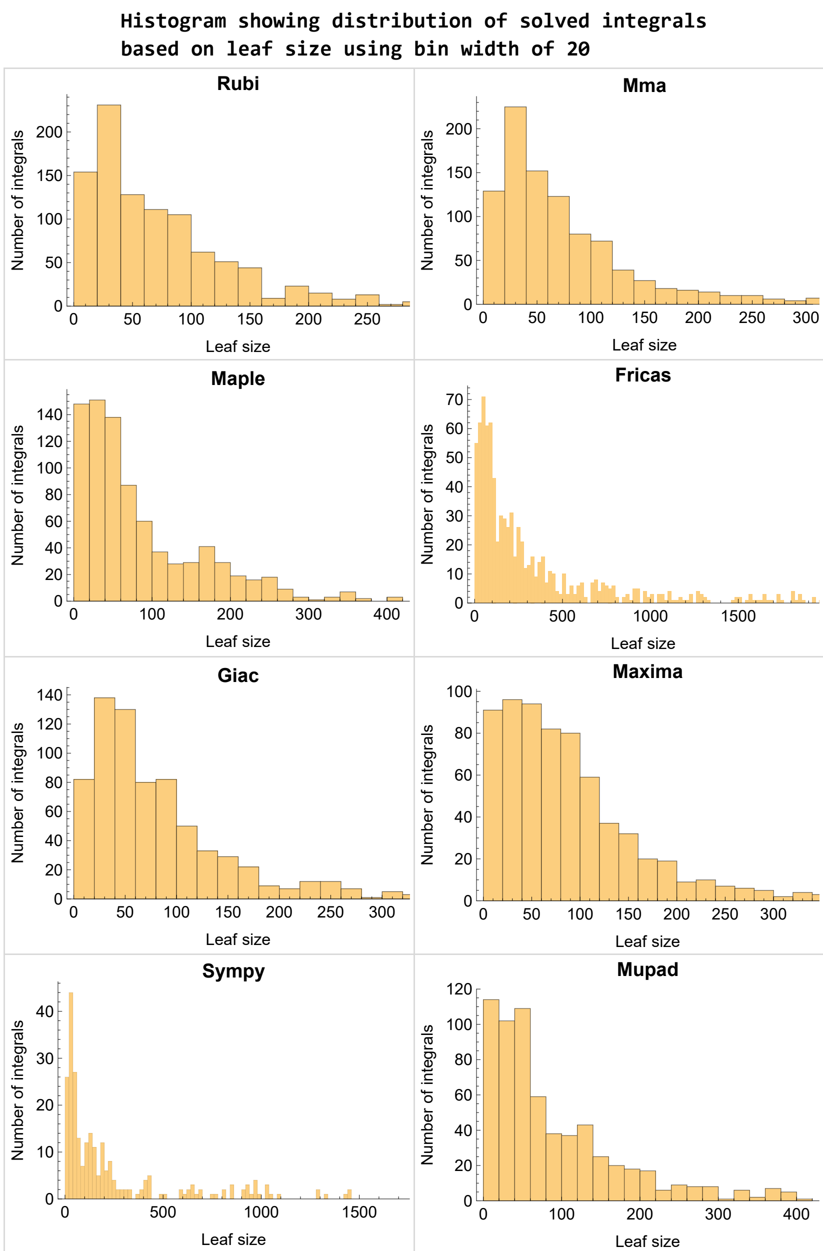


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

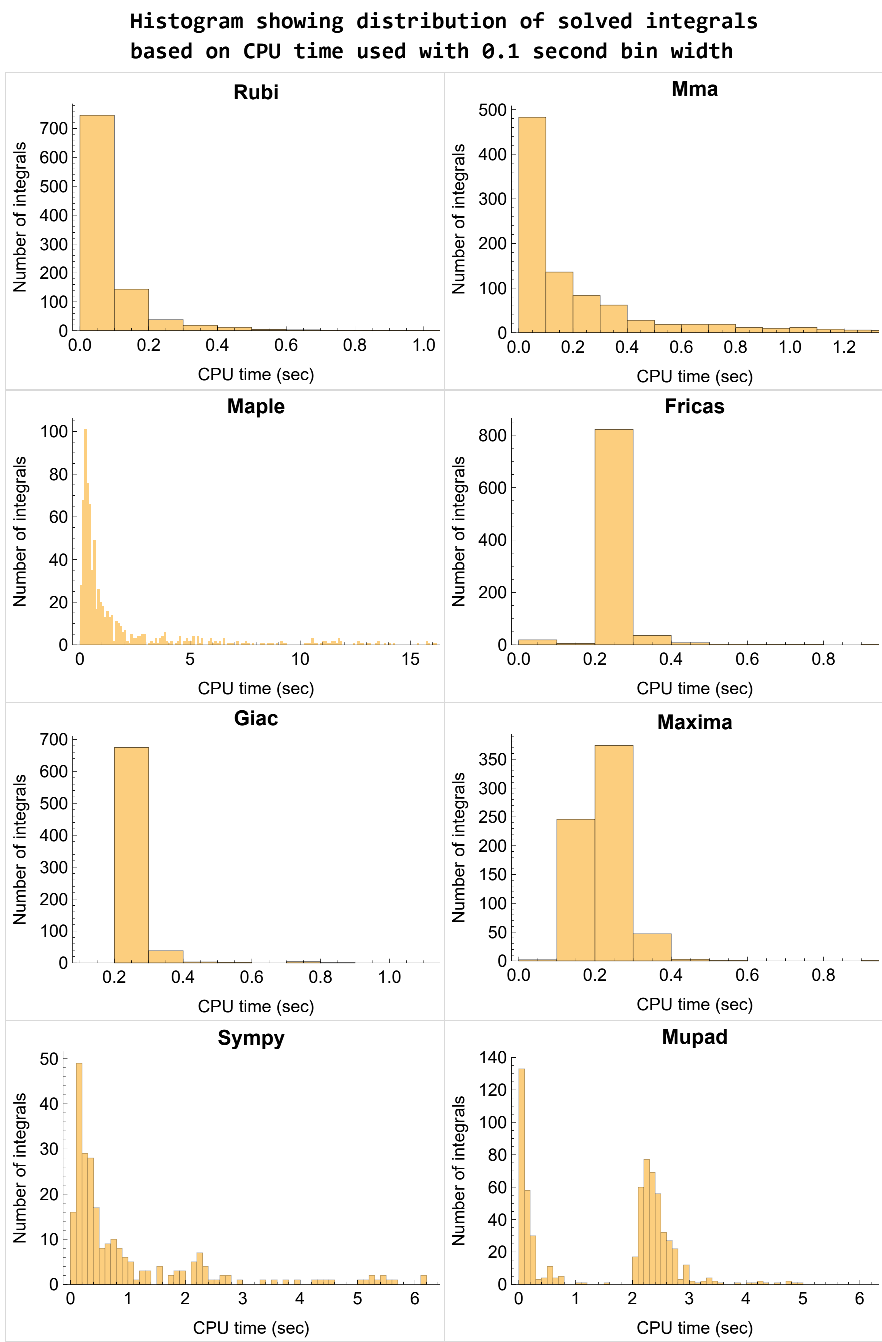


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

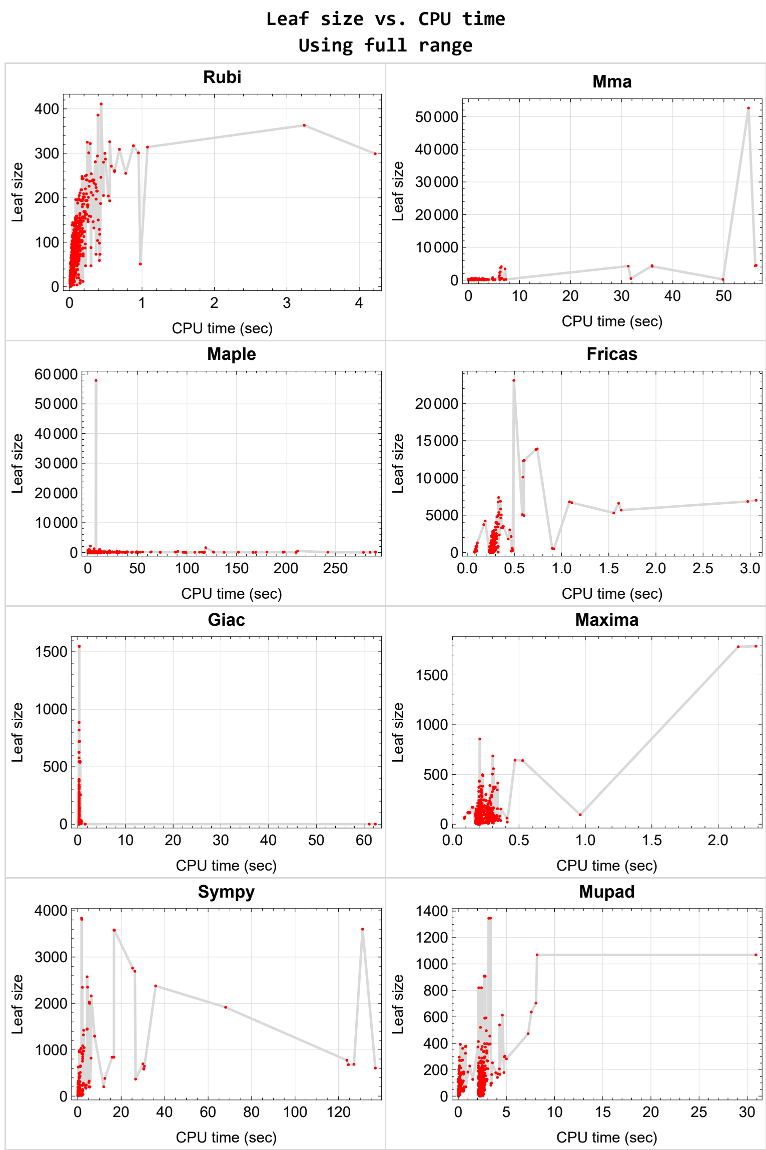


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{334, 339, 340, 341, 346, 347, 348, 353, 354, 355, 360, 361, 362, 367, 368, 369, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 402, 403, 404, 409, 410, 411, 416, 417, 424, 429, 430, 431, 436, 437, 438, 443, 444, 445, 450, 451, 452, 457, 458, 459, 464, 465, 466, 471, 472, 473, 478, 479, 480, 485, 486, 487, 492, 493, 494, 499, 500, 501, 505, 506, 507, 512, 513, 514, 519, 520, 521, 526, 527, 870, 1014, 1015, 1016, 1017}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {622, 761, 762, 763, 765, 766, 767, 768, 769, 774, 775, 776, 777, 819}

Maple {89, 120, 765, 766, 767}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

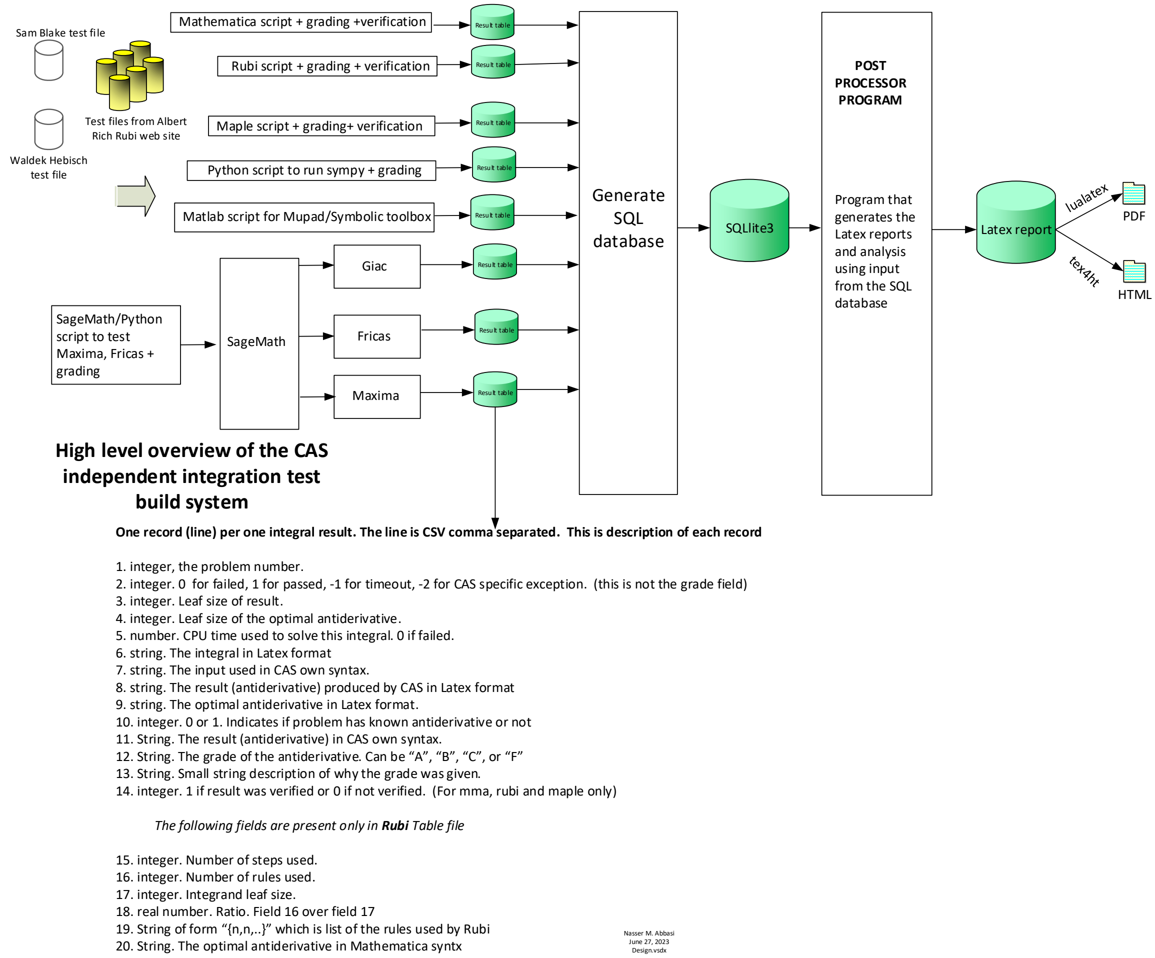
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 342, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 363, 364, 365, 366, 370, 371, 372, 373, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 398, 399, 400, 401, 405, 406, 407, 408, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 432, 433, 434, 435, 439, 440, 441, 442, 446, 447, 448, 449, 453, 454, 455, 456, 460, 461, 462, 463, 467, 468, 469, 470, 474, 475, 476, 477, 481, 482, 483, 484, 488, 489, 490, 491, 495, 496, 497, 498, 502, 503, 504, 508, 509, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 32, 34, 35, 36, 37, 38, 40, 42, 44, 45, 46, 47, 48, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 145, 148, 149, 151, 152, 154, 157, 160, 161, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 237, 239, 240, 241, 242, 243, 244, 246, 248, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 342, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 363, 364, 365, 366, 370, 371, 372, 373, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 398, 399, 400, 401, 405, 406, 407, 408, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 428, 433, 434, 439, 440, 441, 442, 446, 447, 448, 449, 453, 454, 455, 462, 463, 467, 468, 469, 474, 475, 476, 477, 483, 484, 488, 489, 497, 498, 502, 503, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 528, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 543, 544, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 671, 673, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 746, 747, 748, 751, 753, 754, 755, 756, 757, 758, 759, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 809, 810, 811, 812, 813, 814, 815, 816, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 938, 941, 942, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 981, 982, 986, 987, 988, 989, 990, 991, 992, 993, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1054, 1057, 1058, 1059 }

B grade { 8, 24, 100, 121, 122, 123, 127, 143, 144, 150, 155, 156, 162, 189, 234, 245, 254, 427, 432, 456, 460, 461, 470, 481, 482, 490, 495, 496, 508, 509, 563, 575, 584, 614, 616, 648, 658, 668, 674, 677, 687, 702, 704, 745, 749, 750, 752, 760, 808, 882, 949, 980, 983, 984, 994, 998, 999, 1000, 1001, 1002, 1026, 1027, 1053, 1055, 1056 }

C grade { 29, 31, 33, 39, 41, 43, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 146, 147, 153, 158, 159, 165, 211, 213, 215, 229, 230, 231, 232, 233, 236, 238, 247, 249, 435, 491, 504, 529, 531, 540, 542, 545, 547, 556, 558, 576, 590, 591, 592, 593, 594, 595, 620, 622, 670, 672, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 775, 776, 777, 817, 818, 819, 911, 918, 930, 935, 937, 939, 940, 943, 944, 985, 1022 }

F normal fail { }

F(-1) timedout fail { 772, 773, 778, 779 }

F(-2) exception fail { }

Maple

A grade { 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 151, 154, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 200, 206, 207, 212, 216, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 234, 235, 245, 246, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 345, 349, 350, 351, 352, 359, 363, 364, 365, 366, 373, 377, 378, 379, 380, 387, 391, 392, 393, 394, 401, 405, 406, 408, 412, 413, 415, 418, 419, 420, 421, 422, 423, 425, 428, 434, 435, 439, 440, 441, 442, 447, 448, 449, 453, 454, 456, 463, 467, 470, 476, 477, 481, 482, 484, 491, 498, 504, 508, 509, 511, 517, 518, 522, 525, 566, 567, 572, 573, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 623, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 680, 681, 682, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 762, 764, 771, 777, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 792, 795, 798, 801, 802, 803, 804, 805, 806, 807, 809, 810, 811, 812, 813, 814, 815, 816, 818, 819, 822, 823, 824, 825, 832, 833, 834, 835, 836, 837, 848, 855, 856, 857, 864, 866, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 941, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 979, 982, 986, 987, 991, 992, 993, 994, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1038, 1039, 1041, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1055, 1056 }

B grade { 5, 7, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 152, 153, 158, 159, 160, 161, 164, 165, 211, 214, 236, 237, 238, 247, 248, 249, 343, 358, 372, 385, 398, 399, 400, 407, 414, 426, 427, 432, 433, 446, 455, 460, 461, 462, 468, 469, 483, 490, 495, 496, 497, 503, 510, 523, 524, 531, 540, 547, 556, 560, 563, 568, 569, 570, 571, 574, 577, 587, 589, 619, 622, 624, 634, 649, 659, 677, 683, 704, 743, 744, 745, 761, 763, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 778, 779, 790, 791, 793, 794, 796, 797, 799, 800, 817, 820, 821, 830, 838, 839, 842, 843, 844, 845, 846, 847, 851, 852, 853, 858, 859, 860, 861, 862, 863, 865, 867, 868, 869, 983, 984, 990, 997, 1022, 1023, 1040 }

C grade { 2, 3, 4, 6, 89, 120, 143, 144, 145, 150, 155, 156, 157, 162, 201, 202, 203, 204, 208, 209, 210, 213, 215, 217, 218, 219, 220, 231, 232, 233, 240, 241, 242, 243, 244, 344, 386, 808, 826, 827, 828, 829, 831, 840, 841, 937, 938, 939, 940, 942, 943, 944, 980, 981, 985, 988, 989, 1037, 1052, 1053, 1054, 1057, 1058, 1059 }

F normal fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 342, 356, 357, 370, 371, 384, 474, 475, 488, 489, 502, 515, 516, 528, 529, 530, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 561, 562, 564, 565, 678, 679, 684, 685, 849, 850, 854, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 998, 1042 }

F(-1) timedout fail { 97 }

F(-2) exception fail { }

Fricas

A grade { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 71, 75, 78, 102, 106, 109, 167, 173, 179, 182, 193, 195, 196, 199, 216, 224, 225, 226, 228, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 276, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 293, 294, 295, 298, 299, 300, 301, 303, 306, 307, 308, 309, 310, 312, 313, 315, 316, 317, 318, 319, 321, 322, 324, 325, 326, 330, 387, 442, 572, 580, 581, 585, 596, 597, 599, 600, 603, 604, 605, 606, 608, 609, 610, 612, 613, 617, 625, 627, 628, 629, 630, 632, 635, 637, 638, 639, 640, 642, 647, 657, 658, 660, 667, 668, 669, 670, 675, 681, 688, 691, 694, 695, 724, 727, 730, 732, 733, 736, 739, 740, 741, 742, 746, 747, 748, 771, 777, 780, 781, 782, 783, 784, 785, 786, 787, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 811, 812, 813, 814, 815, 816, 823, 824, 825, 838, 839, 842, 855, 856, 857, 864, 872, 879, 880, 886, 887, 893, 894, 901, 903, 910, 915, 920, 922, 927, 948, 955, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 989, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1035, 1036, 1043, 1044, 1055 }

B grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 183, 184, 185, 186, 187, 192, 194, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 263, 272, 275, 277, 284, 290, 291, 292, 296, 297, 302, 304, 305, 311, 314, 320, 323, 327, 328, 329, 331, 332, 333, 338, 342, 343, 344, 345, 350, 351, 352, 356, 357, 358, 359, 365, 366, 370, 371, 372, 373, 380, 384, 385, 386, 394, 398, 399, 400, 401, 405, 406, 407, 408, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 432, 433, 434, 435, 439, 440, 441, 446, 447, 448, 449, 453, 454, 455, 456, 460, 461, 462, 463, 470, 474, 475, 476, 477, 484, 488, 489, 490, 491, 497, 498, 502, 503, 504, 511, 515, 516, 517, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 586, 587, 588, 589, 598, 601, 602, 607, 611, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 626, 631, 633, 634, 636, 641, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 661, 662, 663, 664, 665, 666, 671, 672, 673, 674, 676, 677, 678, 679, 680, 682, 683, 684, 685, 686, 687, 689, 690, 692, 693, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 731, 734, 735, 737, 738, 743, 744, 745, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 768, 769, 770, 772, 773, 774, 775, 776, 778, 779, 788, 790, 791, 793, 794, 796, 797, 799, 800, 801, 808, 809, 810, 817, 818, 819, 820, 821, 822, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 840, 841, 843, 844, 845, 846, 847, 848, 849, 850, 858, 859, 860, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 885, 897, 898, 902, 904, 905, 906, 907, 908, 909, 911, 912, 913, 914, 916, 917, 918, 919, 921, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 944, 945, 946, 947, 952, 953, 954, 959, 960, 961, 962, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1037, 1038, 1039, 1040, 1041, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1056, 1057, 1058, 1059 }

C grade { 335, 336, 337, 349, 363, 364, 377, 378, 379, 391, 392, 393, 467, 468, 469, 481, 482, 483, 495, 496, 508, 509, 510, 522, 523, 524, 590, 591, 592, 593, 594, 595, 761, 762, 763, 764, 765, 766, 767, 851, 852, 853, 937, 938, 939, 940 }

F normal fail { 188, 189, 190, 191, 205, 239, 854, 861, 862, 863, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965 }

F(-1) timedout fail { }

F(-2) exception fail { 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 966, 1042 }

Maxima

A grade { 1, 2, 3, 4, 6, 8, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 29, 30, 42, 70, 75, 77, 79, 82, 84, 85, 86, 106, 113, 115, 116, 117, 139, 140, 143, 149, 150, 161, 162, 206, 216, 235, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 335, 336, 337, 338, 344, 349, 352, 359, 363, 364, 366, 377, 378, 379, 386, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 423, 425, 426, 432, 433, 435, 439, 440, 446, 449, 453, 454, 455, 460, 467, 468, 469, 476, 481, 482, 483, 491, 496, 498, 508, 509, 510, 517, 522, 523, 524, 575, 576, 580, 581, 583, 586, 596, 599, 600, 601, 602, 603, 604, 605, 608, 609, 610, 611, 612, 613, 617, 618, 620, 627, 628, 629, 630, 631, 632, 633, 637, 638, 640, 641, 642, 647, 648, 658, 660, 662, 668, 669, 670, 671, 672, 674, 675, 680, 681, 688, 690, 691, 693, 697, 700, 707, 709, 711, 713, 715, 717, 719, 721, 723, 730, 731, 732, 733, 739, 740, 741, 746, 747, 748, 753, 754, 755, 756, 783, 786, 802, 803, 804, 805, 806, 807, 809, 811, 812, 813, 814, 815, 816, 823, 824, 825, 842, 843, 844, 845, 846, 847, 848, 851, 852, 853, 855, 856, 857, 858, 859, 872, 873, 874, 875, 876, 877, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 967, 968, 969, 973, 974, 975, 979, 981, 982, 986, 993, 994, 1013, 1019, 1020, 1026, 1027, 1028, 1031, 1033, 1034, 1037, 1038, 1043, 1044, 1045, 1046, 1050, 1051, 1055, 1056 }

B grade { 5, 7, 10, 11, 13, 14, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 67, 68, 71, 72, 73, 74, 76, 78, 80, 81, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 192, 193, 194, 196, 197, 198, 200, 201, 207, 208, 211, 212, 217, 218, 221, 222, 223, 225, 226, 227, 229, 230, 234, 236, 240, 241, 245, 246, 247, 262, 277, 283, 302, 320, 329, 345, 350, 351, 365, 373, 380, 387, 394, 401, 408, 415, 421, 422, 427, 428, 434, 441, 442, 447, 448, 456, 461, 462, 463, 470, 477, 484, 495, 497, 504, 511, 518, 525, 560, 561, 562, 563, 564, 565, 566, 567, 570, 573, 574, 577, 579, 582, 584, 588, 597, 598, 606, 607, 614, 615, 616, 619, 621, 622, 623, 624, 625, 626, 634, 635, 636, 639, 643, 644, 645, 646, 649, 651, 653, 654, 655, 656, 657, 659, 661, 663, 664, 665, 666, 667, 673, 676, 677, 678, 679, 682, 683, 684, 685, 686, 687, 702, 703, 704, 705, 735, 749, 750, 751, 752, 768, 769, 770, 774, 775, 776, 808, 810, 817, 818, 819, 820, 821, 822, 830, 838, 839, 840, 841, 860, 980, 983, 984, 987, 990, 991, 992, 995, 996, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1018, 1021, 1022, 1023, 1024, 1025, 1030, 1032, 1040, 1048, 1049, 1053, 1054, 1057, 1058 }

C grade { }

F normal fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 188, 189, 190, 191, 202, 203, 204, 205, 209, 210, 213, 214, 215, 219, 220, 231, 232, 233, 237, 238, 239, 242, 243, 244, 248, 249, 342, 343, 356, 357, 358, 370, 371, 372, 384, 385, 474, 475, 488, 489, 490, 502, 503, 515, 516, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 590, 591, 592, 593, 594, 595, 761, 762, 763, 764, 765, 766, 767, 771, 772, 773, 777, 778, 779, 826, 827, 828, 829, 831, 832, 833, 834, 835, 837, 849, 850, 854, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 985, 988, 989, 997, 998, 999, 1000, 1001, 1002, 1029, 1039, 1041, 1042, 1047, 1052, 1059 }

F(-1) timedout fail { }

F(-2) exception fail { 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 195, 199, 224, 228, 418, 419, 420, 568, 569, 571, 572, 578, 585, 587, 589, 650, 652, 689, 692, 694, 695, 696, 698, 699, 701, 706, 708, 710, 712, 714, 716, 718, 720, 722, 724, 725, 726, 727, 728, 729, 734, 736, 737, 738, 742, 743, 744, 745, 757, 758, 759, 760, 780, 781, 782, 784, 785, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 836, 878, 879, 880, 885, 886, 887, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 1035, 1036 }

Giac

A grade { 1, 2, 3, 4, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 30, 32, 33, 40, 42, 43, 47, 48, 70, 71, 73, 77, 78, 86, 87, 88, 90, 95, 97, 101, 108, 109, 117, 118, 119, 143, 145, 149, 150, 151, 152, 154, 155, 157, 161, 162, 163, 164, 166, 170, 171, 176, 177, 182, 183, 184, 197, 202, 203, 204, 206, 209, 216, 217, 219, 222, 235, 237, 241, 243, 251, 252, 253, 255, 256, 257, 258, 260, 261, 262, 264, 265, 266, 267, 269, 270, 271, 273, 274, 275, 276, 281, 282, 283, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 303, 304, 305, 306, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 326, 327, 328, 330, 331, 332, 333, 359, 366, 387, 408, 435, 498, 566, 567, 568, 569, 572, 573, 575, 576, 578, 579, 585, 586, 587, 588, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 680, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 740, 742, 743, 746, 747, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 795, 796, 798, 799, 801, 802, 803, 804, 805, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 819, 822, 823, 824, 825, 826, 828, 829, 830, 831, 832, 834, 835, 837, 838, 839, 840, 841, 855, 856, 857, 858, 859, 860, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 980, 981, 985, 986, 987, 988, 989, 995, 1020, 1033, 1034, 1035, 1036, 1037, 1043, 1044, 1046, 1052, 1053, 1055, 1056, 1058, 1059 }

B grade { 5, 7, 8, 10, 11, 13, 14, 24, 25, 26, 27, 28, 29, 31, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 69, 72, 74, 75, 76, 79, 80, 81, 82, 84, 85, 91, 93, 94, 96, 98, 99, 100, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 115, 116, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 146, 147, 148, 153, 156, 158, 159, 160, 165, 167, 168, 169, 172, 173, 174, 175, 178, 179, 180, 181, 185, 186, 187, 192, 193, 194, 195, 196, 198, 199, 200, 201, 207, 208, 210, 211, 212, 213, 214, 215, 218, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 238, 240, 242, 244, 245, 246, 247, 248, 249, 254, 263, 272, 277, 278, 279, 284, 302, 311, 320, 329, 338, 344, 345, 350, 351, 352, 365, 373, 380, 386, 394, 401, 415, 418, 427, 428, 434, 441, 442, 447, 448, 449, 456, 463, 470, 477, 484, 491, 497, 504, 511, 518, 525, 570, 571, 574, 577, 580, 581, 582, 583, 584, 589, 596, 605, 614, 616, 644, 646, 648, 658, 659, 668, 674, 675, 676, 681, 682, 686, 687, 702, 703, 704, 705, 739, 741, 744, 745, 748, 749, 750, 751, 752, 753, 768, 769, 770, 771, 774, 775, 776, 777, 791, 794, 797, 800, 810, 817, 818, 820, 821, 836, 893, 894, 979, 982, 983, 984, 990, 991, 992, 993, 994, 996, 997, 998, 999, 1001, 1002, 1003, 1006, 1007, 1010, 1011, 1012, 1013, 1018, 1019, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1030, 1031, 1032, 1040, 1045, 1047, 1054, 1057 }

C grade { 897, 898, 1000, 1050, 1051 }

F normal fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 83, 89, 92, 114, 120, 126, 188, 189, 190, 191, 205, 239, 250, 259, 268, 280, 289, 298, 307, 316, 325, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 475, 476, 481, 482, 483, 488, 489, 490, 495, 496, 502, 503, 508, 509, 510, 515, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 590, 591, 592, 593, 594, 595, 677, 678, 679, 683, 684, 685, 761, 762, 763, 764, 765, 766, 767, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 1004, 1005, 1008, 1009, 1029, 1039, 1041, 1042, 1048, 1049 }

F(-1) timedout fail { 474, 827, 833 }

F(-2) exception fail { 493, 516, 544, 757, 758, 759, 760, 772, 773, 778, 779, 861, 862, 863, 864, 865, 866, 1028, 1038 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 149, 150, 152, 153, 155, 156, 158, 159, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 290, 291, 292, 293, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 326, 327, 328, 329, 338, 344, 345, 350, 351, 352, 359, 365, 366, 373, 380, 386, 387, 394, 401, 408, 415, 418, 427, 428, 434, 435, 441, 442, 447, 448, 449, 456, 463, 470, 477, 484, 491, 497, 498, 504, 511, 518, 525, 560, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 746, 747, 748, 749, 753, 754, 755, 756, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 792, 795, 798, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 830, 836, 838, 839, 840, 841, 855, 856, 857, 858, 859, 872, 873, 874, 875, 876, 877, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059 }

C grade { }

F normal fail { }

F(-1) timedout fail { 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 145, 148, 151, 154, 157, 160, 163, 166, 188, 189, 190, 191, 205, 239, 250, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 335, 336, 337, 342, 343, 349, 356, 357, 358, 363, 364, 370, 371, 372, 377, 378, 379, 384, 385, 391, 392, 393, 398, 399, 400, 405, 406, 407, 412, 413, 414, 419, 420, 421, 422, 423, 425, 426, 432, 433, 439, 440, 446, 453, 454, 455, 460, 461, 462, 467, 468, 469, 474, 475, 476, 481, 482, 483, 488, 489, 490, 495, 496, 502, 503, 508, 509, 510, 515, 516, 517, 522, 523, 524, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 561, 562, 564, 565, 590, 591, 592, 593, 594, 595, 621, 622, 623, 651, 652, 653, 678, 679, 684, 685, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 790, 791, 793, 794, 796, 797, 799, 800, 801, 826, 827, 828, 829, 831, 832, 833, 834, 835, 837, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 871, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 949, 950, 951, 956, 957, 958, 963, 964, 965, 966, 970, 971, 972, 976, 977, 978, 998, 999, 1000, 1001, 1002, 1037, 1038, 1039, 1041, 1050, 1051 }

F(-2) exception fail { }

Sympy

A grade { 1, 8, 82, 96, 97, 113, 132, 193, 194, 196, 197, 198, 199, 222, 223, 224, 225, 226, 227, 251, 252, 253, 254, 260, 261, 262, 263, 269, 270, 271, 272, 281, 282, 283, 284, 299, 300, 301, 302, 308, 309, 310, 311, 317, 318, 319, 320, 329, 580, 596, 599, 600, 605, 606, 608, 609, 618, 628, 638, 648, 668, 674, 680, 730, 731, 732, 739, 740, 741, 746, 747, 748, 749, 755, 756, 855, 857, 1003, 1018, 1026, 1027, 1030, 1031, 1032, 1043, 1045, 1055 }

B grade { 2, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 81, 83, 85, 87, 88, 105, 112, 114, 131, 133, 134, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 195, 221, 228, 290, 291, 292, 293, 326, 327, 328, 568, 569, 574, 575, 576, 577, 581, 582, 583, 584, 585, 586, 597, 598, 601, 602, 607, 610, 611, 619, 621, 623, 629, 631, 633, 639, 641, 643, 658, 688, 689, 691, 692, 697, 700, 703, 705, 706, 715, 724, 727, 733, 736, 753, 754, 783, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 838, 839, 840, 841, 856, 872, 873, 874, 878, 879, 880, 885, 886, 887, 893, 894, 897, 898, 901, 902, 903, 907, 908, 909, 913, 914, 915, 919, 920, 921, 925, 926, 927, 931, 932, 933, 945, 946, 947, 948, 952, 953, 954, 955, 959, 960, 961, 962, 967, 968, 969, 973, 974, 975, 983, 984, 1028, 1044, 1050, 1051 }

C grade { 566, 567, 1052, 1053, 1054 }

F normal fail { 3, 4, 5, 6, 7, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 59, 60, 61, 62, 63, 64, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 188, 189, 190, 191, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 264, 265, 266, 267, 268, 273, 274, 275, 276, 277, 278, 279, 280, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 330, 331, 332, 333, 335, 336, 337, 338, 342, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 363, 364, 365, 366, 370, 371, 372, 373, 377, 378, 379, 380, 384, 385, 386, 387, 392, 393, 394, 398, 399, 400, 401, 405, 406, 407, 408, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 433, 434, 435, 441, 442, 447, 448, 449, 455, 456, 463, 467, 468, 469, 470, 474, 475, 476, 477, 481, 482, 483, 484, 488, 489, 490, 491, 495, 496, 497, 498, 502, 503, 504, 508, 509, 510, 511, 515, 516, 517, 518, 522, 523, 524, 525, 530, 531, 532, 540, 541, 542, 543, 545, 546, 547, 548, 549, 556, 557, 558, 560, 561, 563, 572, 573, 578, 579, 590, 591, 593, 594, 603, 604, 612, 613, 614, 615, 616, 617, 620, 622, 624, 625, 626, 627, 630, 632, 634, 635, 636, 637, 640, 642, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 675, 676, 677, 681, 682, 683, 684, 686, 687, 694, 695, 762, 763, 764, 765, 769, 770, 771, 772, 773, 775, 776, 777, 778, 780, 781, 784, 785, 786, 787, 788, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 862, 863, 864, 865, 866, 868, 869, 871, 875, 876, 877, 881, 882, 883, 884, 888, 889, 890, 891, 892, 895, 896, 899, 900, 904, 922, 937, 938, 939, 940, 941, 942, 943, 944, 949, 966, 970, 971, 972, 976, 977, 978, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059 }

F(-1) timedout fail { 49, 50, 55, 56, 57, 58, 65, 66, 67, 68, 69, 369, 383, 390, 391, 404, 411, 412, 424, 431, 432, 438, 439, 440, 445, 446, 452, 453, 454, 459, 460, 461, 462, 528, 529, 533, 534, 535, 536, 537, 538, 539, 544, 550, 551, 552, 553, 554, 555, 559, 562, 564, 565, 570, 571, 587, 588, 589, 592, 595, 678, 679, 685, 690, 693, 696, 698, 699, 701, 702, 704, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 725, 726, 728, 729, 734, 735, 737, 738, 742, 743, 744, 745, 750, 751, 752, 757, 758, 759, 760, 761, 766, 767, 768, 774, 779, 782, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 854, 858, 859, 860, 861, 867, 905, 906, 910, 911, 912, 916, 917, 918, 923, 924, 928, 929, 930, 934, 935, 936, 950, 951, 956, 957, 958, 963, 964, 965, 1013, 1029, 1042 }

F(-2) exception fail { 250 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	21	34	19	21	21
N.S.	1	1.00	1.00	0.77	0.95	1.55	0.86	0.95	0.95
time (sec)	N/A	0.012	0.067	0.203	0.261	0.253	0.134	0.278	0.110

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	185	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	8.41	0.95	0.95
time (sec)	N/A	0.018	0.090	2.945	0.269	0.245	3.537	0.270	0.095

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	0	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	0.00	0.95	0.95
time (sec)	N/A	0.030	0.383	2.888	0.269	0.249	0.000	0.269	2.203

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	0	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	0.00	0.95	0.95
time (sec)	N/A	0.029	0.271	0.584	0.269	0.265	0.000	0.261	2.151

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	40	69	89	0	51	57
N.S.	1	1.00	1.00	1.82	3.14	4.05	0.00	2.32	2.59
time (sec)	N/A	0.030	0.294	0.545	0.264	0.262	0.000	0.269	0.466

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	44	21	47	0	21	21
N.S.	1	1.00	1.00	2.00	0.95	2.14	0.00	0.95	0.95
time (sec)	N/A	0.030	0.348	0.627	0.266	0.256	0.000	0.261	0.166

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	40	38	89	0	48	57
N.S.	1	1.00	1.00	1.82	1.73	4.05	0.00	2.18	2.59
time (sec)	N/A	0.031	0.525	0.608	0.264	0.257	0.000	0.266	2.506

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13
N.S.	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87
time (sec)	N/A	0.010	0.017	0.257	0.173	0.246	0.079	0.260	0.060

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	35	19
N.S.	1	1.00	1.00	1.05	1.00	3.58	2.58	1.84	1.00
time (sec)	N/A	0.018	0.007	6.837	0.176	0.258	0.334	0.264	2.192

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	54	373	175	638	327	135
N.S.	1	1.00	1.00	1.38	9.56	4.49	16.36	8.38	3.46
time (sec)	N/A	0.033	0.047	197.020	0.307	0.262	1.214	0.334	2.423

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	82	686	379	2574	722	255
N.S.	1	1.00	0.83	1.39	11.63	6.42	43.63	12.24	4.32
time (sec)	N/A	0.037	0.138	0.144	0.303	0.267	4.323	0.379	2.637

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	35	19
N.S.	1	1.00	1.00	1.05	1.00	3.58	2.58	1.84	1.00
time (sec)	N/A	0.020	0.007	4.891	0.172	0.257	0.371	0.297	2.207

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	44	55	293	189	648	325	132
N.S.	1	1.00	1.10	1.38	7.32	4.72	16.20	8.12	3.30
time (sec)	N/A	0.036	0.215	0.132	0.294	0.266	1.216	0.319	0.169

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	77	82	558	407	2351	721	254
N.S.	1	1.00	1.31	1.39	9.46	6.90	39.85	12.22	4.31
time (sec)	N/A	0.045	0.256	0.071	0.307	0.262	4.536	0.365	2.372

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	23	33	39	40	92	32	18
N.S.	1	1.00	0.50	0.72	0.85	0.87	2.00	0.70	0.39
time (sec)	N/A	0.031	0.069	3.213	0.173	0.242	0.162	0.274	0.101

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	61	88	90	136	88	43
N.S.	1	1.00	0.58	0.88	1.28	1.30	1.97	1.28	0.62
time (sec)	N/A	0.053	0.066	50.433	0.178	0.254	0.329	0.284	2.254

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	52	79	110	138	189	116	53
N.S.	1	1.00	0.57	0.86	1.20	1.50	2.05	1.26	0.58
time (sec)	N/A	0.079	0.085	0.049	0.177	0.238	0.682	0.278	2.397

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	40	56	88	90	136	88	42
N.S.	1	1.00	0.60	0.84	1.31	1.34	2.03	1.31	0.63
time (sec)	N/A	0.038	0.035	22.627	0.177	0.248	0.340	0.272	2.259

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	33	74	66	97	189	60	32
N.S.	1	1.00	0.37	0.82	0.73	1.08	2.10	0.67	0.36
time (sec)	N/A	0.064	0.063	0.347	0.175	0.253	0.690	0.266	0.197

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	62	92	132	197	231	144	65
N.S.	1	1.00	0.55	0.81	1.17	1.74	2.04	1.27	0.58
time (sec)	N/A	0.088	0.117	0.056	0.176	0.252	1.304	0.284	2.394

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	52	66	110	138	189	116	53
N.S.	1	1.00	0.59	0.75	1.25	1.57	2.15	1.32	0.60
time (sec)	N/A	0.050	0.059	127.172	0.174	0.261	0.674	0.283	2.416

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	62	84	132	195	231	144	65
N.S.	1	1.00	0.56	0.76	1.19	1.76	2.08	1.30	0.59
time (sec)	N/A	0.074	0.081	0.046	0.178	0.250	1.313	0.278	0.251

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	43	102	86	179	277	88	42
N.S.	1	1.00	0.32	0.76	0.64	1.34	2.07	0.66	0.31
time (sec)	N/A	0.098	0.108	0.085	0.172	0.250	2.623	0.281	2.687

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	50	60	0	41	30
N.S.	1	1.00	2.82	1.09	4.55	5.45	0.00	3.73	2.73
time (sec)	N/A	0.009	0.024	0.388	0.262	0.239	0.000	0.282	2.149

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	23	61	155	0	64	52
N.S.	1	1.00	1.83	1.00	2.65	6.74	0.00	2.78	2.26
time (sec)	N/A	0.018	0.119	0.893	0.177	0.243	0.000	0.274	0.069

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	23	88	371	0	93	78
N.S.	1	1.00	1.33	0.85	3.26	13.74	0.00	3.44	2.89
time (sec)	N/A	0.019	0.027	2.802	0.252	0.263	0.000	0.272	2.140

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	57	33	108	697	0	88	133
N.S.	1	1.00	1.50	0.87	2.84	18.34	0.00	2.32	3.50
time (sec)	N/A	0.023	0.089	7.685	0.178	0.261	0.000	0.265	2.162

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	33	131	1073	0	122	169
N.S.	1	1.00	1.15	0.82	3.28	26.82	0.00	3.05	4.22
time (sec)	N/A	0.023	0.065	21.980	0.265	0.256	0.000	0.280	2.181

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	29	25	43	103	0	54	48
N.S.	1	1.00	1.21	1.04	1.79	4.29	0.00	2.25	2.00
time (sec)	N/A	0.017	0.014	0.677	0.260	0.241	0.000	0.264	2.159

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	13	32	18	81	0	18	18
N.S.	1	1.00	0.57	1.39	0.78	3.52	0.00	0.78	0.78
time (sec)	N/A	0.023	0.051	2.356	0.171	0.248	0.000	0.291	0.080

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	47	90	511	0	102	107
N.S.	1	1.00	0.59	0.96	1.84	10.43	0.00	2.08	2.18
time (sec)	N/A	0.030	0.012	6.325	0.261	0.263	0.000	0.287	2.245

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	44	94	230	0	60	152
N.S.	1	1.00	1.21	1.16	2.47	6.05	0.00	1.58	4.00
time (sec)	N/A	0.028	0.130	16.523	0.178	0.250	0.000	0.280	2.206

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	29	60	136	1183	0	124	210
N.S.	1	1.00	0.41	0.86	1.94	16.90	0.00	1.77	3.00
time (sec)	N/A	0.032	0.015	39.874	0.258	0.258	0.000	0.279	0.079

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	25	91	379	0	93	78
N.S.	1	1.00	1.21	0.89	3.25	13.54	0.00	3.32	2.79
time (sec)	N/A	0.019	0.031	1.220	0.257	0.251	0.000	0.267	0.079

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	43	106	709	0	110	111
N.S.	1	1.00	1.76	0.88	2.16	14.47	0.00	2.24	2.27
time (sec)	N/A	0.034	0.120	4.115	0.175	0.260	0.000	0.282	2.212

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	43	102	774	0	96	96
N.S.	1	1.00	1.42	1.00	2.37	18.00	0.00	2.23	2.23
time (sec)	N/A	0.033	0.060	11.823	0.268	0.263	0.000	0.279	2.311

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	101	53	149	1573	0	128	192
N.S.	1	1.00	1.53	0.80	2.26	23.83	0.00	1.94	2.91
time (sec)	N/A	0.038	0.113	30.047	0.186	0.252	0.000	0.281	2.228

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	53	181	2103	0	171	187
N.S.	1	1.00	0.93	0.91	3.12	36.26	0.00	2.95	3.22
time (sec)	N/A	0.037	0.351	97.623	0.256	0.258	0.000	0.277	0.081

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	33	90	515	0	80	129
N.S.	1	1.00	0.89	0.89	2.43	13.92	0.00	2.16	3.49
time (sec)	N/A	0.021	0.011	2.813	0.267	0.249	0.000	0.269	0.080

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	45	90	229	0	60	153
N.S.	1	1.00	1.22	1.22	2.43	6.19	0.00	1.62	4.14
time (sec)	N/A	0.027	0.110	9.110	0.173	0.240	0.000	0.287	2.130

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	33	65	132	1176	0	124	187
N.S.	1	1.00	0.50	0.98	2.00	17.82	0.00	1.88	2.83
time (sec)	N/A	0.032	0.012	22.996	0.263	0.252	0.000	0.294	2.123

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	45	90	330	0	31	31
N.S.	1	1.00	0.81	0.85	1.70	6.23	0.00	0.58	0.58
time (sec)	N/A	0.029	0.042	72.999	0.172	0.230	0.000	0.277	0.063

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	33	78	178	2092	0	148	291
N.S.	1	1.00	0.37	0.88	2.00	23.51	0.00	1.66	3.27
time (sec)	N/A	0.036	0.013	151.886	0.265	0.264	0.000	0.281	2.161

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	46	33	133	1082	0	122	169
N.S.	1	1.00	1.18	0.85	3.41	27.74	0.00	3.13	4.33
time (sec)	N/A	0.025	0.088	5.368	0.265	0.259	0.000	0.284	0.066

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	123	61	155	1591	0	130	214
N.S.	1	1.00	1.76	0.87	2.21	22.73	0.00	1.86	3.06
time (sec)	N/A	0.038	0.130	15.399	0.176	0.265	0.000	0.286	2.134

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	61	179	2114	0	171	187
N.S.	1	1.00	0.97	1.05	3.09	36.45	0.00	2.95	3.22
time (sec)	N/A	0.036	0.132	36.280	0.265	0.272	0.000	0.283	2.202

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	139	71	195	2802	0	152	295
N.S.	1	1.00	1.56	0.80	2.19	31.48	0.00	1.71	3.31
time (sec)	N/A	0.042	0.132	115.248	0.203	0.267	0.000	0.282	0.098

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	91	70	150	2231	0	124	205
N.S.	1	1.00	1.32	1.01	2.17	32.33	0.00	1.80	2.97
time (sec)	N/A	0.040	0.059	242.163	0.267	0.255	0.000	0.280	2.203

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	59	0	0	997	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	9.41	0.00	0.00	0.00
time (sec)	N/A	0.083	0.048	0.000	0.000	0.273	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	591	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	7.30	0.00	0.00	0.00
time (sec)	N/A	0.062	0.034	0.000	0.000	0.269	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	310	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.056	0.037	0.000	0.000	0.257	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	59	0	0	142	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.029	0.024	0.000	0.000	0.260	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	144	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	2.67	0.00	0.00	0.00
time (sec)	N/A	0.031	0.020	0.000	0.000	0.253	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	57	0	0	311	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	0.054	0.022	0.000	0.000	0.253	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	598	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	7.38	0.00	0.00	0.00
time (sec)	N/A	0.054	0.024	0.000	0.000	0.275	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	59	0	0	1001	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	9.44	0.00	0.00	0.00
time (sec)	N/A	0.079	0.027	0.000	0.000	0.270	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	1042	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	6.72	0.00	0.00	0.00
time (sec)	N/A	0.135	0.040	0.000	0.000	0.277	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	751	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.143	0.042	0.000	0.000	0.280	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	59	0	0	1003	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.186	0.039	0.000	0.000	0.275	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	59	0	0	727	0	0	0
N.S.	1	1.00	0.27	0.00	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.166	0.027	0.000	0.000	0.278	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	59	0	0	572	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	4.47	0.00	0.00	0.00
time (sec)	N/A	0.058	0.023	0.000	0.000	0.279	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	59	0	0	578	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	4.52	0.00	0.00	0.00
time (sec)	N/A	0.061	0.021	0.000	0.000	0.269	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	57	0	0	723	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.157	0.018	0.000	0.000	0.281	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	57	0	0	1013	0	0	0
N.S.	1	1.00	0.23	0.00	0.00	4.17	0.00	0.00	0.00
time (sec)	N/A	0.173	0.023	0.000	0.000	0.275	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	749	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	4.83	0.00	0.00	0.00
time (sec)	N/A	0.087	0.022	0.000	0.000	0.273	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	59	0	0	1056	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	6.81	0.00	0.00	0.00
time (sec)	N/A	0.080	0.024	0.000	0.000	0.272	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	6
N.S.	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.38
time (sec)	N/A	0.020	0.008	0.000	0.278	0.276	0.000	0.000	2.341

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	0
N.S.	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.00
time (sec)	N/A	0.019	0.012	0.000	0.281	0.254	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	0	54	0	17	31
N.S.	1	1.00	1.00	0.70	0.00	5.40	0.00	1.70	3.10
time (sec)	N/A	0.017	0.007	2.121	0.000	0.246	0.000	0.259	2.189

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	21	41	86	0	32	49
N.S.	1	1.00	1.00	0.91	1.78	3.74	0.00	1.39	2.13
time (sec)	N/A	0.011	0.011	0.273	0.273	0.253	0.000	0.275	2.245

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	33	54	31	0	41	49
N.S.	1	1.00	1.00	1.57	2.57	1.48	0.00	1.95	2.33
time (sec)	N/A	0.020	0.093	0.271	0.192	0.253	0.000	0.274	2.387

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	62	91	463	0	97	107
N.S.	1	1.00	1.06	1.27	1.86	9.45	0.00	1.98	2.18
time (sec)	N/A	0.022	0.015	0.892	0.270	0.252	0.000	0.295	2.492

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	98	93	0	67	131
N.S.	1	1.00	1.00	1.38	2.65	2.51	0.00	1.81	3.54
time (sec)	N/A	0.022	0.099	0.389	0.187	0.260	0.000	0.313	2.494

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	56	197	0	60	48
N.S.	1	1.00	0.89	0.89	2.00	7.04	0.00	2.14	1.71
time (sec)	N/A	0.018	0.014	0.342	0.272	0.252	0.000	0.276	0.086

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	64	54	0	71	50
N.S.	1	1.00	0.78	0.98	1.60	1.35	0.00	1.78	1.25
time (sec)	N/A	0.029	0.166	0.338	0.184	0.238	0.000	0.295	2.241

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	41	103	742	0	99	97
N.S.	1	1.00	0.81	0.95	2.40	17.26	0.00	2.30	2.26
time (sec)	N/A	0.031	0.037	0.432	0.276	0.259	0.000	0.324	2.240

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	71	290	0	55	77
N.S.	1	1.00	1.00	0.87	1.87	7.63	0.00	1.45	2.03
time (sec)	N/A	0.021	0.013	0.608	0.275	0.256	0.000	0.271	0.096

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	52	79	63	0	63	78
N.S.	1	1.00	1.05	1.37	2.08	1.66	0.00	1.66	2.05
time (sec)	N/A	0.030	0.110	0.618	0.191	0.256	0.000	0.302	2.186

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	78	81	116	851	0	117	136
N.S.	1	1.00	1.18	1.23	1.76	12.89	0.00	1.77	2.06
time (sec)	N/A	0.032	0.019	1.212	0.273	0.257	0.000	0.297	2.170

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	33	81	457	0	84	77
N.S.	1	1.00	0.85	0.82	2.02	11.42	0.00	2.10	1.92
time (sec)	N/A	0.024	0.025	1.497	0.277	0.267	0.000	0.277	0.122

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	17	23	24
N.S.	1	1.00	1.00	1.09	2.09	4.91	1.55	2.09	2.18
time (sec)	N/A	0.009	0.007	0.195	0.197	0.245	0.095	0.261	2.085

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	84	22	27	13
N.S.	1	1.00	1.00	0.93	0.87	5.60	1.47	1.80	0.87
time (sec)	N/A	0.015	0.009	0.554	0.196	0.243	0.130	0.274	0.081

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	36	115	39	0	31
N.S.	1	1.00	1.00	1.06	2.25	7.19	2.44	0.00	1.94
time (sec)	N/A	0.022	0.070	1.228	0.314	0.273	0.158	0.000	2.225

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	138	0	31	31
N.S.	1	1.00	1.00	0.93	0.87	9.20	0.00	2.07	2.07
time (sec)	N/A	0.022	0.005	0.928	0.188	0.233	0.000	0.277	0.080

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	208	44	37	230
N.S.	1	1.00	1.00	0.93	0.87	13.87	2.93	2.47	15.33
time (sec)	N/A	0.022	0.004	1.452	0.186	0.240	0.251	0.305	0.123

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	69	0	38	42
N.S.	1	1.00	1.00	1.05	1.00	3.63	0.00	2.00	2.21
time (sec)	N/A	0.026	0.014	6.535	0.190	0.269	0.000	0.294	2.205

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	148	172	41	45	48
N.S.	1	1.00	1.00	0.89	5.48	6.37	1.52	1.67	1.78
time (sec)	N/A	0.019	0.092	0.453	0.193	0.240	0.182	0.275	0.085

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	214	345	46	45	251
N.S.	1	1.00	1.00	0.84	6.90	11.13	1.48	1.45	8.10
time (sec)	N/A	0.026	0.144	3.870	0.195	0.248	0.397	0.291	2.319

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	36	36	32	209	345	219	0	0	101
N.S.	1	1.00	0.89	5.81	9.58	6.08	0.00	0.00	2.81
time (sec)	N/A	0.035	0.272	6.181	0.336	0.244	0.000	0.000	2.382

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	56	26	276	304	0	53	270
N.S.	1	1.00	1.81	0.84	8.90	9.81	0.00	1.71	8.71
time (sec)	N/A	0.024	0.115	5.951	0.199	0.235	0.000	0.286	0.137

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	26	352	551	0	148	168
N.S.	1	1.00	0.83	0.74	10.06	15.74	0.00	4.23	4.80
time (sec)	N/A	0.027	0.187	0.412	0.308	0.253	0.000	0.304	2.464

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	73	55	504	180	0	0	115
N.S.	1	1.00	1.82	1.38	12.60	4.50	0.00	0.00	2.88
time (sec)	N/A	0.034	0.637	117.675	0.300	0.266	0.000	0.000	2.326

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	43	66	269	0	76	82
N.S.	1	1.00	1.00	1.26	1.94	7.91	0.00	2.24	2.41
time (sec)	N/A	0.018	0.013	1.013	0.273	0.256	0.000	0.275	0.097

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	74	75	112	814	0	102	186
N.S.	1	1.00	1.35	1.36	2.04	14.80	0.00	1.85	3.38
time (sec)	N/A	0.033	0.017	1.245	0.304	0.250	0.000	0.278	2.231

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	58	110	808	0	98	215
N.S.	1	1.00	1.00	1.05	2.00	14.69	0.00	1.78	3.91
time (sec)	N/A	0.035	0.013	2.919	0.288	0.277	0.000	0.291	2.194

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	191	185	29	35	129
N.S.	1	1.00	1.00	0.86	9.10	8.81	1.38	1.67	6.14
time (sec)	N/A	0.014	0.020	2.926	0.221	0.240	0.289	0.245	2.237

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	371	634	34	35	520
N.S.	1	1.00	1.00	0.00	14.84	25.36	1.36	1.40	20.80
time (sec)	N/A	0.022	0.025	180.000	0.198	0.246	2.418	0.265	2.304

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	48	46	85	925	0	73	200
N.S.	1	1.00	1.26	1.21	2.24	24.34	0.00	1.92	5.26
time (sec)	N/A	0.040	0.008	16.046	0.281	0.252	0.000	0.268	0.061

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	85	925	0	73	206
N.S.	1	1.00	1.00	1.00	2.36	25.69	0.00	2.03	5.72
time (sec)	N/A	0.029	0.008	169.339	0.279	0.251	0.000	0.265	2.166

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	67	26	857	778	0	66	820
N.S.	1	1.00	2.03	0.79	25.97	23.58	0.00	2.00	24.85
time (sec)	N/A	0.023	0.037	0.090	0.205	0.234	0.000	0.262	2.130

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	21	59	113	0	44	53
N.S.	1	1.00	1.83	0.91	2.57	4.91	0.00	1.91	2.30
time (sec)	N/A	0.015	0.050	0.298	0.195	0.247	0.000	0.258	0.075

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	56	31	0	45	49
N.S.	1	1.00	1.00	1.50	2.55	1.41	0.00	2.05	2.23
time (sec)	N/A	0.017	0.011	0.311	0.196	0.254	0.000	0.282	2.089

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	85	62	108	612	0	105	112
N.S.	1	1.00	1.73	1.27	2.20	12.49	0.00	2.14	2.29
time (sec)	N/A	0.028	0.104	0.975	0.203	0.291	0.000	0.293	0.082

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	100	89	0	71	131
N.S.	1	1.00	1.00	1.38	2.70	2.41	0.00	1.92	3.54
time (sec)	N/A	0.019	0.016	0.556	0.196	0.247	0.000	0.288	2.122

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	23	70	203	105	63	49
N.S.	1	1.00	0.93	0.85	2.59	7.52	3.89	2.33	1.81
time (sec)	N/A	0.018	0.017	0.499	0.185	0.253	0.826	0.273	0.078

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	66	60	0	71	50
N.S.	1	1.00	0.78	0.98	1.65	1.50	0.00	1.78	1.25
time (sec)	N/A	0.031	0.175	0.552	0.188	0.252	0.000	0.279	0.090

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	43	120	743	0	102	97
N.S.	1	1.00	0.81	1.00	2.79	17.28	0.00	2.37	2.26
time (sec)	N/A	0.031	0.036	0.772	0.196	0.264	0.000	0.302	2.100

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	60	31	87	357	0	69	81
N.S.	1	1.00	1.58	0.82	2.29	9.39	0.00	1.82	2.13
time (sec)	N/A	0.025	0.088	0.891	0.193	0.258	0.000	0.287	2.071

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	52	79	63	0	67	78
N.S.	1	1.00	1.00	1.37	2.08	1.66	0.00	1.76	2.05
time (sec)	N/A	0.028	0.015	1.293	0.192	0.249	0.000	0.288	0.106

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	103	81	133	1077	0	123	140
N.S.	1	1.00	1.56	1.23	2.02	16.32	0.00	1.86	2.12
time (sec)	N/A	0.036	0.126	2.700	0.189	0.282	0.000	0.318	2.119

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	33	95	457	0	85	77
N.S.	1	1.00	0.90	0.85	2.44	11.72	0.00	2.18	1.97
time (sec)	N/A	0.024	0.024	2.310	0.199	0.257	0.000	0.275	0.112

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	17	25	24
N.S.	1	1.00	1.00	1.09	2.27	5.09	1.55	2.27	2.18
time (sec)	N/A	0.008	0.008	0.144	0.209	0.232	0.779	0.281	0.061

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	86	22	27	13
N.S.	1	1.00	1.00	0.93	0.87	5.73	1.47	1.80	0.87
time (sec)	N/A	0.016	0.010	0.164	0.186	0.247	1.558	0.278	0.077

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	53	115	36	0	31
N.S.	1	1.00	1.00	1.06	3.31	7.19	2.25	0.00	1.94
time (sec)	N/A	0.024	0.016	0.324	0.349	0.254	2.192	0.000	2.096

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	139	0	31	31
N.S.	1	1.00	1.00	0.93	0.87	9.27	0.00	2.07	2.07
time (sec)	N/A	0.021	0.003	0.191	0.187	0.247	0.000	0.309	2.065

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	208	0	37	231
N.S.	1	1.00	1.00	0.93	0.87	13.87	0.00	2.47	15.40
time (sec)	N/A	0.023	0.004	0.214	0.197	0.245	0.000	0.318	2.086

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	70	0	39	43
N.S.	1	1.00	1.00	1.05	1.00	3.50	0.00	1.95	2.15
time (sec)	N/A	0.029	0.017	2.002	0.196	0.255	0.000	0.293	2.137

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	148	171	0	49	48
N.S.	1	1.00	1.00	0.89	5.48	6.33	0.00	1.81	1.78
time (sec)	N/A	0.017	0.011	0.212	0.199	0.250	0.000	0.306	2.082

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	214	343	0	49	252
N.S.	1	1.00	1.00	0.87	6.90	11.06	0.00	1.58	8.13
time (sec)	N/A	0.025	0.018	0.356	0.191	0.243	0.000	0.330	2.147

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	37	37	34	479	414	216	0	0	100
N.S.	1	1.00	0.92	12.95	11.19	5.84	0.00	0.00	2.70
time (sec)	N/A	0.035	0.050	0.855	0.340	0.263	0.000	0.000	2.175

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	75	45	84	387	0	84	87
N.S.	1	1.00	2.21	1.32	2.47	11.38	0.00	2.47	2.56
time (sec)	N/A	0.025	0.088	0.915	0.186	0.258	0.000	0.279	0.083

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	58	129	1109	0	106	219
N.S.	1	1.00	2.05	1.05	2.35	20.16	0.00	1.93	3.98
time (sec)	N/A	0.050	0.125	1.004	0.191	0.262	0.000	0.288	0.114

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	74	133	1114	0	110	190
N.S.	1	1.00	2.05	1.35	2.42	20.25	0.00	2.00	3.45
time (sec)	N/A	0.047	0.110	0.968	0.191	0.264	0.000	0.318	2.050

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	14	149	164	0	30	144
N.S.	1	1.00	1.59	0.82	8.76	9.65	0.00	1.76	8.47
time (sec)	N/A	0.020	0.035	0.476	0.191	0.248	0.000	0.263	2.074

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	139	222	0	29	210
N.S.	1	1.00	1.00	1.29	8.18	13.06	0.00	1.71	12.35
time (sec)	N/A	0.021	0.010	0.500	0.196	0.244	0.000	0.255	2.228

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	33	368	114	0	0	87
N.S.	1	1.00	1.15	1.27	14.15	4.38	0.00	0.00	3.35
time (sec)	N/A	0.028	0.080	34.492	0.321	0.268	0.000	0.000	2.243

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	95	46	98	1260	0	71	214
N.S.	1	1.00	2.50	1.21	2.58	33.16	0.00	1.87	5.63
time (sec)	N/A	0.050	0.028	1.077	0.208	0.247	0.000	0.268	2.090

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	47	20	431	430	0	48	413
N.S.	1	1.00	1.88	0.80	17.24	17.20	0.00	1.92	16.52
time (sec)	N/A	0.029	0.036	1.867	0.202	0.249	0.000	0.276	2.084

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	191	250	0	47	40
N.S.	1	1.00	1.00	0.83	6.59	8.62	0.00	1.62	1.38
time (sec)	N/A	0.014	0.014	0.279	0.199	0.254	0.000	0.285	2.094

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	435	442	0	54	372
N.S.	1	1.00	1.00	0.79	13.18	13.39	0.00	1.64	11.27
time (sec)	N/A	0.025	0.012	1.022	0.200	0.244	0.000	0.268	2.040

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	58	87	58	71	23
N.S.	1	1.00	0.96	0.89	2.15	3.22	2.15	2.63	0.85
time (sec)	N/A	0.020	0.028	0.427	0.207	0.260	0.174	0.271	0.166

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	59	75	61	78	23
N.S.	1	1.00	1.00	0.89	2.19	2.78	2.26	2.89	0.85
time (sec)	N/A	0.022	0.032	0.414	0.208	0.243	0.174	0.265	2.145

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	58	89	58	69	23
N.S.	1	1.00	0.96	0.89	2.15	3.30	2.15	2.56	0.85
time (sec)	N/A	0.015	0.013	0.424	0.199	0.250	0.165	0.267	2.119

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	59	77	58	74	23
N.S.	1	1.00	0.96	0.89	2.19	2.85	2.15	2.74	0.85
time (sec)	N/A	0.015	0.014	0.389	0.202	0.251	0.171	0.280	2.122

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	151	83	259	0	95	115
N.S.	1	1.00	0.78	4.08	2.24	7.00	0.00	2.57	3.11
time (sec)	N/A	0.051	0.347	0.237	0.285	0.261	0.000	0.285	2.641

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	149	87	216	0	86	121
N.S.	1	1.00	0.83	4.14	2.42	6.00	0.00	2.39	3.36
time (sec)	N/A	0.053	0.332	0.235	0.286	0.266	0.000	0.280	2.653

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	155	157	259	0	97	115
N.S.	1	1.00	0.78	4.19	4.24	7.00	0.00	2.62	3.11
time (sec)	N/A	0.025	0.324	0.247	0.216	0.253	0.000	0.281	0.524

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	153	160	216	0	90	121
N.S.	1	1.00	0.89	4.25	4.44	6.00	0.00	2.50	3.36
time (sec)	N/A	0.025	0.313	0.247	0.216	0.266	0.000	0.277	2.560

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	77	68	184	0	79	266
N.S.	1	1.00	0.75	2.14	1.89	5.11	0.00	2.19	7.39
time (sec)	N/A	0.016	0.150	0.431	0.284	0.251	0.000	0.288	3.044

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	75	67	156	0	70	268
N.S.	1	1.00	0.82	2.27	2.03	4.73	0.00	2.12	8.12
time (sec)	N/A	0.017	0.151	0.460	0.282	0.262	0.000	0.272	2.976

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	79	133	184	0	81	266
N.S.	1	1.00	0.78	2.19	3.69	5.11	0.00	2.25	7.39
time (sec)	N/A	0.016	0.157	0.203	0.204	0.256	0.000	0.285	2.332

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	77	129	156	0	74	269
N.S.	1	1.00	0.88	2.33	3.91	4.73	0.00	2.24	8.15
time (sec)	N/A	0.017	0.141	0.247	0.215	0.270	0.000	0.290	0.321

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	57	327	0	49	133
N.S.	1	1.00	2.97	5.76	1.97	11.28	0.00	1.69	4.59
time (sec)	N/A	0.017	0.041	0.343	0.286	0.249	0.000	0.267	2.288

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	205	105	902	0	97	173
N.S.	1	1.00	2.27	4.56	2.33	20.04	0.00	2.16	3.84
time (sec)	N/A	0.040	0.076	0.363	0.285	0.270	0.000	0.264	0.222

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	70	240	149	1737	0	120	0
N.S.	1	1.00	0.97	3.33	2.07	24.12	0.00	1.67	0.00
time (sec)	N/A	0.063	0.232	0.396	0.292	0.277	0.000	0.266	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	94	439	0	93	139
N.S.	1	1.00	3.21	5.34	3.24	15.14	0.00	3.21	4.79
time (sec)	N/A	0.014	0.042	0.327	0.210	0.247	0.000	0.260	0.158

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	110	197	140	1237	0	136	181
N.S.	1	1.00	2.39	4.28	3.04	26.89	0.00	2.96	3.93
time (sec)	N/A	0.033	0.076	0.330	0.212	0.267	0.000	0.261	2.281

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	230	186	2372	0	169	0
N.S.	1	1.00	0.96	3.15	2.55	32.49	0.00	2.32	0.00
time (sec)	N/A	0.067	0.238	0.387	0.197	0.269	0.000	0.266	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	148	49	87	0	49	65
N.S.	1	1.00	1.00	5.69	1.88	3.35	0.00	1.88	2.50
time (sec)	N/A	0.011	0.086	0.416	0.208	0.266	0.000	0.261	0.242

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	83	181	70	405	0	68	150
N.S.	1	1.00	2.37	5.17	2.00	11.57	0.00	1.94	4.29
time (sec)	N/A	0.025	0.068	1.136	0.291	0.258	0.000	0.265	2.245

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	42	120	246	0	51	0
N.S.	1	1.00	0.92	1.11	3.16	6.47	0.00	1.34	0.00
time (sec)	N/A	0.033	0.119	0.988	0.204	0.257	0.000	0.269	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	150	84	86	0	51	65
N.S.	1	1.00	1.00	5.77	3.23	3.31	0.00	1.96	2.50
time (sec)	N/A	0.011	0.080	0.186	0.206	0.272	0.000	0.268	2.243

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	172	103	617	0	104	156
N.S.	1	1.00	2.50	4.78	2.86	17.14	0.00	2.89	4.33
time (sec)	N/A	0.024	0.058	0.311	0.203	0.257	0.000	0.269	0.216

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	57	131	246	0	53	0
N.S.	1	1.00	0.90	1.46	3.36	6.31	0.00	1.36	0.00
time (sec)	N/A	0.034	0.126	0.554	0.199	0.242	0.000	0.274	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	86	167	59	327	0	53	133
N.S.	1	1.00	2.97	5.76	2.03	11.28	0.00	1.83	4.59
time (sec)	N/A	0.016	0.040	0.361	0.276	0.262	0.000	0.267	0.173

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	102	207	103	902	0	97	173
N.S.	1	1.00	2.27	4.60	2.29	20.04	0.00	2.16	3.84
time (sec)	N/A	0.034	0.075	0.392	0.287	0.260	0.000	0.266	0.213

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	115	238	149	1737	0	122	0
N.S.	1	1.00	1.60	3.31	2.07	24.12	0.00	1.69	0.00
time (sec)	N/A	0.065	0.230	0.444	0.303	0.274	0.000	0.270	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	155	90	439	0	91	139
N.S.	1	1.00	3.21	5.34	3.10	15.14	0.00	3.14	4.79
time (sec)	N/A	0.017	0.039	0.270	0.201	0.256	0.000	0.273	0.159

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	110	195	144	1237	0	142	183
N.S.	1	1.00	2.39	4.24	3.13	26.89	0.00	3.09	3.98
time (sec)	N/A	0.035	0.073	0.312	0.221	0.263	0.000	0.268	2.234

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	228	184	2372	0	167	0
N.S.	1	1.00	0.96	3.12	2.52	32.49	0.00	2.29	0.00
time (sec)	N/A	0.070	0.221	0.427	0.203	0.277	0.000	0.277	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	146	51	86	0	50	64
N.S.	1	1.00	1.00	5.62	1.96	3.31	0.00	1.92	2.46
time (sec)	N/A	0.012	0.081	0.434	0.197	0.250	0.000	0.263	0.221

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	83	183	70	405	0	68	148
N.S.	1	1.00	2.37	5.23	2.00	11.57	0.00	1.94	4.23
time (sec)	N/A	0.026	0.069	1.126	0.300	0.253	0.000	0.264	2.230

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	119	248	0	49	0
N.S.	1	1.00	0.92	0.68	3.13	6.53	0.00	1.29	0.00
time (sec)	N/A	0.035	0.124	0.954	0.202	0.249	0.000	0.264	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	152	80	87	0	50	66
N.S.	1	1.00	1.00	5.85	3.08	3.35	0.00	1.92	2.54
time (sec)	N/A	0.012	0.080	0.281	0.204	0.256	0.000	0.260	2.233

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	90	170	105	617	0	106	156
N.S.	1	1.00	2.50	4.72	2.92	17.14	0.00	2.94	4.33
time (sec)	N/A	0.026	0.061	0.441	0.206	0.250	0.000	0.289	2.248

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	36	132	243	0	51	0
N.S.	1	1.00	0.90	0.92	3.38	6.23	0.00	1.31	0.00
time (sec)	N/A	0.036	0.124	0.685	0.206	0.244	0.000	0.269	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	72	153	85	42
N.S.	1	1.00	1.00	0.93	0.00	1.67	3.56	1.98	0.98
time (sec)	N/A	0.033	0.156	0.442	0.000	0.245	0.323	0.288	0.169

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	120	405	120	76
N.S.	1	1.00	1.11	0.92	0.00	1.94	6.53	1.94	1.23
time (sec)	N/A	0.050	0.515	0.816	0.000	0.259	0.722	0.261	0.270

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	0	218	918	179	182
N.S.	1	1.00	0.95	0.92	0.00	2.40	10.09	1.97	2.00
time (sec)	N/A	0.061	0.326	1.496	0.000	0.263	1.911	0.278	0.554

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	89	0	192	1027	156	152
N.S.	1	1.00	1.20	1.01	0.00	2.18	11.67	1.77	1.73
time (sec)	N/A	0.052	0.501	1.815	0.000	0.265	1.557	0.294	2.695

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	414	2001	260	337
N.S.	1	1.00	1.10	0.92	0.00	2.88	13.90	1.81	2.34
time (sec)	N/A	0.100	1.165	4.585	0.000	0.248	5.481	0.282	2.614

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	177	190	0	731	3580	373	906
N.S.	1	1.00	0.91	0.97	0.00	3.75	18.36	1.91	4.65
time (sec)	N/A	0.117	1.082	10.555	0.000	0.273	16.688	0.282	2.725

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	71	153	85	42
N.S.	1	1.00	1.00	0.93	0.00	1.65	3.56	1.98	0.98
time (sec)	N/A	0.029	0.117	0.381	0.000	0.247	0.317	0.265	0.152

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	115	408	120	68
N.S.	1	1.00	1.11	0.92	0.00	1.85	6.58	1.94	1.10
time (sec)	N/A	0.041	0.484	0.808	0.000	0.255	0.703	0.265	0.247

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	217	921	179	180
N.S.	1	1.00	0.93	0.92	0.00	2.38	10.12	1.97	1.98
time (sec)	N/A	0.056	0.285	1.477	0.000	0.287	1.881	0.272	2.533

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	105	89	0	192	1027	156	115
N.S.	1	1.00	1.19	1.01	0.00	2.18	11.67	1.77	1.31
time (sec)	N/A	0.051	0.465	1.715	0.000	0.241	1.582	0.270	2.617

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	397	2008	260	337
N.S.	1	1.00	1.10	0.92	0.00	2.76	13.94	1.81	2.34
time (sec)	N/A	0.077	1.084	4.770	0.000	0.253	5.288	0.277	2.691

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	0	726	3582	373	908
N.S.	1	1.00	0.90	0.97	0.00	3.72	18.37	1.91	4.66
time (sec)	N/A	0.105	1.109	11.099	0.000	0.267	16.835	0.281	2.717

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	0	71	153	85	42
N.S.	1	1.00	1.00	0.93	0.00	1.65	3.56	1.98	0.98
time (sec)	N/A	0.034	0.154	0.461	0.000	0.255	0.327	0.264	0.150

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	0	119	408	120	68
N.S.	1	1.00	1.11	0.92	0.00	1.92	6.58	1.94	1.10
time (sec)	N/A	0.044	0.469	0.836	0.000	0.255	0.707	0.267	0.246

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	0	213	921	179	182
N.S.	1	1.00	0.93	0.92	0.00	2.34	10.12	1.97	2.00
time (sec)	N/A	0.062	0.314	1.619	0.000	0.263	1.956	0.274	2.565

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	74	63	0	114	408	124	76
N.S.	1	1.00	1.09	0.93	0.00	1.68	6.00	1.82	1.12
time (sec)	N/A	0.039	0.512	0.782	0.000	0.260	0.742	0.260	2.297

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	107	89	0	192	1027	156	135
N.S.	1	1.00	1.22	1.01	0.00	2.18	11.67	1.77	1.53
time (sec)	N/A	0.049	0.461	1.965	0.000	0.262	1.558	0.261	2.615

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	0	398	2001	260	337
N.S.	1	1.00	1.10	0.92	0.00	2.76	13.90	1.81	2.34
time (sec)	N/A	0.070	1.013	4.817	0.000	0.264	5.549	0.285	2.618

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	0	243	935	183	183
N.S.	1	1.00	0.93	0.93	0.00	2.51	9.64	1.89	1.89
time (sec)	N/A	0.062	0.371	1.326	0.000	0.249	1.941	0.286	0.552

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	0	443	2030	256	337
N.S.	1	1.00	1.11	0.92	0.00	3.21	14.71	1.86	2.44
time (sec)	N/A	0.085	1.126	4.009	0.000	0.262	5.342	0.278	2.645

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	176	190	0	729	3580	373	908
N.S.	1	1.00	0.90	0.97	0.00	3.74	18.36	1.91	4.66
time (sec)	N/A	0.118	1.044	11.361	0.000	0.268	16.865	0.300	2.795

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.717	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	240	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	2.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	99	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.797	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	103	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	15	15	12	27	17	20	25	6
N.S.	1	1.88	1.88	1.50	3.38	2.12	2.50	3.12	0.75
time (sec)	N/A	0.006	0.026	0.322	0.194	0.253	0.127	0.266	0.071

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	22	20	27	13
N.S.	1	1.00	1.00	0.82	1.59	1.29	1.18	1.59	0.76
time (sec)	N/A	0.007	0.044	0.416	0.197	0.261	0.133	0.268	0.061

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	36	20	27	14
N.S.	1	1.00	1.00	0.82	1.59	2.12	1.18	1.59	0.82
time (sec)	N/A	0.007	0.025	0.361	0.193	0.328	0.131	0.263	0.058

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	78	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	2.23	1.69	0.74
time (sec)	N/A	0.024	0.038	0.324	0.000	0.244	0.253	0.261	0.097

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	19	20	25	11
N.S.	1	1.00	1.00	0.80	1.80	1.27	1.33	1.67	0.73
time (sec)	N/A	0.008	0.031	0.315	0.194	0.246	0.128	0.270	2.187

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	33	20	26	11
N.S.	1	1.00	1.00	0.82	1.59	1.94	1.18	1.53	0.65
time (sec)	N/A	0.008	0.030	0.399	0.187	0.244	0.129	0.258	0.060

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	38	20	27	15
N.S.	1	1.00	1.00	0.82	1.59	2.24	1.18	1.59	0.88
time (sec)	N/A	0.008	0.030	0.413	0.196	0.258	0.127	0.256	2.187

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	37	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	1.06	1.69	0.74
time (sec)	N/A	0.026	0.046	0.324	0.000	0.254	0.265	0.270	2.204

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	53	115	0	36	47
N.S.	1	1.00	1.00	0.84	2.79	6.05	0.00	1.89	2.47
time (sec)	N/A	0.018	0.012	0.177	0.277	0.262	0.000	0.261	2.262

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	60	46	76	0	43	23
N.S.	1	1.00	1.00	3.16	2.42	4.00	0.00	2.26	1.21
time (sec)	N/A	0.030	0.021	0.236	0.284	0.252	0.000	0.269	2.183

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	42	0	213	0	71	71
N.S.	1	1.00	1.00	0.61	0.00	3.09	0.00	1.03	1.03
time (sec)	N/A	0.070	0.097	0.261	0.000	0.271	0.000	0.342	0.788

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	60	0	306	0	81	82
N.S.	1	1.00	0.93	0.69	0.00	3.52	0.00	0.93	0.94
time (sec)	N/A	0.215	0.148	0.306	0.000	0.267	0.000	0.288	3.388

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	84	0	300	0	100	98
N.S.	1	1.00	1.00	0.97	0.00	3.45	0.00	1.15	1.13
time (sec)	N/A	0.184	0.093	0.293	0.000	0.280	0.000	0.273	3.442

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	16	42	0	16	16
N.S.	1	1.00	1.00	0.90	1.60	4.20	0.00	1.60	1.60
time (sec)	N/A	0.018	0.008	0.199	0.271	0.257	0.000	0.256	0.055

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	49	118	0	36	47
N.S.	1	1.00	1.00	0.85	2.45	5.90	0.00	1.80	2.35
time (sec)	N/A	0.019	0.017	0.200	0.272	0.263	0.000	0.269	0.082

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	72	60	128	0	54	52
N.S.	1	1.00	1.00	2.57	2.14	4.57	0.00	1.93	1.86
time (sec)	N/A	0.038	0.023	0.244	0.274	0.257	0.000	0.263	2.153

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	42	0	293	0	75	141
N.S.	1	1.00	0.93	0.51	0.00	3.57	0.00	0.91	1.72
time (sec)	N/A	0.146	0.165	0.256	0.000	0.254	0.000	0.280	4.069

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	102	0	164	0	68	56
N.S.	1	1.00	1.00	2.68	0.00	4.32	0.00	1.79	1.47
time (sec)	N/A	0.054	0.043	0.243	0.000	0.270	0.000	0.274	2.325

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	42	39	42	35	0	38	35
N.S.	1	1.00	2.62	2.44	2.62	2.19	0.00	2.38	2.19
time (sec)	N/A	0.014	0.061	1.183	0.270	0.252	0.000	0.258	0.100

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	26	45	52	0	41	27
N.S.	1	1.00	0.81	1.24	2.14	2.48	0.00	1.95	1.29
time (sec)	N/A	0.022	0.008	1.229	0.268	0.253	0.000	0.260	2.196

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	110	40	0	215	0	115	251
N.S.	1	1.00	1.55	0.56	0.00	3.03	0.00	1.62	3.54
time (sec)	N/A	0.058	0.038	1.228	0.000	0.261	0.000	0.342	2.120

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	101	0	182	0	118	100
N.S.	1	1.00	0.92	1.63	0.00	2.94	0.00	1.90	1.61
time (sec)	N/A	0.062	0.057	1.320	0.000	0.246	0.000	0.291	2.172

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	269	78	0	250	0	154	288
N.S.	1	1.00	3.16	0.92	0.00	2.94	0.00	1.81	3.39
time (sec)	N/A	0.059	0.071	1.254	0.000	0.264	0.000	0.283	2.440

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	6	0	3	3
N.S.	1	1.00	1.00	0.86	1.00	0.86	0.00	0.43	0.43
time (sec)	N/A	0.010	0.001	0.233	0.269	0.249	0.000	0.269	0.048

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	40	39	31	0	19	19
N.S.	1	1.00	1.00	2.67	2.60	2.07	0.00	1.27	1.27
time (sec)	N/A	0.026	0.021	0.251	0.273	0.253	0.000	0.299	0.068

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	62	50	76	0	44	42
N.S.	1	1.00	1.00	2.38	1.92	2.92	0.00	1.69	1.62
time (sec)	N/A	0.019	0.017	0.272	0.269	0.257	0.000	0.280	0.074

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	84	41	0	243	0	68	282
N.S.	1	1.00	1.12	0.55	0.00	3.24	0.00	0.91	3.76
time (sec)	N/A	0.077	0.091	0.295	0.000	0.258	0.000	0.283	4.994

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	92	0	107	0	58	41
N.S.	1	1.00	0.83	2.56	0.00	2.97	0.00	1.61	1.14
time (sec)	N/A	0.031	0.018	0.271	0.000	0.250	0.000	0.265	0.206

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	15	15	12	27	19	20	25	6
N.S.	1	1.88	1.88	1.50	3.38	2.38	2.50	3.12	0.75
time (sec)	N/A	0.007	0.003	0.321	0.198	0.254	0.130	0.249	0.056

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	33	20	26	11
N.S.	1	1.00	1.00	0.82	1.59	1.94	1.18	1.53	0.65
time (sec)	N/A	0.007	0.006	0.428	0.180	0.243	0.140	0.258	2.097

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	36	20	27	14
N.S.	1	1.00	1.00	0.82	1.59	2.12	1.18	1.59	0.82
time (sec)	N/A	0.007	0.004	0.405	0.180	0.237	0.130	0.266	2.089

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	42	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	1.20	1.69	0.74
time (sec)	N/A	0.025	0.025	0.338	0.000	0.247	0.248	0.259	0.083

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	17	20	25	9
N.S.	1	1.00	1.00	0.80	1.80	1.13	1.33	1.67	0.60
time (sec)	N/A	0.006	0.005	0.326	0.185	0.248	0.125	0.265	0.057

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	20	20	27	20
N.S.	1	1.00	1.00	0.82	1.59	1.18	1.18	1.59	1.18
time (sec)	N/A	0.007	0.004	0.434	0.186	0.263	0.130	0.259	2.129

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	27	34	20	27	15
N.S.	1	1.00	1.00	0.82	1.59	2.00	1.18	1.59	0.88
time (sec)	N/A	0.006	0.004	0.411	0.189	0.238	0.129	0.274	2.151

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	0	42	56	59	26
N.S.	1	1.00	0.71	0.80	0.00	1.20	1.60	1.69	0.74
time (sec)	N/A	0.022	0.020	0.312	0.000	0.251	0.255	0.256	0.070

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	51	16	52	73	0	45	48
N.S.	1	1.00	2.68	0.84	2.74	3.84	0.00	2.37	2.53
time (sec)	N/A	0.021	0.042	0.194	0.283	0.252	0.000	0.263	2.132

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	55	17	153	82	0	45	53
N.S.	1	1.00	2.75	0.85	7.65	4.10	0.00	2.25	2.65
time (sec)	N/A	0.021	0.053	0.196	0.287	0.255	0.000	0.259	2.145

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	42	0	213	0	119	133
N.S.	1	1.00	1.64	0.61	0.00	3.09	0.00	1.72	1.93
time (sec)	N/A	0.058	0.033	0.304	0.000	0.260	0.000	0.351	0.078

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	249	42	0	293	0	127	141
N.S.	1	1.00	3.04	0.51	0.00	3.57	0.00	1.55	1.72
time (sec)	N/A	0.096	0.033	0.272	0.000	0.262	0.000	0.303	0.093

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	281	79	0	258	0	157	170
N.S.	1	1.00	3.23	0.91	0.00	2.97	0.00	1.80	1.95
time (sec)	N/A	0.182	0.068	0.303	0.000	0.259	0.000	0.287	0.100

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	25	16	29	52	0	26	29
N.S.	1	1.00	2.50	1.60	2.90	5.20	0.00	2.60	2.90
time (sec)	N/A	0.018	0.021	0.313	0.191	0.249	0.000	0.254	2.121

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	50	57	104	0	55	57
N.S.	1	1.00	1.04	1.11	1.27	2.31	0.00	1.22	1.27
time (sec)	N/A	0.044	0.027	0.257	0.276	0.251	0.000	0.263	0.060

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	73	63	70	101	0	67	71
N.S.	1	1.00	2.61	2.25	2.50	3.61	0.00	2.39	2.54
time (sec)	N/A	0.042	0.051	0.248	0.275	0.266	0.000	0.255	0.071

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	133	190	0	272	0	157	143
N.S.	1	1.00	1.21	1.73	0.00	2.47	0.00	1.43	1.30
time (sec)	N/A	0.127	0.105	0.276	0.000	0.264	0.000	0.296	0.097

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	95	87	0	157	0	89	101
N.S.	1	1.00	2.50	2.29	0.00	4.13	0.00	2.34	2.66
time (sec)	N/A	0.059	0.056	0.285	0.000	0.261	0.000	0.254	0.089

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.321	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	44	43	68	0	39	32
N.S.	1	1.00	1.00	2.93	2.87	4.53	0.00	2.60	2.13
time (sec)	N/A	0.011	0.006	1.274	0.278	0.252	0.000	0.272	2.180

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	40	114	31	0	19	19
N.S.	1	1.00	1.00	2.67	7.60	2.07	0.00	1.27	1.27
time (sec)	N/A	0.024	0.020	1.375	0.277	0.248	0.000	0.259	2.129

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	40	0	215	0	135	126
N.S.	1	1.00	0.94	0.56	0.00	3.03	0.00	1.90	1.77
time (sec)	N/A	0.026	0.072	1.448	0.000	0.259	0.000	0.359	1.501

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	84	41	0	243	0	68	297
N.S.	1	1.00	1.12	0.55	0.00	3.24	0.00	0.91	3.96
time (sec)	N/A	0.094	0.088	1.189	0.000	0.253	0.000	0.282	4.779

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	83	0	282	0	177	206
N.S.	1	1.00	0.95	0.98	0.00	3.32	0.00	2.08	2.42
time (sec)	N/A	0.042	0.055	1.282	0.000	0.266	0.000	0.266	4.266

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	21	8	19	19	0	16	19
N.S.	1	1.00	3.00	1.14	2.71	2.71	0.00	2.29	2.71
time (sec)	N/A	0.011	0.010	0.255	0.204	0.264	0.000	0.256	0.060

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	24	47	52	0	40	29
N.S.	1	1.00	1.00	1.14	2.24	2.48	0.00	1.90	1.38
time (sec)	N/A	0.021	0.008	0.271	0.267	0.252	0.000	0.281	0.070

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	67	53	60	54	0	57	61
N.S.	1	1.00	2.58	2.04	2.31	2.08	0.00	2.19	2.35
time (sec)	N/A	0.023	0.061	0.271	0.275	0.250	0.000	0.276	0.062

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	101	0	180	0	108	104
N.S.	1	1.00	0.92	1.63	0.00	2.90	0.00	1.74	1.68
time (sec)	N/A	0.057	0.068	0.290	0.000	0.259	0.000	0.280	2.217

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	91	77	0	101	0	79	91
N.S.	1	1.00	2.53	2.14	0.00	2.81	0.00	2.19	2.53
time (sec)	N/A	0.039	0.058	0.280	0.000	0.255	0.000	0.264	0.089

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	59	88	0	0	0
N.S.	1	1.00	0.94	0.00	0.84	1.26	0.00	0.00	0.00
time (sec)	N/A	0.086	0.094	0.000	0.089	0.076	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	50	74	86	74	119	73	64
N.S.	1	1.00	0.53	0.79	0.91	0.79	1.27	0.78	0.68
time (sec)	N/A	0.052	0.067	0.883	0.190	0.256	0.306	0.260	2.206

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	39	58	64	62	75	57	46
N.S.	1	1.00	0.61	0.91	1.00	0.97	1.17	0.89	0.72
time (sec)	N/A	0.029	0.045	0.652	0.195	0.257	0.233	0.273	2.162

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	28	42	46	42	56	41	28
N.S.	1	1.00	0.64	0.95	1.05	0.95	1.27	0.93	0.64
time (sec)	N/A	0.018	0.041	0.499	0.204	0.256	0.201	0.258	0.066

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13
N.S.	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87
time (sec)	N/A	0.009	0.004	0.288	0.184	0.252	0.088	0.272	2.136

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	23	37	0	23	0
N.S.	1	1.00	0.93	0.96	0.85	1.37	0.00	0.85	0.00
time (sec)	N/A	0.052	0.014	0.882	0.245	0.250	0.000	0.259	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	42	55	27	65	0	52	0
N.S.	1	1.00	1.08	1.41	0.69	1.67	0.00	1.33	0.00
time (sec)	N/A	0.067	0.051	1.133	0.254	0.257	0.000	0.291	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	61	89	30	104	0	86	0
N.S.	1	1.00	1.02	1.48	0.50	1.73	0.00	1.43	0.00
time (sec)	N/A	0.088	0.101	1.205	0.250	0.246	0.000	0.261	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	121	31	115	0	120	0
N.S.	1	1.00	0.91	1.42	0.36	1.35	0.00	1.41	0.00
time (sec)	N/A	0.105	0.093	1.542	0.253	0.248	0.000	0.265	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	113	162	0	0	0
N.S.	1	1.00	0.85	0.00	0.84	1.21	0.00	0.00	0.00
time (sec)	N/A	0.141	0.180	0.000	0.119	0.082	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	86	141	160	135	146	140	108
N.S.	1	1.00	0.74	1.21	1.37	1.15	1.25	1.20	0.92
time (sec)	N/A	0.087	0.279	2.872	0.210	0.257	0.410	0.271	2.229

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	109	122	105	105	108	69
N.S.	1	1.00	0.78	1.31	1.47	1.27	1.27	1.30	0.83
time (sec)	N/A	0.050	0.152	2.421	0.213	0.271	0.308	0.274	2.205

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	56	84	74	61	76	41
N.S.	1	1.00	1.02	1.24	1.87	1.64	1.36	1.69	0.91
time (sec)	N/A	0.022	0.085	1.899	0.207	0.258	0.254	0.280	0.099

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	38	20	54	13
N.S.	1	1.00	1.00	0.93	0.87	2.53	1.33	3.60	0.87
time (sec)	N/A	0.014	0.003	0.757	0.189	0.244	0.100	0.278	2.147

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	47	42	67	0	42	0
N.S.	1	1.00	0.83	1.00	0.89	1.43	0.00	0.89	0.00
time (sec)	N/A	0.103	0.015	1.376	0.262	0.266	0.000	0.271	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	70	95	50	124	0	90	0
N.S.	1	1.00	0.88	1.19	0.62	1.55	0.00	1.12	0.00
time (sec)	N/A	0.126	0.125	1.773	0.272	0.254	0.000	0.268	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	105	159	58	196	0	156	0
N.S.	1	1.00	0.88	1.34	0.49	1.65	0.00	1.31	0.00
time (sec)	N/A	0.170	0.187	2.607	0.276	0.263	0.000	0.273	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	138	226	58	223	0	223	0
N.S.	1	1.00	0.90	1.47	0.38	1.45	0.00	1.45	0.00
time (sec)	N/A	0.204	0.217	3.464	0.273	0.250	0.000	0.271	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	110	0	117	172	0	0	0
N.S.	1	1.00	0.79	0.00	0.84	1.24	0.00	0.00	0.00
time (sec)	N/A	0.163	0.135	0.000	0.115	0.089	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	91	146	171	191	226	145	125
N.S.	1	1.00	0.59	0.94	1.10	1.23	1.46	0.94	0.81
time (sec)	N/A	0.101	0.420	7.582	0.200	0.264	0.688	0.291	2.348

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	70	114	127	154	150	113	89
N.S.	1	1.00	0.69	1.13	1.26	1.52	1.49	1.12	0.88
time (sec)	N/A	0.056	0.162	5.505	0.214	0.248	0.442	0.273	2.240

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	91	108	110	81	57
N.S.	1	1.00	0.77	1.06	1.40	1.66	1.69	1.25	0.88
time (sec)	N/A	0.032	0.091	3.924	0.206	0.261	0.302	0.268	0.120

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	54	20	57	13
N.S.	1	1.00	1.00	0.93	0.87	3.60	1.33	3.80	0.87
time (sec)	N/A	0.015	0.002	2.097	0.183	0.253	0.146	0.271	2.128

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	0
N.S.	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	0.00
time (sec)	N/A	0.097	0.040	2.880	0.286	0.251	0.000	0.260	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	105	53	139	0	100	0
N.S.	1	1.00	0.90	1.18	0.60	1.56	0.00	1.12	0.00
time (sec)	N/A	0.125	0.126	4.392	0.294	0.244	0.000	0.259	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	112	173	60	227	0	168	0
N.S.	1	1.00	0.90	1.38	0.48	1.82	0.00	1.34	0.00
time (sec)	N/A	0.168	0.371	6.585	0.327	0.246	0.000	0.263	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	241	59	261	0	236	0
N.S.	1	1.00	0.89	1.43	0.35	1.54	0.00	1.40	0.00
time (sec)	N/A	0.206	0.334	9.278	0.288	0.247	0.000	0.261	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	13	13	0	13	0
N.S.	1	1.00	1.00	0.88	1.62	1.62	0.00	1.62	0.00
time (sec)	N/A	0.021	0.004	0.361	0.233	0.245	0.000	0.262	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	24	0	30	0
N.S.	1	1.00	1.00	0.94	0.94	1.50	0.00	1.88	0.00
time (sec)	N/A	0.036	0.005	0.406	0.252	0.242	0.000	0.266	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	13	43	0	48	0
N.S.	1	1.00	1.00	0.89	0.48	1.59	0.00	1.78	0.00
time (sec)	N/A	0.045	0.006	0.451	0.241	0.254	0.000	0.251	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	114	0	113	162	0	0	0
N.S.	1	1.00	0.85	0.00	0.84	1.21	0.00	0.00	0.00
time (sec)	N/A	0.143	0.216	0.000	0.125	0.085	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	84	141	160	135	146	140	119
N.S.	1	1.00	0.72	1.21	1.37	1.15	1.25	1.20	1.02
time (sec)	N/A	0.102	0.275	3.878	0.251	0.270	0.411	0.253	0.167

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	66	109	122	104	105	108	82
N.S.	1	1.00	0.80	1.31	1.47	1.25	1.27	1.30	0.99
time (sec)	N/A	0.063	0.304	2.984	0.215	0.251	0.330	0.306	2.178

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	56	84	76	61	76	44
N.S.	1	1.00	0.84	1.24	1.87	1.69	1.36	1.69	0.98
time (sec)	N/A	0.029	0.094	2.505	0.216	0.281	0.217	0.254	2.153

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	32	20	54	13
N.S.	1	1.00	1.00	0.93	0.87	2.13	1.33	3.60	0.87
time (sec)	N/A	0.024	0.003	1.153	0.201	0.257	0.099	0.258	2.091

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	0
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	0.00
time (sec)	N/A	0.101	0.015	1.663	0.287	0.266	0.000	0.265	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	96	50	126	0	91	0
N.S.	1	1.00	0.85	1.20	0.62	1.58	0.00	1.14	0.00
time (sec)	N/A	0.145	0.141	1.881	0.309	0.242	0.000	0.268	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	107	159	58	195	0	156	0
N.S.	1	1.00	0.90	1.34	0.49	1.64	0.00	1.31	0.00
time (sec)	N/A	0.174	0.153	2.733	0.272	0.251	0.000	0.262	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	138	229	58	224	0	222	0
N.S.	1	1.00	0.90	1.49	0.38	1.45	0.00	1.44	0.00
time (sec)	N/A	0.215	0.174	3.822	0.289	0.248	0.000	0.255	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	106	0	71	122	0	0	0
N.S.	1	1.00	1.25	0.00	0.84	1.44	0.00	0.00	0.00
time (sec)	N/A	0.096	0.121	0.000	0.091	0.088	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	58	79	91	140	250	78	70
N.S.	1	1.00	0.73	1.00	1.15	1.77	3.16	0.99	0.89
time (sec)	N/A	0.085	0.110	11.058	0.214	0.247	0.553	0.274	2.284

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	48	63	69	110	204	62	52
N.S.	1	1.00	0.80	1.05	1.15	1.83	3.40	1.03	0.87
time (sec)	N/A	0.084	0.099	7.806	0.250	0.263	0.440	0.249	2.207

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	51	88	131	46	36
N.S.	1	1.00	1.00	1.15	1.24	2.15	3.20	1.12	0.88
time (sec)	N/A	0.044	0.154	4.937	0.229	0.270	0.311	0.268	0.108

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	23	33	39	40	92	32	18
N.S.	1	1.00	0.50	0.72	0.85	0.87	2.00	0.70	0.39
time (sec)	N/A	0.030	0.011	3.158	0.211	0.255	0.157	0.256	0.071

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	27	41	0	27	0
N.S.	1	1.00	0.97	0.91	0.82	1.24	0.00	0.82	0.00
time (sec)	N/A	0.062	0.262	3.394	0.252	0.255	0.000	0.271	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	54	32	88	0	55	0
N.S.	1	1.00	0.87	1.04	0.62	1.69	0.00	1.06	0.00
time (sec)	N/A	0.089	0.058	4.190	0.265	0.250	0.000	0.266	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	65	92	36	140	0	89	0
N.S.	1	1.00	0.97	1.37	0.54	2.09	0.00	1.33	0.00
time (sec)	N/A	0.105	0.069	5.999	0.253	0.266	0.000	0.271	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	79	122	36	172	0	123	0
N.S.	1	1.00	0.86	1.33	0.39	1.87	0.00	1.34	0.00
time (sec)	N/A	0.128	0.130	8.790	0.271	0.263	0.000	0.269	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	175	0	171	248	0	0	0
N.S.	1	1.00	0.84	0.00	0.82	1.19	0.00	0.00	0.00
time (sec)	N/A	0.233	0.261	0.000	0.154	0.102	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	125	213	245	274	253	212	167
N.S.	1	1.00	0.62	1.05	1.21	1.36	1.25	1.05	0.83
time (sec)	N/A	0.205	0.780	24.677	0.238	0.253	0.753	0.263	0.548

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	105	165	187	209	182	164	123
N.S.	1	1.00	0.71	1.11	1.26	1.41	1.23	1.11	0.83
time (sec)	N/A	0.141	0.188	18.546	0.267	0.249	0.554	0.267	2.570

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	117	129	152	112	116	83
N.S.	1	1.00	0.74	1.24	1.37	1.62	1.19	1.23	0.88
time (sec)	N/A	0.081	0.114	13.598	0.227	0.249	0.420	0.273	0.159

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	26	78	64	44	82	26
N.S.	1	1.00	0.87	0.84	2.52	2.06	1.42	2.65	0.84
time (sec)	N/A	0.026	0.048	8.601	0.201	0.254	0.213	0.272	2.133

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	71	64	103	0	64	0
N.S.	1	1.00	0.84	0.97	0.88	1.41	0.00	0.88	0.00
time (sec)	N/A	0.152	0.060	8.326	0.308	0.253	0.000	0.271	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	104	151	76	214	0	144	0
N.S.	1	1.00	0.84	1.22	0.61	1.73	0.00	1.16	0.00
time (sec)	N/A	0.208	0.212	9.303	0.321	0.291	0.000	0.267	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	162	250	88	338	0	243	0
N.S.	1	1.00	0.88	1.36	0.48	1.84	0.00	1.32	0.00
time (sec)	N/A	0.254	0.304	14.241	0.303	0.254	0.000	0.282	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	212	349	88	397	0	342	0
N.S.	1	1.00	0.89	1.47	0.37	1.67	0.00	1.44	0.00
time (sec)	N/A	0.318	0.287	21.045	0.311	0.263	0.000	0.255	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	117	172	0	0	0
N.S.	1	1.00	0.79	0.00	0.83	1.22	0.00	0.00	0.00
time (sec)	N/A	0.141	0.078	0.000	0.121	0.087	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	95	146	171	191	226	145	126
N.S.	1	1.00	0.61	0.94	1.10	1.23	1.46	0.94	0.81
time (sec)	N/A	0.100	0.422	13.011	0.223	0.249	0.540	0.261	2.334

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	72	114	127	154	150	113	89
N.S.	1	1.00	0.71	1.13	1.26	1.52	1.49	1.12	0.88
time (sec)	N/A	0.054	0.140	10.400	0.238	0.261	0.410	0.266	0.220

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	69	91	108	110	81	55
N.S.	1	1.00	0.77	1.06	1.40	1.66	1.69	1.25	0.85
time (sec)	N/A	0.032	0.087	7.361	0.225	0.244	0.310	0.264	0.163

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	54	20	57	13
N.S.	1	1.00	1.00	0.93	0.87	3.60	1.33	3.80	0.87
time (sec)	N/A	0.014	0.002	4.709	0.224	0.250	0.136	0.262	0.069

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	0
N.S.	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	0.00
time (sec)	N/A	0.097	0.038	4.558	0.299	0.239	0.000	0.264	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	105	53	139	0	100	0
N.S.	1	1.00	0.88	1.18	0.60	1.56	0.00	1.12	0.00
time (sec)	N/A	0.123	0.155	5.328	0.305	0.247	0.000	0.265	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	113	173	60	229	0	168	0
N.S.	1	1.00	0.90	1.38	0.48	1.83	0.00	1.34	0.00
time (sec)	N/A	0.165	0.319	5.542	0.329	0.253	0.000	0.260	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	241	61	264	0	236	0
N.S.	1	1.00	0.89	1.43	0.36	1.56	0.00	1.40	0.00
time (sec)	N/A	0.196	0.319	8.566	0.286	0.247	0.000	0.258	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	174	0	171	248	0	0	0
N.S.	1	1.00	0.83	0.00	0.82	1.19	0.00	0.00	0.00
time (sec)	N/A	0.194	0.218	0.000	0.147	0.083	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	136	213	245	274	253	212	173
N.S.	1	1.00	0.67	1.05	1.21	1.36	1.25	1.05	0.86
time (sec)	N/A	0.188	0.315	39.141	0.221	0.257	0.770	0.265	0.289

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	98	165	187	214	182	164	112
N.S.	1	1.00	0.66	1.11	1.26	1.45	1.23	1.11	0.76
time (sec)	N/A	0.133	0.277	26.872	0.240	0.272	0.542	0.278	0.244

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	70	116	129	153	112	116	71
N.S.	1	1.00	0.74	1.23	1.37	1.63	1.19	1.23	0.76
time (sec)	N/A	0.072	0.116	19.315	0.221	0.259	0.401	0.266	0.156

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	26	78	79	44	82	26
N.S.	1	1.00	0.87	0.84	2.52	2.55	1.42	2.65	0.84
time (sec)	N/A	0.025	0.049	12.892	0.206	0.251	0.209	0.278	2.146

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	71	64	103	0	64	0
N.S.	1	1.00	0.86	0.97	0.88	1.41	0.00	0.88	0.00
time (sec)	N/A	0.134	0.054	11.172	0.307	0.251	0.000	0.315	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	106	147	76	203	0	140	0
N.S.	1	1.00	0.85	1.19	0.61	1.64	0.00	1.13	0.00
time (sec)	N/A	0.183	0.162	12.740	0.321	0.247	0.000	0.262	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	164	246	88	336	0	239	0
N.S.	1	1.00	0.89	1.34	0.48	1.83	0.00	1.30	0.00
time (sec)	N/A	0.245	0.241	13.449	0.309	0.258	0.000	0.292	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	212	349	88	392	0	342	0
N.S.	1	1.00	0.89	1.47	0.37	1.65	0.00	1.44	0.00
time (sec)	N/A	0.309	0.287	21.615	0.333	0.245	0.000	0.273	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	119	0	117	172	0	0	0
N.S.	1	1.00	0.77	0.00	0.75	1.11	0.00	0.00	0.00
time (sec)	N/A	0.173	0.130	0.000	0.131	0.085	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	90	146	171	248	314	145	126
N.S.	1	1.00	0.63	1.02	1.20	1.73	2.20	1.01	0.88
time (sec)	N/A	0.147	0.587	88.148	0.226	0.262	1.017	0.266	2.465

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	72	114	127	202	212	113	89
N.S.	1	1.00	0.69	1.09	1.21	1.92	2.02	1.08	0.85
time (sec)	N/A	0.097	0.151	63.257	0.219	0.279	0.771	0.261	0.347

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	82	91	148	148	81	55
N.S.	1	1.00	0.75	1.22	1.36	2.21	2.21	1.21	0.82
time (sec)	N/A	0.051	0.163	44.818	0.216	0.255	0.545	0.269	0.251

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	35	34	56	72	42	57	26
N.S.	1	1.00	1.13	1.10	1.81	2.32	1.35	1.84	0.84
time (sec)	N/A	0.025	0.050	31.923	0.206	0.243	0.309	0.269	0.109

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	47	50	45	73	0	45	0
N.S.	1	1.00	0.89	0.94	0.85	1.38	0.00	0.85	0.00
time (sec)	N/A	0.103	0.526	28.397	0.286	0.254	0.000	0.272	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	105	53	159	0	100	0
N.S.	1	1.00	0.88	1.18	0.60	1.79	0.00	1.12	0.00
time (sec)	N/A	0.138	0.143	30.955	0.288	0.245	0.000	0.280	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	118	173	61	274	0	168	0
N.S.	1	1.00	0.90	1.32	0.47	2.09	0.00	1.28	0.00
time (sec)	N/A	0.178	0.156	32.460	0.298	0.262	0.000	0.272	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	241	61	315	0	236	0
N.S.	1	1.00	0.89	1.43	0.36	1.86	0.00	1.40	0.00
time (sec)	N/A	0.216	0.212	49.265	0.278	0.250	0.000	0.272	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	100	18	17	18	20
N.S.	1	1.00	1.20	1.60	10.00	1.80	1.70	1.80	2.00
time (sec)	N/A	0.014	0.336	0.132	0.489	0.257	10.115	0.272	2.173

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	88	116	84	257	0	0	0
N.S.	1	1.00	0.97	1.27	0.92	2.82	0.00	0.00	0.00
time (sec)	N/A	0.117	0.178	0.468	0.262	0.273	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	66	94	63	207	0	0	0
N.S.	1	1.00	1.02	1.45	0.97	3.18	0.00	0.00	0.00
time (sec)	N/A	0.098	0.116	0.504	0.259	0.258	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	70	40	141	0	0	0
N.S.	1	1.00	0.98	1.56	0.89	3.13	0.00	0.00	0.00
time (sec)	N/A	0.058	0.088	0.470	0.263	0.257	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	21	37	0	24	11
N.S.	1	1.00	1.00	1.18	1.91	3.36	0.00	2.18	1.00
time (sec)	N/A	0.005	0.001	0.164	0.213	0.245	0.000	0.258	0.063

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	22	18	15	18	20
N.S.	1	1.00	1.20	1.60	2.20	1.80	1.50	1.80	2.00
time (sec)	N/A	0.014	9.075	0.225	0.267	0.248	1.006	0.250	2.189

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	29	18	17	18	20
N.S.	1	1.00	1.20	1.60	2.90	1.80	1.70	1.80	2.00
time (sec)	N/A	0.014	16.638	0.212	0.266	0.255	0.847	0.251	2.212

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	20
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.25
time (sec)	N/A	0.295	48.176	0.123	0.361	0.252	35.580	0.277	2.214

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	130	0	0	672	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	5.95	0.00	0.00	0.00
time (sec)	N/A	0.080	0.217	0.000	0.000	0.267	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	85	154	0	468	0	0	0
N.S.	1	1.00	1.23	2.23	0.00	6.78	0.00	0.00	0.00
time (sec)	N/A	0.044	0.223	0.751	0.000	0.255	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	59	37	116	0	70	49
N.S.	1	1.00	1.33	2.46	1.54	4.83	0.00	2.92	2.04
time (sec)	N/A	0.013	0.036	0.482	0.362	0.258	0.000	0.263	0.076

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	0	23	13
N.S.	1	1.00	1.00	1.09	2.09	4.91	0.00	2.09	1.18
time (sec)	N/A	0.010	0.003	0.245	0.200	0.241	0.000	0.272	0.059

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	60	20	17	20	20
N.S.	1	1.00	1.12	1.12	3.75	1.25	1.06	1.25	1.25
time (sec)	N/A	0.140	6.146	0.209	0.325	0.245	1.448	0.414	2.178

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	64	20	19	20	20
N.S.	1	1.00	1.12	1.12	4.00	1.25	1.19	1.25	1.25
time (sec)	N/A	0.159	6.594	0.212	0.331	0.256	1.520	0.684	2.202

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.11
time (sec)	N/A	0.355	39.727	0.120	0.358	0.252	110.413	0.290	2.259

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	121	110	1113	0	0	0
N.S.	1	1.00	1.04	1.46	1.33	13.41	0.00	0.00	0.00
time (sec)	N/A	0.114	0.978	2.531	0.284	0.269	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	73	94	378	0	142	102
N.S.	1	1.00	1.31	1.74	2.24	9.00	0.00	3.38	2.43
time (sec)	N/A	0.040	0.091	1.640	0.282	0.260	0.000	0.297	2.216

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	131	105	0	184	36
N.S.	1	1.00	1.00	1.43	4.37	3.50	0.00	6.13	1.20
time (sec)	N/A	0.022	0.181	1.152	0.216	0.247	0.000	0.278	2.203

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	84	0	27	13
N.S.	1	1.00	1.00	0.93	1.53	5.60	0.00	1.80	0.87
time (sec)	N/A	0.016	0.001	0.610	0.208	0.241	0.000	0.256	2.176

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	101	20	17	20	20
N.S.	1	1.00	1.11	1.00	5.61	1.11	0.94	1.11	1.11
time (sec)	N/A	0.159	22.239	0.230	0.305	0.245	3.862	0.273	2.209

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	100	20	19	20	20
N.S.	1	1.00	1.11	1.00	5.56	1.11	1.06	1.11	1.11
time (sec)	N/A	0.188	18.928	0.272	0.306	0.253	4.616	0.268	2.219

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.38
time (sec)	N/A	0.075	14.706	0.200	0.375	0.246	38.334	0.263	2.203

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	211	0	0	609	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	3.12	0.00	0.00	0.00
time (sec)	N/A	0.141	0.196	0.000	0.000	0.267	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	153	0	0	477	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	3.53	0.00	0.00	0.00
time (sec)	N/A	0.105	0.144	0.000	0.000	0.267	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	93	162	0	328	0	0	0
N.S.	1	1.00	1.21	2.10	0.00	4.26	0.00	0.00	0.00
time (sec)	N/A	0.052	0.085	0.664	0.000	0.260	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	21	41	86	0	32	49
N.S.	1	1.00	1.00	0.91	1.78	3.74	0.00	1.39	2.13
time (sec)	N/A	0.011	0.003	0.322	0.321	0.252	0.000	0.266	0.063

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	17	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.06	1.25	1.38
time (sec)	N/A	0.056	5.168	0.411	0.430	0.250	1.346	0.319	2.233

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.38
time (sec)	N/A	0.074	4.607	0.335	0.400	0.266	1.407	0.348	2.199

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	144	22	20	22	22
N.S.	1	1.00	1.17	1.67	12.00	1.83	1.67	1.83	1.83
time (sec)	N/A	0.020	0.408	0.233	0.703	0.247	120.276	0.286	2.224

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	99	125	108	721	0	0	0
N.S.	1	1.00	1.11	1.40	1.21	8.10	0.00	0.00	0.00
time (sec)	N/A	0.121	1.069	1.003	0.284	0.262	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	77	99	84	515	0	0	0
N.S.	1	1.00	1.18	1.52	1.29	7.92	0.00	0.00	0.00
time (sec)	N/A	0.080	0.673	0.893	0.268	0.266	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	41	95	185	0	95	45
N.S.	1	1.00	1.48	1.32	3.06	5.97	0.00	3.06	1.45
time (sec)	N/A	0.020	0.115	0.954	0.238	0.257	0.000	0.285	0.106

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	23	17	25	33	0	24	20
N.S.	1	1.00	1.77	1.31	1.92	2.54	0.00	1.85	1.54
time (sec)	N/A	0.007	0.001	0.266	0.217	0.263	0.000	0.289	0.076

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	49	22	19	22	22
N.S.	1	1.00	1.17	1.67	4.08	1.83	1.58	1.83	1.83
time (sec)	N/A	0.021	17.056	0.380	0.297	0.246	3.313	0.284	2.211

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	68	22	20	22	22
N.S.	1	1.00	1.17	1.67	5.67	1.83	1.67	1.83	1.83
time (sec)	N/A	0.021	10.978	0.376	0.299	0.242	3.960	0.285	2.251

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.050	52.673	0.213	0.392	0.253	0.000	0.293	2.282

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	245	0	0	2163	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	9.01	0.00	0.00	0.00
time (sec)	N/A	0.242	2.006	0.000	0.000	0.293	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	180	0	0	1577	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	11.03	0.00	0.00	0.00
time (sec)	N/A	0.158	0.588	0.000	0.000	0.287	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	116	178	0	1064	0	0	0
N.S.	1	1.00	1.27	1.96	0.00	11.69	0.00	0.00	0.00
time (sec)	N/A	0.082	0.638	1.395	0.000	0.287	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	43	66	269	0	76	82
N.S.	1	1.00	1.00	1.26	1.94	7.91	0.00	2.24	2.41
time (sec)	N/A	0.021	0.003	1.033	0.292	0.265	0.000	0.266	2.219

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	131	22	19	22	22
N.S.	1	1.00	1.11	1.11	7.28	1.22	1.06	1.22	1.22
time (sec)	N/A	0.050	15.276	0.351	0.357	0.257	8.664	1.766	2.245

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	133	22	20	22	22
N.S.	1	1.00	1.11	1.11	7.39	1.22	1.11	1.22	1.22
time (sec)	N/A	0.055	11.082	0.407	0.361	0.247	10.785	1.633	2.304

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.103	17.722	0.273	0.442	0.251	113.002	0.279	2.229

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	159	189	181	966	0	0	0
N.S.	1	1.00	0.86	1.02	0.98	5.22	0.00	0.00	0.00
time (sec)	N/A	0.180	0.286	2.301	0.267	0.282	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	122	152	138	789	0	0	0
N.S.	1	1.00	0.94	1.17	1.06	6.07	0.00	0.00	0.00
time (sec)	N/A	0.139	0.251	1.651	0.277	0.277	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	110	95	558	0	0	0
N.S.	1	1.00	0.81	1.24	1.07	6.27	0.00	0.00	0.00
time (sec)	N/A	0.084	0.299	1.242	0.266	0.252	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	56	197	0	60	48
N.S.	1	1.00	0.89	0.89	2.00	7.04	0.00	2.14	1.71
time (sec)	N/A	0.018	0.012	0.731	0.307	0.261	0.000	0.269	0.065

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	46	20	17	20	22
N.S.	1	1.00	1.11	1.00	2.56	1.11	0.94	1.11	1.22
time (sec)	N/A	0.076	12.197	0.419	0.357	0.242	2.823	0.268	2.205

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	53	20	19	20	22
N.S.	1	1.00	1.11	1.00	2.94	1.11	1.06	1.11	1.22
time (sec)	N/A	0.096	10.521	0.379	0.331	0.238	3.444	0.261	2.242

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.123	45.958	0.277	0.440	0.248	0.000	0.281	2.305

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	196	0	0	1225	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	7.56	0.00	0.00	0.00
time (sec)	N/A	0.141	0.878	0.000	0.000	0.281	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	132	205	0	879	0	0	0
N.S.	1	1.00	1.27	1.97	0.00	8.45	0.00	0.00	0.00
time (sec)	N/A	0.091	0.812	0.893	0.000	0.268	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	50	94	81	283	0	102	90
N.S.	1	1.00	1.09	2.04	1.76	6.15	0.00	2.22	1.96
time (sec)	N/A	0.037	0.223	0.717	0.355	0.257	0.000	0.301	0.113

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	33	54	31	0	41	22
N.S.	1	1.00	1.00	1.57	2.57	1.48	0.00	1.95	1.05
time (sec)	N/A	0.019	0.005	0.373	0.208	0.251	0.000	0.271	0.082

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	79	22	19	22	22
N.S.	1	1.00	1.11	1.11	4.39	1.22	1.06	1.22	1.22
time (sec)	N/A	0.098	8.597	0.350	0.352	0.248	8.511	0.876	2.278

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	87	22	20	22	22
N.S.	1	1.00	1.11	1.11	4.83	1.22	1.11	1.22	1.22
time (sec)	N/A	0.109	6.401	0.348	0.350	0.266	10.905	1.227	2.332

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	171	22	0	22	22
N.S.	1	1.00	1.17	1.67	14.25	1.83	0.00	1.83	1.83
time (sec)	N/A	0.022	0.687	0.250	0.735	0.253	0.000	0.312	2.328

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	249	234	236	2207	0	0	0
N.S.	1	1.00	1.36	1.28	1.29	12.06	0.00	0.00	0.00
time (sec)	N/A	0.249	1.539	0.669	0.254	0.276	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	196	164	183	1649	0	0	0
N.S.	1	1.00	1.69	1.41	1.58	14.22	0.00	0.00	0.00
time (sec)	N/A	0.148	1.354	0.661	0.265	0.278	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	111	131	1106	0	0	0
N.S.	1	1.00	0.84	1.35	1.60	13.49	0.00	0.00	0.00
time (sec)	N/A	0.083	0.596	0.575	0.266	0.270	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	61	339	0	65	25
N.S.	1	1.00	1.00	0.85	2.26	12.56	0.00	2.41	0.93
time (sec)	N/A	0.015	0.001	0.372	0.297	0.273	0.000	0.277	2.219

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	111	22	19	22	22
N.S.	1	1.00	1.17	1.67	9.25	1.83	1.58	1.83	1.83
time (sec)	N/A	0.021	13.219	0.405	0.286	0.249	19.856	0.280	2.285

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	143	22	20	22	22
N.S.	1	1.00	1.17	1.67	11.92	1.83	1.67	1.83	1.83
time (sec)	N/A	0.021	8.077	0.394	0.292	0.242	27.048	0.273	2.370

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	102	18	17	18	20
N.S.	1	1.00	1.20	1.60	10.20	1.80	1.70	1.80	2.00
time (sec)	N/A	0.012	5.997	0.120	0.502	0.255	108.551	0.263	2.203

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	91	200	130	216	0	0	0
N.S.	1	1.00	1.05	2.30	1.49	2.48	0.00	0.00	0.00
time (sec)	N/A	0.111	0.003	0.323	0.256	0.262	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	166	96	168	0	0	0
N.S.	1	1.00	1.05	2.63	1.52	2.67	0.00	0.00	0.00
time (sec)	N/A	0.097	0.003	0.313	0.251	0.268	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	122	58	112	0	0	0
N.S.	1	1.00	1.04	2.71	1.29	2.49	0.00	0.00	0.00
time (sec)	N/A	0.062	0.003	0.318	0.240	0.258	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	13	23	37	0	25	11
N.S.	1	1.00	1.73	1.18	2.09	3.36	0.00	2.27	1.00
time (sec)	N/A	0.005	0.003	0.147	0.203	0.247	0.000	0.266	0.070

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	35	18	15	18	20
N.S.	1	1.00	1.20	1.60	3.50	1.80	1.50	1.80	2.00
time (sec)	N/A	0.013	0.277	0.184	0.278	0.247	3.293	0.257	2.182

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	16	46	18	17	18	20
N.S.	1	1.00	1.20	1.60	4.60	1.80	1.70	1.80	2.00
time (sec)	N/A	0.013	0.717	0.242	0.251	0.251	3.826	0.267	2.245

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	0	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	0.00	1.25	1.38
time (sec)	N/A	0.071	17.231	0.253	0.395	0.244	0.000	0.270	2.210

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	175	246	206	511	0	0	0
N.S.	1	1.00	1.06	1.49	1.25	3.10	0.00	0.00	0.00
time (sec)	N/A	0.143	0.166	0.602	0.266	0.267	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	125	196	152	391	0	0	0
N.S.	1	1.00	1.09	1.70	1.32	3.40	0.00	0.00	0.00
time (sec)	N/A	0.098	0.113	0.625	0.264	0.256	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	73	139	94	255	0	0	0
N.S.	1	1.00	1.11	2.11	1.42	3.86	0.00	0.00	0.00
time (sec)	N/A	0.048	0.054	0.629	0.256	0.253	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	21	59	113	0	44	53
N.S.	1	1.00	1.83	0.91	2.57	4.91	0.00	1.91	2.30
time (sec)	N/A	0.014	0.008	0.372	0.196	0.266	0.000	0.270	2.228

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	17	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.06	1.25	1.38
time (sec)	N/A	0.062	8.899	0.353	0.400	0.252	9.383	0.432	2.230

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	19	20	22
N.S.	1	1.00	1.12	1.12	1.25	1.25	1.19	1.25	1.38
time (sec)	N/A	0.078	24.615	0.381	0.400	0.250	10.631	0.440	2.236

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.22
time (sec)	N/A	0.100	16.830	0.277	0.422	0.255	0.000	0.286	2.380

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	236	272	225	876	0	0	0
N.S.	1	1.00	1.31	1.51	1.25	4.87	0.00	0.00	0.00
time (sec)	N/A	0.176	0.457	1.139	0.281	0.259	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	178	222	171	697	0	0	0
N.S.	1	1.00	1.41	1.76	1.36	5.53	0.00	0.00	0.00
time (sec)	N/A	0.141	0.249	1.142	0.273	0.271	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	72	162	113	488	0	0	0
N.S.	1	1.00	0.82	1.84	1.28	5.55	0.00	0.00	0.00
time (sec)	N/A	0.084	0.302	1.018	0.266	0.266	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	23	70	203	0	63	49
N.S.	1	1.00	0.93	0.85	2.59	7.52	0.00	2.33	1.81
time (sec)	N/A	0.018	0.009	0.772	0.199	0.258	0.000	0.275	0.065

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	57	20	17	20	22
N.S.	1	1.00	1.11	1.00	3.17	1.11	0.94	1.11	1.22
time (sec)	N/A	0.080	7.144	0.396	0.323	0.263	21.801	0.281	2.285

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	72	20	19	20	22
N.S.	1	1.00	1.11	1.00	4.00	1.11	1.06	1.11	1.22
time (sec)	N/A	0.095	7.397	0.435	0.330	0.240	26.594	0.279	2.296

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	48	0	336	0	101	48
N.S.	1	1.00	1.00	1.45	0.00	10.18	0.00	3.06	1.45
time (sec)	N/A	0.041	0.038	0.327	0.000	0.272	0.000	0.266	0.130

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	87	0	617	0	0	0
N.S.	1	1.00	0.88	1.19	0.00	8.45	0.00	0.00	0.00
time (sec)	N/A	0.117	0.093	0.334	0.000	0.264	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	85	117	0	875	0	0	0
N.S.	1	1.00	0.83	1.15	0.00	8.58	0.00	0.00	0.00
time (sec)	N/A	0.144	0.098	0.361	0.000	0.279	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	82	146	916	0	0	0
N.S.	1	1.00	0.89	1.30	2.32	14.54	0.00	0.00	0.00
time (sec)	N/A	0.129	0.066	0.428	0.237	0.273	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	135	127	174	1512	0	0	0
N.S.	1	1.00	1.41	1.32	1.81	15.75	0.00	0.00	0.00
time (sec)	N/A	0.216	0.190	0.451	0.240	0.285	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	191	184	238	2067	0	0	0
N.S.	1	1.00	1.21	1.16	1.51	13.08	0.00	0.00	0.00
time (sec)	N/A	0.293	0.638	0.449	0.249	0.270	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	20	20	0	20	20
N.S.	1	1.00	1.12	1.12	1.25	1.25	0.00	1.25	1.25
time (sec)	N/A	0.281	15.271	0.117	0.337	0.249	0.000	0.274	2.291

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	109	174	121	551	0	0	0
N.S.	1	1.00	1.17	1.87	1.30	5.92	0.00	0.00	0.00
time (sec)	N/A	0.077	0.200	0.441	0.266	0.268	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	69	134	83	367	0	0	0
N.S.	1	1.00	1.17	2.27	1.41	6.22	0.00	0.00	0.00
time (sec)	N/A	0.043	0.199	0.407	0.262	0.251	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	114	54	64	169	0	93	53
N.S.	1	1.00	4.56	2.16	2.56	6.76	0.00	3.72	2.12
time (sec)	N/A	0.014	0.038	0.369	0.248	0.249	0.000	0.276	0.131

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	0	25	13
N.S.	1	1.00	1.00	1.09	2.27	5.09	0.00	2.27	1.18
time (sec)	N/A	0.008	0.003	0.155	0.189	0.253	0.000	0.265	2.273

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	79	20	17	20	20
N.S.	1	1.00	1.12	1.12	4.94	1.25	1.06	1.25	1.25
time (sec)	N/A	0.104	22.777	0.231	0.288	0.258	9.118	0.472	2.287

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	18	79	20	19	20	20
N.S.	1	1.00	1.12	1.12	4.94	1.25	1.19	1.25	1.25
time (sec)	N/A	0.143	32.498	0.232	0.293	0.286	11.472	0.748	2.404

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	145	22	0	22	22
N.S.	1	1.00	1.17	1.67	12.08	1.83	0.00	1.83	1.83
time (sec)	N/A	0.022	7.611	0.248	0.724	0.260	0.000	0.281	2.328

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	222	198	146	632	0	0	0
N.S.	1	1.00	2.55	2.28	1.68	7.26	0.00	0.00	0.00
time (sec)	N/A	0.131	0.606	0.990	0.267	0.269	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	117	156	108	453	0	0	0
N.S.	1	1.00	1.80	2.40	1.66	6.97	0.00	0.00	0.00
time (sec)	N/A	0.083	0.953	0.991	0.273	0.260	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	54	115	189	0	98	45
N.S.	1	1.00	1.48	1.74	3.71	6.10	0.00	3.16	1.45
time (sec)	N/A	0.020	0.088	0.862	0.236	0.257	0.000	0.272	2.720

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	27	18	25	33	0	24	20
N.S.	1	1.00	2.08	1.38	1.92	2.54	0.00	1.85	1.54
time (sec)	N/A	0.007	0.002	0.231	0.196	0.273	0.000	0.268	0.079

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	69	22	19	22	22
N.S.	1	1.00	1.17	1.67	5.75	1.83	1.58	1.83	1.83
time (sec)	N/A	0.021	0.126	0.330	0.310	0.260	21.531	0.280	2.362

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	91	22	20	22	22
N.S.	1	1.00	1.17	1.67	7.58	1.83	1.67	1.83	1.83
time (sec)	N/A	0.021	0.729	0.307	0.310	0.252	25.975	0.263	2.330

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.095	26.427	0.255	0.435	0.258	0.000	0.275	2.377

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	219	241	216	1055	0	0	0
N.S.	1	1.00	1.53	1.69	1.51	7.38	0.00	0.00	0.00
time (sec)	N/A	0.140	0.317	0.612	0.279	0.275	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	152	185	157	731	0	0	0
N.S.	1	1.00	1.60	1.95	1.65	7.69	0.00	0.00	0.00
time (sec)	N/A	0.087	0.221	0.594	0.281	0.270	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	79	89	109	367	0	144	95
N.S.	1	1.00	1.68	1.89	2.32	7.81	0.00	3.06	2.02
time (sec)	N/A	0.038	0.321	0.563	0.272	0.260	0.000	0.301	0.100

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	33	56	31	0	45	22
N.S.	1	1.00	1.00	1.50	2.55	1.41	0.00	2.05	1.00
time (sec)	N/A	0.017	0.005	0.330	0.191	0.252	0.000	0.266	2.315

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	94	22	19	22	22
N.S.	1	1.00	1.11	1.11	5.22	1.22	1.06	1.22	1.22
time (sec)	N/A	0.091	18.292	0.308	0.332	0.261	52.124	0.750	2.391

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	102	22	20	22	22
N.S.	1	1.00	1.11	1.11	5.67	1.22	1.11	1.22	1.22
time (sec)	N/A	0.095	14.331	0.312	0.333	0.248	70.250	1.628	2.397

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.338	35.339	0.104	0.367	0.256	0.000	0.281	2.478

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	118	177	130	979	0	0	0
N.S.	1	1.00	1.42	2.13	1.57	11.80	0.00	0.00	0.00
time (sec)	N/A	0.122	1.103	0.535	0.264	0.270	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	72	107	383	0	139	101
N.S.	1	1.00	1.31	1.71	2.55	9.12	0.00	3.31	2.40
time (sec)	N/A	0.040	0.095	0.424	0.268	0.259	0.000	0.275	2.318

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	43	130	107	0	184	36
N.S.	1	1.00	1.00	1.43	4.33	3.57	0.00	6.13	1.20
time (sec)	N/A	0.022	0.178	0.381	0.208	0.250	0.000	0.270	2.367

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	25	86	0	27	13
N.S.	1	1.00	1.00	0.93	1.67	5.73	0.00	1.80	0.87
time (sec)	N/A	0.016	0.003	0.170	0.185	0.250	0.000	0.268	2.380

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	128	20	17	20	20
N.S.	1	1.00	1.11	1.00	7.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.127	15.930	0.213	0.306	0.258	21.179	0.273	2.455

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	127	20	19	20	20
N.S.	1	1.00	1.11	1.00	7.06	1.11	1.06	1.11	1.11
time (sec)	N/A	0.174	19.038	0.201	0.307	0.246	27.851	0.265	2.428

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	22	22	0	22	22
N.S.	1	1.00	1.11	1.11	1.22	1.22	0.00	1.22	1.22
time (sec)	N/A	0.052	164.808	0.201	0.406	0.258	0.000	0.275	2.387

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	280	340	262	1802	0	0	0
N.S.	1	1.00	1.39	1.69	1.30	8.97	0.00	0.00	0.00
time (sec)	N/A	0.260	3.309	1.093	0.282	0.284	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	222	210	197	1311	0	0	0
N.S.	1	1.00	1.80	1.71	1.60	10.66	0.00	0.00	0.00
time (sec)	N/A	0.174	1.835	1.052	0.289	0.279	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	116	156	124	842	0	0	0
N.S.	1	1.00	1.41	1.90	1.51	10.27	0.00	0.00	0.00
time (sec)	N/A	0.091	0.864	1.042	0.277	0.264	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	75	45	84	387	0	84	87
N.S.	1	1.00	2.21	1.32	2.47	11.38	0.00	2.47	2.56
time (sec)	N/A	0.023	0.008	0.779	0.200	0.253	0.000	0.266	2.333

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	157	22	19	22	22
N.S.	1	1.00	1.11	1.11	8.72	1.22	1.06	1.22	1.22
time (sec)	N/A	0.053	47.096	0.348	0.325	0.263	51.835	1.706	2.358

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	20	159	22	20	22	22
N.S.	1	1.00	1.11	1.11	8.83	1.22	1.11	1.22	1.22
time (sec)	N/A	0.055	47.665	0.373	0.326	0.253	61.924	2.492	2.398

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	173	22	0	22	22
N.S.	1	1.00	1.17	1.67	14.42	1.83	0.00	1.83	1.83
time (sec)	N/A	0.026	89.980	0.243	0.771	0.264	0.000	0.358	2.482

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	422	375	302	1985	0	0	0
N.S.	1	1.00	2.36	2.09	1.69	11.09	0.00	0.00	0.00
time (sec)	N/A	0.249	2.144	0.527	0.252	0.292	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	314	246	226	1467	0	0	0
N.S.	1	1.00	2.75	2.16	1.98	12.87	0.00	0.00	0.00
time (sec)	N/A	0.151	2.090	0.525	0.280	0.284	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	131	164	149	975	0	0	0
N.S.	1	1.00	1.60	2.00	1.82	11.89	0.00	0.00	0.00
time (sec)	N/A	0.090	0.937	0.518	0.265	0.279	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	23	79	346	0	66	25
N.S.	1	1.00	1.26	0.85	2.93	12.81	0.00	2.44	0.93
time (sec)	N/A	0.015	0.043	0.315	0.196	0.262	0.000	0.307	2.335

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	144	22	19	22	22
N.S.	1	1.00	1.17	1.67	12.00	1.83	1.58	1.83	1.83
time (sec)	N/A	0.021	0.262	0.332	0.310	0.260	118.960	0.284	2.560

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	20	175	22	20	22	22
N.S.	1	1.00	1.17	1.67	14.58	1.83	1.67	1.83	1.83
time (sec)	N/A	0.022	0.210	0.339	0.317	0.277	159.591	0.274	2.565

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	17	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.06	1.12	1.38
time (sec)	N/A	0.172	8.760	0.100	0.314	0.251	0.342	0.265	2.389

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	150	241	203	448	0	0	0
N.S.	1	1.00	1.01	1.63	1.37	3.03	0.00	0.00	0.00
time (sec)	N/A	0.115	0.255	0.844	0.208	0.259	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	108	186	148	351	0	0	0
N.S.	1	1.00	1.11	1.92	1.53	3.62	0.00	0.00	0.00
time (sec)	N/A	0.085	0.194	0.674	0.189	0.263	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	125	87	224	0	0	0
N.S.	1	1.00	1.16	2.16	1.50	3.86	0.00	0.00	0.00
time (sec)	N/A	0.040	0.164	0.529	0.197	0.275	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	50	60	0	41	30
N.S.	1	1.00	2.82	1.09	4.55	5.45	0.00	3.73	2.73
time (sec)	N/A	0.010	0.005	0.356	0.269	0.267	0.000	0.257	0.069

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.38
time (sec)	N/A	0.027	13.677	0.201	0.314	0.256	0.537	0.298	2.318

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	17	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.06	1.12	1.38
time (sec)	N/A	0.028	13.794	0.199	0.312	0.265	0.385	0.359	2.303

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.282	102.848	0.114	0.302	0.263	0.382	0.264	2.398

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	263	0	0	1280	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	5.66	0.00	0.00	0.00
time (sec)	N/A	0.269	0.267	0.000	0.000	0.303	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	181	0	0	937	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	6.42	0.00	0.00	0.00
time (sec)	N/A	0.173	0.175	0.000	0.000	0.278	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	78	95	90	401	0	0	0
N.S.	1	1.00	1.16	1.42	1.34	5.99	0.00	0.00	0.00
time (sec)	N/A	0.083	0.284	1.309	0.291	0.253	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	23	61	155	0	64	52
N.S.	1	1.00	1.83	1.00	2.65	6.74	0.00	2.78	2.26
time (sec)	N/A	0.019	0.006	0.822	0.189	0.254	0.000	0.277	0.067

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	99	20	17	20	22
N.S.	1	1.00	1.11	1.00	5.50	1.11	0.94	1.11	1.22
time (sec)	N/A	0.156	35.641	0.194	0.320	0.259	0.752	0.968	2.295

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	111	20	19	20	22
N.S.	1	1.00	1.11	1.00	6.17	1.11	1.06	1.11	1.22
time (sec)	N/A	0.174	22.311	0.196	0.341	0.268	0.525	1.229	2.354

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.348	80.004	0.116	0.313	0.252	0.367	0.276	2.311

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	524	359	329	3409	0	0	0
N.S.	1	1.00	2.18	1.50	1.37	14.20	0.00	0.00	0.00
time (sec)	N/A	0.320	6.395	7.581	0.211	0.312	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	380	256	229	2523	0	0	0
N.S.	1	1.00	2.57	1.73	1.55	17.05	0.00	0.00	0.00
time (sec)	N/A	0.189	2.611	5.470	0.213	0.291	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	87	166	142	1543	0	0	0
N.S.	1	1.00	0.92	1.75	1.49	16.24	0.00	0.00	0.00
time (sec)	N/A	0.092	0.379	3.822	0.231	0.288	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	36	23	88	371	0	93	78
N.S.	1	1.00	1.33	0.85	3.26	13.74	0.00	3.44	2.89
time (sec)	N/A	0.022	0.012	2.610	0.269	0.257	0.000	0.268	0.077

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	147	20	17	20	22
N.S.	1	1.00	1.11	1.00	8.17	1.11	0.94	1.11	1.22
time (sec)	N/A	0.172	47.503	0.204	0.329	0.255	0.731	0.456	2.436

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	155	20	19	20	22
N.S.	1	1.00	1.11	1.00	8.61	1.11	1.06	1.11	1.22
time (sec)	N/A	0.199	26.474	0.251	0.335	0.285	0.515	0.543	2.334

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.304	45.165	0.098	0.322	0.258	0.343	0.278	2.351

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	334	0	0	1309	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	5.52	0.00	0.00	0.00
time (sec)	N/A	0.276	1.886	0.000	0.000	0.308	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	252	0	0	966	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	6.15	0.00	0.00	0.00
time (sec)	N/A	0.180	1.033	0.000	0.000	0.277	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	184	179	0	567	0	0	0
N.S.	1	1.00	2.33	2.27	0.00	7.18	0.00	0.00	0.00
time (sec)	N/A	0.083	0.493	0.994	0.000	0.285	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	29	25	43	103	0	54	48
N.S.	1	1.00	1.21	1.04	1.79	4.29	0.00	2.25	2.00
time (sec)	N/A	0.021	0.003	0.615	0.280	0.252	0.000	0.263	2.265

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	106	20	17	20	22
N.S.	1	1.00	1.11	1.00	5.89	1.11	0.94	1.11	1.22
time (sec)	N/A	0.148	31.244	0.223	0.350	0.267	0.565	0.835	2.377

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	114	20	19	0	22
N.S.	1	1.00	1.11	1.00	6.33	1.11	1.06	0.00	1.22
time (sec)	N/A	0.182	24.083	0.256	0.350	0.254	0.403	0.000	2.439

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.366	5.218	0.140	0.334	0.264	0.386	0.272	2.454

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	307	263	180	1924	0	0	0
N.S.	1	1.00	3.61	3.09	2.12	22.64	0.00	0.00	0.00
time (sec)	N/A	0.172	1.477	6.984	0.208	0.286	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	216	199	118	1327	0	0	0
N.S.	1	1.00	3.38	3.11	1.84	20.73	0.00	0.00	0.00
time (sec)	N/A	0.124	0.906	4.993	0.209	0.286	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	62	87	292	0	72	43
N.S.	1	1.00	0.87	2.07	2.90	9.73	0.00	2.40	1.43
time (sec)	N/A	0.039	0.163	3.527	0.203	0.265	0.000	0.268	0.080

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	13	32	18	81	0	18	18
N.S.	1	1.00	0.57	1.39	0.78	3.52	0.00	0.78	0.78
time (sec)	N/A	0.024	0.003	2.729	0.194	0.271	0.000	0.288	2.455

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	101	22	19	22	22
N.S.	1	1.00	1.10	1.00	5.05	1.10	0.95	1.10	1.10
time (sec)	N/A	0.046	24.151	0.246	0.322	0.251	0.566	0.270	2.323

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	105	22	20	22	22
N.S.	1	1.00	1.10	1.00	5.25	1.10	1.00	1.10	1.10
time (sec)	N/A	0.047	18.241	0.241	0.321	0.267	0.390	0.264	2.312

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.409	60.430	0.124	0.325	0.262	0.363	0.280	2.334

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	310	0	0	3825	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	18.57	0.00	0.00	0.00
time (sec)	N/A	0.322	4.856	0.000	0.000	0.333	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	205	232	0	2227	0	0	0
N.S.	1	1.00	1.71	1.93	0.00	18.56	0.00	0.00	0.00
time (sec)	N/A	0.124	3.162	8.926	0.000	0.288	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	47	90	511	0	102	107
N.S.	1	1.00	0.59	0.96	1.84	10.43	0.00	2.08	2.18
time (sec)	N/A	0.030	0.002	6.353	0.279	0.256	0.000	0.289	0.091

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	214	22	19	22	22
N.S.	1	1.00	1.10	1.00	10.70	1.10	0.95	1.10	1.10
time (sec)	N/A	0.209	49.821	0.254	0.393	0.245	0.573	2.065	2.309

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	215	22	20	22	22
N.S.	1	1.00	1.10	1.00	10.75	1.10	1.00	1.10	1.10
time (sec)	N/A	0.268	32.903	0.247	0.404	0.256	0.398	2.812	2.350

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	19	20	22
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.06	1.11	1.22
time (sec)	N/A	0.358	67.847	0.151	0.324	0.258	0.359	0.270	2.379

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	565	417	352	3394	0	0	0
N.S.	1	1.00	2.35	1.74	1.47	14.14	0.00	0.00	0.00
time (sec)	N/A	0.336	6.512	3.828	0.216	0.298	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	388	266	243	2562	0	0	0
N.S.	1	1.00	2.62	1.80	1.64	17.31	0.00	0.00	0.00
time (sec)	N/A	0.172	2.954	2.622	0.231	0.270	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	170	145	1578	0	0	0
N.S.	1	1.00	0.89	1.79	1.53	16.61	0.00	0.00	0.00
time (sec)	N/A	0.092	0.464	1.844	0.225	0.287	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	25	91	379	0	93	78
N.S.	1	1.00	1.21	0.89	3.25	13.54	0.00	3.32	2.79
time (sec)	N/A	0.019	0.018	1.244	0.275	0.256	0.000	0.268	2.491

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	167	20	17	20	22
N.S.	1	1.00	1.11	1.00	9.28	1.11	0.94	1.11	1.22
time (sec)	N/A	0.174	55.269	0.252	0.354	0.246	0.566	0.413	2.380

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	170	20	19	20	22
N.S.	1	1.00	1.11	1.00	9.44	1.11	1.06	1.11	1.22
time (sec)	N/A	0.195	23.517	0.257	0.362	0.254	0.406	0.505	2.293

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.439	60.007	0.124	0.333	0.252	0.359	0.277	2.286

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	317	466	0	0	5356	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	16.90	0.00	0.00	0.00
time (sec)	N/A	0.881	6.438	0.000	0.000	0.334	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	197	341	0	0	3804	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	19.31	0.00	0.00	0.00
time (sec)	N/A	0.366	6.336	0.000	0.000	0.321	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	139	148	166	1694	0	0	0
N.S.	1	1.00	1.28	1.36	1.52	15.54	0.00	0.00	0.00
time (sec)	N/A	0.117	1.368	5.888	0.306	0.283	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	43	106	709	0	110	111
N.S.	1	1.00	1.76	0.88	2.16	14.47	0.00	2.24	2.27
time (sec)	N/A	0.032	0.009	3.900	0.202	0.260	0.000	0.312	0.089

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	226	22	19	22	22
N.S.	1	1.00	1.10	1.00	11.30	1.10	0.95	1.10	1.10
time (sec)	N/A	0.228	69.138	0.267	0.378	0.248	0.564	2.356	2.336

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	226	22	20	3	22
N.S.	1	1.00	1.10	1.00	11.30	1.10	1.00	0.15	1.10
time (sec)	N/A	0.248	43.819	0.186	0.393	0.251	0.406	20.102	2.432

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.398	167.592	0.119	0.325	0.253	0.360	0.286	2.371

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	274	445	381	6764	0	0	0
N.S.	1	1.00	1.14	1.85	1.59	28.18	0.00	0.00	0.00
time (sec)	N/A	0.234	3.337	29.302	0.226	0.328	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	192	299	273	4779	0	0	0
N.S.	1	1.00	1.29	2.01	1.83	32.07	0.00	0.00	0.00
time (sec)	N/A	0.150	3.064	21.848	0.220	0.313	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	106	197	164	3025	0	0	0
N.S.	1	1.00	1.16	2.16	1.80	33.24	0.00	0.00	0.00
time (sec)	N/A	0.083	0.749	15.865	0.222	0.302	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	43	102	774	0	96	96
N.S.	1	1.00	1.42	1.00	2.37	18.00	0.00	2.23	2.23
time (sec)	N/A	0.033	0.006	11.704	0.276	0.258	0.000	0.278	0.091

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	208	22	19	22	22
N.S.	1	1.00	1.10	1.00	10.40	1.10	0.95	1.10	1.10
time (sec)	N/A	0.050	51.986	0.261	0.358	0.257	0.575	0.532	2.250

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	206	22	20	22	22
N.S.	1	1.00	1.10	1.00	10.30	1.10	1.00	1.10	1.10
time (sec)	N/A	0.055	38.386	0.257	0.373	0.247	0.401	0.641	2.271

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.389	0.000	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	142	0	0	0	0	0	0
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	2.332	0.000	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	56	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.262	0.000	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	109	250	0	0	0	0	0
N.S.	1	1.00	2.95	6.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	1.286	0.261	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	69	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.361	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	69	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	65	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	57	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.300	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	100	250	0	0	0	0	0
N.S.	1	1.00	1.75	4.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	1.400	0.287	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	71	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.322	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	125	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	3.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.421	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	143	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	3.954	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	77	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	115	229	0	0	0	0	0
N.S.	1	1.00	1.62	3.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	1.665	0.232	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.277	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	66	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	67	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.357	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	83	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	75	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	70	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	56	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.243	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	100	229	0	0	0	0	0
N.S.	1	1.00	1.41	3.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.898	0.285	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	67	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.457	0.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	111	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	3.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	103	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.495	0.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	42	35	53	0	0	35
N.S.	1	1.00	1.00	3.23	2.69	4.08	0.00	0.00	2.69
time (sec)	N/A	0.036	0.111	0.655	0.310	0.249	0.000	0.000	2.288

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	23	0	69	95	0	0	0
N.S.	1	1.00	0.74	0.00	2.23	3.06	0.00	0.00	0.00
time (sec)	N/A	0.064	0.107	0.000	0.335	0.262	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	29	0	103	253	0	0	0
N.S.	1	1.00	0.58	0.00	2.06	5.06	0.00	0.00	0.00
time (sec)	N/A	0.084	0.140	0.000	0.301	0.259	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	54	55	0	0	23
N.S.	1	1.00	2.69	3.23	4.15	4.23	0.00	0.00	1.77
time (sec)	N/A	0.042	0.106	0.640	0.299	0.245	0.000	0.000	2.234

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	21	0	109	97	0	0	0
N.S.	1	1.00	0.68	0.00	3.52	3.13	0.00	0.00	0.00
time (sec)	N/A	0.074	0.085	0.000	0.300	0.252	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	163	259	0	0	0
N.S.	1	1.00	0.88	0.00	3.26	5.18	0.00	0.00	0.00
time (sec)	N/A	0.098	0.287	0.000	0.308	0.266	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	60	50	122	167	585	87	178
N.S.	1	1.00	1.15	0.96	2.35	3.21	11.25	1.67	3.42
time (sec)	N/A	0.100	0.159	0.687	0.278	0.266	30.481	0.277	2.556

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	52	119	174	586	88	177
N.S.	1	1.00	1.17	0.98	2.25	3.28	11.06	1.66	3.34
time (sec)	N/A	0.102	0.233	0.714	0.297	0.258	30.303	0.275	0.342

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	104	0	289	840	60	198
N.S.	1	1.00	0.98	1.82	0.00	5.07	14.74	1.05	3.47
time (sec)	N/A	0.108	0.091	0.372	0.000	0.265	16.661	0.263	2.513

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	109	0	299	840	62	199
N.S.	1	1.00	0.95	1.85	0.00	5.07	14.24	1.05	3.37
time (sec)	N/A	0.103	0.125	0.413	0.000	0.277	15.822	0.259	2.486

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	62	134	0	96	103
N.S.	1	1.00	1.00	2.32	2.48	5.36	0.00	3.84	4.12
time (sec)	N/A	0.041	0.295	0.621	0.409	0.261	0.000	0.301	2.427

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	55	0	129	0	100	105
N.S.	1	1.00	1.00	2.20	0.00	5.16	0.00	4.00	4.20
time (sec)	N/A	0.044	0.115	0.556	0.000	0.274	0.000	0.281	2.427

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	63	59	0	249	0	53	636
N.S.	1	1.00	1.02	0.95	0.00	4.02	0.00	0.85	10.26
time (sec)	N/A	0.131	0.205	0.510	0.000	0.482	0.000	0.267	7.579

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	77	58	141	172	0	90	539
N.S.	1	1.00	1.33	1.00	2.43	2.97	0.00	1.55	9.29
time (sec)	N/A	0.130	1.195	0.680	0.284	0.467	0.000	0.286	4.291

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	39	64	70	238	41	56
N.S.	1	1.00	1.26	2.05	3.37	3.68	12.53	2.16	2.95
time (sec)	N/A	0.030	0.015	0.206	0.272	0.268	3.320	0.252	2.290

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	17	9	14	17	41	14	14
N.S.	1	1.00	2.12	1.12	1.75	2.12	5.12	1.75	1.75
time (sec)	N/A	0.034	0.022	0.251	0.191	0.246	0.449	0.242	0.046

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	9	14	17	12	14	14
N.S.	1	1.00	2.25	1.12	1.75	2.12	1.50	1.75	1.75
time (sec)	N/A	0.035	0.035	0.243	0.196	0.257	0.369	0.245	0.049

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	39	102	70	61	38	56
N.S.	1	1.00	1.26	2.05	5.37	3.68	3.21	2.00	2.95
time (sec)	N/A	0.029	0.061	0.540	0.287	0.263	1.042	0.255	0.119

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	127	84	0	598	0	71	704
N.S.	1	1.00	1.72	1.14	0.00	8.08	0.00	0.96	9.51
time (sec)	N/A	0.188	0.626	0.605	0.000	0.476	0.000	0.259	8.066

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	106	86	158	401	0	109	613
N.S.	1	1.00	1.54	1.25	2.29	5.81	0.00	1.58	8.88
time (sec)	N/A	0.188	1.624	0.720	0.295	0.480	0.000	0.270	4.577

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	8	23	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.89	2.56	1.00
time (sec)	N/A	0.006	0.036	0.238	0.189	0.257	0.081	0.259	2.321

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	37	46	42	78	74	39
N.S.	1	1.00	0.97	1.00	1.24	1.14	2.11	2.00	1.05
time (sec)	N/A	0.013	0.051	0.999	0.196	0.256	0.086	0.263	2.320

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	63	48	69	97	66	134	53
N.S.	1	1.00	1.80	1.37	1.97	2.77	1.89	3.83	1.51
time (sec)	N/A	0.017	0.117	5.914	0.195	0.259	0.116	0.262	0.101

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	87	90	103	168	265	208	143
N.S.	1	1.00	1.21	1.25	1.43	2.33	3.68	2.89	1.99
time (sec)	N/A	0.028	0.101	24.436	0.202	0.264	0.188	0.274	2.316

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	133	102	191	298	172	344	117
N.S.	1	1.00	2.18	1.67	3.13	4.89	2.82	5.64	1.92
time (sec)	N/A	0.033	0.201	180.655	0.210	0.248	0.249	0.266	0.137

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	148	105	35	35
N.S.	1	1.00	1.21	1.03	0.00	3.89	2.76	0.92	0.92
time (sec)	N/A	0.019	0.027	0.655	0.000	0.265	2.106	0.254	0.106

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	27	29	62	604	26	22
N.S.	1	1.00	1.00	1.59	1.71	3.65	35.53	1.53	1.29
time (sec)	N/A	0.011	0.029	5.051	0.228	0.249	136.944	0.259	2.281

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	96	146	0	1495	0	88	157
N.S.	1	1.00	1.25	1.90	0.00	19.42	0.00	1.14	2.04
time (sec)	N/A	0.039	0.359	32.794	0.000	0.286	0.000	0.260	2.336

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	46	498	527	0	53	47
N.S.	1	1.00	0.96	0.69	7.43	7.87	0.00	0.79	0.70
time (sec)	N/A	0.026	0.098	137.481	0.225	0.256	0.000	0.271	2.428

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	147	462	0	6874	0	236	354
N.S.	1	1.00	1.31	4.12	0.00	61.38	0.00	2.11	3.16
time (sec)	N/A	0.059	0.663	0.288	0.000	0.352	0.000	0.266	2.407

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	206	33	0	57	0	0	0
N.S.	1	1.00	3.17	0.51	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.021	0.514	0.986	0.000	0.078	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	92	171	0	101	0	0	0
N.S.	1	1.00	0.89	1.66	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.039	0.418	0.809	0.000	0.086	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	193	42	0	274	0	0	0
N.S.	1	1.00	1.87	0.41	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.038	0.588	0.912	0.000	0.086	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	97	0	29	0	0	0
N.S.	1	1.00	1.25	1.49	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.021	0.085	0.650	0.000	0.079	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	148	33	0	222	0	0	0
N.S.	1	1.00	1.32	0.29	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.037	0.372	0.587	0.000	0.087	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	37	0	679	0	0	0
N.S.	1	1.00	1.15	0.32	0.00	5.85	0.00	0.00	0.00
time (sec)	N/A	0.041	0.425	0.629	0.000	0.097	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	45	12	23	21	29	56	11
N.S.	1	1.00	1.96	0.52	1.00	0.91	1.26	2.43	0.48
time (sec)	N/A	0.012	0.007	0.343	0.190	0.247	0.090	0.254	0.061

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	18	88	43	44	17	17
N.S.	1	1.00	0.96	0.69	3.38	1.65	1.69	0.65	0.65
time (sec)	N/A	0.013	0.138	1.276	0.199	0.243	0.116	0.254	0.070

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	18	146	64	83	17	17
N.S.	1	1.00	0.96	0.69	5.62	2.46	3.19	0.65	0.65
time (sec)	N/A	0.012	0.208	6.512	0.206	0.237	0.163	0.257	0.074

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	18	34	36	17	17
N.S.	1	1.00	0.92	1.04	0.69	1.31	1.38	0.65	0.65
time (sec)	N/A	0.015	0.077	0.741	0.200	0.247	0.127	0.261	2.404

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	17	23	34	17	17
N.S.	1	1.00	1.00	0.75	0.71	0.96	1.42	0.71	0.71
time (sec)	N/A	0.012	0.174	0.667	0.195	0.254	0.296	0.267	0.073

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	49	66	17	17
N.S.	1	1.00	1.00	0.69	0.65	1.88	2.54	0.65	0.65
time (sec)	N/A	0.013	0.190	3.465	0.197	0.248	0.448	0.263	0.110

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	71	90	17	17
N.S.	1	1.00	1.00	0.69	0.65	2.73	3.46	0.65	0.65
time (sec)	N/A	0.014	0.200	19.502	0.205	0.259	0.613	0.269	2.293

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	23	17	24	0	17	15
N.S.	1	1.00	0.92	0.88	0.65	0.92	0.00	0.65	0.58
time (sec)	N/A	0.015	0.066	0.332	0.190	0.255	0.000	0.252	2.252

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	16	17	42	0	17	15
N.S.	1	1.00	0.92	0.62	0.65	1.62	0.00	0.65	0.58
time (sec)	N/A	0.013	0.079	0.270	0.191	0.255	0.000	0.252	2.226

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	47	16	24	22	29	56	15
N.S.	1	1.00	1.96	0.67	1.00	0.92	1.21	2.33	0.62
time (sec)	N/A	0.012	0.041	0.369	0.190	0.255	0.102	0.271	0.050

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	89	43	44	17	17
N.S.	1	1.00	1.00	0.67	3.30	1.59	1.63	0.63	0.63
time (sec)	N/A	0.012	0.088	1.310	0.190	0.240	0.105	0.257	2.209

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	147	62	83	17	17
N.S.	1	1.00	1.00	0.67	5.44	2.30	3.07	0.63	0.63
time (sec)	N/A	0.012	0.120	6.793	0.213	0.244	0.144	0.261	2.306

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	20	39	37	23	21
N.S.	1	1.00	0.96	1.04	0.71	1.39	1.32	0.82	0.75
time (sec)	N/A	0.012	0.057	0.739	0.202	0.236	0.117	0.255	2.365

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	14	13	20	32	13	13
N.S.	1	1.00	0.92	0.58	0.54	0.83	1.33	0.54	0.54
time (sec)	N/A	0.013	0.007	0.639	0.192	0.247	0.207	0.254	2.252

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	17	41	65	17	17
N.S.	1	1.00	1.00	0.67	0.63	1.52	2.41	0.63	0.63
time (sec)	N/A	0.013	0.114	3.433	0.195	0.251	0.278	0.257	0.101

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	17	59	88	17	17
N.S.	1	1.00	1.00	0.67	0.63	2.19	3.26	0.63	0.63
time (sec)	N/A	0.015	0.137	19.299	0.202	0.250	0.618	0.264	2.298

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	19	17	24	0	17	18
N.S.	1	1.00	0.96	0.70	0.63	0.89	0.00	0.63	0.67
time (sec)	N/A	0.014	0.040	0.450	0.198	0.236	0.000	0.259	2.602

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	19	17	40	0	17	27
N.S.	1	1.00	0.96	0.70	0.63	1.48	0.00	0.63	1.00
time (sec)	N/A	0.013	0.035	0.287	0.201	0.247	0.000	0.261	0.190

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	355	160	279	2040	0	240	495
N.S.	1	1.00	2.86	1.29	2.25	16.45	0.00	1.94	3.99
time (sec)	N/A	0.144	1.301	198.102	0.293	0.266	0.000	0.262	2.909

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	84	210	207	0	110	145
N.S.	1	1.00	0.79	0.84	2.10	2.07	0.00	1.10	1.45
time (sec)	N/A	0.151	0.241	52.869	0.223	0.252	0.000	0.267	2.360

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	194	64	120	502	0	117	233
N.S.	1	1.00	3.34	1.10	2.07	8.66	0.00	2.02	4.02
time (sec)	N/A	0.071	1.299	13.145	0.278	0.269	0.000	0.258	2.505

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	26	43	42	0	31	33
N.S.	1	1.00	0.90	0.90	1.48	1.45	0.00	1.07	1.14
time (sec)	N/A	0.041	0.035	2.971	0.195	0.259	0.000	0.257	2.326

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	30	19	21	40
N.S.	1	1.00	1.00	1.09	1.00	2.73	1.73	1.91	3.64
time (sec)	N/A	0.007	0.006	0.359	0.198	0.251	0.196	0.263	0.073

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	27	28	27	32	22	25
N.S.	1	1.00	1.00	2.45	2.55	2.45	2.91	2.00	2.27
time (sec)	N/A	0.028	0.003	0.431	0.190	0.261	0.154	0.262	0.085

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	502	101	100	362	0	97	132
N.S.	1	1.00	8.10	1.63	1.61	5.84	0.00	1.56	2.13
time (sec)	N/A	0.101	1.588	2.048	0.283	0.257	0.000	0.272	2.439

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	42	78	117	543	651	75	0
N.S.	1	1.00	0.88	1.62	2.44	11.31	13.56	1.56	0.00
time (sec)	N/A	0.058	0.100	11.794	0.203	0.263	1.299	0.264	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	3430	357	375	2978	0	267	0
N.S.	1	1.00	23.49	2.45	2.57	20.40	0.00	1.83	0.00
time (sec)	N/A	0.256	7.169	90.760	0.321	0.286	0.000	0.283	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	176	297	2640	2162	152	0
N.S.	1	1.00	0.87	1.85	3.13	27.79	22.76	1.60	0.00
time (sec)	N/A	0.091	0.167	289.964	0.251	0.283	6.197	0.283	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	62	78	235	94	0	34	90
N.S.	1	1.00	1.55	1.95	5.88	2.35	0.00	0.85	2.25
time (sec)	N/A	0.040	0.041	0.488	0.277	0.255	0.000	0.257	2.438

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	74	60	181	52	0	22	65
N.S.	1	1.00	1.95	1.58	4.76	1.37	0.00	0.58	1.71
time (sec)	N/A	0.074	0.119	0.733	0.195	0.244	0.000	0.255	2.361

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	39	26	73	50	0	21	39
N.S.	1	1.00	1.50	1.00	2.81	1.92	0.00	0.81	1.50
time (sec)	N/A	0.036	0.028	73.087	0.285	0.244	0.000	0.263	2.317

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	15	25	17	0	12	14
N.S.	1	1.00	0.80	0.75	1.25	0.85	0.00	0.60	0.70
time (sec)	N/A	0.054	0.012	4.597	0.220	0.244	0.000	0.255	2.614

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	11	17	18	14
N.S.	1	1.00	1.00	1.00	0.82	1.00	1.55	1.64	1.27
time (sec)	N/A	0.006	0.005	1.164	0.205	0.255	0.229	0.253	0.057

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	15	15	11	22	11	14
N.S.	1	1.00	1.31	1.15	1.15	0.85	1.69	0.85	1.08
time (sec)	N/A	0.025	0.018	1.461	0.196	0.249	0.114	0.258	2.475

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	15	14	17	0	12	14
N.S.	1	1.00	1.55	0.75	0.70	0.85	0.00	0.60	0.70
time (sec)	N/A	0.034	0.024	7.432	0.200	0.252	0.000	0.257	0.098

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	40	26	33	50	432	21	41
N.S.	1	1.00	1.43	0.93	1.18	1.79	15.43	0.75	1.46
time (sec)	N/A	0.037	0.033	166.629	0.201	0.260	0.982	0.265	0.203

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	41	40	52	0	22	67
N.S.	1	1.00	1.97	1.08	1.05	1.37	0.00	0.58	1.76
time (sec)	N/A	0.054	0.066	0.408	0.198	0.252	0.000	0.253	0.192

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	45	68	58	94	1445	34	94
N.S.	1	1.00	1.07	1.62	1.38	2.24	34.40	0.81	2.24
time (sec)	N/A	0.045	0.084	0.396	0.221	0.269	4.262	0.252	2.422

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	62	78	235	94	0	34	94
N.S.	1	1.00	1.48	1.86	5.60	2.24	0.00	0.81	2.24
time (sec)	N/A	0.041	0.037	0.323	0.280	0.259	0.000	0.255	2.336

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	75	26	181	52	0	22	67
N.S.	1	1.00	1.97	0.68	4.76	1.37	0.00	0.58	1.76
time (sec)	N/A	0.073	0.066	278.081	0.202	0.246	0.000	0.255	2.290

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	26	73	50	0	21	41
N.S.	1	1.00	1.39	0.93	2.61	1.79	0.00	0.75	1.46
time (sec)	N/A	0.038	0.027	15.784	0.276	0.257	0.000	0.254	0.166

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	15	25	17	0	12	14
N.S.	1	1.00	0.80	0.75	1.25	0.85	0.00	0.60	0.70
time (sec)	N/A	0.053	0.005	6.089	0.189	0.249	0.000	0.252	0.120

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	9	11	17	18	14
N.S.	1	1.00	1.00	1.00	0.82	1.00	1.55	1.64	1.27
time (sec)	N/A	0.006	0.004	1.622	0.212	0.249	0.212	0.260	2.197

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	15	15	11	22	11	14
N.S.	1	1.00	1.55	1.36	1.36	1.00	2.00	1.00	1.27
time (sec)	N/A	0.026	0.018	1.918	0.209	0.258	0.131	0.258	0.137

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	15	14	17	0	12	14
N.S.	1	1.00	1.55	0.75	0.70	0.85	0.00	0.60	0.70
time (sec)	N/A	0.033	0.025	9.132	0.200	0.248	0.000	0.245	2.241

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	26	33	50	432	21	39
N.S.	1	1.00	1.04	1.00	1.27	1.92	16.62	0.81	1.50
time (sec)	N/A	0.038	0.033	44.040	0.212	0.257	0.972	0.261	0.190

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	74	41	40	52	0	22	65
N.S.	1	1.00	1.95	1.08	1.05	1.37	0.00	0.58	1.71
time (sec)	N/A	0.053	0.052	0.647	0.205	0.246	0.000	0.258	2.370

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	45	68	58	94	1445	34	90
N.S.	1	1.00	1.12	1.70	1.45	2.35	36.12	0.85	2.25
time (sec)	N/A	0.040	0.084	0.511	0.206	0.256	4.418	0.262	2.777

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	143	159	330	2716	0	234	392
N.S.	1	1.00	1.15	1.28	2.66	21.90	0.00	1.89	3.16
time (sec)	N/A	0.171	0.845	55.342	0.197	0.276	0.000	0.268	0.211

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	95	84	214	209	0	112	146
N.S.	1	1.00	0.94	0.83	2.12	2.07	0.00	1.11	1.45
time (sec)	N/A	0.162	0.290	20.292	0.190	0.259	0.000	0.260	2.417

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	78	66	152	674	0	115	169
N.S.	1	1.00	1.32	1.12	2.58	11.42	0.00	1.95	2.86
time (sec)	N/A	0.091	0.478	5.105	0.199	0.279	0.000	0.268	0.110

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	27	45	37	0	29	33
N.S.	1	1.00	0.85	1.00	1.67	1.37	0.00	1.07	1.22
time (sec)	N/A	0.049	0.165	1.339	0.188	0.249	0.000	0.266	2.311

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	30	14	13	29	22	33	35
N.S.	1	1.00	2.50	1.17	1.08	2.42	1.83	2.75	2.92
time (sec)	N/A	0.007	0.032	0.366	0.186	0.246	0.245	0.256	0.070

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	27	26	27	0	19	23
N.S.	1	1.00	1.00	2.45	2.36	2.45	0.00	1.73	2.09
time (sec)	N/A	0.032	0.003	0.445	0.208	0.250	0.000	0.245	2.198

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	96	0	682	0	68	139
N.S.	1	1.00	0.91	1.43	0.00	10.18	0.00	1.01	2.07
time (sec)	N/A	0.099	0.320	2.025	0.000	0.270	0.000	0.262	2.483

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	77	75	111	521	0	66	0
N.S.	1	1.00	1.54	1.50	2.22	10.42	0.00	1.32	0.00
time (sec)	N/A	0.078	0.154	10.248	0.203	0.253	0.000	0.264	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	206	0	5830	0	242	0
N.S.	1	1.00	0.94	1.30	0.00	36.67	0.00	1.52	0.00
time (sec)	N/A	0.268	0.371	40.024	0.000	0.353	0.000	0.269	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	138	174	285	2564	0	135	0
N.S.	1	1.00	1.41	1.78	2.91	26.16	0.00	1.38	0.00
time (sec)	N/A	0.121	0.319	126.517	0.222	0.282	0.000	0.267	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	32	32	236	270	0	33	81
N.S.	1	1.00	1.14	1.14	8.43	9.64	0.00	1.18	2.89
time (sec)	N/A	0.048	0.143	5.023	0.201	0.261	0.000	0.249	2.214

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	183	68	0	22	59
N.S.	1	1.00	1.00	0.77	6.10	2.27	0.00	0.73	1.97
time (sec)	N/A	0.086	0.056	1.820	0.189	0.251	0.000	0.261	2.194

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	22	66	91	0	22	33
N.S.	1	1.00	1.11	1.22	3.67	5.06	0.00	1.22	1.83
time (sec)	N/A	0.040	0.097	0.665	0.190	0.244	0.000	0.247	0.049

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	10	11	25	22	0	10	10
N.S.	1	1.00	0.71	0.79	1.79	1.57	0.00	0.71	0.71
time (sec)	N/A	0.056	0.037	0.245	0.185	0.253	0.000	0.260	2.305

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	23	10	9	13	19	25	11
N.S.	1	1.00	2.56	1.11	1.00	1.44	2.11	2.78	1.22
time (sec)	N/A	0.008	0.003	0.160	0.189	0.242	0.256	0.260	0.038

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	9	12	11	13	0	11	11
N.S.	1	1.00	1.80	2.40	2.20	2.60	0.00	2.20	2.20
time (sec)	N/A	0.023	0.026	0.282	0.208	0.255	0.000	0.252	0.045

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	11	12	20	0	10	10
N.S.	1	1.00	0.83	0.92	1.00	1.67	0.00	0.83	0.83
time (sec)	N/A	0.034	0.028	0.496	0.187	0.245	0.000	0.254	0.183

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	31	89	0	21	33
N.S.	1	1.00	1.29	1.57	2.21	6.36	0.00	1.50	2.36
time (sec)	N/A	0.040	0.032	1.332	0.206	0.244	0.000	0.253	2.179

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	2	38	68	0	22	57
N.S.	1	1.00	1.15	0.08	1.46	2.62	0.00	0.85	2.19
time (sec)	N/A	0.062	0.021	3.747	0.199	0.256	0.000	0.263	2.183

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	32	2	52	266	0	30	79
N.S.	1	1.00	1.45	0.09	2.36	12.09	0.00	1.36	3.59
time (sec)	N/A	0.043	0.028	10.657	0.206	0.253	0.000	0.269	2.161

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	28	238	265	0	28	77
N.S.	1	1.00	1.25	1.17	9.92	11.04	0.00	1.17	3.21
time (sec)	N/A	0.044	0.083	0.481	0.206	0.247	0.000	0.269	2.208

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	23	183	68	0	22	57
N.S.	1	1.00	1.15	0.88	7.04	2.62	0.00	0.85	2.19
time (sec)	N/A	0.079	0.008	0.282	0.192	0.252	0.000	0.248	2.191

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	20	68	88	0	19	31
N.S.	1	1.00	1.12	1.25	4.25	5.50	0.00	1.19	1.94
time (sec)	N/A	0.038	0.058	0.269	0.194	0.258	0.000	0.259	2.162

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	25	20	0	10	10
N.S.	1	1.00	1.50	0.92	2.08	1.67	0.00	0.83	0.83
time (sec)	N/A	0.055	0.002	0.124	0.200	0.248	0.000	0.267	0.063

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	27	12	11	11	19	25	9
N.S.	1	1.00	2.45	1.09	1.00	1.00	1.73	2.27	0.82
time (sec)	N/A	0.007	0.008	0.116	0.175	0.254	0.262	0.257	2.176

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	13	11	0	10	9
N.S.	1	1.00	1.00	1.11	1.44	1.22	0.00	1.11	1.00
time (sec)	N/A	0.028	0.035	0.266	0.192	0.249	0.000	0.253	0.043

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	11	12	22	0	10	10
N.S.	1	1.00	1.71	0.79	0.86	1.57	0.00	0.71	0.71
time (sec)	N/A	0.034	0.001	0.446	0.195	0.256	0.000	0.249	2.342

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	27	20	35	90	0	20	31
N.S.	1	1.00	1.35	1.00	1.75	4.50	0.00	1.00	1.55
time (sec)	N/A	0.040	0.048	1.336	0.194	0.249	0.000	0.243	0.051

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	38	68	0	22	59
N.S.	1	1.00	0.93	0.77	1.27	2.27	0.00	0.73	1.97
time (sec)	N/A	0.059	0.008	3.787	0.204	0.265	0.000	0.260	2.185

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	39	2	58	269	0	31	79
N.S.	1	1.00	1.30	0.07	1.93	8.97	0.00	1.03	2.63
time (sec)	N/A	0.047	0.052	10.357	0.197	0.254	0.000	0.255	2.155

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	19	9	8	53	8	24	27
N.S.	1	1.00	2.38	1.12	1.00	6.62	1.00	3.00	3.38
time (sec)	N/A	0.006	0.005	0.136	0.174	0.246	0.168	0.255	0.045

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	26	32	0	39	26
N.S.	1	1.00	0.82	0.68	1.18	1.45	0.00	1.77	1.18
time (sec)	N/A	0.019	0.004	0.400	0.186	0.253	0.000	0.256	2.226

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	67	28	67	616	0	62	71
N.S.	1	1.00	1.97	0.82	1.97	18.12	0.00	1.82	2.09
time (sec)	N/A	0.037	0.321	1.198	0.184	0.258	0.000	0.250	0.067

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	35	42	54	55	0	0	13
N.S.	1	1.00	2.69	3.23	4.15	4.23	0.00	0.00	1.00
time (sec)	N/A	0.056	0.038	0.931	0.280	0.246	0.000	0.000	2.233

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	21	0	109	97	0	0	0
N.S.	1	1.00	0.68	0.00	3.52	3.13	0.00	0.00	0.00
time (sec)	N/A	0.082	0.020	0.000	0.291	0.246	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	163	259	0	0	0
N.S.	1	1.00	0.88	0.00	3.26	5.18	0.00	0.00	0.00
time (sec)	N/A	0.108	0.223	0.000	0.287	0.251	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	42	10	16	16
N.S.	1	1.00	1.00	1.12	1.00	5.25	1.25	2.00	2.00
time (sec)	N/A	0.005	0.007	0.569	0.181	0.269	0.197	0.250	2.201

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	26	30	0	37	26
N.S.	1	1.00	0.73	0.59	1.18	1.36	0.00	1.68	1.18
time (sec)	N/A	0.018	0.029	0.352	0.194	0.254	0.000	0.253	2.229

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	29	56	486	0	66	57
N.S.	1	1.00	1.09	0.85	1.65	14.29	0.00	1.94	1.68
time (sec)	N/A	0.033	0.025	3.722	0.268	0.260	0.000	0.250	0.064

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	43	39	57	0	0	15
N.S.	1	1.00	1.00	3.07	2.79	4.07	0.00	0.00	1.07
time (sec)	N/A	0.038	0.080	0.605	0.289	0.254	0.000	0.000	2.280

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	24	0	77	99	0	0	0
N.S.	1	1.00	0.73	0.00	2.33	3.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.105	0.000	0.294	0.251	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	30	0	115	257	0	0	0
N.S.	1	1.00	0.57	0.00	2.17	4.85	0.00	0.00	0.00
time (sec)	N/A	0.083	0.148	0.000	0.292	0.268	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	24	35	17	39	96	0	43	39
N.S.	1	1.33	1.94	0.94	2.17	5.33	0.00	2.39	2.17
time (sec)	N/A	0.055	0.034	0.297	0.177	0.248	0.000	0.253	0.047

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	24	50	17	40	96	0	43	41
N.S.	1	1.20	2.50	0.85	2.00	4.80	0.00	2.15	2.05
time (sec)	N/A	0.062	0.034	0.300	0.190	0.241	0.000	0.252	0.042

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	39	40	43	143	43	30
N.S.	1	1.00	0.74	1.00	1.03	1.10	3.67	1.10	0.77
time (sec)	N/A	0.055	0.126	0.351	0.189	0.265	0.265	0.257	0.094

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	89	93	0	435	685	61	157
N.S.	1	1.00	1.20	1.26	0.00	5.88	9.26	0.82	2.12
time (sec)	N/A	0.069	0.270	0.458	0.000	0.262	127.105	0.264	2.470

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	106	87	337	0	114	86
N.S.	1	1.00	0.74	1.05	0.86	3.34	0.00	1.13	0.85
time (sec)	N/A	0.108	0.233	1.254	0.186	0.251	0.000	0.260	2.423

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	38	41	42	150	43	29
N.S.	1	1.00	0.74	0.97	1.05	1.08	3.85	1.10	0.74
time (sec)	N/A	0.048	0.021	0.246	0.182	0.263	0.322	0.265	0.053

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	80	93	0	435	774	61	157
N.S.	1	1.00	1.08	1.26	0.00	5.88	10.46	0.82	2.12
time (sec)	N/A	0.060	0.167	0.384	0.000	0.274	123.841	0.266	2.432

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	104	86	331	0	111	84
N.S.	1	1.00	0.74	1.03	0.85	3.28	0.00	1.10	0.83
time (sec)	N/A	0.092	0.100	0.869	0.194	0.265	0.000	0.266	2.403

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	200	0	48	164
N.S.	1	1.00	1.20	1.08	0.00	4.00	0.00	0.96	3.28
time (sec)	N/A	0.071	0.134	0.971	0.000	0.276	0.000	0.253	4.320

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	68	53	0	239	0	60	177
N.S.	1	1.00	1.33	1.04	0.00	4.69	0.00	1.18	3.47
time (sec)	N/A	0.075	0.103	1.109	0.000	0.274	0.000	0.274	2.610

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	125	99	0	594	0	72	183
N.S.	1	1.00	1.89	1.50	0.00	9.00	0.00	1.09	2.77
time (sec)	N/A	0.047	0.282	1.663	0.000	0.257	0.000	0.262	2.575

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	101	104	348	983	113	108
N.S.	1	1.00	0.90	1.49	1.53	5.12	14.46	1.66	1.59
time (sec)	N/A	0.100	0.371	0.400	0.214	0.261	0.823	0.269	2.743

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	301	205	121	0	1633	0	174	255
N.S.	1	1.54	1.05	0.62	0.00	8.37	0.00	0.89	1.31
time (sec)	N/A	0.952	0.447	1.045	0.000	0.287	0.000	0.271	2.569

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	124	98	0	596	0	72	183
N.S.	1	1.00	1.94	1.53	0.00	9.31	0.00	1.12	2.86
time (sec)	N/A	0.045	0.102	1.089	0.000	0.258	0.000	0.263	2.420

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	66	104	104	348	952	114	104
N.S.	1	1.00	0.99	1.55	1.55	5.19	14.21	1.70	1.55
time (sec)	N/A	0.093	0.209	0.400	0.205	0.260	0.760	0.249	0.453

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	193	204	124	0	1645	0	174	255
N.S.	1	1.45	1.53	0.93	0.00	12.37	0.00	1.31	1.92
time (sec)	N/A	0.553	0.326	0.679	0.000	0.287	0.000	0.257	2.597

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	54	31	167	216	0	50	42
N.S.	1	1.00	2.84	1.63	8.79	11.37	0.00	2.63	2.21
time (sec)	N/A	0.022	0.257	12.404	0.204	0.239	0.000	0.264	2.419

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	117	160	289	1268	3813	251	159
N.S.	1	1.00	1.12	1.54	2.78	12.19	36.66	2.41	1.53
time (sec)	N/A	0.174	0.688	0.552	0.219	0.288	1.861	0.273	2.555

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	40	41	167	216	0	48	42
N.S.	1	1.00	2.11	2.16	8.79	11.37	0.00	2.53	2.21
time (sec)	N/A	0.023	0.069	5.859	0.198	0.250	0.000	0.254	2.262

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	119	158	292	1269	3840	251	159
N.S.	1	1.00	1.14	1.52	2.81	12.20	36.92	2.41	1.53
time (sec)	N/A	0.167	0.631	0.542	0.213	0.263	1.771	0.266	2.347

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	427	678	60	157
N.S.	1	1.00	1.10	1.28	0.00	5.93	9.42	0.83	2.18
time (sec)	N/A	0.070	0.159	0.247	0.000	0.282	124.528	0.277	2.383

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	98	83	334	0	101	81
N.S.	1	1.00	0.72	0.96	0.81	3.27	0.00	0.99	0.79
time (sec)	N/A	0.115	0.144	0.587	0.197	0.254	0.000	0.259	2.451

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	180	166	0	1861	0	163	261
N.S.	1	1.00	1.31	1.21	0.00	13.58	0.00	1.19	1.91
time (sec)	N/A	0.141	0.781	3.858	0.000	0.291	0.000	0.250	2.530

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	99	84	334	0	102	81
N.S.	1	1.00	0.72	0.97	0.82	3.27	0.00	1.00	0.79
time (sec)	N/A	0.116	0.372	0.617	0.194	0.259	0.000	0.257	2.413

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	179	168	0	1847	0	159	260
N.S.	1	1.00	1.47	1.38	0.00	15.14	0.00	1.30	2.13
time (sec)	N/A	0.164	0.792	2.379	0.000	0.279	0.000	0.266	2.473

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	128	160	153	1158	0	199	127
N.S.	1	1.00	0.66	0.82	0.79	5.97	0.00	1.03	0.65
time (sec)	N/A	0.241	0.433	10.944	0.205	0.267	0.000	0.255	2.699

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	167	174	0	1829	0	163	259
N.S.	1	1.00	1.22	1.27	0.00	13.35	0.00	1.19	1.89
time (sec)	N/A	0.131	0.865	1.922	0.000	0.297	0.000	0.257	2.608

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	126	161	150	1162	0	202	129
N.S.	1	1.00	0.65	0.83	0.77	5.99	0.00	1.04	0.66
time (sec)	N/A	0.222	0.379	7.100	0.200	0.281	0.000	0.281	2.665

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	325	266	0	4935	0	325	371
N.S.	1	1.00	1.53	1.25	0.00	23.28	0.00	1.53	1.75
time (sec)	N/A	0.299	1.631	31.595	0.000	0.324	0.000	0.276	2.810

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	93	107	376	962	128	98
N.S.	1	1.00	0.65	1.00	1.15	4.04	10.34	1.38	1.05
time (sec)	N/A	0.145	0.137	0.279	0.210	0.266	0.835	0.268	2.766

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	222	128	0	1819	0	179	397
N.S.	1	1.00	1.35	0.78	0.00	11.02	0.00	1.08	2.41
time (sec)	N/A	0.218	0.798	0.675	0.000	0.288	0.000	0.260	2.733

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	176	177	241	1655	0	238	127
N.S.	1	1.00	0.82	0.82	1.12	7.70	0.00	1.11	0.59
time (sec)	N/A	0.382	0.804	2.153	0.209	0.285	0.000	0.285	2.705

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	264	130	0	1805	0	179	397
N.S.	1	1.00	1.62	0.80	0.00	11.07	0.00	1.10	2.44
time (sec)	N/A	0.236	0.662	0.687	0.000	0.289	0.000	0.256	2.994

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	174	190	244	1726	0	232	132
N.S.	1	1.00	0.85	0.93	1.19	8.42	0.00	1.13	0.64
time (sec)	N/A	0.468	1.348	1.681	0.207	0.286	0.000	0.265	2.670

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	474	208	0	5061	0	310	592
N.S.	1	1.00	1.82	0.80	0.00	19.39	0.00	1.19	2.27
time (sec)	N/A	0.623	2.783	6.113	0.000	0.332	0.000	0.269	2.882

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	183	178	240	1661	0	240	127
N.S.	1	1.00	0.85	0.83	1.12	7.73	0.00	1.12	0.59
time (sec)	N/A	0.374	0.866	1.681	0.198	0.284	0.000	0.257	2.590

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	481	198	0	5031	0	310	590
N.S.	1	1.00	1.86	0.76	0.00	19.42	0.00	1.20	2.28
time (sec)	N/A	0.620	1.465	4.435	0.000	0.361	0.000	0.278	2.742

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	366	276	384	4001	0	384	173
N.S.	1	1.00	1.17	0.88	1.22	12.74	0.00	1.22	0.55
time (sec)	N/A	1.076	1.031	17.210	0.211	0.315	0.000	0.267	2.759

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	126	0	233	367	80	178
N.S.	1	1.00	0.98	1.58	0.00	2.91	4.59	1.00	2.22
time (sec)	N/A	0.055	0.608	0.844	0.000	0.281	26.639	0.263	4.745

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	155	115	0	679	0	83	168
N.S.	1	1.00	1.89	1.40	0.00	8.28	0.00	1.01	2.05
time (sec)	N/A	0.057	0.484	5.330	0.000	0.272	0.000	0.261	2.637

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	134	187	0	1855	0	152	217
N.S.	1	1.00	1.09	1.52	0.00	15.08	0.00	1.24	1.76
time (sec)	N/A	0.088	1.026	31.777	0.000	0.313	0.000	0.267	2.553

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	78	126	0	234	697	80	177
N.S.	1	1.00	0.98	1.58	0.00	2.92	8.71	1.00	2.21
time (sec)	N/A	0.043	0.150	0.611	0.000	0.267	30.011	0.260	3.893

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	151	116	0	680	0	83	168
N.S.	1	1.00	1.94	1.49	0.00	8.72	0.00	1.06	2.15
time (sec)	N/A	0.045	0.221	2.604	0.000	0.274	0.000	0.256	2.608

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	134	211	0	1855	0	152	216
N.S.	1	1.00	1.12	1.76	0.00	15.46	0.00	1.27	1.80
time (sec)	N/A	0.088	0.759	13.826	0.000	0.299	0.000	0.258	2.395

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	7	6	16	8	6	6
N.S.	1	1.00	1.55	0.64	0.55	1.45	0.73	0.55	0.55
time (sec)	N/A	0.030	0.032	0.346	0.194	0.243	0.215	0.269	0.068

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	7	6	19	10	6	6
N.S.	1	1.00	1.55	0.64	0.55	1.73	0.91	0.55	0.55
time (sec)	N/A	0.025	0.024	0.306	0.189	0.256	0.177	0.261	2.236

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	15	17	10	13	12	13	16
N.S.	1	1.00	1.07	1.21	0.71	0.93	0.86	0.93	1.14
time (sec)	N/A	0.020	0.051	0.694	0.190	0.253	0.075	0.254	0.095

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	56	87	60	326	54	53
N.S.	1	1.00	0.81	1.06	1.64	1.13	6.15	1.02	1.00
time (sec)	N/A	0.036	0.201	0.444	0.204	0.256	0.339	0.257	2.365

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	118	0	749	0	88	199
N.S.	1	1.00	1.12	1.51	0.00	9.60	0.00	1.13	2.55
time (sec)	N/A	0.050	0.305	1.653	0.000	0.285	0.000	0.253	2.733

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	63	337	232	0	70	67
N.S.	1	1.00	0.99	0.89	4.75	3.27	0.00	0.99	0.94
time (sec)	N/A	0.047	0.255	14.047	0.218	0.253	0.000	0.263	2.495

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	90	175	0	264	643	89	302
N.S.	1	1.00	0.98	1.90	0.00	2.87	6.99	0.97	3.28
time (sec)	N/A	0.053	0.540	0.641	0.000	0.273	30.671	0.252	4.825

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	106	133	0	799	0	95	210
N.S.	1	1.00	1.20	1.51	0.00	9.08	0.00	1.08	2.39
time (sec)	N/A	0.049	0.482	2.408	0.000	0.287	0.000	0.253	2.575

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	146	228	0	1931	0	183	224
N.S.	1	1.00	1.08	1.69	0.00	14.30	0.00	1.36	1.66
time (sec)	N/A	0.105	0.920	12.656	0.000	0.298	0.000	0.258	2.497

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	116	97	137	160	196	219	131
N.S.	1	1.00	0.97	0.82	1.15	1.34	1.65	1.84	1.10
time (sec)	N/A	0.094	0.131	4.824	0.192	0.258	0.145	0.250	0.164

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	54	63	59	100	83	55
N.S.	1	1.00	0.92	0.92	1.07	1.00	1.69	1.41	0.93
time (sec)	N/A	0.026	0.067	1.001	0.188	0.245	0.095	0.247	2.353

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	26	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	2.17	1.00
time (sec)	N/A	0.007	0.038	0.233	0.196	0.245	0.067	0.266	0.055

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	53	0	248	0	46	78
N.S.	1	1.00	1.06	1.04	0.00	4.86	0.00	0.90	1.53
time (sec)	N/A	0.051	0.060	0.572	0.000	0.273	0.000	0.258	0.223

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	105	191	0	1268	0	111	0
N.S.	1	1.00	1.17	2.12	0.00	14.09	0.00	1.23	0.00
time (sec)	N/A	0.068	0.193	3.235	0.000	0.274	0.000	0.262	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	183	577	0	7379	0	304	0
N.S.	1	1.00	1.25	3.95	0.00	50.54	0.00	2.08	0.00
time (sec)	N/A	0.124	0.356	21.914	0.000	0.332	0.000	0.273	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	488	1588	0	23093	0	717	0
N.S.	1	1.00	2.22	7.22	0.00	104.97	0.00	3.26	0.00
time (sec)	N/A	0.219	0.628	118.960	0.000	0.494	0.000	0.297	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	112	96	137	144	189	186	131
N.S.	1	1.00	1.07	0.91	1.30	1.37	1.80	1.77	1.25
time (sec)	N/A	0.081	0.127	4.421	0.187	0.255	0.122	0.258	0.161

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	55	63	57	100	81	51
N.S.	1	1.00	0.96	0.96	1.11	1.00	1.75	1.42	0.89
time (sec)	N/A	0.027	0.051	0.971	0.193	0.251	0.082	0.249	2.308

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	26	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	2.17	1.00
time (sec)	N/A	0.007	0.003	0.223	0.182	0.245	0.053	0.263	2.212

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	35	14	36	32	17	39	46
N.S.	1	1.00	2.33	0.93	2.40	2.13	1.13	2.60	3.07
time (sec)	N/A	0.014	0.030	0.521	0.192	0.250	0.255	0.259	0.174

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	49	86	236	0	84	0
N.S.	1	1.00	2.02	1.14	2.00	5.49	0.00	1.95	0.00
time (sec)	N/A	0.029	0.232	2.254	0.200	0.261	0.000	0.261	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	148	103	248	1504	0	205	0
N.S.	1	1.00	1.66	1.16	2.79	16.90	0.00	2.30	0.00
time (sec)	N/A	0.067	0.370	11.128	0.215	0.266	0.000	0.258	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	300	178	487	4015	0	377	0
N.S.	1	1.00	2.14	1.27	3.48	28.68	0.00	2.69	0.00
time (sec)	N/A	0.147	0.413	44.907	0.228	0.303	0.000	0.270	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	208	321	277	1293	626	390	361
N.S.	1	1.00	1.11	1.71	1.47	6.88	3.33	2.07	1.92
time (sec)	N/A	0.112	0.325	1.935	0.201	0.273	0.306	0.276	0.454

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	134	161	161	664	298	194	144
N.S.	1	1.00	0.99	1.18	1.18	4.88	2.19	1.43	1.06
time (sec)	N/A	0.067	0.197	0.634	0.198	0.265	0.210	0.255	2.278

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	72	80	79	238	122	96	70
N.S.	1	1.00	0.80	0.89	0.88	2.64	1.36	1.07	0.78
time (sec)	N/A	0.037	0.100	0.443	0.182	0.261	0.119	0.251	2.252

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	61	20	36	22
N.S.	1	1.00	1.00	0.96	0.92	2.54	0.83	1.50	0.92
time (sec)	N/A	0.008	0.006	0.199	0.188	0.252	0.069	0.265	0.068

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	25	0	88	0	0	0
N.S.	1	1.00	1.06	0.74	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.026	0.066	0.401	0.000	0.268	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	47	0	660	0	0	0
N.S.	1	1.00	0.68	0.47	0.00	6.60	0.00	0.00	0.00
time (sec)	N/A	0.060	0.116	0.559	0.000	0.288	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	184	92	0	3035	0	0	0
N.S.	1	1.00	1.26	0.63	0.00	20.79	0.00	0.00	0.00
time (sec)	N/A	0.093	0.285	0.684	0.000	0.451	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	425	184	0	6590	0	0	0
N.S.	1	1.00	2.15	0.93	0.00	33.28	0.00	0.00	0.00
time (sec)	N/A	0.128	0.619	0.883	0.000	1.605	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	3775	899	0	928	0	0	0
N.S.	1	1.00	12.84	3.06	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	0.385	6.275	1.742	0.000	0.101	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	2292	321	0	463	0	0	0
N.S.	1	1.00	9.20	1.29	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.220	6.116	0.794	0.000	0.088	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	1401	317	0	314	0	0	0
N.S.	1	1.00	13.74	3.11	0.00	3.08	0.00	0.00	0.00
time (sec)	N/A	0.061	6.090	1.824	0.000	0.088	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	237	251	0	108	0	0	0
N.S.	1	1.00	2.32	2.46	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.060	0.407	0.742	0.000	0.075	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	156	156	806	888	0	798	0	0	0
N.S.	1	1.00	5.17	5.69	0.00	5.12	0.00	0.00	0.00
time (sec)	N/A	0.084	6.019	1.023	0.000	0.091	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	322	322	2492	2160	0	3730	0	0	0
N.S.	1	1.00	7.74	6.71	0.00	11.58	0.00	0.00	0.00
time (sec)	N/A	0.286	6.234	2.506	0.000	0.175	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	411	411	4093	57909	0	13897	0	0	0
N.S.	1	1.00	9.96	140.90	0.00	33.81	0.00	0.00	0.00
time (sec)	N/A	0.436	6.424	8.225	0.000	0.742	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	4500	289	1783	784	0	315	0
N.S.	1	1.00	32.14	2.06	12.74	5.60	0.00	2.25	0.00
time (sec)	N/A	0.084	56.315	0.604	2.151	0.323	0.000	0.304	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	4392	189	640	329	0	183	0
N.S.	1	1.00	47.74	2.05	6.96	3.58	0.00	1.99	0.00
time (sec)	N/A	0.056	35.933	0.210	0.527	0.300	0.000	0.278	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	455	125	153	143	0	81	0
N.S.	1	1.00	12.30	3.38	4.14	3.86	0.00	2.19	0.00
time (sec)	N/A	0.026	31.802	0.215	0.363	0.276	0.000	0.269	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	211	129	0	681	0	538	0
N.S.	1	1.00	2.13	1.30	0.00	6.88	0.00	5.43	0.00
time (sec)	N/A	0.086	49.812	0.387	0.000	0.309	0.000	0.485	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	0	417	0	1801	0	0	0
N.S.	1	1.00	0.00	2.69	0.00	11.62	0.00	0.00	0.00
time (sec)	N/A	0.100	0.000	0.238	0.000	0.434	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	0	817	0	5297	0	0	0
N.S.	1	1.00	0.00	3.99	0.00	25.84	0.00	0.00	0.00
time (sec)	N/A	0.132	0.000	0.268	0.000	1.553	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	4368	275	1789	784	0	315	0
N.S.	1	1.00	29.92	1.88	12.25	5.37	0.00	2.16	0.00
time (sec)	N/A	0.095	56.146	0.602	2.284	0.329	0.000	0.318	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	4260	189	644	329	0	184	0
N.S.	1	1.00	44.38	1.97	6.71	3.43	0.00	1.92	0.00
time (sec)	N/A	0.063	31.280	0.221	0.470	0.291	0.000	0.280	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	4196	131	156	143	0	81	0
N.S.	1	1.00	107.59	3.36	4.00	3.67	0.00	2.08	0.00
time (sec)	N/A	0.026	35.969	0.227	0.337	0.279	0.000	0.295	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	52609	129	0	680	0	546	0
N.S.	1	1.00	515.77	1.26	0.00	6.67	0.00	5.35	0.00
time (sec)	N/A	0.077	54.832	0.405	0.000	0.324	0.000	0.524	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	0	415	0	2137	0	0	0
N.S.	1	1.00	0.00	2.61	0.00	13.44	0.00	0.00	0.00
time (sec)	N/A	0.099	0.000	0.234	0.000	0.465	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	0	817	0	5675	0	0	0
N.S.	1	1.00	0.00	3.87	0.00	26.90	0.00	0.00	0.00
time (sec)	N/A	0.144	0.000	0.287	0.000	1.632	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	86	177	0	429	0	106	472
N.S.	1	1.00	0.80	1.65	0.00	4.01	0.00	0.99	4.41
time (sec)	N/A	0.100	0.211	0.431	0.000	0.265	0.000	0.265	7.253

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	178	0	438	0	106	324
N.S.	1	1.00	0.76	1.58	0.00	3.88	0.00	0.94	2.87
time (sec)	N/A	0.114	0.423	0.280	0.000	0.284	0.000	0.270	0.742

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	86	181	0	455	0	106	325
N.S.	1	1.00	0.83	1.74	0.00	4.38	0.00	1.02	3.12
time (sec)	N/A	0.083	0.386	0.296	0.000	0.275	0.000	0.259	2.951

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	11	10	19	34	10	10
N.S.	1	1.00	1.00	0.61	0.56	1.06	1.89	0.56	0.56
time (sec)	N/A	0.018	0.114	0.108	0.206	0.247	0.319	0.267	0.048

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	53	0	234	0	46	78
N.S.	1	1.00	1.00	0.98	0.00	4.33	0.00	0.85	1.44
time (sec)	N/A	0.052	0.059	0.265	0.000	0.273	0.000	0.259	0.196

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	96	167	0	486	0	126	1069
N.S.	1	1.00	0.66	1.14	0.00	3.33	0.00	0.86	7.32
time (sec)	N/A	0.325	0.318	2.089	0.000	0.919	0.000	0.252	30.903

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	12	11	17	0	13	11
N.S.	1	1.00	1.47	0.63	0.58	0.89	0.00	0.68	0.58
time (sec)	N/A	0.033	0.224	0.127	0.199	0.252	0.000	0.254	0.063

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	54	53	0	244	0	46	78
N.S.	1	1.00	1.08	1.06	0.00	4.88	0.00	0.92	1.56
time (sec)	N/A	0.067	0.069	0.244	0.000	0.265	0.000	0.254	0.219

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	108	124	0	546	0	122	1069
N.S.	1	1.00	0.92	1.05	0.00	4.63	0.00	1.03	9.06
time (sec)	N/A	0.416	1.565	0.429	0.000	0.900	0.000	0.274	8.181

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	200	0	505	0	122	377
N.S.	1	1.00	0.87	1.67	0.00	4.21	0.00	1.02	3.14
time (sec)	N/A	0.088	1.008	0.371	0.000	0.279	0.000	0.257	0.783

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	130	224	0	2211	0	179	0
N.S.	1	1.00	1.20	2.07	0.00	20.47	0.00	1.66	0.00
time (sec)	N/A	0.077	0.540	1.709	0.000	0.305	0.000	0.274	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	373	836	0	12285	0	625	0
N.S.	1	1.00	1.88	4.22	0.00	62.05	0.00	3.16	0.00
time (sec)	N/A	0.186	0.997	11.623	0.000	0.594	0.000	0.277	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	104	197	0	508	0	122	375
N.S.	1	1.00	0.86	1.63	0.00	4.20	0.00	1.01	3.10
time (sec)	N/A	0.092	0.211	0.345	0.000	0.276	0.000	0.262	0.730

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	125	225	0	2228	0	177	0
N.S.	1	1.00	1.16	2.08	0.00	20.63	0.00	1.64	0.00
time (sec)	N/A	0.072	0.237	1.701	0.000	0.314	0.000	0.257	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	336	857	0	12366	0	625	0
N.S.	1	1.00	1.73	4.42	0.00	63.74	0.00	3.22	0.00
time (sec)	N/A	0.177	0.504	11.589	0.000	0.603	0.000	0.278	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	107	226	0	583	0	125	376
N.S.	1	1.00	0.86	1.81	0.00	4.66	0.00	1.00	3.01
time (sec)	N/A	0.093	0.838	0.375	0.000	0.285	0.000	0.256	0.765

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	123	225	0	2119	0	179	0
N.S.	1	1.00	1.14	2.08	0.00	19.62	0.00	1.66	0.00
time (sec)	N/A	0.080	0.494	1.722	0.000	0.306	0.000	0.263	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	319	800	0	10107	0	577	0
N.S.	1	1.00	1.64	4.12	0.00	52.10	0.00	2.97	0.00
time (sec)	N/A	0.157	1.039	11.484	0.000	0.589	0.000	0.273	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	119	237	0	605	0	136	454
N.S.	1	1.00	0.87	1.73	0.00	4.42	0.00	0.99	3.31
time (sec)	N/A	0.132	0.941	0.413	0.000	0.296	0.000	0.279	3.270

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	143	249	0	2541	0	207	0
N.S.	1	1.00	1.18	2.06	0.00	21.00	0.00	1.71	0.00
time (sec)	N/A	0.094	0.713	1.736	0.000	0.309	0.000	0.280	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	465	1084	0	13813	0	819	0
N.S.	1	1.00	2.00	4.65	0.00	59.28	0.00	3.52	0.00
time (sec)	N/A	0.337	1.317	11.799	0.000	0.728	0.000	0.296	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	34	36	0	55	0	35	0
N.S.	1	1.00	1.55	1.64	0.00	2.50	0.00	1.59	0.00
time (sec)	N/A	0.059	0.148	1.692	0.000	0.264	0.000	0.274	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	72	58	107	753	58	57
N.S.	1	1.00	1.21	1.01	0.82	1.51	10.61	0.82	0.80
time (sec)	N/A	0.035	0.349	0.245	0.195	0.265	2.205	0.267	2.327

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	84	72	57	110	806	58	57
N.S.	1	1.00	1.09	0.94	0.74	1.43	10.47	0.75	0.74
time (sec)	N/A	0.031	0.150	0.237	0.194	0.258	2.263	0.276	2.416

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	103	121	99	134	1321	79	75
N.S.	1	1.00	1.20	1.41	1.15	1.56	15.36	0.92	0.87
time (sec)	N/A	0.052	0.376	0.253	0.199	0.264	2.604	0.267	2.394

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	86	62	65	59	852	49	48
N.S.	1	1.00	1.12	0.81	0.84	0.77	11.06	0.64	0.62
time (sec)	N/A	0.033	0.374	0.247	0.198	0.266	2.283	0.268	0.128

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	62	62	56	904	48	47
N.S.	1	1.00	1.10	0.79	0.79	0.72	11.59	0.62	0.60
time (sec)	N/A	0.030	0.138	0.262	0.211	0.260	2.354	0.267	2.295

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	102	108	105	70	1420	69	64
N.S.	1	1.00	1.26	1.33	1.30	0.86	17.53	0.85	0.79
time (sec)	N/A	0.048	0.366	0.239	0.214	0.261	2.719	0.278	2.360

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	9	24	35	19	172	5	5
N.S.	1	1.00	3.00	8.00	11.67	6.33	57.33	1.67	1.67
time (sec)	N/A	0.012	0.004	1.046	0.284	0.256	3.771	0.270	0.050

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	8	11	8	40	48	10	10
N.S.	1	1.00	0.73	1.00	0.73	3.64	4.36	0.91	0.91
time (sec)	N/A	0.017	0.004	30.154	0.201	0.254	0.681	0.265	2.255

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	2	64	304	3602	46	28
N.S.	1	1.00	0.85	0.08	2.46	11.69	138.54	1.77	1.08
time (sec)	N/A	0.019	0.006	29.067	0.297	0.266	131.065	0.279	2.287

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.606	0.200	0.248	0.224	0.263	2.365

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	22	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.012	0.000	11.536	0.190	0.253	0.431	0.262	0.039

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	34	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.013	0.001	98.956	0.197	0.238	0.742	0.275	0.020

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.011	0.001	1.068	0.207	0.246	0.000	0.265	2.257

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.011	0.000	38.338	0.189	0.249	0.000	0.264	2.194

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.001	50.523	0.201	0.276	0.000	0.268	2.219

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	529	39	64	70	0	41	56
N.S.	1	1.00	27.84	2.05	3.37	3.68	0.00	2.16	2.95
time (sec)	N/A	0.022	0.667	0.464	0.281	0.278	0.000	0.272	0.185

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	549	2	88	266	0	63	78
N.S.	1	1.00	17.71	0.06	2.84	8.58	0.00	2.03	2.52
time (sec)	N/A	0.038	6.246	16.123	0.290	0.287	0.000	0.274	2.242

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	765	2	114	717	0	77	114
N.S.	1	1.00	14.17	0.04	2.11	13.28	0.00	1.43	2.11
time (sec)	N/A	0.043	4.747	210.079	0.279	0.281	0.000	0.276	0.071

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	37	36	67	0	36	54
N.S.	1	1.00	1.00	2.06	2.00	3.72	0.00	2.00	3.00
time (sec)	N/A	0.025	0.085	0.834	0.279	0.261	0.000	0.270	0.158

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	64	61	60	262	0	60	77
N.S.	1	1.00	2.00	1.91	1.88	8.19	0.00	1.88	2.41
time (sec)	N/A	0.036	0.117	26.167	0.298	0.275	0.000	0.271	0.060

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	66	2	84	715	0	72	112
N.S.	1	1.00	1.22	0.04	1.56	13.24	0.00	1.33	2.07
time (sec)	N/A	0.057	0.188	37.526	0.316	0.285	0.000	0.273	2.232

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.001	0.372	0.198	0.247	0.000	0.267	0.072

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	10.525	0.199	0.240	0.000	0.262	0.027

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	116.966	0.206	0.272	0.000	0.289	2.299

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	217	74	0	3313	0	1	0
N.S.	1	1.00	0.80	0.27	0.00	12.23	0.00	0.00	0.00
time (sec)	N/A	0.577	1.273	0.234	0.000	0.375	0.000	62.360	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	244	70	0	3309	0	0	0
N.S.	1	1.00	0.87	0.25	0.00	11.82	0.00	0.00	0.00
time (sec)	N/A	0.468	2.104	0.201	0.000	0.363	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	283	108	0	4943	0	5	0
N.S.	1	1.00	0.92	0.35	0.00	16.00	0.00	0.02	0.00
time (sec)	N/A	0.688	2.600	0.899	0.000	0.600	0.000	0.762	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	326	144	0	6680	0	24	0
N.S.	1	1.00	0.90	0.40	0.00	18.40	0.00	0.07	0.00
time (sec)	N/A	3.240	0.619	1.297	0.000	1.109	0.000	0.815	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	28	225	57	0	28	48
N.S.	1	1.00	1.17	2.33	18.75	4.75	0.00	2.33	4.00
time (sec)	N/A	0.065	0.173	0.187	0.283	0.245	0.000	0.288	2.656

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	258	79	0	6841	0	1	0
N.S.	1	1.00	0.86	0.26	0.00	22.80	0.00	0.00	0.00
time (sec)	N/A	0.487	1.734	3.666	0.000	2.974	0.000	1.554	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	198	208	0	3485	0	1	0
N.S.	1	1.00	0.89	0.93	0.00	15.63	0.00	0.00	0.00
time (sec)	N/A	0.366	0.786	0.536	0.000	0.380	0.000	61.080	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	227	204	0	3505	0	0	0
N.S.	1	1.00	0.99	0.89	0.00	15.24	0.00	0.00	0.00
time (sec)	N/A	0.360	0.817	0.523	0.000	0.382	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	264	274	0	5079	0	5	0
N.S.	1	1.00	1.04	1.07	0.00	19.92	0.00	0.02	0.00
time (sec)	N/A	0.775	0.750	0.908	0.000	0.584	0.000	0.728	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	309	354	0	6794	0	24	0
N.S.	1	1.00	1.03	1.18	0.00	22.72	0.00	0.08	0.00
time (sec)	N/A	4.219	0.433	1.499	0.000	1.083	0.000	0.795	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	54	0	26	51
N.S.	1	1.00	1.00	1.09	0.00	4.91	0.00	2.36	4.64
time (sec)	N/A	0.058	0.015	0.165	0.000	0.253	0.000	0.260	2.675

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	241	254	0	6997	0	1	0
N.S.	1	1.00	0.98	1.03	0.00	28.44	0.00	0.00	0.00
time (sec)	N/A	0.432	0.768	3.684	0.000	3.063	0.000	1.564	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	34	92	74	367	202	46	209
N.S.	1	1.00	0.87	2.36	1.90	9.41	5.18	1.18	5.36
time (sec)	N/A	0.111	0.315	0.178	0.295	0.298	0.589	0.270	2.913

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	92	74	363	224	45	208
N.S.	1	1.00	0.87	2.42	1.95	9.55	5.89	1.18	5.47
time (sec)	N/A	0.083	0.060	0.155	0.287	0.286	0.581	0.254	2.673

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	47	73	95	136	33	25
N.S.	1	1.00	1.05	1.24	1.92	2.50	3.58	0.87	0.66
time (sec)	N/A	0.107	5.035	0.167	0.292	0.279	0.502	0.255	2.484

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	47	73	95	136	33	25
N.S.	1	1.00	1.05	1.24	1.92	2.50	3.58	0.87	0.66
time (sec)	N/A	0.067	5.034	0.153	0.285	0.271	0.493	0.260	0.095

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	47	136	36	91	0	0	0
N.S.	1	1.00	0.80	2.31	0.61	1.54	0.00	0.00	0.00
time (sec)	N/A	0.412	0.119	0.141	0.295	0.262	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	71	209	60	188	0	0	0
N.S.	1	1.00	0.68	2.01	0.58	1.81	0.00	0.00	0.00
time (sec)	N/A	0.387	0.106	0.094	0.307	0.275	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	93	281	80	272	0	0	0
N.S.	1	1.00	0.62	1.87	0.53	1.81	0.00	0.00	0.00
time (sec)	N/A	0.396	0.109	0.092	0.305	0.263	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	43	175	43	152	0	0	0
N.S.	1	1.00	0.59	2.40	0.59	2.08	0.00	0.00	0.00
time (sec)	N/A	0.365	0.033	0.147	0.316	0.290	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	253	67	294	0	0	0
N.S.	1	1.00	0.58	2.58	0.68	3.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.037	0.100	0.310	0.276	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	68	329	87	427	0	0	0
N.S.	1	1.00	0.53	2.55	0.67	3.31	0.00	0.00	0.00
time (sec)	N/A	0.411	0.043	0.110	0.301	0.278	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	66	150	60	351	0	0	0
N.S.	1	1.00	0.75	1.70	0.68	3.99	0.00	0.00	0.00
time (sec)	N/A	0.296	0.074	0.137	0.283	0.294	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	128	0	0	786	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	4.20	0.00	0.00	0.00
time (sec)	N/A	0.428	0.181	0.000	0.000	0.280	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	287	180	0	0	1202	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.495	0.263	0.000	0.000	0.294	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	71	252	92	1757	0	0	0
N.S.	1	1.00	0.54	1.91	0.70	13.31	0.00	0.00	0.00
time (sec)	N/A	0.335	0.245	0.127	0.293	0.273	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	113	441	154	3431	0	0	0
N.S.	1	1.00	0.55	2.16	0.75	16.82	0.00	0.00	0.00
time (sec)	N/A	0.535	0.366	0.130	0.295	0.322	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	149	602	207	4629	0	0	0
N.S.	1	1.00	0.46	1.85	0.63	14.20	0.00	0.00	0.00
time (sec)	N/A	0.554	0.615	0.124	0.289	0.355	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	162	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.635	0.000	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	77	99	126	164	190	138	79
N.S.	1	1.00	0.71	0.91	1.16	1.50	1.74	1.27	0.72
time (sec)	N/A	0.076	0.654	88.697	0.197	0.242	0.363	0.267	2.633

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	66	63	80	129	81	44
N.S.	1	1.00	0.79	1.05	1.00	1.27	2.05	1.29	0.70
time (sec)	N/A	0.030	0.390	7.153	0.191	0.252	0.174	0.271	2.485

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	31	24	34	18
N.S.	1	1.00	1.90	0.95	0.90	1.55	1.20	1.70	0.90
time (sec)	N/A	0.013	0.043	0.241	0.191	0.248	0.088	0.269	0.060

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	48	133	73	299	0	79	343
N.S.	1	1.00	1.09	3.02	1.66	6.80	0.00	1.80	7.80
time (sec)	N/A	0.055	0.240	1.060	0.287	0.258	0.000	0.373	2.946

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	213	150	765	0	140	229
N.S.	1	1.00	1.01	2.39	1.69	8.60	0.00	1.57	2.57
time (sec)	N/A	0.081	0.497	15.759	0.282	0.257	0.000	0.421	2.793

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	121	481	327	2439	0	256	0
N.S.	1	1.00	0.85	3.36	2.29	17.06	0.00	1.79	0.00
time (sec)	N/A	0.130	0.706	211.638	0.299	0.293	0.000	0.596	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	239	1260	0	0	0	0	0
N.S.	1	1.00	0.79	4.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	1.473	6.296	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	202	935	0	0	0	0	0
N.S.	1	1.00	0.81	3.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.173	0.963	0.645	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	94	352	0	0	0	0	0
N.S.	1	1.00	0.98	3.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.291	0.464	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	90	181	0	143	0	0	0
N.S.	1	1.00	0.94	1.89	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.059	0.310	0.247	0.000	0.085	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	119	630	0	1283	0	0	0
N.S.	1	1.00	0.75	3.99	0.00	8.12	0.00	0.00	0.00
time (sec)	N/A	0.092	0.671	0.411	0.000	0.107	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	237	641	0	4231	0	0	0
N.S.	1	1.00	0.73	1.97	0.00	13.02	0.00	0.00	0.00
time (sec)	N/A	0.245	1.704	0.753	0.000	0.190	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	279	687	0	1488	0	0	0
N.S.	1	1.00	0.72	1.78	0.00	3.85	0.00	0.00	0.00
time (sec)	N/A	0.393	0.133	1.554	0.000	0.287	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	210	530	0	1122	0	0	0
N.S.	1	1.00	0.75	1.89	0.00	3.99	0.00	0.00	0.00
time (sec)	N/A	0.356	0.063	0.939	0.000	0.282	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	143	376	0	754	0	0	0
N.S.	1	1.00	0.77	2.02	0.00	4.05	0.00	0.00	0.00
time (sec)	N/A	0.197	0.048	0.913	0.000	0.277	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.07	1.00	1.14	1.14
time (sec)	N/A	0.060	0.964	0.097	0.265	0.264	66.010	0.256	2.292

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	55	53	105	124	54	45
N.S.	1	1.00	0.76	0.83	0.80	1.59	1.88	0.82	0.68
time (sec)	N/A	0.024	0.029	0.347	0.182	0.252	0.876	0.261	0.278

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	32	33	37	92	139	32	32
N.S.	1	1.00	0.80	0.82	0.92	2.30	3.48	0.80	0.80
time (sec)	N/A	0.022	0.015	0.154	0.188	0.270	0.439	0.283	2.370

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	25	27	26	54	54	26	25
N.S.	1	1.00	0.78	0.84	0.81	1.69	1.69	0.81	0.78
time (sec)	N/A	0.012	0.011	0.096	0.186	0.263	0.210	0.259	0.071

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	40	45	53	0	34	39
N.S.	1	1.00	0.92	1.54	1.73	2.04	0.00	1.31	1.50
time (sec)	N/A	0.013	0.015	0.105	0.186	0.262	0.000	0.257	0.082

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	43	62	104	0	44	37
N.S.	1	1.00	0.88	1.02	1.48	2.48	0.00	1.05	0.88
time (sec)	N/A	0.029	0.034	0.168	0.190	0.273	0.000	0.271	2.338

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	67	88	388	0	64	90
N.S.	1	1.00	0.84	0.92	1.21	5.32	0.00	0.88	1.23
time (sec)	N/A	0.036	0.041	0.365	0.182	0.252	0.000	0.272	2.325

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	108	85	0	316	976	84	127
N.S.	1	1.00	0.78	0.61	0.00	2.27	7.02	0.60	0.91
time (sec)	N/A	0.051	0.377	0.408	0.000	0.247	2.350	0.281	2.920

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	58	57	0	149	428	56	105
N.S.	1	1.00	0.66	0.65	0.00	1.69	4.86	0.64	1.19
time (sec)	N/A	0.027	0.116	0.215	0.000	0.257	0.845	0.263	0.254

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	37	0	67	201	40	54
N.S.	1	1.00	0.70	0.69	0.00	1.24	3.72	0.74	1.00
time (sec)	N/A	0.013	0.052	0.122	0.000	0.257	0.383	0.259	0.108

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	49	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	137	0	0	0	0	0	0
N.S.	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	1.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	82	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	1.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	106	85	0	313	1085	84	125
N.S.	1	1.00	0.76	0.61	0.00	2.25	7.81	0.60	0.90
time (sec)	N/A	0.036	0.366	0.575	0.000	0.272	2.294	0.266	2.913

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	57	0	146	432	56	68
N.S.	1	1.00	0.64	0.65	0.00	1.66	4.91	0.64	0.77
time (sec)	N/A	0.024	0.114	0.233	0.000	0.252	0.841	0.267	2.543

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	37	0	66	167	40	53
N.S.	1	1.00	0.70	0.69	0.00	1.22	3.09	0.74	0.98
time (sec)	N/A	0.012	0.053	0.129	0.000	0.261	0.381	0.279	0.080

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	51	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	80	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	89	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	196	195	189	441	2377	1546	252
N.S.	1	1.00	0.77	0.77	0.74	1.74	9.36	6.09	0.99
time (sec)	N/A	0.299	7.343	0.863	0.206	0.261	35.866	0.333	3.538

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	91	88	135	510	885	88
N.S.	1	1.00	0.88	0.86	0.83	1.27	4.81	8.35	0.83
time (sec)	N/A	0.136	0.720	0.238	0.199	0.263	1.190	0.294	2.669

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	1.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	250	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	1.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	230	192	187	2340	2346	1548	288
N.S.	1	1.00	0.92	0.76	0.75	9.32	9.35	6.17	1.15
time (sec)	N/A	0.233	0.579	0.480	0.218	0.339	2.141	0.331	2.704

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	88	87	430	488	886	134
N.S.	1	1.00	0.87	0.87	0.86	4.26	4.83	8.77	1.33
time (sec)	N/A	0.117	0.222	0.179	0.206	0.263	0.649	0.296	2.484

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	127	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.246	0.000	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	44	56	111	139	52	50
N.S.	1	1.00	0.74	0.64	0.81	1.61	2.01	0.75	0.72
time (sec)	N/A	0.032	0.051	284.720	0.196	0.249	2.172	0.271	0.541

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	47	50	95	177	57	43
N.S.	1	1.00	0.79	0.82	0.88	1.67	3.11	1.00	0.75
time (sec)	N/A	0.033	0.052	19.050	0.188	0.250	0.934	0.260	2.540

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	26	29	53	76	26	26
N.S.	1	1.00	0.80	0.74	0.83	1.51	2.17	0.74	0.74
time (sec)	N/A	0.017	0.005	1.799	0.181	0.243	0.391	0.258	2.364

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	27	38	49	0	32	38
N.S.	1	1.00	0.88	1.08	1.52	1.96	0.00	1.28	1.52
time (sec)	N/A	0.013	0.007	0.063	0.189	0.270	0.000	0.261	0.065

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	25	45	103	0	46	36
N.S.	1	1.00	0.83	0.61	1.10	2.51	0.00	1.12	0.88
time (sec)	N/A	0.029	0.039	0.118	0.185	0.261	0.000	0.260	2.365

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	59	55	78	387	0	62	102
N.S.	1	1.00	0.84	0.79	1.11	5.53	0.00	0.89	1.46
time (sec)	N/A	0.036	0.063	0.441	0.200	0.253	0.000	0.262	0.148

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	89	77	167	294	81	65
N.S.	1	1.00	0.74	0.98	0.85	1.84	3.23	0.89	0.71
time (sec)	N/A	0.049	0.071	0.032	0.183	0.261	5.104	0.272	0.630

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	40	41	38	90	144	36	36
N.S.	1	1.00	0.82	0.84	0.78	1.84	2.94	0.73	0.73
time (sec)	N/A	0.036	0.024	96.043	0.199	0.258	2.201	0.254	0.520

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	47	50	96	175	57	42
N.S.	1	1.00	0.75	0.82	0.88	1.68	3.07	1.00	0.74
time (sec)	N/A	0.031	0.050	10.561	0.191	0.253	0.881	0.276	0.287

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	41	50	72	0	39	35
N.S.	1	1.00	0.88	0.98	1.19	1.71	0.00	0.93	0.83
time (sec)	N/A	0.028	0.026	0.372	0.203	0.271	0.000	0.263	2.359

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	179	48	62	198	0	56	62
N.S.	1	1.00	3.38	0.91	1.17	3.74	0.00	1.06	1.17
time (sec)	N/A	0.026	1.167	0.081	0.202	0.268	0.000	0.251	2.389

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	35	69	262	0	59	65
N.S.	1	1.00	0.74	0.56	1.11	4.23	0.00	0.95	1.05
time (sec)	N/A	0.052	0.070	0.147	0.197	0.254	0.000	0.263	2.461

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	88	54	154	202	52	50
N.S.	1	1.00	0.74	1.28	0.78	2.23	2.93	0.75	0.72
time (sec)	N/A	0.037	0.032	0.047	0.190	0.254	11.958	0.272	2.708

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	84	77	165	325	81	65
N.S.	1	1.00	0.74	0.92	0.85	1.81	3.57	0.89	0.71
time (sec)	N/A	0.053	0.075	0.028	0.197	0.250	5.273	0.257	0.622

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	52	56	111	139	52	50
N.S.	1	1.00	0.74	0.75	0.81	1.61	2.01	0.75	0.72
time (sec)	N/A	0.032	0.030	48.589	0.201	0.253	2.211	0.247	0.526

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	68	50	65	170	0	57	66
N.S.	1	1.00	1.15	0.85	1.10	2.88	0.00	0.97	1.12
time (sec)	N/A	0.042	0.057	5.153	0.190	0.253	0.000	0.263	2.372

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	56	68	213	0	63	53
N.S.	1	1.00	0.83	0.89	1.08	3.38	0.00	1.00	0.84
time (sec)	N/A	0.048	0.083	0.802	0.204	0.252	0.000	0.252	2.474

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	286	77	88	459	0	72	97
N.S.	1	1.00	3.53	0.95	1.09	5.67	0.00	0.89	1.20
time (sec)	N/A	0.039	1.055	0.139	0.192	0.265	0.000	0.267	0.081

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	47	52	152	235	46	42
N.S.	1	1.00	0.75	0.82	0.91	2.67	4.12	0.81	0.74
time (sec)	N/A	0.035	0.078	290.280	0.208	0.260	2.192	0.248	0.575

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	55	53	105	128	54	47
N.S.	1	1.00	0.77	0.83	0.80	1.59	1.94	0.82	0.71
time (sec)	N/A	0.032	0.068	21.420	0.189	0.268	0.873	0.262	0.251

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	24	91	117	18	18
N.S.	1	1.00	1.09	0.83	1.04	3.96	5.09	0.78	0.78
time (sec)	N/A	0.017	0.014	1.926	0.189	0.251	0.430	0.264	2.431

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	57	64	0	30	30
N.S.	1	1.00	0.92	1.03	1.54	1.73	0.00	0.81	0.81
time (sec)	N/A	0.022	0.028	0.104	0.190	0.263	0.000	0.258	0.064

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	66	65	76	200	0	55	63
N.S.	1	1.00	1.22	1.20	1.41	3.70	0.00	1.02	1.17
time (sec)	N/A	0.029	0.092	0.162	0.199	0.267	0.000	0.262	2.382

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	56	86	262	0	48	66
N.S.	1	1.00	0.76	0.89	1.37	4.16	0.00	0.76	1.05
time (sec)	N/A	0.046	0.061	0.430	0.197	0.265	0.000	0.260	2.420

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	108	78	175	197	82	69
N.S.	1	1.00	0.73	1.08	0.78	1.75	1.97	0.82	0.69
time (sec)	N/A	0.048	0.076	0.024	0.199	0.271	5.636	0.265	0.609

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	38	44	42	108	128	43	39
N.S.	1	1.00	0.73	0.85	0.81	2.08	2.46	0.83	0.75
time (sec)	N/A	0.039	0.028	107.880	0.197	0.264	2.350	0.266	0.542

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	55	53	105	128	54	47
N.S.	1	1.00	0.82	0.83	0.80	1.59	1.94	0.82	0.71
time (sec)	N/A	0.032	0.032	10.725	0.195	0.260	1.011	0.275	2.527

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	58	54	61	98	0	46	53
N.S.	1	1.00	1.29	1.20	1.36	2.18	0.00	1.02	1.18
time (sec)	N/A	0.022	0.097	0.515	0.191	0.254	0.000	0.261	0.089

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	57	86	195	0	56	51
N.S.	1	1.00	0.83	0.97	1.46	3.31	0.00	0.95	0.86
time (sec)	N/A	0.033	0.050	0.218	0.193	0.266	0.000	0.289	2.375

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	247	78	96	459	0	72	98
N.S.	1	1.00	2.91	0.92	1.13	5.40	0.00	0.85	1.15
time (sec)	N/A	0.049	3.665	0.292	0.197	0.270	0.000	0.281	2.492

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	61	52	186	382	46	46
N.S.	1	1.00	0.79	1.07	0.91	3.26	6.70	0.81	0.81
time (sec)	N/A	0.047	0.040	0.025	0.202	0.260	12.490	0.255	2.549

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	108	76	176	197	82	69
N.S.	1	1.00	0.73	1.08	0.76	1.76	1.97	0.82	0.69
time (sec)	N/A	0.051	0.072	0.043	0.196	0.271	5.490	0.265	0.602

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	47	52	152	233	46	42
N.S.	1	1.00	0.75	0.82	0.91	2.67	4.09	0.81	0.74
time (sec)	N/A	0.034	0.044	48.701	0.192	0.256	2.334	0.262	0.576

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	55	70	127	0	50	49
N.S.	1	1.00	0.81	0.93	1.19	2.15	0.00	0.85	0.83
time (sec)	N/A	0.050	0.040	5.520	0.196	0.277	0.000	0.282	0.102

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	220	79	87	272	0	69	77
N.S.	1	1.00	3.01	1.08	1.19	3.73	0.00	0.95	1.05
time (sec)	N/A	0.038	1.011	1.105	0.200	0.280	0.000	0.264	2.517

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	61	70	106	398	0	70	80
N.S.	1	1.00	0.76	0.88	1.32	4.98	0.00	0.88	1.00
time (sec)	N/A	0.045	0.102	0.334	0.199	0.272	0.000	0.276	0.078

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	42	40	90	373	0	90	91
N.S.	1	1.00	0.37	0.35	0.80	3.30	0.00	0.80	0.81
time (sec)	N/A	0.065	0.029	0.635	0.283	0.263	0.000	0.261	0.279

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	120	44	101	882	0	95	122
N.S.	1	1.00	0.93	0.34	0.78	6.84	0.00	0.74	0.95
time (sec)	N/A	0.079	0.075	5.142	0.278	0.278	0.000	0.263	2.653

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	58	48	105	920	0	99	112
N.S.	1	1.00	0.45	0.37	0.81	7.08	0.00	0.76	0.86
time (sec)	N/A	0.084	0.053	1.463	0.284	0.269	0.000	0.269	2.674

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	64	50	115	1616	0	103	154
N.S.	1	1.00	0.43	0.34	0.77	10.85	0.00	0.69	1.03
time (sec)	N/A	0.091	0.065	13.555	0.276	0.286	0.000	0.266	2.831

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	24	34	202	0	35	38
N.S.	1	1.00	0.91	0.71	1.00	5.94	0.00	1.03	1.12
time (sec)	N/A	0.024	0.039	0.176	0.272	0.270	0.000	0.261	0.216

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	54	47	522	0	42	80
N.S.	1	1.00	1.02	1.02	0.89	9.85	0.00	0.79	1.51
time (sec)	N/A	0.028	0.064	0.527	0.266	0.272	0.000	0.261	2.713

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	161	56	47	557	0	42	62
N.S.	1	1.00	2.93	1.02	0.85	10.13	0.00	0.76	1.13
time (sec)	N/A	0.039	2.620	0.238	0.283	0.271	0.000	0.267	2.700

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	310	60	59	992	0	48	114
N.S.	1	1.00	4.13	0.80	0.79	13.23	0.00	0.64	1.52
time (sec)	N/A	0.044	4.683	0.470	0.266	0.279	0.000	0.259	2.924

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	505	1295	93	228
N.S.	1	1.00	0.63	1.47	0.00	3.69	9.45	0.68	1.66
time (sec)	N/A	0.077	0.776	0.040	0.000	0.270	7.773	0.261	1.196

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	379	972	86	126
N.S.	1	1.00	0.63	1.40	0.00	2.98	7.65	0.68	0.99
time (sec)	N/A	0.069	0.756	24.584	0.000	0.264	2.742	0.266	2.997

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	47	61	0	142	304	47	58
N.S.	1	1.00	0.71	0.92	0.00	2.15	4.61	0.71	0.88
time (sec)	N/A	0.033	0.062	2.071	0.000	0.276	0.970	0.255	2.561

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	37	0	97	184	40	54
N.S.	1	1.00	0.70	0.69	0.00	1.80	3.41	0.74	1.00
time (sec)	N/A	0.012	0.019	0.120	0.000	0.265	0.457	0.260	2.516

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	120	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.519	0.000	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	105	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	159	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	1.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	117	278	0	919	2693	132	395
N.S.	1	1.00	0.60	1.43	0.00	4.71	13.81	0.68	2.03
time (sec)	N/A	0.103	0.784	0.027	0.000	0.277	26.295	0.271	3.311

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	58	97	0	303	819	58	96
N.S.	1	1.00	0.70	1.17	0.00	3.65	9.87	0.70	1.16
time (sec)	N/A	0.057	0.271	112.680	0.000	0.262	6.137	0.265	3.389

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	80	178	0	381	972	86	126
N.S.	1	1.00	0.63	1.40	0.00	3.00	7.65	0.68	0.99
time (sec)	N/A	0.064	0.390	11.292	0.000	0.263	2.598	0.255	2.904

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	55	60	0	176	435	58	68
N.S.	1	1.00	0.58	0.63	0.00	1.85	4.58	0.61	0.72
time (sec)	N/A	0.028	0.120	0.217	0.000	0.264	1.052	0.268	0.267

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	85	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	148	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.721	0.000	0.000	0.000	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	111	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.994	0.000	0.000	0.000	0.000	0.000	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	113	202	0	676	1916	93	182
N.S.	1	1.00	0.82	1.47	0.00	4.93	13.99	0.68	1.33
time (sec)	N/A	0.088	0.719	0.030	0.000	0.269	68.042	0.277	1.003

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	118	278	0	917	2761	132	393
N.S.	1	1.00	0.61	1.43	0.00	4.70	14.16	0.68	2.02
time (sec)	N/A	0.103	0.808	0.025	0.000	0.258	25.243	0.275	3.204

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	86	202	0	501	1292	93	163
N.S.	1	1.00	0.63	1.47	0.00	3.66	9.43	0.68	1.19
time (sec)	N/A	0.071	0.596	63.865	0.000	0.266	7.786	0.264	3.467

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	106	100	0	381	1046	86	125
N.S.	1	1.00	0.74	0.69	0.00	2.65	7.26	0.60	0.87
time (sec)	N/A	0.042	0.348	0.532	0.000	0.256	2.951	0.262	2.924

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	172	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.844	0.000	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	145	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.966	0.000	0.000	0.000	0.000	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	176	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	1.818	0.000	0.000	0.000	0.000	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	51	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	4.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	36	30	16
N.S.	1	1.00	1.00	1.00	0.94	1.53	2.12	1.76	0.94
time (sec)	N/A	0.014	0.034	2.457	0.181	0.257	0.139	0.281	0.101

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	22	35	49	38	38
N.S.	1	1.00	1.00	1.00	1.00	1.59	2.23	1.73	1.73
time (sec)	N/A	0.012	0.100	0.174	0.190	0.252	1.399	0.273	2.488

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	22	35	48	38	38
N.S.	1	1.00	0.96	0.96	0.96	1.52	2.09	1.65	1.65
time (sec)	N/A	0.012	0.021	0.105	0.195	0.258	0.311	0.293	2.413

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	0	13	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.053	0.000	0.000	0.243	0.000	0.000	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	0	19	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.017	0.083	0.000	0.000	0.257	0.000	0.000	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	18	0	0	19	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.037	0.000	0.000	0.253	0.000	0.000	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	36	30	16
N.S.	1	1.00	1.00	1.00	0.94	1.53	2.12	1.76	0.94
time (sec)	N/A	0.011	0.013	1.490	0.186	0.249	0.169	0.268	0.073

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	22	35	49	38	38
N.S.	1	1.00	1.00	1.00	1.00	1.59	2.23	1.73	1.73
time (sec)	N/A	0.011	0.097	0.224	0.194	0.254	1.874	0.278	2.423

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	22	35	48	38	38
N.S.	1	1.00	0.96	0.96	0.96	1.52	2.09	1.65	1.65
time (sec)	N/A	0.011	0.023	0.158	0.192	0.243	0.380	0.264	2.369

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	0	13	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.071	0.000	0.000	0.258	0.000	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	0	19	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.021	0.095	0.000	0.000	0.259	0.000	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	18	0	0	19	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.035	0.000	0.000	0.247	0.000	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	42	0	45	50
N.S.	1	1.00	1.00	1.09	1.00	3.82	0.00	4.09	4.55
time (sec)	N/A	0.031	0.009	1.499	0.198	0.260	0.000	0.251	0.188

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	9	24	35	19	0	5	5
N.S.	1	1.00	3.00	8.00	11.67	6.33	0.00	1.67	1.67
time (sec)	N/A	0.031	0.001	1.429	0.341	0.249	0.000	0.258	0.076

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	26	11	21	0	11	11
N.S.	1	1.00	0.82	2.36	1.00	1.91	0.00	1.00	1.00
time (sec)	N/A	0.025	0.061	1.617	0.307	0.246	0.000	0.245	2.355

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	69	0	39	54
N.S.	1	1.00	1.00	1.05	1.00	3.63	0.00	2.05	2.84
time (sec)	N/A	0.038	0.142	30.523	0.192	0.275	0.000	0.258	2.532

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	11	13	12	14	29	12	12
N.S.	1	1.00	2.75	3.25	3.00	3.50	7.25	3.00	3.00
time (sec)	N/A	0.048	0.004	1.013	0.190	0.242	0.318	0.251	2.453

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	11	13	12	14	29	12	12
N.S.	1	1.00	2.75	3.25	3.00	3.50	7.25	3.00	3.00
time (sec)	N/A	0.061	0.001	0.901	0.227	0.249	0.311	0.251	2.391

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	27	26	0	23	0	9	9
N.S.	1	1.00	5.40	5.20	0.00	4.60	0.00	1.80	1.80
time (sec)	N/A	0.035	0.041	2.361	0.000	0.239	0.000	0.254	2.384

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	24	29	77	0	29	23
N.S.	1	1.00	0.73	1.60	1.93	5.13	0.00	1.93	1.53
time (sec)	N/A	0.043	0.011	1.735	0.205	0.246	0.000	0.262	0.094

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	21	32	53	0	26	20
N.S.	1	1.00	0.73	1.40	2.13	3.53	0.00	1.73	1.33
time (sec)	N/A	0.045	0.078	1.949	0.228	0.275	0.000	0.286	2.410

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	74	31	0	309	0	1	169
N.S.	1	1.00	0.73	0.30	0.00	3.03	0.00	0.01	1.66
time (sec)	N/A	0.094	0.232	2.774	0.000	0.272	0.000	0.265	4.110

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	40	0	32	0	19	19
N.S.	1	1.00	1.00	1.82	0.00	1.45	0.00	0.86	0.86
time (sec)	N/A	0.050	0.099	2.085	0.000	0.254	0.000	0.283	0.109

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	75	66	172	0	113	297
N.S.	1	1.00	0.93	2.68	2.36	6.14	0.00	4.04	10.61
time (sec)	N/A	0.076	0.204	1.825	0.282	0.254	0.000	0.285	2.759

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	60	151	688	0	264	107
N.S.	1	1.00	0.98	1.13	2.85	12.98	0.00	4.98	2.02
time (sec)	N/A	0.115	0.281	4.509	0.290	0.266	0.000	0.286	2.785

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	122	116	276	1975	0	543	1347
N.S.	1	1.00	1.56	1.49	3.54	25.32	0.00	6.96	17.27
time (sec)	N/A	0.117	0.485	11.445	0.295	0.286	0.000	0.296	3.338

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	73	0	32	32
N.S.	1	1.00	1.00	0.92	0.83	6.08	0.00	2.67	2.67
time (sec)	N/A	0.070	0.104	11.843	0.189	0.238	0.000	0.287	2.539

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	67	26	25	778	0	66	820
N.S.	1	1.00	2.03	0.79	0.76	23.58	0.00	2.00	24.85
time (sec)	N/A	0.084	0.014	0.035	0.190	0.245	0.000	0.296	2.408

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	24	122	50	0	28	47
N.S.	1	1.00	1.04	0.92	4.69	1.92	0.00	1.08	1.81
time (sec)	N/A	0.070	3.093	3.658	0.284	0.251	0.000	0.272	0.528

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	12	14	0	12	12
N.S.	1	1.00	1.00	1.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.016	0.001	0.142	0.188	0.243	0.000	0.271	2.368

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	36	0	67	0	35	50
N.S.	1	1.00	1.00	1.80	0.00	3.35	0.00	1.75	2.50
time (sec)	N/A	0.099	0.090	0.333	0.000	0.248	0.000	0.267	0.179

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	43	0	0	112	0	44	0
N.S.	1	1.00	4.78	0.00	0.00	12.44	0.00	4.89	0.00
time (sec)	N/A	0.037	0.040	0.000	0.000	0.243	0.000	0.264	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	47	8	0	118	0	44	0
N.S.	1	1.00	5.22	0.89	0.00	13.11	0.00	4.89	0.00
time (sec)	N/A	0.037	0.045	0.154	0.000	0.246	0.000	0.266	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	46	13	0	154	0	85	0
N.S.	1	1.00	3.29	0.93	0.00	11.00	0.00	6.07	0.00
time (sec)	N/A	0.044	0.034	0.156	0.000	0.249	0.000	0.283	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	51	32	0	219	0	120	0
N.S.	1	1.00	2.68	1.68	0.00	11.53	0.00	6.32	0.00
time (sec)	N/A	0.038	0.309	0.161	0.000	0.250	0.000	0.285	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	55	19	0	334	0	145	0
N.S.	1	1.00	2.29	0.79	0.00	13.92	0.00	6.04	0.00
time (sec)	N/A	0.031	0.072	0.147	0.000	0.249	0.000	0.279	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	25	14	34	340	19	41	375
N.S.	1	1.00	1.47	0.82	2.00	20.00	1.12	2.41	22.06
time (sec)	N/A	0.057	0.012	0.023	0.199	0.248	0.920	0.266	2.481

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	61	104	73	0	0	108
N.S.	1	1.00	0.65	1.42	2.42	1.70	0.00	0.00	2.51
time (sec)	N/A	0.027	0.087	0.801	0.258	0.249	0.000	0.000	2.524

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	61	104	73	0	0	108
N.S.	1	1.00	0.65	1.42	2.42	1.70	0.00	0.00	2.51
time (sec)	N/A	0.029	0.010	0.654	0.270	0.257	0.000	0.000	0.002

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	65	117	91	0	255	127
N.S.	1	1.00	0.56	1.02	1.83	1.42	0.00	3.98	1.98
time (sec)	N/A	0.028	0.095	0.802	0.264	0.247	0.000	0.326	2.571

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	65	117	91	0	255	127
N.S.	1	1.00	0.56	1.02	1.83	1.42	0.00	3.98	1.98
time (sec)	N/A	0.028	0.010	0.644	0.263	0.249	0.000	0.319	0.003

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	59	103	73	0	0	107
N.S.	1	1.00	0.65	1.37	2.40	1.70	0.00	0.00	2.49
time (sec)	N/A	0.027	0.076	0.770	0.258	0.252	0.000	0.000	2.449

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	28	59	103	73	0	0	107
N.S.	1	1.00	0.65	1.37	2.40	1.70	0.00	0.00	2.49
time (sec)	N/A	0.029	0.010	0.649	0.256	0.269	0.000	0.000	0.002

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	63	117	91	0	254	127
N.S.	1	1.00	0.56	0.98	1.83	1.42	0.00	3.97	1.98
time (sec)	N/A	0.030	0.087	0.816	0.264	0.261	0.000	0.303	2.430

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	36	63	117	91	0	254	127
N.S.	1	1.00	0.56	0.98	1.83	1.42	0.00	3.97	1.98
time (sec)	N/A	0.029	0.010	0.652	0.260	0.264	0.000	0.295	0.002

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	95	12	0	20	21
N.S.	1	1.00	1.00	0.89	10.56	1.33	0.00	2.22	2.33
time (sec)	N/A	0.018	0.006	1.721	0.961	0.258	0.000	0.275	2.409

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	20	21
N.S.	1	1.00	1.00	0.89	0.78	1.33	0.00	2.22	2.33
time (sec)	N/A	0.015	0.009	0.139	0.190	0.247	0.000	0.263	2.364

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.010	0.068	0.025	0.294	0.240	0.627	0.261	2.391

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.010	0.047	0.022	0.298	0.254	0.538	0.259	2.344

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.015	0.091	0.031	0.289	0.253	0.817	0.262	2.440

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.017	0.090	0.030	0.293	0.257	3.170	0.271	2.425

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	23	87	12	24	26
N.S.	1	1.00	1.00	0.92	1.77	6.69	0.92	1.85	2.00
time (sec)	N/A	0.024	0.010	1.414	0.191	0.248	0.150	0.272	2.491

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	51
N.S.	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	4.25
time (sec)	N/A	0.030	0.026	0.301	0.193	0.273	0.000	0.265	0.200

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	70	0	40	55
N.S.	1	1.00	1.00	1.05	1.00	3.50	0.00	2.00	2.75
time (sec)	N/A	0.031	0.293	19.686	0.193	0.272	0.000	0.262	2.515

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	12	14	0	12	12
N.S.	1	1.00	1.00	1.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.014	0.003	0.125	0.211	0.270	0.000	0.256	0.052

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	19	13	12	14	0	12	12
N.S.	1	1.00	4.75	3.25	3.00	3.50	0.00	3.00	3.00
time (sec)	N/A	0.047	0.005	0.175	0.195	0.254	0.000	0.260	2.411

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	56	75	77	174	0	113	297
N.S.	1	1.00	2.00	2.68	2.75	6.21	0.00	4.04	10.61
time (sec)	N/A	0.066	2.932	0.523	0.213	0.255	0.000	0.254	2.722

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	62	177	694	0	265	107
N.S.	1	1.00	1.17	1.17	3.34	13.09	0.00	5.00	2.02
time (sec)	N/A	0.096	3.515	0.832	0.204	0.286	0.000	0.276	2.755

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	136	118	316	1980	0	544	1346
N.S.	1	1.00	1.74	1.51	4.05	25.38	0.00	6.97	17.26
time (sec)	N/A	0.104	5.200	1.854	0.219	0.306	0.000	0.273	3.166

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	136	40	39	386	44	224	39
N.S.	1	1.00	3.78	1.11	1.08	10.72	1.22	6.22	1.08
time (sec)	N/A	0.067	0.278	0.035	0.200	0.282	0.950	0.262	2.445

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	114	40	39	386	44	224	39
N.S.	1	1.00	3.17	1.11	1.08	10.72	1.22	6.22	1.08
time (sec)	N/A	0.064	0.430	0.036	0.190	0.248	1.020	0.264	0.212

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	154	46	0	15
N.S.	1	1.00	1.00	0.84	0.79	8.11	2.42	0.00	0.79
time (sec)	N/A	0.042	0.009	0.122	0.191	0.260	0.258	0.000	2.573

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	41	22	0	53	0	0	21
N.S.	1	1.00	1.52	0.81	0.00	1.96	0.00	0.00	0.78
time (sec)	N/A	0.124	0.029	0.119	0.000	0.264	0.000	0.000	2.584

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	37	8	17	8
N.S.	1	1.00	1.00	0.88	2.12	4.62	1.00	2.12	1.00
time (sec)	N/A	0.145	0.017	0.049	0.188	0.281	1.735	0.268	2.325

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	6	17	7	17	6
N.S.	1	1.00	1.50	0.88	0.75	2.12	0.88	2.12	0.75
time (sec)	N/A	0.010	0.006	0.042	0.187	0.258	0.114	0.254	2.343

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	15	37	8	15	8
N.S.	1	1.00	1.00	0.88	1.88	4.62	1.00	1.88	1.00
time (sec)	N/A	0.143	0.017	0.034	0.200	0.255	0.128	0.258	2.300

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	68	59	86	85	251	0	92	273
N.S.	1	1.31	1.13	1.65	1.63	4.83	0.00	1.77	5.25
time (sec)	N/A	0.121	0.121	1.449	0.279	0.273	0.000	0.261	0.443

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	68	59	78	85	251	0	92	137
N.S.	1	1.31	1.13	1.50	1.63	4.83	0.00	1.77	2.63
time (sec)	N/A	0.092	0.077	1.316	0.298	0.269	0.000	0.269	2.528

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	61	0	303	0	47	153
N.S.	1	1.00	0.92	1.17	0.00	5.83	0.00	0.90	2.94
time (sec)	N/A	0.109	0.115	1.344	0.000	0.265	0.000	0.254	2.732

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	63	0	297	0	49	120
N.S.	1	1.00	0.96	1.21	0.00	5.71	0.00	0.94	2.31
time (sec)	N/A	0.081	0.084	1.322	0.000	0.278	0.000	0.255	0.264

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	39	18	335	0	36	0
N.S.	1	1.00	1.03	1.30	0.60	11.17	0.00	1.20	0.00
time (sec)	N/A	0.028	0.027	0.155	0.297	0.273	0.000	0.264	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	40	174	0	0	0
N.S.	1	1.00	1.00	1.00	1.29	5.61	0.00	0.00	0.00
time (sec)	N/A	0.030	0.046	0.148	0.321	0.274	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	68	0	124	0	0	0
N.S.	1	1.00	1.40	1.58	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.055	0.032	0.418	0.000	0.256	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	33	91	0	47	25
N.S.	1	1.00	1.00	2.17	2.75	7.58	0.00	3.92	2.08
time (sec)	N/A	0.027	0.018	1.472	0.306	0.249	0.000	0.258	2.394

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	68	75	0	408	0	0	0
N.S.	1	1.00	1.28	1.42	0.00	7.70	0.00	0.00	0.00
time (sec)	N/A	0.103	0.095	3.754	0.000	0.269	0.000	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	51
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.55
time (sec)	N/A	0.127	2.009	0.000	0.000	0.000	0.000	0.000	2.479

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	11	22	29	31	10	10
N.S.	1	1.00	1.00	0.50	1.00	1.32	1.41	0.45	0.45
time (sec)	N/A	0.026	0.002	0.207	0.182	0.261	0.072	0.243	0.059

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	37	28	29	95	381	27	27
N.S.	1	1.00	0.54	0.41	0.42	1.38	5.52	0.39	0.39
time (sec)	N/A	0.148	0.050	0.408	0.185	0.255	0.664	0.272	0.057

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	120	194	213	396	241	386	213
N.S.	1	1.00	0.93	1.50	1.65	3.07	1.87	2.99	1.65
time (sec)	N/A	0.105	0.298	0.223	0.202	0.256	3.973	0.267	0.251

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	28	27	95	0	27	27
N.S.	1	1.00	0.85	0.72	0.69	2.44	0.00	0.69	0.69
time (sec)	N/A	0.033	0.020	0.158	0.183	0.261	0.000	0.256	0.057

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	30	18	0	431	0	46	148
N.S.	1	1.00	1.20	0.72	0.00	17.24	0.00	1.84	5.92
time (sec)	N/A	0.068	0.208	0.116	0.000	0.267	0.000	0.276	2.451

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	26	389	271	0	0	207
N.S.	1	1.00	0.92	0.70	10.51	7.32	0.00	0.00	5.59
time (sec)	N/A	0.069	0.044	0.128	0.281	0.285	0.000	0.000	3.082

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	21	13	0	0	4
N.S.	1	1.00	1.00	1.25	5.25	3.25	0.00	0.00	1.00
time (sec)	N/A	0.104	0.075	12.679	0.411	0.254	0.000	0.000	2.577

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	27	28	70	65	274	0
N.S.	1	1.00	1.22	1.00	1.04	2.59	2.41	10.15	0.00
time (sec)	N/A	0.066	0.065	5.315	0.198	0.258	0.430	0.294	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	37	37	36	76	63	282	0
N.S.	1	1.00	1.16	1.16	1.12	2.38	1.97	8.81	0.00
time (sec)	N/A	0.068	0.092	5.187	0.210	0.257	0.454	0.270	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	25	74	0	192	97	34	77
N.S.	1	1.00	0.49	1.45	0.00	3.76	1.90	0.67	1.51
time (sec)	N/A	0.120	0.026	0.280	0.000	0.277	2.222	0.730	0.220

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	115	68	93	127	202	37	48
N.S.	1	1.00	2.45	1.45	1.98	2.70	4.30	0.79	1.02
time (sec)	N/A	0.220	6.936	0.351	0.292	0.277	5.099	0.330	2.386

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	30	49	38	53	44	25
N.S.	1	1.00	1.55	2.73	4.45	3.45	4.82	4.00	2.27
time (sec)	N/A	0.052	0.006	0.291	0.294	0.245	0.445	0.280	2.272

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	65	15	14	40	37	14	14
N.S.	1	1.00	2.95	0.68	0.64	1.82	1.68	0.64	0.64
time (sec)	N/A	0.036	0.050	0.141	0.195	0.257	0.250	0.266	0.093

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	65	15	14	40	0	14	14
N.S.	1	1.00	4.64	1.07	1.00	2.86	0.00	1.00	1.00
time (sec)	N/A	0.148	0.037	0.206	0.204	0.273	0.000	0.289	0.083

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	17	30	50	37	0	43	26
N.S.	1	1.00	1.42	2.50	4.17	3.08	0.00	3.58	2.17
time (sec)	N/A	0.184	0.005	0.247	0.292	0.262	0.000	0.285	2.331

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	68	93	127	0	37	48
N.S.	1	1.00	1.11	1.45	1.98	2.70	0.00	0.79	1.02
time (sec)	N/A	0.296	6.127	0.431	0.292	0.252	0.000	0.309	2.379

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	26	74	0	192	0	34	77
N.S.	1	1.00	0.51	1.45	0.00	3.76	0.00	0.67	1.51
time (sec)	N/A	0.977	0.018	0.299	0.000	0.250	0.000	0.428	2.384

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [423] had the largest ratio of [1.25000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	14	0.214
2	A	2	1	1.00	21	0.048
3	A	2	2	1.00	23	0.087
4	A	2	2	1.00	21	0.095
5	A	2	2	1.00	23	0.087
6	A	2	2	1.00	23	0.087
7	A	2	2	1.00	23	0.087
8	A	2	2	1.00	13	0.154
9	A	2	2	1.00	15	0.133
10	A	3	2	1.00	17	0.118
11	A	3	2	1.00	17	0.118
12	A	2	2	1.00	15	0.133
13	A	3	2	1.00	17	0.118
14	A	3	2	1.00	17	0.118
15	A	3	3	1.00	17	0.176
16	A	4	3	1.00	17	0.176
17	A	5	3	1.00	17	0.176
18	A	4	3	1.00	17	0.176
19	A	5	3	1.00	17	0.176
20	A	6	3	1.00	17	0.176
21	A	5	3	1.00	17	0.176
22	A	6	3	1.00	17	0.176
23	A	7	3	1.00	17	0.176
24	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	3	1.00	15	0.200
26	A	3	2	1.00	15	0.133
27	A	4	3	1.00	15	0.200
28	A	4	3	1.00	15	0.200
29	A	3	3	1.00	15	0.200
30	A	3	2	1.00	17	0.118
31	A	4	4	1.00	17	0.235
32	A	3	2	1.00	17	0.118
33	A	5	4	1.00	17	0.235
34	A	3	2	1.00	15	0.133
35	A	4	4	1.00	17	0.235
36	A	4	3	1.00	17	0.176
37	A	5	4	1.00	17	0.235
38	A	4	3	1.00	17	0.176
39	A	4	3	1.00	15	0.200
40	A	3	2	1.00	17	0.118
41	A	5	4	1.00	17	0.235
42	A	3	2	1.00	17	0.118
43	A	6	4	1.00	17	0.235
44	A	4	3	1.00	15	0.200
45	A	5	4	1.00	17	0.235
46	A	4	3	1.00	17	0.176
47	A	6	4	1.00	17	0.235
48	A	4	3	1.00	17	0.176
49	A	6	5	1.00	21	0.238
50	A	5	5	1.00	21	0.238
51	A	5	5	1.00	21	0.238
52	A	4	4	1.00	21	0.190
53	A	4	4	1.00	21	0.190
54	A	5	5	1.00	21	0.238
55	A	5	5	1.00	21	0.238
56	A	6	5	1.00	21	0.238
57	A	9	9	1.00	21	0.429
58	A	9	9	1.00	21	0.429
59	A	12	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	11	7	1.00	21	0.333
61	A	8	8	1.00	21	0.381
62	A	8	8	1.00	21	0.381
63	A	11	7	1.00	21	0.333
64	A	12	8	1.00	21	0.381
65	A	9	9	1.00	21	0.429
66	A	9	9	1.00	21	0.429
67	A	1	1	1.00	13	0.077
68	A	1	1	1.00	13	0.077
69	A	2	2	1.00	9	0.222
70	A	3	3	1.00	13	0.231
71	A	3	2	1.00	15	0.133
72	A	4	4	1.00	15	0.267
73	A	3	2	1.00	15	0.133
74	A	3	2	1.00	15	0.133
75	A	4	4	1.00	17	0.235
76	A	4	3	1.00	17	0.176
77	A	4	3	1.00	15	0.200
78	A	3	2	1.00	17	0.118
79	A	5	4	1.00	17	0.235
80	A	4	3	1.00	15	0.200
81	A	2	2	1.00	13	0.154
82	A	2	2	1.00	15	0.133
83	A	2	2	1.00	17	0.118
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	17	0.118
86	A	2	2	1.00	17	0.118
87	A	2	1	1.00	15	0.067
88	A	3	2	1.00	17	0.118
89	A	3	2	1.00	19	0.105
90	A	3	2	1.00	17	0.118
91	A	3	2	1.00	19	0.105
92	A	3	2	1.00	17	0.118
93	A	2	2	1.00	15	0.133
94	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	3	1.00	17	0.176
96	A	3	2	1.00	7	0.286
97	A	3	2	1.00	9	0.222
98	A	4	3	1.00	9	0.333
99	A	4	3	1.00	9	0.333
100	A	3	2	1.00	9	0.222
101	A	3	3	1.00	13	0.231
102	A	3	2	1.00	15	0.133
103	A	4	4	1.00	15	0.267
104	A	3	2	1.00	15	0.133
105	A	3	2	1.00	15	0.133
106	A	4	4	1.00	17	0.235
107	A	4	3	1.00	17	0.176
108	A	4	3	1.00	15	0.200
109	A	3	2	1.00	17	0.118
110	A	5	4	1.00	17	0.235
111	A	4	3	1.00	15	0.200
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	15	0.133
114	A	2	2	1.00	17	0.118
115	A	2	2	1.00	17	0.118
116	A	2	2	1.00	17	0.118
117	A	2	2	1.00	17	0.118
118	A	2	1	1.00	15	0.067
119	A	3	2	1.00	17	0.118
120	A	3	2	1.00	19	0.105
121	A	2	2	1.00	15	0.133
122	A	3	3	1.00	17	0.176
123	A	3	2	1.00	15	0.133
124	A	3	2	1.00	9	0.222
125	A	3	2	1.00	9	0.222
126	A	3	2	1.00	9	0.222
127	A	4	3	1.00	9	0.333
128	A	3	2	1.00	9	0.222
129	A	3	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	2	1.00	9	0.222
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	14	0.143
133	A	3	2	1.00	13	0.154
134	A	3	2	1.00	14	0.143
135	A	4	3	1.00	13	0.231
136	A	4	3	1.00	14	0.214
137	A	4	3	1.00	13	0.231
138	A	4	3	1.00	14	0.214
139	A	3	2	1.00	13	0.154
140	A	3	2	1.00	14	0.143
141	A	3	2	1.00	13	0.154
142	A	3	2	1.00	14	0.143
143	A	3	3	1.00	13	0.231
144	A	6	6	1.00	15	0.400
145	A	9	7	1.00	15	0.467
146	A	3	3	1.00	13	0.231
147	A	6	6	1.00	15	0.400
148	A	9	7	1.00	15	0.467
149	A	3	3	1.00	13	0.231
150	A	4	4	1.00	15	0.267
151	A	5	5	1.00	15	0.333
152	A	3	3	1.00	13	0.231
153	A	4	4	1.00	15	0.267
154	A	5	5	1.00	15	0.333
155	A	3	3	1.00	13	0.231
156	A	6	6	1.00	15	0.400
157	A	9	7	1.00	15	0.467
158	A	3	3	1.00	13	0.231
159	A	6	6	1.00	15	0.400
160	A	9	7	1.00	15	0.467
161	A	3	3	1.00	13	0.231
162	A	4	4	1.00	15	0.267
163	A	5	5	1.00	15	0.333
164	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	4	4	1.00	15	0.267
166	A	5	5	1.00	15	0.333
167	A	4	2	1.00	13	0.154
168	A	5	2	1.00	15	0.133
169	A	6	2	1.00	15	0.133
170	A	6	2	1.00	17	0.118
171	A	8	2	1.00	17	0.118
172	A	10	2	1.00	17	0.118
173	A	4	2	1.00	13	0.154
174	A	5	2	1.00	15	0.133
175	A	6	2	1.00	15	0.133
176	A	6	2	1.00	17	0.118
177	A	8	2	1.00	17	0.118
178	A	10	2	1.00	17	0.118
179	A	4	2	1.00	13	0.154
180	A	5	2	1.00	15	0.133
181	A	6	2	1.00	15	0.133
182	A	5	2	1.00	15	0.133
183	A	6	2	1.00	17	0.118
184	A	8	2	1.00	17	0.118
185	A	6	2	1.00	15	0.133
186	A	8	2	1.00	17	0.118
187	A	10	2	1.00	17	0.118
188	A	6	3	1.00	13	0.231
189	A	6	3	1.00	13	0.231
190	A	6	3	1.00	13	0.231
191	A	6	3	1.00	13	0.231
192	A	1	1	1.88	7	0.143
193	A	1	1	1.00	7	0.143
194	A	1	1	1.00	7	0.143
195	A	4	2	1.00	7	0.286
196	A	1	1	1.00	7	0.143
197	A	1	1	1.00	7	0.143
198	A	1	1	1.00	7	0.143
199	A	4	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	3	1.00	7	0.429
201	A	5	3	1.00	7	0.429
202	A	6	4	1.00	7	0.571
203	A	9	4	1.00	7	0.571
204	A	10	5	1.00	7	0.714
205	A	6	3	1.00	7	0.429
206	A	3	2	1.00	7	0.286
207	A	3	2	1.00	7	0.286
208	A	6	3	1.00	7	0.429
209	A	6	3	1.00	7	0.429
210	A	7	3	1.00	7	0.429
211	A	2	2	1.00	7	0.286
212	A	5	5	1.00	7	0.714
213	A	4	3	1.00	7	0.429
214	A	7	6	1.00	7	0.857
215	A	7	4	1.00	7	0.571
216	A	2	2	1.00	7	0.286
217	A	2	1	1.00	7	0.143
218	A	4	2	1.00	7	0.286
219	A	4	2	1.00	7	0.286
220	A	7	3	1.00	7	0.429
221	A	1	1	1.88	7	0.143
222	A	1	1	1.00	7	0.143
223	A	1	1	1.00	7	0.143
224	A	4	2	1.00	7	0.286
225	A	1	1	1.00	7	0.143
226	A	1	1	1.00	7	0.143
227	A	1	1	1.00	7	0.143
228	A	4	2	1.00	7	0.286
229	A	4	3	1.00	7	0.429
230	A	3	2	1.00	7	0.286
231	A	6	4	1.00	7	0.571
232	A	6	3	1.00	7	0.429
233	A	10	5	1.00	7	0.714
234	A	4	3	1.00	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	9	4	1.00	7	0.571
236	A	6	3	1.00	7	0.429
237	A	10	4	1.00	7	0.571
238	A	7	3	1.00	7	0.429
239	A	6	3	1.00	7	0.429
240	A	2	2	1.00	7	0.286
241	A	2	1	1.00	7	0.143
242	A	4	3	1.00	7	0.429
243	A	4	2	1.00	7	0.286
244	A	7	4	1.00	7	0.571
245	A	2	2	1.00	7	0.286
246	A	5	5	1.00	7	0.714
247	A	4	2	1.00	7	0.286
248	A	7	6	1.00	7	0.857
249	A	7	3	1.00	7	0.429
250	A	5	4	1.00	16	0.250
251	A	5	5	1.00	16	0.312
252	A	3	3	1.00	16	0.188
253	A	3	3	1.00	14	0.214
254	A	2	2	1.00	13	0.154
255	A	5	5	1.00	16	0.312
256	A	6	6	1.00	16	0.375
257	A	7	6	1.00	16	0.375
258	A	8	6	1.00	16	0.375
259	A	8	3	1.00	18	0.167
260	A	7	5	1.00	18	0.278
261	A	4	4	1.00	18	0.222
262	A	3	2	1.00	16	0.125
263	A	2	2	1.00	15	0.133
264	A	8	4	1.00	18	0.222
265	A	10	5	1.00	18	0.278
266	A	12	5	1.00	18	0.278
267	A	14	5	1.00	18	0.278
268	A	8	3	1.00	18	0.167
269	A	9	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	4	3	1.00	18	0.167
271	A	4	3	1.00	16	0.188
272	A	2	2	1.00	15	0.133
273	A	8	4	1.00	18	0.222
274	A	10	5	1.00	18	0.278
275	A	12	5	1.00	18	0.278
276	A	14	5	1.00	18	0.278
277	A	3	3	1.00	8	0.375
278	A	4	4	1.00	8	0.500
279	A	5	4	1.00	8	0.500
280	A	8	3	1.00	18	0.167
281	A	7	5	1.00	18	0.278
282	A	4	4	1.00	18	0.222
283	A	3	2	1.00	16	0.125
284	A	2	2	1.00	15	0.133
285	A	8	4	1.00	18	0.222
286	A	10	5	1.00	18	0.278
287	A	12	5	1.00	18	0.278
288	A	14	5	1.00	18	0.278
289	A	5	3	1.00	20	0.150
290	A	6	3	1.00	20	0.150
291	A	5	3	1.00	20	0.150
292	A	4	3	1.00	18	0.167
293	A	3	3	1.00	17	0.176
294	A	5	4	1.00	20	0.200
295	A	6	5	1.00	20	0.250
296	A	7	5	1.00	20	0.250
297	A	8	5	1.00	20	0.250
298	A	11	3	1.00	20	0.150
299	A	14	3	1.00	20	0.150
300	A	11	3	1.00	20	0.150
301	A	8	3	1.00	18	0.167
302	A	3	2	1.00	17	0.118
303	A	11	4	1.00	20	0.200
304	A	14	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	17	5	1.00	20	0.250
306	A	20	5	1.00	20	0.250
307	A	8	3	1.00	18	0.167
308	A	9	5	1.00	18	0.278
309	A	4	3	1.00	18	0.167
310	A	4	3	1.00	16	0.188
311	A	2	2	1.00	15	0.133
312	A	8	4	1.00	18	0.222
313	A	10	5	1.00	18	0.278
314	A	12	5	1.00	18	0.278
315	A	14	5	1.00	18	0.278
316	A	11	3	1.00	20	0.150
317	A	14	3	1.00	20	0.150
318	A	11	3	1.00	20	0.150
319	A	8	3	1.00	18	0.167
320	A	3	2	1.00	17	0.118
321	A	11	4	1.00	20	0.200
322	A	14	5	1.00	20	0.250
323	A	17	5	1.00	20	0.250
324	A	20	5	1.00	20	0.250
325	A	8	3	1.00	20	0.150
326	A	10	3	1.00	20	0.150
327	A	8	3	1.00	20	0.150
328	A	6	3	1.00	18	0.167
329	A	3	2	1.00	17	0.118
330	A	8	4	1.00	20	0.200
331	A	10	5	1.00	20	0.250
332	A	12	5	1.00	20	0.250
333	A	14	5	1.00	20	0.250
334	N/A	0	0	1.00	10	0.000
335	A	6	6	1.00	10	0.600
336	A	5	5	1.00	10	0.500
337	A	4	4	1.00	8	0.500
338	A	1	1	1.00	6	0.167
339	N/A	0	0	1.00	10	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	N/A	0	0	1.00	10	0.000
341	N/A	0	0	1.00	16	0.000
342	A	8	5	1.00	16	0.312
343	A	6	4	1.00	16	0.250
344	A	2	2	1.00	14	0.143
345	A	2	2	1.00	13	0.154
346	N/A	0	0	1.00	16	0.000
347	N/A	0	0	1.00	16	0.000
348	N/A	0	0	1.00	18	0.000
349	A	6	6	1.00	18	0.333
350	A	3	3	1.00	18	0.167
351	A	3	3	1.00	16	0.188
352	A	2	2	1.00	15	0.133
353	N/A	0	0	1.00	18	0.000
354	N/A	0	0	1.00	18	0.000
355	N/A	0	0	1.00	16	0.000
356	A	14	8	1.00	16	0.500
357	A	11	7	1.00	16	0.438
358	A	8	6	1.00	14	0.429
359	A	3	3	1.00	13	0.231
360	N/A	0	0	1.00	16	0.000
361	N/A	0	0	1.00	16	0.000
362	N/A	0	0	1.00	12	0.000
363	A	7	7	1.00	12	0.583
364	A	6	6	1.00	12	0.500
365	A	3	3	1.00	10	0.300
366	A	2	2	1.00	8	0.250
367	N/A	0	0	1.00	12	0.000
368	N/A	0	0	1.00	12	0.000
369	N/A	0	0	1.00	18	0.000
370	A	25	9	1.00	18	0.500
371	A	17	7	1.00	18	0.389
372	A	12	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	2	2	1.00	15	0.133
374	N/A	0	0	1.00	18	0.000
375	N/A	0	0	1.00	18	0.000
376	N/A	0	0	1.00	18	0.000
377	A	12	12	1.00	18	0.667
378	A	9	9	1.00	18	0.500
379	A	8	8	1.00	16	0.500
380	A	3	2	1.00	15	0.133
381	N/A	0	0	1.00	18	0.000
382	N/A	0	0	1.00	18	0.000
383	N/A	0	0	1.00	18	0.000
384	A	13	8	1.00	18	0.444
385	A	10	7	1.00	18	0.389
386	A	5	5	1.00	16	0.312
387	A	3	2	1.00	15	0.133
388	N/A	0	0	1.00	18	0.000
389	N/A	0	0	1.00	18	0.000
390	N/A	0	0	1.00	12	0.000
391	A	13	10	1.00	12	0.833
392	A	9	8	1.00	12	0.667
393	A	7	7	1.00	10	0.700
394	A	2	2	1.00	8	0.250
395	N/A	0	0	1.00	12	0.000
396	N/A	0	0	1.00	12	0.000
397	N/A	0	0	1.00	10	0.000
398	A	6	6	1.00	10	0.600
399	A	5	5	1.00	10	0.500
400	A	4	4	1.00	8	0.500
401	A	1	1	1.00	6	0.167
402	N/A	0	0	1.00	10	0.000
403	N/A	0	0	1.00	10	0.000
404	N/A	0	0	1.00	16	0.000
405	A	14	8	1.00	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	11	7	1.00	16	0.438
407	A	8	6	1.00	14	0.429
408	A	3	3	1.00	13	0.231
409	N/A	0	0	1.00	16	0.000
410	N/A	0	0	1.00	16	0.000
411	N/A	0	0	1.00	18	0.000
412	A	12	12	1.00	18	0.667
413	A	9	9	1.00	18	0.500
414	A	8	8	1.00	16	0.500
415	A	3	2	1.00	15	0.133
416	N/A	0	0	1.00	18	0.000
417	N/A	0	0	1.00	18	0.000
418	A	6	5	1.00	10	0.500
419	A	11	10	1.00	12	0.833
420	A	12	10	1.00	12	0.833
421	A	16	10	1.00	10	1.000
422	A	19	11	1.00	12	0.917
423	A	26	15	1.00	12	1.250
424	N/A	0	0	1.00	16	0.000
425	A	8	5	1.00	16	0.312
426	A	6	4	1.00	16	0.250
427	A	2	2	1.00	14	0.143
428	A	2	2	1.00	13	0.154
429	N/A	0	0	1.00	16	0.000
430	N/A	0	0	1.00	16	0.000
431	N/A	0	0	1.00	12	0.000
432	A	7	7	1.00	12	0.583
433	A	6	6	1.00	12	0.500
434	A	3	3	1.00	10	0.300
435	A	2	2	1.00	8	0.250
436	N/A	0	0	1.00	12	0.000
437	N/A	0	0	1.00	12	0.000
438	N/A	0	0	1.00	18	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
439	A	13	8	1.00	18	0.444
440	A	10	7	1.00	18	0.389
441	A	5	5	1.00	16	0.312
442	A	3	2	1.00	15	0.133
443	N/A	0	0	1.00	18	0.000
444	N/A	0	0	1.00	18	0.000
445	N/A	0	0	1.00	18	0.000
446	A	6	6	1.00	18	0.333
447	A	3	3	1.00	18	0.167
448	A	3	3	1.00	16	0.188
449	A	2	2	1.00	15	0.133
450	N/A	0	0	1.00	18	0.000
451	N/A	0	0	1.00	18	0.000
452	N/A	0	0	1.00	18	0.000
453	A	25	9	1.00	18	0.500
454	A	17	7	1.00	18	0.389
455	A	12	5	1.00	16	0.312
456	A	2	2	1.00	15	0.133
457	N/A	0	0	1.00	18	0.000
458	N/A	0	0	1.00	18	0.000
459	N/A	0	0	1.00	12	0.000
460	A	13	10	1.00	12	0.833
461	A	9	8	1.00	12	0.667
462	A	7	7	1.00	10	0.700
463	A	2	2	1.00	8	0.250
464	N/A	0	0	1.00	12	0.000
465	N/A	0	0	1.00	12	0.000
466	N/A	0	0	1.00	16	0.000
467	A	10	6	1.00	16	0.375
468	A	8	5	1.00	16	0.312
469	A	6	4	1.00	14	0.286
470	A	2	2	1.00	13	0.154
471	N/A	0	0	1.00	16	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	N/A	0	0	1.00	16	0.000
473	N/A	0	0	1.00	18	0.000
474	A	21	13	1.00	18	0.722
475	A	17	14	1.00	18	0.778
476	A	10	10	1.00	16	0.625
477	A	3	3	1.00	15	0.200
478	N/A	0	0	1.00	18	0.000
479	N/A	0	0	1.00	18	0.000
480	N/A	0	0	1.00	18	0.000
481	A	20	16	1.00	18	0.889
482	A	15	12	1.00	18	0.667
483	A	11	10	1.00	16	0.625
484	A	3	2	1.00	15	0.133
485	N/A	0	0	1.00	18	0.000
486	N/A	0	0	1.00	18	0.000
487	N/A	0	0	1.00	18	0.000
488	A	21	13	1.00	18	0.722
489	A	17	14	1.00	18	0.778
490	A	10	10	1.00	16	0.625
491	A	3	3	1.00	15	0.200
492	N/A	0	0	1.00	18	0.000
493	N/A	0	0	1.00	18	0.000
494	N/A	0	0	1.00	20	0.000
495	A	7	7	1.00	20	0.350
496	A	6	6	1.00	20	0.300
497	A	3	3	1.00	18	0.167
498	A	3	2	1.00	17	0.118
499	N/A	0	0	1.00	20	0.000
500	N/A	0	0	1.00	20	0.000
501	N/A	0	0	1.00	20	0.000
502	A	29	18	1.00	20	0.900
503	A	13	12	1.00	18	0.667
504	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	N/A	0	0	1.00	20	0.000
506	N/A	0	0	1.00	20	0.000
507	N/A	0	0	1.00	18	0.000
508	A	20	16	1.00	18	0.889
509	A	15	12	1.00	18	0.667
510	A	11	10	1.00	16	0.625
511	A	3	2	1.00	15	0.133
512	N/A	0	0	1.00	18	0.000
513	N/A	0	0	1.00	18	0.000
514	N/A	0	0	1.00	20	0.000
515	A	40	19	1.00	20	0.950
516	A	29	19	1.00	20	0.950
517	A	13	12	1.00	18	0.667
518	A	4	4	1.00	17	0.235
519	N/A	0	0	1.00	20	0.000
520	N/A	0	0	1.00	20	0.000
521	N/A	0	0	1.00	20	0.000
522	A	16	9	1.00	20	0.450
523	A	10	7	1.00	20	0.350
524	A	7	5	1.00	18	0.278
525	A	4	3	1.00	17	0.176
526	N/A	0	0	1.00	20	0.000
527	N/A	0	0	1.00	20	0.000
528	A	4	3	1.00	18	0.167
529	A	3	3	1.00	18	0.167
530	A	3	3	1.00	18	0.167
531	A	2	2	1.00	18	0.111
532	A	2	2	1.00	18	0.111
533	A	3	3	1.00	18	0.167
534	A	3	3	1.00	18	0.167
535	A	4	3	1.00	18	0.167
536	A	5	4	1.00	18	0.222
537	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
538	A	4	4	1.00	18	0.222
539	A	3	3	1.00	18	0.167
540	A	3	3	1.00	18	0.167
541	A	4	4	1.00	18	0.222
542	A	4	4	1.00	18	0.222
543	A	5	4	1.00	18	0.222
544	A	5	4	1.00	18	0.222
545	A	4	4	1.00	18	0.222
546	A	4	4	1.00	18	0.222
547	A	3	3	1.00	18	0.167
548	A	3	3	1.00	18	0.167
549	A	4	4	1.00	18	0.222
550	A	4	4	1.00	18	0.222
551	A	5	4	1.00	18	0.222
552	A	5	4	1.00	18	0.222
553	A	4	4	1.00	18	0.222
554	A	4	4	1.00	18	0.222
555	A	3	3	1.00	18	0.167
556	A	3	3	1.00	18	0.167
557	A	4	4	1.00	18	0.222
558	A	4	4	1.00	18	0.222
559	A	5	4	1.00	18	0.222
560	A	3	3	1.00	9	0.333
561	A	4	4	1.00	9	0.444
562	A	5	5	1.00	9	0.556
563	A	3	3	1.00	9	0.333
564	A	4	4	1.00	9	0.444
565	A	5	5	1.00	9	0.556
566	A	7	6	1.00	14	0.429
567	A	7	6	1.00	15	0.400
568	A	6	5	1.00	14	0.357
569	A	6	5	1.00	15	0.333
570	A	3	3	1.00	17	0.176
571	A	3	3	1.00	16	0.188
572	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
573	A	6	6	1.00	15	0.400
574	A	3	3	1.00	17	0.176
575	A	4	4	1.00	17	0.235
576	A	4	4	1.00	17	0.235
577	A	3	3	1.00	17	0.176
578	A	6	6	1.00	17	0.353
579	A	7	7	1.00	17	0.412
580	A	3	2	1.00	9	0.222
581	A	2	2	1.00	11	0.182
582	A	2	1	1.00	11	0.091
583	A	3	2	1.00	11	0.182
584	A	3	2	1.00	11	0.182
585	A	2	2	1.00	11	0.182
586	A	1	1	1.00	11	0.091
587	A	3	3	1.00	11	0.273
588	A	2	2	1.00	11	0.182
589	A	4	3	1.00	11	0.273
590	A	2	2	1.00	13	0.154
591	A	3	3	1.00	13	0.231
592	A	3	3	1.00	13	0.231
593	A	2	2	1.00	13	0.154
594	A	3	3	1.00	13	0.231
595	A	3	3	1.00	13	0.231
596	A	3	2	1.00	17	0.118
597	A	1	1	1.00	19	0.053
598	A	1	1	1.00	19	0.053
599	A	1	1	1.00	19	0.053
600	A	1	1	1.00	19	0.053
601	A	1	1	1.00	19	0.053
602	A	1	1	1.00	19	0.053
603	A	1	1	1.00	21	0.048
604	A	1	1	1.00	21	0.048
605	A	3	2	1.00	18	0.111
606	A	1	1	1.00	20	0.050
607	A	1	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
608	A	1	1	1.00	20	0.050
609	A	1	1	1.00	20	0.050
610	A	1	1	1.00	20	0.050
611	A	1	1	1.00	20	0.050
612	A	1	1	1.00	22	0.045
613	A	1	1	1.00	22	0.045
614	A	8	8	1.00	11	0.727
615	A	4	4	1.00	11	0.364
616	A	7	7	1.00	11	0.636
617	A	4	3	1.00	11	0.273
618	A	3	2	1.00	9	0.222
619	A	3	3	1.00	11	0.273
620	A	6	6	1.00	11	0.546
621	A	4	3	1.00	11	0.273
622	A	8	8	1.00	11	0.727
623	A	4	3	1.00	11	0.273
624	A	4	3	1.00	11	0.273
625	A	5	4	1.00	11	0.364
626	A	4	3	1.00	11	0.273
627	A	4	4	1.00	11	0.364
628	A	3	2	1.00	9	0.222
629	A	3	3	1.00	11	0.273
630	A	3	3	1.00	11	0.273
631	A	4	3	1.00	11	0.273
632	A	4	3	1.00	11	0.273
633	A	4	3	1.00	11	0.273
634	A	4	3	1.00	11	0.273
635	A	5	4	1.00	11	0.364
636	A	4	3	1.00	11	0.273
637	A	4	4	1.00	11	0.364
638	A	3	2	1.00	9	0.222
639	A	3	3	1.00	11	0.273
640	A	3	3	1.00	11	0.273
641	A	4	3	1.00	11	0.273
642	A	4	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
643	A	4	3	1.00	11	0.273
644	A	8	8	1.00	11	0.727
645	A	4	4	1.00	11	0.364
646	A	7	7	1.00	11	0.636
647	A	4	3	1.00	11	0.273
648	A	3	2	1.00	9	0.222
649	A	3	3	1.00	11	0.273
650	A	5	5	1.00	11	0.454
651	A	4	3	1.00	11	0.273
652	A	7	7	1.00	11	0.636
653	A	4	3	1.00	11	0.273
654	A	4	3	1.00	7	0.429
655	A	5	4	1.00	7	0.571
656	A	4	3	1.00	7	0.429
657	A	4	4	1.00	7	0.571
658	A	3	2	1.00	5	0.400
659	A	3	3	1.00	7	0.429
660	A	3	3	1.00	7	0.429
661	A	4	3	1.00	7	0.429
662	A	4	3	1.00	7	0.429
663	A	4	3	1.00	7	0.429
664	A	4	3	1.00	9	0.333
665	A	5	4	1.00	9	0.444
666	A	4	3	1.00	9	0.333
667	A	4	4	1.00	9	0.444
668	A	3	2	1.00	7	0.286
669	A	3	3	1.00	9	0.333
670	A	3	3	1.00	9	0.333
671	A	4	3	1.00	9	0.333
672	A	4	3	1.00	9	0.333
673	A	4	3	1.00	9	0.333
674	A	3	2	1.00	5	0.400
675	A	4	3	1.00	7	0.429
676	A	6	5	1.00	7	0.714
677	A	4	4	1.00	9	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
678	A	5	5	1.00	9	0.556
679	A	6	6	1.00	9	0.667
680	A	3	2	1.00	7	0.286
681	A	4	3	1.00	9	0.333
682	A	6	5	1.00	9	0.556
683	A	3	3	1.00	11	0.273
684	A	4	4	1.00	11	0.364
685	A	5	5	1.00	11	0.454
686	A	6	6	1.33	7	0.857
687	A	6	6	1.20	9	0.667
688	A	2	2	1.00	14	0.143
689	A	4	4	1.00	16	0.250
690	A	5	5	1.00	16	0.312
691	A	2	2	1.00	14	0.143
692	A	4	4	1.00	16	0.250
693	A	5	5	1.00	16	0.312
694	A	5	4	1.00	14	0.286
695	A	5	4	1.00	14	0.286
696	A	3	3	1.00	14	0.214
697	A	4	4	1.00	16	0.250
698	A	16	10	1.54	16	0.625
699	A	3	3	1.00	14	0.214
700	A	4	4	1.00	16	0.250
701	A	8	4	1.45	16	0.250
702	A	2	2	1.00	14	0.143
703	A	5	5	1.00	16	0.312
704	A	2	2	1.00	14	0.143
705	A	5	5	1.00	16	0.312
706	A	5	5	1.00	16	0.312
707	A	7	7	1.00	18	0.389
708	A	9	8	1.00	18	0.444
709	A	7	7	1.00	18	0.389
710	A	10	8	1.00	20	0.400
711	A	13	8	1.00	20	0.400
712	A	9	8	1.00	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
713	A	13	9	1.00	20	0.450
714	A	17	9	1.00	20	0.450
715	A	6	5	1.00	16	0.312
716	A	13	8	1.00	18	0.444
717	A	17	13	1.00	18	0.722
718	A	13	8	1.00	18	0.444
719	A	21	10	1.00	20	0.500
720	A	33	12	1.00	20	0.600
721	A	17	13	1.00	18	0.722
722	A	33	12	1.00	20	0.600
723	A	48	12	1.00	20	0.600
724	A	3	3	1.00	18	0.167
725	A	3	3	1.00	18	0.167
726	A	4	4	1.00	18	0.222
727	A	3	3	1.00	18	0.167
728	A	3	3	1.00	18	0.167
729	A	4	4	1.00	18	0.222
730	A	1	1	1.00	15	0.067
731	A	1	1	1.00	15	0.067
732	A	1	1	1.00	21	0.048
733	A	1	1	1.00	21	0.048
734	A	3	3	1.00	21	0.143
735	A	3	3	1.00	21	0.143
736	A	3	3	1.00	22	0.136
737	A	3	3	1.00	22	0.136
738	A	4	4	1.00	22	0.182
739	A	5	4	1.00	12	0.333
740	A	4	3	1.00	12	0.250
741	A	3	2	1.00	10	0.200
742	A	3	3	1.00	12	0.250
743	A	5	5	1.00	12	0.417
744	A	5	5	1.00	12	0.417
745	A	6	6	1.00	12	0.500
746	A	5	4	1.00	12	0.333
747	A	4	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
748	A	3	2	1.00	10	0.200
749	A	2	2	1.00	12	0.167
750	A	4	4	1.00	12	0.333
751	A	4	4	1.00	12	0.333
752	A	5	5	1.00	12	0.417
753	A	6	3	1.00	24	0.125
754	A	5	3	1.00	24	0.125
755	A	4	3	1.00	24	0.125
756	A	3	2	1.00	22	0.091
757	A	1	1	1.00	24	0.042
758	A	2	2	1.00	24	0.083
759	A	3	2	1.00	24	0.083
760	A	4	2	1.00	24	0.083
761	A	7	7	1.00	14	0.500
762	A	6	6	1.00	14	0.429
763	A	2	2	1.00	14	0.143
764	A	2	2	1.00	14	0.143
765	A	3	3	1.00	14	0.214
766	A	7	7	1.00	14	0.500
767	A	8	7	1.00	14	0.500
768	A	3	2	1.00	26	0.077
769	A	2	2	1.00	26	0.077
770	A	1	1	1.00	26	0.038
771	A	3	3	1.00	26	0.115
772	A	4	4	1.00	26	0.154
773	A	5	4	1.00	26	0.154
774	A	3	2	1.00	28	0.071
775	A	2	2	1.00	28	0.071
776	A	1	1	1.00	28	0.036
777	A	3	3	1.00	28	0.107
778	A	4	4	1.00	28	0.143
779	A	5	4	1.00	28	0.143
780	A	5	5	1.00	12	0.417
781	A	5	5	1.00	12	0.417
782	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
783	A	1	1	1.00	11	0.091
784	A	4	4	1.00	15	0.267
785	A	10	9	1.00	17	0.529
786	A	4	4	1.00	15	0.267
787	A	4	4	1.00	15	0.267
788	A	9	7	1.00	17	0.412
789	A	4	4	1.00	19	0.210
790	A	4	4	1.00	19	0.210
791	A	5	5	1.00	19	0.263
792	A	4	4	1.00	19	0.210
793	A	4	4	1.00	19	0.210
794	A	5	5	1.00	19	0.263
795	A	4	4	1.00	22	0.182
796	A	4	4	1.00	22	0.182
797	A	5	5	1.00	22	0.227
798	A	4	4	1.00	23	0.174
799	A	4	4	1.00	23	0.174
800	A	5	5	1.00	23	0.217
801	A	1	1	1.00	32	0.031
802	A	1	1	1.00	19	0.053
803	A	1	1	1.00	19	0.053
804	A	1	1	1.00	23	0.043
805	A	1	1	1.00	20	0.050
806	A	1	1	1.00	20	0.050
807	A	1	1	1.00	24	0.042
808	A	2	1	1.00	11	0.091
809	A	2	1	1.00	11	0.091
810	A	4	3	1.00	11	0.273
811	A	2	2	1.00	13	0.154
812	A	2	2	1.00	13	0.154
813	A	2	2	1.00	13	0.154
814	A	2	2	1.00	11	0.182
815	A	2	2	1.00	11	0.182
816	A	2	2	1.00	11	0.182
817	A	4	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
818	A	6	4	1.00	13	0.308
819	A	6	4	1.00	13	0.308
820	A	4	2	1.00	11	0.182
821	A	6	4	1.00	11	0.364
822	A	6	4	1.00	11	0.364
823	A	2	2	1.00	13	0.154
824	A	2	2	1.00	13	0.154
825	A	2	2	1.00	13	0.154
826	A	7	4	1.00	14	0.286
827	A	8	4	1.00	17	0.235
828	A	9	5	1.00	19	0.263
829	A	10	6	1.00	19	0.316
830	A	3	3	1.00	27	0.111
831	A	7	4	1.00	21	0.190
832	A	5	3	1.00	14	0.214
833	A	6	3	1.00	17	0.176
834	A	7	4	1.00	19	0.210
835	A	8	5	1.00	19	0.263
836	A	3	3	1.00	27	0.111
837	A	5	3	1.00	21	0.143
838	A	4	3	1.00	20	0.150
839	A	4	3	1.00	20	0.150
840	A	6	4	1.00	16	0.250
841	A	6	4	1.00	16	0.250
842	A	6	4	1.00	16	0.250
843	A	8	5	1.00	18	0.278
844	A	10	6	1.00	18	0.333
845	A	5	5	1.00	16	0.312
846	A	6	6	1.00	18	0.333
847	A	7	7	1.00	18	0.389
848	A	10	10	1.00	16	0.625
849	A	17	14	1.00	18	0.778
850	A	21	13	1.00	18	0.722
851	A	12	11	1.00	16	0.688
852	A	16	13	1.00	18	0.722

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
853	A	21	17	1.00	18	0.944
854	A	4	4	1.00	18	0.222
855	A	3	3	1.00	18	0.167
856	A	2	2	1.00	18	0.111
857	A	3	2	1.00	16	0.125
858	A	4	4	1.00	18	0.222
859	A	6	6	1.00	18	0.333
860	A	7	7	1.00	18	0.389
861	A	8	8	1.00	20	0.400
862	A	7	7	1.00	20	0.350
863	A	3	3	1.00	20	0.150
864	A	3	3	1.00	20	0.150
865	A	5	5	1.00	20	0.250
866	A	8	8	1.00	20	0.400
867	A	13	8	1.00	14	0.571
868	A	11	7	1.00	14	0.500
869	A	9	6	1.00	12	0.500
870	N/A	0	0	1.00	14	0.000
871	A	2	2	1.00	18	0.111
872	A	4	3	1.00	18	0.167
873	A	5	4	1.00	18	0.222
874	A	3	2	1.00	16	0.125
875	A	4	4	1.00	16	0.250
876	A	5	4	1.00	18	0.222
877	A	5	4	1.00	18	0.222
878	A	2	2	1.00	16	0.125
879	A	2	2	1.00	16	0.125
880	A	1	1	1.00	14	0.071
881	A	1	1	1.00	14	0.071
882	A	1	1	1.00	16	0.062
883	A	2	2	1.00	16	0.125
884	A	2	2	1.00	18	0.111
885	A	2	2	1.00	16	0.125
886	A	2	2	1.00	16	0.125
887	A	1	1	1.00	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
888	A	1	1	1.00	14	0.071
889	A	1	1	1.00	16	0.062
890	A	2	2	1.00	16	0.125
891	A	2	2	1.00	18	0.111
892	A	2	2	1.00	18	0.111
893	A	8	6	1.00	25	0.240
894	A	6	5	1.00	23	0.217
895	A	2	2	1.00	25	0.080
896	A	3	3	1.00	25	0.120
897	A	8	6	1.00	22	0.273
898	A	6	5	1.00	20	0.250
899	A	2	2	1.00	22	0.091
900	A	3	3	1.00	22	0.136
901	A	4	3	1.00	22	0.136
902	A	5	4	1.00	22	0.182
903	A	4	3	1.00	20	0.150
904	A	3	3	1.00	14	0.214
905	A	5	4	1.00	20	0.200
906	A	5	5	1.00	22	0.227
907	A	5	4	1.00	24	0.167
908	A	4	3	1.00	24	0.125
909	A	5	4	1.00	22	0.182
910	A	5	4	1.00	20	0.200
911	A	5	4	1.00	16	0.250
912	A	5	4	1.00	22	0.182
913	A	4	3	1.00	24	0.125
914	A	5	4	1.00	24	0.167
915	A	4	3	1.00	22	0.136
916	A	5	4	1.00	22	0.182
917	A	5	4	1.00	22	0.182
918	A	7	6	1.00	16	0.375
919	A	5	4	1.00	24	0.167
920	A	4	3	1.00	24	0.125
921	A	4	3	1.00	22	0.136
922	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
923	A	5	5	1.00	22	0.227
924	A	5	4	1.00	24	0.167
925	A	4	3	1.00	26	0.115
926	A	4	3	1.00	26	0.115
927	A	4	3	1.00	24	0.125
928	A	5	4	1.00	22	0.182
929	A	4	3	1.00	18	0.167
930	A	6	6	1.00	24	0.250
931	A	5	4	1.00	26	0.154
932	A	4	3	1.00	26	0.115
933	A	5	4	1.00	24	0.167
934	A	5	4	1.00	24	0.167
935	A	6	5	1.00	24	0.208
936	A	4	3	1.00	18	0.167
937	A	12	9	1.00	12	0.750
938	A	13	10	1.00	14	0.714
939	A	13	10	1.00	14	0.714
940	A	14	11	1.00	16	0.688
941	A	6	6	1.00	12	0.500
942	A	7	7	1.00	14	0.500
943	A	7	7	1.00	14	0.500
944	A	8	8	1.00	16	0.500
945	A	4	2	1.00	22	0.091
946	A	4	2	1.00	22	0.091
947	A	3	3	1.00	20	0.150
948	A	1	1	1.00	14	0.071
949	A	4	3	1.00	14	0.214
950	A	4	2	1.00	20	0.100
951	A	4	2	1.00	22	0.091
952	A	5	2	1.00	24	0.083
953	A	4	3	1.00	24	0.125
954	A	4	2	1.00	22	0.091
955	A	2	2	1.00	16	0.125
956	A	6	4	1.00	20	0.200
957	A	5	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
958	A	5	2	1.00	22	0.091
959	A	4	2	1.00	24	0.083
960	A	5	2	1.00	24	0.083
961	A	4	2	1.00	22	0.091
962	A	2	2	1.00	16	0.125
963	A	8	4	1.00	22	0.182
964	A	7	4	1.00	22	0.182
965	A	6	3	1.00	16	0.188
966	A	10	5	1.00	56	0.089
967	A	2	2	1.00	17	0.118
968	A	2	2	1.00	22	0.091
969	A	2	2	1.00	22	0.091
970	A	2	2	1.00	17	0.118
971	A	2	2	1.00	22	0.091
972	A	2	2	1.00	22	0.091
973	A	2	2	1.00	17	0.118
974	A	2	2	1.00	22	0.091
975	A	2	2	1.00	22	0.091
976	A	2	2	1.00	17	0.118
977	A	2	2	1.00	22	0.091
978	A	2	2	1.00	22	0.091
979	A	2	2	1.00	13	0.154
980	A	2	2	1.00	13	0.154
981	A	2	2	1.00	13	0.154
982	A	2	2	1.00	13	0.154
983	A	3	1	1.00	17	0.059
984	A	4	3	1.00	23	0.130
985	A	3	3	1.00	17	0.176
986	A	3	2	1.00	16	0.125
987	A	3	2	1.00	18	0.111
988	A	7	7	1.00	15	0.467
989	A	3	3	1.00	19	0.158
990	A	3	2	1.00	19	0.105
991	A	3	2	1.00	21	0.095
992	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
993	A	2	2	1.00	17	0.118
994	A	4	3	1.00	19	0.158
995	A	5	5	1.00	19	0.263
996	A	2	2	1.00	11	0.182
997	A	3	2	1.00	17	0.118
998	A	2	2	1.00	17	0.118
999	A	2	2	1.00	17	0.118
1000	A	3	3	1.00	15	0.200
1001	A	3	3	1.00	15	0.200
1002	A	3	3	1.00	15	0.200
1003	A	4	3	1.00	15	0.200
1004	A	4	3	1.00	20	0.150
1005	A	4	3	1.00	19	0.158
1006	A	4	3	1.00	24	0.125
1007	A	4	3	1.00	21	0.143
1008	A	4	3	1.00	20	0.150
1009	A	4	3	1.00	19	0.158
1010	A	4	3	1.00	24	0.125
1011	A	4	3	1.00	21	0.143
1012	A	1	3	1.00	8	0.375
1013	A	1	2	1.00	8	0.250
1014	N/A	0	0	1.00	18	0.000
1015	N/A	0	0	1.00	18	0.000
1016	N/A	0	0	1.00	20	0.000
1017	N/A	0	0	1.00	20	0.000
1018	A	3	2	1.00	13	0.154
1019	A	2	2	1.00	13	0.154
1020	A	2	2	1.00	13	0.154
1021	A	2	2	1.00	11	0.182
1022	A	3	2	1.00	19	0.105
1023	A	3	2	1.00	19	0.105
1024	A	3	2	1.00	21	0.095
1025	A	3	2	1.00	21	0.095
1026	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1027	A	4	3	1.00	17	0.176
1028	A	2	2	1.00	17	0.118
1029	A	3	3	1.00	16	0.188
1030	A	3	3	1.00	18	0.167
1031	A	1	1	1.00	18	0.056
1032	A	3	3	1.00	18	0.167
1033	A	9	7	1.31	15	0.467
1034	A	8	7	1.31	15	0.467
1035	A	4	2	1.00	15	0.133
1036	A	4	2	1.00	15	0.133
1037	A	3	3	1.00	21	0.143
1038	A	3	3	1.00	21	0.143
1039	A	12	7	1.00	8	0.875
1040	A	5	4	1.00	10	0.400
1041	A	19	6	1.00	10	0.600
1042	A	8	5	1.00	18	0.278
1043	A	6	5	1.00	8	0.625
1044	A	5	3	1.00	15	0.200
1045	A	7	4	1.00	22	0.182
1046	A	4	3	1.00	12	0.250
1047	A	6	5	1.00	15	0.333
1048	A	5	4	1.00	15	0.267
1049	A	3	2	1.00	13	0.154
1050	A	4	4	1.00	23	0.174
1051	A	4	4	1.00	25	0.160
1052	A	6	3	1.00	39	0.077
1053	A	5	3	1.00	39	0.077
1054	A	3	2	1.00	39	0.051
1055	A	1	1	1.00	31	0.032
1056	A	2	1	1.00	31	0.032
1057	A	2	1	1.00	39	0.026
1058	A	5	3	1.00	39	0.077
1059	A	6	3	1.00	39	0.077

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{2}{-1+3\cosh(4+6x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 2738, 212}

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[In] Int[2/(-1 + 3*Cosh[4 + 6*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{-1+3\cosh(4+6x)}\,dx \\ &= -\left(\frac{2}{3}i\text{Subst}\left(\int \frac{1}{2-4x^2}\,dx, x, \tan\left(\frac{1}{2}(4i+6ix)\right)\right)\right) \\ &= \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[In] Integrate[2/(-1 + 3*Cosh[4 + 6*x]),x]
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{2}\,\tanh(2+3x)\right)\sqrt{2}}{6}$	17
default	$\frac{\arctan\left(\sqrt{2}\,\tanh(2+3x)\right)\sqrt{2}}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44

```
[In] int(2/(-1+3*cosh(4+6*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{1}{6}\,\sqrt{2}\arctan\left(\frac{3}{4}\,\sqrt{2}\cosh\left(6\,x+4\right)+\frac{3}{4}\,\sqrt{2}\sinh\left(6\,x+4\right)-\frac{1}{4}\,\sqrt{2}\right)$$

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="fricas")
[Out] 1/6*sqrt(2)*arctan(3/4*sqrt(2)*cosh(6*x + 4) + 3/4*sqrt(2)*sinh(6*x + 4) -
1/4*sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tanh\left(3x+2\right)\right)}{6}$$

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x)
[Out] sqrt(2)*atan(sqrt(2)*tanh(3*x + 2))/6
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = -\frac{1}{6}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\left(3\,e^{(-6\,x-4)}-1\right)\right)$$

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3\,e^{(6\,x+4)}-1\right)\right)$$

```
[In] integrate(2/(-1+3*cosh(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{2}{-1+3\cosh(4+6x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

```
[In] int(2/(3*cosh(6*x + 4) - 1),x)
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.2 $\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {209}

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[In] Int[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tanh(2+3x)\right)\\&=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[In] Integrate[(Cosh[2 + 3*x]^2 + 2*Sinh[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92
default	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92

```
[In] int(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*I*2^(1/2)*ln(exp(4+6*x)-1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*x
)-1/3-2/3*I*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(3x+2)+2\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="fricas")
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(19) = 38$.

Time = 3.54 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.41

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{2093258\sqrt{5-2\sqrt{6}}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{1152360\sqrt{6}+2822694} + \frac{854569\sqrt{6}\sqrt{5-2\sqrt{6}}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{5-2\sqrt{6}}}\right)}{1152360\sqrt{6}+2822694} - \frac{86329\sqrt{6}\sqrt{2\sqrt{6}+5}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{6}+5}}\right)}{1152360\sqrt{6}+2822694} - \frac{211462\sqrt{2\sqrt{6}+5}\operatorname{atan}\left(\frac{\tanh\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{6}+5}}\right)}{1152360\sqrt{6}+2822694}$$

```
[In] integrate(1/(cosh(2+3*x)**2+2*sinh(2+3*x)**2),x)
```

```
[Out] 2093258*sqrt(5 - 2*sqrt(6))*atan(tanh(3*x/2 + 1)/sqrt(5 - 2*sqrt(6)))/(1152
360*sqrt(6) + 2822694) + 854569*sqrt(6)*sqrt(5 - 2*sqrt(6))*atan(tanh(3*x/2
 + 1)/sqrt(5 - 2*sqrt(6)))/(1152360*sqrt(6) + 2822694) - 86329*sqrt(6)*sqrt
(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6) + 5))/(1152360*sqrt(6)
+ 2822694) - 211462*sqrt(2*sqrt(6) + 5)*atan(tanh(3*x/2 + 1)/sqrt(2*sqrt(6)
 + 5))/(1152360*sqrt(6) + 2822694)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{1}{6}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\big(3\,e^{(6\,x+4)}-1\big)\right)$$

```
[In] integrate(1/(cosh(2+3*x)^2+2*sinh(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cosh^2(2+3x)+2\sinh^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

```
[In] int(1/(2*sinh(3*x + 2)^2 + cosh(3*x + 2)^2),x)
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.3 $\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 209}

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[In] Int[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tanh(2+3x)\right)\\ &= \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[In] Integrate[Sech[2 + 3*x]^2/(1 + 2*Tanh[2 + 3*x]^2),x]
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92
default	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92

```
[In] int(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*I*2^(1/2)*ln(exp(4+6*x)-1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*x
)-1/3-2/3*I*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(3x+2)+2\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="fricas")
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \int \frac{\operatorname{sech}^2(3x+2)}{2\tanh^2(3x+2)+1}\,dx$$

```
[In] integrate(sech(2+3*x)**2/(1+2*tanh(2+3*x)**2),x)
[Out] Integral(sech(3*x + 2)**2/(2*tanh(3*x + 2)**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}-1\right)\right)$$

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

```
[In] integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)}\,dx=\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\left(3\,e^{6\,x+4}-1\right)}{4}\right)}{6}$$

```
[In] int(1/(cosh(3*x + 2)^2*(2*tanh(3*x + 2)^2 + 1)),x)
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.4 $\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)} \, dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3756, 209}

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)} \, dx = \frac{\arctan\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[In] Int[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \coth(2+3x)\right)\right)\\ &= \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[In] Integrate[Csch[2 + 3*x]^2/(2 + Coth[2 + 3*x]^2),x]
[Out] ArcTan[Sqrt[2]*Tanh[2 + 3*x]]/(3*Sqrt[2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92
default	$-\frac{\left(3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}+2\sqrt{2}}\right)}{9\left(2\sqrt{3}+2\sqrt{2}\right)}-\frac{\left(-3+\sqrt{6}\right)\sqrt{6}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{2\sqrt{3}-2\sqrt{2}}\right)}{9\left(2\sqrt{3}-2\sqrt{2}\right)}$	92

```
[In] int(csch(2+3*x)^2/(2+coth(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*I*2^(1/2)*ln(exp(4+6*x)-1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*x
)-1/3-2/3*I*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = -\frac{1}{6}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh\left(3\,x+2\right)+2\,\sqrt{2}\sinh\left(3\,x+2\right)}{2\left(\cosh\left(3\,x+2\right)-\sinh\left(3\,x+2\right)\right)}\right)$$

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="fricas")
[Out] -1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*cosh(3*x + 2) + 2*sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = \int \frac{\operatorname{csch}^2\left(3x+2\right)}{\coth^2\left(3x+2\right)+2}\,dx$$

```
[In] integrate(csch(2+3*x)**2/(2+coth(2+3*x)**2),x)
[Out] Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 + 2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = -\frac{1}{6}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\left(3\,e^{(-6\,x-4)}-1\right)\right)$$

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx = \frac{1}{6}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\left(3\,e^{(6\,x+4)}-1\right)\right)$$

```
[In] integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\coth^2(2+3x)}\,dx=\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\left(3\,\mathrm{e}^{6\,x+4}-1\right)}{4}\right)}{6}$$

```
[In] int(1/(sinh(3*x + 2)^2*(coth(3*x + 2)^2 + 2)),x)
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) - 1))/4))/6
```

3.5 $\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[Out] -1/6*arctanh(2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 212}

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(3x+2)\right)}{3\sqrt{2}}$$

```
[In] Int[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tanh[2 + 3*x]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= -\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(2+3x)\right)}{3\sqrt{2}}$$

```
[In] Integrate[Csch[2 + 3*x]^2/(2 - Coth[2 + 3*x]^2),x]
[Out] -1/3*ArcTanh[Sqrt[2]*Tanh[2 + 3*x]]/Sqrt[2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(e^{4+6x}-3-2\sqrt{2}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{4+6x}-3+2\sqrt{2}\right)}{12}$	40
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{6}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{6}$	44
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{6}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(1+\frac{3x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{6}$	44

```
[In] int(csch(2+3*x)^2/(2-coth(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*2^(1/2)*ln(exp(4+6*x)-3-2*2^(1/2))-1/12*2^(1/2)*ln(exp(4+6*x)-3+2*2^(1
/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^2-4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right)+3\left(2\sqrt{2}+3\right)\sinh\left(\right.}{\cosh\left(3x+2\right)^2+\sinh\left(3x+2\right)^2-3}\right.$$

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*cos
h(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 - 2*sqrt(2) -
3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 - 3))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\int \frac{\operatorname{csch}^2(3x+2)}{\coth^2(3x+2)-2}\,dx$$

```
[In] integrate(csch(2+3*x)**2/(2-coth(2+3*x)**2),x)
```

```
[Out] -Integral(csch(3*x + 2)**2/(coth(3*x + 2)**2 - 2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}+1}{\sqrt{2}+e^{(-3x-2)}-1}\right) + \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-3x-2)}-1}{\sqrt{2}+e^{(-3x-2)}+1}\right)$$

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) + 1)/(sqrt(2) + e^(-3*x - 2) - 1
)) + 1/12*sqrt(2)*log(-(sqrt(2) - e^(-3*x - 2) - 1)/(sqrt(2) + e^(-3*x - 2)
 + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}e^4 - 6\,e^4 + 2\,e^{(6\,x+8)}\right|}{\left|4\sqrt{2}e^4 - 6\,e^4 + 2\,e^{(6\,x+8)}\right|}\right)$$

```
[In] integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-4*sqrt(2)*e^4 - 6*e^4 + 2*e^(6*x + 8))/abs(4*sqrt(2)*
e^4 - 6*e^4 + 2*e^(6*x + 8)))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\coth^2(2+3x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\left(3\,e^{6\,x+4}-1\right)-4\,e^{6\,x+4}\right)-\ln\left(-4\,e^{6\,x+4}-\sqrt{2}\left(3\,e^{6\,x+4}-1\right)\right)\right)}{12}$$

```
[In] int(-1/(sinh(3*x + 2)^2*(coth(3*x + 2)^2 - 2)),x)
```

```
[Out] (2^(1/2)*(log(2^(1/2)*(3*exp(6*x + 4) - 1) - 4*exp(6*x + 4)) - log(- 4*exp(
6*x + 4) - 2^(1/2)*(3*exp(6*x + 4) - 1))))/12
```

3.6 $\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx=\frac{\arctan\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] 1/6*arctan(1/2*2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 209}

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx=\frac{\arctan\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[In] Int[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \coth(2+3x)\right)\right)\\ &= \frac{\arctan\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = \frac{\arctan\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[In] Integrate[Csch[2 + 3*x]^2/(1 + 2*Coth[2 + 3*x]^2),x]
[Out] ArcTan[Tanh[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4+6x}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	44
derivativedivides	$-\frac{\sqrt{3}\left(3+\sqrt{3}\right)\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}+\sqrt{2}}\right)}{9\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)}{9\left(\sqrt{6}-\sqrt{2}\right)}$	80
default	$-\frac{\sqrt{3}\left(3+\sqrt{3}\right)\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}+\sqrt{2}}\right)}{9\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2\tanh\left(1+\frac{3x}{2}\right)}{\sqrt{6}-\sqrt{2}}\right)}{9\left(\sqrt{6}-\sqrt{2}\right)}$	80

```
[In] int(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*I*2^(1/2)*ln(exp(4+6*x)+1/3+2/3*I*2^(1/2))-1/12*I*2^(1/2)*ln(exp(4+6*x
)+1/3-2/3*I*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(-\frac{2\sqrt{2}\cosh(3x+2)+\sqrt{2}\sinh(3x+2)}{2(\cosh(3x+2)-\sinh(3x+2))}\right)$$

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="fricas")
[Out] -1/6*sqrt(2)*arctan(-1/2*(2*sqrt(2)*cosh(3*x + 2) + sqrt(2)*sinh(3*x + 2))/
(cosh(3*x + 2) - sinh(3*x + 2)))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = \int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)+1}\,dx$$

```
[In] integrate(csch(2+3*x)**2/(1+2*coth(2+3*x)**2),x)
[Out] Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(-6x-4)}+1\right)\right)$$

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(-6*x - 4) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}+1\right)\right)$$

```
[In] integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/4*sqrt(2)*(3*e^(6*x + 4) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\coth^2(2+3x)}\,dx=\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\left(3\,{\mathrm{e}}^{6\,x+4}+1\right)}{4}\right)}{6}$$

```
[In] int(1/(sinh(3*x + 2)^2*(2*coth(3*x + 2)^2 + 1)),x)
[Out] (2^(1/2)*atan((2^(1/2)*(3*exp(6*x + 4) + 1))/4))/6
```

3.7 $\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] -1/6*arctanh(1/2*2^(1/2)*tanh(2+3*x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 212}

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tanh(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[In] Int[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Tanh[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(2+3x)\right)\right)$$
$$= -\frac{\operatorname{arctanh}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tanh(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[In] Integrate[Csch[2 + 3*x]^2/(1 - 2*Coth[2 + 3*x]^2),x]
[Out] -1/3*ArcTanh[Tanh[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.61 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e^{4+6x}+3+2\sqrt{2}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{4+6x}+3-2\sqrt{2}\right)}{12}$
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(1+\frac{3x}{2}\right)^2+\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}{\tanh\left(1+\frac{3x}{2}\right)^2-\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}-1\right)\right)}{24}+\frac{\sqrt{2}\,(}{}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(1+\frac{3x}{2}\right)^2+\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}{\tanh\left(1+\frac{3x}{2}\right)^2-\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(1+\frac{3x}{2}\right)\sqrt{2}-1\right)\right)}{24}+\frac{\sqrt{2}\,(}{}$

```
[In] int(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/12*2^(1/2)*ln(exp(4+6*x)+3+2*2^(1/2))-1/12*2^(1/2)*ln(exp(4+6*x)+3-2*2^(1
/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(3x+2\right)^2-4\left(3\sqrt{2}+4\right)\cosh\left(3x+2\right)\sinh\left(3x+2\right)+3\left(2\sqrt{2}+3\right)\sinh(}{\cosh\left(3x+2\right)^2+\sinh\left(3x+2\right)^2+3}\right.$$

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(3*x + 2)^2 - 4*(3*sqrt(2) + 4)*cos
h(3*x + 2)*sinh(3*x + 2) + 3*(2*sqrt(2) + 3)*sinh(3*x + 2)^2 + 2*sqrt(2) +
3)/(cosh(3*x + 2)^2 + sinh(3*x + 2)^2 + 3))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\int \frac{\operatorname{csch}^2(3x+2)}{2\coth^2(3x+2)-1}\,dx$$

```
[In] integrate(csch(2+3*x)**2/(1-2*coth(2+3*x)**2),x)
```

```
[Out] -Integral(csch(3*x + 2)**2/(2*coth(3*x + 2)**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-6x-4)}-3}{2\sqrt{2}+e^{(-6x-4)}+3}\right)$$

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(2*sqrt(2) - e^(-6*x - 4) - 3)/(2*sqrt(2) + e^(-6*x - 4)
+ 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}e^4 - 3e^4 - e^{(6x+8)}}{2\sqrt{2}e^4 + 3e^4 + e^{(6x+8)}}\right)$$

```
[In] integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(-(2*sqrt(2)*e^4 - 3*e^4 - e^(6*x + 8))/(2*sqrt(2)*e^4 + 3
*e^4 + e^(6*x + 8)))
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\coth^2(2+3x)}\,dx = -\frac{\sqrt{2}\left(\ln\left(4\,\mathrm{e}^{6x+4}+\sqrt{2}\left(3\,\mathrm{e}^{6x+4}+1\right)\right)-\ln\left(4\,\mathrm{e}^{6x+4}-\sqrt{2}\left(3\,\mathrm{e}^{6x+4}+1\right)\right)\right)}{12}$$

```
[In] int(-1/(sinh(3*x + 2)^2*(2*coth(3*x + 2)^2 - 1)),x)
```

```
[Out] -(2^(1/2)*(log(4*exp(6*x + 4) + 2^(1/2)*(3*exp(6*x + 4) + 1)) - log(4*exp(6
*x + 4) - 2^(1/2)*(3*exp(6*x + 4) + 1))))/12
```

3.8 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{\sinh^2(a+bx)}{2b}$$

```
[Out] 1/2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2644, 30}

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{\sinh^2(a+bx)}{2b}$$

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}(\int x\,dx, x, i\sinh(a+bx))}{b}$$
$$= \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{1}{2}\left(\frac{\cosh(2a)\cosh(2bx)}{2b} + \frac{\sinh(2a)\sinh(2bx)}{2b}\right)$$

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] ((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^2}{2b}$	14
default	$\frac{\cosh(bx+a)^2}{2b}$	14
risch	$\frac{e^{2bx+2a}}{8b} + \frac{e^{-2bx-2a}}{8b}$	30

```
[In] int(cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*cosh(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2 + \sinh(bx+a)^2}{4\,b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((sinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/2*cosh(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^2}{2\,b}$$

```
[In] int(cosh(a + b*x)*sinh(a + b*x),x)
[Out] cosh(a + b*x)^2/(2*b)
```

3.9 $\int \cosh(a+bx)\sinh^n(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{1+n}(a+bx)}{b(1+n)}$$

```
[Out] sinh(b*x+a)^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2644, 30}

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{n+1}(a+bx)}{b(n+1)}$$

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^n \, dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^{1+n}(a+bx)}{b(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{1+n}(a+bx)}{b(1+n)}$$

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^n,x]
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 6.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^{n+1}}{b(n+1)}$	20
default	$\frac{\sinh(bx+a)^{n+1}}{b(n+1)}$	20

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] sinh(b*x+a)^(n+1)/b/(n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\cosh\left(n\log\left(\sinh\left(bx+a\right)\right)\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)\sinh\left(n\log\left(\sinh\left(bx+a\right)\right)\right)}{(bn+b)\cosh\left(bx+a\right)^2-(bn+b)\sinh\left(bx+a\right)^2}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="fricas")
[Out] (cosh(n*log(sinh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sinh(n*log(sinh(b
*x + a))))/((b*n + b)*cosh(b*x + a)^2 - (b*n + b)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \begin{cases} \frac{x\cosh(a)}{\sinh(a)} & \text{for } b=0 \wedge n=-1 \\ x\sinh^n(a)\cosh(a) & \text{for } b=0 \\ \frac{\log(\sinh(a+bx))}{b} & \text{for } n=-1 \\ \frac{\sinh(a+bx)\sinh^n(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*cosh(a)/sinh(a), Eq(b, 0) & Eq(n, -1)), (x*sinh(a)**n*cosh(a),
 Eq(b, 0)), (log(sinh(a + b*x))/b, Eq(n, -1)), (sinh(a + b*x)*sinh(a + b*x)
**n/(b*n + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh(bx+a)^{n+1}}{b(n+1)}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] sinh(b*x + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx = \frac{\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)^{n+1}}{b(n+1)}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh(a+b\,x)^{n+1}}{b\,\left(n+1\right)}$$

```
[In] int(cosh(a + b*x)*sinh(a + b*x)^n,x)
[Out] sinh(a + b*x)^(n + 1)/(b*(n + 1))
```

3.10 $\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{1+n}(a+bx)}{b(1+n)} + \frac{\sinh^{3+n}(a+bx)}{b(3+n)}$$

```
[Out] sinh(b*x+a)^(1+n)/b/(1+n)+sinh(b*x+a)^(3+n)/b/(3+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 14}

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{n+1}(a+bx)}{b(n+1)} + \frac{\sinh^{n+3}(a+bx)}{b(n+3)}$$

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^n(1+x^2)\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^n+x^{2+n})\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^{1+n}(a+bx)}{b(1+n)}+\frac{\sinh^{3+n}(a+bx)}{b(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{1+n}(a+bx)}{b(1+n)}+\frac{\sinh^{3+n}(a+bx)}{b(3+n)}$$

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^n,x]
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + Sinh[a + b*x]^(3 + n)/(b*(3 + n))
```

Maple [A] (verified)

Time = 197.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^3 \mathrm{e}^{n\ln(\sinh(bx+a))}}{b(3+n)}+\frac{\sinh(bx+a)\mathrm{e}^{n\ln(\sinh(bx+a))}}{b(n+1)}$	54
default	$\frac{\sinh(bx+a)^3 \mathrm{e}^{n\ln(\sinh(bx+a))}}{b(3+n)}+\frac{\sinh(bx+a)\mathrm{e}^{n\ln(\sinh(bx+a))}}{b(n+1)}$	54

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] 1/b/(3+n)*sinh(b*x+a)^3*exp(n*ln(sinh(b*x+a)))+1/b/(n+1)*sinh(b*x+a)*exp(n*
ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.49

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$$
$$= \frac{\left((n+1)\sinh\left(bx+a\right)^3 + \left(3\left(n+1\right)\cosh\left(bx+a\right)^2 + n + 9\right)\sinh\left(bx+a\right)\right)\cosh\left(n\log\left(\sinh\left(bx+a\right)\right)\right) + ((}{4\left(\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx+a\right)^4 - 2\left(bn^2+4\,bn+3\,b\right)\cosh\left(bx\right.\right.}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 1/4*(((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*cosh(b*x + a)^2 + n + 9)*sinh(b*
x + a))*cosh(n*log(sinh(b*x + a))) + ((n + 1)*sinh(b*x + a)^3 + (3*(n + 1)*
cosh(b*x + a)^2 + n + 9)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^2
 + 4*b*n + 3*b)*cosh(b*x + a)^4 - 2*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)^2*s
inh(b*x + a)^2 + (b*n^2 + 4*b*n + 3*b)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(29) = 58$.

Time = 1.21 (sec) , antiderivative size = 638, normalized size of antiderivative = 16.36

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$$
$$= \begin{cases} x\sinh^n(a)\cosh^3(a) \\ \frac{\log\left(\sinh\left(a+bx\right)\right)}{b} - \frac{\cosh^2\left(a+bx\right)}{2b\sinh^2\left(a+bx\right)} \\ \frac{bx\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2bx\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{bx}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2\log\left(\tanh\right.}{b\tanh^4\left(\frac{a}{2}\right.} \\ \frac{n\sinh\left(a+bx\right)\sinh^n\left(a+bx\right)\cosh^2\left(a+bx\right)}{bn^2+4bn+3b} - \frac{2\sinh^3\left(a+bx\right)\sinh^n\left(a+bx\right)}{bn^2+4bn+3b} + \frac{3\sinh\left(a+bx\right)\sinh^n\left(a+bx\right)\cosh^2\left(a+bx\right)}{bn^2+4bn+3b} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*sinh(a)**n*cosh(a)**3, Eq(b, 0)), (log(sinh(a + b*x))/b - cosh
(a + b*x)**2/(2*b*sinh(a + b*x)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**4/
(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*b*x*tanh(a/2 +
b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + b*x/(b*
tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x
/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/
2)**2 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2
+ b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)/
(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*
```

```
x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)*
*2 + b) - 2*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2
)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(b*tanh(a/2 +
 b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*tanh(a/2 + b*x/2)**2/(b*tanh
(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)), (n*sinh(a + b
*x)*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b) - 2*sinh(a + b
*x)**3*sinh(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) + 3*sinh(a + b*x)*sinh(a + b
*x)**n*cosh(a + b*x)**2/(b*n**2 + 4*b*n + 3*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(39) = 78$.

Time = 0.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 9.56

$$\begin{aligned}
\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx
&= \frac{ne^{\left((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} \\
&\quad + \frac{(n+9)e^{\left((bx+a)n+bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+a\right)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} \\
&\quad - \frac{(n+9)e^{\left((bx+a)n-bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-a\right)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} \\
&\quad - \frac{(n+1)e^{\left((bx+a)n-3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-3\,a\right)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b} \\
&\quad + \frac{e^{\left((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{8\,(2^n n^2+2^{n+2}n+3\cdot 2^n)b}
\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 1/8*n*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a
) + 1) + 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*(n + 9)*e^((b*x + a
)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + a)/((2^n*n
^2 + 2^(n + 2)*n + 3*2^n)*b) - 1/8*(n + 9)*e^((b*x + a)*n - b*x + n*log(e^(
-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) - a)/((2^n*n^2 + 2^(n + 2)*n + 3*
2^n)*b) - 1/8*(n + 1)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*
log(-e^(-b*x - a) + 1) - 3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b) + 1/8*e^(
(b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) +
3*a)/((2^n*n^2 + 2^(n + 2)*n + 3*2^n)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(39) = 78$.

Time = 0.33 (sec) , antiderivative size = 327, normalized size of antiderivative = 8.38

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx$$
$$= \frac{ne^{\left(7\,bx+n\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+7\,a\right)}+ne^{\left(5\,bx+n\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)+5\,a\right)}-ne^{\left(3\,bx+n\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-b}\right.\right.}}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] 1/8*(n*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) + n*
e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - n*e^(3*b*
x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - n*e^(b*x + n*log
(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + e^(7*b*x + n*log(1/2*(e^(2*
b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) + 9*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2
*a) - 1)*e^(-b*x - a)) + 5*a) - 9*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1
)*e^(-b*x - a)) + 3*a) - e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x -
 a)) + a))/(b*n^2*e^(4*b*x + 4*a) + 4*b*n*e^(4*b*x + 4*a) + 3*b*e^(4*b*x +
4*a))
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.46

$$\int \cosh^3(a+bx)\sinh^n(a+bx)\,dx = -\left(\frac{1}{2}\right)^n \mathrm{e}^{-3\,a-3\,b\,x}\,\left(\mathrm{e}^{a+b\,x}-\mathrm{e}^{-a-b\,x}\right)^n\,\left(\frac{\frac{n}{8}+\frac{1}{8}}{b\,\left(n^2+4\,n+3\right)} + \frac{\mathrm{e}^{2\,a+2\,b\,x}\,\left(n+9\right)}{8\,b\,\left(n^2+4\,n+3\right)} - \frac{\mathrm{e}^{6\,a+6\,b\,x}\,\left(n+1\right)}{8\,b\,\left(n^2+4\,n+3\right)} - \frac{\mathrm{e}^{4\,a+4\,b\,x}\,\left(n+9\right)}{8\,b\,\left(n^2+4\,n+3\right)}\right)$$

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x)^n,x)
```

```
[Out] -(1/2)^n*exp(- 3*a - 3*b*x)*(exp(a + b*x) - exp(- a - b*x))^n*((n/8 + 1/8)/
(b*(4*n + n^2 + 3)) + (exp(2*a + 2*b*x)*(n + 9))/(8*b*(4*n + n^2 + 3)) - (e
xp(6*a + 6*b*x)*(n + 1))/(8*b*(4*n + n^2 + 3)) - (exp(4*a + 4*b*x)*(n + 9))
/(8*b*(4*n + n^2 + 3)))
```

3.11 $\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{1+n}(a+bx)}{b(1+n)}+\frac{2\sinh^{3+n}(a+bx)}{b(3+n)}+\frac{\sinh^{5+n}(a+bx)}{b(5+n)}$$

```
[Out] sinh(b*x+a)^(1+n)/b/(1+n)+2*sinh(b*x+a)^(3+n)/b/(3+n)+sinh(b*x+a)^(5+n)/b/(
5+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 276}

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx=\frac{\sinh^{n+1}(a+bx)}{b(n+1)}+\frac{2\sinh^{n+3}(a+bx)}{b(n+3)}+\frac{\sinh^{n+5}(a+bx)}{b(n+5)}$$

```
[In] Int[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
```

```
[Out] Sinh[a + b*x]^(1 + n)/(b*(1 + n)) + (2*Sinh[a + b*x]^(3 + n))/(b*(3 + n)) +
 Sinh[a + b*x]^(5 + n)/(b*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^n(1+x^2)^2\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^n+2x^{2+n}+x^{4+n}\right)\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{\sinh^{1+n}(a+bx)}{b(1+n)}+\frac{2\sinh^{3+n}(a+bx)}{b(3+n)}+\frac{\sinh^{5+n}(a+bx)}{b(5+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \frac{\sinh^{1+n}(a+bx)\left(\frac{1}{1+n}+\frac{2\sinh^2(a+bx)}{3+n}+\frac{\sinh^4(a+bx)}{5+n}\right)}{b}$$

```
[In] Integrate[Cosh[a + b*x]^5*Sinh[a + b*x]^n,x]
[Out] (Sinh[a + b*x]^(1 + n)*((1 + n)^(-1) + (2*Sinh[a + b*x]^2)/(3 + n) + Sinh[a
 + b*x]^4/(5 + n)))/b
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\frac{\sinh\left(bx+a\right)^5 e^{n\ln(\sinh(bx+a))}}{b\left(5+n\right)}+\frac{\sinh\left(bx+a\right) e^{n\ln(\sinh(bx+a))}}{b\left(n+1\right)}+\frac{2\sinh\left(bx+a\right)^3 e^{n\ln(\sinh(bx+a))}}{b\left(3+n\right)}$$

```
[In] int(cosh(b*x+a)^5*sinh(b*x+a)^n,x)
[Out] 1/b/(5+n)*sinh(b*x+a)^5*exp(n*ln(sinh(b*x+a)))+1/b/(n+1)*sinh(b*x+a)*exp(n*
ln(sinh(b*x+a)))+2/b/(3+n)*sinh(b*x+a)^3*exp(n*ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(59) = 118.

Time = 0.27 (sec) , antiderivative size = 379, normalized size of antiderivative = 6.42

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx$$
$$= \frac{\left((n^2+4n+3)\sinh\left(bx+a\right)^5+\left(10\left(n^2+4n+3\right)\cosh\left(bx+a\right)^2+3n^2+28n+25\right)\sinh\left(bx+a\right)^3+\left(5\right.}{}$$

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 1/16*(((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2 + 4*n + 3)*cosh(b*x + a)^
2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2 + 4*n + 3)*cosh(b*x + a)^4
 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2 + 24*n + 150)*sinh(b*x + a
))*cosh(n*log(sinh(b*x + a))) + ((n^2 + 4*n + 3)*sinh(b*x + a)^5 + (10*(n^2
 + 4*n + 3)*cosh(b*x + a)^2 + 3*n^2 + 28*n + 25)*sinh(b*x + a)^3 + (5*(n^2
+ 4*n + 3)*cosh(b*x + a)^4 + 3*(3*n^2 + 28*n + 25)*cosh(b*x + a)^2 + 2*n^2
+ 24*n + 150)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a))))/((b*n^3 + 9*b*n^2
+ 23*b*n + 15*b)*cosh(b*x + a)^6 - 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cosh
(b*x + a)^4*sinh(b*x + a)^2 + 3*(b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cosh(b*x
+ a)^2*sinh(b*x + a)^4 - (b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*sinh(b*x + a)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2574 vs. 2(46) = 92.

Time = 4.32 (sec) , antiderivative size = 2574, normalized size of antiderivative = 43.63

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)**5*sinh(b*x+a)**n,x)
```

```
[Out] Piecewise((x*sinh(a)**n*cosh(a)**5, Eq(b, 0)), (log(sinh(a + b*x))/b - cosh
(a + b*x)**2/(2*b*sinh(a + b*x)**2) - cosh(a + b*x)**4/(4*b*sinh(a + b*x)**
4), Eq(n, -5)), (16*b*x*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16
*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*b*x*tanh(a/2 + b*x
/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2
 + b*x/2)**2) + 16*b*x*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*
b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/
2) + 1)*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*
x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 64*log(tanh(a/2 + b*x/2) + 1)*tanh(a/
2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*t
anh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(
```

8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2
)**2) + 16*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/
2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh
(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a
/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 16*log(tanh(a/2 + b*x/2))*tanh
(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*
b*tanh(a/2 + b*x/2)**2) - tanh(a/2 + b*x/2)**8/(8*b*tanh(a/2 + b*x/2)**6 -
16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 18*tanh(a/2 + b*x/2
)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 +
b*x/2)**2) - 1/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b
*tanh(a/2 + b*x/2)**2), Eq(n, -3)), (b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 +
b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh
(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8
- 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/
2)**2 + b) + 6*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(
a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b)
- 4*b*x*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2
)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + b*x/(b*ta
nh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 -
4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x
/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 +
b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*log(tanh(a/2 + b*x/2) + 1)*ta
nh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b
*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 12*log(tanh(a/2 + b
*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*
x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*log(
tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*t
anh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 +
b) - 2*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 +
b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + log
(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh
(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b)
- 4*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 -
4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)
2 + b) + 6*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)4/(b*tanh(a/2 + b*x/
2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2
+ b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(a/
2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*t
anh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(b*tanh(a/2 + b*x/2)**8 -
4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2
)**2 + b) + 4*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 +
b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*t
anh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*
b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 4*tanh(a/2 + b*x/2
)**2/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*

```
x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b), Eq(n, -1)), (n**2*sinh(a + b*x)*si
nh(a + b*x)**n*cosh(a + b*x)**4/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 4*n*s
inh(a + b*x)**3*sinh(a + b*x)**n*cosh(a + b*x)**2/(b*n**3 + 9*b*n**2 + 23*b
*n + 15*b) + 8*n*sinh(a + b*x)*sinh(a + b*x)**n*cosh(a + b*x)**4/(b*n**3 +
9*b*n**2 + 23*b*n + 15*b) + 8*sinh(a + b*x)**5*sinh(a + b*x)**n/(b*n**3 + 9
*b*n**2 + 23*b*n + 15*b) - 20*sinh(a + b*x)**3*sinh(a + b*x)**n*cosh(a + b*
x)**2/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) + 15*sinh(a + b*x)*sinh(a + b*x)*
*n*cosh(a + b*x)**4/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(59) = 118$.

Time = 0.30 (sec) , antiderivative size = 686, normalized size of antiderivative = 11.63

$$\begin{aligned}\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx &= \frac{n^2e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{32\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &+ \frac{ne^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{8\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &+ \frac{(3\,n^2+28\,n+25)e^{\left((bx+a)n+3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{32\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &+ \frac{(n^2+12\,n+75)e^{\left((bx+a)n+bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+a\right)}}{16\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &- \frac{(n^2+12\,n+75)e^{\left((bx+a)n-bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-a\right)}}{16\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &- \frac{(3\,n^2+28\,n+25)e^{\left((bx+a)n-3\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-3\,a\right)}}{32\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &- \frac{(n^2+4\,n+3)e^{\left((bx+a)n-5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-5\,a\right)}}{32\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b} \\ &+ \frac{3\,e^{\left((bx+a)n+5\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)+5\,a\right)}}{32\,(2^nn^3+9\cdot 2^nn^2+23\cdot 2^nn+15\cdot 2^n)b}\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 1/32*n^2*e^((b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
- a) + 1) + 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/8*n*e^((
b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + 5
*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/32*(3*n^2 + 28*n + 25
)*e^((b*x + a)*n + 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) +
1) + 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 1/16*(n^2 + 12*n
+ 75)*e^((b*x + a)*n + b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
```

```
+ 1) + a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/16*(n^2 + 12*n
+ 75)*e^((b*x + a)*n - b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a)
+ 1) - a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(3*n^2 + 28*
n + 25)*e^((b*x + a)*n - 3*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x -
 a) + 1) - 3*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) - 1/32*(n^2 +
 4*n + 3)*e^((b*x + a)*n - 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x
 - a) + 1) - 5*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b) + 3/32*e^((
b*x + a)*n + 5*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1) + 5
*a)/((2^n*n^3 + 9*2^n*n^2 + 23*2^n*n + 15*2^n)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(59) = 118$.

Time = 0.38 (sec) , antiderivative size = 722, normalized size of antiderivative = 12.24

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)^5*sinh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] 1/32*(n^2*e^(11*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 11*a)
 + 3*n^2*e^(9*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 9*a) +
2*n^2*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) - 2*n
^2*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 3*n^2*
e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - n^2*e^(b*
x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a) + 4*n*e^(11*b*x + n*
log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 11*a) + 28*n*e^(9*b*x + n*log
(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 9*a) + 24*n*e^(7*b*x + n*log(1/2
*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 7*a) - 24*n*e^(5*b*x + n*log(1/2*(e^
(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 5*a) - 28*n*e^(3*b*x + n*log(1/2*(e^(2*b
*x + 2*a) - 1)*e^(-b*x - a)) + 3*a) - 4*n*e^(b*x + n*log(1/2*(e^(2*b*x + 2*
a) - 1)*e^(-b*x - a)) + a) + 3*e^(11*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*
e^(-b*x - a)) + 11*a) + 25*e^(9*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b
*x - a)) + 9*a) + 150*e^(7*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x -
a)) + 7*a) - 150*e^(5*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) +
 5*a) - 25*e^(3*b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + 3*a)
- 3*e^(b*x + n*log(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) + a))/(b*n^3*e^(
6*b*x + 6*a) + 9*b*n^2*e^(6*b*x + 6*a) + 23*b*n*e^(6*b*x + 6*a) + 15*b*e^(6
*b*x + 6*a))
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.32

$$\int \cosh^5(a+bx)\sinh^n(a+bx)\,dx = -e^{-5a-5bx}\left(\frac{e^{a+bx}}{2}-\frac{e^{-a-bx}}{2}\right)^n\left(\frac{n^2+4n+3}{32b\,(n^3+9n^2+23n+15)} - \frac{e^{10a+10bx}\,(n^2+4n+3)}{32b\,(n^3+9n^2+23n+15)} + \frac{e^{2a+2bx}\,(3n^2+28n+25)}{32b\,(n^3+9n^2+23n+15)} - \frac{e^{8a+8bx}\,(3n^2+28n+25)}{32b\,(n^3+9n^2+23n+15)} + \frac{e^{4a+4bx}\,(2n^2+24n+150)}{32b\,(n^3+9n^2+23n+15)} - \frac{e^{6a+6bx}\,(2n^2+24n+150)}{32b\,(n^3+9n^2+23n+15)}\right)$$

```
[In] int(cosh(a + b*x)^5*sinh(a + b*x)^n,x)
```

```
[Out] -exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 - exp(- a - b*x)/2)^n*((4*n + n^2 + 3)/
(32*b*(23*n + 9*n^2 + n^3 + 15)) - (exp(10*a + 10*b*x)*(4*n + n^2 + 3))/(32
*b*(23*n + 9*n^2 + n^3 + 15)) + (exp(2*a + 2*b*x)*(28*n + 3*n^2 + 25))/(32*
b*(23*n + 9*n^2 + n^3 + 15)) - (exp(8*a + 8*b*x)*(28*n + 3*n^2 + 25))/(32*b
*(23*n + 9*n^2 + n^3 + 15)) + (exp(4*a + 4*b*x)*(24*n + 2*n^2 + 150))/(32*b
*(23*n + 9*n^2 + n^3 + 15)) - (exp(6*a + 6*b*x)*(24*n + 2*n^2 + 150))/(32*b
*(23*n + 9*n^2 + n^3 + 15)))
```

3.12 $\int \cosh^m(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^{1+m}(a+bx)}{b(1+m)}$$

```
[Out] cosh(b*x+a)^(1+m)/b/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2645, 30}

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^m \, dx, x, \cosh(a+bx)\right)}{b} \\ &= \frac{\cosh^{1+m}(a+bx)}{b(1+m)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^m(a+bx) \sinh(a+bx) \, dx = \frac{\cosh^{1+m}(a+bx)}{b(1+m)}$$

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x],x]
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m))
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\cosh(bx+a)^{1+m}}{b(1+m)}$
default	$\frac{\cosh(bx+a)^{1+m}}{b(1+m)}$
risch	$\frac{\left(\mathrm{e}^{bx+a}\right)^{-m}\left(1+\mathrm{e}^{2bx+2a}\right)^{m}\left(\frac{1}{2}\right)^{m}\left(\mathrm{e}^{2bx+2a}\mathrm{e}^{-\frac{i\,\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{-bx-a}\right)^{3}\pi m}{2}}\mathrm{e}^{\frac{i\,\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{-bx-a}\right)^{2}\mathrm{csgn}\left(i\mathrm{e}^{-bx}\right.}{2}}\right.}{}$

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] cosh(b*x+a)^(1+m)/b/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)\cosh(m\log(\cosh(bx+a))) + \cosh(bx+a)\sinh(m\log(\cosh(bx+a)))}{(bm+b)\cosh(bx+a)^2 - (bm+b)\sinh(bx+a)^2}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)*cosh(m*log(cosh(b*x + a))) + cosh(b*x + a)*sinh(m*log(cosh(b
*x + a))))/((b*m + b)*cosh(b*x + a)^2 - (b*m + b)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x\sinh(a)}{\cosh(a)} & \text{for } b=0 \wedge m=-1 \\ x\sinh(a)\cosh^m(a) & \text{for } b=0 \\ \frac{\log(\cosh(a+bx))}{b} & \text{for } m=-1 \\ \frac{\cosh(a+bx)\cosh^m(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)/cosh(a), Eq(b, 0) & Eq(m, -1)), (x*sinh(a)*cosh(a)**m,
 Eq(b, 0)), (log(cosh(a + b*x))/b, Eq(m, -1)), (cosh(a + b*x)*cosh(a + b*x)
**m/(b*m + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^{m+1}}{b(m+1)}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)^(m + 1)/(b*(m + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)^{m+1}}{b(m+1)}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a),x, algorithm="giac")
[Out] (1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a))^(m + 1)/(b*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^m(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^{m+1}}{b\,(m+1)}$$

```
[In] int(cosh(a + b*x)^m*sinh(a + b*x),x)
[Out] cosh(a + b*x)^(m + 1)/(b*(m + 1))
```

3.13 $\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = -\frac{\cosh^{1+m}(a+bx)}{b(1+m)} + \frac{\cosh^{3+m}(a+bx)}{b(3+m)}$$

```
[Out] -cosh(b*x+a)^(1+m)/b/(1+m)+cosh(b*x+a)^(3+m)/b/(3+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 14}

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh^{m+3}(a+bx)}{b(m+3)} - \frac{\cosh^{m+1}(a+bx)}{b(m+1)}$$

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]^(1 + m)/(b*(1 + m))) + Cosh[a + b*x]^(3 + m)/(b*(3 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^m(1-x^2)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int (x^m - x^{2+m})\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh^{1+m}(a+bx)}{b(1+m)} + \frac{\cosh^{3+m}(a+bx)}{b(3+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh^{1+m}(a+bx)(-5-m+(1+m)\cosh(2(a+bx)))}{2b(1+m)(3+m)}$$

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^3,x]
[Out] (Cosh[a + b*x]^(1 + m)*(-5 - m + (1 + m)*Cosh[2*(a + b*x)]))/(2*b*(1 + m)*(
3 + m))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\frac{\cosh(bx+a)^3\,\mathrm{e}^{m\ln(\cosh(bx+a))}}{b\,(3+m)} - \frac{\cosh(bx+a)\,\mathrm{e}^{m\ln(\cosh(bx+a))}}{b\,(1+m)}$$

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a)^3,x)
[Out] 1/b/(3+m)*cosh(b*x+a)^3*exp(m*ln(cosh(b*x+a)))-1/b/(1+m)*cosh(b*x+a)*exp(m*
ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(40) = 80.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.72

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{((m+1)\cosh(bx+a)^3 + 3(m+1)\cosh(bx+a)\sinh(bx+a)^2 - (m+9)\cosh(bx+a))\cosh(m\log(\mathrm{co}}{4\left((bm^2+4bm+3b)\cosh(bx+a)^4 - 2(bm^2+\right.}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(((m + 1)*cosh(b*x + a)^3 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (
m + 9)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m + 1)*cosh(b*x + a)^3
 + 3*(m + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (m + 9)*cosh(b*x + a))*sinh(m*
log(cosh(b*x + a))))/((b*m^2 + 4*b*m + 3*b)*cosh(b*x + a)^4 - 2*(b*m^2 + 4*
b*m + 3*b)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (b*m^2 + 4*b*m + 3*b)*sinh(b*x
 + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(29) = 58$.

Time = 1.22 (sec) , antiderivative size = 648, normalized size of antiderivative = 16.20

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx$$

$$= \begin{cases} x\sinh^3(a)\cosh^m(a) \\ \frac{\log(\cosh(a+bx))}{b} - \frac{\sinh^2(a+bx)}{2b\cosh^2(a+bx)} \\ -\frac{bx\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{2bx\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{bx}{b\tanh^4\left(\frac{a}{2}+\frac{bx}{2}\right)-2b\tanh^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{2\log\left(\mathrm{ta}\right.}{b\tanh^4} \\ \frac{m\sinh^2(a+bx)\cosh(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} + \frac{3\sinh^2(a+bx)\cosh(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} - \frac{2\cosh^3(a+bx)\cosh^m(a+bx)}{bm^2+4bm+3b} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sinh
(a + b*x)**2/(2*b*cosh(a + b*x)**2), Eq(m, -3)), (-b*x*tanh(a/2 + b*x/2)**4
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*b*x*tanh(a/2 +
 b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - b*x/(b
*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*
x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x
/2)**2 + b) - 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2
 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2) + 1)
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(a/2 + b
*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 +
 b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(a/2 + b*x/2)
**2 + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*tanh(a
/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(
m, -1)), (m*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**2 + 4*b*m
 + 3*b) + 3*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**2 + 4*b*m
 + 3*b) - 2*cosh(a + b*x)**3*cosh(a + b*x)**m/(b*m**2 + 4*b*m + 3*b), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.32

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \frac{me^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} - \frac{(m+9)e^{\left((bx+a)m+bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} - \frac{(m+9)e^{\left((bx+a)m-bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} + \frac{(m+1)e^{\left((bx+a)m-3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-3\,a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b} + \frac{e^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\,(2^m m^2+2^{m+2}m+3\cdot 2^m)b}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*m*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2
 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^((b*x + a)*m + b*x + m*log(e^(-2
*b*x - 2*a) + 1) + a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) - 1/8*(m + 9)*e^(
(b*x + a)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^2 + 2^(m + 2)*
m + 3*2^m)*b) + 1/8*(m + 1)*e^((b*x + a)*m - 3*b*x + m*log(e^(-2*b*x - 2*a)
 + 1) - 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b) + 1/8*e^((b*x + a)*m + 3*b
*x + m*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^m*m^2 + 2^(m + 2)*m + 3*2^m)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(40) = 80$.

Time = 0.32 (sec) , antiderivative size = 325, normalized size of antiderivative = 8.12

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \frac{me^{\left(7\,bx+m\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+7\,a\right)} - me^{\left(5\,bx+m\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)+5\,a\right)} - me^{\left(3\,bx+m\log\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}+1\right.\right.\right.}}{}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(m*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) - m*
e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - m*e^(3*b*
x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + m*e^(b*x + m*log
(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + e^(7*b*x + m*log(1/2*(e^(2*
b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) - 9*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2
```

```
*a) + 1)*e^(-b*x - a)) + 5*a) - 9*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1
)*e^(-b*x - a)) + 3*a) + e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x -
 a)) + a))/(b*m^2*e^(4*b*x + 4*a) + 4*b*m*e^(4*b*x + 4*a) + 3*b*e^(4*b*x +
4*a))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.30

$$\int \cosh^m(a+bx)\sinh^3(a+bx)\,dx = \left(\frac{1}{2}\right)^m e^{-3a-3bx}\left(e^{a+bx}+e^{-a-bx}\right)^m\left(\frac{\frac{m}{8}+\frac{1}{8}}{b\,(m^2+4m+3)} - \frac{e^{2a+2bx}\,(m+9)}{8b\,(m^2+4m+3)} + \frac{e^{6a+6bx}\,(m+1)}{8b\,(m^2+4m+3)} - \frac{e^{4a+4bx}\,(m+9)}{8b\,(m^2+4m+3)}\right)$$

```
[In] int(cosh(a + b*x)^m*sinh(a + b*x)^3,x)
```

```
[Out] (1/2)^m*exp(- 3*a - 3*b*x)*(exp(a + b*x) + exp(- a - b*x))^m*((m/8 + 1/8)/(
b*(4*m + m^2 + 3)) - (exp(2*a + 2*b*x)*(m + 9))/(8*b*(4*m + m^2 + 3)) + (ex
p(6*a + 6*b*x)*(m + 1))/(8*b*(4*m + m^2 + 3)) - (exp(4*a + 4*b*x)*(m + 9))/
(8*b*(4*m + m^2 + 3)))
```

3.14 $\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx=\frac{\cosh^{1+m}(a+bx)}{b(1+m)}-\frac{2\cosh^{3+m}(a+bx)}{b(3+m)}+\frac{\cosh^{5+m}(a+bx)}{b(5+m)}$$

```
[Out] cosh(b*x+a)^(1+m)/b/(1+m)-2*cosh(b*x+a)^(3+m)/b/(3+m)+cosh(b*x+a)^(5+m)/b/(
5+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 276}

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx=\frac{\cosh^{m+1}(a+bx)}{b(m+1)}-\frac{2\cosh^{m+3}(a+bx)}{b(m+3)}+\frac{\cosh^{m+5}(a+bx)}{b(m+5)}$$

```
[In] Int[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]^(1 + m)/(b*(1 + m)) - (2*Cosh[a + b*x]^(3 + m))/(b*(3 + m)) +
 Cosh[a + b*x]^(5 + m)/(b*(5 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^m(1-x^2)^2\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^m - 2x^{2+m} + x^{4+m})\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh^{1+m}(a+bx)}{b(1+m)} - \frac{2\cosh^{3+m}(a+bx)}{b(3+m)} + \frac{\cosh^{5+m}(a+bx)}{b(5+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \frac{\cosh^{1+m}(a+bx)\left(89+28m+3m^2-4(7+8m+m^2)\cosh(2(a+bx))+(3+4m+m^2)\cosh(4(a+bx))\right)}{8b(1+m)(3+m)(5+m)}$$

```
[In] Integrate[Cosh[a + b*x]^m*Sinh[a + b*x]^5,x]
[Out] (Cosh[a + b*x]^(1 + m)*(89 + 28*m + 3*m^2 - 4*(7 + 8*m + m^2)*Cosh[2*(a + b
*x)] + (3 + 4*m + m^2)*Cosh[4*(a + b*x)]))/(8*b*(1 + m)*(3 + m)*(5 + m))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\frac{\cosh(bx+a)\,\mathrm{e}^{m\ln(\cosh(bx+a))}}{b(1+m)} + \frac{\cosh(bx+a)^5\,\mathrm{e}^{m\ln(\cosh(bx+a))}}{b(5+m)} - \frac{2\cosh(bx+a)^3\,\mathrm{e}^{m\ln(\cosh(bx+a))}}{b(3+m)}$$

```
[In] int(cosh(b*x+a)^m*sinh(b*x+a)^5,x)
[Out] 1/b/(1+m)*cosh(b*x+a)*exp(m*ln(cosh(b*x+a)))+1/b/(5+m)*cosh(b*x+a)^5*exp(m*
ln(cosh(b*x+a)))-2/b/(3+m)*cosh(b*x+a)^3*exp(m*ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \frac{\left((m^2+4m+3)\cosh(bx+a)^5+5(m^2+4m+3)\cosh(bx+a)\sinh(bx+a)^4-(3m^2+28m+25)\cos\right.}{}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(((m^2 + 4*m + 3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*si
nh(b*x + a)^4 - (3*m^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*c
osh(b*x + a)^3 - 3*(3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(
m^2 + 12*m + 75)*cosh(b*x + a))*cosh(m*log(cosh(b*x + a))) + ((m^2 + 4*m +
3)*cosh(b*x + a)^5 + 5*(m^2 + 4*m + 3)*cosh(b*x + a)*sinh(b*x + a)^4 - (3*m
^2 + 28*m + 25)*cosh(b*x + a)^3 + (10*(m^2 + 4*m + 3)*cosh(b*x + a)^3 - 3*(
3*m^2 + 28*m + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(m^2 + 12*m + 75)*cos
h(b*x + a))*sinh(m*log(cosh(b*x + a))))/((b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*
cosh(b*x + a)^6 - 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^4*sinh(
b*x + a)^2 + 3*(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*cosh(b*x + a)^2*sinh(b*x +
 a)^4 - (b*m^3 + 9*b*m^2 + 23*b*m + 15*b)*sinh(b*x + a)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. $2(46) = 92$.

Time = 4.54 (sec) , antiderivative size = 2351, normalized size of antiderivative = 39.85

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)**m*sinh(b*x+a)**5,x)
```

```
[Out] Piecewise((x*sinh(a)**5*cosh(a)**m, Eq(b, 0)), (log(cosh(a + b*x))/b - sinh
(a + b*x)**4/(4*b*cosh(a + b*x)**4) - sinh(a + b*x)**2/(2*b*cosh(a + b*x)**
2), Eq(m, -5)), (-2*b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*
tanh(a/2 + b*x/2)**4 + b) + 4*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)
**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*b*x/(b*tanh(a/2 + b*x/2)**8 - 2*b*t
anh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**
8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 8*log(tanh(a/2
+ b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 +
 b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2)**8 - 2*
b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*
```

```
x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*log(tan
h(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*t
anh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*
x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)**6/(b*tanh(a/
2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b), Eq(m, -3)), (b*x*tanh(
a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*ta
nh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2
)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*
x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*b*x*tanh(a/2 + b*x/2)**4/(b*tan
h(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4
*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*x*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x
/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2
 + b*x/2)**2 + b) + b*x/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2
+ b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*
log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4
*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)*
*2 + b) - 12*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 +
b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(
a/2 + b*x/2)**2 + b) + 8*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b
*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4
 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2
 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*ta
nh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**
8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2
)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2)**2 + 1)*tanh
(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*t
anh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*log(tanh(a/2 + b*x/
2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*
x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*log(
tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 4*
b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**
2 + b) + log(tanh(a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a
/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) -
 2*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 8*tanh(a/2 + b
*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2
+ b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 2*tanh(a/2 + b*x/2)**2/(b*tan
h(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4
*b*tanh(a/2 + b*x/2)**2 + b), Eq(m, -1)), (m**2*sinh(a + b*x)**4*cosh(a + b
*x)*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*m*sinh(a + b*x
)**4*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) - 4
*m*sinh(a + b*x)**2*cosh(a + b*x)**3*cosh(a + b*x)**m/(b*m**3 + 9*b*m**2 +
```

```
23*b*m + 15*b) + 15*sinh(a + b*x)**4*cosh(a + b*x)*cosh(a + b*x)**m/(b*m**3
 + 9*b*m**2 + 23*b*m + 15*b) - 20*sinh(a + b*x)**2*cosh(a + b*x)**3*cosh(a
+ b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*cosh(a + b*x)**5*cosh(a +
 b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(59) = 118$.

Time = 0.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 9.46

$$\begin{aligned}
\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx
&= \frac{m^2 e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad+\frac{m e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{8\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad-\frac{(3\,m^2+28\,m+25)e^{\left((bx+a)m+3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad+\frac{(m^2+12\,m+75)e^{\left((bx+a)m+bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{16\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad+\frac{(m^2+12\,m+75)e^{\left((bx+a)m-bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{16\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad-\frac{(3\,m^2+28\,m+25)e^{\left((bx+a)m-3\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-3\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad+\frac{(m^2+4\,m+3)e^{\left((bx+a)m-5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)-5\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b} \\
&\quad+\frac{3\,e^{\left((bx+a)m+5\,bx+m\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\,(2^m m^3+9\cdot 2^m m^2+23\cdot 2^m m+15\cdot 2^m)b}
\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/32*m^2*e^((b*x + a)*m + 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*
m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/8*m*e^((b*x + a)*m + 5*b*x + m*
log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)
*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m + 3*b*x + m*log(e^(-2*b*x - 2
*a) + 1) + 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 +
 12*m + 75)*e^((b*x + a)*m + b*x + m*log(e^(-2*b*x - 2*a) + 1) + a)/((2^m*m
^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 1/16*(m^2 + 12*m + 75)*e^((b*x + a
)*m - b*x + m*log(e^(-2*b*x - 2*a) + 1) - a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m
*m + 15*2^m)*b) - 1/32*(3*m^2 + 28*m + 25)*e^((b*x + a)*m - 3*b*x + m*log(e
^(-2*b*x - 2*a) + 1) - 3*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) +
 1/32*(m^2 + 4*m + 3)*e^((b*x + a)*m - 5*b*x + m*log(e^(-2*b*x - 2*a) + 1)
```

```
- 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m*m + 15*2^m)*b) + 3/32*e^((b*x + a)*m
+ 5*b*x + m*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^m*m^3 + 9*2^m*m^2 + 23*2^m
*m + 15*2^m)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(59) = 118$.

Time = 0.36 (sec) , antiderivative size = 721, normalized size of antiderivative = 12.22

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)^m*sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/32*(m^2*e^(11*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 11*a)
 - 3*m^2*e^(9*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 9*a) +
2*m^2*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) + 2*m
^2*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 3*m^2*
e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + m^2*e^(b*
x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a) + 4*m*e^(11*b*x + m*
log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 11*a) - 28*m*e^(9*b*x + m*log
(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 9*a) + 24*m*e^(7*b*x + m*log(1/2
*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 7*a) + 24*m*e^(5*b*x + m*log(1/2*(e^
(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 5*a) - 28*m*e^(3*b*x + m*log(1/2*(e^(2*b
*x + 2*a) + 1)*e^(-b*x - a)) + 3*a) + 4*m*e^(b*x + m*log(1/2*(e^(2*b*x + 2*
a) + 1)*e^(-b*x - a)) + a) + 3*e^(11*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*
e^(-b*x - a)) + 11*a) - 25*e^(9*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b
*x - a)) + 9*a) + 150*e^(7*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x -
a)) + 7*a) + 150*e^(5*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) +
 5*a) - 25*e^(3*b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + 3*a)
+ 3*e^(b*x + m*log(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)) + a))/(b*m^3*e^(
6*b*x + 6*a) + 9*b*m^2*e^(6*b*x + 6*a) + 23*b*m*e^(6*b*x + 6*a) + 15*b*e^(6
*b*x + 6*a))
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.31

$$\int \cosh^m(a+bx)\sinh^5(a+bx)\,dx$$
$$= e^{-5a-5bx}\left(\frac{e^{a+bx}}{2}+\frac{e^{-a-bx}}{2}\right)^m\left(\frac{m^2+4m+3}{32b\,(m^3+9m^2+23m+15)}+\frac{e^{10a+10bx}\,(m^2+4m+3)}{32b\,(m^3+9m^2+23m+15)}-\frac{e^{2a+2bx}\,(3m^2+28m+25)}{32b\,(m^3+9m^2+23m+15)}-\frac{e^{8a+8bx}\,(3m^2+28m+25)}{32b\,(m^3+9m^2+23m+15)}+\frac{e^{4a+4bx}\,(2m^2+24m+150)}{32b\,(m^3+9m^2+23m+15)}+\frac{e^{6a+6bx}\,(2m^2+24m+150)}{32b\,(m^3+9m^2+23m+15)}\right)$$

```
[In] int(cosh(a + b*x)^m*sinh(a + b*x)^5,x)
```

```
[Out] exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 + exp(- a - b*x)/2)^m*((4*m + m^2 + 3)/(
32*b*(23*m + 9*m^2 + m^3 + 15)) + (exp(10*a + 10*b*x)*(4*m + m^2 + 3))/(32*
b*(23*m + 9*m^2 + m^3 + 15)) - (exp(2*a + 2*b*x)*(28*m + 3*m^2 + 25))/(32*b
*(23*m + 9*m^2 + m^3 + 15)) - (exp(8*a + 8*b*x)*(28*m + 3*m^2 + 25))/(32*b*
(23*m + 9*m^2 + m^3 + 15)) + (exp(4*a + 4*b*x)*(24*m + 2*m^2 + 150))/(32*b*
(23*m + 9*m^2 + m^3 + 15)) + (exp(6*a + 6*b*x)*(24*m + 2*m^2 + 150))/(32*b*
(23*m + 9*m^2 + m^3 + 15)))
```

3.15 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{8}-\frac{\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

```
[Out] -1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/8*x - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*
x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{1}{4}\int \cosh^2(a+bx)\,dx \\
&= -\frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{\int 1\,dx}{8} \\
&= -\frac{x}{8} - \frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{-4(a+bx)+\sinh(4(a+bx))}{32b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{x}{8}+\frac{e^{4bx+4a}}{64b}-\frac{e^{-4bx-4a}}{64b}$	33
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}}{b}$	43
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}}{b}$	43

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/8*x+1/64*exp(4*b*x+4*a)/b-1/64*exp(-4*b*x-4*a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \neq \\ x\sinh^2(a)\cosh^2(a) & \text{otherw} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)
*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{bx+a}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(4\,a+4\,b\,x)}{32\,b} - \frac{x}{8}$$

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
[Out] sinh(4*a + 4*b*x)/(32*b) - x/8
```

3.16 $\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{x}{16}+\frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b}$$

```
[Out] 1/16*x+1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/8*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6*c
osh(b*x+a)^3*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2648, 2715, 8}

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b} - \frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} + \frac{\sinh(a+bx)\cosh(a+bx)}{16b} + \frac{x}{16}$$

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

```
[Out] x/16 + (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a + b*x
])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} - \frac{1}{2}\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx \\
&= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} + \frac{1}{8}\int \cosh^2(a+bx)\,dx \\
&= \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} \\
&\quad + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b} + \frac{\int 1\,dx}{16} \\
&= \frac{x}{16} + \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{12bx - 3\sinh(2(a+bx)) - 3\sinh(4(a+bx)) + \sinh(6(a+bx))}{192b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
[Out] (12*b*x - 3*Sinh[2*(a + b*x)] - 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(1
92*b)
```

Maple [A] (verified)

Time = 50.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^3}{6}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$	61
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^3}{6}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$	61
risch	$\frac{x}{16}+\frac{e^{6bx+6a}}{384b}-\frac{e^{4bx+4a}}{128b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}+\frac{e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*cos
h(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{3\cosh(bx+a)\sinh(bx+a)^5+2\left(5\cosh(bx+a)^3-3\cosh(bx+a)\right)\sinh(bx+a)^3+6\,bx+3\left(\cosh(bx\right.}{96\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*x +
 a))*sinh(b*x + a)^3 + 6*b*x + 3*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(58) = 116.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= \begin{cases} -\frac{x\sinh^6(a+bx)}{16}+\frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16}-\frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16}+\frac{x\cosh^6(a+bx)}{16}+\frac{\sinh^5(a+bx)\cosh(a+bx)}{16b}+\frac{\sinh}{} \\ x\sinh^4(a)\cosh^2(a) \end{cases}$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**6/16 + 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/1
6 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + x*cosh(a + b*x)**6/16 + sinh
(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*b)
 - sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)*
*2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = -\frac{\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b} + \frac{bx+a}{16\,b} + \frac{3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b + 1/
16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - e^(-6*b*x
 - 6*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{1}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} + \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/384*e^(6*b*x + 6*a)/b - 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b*x
 + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b + 1/128*e^(-4*b*x - 4*a)/b - 1/384*e^(
-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{x}{16} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(4\,a+4\,b\,x)}{64} - \frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^4,x)
```

```
[Out] x/16 - (sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/64 - sinh(6*a + 6*b*x)/192
)/b
```

3.17 $\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{5x}{128}-\frac{5\cosh(a+bx)\sinh(a+bx)}{128b}+\frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b}-\frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b}+\frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b}$$

```
[Out] -5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b+5/64*cosh(b*x+a)^3*sinh(b*x+a)/b-5
/48*cosh(b*x+a)^3*sinh(b*x+a)^3/b+1/8*cosh(b*x+a)^3*sinh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2648, 2715, 8}

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{48b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{64b}-\frac{5\sinh(a+bx)\cosh(a+bx)}{128b}-\frac{5x}{128}$$

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(64*b) - (5*Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(48*b) + (Cosh[a
 + b*x]^3*Sinh[a + b*x]^5)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} - \frac{5}{8}\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx \\
&= -\frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} \\
&\quad + \frac{5}{16}\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx \\
&= \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} \\
&\quad + \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} - \frac{5}{64}\int \cosh^2(a+bx)\,dx \\
&= -\frac{5\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} \\
&\quad - \frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b} - \frac{5\int 1\,dx}{128} \\
&= -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b} \\
&\quad - \frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b} + \frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{-120bx+48\sinh(2(a+bx))+24\sinh(4(a+bx))-16\sinh(6(a+bx))+3\sinh(8(a+bx))}{3072b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*b*x + 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] - 16*Sinh[6*(a + b*
x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^3}{8}-\frac{5\cosh(bx+a)^3\sinh(bx+a)^3}{48}+\frac{5\cosh(bx+a)^3\sinh(bx+a)}{64}-\frac{5\cosh(bx+a)\sinh(bx+a)}{128}-\frac{5bx}{128}-\frac{5a}{128}}{b}$$

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/8*sinh(b*x+a)^5*cosh(b*x+a)^3-5/48*cosh(b*x+a)^3*sinh(b*x+a)^3+5/64*
cosh(b*x+a)^3*sinh(b*x+a)-5/128*cosh(b*x+a)*sinh(b*x+a)-5/128*b*x-5/128*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.50

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{3\cosh(bx+a)\sinh(bx+a)^7+3\left(7\cosh(bx+a)^3-4\cosh(bx+a)\right)\sinh(bx+a)^5+\left(21\cosh(bx+a)\right.}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 - 40*cosh(b*x + a)^3 + 12*cosh(
b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 - 4*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(85) = 170.

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+bx)}{128} \\ x\sinh^6(a)\cosh^2(a) \end{cases}$$

[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**6,x)

[Out] Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)**2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*cosh(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a + b*x)/(128*b) + 73*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) - 55*sinh(a + b*x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= -\frac{\left(16\,e^{(-2\,bx-2\,a)} - 24\,e^{(-4\,bx-4\,a)} - 48\,e^{(-6\,bx-6\,a)} - 3\right)e^{(8\,bx+8\,a)}}{6144\,b} - \frac{5\,(bx+a)}{128\,b} - \frac{48\,e^{(-2\,bx-2\,a)} + 24\,e^{(-4\,bx-4\,a)} - 16\,e^{(-6\,bx-6\,a)} + 3\,e^{(-8\,bx-8\,a)}}{6144\,b}$$

[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="maxima")

[Out] -1/6144*(16*e^(-2*b*x - 2*a) - 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) - 3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b - 1/6144*(48*e^(-2*b*x - 2*a) + 24*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a))/b

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{5}{128}\,x + \frac{e^{(8\,bx+8\,a)}}{2048\,b} - \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{e^{(4\,bx+4\,a)}}{256\,b} + \frac{e^{(2\,bx+2\,a)}}{128\,b} - \frac{e^{(-2\,bx-2\,a)}}{128\,b} - \frac{e^{(-4\,bx-4\,a)}}{256\,b} + \frac{e^{(-6\,bx-6\,a)}}{384\,b} - \frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4*
b*x + 4*a)/b + 1/128*e^(2*b*x + 2*a)/b - 1/128*e^(-2*b*x - 2*a)/b - 1/256*e
^(-4*b*x - 4*a)/b + 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{\frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(6\,a+6\,b\,x)}{192} + \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b} - \frac{5\,x}{128}$$

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^6,x)
```

```
[Out] (sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/128 - sinh(6*a + 6*b*x)/192 + sin
h(8*a + 8*b*x)/1024)/b - (5*x)/128
```

3.18 $\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{16}-\frac{\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

```
[Out] -1/16*x-1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6
*cosh(b*x+a)^5*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{\sinh(a+bx)\cosh^3(a+bx)}{24b}-\frac{\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{x}{16}$$

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

```
[Out] -1/16*x - (Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (Cosh[a + b*x]^3*Sinh[a +
b*x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{1}{6}\int \cosh^4(a+bx)\,dx \\
&= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{1}{8}\int \cosh^2(a+bx)\,dx \\
&= -\frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} \\
&\quad + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} - \frac{\int 1\,dx}{16} \\
&= -\frac{x}{16} - \frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{-12bx - 3\sinh(2(a+bx)) + 3\sinh(4(a+bx)) + \sinh(6(a+bx))}{192b}$$

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
[Out] (-12*b*x - 3*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(
192*b)
```

Maple [A] (verified)

Time = 22.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^5}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$	56
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^5}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$	56
risch	$-\frac{x}{16}+\frac{e^{6bx+6a}}{384b}+\frac{e^{4bx+4a}}{128b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}-\frac{e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6*sinh(b*x+a)*cosh(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*
sinh(b*x+a)-1/16*b*x-1/16*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)\sinh(bx+a)^5+2\left(5\cosh(bx+a)^3+3\cosh(bx+a)\right)\sinh(bx+a)^3-6\,bx+3\left(\cosh(bx\right.}{96\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x +
 a))*sinh(b*x + a)^3 - 6*b*x + 3*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(56) = 112.

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{x\sinh^6(a+bx)}{16}-\frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16}+\frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16}-\frac{x\cosh^6(a+bx)}{16}-\frac{\sinh^5(a+bx)\cosh(a+bx)}{16b}+\frac{\sinh^3(\ldots}{} \\ x\sinh^2(a)\cosh^4(a)\end{cases}$$

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**6/16 - 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/16
 + 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - x*cosh(a + b*x)**6/16 - sinh(
a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*b)
+ sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**
4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + 1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{bx+a}{16\,b} + \frac{3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b - 1/1
6*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - e^(-6*b*x
- 6*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} - \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*x + 1/384*e^(6*b*x + 6*a)/b + 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b*
x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/128*e^(-4*b*x - 4*a)/b - 1/384*e^
(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4\,a+4\,b\,x)}{64}-\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}}{b}-\frac{x}{16}$$

[In] int(cosh(a + b*x)^4*sinh(a + b*x)^2,x)

[Out] (sinh(4*a + 4*b*x)/64 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192)/b - x
/16

3.19 $\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3x}{128}+\frac{3\cosh(a+bx)\sinh(a+bx)}{128b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{64b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{16b}+\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b}$$

```
[Out] 3/128*x+3/128*cosh(b*x+a)*sinh(b*x+a)/b+1/64*cosh(b*x+a)^3*sinh(b*x+a)/b-1/
16*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^5*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{\sinh(a+bx)\cosh^5(a+bx)}{16b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{64b}+\frac{3\sinh(a+bx)\cosh(a+bx)}{128b}+\frac{3x}{128}$$

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(64*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(16*b) + (Cosh[a + b*x]
^5*Sinh[a + b*x]^3)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} - \frac{3}{8}\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx \\
&= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} + \frac{1}{16}\int \cosh^4(a+bx)\,dx \\
&= \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} \\
&\quad + \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} + \frac{3}{64}\int \cosh^2(a+bx)\,dx \\
&= \frac{3\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} \\
&\quad - \frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b} + \frac{3\int 1\,dx}{128} \\
&= \frac{3x}{128} + \frac{3\cosh(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{64b} \\
&\quad - \frac{\cosh^5(a+bx)\sinh(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx=\frac{24(a+bx)-8\sinh(4(a+bx))+\sinh(8(a+bx))}{1024b}$$

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
[Out] (24*(a + b*x) - 8*Sinh[4*(a + b*x)] + Sinh[8*(a + b*x)])/(1024*b)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^5}{8}-\frac{\sinh(bx+a)\cosh(bx+a)^5}{16}+\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{16}+\frac{3bx}{128}+\frac{3a}{128}}{b}$$

```
[In] int(cosh(b*x+a)^4*sinh(b*x+a)^4,x)
[Out] 1/b*(1/8*sinh(b*x+a)^3*cosh(b*x+a)^5-1/16*sinh(b*x+a)*cosh(b*x+a)^5+1/16*(1
/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/128*b*x+3/128*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$=\frac{7\cosh(bx+a)^3\sinh(bx+a)^5+\cosh(bx+a)\sinh(bx+a)^7+\left(7\cosh(bx+a)^5-4\cosh(bx+a)\right)\sinh}{128\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="fricas")
[Out] 1/128*(7*cosh(b*x + a)^3*sinh(b*x + a)^5 + cosh(b*x + a)*sinh(b*x + a)^7 +
(7*cosh(b*x + a)^5 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x +
 a)^7 - 4*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(80) = 160.

Time = 0.69 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sinh^8(a+bx)}{128} - \frac{3x\sinh^6(a+bx)\cosh^2(a+bx)}{32} + \frac{9x\sinh^4(a+bx)\cosh^4(a+bx)}{64} - \frac{3x\sinh^2(a+bx)\cosh^6(a+bx)}{32} + \frac{3x\cosh^8(a+bx)}{128} - \\ x\sinh^4(a)\cosh^4(a) \end{cases}$$

```
[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**8/128 - 3*x*sinh(a + b*x)**6*cosh(a + b*x)**2
/32 + 9*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 - 3*x*sinh(a + b*x)**2*cosh(
a + b*x)**6/32 + 3*x*cosh(a + b*x)**8/128 - 3*sinh(a + b*x)**7*cosh(a + b*x
)/(128*b) + 11*sinh(a + b*x)**5*cosh(a + b*x)**3/(128*b) + 11*sinh(a + b*x)
**3*cosh(a + b*x)**5/(128*b) - 3*sinh(a + b*x)*cosh(a + b*x)**7/(128*b), Ne
(b, 0)), (x*sinh(a)**4*cosh(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = -\frac{\left(8\,e^{(-4\,bx-4\,a)}-1\right)e^{(8\,bx+8\,a)}}{2048\,b} + \frac{3\,(bx+a)}{128\,b} + \frac{8\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}}{2048\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2048*(8*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/128*(b*x + a)/b + 1/
2048*(8*e^(-4*b*x - 4*a) - e^(-8*b*x - 8*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3}{128}\,x + \frac{e^{(8\,bx+8\,a)}}{2048\,b} - \frac{e^{(4\,bx+4\,a)}}{256\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b} - \frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/256*e^(4*b*x + 4*a)/b + 1/256*e^(-4*
b*x - 4*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3\,x}{128} - \frac{\frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b}$$

```
[In] int(cosh(a + b*x)^4*sinh(a + b*x)^4,x)
[Out] (3*x)/128 - (sinh(4*a + 4*b*x)/128 - sinh(8*a + 8*b*x)/1024)/b
```

3.20 $\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{3x}{256}-\frac{3\cosh(a+bx)\sinh(a+bx)}{256b}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{32b}-\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b}+\frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b}$$

```
[Out] -3/256*x-3/256*cosh(b*x+a)*sinh(b*x+a)/b-1/128*cosh(b*x+a)^3*sinh(b*x+a)/b+
1/32*cosh(b*x+a)^5*sinh(b*x+a)/b-1/16*cosh(b*x+a)^5*sinh(b*x+a)^3/b+1/10*co
sh(b*x+a)^5*sinh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used

= {2648, 2715, 8}

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b} - \frac{\sinh^3(a+bx)\cosh^5(a+bx)}{16b} + \frac{\sinh(a+bx)\cosh^5(a+bx)}{32b} - \frac{\sinh(a+bx)\cosh^3(a+bx)}{128b} - \frac{3\sinh(a+bx)\cosh(a+bx)}{256b} - \frac{3x}{256}$$

```
[In] Int[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

```
[Out] (-3*x)/256 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) - (Cosh[a + b*x]^3*Sin
h[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(32*b) - (Cosh[a + b*
x]^5*Sinh[a + b*x]^3)/(16*b) + (Cosh[a + b*x]^5*Sinh[a + b*x]^5)/(10*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} - \frac{1}{2}\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$

$$= -\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} + \frac{3}{16}\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$

$$= \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} - \frac{1}{32}\int \cosh^4(a+bx)\,dx$$

$$= -\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} - \frac{3}{128}\int \cosh^2(a+bx)\,dx$$

$$= -\frac{3\cosh(a+bx)\sinh(a+bx)}{256b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b} - \frac{3\int 1\,dx}{256}$$

$$= -\frac{3x}{256} - \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{32b} - \frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b} + \frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{-120bx + 20\sinh(2(a+bx)) + 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) - 5\sinh(8(a+bx)) + 2\sinh(10(a}{10240b}$$

```
[In] Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

```
[Out] (-120*b*x + 20*Sinh[2*(a + b*x)] + 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*
x)] - 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^5}{10}-\frac{\sinh(bx+a)^3\cosh(bx+a)^5}{16}+\frac{\sinh(bx+a)\cosh(bx+a)^5}{32}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{32}-\frac{3bx}{256}-\frac{3a}{256}}{b}$$

[In] int(cosh(b*x+a)^4*sinh(b*x+a)^6,x)

[Out] 1/b*(1/10*sinh(b*x+a)^5*cosh(b*x+a)^5-1/16*sinh(b*x+a)^3*cosh(b*x+a)^5+1/32 *sinh(b*x+a)*cosh(b*x+a)^5-1/32*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b* x+a)-3/256*b*x-3/256*a)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.74

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{5\cosh(bx+a)\sinh(bx+a)^9+10\left(6\cosh(bx+a)^3-\cosh(bx+a)\right)\sinh(bx+a)^7+\left(126\cosh(bx+a)\right.}{}$$

[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="fricas")

[Out] 1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 15*cosh (b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5* cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 5*(cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a))/b

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(100) = 200.

Time = 1.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.04

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \begin{cases} \frac{3x\sinh^{10}(a+bx)}{256}-\frac{15x\sinh^8(a+bx)\cosh^2(a+bx)}{256}+\frac{15x\sinh^6(a+bx)\cosh^4(a+bx)}{128}-\frac{15x\sinh^4(a+bx)\cosh^6(a+bx)}{128}+\frac{15x\sinh^2(a+\ldots}{} \\ x\sinh^6(a)\cosh^4(a) \end{cases}$$

[In] integrate(cosh(b*x+a)**4*sinh(b*x+a)**6,x)

```
[Out] Piecewise((3*x*sinh(a + b*x)**10/256 - 15*x*sinh(a + b*x)**8*cosh(a + b*x)*
*2/256 + 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 - 15*x*sinh(a + b*x)**4
*cosh(a + b*x)**6/128 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 - 3*x*co
sh(a + b*x)**10/256 - 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) + 7*sinh(a +
 b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/(10*b
) - 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) + 3*sinh(a + b*x)*cosh(a +
b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}-40\,e^{(-6\,bx-6\,a)}-20\,e^{(-8\,bx-8\,a)}-2\right)e^{(10\,bx+10\,a)}}{20480\,b} - \frac{3\,(bx+a)}{256\,b} - \frac{20\,e^{(-2\,bx-2\,a)}+40\,e^{(-4\,bx-4\,a)}-10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+2\,e^{(-10\,bx-10\,a)}}{20480\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20480*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) -
20*e^(-8*b*x - 8*a) - 2)*e^(10*b*x + 10*a)/b - 3/256*(b*x + a)/b - 1/20480*
(20*e^(-2*b*x - 2*a) + 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) - 5*e^(-8*
b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{3}{256}\,x + \frac{e^{(10\,bx+10\,a)}}{10240\,b} - \frac{e^{(8\,bx+8\,a)}}{4096\,b} - \frac{e^{(6\,bx+6\,a)}}{2048\,b} + \frac{e^{(4\,bx+4\,a)}}{512\,b} + \frac{e^{(2\,bx+2\,a)}}{1024\,b} - \frac{e^{(-2\,bx-2\,a)}}{1024\,b} - \frac{e^{(-4\,bx-4\,a)}}{512\,b} + \frac{e^{(-6\,bx-6\,a)}}{2048\,b} + \frac{e^{(-8\,bx-8\,a)}}{4096\,b} - \frac{e^{(-10\,bx-10\,a)}}{10240\,b}$$

```
[In] integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -3/256*x + 1/10240*e^(10*b*x + 10*a)/b - 1/4096*e^(8*b*x + 8*a)/b - 1/2048*
e^(6*b*x + 6*a)/b + 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b - 1/
1024*e^(-2*b*x - 2*a)/b - 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x - 6*a
)/b + 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{20\sinh(2\,a+2\,b\,x)+40\sinh(4\,a+4\,b\,x)-10\sinh(6\,a+6\,b\,x)-5\sinh(8\,a+8\,b\,x)+2\sinh(10\,a+10\,b\,x}{10240\,b}$$

```
[In] int(cosh(a + b*x)^4*sinh(a + b*x)^6,x)
```

```
[Out] (20*sinh(2*a + 2*b*x) + 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) - 5*sin
h(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) - 120*b*x)/(10240*b)
```

3.21 $\int \cosh^6(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^6(a+bx) \sinh^2(a+bx) \, dx = -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b}$$

```
[Out] -5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b-5/192*cosh(b*x+a)^3*sinh(b*x+a)/b-
1/48*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^7*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2648, 2715, 8}

$$\int \cosh^6(a+bx) \sinh^2(a+bx) \, dx = \frac{\sinh(a+bx)\cosh^7(a+bx)}{8b} - \frac{\sinh(a+bx)\cosh^5(a+bx)}{48b} - \frac{5\sinh(a+bx)\cosh^3(a+bx)}{192b} - \frac{5\sinh(a+bx)\cosh(a+bx)}{128b} - \frac{5x}{128}$$

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*x)/128 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(128*b) - (5*Cosh[a + b*x]^3*S
inh[a + b*x])/(192*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(48*b) + (Cosh[a +
b*x]^7*Sinh[a + b*x])/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{1}{8}\int \cosh^6(a+bx)\,dx \\
&= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{5}{48}\int \cosh^4(a+bx)\,dx \\
&= -\frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} \\
&\quad + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{5}{64}\int \cosh^2(a+bx)\,dx \\
&= -\frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} \\
&\quad - \frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b} - \frac{5\int 1\,dx}{128} \\
&= -\frac{5x}{128} - \frac{5\cosh(a+bx)\sinh(a+bx)}{128b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b} \\
&\quad - \frac{\cosh^5(a+bx)\sinh(a+bx)}{48b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{-120bx-48\sinh(2(a+bx))+24\sinh(4(a+bx))+16\sinh(6(a+bx))+3\sinh(8(a+bx))}{3072b}$$

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
[Out] (-120*b*x - 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] + 16*Sinh[6*(a + b*
x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

Maple [A] (verified)

Time = 127.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result	siz
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^7}{8}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{8}-\frac{5bx}{128}-\frac{5a}{128}}{b}$	66
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^7}{8}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{8}-\frac{5bx}{128}-\frac{5a}{128}}{b}$	66
risch	$-\frac{5x}{128}+\frac{e^{8bx+8a}}{2048b}+\frac{e^{6bx+6a}}{384b}+\frac{e^{4bx+4a}}{256b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}-\frac{e^{-4bx-4a}}{256b}-\frac{e^{-6bx-6a}}{384b}-\frac{e^{-8bx-8a}}{2048b}$	11

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/8*sinh(b*x+a)*cosh(b*x+a)^7-1/8*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^
3+5/16*cosh(b*x+a))*sinh(b*x+a)-5/128*b*x-5/128*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)\sinh(bx+a)^7+3\left(7\cosh(bx+a)^3+4\cosh(bx+a)\right)\sinh(bx+a)^5+\left(21\cosh(bx+a)^5\right.}{}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 + 4*cosh(b*x
+ a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 + 40*cosh(b*x + a)^3 + 12*cosh(
```

```
b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 + 4*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(80) = 160$.

Time = 0.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+bx)}{128} \\ x\sinh^2(a)\cosh^6(a) \end{cases}$$

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a + b
*x)/(128*b) - 55*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) + 73*sinh(a + b*
x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128*b),
Ne(b, 0)), (x*sinh(a)**2*cosh(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{\left(16\,e^{(-2\,bx-2\,a)} + 24\,e^{(-4\,bx-4\,a)} - 48\,e^{(-6\,bx-6\,a)} + 3\right)e^{(8\,bx+8\,a)}}{6144\,b} - \frac{5\,(bx+a)}{128\,b} + \frac{48\,e^{(-2\,bx-2\,a)} - 24\,e^{(-4\,bx-4\,a)} - 16\,e^{(-6\,bx-6\,a)} - 3\,e^{(-8\,bx-8\,a)}}{6144\,b}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6144*(16*e^(-2*b*x - 2*a) + 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) + 3
)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b + 1/6144*(48*e^(-2*b*x - 2*a) - 24*
e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = -\frac{5}{128}\,x + \frac{e^{(8\,bx+8\,a)}}{2048\,b} + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{e^{(4\,bx+4\,a)}}{256\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} - \frac{e^{(-4\,bx-4\,a)}}{256\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b} - \frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -5/128*x + 1/2048*e^(8*b*x + 8*a)/b + 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4*
b*x + 4*a)/b - 1/128*e^(2*b*x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/256*e
^(-4*b*x - 4*a)/b - 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4\,a+4\,b\,x)}{128} - \frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192} + \frac{\sinh(8\,a+8\,b\,x)}{1024}}{b} - \frac{5\,x}{128}$$

```
[In] int(cosh(a + b*x)^6*sinh(a + b*x)^2,x)
```

```
[Out] (sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192 + sin
h(8*a + 8*b*x)/1024)/b - (5*x)/128
```

3.22 $\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{3x}{256}+\frac{3\cosh(a+bx)\sinh(a+bx)}{256b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{160b}-\frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b}+\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b}$$

```
[Out] 3/256*x+3/256*cosh(b*x+a)*sinh(b*x+a)/b+1/128*cosh(b*x+a)^3*sinh(b*x+a)/b+1
/160*cosh(b*x+a)^5*sinh(b*x+a)/b-3/80*cosh(b*x+a)^7*sinh(b*x+a)/b+1/10*cosh
(b*x+a)^7*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used

$= \{2648, 2715, 8\}$

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b} - \frac{3\sinh(a+bx)\cosh^7(a+bx)}{80b} + \frac{\sinh(a+bx)\cosh^5(a+bx)}{160b} + \frac{\sinh(a+bx)\cosh^3(a+bx)}{128b} + \frac{3\sinh(a+bx)\cosh(a+bx)}{256b} + \frac{3x}{256}$$

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(256*b) + (Cosh[a + b*x]^3*Sinh
[a + b*x])/(128*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(160*b) - (3*Cosh[a +
b*x]^7*Sinh[a + b*x])/(80*b) + (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(10*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} - \frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$

$$= -\frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} + \frac{3}{80}\int \cosh^6(a+bx)\,dx$$

$$= \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} + \frac{1}{32}\int \cosh^4(a+bx)\,dx$$

$$= \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} + \frac{3}{128}\int \cosh^2(a+bx)\,dx$$

$$= \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b} + \frac{3\int 1\,dx}{256}$$

$$= \frac{3x}{256} + \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{120bx + 20\sinh(2(a+bx)) - 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) + 5\sinh(8(a+bx)) + 2\sinh(10(a+ }{10240b}$$

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sinh[2*(a + b*x)] - 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*x
)] + 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^7}{10}-\frac{3\sinh(bx+a)\cosh(bx+a)^7}{80}+\frac{3\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{80}+\frac{3bx}{256}+\frac{3a}{256}}{b}$$

[In] int(cosh(b*x+a)^6*sinh(b*x+a)^4,x)

[Out] 1/b*(1/10*sinh(b*x+a)^3*cosh(b*x+a)^7-3/80*sinh(b*x+a)*cosh(b*x+a)^7+3/80*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+3/256*b*x+3/256*a)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.76

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{5\cosh(bx+a)\sinh(bx+a)^9+10\left(6\cosh(bx+a)^3+\cosh(bx+a)\right)\sinh(bx+a)^7+\left(126\cosh(bx+a)^5\right.}{}$$

[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="fricas")

[Out] 1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 30*b*x + 5*(cosh(b*x + a)^9 + 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a))/b

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(100) = 200.

Time = 1.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \begin{cases} -\frac{3x\sinh^{10}(a+bx)}{256}+\frac{15x\sinh^8(a+bx)\cosh^2(a+bx)}{256}-\frac{15x\sinh^6(a+bx)\cosh^4(a+bx)}{128}+\frac{15x\sinh^4(a+bx)\cosh^6(a+bx)}{128}-\frac{15x\sinh^2(a+}{} \\ x\sinh^4(a)\cosh^6(a) \end{cases}$$

[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**4,x)

```
[Out] Piecewise((-3*x*sinh(a + b*x)**10/256 + 15*x*sinh(a + b*x)**8*cosh(a + b*x)
**2/256 - 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 + 15*x*sinh(a + b*x)**
4*cosh(a + b*x)**6/128 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 + 3*x*c
osh(a + b*x)**10/256 + 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) - 7*sinh(a
+ b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/(10*
b) + 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) - 3*sinh(a + b*x)*cosh(a +
 b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}-40\,e^{(-6\,bx-6\,a)}+20\,e^{(-8\,bx-8\,a)}+2\right)e^{(10\,bx+10\,a)}}{20480\,b}+\frac{3\,(bx+a)}{256\,b} - \frac{20\,e^{(-2\,bx-2\,a)}-40\,e^{(-4\,bx-4\,a)}-10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+2\,e^{(-10\,bx-10\,a)}}{20480\,b}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/20480*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) + 2
0*e^(-8*b*x - 8*a) + 2)*e^(10*b*x + 10*a)/b + 3/256*(b*x + a)/b - 1/20480*(
20*e^(-2*b*x - 2*a) - 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) + 5*e^(-8*b
*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{3}{256}\,x+\frac{e^{(10\,bx+10\,a)}}{10240\,b}+\frac{e^{(8\,bx+8\,a)}}{4096\,b}-\frac{e^{(6\,bx+6\,a)}}{2048\,b} - \frac{e^{(4\,bx+4\,a)}}{512\,b}+\frac{e^{(2\,bx+2\,a)}}{1024\,b}-\frac{e^{(-2\,bx-2\,a)}}{1024\,b}+\frac{e^{(-4\,bx-4\,a)}}{512\,b} + \frac{e^{(-6\,bx-6\,a)}}{2048\,b}-\frac{e^{(-8\,bx-8\,a)}}{4096\,b}-\frac{e^{(-10\,bx-10\,a)}}{10240\,b}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/10240*e^(10*b*x + 10*a)/b + 1/4096*e^(8*b*x + 8*a)/b - 1/2048*e
^(6*b*x + 6*a)/b - 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b - 1/1
024*e^(-2*b*x - 2*a)/b + 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x - 6*a)
/b - 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$=\frac{20\sinh(2\,a+2\,b\,x)-40\sinh(4\,a+4\,b\,x)-10\sinh(6\,a+6\,b\,x)+5\sinh(8\,a+8\,b\,x)+2\sinh(10\,a+10\,b\,x)}{10240\,b}$$

```
[In] int(cosh(a + b*x)^6*sinh(a + b*x)^4,x)
```

```
[Out] (20*sinh(2*a + 2*b*x) - 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) + 5*sin
h(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) + 120*b*x)/(10240*b)
```

3.23 $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{5x}{1024}-\frac{5\cosh(a+bx)\sinh(a+bx)}{1024b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b}$$

```
[Out] -5/1024*x-5/1024*cosh(b*x+a)*sinh(b*x+a)/b-5/1536*cosh(b*x+a)^3*sinh(b*x+a)
/b-1/384*cosh(b*x+a)^5*sinh(b*x+a)/b+1/64*cosh(b*x+a)^7*sinh(b*x+a)/b-1/24*
cosh(b*x+a)^7*sinh(b*x+a)^3/b+1/12*cosh(b*x+a)^7*sinh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used

$= \{2648, 2715, 8\}$

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{\sinh^3(a+bx)\cosh^7(a+bx)}{24b} + \frac{\sinh(a+bx)\cosh^7(a+bx)}{64b} - \frac{\sinh(a+bx)\cosh^5(a+bx)}{384b} - \frac{5\sinh(a+bx)\cosh^3(a+bx)}{1536b} - \frac{5\sinh(a+bx)\cosh(a+bx)}{1024b} - \frac{5x}{1024}$$

```
[In] Int[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/1024 - (5*Cosh[a + b*x]*Sinh[a + b*x])/(1024*b) - (5*Cosh[a + b*x]^3
*Sinh[a + b*x])/(1536*b) - (Cosh[a + b*x]^5*Sinh[a + b*x])/(384*b) + (Cosh[
a + b*x]^7*Sinh[a + b*x])/(64*b) - (Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(24*b)
 + (Cosh[a + b*x]^7*Sinh[a + b*x]^5)/(12*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\text{integral} = \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} - \frac{5}{12}\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$

$$= -\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} + \frac{1}{8}\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$

$$= \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} - \frac{1}{64}\int \cosh^6(a+bx)\,dx$$

$$= -\frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} - \frac{5}{384}\int \cosh^4(a+bx)\,dx$$

$$= -\frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} - \frac{5}{512}\int \cosh^2(a+bx)\,dx$$

$$= -\frac{5\cosh(a+bx)\sinh(a+bx)}{1024b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b} - \frac{5\int 1\,dx}{1024}$$

$$= -\frac{5x}{1024} - \frac{5\cosh(a+bx)\sinh(a+bx)}{1024b} - \frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b} - \frac{\cosh^5(a+bx)\sinh(a+bx)}{384b} + \frac{\cosh^7(a+bx)\sinh(a+bx)}{64b} - \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{-120a-120bx+45\sinh(4(a+bx))-9\sinh(8(a+bx))+\sinh(12(a+bx))}{24576b}$$

```
[In] Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
[Out] (-120*a - 120*b*x + 45*Sinh[4*(a + b*x)] - 9*Sinh[8*(a + b*x)] + Sinh[12*(a
 + b*x)])/(24576*b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^7}{12}-\frac{\sinh(bx+a)^3\cosh(bx+a)^7}{24}+\frac{\sinh(bx+a)\cosh(bx+a)^7}{64}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{64}-}{b}$$

```
[In] int(cosh(b*x+a)^6*sinh(b*x+a)^6,x)
```

```
[Out] 1/b*(1/12*sinh(b*x+a)^5*cosh(b*x+a)^7-1/24*sinh(b*x+a)^3*cosh(b*x+a)^7+1/64
*sinh(b*x+a)*cosh(b*x+a)^7-1/64*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*
cosh(b*x+a))*sinh(b*x+a)-5/1024*b*x-5/1024*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$$
$$=\frac{55\cosh(bx+a)^3\sinh(bx+a)^9+3\cosh(bx+a)\sinh(bx+a)^{11}+18\left(11\cosh(bx+a)^5-\cosh(bx+a)\right)}{}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/6144*(55*cosh(b*x + a)^3*sinh(b*x + a)^9 + 3*cosh(b*x + a)*sinh(b*x + a)^
11 + 18*(11*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^7 + 18*(11*cosh(
b*x + a)^7 - 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + (55*cosh(b*x + a)^9 - 126
*cosh(b*x + a)^5 + 45*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x
 + a)^11 - 6*cosh(b*x + a)^7 + 15*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(121) = 242.

Time = 2.62 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$$
$$=\begin{cases}-\frac{5x\sinh^{12}(a+bx)}{1024}+\frac{15x\sinh^{10}(a+bx)\cosh^2(a+bx)}{512}-\frac{75x\sinh^8(a+bx)\cosh^4(a+bx)}{1024}+\frac{25x\sinh^6(a+bx)\cosh^6(a+bx)}{256}-\frac{75x\sinh^4(a-}{} \\ x\sinh^6(a)\cosh^6(a)\end{cases}$$

```
[In] integrate(cosh(b*x+a)**6*sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**12/1024 + 15*x*sinh(a + b*x)**10*cosh(a + b*
x)**2/512 - 75*x*sinh(a + b*x)**8*cosh(a + b*x)**4/1024 + 25*x*sinh(a + b*x
)**6*cosh(a + b*x)**6/256 - 75*x*sinh(a + b*x)**4*cosh(a + b*x)**8/1024 + 1
5*x*sinh(a + b*x)**2*cosh(a + b*x)**10/512 - 5*x*cosh(a + b*x)**12/1024 + 5
*sinh(a + b*x)**11*cosh(a + b*x)/(1024*b) - 85*sinh(a + b*x)**9*cosh(a + b*
x)**3/(3072*b) + 33*sinh(a + b*x)**7*cosh(a + b*x)**5/(512*b) + 33*sinh(a +
 b*x)**5*cosh(a + b*x)**7/(512*b) - 85*sinh(a + b*x)**3*cosh(a + b*x)**9/(3
072*b) + 5*sinh(a + b*x)*cosh(a + b*x)**11/(1024*b), Ne(b, 0)), (x*sinh(a)*
*6*cosh(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{\left(9\,e^{(-4\,bx-4\,a)} - 45\,e^{(-8\,bx-8\,a)} - 1\right)e^{(12\,bx+12\,a)}}{49152\,b} - \frac{5\,(bx+a)}{1024\,b} - \frac{45\,e^{(-4\,bx-4\,a)} - 9\,e^{(-8\,bx-8\,a)} + e^{(-12\,bx-12\,a)}}{49152\,b}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/49152*(9*e^(-4*b*x - 4*a) - 45*e^(-8*b*x - 8*a) - 1)*e^(12*b*x + 12*a)/b
 - 5/1024*(b*x + a)/b - 1/49152*(45*e^(-4*b*x - 4*a) - 9*e^(-8*b*x - 8*a) +
 e^(-12*b*x - 12*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{5}{1024}\,x + \frac{e^{(12\,bx+12\,a)}}{49152\,b} - \frac{3\,e^{(8\,bx+8\,a)}}{16384\,b} + \frac{15\,e^{(4\,bx+4\,a)}}{16384\,b} - \frac{15\,e^{(-4\,bx-4\,a)}}{16384\,b} + \frac{3\,e^{(-8\,bx-8\,a)}}{16384\,b} - \frac{e^{(-12\,bx-12\,a)}}{49152\,b}$$

```
[In] integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/1024*x + 1/49152*e^(12*b*x + 12*a)/b - 3/16384*e^(8*b*x + 8*a)/b + 15/16
384*e^(4*b*x + 4*a)/b - 15/16384*e^(-4*b*x - 4*a)/b + 3/16384*e^(-8*b*x - 8
*a)/b - 1/49152*e^(-12*b*x - 12*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.31

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{\frac{15\sinh(4\,a+4\,b\,x)}{8192}-\frac{3\sinh(8\,a+8\,b\,x)}{8192}+\frac{\sinh(12\,a+12\,b\,x)}{24576}}{b}-\frac{5\,x}{1024}$$

```
[In] int(cosh(a + b*x)^6*sinh(a + b*x)^6,x)
```

```
[Out] ((15*sinh(4*a + 4*b*x))/8192 - (3*sinh(8*a + 8*b*x))/8192 + sinh(12*a + 12*
b*x)/24576)/b - (5*x)/1024
```

3.24 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log(\tanh(a+bx))}{b}$$

```
[Out] ln(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2700, 29}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Log[Tanh[a + b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = 2\left(-\frac{\log(\cosh(a+bx))}{2b} + \frac{\log(\sinh(a+bx))}{2b}\right)$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x],x]
[Out] 2*(-1/2*Log[Cosh[a + b*x]]/b + Log[Sinh[a + b*x]]/(2*b))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tanh(bx+a))}{b}$	12
default	$\frac{\ln(\tanh(bx+a))}{b}$	12
risch	$\frac{\ln\left(e^{2bx+2a}-1\right)}{b} - \frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	35

```
[In] int(csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right) - \log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
[Out] -(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x + a
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x)
[Out] Integral(csch(a + b*x)*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(2\,bx+2\,a)}+1\right) - \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] -(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1
)))/b
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\,\operatorname{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
[Out] -(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.25 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2702, 327, 213}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{\operatorname{sech}(a+bx)}{b}-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= \frac{\text{sech}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= -\frac{\text{arctanh}(\cosh(a+bx))}{b} + \frac{\text{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
[Out] -(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{1}{\cosh(bx+a)}-2\,\text{arctanh}\left(e^{bx+a}\right)}{b}$	23
default	$\frac{\frac{1}{\cosh(bx+a)}-2\,\text{arctanh}\left(e^{bx+a}\right)}{b}$	23
risch	$\frac{2\,e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}-\frac{\ln\left(e^{bx+a}+1\right)}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}$	53

```
[In] int(csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(23) = 46.

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+\right.}{b\cosh\left(bx+\right.}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(23) = 46.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}+1)}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^(
-2*b*x - 2*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}$$

[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")

[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2) + log(e^(b*x + a) + e^(-b*x - a) - 2))/b

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)

[Out] (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)

3.26 $\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2700, 14}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{2\log(\cosh(a+bx)) - 2\log(\sinh(a+bx)) - \text{sech}^2(a+bx)}{2b}$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
[Out] -1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
default	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
risch	$\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2} - \frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b} + \frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	62

```
[In] int(csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 13.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 \right.\right.}{}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)
*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x + a
)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)
*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2bx-2a)}+1\right)}{b} + \frac{2e^{(-2bx-2a)}}{b(2e^{(-2bx-2a)}+e^{(-4bx-4a)}+1)}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

3.27 $\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2702, 308, 213}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b + Sech[a + b*x]^3/(3*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \text{sech}(a+bx)\right)}{b}\\
&= \frac{\text{sech}(a+bx)}{b}+\frac{\text{sech}^3(a+bx)}{3b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\
&= -\frac{\text{arctanh}(\cosh(a+bx))}{b}+\frac{\text{sech}(a+bx)}{b}+\frac{\text{sech}^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \text{csch}(a+bx)\text{sech}^4(a+bx)\,dx = -\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\text{sech}(a+bx)}{b}+\frac{\text{sech}^3(a+bx)}{3b}$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^4,x]
[Out] -(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b +
Sech[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 7.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	33
default	$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	33
risch	$\frac{2\,\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+10\,\mathrm{e}^{2bx+2a}+3\right)}{3b\left(1+\mathrm{e}^{2bx+2a}\right)^3}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	77

```
[In] int(csch(b*x+a)*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3/cosh(b*x+a)^3+1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 697, normalized size of antiderivative = 18.34

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$$
$$= \frac{6\,\cosh(bx+a)^5+30\,\cosh(bx+a)\sinh(bx+a)^4+6\,\sinh(bx+a)^5+20\left(3\,\cosh(bx+a)^2+1\right)\sinh(bx+$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a)
^5 + 20*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 20*cosh(b*x + a)^3 + 60*(
cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(c
osh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a
) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5
 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*si
nh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3
```

```
 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x +
 a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**4,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.84

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\left(3e^{(-bx-a)}+10e^{(-3bx-3a)}+3e^{(-5bx-5a)}\right)}{3b(3e^{(-2bx-2a)}+3e^{(-4bx-4a)}+e^{(-6bx-6a)}+1)}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2/3*(3*e^(-b*x - a) +
10*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*
b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}-3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{6b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
[Out] 1/6*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))^
3 - 3*log(e^(b*x + a) + e^(-b*x - a) + 2) + 3*log(e^(b*x + a) + e^(-b*x - a
) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} + \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)),x)
```

```
[Out] (8*exp(a + b*x))/(3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*ata
n((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(a + b*x))/(3*b*(
3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (2*exp(a
 + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.28 $\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{b}+\frac{\tanh^4(a+bx)}{4b}$$

```
[Out] ln(tanh(b*x+a))/b-tanh(b*x+a)^2/b+1/4*tanh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}-\frac{\tanh^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/b + Tanh[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x}+x\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{b} + \frac{\tanh^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{4\log(\cosh(a+bx)) - 4\log(\sinh(a+bx)) - 2\operatorname{sech}^2(a+bx) - \operatorname{sech}^4(a+bx)}{4b}$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^5,x]
[Out] -1/4*(4*Log[Cosh[a + b*x]] - 4*Log[Sinh[a + b*x]] - 2*Sech[a + b*x]^2 - Sec
h[a + b*x]^4)/b
```

Maple [A] (verified)

Time = 21.98 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{1}{4\cosh(bx+a)^4}+\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
default	$\frac{\frac{1}{4\cosh(bx+a)^4}+\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
risch	$\frac{2\,e^{2bx+2a}\left(e^{4bx+4a}+4\,e^{2bx+2a}+1\right)}{b\left(1+e^{2bx+2a}\right)^4}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}-\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	84

```
[In] int(csch(b*x+a)*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4/cosh(b*x+a)^4+1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 1073, normalized size of antiderivative = 26.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
[Out] (2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6 +
 2*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 8*(5*cosh
(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 + 24
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x + a)^
8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^
2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*cosh(
b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 + 3)
*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 15*cosh(b
*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b*x + a))*s
inh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (c
osh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*
x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 +
10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^
6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*
x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cos
h(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 4*(3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*
x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^
```

```
6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(
b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x
 + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cos
h(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a
)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x
 + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.28

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\left(e^{(-2\,bx-2\,a)}+4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1)}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*(e^(-2*b*x - 2*a) + 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e
^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a
) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.05

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2+20\,e^{(2bx+2a)}+20\,e^{(-2bx-2a)}+44}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^2} - 2\,\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+2\,\log\left(e^{(2bx+2a)}+\right.}{4\,b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 20*e^(2*b*x + 2*a) + 20*e^
(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)^2 - 2*log(e^(
2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 2*log(e^(2*b*x + 2*a) + e^(-2*b*x -
2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.22

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} + \frac{4}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) + 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - 8/(b*
(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + 4/(b*(4
*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b
*x) + 1))
```

3.29 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2701, 327, 213}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x],x]
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= -\frac{\text{csch}(a+bx)}{b} - \frac{i\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= -\frac{\arctan(\sinh(a+bx))}{b} - \frac{\text{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{\text{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\sinh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	58

```
[In] int(csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(24) = 48.

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\arctan(\cosh(bx+a)+\sinh(bx}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
[Out] -2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(b*
cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a),x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{2e^{(-bx-a)}}{b(e^{(-2bx-2a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
[Out] 2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\pi + \frac{4}{e^{(bx+a)}-e^{(-bx-a)}} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{2b}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
[Out] -1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a) -
 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+bx}}{b\,\left(e^{2a+2bx}-1\right)}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(2*a + 2*b*x) - 1))
```

3.30 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{b}-\frac{\tanh(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b-tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 14}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b} \\
&= \frac{i\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, i\tanh(a+bx)\right)}{b} \\
&= -\frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{2\coth(2(a+bx))}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
[Out] (-2*Coth[2*(a + b*x)])/b
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
risch	$-\frac{4}{b\left(\mathrm{e}^{2bx+2a}-1\right)\left(1+\mathrm{e}^{2bx+2a}\right)}$	32

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\cosh\left(bx+a\right)^4 + 4\,b\cosh\left(bx+a\right)^3\sinh\left(bx+a\right) + 6\,b\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2 + 4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + b\sinh\left(bx+a\right)^4 - b}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{4}{b(e^{(-4\,bx-4\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/(b*(e^(-4*b*x - 4*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b(e^{(4\,bx+4\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] -4/(b*(e^(4*b*x + 4*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] -4/(b*(exp(4*a + 4*b*x) - 1))
```

3.31 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+a))/b-3/2*csch(b*x+a)/b+1/2*csch(b*x+a)*sech(b*x+a)^2/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2701, 294, 327, 213}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b} \\
&= \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b} \\
&= -\frac{3\text{csch}(a+bx)}{2b} + \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b} \\
&= -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\text{csch}(a+bx)}{2b} + \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)}{b}$	47
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)}{b}$	47
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}+2\,e^{2bx+2a}+3\right)}{b\left(1+e^{2bx+2a}\right)^2\left(e^{2bx+2a}-1\right)}+\frac{3i\ln\left(e^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(e^{bx+a}+i\right)}{2b}$	95

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp(
b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 511, normalized size of antiderivative = 10.43

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2+1\right)\sinh(b}{}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
[Out] -(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
+ 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*cos
```

```
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*cosh(b*x
+ a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(b*cosh(b*
x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + b*cosh(b
*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*
x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{3\,e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} + 1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
[Out] 3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*b
*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.08

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4e^{(bx+a)}-4e^{(-bx-a)}} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{4b}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-b
*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2*a
) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan(
(exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(2*
a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.32 $\int \text{csch}^2(a+bx)\text{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \text{csch}^2(a+bx)\text{sech}^4(a+bx)\,dx=-\frac{\coth(a+bx)}{b}-\frac{2\tanh(a+bx)}{b}+\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] -coth(b*x+a)/b-2*tanh(b*x+a)/b+1/3*tanh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 276}

$$\int \text{csch}^2(a+bx)\text{sech}^4(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}-\frac{2\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

```
[Out] -(Coth[a + b*x]/b) - (2*Tanh[a + b*x])/b + Tanh[a + b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{i\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}-\frac{2\tanh(a+bx)}{b}+\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\coth(a+bx)}{b}-\frac{5\tanh(a+bx)}{3b}-\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
[Out] -(Coth[a + b*x]/b) - (5*Tanh[a + b*x])/(3*b) - (Sech[a + b*x]^2*Tanh[a + b*
x])/(3*b)
```

Maple [A] (verified)

Time = 16.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	44
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	44
risch	$-\frac{16\left(2\,e^{2bx+2a}+1\right)}{3b\left(e^{2bx+2a}-1\right)\left(1+e^{2bx+2a}\right)^3}$	45

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^3-4*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 6.05

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+2\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\right.\right.}$$

[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="fricas")

```
[Out] -16/3*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x +
 a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cosh(
b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*x +
 a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^2)*sinh(b
*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3)*sinh(b*x + a)^2 -
 3*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 - b)*sinh(
b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**4,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(36) = 72.

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1)} - \frac{16}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}+1)}$$

[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="maxima")

```
[Out] -32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8*
b*x - 8*a) + 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8
*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}-1}-\frac{3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}+1\right)^3}\right)}{3\,b}$$

[In] integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="giac")

[Out] -2/3*(3/(e^(2*b*x + 2*a) - 1) - (3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 5
)/(e^(2*b*x + 2*a) + 1)^3)/b

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{2}{3\,b}+\frac{4\,e^{2\,a+2\,b\,x}}{b}+\frac{2\,e^{4\,a+4\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1}+\frac{\frac{2}{b}+\frac{2\,e^{2\,a+2\,b\,x}}{3\,b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}-\frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{2}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^2),x)

[Out] (2/(3*b) + (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
+ 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1) + (2/b + (2*exp(2*a +
2*b*x))/(3*b))/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1) - 2/(b*(exp(2*a
+ 2*b*x) - 1)) + 2/(3*b*(exp(2*a + 2*b*x) + 1))

3.33 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{15\arctan(\sinh(a+bx))}{8b} - \frac{15\operatorname{csch}(a+bx)}{8b} + \frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b}$$

```
[Out] -15/8*arctan(sinh(b*x+a))/b-15/8*csch(b*x+a)/b+5/8*csch(b*x+a)*sech(b*x+a)^
2/b+1/4*csch(b*x+a)*sech(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2701, 294, 327, 213}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{15\arctan(\sinh(a+bx))}{8b} - \frac{15\operatorname{csch}(a+bx)}{8b} + \frac{\operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{5\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{8b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

```
[Out] (-15*ArcTan[Sinh[a + b*x]])/(8*b) - (15*Csch[a + b*x])/(8*b) + (5*Csch[a +
b*x]*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]*Sech[a + b*x]^4)/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\
&= \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b} - \frac{(5i)\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, -i\text{csch}(a+bx)\right)}{4b}\\
&= \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b}\\
&\quad - \frac{(15i)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{8b}\\
&= -\frac{15\text{csch}(a+bx)}{8b} + \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b}\\
&\quad + \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b} - \frac{(15i)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{8b}
\end{aligned}$$

$$= -\frac{15\arctan(\sinh(a+bx))}{8b} - \frac{15\text{csch}(a+bx)}{8b} + \frac{5\text{csch}(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}(a+bx)\text{sech}^4(a+bx)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \text{csch}^2(a+bx)\text{sech}^5(a+bx)\,dx = -\frac{\text{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 39.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^4}-5\left(\frac{\text{sech}(bx+a)^3}{4}+\frac{3\,\text{sech}(bx+a)}{8}\right)\tanh(bx+a)-\frac{15\arctan\left(e^{bx+a}\right)}{4}}{b}$	60
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^4}-5\left(\frac{\text{sech}(bx+a)^3}{4}+\frac{3\,\text{sech}(bx+a)}{8}\right)\tanh(bx+a)-\frac{15\arctan\left(e^{bx+a}\right)}{4}}{b}$	60
risch	$-\frac{e^{bx+a}\left(15\,e^{8bx+8a}+40\,e^{6bx+6a}+18\,e^{4bx+4a}+40\,e^{2bx+2a}+15\right)}{4b\left(1+e^{2bx+2a}\right)^4\left(e^{2bx+2a}-1\right)}+\frac{15i\ln\left(e^{bx+a}-i\right)}{8b}-\frac{15i\ln\left(e^{bx+a}+i\right)}{8b}$	117

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^4-5*(1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*tan
h(b*x+a)-15/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 1183, normalized size of antiderivative = 16.90

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
+ a)^9 + 20*(27*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7 +
 140*(9*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^6 + 6*(315*cosh(b*
x + a)^4 + 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 18*cosh(b*x + a)^5 +
10*(189*cosh(b*x + a)^5 + 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x +
 a)^4 + 20*(63*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 2
)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 60*(9*cosh(b*x + a)^7 + 14*cosh(b*
x + a)^5 + 3*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(
b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + 3*(15*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(5*cosh(b*x +
a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 42*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh(b*x + a)^5
+ 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a
)^6 + 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cos
h(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 21*cosh(b*x + a)^5 + 5*cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8 + 28*cosh(b*x +
 a)^6 + 10*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cos
h(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 6*cosh(b*x + a)^
5 - 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x
 + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 + 56*cosh(b*x + a)^6 + 18*co
sh(b*x + a)^4 + 24*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 15*cosh(b*x + a))/(
b*cosh(b*x + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^1
0 + 3*b*cosh(b*x + a)^8 + 3*(15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 24
*(5*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 + 2*b*cosh(b*x + a
)^6 + 2*(105*b*cosh(b*x + a)^4 + 42*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6
+ 12*(21*b*cosh(b*x + a)^5 + 14*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a)^5 - 2*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 + 105*b*cosh(b*x
 + a)^4 + 15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)
^7 + 21*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
 + a)^3 - 3*b*cosh(b*x + a)^2 + 3*(15*b*cosh(b*x + a)^8 + 28*b*cosh(b*x + a
)^6 + 10*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(
5*b*cosh(b*x + a)^9 + 12*b*cosh(b*x + a)^7 + 6*b*cosh(b*x + a)^5 - 4*b*cosh
(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}^2{(a+bx)}\operatorname{sech}^5{(a+bx)}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**5,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\begin{aligned}&\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx\\&=\frac{15\arctan\left(e^{(-bx-a)}\right)}{4\,b}\\&\quad-\frac{15\,e^{(-bx-a)}+40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}+40\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{4\,b(3\,e^{(-2\,bx-2\,a)}+2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}-e^{(-10\,bx-10\,a)}+1)}\end{aligned}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 15/4*arctan(e^(-b*x - a))/b - 1/4*(15*e^(-b*x - a) + 40*e^(-3*b*x - 3*a) +
18*e^(-5*b*x - 5*a) + 40*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(3*e^(-
2*b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a)
 - e^(-10*b*x - 10*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx=-\frac{15\,\pi+\frac{4\left(7\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2}+\frac{32}{e^{(bx+a)}-e^{(-bx-a)}}+30\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/16*(15*pi + 4*(7*(e^(b*x + a) - e^(-b*x - a))^3 + 36*e^(b*x + a) - 36*e^
(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32/(e^(b*x + a) - e^(-
b*x - a)) + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{3\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{15\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}-\frac{4\,\mathrm{e}^{a+b\,x}}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{7\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^2),x)
```

```
[Out] (3*exp(a + b*x))/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (15*at
an((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (6*exp(a + b*x))/(b*
(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*exp(
a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x)
+ exp(8*a + 8*b*x) + 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1)) - (7
*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.34 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2700, 14}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b - Log[Tanh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{\text{csch}^2(a+bx) - 2\log(\cosh(a+bx)) + 2\log(\sinh(a+bx))}{2b}$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
[Out] -1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
default	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
risch	$-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{2bx+2a}-1\right)}{b}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	62

```
[In] int(csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sinh(b*x+a)^2-ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 379, normalized size of antiderivative = 13.54

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+\right.\right.}{}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x +
a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a
)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b
*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*si
nh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2}+\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)-\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*a
 + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.35 $\int \mathbf{csch}^3(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(cosh(b*x+a))/b-3/2*sech(b*x+a)/b-1/2*csch(b*x+a)^2*sech(b*x+a)/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2702, 294, 327, 213}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\
&= -\frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b}\\
&= -\frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b}\\
&= \frac{3\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b + (3*Log[Cosh[(a + b*x)/2]])/(2*b) - (3*Log[Sinh
[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	43
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}-2\,e^{2bx+2a}+3\right)}{b\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)}-\frac{3\ln\left(e^{bx+a}-1\right)}{2b}+\frac{3\ln\left(e^{bx+a}+1\right)}{2b}$	91

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(43) = 86$.

Time = 0.26 (sec) , antiderivative size = 709, normalized size of antiderivative = 14.47

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 4\left(15\cosh(bx+a)^2 - 1\right)\sinh(}{}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 6*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4
+ 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x +
a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2
*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a
) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(43) = 86.

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{3\log\left(e^{(-bx-a)}+1\right)}{2b}-\frac{3\log\left(e^{(-bx-a)}-1\right)}{2b}+\frac{3e^{(-bx-a)}-2e^{(-3bx-3a)}+3e^{(-5bx-5a)}}{b\left(e^{(-2bx-2a)}+e^{(-4bx-4a)}-e^{(-6bx-6a)}-1\right)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
[Out] 3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a)
 - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*b*
x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4\,e^{(bx+a)}-4\,e^{(-bx-a)}} - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a)
)^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a) +
2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.36 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}+\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b} \\
&= -\frac{\coth^2(a+bx)}{2b} - \frac{2\log(\tanh(a+bx))}{b} + \frac{\tanh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = 8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b} + \frac{\log(\cosh(a+bx))}{4b} - \frac{\log(\sinh(a+bx))}{4b} - \frac{\operatorname{sech}^2(a+bx)}{16b}\right)$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
[Out] 8*(-1/16*Csch[a + b*x]^2/b + Log[Cosh[a + b*x]]/(4*b) - Log[Sinh[a + b*x]]/
(4*b) - Sech[a + b*x]^2/(16*b))
```

Maple [A] (verified)

Time = 11.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{2}}-\frac{1}{\cosh(bx+a)^{2}}-2\ln(\tanh(bx+a))}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{2}}-\frac{1}{\cosh(bx+a)^{2}}-2\ln(\tanh(bx+a))}{b}$	43
risch	$-\frac{4\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}+\frac{2\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}-\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	87

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^2-1/cosh(b*x+a)^2-2*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 774, normalized size of antiderivative = 18.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
[Out] -2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b
*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)
*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^8 +
56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8
*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 -
1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x + a)^5 + cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x + a)^3*sinh(b*x +
a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x + a)*sinh(b*x +
a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 -
b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a
)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b*
cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{2\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{2\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\log\left(e^{(-2bx-2a)}+1\right)}{b} + \frac{4\left(e^{(-2bx-2a)}+e^{(-6bx-6a)}\right)}{b(2e^{(-4bx-4a)}-e^{(-8bx-8a)}-1)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4*a
) - e^(-8*b*x - 8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(39) = 78$.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-4} - \log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right) + \log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*
a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x + 2
*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)} - \frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] (4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8*b
*x) - 2*exp(4*a + 4*b*x) + 1))
```

3.37 $\int \mathrm{csch}^3(a+bx)\mathrm{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \mathrm{csch}^3(a+bx)\mathrm{sech}^4(a+bx)\,dx = \frac{5\,\mathrm{arctanh}(\cosh(a+bx))}{2b} - \frac{5\,\mathrm{sech}(a+bx)}{2b} - \frac{5\,\mathrm{sech}^3(a+bx)}{6b} - \frac{\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)}{2b}$$

```
[Out] 5/2*arctanh(cosh(b*x+a))/b-5/2*sech(b*x+a)/b-5/6*sech(b*x+a)^3/b-1/2*csch(b
*x+a)^2*sech(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2702, 294, 308, 213}

$$\int \mathrm{csch}^3(a+bx)\mathrm{sech}^4(a+bx)\,dx = \frac{5\,\mathrm{arctanh}(\cosh(a+bx))}{2b} - \frac{5\,\mathrm{sech}^3(a+bx)}{6b} - \frac{5\,\mathrm{sech}(a+bx)}{2b} - \frac{\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

```
[Out] (5*ArcTanh[Cosh[a + b*x]])/(2*b) - (5*Sech[a + b*x])/(2*b) - (5*Sech[a + b*
x]^3)/(6*b) - (Csch[a + b*x]^2*Sech[a + b*x]^3)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{b}\\
&= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= -\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= -\frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}\\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(a+bx)\right)}{2b}\\
&= \frac{5\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{5\operatorname{sech}(a+bx)}{2b} - \frac{5\operatorname{sech}^3(a+bx)}{6b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{2\operatorname{sech}(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b + (5*Log[Cosh[(a + b*x)/2]])/(2*b) - (5*Log[Sinh
[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - (2*Sech[a + b*x])/b - S
ech[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 30.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^3}-\frac{5}{6\cosh(bx+a)^3}-\frac{5}{2\cosh(bx+a)}+5\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	53
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^3}-\frac{5}{6\cosh(bx+a)^3}-\frac{5}{2\cosh(bx+a)}+5\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	53
risch	$-\frac{e^{bx+a}\left(15\,e^{8bx+8a}+20\,e^{6bx+6a}-22\,e^{4bx+4a}+20\,e^{2bx+2a}+15\right)}{3b\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)^3}-\frac{5\ln\left(e^{bx+a}-1\right)}{2b}+\frac{5\ln\left(e^{bx+a}+1\right)}{2b}$	113

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^3-5/6/cosh(b*x+a)^3-5/2/cosh(b*x+a)+5*a
rctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1573 vs. $2(58) = 116$.

Time = 0.25 (sec) , antiderivative size = 1573, normalized size of antiderivative = 23.83

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
+ a)^9 + 40*(27*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7 +
```

280*(9*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^6 + 4*(945*cosh(b*x + a)^4 + 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 44*cosh(b*x + a)^5 + 20*(189*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^4 + 40*(63*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 40*(27*cosh(b*x + a)^7 + 21*cosh(b*x + a)^5 - 11*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 + 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 + 28*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 + 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 + 28*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 10*(27*cosh(b*x + a)^8 + 28*cosh(b*x + a)^6 - 22*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 30*cosh(b*x + a))/(b*cosh(b*x + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 + b*cosh(b*x + a)^8 + (45*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 8*(15*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^6 + 2*(105*b*cosh(b*x + a)^4 + 14*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 4*(63*b*cosh(b*x + a)^5 + 14*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - 2*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 + 35*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)^5 - 5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (45*b*cosh(b*x + a)^8 + 28*b*cosh(b*x + a)^6 - 30*b*cosh(b*x + a)^4 - 12*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^9 + 4*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 - 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**4,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(58) = 116$.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\log\left(e^{(-bx-a)}+1\right)}{2b} - \frac{5\log\left(e^{(-bx-a)}-1\right)}{2b} - \frac{15e^{(-bx-a)}+20e^{(-3bx-3a)}-22e^{(-5bx-5a)}+20e^{(-7bx-7a)}+15e^{(-9bx-9a)}}{3b\left(e^{(-2bx-2a)}-2e^{(-4bx-4a)}-2e^{(-6bx-6a)}+e^{(-8bx-8a)}+e^{(-10bx-10a)}+1\right)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="maxima")
[Out] 5/2*log(e^(-b*x - a) + 1)/b - 5/2*log(e^(-b*x - a) - 1)/b - 1/3*(15*e^(-b*x
 - a) + 20*e^(-3*b*x - 3*a) - 22*e^(-5*b*x - 5*a) + 20*e^(-7*b*x - 7*a) + 1
5*e^(-9*b*x - 9*a))/(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x
 - 6*a) + e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(58) = 116$.

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\frac{12\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + \frac{16\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3} - 15\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 15\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{12b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="giac")
[Out] -1/12*(12*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
 + 16*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 2)/(e^(b*x + a) + e^(-b*x - a))^3
 - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x -
a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.91

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+bx}}{b\,\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)} - \frac{8\,e^{a+bx}}{3\,b\,\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)} + \frac{8\,e^{a+bx}}{3\,b\,\left(3\,e^{2a+2bx}+3\,e^{4a+4bx}+e^{6a+6bx}+1\right)} - \frac{e^{a+bx}}{b\,\left(e^{2a+2bx}-1\right)} - \frac{4\,e^{a+bx}}{b\,\left(e^{2a+2bx}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^3),x)
```

```
[Out] (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp(2*
a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - exp(a + b*x)/(b*
(exp(2*a + 2*b*x) - 1)) - (4*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.38 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\tanh^4(a+bx)}{4b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-3*ln(tanh(b*x+a))/b+3/2*tanh(b*x+a)^2/b-1/4*tanh(b*x+a
)^4/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\tanh^4(a+bx)}{4b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b - (3*Log[Tanh[a + b*x]])/b + (3*Tanh[a + b*x]^2)/(2*
b) - Tanh[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\ &= -\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\ &= -\frac{\coth^2(a+bx)}{2b}-\frac{3\log(\tanh(a+bx))}{b}+\frac{3\tanh^2(a+bx)}{2b}-\frac{\tanh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{2\operatorname{csch}^2(a+bx)-12\log(\cosh(a+bx))+12\log(\sinh(a+bx))+4\operatorname{sech}^2(a+bx)+\operatorname{sech}^4(a+bx)}{4b}$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
[Out] -1/4*(2*Csch[a + b*x]^2 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[a + b*x]] + 4
*Sech[a + b*x]^2 + Sech[a + b*x]^4)/b
```

Maple [A] (verified)

Time = 97.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{4}}-\frac{3}{4\cosh(bx+a)^{4}}-\frac{3}{2\cosh(bx+a)^{2}}-3\ln(\tanh(bx+a))}{b}$	53
default	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{4}}-\frac{3}{4\cosh(bx+a)^{4}}-\frac{3}{2\cosh(bx+a)^{2}}-3\ln(\tanh(bx+a))}{b}$	53
risch	$-\frac{2\,e^{2bx+2a}\left(3\,e^{8bx+8a}+6\,e^{6bx+6a}-2\,e^{4bx+4a}+6\,e^{2bx+2a}+3\right)}{b\left(1+e^{2bx+2a}\right)^{4}\left(e^{2bx+2a}-1\right)^{2}}+\frac{3\ln\left(1+e^{2bx+2a}\right)}{b}-\frac{3\ln\left(e^{2bx+2a}-1\right)}{b}$	122

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^4-3/4/cosh(b*x+a)^4-3/2/cosh(b*x+a)^2-3
*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 2103, normalized size of antiderivative = 36.26

$$\int \mathrm{csch}^3(a+bx)\mathrm{sech}^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="fricas")
[Out] -(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^1
0 + 6*(45*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^8 + 12*cosh(b*x + a)^8 + 48*(1
5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x + a)
^4 + 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(63*c
osh(b*x + a)^5 + 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 + 12*(
105*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^4 + 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b*x + a)
^8 + 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*si
nh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 -
 cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 + 30*cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 105*cosh(b*x + a)^4 - 7*cosh
(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^
7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)
^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 60*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)
```

^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 + 45*cosh(b*x + a)^8 - 14*
cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 + 5*cosh(b*x + a)^9 - 2*cosh
(b*x + a)^7 - 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
 + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b
*x + a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x
 + a)^3 + cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 +
30*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^
6 + 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b
x + a)^6 + 8(99*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x +
a)^6 - 70*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(
b*x + a)^4 + 4*(55*cosh(b*x + a)^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^
5 - 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x +
a)^10 + 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)
^11 + 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 - cosh(b*x
+ a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x +
a) - sinh(b*x + a))) + 12*(5*cosh(b*x + a)^9 + 8*cosh(b*x + a)^7 - 2*cosh(b
*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x +
 a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11 + b*sinh(b*x + a)^12 + 2*b*cos
h(b*x + a)^10 + 2*(33*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^10 + 20*(11*b*co
sh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^9 - b*cosh(b*x + a)^8 + (495
*b*cosh(b*x + a)^4 + 90*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(99*b*co
sh(b*x + a)^5 + 30*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 - 4
*b*cosh(b*x + a)^6 + 4*(231*b*cosh(b*x + a)^6 + 105*b*cosh(b*x + a)^4 - 7*b
*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 8*(99*b*cosh(b*x + a)^7 + 63*b*cosh
(b*x + a)^5 - 7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - b*
cosh(b*x + a)^4 + (495*b*cosh(b*x + a)^8 + 420*b*cosh(b*x + a)^6 - 70*b*cos
h(b*x + a)^4 - 60*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(55*b*cosh(b*x
 + a)^9 + 60*b*cosh(b*x + a)^7 - 14*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^
3 - b*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + 2*(33*b*cosh(b
*x + a)^10 + 45*b*cosh(b*x + a)^8 - 14*b*cosh(b*x + a)^6 - 30*b*cosh(b*x +
a)^4 - 3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(3*b*cosh(b*x + a)^11 +
 5*b*cosh(b*x + a)^9 - 2*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 - b*cosh(b
*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**5,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.12

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= -\frac{3\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{3\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{3\log\left(e^{(-2bx-2a)}+1\right)}{b} - \frac{2\left(3e^{(-2bx-2a)}+6e^{(-4bx-4a)}-2e^{(-6bx-6a)}+6e^{(-8bx-8a)}+3e^{(-10bx-10a)}\right)}{b(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-4e^{(-6bx-6a)}-e^{(-8bx-8a)}+2e^{(-10bx-10a)}+e^{(-12bx-12a)}+1)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="maxima")
[Out] -3*log(e^(-b*x - a) + 1)/b - 3*log(e^(-b*x - a) - 1)/b + 3*log(e^(-2*b*x -
2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*
a) + 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) - e^
(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + 2*e^(-10*b*x - 10*
a) + e^(-12*b*x - 12*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{\frac{2\left(3e^{(2bx+2a)}+3e^{(-2bx-2a)}-10\right)}{e^{(2bx+2a)}+e^{(-2bx-2a)}-2} - \frac{9\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2+52e^{(2bx+2a)}+52e^{(-2bx-2a)}+84}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^2} + 6\log\left(e^{(2bx+2a)}+e^{(-2bx-2}\right.}{4b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="giac")
[Out] 1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) - 10)/(e^(2*b*x + 2*a) + e^(
-2*b*x - 2*a) - 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 52*e^(2*b*
x + 2*a) + 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) +
2)^2 + 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 6*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.22

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{6\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} + \frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} - \frac{4}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^3),x)
```

```
[Out] (6*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 4/(b*(exp(2*a
 + 2*b*x) + 1)) - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2
*exp(2*a + 2*b*x) + 1)) + 8/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + e
xp(6*a + 6*b*x) + 1)) - 4/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*e
xp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.39 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}+\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

```
[Out] arctan(sinh(b*x+a))/b+csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2701, 308, 213}

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b + Csch[a + b*x]/b - Csch[a + b*x]^3/(3*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= \frac{i\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= \frac{\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{i\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= \frac{\arctan(\sinh(a+bx))}{b}+\frac{\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \text{csch}^4(a+bx)\text{sech}(a+bx)\,dx = -\frac{\text{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x],x]
[Out] -1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Sinh[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^{3}}+\frac{1}{\sinh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	33
default	$\frac{-\frac{1}{3\sinh(bx+a)^{3}}+\frac{1}{\sinh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{2\,e^{bx+a}\left(3\,e^{4bx+4a}-10\,e^{2bx+2a}+3\right)}{3b\left(e^{2bx+2a}-1\right)^{3}}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	82

```
[In] int(csch(b*x+a)^4*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3/sinh(b*x+a)^3+1/sinh(b*x+a)+2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 515, normalized size of antiderivative = 13.92

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\left(3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+10\left(3\cosh(bx+a)^2-1\right)\sinh(\right.}{}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="fricas")
[Out] 2/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)
^5 + 10*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 30*(
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*
cosh(b*x + a)^4 - 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/
(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
- 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*
b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^
2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(
b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) -
b)
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a),x)
[Out] Integral(csch(a + b*x)**4*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{2\left(3e^{(-bx-a)} - 10e^{(-3bx-3a)} + 3e^{(-5bx-5a)}\right)}{3b\left(3e^{(-2bx-2a)} - 3e^{(-4bx-4a)} + e^{(-6bx-6a)} - 1\right)}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="maxima")
[Out] -2*arctan(e^(-b*x - a))/b - 2/3*(3*e^(-b*x - a) - 10*e^(-3*b*x - 3*a) + 3*e
^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{3\pi + \frac{4\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 - 4\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)} - 1\right)e^{(-bx-a)}\right)}{6b}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="giac")
[Out] 1/6*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*x
 - a))^3 + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.49

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)} + \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^4),x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(3
*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*
exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (2*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.40 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{2\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

```
[Out] 2*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 276}

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{2\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

```
[Out] (2*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + Tanh[a + b*x]/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= -\frac{i\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{2\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{5\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
[Out] (5*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b) + Tanh[a +
b*x]/b
```

Maple [A] (verified)

Time = 9.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{16\left(2\,\mathrm{e}^{2bx+2a}-1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^{3}\left(1+\mathrm{e}^{2bx+2a}\right)}$	45
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^{3}\cosh(bx+a)}+\frac{4}{3\sinh(bx+a)\cosh(bx+a)}+\frac{8\tanh(bx+a)}{3}}{b}$	50
default	$\frac{-\frac{1}{3\sinh(bx+a)^{3}\cosh(bx+a)}+\frac{4}{3\sinh(bx+a)\cosh(bx+a)}+\frac{8\tanh(bx+a)}{3}}{b}$	50

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -16/3*(2*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^3/(1+exp(2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(35) = 70$.

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 6.19

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7-2\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\left(\right.\right.\right.}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -16/3*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x +
 a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5 + (21*b*cosh(
b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 - 2*b*cosh(b*x +
 a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2)*sinh(b
*x + a)^3 + (21*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^3)*sinh(b*x + a)^2 +
 b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 10*b*cosh(b*x + a)^4 + 3*b)*sinh(
b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^4\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{32\,e^{(-2\,bx-2\,a)}}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1)} - \frac{16}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*b
*x - 8*a) - 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*
b*x - 8*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}+1} - \frac{3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}-1\right)^3}\right)}{3\,b}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="giac")
[Out] -2/3*(3/(e^(2*b*x + 2*a) + 1) - (3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 5
)/(e^(2*b*x + 2*a) - 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.14

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{2}{3\,b} - \frac{4\,e^{2\,a+2\,b\,x}}{b} + \frac{2\,e^{4\,a+4\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x} - 3\,e^{4\,a+4\,b\,x} + e^{6\,a+6\,b\,x} - 1} - \frac{\frac{2}{b} - \frac{2\,e^{2\,a+2\,b\,x}}{3\,b}}{e^{4\,a+4\,b\,x} - 2\,e^{2\,a+2\,b\,x} + 1} + \frac{2}{3\,b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^4),x)
[Out] (2/(3*b) - (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
+ 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1) - (2/b - (2*exp(2*a +
 2*b*x))/(3*b))/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) + 2/(3*b*(exp(2
*a + 2*b*x) - 1)) - 2/(b*(exp(2*a + 2*b*x) + 1))
```

3.41 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} + \frac{5\operatorname{csch}(a+bx)}{2b} - \frac{5\operatorname{csch}^3(a+bx)}{6b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] 5/2*arctan(sinh(b*x+a))/b+5/2*csch(b*x+a)/b-5/6*csch(b*x+a)^3/b+1/2*csch(b*
x+a)^3*sech(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2701, 294, 308, 213}

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\operatorname{csch}^3(a+bx)}{6b} + \frac{5\operatorname{csch}(a+bx)}{2b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) + (5*Csch[a + b*x])/(2*b) - (5*Csch[a + b*x
]^3)/(6*b) + (Csch[a + b*x]^3*Sech[a + b*x]^2)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\
&= \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{2b} + \frac{(5i)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b}\\
&= \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{2b} + \frac{(5i)\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, -i\text{csch}(a+bx)\right)}{2b}\\
&= \frac{5\text{csch}(a+bx)}{2b} - \frac{5\text{csch}^3(a+bx)}{6b} + \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{2b}\\
&\quad + \frac{(5i)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b}\\
&= \frac{5\arctan(\sinh(a+bx))}{2b} + \frac{5\text{csch}(a+bx)}{2b} - \frac{5\text{csch}^3(a+bx)}{6b} + \frac{\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.50

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= -\frac{\operatorname{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
[Out] -1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, -Sinh[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 23.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^2}+\frac{5}{3\sinh(bx+a)\cosh(bx+a)^2}+\frac{5\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(e^{bx+a}\right)}{b}$	65
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^2}+\frac{5}{3\sinh(bx+a)\cosh(bx+a)^2}+\frac{5\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(e^{bx+a}\right)}{b}$	65
risch	$\frac{e^{bx+a}\left(15\,e^{8bx+8a}-20\,e^{6bx+6a}-22\,e^{4bx+4a}-20\,e^{2bx+2a}+15\right)}{3b\left(1+e^{2bx+2a}\right)^2\left(e^{2bx+2a}-1\right)^3}+\frac{5i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{5i\ln\left(e^{bx+a}-i\right)}{2b}$	117

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^2+5/3/sinh(b*x+a)/cosh(b*x+a)^2+5/2*sec
h(b*x+a)*tanh(b*x+a)+5*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. 2(58) = 116.

Time = 0.25 (sec) , antiderivative size = 1176, normalized size of antiderivative = 17.82

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="fricas")
[Out] 1/3*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x +
 a)^9 + 20*(27*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^7 - 20*cosh(b*x + a)^7 +
140*(9*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^6 + 2*(945*cosh(b*x +
 a)^4 - 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10
```

```
*(189*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a
)^4 + 20*(63*cosh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^3 - 20*cosh(b*x + a)^3 + 20*(27*cosh(b*x + a)^7 - 21*cosh(b*
x + a)^5 - 11*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh
(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + (45*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(15*cosh(b*x + a)^
3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 14*cosh(b*x +
 a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x + a)^5 - 14
*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x + a)^
6 - 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 2*cosh(b
*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6
- 30*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x +
 a)^2 + 2*(5*cosh(b*x + a)^9 - 4*cosh(b*x + a)^7 - 6*cosh(b*x + a)^5 + 4*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + si
nh(b*x + a)) + 5*(27*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6 - 22*cosh(b*x + a
)^4 - 12*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 15*cosh(b*x + a))/(b*cosh(b*x
 + a)^10 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 - b*cosh
(b*x + a)^8 + (45*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(15*b*cosh(b*x
 + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^6 + 2*(105*b
*cosh(b*x + a)^4 - 14*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 4*(63*b*cosh
(b*x + a)^5 - 14*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*b*cosh(b*x + a)^4 + 2*(105*b*cosh(b*x + a)^6 - 35*b*cosh(b*x + a)^4 - 15*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 - 7*b*cosh(
b*x + a)^5 - 5*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 + b*cos
h(b*x + a)^2 + (45*b*cosh(b*x + a)^8 - 28*b*cosh(b*x + a)^6 - 30*b*cosh(b*x
 + a)^4 + 12*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^
9 - 4*b*cosh(b*x + a)^7 - 6*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 + b*cos
h(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^4\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= -\frac{5 \arctan\left(e^{(-bx-a)}\right)}{b} - \frac{15\,e^{(-bx-a)} - 20\,e^{(-3\,bx-3\,a)} - 22\,e^{(-5\,bx-5\,a)} - 20\,e^{(-7\,bx-7\,a)} + 15\,e^{(-9\,bx-9\,a)}}{3\,b(e^{(-2\,bx-2\,a)} + 2\,e^{(-4\,bx-4\,a)} - 2\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -5*arctan(e^(-b*x - a))/b - 1/3*(15*e^(-b*x - a) - 20*e^(-3*b*x - 3*a) - 22
*e^(-5*b*x - 5*a) - 20*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(e^(-2*b*
x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + e^(
-10*b*x - 10*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{15\,\pi + \frac{12\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + \frac{16\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 - 2\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3} + 30 \arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{12\,b}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(15*pi + 12*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))
^2 + 4) + 16*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 2)/(e^(b*x + a) - e^(-b*x
- a))^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} + \frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^4),x)
```

```
[Out] (5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(3
*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(2*ex
p(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2*a
+ 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (4*exp(a + b*x))/(
b*(exp(2*a + 2*b*x) - 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.42 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx=\frac{3\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{3\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] 3*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+3*tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 276}

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\tanh^3(a+bx)}{3b}+\frac{3\tanh(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{3\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

```
[Out] (3*Coth[a + b*x])/b - Coth[a + b*x]^3/(3*b) + (3*Tanh[a + b*x])/b - Tanh[a
+ b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4}\,dx, x, i\tanh(a+bx)\right)}{b} \\
&= -\frac{i\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)\,dx, x, i\tanh(a+bx)\right)}{b} \\
&= \frac{3\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} + \frac{3\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \text{csch}^4(a+bx)\text{sech}^4(a+bx)\,dx = 16\left(\frac{\coth(2(a+bx))}{3b} - \frac{\coth(2(a+bx))\text{csch}^2(2(a+bx))}{6b}\right)$$

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
[Out] 16*(Coth[2*(a + b*x)]/(3*b) - (Coth[2*(a + b*x)]*Csch[2*(a + b*x)]^2)/(6*b)
)
```

Maple [A] (verified)

Time = 73.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{32(3\,e^{4bx+4a}-1)}{3b(1+e^{2bx+2a})^3(e^{2bx+2a}-1)^3}$	45
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^3}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^3}+8\left(\frac{2}{3}+\frac{\text{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	62
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^3}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^3}+8\left(\frac{2}{3}+\frac{\text{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	62

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -32/3*(3*exp(4*b*x+4*a)-1)/b/(1+exp(2*b*x+2*a))^3/(exp(2*b*x+2*a)-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

Time = 0.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 6.23

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{10}+120\,b\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{7}+45\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{8}+10\,b\cosh\left(\right.\right.}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -64/3*(cosh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(
b*cosh(b*x + a)^10 + 120*b*cosh(b*x + a)^3*sinh(b*x + a)^7 + 45*b*cosh(b*x
+ a)^2*sinh(b*x + a)^8 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x +
a)^10 - 3*b*cosh(b*x + a)^6 + 3*(70*b*cosh(b*x + a)^4 - b)*sinh(b*x + a)^6
+ 18*(14*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(14*b*co
sh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 60*(2*b*cosh(b*x + a
)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + (45*b*cosh
(b*x + a)^8 - 45*b*cosh(b*x + a)^4 + 2*b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x
 + a)^9 - 9*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{32\,e^{(-4\,bx-4\,a)}}{b(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1)} - \frac{32}{3\,b(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 32*e^(-4*b*x - 4*a)/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12*b*
x - 12*a) - 1)) - 32/3/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12
*b*x - 12*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\left(3\,e^{(4\,bx+4\,a)}-1\right)}{3\,b(e^{(4\,bx+4\,a)}-1)^3}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="giac")
[Out] -32/3*(3*e^(4*b*x + 4*a) - 1)/(b*(e^(4*b*x + 4*a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-1\right)}{3\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)^3}$$

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^4),x)
[Out] -(32*(3*exp(4*a + 4*b*x) - 1))/(3*b*(exp(4*a + 4*b*x) - 1)^3)
```

3.43 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{35\arctan(\sinh(a+bx))}{8b} + \frac{35\operatorname{csch}(a+bx)}{8b} - \frac{35\operatorname{csch}^3(a+bx)}{24b} + \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b}$$

```
[Out] 35/8*arctan(sinh(b*x+a))/b+35/8*csch(b*x+a)/b-35/24*csch(b*x+a)^3/b+7/8*csc
h(b*x+a)^3*sech(b*x+a)^2/b+1/4*csch(b*x+a)^3*sech(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2701, 294, 308, 213}

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{35\arctan(\sinh(a+bx))}{8b} - \frac{35\operatorname{csch}^3(a+bx)}{24b} + \frac{35\operatorname{csch}(a+bx)}{8b} + \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)}{4b} + \frac{7\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{8b}$$

```
[In] Int[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

```
[Out] (35*ArcTan[Sinh[a + b*x]])/(8*b) + (35*Csch[a + b*x])/(8*b) - (35*Csch[a +
b*x]^3)/(24*b) + (7*Csch[a + b*x]^3*Sech[a + b*x]^2)/(8*b) + (Csch[a + b*x]
^3*Sech[a + b*x]^4)/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\
&= \frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b} + \frac{(7i)\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, -i\text{csch}(a+bx)\right)}{4b}\\
&= \frac{7\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b}\\
&\quad + \frac{(35i)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{8b}\\
&= \frac{7\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b}\\
&\quad + \frac{(35i)\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, -i\text{csch}(a+bx)\right)}{8b}
\end{aligned}
$$

$$= \frac{35\text{csch}(a+bx)}{8b} - \frac{35\text{csch}^3(a+bx)}{24b} + \frac{7\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b} + \frac{(35i)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{8b}$$

$$= \frac{35\arctan(\sinh(a+bx))}{8b} + \frac{35\text{csch}(a+bx)}{8b} - \frac{35\text{csch}^3(a+bx)}{24b} + \frac{7\text{csch}^3(a+bx)\text{sech}^2(a+bx)}{8b} + \frac{\text{csch}^3(a+bx)\text{sech}^4(a+bx)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \text{csch}^4(a+bx)\text{sech}^5(a+bx)\,dx = -\frac{\text{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 3, -\frac{1}{2}, -\sinh^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
[Out] -1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, -Sinh[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 151.89 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\left(\frac{\text{sech}(bx+a)^3}{4}+\frac{3\,\text{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{35\arctan\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\left(\frac{\text{sech}(bx+a)^3}{4}+\frac{3\,\text{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{35\arctan\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(105\,e^{12bx+12a}+70\,e^{10bx+10a}-329\,e^{8bx+8a}-204\,e^{6bx+6a}-329\,e^{4bx+4a}+70\,e^{2bx+2a}+105\right)}{12b\left(1+e^{2bx+2a}\right)^4\left(e^{2bx+2a}-1\right)^3}+\frac{35i\ln\left(e^{bx+a}+i\right)}{8b}-$

```
[In] int(csch(b*x+a)^4*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^4+7/3/sinh(b*x+a)/cosh(b*x+a)^4+35/3*(1
/4*sech(b*x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+35/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2092 vs. $2(79) = 158$.

Time = 0.26 (sec) , antiderivative size = 2092, normalized size of antiderivative = 23.51

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="fricas")
[Out] 1/12*(105*cosh(b*x + a)^13 + 1365*cosh(b*x + a)*sinh(b*x + a)^12 + 105*sinh
(b*x + a)^13 + 70*(117*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^11 + 70*cosh(b*x
+ a)^11 + 770*(39*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^10 + 7*(10
725*cosh(b*x + a)^4 + 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 329*cosh(
b*x + a)^9 + 21*(6435*cosh(b*x + a)^5 + 550*cosh(b*x + a)^3 - 141*cosh(b*x
+ a))*sinh(b*x + a)^8 + 12*(15015*cosh(b*x + a)^6 + 1925*cosh(b*x + a)^4 -
987*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^7 - 204*cosh(b*x + a)^7 + 84*(2145*
cosh(b*x + a)^7 + 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 - 17*cosh(b*x +
 a))*sinh(b*x + a)^6 + 7*(19305*cosh(b*x + a)^8 + 4620*cosh(b*x + a)^6 - 59
22*cosh(b*x + a)^4 - 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 - 329*cosh(b
*x + a)^5 + 7*(10725*cosh(b*x + a)^9 + 3300*cosh(b*x + a)^7 - 5922*cosh(b*x
 + a)^5 - 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x + a)^4 + 14*(2
145*cosh(b*x + a)^10 + 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a)^6 - 510*cos
h(b*x + a)^4 - 235*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 70*cosh(b*x + a)^
3 + 14*(585*cosh(b*x + a)^11 + 275*cosh(b*x + a)^9 - 846*cosh(b*x + a)^7 -
306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 + 15*cosh(b*x + a))*sinh(b*x + a)
^2 + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + sinh(b*x +
 a)^14 + (91*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^12 + cosh(b*x + a)^12 + 4*(
91*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh(b*x + a
)^4 + 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10 + 2*(10
01*cosh(b*x + a)^5 + 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b*x + a)^
9 + 3*(1001*cosh(b*x + a)^6 + 165*cosh(b*x + a)^4 - 45*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 + 33*cosh(b*
x + a)^5 - 15*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 3*(1001*co
sh(b*x + a)^8 + 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 - 28*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a)^9 + 39
6*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 - 84*cosh(b*x + a)^3 + 9*cosh(b*x +
 a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 + 495*cosh(b*x + a)^8 - 630*c
osh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 + 3)*sinh(b*x + a
)^4 + 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 + 55*cosh(b*x + a)^9 - 90*
cosh(b*x + a)^7 - 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 + 3*cosh(b*x + a)
)*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 + 66*cosh(b*x + a)^10 - 135*cosh(b
*x + a)^8 - 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 -
1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 + 6*cosh(b*x +
 a)^11 - 15*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^5 + 6*co
```

```
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + si
nh(b*x + a)) + 7*(195*cosh(b*x + a)^12 + 110*cosh(b*x + a)^10 - 423*cosh(b*
x + a)^8 - 204*cosh(b*x + a)^6 - 235*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 +
 15)*sinh(b*x + a) + 105*cosh(b*x + a))/(b*cosh(b*x + a)^14 + 14*b*cosh(b*x
 + a)*sinh(b*x + a)^13 + b*sinh(b*x + a)^14 + b*cosh(b*x + a)^12 + (91*b*co
sh(b*x + a)^2 + b)*sinh(b*x + a)^12 + 4*(91*b*cosh(b*x + a)^3 + 3*b*cosh(b*
x + a))*sinh(b*x + a)^11 - 3*b*cosh(b*x + a)^10 + (1001*b*cosh(b*x + a)^4 +
 66*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^10 + 2*(1001*b*cosh(b*x + a)^5 +
 110*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^9 - 3*b*cosh(b*x
 + a)^8 + 3*(1001*b*cosh(b*x + a)^6 + 165*b*cosh(b*x + a)^4 - 45*b*cosh(b*x
 + a)^2 - b)*sinh(b*x + a)^8 + 24*(143*b*cosh(b*x + a)^7 + 33*b*cosh(b*x +
a)^5 - 15*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 + 3*b*cosh(b
*x + a)^6 + 3*(1001*b*cosh(b*x + a)^8 + 308*b*cosh(b*x + a)^6 - 210*b*cosh(
b*x + a)^4 - 28*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 2*(1001*b*cosh(b*x
 + a)^9 + 396*b*cosh(b*x + a)^7 - 378*b*cosh(b*x + a)^5 - 84*b*cosh(b*x + a
)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^4 + (1001*b*co
sh(b*x + a)^10 + 495*b*cosh(b*x + a)^8 - 630*b*cosh(b*x + a)^6 - 210*b*cosh
(b*x + a)^4 + 45*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 4*(91*b*cosh(b*
x + a)^11 + 55*b*cosh(b*x + a)^9 - 90*b*cosh(b*x + a)^7 - 42*b*cosh(b*x + a
)^5 + 15*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*
x + a)^2 + (91*b*cosh(b*x + a)^12 + 66*b*cosh(b*x + a)^10 - 135*b*cosh(b*x
+ a)^8 - 84*b*cosh(b*x + a)^6 + 45*b*cosh(b*x + a)^4 + 18*b*cosh(b*x + a)^2
 - b)*sinh(b*x + a)^2 + 2*(7*b*cosh(b*x + a)^13 + 6*b*cosh(b*x + a)^11 - 15
*b*cosh(b*x + a)^9 - 12*b*cosh(b*x + a)^7 + 9*b*cosh(b*x + a)^5 + 6*b*cosh(
b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**4*sech(b*x+a)**5,x)
[Out] Integral(csch(a + b*x)**4*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{35\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{105\,e^{(-bx-a)} + 70\,e^{(-3\,bx-3\,a)} - 329\,e^{(-5\,bx-5\,a)} - 204\,e^{(-7\,bx-7\,a)} - 329\,e^{(-9\,bx-9\,a)} + 70\,e^{(-11\,bx-11\,a)} + 105\,e^{(}}{12\,b(e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} - 3\,e^{(-6\,bx-6\,a)} + 3\,e^{(-8\,bx-8\,a)} + 3\,e^{(-10\,bx-10\,a)} - e^{(-12\,bx-12\,a)} - e^{(-14\,bx-1}}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -35/4*arctan(e^(-b*x - a))/b + 1/12*(105*e^(-b*x - a) + 70*e^(-3*b*x - 3*a)
 - 329*e^(-5*b*x - 5*a) - 204*e^(-7*b*x - 7*a) - 329*e^(-9*b*x - 9*a) + 70*
e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a))/(b*(e^(-2*b*x - 2*a) - 3*e^(-4
*b*x - 4*a) - 3*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a
) - e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{105\,\pi + \frac{12\left(11\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + \frac{32\left(9\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 210\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-\right.\right.}{48\,b}$$

```
[In] integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/48*(105*pi + 12*(11*(e^(b*x + a) - e^(-b*x - a))^3 + 52*e^(b*x + a) - 52*
e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32*(9*(e^(b*x + a) -
 e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*x - a))^3 + 210*arctan(1/2*(e^(2
*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.27

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{35\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} - \frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{7\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)} - \frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} + \frac{4\,\mathrm{e}^{a+b\,x}}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)} + \frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} + \frac{11\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^4),x)
```

```
[Out] (35*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) - (8*exp(a + b*x
))/(3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (7*exp(a + b*x))/(2*
b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*e
xp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (6*exp(a +
b*x))/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1))
+ (4*exp(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a
+ 6*b*x) + exp(8*a + 8*b*x) + 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2*b*x) -
 1)) + (11*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.44 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\coth^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+ln(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\coth^4(a+bx)}{4b}+\frac{\coth^2(a+bx)}{b}+\frac{\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x],x]
```

```
[Out] Coth[a + b*x]^2/b - Coth[a + b*x]^4/(4*b) + Log[Tanh[a + b*x]]/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= \frac{\coth^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{\log(\tanh(a+bx))}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$
\begin{aligned}
&\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx\\
&= \frac{2\operatorname{csch}^2(a+bx)-\operatorname{csch}^4(a+bx)-4\log(\cosh(a+bx))+4\log(\sinh(a+bx))}{4b}
\end{aligned}
$$

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x],x]
[Out] (2*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 4*Log[Cosh[a + b*x]] + 4*Log[Sinh[a
+ b*x]])/(4*b)
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4}+\frac{1}{2\sinh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
default	$\frac{-\frac{1}{4\sinh(bx+a)^4}+\frac{1}{2\sinh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
risch	$\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}-4\,\mathrm{e}^{2bx+2a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}-\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	84

```
[In] int(csch(b*x+a)^5*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/4/sinh(b*x+a)^4+1/2/sinh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 1082, normalized size of antiderivative = 27.74

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="fricas")
[Out] (2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6 +
 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^4 - 8*cosh(b*x + a)^4 + 8*(5*cosh
(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 - 24
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x + a)^
8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3*cosh(
b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 3)
*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b
*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*
(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (c
osh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*c
osh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a
)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*
x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^
6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*
x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*
x + a))) + 4*(3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*
x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^
```

```
6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(
b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x
 + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 - 15*b*cos
h(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a
)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
 + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a),x)
[Out] Integral(csch(a + b*x)**5*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} - \frac{2\left(e^{(-2\,bx-2\,a)}-4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="maxima")
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b - 2*(e^(-2*b*x - 2*a) - 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e
^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a
) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.13

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\frac{3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-20e^{(2bx+2a)}-20e^{(-2bx-2a)}+44}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)^2}+2\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)-2\log\left(e^{(2bx+2a)}\right.}{4b}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 20*e^(2*b*x + 2*a) - 20*e
^(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2)^2 + 2*log(e^
(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 2*log(e^(2*b*x + 2*a) + e^(-2*b*x -
 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.33

$$\begin{aligned}\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx &= \frac{2}{b\left(e^{2a+2bx}-1\right)}-\frac{2\operatorname{atan}\left(\frac{e^{2a}e^{2bx}\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}\\&\quad-\frac{2}{b\left(e^{4a+4bx}-2e^{2a+2bx}+1\right)}\\&\quad-\frac{8}{b\left(3e^{2a+2bx}-3e^{4a+4bx}+e^{6a+6bx}-1\right)}\\&\quad-\frac{4}{b\left(6e^{4a+4bx}-4e^{2a+2bx}-4e^{6a+6bx}+e^{8a+8bx}+1\right)}\end{aligned}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^5),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) - 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - 8/(b*
(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - 4/(b*(6
*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b
*x) + 1))
```

3.45 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{15\operatorname{arctanh}(\cosh(a+bx))}{8b}+\frac{15\operatorname{sech}(a+bx)}{8b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}$$

```
[Out] -15/8*arctanh(cosh(b*x+a))/b+15/8*sech(b*x+a)/b+5/8*csch(b*x+a)^2*sech(b*x+
a)/b-1/4*csch(b*x+a)^4*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2702, 294, 327, 213}

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{15\operatorname{arctanh}(\cosh(a+bx))}{8b}+\frac{15\operatorname{sech}(a+bx)}{8b}-\frac{\operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)}{4b}+\frac{5\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cosh[a + b*x]])/(8*b) + (15*Sech[a + b*x])/(8*b) + (5*Csch[a +
 b*x]^2*Sech[a + b*x])/(8*b) - (Csch[a + b*x]^4*Sech[a + b*x])/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \text{sech}(a+bx)\right)}{b} \\
&= -\frac{\text{csch}^4(a+bx)\text{sech}(a+bx)}{4b} + \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \text{sech}(a+bx)\right)}{4b} \\
&= \frac{5\text{csch}^2(a+bx)\text{sech}(a+bx)}{8b} - \frac{\text{csch}^4(a+bx)\text{sech}(a+bx)}{4b} \\
&\quad + \frac{15\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{8b} \\
&= \frac{15\text{sech}(a+bx)}{8b} + \frac{5\text{csch}^2(a+bx)\text{sech}(a+bx)}{8b} \\
&\quad - \frac{\text{csch}^4(a+bx)\text{sech}(a+bx)}{4b} + \frac{15\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{8b}
\end{aligned}
$$

$$= -\frac{15\text{arctanh}(\cosh(a+bx))}{8b} + \frac{15\text{sech}(a+bx)}{8b} + \frac{5\text{csch}^2(a+bx)\text{sech}(a+bx)}{8b} - \frac{\text{csch}^4(a+bx)\text{sech}(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \text{csch}^5(a+bx)\text{sech}^2(a+bx)\,dx = \frac{7\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{15\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{15\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{7\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

```
[Out] (7*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (15*Log[Cosh[
(a + b*x)/2]])/(8*b) + (15*Log[Sinh[(a + b*x)/2]])/(8*b) + (7*Sech[(a + b*x
)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b) + Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 15.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\text{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	61
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\text{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	61
risch	$\frac{e^{bx+a}\left(15\,e^{8bx+8a}-40\,e^{6bx+6a}+18\,e^{4bx+4a}-40\,e^{2bx+2a}+15\right)}{4b\left(e^{2bx+2a}-1\right)^4\left(1+e^{2bx+2a}\right)} - \frac{15\ln\left(e^{bx+a}+1\right)}{8b} + \frac{15\ln\left(e^{bx+a}-1\right)}{8b}$	113

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)+5/8/sinh(b*x+a)^2/cosh(b*x+a)+15/8/cosh
(b*x+a)-15/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 1591, normalized size of antiderivative = 22.73

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x +
 a)^9 + 40*(27*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^7 - 80*cosh(b*x + a)^7 +
280*(9*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^6 + 12*(315*cosh(b*
x + a)^4 - 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 36*cosh(b*x + a)^5 +
20*(189*cosh(b*x + a)^5 - 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x +
 a)^4 + 40*(63*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 2
)*sinh(b*x + a)^3 - 80*cosh(b*x + a)^3 + 120*(9*cosh(b*x + a)^7 - 14*cosh(b
*x + a)^5 + 3*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^2 - 15*(cosh
(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10 + 3*(15*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(5*cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 42*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh(b*x + a)^5
 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*cosh(b*x +
a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 2*co
sh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 21*cosh(b*x + a)^5 + 5*cosh(b*x + a
)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8 - 28*cosh(b*x
+ a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*co
sh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7 + 6*cosh(b*x + a)
^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x +
 a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x
 + a)^9 + sinh(b*x + a)^10 + 3*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3
*cosh(b*x + a)^8 + 24*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 +
 2*(105*cosh(b*x + a)^4 - 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(
b*x + a)^6 + 12*(21*cosh(b*x + a)^5 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*s
inh(b*x + a)^5 + 2*(105*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x
 + a)^2 + 1)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 -
21*cosh(b*x + a)^5 + 5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3
*(15*cosh(b*x + a)^8 - 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 1
2*cosh(b*x + a)^7 + 6*cosh(b*x + a)^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a)
)*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 10*(27*cosh(b
*x + a)^8 - 56*cosh(b*x + a)^6 + 18*cosh(b*x + a)^4 - 24*cosh(b*x + a)^2 +
3)*sinh(b*x + a) + 30*cosh(b*x + a))/(b*cosh(b*x + a)^10 + 10*b*cosh(b*x +
a)*sinh(b*x + a)^9 + b*sinh(b*x + a)^10 - 3*b*cosh(b*x + a)^8 + 3*(15*b*cos
h(b*x + a)^2 - b)*sinh(b*x + a)^8 + 24*(5*b*cosh(b*x + a)^3 - b*cosh(b*x +
```

```
a))*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^6 + 2*(105*b*cosh(b*x + a)^4 - 42*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 12*(21*b*cosh(b*x + a)^5 - 14*b*cos
h(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^5 + 2*b*cosh(b*x + a)^4 + 2*(
105*b*cosh(b*x + a)^6 - 105*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + b)*s
inh(b*x + a)^4 + 8*(15*b*cosh(b*x + a)^7 - 21*b*cosh(b*x + a)^5 + 5*b*cosh(
b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - 3*b*cosh(b*x + a)^2 + 3*(15
*b*cosh(b*x + a)^8 - 28*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 + 4*b*cosh
(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(5*b*cosh(b*x + a)^9 - 12*b*cosh(b*x +
 a)^7 + 6*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh
(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(62) = 124$.

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{15\,\log\left(e^{(-bx-a)}+1\right)}{8\,b} + \frac{15\,\log\left(e^{(-bx-a)}-1\right)}{8\,b} - \frac{15\,e^{(-bx-a)} - 40\,e^{(-3\,bx-3\,a)} + 18\,e^{(-5\,bx-5\,a)} - 40\,e^{(-7\,bx-7\,a)} + 15\,e^{(-9\,bx-9\,a)}}{4\,b(3\,e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} - 2\,e^{(-6\,bx-6\,a)} + 3\,e^{(-8\,bx-8\,a)} - e^{(-10\,bx-10\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(-b*x - a) + 1)/b + 15/8*log(e^(-b*x - a) - 1)/b - 1/4*(15*e^(-
b*x - a) - 40*e^(-3*b*x - 3*a) + 18*e^(-5*b*x - 5*a) - 40*e^(-7*b*x - 7*a)
+ 15*e^(-9*b*x - 9*a))/(b*(3*e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-
6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) - e^(-10*b*x - 10*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.86

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4\left(7\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}+\frac{32}{e^{(bx+a)}+e^{(-bx-a)}}-15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*(7*(e^(b*x + a) + e^(-b*x - a))^3 - 36*e^(b*x + a) - 36*e^(-b*x - a
))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32/(e^(b*x + a) + e^(-b*x - a))
 - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x -
a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{15\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}-\frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}+\frac{7\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^5),x)
```

```
[Out] (3*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (15*at
an((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (6*exp(a + b*x))/(
b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*ex
p(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x
) + exp(8*a + 8*b*x) + 1)) + (7*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1))
+ (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.46 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{3\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] 3/2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+3*ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)
^2/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\coth^2(a+bx)}{2b}+\frac{3\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

```
[Out] (3*Coth[a + b*x]^2)/(2*b) - Coth[a + b*x]^4/(4*b) + (3*Log[Tanh[a + b*x]])/
b - Tanh[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\ &= \frac{3\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4\operatorname{csch}^2(a+bx)-\operatorname{csch}^4(a+bx)-12\log(\cosh(a+bx))+12\log(\sinh(a+bx))+2\operatorname{sech}^2(a+bx)}{4b}$$

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
[Out] (4*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[
a + b*x]] + 2*Sech[a + b*x]^2)/(4*b)
```

Maple [A] (verified)

Time = 36.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2\cosh(bx+a)^2}+3\ln(\tanh(bx+a))}{b}$	61
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2\cosh(bx+a)^2}+3\ln(\tanh(bx+a))}{b}$	61
risch	$\frac{2\,\mathrm{e}^{2bx+2a}\left(3\,\mathrm{e}^{8bx+8a}-6\,\mathrm{e}^{6bx+6a}-2\,\mathrm{e}^{4bx+4a}-6\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{3\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}-\frac{3\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	122

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^2+3/4/sinh(b*x+a)^2/cosh(b*x+a)^2+3/2/c
osh(b*x+a)^2+3*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 2114, normalized size of antiderivative = 36.45

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^10
 + 6*(45*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^8 - 12*cosh(b*x + a)^8 + 48*(15
*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x + a)^
4 - 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(63*co
sh(b*x + a)^5 - 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 + 12*(1
05*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^4 - 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 - 42*cosh(b*x + a)^5 -
5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b*x + a)^
8 - 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*x + a)*sin
h(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
 + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 -
cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 30*cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 - 7*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(99*cosh(b*x + a)^7
 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^
5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 60*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(55*cosh(b*x + a)^
```

```
9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a)^10 - 45*cosh(b*x + a)^8 - 14*c
osh(b*x + a)^6 + 30*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^
2 - 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^11 - 5*cosh(b*x + a)^9 - 2*cosh(
b*x + a)^7 + 6*cosh(b*x + a)^5 - cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x
+ a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*
x + a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*
x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 3
0*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6
 - 105*cosh(b*x + a)^4 - 7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*
x + a)^6 + 8*(99*cosh(b*x + a)^7 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 +
 3*cosh(b*x + a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a
)^6 - 70*cosh(b*x + a)^4 + 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(55*cosh(b*x + a)^9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5
 + 20*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 2*(33*cosh(b*x + a
)^10 - 45*cosh(b*x + a)^8 - 14*cosh(b*x + a)^6 + 30*cosh(b*x + a)^4 - 3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(3*cosh(b*x + a)^
11 - 5*cosh(b*x + a)^9 - 2*cosh(b*x + a)^7 + 6*cosh(b*x + a)^5 - cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 12*(5*cosh(b*x + a)^9 - 8*cosh(b*x + a)^7 - 2*cosh(b*
x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x +
a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11 + b*sinh(b*x + a)^12 - 2*b*cosh
(b*x + a)^10 + 2*(33*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^10 + 20*(11*b*cos
h(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^9 - b*cosh(b*x + a)^8 + (495*
b*cosh(b*x + a)^4 - 90*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^8 + 8*(99*b*cos
h(b*x + a)^5 - 30*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^7 + 4*
b*cosh(b*x + a)^6 + 4*(231*b*cosh(b*x + a)^6 - 105*b*cosh(b*x + a)^4 - 7*b*
cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*(99*b*cosh(b*x + a)^7 - 63*b*cosh(
b*x + a)^5 - 7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 - b*c
osh(b*x + a)^4 + (495*b*cosh(b*x + a)^8 - 420*b*cosh(b*x + a)^6 - 70*b*cosh
(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(55*b*cosh(b*x
+ a)^9 - 60*b*cosh(b*x + a)^7 - 14*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3
 - b*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b*cosh(b*x + a)^2 + 2*(33*b*cosh(b*
x + a)^10 - 45*b*cosh(b*x + a)^8 - 14*b*cosh(b*x + a)^6 + 30*b*cosh(b*x + a
)^4 - 3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(3*b*cosh(b*x + a)^11 -
5*b*cosh(b*x + a)^9 - 2*b*cosh(b*x + a)^7 + 6*b*cosh(b*x + a)^5 - b*cosh(b*
x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**3,x)
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.09

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{3\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{3\log\left(e^{(-2bx-2a)}+1\right)}{b} - \frac{2\left(3e^{(-2bx-2a)} - 6e^{(-4bx-4a)} - 2e^{(-6bx-6a)} - 6e^{(-8bx-8a)} + 3e^{(-10bx-10a)}\right)}{b(2e^{(-2bx-2a)} + e^{(-4bx-4a)} - 4e^{(-6bx-6a)} + e^{(-8bx-8a)} + 2e^{(-10bx-10a)} - e^{(-12bx-12a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="maxima")
[Out] 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 3*log(e^(-2*b*x - 2
*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a
) - 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) + e^(
-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 2*e^(-10*b*x - 10*a
) - e^(-12*b*x - 12*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{\frac{2\left(3e^{(2bx+2a)}+3e^{(-2bx-2a)}+10\right)}{e^{(2bx+2a)}+e^{(-2bx-2a)}+2} - \frac{9\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-52e^{(2bx+2a)}-52e^{(-2bx-2a)}+84}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)^2} - 6\log\left(e^{(2bx+2a)}+e^{(-2bx-2a}\right.}{4b}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="giac")
[Out] 1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) + 10)/(e^(2*b*x + 2*a) + e^(
-2*b*x - 2*a) + 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 52*e^(2*b*
x + 2*a) - 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) -
2)^2 - 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 6*log(e^(2*b*x + 2*a
) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.22

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{6\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^5),x)
```

```
[Out] 4/(b*(exp(2*a + 2*b*x) - 1)) + 2/(b*(exp(2*a + 2*b*x) + 1)) - (6*atan((exp(
2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) +
 exp(4*a + 4*b*x) + 1)) - 8/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + e
xp(6*a + 6*b*x) - 1)) - 4/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*e
xp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.47 $\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx = -\frac{35\mathrm{arctanh}(\cosh(a+bx))}{8b}+\frac{35\mathrm{sech}(a+bx)}{8b}+\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}$$

```
[Out] -35/8*arctanh(cosh(b*x+a))/b+35/8*sech(b*x+a)/b+35/24*sech(b*x+a)^3/b+7/8*c
sch(b*x+a)^2*sech(b*x+a)^3/b-1/4*csch(b*x+a)^4*sech(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2702, 294, 308, 213}

$$\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx = -\frac{35\mathrm{arctanh}(\cosh(a+bx))}{8b}+\frac{35\mathrm{sech}^3(a+bx)}{24b}+\frac{35\mathrm{sech}(a+bx)}{8b}-\frac{\mathrm{csch}^4(a+bx)\mathrm{sech}^3(a+bx)}{4b}+\frac{7\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)}{8b}$$

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cosh[a + b*x]])/(8*b) + (35*Sech[a + b*x])/(8*b) + (35*Sech[a
+ b*x]^3)/(24*b) + (7*Csch[a + b*x]^2*Sech[a + b*x]^3)/(8*b) - (Csch[a + b*
x]^4*Sech[a + b*x]^3)/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \text{sech}(a+bx)\right)}{b} \\
&= -\frac{\text{csch}^4(a+bx)\text{sech}^3(a+bx)}{4b} + \frac{7\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \text{sech}(a+bx)\right)}{4b} \\
&= \frac{7\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{8b} - \frac{\text{csch}^4(a+bx)\text{sech}^3(a+bx)}{4b} \\
&\quad + \frac{35\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{8b} \\
&= \frac{7\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{8b} - \frac{\text{csch}^4(a+bx)\text{sech}^3(a+bx)}{4b} \\
&\quad + \frac{35\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \text{sech}(a+bx)\right)}{8b}
\end{aligned}$$

$$=\frac{35\text{sech}(a+bx)}{8b}+\frac{35\text{sech}^3(a+bx)}{24b}+\frac{7\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{8b}-\frac{\text{csch}^4(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{35\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\text{sech}(a+bx)\right)}{8b}$$

$$=-\frac{35\text{arctanh}(\cosh(a+bx))}{8b}+\frac{35\text{sech}(a+bx)}{8b}+\frac{35\text{sech}^3(a+bx)}{24b}+\frac{7\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{8b}-\frac{\text{csch}^4(a+bx)\text{sech}^3(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int\text{csch}^5(a+bx)\text{sech}^4(a+bx)\,dx=\frac{11\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{35\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{35\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{11\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\text{sech}(a+bx)}{b}+\frac{\text{sech}^3(a+bx)}{3b}$$

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

```
[Out] (11*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (35*Log[Cosh
[(a + b*x)/2]])/(8*b) + (35*Log[Sinh[(a + b*x)/2]])/(8*b) + (11*Sech[(a + b
*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b) + (3*Sech[a + b*x])/b + Sech[
a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 115.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\text{arctanh}\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\text{arctanh}\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(105\,e^{12bx+12a}-70\,e^{10bx+10a}-329\,e^{8bx+8a}+204\,e^{6bx+6a}-329\,e^{4bx+4a}-70\,e^{2bx+2a}+105\right)}{12b\left(e^{2bx+2a}-1\right)^4\left(1+e^{2bx+2a}\right)^3}+\frac{35\ln\left(e^{bx+a}-1\right)}{8b}-$

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^3+7/8/sinh(b*x+a)^2/cosh(b*x+a)^3+35/24
/cosh(b*x+a)^3+35/8/cosh(b*x+a)-35/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2802 vs. $2(79) = 158$.

Time = 0.27 (sec) , antiderivative size = 2802, normalized size of antiderivative = 31.48

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(210*cosh(b*x + a)^13 + 2730*cosh(b*x + a)*sinh(b*x + a)^12 + 210*sinh
(b*x + a)^13 + 140*(117*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^11 - 140*cosh(b*
x + a)^11 + 1540*(39*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^10 + 14
*(10725*cosh(b*x + a)^4 - 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 658*c
osh(b*x + a)^9 + 42*(6435*cosh(b*x + a)^5 - 550*cosh(b*x + a)^3 - 141*cosh(
b*x + a))*sinh(b*x + a)^8 + 24*(15015*cosh(b*x + a)^6 - 1925*cosh(b*x + a)^
4 - 987*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^7 + 408*cosh(b*x + a)^7 + 168*(
2145*cosh(b*x + a)^7 - 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 + 17*cosh(
b*x + a))*sinh(b*x + a)^6 + 14*(19305*cosh(b*x + a)^8 - 4620*cosh(b*x + a)^
6 - 5922*cosh(b*x + a)^4 + 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 - 658*
cosh(b*x + a)^5 + 14*(10725*cosh(b*x + a)^9 - 3300*cosh(b*x + a)^7 - 5922*c
osh(b*x + a)^5 + 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x + a)^4
+ 28*(2145*cosh(b*x + a)^10 - 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a)^6 +
510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^3 - 140*cosh(b
*x + a)^3 + 28*(585*cosh(b*x + a)^11 - 275*cosh(b*x + a)^9 - 846*cosh(b*x +
 a)^7 + 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(
b*x + a)^2 - 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + si
nh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^12 - cosh(b*x + a)^
12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh
(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10
 + 2*(1001*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b
*x + a)^9 + 3*(1001*cosh(b*x + a)^6 - 165*cosh(b*x + a)^4 - 45*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 - 33
*cosh(b*x + a)^5 - 15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 3*
(1001*cosh(b*x + a)^8 - 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 28*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a
)^9 - 396*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 + 84*cosh(b*x + a)^3 + 9*co
sh(b*x + a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 - 495*cosh(b*x + a)^8
 - 630*cosh(b*x + a)^6 + 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 - 3)*sinh
(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 - 55*cosh(b*x + a)
^9 - 90*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 - 3*cosh(
b*x + a))*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 - 66*cosh(b*x + a)^10 - 13
```

5*cosh(b*x + a)^8 + 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 - 18*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 - 6*cosh(b*x + a)^11 - 15*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^12 - cosh(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^11 + (1001*cosh(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*cosh(b*x + a)^10 + 2*(1001*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 15*cosh(b*x + a))*sinh(b*x + a)^9 + 3*(1001*cosh(b*x + a)^6 - 165*cosh(b*x + a)^4 - 45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(143*cosh(b*x + a)^7 - 33*cosh(b*x + a)^5 - 15*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 3*(1001*cosh(b*x + a)^8 - 308*cosh(b*x + a)^6 - 210*cosh(b*x + a)^4 + 28*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 3*cosh(b*x + a)^6 + 2*(1001*cosh(b*x + a)^9 - 396*cosh(b*x + a)^7 - 378*cosh(b*x + a)^5 + 84*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5 + (1001*cosh(b*x + a)^10 - 495*cosh(b*x + a)^8 - 630*cosh(b*x + a)^6 + 210*cosh(b*x + a)^4 + 45*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(91*cosh(b*x + a)^11 - 55*cosh(b*x + a)^9 - 90*cosh(b*x + a)^7 + 42*cosh(b*x + a)^5 + 15*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + (91*cosh(b*x + a)^12 - 66*cosh(b*x + a)^10 - 135*cosh(b*x + a)^8 + 84*cosh(b*x + a)^6 + 45*cosh(b*x + a)^4 - 18*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(7*cosh(b*x + a)^13 - 6*cosh(b*x + a)^11 - 15*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7 + 9*cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 14*(195*cosh(b*x + a)^12 - 110*cosh(b*x + a)^10 - 423*cosh(b*x + a)^8 + 204*cosh(b*x + a)^6 - 235*cosh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 15)*sinh(b*x + a) + 210*cosh(b*x + a))/(b*cosh(b*x + a)^14 + 14*b*cosh(b*x + a)*sinh(b*x + a)^13 + b*sinh(b*x + a)^14 - b*cosh(b*x + a)^12 + (91*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^12 + 4*(91*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^11 - 3*b*cosh(b*x + a)^10 + (1001*b*cosh(b*x + a)^4 - 66*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^10 + 2*(1001*b*cosh(b*x + a)^5 - 110*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^9 + 3*b*cosh(b*x + a)^8 + 3*(1001*b*cosh(b*x + a)^6 - 165*b*cosh(b*x + a)^4 - 45*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^8 + 24*(143*b*cosh(b*x + a)^7 - 33*b*cosh(b*x + a)^5 - 15*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^7 + 3*b*cosh(b*x + a)^6 + 3*(1001*b*cosh(b*x + a)^8 - 308*b*cosh(b*x + a)^6 - 210*b*cosh(b*x + a)^4 + 28*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 2*(1001*b*cosh(b*x + a)^9 - 396*b*cosh(b*x + a)^7 - 378*b*cosh(b*x + a)^5 + 84*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^4 + (1001*b*cosh(b*x + a)^10 - 495*b*cosh(b*x + a)^8 - 630*b*cosh(b*x + a)^6 + 210*b*cosh(b*x + a)^4 + 45*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^4 + 4*(91*b*cosh(b*x + a)^11 - 55*b*cosh(b*x + a)^9 - 90*b*cosh(b*x + a)^7 + 42*b*cosh(b*x + a)^5 + 15*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (91*b*cosh(b*x + a)^12 - 66*b*cosh(b*x + a)^10 - 135*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 45*b*cosh(b*x + a)^4 -

```
 18*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(7*b*cosh(b*x + a)^13 - 6*b*
cosh(b*x + a)^11 - 15*b*cosh(b*x + a)^9 + 12*b*cosh(b*x + a)^7 + 9*b*cosh(b
*x + a)^5 - 6*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx=\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**4,x)
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(79)=158$.

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{35\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{35\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{105\,e^{(-bx-a)}-70\,e^{(-3\,bx-3\,a)}-329\,e^{(-5\,bx-5\,a)}+204\,e^{(-7\,bx-7\,a)}-329\,e^{(-9\,bx-9\,a)}-70\,e^{(-11\,bx-11\,a)}+105}{12\,b(e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-3\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}+3\,e^{(-10\,bx-10\,a)}+e^{(-12\,bx-12\,a)}-e^{(-14\,bx-}}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="maxima")
[Out] -35/8*log(e^(-b*x - a) + 1)/b + 35/8*log(e^(-b*x - a) - 1)/b - 1/12*(105*e^
(-b*x - a) - 70*e^(-3*b*x - 3*a) - 329*e^(-5*b*x - 5*a) + 204*e^(-7*b*x - 7
*a) - 329*e^(-9*b*x - 9*a) - 70*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a)
)/(b*(e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) - 3*e^(-8*
b*x - 8*a) + 3*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a)
 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx=\frac{\frac{12\left(11\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}+\frac{32\left(9\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}-105\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+}{48\,b}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/48*(12*(11*(e^(b*x + a) + e^(-b*x - a))^3 - 52*e^(b*x + a) - 52*e^(-b*x -
 a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32*(9*(e^(b*x + a) + e^(-b*x
- a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))^3 - 105*log(e^(b*x + a) + e^(-b*x
 - a) + 2) + 105*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.31

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{7\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{35\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}+\frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}-\frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}+\frac{11\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{6\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^4*sinh(a + b*x)^5),x)
```

```
[Out] (7*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (35*at
an((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) + (8*exp(a + b*x))/(
3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b*(3*e
xp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (8*exp(a +
b*x))/(3*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)
) - (4*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*
a + 6*b*x) + exp(8*a + 8*b*x) + 1)) + (11*exp(a + b*x))/(4*b*(exp(2*a + 2*b
*x) - 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.48 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{2\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{6\log(\tanh(a+bx))}{b} - \frac{2\tanh^2(a+bx)}{b} + \frac{\tanh^4(a+bx)}{4b}$$

```
[Out] 2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+6*ln(tanh(b*x+a))/b-2*tanh(b*x+a)^2/b
+1/4*tanh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\tanh^4(a+bx)}{4b} - \frac{2\tanh^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{2\coth^2(a+bx)}{b} + \frac{6\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

```
[Out] (2*Coth[a + b*x]^2)/b - Coth[a + b*x]^4/(4*b) + (6*Log[Tanh[a + b*x]])/b -
(2*Tanh[a + b*x]^2)/b + Tanh[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^5}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x^3}\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^3}+\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}$$
$$= \frac{2\coth^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{6\log(\tanh(a+bx))}{b}-\frac{2\tanh^2(a+bx)}{b}+\frac{\tanh^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \text{csch}^5(a+bx)\text{sech}^5(a+bx)\,dx = 32\left(\frac{3\text{csch}^2(a+bx)}{64b}-\frac{\text{csch}^4(a+bx)}{128b}-\frac{3\log(\cosh(a+bx))}{16b}+\frac{3\log(\sinh(a+bx))}{16b}+\frac{3\text{sech}^2(a+bx)}{64b}+\frac{\text{sech}^4(a+bx)}{128b}\right)$$

```
[In] Integrate[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

```
[Out] 32*((3*Csch[a + b*x]^2)/(64*b) - Csch[a + b*x]^4/(128*b) - (3*Log[Cosh[a +
b*x]])/(16*b) + (3*Log[Sinh[a + b*x]])/(16*b) + (3*Sech[a + b*x]^2)/(64*b)
+ Sech[a + b*x]^4/(128*b))
```

Maple [A] (verified)

Time = 242.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2\cosh(bx+a)^4}+\frac{3}{\cosh(bx+a)^2}+6\ln(\tanh(bx+a))}{b}$	70
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2\cosh(bx+a)^4}+\frac{3}{\cosh(bx+a)^2}+6\ln(\tanh(bx+a))}{b}$	70
risch	$\frac{4\,\mathrm{e}^{2bx+2a}\left(3\,\mathrm{e}^{12bx+12a}-11\,\mathrm{e}^{8bx+8a}-11\,\mathrm{e}^{4bx+4a}+3\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^4\left(\mathrm{e}^{2bx+2a}-1\right)^4}+\frac{6\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}-\frac{6\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	111

```
[In] int(csch(b*x+a)^5*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^4+1/sinh(b*x+a)^2/cosh(b*x+a)^4+3/2/cos
h(b*x+a)^4+3/cosh(b*x+a)^2+6*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 2231, normalized size of antiderivative = 32.33

$$\int \mathrm{csch}^5(a+bx)\mathrm{sech}^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 2*(6*cosh(b*x + a)^14 + 2184*cosh(b*x + a)^3*sinh(b*x + a)^11 + 546*cosh(b*
x + a)^2*sinh(b*x + a)^12 + 84*cosh(b*x + a)*sinh(b*x + a)^13 + 6*sinh(b*x
+ a)^14 + 22*(273*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^10 - 22*cosh(b*x + a)^
10 + 44*(273*cosh(b*x + a)^5 - 5*cosh(b*x + a))*sinh(b*x + a)^9 + 198*(91*c
osh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^8 + 528*(39*cosh(b*x + a)
^7 - 5*cosh(b*x + a)^3)*sinh(b*x + a)^7 + 22*(819*cosh(b*x + a)^8 - 210*cos
h(b*x + a)^4 - 1)*sinh(b*x + a)^6 - 22*cosh(b*x + a)^6 + 132*(91*cosh(b*x +
 a)^9 - 42*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^5 + 66*(91*cosh(b
*x + a)^10 - 70*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(2
73*cosh(b*x + a)^11 - 330*cosh(b*x + a)^7 - 55*cosh(b*x + a)^3)*sinh(b*x +
a)^3 + 6*(91*cosh(b*x + a)^12 - 165*cosh(b*x + a)^8 - 55*cosh(b*x + a)^4 +
1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^16 + 560*cosh(b*x
 + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a)^14 + 16*cosh(b
*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b*x + a)^4 - 1)*s
inh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x
 + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a)^9 + 6*(
2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^8 + 6*cosh(b*
x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5 + 3*cosh(b*x + a))
```

*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cosh(b*x + a)^6 + 3*cosh(b
*x + a)^2)*sinh(b*x + a)^6 + 48*(91*cosh(b*x + a)^11 - 66*cosh(b*x + a)^7 +
 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + 4*(455*cosh(b*x + a)^12 - 495*cosh(b*
x + a)^8 + 105*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 4*cosh(b*x + a)^4 + 1
6*(35*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 + 21*cosh(b*x + a)^5 - cosh(b*x
 + a))*sinh(b*x + a)^3 + 24*(5*cosh(b*x + a)^14 - 11*cosh(b*x + a)^10 + 7*c
osh(b*x + a)^6 - cosh(b*x + a)^2)*sinh(b*x + a)^2 + 16*(cosh(b*x + a)^15 -
3*cosh(b*x + a)^11 + 3*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1
)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 3*(cosh(b*x + a)^1
6 + 560*cosh(b*x + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a
)^14 + 16*cosh(b*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)
^5 - cosh(b*x + a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(
b*x + a)^9 + 6*(2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x +
a)^8 + 6*cosh(b*x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5 +
3*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cosh(b*x +
 a)^6 + 3*cosh(b*x + a)^2)*sinh(b*x + a)^6 + 48*(91*cosh(b*x + a)^11 - 66*c
osh(b*x + a)^7 + 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + 4*(455*cosh(b*x + a)^
12 - 495*cosh(b*x + a)^8 + 105*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 4*cos
h(b*x + a)^4 + 16*(35*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 + 21*cosh(b*x +
 a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 24*(5*cosh(b*x + a)^14 - 11*cosh(b
*x + a)^10 + 7*cosh(b*x + a)^6 - cosh(b*x + a)^2)*sinh(b*x + a)^2 + 16*(cos
h(b*x + a)^15 - 3*cosh(b*x + a)^11 + 3*cosh(b*x + a)^7 - cosh(b*x + a)^3)*s
inh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*
(21*cosh(b*x + a)^13 - 55*cosh(b*x + a)^9 - 33*cosh(b*x + a)^5 + 3*cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^16 + 560*b*cosh(b*x + a)^3*sinh(b*x
+ a)^13 + 120*b*cosh(b*x + a)^2*sinh(b*x + a)^14 + 16*b*cosh(b*x + a)*sinh(
b*x + a)^15 + b*sinh(b*x + a)^16 - 4*b*cosh(b*x + a)^12 + 4*(455*b*cosh(b*x
 + a)^4 - b)*sinh(b*x + a)^12 + 48*(91*b*cosh(b*x + a)^5 - b*cosh(b*x + a))
*sinh(b*x + a)^11 + 88*(91*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*
x + a)^10 + 880*(13*b*cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^9
+ 6*b*cosh(b*x + a)^8 + 6*(2145*b*cosh(b*x + a)^8 - 330*b*cosh(b*x + a)^4 +
 b)*sinh(b*x + a)^8 + 16*(715*b*cosh(b*x + a)^9 - 198*b*cosh(b*x + a)^5 + 3
*b*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*b*cosh(b*x + a)^10 - 66*b*cosh(
b*x + a)^6 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^6 + 48*(91*b*cosh(b*x + a)^
11 - 66*b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)^3)*sinh(b*x + a)^5 - 4*b*cosh
(b*x + a)^4 + 4*(455*b*cosh(b*x + a)^12 - 495*b*cosh(b*x + a)^8 + 105*b*cos
h(b*x + a)^4 - b)*sinh(b*x + a)^4 + 16*(35*b*cosh(b*x + a)^13 - 55*b*cosh(b
*x + a)^9 + 21*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^3 + 24*(5
*b*cosh(b*x + a)^14 - 11*b*cosh(b*x + a)^10 + 7*b*cosh(b*x + a)^6 - b*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 16*(b*cosh(b*x + a)^15 - 3*b*cosh(b*x + a)^11
 + 3*b*cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**5*sech(b*x+a)**5,x)
[Out] Integral(csch(a + b*x)**5*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.17

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{6\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{6\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{6\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} - \frac{4\left(3\,e^{(-2\,bx-2\,a)} - 11\,e^{(-6\,bx-6\,a)} - 11\,e^{(-10\,bx-10\,a)} + 3\,e^{(-14\,bx-14\,a)}\right)}{b(4\,e^{(-4\,bx-4\,a)} - 6\,e^{(-8\,bx-8\,a)} + 4\,e^{(-12\,bx-12\,a)} - e^{(-16\,bx-16\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="maxima")
[Out] 6*log(e^(-b*x - a) + 1)/b + 6*log(e^(-b*x - a) - 1)/b - 6*log(e^(-2*b*x - 2
*a) + 1)/b - 4*(3*e^(-2*b*x - 2*a) - 11*e^(-6*b*x - 6*a) - 11*e^(-10*b*x -
10*a) + 3*e^(-14*b*x - 14*a))/(b*(4*e^(-4*b*x - 4*a) - 6*e^(-8*b*x - 8*a) +
 4*e^(-12*b*x - 12*a) - e^(-16*b*x - 16*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.80

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\frac{4\left(3\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^3-20\,e^{(2\,bx+2\,a)}-20\,e^{(-2\,bx-2\,a)}\right)}{\left(\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4\right)^2} - 3\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + 3\,\log\left(e^{(2\,bx+2\,a)}+\right.}{b}$$

```
[In] integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="giac")
[Out] (4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^3 - 20*e^(2*b*x + 2*a) - 20*e^(-
2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 4)^2 - 3*log(e^(2*b
*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 3*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a
) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.97

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{12\,e^{2\,a+2\,b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-1\right)}-\frac{12\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{8\,e^{2\,a+2\,b\,x}}{b\,\left(e^{8\,a+8\,b\,x}-2\,e^{4\,a+4\,b\,x}+1\right)} - \frac{32\,e^{2\,a+2\,b\,x}}{b\,\left(3\,e^{4\,a+4\,b\,x}-3\,e^{8\,a+8\,b\,x}+e^{12\,a+12\,b\,x}-1\right)} - \frac{64\,e^{6\,a+6\,b\,x}}{b\,\left(6\,e^{8\,a+8\,b\,x}-4\,e^{4\,a+4\,b\,x}-4\,e^{12\,a+12\,b\,x}+e^{16\,a+16\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^5*sinh(a + b*x)^5),x)
```

```
[Out] (12*exp(2*a + 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (12*atan((exp(2*a)*exp(2
*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8
*b*x) - 2*exp(4*a + 4*b*x) + 1)) - (32*exp(2*a + 2*b*x))/(b*(3*exp(4*a + 4*
b*x) - 3*exp(8*a + 8*b*x) + exp(12*a + 12*b*x) - 1)) - (64*exp(6*a + 6*b*x)
)/(b*(6*exp(8*a + 8*b*x) - 4*exp(4*a + 4*b*x) - 4*exp(12*a + 12*b*x) + exp(
16*a + 16*b*x) + 1))
```

3.49 $\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b-2/5*sinh(b*x+a)^(5/2)/b/cosh(b*x+a)^(5/2)-2*sinh(b*x+a)^
(1/2)/b/cosh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2646, 2655, 304, 209, 212}

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]]) - (2*Sinh[a + b*x]^(5/2))/(5*b*Cosh[a + b*x]^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\int\frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx\\
&= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\int\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx
\end{aligned}
$$

$$= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{9}{4},\frac{13}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{9}{2}}(a+bx)}{9b\sqrt{\cosh(a+bx)}}$$

```
[In] Integrate[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(9/2))/(9*b*Sqrt[Cosh[a + b*x]])
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{7}{2}}}{\cosh(bx+a)^{\frac{7}{2}}}dx$$

```
[In] int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
[Out] int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 997, normalized size of antiderivative = 9.41

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*x
 + a)^6 + 72*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 72*cosh(b*x + a)^4 +
 96*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b*x
+ a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 - 10*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*s
inh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)^2 + 2)*sinh(b*x +
a)^3 + 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x
 + a)^2 + 3*(5*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*c
osh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 144*(cosh(b*x + a)^
5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 24)/(b*cosh(b*x + a)
^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x +
 a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^
3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(
b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(b*x+a)**(7/2)/cosh(b*x+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{7/2}}{{\mathrm{cosh}\left(a+b\,x\right)}^{7/2}}\,dx$$

```
[In] int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2),x)
[Out] int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2), x)
```

3.50 $\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b-2/3*sinh(b*x+a)^(3/2)/b/cosh(b*x+a)^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2646, 2654, 304, 209, 212}

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]
^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} + \int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx \\
&= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{7}{4},\frac{11}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{7}{2}}(a+bx)}{7b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
[Out] (2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(7/2))/(7*b*Cosh[a + b*x]^(3/2))
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
[Out] int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 591, normalized size of antiderivative = 7.30

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{4\cosh(bx+a)^4 + 16\cosh(bx+a)\sinh(bx+a)^3 + 4\sinh(bx+a)^4 + 8\left(3\cosh(bx+a)^2+1\right)\sinh(bx}{}$$

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
[Out] -1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 8*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*(cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*si
nh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a)
)*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) -
```

```
sinh(b*x + a)^2) + 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a
) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)
^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*sqrt(cosh(b*x +
 a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x +
a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(b*x+a)**(5/2)/cosh(b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{5}{2}}}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{5}{2}}}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{5/2}}{\cosh\left(a+b\,x\right)^{5/2}}\,dx$$

```
[In] int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2),x)
[Out] int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2), x)
```

3.51 $\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b-2*sinh(b*x+a)^(1/2)/b/cosh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2646, 2655, 304, 209, 212}

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} + \int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$$

$$= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{5}{2}}(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

```
[In] Integrate[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(5/2))/(5*b*Sqrt[Cosh[a + b*x]])
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}dx$$

```
[In] int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.92

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)\arctan\left(-\cosh(bx+a)^2+2\,(\cosh\right.}{}$$

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) - 4*cosh(b*x + a)^2 - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
+ a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x +
```

```
a) - sinh(b*x + a)^2) - 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a
))*sqrt(sinh(b*x + a)) - 8*cosh(b*x + a)*sinh(b*x + a) - 4*sinh(b*x + a)^2
- 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2 + b)
```

Sympy [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(sinh(b*x+a)**(3/2)/cosh(b*x+a)**(3/2),x)
[Out] Integral(sinh(a + b*x)**(3/2)/cosh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{3/2}}{\cosh\left(a+b\,x\right)^{3/2}}\,dx$$

```
[In] int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2),x)
[Out] int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2), x)
```

3.52 $\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2654, 304, 209, 212}

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

```
[In] Int[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
 + b*x]]/Sqrt[Cosh[a + b*x]]]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
[Out] (2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]^(3/2))
```

Maple [F]

$$\int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}dx$$

[In] int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)

[Out] int(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2\arctan\left(-\cosh(bx+a)^2+2\left(\cosh(bx+a)+\sinh(bx+a)\right)\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)}-2\cosh(\right.}{}$$

[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="fricas")

[Out] -1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2))/b

Sympy [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$$

[In] integrate(sinh(b*x+a)**(1/2)/cosh(b*x+a)**(1/2),x)

[Out] Integral(sqrt(sinh(a + b*x))/sqrt(cosh(a + b*x)), x)

Maxima [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(a+b\,x\right)}}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

```
[In] int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2),x)
[Out] int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2), x)
```

3.53 $\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2655, 304, 209, 212}

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

```
[In] Int[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} \\
&= -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} + \frac{\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$
\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\sinh^2(a+bx)\right)\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}
$$

```
[In] Integrate[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -Sinh[a + b*x]^
2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b*x]])
```

Maple [F]

$$\int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}dx$$

[In] int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)

[Out] int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.67

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\arctan\left(-\cosh(bx+a)^2 + 2(\cosh(bx+a)+\sinh(bx+a))\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)} - 2\cosh(bx$$

[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2) - log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2))/b

Sympy [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$$

[In] integrate(cosh(b*x+a)**(1/2)/sinh(b*x+a)**(1/2),x)

[Out] Integral(sqrt(cosh(a + b*x))/sqrt(sinh(a + b*x)), x)

Maxima [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh(a+b\,x)}}{\sqrt{\sinh(a+b\,x)}}\,dx$$

```
[In] int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2),x)
[Out] int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2), x)
```

3.54 $\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)} dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b-2*cosh(b*x+a)^(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2647, 2654, 304, 209, 212}

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)} dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[In] Int[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\int\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx\\
&= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}-\frac{2\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}\\
&= -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},-\sinh^2(a+bx)\right)}{b\cosh^{\frac{3}{2}}(a+bx)\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
[Out] (-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -Sinh[a + b*
x]^2])/(b*Cosh[a + b*x]^(3/2)*Sqrt[Sinh[a + b*x]])
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{3}{2}}}{\sinh(bx+a)^{\frac{3}{2}}}dx$$

```
[In] int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
[Out] int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(67) = 134.

Time = 0.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.94

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx =$$

$$-\frac{2\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\arctan\left(-\cosh(bx+a)^2+2\,(\cos\right.}{}$$

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(
b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
a)^2) + 4*cosh(b*x + a)^2 + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x +
 a) - sinh(b*x + a)^2) + 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x +
a))*sqrt(sinh(b*x + a)) + 8*cosh(b*x + a)*sinh(b*x + a) + 4*sinh(b*x + a)^2
 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2 - b)
```

Sympy [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(cosh(b*x+a)**(3/2)/sinh(b*x+a)**(3/2),x)
[Out] Integral(cosh(a + b*x)**(3/2)/sinh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{3}{2}}}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{3}{2}}}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{3/2}}{\sinh(a+b\,x)^{3/2}}\,dx$$

```
[In] int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2),x)
[Out] int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2), x)
```

3.55 $\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[Out] -arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/si
nh(b*x+a)^(1/2))/b-2/3*cosh(b*x+a)^(3/2)/b/sinh(b*x+a)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2647, 2655, 304, 209, 212}

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/b) + ArcTanh[Sqrt[Cosh[a
+ b*x]]/Sqrt[Sinh[a + b*x]]]/b - (2*Cosh[a + b*x]^(3/2))/(3*b*Sinh[a + b*x]
^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\int\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx\\
&= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}\\
&= -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)} dx = -\frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{3}{4},\frac{1}{4},-\sinh^2(a+bx)\right)}{3b\sqrt{\cosh(a+bx)}\sinh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -Sinh[a + b*
x]^2])/(3*b*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x]^(3/2))
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}} dx$$

```
[In] int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 598, normalized size of antiderivative = 7.38

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)} dx =$$

$$-\frac{4\cosh(bx+a)^4+16\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4+8\left(3\cosh(bx+a)^2-1\right)\sinh(bx}{}$$

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a
)^4 + 8*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a)
)*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) -
 sinh(b*x + a)^2) - 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
```

```
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b
*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a
)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)
^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*sqrt(cosh(b*x +
 a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x +
a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(b*x+a)**(5/2)/sinh(b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{5/2}}{\sinh\left(a+b\,x\right)^{5/2}}\,dx$$

```
[In] int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2),x)
[Out] int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2), x)
```

3.56 $\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)} dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[Out] -arctan(sinh(b*x+a)^(1/2)/cosh(b*x+a)^(1/2))/b+arctanh(sinh(b*x+a)^(1/2)/co
sh(b*x+a)^(1/2))/b-2/5*cosh(b*x+a)^(5/2)/b/sinh(b*x+a)^(5/2)-2*cosh(b*x+a)^
(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2647, 2654, 304, 209, 212}

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)} dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[In] Int[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/b) + ArcTanh[Sqrt[Sinh[a
+ b*x]]/Sqrt[Cosh[a + b*x]]]/b - (2*Cosh[a + b*x]^(5/2))/(5*b*Sinh[a + b*x]
^(5/2)) - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\int\frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx\\ &= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\int\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx\end{aligned}$$

$$= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4}, -\frac{5}{4}, -\frac{1}{4}, -\sinh^2(a+bx)\right)}{5b\cosh^{\frac{3}{2}}(a+bx)\sinh^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
[Out] (-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, -Sinh[a + b
*x]^2])/(5*b*Cosh[a + b*x]^(3/2)*Sinh[a + b*x]^(5/2))
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{7}{2}}}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
[Out] int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 1001, normalized size of antiderivative = 9.44

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*x
 + a)^6 + 72*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 72*cosh(b*x + a)^4 +
 96*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b*x
+ a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 10*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a
))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 -
1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*si
nh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*s
inh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)^2 - 2)*sinh(b*x +
a)^3 - 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x
 + a)^2 + 3*(5*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*c
osh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 144*(cosh(b*x + a)^
5 - 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 24)/(b*cosh(b*x + a)
^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x +
 a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^
3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(
b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(b*x+a)**(7/2)/sinh(b*x+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{{\mathrm{cosh}\left(a+b\,x\right)}^{7/2}}{{\mathrm{sinh}\left(a+b\,x\right)}^{7/2}}\,dx$$

```
[In] int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2),x)
[Out] int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2), x)
```

3.57 $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

```
[Out] -1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3)
/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/4*sinh(b*x+a)^(
4/3)/b/cosh(b*x+a)^(4/3)-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{2646, 2654, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt
[3]])/b - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx \\
&= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= -\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}
\end{aligned}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{5}{3},\frac{8}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{10}{3}}(a+bx)}{10b\cosh^{\frac{4}{3}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(10/3))/(10*b*Cosh[a + b*x]^(4/3))
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{\frac{7}{3}}}{\cosh\left(bx+a\right)^{\frac{7}{3}}}dx$$

[In] int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)

[Out] int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 1042, normalized size of antiderivative = 6.72

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \text{Too large to display}$$

[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="fricas")

[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3 + sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 + sqrt(3))*sinh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 + sqrt(3)*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a

```
)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^
(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(b*x+a)**(7/3)/cosh(b*x+a)**(7/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{3}}}{\cosh\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{3}}}{\cosh\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{7/3}}{\cosh\left(a+b\,x\right)^{7/3}}\,dx$$

```
[In] int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3),x)
[Out] int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3), x)
```

3.58 $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)} dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

```
[Out] -1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3)
/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/2*sinh(b*x+a)^(
2/3)/b/cosh(b*x+a)^(2/3)-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{2646, 2655, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt
[3]])/b - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{4}{3},\frac{7}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{8}{3}}(a+bx)}{8b\cosh^{\frac{2}{3}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(8/3))/(8*b*Cosh[a + b*x]^(2/3))
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}dx$$

[In] int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)

[Out] int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.85

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \text{Too large to display}$$

[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="fricas")

[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 12*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(b*x+a)**(5/3)/cosh(b*x+a)**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{5}{3}}}{\cosh\left(bx+a\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{5}{3}}}{\cosh\left(bx+a\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{5/3}}{{\mathrm{cosh}\left(a+b\,x\right)}^{5/3}}\,dx$$

```
[In] int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3),x)
[Out] int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3), x)
```

3.59 $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 243

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

```
[Out] arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-1/4*ln(1+cosh(b*x+a)^(2/3)/s
inh(b*x+a)^(2/3)-cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/4*ln(1+cosh(b*x+a
)^(2/3)/sinh(b*x+a)^(2/3)+cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-3*sinh(b*x
+a)^(1/3)/b/cosh(b*x+a)^(1/3)+1/2*arctan(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*
x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b
*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2646, 2655, 302, 648, 632, 210, 642, 212}

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} - \frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}} + 1\right)}{4b} + \frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}} + 1\right)}{4b}$$

```
[In] Int[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b) - (3*Sinh[a + b*x]^(1/3))/(b*Cosh
[a + b*x]^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \frac{3\text{Subst}\left(\int \frac{x^4}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b}$$

$$
\begin{aligned}
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} \\
&+ \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}} \\
&+ \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{2b} \\
&+ \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{2b} \\
&= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} \\
&+ \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} \\
&+ \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$
\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{7}{6},\frac{13}{6},-\sinh^2(a+bx)\right)\sinh^{\frac{7}{3}}(a+bx)}{7b\sqrt[3]{\cosh(a+bx)}}
$$

```
[In] Integrate[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
[Out] (3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(7/3))/(7*b*Cosh[a + b*x]^(1/3))
```

Maple [F]

$$\int \frac{\sinh \left(bx + a\right)^{\frac{4}{3}}}{\cosh \left(bx + a\right)^{\frac{4}{3}}} dx$$

[In] int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)

[Out] int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 1003, normalized size of antiderivative = 4.13

$$\int \frac{\sinh^{\frac{4}{3}}(a + bx)}{\cosh^{\frac{4}{3}}(a + bx)} dx = \text{Too large to display}$$

[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="fricas")

[Out] 1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + s
qrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*s
qrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*c
osh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 + 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x
 + a) + sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(-1/3*(sqrt(3)*cosh(b*x +
a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*
(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 + 1)) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x

```
+ a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x +
a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 + 1)) - 24*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)
^(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
+ a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx=\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$$

```
[In] integrate(sinh(b*x+a)**(4/3)/cosh(b*x+a)**(4/3),x)
[Out] Integral(sinh(a + b*x)**(4/3)/cosh(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx=\int \frac{\sinh{(bx+a)}^{\frac{4}{3}}}{\cosh{(bx+a)}^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx=\int \frac{\sinh{(bx+a)}^{\frac{4}{3}}}{\cosh{(bx+a)}^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{4/3}}{\cosh\left(a+b\,x\right)^{4/3}}\,dx$$

```
[In] int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3),x)
[Out] int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3), x)
```

3.60 $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

```
[Out] arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b-1/4*ln(1-sinh(b*x+a)^(1/3)/c
osh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a
)^(1/3)/cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/2*arctan
(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arcta
n(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2654, 302, 648, 632, 210, 642, 212}

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} + 1\right)}{4b} + \frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} + 1\right)}{4b}$$

```
[In] Int[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{2b}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\text{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.27

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\cosh^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},-\sinh^2(a+bx)\right)\sinh^{\frac{5}{3}}(a+bx)}{5b\cosh^{\frac{5}{3}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(5/3))/(5*b*Cosh[a + b*x]^(5/3))
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

```
[In] int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

```
[Out] int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(176) = 352$.

Time = 0.28 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.33

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a
)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt
(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*sqr
t(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*
x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + log((cosh(b*x
+ a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
```

```
)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - log((cosh(b
*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x
 + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 2*lo
g(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)
*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/
b
```

Sympy [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

```
[In] integrate(sinh(b*x+a)**(2/3)/cosh(b*x+a)**(2/3),x)
[Out] Integral(sinh(a + b*x)**(2/3)/cosh(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{2}{3}}}{\cosh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{2/3}}{\cosh\left(a+b\,x\right)^{2/3}}\,dx$$

```
[In] int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3),x)
[Out] int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3), x)
```

3.61 $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

```
[Out] -1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3)
/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-1/2*arctan(1/3*(1
+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2654, 281, 298, 31, 648, 632, 210, 642}

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt
[3]])/b - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{1}{3}}}{\cosh(bx+a)^{\frac{1}{3}}}dx$$

```
[In] int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

```
[Out] int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.47

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2+4\left(\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)\right)\cosh(bx+a)^{\frac{1}{3}}\sinh}{3\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)}\right.}{}$$

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqr
t(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a
```

```
)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3
 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/
3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(b*x + a)^2 - 2
*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx=\int \frac{\sqrt[3]{\sinh{(a+bx)}}}{\sqrt[3]{\cosh{(a+bx)}}}\,dx$$

```
[In] integrate(sinh(b*x+a)**(1/3)/cosh(b*x+a)**(1/3),x)
[Out] Integral(sinh(a + b*x)**(1/3)/cosh(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx=\int \frac{\sinh{(bx+a)}^{\frac{1}{3}}}{\cosh{(bx+a)}^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx=\int \frac{\sinh{(bx+a)}^{\frac{1}{3}}}{\cosh{(bx+a)}^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sinh(a+b\,x)^{1/3}}{\cosh\left(a+b\,x\right)^{1/3}}\,dx$$

```
[In] int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3),x)
[Out] int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3), x)
```

3.62 $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

```
[Out] -1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3)
/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-1/2*arctan(1/3*(1
+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2655, 281, 298, 31, 648, 632, 210, 642}

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt
[3]])/b - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{3\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

```
[In] Integrate[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{1}{3}}}{\sinh(bx+a)^{\frac{1}{3}}}dx$$

```
[In] int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

```
[Out] int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(103) = 206.

Time = 0.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\cosh(bx+a)^2+2\sqrt{3}\cosh(bx+a)\sinh(bx+a)+\sqrt{3}\sinh(bx+a)^2+4\left(\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(bx+a)\right)\cosh(bx+a)^{\frac{2}{3}}\sinh}{3\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)}\right.}{}$$

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqr
t(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a
```

```
)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/
3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3
 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/
3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)
+ 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(b*x + a)^2 - 2
*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

```
[In] integrate(cosh(b*x+a)**(1/3)/sinh(b*x+a)**(1/3),x)
[Out] Integral(cosh(a + b*x)**(1/3)/sinh(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh(bx+a)^{\frac{1}{3}}}{\sinh(bx+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh(bx+a)^{\frac{1}{3}}}{\sinh(bx+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh(a+b\,x)^{1/3}}{\sinh\left(a+b\,x\right)^{1/3}}\,dx$$

```
[In] int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3),x)
[Out] int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3), x)
```

3.63 $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 218

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b}$$

```
[Out] arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b-1/4*ln(1+cosh(b*x+a)^(2/3)/s
inh(b*x+a)^(2/3)-cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/4*ln(1+cosh(b*x+a
)^(2/3)/sinh(b*x+a)^(2/3)+cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/2*arctan
(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arcta
n(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2655, 302, 648, 632, 210, 642, 212}

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}} + 1\right)}{4b} + \frac{\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}} + 1\right)}{4b}$$

```
[In] Int[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/b - Log[
1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a +
b*x]^(1/3)]/(4*b) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh[
a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/(4*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{x^4}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{2b}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\text{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} - \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b} + \frac{\log\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.26

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},-\sinh^2(a+bx)\right)\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

```
[In] Integrate[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
[Out] (3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -Sinh[a + b*x]^
2]*Sinh[a + b*x]^(1/3))/(b*Cosh[a + b*x]^(1/3))
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}dx$$

```
[In] int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
[Out] int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(176) = 352.

Time = 0.28 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.32

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="fricas")
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a
)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt
(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*sqr
t(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*
x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + log((cosh(b*x
+ a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
```

```
)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x +
a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*log((cosh
(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - log((co
sh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(
b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - 2*lo
g(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)
*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/
b
```

Sympy [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

```
[In] integrate(cosh(b*x+a)**(2/3)/sinh(b*x+a)**(2/3),x)
[Out] Integral(cosh(a + b*x)**(2/3)/sinh(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{2}{3}}}{\sinh\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{2/3}}{\sinh\left(a+b\,x\right)^{2/3}}\,dx$$

```
[In] int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3),x)
[Out] int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3), x)
```

3.64 $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 243

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

```
[Out] arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b-1/4*ln(1-sinh(b*x+a)^(1/3)/c
osh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a
)^(1/3)/cosh(b*x+a)^(1/3)+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b-3*cosh(b*x
+a)^(1/3)/b/sinh(b*x+a)^(1/3)+1/2*arctan(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*
x+a)^(1/3))*3^(1/2))*3^(1/2)/b-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b
*x+a)^(1/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2647, 2654, 302, 648, 632, 210, 642, 212}

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} - \frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)} - \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} + 1\right)}{4b} + \frac{\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)} + \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} + 1\right)}{4b}$$

```
[In] Int[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]])
/(2*b) - (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/
Sqrt[3]])/(2*b) + ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)]/b - Log[
1 - Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a +
b*x]^(2/3)]/(4*b) + Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[
a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(1/3))/(b*Sinh
[a + b*x]^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} + \int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} - \frac{3\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b}$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{4b}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{2b}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} - \frac{\log\left(1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cosh^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},-\frac{1}{6},\frac{5}{6},-\sinh^2(a+bx)\right)}{b\cosh^{\frac{5}{3}}(a+bx)\sqrt[3]{\sinh(a+bx)}}$$

```
[In] Integrate[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
[Out] (-3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, -Sinh[a + b*
x]^2])/(b*Cosh[a + b*x]^(5/3)*Sinh[a + b*x]^(1/3))
```

Maple [F]

$$\int \frac{\cosh\left(bx+a\right)^{\frac{4}{3}}}{\sinh\left(bx+a\right)^{\frac{4}{3}}}dx$$

[In] int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)

[Out] int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(197) = 394$.

Time = 0.28 (sec) , antiderivative size = 1013, normalized size of antiderivative = 4.17

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="fricas")

```
[Out] 1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + s
qrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*s
qrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*c
osh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/
3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 - 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x
 + a) + sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(-1/3*(sqrt(3)*cosh(b*x +
a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*
(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a
))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x +
a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x +
 a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
 sinh(b*x + a)^2 - 1)) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x
```

```
+ a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x +
a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
+ sinh(b*x + a)^2 - 1)) - 24*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)
^(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
+ a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$$

```
[In] integrate(cosh(b*x+a)**(4/3)/sinh(b*x+a)**(4/3),x)
[Out] Integral(cosh(a + b*x)**(4/3)/sinh(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{4/3}}{\sinh\left(a+b\,x\right)^{4/3}}\,dx$$

```
[In] int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3),x)
[Out] int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3), x)
```

3.65 $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

```
[Out] -1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2/3)
/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/2*cosh(b*x+a)^(
2/3)/b/sinh(b*x+a)^(2/3)-1/2*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{2647, 2654, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt
[3]])/b - Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3) + Sinh[a + b*x]^(4/3)/Cosh[a + b*x]
^(4/3)]/(4*b) - (3*Cosh[a + b*x]^(2/3))/(2*b*Sinh[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx \\
&= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} \\
&= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= -\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}
\end{aligned}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\sinh^2(a+bx)\right)}{2b\cosh^{\frac{4}{3}}(a+bx)\sinh^{\frac{2}{3}}(a+bx)}$$

```
[In] Integrate[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
[Out] (-3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -Sinh[a + b*
x]^2])/(2*b*Cosh[a + b*x]^(4/3)*Sinh[a + b*x]^(2/3))
```

Maple [F]

$$\int \frac{\cosh\left(bx+a\right)^{\frac{5}{3}}}{\sinh\left(bx+a\right)^{\frac{5}{3}}}dx$$

[In] int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)

[Out] int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(124) = 248$.

Time = 0.27 (sec) , antiderivative size = 749, normalized size of antiderivative = 4.83

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="fricas")

[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 12*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(b*x+a)**(5/3)/sinh(b*x+a)**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{3}}}{\sinh(bx+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{5/3}}{\sinh(a+b\,x)^{5/3}}\,dx$$

```
[In] int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3),x)
[Out] int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3), x)
```

3.66 $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

```
[Out] -1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4/3)
/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/4*cosh(b*x+a)^(
4/3)/b/sinh(b*x+a)^(4/3)-1/2*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^
(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{2647, 2655, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)}{2b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt
[3]])/b - Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)]/(2*b) + Log[1 +
Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3) + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]
^(2/3)]/(4*b) - (3*Cosh[a + b*x]^(4/3))/(4*b*Sinh[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x^3}{1-x^6}\,dx, x, \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b}$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\sinh^2(a+bx)\right)}{4b\cosh^{\frac{2}{3}}(a+bx)\sinh^{\frac{4}{3}}(a+bx)}$$

```
[In] Integrate[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

```
[Out] (-3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -Sinh[a + b*
x]^2])/(4*b*Cosh[a + b*x]^(2/3)*Sinh[a + b*x]^(4/3))
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}dx$$

[In] int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)

[Out] int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(124) = 248$.

Time = 0.27 (sec) , antiderivative size = 1056, normalized size of antiderivative = 6.81

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="fricas")

[Out] -1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3 + sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 - sqrt(3))*sinh(b*x + a)^2 - 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 - sqrt(3)*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a

```
)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x + a)^
(1/3)*sinh(b*x + a)^(2/3))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x
+ a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 -
 b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(b*x+a)**(7/3)/sinh(b*x+a)**(7/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{3}}}{\sinh\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{3}}}{\sinh\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx=\int \frac{{\cosh(a+b\,x)}^{7/3}}{{\sinh\left(a+b\,x\right)}^{7/3}}\,dx$$

```
[In] int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3),x)
[Out] int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3), x)
```

3.67 $\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

```
[Out] -3/5*cosh(x)^(5/3)/sinh(x)^(5/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2643}

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

```
[In] Int[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

```
[In] Integrate[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
[Out] (-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Maple [F]

$$\int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}\,dx$$

```
[In] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
[Out] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{6\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 + 1\right)\sinh(x) + \cosh(x)\right)\cosh}{5\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 4\left(\cosh(x\right.\right.}$$

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="fricas")
[Out] -6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(
x) + cosh(x))*cosh(x)^(2/3)*sinh(x)^(1/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
+ sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 -
cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**(2/3)/sinh(x)**(8/3),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}} - \frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}$$

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="maxima")
[Out] 3/5*(e^(-2*x) + 1)^(2/3)*e^(-4*x)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
- 3/5*(e^(-2*x) + 1)^(2/3)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
```

Giac [F]

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="giac")
[Out] integrate(cosh(x)^(2/3)/sinh(x)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\coth(x)^{5/3}}{5}$$

```
[In] int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
[Out] -(3*coth(x)^(5/3))/5
```

3.68 $\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)} dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

```
[Out] 3/5*sinh(x)^(5/3)/cosh(x)^(5/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2643}

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)} dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

```
[In] Int[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

```
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

```
[In] Integrate[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
[Out] (3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Maple [F]

$$\int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}dx$$

```
[In] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
[Out] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{6\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)\right)\cosh(x)}{5\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)^3 + \right.\right.}$$

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="fricas")
[Out] 6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x
) - cosh(x))*cosh(x)^(1/3)*sinh(x)^(2/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + c
osh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**(2/3)/cosh(x)**(8/3),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = -\frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}} + \frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}}$$

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="maxima")
[Out] -3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)*e^(-4*x)/(e^(-2*x) + 1)^(8/3) +
 3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)/(e^(-2*x) + 1)^(8/3)
```

Giac [F]

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{\sinh\left(x\right)^{\frac{2}{3}}}{\cosh\left(x\right)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="giac")
[Out] integrate(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{{\mathrm{sinh}(x)}^{2/3}}{{\mathrm{cosh}\left(x\right)}^{8/3}}\,dx$$

```
[In] int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
[Out] int(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

3.69 $\int \cosh(x)\mathbf{csch}^{\frac{7}{3}}(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

```
[Out] -3/4*csch(x)^(4/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2701, 30}

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

```
[In] Int[Cosh[x]*Csch[x]^(7/3),x]
```

```
[Out] (-3*Csch[x]^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \sqrt[3]{x}\,dx, x, \text{csch}(x)\right)\\ &= -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

```
[In] Integrate[Cosh[x]*Csch[x]^(7/3),x]
[Out] (-3*Csch[x]^(4/3))/4
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{3\,\text{csch}(x)^{\frac{4}{3}}}{4}$	7
default	$-\frac{3\,\text{csch}(x)^{\frac{4}{3}}}{4}$	7

```
[In] int(cosh(x)*csch(x)^(7/3),x,method=_RETURNVERBOSE)
[Out] -3/4*csch(x)^(4/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 5.40

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{\cosh(x)+\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}\right)^{\frac{1}{3}}(\cosh(x)+\sinh(x))}{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}$$

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="fricas")
[Out] -3/2*2^(1/3)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1))^(1/3)*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)*csch(x)**(7/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = \int \cosh(x)\,\text{csch}(x)^{\frac{7}{3}}\,dx$$

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="maxima")
[Out] integrate(cosh(x)*csch(x)^(7/3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3 \cdot 2^{\frac{1}{3}} e^{\left(\frac{4}{3}\,x\right)}}{2\,\left(e^{(2\,x)} - 1\right)^{\frac{4}{3}}}$$

```
[In] integrate(cosh(x)*csch(x)^(7/3),x, algorithm="giac")
[Out] -3/2*2^(1/3)*e^(4/3*x)/(e^(2*x) - 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\,e^x\,\left(-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}\right)^{1/3}}{2\,\left(e^{2\,x}-1\right)}$$

```
[In] int(cosh(x)*(1/sinh(x))^(7/3),x)
[Out] -(3*exp(x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/3))/(2*(exp(2*x) - 1))
```

3.70 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 327, 209}

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=\frac{\sinh(a+bx)}{b}-\frac{\arctan(\sinh(a+bx))}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b} \\ &= \frac{\sinh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b} \\ &= -\frac{\arctan(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{\arctan(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\sinh(bx+a)-2\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	21
default	$\frac{\sinh(bx+a)-2\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	21
risch	$\frac{\mathrm{e}^{bx+a}}{2b} - \frac{\mathrm{e}^{-bx-a}}{2b} + \frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{b} - \frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{b}$	59

```
[In] int(sinh(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\cosh\left(bx+ \right.}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
[Out] -1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a
)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \int \sinh\left(a+bx\right)\tanh\left(a+bx\right)dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*tanh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
[Out] 2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\arctan\left(e^{(bx+a)}\right) - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="giac")
[Out] -1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-b\,x}}{2\,b}$$

```
[In] int(sinh(a + b*x)*tanh(a + b*x),x)
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
- exp(- a - b*x)/(2*b)
```

3.71 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b+sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}+6\,e^{bx+a}+e^{-bx-a}}{2b\left(1+e^{2bx+2a}\right)}$	46

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(bx+a)^2 + \sinh(bx+a)^2 + 3}{2\,b\cosh(bx+a)}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \sinh(a+bx)\tanh^2(a+bx)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**2,x)
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} + \frac{5\,e^{(-2\,bx-2\,a)} + 1}{2\,b(e^{(-bx-a)} + e^{(-3\,bx-3\,a)})}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3*
b*x - 3*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} + e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) + e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{-a-b\,x}\,\left(6\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(sinh(a + b*x)*tanh(a + b*x)^2,x)
[Out] (exp(- a - b*x)*(6*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))/(2*b*(exp(2*a
+ 2*b*x) + 1))
```

3.72 $\int \sinh(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} + \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+a))/b+3/2*sinh(b*x+a)/b-1/2*sinh(b*x+a)*tanh(b*x+a)^2/
b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2672, 294, 327, 209}

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} + \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sinh[a + b*x])/(2*b) - (Sinh[a + b*x]
*Tanh[a + b*x]^2)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \sinh(a+bx)\right)}{b}\\
&= -\frac{\sinh(a+bx)\tanh^2(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}\\
&= \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}\\
&= -\frac{3\arctan(\sinh(a+bx))}{2b} + \frac{3\sinh(a+bx)}{2b} - \frac{\sinh(a+bx)\tanh^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} + \frac{3\mathrm{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{\sinh(a+bx)\tanh^2(a+bx)}{b}$$

[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^3,x]

[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(2*b) + (Sinh[a + b*x]*Tanh[a + b*x]^2)/b

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)}{b}$	62
default	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)}{b}$	62
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2bx+2a}-1\right)}{b\left(1+e^{2bx+2a}\right)^2}+\frac{3i\ln\left(e^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(e^{bx+a}+i\right)}{2b}$	93

[In] int(sinh(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/b*(sinh(b*x+a)^3/cosh(b*x+a)^2+3/cosh(b*x+a)^2*sinh(b*x+a)-3/2*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp(b*x+a)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(43) = 86$.

Time = 0.25 (sec) , antiderivative size = 463, normalized size of antiderivative = 9.45

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a)^4}{}$$

[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b

```
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*si
nh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3
 + 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a
)^2 + (5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x
+ a))*arctan(cosh(b*x + a) + sinh(b*x + a)) - 3*cosh(b*x + a)^2 + 6*(cosh(b
*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b
*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 2*b*cos
h(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + 2*(5*b*cosh(b*
x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a) + (5*b*cosh
(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \int \sinh{\left(a+bx \right)}\tanh^3{\left(a+bx \right)}\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**3,x)
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.86

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)}}{2b} + \frac{4e^{(-2bx-2a)} - e^{(-4bx-4a)} + 1}{2b(e^{(-bx-a)} + 2e^{(-3bx-3a)} + e^{(-5bx-5a)})}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
[Out] 3*arctan(e^(-b*x - a))/b - 1/2*e^(-b*x - a)/b + 1/2*(4*e^(-2*b*x - 2*a) - e
^(-4*b*x - 4*a) + 1)/(b*(e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + e^(-5*b*x - 5*
a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\,\pi - \frac{4\,\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 6\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right) - 2\,e^{(bx+a)} + 2\,e^{(-bx-a)}}{4\,b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2
 + 4) + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) - 2*e^(b*x + a) +
2*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-b\,x}}{2\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(sinh(a + b*x)*tanh(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
- exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a
+ 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.73 $\int \sinh(a+bx)\tanh^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{2\operatorname{sech}(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[Out] cosh(b*x+a)/b+2*sech(b*x+a)/b-1/3*sech(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 276}

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{2\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
[Out] Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{\cosh(bx+a)^3}+\frac{4\sinh(bx+a)^2}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$	51
default	$\frac{\frac{\sinh(bx+a)^4}{\cosh(bx+a)^3}+\frac{4\sinh(bx+a)^2}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$	51
risch	$\frac{3\,e^{7bx+7a}+36\,e^{5bx+5a}+50\,e^{3bx+3a}+36\,e^{bx+a}+3\,e^{-bx-a}}{6b\left(1+e^{2bx+2a}\right)^3}$	72

```
[In] int(sinh(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)^4/cosh(b*x+a)^3+4*sinh(b*x+a)^2/cosh(b*x+a)^3+8/3/cosh(b*x
+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.51

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{3\cosh(bx+a)^4 + 3\sinh(bx+a)^4 + 18\left(\cosh(bx+a)^2+2\right)\sinh(bx+a)^2 + 36\cosh(bx+a)^2 + 25}{6\left(b\cosh(bx+a)^3 + 3\,b\cosh(bx+a)\sinh(bx+a)^2 + 3\,b\cosh(bx+a)\right)}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 + 2)*sinh(
b*x + a)^2 + 36*cosh(b*x + a)^2 + 25)/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a
)*sinh(b*x + a)^2 + 3*b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \int \sinh(a+bx)\tanh^4(a+bx)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)**4,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(35) = 70$.

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.65

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} + \frac{33\,e^{(-2\,bx-2\,a)} + 41\,e^{(-4\,bx-4\,a)} + 27\,e^{(-6\,bx-6\,a)} + 3}{6\,b\left(e^{(-bx-a)} + 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)}\right)}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b + 1/6*(33*e^(-2*b*x - 2*a) + 41*e^(-4*b*x - 4*a) + 27*e^
(-6*b*x - 6*a) + 3)/(b*(e^(-b*x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5
*a) + e^(-7*b*x - 7*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{\frac{8\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}+3\,e^{(bx+a)}+3\,e^{(-bx-a)}}{6\,b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 2)/(e^(b*x + a) + e^(-b*x - a))^
3 + 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{2\,b}+\frac{\mathrm{e}^{-a-b\,x}}{2\,b}-\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}+\frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}+\frac{4\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(sinh(a + b*x)*tanh(a + b*x)^4,x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(2*exp(2*
a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp(2*a + 2*
b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*exp(a + b*x))/(b*(e
xp(2*a + 2*b*x) + 1))
```

3.74 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] 1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^2(a+bx)}{2b} - \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{-\frac{1}{2}\cosh^2(a+bx)+\log(\cosh(a+bx))}{b}$$

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
[Out] -((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
default	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
risch	$x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{2a}{b}-\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	54

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*sinh(b*x+a)^2-ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.04

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{8\,bx\cosh\left(bx+a\right)^2 + \cosh\left(bx+a\right)^4 + 4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + \sinh\left(bx+a\right)^4 + 2\,\left(4\,bx + 3\,\cosh\left(bx\right.\right.}{8}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a
)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*co
sh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) + cos
h(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sin
h(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \sinh^2\left(a+bx\right)\tanh\left(a+bx\right)dx$$

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*b
*x - 2*a) + 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{8\,bx-\left(4\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+8\,a+e^{(2\,bx+2\,a)}-8\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*a + e^(2*b*x + 2*
a) - 8*log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = x-\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b}+\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b}$$

```
[In] int(sinh(a + b*x)^2*tanh(a + b*x),x)
```

```
[Out] x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

3.75 $\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx=-\frac{3x}{2}+\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] -3/2*x+3/2*tanh(b*x+a)/b+1/2*sinh(b*x+a)^2*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 327, 212}

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx=\frac{3\tanh(a+bx)}{2b}+\frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}-\frac{3x}{2}$$

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 + (3*Tanh[a + b*x])/(2*b) + (Sinh[a + b*x]^2*Tanh[a + b*x])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= \frac{3\tanh(a+bx)}{2b} + \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= -\frac{3x}{2} + \frac{3\tanh(a+bx)}{2b} + \frac{\sinh^2(a+bx)\tanh(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$
\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{-6(a+bx)+\sinh(2(a+bx))+4\tanh(a+bx)}{4b}
$$

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
[Out] (-6*(a + b*x) + Sinh[2*(a + b*x)] + 4*Tanh[a + b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{2\cosh(bx+a)}-\frac{3bx}{2}-\frac{3a}{2}+\frac{3\tanh(bx+a)}{2}}{b}$	39
default	$\frac{\frac{\sinh(bx+a)^3}{2\cosh(bx+a)}-\frac{3bx}{2}-\frac{3a}{2}+\frac{3\tanh(bx+a)}{2}}{b}$	39
risch	$-\frac{3x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{2}{b(1+e^{2bx+2a})}$	51

[In] int(sinh(b*x+a)^2*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/b*(1/2*sinh(b*x+a)^3/cosh(b*x+a)-3/2*b*x-3/2*a+3/2*tanh(b*x+a))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\sinh(bx+a)^3-4(3bx+2)\cosh(bx+a)+3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{8b\cosh(bx+a)}$$

[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")

[Out] 1/8*(sinh(b*x + a)^3 - 4*(3*b*x + 2)*cosh(b*x + a) + 3*(cosh(b*x + a)^2 + 3
)*sinh(b*x + a))/(b*cosh(b*x + a))

Sympy [F]

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$$

[In] integrate(sinh(b*x+a)**2*tanh(b*x+a)**2,x)

[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{17\,e^{(-2\,bx-2\,a)}+1}{8\,b\big(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}\big)}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b + 1/8*(17*e^(-2*b*x - 2*a) + 1)/(
b*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(34) = 68.

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{12\,bx+12\,a-\frac{3\,e^{(4\,bx+4\,a)}-14\,e^{(2\,bx+2\,a)}-1}{e^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}}-e^{(2\,bx+2\,a)}}{8\,b}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(12*b*x + 12*a - (3*e^(4*b*x + 4*a) - 14*e^(2*b*x + 2*a) - 1)/(e^(4*b*
x + 4*a) + e^(2*b*x + 2*a)) - e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{2\,a+2\,b\,x}}{8\,b} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{-2\,a-2\,b\,x}}{8\,b} - \frac{3\,x}{2}$$

```
[In] int(sinh(a + b*x)^2*tanh(a + b*x)^2,x)
```

```
[Out] exp(2*a + 2*b*x)/(8*b) - 2/(b*(exp(2*a + 2*b*x) + 1)) - exp(- 2*a - 2*b*x)/
(8*b) - (3*x)/2
```

3.76 $\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] 1/2*cosh(b*x+a)^2/b-2*ln(cosh(b*x+a))/b-1/2*sech(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2670, 272, 45}

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\operatorname{sech}^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}$$

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]^2/(2*b) - (2*Log[Cosh[a + b*x]])/b - Sech[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, \cosh(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \cosh^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, \cosh^2(a+bx)\right)}{2b} \\
&= \frac{\cosh^2(a+bx)}{2b} - \frac{2\log(\cosh(a+bx))}{b} - \frac{\operatorname{sech}^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{4\log(\cosh(a+bx)) + \operatorname{sech}^2(a+bx) - \sinh^2(a+bx)}{2b}$$

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
[Out] -1/2*(4*Log[Cosh[a + b*x]] + Sech[a + b*x]^2 - Sinh[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{2\cosh(bx+a)^2}-2\ln(\cosh(bx+a))+\tanh(bx+a)^2}{b}$	41
default	$\frac{\frac{\sinh(bx+a)^4}{2\cosh(bx+a)^2}-2\ln(\cosh(bx+a))+\tanh(bx+a)^2}{b}$	41
risch	$2x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{4a}{b}-\frac{2\,e^{2bx+2a}}{b\left(1+e^{2bx+2a}\right)^2}-\frac{2\ln\left(1+e^{2bx+2a}\right)}{b}$	83

```
[In] int(sinh(b*x+a)^2*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*sinh(b*x+a)^4/cosh(b*x+a)^2-2*ln(cosh(b*x+a))+tanh(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 742, normalized size of antiderivative = 17.26

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$$

$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 2\,(8\,bx+1)\cosh(bx+a)^6 + 2\,(8\,bx+14}{}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
2*(8*b*x + 1)*cosh(b*x + a)^6 + 2*(8*b*x + 14*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^6 + 4*(14*cosh(b*x + a)^3 + 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x + a
)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 + 15*(8*b*x +
1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 5
*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x + a)^3
+ 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 + 15*(8*b*x + 1)*co
sh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 + 8*b*x + 1)*sinh(b*x + a)^2
 - 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ (15*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 12*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5
 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*cosh(b*x + a)/(c
osh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 + 3*(8*b*x + 1)*cosh(
b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 + (8*b*x + 1)*cosh(b*x + a))*si
nh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b
*sinh(b*x + a)^6 + 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 2*b)*sinh(
b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*sinh(b*x + a)^3 +
b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 12*b*cosh(b*x + a)^2 + b)*sinh(
b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \int \sinh^2{(a+bx)}\tanh^3{(a+bx)}\,dx$$

```
[In] integrate(sinh(b*x+a)**2*tanh(b*x+a)**3,x)
[Out] Integral(sinh(a + b*x)**2*tanh(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\,(bx+a)}{b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)} - 15\,e^{(-4\,bx-4\,a)} + 1}{8\,b(e^{(-2\,bx-2\,a)} + 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)})}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
[Out] -2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b - 2*log(e^(-2*b*x - 2*a) + 1)/b + 1
/8*(2*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 1)/(b*(e^(-2*b*x - 2*a) + 2*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(39) = 78$.

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{16\,bx - \left(8\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 16\,a + \frac{8\,\left(3\,e^{(4\,bx+4\,a)} + 4\,e^{(2\,bx+2\,a)} + 3\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2} + e^{(2\,bx+2\,a)} - 16\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

```
[In] integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
[Out] 1/8*(16*b*x - (8*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 16*a + 8*(3*e^(4*b
*x + 4*a) + 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) + 1)^2 + e^(2*b*x + 2*a
) - 16*log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = 2\,x - \frac{2\,\ln\left(e^{2\,a}\,e^{2\,b\,x}+1\right)}{b} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} + \frac{2}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

```
[In] int(sinh(a + b*x)^2*tanh(a + b*x)^3,x)
```

```
[Out] 2*x - (2*log(exp(2*a)*exp(2*b*x) + 1))/b - 2/(b*(exp(2*a + 2*b*x) + 1)) + 2
/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*b)
 + exp(2*a + 2*b*x)/(8*b)
```

3.77 $\int \sinh^3(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] arctan(sinh(b*x+a))/b-sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2672, 308, 209}

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh^3(a+bx)}{3b}-\frac{\sinh(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \sinh(a+bx)\right)}{b} \\ &= -\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b} \\ &= \frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x],x]
[Out] ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{3}-\sinh(bx+a)+2\arctan\left(e^{bx+a}\right)}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^3}{3}-\sinh(bx+a)+2\arctan\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{e^{3bx+3a}}{24b}-\frac{5\,e^{bx+a}}{8b}+\frac{5\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	87

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*sinh(b*x+a)^3-sinh(b*x+a)+2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 290, normalized size of antiderivative = 7.63

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{\cosh{(bx+a)}^6 + 6\cosh{(bx+a)}\sinh{(bx+a)}^5 + \sinh{(bx+a)}^6 + 15\left(\cosh{(bx+a)}^2-1\right)\sinh{(bx+a)}^4}{}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="fricas")
[Out] 1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 15*(cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 15*cosh(b*x + a)^4 + 20*(cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 48*(cosh(b*x + a)^3 + 3*cosh(b*x + a)^2*
sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3)*arctan(c
osh(b*x + a) + sinh(b*x + a)) + 15*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 1
0*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^3
+ 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b
*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \int \sinh^3{(a+bx)}\tanh{(a+bx)}\,dx$$

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a),x)
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = -\frac{\left(15\,e^{(-2\,bx-2\,a)}-1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b} - \frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(15*e^(-2*b*x - 2*a) - 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) -
 e^(-3*b*x - 3*a))/b - 2*arctan(e^(-b*x - a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\left(15\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+48\,\arctan\left(e^{(bx+a)}\right)+e^{(3\,bx+3\,a)}-15\,e^{(bx+a)}}{24\,b}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/24*((15*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 48*arctan(e^(b*x + a)) +
e^(3*b*x + 3*a) - 15*e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{5\,\mathrm{e}^{a+b\,x}}{8\,b} + \frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b} - \frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

```
[In] int(sinh(a + b*x)^3*tanh(a + b*x),x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (5*exp(a + b*x))/(8
*b) + (5*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*b*
x)/(24*b)
```

3.78 $\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -2*cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b-sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh^3(a+bx)}{3b}-\frac{2\cosh(a+bx)}{b}-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x])/b + Cosh[a + b*x]^3/(3*b) - Sech[a + b*x]/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, \cosh(a+bx)\right)}{b} \\ &= -\frac{2\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\text{sech}(a+bx)}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{7\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
[Out] (-7*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{3\cosh(bx+a)}-\frac{4\sinh(bx+a)^2}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$	52
default	$\frac{\frac{\sinh(bx+a)^4}{3\cosh(bx+a)}-\frac{4\sinh(bx+a)^2}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$	52
risch	$\frac{e^{5bx+5a}-20\,e^{3bx+3a}-90\,e^{bx+a}-20\,e^{-bx-a}+e^{-3bx-3a}}{24b\left(1+e^{2bx+2a}\right)}$	68

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*sinh(b*x+a)^4/cosh(b*x+a)-4/3*sinh(b*x+a)^2/cosh(b*x+a)-8/3/cosh(b
*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-10\right)\sinh(bx+a)^2-20\cosh(bx+a)^2-45}{24\,b\cosh(bx+a)}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b
*x + a)^2 - 20*cosh(b*x + a)^2 - 45)/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a)**2,x)
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{21\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b} - \frac{20\,e^{(-2\,bx-2\,a)}+69\,e^{(-4\,bx-4\,a)}-1}{24\,b\left(e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/24*(21*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 1/24*(20*e^(-2*b*x - 2*a) +
69*e^(-4*b*x - 4*a) - 1)/(b*(e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3 - \frac{48}{e^{(bx+a)}+e^{(-bx-a)}} - 24\,e^{(bx+a)} - 24\,e^{(-bx-a)}}{24\,b}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
[Out] 1/24*((e^(b*x + a) + e^(-b*x - a))^3 - 48/(e^(b*x + a) + e^(-b*x - a)) - 24
*e^(b*x + a) - 24*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{-3\,a-3\,b\,x}}{24\,b} - \frac{7\,e^{-a-b\,x}}{8\,b} - \frac{7\,e^{a+b\,x}}{8\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(sinh(a + b*x)^3*tanh(a + b*x)^2,x)
[Out] exp(- 3*a - 3*b*x)/(24*b) - (7*exp(- a - b*x))/(8*b) - (7*exp(a + b*x))/(8*
b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.79 $\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}$$

```
[Out] 5/2*arctan(sinh(b*x+a))/b-5/2*sinh(b*x+a)/b+5/6*sinh(b*x+a)^3/b-1/2*sinh(b*
x+a)^3*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2672, 294, 308, 209}

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{5\sinh(a+bx)}{2b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}$$

```
[In] Int[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sinh[a + b*x])/(2*b) + (5*Sinh[a + b*x
]^3)/(6*b) - (Sinh[a + b*x]^3*Tanh[a + b*x]^2)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \sinh(a+bx)\right)}{b}\\
&= -\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}\\
&= -\frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b} + \frac{5\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \sinh(a+bx)\right)}{2b}\\
&= -\frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{2b}\\
&= \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\sinh(a+bx)}{2b} + \frac{5\sinh^3(a+bx)}{6b} - \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{5\sinh(a+bx)\tanh^2(a+bx)}{3b} + \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{3b}$$

```
[In] Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sech[a + b*x]*Tanh[a + b*x])/(2*b) - (
5*Sinh[a + b*x]*Tanh[a + b*x]^2)/(3*b) + (Sinh[a + b*x]^3*Tanh[a + b*x]^2)/
(3*b)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result	siz
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)^3}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{5\,\text{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan(e^{bx+a})}{b}$	81
default	$\frac{\frac{\sinh(bx+a)^5}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)^3}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{5\,\text{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan(e^{bx+a})}{b}$	81
risch	$\frac{e^{3bx+3a}}{24b}-\frac{9\,e^{bx+a}}{8b}+\frac{9\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}-\frac{e^{bx+a}(e^{2bx+2a}-1)}{b(1+e^{2bx+2a})^2}+\frac{5i\ln(e^{bx+a}+i)}{2b}-\frac{5i\ln(e^{bx+a}-i)}{2b}$	12

```
[In] int(sinh(b*x+a)^3*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*sinh(b*x+a)^5/cosh(b*x+a)^2-5/3*sinh(b*x+a)^3/cosh(b*x+a)^2-5/cosh
(b*x+a)^2*sinh(b*x+a)+5/2*sech(b*x+a)*tanh(b*x+a)+5*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 851, normalized size of antiderivative = 12.89

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + 5*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^8 - 25*cosh(b*x + a)^8 + 40*(3*
```

```
cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a)^4
 - 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63*cos
h(b*x + a)^5 - 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5 + 10
*(21*cosh(b*x + a)^6 - 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 + 5)*sinh(b
*x + a)^4 + 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 - 35*cosh(b*x + a)^5
 - 25*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cosh(b*x +
a)^8 - 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 + 60*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^2 + 120*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 +
sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^5 + 2*cosh(b*x + a
)^5 + 5*(7*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*
x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*
cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (
7*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 3*cosh(b*x + a)^2)*sinh(b*x + a))*
arctan(cosh(b*x + a) + sinh(b*x + a)) + 25*cosh(b*x + a)^2 + 10*(cosh(b*x +
 a)^9 - 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 5*co
sh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x + a)*sinh
(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cosh(b*x + a)
^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*sin
h(b*x + a)^4 + b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a)^4 + 20*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 +
3*b*cosh(b*x + a))*sinh(b*x + a)^2 + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x +
 a)^4 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$

```
[In] integrate(sinh(b*x+a)**3*tanh(b*x+a)**3,x)
[Out] Integral(sinh(a + b*x)**3*tanh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{27\,e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{24\,b} - \frac{5\,\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{25\,e^{(-2\,bx-2\,a)} + 77\,e^{(-4\,bx-4\,a)} + 3\,e^{(-6\,bx-6\,a)} - 1}{24\,b\left(e^{(-3\,bx-3\,a)} + 2\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)}\right)}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(27*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 5*arctan(e^(-b*x - a))/b - 1/
24*(25*e^(-2*b*x - 2*a) + 77*e^(-4*b*x - 4*a) + 3*e^(-6*b*x - 6*a) - 1)/(b*
(e^(-3*b*x - 3*a) + 2*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$
$$= \frac{30\,\pi + \left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 - \frac{24\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2+4} + 60\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right) - 24\,e^{(bx+a)} + 2}{24\,b}$$

```
[In] integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*(30*pi + (e^(b*x + a) - e^(-b*x - a))^3 - 24*(e^(b*x + a) - e^(-b*x -
a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4) + 60*arctan(1/2*(e^(2*b*x + 2*a) -
 1)*e^(-b*x - a)) - 24*e^(b*x + a) + 24*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.06

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{9\,\mathrm{e}^{a+b\,x}}{8\,b} + \frac{9\,\mathrm{e}^{-a-b\,x}}{8\,b} - \frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b} + \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(sinh(a + b*x)^3*tanh(a + b*x)^3,x)
```

```
[Out] (5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (9*exp(a + b*x))/(8
*b) + (9*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*b*
x)/(24*b) + (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)
) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.80 $\int \sinh^4(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx=-\frac{\cosh^2(a+bx)}{b}+\frac{\cosh^4(a+bx)}{4b}+\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] -cosh(b*x+a)^2/b+1/4*cosh(b*x+a)^4/b+ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 272, 45}

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^4(a+bx)}{4b}-\frac{\cosh^2(a+bx)}{b}+\frac{\log(\cosh(a+bx))}{b}$$

```
[In] Int[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

```
[Out] -(Cosh[a + b*x]^2/b) + Cosh[a + b*x]^4/(4*b) + Log[Cosh[a + b*x]]/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x}\,dx, x, \cosh(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \cosh^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, \cosh^2(a+bx)\right)}{2b} \\
&= -\frac{\cosh^2(a+bx)}{b}+\frac{\cosh^4(a+bx)}{4b}+\frac{\log(\cosh(a+bx))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{-\cosh^2(a+bx)+\frac{1}{4}\cosh^4(a+bx)+\log(\cosh(a+bx))}{b}$$

```
[In] Integrate[Sinh[a + b*x]^4*Tanh[a + b*x],x]
[Out] (-Cosh[a + b*x]^2 + Cosh[a + b*x]^4/4 + Log[Cosh[a + b*x]])/b
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{4}-\frac{\sinh(bx+a)^2}{2}+\ln(\cosh(bx+a))}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^4}{4}-\frac{\sinh(bx+a)^2}{2}+\ln(\cosh(bx+a))}{b}$	33
risch	$-x+\frac{e^{4bx+4a}}{64b}-\frac{3\,e^{2bx+2a}}{16b}-\frac{3\,e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}-\frac{2a}{b}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	83

```
[In] int(sinh(b*x+a)^4*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4*sinh(b*x+a)^4-1/2*sinh(b*x+a)^2+ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 457, normalized size of antiderivative = 11.42

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2-3\right)\sinh(bx+a)^6 -}{}$$

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="fricas")
[Out] 1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 4*(7*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 - 12*co
sh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 9*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*(35*cosh(b*x + a)^4 - 32*b*x - 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) - 30*cosh(b*x + a)^3)*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 - 45*cosh(b*x + a)^4 - 3)
*sinh(b*x + a)^2 - 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*si
nh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(
b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 - 9*cosh(b*x + a)^
5 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x +
 a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x +
a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

Sympy [F]

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

```
[In] integrate(sinh(b*x+a)**4*tanh(b*x+a),x)
[Out] Integral(sinh(a + b*x)**4*tanh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.02

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = -\frac{\left(12\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{64\,b}+\frac{bx+a}{b}-\frac{12\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}}{64\,b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b - 1/64*(12*
e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a))/b + log(e^(-2*b*x - 2*a) + 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = -\frac{64\,bx-\left(48\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}+64\,a-e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}-64\,\log\left(e^{(2\,bx+2\,a)}\right.}{64\,b}$$

```
[In] integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(64*b*x - (48*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4
*a) + 64*a - e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) - 64*log(e^(2*b*x + 2*a)
+ 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b}-x-\frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}}{16\,b}-\frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16\,b}+\frac{\mathrm{e}^{-4\,a-4\,b\,x}}{64\,b}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{64\,b}$$

```
[In] int(sinh(a + b*x)^4*tanh(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) + 1)/b - x - (3*exp(- 2*a - 2*b*x))/(16*b) - (3*exp
(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*b)
```

3.81 $\int \mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] -sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2686, 8}

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}(\int 1\,dx, x, \text{sech}(a+bx))}{b}\\ &= -\frac{\text{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
[Out] -(Sech[a + b*x]/b)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\text{sech}(bx+a)}{b}$	12
default	$-\frac{\text{sech}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b(1+\mathrm{e}^{2bx+2a})}$	25

```
[In] int(sech(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -sech(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+b}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \begin{cases} -\frac{\operatorname{sech}(a+bx)}{b} & \text{for } b \neq 0 \\ x\tanh(a)\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x)
[Out] Piecewise((-sech(a + b*x)/b, Ne(b, 0)), (x*tanh(a)*sech(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)} + e^{(-bx-a)}\right)}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)} + e^{(-bx-a)}\right)}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="giac")
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(tanh(a + b*x)/cosh(a + b*x),x)
[Out] -(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.82 $\int \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -1/2*sech(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*Sech[a + b*x]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}(\int x\,dx, x, \text{sech}(a+bx))}{b}$$
$$= -\frac{\text{sech}^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{\text{sech}^2(a+bx)}{2b}$$

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] -1/2*Sech[a + b*x]^2/b
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^2}{2b}$	14
default	$\frac{\tanh(bx+a)^2}{2b}$	14
risch	$-\frac{2\,e^{2bx+2a}}{b(1+e^{2bx+2a})^2}$	28

```
[In] int(sech(b*x+a)^2*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*tanh(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx-\right.\right.}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x + a)
^2 + b)*sinh(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \begin{cases} -\frac{\operatorname{sech}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tanh(a)\operatorname{sech}^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a),x)
[Out] Piecewise((-sech(a + b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)*sech(a)**2, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{\tanh(bx+a)^2}{2\,b}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
[Out] 1/2*tanh(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,a)}+1)^2}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{1}{2\,b\cosh\left(a+b\,x\right)^2}$$

```
[In] int(tanh(a + b*x)/cosh(a + b*x)^2,x)
[Out] -1/(2*b*cosh(a + b*x)^2)
```

3.83 $\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

```
[Out] -sech(b*x+a)^n/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2702, 30}

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

```
[In] Int[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]^n/(b*n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x^{-1+n}\,dx, x, \text{sech}(a+bx)\right)}{b} \\ &= -\frac{\text{sech}^n(a+bx)}{bn} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \text{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\text{sech}^n(a+bx)}{bn}$$

```
[In] Integrate[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
[Out] -(Sech[a + b*x]^n/(b*n))
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\text{sech}(bx+a)^n}{bn}$
default	$-\frac{\text{sech}(bx+a)^n}{bn}$
risch	$-\frac{2^n\left(e^{bx+a}\right)^n\left(1+e^{2bx+2a}\right)^{-n}e^{-\frac{i\,\text{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)\pi n\left(-\text{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)+\text{csgn}\left(\frac{i}{1+e^{2bx+2a}}\right)\right)\left(-\text{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)+\right.}{2}}}{bn}$

```
[In] int(sech(b*x+a)^n*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -sech(b*x+a)^n/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int \text{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)+\sinh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+\right.}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a\right.}\right.\right.}{bn}$$

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))) + sinh(n*log(2*(cosh(b*x + a)
 + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 + 1))))/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \begin{cases} x\tanh{(a)} & \text{for}\: b = 0 \wedge n = 0 \\ x\tanh{(a)}\operatorname{sech}^{n}{(a)} & \text{for}\: b = 0 \\ x - \frac{\log{(\tanh{(a+bx)}+1)}}{b} & \text{for}\: n = 0 \\ -\frac{\operatorname{sech}^{n}{(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)**n*tanh(b*x+a),x)
```

```
[Out] Piecewise((x*tanh(a), Eq(b, 0) & Eq(n, 0)), (x*tanh(a)*sech(a)**n, Eq(b, 0)
), (x - log(tanh(a + b*x) + 1)/b, Eq(n, 0)), (-sech(a + b*x)**n/(b*n), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{2^{n}e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{bn}$$

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2^n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/(b*n)
```

Giac [F]

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \int \operatorname{sech}\left(bx+a\right)^{n}\tanh\left(bx+a\right)\,dx$$

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^n*tanh(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\left(\frac{2\,e^{a+b\,x}}{e^{2\,a+2\,b\,x}+1}\right)^n}{b\,n}$$

```
[In] int(tanh(a + b*x)*(1/cosh(a + b*x))^n,x)
[Out] -((2*exp(a + b*x))/(exp(2*a + 2*b*x) + 1))^n/(b*n)
```

3.84 $\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] 1/3*tanh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}$$

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int x^2\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\tanh^3(a+bx)}{3b}$$

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
[Out] Tanh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^3}{3b}$	14
default	$\frac{\tanh(bx+a)^3}{3b}$	14
risch	$-\frac{2(3\,e^{4bx+4a}+1)}{3b(1+e^{2bx+2a})^3}$	32

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*tanh(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(13) = 26.

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 9.20

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx =$$
$$-\frac{8\left(\cosh(bx+a)^2+\cosh(bx+a)\sinh\right.}{3\left(b\cosh(bx+a)^4+4\,b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4+4\,b\cosh(bx+a)^2+2\left(3\,b\cosh\right.\right.}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*
b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*co
sh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**2,x)
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\tanh(bx+a)^3}{3\,b}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*tanh(b*x + a)^3/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}+1)^3}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
[Out] -2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\left(3\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)^3}$$

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x)^2,x)
[Out] -(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

3.85 $\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}$$

```
[Out] 1/4*tanh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}$$

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

```
[Out] Tanh[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^3\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\tanh^4(a+bx)}{4b}$$

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
[Out] Tanh[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^4}{4b}$	14
default	$\frac{\tanh(bx+a)^4}{4b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^4}$	39

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*tanh(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 13.87

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx =$$
$$-\frac{2\left(\cosh\left(bx+a\right)^3+3\,\cosh\left(b\right.\right.}{b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)^3+\left(10\,b\cosh\left(b\right.\right.}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5 +
5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 5*b*cosh(b*x + a)^3
 + (10*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + 5*(2*b*cosh(b*x + a)^3 +
3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x + a) + (5*b*cosh(b*x + a
)^4 + 9*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}^2(a+bx)}{4b} - \frac{\operatorname{sech}^2(a+bx)}{4b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**2/(4*b) - sech(a + b*x)**2/(4*b
), Ne(b, 0)), (x*tanh(a)**3*sech(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\tanh(bx+a)^4}{4\,b}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*tanh(b*x + a)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(13) = 26$.

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(e^{(6\,bx+6\,a)} + e^{(2\,bx+2\,a)}\right)}{b(e^{(2\,bx+2\,a)}+1)^4}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 15.33

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\frac{1}{2b}-\frac{3\,e^{2\,a+2\,b\,x}}{2\,b}+\frac{3\,e^{4\,a+4\,b\,x}}{2\,b}-\frac{e^{6\,a+6\,b\,x}}{2\,b}}{4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{\frac{1}{2\,b}-\frac{e^{2\,a+2\,b\,x}}{b}+\frac{e^{4\,a+4\,b\,x}}{2\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1} + \frac{\frac{1}{2\,b}-\frac{e^{2\,a+2\,b\,x}}{2\,b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1} - \frac{1}{2\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(tanh(a + b*x)^3/cosh(a + b*x)^2,x)
```

```
[Out] (1/(2*b) - (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) - exp(6*
a + 6*b*x)/(2*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*
b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) - exp(2*a + 2*b*x)/b + exp(4*a + 4*
b*x)/(2*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1
) + (1/(2*b) - exp(2*a + 2*b*x)/(2*b))/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*
x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) + 1))
```

3.86 $\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

```
[Out] tanh(b*x+a)^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 32}

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (-ix)^n\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\tanh(bx+a)^{n+1}}{b(n+1)}$
default	$\frac{\tanh(bx+a)^{n+1}}{b(n+1)}$
risch	$\frac{(e^{2bx+2a}-1)\,(e^{bx+a}-1)^n\,(e^{bx+a}+1)^n\,(1+e^{2bx+2a})^{-n}\,e^{-\frac{i\pi n\left(-\operatorname{csgn}\left(\frac{i}{1+e^{2bx+2a}}\right)\operatorname{csgn}\left(\frac{i(e^{bx+a}+1)}{1+e^{2bx+2a}}\right)^2+\operatorname{csgn}\left(\frac{i}{1+e^{2bx+2a}}\right.}{}}}{}$

```
[In] int(sech(b*x+a)^2*tanh(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] tanh(b*x+a)^(1+1)/b/(n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)\sinh(bx+a)+\sinh(bx+a)\sinh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)}{(bn+b)\cosh(bx+a)}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (cosh(n*log(sinh(b*x + a)/cosh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sin
h(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n + b)*cosh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \int \tanh^n(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**2*tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh(bx+a)^{n+1}}{b(n+1)}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] tanh(b*x + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right)^{n+1}}{b(n+1)}$$

```
[In] integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="giac")
[Out] ((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh(a+b\,x)\,\left(\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}\right)^n}{b\,\left(n+1\right)}$$

```
[In] int(tanh(a + b*x)^n/cosh(a + b*x)^2,x)
[Out] (tanh(a + b*x)*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(b*(n + 1
))
```

3.87 $\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[Out] -sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2686}

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=\frac{\operatorname{sech}^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

```
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\operatorname{sech}(a+bx)\right)}{b}\\&=-\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^3,x]
[Out] -(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)^3}{3}-\operatorname{sech}(bx+a)}{b}$	24
default	$\frac{\frac{\operatorname{sech}(bx+a)^3}{3}-\operatorname{sech}(bx+a)}{b}$	24
risch	$-\frac{2\,e^{bx+a}\left(3\,e^{4bx+4a}+2\,e^{2bx+2a}+3\right)}{3b\left(1+e^{2bx+2a}\right)^3}$	49

```
[In] int(sech(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*sech(b*x+a)^3-sech(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.37

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=$$

$$-\frac{2\left(3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2+3\sinh(bx+a)^3\right.}{3\left(b\cosh(bx+a)^4+4\,b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4+4\,b\cosh(bx+a)^2+2\left(3\,b\cosh\right.\right.}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
[Out] -2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a)
^3 + (9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*b*cosh(b*
x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}(a+bx)}{3b} - \frac{2\operatorname{sech}(a+bx)}{3b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**3,x)
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)/(3*b) - 2*sech(a + b*x)/(3*b), N
e(b, 0)), (x*tanh(a)**3*sech(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\begin{aligned}\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = & -\frac{2\,e^{(-bx-a)}}{b(3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} + 1)} \\ & -\frac{4\,e^{(-3\,bx-3\,a)}}{3\,b(3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} + 1)} \\ & -\frac{2\,e^{(-5\,bx-5\,a)}}{b(3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} + 1)}\end{aligned}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
[Out] -2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6
*a) + 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a
) + e^(-6*b*x - 6*a) + 1)) - 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) + 3*
e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(3\left(e^{(bx+a)} + e^{(-bx-a)}\right)^2 - 4\right)}{3\,b\left(e^{(bx+a)} + e^{(-bx-a)}\right)^3}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
[Out] -2/3*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 4)/(b*(e^(b*x + a) + e^(-b*x - a))
^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=-\frac{2\,\mathrm{e}^{a+b\,x}\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+3\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^3}$$

```
[In] int(tanh(a + b*x)^3/cosh(a + b*x),x)
```

```
[Out] -(2*exp(a + b*x)*(2*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + 3))/(3*b*(exp(2
*a + 2*b*x) + 1)^3)
```

3.88 $\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}^3(a+bx)}{3b}+\frac{\operatorname{sech}^5(a+bx)}{5b}$$

```
[Out] -1/3*sech(b*x+a)^3/b+1/5*sech(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 14}

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx=\frac{\operatorname{sech}^5(a+bx)}{5b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

```
[In] Int[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

```
[Out] -1/3*Sech[a + b*x]^3/b + Sech[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2(-1+x^2)\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (-x^2+x^4)\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= -\frac{\text{sech}^3(a+bx)}{3b}+\frac{\text{sech}^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \text{sech}^3(a+bx)\tanh^3(a+bx)\,dx = -\frac{\text{sech}^3(a+bx)}{3b}+\frac{\text{sech}^5(a+bx)}{5b}$$

```
[In] Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
[Out] -1/3*Sech[a + b*x]^3/b + Sech[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\text{sech}(bx+a)^5}{5}-\frac{\text{sech}(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\text{sech}(bx+a)^5}{5}-\frac{\text{sech}(bx+a)^3}{3}}{b}$	26
risch	$-\frac{8\,e^{3bx+3a}\left(5\,e^{4bx+4a}-2\,e^{2bx+2a}+5\right)}{15b\left(1+e^{2bx+2a}\right)^5}$	52

```
[In] int(sech(b*x+a)^3*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*sech(b*x+a)^5-1/3*sech(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 345, normalized size of antiderivative = 11.13

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+5\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\right.\right.}$$

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
a)^4 + 2*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 +
5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x
+ a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^
5 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(
b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a)
^2 + 5*b)*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}^3(a+bx)}{5b} - \frac{2\operatorname{sech}^3(a+bx)}{15b} & \text{for } b\neq 0 \\ x\tanh^3(a)\operatorname{sech}^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)**3*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**3/(5*b) - 2*sech(a + b*x)**3/(1
5*b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.90

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = -\frac{8\,e^{(-3\,bx-3\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} + \frac{16\,e^{(-5\,bx-5\,a)}}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} - \frac{8\,e^{(-7\,bx-7\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15*e^(-5
*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 8/3*e^(-7*b*x - 7*a)/
(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = -\frac{8\left(5\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-12\right)}{15\,b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^5}$$

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(5*(e^(b*x + a) + e^(-b*x - a))^2 - 12)/(b*(e^(b*x + a) + e^(-b*x - a
))^5)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 8.10

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$$
$$=\frac{\frac{4\,e^{a+b\,x}}{5\,b}-\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}-\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}+5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}+1}$$
$$-\frac{28\,e^{a+b\,x}}{15\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{64\,e^{a+b\,x}}{15\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}$$
$$-\frac{16\,e^{a+b\,x}}{5\,b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

```
[In] int(tanh(a + b*x)^3/cosh(a + b*x)^3,x)
```

```
[Out] ((4*exp(a + b*x))/(5*b) - (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*b*x
))/(5*b) - (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) + 10*exp(4*a + 4
*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x) + 1)
- (28*exp(a + b*x))/(15*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (6
4*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a +
6*b*x) + 1)) - (16*exp(a + b*x))/(5*b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b
*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.89 $\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=-\frac{\operatorname{sech}^n(a+bx)}{bn}+\frac{\operatorname{sech}^{2+n}(a+bx)}{b(2+n)}$$

```
[Out] -sech(b*x+a)^n/b/n+sech(b*x+a)^(2+n)/b/(2+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 14}

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=\frac{\operatorname{sech}^{n+2}(a+bx)}{b(n+2)}-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

```
[In] Int[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
[Out] -(Sech[a + b*x]^n/(b*n)) + Sech[a + b*x]^(2 + n)/(b*(2 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^{-1+n}(-1+x^2)\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^{-1+n}+x^{1+n}\right)\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= -\frac{\text{sech}^n(a+bx)}{bn}+\frac{\text{sech}^{2+n}(a+bx)}{b(2+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \text{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \frac{\text{sech}^n(a+bx)\left(-\frac{1}{n}+\frac{\text{sech}^2(a+bx)}{2+n}\right)}{b}$$

```
[In] Integrate[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
[Out] (Sech[a + b*x]^n*(-n^(-1) + Sech[a + b*x]^2/(2 + n)))/b
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.81

method	result
risch	$-\frac{\left(n\,e^{4bx+4a}+2\,e^{4bx+4a}-2n\,e^{2bx+2a}+4\,e^{2bx+2a}+n+2\right)2^n\left(e^{bx+a}\right)^n\left(1+e^{2bx+2a}\right)^{-n}e^{-\frac{i\,\text{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)\pi n\left(-\text{csgn}\left(\frac{ie^{bx+a}}{1+e^{2bx+2a}}\right)+\text{csg}\right.}{}}}{bn(n+2)\left(1+e^{2bx+2a}\right)^2}$

```
[In] int(sech(b*x+a)^n*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)-2*n*exp(2*b*x+2*a)+4*exp(2*b*x+2*a)+n+2
)/b/n/(n+2)/(1+exp(2*b*x+2*a))^2*2^n*exp(b*x+a)^n*(1+exp(2*b*x+2*a))^(-n)*e
xp(-1/2*I*csgn(I*exp(b*x+a)/(1+exp(2*b*x+2*a)))*Pi*n*(-csgn(I*exp(b*x+a)/(1
+exp(2*b*x+2*a)))+csgn(I/(1+exp(2*b*x+2*a))))*(-csgn(I*exp(b*x+a)/(1+exp(2*
b*x+2*a)))+csgn(I*exp(b*x+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 6.08

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left((n+2)\cosh\left(bx+a\right)^2+(n+2)\sinh\left(bx+a\right)^2-n+2\right)\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\mathrm{si}}\right.\right.}{bn^2+\left(bn^2+2\,bn\right)\mathrm{c}}$$

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 - n + 2)*cosh(n*log(2*
(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 + 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*
x + a)^2 - n + 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x +
a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))))/(b*n^2 + (b*
n^2 + 2*b*n)*cosh(b*x + a)^2 + (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n)
```

Sympy [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} x\tanh^3(a)\operatorname{sech}^n(a) & \text{for } b=0 \\ \int \frac{\tanh^3(a+bx)}{\operatorname{sech}^2(a+bx)}\,dx & \text{for } n=-2 \\ x-\frac{\log(\tanh(a+bx)+1)}{b}-\frac{\tanh^2(a+bx)}{2b} & \text{for } n=0 \\ -\frac{n\tanh^2(a+bx)\operatorname{sech}^n(a+bx)}{bn^2+2bn}-\frac{2\operatorname{sech}^n(a+bx)}{bn^2+2bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a)**n*tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*tanh(a)**3*sech(a)**n, Eq(b, 0)), (Integral(tanh(a + b*x)**3/s
ech(a + b*x)**2, x), Eq(n, -2)), (x - log(tanh(a + b*x) + 1)/b - tanh(a + b
*x)**2/(2*b), Eq(n, 0)), (-n*tanh(a + b*x)**2*sech(a + b*x)**n/(b*n**2 + 2*
b*n) - 2*sech(a + b*x)**n/(b*n**2 + 2*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(36) = 72$.

Time = 0.34 (sec) , antiderivative size = 345, normalized size of antiderivative = 9.58

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$$
$$= -\frac{2^n n e^{\left(-(bx+a)n - n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2 + 2\,(n^2+2\,n)e^{(-2\,bx-2\,a)} + (n^2+2\,n)e^{(-4\,bx-4\,a)} + 2\,n\right)b}$$
$$+ \frac{(2^{n+1}n - 2^{n+2})e^{\left(-(bx+a)n - 2\,bx - n\log\left(e^{(-2\,bx-2\,a)}+1\right) - 2\,a\right)}}{\left(n^2 + 2\,(n^2+2\,n)e^{(-2\,bx-2\,a)} + (n^2+2\,n)e^{(-4\,bx-4\,a)} + 2\,n\right)b}$$
$$- \frac{(2^n n + 2^{n+1})e^{\left(-(bx+a)n - 4\,bx - n\log\left(e^{(-2\,bx-2\,a)}+1\right) - 4\,a\right)}}{\left(n^2 + 2\,(n^2+2\,n)e^{(-2\,bx-2\,a)} + (n^2+2\,n)e^{(-4\,bx-4\,a)} + 2\,n\right)b}$$
$$- \frac{2^{n+1}e^{\left(-(bx+a)n - n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2 + 2\,(n^2+2\,n)e^{(-2\,bx-2\,a)} + (n^2+2\,n)e^{(-4\,bx-4\,a)} + 2\,n\right)b}$$

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2^n*n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 2*(n^2 + 2*n)
*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) + (2^(n + 1)*n -
 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((
n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*
b) - (2^n*n + 2^(n + 1))*e^(-(b*x + a)*n - 4*b*x - n*log(e^(-2*b*x - 2*a) +
 1) - 4*a)/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x -
 4*a) + 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/
((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n
)*b)
```

Giac [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \int \operatorname{sech}\left(bx+a\right)^n \tanh\left(bx+a\right)^3\,dx$$

```
[In] integrate(sech(b*x+a)^n*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^n*tanh(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=-\frac{\left(\frac{1}{\frac{e^{a+b\,x}}{2}+\frac{e^{-a-b\,x}}{2}}\right)^n\left(\frac{1}{b\,n}+\frac{e^{4\,a+4\,b\,x}}{b\,n}-\frac{e^{2\,a+2\,b\,x}\,(2\,n-4)}{b\,n\,(n+2)}\right)}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}$$

```
[In] int(tanh(a + b*x)^3*(1/cosh(a + b*x))^n,x)
```

```
[Out] -((1/(exp(a + b*x)/2 + exp(- a - b*x)/2))^n*(1/(b*n) + exp(4*a + 4*b*x)/(b*
n) - (exp(2*a + 2*b*x)*(2*n - 4))/(b*n*(n + 2))))/(2*exp(2*a + 2*b*x) + exp
(4*a + 4*b*x) + 1)
```

3.90 $\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

```
[Out] 1/3*tanh(b*x+a)^3/b-1/5*tanh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 14}

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

```
[In] Int[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{i\text{Subst}\left(\int x^2(1+x^2)\,dx, x, i\tanh(a+bx)\right)}{b} \\ &= \frac{i\text{Subst}\left(\int (x^2+x^4)\,dx, x, i\tanh(a+bx)\right)}{b} \\ &= \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \text{sech}^4(a+bx)\tanh^2(a+bx)\,dx = \frac{2\tanh(a+bx)}{15b} + \frac{\text{sech}^2(a+bx)\tanh(a+bx)}{15b} - \frac{\text{sech}^4(a+bx)\tanh(a+bx)}{5b}$$

```
[In] Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
[Out] (2*Tanh[a + b*x])/(15*b) + (Sech[a + b*x]^2*Tanh[a + b*x])/(15*b) - (Sech[a
 + b*x]^4*Tanh[a + b*x])/(5*b)
```

Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\tanh(bx+a)^5}{5}+\frac{\tanh(bx+a)^3}{3}}{b}$	26
default	$\frac{-\frac{\tanh(bx+a)^5}{5}+\frac{\tanh(bx+a)^3}{3}}{b}$	26
risch	$-\frac{4\left(15\,e^{6bx+6a}-5\,e^{4bx+4a}+5\,e^{2bx+2a}+1\right)}{15b\left(1+e^{2bx+2a}\right)^5}$	54

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/5*tanh(b*x+a)^5+1/3*tanh(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 304, normalized size of antiderivative = 9.81

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+5\,b\cosh\left(bx+a\right)^5+\left(21\,b\cosh\right.\right.}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="fricas")
[Out] -8/15*(8*cosh(b*x + a)^3 + 24*cosh(b*x + a)*sinh(b*x + a)^2 + 7*sinh(b*x +
a)^3 + (21*cosh(b*x + a)^2 - 5)*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cos
h(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5 + (21*
b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 5*b*cos
h(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a)^4
+ 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 50*
b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(b*x + a
) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a)^2 + 5*
b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**2,x)
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 8.90

$$\begin{aligned}&\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx\\&=\frac{4\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}\\&\quad-\frac{4\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}\\&\quad+\frac{4\,e^{(-6\,bx-6\,a)}}{b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}\\&\quad+\frac{4}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}\end{aligned}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-
6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 4/3*e^(-4*b*
x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a)
 + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4*e^(-6*b*x - 6*a)/(b*(5
*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x
 - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4/15/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4
*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a)
 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = -\frac{4\left(15\,e^{(6\,bx+6\,a)} - 5\,e^{(4\,bx+4\,a)} + 5\,e^{(2\,bx+2\,a)} + 1\right)}{15\,b\left(e^{(2\,bx+2\,a)} + 1\right)^5}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4/15*(15*e^(6*b*x + 6*a) - 5*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(
e^(2*b*x + 2*a) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 270, normalized size of antiderivative = 8.71

$$\begin{aligned}
&\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx \\
&= \frac{\frac{8}{15\,b} - \frac{4\,e^{2\,a+2\,b\,x}}{5\,b}}{3\,e^{2\,a+2\,b\,x} + 3\,e^{4\,a+4\,b\,x} + e^{6\,a+6\,b\,x} + 1} - \frac{\frac{2}{5\,b} - \frac{8\,e^{2\,a+2\,b\,x}}{5\,b} + \frac{6\,e^{4\,a+4\,b\,x}}{5\,b}}{4\,e^{2\,a+2\,b\,x} + 6\,e^{4\,a+4\,b\,x} + 4\,e^{6\,a+6\,b\,x} + e^{8\,a+8\,b\,x} + 1} \\
&\quad - \frac{\frac{8\,e^{2\,a+2\,b\,x}}{5\,b} - \frac{16\,e^{4\,a+4\,b\,x}}{5\,b} + \frac{8\,e^{6\,a+6\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x} + 10\,e^{4\,a+4\,b\,x} + 10\,e^{6\,a+6\,b\,x} + 5\,e^{8\,a+8\,b\,x} + e^{10\,a+10\,b\,x} + 1} \\
&\quad - \frac{2}{5\,b\,\left(2\,e^{2\,a+2\,b\,x} + e^{4\,a+4\,b\,x} + 1\right)}
\end{aligned}$$

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x)^4,x)
```

```
[Out] (8/(15*b) - (4*exp(2*a + 2*b*x))/(5*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4
*b*x) + exp(6*a + 6*b*x) + 1) - (2/(5*b) - (8*exp(2*a + 2*b*x))/(5*b) + (6*
exp(4*a + 4*b*x))/(5*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6
*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - ((8*exp(2*a + 2*b*x))/(5*b) - (16*exp
(4*a + 4*b*x))/(5*b) + (8*exp(6*a + 6*b*x))/(5*b))/(5*exp(2*a + 2*b*x) + 10
*exp(4*a + 4*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10*a + 1
0*b*x) + 1) - 2/(5*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

3.91 $\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] 2/3*tanh(b*x+a)^(3/2)/b-2/7*tanh(b*x+a)^(7/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2687, 14}

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[In] Int[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

```
[Out] (2*Tanh[a + b*x]^(3/2))/(3*b) - (2*Tanh[a + b*x]^(7/2))/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{i\text{Subst}\left(\int\sqrt{-ix}(1+x^2)\,dx,x,i\tanh(a+bx)\right)}{b}\\&=-\frac{i\text{Subst}\left(\int\left(\sqrt{-ix}-(-ix)^{5/2}\right)\,dx,x,i\tanh(a+bx)\right)}{b}\\&=\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int\text{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx=\frac{2\left(4+3\text{sech}^2(a+bx)\right)\tanh^{\frac{3}{2}}(a+bx)}{21b}$$

```
[In] Integrate[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
[Out] (2*(4 + 3*Sech[a + b*x]^2)*Tanh[a + b*x]^(3/2))/(21*b)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{-\frac{2\tanh(bx+a)^{\frac{7}{2}}}{7}+\frac{2\tanh(bx+a)^{\frac{3}{2}}}{3}}{b}$	26
default	$\frac{-\frac{2\tanh(bx+a)^{\frac{7}{2}}}{7}+\frac{2\tanh(bx+a)^{\frac{3}{2}}}{3}}{b}$	26

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/b*(-2/7*tanh(b*x+a)^(7/2)+2/3*tanh(b*x+a)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(27) = 54.

Time = 0.25 (sec) , antiderivative size = 551, normalized size of antiderivative = 15.74

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$$
$$= \frac{8\left(\cosh\left(bx+a\right)^6+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+\sinh\left(bx+a\right)^6+3\left(5\,\cosh\left(bx+a\right)^2+1\right)\sinh\left(bx+a\right)\right.}{}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 8/21*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5
 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + (cosh(b*x + a)^6 + 6*
cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 4)*
sinh(b*x + a)^4 + 4*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 4*cosh(b*x + a
))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 24*cosh(b*x + a)^2 - 4)*sinh(b*x
 + a)^2 - 4*cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 - 4*
cosh(b*x + a))*sinh(b*x + a) - 1)*sqrt(sinh(b*x + a)/cosh(b*x + a)) + 1)/(b
*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 +
3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*
cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2
+ 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*
cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \int \sqrt{\tanh\left(a+bx\right)}\operatorname{sech}^4\left(a+bx\right)dx$$

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(a + b*x))*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(27) = 54$.

Time = 0.31 (sec) , antiderivative size = 352, normalized size of antiderivative = 10.06

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-2\,bx-2\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}} - \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-4\,bx-4\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}} - \frac{8\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-6\,bx-6\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}} + \frac{8\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-2*b*x - 2*a)/(b*(3
*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b
*x - 2*a) + 1)) - 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-
4*b*x - 4*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)
 + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) - 8/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b
*x - a) + 1)*e^(-6*b*x - 6*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) +
 e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) + 8/21*sqrt(e^(-b*x - a)
 + 1)*sqrt(-e^(-b*x - a) + 1)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) +
 e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(27) = 54$.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.23

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{16\left(21\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}\right)^5-7\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}\right)^4+28\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a}\right.\right.}{21\,b\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}-1\right)^7}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 16/21*(21*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a))^5 - 7*(sqrt(e^(4*b*
x + 4*a) - 1) - e^(2*b*x + 2*a))^4 + 28*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b
*x + 2*a))^3 + 7*sqrt(e^(4*b*x + 4*a) - 1) - 7*e^(2*b*x + 2*a) - 1)/(b*(sqr
t(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.80

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{8\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{21\,b} + \frac{8\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{21\,b\,\left(e^{2a+2bx}+1\right)} - \frac{24\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{7\,b\,\left(e^{2a+2bx}+1\right)^2} + \frac{16\sqrt{\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}}}{7\,b\,\left(e^{2a+2bx}+1\right)^3}$$

```
[In] int(tanh(a + b*x)^(1/2)/cosh(a + b*x)^4,x)
```

```
[Out] (8*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b) + (8*((exp
(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b*(exp(2*a + 2*b*x) +
 1)) - (24*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(7*b*(exp
(2*a + 2*b*x) + 1)^2) + (16*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))
^(1/2))/(7*b*(exp(2*a + 2*b*x) + 1)^3)
```

3.92 $\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}-\frac{\tanh^{3+n}(a+bx)}{b(3+n)}$$

```
[Out] tanh(b*x+a)^(1+n)/b/(1+n)-tanh(b*x+a)^(3+n)/b/(3+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 14}

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{n+1}(a+bx)}{b(n+1)}-\frac{\tanh^{n+3}(a+bx)}{b(n+3)}$$

```
[In] Int[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

```
[Out] Tanh[a + b*x]^(1 + n)/(b*(1 + n)) - Tanh[a + b*x]^(3 + n)/(b*(3 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{i\text{Subst}\left(\int (-ix)^n\,(1+x^2)\;dx, x, i\tanh(a+bx)\right)}{b} \\ &= -\frac{i\text{Subst}\left(\int \left((-ix)^n - (-ix)^{2+n}\right)\;dx, x, i\tanh(a+bx)\right)}{b} \\ &= \frac{\tanh^{1+n}(a+bx)}{b(1+n)} - \frac{\tanh^{3+n}(a+bx)}{b(3+n)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh^{-1+n}(a+bx)\left((2+n+\cosh(2(a+bx)))\operatorname{sech}^2(a+bx)\tanh^2(a+bx) - 2\tanh^2(a+bx)^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

```
[In] Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

```
[Out] (Tanh[a + b*x]^(-1 + n)*((2 + n + Cosh[2*(a + b*x)])*Sech[a + b*x]^2*Tanh[a
 + b*x]^2 - 2*(Tanh[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

Maple [A] (verified)

Time = 117.68 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\tanh(bx+a)\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(n+1)} - \frac{\tanh(bx+a)^3\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(3+n)}$
default	$\frac{\tanh(bx+a)\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(n+1)} - \frac{\tanh(bx+a)^3\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(3+n)}$
risch	$2\left(\mathrm{e}^{6bx+6a}+2n\,\mathrm{e}^{4bx+4a}+3\,\mathrm{e}^{4bx+4a}-2n\,\mathrm{e}^{2bx+2a}-3\,\mathrm{e}^{2bx+2a}-1\right)\left(\mathrm{e}^{bx+a}-1\right)^n\left(\mathrm{e}^{bx+a}+1\right)^n\left(1+\mathrm{e}^{2bx+2a}\right)^{-n}\mathrm{e}^{-\frac{i\pi n\left(-\operatorname{csgn}\left(\frac{}{1-}\right.\right.}{}}$

```
[In] int(sech(b*x+a)^4*tanh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(n+1)*tanh(b*x+a)*exp(n*ln(tanh(b*x+a)))-1/b/(3+n)*tanh(b*x+a)^3*exp(n*
ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(40) = 80.

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.50

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{2\left(\left(\sinh(bx+a)^3+\left(3\cosh(bx+a)^2+2n+3\right)\sinh(bx+a)\right)\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)+\left(\sinh(bx+a\right.\right.}{(bn^2+4bn+3b)\cosh(bx+a)^3+3(bn^2+4bn+3b)\cosh(bx+a)\sinh(b}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] 2*((sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 2*n + 3)*sinh(b*x + a))*cosh(n*l
og(sinh(b*x + a)/cosh(b*x + a))) + (sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 +
2*n + 3)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n^2 +
 4*b*n + 3*b)*cosh(b*x + a)^3 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)*sinh(
b*x + a)^2 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \int \tanh^n(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**4*tanh(b*x+a)**n,x)
```

```
[Out] Integral(tanh(a + b*x)**n*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(40) = 80.

Time = 0.30 (sec) , antiderivative size = 504, normalized size of antiderivative = 12.60

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{2(2n+3)e^{\left(-2bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2bx-2a)}+1\right)-2a\right)}}{\left(n^2+3(n^2+4n+3)e^{(-2bx-2a)}+3(n^2+4n+3)e^{(-4bx-4a)}+(n^2+4n+3)e^{(-6bx-6a)}+4n+3\right)b} - \frac{2(2n+3)e^{\left(-4bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2bx-2a)}+1\right)-4a\right)}}{\left(n^2+3(n^2+4n+3)e^{(-2bx-2a)}+3(n^2+4n+3)e^{(-4bx-4a)}+(n^2+4n+3)e^{(-6bx-6a)}+4n+3\right)b} - \frac{2e^{\left(-6bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2bx-2a)}+1\right)-6a\right)}}{\left(n^2+3(n^2+4n+3)e^{(-2bx-2a)}+3(n^2+4n+3)e^{(-4bx-4a)}+(n^2+4n+3)e^{(-6bx-6a)}+4n+3\right)b} + \frac{2e^{\left(n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2bx-2a)}+1\right)\right)}}{\left(n^2+3(n^2+4n+3)e^{(-2bx-2a)}+3(n^2+4n+3)e^{(-4bx-4a)}+(n^2+4n+3)e^{(-6bx-6a)}+4n+3\right)b}$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] 2*(2*n + 3)*e^(-2*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1)
- n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((n^2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x -
2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n + 3)*e^(-6*b*x - 6*a
) + 4*n + 3)*b) - 2*(2*n + 3)*e^(-4*b*x + n*log(e^(-b*x - a) + 1) + n*log(-
e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 4*a)/((n^2 + 3*(n^2 + 4*n
 + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n +
3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) - 2*e^(-6*b*x + n*log(e^(-b*x - a) + 1) +
 n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 6*a)/((n^2 + 3*(n
^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2
+ 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) + 2*e^(n*log(e^(-b*x - a) + 1) +
n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 3*(n^2 + 4*
n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n +
 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b)
```

Giac [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\int \tanh\left(bx+a\right)^n \operatorname{sech}\left(bx+a\right)^4\,dx$$

```
[In] integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(tanh(b*x + a)^n*sech(b*x + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{e^{-3a-3bx}\left(\frac{4e^{3a+3bx}\sinh(3a+3bx)}{b(n^2+4n+3)}+\frac{2e^{3a+3bx}\sinh(a+bx)(4n+6)}{b(n^2+4n+3)}\right)\left(\frac{e^{2a+2bx}-1}{e^{2a+2bx}+1}\right)^n}{8\cosh\left(a+bx\right)^3}$$

```
[In] int(tanh(a + b*x)^n/cosh(a + b*x)^4,x)
```

```
[Out] (exp(- 3*a - 3*b*x)*((4*exp(3*a + 3*b*x)*sinh(3*a + 3*b*x))/(b*(4*n + n^2 +
 3)) + (2*exp(3*a + 3*b*x)*sinh(a + b*x)*(4*n + 6))/(b*(4*n + n^2 + 3)))*((
exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(8*cosh(a + b*x)^3)
```

3.93 $\int \mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{2b} - \frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{2b} - \frac{\tanh(a+bx)\mathrm{sech}(a+bx)}{2b}$$

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{1}{2}\int \operatorname{sech}(a+bx)\,dx \\ &= \frac{\arctan(\sinh(a+bx))}{2b} - \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{2b} - \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan(e^{bx+a})}{b}$	43
default	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan(e^{bx+a})}{b}$	43
risch	$-\frac{e^{bx+a}(e^{2bx+2a}-1)}{b(1+e^{2bx+2a})^2}+\frac{i\ln(e^{bx+a}+i)}{2b}-\frac{i\ln(e^{bx+a}-i)}{2b}$	69

```
[In] int(sech(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/cosh(b*x+a)^2*sinh(b*x+a)+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*
x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 7.91

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - (\cosh(bx+a)^4 + 4\cosh(bx+a)\sin}{b\cosh(bx+a)^4 + 4\,b\cosh}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
+ (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a
)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 +
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**2,x)
```

```
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi - \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{4b}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,e^{a+bx}}{b\,\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)} - \frac{e^{a+bx}}{b\,\left(e^{2a+2bx}+1\right)}$$

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x),x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*
x) + 1))
```

3.94 $\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}-\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}$$

```
[Out] 3/8*arctan(sinh(b*x+a))/b-3/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)*tan
h(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}-\frac{\tanh^3(a+bx)\operatorname{sech}(a+bx)}{4b}-\frac{3\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) - (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (
Sech[a + b*x]*Tanh[a + b*x]^3)/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}+\frac{3}{4}\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx \\
&= -\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}+\frac{3}{8}\int \operatorname{sech}(a+bx)\,dx \\
&= \frac{3\arctan(\sinh(a+bx))}{8b}-\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\arctan(\sinh(a+bx))}{8b}+\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{3\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b} - \frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{b}$$

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^4,x]
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (
3*Sech[a + b*x]^3*Tanh[a + b*x])/(4*b) - (Sech[a + b*x]*Tanh[a + b*x]^3)/b
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)^3}{\cosh(bx+a)^4}-\frac{\sinh(bx+a)}{\cosh(bx+a)^4}+\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$	75
default	$\frac{-\frac{\sinh(bx+a)^3}{\cosh(bx+a)^4}-\frac{\sinh(bx+a)}{\cosh(bx+a)^4}+\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$	75
risch	$-\frac{e^{bx+a}\left(5\,e^{6bx+6a}-3\,e^{4bx+4a}+3\,e^{2bx+2a}-5\right)}{4b\left(1+e^{2bx+2a}\right)^4}+\frac{3i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{3i\ln\left(e^{bx+a}-i\right)}{8b}$	93

```
[In] int(sech(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-sinh(b*x+a)^3/cosh(b*x+a)^4-1/cosh(b*x+a)^4*sinh(b*x+a)+(1/4*sech(b*x
+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+3/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(49) = 98$.

Time = 0.25 (sec) , antiderivative size = 814, normalized size of antiderivative = 14.80

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
[Out] -1/4*(5*cosh(b*x + a)^7 + 35*cosh(b*x + a)*sinh(b*x + a)^6 + 5*sinh(b*x + a
)^7 + 3*(35*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 3*cosh(b*x + a)^5 + 5*(3
5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^4 + (175*cosh(b*x + a)^4
 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + 3*(35*cosh
(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(co
sh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)
^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x
 + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 1
0*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6
 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh
(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (35*c
osh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)
- 5*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
 b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh
(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 +
 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b
)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cos
h(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6
+ 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*co
```

```
sh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \int \tanh^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)**4,x)
[Out] Integral(tanh(a + b*x)**4*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = -\frac{3\arctan\left(e^{(-bx-a)}\right)}{4b} - \frac{5e^{(-bx-a)} - 3e^{(-3bx-3a)} + 3e^{(-5bx-5a)} - 5e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)} + 6e^{(-4bx-4a)} + 4e^{(-6bx-6a)} + e^{(-8bx-8a)} + 1)}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
[Out] -3/4*arctan(e^(-b*x - a))/b - 1/4*(5*e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*
e^(-5*b*x - 5*a) - 5*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x
 - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\pi - \frac{4\left(5\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+12e^{(bx+a)}-12e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{16b}$$

```
[In] integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
[Out] 1/16*(3*pi - 4*(5*(e^(b*x + a) - e^(-b*x - a))^3 + 12*e^(b*x + a) - 12*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.38

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} + \frac{9\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} + \frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} - \frac{5\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(tanh(a + b*x)^4/cosh(a + b*x),x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (9*exp(a + b*x)
)/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b*(
3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*exp(a
 + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) +
 exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.95 $\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{8b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

```
[Out] 1/8*arctan(sinh(b*x+a))/b+1/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)^3*t
anh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2691, 3853, 3855}

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{8b}-\frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[In] Int[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{1}{4}\int\operatorname{sech}^3(a+bx)\,dx\\ &= \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{1}{8}\int\operatorname{sech}(a+bx)\,dx\\ &= \frac{\arctan(\sinh(a+bx))}{8b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int\operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{8b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

```
[In] Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
[Out] ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[a
 + b*x]^3*Tanh[a + b*x])/(4*b)
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{3\cosh(bx+a)^4}+\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\,\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	58
default	$\frac{-\frac{\sinh(bx+a)}{3\cosh(bx+a)^4}+\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\,\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	58
risch	$\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{6bx+6a}-7\,\mathrm{e}^{4bx+4a}+7\,\mathrm{e}^{2bx+2a}-1\right)}{4b\left(1+\mathrm{e}^{2bx+2a}\right)^4}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{8b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{8b}$	91

```
[In] int(sech(b*x+a)^3*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/cosh(b*x+a)^4*sinh(b*x+a)+1/3*(1/4*sech(b*x+a)^3+3/8*sech(b*x+a))
*tanh(b*x+a)+1/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 808, normalized size of antiderivative = 14.69

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 +
7*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 7*cosh(b*x + a)^5 + 35*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^4 + 7*(5*cosh(b*x + a)^4 - 10*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^3 + 7*cosh(b*x + a)^3 + 7*(3*cosh(b*x + a)^5
- 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (cosh(b*x + a)^8
+ 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*cosh(b*
x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^2 + 3)*s
inh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 15*cosh(b*x
 + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x + a)^2 + 8*(c
osh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (7*cosh(b*x + a)^6
- 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a
))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)
^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*
(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
```

```
a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^
4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh
(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cosh(b*x
+ a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 +
3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
+ b)
```

Sympy [F]

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

```
[In] integrate(sech(b*x+a)**3*tanh(b*x+a)**2,x)
[Out] Integral(tanh(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\begin{aligned}&\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx\\&= -\frac{\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{e^{(-bx-a)} - 7\,e^{(-3\,bx-3\,a)} + 7\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}\end{aligned}$$

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*arctan(e^(-b*x - a))/b + 1/4*(e^(-b*x - a) - 7*e^(-3*b*x - 3*a) + 7*e^
(-5*b*x - 5*a) - e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4
*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\begin{aligned}&\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx\\&= \frac{\pi + \frac{4\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3-4\,e^{(bx+a)}+4\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}\end{aligned}$$

```
[In] integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(pi + 4*((e^(b*x + a) - e^(-b*x - a))^3 - 4*e^(b*x + a) + 4*e^(-b*x -
a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 2*arctan(1/2*(e^(2*b*x + 2*a)
- 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.91

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} - \frac{\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{e}^{3\,a+3\,b\,x}}{b}+\frac{\mathrm{e}^{5\,a+5\,b\,x}}{b}}{4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1} - \frac{3\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} + \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} + \frac{\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(tanh(a + b*x)^2/cosh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(4*(b^2)^(1/2)) - (exp(a + b*x)/b - (
2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(4*exp(2*a + 2*b*x) + 6*exp(4*a
 + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*x))/(
2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (2*exp(a + b*x))/(b*(3*e
xp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a + b*x
)/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.96 $\int \mathbf{sech}(x) \tanh^5(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \operatorname{sech}(x) \tanh^5(x) \, dx = -\operatorname{sech}(x) + \frac{2\operatorname{sech}^3(x)}{3} - \frac{\operatorname{sech}^5(x)}{5}$$

```
[Out] -sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2686, 200}

$$\int \operatorname{sech}(x) \tanh^5(x) \, dx = -\frac{1}{5}\operatorname{sech}^5(x) + \frac{2\operatorname{sech}^3(x)}{3} - \operatorname{sech}(x)$$

```
[In] Int[Sech[x]*Tanh[x]^5,x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \left(-1+x^2\right)^2 \, dx, x, \text{sech}(x)\right)\\ &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right) \, dx, x, \text{sech}(x)\right)\\ &= -\text{sech}(x)+\frac{2\text{sech}^3(x)}{3}-\frac{\text{sech}^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \text{sech}(x)\tanh^5(x)\,dx = -\text{sech}(x)+\frac{2\text{sech}^3(x)}{3}-\frac{\text{sech}^5(x)}{5}$$

```
[In] Integrate[Sech[x]*Tanh[x]^5,x]
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\operatorname{sech}(x)+\frac{2\operatorname{sech}(x)^3}{3}-\frac{\operatorname{sech}(x)^5}{5}$	18
default	$-\operatorname{sech}(x)+\frac{2\operatorname{sech}(x)^3}{3}-\frac{\operatorname{sech}(x)^5}{5}$	18
risch	$-\frac{2\,\mathrm{e}^{x}\left(15\,\mathrm{e}^{8x}+20\,\mathrm{e}^{6x}+58\,\mathrm{e}^{4x}+20\,\mathrm{e}^{2x}+15\right)}{15\left(1+\mathrm{e}^{2x}\right)^{5}}$	39

```
[In] int(sech(x)*tanh(x)^5,x,method=_RETURNVERBOSE)
[Out] -sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.81

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx =$$

$$-\frac{2\left(15\cosh(x)^5+75\cosh(x)\sinh(x)^4+15\sinh(x)^5+5\left(30\cosh(x)^2+1\right)\sin\right.}{15\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2+2\right)\sinh(x)^4+6\cosh(x)^4+4\left(5\cos\right.\right.}$$

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -2/15*(15*cosh(x)^5 + 75*cosh(x)*sinh(x)^4 + 15*sinh(x)^5 + 5*(30*cosh(x)^2
 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 15*(10*cosh(x)^3 + 7*cosh(x))*sinh(x)^2 +
(75*cosh(x)^4 + 15*cosh(x)^2 + 38)*sinh(x) + 78*cosh(x))/(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 2)*sinh(x)^4 + 6*cosh(x)^4 +
4*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 12*cosh(x)^2 + 5)*
sinh(x)^2 + 15*cosh(x)^2 + 2*(3*cosh(x)^5 + 8*cosh(x)^3 + 5*cosh(x))*sinh(x
) + 10)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{\tanh^4(x)\operatorname{sech}(x)}{5} - \frac{4\tanh^2(x)\operatorname{sech}(x)}{15} - \frac{8\operatorname{sech}(x)}{15}$$

```
[In] integrate(sech(x)*tanh(x)**5,x)
```

```
[Out] -tanh(x)**4*sech(x)/5 - 4*tanh(x)**2*sech(x)/15 - 8*sech(x)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(17) = 34$.

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 9.10

$$\begin{aligned}\int \operatorname{sech}(x)\tanh^5(x)\,dx = &-\frac{2e^{(-x)}}{5e^{(-2x)}+10e^{(-4x)}+10e^{(-6x)}+5e^{(-8x)}+e^{(-10x)}+1}\\ &-\frac{8e^{(-3x)}}{3\left(5e^{(-2x)}+10e^{(-4x)}+10e^{(-6x)}+5e^{(-8x)}+e^{(-10x)}+1\right)}\\ &-\frac{116e^{(-5x)}}{15\left(5e^{(-2x)}+10e^{(-4x)}+10e^{(-6x)}+5e^{(-8x)}+e^{(-10x)}+1\right)}\\ &-\frac{8e^{(-7x)}}{3\left(5e^{(-2x)}+10e^{(-4x)}+10e^{(-6x)}+5e^{(-8x)}+e^{(-10x)}+1\right)}\\ &-\frac{2e^{(-9x)}}{5e^{(-2x)}+10e^{(-4x)}+10e^{(-6x)}+5e^{(-8x)}+e^{(-10x)}+1}\end{aligned}$$

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x)
+ 1) - 8/3*e^(-3*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) +
e^(-10*x) + 1) - 116/15*e^(-5*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) +
5*e^(-8*x) + e^(-10*x) + 1) - 8/3*e^(-7*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e
^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 2*e^(-9*x)/(5*e^(-2*x) + 10*e^(-4*x
) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{2\left(15\left(e^{(-x)}+e^{x}\right)^4-40\left(e^{(-x)}+e^{x}\right)^2+48\right)}{15\left(e^{(-x)}+e^{x}\right)^5}$$

```
[In] integrate(sech(x)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -2/15*(15*(e^(-x) + e^x)^4 - 40*(e^(-x) + e^x)^2 + 48)/(e^(-x) + e^x)^5
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\begin{aligned}\int \operatorname{sech}(x)\tanh^5(x)\,dx = {} & \frac{64\,e^{x}}{5\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)} \\ & -\frac{2\,e^{x}}{e^{2\,x}+1}-\frac{176\,e^{x}}{15\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} \\ & -\frac{32\,e^{x}}{5\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)} \\ & +\frac{16\,e^{x}}{3\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}\end{aligned}$$

```
[In] int(tanh(x)^5/cosh(x),x)
```

```
[Out] (64*exp(x))/(5*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2*
exp(x))/(exp(2*x) + 1) - (176*exp(x))/(15*(3*exp(2*x) + 3*exp(4*x) + exp(6*
x) + 1)) - (32*exp(x))/(5*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8
*x) + exp(10*x) + 1)) + (16*exp(x))/(3*(2*exp(2*x) + exp(4*x) + 1))
```

3.97 $\int \operatorname{sech}^7(x) \tanh^5(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \operatorname{sech}^7(x) \tanh^5(x) \, dx = -\frac{1}{7}\operatorname{sech}^7(x) + \frac{2\operatorname{sech}^9(x)}{9} - \frac{\operatorname{sech}^{11}(x)}{11}$$

```
[Out] -1/7*sech(x)^7+2/9*sech(x)^9-1/11*sech(x)^11
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 276}

$$\int \operatorname{sech}^7(x) \tanh^5(x) \, dx = -\frac{1}{11}\operatorname{sech}^{11}(x) + \frac{2\operatorname{sech}^9(x)}{9} - \frac{\operatorname{sech}^7(x)}{7}$$

```
[In] Int[Sech[x]^7*Tanh[x]^5,x]
```

```
[Out] -1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx, x, \text{sech}(x)\right) \\ &= -\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx, x, \text{sech}(x)\right) \\ &= -\frac{1}{7}\text{sech}^7(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^{11}(x)}{11} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \text{sech}^7(x)\tanh^5(x)\,dx = -\frac{1}{7}\text{sech}^7(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^{11}(x)}{11}$$

```
[In] Integrate[Sech[x]^7*Tanh[x]^5,x]
[Out] -1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int(sech(x)^7*tanh(x)^5,x)
[Out] int(sech(x)^7*tanh(x)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 634, normalized size of antiderivative = 25.36

$$\int \text{sech}^7(x)\tanh^5(x)\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="fricas")
[Out] -128/693*(99*cosh(x)^8 + 792*cosh(x)*sinh(x)^7 + 99*sinh(x)^8 + 44*(63*cosh
(x)^2 - 5)*sinh(x)^6 - 220*cosh(x)^6 + 264*(21*cosh(x)^3 - 5*cosh(x))*sinh(
x)^5 + 10*(693*cosh(x)^4 - 330*cosh(x)^2 + 37)*sinh(x)^4 + 370*cosh(x)^4 +
```

```
8*(693*cosh(x)^5 - 550*cosh(x)^3 + 185*cosh(x))*sinh(x)^3 + 4*(693*cosh(x)^
6 - 825*cosh(x)^4 + 555*cosh(x)^2 - 55)*sinh(x)^2 - 220*cosh(x)^2 + 8*(99*c
osh(x)^7 - 165*cosh(x)^5 + 185*cosh(x)^3 - 55*cosh(x))*sinh(x) + 99)/(cosh(
x)^15 + 15*cosh(x)*sinh(x)^14 + sinh(x)^15 + (105*cosh(x)^2 + 11)*sinh(x)^1
3 + 11*cosh(x)^13 + 13*(35*cosh(x)^3 + 11*cosh(x))*sinh(x)^12 + (1365*cosh(
x)^4 + 858*cosh(x)^2 + 55)*sinh(x)^11 + 55*cosh(x)^11 + 11*(273*cosh(x)^5 +
 286*cosh(x)^3 + 55*cosh(x))*sinh(x)^10 + 55*(91*cosh(x)^6 + 143*cosh(x)^4
+ 55*cosh(x)^2 + 3)*sinh(x)^9 + 165*cosh(x)^9 + 33*(195*cosh(x)^7 + 429*cos
h(x)^5 + 275*cosh(x)^3 + 45*cosh(x))*sinh(x)^8 + (6435*cosh(x)^8 + 18876*co
sh(x)^6 + 18150*cosh(x)^4 + 5940*cosh(x)^2 + 329)*sinh(x)^7 + 331*cosh(x)^7
 + (5005*cosh(x)^9 + 18876*cosh(x)^7 + 25410*cosh(x)^5 + 13860*cosh(x)^3 +
2317*cosh(x))*sinh(x)^6 + (3003*cosh(x)^10 + 14157*cosh(x)^8 + 25410*cosh(x
)^6 + 20790*cosh(x)^4 + 6909*cosh(x)^2 + 451)*sinh(x)^5 + 473*cosh(x)^5 + 5
*(273*cosh(x)^11 + 1573*cosh(x)^9 + 3630*cosh(x)^7 + 4158*cosh(x)^5 + 2317*
cosh(x)^3 + 473*cosh(x))*sinh(x)^4 + (455*cosh(x)^12 + 3146*cosh(x)^10 + 90
75*cosh(x)^8 + 13860*cosh(x)^6 + 11515*cosh(x)^4 + 4510*cosh(x)^2 + 407)*si
nh(x)^3 + 517*cosh(x)^3 + (105*cosh(x)^13 + 858*cosh(x)^11 + 3025*cosh(x)^9
 + 5940*cosh(x)^7 + 6951*cosh(x)^5 + 4730*cosh(x)^3 + 1551*cosh(x))*sinh(x)
^2 + (15*cosh(x)^14 + 143*cosh(x)^12 + 605*cosh(x)^10 + 1485*cosh(x)^8 + 23
03*cosh(x)^6 + 2255*cosh(x)^4 + 1221*cosh(x)^2 + 165)*sinh(x) + 495*cosh(x)
)
```

Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{\tanh^4(x)\operatorname{sech}^7(x)}{11} - \frac{4\tanh^2(x)\operatorname{sech}^7(x)}{99} - \frac{8\operatorname{sech}^7(x)}{693}$$

```
[In] integrate(sech(x)**7*tanh(x)**5,x)
[Out] -tanh(x)**4*sech(x)**7/11 - 4*tanh(x)**2*sech(x)**7/99 - 8*sech(x)**7/693
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 14.84

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx =$$

$$-\frac{128\,e^{(-7\,x)}}{7\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}}$$
$$+\frac{2560\,e^{(-9\,x)}}{63\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}}$$
$$-\frac{47360\,e^{(-11\,x)}}{693\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16}}$$
$$+\frac{2560\,e^{(-13\,x)}}{63\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}}$$
$$-\frac{128\,e^{(-15\,x)}}{7\,(11\,e^{(-2\,x)}+55\,e^{(-4\,x)}+165\,e^{(-6\,x)}+330\,e^{(-8\,x)}+462\,e^{(-10\,x)}+462\,e^{(-12\,x)}+330\,e^{(-14\,x)}+165\,e^{(-16\,x)}}$$

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] -128/7*e^(-7*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x
) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-9*x)/(11*e^(-2*x) + 55*e^(-
4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^
(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) - 47
360/693*e^(-11*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x)
+ 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18
*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-13*x)/(11*e^(-2*x) + 55*e
^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330
*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) -
 128/7*e^(-15*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*
x) + 11*e^(-20*x) + e^(-22*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{128\left(99\left(e^{(-x)}+e^{x}\right)^4-616\left(e^{(-x)}+e^{x}\right)^2+1008\right)}{693\left(e^{(-x)}+e^{x}\right)^{11}}$$

```
[In] integrate(sech(x)^7*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -128/693*(99*(e^(-x) + e^x)^4 - 616*(e^(-x) + e^x)^2 + 1008)/(e^(-x) + e^x)
^11
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 520, normalized size of antiderivative = 20.80

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = \frac{\frac{64\,e^{5\,x}}{11}-\frac{320\,e^{7\,x}}{11}+\frac{640\,e^{9\,x}}{11}-\frac{640\,e^{11\,x}}{11}+\frac{320\,e^{13\,x}}{11}-\frac{64\,e^{15\,x}}{11}}{11\,e^{2\,x}+55\,e^{4\,x}+165\,e^{6\,x}+330\,e^{8\,x}+462\,e^{10\,x}+462\,e^{12\,x}+330\,e^{14\,x}+165\,e^{16\,x}+55\,e^{18\,x}+11\,e^{20\,x}+e^{22\,x}} - \frac{38464\,e^{x}}{693\,\left(6\,e^{2\,x}+15\,e^{4\,x}+20\,e^{6\,x}+15\,e^{8\,x}+6\,e^{10\,x}+e^{12\,x}+1\right)} - \frac{640\,e^{x}}{33\,\left(8\,e^{2\,x}+28\,e^{4\,x}+56\,e^{6\,x}+70\,e^{8\,x}+56\,e^{10\,x}+28\,e^{12\,x}+8\,e^{14\,x}+e^{16\,x}+1\right)} - \frac{104\,e^{x}}{21\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)} + \frac{1664\,e^{x}}{63\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)} + \frac{4096\,e^{x}}{77\,\left(7\,e^{2\,x}+21\,e^{4\,x}+35\,e^{6\,x}+35\,e^{8\,x}+21\,e^{10\,x}+7\,e^{12\,x}+e^{14\,x}+1\right)} + \frac{\frac{16\,e^{3\,x}}{11}-\frac{112\,e^{5\,x}}{11}+\frac{288\,e^{7\,x}}{11}-32\,e^{9\,x}+\frac{208\,e^{11\,x}}{11}-\frac{48\,e^{13\,x}}{11}}{10\,e^{2\,x}+45\,e^{4\,x}+120\,e^{6\,x}+210\,e^{8\,x}+252\,e^{10\,x}+210\,e^{12\,x}+120\,e^{14\,x}+45\,e^{16\,x}+10\,e^{18\,x}+e^{20\,x}+1} - \frac{\frac{280\,e^{3\,x}}{99}-\frac{112\,e^{5\,x}}{11}+16\,e^{7\,x}-\frac{104\,e^{9\,x}}{9}+\frac{104\,e^{11\,x}}{33}-\frac{8\,e^{x}}{33}}{9\,e^{2\,x}+36\,e^{4\,x}+84\,e^{6\,x}+126\,e^{8\,x}+126\,e^{10\,x}+84\,e^{12\,x}+36\,e^{14\,x}+9\,e^{16\,x}+e^{18\,x}+1}$$

```
[In] int(tanh(x)^5/cosh(x)^7,x)
```

```
[Out] ((64*exp(5*x))/11 - (320*exp(7*x))/11 + (640*exp(9*x))/11 - (640*exp(11*x))
/11 + (320*exp(13*x))/11 - (64*exp(15*x))/11)/(11*exp(2*x) + 55*exp(4*x) +
165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(12*x) + 330*exp(14*x)
 + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(22*x) + 1) - (38464*ex
p(x))/(693*(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10
*x) + exp(12*x) + 1)) - (640*exp(x))/(33*(8*exp(2*x) + 28*exp(4*x) + 56*exp
(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + exp(16*x)
 + 1)) - (104*exp(x))/(21*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x)
+ 1)) + (1664*exp(x))/(63*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8
*x) + exp(10*x) + 1)) + (4096*exp(x))/(77*(7*exp(2*x) + 21*exp(4*x) + 35*ex
p(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12*x) + exp(14*x) + 1)) + ((16*
exp(3*x))/11 - (112*exp(5*x))/11 + (288*exp(7*x))/11 - 32*exp(9*x) + (208*e
xp(11*x))/11 - (48*exp(13*x))/11)/(10*exp(2*x) + 45*exp(4*x) + 120*exp(6*x)
 + 210*exp(8*x) + 252*exp(10*x) + 210*exp(12*x) + 120*exp(14*x) + 45*exp(16
*x) + 10*exp(18*x) + exp(20*x) + 1) - ((280*exp(3*x))/99 - (112*exp(5*x))/1
1 + 16*exp(7*x) - (104*exp(9*x))/9 + (104*exp(11*x))/33 - (8*exp(x))/33)/(9
*exp(2*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*e
xp(12*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1)
```

3.98 $\int \operatorname{sech}^3(x)\tanh^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{16}\arctan(\sinh(x)) + \frac{1}{16}\operatorname{sech}(x)\tanh(x) - \frac{1}{8}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)$$

```
[Out] 1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)-1/8*sech(x)^3*tanh(x)-1/6*sech(x)
^3*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2691, 3853, 3855}

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{16}\arctan(\sinh(x)) - \frac{1}{6}\tanh^3(x)\operatorname{sech}^3(x) - \frac{1}{8}\tanh(x)\operatorname{sech}^3(x) + \frac{1}{16}\tanh(x)\operatorname{sech}(x)$$

```
[In] Int[Sech[x]^3*Tanh[x]^4,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 - (Sech[x]^3*Tanh[x])/8 - (Sech[x
]^3*Tanh[x]^3)/6
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)+\frac{1}{2}\int \operatorname{sech}^3(x)\tanh^2(x)\,dx\\ &= -\frac{1}{8}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)+\frac{1}{8}\int \operatorname{sech}^3(x)\,dx\\ &= \frac{1}{16}\operatorname{sech}(x)\tanh(x)-\frac{1}{8}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)+\frac{1}{16}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{16}\arctan(\sinh(x))+\frac{1}{16}\operatorname{sech}(x)\tanh(x)-\frac{1}{8}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{16}\arctan(\sinh(x))+\frac{1}{16}\operatorname{sech}(x)\tanh(x)+\frac{1}{24}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^5(x)\tanh(x)-\frac{1}{3}\operatorname{sech}^3(x)\tanh^3(x)$$

```
[In] Integrate[Sech[x]^3*Tanh[x]^4,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6 - (Sech[x]^3*Tanh[x]^3)/3
```

Maple [A] (verified)

Time = 16.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\sinh(x)^3}{3\cosh(x)^6}-\frac{\sinh(x)}{5\cosh(x)^6}+\frac{\left(\frac{\operatorname{sech}(x)^5}{6}+\frac{5\operatorname{sech}(x)^3}{24}+\frac{5\operatorname{sech}(x)}{16}\right)\tanh(x)}{5}+\frac{\arctan(e^x)}{8}$	46
risch	$\frac{e^x(3e^{10x}-47e^{8x}+78e^{6x}-78e^{4x}+47e^{2x}-3)}{24(1+e^{2x})^6}+\frac{i\ln(e^x+i)}{16}-\frac{i\ln(e^x-i)}{16}$	64

```
[In] int(sech(x)^3*tanh(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*sinh(x)^3/cosh(x)^6-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech
(x)^3+5/16*sech(x))*tanh(x)+1/8*arctan(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 24.34

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
- 47)*sinh(x)^9 - 47*cosh(x)^9 + 9*(55*cosh(x)^3 - 47*cosh(x))*sinh(x)^8 +
6*(165*cosh(x)^4 - 282*cosh(x)^2 + 13)*sinh(x)^7 + 78*cosh(x)^7 + 42*(33*co
sh(x)^5 - 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 - 987*cos
h(x)^4 + 273*cosh(x)^2 - 13)*sinh(x)^5 - 78*cosh(x)^5 + 6*(165*cosh(x)^7 -
987*cosh(x)^5 + 455*cosh(x)^3 - 65*cosh(x))*sinh(x)^4 + (495*cosh(x)^8 - 39
48*cosh(x)^6 + 2730*cosh(x)^4 - 780*cosh(x)^2 + 47)*sinh(x)^3 + 47*cosh(x)^
3 + 3*(55*cosh(x)^9 - 564*cosh(x)^7 + 546*cosh(x)^5 - 260*cosh(x)^3 + 47*co
sh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(
11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))
*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8
+ 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6
 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cos
h(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x
)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x
)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(
x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(
x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)*ar
ctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 - 141*cosh(x)^8 + 182*cosh(x)^6
```

```
- 130*cosh(x)^4 + 47*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/(cosh(x)^12 + 12*c
osh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x
)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh
(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*c
osh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*s
inh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 +
5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*
cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 +
 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45
*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*
cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*
cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \int \tanh^4(x)\operatorname{sech}^3(x)\,dx$$

```
[In] integrate(sech(x)**3*tanh(x)**4,x)
```

```
[Out] Integral(tanh(x)**4*sech(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{3\,e^{(-x)} - 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} - 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) - 47*e^(-3*x) + 78*e^(-5*x) - 78*e^(-7*x) + 47*e^(-9*x) - 3*
e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-10
*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{32}\,\pi - \frac{3\left(e^{(-x)} - e^x\right)^5 - 32\left(e^{(-x)} - e^x\right)^3 - 48\,e^{(-x)} + 48\,e^x}{24\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)^3} + \frac{1}{16}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

```
[In] integrate(sech(x)^3*tanh(x)^4,x, algorithm="giac")
```

```
[Out] 1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 - 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*e
^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.26

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{\operatorname{atan}(e^x)}{8} - \frac{10\,e^x}{4\,e^{2x} + 6\,e^{4x} + 4\,e^{6x} + e^{8x} + 1} + \frac{e^x}{8\,\left(e^{2x} + 1\right)} + \frac{7\,e^x}{3\,e^{2x} + 3\,e^{4x} + e^{6x} + 1} - \frac{4\,e^{5x} - \frac{8\,e^{3x}}{3} - \frac{8\,e^{7x}}{3} + \frac{2\,e^{9x}}{3} + \frac{2\,e^x}{3}}{6\,e^{2x} + 15\,e^{4x} + 20\,e^{6x} + 15\,e^{8x} + 6\,e^{10x} + e^{12x} + 1} + \frac{16\,e^x}{3\,\left(5\,e^{2x} + 10\,e^{4x} + 10\,e^{6x} + 5\,e^{8x} + e^{10x} + 1\right)} - \frac{23\,e^x}{12\,\left(2\,e^{2x} + e^{4x} + 1\right)}$$

```
[In] int(tanh(x)^4/cosh(x)^3,x)
```

```
[Out] atan(exp(x))/8 - (10*exp(x))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*
x) + 1) + exp(x)/(8*(exp(2*x) + 1)) + (7*exp(x))/(3*exp(2*x) + 3*exp(4*x) +
 exp(6*x) + 1) - (4*exp(5*x) - (8*exp(3*x))/3 - (8*exp(7*x))/3 + (2*exp(9*x
))/3 + (2*exp(x))/3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x)
+ 6*exp(10*x) + exp(12*x) + 1) + (16*exp(x))/(3*(5*exp(2*x) + 10*exp(4*x) +
 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (23*exp(x))/(12*(2*exp(2*x) +
 exp(4*x) + 1))
```

3.99 $\int \operatorname{sech}^5(x) \tanh^2(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \operatorname{sech}^5(x) \tanh^2(x) \, dx = \frac{1}{16} \arctan(\sinh(x)) + \frac{1}{16} \operatorname{sech}(x) \tanh(x) + \frac{1}{24} \operatorname{sech}^3(x) \tanh(x) - \frac{1}{6} \operatorname{sech}^5(x) \tanh(x)$$

```
[Out] 1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)+1/24*sech(x)^3*tanh(x)-1/6*sech(x
)^5*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2691, 3853, 3855}

$$\int \operatorname{sech}^5(x) \tanh^2(x) \, dx = \frac{1}{16} \arctan(\sinh(x)) - \frac{1}{6} \tanh(x) \operatorname{sech}^5(x) + \frac{1}{24} \tanh(x) \operatorname{sech}^3(x) + \frac{1}{16} \tanh(x) \operatorname{sech}(x)$$

```
[In] Int[Sech[x]^5*Tanh[x]^2,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)+\frac{1}{6}\int \operatorname{sech}^5(x)\,dx\\
&= \frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)+\frac{1}{8}\int \operatorname{sech}^3(x)\,dx\\
&= \frac{1}{16}\operatorname{sech}(x)\tanh(x)+\frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)+\frac{1}{16}\int \operatorname{sech}(x)\,dx\\
&= \frac{1}{16}\arctan(\sinh(x))+\frac{1}{16}\operatorname{sech}(x)\tanh(x)+\frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx=\frac{1}{16}\arctan(\sinh(x))+\frac{1}{16}\operatorname{sech}(x)\tanh(x)+\frac{1}{24}\operatorname{sech}^3(x)\tanh(x)-\frac{1}{6}\operatorname{sech}^5(x)\tanh(x)$$

```
[In] Integrate[Sech[x]^5*Tanh[x]^2,x]
```

```
[Out] ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech[
x]^5*Tanh[x])/6
```

Maple [A] (verified)

Time = 169.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sinh(x)}{5\cosh(x)^{6}}+\frac{\left(\frac{\operatorname{sech}(x)^{5}}{6}+\frac{5\operatorname{sech}(x)^{3}}{24}+\frac{5\operatorname{sech}(x)}{16}\right)\tanh(x)}{5}+\frac{\arctan(e^{x})}{8}$	36
risch	$\frac{e^{x}(3e^{10x}+17e^{8x}-114e^{6x}+114e^{4x}-17e^{2x}-3)}{24(1+e^{2x})^{6}}+\frac{i\ln(e^{x}+i)}{16}-\frac{i\ln(e^{x}-i)}{16}$	64

```
[In] int(sech(x)^5*tanh(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech(x)^3+5/16*sech(x))*tanh
(x)+1/8*arctan(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 925, normalized size of antiderivative = 25.69

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="fricas")
[Out] 1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
+ 17)*sinh(x)^9 + 17*cosh(x)^9 + 9*(55*cosh(x)^3 + 17*cosh(x))*sinh(x)^8 +
6*(165*cosh(x)^4 + 102*cosh(x)^2 - 19)*sinh(x)^7 - 114*cosh(x)^7 + 42*(33*c
osh(x)^5 + 34*cosh(x)^3 - 19*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 + 357*co
sh(x)^4 - 399*cosh(x)^2 + 19)*sinh(x)^5 + 114*cosh(x)^5 + 6*(165*cosh(x)^7
+ 357*cosh(x)^5 - 665*cosh(x)^3 + 95*cosh(x))*sinh(x)^4 + (495*cosh(x)^8 +
1428*cosh(x)^6 - 3990*cosh(x)^4 + 1140*cosh(x)^2 - 17)*sinh(x)^3 - 17*cosh(
x)^3 + 3*(55*cosh(x)^9 + 204*cosh(x)^7 - 798*cosh(x)^5 + 380*cosh(x)^3 - 17
*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 +
6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(
x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)
^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x
)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*
cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cos
h(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cos
h(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*
cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*co
sh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*co
sh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 + 51*cosh(x)^8 - 266*cosh(x)^
```

```
6 + 190*cosh(x)^4 - 17*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/(cosh(x)^12 + 12
*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh
(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*co
sh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5
*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)
*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3
+ 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 2
0*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7
 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 +
45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 +
6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 +
5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \int \tanh^2(x)\operatorname{sech}^5(x)\,dx$$

```
[In] integrate(sech(x)**5*tanh(x)**2,x)
```

```
[Out] Integral(tanh(x)**2*sech(x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{3\,e^{(-x)} + 17\,e^{(-3\,x)} - 114\,e^{(-5\,x)} + 114\,e^{(-7\,x)} - 17\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 17*e^(-3*x) - 114*e^(-5*x) + 114*e^(-7*x) - 17*e^(-9*x) -
3*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-
10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{1}{32}\,\pi - \frac{3\,\left(e^{(-x)} - e^{x}\right)^5 + 32\,\left(e^{(-x)} - e^{x}\right)^3 - 48\,e^{(-x)} + 48\,e^{x}}{24\,\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^3} + \frac{1}{16}\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

```
[In] integrate(sech(x)^5*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 + 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*e
^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.72

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{\operatorname{atan}(e^{x})}{8} + \frac{34\,e^{x}}{15\,\left(4\,e^{2\,x} + 6\,e^{4\,x} + 4\,e^{6\,x} + e^{8\,x} + 1\right)} + \frac{e^{x}}{8\,\left(e^{2\,x} + 1\right)} - \frac{9\,e^{x}}{5\,\left(3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1\right)} - \frac{\frac{8\,e^{3\,x}}{3} - \frac{16\,e^{5\,x}}{3} + \frac{8\,e^{7\,x}}{3}}{6\,e^{2\,x} + 15\,e^{4\,x} + 20\,e^{6\,x} + 15\,e^{8\,x} + 6\,e^{10\,x} + e^{12\,x} + 1} - \frac{\frac{28\,e^{5\,x}}{15} - \frac{8\,e^{3\,x}}{3} + \frac{4\,e^{x}}{5}}{5\,e^{2\,x} + 10\,e^{4\,x} + 10\,e^{6\,x} + 5\,e^{8\,x} + e^{10\,x} + 1} + \frac{e^{x}}{12\,\left(2\,e^{2\,x} + e^{4\,x} + 1\right)}$$

```
[In] int(tanh(x)^2/cosh(x)^5,x)
```

```
[Out] atan(exp(x))/8 + (34*exp(x))/(15*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + ex
p(8*x) + 1)) + exp(x)/(8*(exp(2*x) + 1)) - (9*exp(x))/(5*(3*exp(2*x) + 3*ex
p(4*x) + exp(6*x) + 1)) - ((8*exp(3*x))/3 - (16*exp(5*x))/3 + (8*exp(7*x))/
3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + ex
p(12*x) + 1) - ((28*exp(5*x))/15 - (8*exp(3*x))/3 + (4*exp(x))/5)/(5*exp(2*
x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) + exp(x)/(12*(
2*exp(2*x) + exp(4*x) + 1))
```

3.100 $\int \operatorname{sech}^8(x)\tanh^6(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = \frac{\tanh^7(x)}{7} - \frac{\tanh^9(x)}{3} + \frac{3\tanh^{11}(x)}{11} - \frac{\tanh^{13}(x)}{13}$$

```
[Out] 1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 276}

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = -\frac{1}{13}\tanh^{13}(x) + \frac{3\tanh^{11}(x)}{11} - \frac{\tanh^9(x)}{3} + \frac{\tanh^7(x)}{7}$$

```
[In] Int[Sech[x]^8*Tanh[x]^6,x]
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\text{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx, x, i\tanh(x)\right)\\ &= i\text{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)\,dx, x, i\tanh(x)\right)\\ &= \frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(33) = 66.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \operatorname{sech}^8(x)\tanh^6(x)\,dx &= \frac{16\tanh(x)}{3003}+\frac{8\operatorname{sech}^2(x)\tanh(x)}{3003}+\frac{2\operatorname{sech}^4(x)\tanh(x)}{1001}\\ &\quad+\frac{5\operatorname{sech}^6(x)\tanh(x)}{3003}-\frac{53}{429}\operatorname{sech}^8(x)\tanh(x)\\ &\quad+\frac{27}{143}\operatorname{sech}^{10}(x)\tanh(x)-\frac{1}{13}\operatorname{sech}^{12}(x)\tanh(x)\end{aligned}$$

```
[In] Integrate[Sech[x]^8*Tanh[x]^6,x]
```

```
[Out] (16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/1001
 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^10
*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\frac{\tanh\left(x\right)^{7}}{7}-\frac{\tanh\left(x\right)^{9}}{3}+\frac{3\tanh\left(x\right)^{11}}{11}-\frac{\tanh\left(x\right)^{13}}{13}$$

```
[In] int(sech(x)^8*tanh(x)^6,x)
```

```
[Out] 1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(25) = 50.

Time = 0.23 (sec) , antiderivative size = 778, normalized size of antiderivative = 23.58

$$\int \operatorname{sech}^8(x) \tanh^6(x) \, dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="fricas")
```

```
[Out] -64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (5403
6*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 2249*
cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x)^5
 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x))*s
inh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 8294)*s
inh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 15080*cos
h(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^6 + 448
50*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cosh(x)^17
+ 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x)^15 + 13
*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*cosh(x)^4 +
 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(x)^5 + 455*
cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*cosh(x)^4 + 4
68*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh(x)^7 + 273*c
osh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*cosh(x)^8 + 6506
5*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(x)^9 + 716*cosh
(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)^5 + 47190*cosh(
x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655*cosh(x)^8 + 1338
48*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*sinh(x)^7 + 1300*c
osh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*cosh(x)^7 + 132132*
cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^6 + (6188*cosh(x)^12 +
39039*cosh(x)^10 + 100386*cosh(x)^8 + 132132*cosh(x)^6 + 89964*cosh(x)^4 +
26754*cosh(x)^2 + 1638)*sinh(x)^5 + 1794*cosh(x)^5 + (2380*cosh(x)^13 + 177
45*cosh(x)^11 + 55770*cosh(x)^9 + 94380*cosh(x)^7 + 90216*cosh(x)^5 + 45500
*cosh(x)^3 + 8970*cosh(x))*sinh(x)^4 + (680*cosh(x)^14 + 5915*cosh(x)^12 +
22308*cosh(x)^10 + 47190*cosh(x)^8 + 59976*cosh(x)^6 + 44590*cosh(x)^4 + 16
380*cosh(x)^2 + 1430)*sinh(x)^3 + 2002*cosh(x)^3 + (136*cosh(x)^15 + 1365*c
osh(x)^13 + 6084*cosh(x)^11 + 15730*cosh(x)^9 + 25776*cosh(x)^7 + 27300*cos
h(x)^5 + 17940*cosh(x)^3 + 6006*cosh(x))*sinh(x)^2 + (17*cosh(x)^16 + 195*c
osh(x)^14 + 1014*cosh(x)^12 + 3146*cosh(x)^10 + 6426*cosh(x)^8 + 8918*cosh(
x)^6 + 8190*cosh(x)^4 + 4290*cosh(x)^2 + 572)*sinh(x) + 2002*cosh(x))
```

Sympy [F]

$$\int \operatorname{sech}^8(x) \tanh^6(x)\, dx = \int \tanh^6(x) \operatorname{sech}^8(x)\, dx$$

```
[In] integrate(sech(x)**8*tanh(x)**6,x)
```

```
[Out] Integral(tanh(x)**6*sech(x)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 857, normalized size of antiderivative = 25.97

$$\int \operatorname{sech}^8(x) \tanh^6(x)\, dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="maxima")
```

```
[Out] 32/231*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 64/
77*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287
*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*
x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 64/21*e
^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(
-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) +
 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 512/21*e^(-
8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10
*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 28
6*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 768/7*e^(-10*x
)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x)
 + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e
^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 1216/7*e^(-12*x)/
(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) +
 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(
-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 192*e^(-14*x)/(13*e
^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716
*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x
) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 96*e^(-16*x)/(13*e^(-2*x
) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-1
2*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78
*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 32*e^(-18*x)/(13*e^(-2*x) + 78
*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
```

```
 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-2
2*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 32/3003/(13*e^(-2*x) + 78*e^(-4*x) +
 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-1
4*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e
^(-24*x) + e^(-26*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = -\frac{32\left(3003\,e^{(18\,x)} - 9009\,e^{(16\,x)} + 18018\,e^{(14\,x)} - 16302\,e^{(12\,x)} + 10296\,e^{(10\,x)} - 2288\,e^{(8\,x)} + 286\,e^{(6\,x)} + 78\,e^{(4\,x)} + 13\,e^{(2\,x)} + 1\right)}{3003\left(e^{(2\,x)} + 1\right)^{13}}$$

```
[In] integrate(sech(x)^8*tanh(x)^6,x, algorithm="giac")
```

```
[Out] -32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x) +
 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) + 1)
/(e^(2*x) + 1)^13
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 820, normalized size of antiderivative = 24.85

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = \text{Too large to display}$$

```
[In] int(tanh(x)^6/cosh(x)^8,x)
```

```
[Out] - ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4*
x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(2*
x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143 +
(6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*exp(
2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(
12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(2
2*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x))/143 -
 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(1
0*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*exp(8*x))/1
3 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/13 - (768*
exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) + 286*exp(6*
x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(14*x) + 1287
*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13*exp(24*x) +
```

```
exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (1792*exp(6*x))
/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x)
 + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1)
 - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6*x))/429 - (3584*
exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2*x) + 36*exp(4*x)
+ 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12*x) + 36*exp(14*x)
+ 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 - (256*exp(6*x))/13 + (8
00*exp(8*x))/13 - (1280*exp(10*x))/13 + (1120*exp(12*x))/13 - (512*exp(14*x
))/13 + (96*exp(16*x))/13)/(12*exp(2*x) + 66*exp(4*x) + 220*exp(6*x) + 495*
exp(8*x) + 792*exp(10*x) + 924*exp(12*x) + 792*exp(14*x) + 495*exp(16*x) +
220*exp(18*x) + 66*exp(20*x) + 12*exp(22*x) + exp(24*x) + 1) - ((128*exp(2*
x))/715 - 256/2145)/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) +
exp(10*x) + 1) - 32/(715*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) +
 1)) - ((960*exp(4*x))/143 - (768*exp(2*x))/715 - (2560*exp(6*x))/143 + (33
60*exp(8*x))/143 - (10752*exp(10*x))/715 + (2688*exp(12*x))/715 + 32/715)/(
10*exp(2*x) + 45*exp(4*x) + 120*exp(6*x) + 210*exp(8*x) + 252*exp(10*x) + 2
10*exp(12*x) + 120*exp(14*x) + 45*exp(16*x) + 10*exp(18*x) + exp(20*x) + 1)
```

3.101 $\int \cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cosh(a+bx)\coth(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2672, 327, 212}

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\text{arctanh}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
[Out] Cosh[a + b*x]/b - Log[Cosh[(a + b*x)/2]]/b + Log[Sinh[(a + b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	21
default	$\frac{\cosh(bx+a)-2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	21
risch	$\frac{\mathrm{e}^{bx+a}}{2b}+\frac{\mathrm{e}^{-bx-a}}{2b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	54

```
[In] int(cosh(b*x+a)*coth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^2 - 2(\cosh(bx+a) + \sinh(bx+a))\log(\cosh(bx+a) + \sinh(bx+a) + 1) + 2(\cosh(bx+a}{2(b\cosh(bx+a}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \int \cosh(a+bx)\coth(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(23) = 46.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-
b*x - a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{e^{(bx+a)} + e^{(-bx-a)} - 2\,\log\left(e^{(bx+a)} + 1\right) + 2\,\log\left(\left|e^{(bx+a)} - 1\right|\right)}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*x
 + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-b\,x}}{2\,b}$$

```
[In] int(cosh(a + b*x)*coth(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) + exp(- a - b*x)/(2*b)
```

3.102 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{i\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\operatorname{csch}(a+bx)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)}{b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}-6\,e^{bx+a}+e^{-bx-a}}{2b\left(e^{2bx+2a}-1\right)}$	46

```
[In] int(cosh(b*x+a)*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/sinh(b*x+a)*cosh(b*x+a)^2-2/sinh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^2 + \sinh(bx+a)^2 - 3}{2\,b\sinh(bx+a)}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh(a+bx)\coth^2(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**2,x)
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{5\,e^{(-2\,bx-2\,a)} - 1}{2\,b(e^{(-bx-a)} - e^{(-3\,bx-3\,a)})}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
[Out] -1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-3
*b*x - 3*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{4}{e^{(bx+a)}-e^{(-bx-a)}} - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(4/(e^(b*x + a) - e^(-b*x - a)) - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^{-a-bx}\left(e^{4a+4bx}-6\,e^{2a+2bx}+1\right)}{2\,b\,\left(e^{2a+2bx}-1\right)}$$

```
[In] int(cosh(a + b*x)*coth(a + b*x)^2,x)
[Out] (exp(- a - b*x)*(exp(4*a + 4*b*x) - 6*exp(2*a + 2*b*x) + 1))/(2*b*(exp(2*a
+ 2*b*x) - 1))
```

3.103 $\int \cosh(a+bx)\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{3\text{arctanh}(\cosh(a+bx))}{2b}+\frac{3\cosh(a+bx)}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

```
[Out] -3/2*arctanh(cosh(b*x+a))/b+3/2*cosh(b*x+a)/b-1/2*cosh(b*x+a)*coth(b*x+a)^2
/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2672, 294, 327, 212}

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{3\text{arctanh}(\cosh(a+bx))}{2b}+\frac{3\cosh(a+bx)}{2b}-\frac{\cosh(a+bx)\coth^2(a+bx)}{2b}$$

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(2*b) + (3*Cosh[a + b*x])/(2*b) - (Cosh[a + b*x
]*Coth[a + b*x]^2)/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cosh(a+bx)\right)}{b}\\
&= -\frac{\cosh(a+bx)\coth^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}\\
&= \frac{3\cosh(a+bx)}{2b} - \frac{\cosh(a+bx)\coth^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}\\
&= -\frac{3\text{arctanh}(\cosh(a+bx))}{2b} + \frac{3\cosh(a+bx)}{2b} - \frac{\cosh(a+bx)\coth^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^3,x]
[Out] Cosh[a + b*x]/b - Csch[(a + b*x)/2]^2/(8*b) - (3*Log[Cosh[(a + b*x)/2]])/(2
*b) + (3*Log[Sinh[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	62
default	$\frac{\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	62
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(e^{bx+a}-1\right)}{2b}-\frac{3\ln\left(e^{bx+a}+1\right)}{2b}$	90

```
[In] int(cosh(b*x+a)*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/sinh(b*x+a)^2*cosh(b*x+a)^3-3*cosh(b*x+a)/sinh(b*x+a)^2+3/2*coth(b*x
+a)*csch(b*x+a)-3*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 612, normalized size of antiderivative = 12.49

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 3\left(5\cosh(bx+a)^2-1\right)\sinh(bx+a)^4 -}{}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^5
+ 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(
b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2
 + 1)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) +
1)/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)
^5 - 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + 2*
(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a)
 + (5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \int \cosh{(a+bx)}\coth^3{(a+bx)}\,dx$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} - \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{4\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} - 1}{2\,b(e^{(-bx-a)} - 2\,e^{(-3\,bx-3\,a)} + e^{(-5\,bx-5\,a)})}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^(-b*x - a)/b - 3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1
)/b - 1/2*(4*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) - 1)/(b*(e^(-b*x - a) - 2*
e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} - 2\,e^{(bx+a)} - 2\,e^{(-bx-a)} + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) -
 2*e^(b*x + a) - 2*e^(-b*x - a) + 3*log(e^(b*x + a) + e^(-b*x - a) + 2) - 3
*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-b\,x}}{2\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(cosh(a + b*x)*coth(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) + exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*
a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

3.104 $\int \cosh(a+bx)\coth^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx=-\frac{2\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -2*csch(b*x+a)/b-1/3*csch(b*x+a)^3/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 276}

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx=\frac{\sinh(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}-\frac{2\text{csch}(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

```
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{i\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= -\frac{2\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{2\text{csch}(a+bx)}{b}-\frac{\text{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^4,x]
[Out] (-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{\sinh(bx+a)^3}-\frac{4\cosh(bx+a)^2}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$	51
default	$\frac{\frac{\cosh(bx+a)^4}{\sinh(bx+a)^3}-\frac{4\cosh(bx+a)^2}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$	51
risch	$-\frac{-3\,\mathrm{e}^{7bx+7a}+36\,\mathrm{e}^{5bx+5a}-50\,\mathrm{e}^{3bx+3a}+36\,\mathrm{e}^{bx+a}-3\,\mathrm{e}^{-bx-a}}{6b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	72

```
[In] int(cosh(b*x+a)*coth(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(cosh(b*x+a)^4/sinh(b*x+a)^3-4*cosh(b*x+a)^2/sinh(b*x+a)^3+8/3/sinh(b*x
+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{3\cosh(bx+a)^4 + 3\sinh(bx+a)^4 + 18\left(\cosh(bx+a)^2 - 2\right)\sinh(bx+a)^2 - 36\cosh(bx+a)^2 + 25}{6\left(b\sinh(bx+a)^3 + 3\left(b\cosh(bx+a)^2 - b\right)\sinh(bx+a)\right)}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 - 2)*sinh(
b*x + a)^2 - 36*cosh(b*x + a)^2 + 25)/(b*sinh(b*x + a)^3 + 3*(b*cosh(b*x +
a)^2 - b)*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \int \cosh(a+bx)\coth^4(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)**4,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(35) = 70$.

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.70

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{33\,e^{(-2\,bx-2\,a)} - 41\,e^{(-4\,bx-4\,a)} + 27\,e^{(-6\,bx-6\,a)} - 3}{6\,b(e^{(-bx-a)} - 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)})}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-b*x - a)/b - 1/6*(33*e^(-2*b*x - 2*a) - 41*e^(-4*b*x - 4*a) + 27*e
^(-6*b*x - 6*a) - 3)/(b*(e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x -
5*a) - e^(-7*b*x - 7*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(35) = 70.

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{\frac{8\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3}-3\,e^{(bx+a)}+3\,e^{(-bx-a)}}{6\,b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(8*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 2)/(e^(b*x + a) - e^(-b*x - a))
^3 - 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(cosh(a + b*x)*coth(a + b*x)^4,x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(exp(4*a
+ 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2*a + 2*
b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(a + b*x))/(b*(e
xp(2*a + 2*b*x) - 1))
```

3.105 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

```
[Out] ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\sinh^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{2\log(\sinh(a+bx)) + \sinh^2(a+bx)}{2b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] (2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
risch	$-x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	55

```
[In] int(cosh(b*x+a)^2*coth(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.52

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx =$$
$$-\frac{8\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)^4-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\sinh\left(bx+a\right)^4+2\,\left(4\,bx-3\,\cos\right.}{}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*s
inh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) - co
sh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*si
nh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(20) = 40$.

Time = 0.83 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$
$$= \begin{cases} \tilde{\infty}x \\ x\cosh^2(a)\coth(a) \\ \tilde{\infty}x \\ -\frac{x\sinh^2(a+bx)\coth(a+bx)}{2}+\frac{x\cosh^2(a+bx)\coth(a+bx)}{2}-\frac{x\cosh(a+bx)}{2\sinh(a+bx)}+\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)\cosh(a+bx)\coth(a+b}{2b} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cosh(a)**2*coth(a), Eq(b, 0)), (
zoo*x, Eq(a, -b*x)), (-x*sinh(a + b*x)**2*coth(a + b*x)/2 + x*cosh(a + b*x)
**2*coth(a + b*x)/2 - x*cosh(a + b*x)/(2*sinh(a + b*x)) + log(sinh(a + b*x)
)/b + sinh(a + b*x)*cosh(a + b*x)*coth(a + b*x)/(2*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
- a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 8\,a - e^{(2\,bx+2\,a)} - 8\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*a - e^(2*b*x + 2
*a) - 8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - x + \frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b}$$

```
[In] int(cosh(a + b*x)^2*coth(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

3.106 $\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\frac{3x}{2}-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}$$

```
[Out] 3/2*x-3/2*coth(b*x+a)/b+1/2*cosh(b*x+a)^2*coth(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 327, 212}

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3x}{2}$$

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

```
[Out] (3*x)/2 - (3*Coth[a + b*x])/(2*b) + (Cosh[a + b*x]^2*Coth[a + b*x])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \coth(a+bx)\right)}{b}\\
&= \frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \coth(a+bx)\right)}{2b}\\
&= -\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}+\frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \coth(a+bx)\right)}{2b}\\
&= \frac{3x}{2}-\frac{3\coth(a+bx)}{2b}+\frac{\cosh^2(a+bx)\coth(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\frac{6(a+bx)-4\coth(a+bx)+\sinh(2(a+bx))}{4b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
[Out] (6*(a + b*x) - 4*Coth[a + b*x] + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	39
default	$\frac{\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	39
risch	$\frac{3x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{2}{b(e^{2bx+2a}-1)}$	51

```
[In] int(cosh(b*x+a)^2*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/sinh(b*x+a)*cosh(b*x+a)^3+3/2*b*x+3/2*a-3/2*coth(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2+4(3bx+2)\sinh(bx+a)-9\cosh(bx+a)}{8b\sinh(bx+a)}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 4*(3*b*x + 2)*sinh
(b*x + a) - 9*cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a)**2,x)
[Out] Integral(cosh(a + b*x)**2*coth(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{17\,e^{(-2\,bx-2\,a)}-1}{8\,b(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)})}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b - 1/8*(17*e^(-2*b*x - 2*a) - 1)/(b
*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{12\,bx+12\,a-\frac{3\,e^{(4\,bx+4\,a)}+14\,e^{(2\,bx+2\,a)}-1}{e^{(4\,bx+4\,a)}-e^{(2\,bx+2\,a)}}+e^{(2\,bx+2\,a)}}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(12*b*x + 12*a - (3*e^(4*b*x + 4*a) + 14*e^(2*b*x + 2*a) - 1)/(e^(4*b*x
 + 4*a) - e^(2*b*x + 2*a)) + e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{3\,x}{2} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

```
[In] int(cosh(a + b*x)^2*coth(a + b*x)^2,x)
```

```
[Out] (3*x)/2 - 2/(b*(exp(2*a + 2*b*x) - 1)) - exp(- 2*a - 2*b*x)/(8*b) + exp(2*a
 + 2*b*x)/(8*b)
```

3.107 $\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

```
[Out] -1/2*csch(b*x+a)^2/b+2*ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2670, 272, 45}

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx=\frac{\sinh^2(a+bx)}{2b}-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}$$

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

```
[Out] -1/2*Csch[a + b*x]^2/b + (2*Log[Sinh[a + b*x]])/b + Sinh[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, -\sinh^2(a+bx)\right)}{2b}\\ &= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, -\sinh^2(a+bx)\right)}{2b}\\ &= -\frac{\text{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)-4\log(\sinh(a+bx))-\sinh^2(a+bx)}{2b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
[Out] -1/2*(Csch[a + b*x]^2 - 4*Log[Sinh[a + b*x]] - Sinh[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{2\sinh(bx+a)^2}+2\ln(\sinh(bx+a))-\coth(bx+a)^2}{b}$	43
default	$\frac{\frac{\cosh(bx+a)^4}{2\sinh(bx+a)^2}+2\ln(\sinh(bx+a))-\coth(bx+a)^2}{b}$	43
risch	$-2x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{4a}{b}-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}-1\right)^2}+\frac{2\ln\left(e^{2bx+2a}-1\right)}{b}$	83

```
[In] int(cosh(b*x+a)^2*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)^4/sinh(b*x+a)^2+2*ln(sinh(b*x+a))-coth(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 743, normalized size of antiderivative = 17.28

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$$

$$= \frac{\cosh{(bx+a)}^8 + 8\cosh{(bx+a)}\sinh{(bx+a)}^7 + \sinh{(bx+a)}^8 - 2\,(8\,bx+1)\cosh{(bx+a)}^6 - 2\,(8\,bx -}{}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 -
2*(8*b*x + 1)*cosh(b*x + a)^6 - 2*(8*b*x - 14*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^6 + 4*(14*cosh(b*x + a)^3 - 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x + a
)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 - 15*(8*b*x +
1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 5
*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x + a)^3
- 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 - 15*(8*b*x + 1)*co
sh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 - 8*b*x - 1)*sinh(b*x + a)^2
 + 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ (15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(
b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5
 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*sinh(b*x + a)/(c
osh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 - 3*(8*b*x + 1)*cosh(
b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 - (8*b*x + 1)*cosh(b*x + a))*si
nh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b
*sinh(b*x + a)^6 - 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 2*b)*sinh(
b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 2*b*cosh(b*x + a))*sinh(b*x + a)^3 +
b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 12*b*cosh(b*x + a)^2 + b)*sinh(
b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 4*b*cosh(b*x + a)^3 + b*cosh(b*x + a)
)*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \int \cosh^2\left(a+bx\right)\coth^3\left(a+bx\right)dx$$

```
[In] integrate(cosh(b*x+a)**2*coth(b*x+a)**3,x)
[Out] Integral(cosh(a + b*x)**2*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(39) = 78$.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{2\,(bx+a)}{b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2\,e^{(-2\,bx-2\,a)} + 15\,e^{(-4\,bx-4\,a)} - 1}{8\,b(e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)})}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="maxima")
[Out] 2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(
e^(-b*x - a) - 1)/b - 1/8*(2*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 1)/(b
*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = -\frac{16\,bx - \left(8\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 16\,a + \frac{8\,\left(3\,e^{(4\,bx+4\,a)} - 4\,e^{(2\,bx+2\,a)} + 3\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - e^{(2\,bx+2\,a)} - 16\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right.\right.}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="giac")
[Out] -1/8*(16*b*x - (8*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 16*a + 8*(3*e^(4*
b*x + 4*a) - 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) - 1)^2 - e^(2*b*x + 2*
a) - 16*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{2\ln\left(e^{2a}\,e^{2bx}-1\right)}{b} - 2x - \frac{2}{b\left(e^{2a+2bx}-1\right)} - \frac{2}{b\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)} + \frac{e^{-2a-2bx}}{8b} + \frac{e^{2a+2bx}}{8b}$$

```
[In] int(cosh(a + b*x)^2*coth(a + b*x)^3,x)
```

```
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2*x - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*b)
 + exp(2*a + 2*b*x)/(8*b)
```

3.108 $\int \cosh^3(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2672, 308, 212}

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b + Cosh[a + b*x]^3/(3*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cosh(a+bx)\right)}{b}\\
&= \frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\
&= -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{5\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x],x]
[Out] (5*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Log[Cosh[(a + b*x)/2]]
/b + Log[Sinh[(a + b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^{3}}{3}+\cosh(bx+a)-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	31
default	$\frac{\frac{\cosh(bx+a)^{3}}{3}+\cosh(bx+a)-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	31
risch	$\frac{\mathrm{e}^{3bx+3a}}{24b}+\frac{5\,\mathrm{e}^{bx+a}}{8b}+\frac{5\,\mathrm{e}^{-bx-a}}{8b}+\frac{\mathrm{e}^{-3bx-3a}}{24b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	82

```
[In] int(cosh(b*x+a)^3*coth(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.39

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^6 + 6\,\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2+1\right)\sinh(bx+a)^4 +}{}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="fricas")
[Out] 1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 15*(cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*cosh(b*x + a)^4 + 20*(cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 + 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 15*cosh(b*x + a)^2 - 24*(cosh(b*x + a)^3
 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh
(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 24*(cosh(b*x + a)^3 +
 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(cosh(b*x + a)^5 + 10
*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^3 +
 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*
sinh(b*x + a)^3)
```

Sympy [F]

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \int \cosh^3{(a+bx)}\coth{(a+bx)}\,dx$$

[In] integrate(cosh(b*x+a)**3*coth(b*x+a),x)

[Out] Integral(cosh(a + b*x)**3*coth(a + b*x), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\left(15\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="maxima")

[Out] 1/24*(15*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) + e^(-3*b*x - 3*a))/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\left(15\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+e^{(3\,bx+3\,a)}+15\,e^{(bx+a)}-24\,\log\left(e^{(bx+a)}+1\right)+24\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{24\,b}$$

[In] integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="giac")

[Out] 1/24*((15*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + e^(3*b*x + 3*a) + 15*e^(b*x + a) - 24*log(e^(b*x + a) + 1) + 24*log(abs(e^(b*x + a) - 1)))/b

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{5\,\mathrm{e}^{a+b\,x}}{8\,b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b} + \frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

```
[In] int(cosh(a + b*x)^3*coth(a + b*x),x)
```

```
[Out] (5*exp(a + b*x))/(8*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + (5*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b)
```

3.109 $\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] -csch(b*x+a)/b+2*sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\sinh^3(a+bx)}{3b}+\frac{2\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b} \\ &= -\frac{i\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, -i\sinh(a+bx)\right)}{b} \\ &= -\frac{\text{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
[Out] -(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{3\sinh(bx+a)}+\frac{4\cosh(bx+a)^2}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$	52
default	$\frac{\frac{\cosh(bx+a)^4}{3\sinh(bx+a)}+\frac{4\cosh(bx+a)^2}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$	52
risch	$\frac{e^{5bx+5a}+20\,e^{3bx+3a}-90\,e^{bx+a}+20\,e^{-bx-a}+e^{-3bx-3a}}{24b\left(e^{2bx+2a}-1\right)}$	68

```
[In] int(cosh(b*x+a)^3*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3/sinh(b*x+a)*cosh(b*x+a)^4+4/3/sinh(b*x+a)*cosh(b*x+a)^2-8/3/sinh(b
*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+10\right)\sinh(bx+a)^2+20\cosh(bx+a)^2-45}{24\,b\sinh(bx+a)}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="fricas")
[Out] 1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b
*x + a)^2 + 20*cosh(b*x + a)^2 - 45)/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \int \cosh^3(a+bx)\coth^2(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a)**2,x)
[Out] Integral(cosh(a + b*x)**3*coth(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = -\frac{21\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b} + \frac{20\,e^{(-2\,bx-2\,a)}-69\,e^{(-4\,bx-4\,a)}+1}{24\,b\left(e^{(-3\,bx-3\,a)}-e^{(-5\,bx-5\,a)}\right)}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="maxima")
[Out] -1/24*(21*e^(-b*x - a) + e^(-3*b*x - 3*a))/b + 1/24*(20*e^(-2*b*x - 2*a) -
69*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3-\frac{48}{e^{(bx+a)}-e^{(-bx-a)}}+24\,e^{(bx+a)}-24\,e^{(-bx-a)}}{24\,b}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*((e^(b*x + a) - e^(-b*x - a))^3 - 48/(e^(b*x + a) - e^(-b*x - a)) + 24
*e^(b*x + a) - 24*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{7\,\mathrm{e}^{a+b\,x}}{8\,b}-\frac{7\,\mathrm{e}^{-a-b\,x}}{8\,b}-\frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(cosh(a + b*x)^3*coth(a + b*x)^2,x)
```

```
[Out] (7*exp(a + b*x))/(8*b) - (7*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*
b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.110 $\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = -\frac{5\operatorname{arctanh}(\cosh(a+bx))}{2b}+\frac{5\cosh(a+bx)}{2b}+\frac{5\cosh^3(a+bx)}{6b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}$$

```
[Out] -5/2*arctanh(cosh(b*x+a))/b+5/2*cosh(b*x+a)/b+5/6*cosh(b*x+a)^3/b-1/2*cosh(
b*x+a)^3*coth(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2672, 294, 308, 212}

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = -\frac{5\operatorname{arctanh}(\cosh(a+bx))}{2b}+\frac{5\cosh^3(a+bx)}{6b}+\frac{5\cosh(a+bx)}{2b}-\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}$$

```
[In] Int[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

```
[Out] (-5*ArcTanh[Cosh[a + b*x]])/(2*b) + (5*Cosh[a + b*x])/(2*b) + (5*Cosh[a + b
*x]^3)/(6*b) - (Cosh[a + b*x]^3*Coth[a + b*x]^2)/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cosh(a+bx)\right)}{b}\\
&= -\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}\\
&= -\frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cosh(a+bx)\right)}{2b}\\
&= \frac{5\cosh(a+bx)}{2b} + \frac{5\cosh^3(a+bx)}{6b} - \frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}\\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{2b}\\
&= -\frac{5\text{arctanh}(\cosh(a+bx))}{2b} + \frac{5\cosh(a+bx)}{2b} + \frac{5\cosh^3(a+bx)}{6b} - \frac{\cosh^3(a+bx)\coth^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{9\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

```
[In] Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Csch[(a + b*x)/2]^2/(8
*b) - (5*Log[Cosh[(a + b*x)/2]])/(2*b) + (5*Log[Sinh[(a + b*x)/2]])/(2*b) -
 Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{3\sinh(bx+a)^2}+\frac{5\cosh(bx+a)^3}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\coth(bx+a)\operatorname{csch}(bx+a)}{2}-5\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	81
default	$\frac{\frac{\cosh(bx+a)^5}{3\sinh(bx+a)^2}+\frac{5\cosh(bx+a)^3}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\coth(bx+a)\operatorname{csch}(bx+a)}{2}-5\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	81
risch	$\frac{e^{3bx+3a}}{24b}+\frac{9\,e^{bx+a}}{8b}+\frac{9\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{5\ln\left(e^{bx+a}+1\right)}{2b}+\frac{5\ln\left(e^{bx+a}-1\right)}{2b}$	118

```
[In] int(cosh(b*x+a)^3*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/sinh(b*x+a)^2*cosh(b*x+a)^5+5/3/sinh(b*x+a)^2*cosh(b*x+a)^3-5*cosh
(b*x+a)/sinh(b*x+a)^2+5/2*coth(b*x+a)*csch(b*x+a)-5*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 1077, normalized size of antiderivative = 16.32

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + 5*(9*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^8 + 25*cosh(b*x + a)^8 + 40*(3*
cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a)^4
 + 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63*cos
h(b*x + a)^5 + 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5 + 10
*(21*cosh(b*x + a)^6 + 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 - 5)*sinh(b
*x + a)^4 - 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 + 35*cosh(b*x + a)^5
 - 25*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cosh(b*x +
a)^8 + 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 - 60*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^2 + 25*cosh(b*x + a)^2 - 60*(cosh(b*x + a)^7 + 7*cosh(b*x + a
)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 - 2)*sinh(b*x + a
)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x
+ a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + co
sh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))
*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 3*cosh(b*x + a
)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 60*(cosh(b*x +
 a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x +
a)^2 - 2)*sinh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*co
sh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a
)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x +
 a)^4 + 3*cosh(b*x + a)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a)
 - 1) + 10*(cosh(b*x + a)^9 + 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 - 20*
cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 -
2*b*cosh(b*x + a))*sinh(b*x + a)^4 + b*cosh(b*x + a)^3 + (35*b*cosh(b*x + a
)^4 - 20*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 - 2
0*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + (7*b*cosh(b*x +
a)^6 - 10*b*cosh(b*x + a)^4 + 3*b*cosh(b*x + a)^2)*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \int \cosh^3{(a+bx)}\coth^3{(a+bx)}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*coth(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**3*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(58) = 116$.

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.02

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{27\,e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{24\,b} - \frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{25\,e^{(-2\,bx-2\,a)} - 77\,e^{(-4\,bx-4\,a)} + 3\,e^{(-6\,bx-6\,a)} + 1}{24\,b(e^{(-3\,bx-3\,a)} - 2\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)})}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(27*e^(-b*x - a) + e^(-3*b*x - 3*a))/b - 5/2*log(e^(-b*x - a) + 1)/b +
 5/2*log(e^(-b*x - a) - 1)/b + 1/24*(25*e^(-2*b*x - 2*a) - 77*e^(-4*b*x - 4
*a) + 3*e^(-6*b*x - 6*a) + 1)/(b*(e^(-3*b*x - 3*a) - 2*e^(-5*b*x - 5*a) + e
^(-7*b*x - 7*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(58) = 116$.

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{\left(e^{(bx+a)} + e^{(-bx-a)}\right)^3 - \frac{24\,\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + 24\,e^{(bx+a)} + 24\,e^{(-bx-a)} - 30\,\log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right) + 30}{24\,b}$$

```
[In] integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*((e^(b*x + a) + e^(-b*x - a))^3 - 24*(e^(b*x + a) + e^(-b*x - a))/((e^
(b*x + a) + e^(-b*x - a))^2 - 4) + 24*e^(b*x + a) + 24*e^(-b*x - a) - 30*lo
g(e^(b*x + a) + e^(-b*x - a) + 2) + 30*log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.12

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{9\,e^{a+b\,x}}{8\,b} - \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{9\,e^{-a-b\,x}}{8\,b} + \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(cosh(a + b*x)^3*coth(a + b*x)^3,x)
```

```
[Out] (9*exp(a + b*x))/(8*b) - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + (9*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) +
1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

3.111 $\int \cosh^4(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{b}+\frac{\sinh^4(a+bx)}{4b}$$

```
[Out] ln(sinh(b*x+a))/b+sinh(b*x+a)^2/b+1/4*sinh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 272, 45}

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx=\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^2(a+bx)}{b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[In] Int[Cosh[a + b*x]^4*Coth[a + b*x],x]
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/b + Sinh[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, -\sinh^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, -\sinh^2(a+bx)\right)}{2b}\\ &= \frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{b}+\frac{\sinh^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{4\log(\sinh(a+bx))+4\sinh^2(a+bx)+\sinh^4(a+bx)}{4b}$$

```
[In] Integrate[Cosh[a + b*x]^4*Coth[a + b*x],x]
[Out] (4*Log[Sinh[a + b*x]] + 4*Sinh[a + b*x]^2 + Sinh[a + b*x]^4)/(4*b)
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{4}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^4}{4}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	33
risch	$-x+\frac{e^{4bx+4a}}{64b}+\frac{3\,e^{2bx+2a}}{16b}+\frac{3\,e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	83

```
[In] int(cosh(b*x+a)^4*coth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^4+1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 457, normalized size of antiderivative = 11.72

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2+3\right)\sinh(bx+a)^6}{}$$

```
[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 4*(7*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 + 12*co
sh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5 + 2
*(35*cosh(b*x + a)^4 - 32*b*x + 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) + 30*cosh(b*x + a)^3)*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 + 45*cosh(b*x + a)^4 + 3)
*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh(b*x +
a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*si
nh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(
b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 + 9*cosh(b*x + a)^
5 + 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x +
 a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x +
a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

Sympy [F]

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \int \cosh^4(a+bx)\coth(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**4*coth(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**4*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\left(12\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{64\,b} + \frac{bx+a}{b} + \frac{12\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}}{64\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="maxima")

[Out] 1/64*(12*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b + 1/64*(12*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = -\frac{64\,bx-\left(48\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}+64\,a-e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}-64\,\log\left(\left|e^{(2\,bx+2\,a)}\right.\right.}{64\,b}$$

[In] integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="giac")

[Out] -1/64*(64*b*x - (48*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a) + 64*a - e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) - 64*log(abs(e^(2*b*x + 2*a) - 1)))/b

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.97

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - x + \frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}}{16\,b} + \frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16\,b} + \frac{\mathrm{e}^{-4\,a-4\,b\,x}}{64\,b} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{64\,b}$$

[In] int(cosh(a + b*x)^4*coth(a + b*x),x)

[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x + (3*exp(- 2*a - 2*b*x))/(16*b) + (3*exp(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*b)

3.112 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \coth(a+bx)\mathrm{csch}(a+bx)\,dx = -\frac{\mathrm{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2686, 8}

$$\int \coth(a+bx)\mathrm{csch}(a+bx)\,dx = -\frac{\mathrm{csch}(a+bx)}{b}$$

```
[In] Int[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}(\int 1\,dx, x, -i\text{csch}(a+bx))}{b}\\ &= -\frac{\text{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x],x]
[Out] -(Csch[a + b*x]/b)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\text{csch}(bx+a)}{b}$	12
default	$-\frac{\text{csch}(bx+a)}{b}$	12
risch	$-\frac{2\,e^{bx+a}}{b(e^{2bx+2a}-1)}$	25

```
[In] int(coth(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] -csch(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \begin{cases} -\frac{\operatorname{csch}(a+bx)}{b} & \text{for } b\neq 0 \\ x\coth(a)\operatorname{csch}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x)
[Out] Piecewise((-csch(a + b*x)/b, Ne(b, 0)), (x*coth(a)*csch(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)}-e^{(-bx-a)})}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)}-e^{(-bx-a)})}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)/sinh(a + b*x),x)
[Out] -(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.113 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\operatorname{csch}^2(a+bx)}{2b}$$

```
[Out] -1/2*csch(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\operatorname{csch}^2(a+bx)}{2b}$$

```
[In] Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*Csch[a + b*x]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, -i\text{csch}(a+bx))}{b}$$
$$= -\frac{\text{csch}^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] -1/2*Csch[a + b*x]^2/b
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^2}{2b}$	14
default	$-\frac{\coth(bx+a)^2}{2b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b(\mathrm{e}^{2bx+2a}-1)^2}$	28

```
[In] int(coth(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*coth(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.73

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx =$$
$$-\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3-b\cosh(bx+a)+3\,(b\cosh(bx+a$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)^2
 - b)*sinh(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \begin{cases} -\frac{\operatorname{csch}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\coth(a)\operatorname{csch}^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Piecewise((-csch(a + b*x)**2/(2*b), Ne(b, 0)), (x*coth(a)*csch(a)**2, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^2}{2\,b}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*coth(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,a)}-1)^2}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{1}{2\,b\,\sinh\left(a+b\,x\right)^2}$$

```
[In] int(coth(a + b*x)/sinh(a + b*x)^2,x)
[Out] -1/(2*b*sinh(a + b*x)^2)
```

3.114 $\int \cosh(a+bx)\mathbf{csch}^{1+n}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = -\frac{\text{csch}^n(a+bx)}{bn}$$

```
[Out] -csch(b*x+a)^n/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2701, 30}

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = -\frac{\text{csch}^n(a+bx)}{bn}$$

```
[In] Int[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^{-1+n}\,dx, x, \text{csch}(a+bx)\right)}{b}$$
$$= -\frac{\text{csch}^n(a+bx)}{bn}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = -\frac{\text{csch}^n(a+bx)}{bn}$$

```
[In] Integrate[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
[Out] -(Csch[a + b*x]^n/(b*n))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\text{csch}(bx+a)^n}{bn}$
default	$-\frac{\text{csch}(bx+a)^n}{bn}$
risch	$-\frac{2^n\left(e^{bx+a}\right)^n\left(e^{bx+a}-1\right)^{-n}\left(e^{bx+a}+1\right)^{-n}e^{-\frac{i\pi n\left(\text{csgn}\left(\frac{i}{e^{bx+a}-1}\right)\text{csgn}\left(\frac{i}{e^{bx+a}+1}\right)\text{csgn}\left(\frac{i}{\left(e^{bx+a}-1\right)\left(e^{bx+a}+1\right)}\right)-\text{csgn}\left(\frac{i}{e^{b}}\right.\right.}{}}}{}$

```
[In] int(coth(b*x+a)*csch(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] -csch(b*x+a)^n/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx =$$
$$-\frac{\cosh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right)\right)+\sinh\left(n\log\left(\frac{2\left(\cosh(bx+a)+\sinh(bx+a\right.}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+}\right.\right.}{bn}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(cosh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))) + sinh(n*log(2*(cosh(b*x + a)
 + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 - 1))))/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.

Time = 2.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = \begin{cases} x\coth(a)\operatorname{csch}^n(a) & \text{for } b=0 \\ \begin{cases} x\coth(a) & \text{for } b=0 \\ \frac{\log(\sinh(a+bx))}{b} & \text{otherwise} \end{cases} & \text{for } n=0 \\ -\frac{\operatorname{csch}^n(a+bx)}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)**n,x)
```

```
[Out] Piecewise((x*coth(a)*csch(a)**n, Eq(b, 0)), (Piecewise((x*coth(a), Eq(b, 0)
), (log(sinh(a + b*x))/b, True)), Eq(n, 0)), (-csch(a + b*x)**n/(b*n), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(16) = 32$.

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = -\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{bn}$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] -2^n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1))/
(b*n)
```

Giac [F]

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^n \coth\left(bx+a\right)\,dx$$

```
[In] integrate(coth(b*x+a)*csch(b*x+a)^n,x, algorithm="giac")
[Out] integrate(csch(b*x + a)^n*coth(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = -\frac{\left(\frac{2\,e^{a+b\,x}}{e^{2\,a+2\,b\,x}-1}\right)^n}{b\,n}$$

```
[In] int(coth(a + b*x)*(1/sinh(a + b*x))^n,x)
[Out] -((2*exp(a + b*x))/(exp(2*a + 2*b*x) - 1))^n/(b*n)
```

3.115 $\int \coth^2(a+bx)\mathrm{\mathbf{csch}}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\coth^3(a+bx)}{3b}$$

```
[Out] -1/3*coth(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\coth^3(a+bx)}{3b}$$

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
[Out] -1/3*Coth[a + b*x]^3/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int x^2\,dx, x, i\coth(a+bx)\right)}{b}$$
$$= -\frac{\coth^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^3(a+bx)}{3b}$$

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
[Out] -1/3*Coth[a + b*x]^3/b
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^3}{3b}$	14
default	$-\frac{\coth(bx+a)^3}{3b}$	14
risch	$-\frac{2\left(3\,\mathrm{e}^{4bx+4a}+1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	32

```
[In] int(csch(b*x+a)^2*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*coth(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 9.27

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx =$$
$$-\frac{8\left(\cosh(bx+a)^2+\cosh(bx+a)\sinh(b\right.}{3\left(b\cosh(bx+a)^4+4\,b\cosh(bx+a)\sinh(bx+a)^3+b\sinh(bx+a)^4-4\,b\cosh(bx+a)^2+2\left(3\,b\cosh\right.\right.}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*c
osh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*
b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*co
sh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F]

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = \int \coth^2(a+bx)\,\text{csch}^2(a+bx)\,dx$$

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a)**2,x)
[Out] Integral(coth(a + b*x)**2*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^3}{3\,b}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -1/3*coth(b*x + a)^3/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^3}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
[Out] -2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\,\left(3\,e^{4\,a+4\,b\,x}+1\right)}{3\,b\,\left(e^{2\,a+2\,b\,x}-1\right)^3}$$

```
[In] int(coth(a + b*x)^2/sinh(a + b*x)^2,x)
[Out] -(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

3.116 $\int \coth^3(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

```
[Out] -1/4*coth(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

```
[Out] -1/4*Coth[a + b*x]^4/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^3\,dx, x, i\coth(a+bx)\right)}{b}$$
$$= -\frac{\coth^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
[Out] -1/4*Coth[a + b*x]^4/b
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^4}{4b}$	14
default	$-\frac{\coth(bx+a)^4}{4b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4}$	39

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/4*coth(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 13.87

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx =$$
$$-\frac{2\left(\cosh(bx+a)^3+3\,\cosh(bx+\right.}{b\cosh(bx+a)^5+5\,b\cosh(bx+a)\sinh(bx+a)^4+b\sinh(bx+a)^5-3\,b\cosh(bx+a)^3+5\left(2\,b\cosh(bx\right.}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5 +
5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3
 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*
b*cosh(b*x + a))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + 5*(b*cosh(b*x + a)^4
 - 3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = \int \coth^3(a+bx)\,\text{csch}^2(a+bx)\,dx$$

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^4}{4\,b}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*coth(b*x + a)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(e^{(6\,bx+6\,a)}+e^{(2\,bx+2\,a)}\right)}{b(e^{(2\,bx+2\,a)}-1)^4}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 15.40

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\frac{1}{2b}+\frac{3\,e^{2a+2bx}}{2b}+\frac{3\,e^{4a+4bx}}{2b}+\frac{e^{6a+6bx}}{2b}}{6\,e^{4a+4bx}-4\,e^{2a+2bx}-4\,e^{6a+6bx}+e^{8a+8bx}+1} - \frac{\frac{1}{2b}+\frac{e^{2a+2bx}}{b}+\frac{e^{4a+4bx}}{2b}}{3\,e^{2a+2bx}-3\,e^{4a+4bx}+e^{6a+6bx}-1} - \frac{\frac{1}{2b}+\frac{e^{2a+2bx}}{2b}}{e^{4a+4bx}-2\,e^{2a+2bx}+1} - \frac{1}{2b\,\left(e^{2a+2bx}-1\right)}$$

```
[In] int(coth(a + b*x)^3/sinh(a + b*x)^2,x)
```

```
[Out] - (1/(2*b) + (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) + exp(
6*a + 6*b*x)/(2*b))/(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a +
6*b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) + exp(2*a + 2*b*x)/b + exp(4*a +
4*b*x)/(2*b))/(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) -
 1) - (1/(2*b) + exp(2*a + 2*b*x)/(2*b))/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*
b*x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) - 1))
```

3.117 $\int \coth^n(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth^{1+n}(a+bx)}{b(1+n)}$$

```
[Out] -coth(b*x+a)^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 32}

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

```
[In] Int[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int (-ix)^n \, dx, x, i \coth(a + bx)\right)}{b}$$
$$= -\frac{\coth^{1+n}(a + bx)}{b(1 + n)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^n(a + bx)\text{csch}^2(a + bx) \, dx = -\frac{\coth^{1+n}(a + bx)}{b(1 + n)}$$

```
[In] Integrate[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
[Out] -(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\coth(bx+a)^{n+1}}{b(n+1)}$
default	$-\frac{\coth(bx+a)^{n+1}}{b(n+1)}$
risch	$-\frac{\left(1+e^{2bx+2a}\right)\left(e^{bx+a}-1\right)^{-n}\left(e^{bx+a}+1\right)^{-n}\left(1+e^{2bx+2a}\right)^{n} e^{-\frac{i\pi n\left(\operatorname{csgn}\left(\frac{i\left(1+e^{2bx+2a}\right)}{\left(e^{bx+a}-1\right)\left(e^{bx+a}+1\right)}\right)^{3}-\operatorname{csgn}\left(\frac{i\left(1+e^{2bx+2a}\right)}{\left(e^{bx+a}-1\right)\left(e^{bx+\cdots}\right.}\right.\right.}{}}}{}$

```
[In] int(coth(b*x+a)^n*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -coth(b*x+a)^(n+1)/b/(n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\cosh(bx+a)\cosh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)+\cosh(bx+a)\sinh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)}{(bn+b)\sinh(bx+a)}$$

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="fricas")
[Out] -(cosh(b*x + a)*cosh(n*log(cosh(b*x + a)/sinh(b*x + a))) + cosh(b*x + a)*si
nh(n*log(cosh(b*x + a)/sinh(b*x + a))))/((b*n + b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

```
[In] integrate(coth(b*x+a)**n*csch(b*x+a)**2,x)
[Out] Integral(coth(a + b*x)**n*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^{n+1}}{b(n+1)}$$

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -coth(b*x + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\left(\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}\right)^{n+1}}{b(n+1)}$$

```
[In] integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="giac")
[Out] -((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(a+b\,x)\,\left(\frac{\mathrm{e}^{2\,a+2\,b\,x}+1}{\mathrm{e}^{2\,a+2\,b\,x}-1}\right)^n}{b\,(n+1)}$$

```
[In] int(coth(a + b*x)^n/sinh(a + b*x)^2,x)
[Out] -(coth(a + b*x)*((exp(2*a + 2*b*x) + 1)/(exp(2*a + 2*b*x) - 1))^n)/(b*(n +
1))
```

3.118 $\int \coth^3(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

```
[Out] -csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2686}

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}^3(a+bx)}{3b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x],x]
[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{i\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\operatorname{csch}(a+bx)\right)}{b} \\ &= -\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x],x]
[Out] -(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{csch}(bx+a)^3}{3}-\operatorname{csch}(bx+a)}{b}$	24
default	$\frac{-\frac{\operatorname{csch}(bx+a)^3}{3}-\operatorname{csch}(bx+a)}{b}$	24
risch	$-\frac{2\,\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}-2\,\mathrm{e}^{2bx+2a}+3\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	49

```
[In] int(coth(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3*csch(b*x+a)^3-csch(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx =$$

$$-\frac{2\left(3\cosh\left(bx+a\right)^3+9\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\sinh\left(bx+a\right)^3\right.}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4-4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cosh\left(\right.\right.\right.}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
[Out] -2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a)
^3 + (9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a
)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*b*cosh(b*x
+ a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)
^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F]

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = \int \coth^3(a+bx)\,\text{csch}(a+bx)\,dx$$

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a),x)
[Out] Integral(coth(a + b*x)**3*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\begin{aligned}\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = & \frac{2\,e^{(-bx-a)}}{b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)} \\ & - \frac{4\,e^{(-3\,bx-3\,a)}}{3\,b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)} \\ & + \frac{2\,e^{(-5\,bx-5\,a)}}{b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)}\end{aligned}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] 2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*
a) - 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a)
 + e^(-6*b*x - 6*a) - 1)) + 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e
^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right)}{3\,b(e^{(bx+a)} - e^{(-bx-a)})^3}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] -2/3*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 4)/(b*(e^(b*x + a) - e^(-b*x - a))
^3)
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+3\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}^3}$$

```
[In] int(coth(a + b*x)^3/sinh(a + b*x),x)
```

```
[Out] -(2*exp(a + b*x)*(3*exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 3))/(3*b*(exp(2
*a + 2*b*x) - 1)^3)
```

3.119 $\int \coth^3(a+bx)\mathbf{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx=-\frac{\text{csch}^3(a+bx)}{3b}-\frac{\text{csch}^5(a+bx)}{5b}$$

```
[Out] -1/3*csch(b*x+a)^3/b-1/5*csch(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 14}

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx=-\frac{\text{csch}^5(a+bx)}{5b}-\frac{\text{csch}^3(a+bx)}{3b}$$

```
[In] Int[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

```
[Out] -1/3*Csch[a + b*x]^3/b - Csch[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{i\text{Subst}\left(\int x^2(-1+x^2)\,dx, x, -i\text{csch}(a+bx)\right)}{b} \\ &= -\frac{i\text{Subst}\left(\int (-x^2+x^4)\,dx, x, -i\text{csch}(a+bx)\right)}{b} \\ &= -\frac{\text{csch}^3(a+bx)}{3b} - \frac{\text{csch}^5(a+bx)}{5b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx = -\frac{\text{csch}^3(a+bx)}{3b} - \frac{\text{csch}^5(a+bx)}{5b}$$

```
[In] Integrate[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
[Out] -1/3*Csch[a + b*x]^3/b - Csch[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{\frac{\text{csch}(bx+a)^5}{5}+\frac{\text{csch}(bx+a)^3}{3}}{b}$	27
default	$-\frac{\frac{\text{csch}(bx+a)^5}{5}+\frac{\text{csch}(bx+a)^3}{3}}{b}$	27
risch	$-\frac{8\,e^{3bx+3a}\left(5\,e^{4bx+4a}+2\,e^{2bx+2a}+5\right)}{15b\left(e^{2bx+2a}-1\right)^5}$	52

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/b*(1/5*csch(b*x+a)^5+1/3*csch(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 11.06

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7-5\,b\cosh\left(bx+a\right)^5+\left(21\,b\cos\right.\right.}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
[Out] -8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
a)^4 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7 +
7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 5*b*cosh(b*x + a)^5
 + (21*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 -
5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 9*b*cosh(b*x + a)^3 + (35*b*cosh(b*x +
 a)^4 - 50*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^
5 - 50*b*cosh(b*x + a)^3 + 27*b*cosh(b*x + a))*sinh(b*x + a)^2 - 5*b*cosh(b
*x + a) + (7*b*cosh(b*x + a)^6 - 25*b*cosh(b*x + a)^4 + 33*b*cosh(b*x + a)^
2 - 15*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = \int \coth^3\left(a+bx\right)\operatorname{csch}^3\left(a+bx\right)dx$$

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.90

$$\begin{aligned}\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx &= \frac{8\,e^{(-3\,bx-3\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)}\\&\quad+\frac{16\,e^{(-5\,bx-5\,a)}}{15\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)}\\&\quad+\frac{8\,e^{(-7\,bx-7\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)}\end{aligned}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-
6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15*e^(-5*
b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 8/3*e^(-7*b*x - 7*a)/(
b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8
*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx = -\frac{8\left(5\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+12\right)}{15\,b(e^{(bx+a)}-e^{(-bx-a)})^5}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -8/15*(5*(e^(b*x + a) - e^(-b*x - a))^2 + 12)/(b*(e^(b*x + a) - e^(-b*x - a
))^5)
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 8.13

$$\begin{aligned}\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx &= -\frac{\frac{4\,e^{a+b\,x}}{5\,b}+\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}+\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}-10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}-5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}-1}\\&\quad-\frac{28\,e^{a+b\,x}}{15\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{64\,e^{a+b\,x}}{15\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}\\&\quad-\frac{16\,e^{a+b\,x}}{5\,b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}\end{aligned}$$

```
[In] int(coth(a + b*x)^3/sinh(a + b*x)^3,x)
```

```
[Out] - ((4*exp(a + b*x))/(5*b) + (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*b
*x))/(5*b) + (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) - 10*exp(4*a +
 4*b*x) + 10*exp(6*a + 6*b*x) - 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x) - 1
) - (28*exp(a + b*x))/(15*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) -
(64*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a
+ 6*b*x) - 1)) - (16*exp(a + b*x))/(5*b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2
*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.120 $\int \cosh^3(a+bx)\mathbf{csch}^{3+n}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx=-\frac{\text{csch}^n(a+bx)}{bn}-\frac{\text{csch}^{2+n}(a+bx)}{b(2+n)}$$

```
[Out] -csch(b*x+a)^n/b/n-csch(b*x+a)^(2+n)/b/(2+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2701, 14}

$$\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx=-\frac{\text{csch}^{n+2}(a+bx)}{b(n+2)}-\frac{\text{csch}^n(a+bx)}{bn}$$

```
[In] Int[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

```
[Out] -(Csch[a + b*x]^n/(b*n)) - Csch[a + b*x]^(2 + n)/(b*(2 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^{-1+n}(-1-x^2)\,dx, x, \operatorname{csch}(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \left(-x^{-1+n}-x^{1+n}\right)dx, x, \operatorname{csch}(a+bx)\right)}{b} \\ &= -\frac{\operatorname{csch}^n(a+bx)}{bn} - \frac{\operatorname{csch}^{2+n}(a+bx)}{b(2+n)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = -\frac{\operatorname{csch}^n(a+bx)\left(2+n+n\operatorname{csch}^2(a+bx)\right)}{bn(2+n)}$$

```
[In] Integrate[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
[Out] -((Csch[a + b*x]^n*(2 + n + n*Csch[a + b*x]^2))/(b*n*(2 + n)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.86 (sec) , antiderivative size = 479, normalized size of antiderivative = 12.95

method	result
risch	$-\frac{\left(n\,e^{4bx+4a}+2\,e^{4bx+4a}+2n\,e^{2bx+2a}-4\,e^{2bx+2a}+n+2\right)2^n\left(e^{bx+a}-1\right)^{-n}\left(e^{bx+a}\right)^n\left(e^{bx+a}+1\right)^{-n}e^{-\frac{i\pi n\left(\operatorname{csgn}\left(\frac{i}{e^{bx+a}-1}\right)\operatorname{csgn}\left(\frac{i}{e^{bx+a}+\cdots}\right.\right.}{\cdots}}}{\cdots}$

```
[In] int(coth(b*x+a)^3*csch(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] -(n*exp(4*b*x+4*a)+2*exp(4*b*x+4*a)+2*n*exp(2*b*x+2*a)-4*exp(2*b*x+2*a)+n+2
)/b/n/(n+2)/(exp(2*b*x+2*a)-1)^2*2^n*(exp(b*x+a)-1)^(-n)*exp(b*x+a)^n*(exp(
b*x+a)+1)^(-n)*exp(-1/2*I*Pi*n*(csgn(I/(exp(b*x+a)-1))*csgn(I/(exp(b*x+a)+1
))*csgn(I/(exp(b*x+a)-1)/(exp(b*x+a)+1))-csgn(I/(exp(b*x+a)-1))*csgn(I/(exp
(b*x+a)-1)/(exp(b*x+a)+1))^2+csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1
))^3-csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1))^2*csgn(I*exp(b*x+a))-
csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1))^2*csgn(I/(exp(b*x+a)-1)/(e
xp(b*x+a)+1))+csgn(I*exp(b*x+a)/(exp(b*x+a)+1)/(exp(b*x+a)-1))*csgn(I*exp(b
```

```
*x+a))*csgn(I/(exp(b*x+a)-1)/(exp(b*x+a)+1))-csgn(I/(exp(b*x+a)+1))*csgn(I/
(exp(b*x+a)-1)/(exp(b*x+a)+1))^2+csgn(I/(exp(b*x+a)-1)/(exp(b*x+a)+1))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 5.84

$$\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx = \frac{\left((n+2)\cosh{(bx+a)}^2+(n+2)\sinh{(bx+a)}^2+n-2\right)\cosh\left(n\log\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(}\right.\right.}{bn^2-(bn^2+2\,bn)\cosh}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x + a)^2 + n - 2)*cosh(n*log(2*(
cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 - 1))) + ((n + 2)*cosh(b*x + a)^2 + (n + 2)*sinh(b*x
 + a)^2 + n - 2)*sinh(n*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))))/(b*n^2 - (b*n
^2 + 2*b*n)*cosh(b*x + a)^2 - (b*n^2 + 2*b*n)*sinh(b*x + a)^2 + 2*b*n)
```

Sympy [F]

$$\int \cosh^3(a+bx)\text{csch}^{3+n}(a+bx)\,dx = \int \coth^3{(a+bx)}\,\text{csch}^n{(a+bx)}\,dx$$

```
[In] integrate(coth(b*x+a)**3*csch(b*x+a)**n,x)
```

```
[Out] Integral(coth(a + b*x)**3*csch(a + b*x)**n, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(37) = 74$.

Time = 0.34 (sec) , antiderivative size = 414, normalized size of antiderivative = 11.19

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx$$
$$= -\frac{2^n n e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b}$$
$$-\frac{(2^{n+1}n-2^{n+2})e^{\left(-(bx+a)n-2\,bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)-2\,a\right)}}{(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b}$$
$$-\frac{(2^n n+2^{n+1})e^{\left(-(bx+a)n-4\,bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)-4\,a\right)}}{(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b}$$
$$-\frac{2^{n+1}e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{(n^2-2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b}$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] -2^n*n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1)
)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2
*n)*b) - (2^(n + 1)*n - 2^(n + 2))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-b*x
- a) + 1) - n*log(-e^(-b*x - a) + 1) - 2*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x
 - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - (2^n*n + 2^(n + 1))*e^(-
(b*x + a)*n - 4*b*x - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1) -
4*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a)
+ 2*n)*b) - 2^(n + 1)*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^
(-b*x - a) + 1))/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4
*b*x - 4*a) + 2*n)*b)
```

Giac [F]

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^n\coth\left(bx+a\right)^3\,dx$$

```
[In] integrate(coth(b*x+a)^3*csch(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^n*coth(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.70

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = -\frac{\left(\frac{1}{\frac{e^{a+b\,x}}{2}-\frac{e^{-a-b\,x}}{2}}\right)^n\left(\frac{1}{b\,n}+\frac{e^{4\,a+4\,b\,x}}{b\,n}+\frac{e^{2\,a+2\,b\,x}\,(2\,n-4)}{b\,n\,(n+2)}\right)}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1}$$

```
[In] int(coth(a + b*x)^3*(1/sinh(a + b*x))^n,x)
```

```
[Out] -((1/(exp(a + b*x)/2 - exp(- a - b*x)/2))^n*(1/(b*n) + exp(4*a + 4*b*x)/(b*
n) + (exp(2*a + 2*b*x)*(2*n - 4))/(b*n*(n + 2))))/(exp(4*a + 4*b*x) - 2*exp
(2*a + 2*b*x) + 1)
```

3.121 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] -1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/2*ArcTanh[Cosh[a + b*x]]/b - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{1}{2}\int \text{csch}(a+bx)\,dx \\ &= -\frac{\text{arctanh}(\cosh(a+bx))}{2b} - \frac{\coth(a+bx)\text{csch}(a+bx)}{2b} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] -1/8*Csch[(a + b*x)/2]^2/b - Log[Cosh[(a + b*x)/2]]/(2*b) + Log[Sinh[(a + b
*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\,\text{csch}(bx+a)}{2}-\text{arctanh}\left(e^{bx+a}\right)}{b}$	45
default	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\,\text{csch}(bx+a)}{2}-\text{arctanh}\left(e^{bx+a}\right)}{b}$	45
risch	$-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{bx+a}+1\right)}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	65

```
[In] int(csch(b*x+a)*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-arctanh(exp(b*x
+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,\cosh\left(bx+a\right)^3 + 6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2 + 2\,\sinh\left(bx+a\right)^3 + \left(\cosh\left(bx+a\right)^4 + 4\,\cosh\left(bx+a\right)\right.}{}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \coth^2\left(a+bx\right)\operatorname{csch}\left(a+bx\right)\,dx$$

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4}+\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)-\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) +
 log(e^(b*x + a) + e^(-b*x - a) + 2) - log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)^2/sinh(a + b*x),x)
```

```
[Out] - atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b
*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a +
2*b*x) - 1))
```

3.122 $\int \coth^2(a+bx)\mathbf{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}$$

```
[Out] 1/8*arctanh(cosh(b*x+a))/b-1/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*cs
ch(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2691, 3853, 3855}

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(8*b) - (Coth[a + b*x]*Csch[a + b*x])/(8*b) - (Coth[
a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b}+\frac{1}{4}\int \text{csch}^3(a+bx)\,dx\\
&= -\frac{\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{8}\int \text{csch}(a+bx)\,dx\\
&= \frac{\text{arctanh}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\text{csch}^3(a+bx)}{4b}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\begin{aligned}
\int \coth^2(a+bx)\text{csch}^3(a+bx)\,dx = &-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}\\
&+\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}\\
&-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}
\end{aligned}$$

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

```
[Out] -1/32*Csch[(a + b*x)/2]^2/b - Csch[(a + b*x)/2]^4/(64*b) + Log[Cosh[(a + b*
x)/2]]/(8*b) - Log[Sinh[(a + b*x)/2]]/(8*b) - Sech[(a + b*x)/2]^2/(32*b) +
Sech[(a + b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	58
default	$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	58
risch	$-\frac{e^{bx+a}\left(e^{6bx+6a}+7e^{4bx+4a}+7e^{2bx+2a}+1\right)}{4b\left(e^{2bx+2a}-1\right)^4}+\frac{\ln\left(e^{bx+a}+1\right)}{8b}-\frac{\ln\left(e^{bx+a}-1\right)}{8b}$	87

```
[In] int(coth(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/sinh(b*x+a)^4*cosh(b*x+a)-1/3*(-1/4*csch(b*x+a)^3+3/8*csch(b*x+a)
)*coth(b*x+a)+1/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 1109, normalized size of antiderivative = 20.16

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(b*x + a)^7 + 14*cosh(b*x + a)*sinh(b*x + a)^6 + 2*sinh(b*x + a
)^7 + 14*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 14*cosh(b*x + a)^5 + 70*
(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^4 + 14*(5*cosh(b*x + a)^4 +
 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 14*(3*cosh(
b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - (cosh(
b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x +
a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*c
osh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 -
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*
x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b
*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 -
 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a
)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*co
```

```
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 1
5*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a
)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(7*cos
h(b*x + a)^6 + 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 1)*sinh(b*x + a) +
 2*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(
b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 +
6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)
*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh
(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 -
 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cos
h(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))
*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

```
[In] integrate(coth(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(coth(a + b*x)**2*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.35

$$\begin{aligned}
&\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx \\
&= \frac{\log\left(e^{(-bx-a)}+1\right)}{8\,b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{8\,b} \\
&\quad + \frac{e^{(-bx-a)}+7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}
\end{aligned}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*log(e^(-b*x - a) + 1)/b - 1/8*log(e^(-b*x - a) - 1)/b + 1/4*(e^(-b*x -
a) + 7*e^(-3*b*x - 3*a) + 7*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a))/(b*(4*e^(-
2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) -
 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.93

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{\frac{4\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3+4\,e^{(bx+a)}+4\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}-\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

```
[In] integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16*(4*((e^(b*x + a) + e^(-b*x - a))^3 + 4*e^(b*x + a) + 4*e^(-b*x - a))/
((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - log(e^(b*x + a) + e^(-b*x - a) + 2
) + log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.98

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} - \frac{\frac{e^{a+b\,x}}{b}+\frac{2\,e^{3\,a+3\,b\,x}}{b}+\frac{e^{5\,a+5\,b\,x}}{b}}{6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{3\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)^2/sinh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(4*(-b^2)^(1/2)) - (exp(a + b*x)/b +
 (2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(6*exp(4*a + 4*b*x) - 4*exp(2
*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*x))
/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(3
*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(a + b
*x)/(4*b*(exp(2*a + 2*b*x) - 1))
```

3.123 $\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}$$

```
[Out] -3/8*arctanh(cosh(b*x+a))/b-3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)^3
*csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b} - \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

```
[In] Int[Coth[a + b*x]^4*Csch[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) - (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]^3*Csch[a + b*x])/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}+\frac{3}{4}\int \coth^2(a+bx)\text{csch}(a+bx)\,dx\\ &= -\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}+\frac{3}{8}\int \text{csch}(a+bx)\,dx\\ &= -\frac{3\text{arctanh}(\cosh(a+bx))}{8b}-\frac{3\coth(a+bx)\text{csch}(a+bx)}{8b}-\frac{\coth^3(a+bx)\text{csch}(a+bx)}{4b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = &-\frac{5\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}\\ &-\frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}\\ &-\frac{5\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}\end{aligned}$$

```
[In] Integrate[Coth[a + b*x]^4*Csch[a + b*x],x]
```

```
[Out] (-5*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (3*Log[Cosh[
(a + b*x)/2]])/(8*b) + (3*Log[Sinh[(a + b*x)/2]])/(8*b) - (5*Sech[(a + b*x)
/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)^3}{\sinh(bx+a)^4}+\frac{\cosh(bx+a)}{\sinh(bx+a)^4}+\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	74
default	$\frac{-\frac{\cosh(bx+a)^3}{\sinh(bx+a)^4}+\frac{\cosh(bx+a)}{\sinh(bx+a)^4}+\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	74
risch	$-\frac{e^{bx+a}\left(5\,e^{6bx+6a}+3\,e^{4bx+4a}+3\,e^{2bx+2a}+5\right)}{4b\left(e^{2bx+2a}-1\right)^4}+\frac{3\ln\left(e^{bx+a}-1\right)}{8b}-\frac{3\ln\left(e^{bx+a}+1\right)}{8b}$	89

```
[In] int(coth(b*x+a)^4*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-cosh(b*x+a)^3/sinh(b*x+a)^4+1/sinh(b*x+a)^4*cosh(b*x+a)+(-1/4*csch(b*
x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 1114, normalized size of antiderivative = 20.25

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(10*cosh(b*x + a)^7 + 70*cosh(b*x + a)*sinh(b*x + a)^6 + 10*sinh(b*x +
 a)^7 + 6*(35*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 6*cosh(b*x + a)^5 + 10
*(35*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(175*cosh(b*x +
 a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 6*cosh(b*x + a)^3 + 6*(35
*cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 +
3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*
(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x
 + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cos
h(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x +
 a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cos
h(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4
*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*co
sh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x
```

```
+ a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*co
sh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
+ 2*(35*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 5)*sinh(
b*x + a) + 10*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*
x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2
 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*
x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x +
a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3
 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b
*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cos
h(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = \int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$$

```
[In] integrate(coth(b*x+a)**4*csch(b*x+a),x)
[Out] Integral(coth(a + b*x)**4*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(49) = 98.

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{3\log\left(e^{(-bx-a)}+1\right)}{8b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{8b} \\ &\quad + \frac{5e^{(-bx-a)}+3e^{(-3bx-3a)}+3e^{(-5bx-5a)}+5e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)}-6e^{(-4bx-4a)}+4e^{(-6bx-6a)}-e^{(-8bx-8a)}-1)}\end{aligned}$$

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="maxima")
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b + 1/4*(5*e^(-b*x
 - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/(b*(4
*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8
*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\frac{4\left(5\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-12\,e^{(bx+a)}-12\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}+3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)-3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

```
[In] integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(5*(e^(b*x + a) + e^(-b*x - a))^3 - 12*e^(b*x + a) - 12*e^(-b*x -
a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 3*log(e^(b*x + a) + e^(-b*x -
a) + 2) - 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.45

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} - \frac{9\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} - \frac{5\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)^4/sinh(a + b*x),x)
```

```
[Out] - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (9*exp(a +
b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (6*exp(a + b*x))/
(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*e
xp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*
x) + exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1))
```

3.124 $\int \coth^2(x)\mathbf{csch}^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \coth^2(x)\operatorname{csch}^4(x)\,dx = \frac{\coth^3(x)}{3} - \frac{\coth^5(x)}{5}$$

```
[Out] 1/3*coth(x)^3-1/5*coth(x)^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 14}

$$\int \coth^2(x)\operatorname{csch}^4(x)\,dx = \frac{\coth^3(x)}{3} - \frac{\coth^5(x)}{5}$$

```
[In] Int[Coth[x]^2*Csch[x]^4,x]
```

```
[Out] Coth[x]^3/3 - Coth[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= i\text{Subst}\left(\int x^2(1+x^2)\ dx, x, i\coth(x)\right) \\ &= i\text{Subst}\left(\int (x^2+x^4)\ dx, x, i\coth(x)\right) \\ &= \frac{\coth^3(x)}{3} - \frac{\coth^5(x)}{5} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{2\coth(x)}{15} - \frac{1}{15}\coth(x)\text{csch}^2(x) - \frac{1}{5}\coth(x)\text{csch}^4(x)$$

```
[In] Integrate[Coth[x]^2*Csch[x]^4,x]
[Out] (2*Coth[x])/15 - (Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\coth(x)^3}{3} - \frac{\coth(x)^5}{5}$	14
default	$\frac{\coth(x)^3}{3} - \frac{\coth(x)^5}{5}$	14
risch	$-\frac{4(15\,e^{6x}+5\,e^{4x}+5\,e^{2x}-1)}{15(e^{2x}-1)^5}$	31

```
[In] int(coth(x)^2*csch(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/3*coth(x)^3-1/5*coth(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 9.65

$$\int \coth^2(x)\text{csch}^4(x)\,dx =$$

$$-\frac{}{15\left(\cosh(x)^7 + 7\cosh(x)\sinh(x)^6 + \sinh(x)^7 + \left(21\cosh(x)^2 - 5\right)\sinh(x)^5 - 5\cosh(x)^5 + 5\left(7\cosh\right.\right.}$$

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="fricas")
[Out] -8/15*(7*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 21*cosh(x)*sinh(x)^2 + 8*sinh(x
)^3 + 5*cosh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)
^2 - 5)*sinh(x)^5 - 5*cosh(x)^5 + 5*(7*cosh(x)^3 - 5*cosh(x))*sinh(x)^4 + (
35*cosh(x)^4 - 50*cosh(x)^2 + 11)*sinh(x)^3 + 9*cosh(x)^3 + (21*cosh(x)^5 -
 50*cosh(x)^3 + 27*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 25*cosh(x)^4 + 33*co
sh(x)^2 - 15)*sinh(x) - 5*cosh(x))
```

Sympy [F]

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \int \coth^2(x)\,\text{csch}^4(x)\,dx$$

```
[In] integrate(coth(x)**2*csch(x)**4,x)
[Out] Integral(coth(x)**2*csch(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 8.76

$$\begin{aligned}\int \coth^2(x)\text{csch}^4(x)\,dx &= \frac{4\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} \\ &+ \frac{4\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} \\ &+ \frac{4\,e^{(-6\,x)}}{5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1} \\ &- \frac{4}{15\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)}\end{aligned}$$

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*
x) - 1) + 4/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 4*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*
e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
5*e^(-8*x) + e^(-10*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \coth^2(x)\operatorname{csch}^4(x)\,dx = -\frac{4\left(15\,e^{(6\,x)} + 5\,e^{(4\,x)} + 5\,e^{(2\,x)} - 1\right)}{15\,\left(e^{(2\,x)} - 1\right)^5}$$

```
[In] integrate(coth(x)^2*csch(x)^4,x, algorithm="giac")
```

```
[Out] -4/15*(15*e^(6*x) + 5*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 8.47

$$\begin{aligned}\int \coth^2(x)\operatorname{csch}^4(x)\,dx = & -\frac{\frac{8\,e^{2\,x}}{5} + \frac{16\,e^{4\,x}}{5} + \frac{8\,e^{6\,x}}{5}}{5\,e^{2\,x} - 10\,e^{4\,x} + 10\,e^{6\,x} - 5\,e^{8\,x} + e^{10\,x} - 1} \\ & - \frac{\frac{4\,e^{2\,x}}{5} + \frac{8}{15}}{3\,e^{2\,x} - 3\,e^{4\,x} + e^{6\,x} - 1} - \frac{2}{5\,\left(e^{4\,x} - 2\,e^{2\,x} + 1\right)} \\ & - \frac{\frac{8\,e^{2\,x}}{5} + \frac{6\,e^{4\,x}}{5} + \frac{2}{5}}{6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1}\end{aligned}$$

```
[In] int(coth(x)^2/sinh(x)^4,x)
```

```
[Out] - ((8*exp(2*x))/5 + (16*exp(4*x))/5 + (8*exp(6*x))/5)/(5*exp(2*x) - 10*exp(
4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))/5 + 8/15)/
(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 2/(5*(exp(4*x) - 2*exp(2*x) + 1)
) - ((8*exp(2*x))/5 + (6*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*exp(2*x) - 4*ex
p(6*x) + exp(8*x) + 1)
```

3.125 $\int \coth^3(x)\mathbf{csch}^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \coth^3(x)\text{csch}^4(x)\,dx=-\frac{1}{4}\text{csch}^4(x)-\frac{\text{csch}^6(x)}{6}$$

```
[Out] -1/4*csch(x)^4-1/6*csch(x)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 14}

$$\int \coth^3(x)\text{csch}^4(x)\,dx=-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

```
[In] Int[Coth[x]^3*Csch[x]^4,x]
```

```
[Out] -1/4*Csch[x]^4 - Csch[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx, x, -i\text{csch}(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^3+x^5\right)\,dx, x, -i\text{csch}(x)\right)$$
$$= -\frac{1}{4}\text{csch}^4(x) - \frac{\text{csch}^6(x)}{6}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \coth^3(x)\text{csch}^4(x)\,dx = -\frac{1}{4}\text{csch}^4(x) - \frac{\text{csch}^6(x)}{6}$$

```
[In] Integrate[Coth[x]^3*Csch[x]^4,x]
[Out] -1/4*Csch[x]^4 - Csch[x]^6/6
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$-\frac{\left(1+\text{csch}(x)^2\right)^3}{6}+\frac{\left(1+\text{csch}(x)^2\right)^2}{4}$	22
default	$-\frac{\left(1+\text{csch}(x)^2\right)^3}{6}+\frac{\left(1+\text{csch}(x)^2\right)^2}{4}$	22
risch	$-\frac{4\,e^{4x}\left(3\,e^{4x}+2\,e^{2x}+3\right)}{3\left(e^{2x}-1\right)^6}$	29

```
[In] int(coth(x)^3*csch(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/6*(1+csch(x)^2)^3+1/4*(1+csch(x)^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 13.06

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx =$$

$$-\frac{}{3\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 2\left(14\cosh(x)^2 - 3\right)\sinh(x)^6 - 6\cosh(x)^6 + 4\left(14\,\mathrm{cosh}\right.\right.}$$

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="fricas")
```

```
[Out] -4/3*(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(3*cosh(x)^3 + cosh(x))*sinh(x) + 3)/(cosh(x)
^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(14*cosh(x)^2 - 3)*sinh(x)^6 - 6*c
osh(x)^6 + 4*(14*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 45*co
sh(x)^2 + 8)*sinh(x)^4 + 16*cosh(x)^4 + 8*(7*cosh(x)^5 - 15*cosh(x)^3 + 7*c
osh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 45*cosh(x)^4 + 48*cosh(x)^2 - 13)*sin
h(x)^2 - 26*cosh(x)^2 + 4*(2*cosh(x)^7 - 9*cosh(x)^5 + 14*cosh(x)^3 - 7*cos
h(x))*sinh(x) + 15)
```

Sympy [F]

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = \int \coth^3(x)\operatorname{csch}^4(x)\,dx$$

```
[In] integrate(coth(x)**3*csch(x)**4,x)
```

```
[Out] Integral(coth(x)**3*csch(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(13) = 26$.

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 8.18

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx$$
$$= \frac{4\,e^{(-4\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1}$$
$$+ \frac{8\,e^{(-6\,x)}}{3\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1\right)}$$
$$+ \frac{4\,e^{(-8\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1}$$

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-4*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10
*x) - e^(-12*x) - 1) + 8/3*e^(-6*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x)
 - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1) + 4*e^(-8*x)/(6*e^(-2*x) - 15
*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = -\frac{4\left(3\,e^{(8\,x)} + 2\,e^{(6\,x)} + 3\,e^{(4\,x)}\right)}{3\left(e^{(2\,x)} - 1\right)^6}$$

```
[In] integrate(coth(x)^3*csch(x)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(8*x) + 2*e^(6*x) + 3*e^(4*x))/(e^(2*x) - 1)^6
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 12.35

$$\begin{aligned}\int \coth^3(x)\operatorname{csch}^4(x)\,dx = &-\frac{\frac{8\,e^{2\,x}}{5}+\frac{12\,e^{4\,x}}{5}+\frac{16\,e^{6\,x}}{15}+\frac{4}{15}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1}\\&-\frac{\frac{4\,e^{2\,x}}{3}+4\,e^{4\,x}+4\,e^{6\,x}+\frac{4\,e^{8\,x}}{3}}{15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1}\\&-\frac{\frac{8\,e^{2\,x}}{15}+\frac{2}{5}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}-\frac{4}{15\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}\\&-\frac{\frac{6\,e^{2\,x}}{5}+\frac{4\,e^{4\,x}}{5}+\frac{2}{5}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}\end{aligned}$$

```
[In] int(coth(x)^3/sinh(x)^4,x)
```

```
[Out] - ((8*exp(2*x))/5 + (12*exp(4*x))/5 + (16*exp(6*x))/15 + 4/15)/(5*exp(2*x)
- 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))/3
 + 4*exp(4*x) + 4*exp(6*x) + (4*exp(8*x))/3)/(15*exp(4*x) - 6*exp(2*x) - 20
*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2*x))/15 +
 2/5)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 4/(15*(exp(4*x) - 2*exp(2*
x) + 1)) - ((6*exp(2*x))/5 + (4*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*exp(2*x)
 - 4*exp(6*x) + exp(8*x) + 1)
```

3.126 $\int \coth^n(x)\operatorname{csch}^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx=\frac{\coth^{1+n}(x)}{1+n}-\frac{\coth^{3+n}(x)}{3+n}$$

```
[Out] coth(x)^(1+n)/(1+n)-coth(x)^(3+n)/(3+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2687, 14}

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx=\frac{\coth^{n+1}(x)}{n+1}-\frac{\coth^{n+3}(x)}{n+3}$$

```
[In] Int[Coth[x]^n*Csch[x]^4,x]
```

```
[Out] Coth[x]^(1 + n)/(1 + n) - Coth[x]^(3 + n)/(3 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\text{Subst}\left(\int (-ix)^n \left(1+x^2\right) \, dx, x, i\coth(x)\right)\right)\\ &= -\left(i\text{Subst}\left(\int \left((-ix)^n-(-ix)^{2+n}\right) \, dx, x, i\coth(x)\right)\right)\\ &= \frac{\coth^{1+n}(x)}{1+n}-\frac{\coth^{3+n}(x)}{3+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \coth^n(x)\text{csch}^4(x) \, dx = \frac{(-2-n+\cosh(2x))\coth^{1+n}(x)\text{csch}^2(x)}{(1+n)(3+n)}$$

```
[In] Integrate[Coth[x]^n*Csch[x]^4,x]
[Out] ((-2 - n + Cosh[2*x])*Coth[x]^(1 + n)*Csch[x]^2)/((1 + n)*(3 + n))
```

Maple [A] (verified)

Time = 34.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\coth(x)\mathrm{e}^{n\ln(\coth(x))}}{n+1}-\frac{\coth(x)^3\mathrm{e}^{n\ln(\coth(x))}}{3+n}$
default	$\frac{\coth(x)\mathrm{e}^{n\ln(\coth(x))}}{n+1}-\frac{\coth(x)^3\mathrm{e}^{n\ln(\coth(x))}}{3+n}$
risch	$-\frac{2\left(-\mathrm{e}^{6x}+2n\,\mathrm{e}^{4x}+3\,\mathrm{e}^{4x}+2n\,\mathrm{e}^{2x}+3\,\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}-1\right)^{-n}\left(\mathrm{e}^{x}+1\right)^{-n}\left(1+\mathrm{e}^{2x}\right)^{n}\mathrm{e}^{-\frac{i\pi n\left(-\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\mathrm{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{x}+1}\right)^{2}+\right.}{}}}{}$

```
[In] int(coth(x)^n*csch(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/(n+1)*coth(x)*exp(n*ln(coth(x)))-1/(3+n)*coth(x)^3*exp(n*ln(coth(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.38

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \frac{2\left(\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 - (2n+3)\cosh(x)\right)\cosh\left(n\log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right) + \left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 - \dots\right.}{(n^2+4n+3)\sinh(x)^3 + 3\left((n^2+4n+3)\cosh(x)^2 - n^2 - 4n - \dots\right.}$$

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*cosh(n*log(cosh(x)
/sinh(x))) + (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*sinh(n*l
og(cosh(x)/sinh(x))))/((n^2 + 4*n + 3)*sinh(x)^3 + 3*((n^2 + 4*n + 3)*cosh(
x)^2 - n^2 - 4*n - 3)*sinh(x))
```

Sympy [F]

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \int \coth^n(x)\,\text{csch}^4(x)\,dx$$

```
[In] integrate(coth(x)**n*csch(x)**4,x)
```

```
[Out] Integral(coth(x)**n*csch(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(26) = 52.

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 14.15

$$\begin{aligned}\int \coth^n(x)\text{csch}^4(x)\,dx &= -\frac{2(2n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2x)}+1\right)-2x\right)}}{n^2-3(n^2+4n+3)e^{(-2x)}+3(n^2+4n+3)e^{(-4x)}-(n^2+4n+3)e^{(-6x)}+4n+3}\\ &\quad-\frac{2(2n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2x)}+1\right)-4x\right)}}{n^2-3(n^2+4n+3)e^{(-2x)}+3(n^2+4n+3)e^{(-4x)}-(n^2+4n+3)e^{(-6x)}+4n+3}\\ &\quad+\frac{2e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2x)}+1\right)-6x\right)}}{n^2-3(n^2+4n+3)e^{(-2x)}+3(n^2+4n+3)e^{(-4x)}-(n^2+4n+3)e^{(-6x)}+4n+3}\\ &\quad+\frac{2e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2x)}+1\right)\right)}}{n^2-3(n^2+4n+3)e^{(-2x)}+3(n^2+4n+3)e^{(-4x)}-(n^2+4n+3)e^{(-6x)}+4n+3}\end{aligned}$$

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="maxima")
```

```
[Out] -2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) +
1) - 2*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) -
(n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) - 2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n
*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 4*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(
-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) +
2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 6*x)/(
n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n
+ 3)*e^(-6*x) + 4*n + 3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n
*log(e^(-2*x) + 1))/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e
^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3)
```

Giac [F]

$$\int \coth^n(x)\text{csch}^4(x)\,dx=\int \coth\left(x\right)^{n}\operatorname{csch}\left(x\right)^{4}\,dx$$

```
[In] integrate(coth(x)^n*csch(x)^4,x, algorithm="giac")
```

```
[Out] integrate(coth(x)^n*csch(x)^4, x)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \coth^n(x)\text{csch}^4(x)\,dx=\frac{\left(\frac{4\cosh(3\,x)}{n^2+4\,n+3}-\frac{2\cosh(x)\,(4\,n+6)}{n^2+4\,n+3}\right)\,\left(\frac{\mathrm{e}^{2\,x}+1}{\mathrm{e}^{2\,x}-1}\right)^n}{2\sinh\left(3\,x\right)-\frac{2\sinh(x)\,(3\,n^2+12\,n+9)}{n^2+4\,n+3}}$$

```
[In] int(coth(x)^n/sinh(x)^4,x)
```

```
[Out] (((4*cosh(3*x))/(4*n + n^2 + 3) - (2*cosh(x)*(4*n + 6))/(4*n + n^2 + 3))*((
exp(2*x) + 1)/(exp(2*x) - 1))^n)/(2*sinh(3*x) - (2*sinh(x)*(12*n + 3*n^2 +
9))/(4*n + n^2 + 3))
```

3.127 $\int \coth^4(x)\mathbf{csch}^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \coth^4(x)\text{csch}^3(x)\,dx = \frac{1}{16}\text{arctanh}(\cosh(x)) - \frac{1}{16}\coth(x)\text{csch}(x) - \frac{1}{8}\coth(x)\text{csch}^3(x) - \frac{1}{6}\coth^3(x)\text{csch}^3(x)$$

```
[Out] 1/16*arctanh(cosh(x))-1/16*coth(x)*csch(x)-1/8*coth(x)*csch(x)^3-1/6*coth(x
)^3*csch(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2691, 3853, 3855}

$$\int \coth^4(x)\text{csch}^3(x)\,dx = \frac{1}{16}\text{arctanh}(\cosh(x)) - \frac{1}{6}\coth^3(x)\text{csch}^3(x) - \frac{1}{8}\coth(x)\text{csch}^3(x) - \frac{1}{16}\coth(x)\text{csch}(x)$$

```
[In] Int[Coth[x]^4*Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/16 - (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/8 - (Coth[
x]^3*Csch[x]^3)/6
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{6}\coth^3(x)\text{csch}^3(x)+\frac{1}{2}\int\coth^2(x)\text{csch}^3(x)\,dx\\ &= -\frac{1}{8}\coth(x)\text{csch}^3(x)-\frac{1}{6}\coth^3(x)\text{csch}^3(x)+\frac{1}{8}\int\text{csch}^3(x)\,dx\\ &= -\frac{1}{16}\coth(x)\text{csch}(x)-\frac{1}{8}\coth(x)\text{csch}^3(x)-\frac{1}{6}\coth^3(x)\text{csch}^3(x)-\frac{1}{16}\int\text{csch}(x)\,dx\\ &= \frac{1}{16}\text{arctanh}(\cosh(x))-\frac{1}{16}\coth(x)\text{csch}(x)-\frac{1}{8}\coth(x)\text{csch}^3(x)-\frac{1}{6}\coth^3(x)\text{csch}^3(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. $2(38) = 76$.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\begin{aligned}\int\coth^4(x)\text{csch}^3(x)\,dx = &-\frac{1}{64}\text{csch}^2\left(\frac{x}{2}\right)-\frac{1}{64}\text{csch}^4\left(\frac{x}{2}\right)-\frac{1}{384}\text{csch}^6\left(\frac{x}{2}\right)+\frac{1}{16}\log\left(\cosh\left(\frac{x}{2}\right)\right)\\ &-\frac{1}{16}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{64}\text{sech}^2\left(\frac{x}{2}\right)+\frac{1}{64}\text{sech}^4\left(\frac{x}{2}\right)-\frac{1}{384}\text{sech}^6\left(\frac{x}{2}\right)\end{aligned}$$

```
[In] Integrate[Coth[x]^4*Csch[x]^3,x]
[Out] -1/64*Csch[x/2]^2 - Csch[x/2]^4/64 - Csch[x/2]^6/384 + Log[Cosh[x/2]]/16 -
Log[Sinh[x/2]]/16 - Sech[x/2]^2/64 + Sech[x/2]^4/64 - Sech[x/2]^6/384
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\cosh(x)^3}{3\sinh(x)^6}+\frac{\cosh(x)}{5\sinh(x)^6}+\frac{\left(-\frac{\operatorname{csch}(x)^5}{6}+\frac{5\operatorname{csch}(x)^3}{24}-\frac{5\operatorname{csch}(x)}{16}\right)\coth(x)}{5}+\frac{\operatorname{arctanh}(e^x)}{8}$	46
risch	$-\frac{e^x(3e^{10x}+47e^{8x}+78e^{6x}+78e^{4x}+47e^{2x}+3)}{24(e^{2x}-1)^6}-\frac{\ln(e^x-1)}{16}+\frac{\ln(e^x+1)}{16}$	60

```
[In] int(coth(x)^4*csch(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/sinh(x)^6*cosh(x)^3+1/5/sinh(x)^6*cosh(x)+1/5*(-1/6*csch(x)^5+5/24*csc
h(x)^3-5/16*csch(x))*coth(x)+1/8*arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. 2(30) = 60.

Time = 0.25 (sec) , antiderivative size = 1260, normalized size of antiderivative = 33.16

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cosh(x)^11 + 66*cosh(x)*sinh(x)^10 + 6*sinh(x)^11 + 2*(165*cosh(x)
^2 + 47)*sinh(x)^9 + 94*cosh(x)^9 + 18*(55*cosh(x)^3 + 47*cosh(x))*sinh(x)^
8 + 12*(165*cosh(x)^4 + 282*cosh(x)^2 + 13)*sinh(x)^7 + 156*cosh(x)^7 + 84*
(33*cosh(x)^5 + 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 12*(231*cosh(x)^6 +
987*cosh(x)^4 + 273*cosh(x)^2 + 13)*sinh(x)^5 + 156*cosh(x)^5 + 12*(165*cos
h(x)^7 + 987*cosh(x)^5 + 455*cosh(x)^3 + 65*cosh(x))*sinh(x)^4 + 2*(495*cos
h(x)^8 + 3948*cosh(x)^6 + 2730*cosh(x)^4 + 780*cosh(x)^2 + 47)*sinh(x)^3 +
94*cosh(x)^3 + 6*(55*cosh(x)^9 + 564*cosh(x)^7 + 546*cosh(x)^5 + 260*cosh(x
)^3 + 47*cosh(x))*sinh(x)^2 - 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(
x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 -
 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15
*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(23
1*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 +
 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15
*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4
+ 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x
)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6
 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^1
1 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11
```

```
 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cos
h(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x
)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7
 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cos
h(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x
)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*si
nh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 2
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*c
osh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(c
osh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(
x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(11*cosh(x)^10 + 141*cosh(x
)^8 + 182*cosh(x)^6 + 130*cosh(x)^4 + 47*cosh(x)^2 + 1)*sinh(x) + 6*cosh(x)
)/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*s
inh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33
*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5
- 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 +
105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)
^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6
+ 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x
)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6
*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2
 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7
 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \int \coth^4(x)\operatorname{csch}^3(x)\,dx$$

```
[In] integrate(coth(x)**4*csch(x)**3,x)
[Out] Integral(coth(x)**4*csch(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(30) = 60$.

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{3\,e^{(-x)} + 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} + 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} + 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1)} + \frac{1}{16}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{16}\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 47*e^(-3*x) + 78*e^(-5*x) + 78*e^(-7*x) + 47*e^(-9*x) + 3*
e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10
*x) - e^(-12*x) - 1) + 1/16*log(e^(-x) + 1) - 1/16*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \coth^4(x)\text{csch}^3(x)\,dx = -\frac{3\left(e^{(-x)}+e^x\right)^5+32\left(e^{(-x)}+e^x\right)^3-48\,e^{(-x)}-48\,e^x}{24\left(\left(e^{(-x)}+e^x\right)^2-4\right)^3} + \frac{1}{32}\log\left(e^{(-x)}+e^x+2\right)-\frac{1}{32}\log\left(e^{(-x)}+e^x-2\right)$$

```
[In] integrate(coth(x)^4*csch(x)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(e^(-x) + e^x)^5 + 32*(e^(-x) + e^x)^3 - 48*e^(-x) - 48*e^x)/((e^(
-x) + e^x)^2 - 4)^3 + 1/32*log(e^(-x) + e^x + 2) - 1/32*log(e^(-x) + e^x -
2)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 5.63

$$\int \coth^4(x)\text{csch}^3(x)\,dx = \frac{\ln\left(\frac{e^x}{8}+\frac{1}{8}\right)}{16}-\frac{\ln\left(\frac{e^x}{8}-\frac{1}{8}\right)}{16}-\frac{10\,e^x}{6\,e^{4x}-4\,e^{2x}-4\,e^{6x}+e^{8x}+1} - \frac{e^x}{8\left(e^{2x}-1\right)}-\frac{7\,e^x}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1} - \frac{\frac{8\,e^{3x}}{3}+4\,e^{5x}+\frac{8\,e^{7x}}{3}+\frac{2\,e^{9x}}{3}+\frac{2\,e^x}{3}}{15\,e^{4x}-6\,e^{2x}-20\,e^{6x}+15\,e^{8x}-6\,e^{10x}+e^{12x}+1} - \frac{16\,e^x}{3\left(5\,e^{2x}-10\,e^{4x}+10\,e^{6x}-5\,e^{8x}+e^{10x}-1\right)} - \frac{23\,e^x}{12\left(e^{4x}-2\,e^{2x}+1\right)}$$

```
[In] int(coth(x)^4/sinh(x)^3,x)
```

```
[Out] log(exp(x)/8 + 1/8)/16 - log(exp(x)/8 - 1/8)/16 - (10*exp(x))/(6*exp(4*x) -
 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - exp(x)/(8*(exp(2*x) - 1)) - (7*e
xp(x))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(3*x))/3 + 4*exp(5
*x) + (8*exp(7*x))/3 + (2*exp(9*x))/3 + (2*exp(x))/3)/(15*exp(4*x) - 6*exp(
2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - (16*exp(x
))/(3*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1)
) - (23*exp(x))/(12*(exp(4*x) - 2*exp(2*x) + 1))
```

3.128 $\int \coth^4(x)\mathrm{csch}^6(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx = -\frac{1}{5}\coth^5(x) + \frac{2\coth^7(x)}{7} - \frac{\coth^9(x)}{9}$$

```
[Out] -1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2687, 276}

$$\int \coth^4(x)\mathrm{csch}^6(x)\,dx = -\frac{1}{9}\coth^9(x) + \frac{2\coth^7(x)}{7} - \frac{\coth^5(x)}{5}$$

```
[In] Int[Coth[x]^4*Csch[x]^6,x]
```

```
[Out] -1/5*Coth[x]^5 + (2*Coth[x]^7)/7 - Coth[x]^9/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = i\text{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx, x, i\coth(x)\right)$$
$$= i\text{Subst}\left(\int \left(x^4+2x^6+x^8\right)\,dx, x, i\coth(x)\right)$$
$$= -\frac{1}{5}\coth^5(x)+\frac{2\coth^7(x)}{7}-\frac{\coth^9(x)}{9}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \coth^4(x)\text{csch}^6(x)\,dx = -\frac{8\coth(x)}{315}+\frac{4}{315}\coth(x)\text{csch}^2(x)-\frac{1}{105}\coth(x)\text{csch}^4(x) - \frac{10}{63}\coth(x)\text{csch}^6(x)-\frac{1}{9}\coth(x)\text{csch}^8(x)$$

```
[In] Integrate[Coth[x]^4*Csch[x]^6,x]
[Out] (-8*Coth[x])/315 + (4*Coth[x]*Csch[x]^2)/315 - (Coth[x]*Csch[x]^4)/105 - (1
0*Coth[x]*Csch[x]^6)/63 - (Coth[x]*Csch[x]^8)/9
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\coth(x)^5}{5}+\frac{2\coth(x)^7}{7}-\frac{\coth(x)^9}{9}$	20
default	$-\frac{\coth(x)^5}{5}+\frac{2\coth(x)^7}{7}-\frac{\coth(x)^9}{9}$	20
risch	$-\frac{16\left(210\,e^{12x}+315\,e^{10x}+441\,e^{8x}+126\,e^{6x}+36\,e^{4x}-9\,e^{2x}+1\right)}{315\left(e^{2x}-1\right)^9}$	49

```
[In] int(coth(x)^4*csch(x)^6,x,method=_RETURNVERBOSE)
[Out] -1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 430, normalized size of antiderivative = 17.20

$$\int \coth^4(x)\text{csch}^6(x)\,dx =$$

$$-\frac{}{315\left(\cosh\left(x\right)^{12}+12\,\cosh\left(x\right)\sinh\left(x\right)^{11}+\sinh\left(x\right)^{12}+3\left(22\,\cosh\left(x\right)^{2}-3\right)\sinh\left(x\right)^{10}-9\,\cosh\left(x\right)^{10}+\right.}$$

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="fricas")
```

```
[Out] -16/315*(211*cosh(x)^6 + 1254*cosh(x)*sinh(x)^5 + 211*sinh(x)^6 + 3*(1055*c
osh(x)^2 + 102)*sinh(x)^4 + 306*cosh(x)^4 + 4*(1045*cosh(x)^3 + 324*cosh(x)
)*sinh(x)^3 + 3*(1055*cosh(x)^4 + 612*cosh(x)^2 + 159)*sinh(x)^2 + 477*cosh
(x)^2 + 6*(209*cosh(x)^5 + 216*cosh(x)^3 + 135*cosh(x))*sinh(x) + 126)/(cos
h(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(22*cosh(x)^2 - 3)*sinh(x)
^10 - 9*cosh(x)^10 + 10*(22*cosh(x)^3 - 9*cosh(x))*sinh(x)^9 + 9*(55*cosh(x
)^4 - 45*cosh(x)^2 + 4)*sinh(x)^8 + 36*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + 4*cosh(x))*sinh(x)^7 + (924*cosh(x)^6 - 1890*cosh(x)^4 + 1008*cos
h(x)^2 - 85)*sinh(x)^6 - 85*cosh(x)^6 + 6*(132*cosh(x)^7 - 378*cosh(x)^5 +
336*cosh(x)^3 - 83*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 126*cosh(x)^6 +
168*cosh(x)^4 - 85*cosh(x)^2 + 9)*sinh(x)^4 + 135*cosh(x)^4 + 4*(55*cosh(x)
^9 - 270*cosh(x)^7 + 504*cosh(x)^5 - 415*cosh(x)^3 + 117*cosh(x))*sinh(x)^3
 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 + 336*cosh(x)^6 - 425*cosh(x)^4 + 270*c
osh(x)^2 - 54)*sinh(x)^2 - 162*cosh(x)^2 + 6*(2*cosh(x)^11 - 15*cosh(x)^9 +
 48*cosh(x)^7 - 83*cosh(x)^5 + 78*cosh(x)^3 - 30*cosh(x))*sinh(x) + 84)
```

Sympy [F]

$$\int \coth^4(x)\text{csch}^6(x)\,dx = \int \coth^4\left(x\right)\text{csch}^6\left(x\right)\,dx$$

```
[In] integrate(coth(x)**4*csch(x)**6,x)
```

```
[Out] Integral(coth(x)**4*csch(x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 17.24

$$\int \coth^4(x)\operatorname{csch}^6(x)\,dx =$$

$$-\frac{16\,e^{(-2\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1} \cdots}$$
$$+\frac{64\,e^{(-4\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1} \cdots}$$
$$+\frac{32\,e^{(-6\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18} \cdots}$$
$$+\frac{112\,e^{(-8\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18} \cdots}$$
$$+\frac{16\,e^{(-10\,x)}}{9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18\,x)} \cdots}$$
$$+\frac{32\,e^{(-12\,x)}}{3\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18} \cdots}$$
$$+\frac{16}{315\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-} \cdots}$$

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="maxima")
```

```
[Out] -16/35*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 12
6*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) +
64/35*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126
*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 3
2/5*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 112
/5*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^
(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16*e
^(-10*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-1
0*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 32/3*e^
(-12*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10
*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16/315/(
9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*
e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \coth^4(x)\operatorname{csch}^6(x)\,dx = -\frac{16\left(210\,e^{(12\,x)}+315\,e^{(10\,x)}+441\,e^{(8\,x)}+126\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\left(e^{(2\,x)}-1\right)^9}$$

```
[In] integrate(coth(x)^4*csch(x)^6,x, algorithm="giac")
```

```
[Out] -16/315*(210*e^(12*x) + 315*e^(10*x) + 441*e^(8*x) + 126*e^(6*x) + 36*e^(4*
x) - 9*e^(2*x) + 1)/(e^(2*x) - 1)^9
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 413, normalized size of antiderivative = 16.52

$$\begin{aligned}\int \coth^4(x)\operatorname{csch}^6(x)\,dx &= -\frac{\frac{8\,e^{2\,x}}{9}+\frac{16\,e^{4\,x}}{3}+\frac{32\,e^{6\,x}}{3}+\frac{80\,e^{8\,x}}{9}+\frac{8\,e^{10\,x}}{3}}{28\,e^{4\,x}-8\,e^{2\,x}-56\,e^{6\,x}+70\,e^{8\,x}-56\,e^{10\,x}+28\,e^{12\,x}-8\,e^{14\,x}+e^{16\,x}+1}\\
&\quad-\frac{\frac{8\,e^{2\,x}}{21}+\frac{16}{63}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}-\frac{8}{63\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}\\
&\quad-\frac{\frac{64\,e^{2\,x}}{63}+\frac{16\,e^{4\,x}}{21}+\frac{32}{105}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1}\\
&\quad-\frac{\frac{32\,e^{2\,x}}{21}+\frac{160\,e^{4\,x}}{63}+\frac{80\,e^{6\,x}}{63}+\frac{16}{63}}{15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1}\\
&\quad-\frac{\frac{32\,e^{4\,x}}{9}+\frac{128\,e^{6\,x}}{9}+\frac{64\,e^{8\,x}}{3}+\frac{128\,e^{10\,x}}{9}+\frac{32\,e^{12\,x}}{9}}{9\,e^{2\,x}-36\,e^{4\,x}+84\,e^{6\,x}-126\,e^{8\,x}+126\,e^{10\,x}-84\,e^{12\,x}+36\,e^{14\,x}-9\,e^{16\,x}+e^{18\,x}-1}\\
&\quad-\frac{\frac{32\,e^{2\,x}}{21}+\frac{32\,e^{4\,x}}{7}+\frac{320\,e^{6\,x}}{63}+\frac{40\,e^{8\,x}}{21}+\frac{8}{63}}{7\,e^{2\,x}-21\,e^{4\,x}+35\,e^{6\,x}-35\,e^{8\,x}+21\,e^{10\,x}-7\,e^{12\,x}+e^{14\,x}-1}\end{aligned}$$

```
[In] int(coth(x)^4/sinh(x)^6,x)
```

```
[Out] - ((8*exp(2*x))/9 + (16*exp(4*x))/3 + (32*exp(6*x))/3 + (80*exp(8*x))/9 + (
8*exp(10*x))/3)/(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x) - 56*
exp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1) - ((8*exp(2*x))/21
+ 16/63)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - 8/(63*(3*e
xp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - ((64*exp(2*x))/63 + (16*exp(4*x))/2
1 + 32/105)/(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x
) - 1) - ((32*exp(2*x))/21 + (160*exp(4*x))/63 + (80*exp(6*x))/63 + 16/63)/
```

```
(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(1
2*x) + 1) - ((32*exp(4*x))/9 + (128*exp(6*x))/9 + (64*exp(8*x))/3 + (128*ex
p(10*x))/9 + (32*exp(12*x))/9)/(9*exp(2*x) - 36*exp(4*x) + 84*exp(6*x) - 12
6*exp(8*x) + 126*exp(10*x) - 84*exp(12*x) + 36*exp(14*x) - 9*exp(16*x) + ex
p(18*x) - 1) - ((32*exp(2*x))/21 + (32*exp(4*x))/7 + (320*exp(6*x))/63 + (4
0*exp(8*x))/21 + 8/63)/(7*exp(2*x) - 21*exp(4*x) + 35*exp(6*x) - 35*exp(8*x
) + 21*exp(10*x) - 7*exp(12*x) + exp(14*x) - 1)
```

3.129 $\int \coth^5(6x)\textbf{csch}(6x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \coth^5(6x)\text{csch}(6x)\,dx=-\frac{1}{6}\text{csch}(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{30}\text{csch}^5(6x)$$

```
[Out] -1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2686, 200}

$$\int \coth^5(6x)\text{csch}(6x)\,dx=-\frac{1}{30}\text{csch}^5(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{6}\text{csch}(6x)$$

```
[In] Int[Coth[6*x]^5*Csch[6*x],x]
```

```
[Out] -1/6*Csch[6*x] - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{6}i\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, -i\text{csch}(6x)\right)\right)\\ &= -\left(\frac{1}{6}i\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\text{csch}(6x)\right)\right)\\ &= -\frac{1}{6}\text{csch}(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{30}\text{csch}^5(6x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \coth^5(6x)\text{csch}(6x)\,dx = -\frac{1}{6}\text{csch}(6x)-\frac{1}{9}\text{csch}^3(6x)-\frac{1}{30}\text{csch}^5(6x)$$

```
[In] Integrate[Coth[6*x]^5*Csch[6*x],x]
[Out] -1/6*Csch[6*x] - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\text{csch}(6x)}{6}-\frac{\text{csch}(6x)^3}{9}-\frac{\text{csch}(6x)^5}{30}$	24
default	$-\frac{\text{csch}(6x)}{6}-\frac{\text{csch}(6x)^3}{9}-\frac{\text{csch}(6x)^5}{30}$	24
risch	$-\frac{e^{6x}\left(15\,e^{48x}-20\,e^{36x}+58\,e^{24x}-20\,e^{12x}+15\right)}{45\left(e^{12x}-1\right)^5}$	41

```
[In] int(coth(6*x)^5*csch(6*x),x,method=_RETURNVERBOSE)
[Out] -1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 8.62

$$\int \coth^5(6x)\mathrm{csch}(6x)\,dx = -\frac{15\cosh(6x)^5+75\cosh(6x)\sinh(6x)^4+15\sinh(6x)^5+5\,(30\cosh(6}{45\,(\cosh(6x)^6+6\cosh(6x)\sinh(6x)^5+\sinh(6x)^6+3\,(5\cosh(6x)^2-2)\sinh(6x)^4-6\cosh(6x)^4}$$

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="fricas")
```

```
[Out] -1/45*(15*cosh(6*x)^5 + 75*cosh(6*x)*sinh(6*x)^4 + 15*sinh(6*x)^5 + 5*(30*c
osh(6*x)^2 - 7)*sinh(6*x)^3 - 5*cosh(6*x)^3 + 15*(10*cosh(6*x)^3 - cosh(6*x
))*sinh(6*x)^2 + 3*(25*cosh(6*x)^4 - 35*cosh(6*x)^2 + 26)*sinh(6*x) + 38*co
sh(6*x))/(cosh(6*x)^6 + 6*cosh(6*x)*sinh(6*x)^5 + sinh(6*x)^6 + 3*(5*cosh(6
*x)^2 - 2)*sinh(6*x)^4 - 6*cosh(6*x)^4 + 4*(5*cosh(6*x)^3 - 4*cosh(6*x))*si
nh(6*x)^3 + 3*(5*cosh(6*x)^4 - 12*cosh(6*x)^2 + 5)*sinh(6*x)^2 + 15*cosh(6*
x)^2 + 2*(3*cosh(6*x)^5 - 8*cosh(6*x)^3 + 5*cosh(6*x))*sinh(6*x) - 10)
```

Sympy [F]

$$\int \coth^5(6x)\mathrm{csch}(6x)\,dx = \int \coth^5(6x)\,\mathrm{csch}\,(6x)\,dx$$

```
[In] integrate(coth(6*x)**5*csch(6*x),x)
```

```
[Out] Integral(coth(6*x)**5*csch(6*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(23) = 46.

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 6.59

$$\begin{aligned}\int \coth^5(6x)\mathrm{csch}(6x)\,dx =& \frac{e^{(-6x)}}{3\,(5\,e^{(-12x)}-10\,e^{(-24x)}+10\,e^{(-36x)}-5\,e^{(-48x)}+e^{(-60x)}-1)}\\ &-\frac{4\,e^{(-18x)}}{9\,(5\,e^{(-12x)}-10\,e^{(-24x)}+10\,e^{(-36x)}-5\,e^{(-48x)}+e^{(-60x)}-1)}\\ &+\frac{58\,e^{(-30x)}}{45\,(5\,e^{(-12x)}-10\,e^{(-24x)}+10\,e^{(-36x)}-5\,e^{(-48x)}+e^{(-60x)}-1)}\\ &-\frac{4\,e^{(-42x)}}{9\,(5\,e^{(-12x)}-10\,e^{(-24x)}+10\,e^{(-36x)}-5\,e^{(-48x)}+e^{(-60x)}-1)}\\ &+\frac{e^{(-54x)}}{3\,(5\,e^{(-12x)}-10\,e^{(-24x)}+10\,e^{(-36x)}-5\,e^{(-48x)}+e^{(-60x)}-1)}\end{aligned}$$

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="maxima")
```

```
[Out] 1/3*e^(-6*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(
-60*x) - 1) - 4/9*e^(-18*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*
e^(-48*x) + e^(-60*x) - 1) + 58/45*e^(-30*x)/(5*e^(-12*x) - 10*e^(-24*x) +
10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) - 4/9*e^(-42*x)/(5*e^(-12*x) -
10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) + 1/3*e^(-54*x)/
(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{15\left(e^{(6x)} - e^{(-6x)}\right)^4 + 40\left(e^{(6x)} - e^{(-6x)}\right)^2 + 48}{45\left(e^{(6x)} - e^{(-6x)}\right)^5}$$

```
[In] integrate(coth(6*x)^5*csch(6*x),x, algorithm="giac")
```

```
[Out] -1/45*(15*(e^(6*x) - e^(-6*x))^4 + 40*(e^(6*x) - e^(-6*x))^2 + 48)/(e^(6*x)
 - e^(-6*x))^5
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{e^{6x}\left(58\,e^{24x} - 20\,e^{12x} - 20\,e^{36x} + 15\,e^{48x} + 15\right)}{45\left(e^{12x} - 1\right)^5}$$

```
[In] int(coth(6*x)^5/sinh(6*x),x)
```

```
[Out] -(exp(6*x)*(58*exp(24*x) - 20*exp(12*x) - 20*exp(36*x) + 15*exp(48*x) + 15)
)/(45*(exp(12*x) - 1)^5)
```

3.130 $\int \coth^7(x)\mathbf{csch}^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \coth^7(x)\text{csch}^3(x)\,dx = -\frac{1}{3}\text{csch}^3(x) - \frac{3\text{csch}^5(x)}{5} - \frac{3\text{csch}^7(x)}{7} - \frac{\text{csch}^9(x)}{9}$$

```
[Out] -1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 276}

$$\int \coth^7(x)\text{csch}^3(x)\,dx = -\frac{1}{9}\text{csch}^9(x) - \frac{3\text{csch}^7(x)}{7} - \frac{3\text{csch}^5(x)}{5} - \frac{\text{csch}^3(x)}{3}$$

```
[In] Int[Coth[x]^7*Csch[x]^3,x]
```

```
[Out] -1/3*Csch[x]^3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\text{Subst}\left(\int x^2(-1+x^2)^3\,dx, x, -i\text{csch}(x)\right)\right)\\ &= -\left(i\text{Subst}\left(\int (-x^2+3x^4-3x^6+x^8)\,dx, x, -i\text{csch}(x)\right)\right)\\ &= -\frac{1}{3}\text{csch}^3(x) - \frac{3\text{csch}^5(x)}{5} - \frac{3\text{csch}^7(x)}{7} - \frac{\text{csch}^9(x)}{9}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \coth^7(x)\text{csch}^3(x)\,dx = -\frac{1}{3}\text{csch}^3(x) - \frac{3\text{csch}^5(x)}{5} - \frac{3\text{csch}^7(x)}{7} - \frac{\text{csch}^9(x)}{9}$$

```
[In] Integrate[Coth[x]^7*Csch[x]^3,x]
[Out] -1/3*Csch[x]^3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{\text{csch}(x)^3}{3}-\frac{3\,\text{csch}(x)^5}{5}-\frac{3\,\text{csch}(x)^7}{7}-\frac{\text{csch}(x)^9}{9}$	26
default	$-\frac{\text{csch}(x)^3}{3}-\frac{3\,\text{csch}(x)^5}{5}-\frac{3\,\text{csch}(x)^7}{7}-\frac{\text{csch}(x)^9}{9}$	26
risch	$-\frac{8\,e^{3x}(105\,e^{12x}+126\,e^{10x}+711\,e^{8x}+356\,e^{6x}+711\,e^{4x}+126\,e^{2x}+105)}{315(e^{2x}-1)^9}$	53

```
[In] int(coth(x)^7*csch(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 13.39

$$\int \coth^7(x)\text{csch}^3(x)\,dx =$$

$$-\frac{}{315\left(\cosh(x)^{11} + 11\cosh(x)\sinh(x)^{10} + \sinh(x)^{11} + \left(55\cosh(x)^2 - 9\right)\sinh(x)^9 - 9\cosh(x)^9 + 3\left(5\right.}$$

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(105*cosh(x)^8 + 840*cosh(x)*sinh(x)^7 + 105*sinh(x)^8 + 42*(70*cosh
(x)^2 + 3)*sinh(x)^6 + 126*cosh(x)^6 + 84*(70*cosh(x)^3 + 9*cosh(x))*sinh(x
)^5 + 6*(1225*cosh(x)^4 + 315*cosh(x)^2 + 136)*sinh(x)^4 + 816*cosh(x)^4 +
24*(245*cosh(x)^5 + 105*cosh(x)^3 + 101*cosh(x))*sinh(x)^3 + 2*(1470*cosh(x
)^6 + 945*cosh(x)^4 + 2448*cosh(x)^2 + 241)*sinh(x)^2 + 482*cosh(x)^2 + 4*(
210*cosh(x)^7 + 189*cosh(x)^5 + 606*cosh(x)^3 + 115*cosh(x))*sinh(x) + 711)
/(cosh(x)^11 + 11*cosh(x)*sinh(x)^10 + sinh(x)^11 + (55*cosh(x)^2 - 9)*sinh
(x)^9 - 9*cosh(x)^9 + 3*(55*cosh(x)^3 - 27*cosh(x))*sinh(x)^8 + (330*cosh(x
)^4 - 324*cosh(x)^2 + 37)*sinh(x)^7 + 35*cosh(x)^7 + 7*(66*cosh(x)^5 - 108*
cosh(x)^3 + 35*cosh(x))*sinh(x)^6 + 3*(154*cosh(x)^6 - 378*cosh(x)^4 + 259*
cosh(x)^2 - 31)*sinh(x)^5 - 75*cosh(x)^5 + (330*cosh(x)^7 - 1134*cosh(x)^5
+ 1225*cosh(x)^3 - 375*cosh(x))*sinh(x)^4 + (165*cosh(x)^8 - 756*cosh(x)^6
+ 1295*cosh(x)^4 - 930*cosh(x)^2 + 162)*sinh(x)^3 + 90*cosh(x)^3 + (55*cosh
(x)^9 - 324*cosh(x)^7 + 735*cosh(x)^5 - 750*cosh(x)^3 + 270*cosh(x))*sinh(x
)^2 + (11*cosh(x)^10 - 81*cosh(x)^8 + 259*cosh(x)^6 - 465*cosh(x)^4 + 486*c
osh(x)^2 - 210)*sinh(x) - 42*cosh(x))
```

Sympy [F]

$$\int \coth^7(x)\text{csch}^3(x)\,dx = \int \coth^7(x)\,\text{csch}^3(x)\,dx$$

```
[In] integrate(coth(x)**7*csch(x)**3,x)
```

```
[Out] Integral(coth(x)**7*csch(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 435, normalized size of antiderivative = 13.18

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx$$

$$= \frac{8\,e^{(-3\,x)}}{3\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18\,x)}}$$
$$+ \frac{16\,e^{(-5\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18}}$$
$$+ \frac{632\,e^{(-7\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1}}$$
$$+ \frac{2848\,e^{(-9\,x)}}{315\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-}}$$
$$+ \frac{632\,e^{(-11\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1}}$$
$$+ \frac{16\,e^{(-13\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18}}$$
$$+ \frac{8\,e^{(-15\,x)}}{3\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18}}$$

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="maxima")
```

```
[Out] 8/3*e^(-3*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16/
5*e^(-5*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 632/3
5*e^(-7*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 2848/
315*e^(-9*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 632
/35*e^(-11*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 16
/5*e^(-13*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e
^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 8/3
*e^(-15*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(
-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \coth^7(x)\text{csch}^3(x)\,dx = \frac{8\left(105\left(e^{(-x)} - e^x\right)^6 + 756\left(e^{(-x)} - e^x\right)^4 + 2160\left(e^{(-x)} - e^x\right)^2 + 2240\right)}{315\left(e^{(-x)} - e^x\right)^9}$$

```
[In] integrate(coth(x)^7*csch(x)^3,x, algorithm="giac")
[Out] 8/315*(105*(e^(-x) - e^x)^6 + 756*(e^(-x) - e^x)^4 + 2160*(e^(-x) - e^x)^2
+ 2240)/(e^(-x) - e^x)^9
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 372, normalized size of antiderivative = 11.27

$$\begin{aligned}
\int \coth^7(x)\text{csch}^3(x)\,dx &= -\frac{5872\,e^x}{105\,\left(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1\right)} \\
&\quad -\frac{\frac{28\,e^{3\,x}}{9}+\frac{28\,e^{5\,x}}{3}+\frac{140\,e^{7\,x}}{9}+\frac{140\,e^{9\,x}}{9}+\frac{28\,e^{11\,x}}{3}+\frac{28\,e^{13\,x}}{9}+\frac{4\,e^{15\,x}}{9}+\frac{4\,e^{x}}{9}}{9\,e^{2\,x}-36\,e^{4\,x}+84\,e^{6\,x}-126\,e^{8\,x}+126\,e^{10\,x}-84\,e^{12\,x}+36\,e^{14\,x}-9\,e^{16\,x}+e^{18\,x}-1} \\
&\quad -\frac{3008\,e^x}{21\,\left(15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1\right)} \\
&\quad -\frac{704\,e^x}{45\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} \\
&\quad -\frac{256\,e^x}{9\,\left(28\,e^{4\,x}-8\,e^{2\,x}-56\,e^{6\,x}+70\,e^{8\,x}-56\,e^{10\,x}+28\,e^{12\,x}-8\,e^{14\,x}+e^{16\,x}+1\right)} \\
&\quad -\frac{36608\,e^x}{315\,\left(5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1\right)}-\frac{20\,e^x}{9\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)} \\
&\quad -\frac{2048\,e^x}{21\,\left(7\,e^{2\,x}-21\,e^{4\,x}+35\,e^{6\,x}-35\,e^{8\,x}+21\,e^{10\,x}-7\,e^{12\,x}+e^{14\,x}-1\right)}
\end{aligned}$$

```
[In] int(coth(x)^7/sinh(x)^3,x)
[Out] - (5872*exp(x))/(105*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))
 - ((28*exp(3*x))/9 + (28*exp(5*x))/3 + (140*exp(7*x))/9 + (140*exp(9*x))/9
 + (28*exp(11*x))/3 + (28*exp(13*x))/9 + (4*exp(15*x))/9 + (4*exp(x))/9)/(9
*exp(2*x) - 36*exp(4*x) + 84*exp(6*x) - 126*exp(8*x) + 126*exp(10*x) - 84*e
xp(12*x) + 36*exp(14*x) - 9*exp(16*x) + exp(18*x) - 1) - (3008*exp(x))/(21*
```

```
(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(1
2*x) + 1)) - (704*exp(x))/(45*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (
256*exp(x))/(9*(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x) - 56*e
xp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1)) - (36608*exp(x))/(3
15*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1)) -
 (20*exp(x))/(9*(exp(4*x) - 2*exp(2*x) + 1)) - (2048*exp(x))/(21*(7*exp(2*x
) - 21*exp(4*x) + 35*exp(6*x) - 35*exp(8*x) + 21*exp(10*x) - 7*exp(12*x) +
exp(14*x) - 1))
```

3.131 $\int \sinh(a+bx)\sinh(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sinh(a+bx)\sinh(c+bx)\,dx=-\frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}$$

```
[Out] -1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5732, 2717}

$$\int \sinh(a+bx)\sinh(c+bx)\,dx=\frac{\sinh(a+2bx+c)}{4b}-\frac{1}{2}x\cosh(a-c)$$

```
[In] Int[Sinh[a + b*x]*Sinh[c + b*x],x]
[Out] -1/2*(x*Cosh[a - c]) + Sinh[a + c + 2*b*x]/(4*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}\cosh(a-c)+\frac{1}{2}\cosh(a+c+2bx)\right)\,dx\\
&= -\frac{1}{2}x\cosh(a-c)+\frac{1}{2}\int \cosh(a+c+2bx)\,dx\\
&= -\frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$
\int \sinh(a+bx)\sinh(c+bx)\,dx = \frac{-2bx\cosh(a-c)+\sinh(a+c+2bx)}{4b}
$$

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + b*x],x]
[Out] (-2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$-\frac{x\cosh(a-c)}{2}+\frac{\sinh(2bx+a+c)}{4b}$
risch	$-\frac{x\,e^{a-c}}{4}-\frac{x\,e^{-a+c}}{4}+\frac{e^{2bx+a+c}}{8b}-\frac{e^{-2bx-a-c}}{8b}$
parallelrisch	$\frac{-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}+2\left(2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

```
[In] int(sinh(b*x+a)*sinh(b*x+c),x,method=_RETURNVERBOSE)
[Out] -1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = -\frac{2\,bx\cosh\left(-a+c\right) - 2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right) + \cosh\left(bx+c\right)^2\sinh\left(-a+c\right) + \sinh\left(\right.}{4\left(b\cosh\left(-a+c\right)^2 - b\sinh\left(-a+c\right)^2\right)}$$

[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="fricas")

[Out] -1/4*(2*b*x*cosh(-a + c) - 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) + cos
h(b*x + c)^2*sinh(-a + c) + sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^2
 - b*sinh(-a + c)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = \begin{cases} \frac{x\sinh\left(a+bx\right)\sinh\left(bx+c\right)}{2} - \frac{x\cosh\left(a+bx\right)\cosh\left(bx+c\right)}{2} + \frac{\sinh\left(a+bx\right)\cosh\left(bx+c\right)}{2b} & \text{for } b \neq 0 \\ x\sinh\left(a\right)\sinh\left(c\right) & \text{otherwise} \end{cases}$$

[In] integrate(sinh(b*x+a)*sinh(b*x+c),x)

[Out] Piecewise((x*sinh(a + b*x)*sinh(b*x + c)/2 - x*cosh(a + b*x)*cosh(b*x + c)/
2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*sinh(a)*sinh(c), True)
)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(23) = 46$.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = -\frac{(bx+a)\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}}{4\,b} + \frac{e^{(2\,bx+a+c)}}{8\,b} - \frac{e^{(-2\,bx-a-c)}}{8\,b}$$

[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="maxima")

[Out] -1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
 1/8*e^(-2*b*x - a - c)/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \sinh(a+bx)\sinh(c+bx)\,dx$$
$$= -\frac{2\,bx\left(e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}-\left(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}-1\right)e^{(-2\,bx-a-c)}-e^{(2\,bx+a+c)}}{8\,b}$$

```
[In] integrate(sinh(b*x+a)*sinh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x +
2*c) - 1)*e^(-2*b*x - a - c) - e^(2*b*x + a + c))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sinh(a+bx)\sinh(c+bx)\,dx = \frac{\sinh(a+c+2\,b\,x)}{4\,b}-\frac{x\cosh(a-c)}{2}$$

```
[In] int(sinh(a + b*x)*sinh(c + b*x),x)
```

```
[Out] sinh(a + c + 2*b*x)/(4*b) - (x*cosh(a - c))/2
```

3.132 $\int \sinh(c-bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{1}{2}x\cosh(a+c) - \frac{\sinh(a-c+2bx)}{4b}$$

```
[Out] 1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5732, 2717}

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{1}{2}x\cosh(a+c) - \frac{\sinh(a+2bx-c)}{4b}$$

```
[In] Int[Sinh[c - b*x]*Sinh[a + b*x],x]
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{2}\cosh(a+c) - \frac{1}{2}\cosh(a-c+2bx)\right)\,dx \\ &= \frac{1}{2}x\cosh(a+c) - \frac{1}{2}\int \cosh(a-c+2bx)\,dx \\ &= \frac{1}{2}x\cosh(a+c) - \frac{\sinh(a-c+2bx)}{4b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{1}{2}x\cosh(a+c) - \frac{\sinh(a-c+2bx)}{4b}$$

```
[In] Integrate[Sinh[c - b*x]*Sinh[a + b*x],x]
[Out] (x*Cosh[a + c])/2 - Sinh[a - c + 2*b*x]/(4*b)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cosh(a+c)}{2}-\frac{\sinh(2bx+a-c)}{4b}$
risch	$\frac{x\,\mathrm{e}^{-a-c}}{4}+\frac{x\,\mathrm{e}^{a+c}}{4}-\frac{\mathrm{e}^{2bx+a-c}}{8b}+\frac{\mathrm{e}^{-2bx-a+c}}{8b}$
parallelrisch	$-\frac{-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}+2\left(2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)-bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right.}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

```
[In] int(-sinh(b*x-c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x*cosh(a+c)-1/4*sinh(2*b*x+a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(23) = 46$.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{2\,bx\cosh\left(a+c\right) - 2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right) + \cosh\left(bx+a\right)^{2}\sinh\left(a+c\right) + \sinh\left(bx+a\right)}{4\,\left(b\cosh\left(a+c\right)^{2} - b\sinh\left(a+c\right)^{2}\right)}$$

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(a + c) - 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) + cosh(b
*x + a)^2*sinh(a + c) + sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b*s
inh(a + c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = -\begin{cases} \frac{x\sinh\left(a+bx\right)\sinh\left(bx-c\right)}{2} - \frac{x\cosh\left(a+bx\right)\cosh\left(bx-c\right)}{2} + \frac{\sinh\left(a+bx\right)\cosh\left(bx-c\right)}{2b} & \text{for } b \neq 0 \\ -x\sinh\left(a\right)\sinh\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x)
```

```
[Out] -Piecewise((x*sinh(a + b*x)*sinh(b*x - c)/2 - x*cosh(a + b*x)*cosh(b*x - c)
/2 + sinh(a + b*x)*cosh(b*x - c)/(2*b), Ne(b, 0)), (-x*sinh(a)*sinh(c), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{(bx+a)\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}}{4\,b} - \frac{e^{(2\,bx+a-c)}}{8\,b} + \frac{e^{(-2\,bx-a+c)}}{8\,b}$$

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b - 1/8*e^(2*b*x + a - c)/b +
1/8*e^(-2*b*x - a + c)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{2\,bx\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)} - \left(e^{(2\,bx)}+e^{(2\,bx+2\,a+2\,c)}-e^{(2\,c)}\right)e^{(-2\,bx-a-c)} - e^{(2\,bx+a-c)}}{8\,b}$$

```
[In] integrate(-sinh(b*x-c)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a + 2
*c) - e^(2*c))*e^(-2*b*x - a - c) - e^(2*b*x + a - c))/b
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sinh(c-bx)\sinh(a+bx)\,dx = \frac{x\cosh(a+c)}{2} - \frac{\sinh(a-c+2\,b\,x)}{4\,b}$$

```
[In] int(sinh(a + b*x)*sinh(c - b*x),x)
[Out] (x*cosh(a + c))/2 - sinh(a - c + 2*b*x)/(4*b)
```

3.133 $\int \cosh(a+bx)\cosh(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}$$

```
[Out] 1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5733, 2717}

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{\sinh(a+2bx+c)}{4b}+\frac{1}{2}x\cosh(a-c)$$

```
[In] Int[Cosh[a + b*x]*Cosh[c + b*x],x]
```

```
[Out] (x*Cosh[a - c])/2 + Sinh[a + c + 2*b*x]/(4*b)
```

<u>Rule 2717</u>

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

<u>Rule 5733</u>

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}\cosh(a-c)+\frac{1}{2}\cosh(a+c+2bx)\right)\,dx \\
&= \frac{1}{2}x\cosh(a-c)+\frac{1}{2}\int \cosh(a+c+2bx)\,dx \\
&= \frac{1}{2}x\cosh(a-c)+\frac{\sinh(a+c+2bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$
\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{2bx\cosh(a-c)+\sinh(a+c+2bx)}{4b}
$$

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + b*x],x]
[Out] (2*b*x*Cosh[a - c] + Sinh[a + c + 2*b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cosh(a-c)}{2}+\frac{\sinh(2bx+a+c)}{4b}$
risch	$\frac{x\,\mathrm{e}^{a-c}}{4}+\frac{x\,\mathrm{e}^{-a+c}}{4}+\frac{\mathrm{e}^{2bx+a+c}}{8b}-\frac{\mathrm{e}^{-2bx-a-c}}{8b}$
parallelrisch	$\frac{bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}+2\left(-2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)+bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

```
[In] int(cosh(b*x+a)*cosh(b*x+c),x,method=_RETURNVERBOSE)
[Out] 1/2*x*cosh(a-c)+1/4*sinh(2*b*x+a+c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.30

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{2\,bx\cosh\left(-a+c\right) + 2\,\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(bx+c\right) - \cosh\left(bx+c\right)^{2}\sinh\left(-a+c\right) - \sinh\left(bx\right.}{4\left(b\cosh\left(-a+c\right)^{2} - b\sinh\left(-a+c\right)^{2}\right)}$$

[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="fricas")

[Out] 1/4*(2*b*x*cosh(-a + c) + 2*cosh(b*x + c)*cosh(-a + c)*sinh(b*x + c) - cosh
(b*x + c)^2*sinh(-a + c) - sinh(b*x + c)^2*sinh(-a + c))/(b*cosh(-a + c)^2
- b*sinh(-a + c)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(20) = 40.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \begin{cases} -\frac{x\sinh\left(a+bx\right)\sinh\left(bx+c\right)}{2} + \frac{x\cosh\left(a+bx\right)\cosh\left(bx+c\right)}{2} + \frac{\sinh\left(a+bx\right)\cosh\left(bx+c\right)}{2b} & \text{for } b \neq 0 \\ x\cosh\left(a\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

[In] integrate(cosh(b*x+a)*cosh(b*x+c),x)

[Out] Piecewise((-x*sinh(a + b*x)*sinh(b*x + c)/2 + x*cosh(a + b*x)*cosh(b*x + c)
/2 + sinh(a + b*x)*cosh(b*x + c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), True
))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(23) = 46.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{(bx+a)\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}}{4\,b} + \frac{e^{(2\,bx+a+c)}}{8\,b} - \frac{e^{(-2\,bx-a-c)}}{8\,b}$$

[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="maxima")

[Out] 1/4*(b*x + a)*(e^(2*a) + e^(2*c))*e^(-a - c)/b + 1/8*e^(2*b*x + a + c)/b -
1/8*e^(-2*b*x - a - c)/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \cosh(a+bx)\cosh(c+bx)\,dx$$
$$= \frac{2\,bx\big(e^{(2\,a)}+e^{(2\,c)}\big)e^{(-a-c)}-\big(e^{(2\,bx+2\,a)}+e^{(2\,bx+2\,c)}+1\big)e^{(-2\,bx-a-c)}+e^{(2\,bx+a+c)}}{8\,b}$$

```
[In] integrate(cosh(b*x+a)*cosh(b*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x*(e^(2*a) + e^(2*c))*e^(-a - c) - (e^(2*b*x + 2*a) + e^(2*b*x + 2
*c) + 1)*e^(-2*b*x - a - c) + e^(2*b*x + a + c))/b
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cosh(a+bx)\cosh(c+bx)\,dx = \frac{x\cosh(a-c)}{2}+\frac{\sinh(a+c+2\,b\,x)}{4\,b}$$

```
[In] int(cosh(a + b*x)*cosh(c + b*x),x)
```

```
[Out] (x*cosh(a - c))/2 + sinh(a + c + 2*b*x)/(4*b)
```

3.134 $\int \cosh(c-bx)\cosh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \cosh(c-bx)\cosh(a+bx)\,dx=\frac{1}{2}x\cosh(a+c)+\frac{\sinh(a-c+2bx)}{4b}$$

```
[Out] 1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5733, 2717}

$$\int \cosh(c-bx)\cosh(a+bx)\,dx=\frac{\sinh(a+2bx-c)}{4b}+\frac{1}{2}x\cosh(a+c)$$

```
[In] Int[Cosh[c - b*x]*Cosh[a + b*x],x]
```

```
[Out] (x*Cosh[a + c])/2 + Sinh[a - c + 2*b*x]/(4*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}\cosh(a+c)+\frac{1}{2}\cosh(a-c+2bx)\right)\,dx\\ &= \frac{1}{2}x\cosh(a+c)+\frac{1}{2}\int\cosh(a-c+2bx)\,dx\\ &= \frac{1}{2}x\cosh(a+c)+\frac{\sinh(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int\cosh(c-bx)\cosh(a+bx)\,dx=\frac{2bx\cosh(a+c)+\sinh(a-c+2bx)}{4b}$$

```
[In] Integrate[Cosh[c - b*x]*Cosh[a + b*x],x]
[Out] (2*b*x*Cosh[a + c] + Sinh[a - c + 2*b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cosh(a+c)}{2}+\frac{\sinh(2bx+a-c)}{4b}$
risch	$\frac{x\,e^{a+c}}{4}+\frac{x\,e^{-a-c}}{4}+\frac{e^{2bx+a-c}}{8b}-\frac{e^{-2bx-a+c}}{8b}$
parallelrisch	$\frac{bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}+2\left(-2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)xb+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)+bx\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{2b\left(1+\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\tanh\left(\frac{bx}{2}-\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$

```
[In] int(cosh(b*x-c)*cosh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x*cosh(a+c)+1/4*sinh(2*b*x+a-c)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \cosh(c-bx)\cosh(a+bx)\,dx$$
$$= \frac{2\,bx\cosh\left(a+c\right)+2\,\cosh\left(bx+a\right)\cosh\left(a+c\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)^{2}\sinh\left(a+c\right)-\sinh\left(bx+a\right)}{4\left(b\cosh\left(a+c\right)^{2}-b\sinh\left(a+c\right)^{2}\right)}$$

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x*cosh(a + c) + 2*cosh(b*x + a)*cosh(a + c)*sinh(b*x + a) - cosh(b
*x + a)^2*sinh(a + c) - sinh(b*x + a)^2*sinh(a + c))/(b*cosh(a + c)^2 - b*s
inh(a + c)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cosh(c-bx)\cosh(a+bx)\,dx$$
$$= \begin{cases} -\frac{x\sinh\left(a+bx\right)\sinh\left(bx-c\right)}{2}+\frac{x\cosh\left(a+bx\right)\cosh\left(bx-c\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(bx-c\right)}{2b} & \text{for}\ b\neq 0 \\ x\cosh\left(a\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)*sinh(b*x - c)/2 + x*cosh(a + b*x)*cosh(b*x - c)
/2 + sinh(a + b*x)*cosh(b*x - c)/(2*b), Ne(b, 0)), (x*cosh(a)*cosh(c), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \cosh(c-bx)\cosh(a+bx)\,dx = \frac{(bx+a)\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}}{4\,b}+\frac{e^{(2\,bx+a-c)}}{8\,b}-\frac{e^{(-2\,bx-a+c)}}{8\,b}$$

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x + a)*(e^(2*a + 2*c) + 1)*e^(-a - c)/b + 1/8*e^(2*b*x + a - c)/b -
1/8*e^(-2*b*x - a + c)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(23) = 46.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \cosh(c-bx)\cosh(a+bx)\,dx = \frac{2\,bx\left(e^{(2\,a+2\,c)}+1\right)e^{(-a-c)}-\left(e^{(2\,bx)}+e^{(2\,bx+2\,a+2\,c)}+e^{(2\,c)}\right)e^{(-2\,bx-a-c)}+e^{(2\,bx+a-c)}}{8\,b}$$

```
[In] integrate(cosh(b*x-c)*cosh(b*x+a),x, algorithm="giac")
[Out] 1/8*(2*b*x*(e^(2*a + 2*c) + 1)*e^(-a - c) - (e^(2*b*x) + e^(2*b*x + 2*a + 2
*c) + e^(2*c))*e^(-2*b*x - a - c) + e^(2*b*x + a - c))/b
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cosh(c-bx)\cosh(a+bx)\,dx = \frac{\sinh(a-c+2\,b\,x)}{4\,b}+\frac{x\cosh(a+c)}{2}$$

```
[In] int(cosh(a + b*x)*cosh(c - b*x),x)
[Out] sinh(a - c + 2*b*x)/(4*b) + (x*cosh(a + c))/2
```

3.135 $\int \tanh(a+bx)\tanh(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(c+bx))}{b}$$

```
[Out] x-coth(a-c)*ln(cosh(b*x+a))/b+coth(a-c)*ln(cosh(b*x+c))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5765, 5763, 3556}

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = -\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(bx+c))}{b}+x$$

```
[In] Int[Tanh[a + b*x]*Tanh[c + b*x],x]
[Out] x - (Coth[a - c]*Log[Cosh[a + b*x]])/b + (Coth[a - c]*Log[Cosh[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5763

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 5765

```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] - Dist[(b/d)*Cosh[(b*c - a*d)/d], Int[Sech[a + b*x]*Sech[c + d*x],
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x-\cosh(a-c)\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx\\ &= x-\coth(a-c)\int \tanh(a+bx)\,dx+\coth(a-c)\int \tanh(c+bx)\,dx\\ &= x-\frac{\coth(a-c)\log(\cosh(a+bx))}{b}+\frac{\coth(a-c)\log(\cosh(c+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x+\frac{\coth(a-c)(-\log(\cosh(a+bx))+\log(\cosh(c+bx)))}{b}$$

```
[In] Integrate[Tanh[a + b*x]*Tanh[c + b*x],x]
[Out] x + (Coth[a - c]*(-Log[Cosh[a + b*x]] + Log[Cosh[c + b*x]]))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 4.08

method	result	size
risch	$x-\frac{\ln(1+e^{2bx+2a})e^{2a}}{b(e^{2a}-e^{2c})}-\frac{\ln(1+e^{2bx+2a})e^{2c}}{b(e^{2a}-e^{2c})}+\frac{\ln(e^{2bx+2a}+e^{2a-2c})e^{2a}}{b(e^{2a}-e^{2c})}+\frac{\ln(e^{2bx+2a}+e^{2a-2c})e^{2c}}{b(e^{2a}-e^{2c})}$	151

```
[In] int(tanh(b*x+a)*tanh(b*x+c),x,method=_RETURNVERBOSE)
[Out] x-1/b/(exp(2*a)-exp(2*c))*ln(1+exp(2*b*x+2*a))*exp(2*a)-1/b/(exp(2*a)-exp(2
*c))*ln(1+exp(2*b*x+2*a))*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a
```

```
)+exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)+exp(2*a-
2*c))*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 7.00

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = \frac{bx\cosh(-a+c)^2 - 2\,bx\cosh(-a+c)\sinh(-a+c) + bx\sinh(-a+c)^2 - bx - \left(\cosh(-a+c)^2 - 2\cosh\right.}{}$$

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
- b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)
*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b*x + c)*sinh(-a + c))/(cosh(b*x
+ c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(b*x
+ c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(
-a + c)^2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cos
h(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F]

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = \int \tanh(a+bx)\tanh(bx+c)\,dx$$

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] Integral(tanh(a + b*x)*tanh(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.24

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x + \frac{a}{b} - \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)\log\left(e^{(-2\,bx-2\,a)} + 1\right)}{b\left(e^{(2\,a)} - e^{(2\,c)}\right)} + \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)\log\left(e^{(-2\,bx)} + e^{(2\,c)}\right)}{b\left(e^{(2\,a)} - e^{(2\,c)}\right)}$$

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] x + a/b - (e^(2*a) + e^(2*c))*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*
c))) + (e^(2*a) + e^(2*c))*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c))
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = \frac{bx - \frac{\left(e^{(4\,a)}+e^{(2\,a+2\,c)}\right)\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(4\,a)}-e^{(2\,a+2\,c)}} + \frac{\left(e^{(2\,a+2\,c)}+e^{(4\,c)}\right)\log\left(e^{(2\,bx+2\,c)}+1\right)}{e^{(2\,a+2\,c)}-e^{(4\,c)}}}{b}$$

```
[In] integrate(tanh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] (b*x - (e^(4*a) + e^(2*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(4*a) - e^(2*a
 + 2*c)) + (e^(2*a + 2*c) + e^(4*c))*log(e^(2*b*x + 2*c) + 1)/(e^(2*a + 2*c
) - e^(4*c)))/b
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.11

$$\int \tanh(a+bx)\tanh(c+bx)\,dx = x - \frac{\ln\left(4\,e^{4\,a}+4\,e^{6\,a}\,e^{2\,b\,x}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth(a-c)}{b} + \frac{\ln\left(4\,e^{4\,a}+4\,e^{2\,a}\,e^{2\,c}+4\,e^{2\,a}\,e^{4\,c}\,e^{2\,b\,x}+4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth(a-c)}{b}$$

```
[In] int(tanh(a + b*x)*tanh(c + b*x),x)
```

```
[Out] x - (log(4*exp(4*a) + 4*exp(6*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) + 4*exp(4
*a)*exp(2*c)*exp(2*b*x))*coth(a - c))/b + (log(4*exp(4*a) + 4*exp(2*a)*exp(
2*c) + 4*exp(2*a)*exp(4*c)*exp(2*b*x) + 4*exp(4*a)*exp(2*c)*exp(2*b*x))*cot
h(a - c))/b
```

3.136 $\int \tanh(c-bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x-\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}$$

```
[Out] -x-coth(a+c)*ln(cosh(b*x-c))/b+coth(a+c)*ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5765, 5763, 3556}

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -\frac{\coth(a+c)\log(\cosh(c-bx))}{b}+\frac{\coth(a+c)\log(\cosh(a+bx))}{b}-x$$

```
[In] Int[Tanh[c - b*x]*Tanh[a + b*x],x]
```

```
[Out] -x - (Coth[a + c]*Log[Cosh[c - b*x]])/b + (Coth[a + c]*Log[Cosh[a + b*x]])/
b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5763

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rule 5765

```
Int[Tanh[(a_.) + (b_.)*(x_)]*Tanh[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] - Dist[(b/d)*Cosh[(b*c - a*d)/d], Int[Sech[a + b*x]*Sech[c + d*x],
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -x + \cosh(a+c)\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx \\
&= -x + \coth(a+c)\int \tanh(c-bx)\,dx + \coth(a+c)\int \tanh(a+bx)\,dx \\
&= -x - \frac{\coth(a+c)\log(\cosh(c-bx))}{b} + \frac{\coth(a+c)\log(\cosh(a+bx))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x + \frac{\coth(a+c)(-\log(\cosh(c-bx)) + \log(\cosh(a+bx)))}{b}$$

```
[In] Integrate[Tanh[c - b*x]*Tanh[a + b*x],x]
[Out] -x + (Coth[a + c]*(-Log[Cosh[c - b*x]] + Log[Cosh[a + b*x]]))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.14

method	result	size
risch	$-x-\frac{\ln\left(\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{2a+2c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}-\frac{\ln\left(\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)}{b\left(\mathrm{e}^{2a+2c}-1\right)}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{2a+2c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b\left(\mathrm{e}^{2a+2c}-1\right)}$	149

```
[In] int(-tanh(b*x-c)*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -x-1/b/(exp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(e
xp(2*a+2*c)-1)*ln(exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(1+ex
p(2*b*x+2*a))*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(1+exp(2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(37) = 74.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 6.00

$$\int \tanh(c-bx)\tanh(a+bx)\,dx =$$

$$-\frac{bx\cosh\left(a+c\right)^2-2\,bx\cosh\left(a+c\right)\sinh\left(a+c\right)+bx\sinh\left(a+c\right)^2-bx-\left(\cosh\left(a+c\right)^2-2\,\cosh\left(a+c\right.\right.}{}$$

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 - b
*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(2*
(cosh(b*x + a)*cosh(a + c) - sinh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*cosh
(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sinh(a
+ c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*lo
g(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b*
cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F]

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -\int \tanh\left(a+bx\right)\tanh\left(bx-c\right)dx$$

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x)
```

```
[Out] -Integral(tanh(a + b*x)*tanh(b*x - c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.42

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -x-\frac{a}{b}+\frac{\left(e^{(2\,a+2\,c)}+1\right)\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b(e^{(2\,a+2\,c)}-1)} - \frac{\left(e^{(2\,a+2\,c)}+1\right)\log\left(e^{(-2\,bx+2\,c)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}$$

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] -x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c)
- 1)) - (e^(2*a + 2*c) + 1)*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = -\frac{bx + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(2bx)}+e^{(2c)}\right)}{e^{(2a+2c)}-1} + \frac{\left(e^{(2a)}+e^{(4a+2c)}\right)\log\left(e^{(2bx+2a)}+1\right)}{e^{(2a)}-e^{(4a+2c)}}}{b}$$

```
[In] integrate(-tanh(b*x-c)*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + (e^(2*a + 2*c) + 1)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) +
(e^(2*a) + e^(4*a + 2*c))*log(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)
))/b
```

Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \tanh(c-bx)\tanh(a+bx)\,dx = \frac{\coth(a+c)\ \ln\left(4\,e^{2a}\,e^{2c} + 4\,e^{4a}\,e^{4c} + 4\,e^{4a}\,e^{2c}\,e^{2bx} + 4\,e^{6a}\,e^{4c}\,e^{2bx}\right)}{b} - \frac{\coth(a+c)\ \ln\left(4\,e^{2a}\,e^{2bx} + 4\,e^{2a}\,e^{2c} + 4\,e^{4a}\,e^{4c} + 4\,e^{4a}\,e^{2c}\,e^{2bx}\right)}{b} - x$$

```
[In] int(tanh(a + b*x)*tanh(c - b*x),x)
```

```
[Out] (coth(a + c)*log(4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) + 4*exp(4*a)*exp
(2*c)*exp(2*b*x) + 4*exp(6*a)*exp(4*c)*exp(2*b*x)))/b - (coth(a + c)*log(4*
exp(2*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) + 4*exp(4*a
)*exp(2*c)*exp(2*b*x)))/b - x
```

3.137 $\int \coth(a+bx)\coth(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \coth(a+bx)\coth(c+bx)\,dx = x - \frac{\coth(a-c)\log(\sinh(a+bx))}{b} + \frac{\coth(a-c)\log(\sinh(c+bx))}{b}$$

```
[Out] x-coth(a-c)*ln(sinh(b*x+a))/b+coth(a-c)*ln(sinh(b*x+c))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5766, 5764, 3556}

$$\int \coth(a+bx)\coth(c+bx)\,dx = -\frac{\coth(a-c)\log(\sinh(a+bx))}{b} + \frac{\coth(a-c)\log(\sinh(bx+c))}{b} + x$$

```
[In] Int[Coth[a + b*x]*Coth[c + b*x],x]
[Out] x - (Coth[a - c]*Log[Sinh[a + b*x]])/b + (Coth[a - c]*Log[Sinh[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5764

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 5766

```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] + Dist[Cosh[(b*c - a*d)/d], Int[Csch[a + b*x]*Csch[c + d*x], x], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x+\cosh(a-c)\int \text{csch}(a+bx)\text{csch}(c+bx)\,dx\\ &= x-\coth(a-c)\int \coth(a+bx)\,dx+\coth(a-c)\int \coth(c+bx)\,dx\\ &= x-\frac{\coth(a-c)\log(\sinh(a+bx))}{b}+\frac{\coth(a-c)\log(\sinh(c+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \coth(a+bx)\coth(c+bx)\,dx = x+\frac{\coth(a-c)(-\log(\sinh(a+bx))+\log(\sinh(c+bx)))}{b}$$

```
[In] Integrate[Coth[a + b*x]*Coth[c + b*x],x]
[Out] x + (Coth[a - c]*(-Log[Sinh[a + b*x]] + Log[Sinh[c + b*x]]))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(37) = 74.

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.19

method	result	size
risch	$x-\frac{\ln\left(e^{2bx+2a}-1\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}-\frac{\ln\left(e^{2bx+2a}-1\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{2a}}{b\left(e^{2a}-e^{2c}\right)}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{2c}}{b\left(e^{2a}-e^{2c}\right)}$	155

```
[In] int(coth(b*x+a)*coth(b*x+c),x,method=_RETURNVERBOSE)
[Out] x-1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-1)*exp(2*a)-1/b/(exp(2*a)-exp(2
*c))*ln(exp(2*b*x+2*a)-1)*exp(2*c)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a
```

```
)-exp(2*a-2*c))*exp(2*a)+1/b/(exp(2*a)-exp(2*c))*ln(exp(2*b*x+2*a)-exp(2*a-
2*c))*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(37) = 74.

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 7.00

$$\int \coth(a+bx)\coth(c+bx)\,dx = \frac{bx\cosh\left(-a+c\right)^2 - 2\,bx\cosh\left(-a+c\right)\sinh\left(-a+c\right) + bx\sinh\left(-a+c\right)^2 - bx - \left(\cosh\left(-a+c\right)^2 - 2\cosh\right.}{}$$

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="fricas")
[Out] (b*x*cosh(-a + c)^2 - 2*b*x*cosh(-a + c)*sinh(-a + c) + b*x*sinh(-a + c)^2
- b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)
*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(b*x + c)*sinh(-a + c))/(cosh(b*x
+ c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a + c))*sinh(b*x + c) + cosh(b*x
+ c)*sinh(-a + c))) + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(
-a + c)^2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cos
h(-a + c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F]

$$\int \coth(a+bx)\coth(c+bx)\,dx = \int \coth\left(a+bx\right)\coth\left(bx+c\right)dx$$

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x)
[Out] Integral(coth(a + b*x)*coth(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(37) = 74.

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 4.24

$$\begin{aligned}\int \coth(a+bx)\coth(c+bx)\,dx = x + \frac{a}{b} &- \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)\log\left(e^{(-bx-a)}+1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)} \\ &- \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)\log\left(e^{(-bx-a)}-1\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)} \\ &+ \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)\log\left(e^{(-bx)}+e^{c}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)} \\ &+ \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)\log\left(e^{(-bx)}-e^{c}\right)}{b\left(e^{(2\,a)}-e^{(2\,c)}\right)}\end{aligned}$$

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

```
[Out] x + a/b - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c)))
 - (e^(2*a) + e^(2*c))*log(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + (e^(
2*a) + e^(2*c))*log(e^(-b*x) + e^c)/(b*(e^(2*a) - e^(2*c))) + (e^(2*a) + e^
(2*c))*log(e^(-b*x) - e^c)/(b*(e^(2*a) - e^(2*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.62

$$\int \coth(a+bx)\coth(c+bx)\,dx = \frac{bx - \frac{\left(e^{(4\,a)}+e^{(2\,a+2\,c)}\right)\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{e^{(4\,a)}-e^{(2\,a+2\,c)}} + \frac{\left(e^{(2\,a+2\,c)}+e^{(4\,c)}\right)\log\left(\left|e^{(2\,bx+2\,c)}-1\right|\right)}{e^{(2\,a+2\,c)}-e^{(4\,c)}}}{b}$$

```
[In] integrate(coth(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

```
[Out] (b*x - (e^(4*a) + e^(2*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(4*a) - e
^(2*a + 2*c)) + (e^(2*a + 2*c) + e^(4*c))*log(abs(e^(2*b*x + 2*c) - 1))/(e^
(2*a + 2*c) - e^(4*c)))/b
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.11

$$\int \coth(a+bx)\coth(c+bx)\,dx = x - \frac{\ln\left(4\,e^{4\,a} - 4\,e^{6\,a}\,e^{2\,b\,x} + 4\,e^{2\,a}\,e^{2\,c} - 4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth(a-c)}{b} + \frac{\ln\left(4\,e^{4\,a} + 4\,e^{2\,a}\,e^{2\,c} - 4\,e^{2\,a}\,e^{4\,c}\,e^{2\,b\,x} - 4\,e^{4\,a}\,e^{2\,c}\,e^{2\,b\,x}\right)\,\coth(a-c)}{b}$$

```
[In] int(coth(a + b*x)*coth(c + b*x),x)
```

```
[Out] x - (log(4*exp(4*a) - 4*exp(6*a)*exp(2*b*x) + 4*exp(2*a)*exp(2*c) - 4*exp(4
*a)*exp(2*c)*exp(2*b*x))*coth(a - c))/b + (log(4*exp(4*a) + 4*exp(2*a)*exp(
2*c) - 4*exp(2*a)*exp(4*c)*exp(2*b*x) - 4*exp(4*a)*exp(2*c)*exp(2*b*x))*cot
h(a - c))/b
```

3.138 $\int \coth(c-bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x-\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}$$

```
[Out] -x-coth(a+c)*ln(-sinh(b*x-c))/b+coth(a+c)*ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5766, 5764, 3556}

$$\int \coth(c-bx)\coth(a+bx)\,dx = -\frac{\coth(a+c)\log(\sinh(c-bx))}{b}+\frac{\coth(a+c)\log(\sinh(a+bx))}{b}-x$$

```
[In] Int[Coth[c - b*x]*Coth[a + b*x],x]
```

```
[Out] -x - (Coth[a + c]*Log[Sinh[c - b*x]])/b + (Coth[a + c]*Log[Sinh[a + b*x]])/
b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5764

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rule 5766

```
Int[Coth[(a_.) + (b_.)*(x_)]*Coth[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[b*(
x/d), x] + Dist[Cosh[(b*c - a*d)/d], Int[Csch[a + b*x]*Csch[c + d*x], x], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -x + \cosh(a+c)\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx \\
&= -x + \coth(a+c)\int \coth(c-bx)\,dx + \coth(a+c)\int \coth(a+bx)\,dx \\
&= -x - \frac{\coth(a+c)\log(\sinh(c-bx))}{b} + \frac{\coth(a+c)\log(\sinh(a+bx))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x + \frac{\coth(a+c)(-\log(\sinh(c-bx)) + \log(-\sinh(a+bx)))}{b}$$

```
[In] Integrate[Coth[c - b*x]*Coth[a + b*x],x]
[Out] -x + (Coth[a + c]*(-Log[Sinh[c - b*x]] + Log[-Sinh[a + b*x]]))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(39) = 78.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.25

method	result	size
risch	$-x-\frac{\ln\left(-\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{2a+2c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}-\frac{\ln\left(-\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)}{b\left(\mathrm{e}^{2a+2c}-1\right)}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{2a+2c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(\mathrm{e}^{2a+2c}-1\right)}$	153

```
[In] int(-coth(b*x-c)*coth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -x-1/b/(exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(2*a+2*c)-1/b/(
exp(2*a+2*c)-1)*ln(-exp(2*a+2*c)+exp(2*b*x+2*a))+1/b/(exp(2*a+2*c)-1)*ln(ex
p(2*b*x+2*a)-1)*exp(2*a+2*c)+1/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(39) = 78$.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 6.00

$$\int \coth(c-bx)\coth(a+bx)\,dx =$$

$$-\frac{bx\cosh\left(a+c\right)^2 - 2\,bx\cosh\left(a+c\right)\sinh\left(a+c\right) + bx\sinh\left(a+c\right)^2 - bx - \left(\cosh\left(a+c\right)^2 - 2\,\cosh\left(a+c\right.\right.}{}$$

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(a + c)^2 - 2*b*x*cosh(a + c)*sinh(a + c) + b*x*sinh(a + c)^2 - b
*x - (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*log(2*
(cosh(a + c)*sinh(b*x + a) - cosh(b*x + a)*sinh(a + c))/(cosh(b*x + a)*cosh
(a + c) - (cosh(a + c) + sinh(a + c))*sinh(b*x + a) + cosh(b*x + a)*sinh(a
+ c))) + (cosh(a + c)^2 - 2*cosh(a + c)*sinh(a + c) + sinh(a + c)^2 + 1)*lo
g(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b*
cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F]

$$\int \coth(c-bx)\coth(a+bx)\,dx = -\int \coth\left(a+bx\right)\coth\left(bx-c\right)dx$$

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x)
```

```
[Out] -Integral(coth(a + b*x)*coth(b*x - c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(39) = 78$.

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int \coth(c-bx)\coth(a+bx)\,dx = -x - \frac{a}{b} + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}+1\right)}{b(e^{(2a+2c)}-1)} + \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx-a)}-1\right)}{b(e^{(2a+2c)}-1)} - \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx+c)}+1\right)}{b(e^{(2a+2c)}-1)} - \frac{\left(e^{(2a+2c)}+1\right)\log\left(e^{(-bx+c)}-1\right)}{b(e^{(2a+2c)}-1)}$$

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="maxima")
```

```
[Out] -x - a/b + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)
) + (e^(2*a + 2*c) + 1)*log(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - (e^
(2*a + 2*c) + 1)*log(e^(-b*x + c) + 1)/(b*(e^(2*a + 2*c) - 1)) - (e^(2*a +
2*c) + 1)*log(e^(-b*x + c) - 1)/(b*(e^(2*a + 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(39) = 78$.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\int \coth(c-bx)\coth(a+bx)\,dx = -\frac{bx + \frac{\left(e^{(2a+2c)}+1\right)\log\left(\left|e^{(2bx)}-e^{(2c)}\right|\right)}{e^{(2a+2c)}-1} + \frac{\left(e^{(2a)}+e^{(4a+2c)}\right)\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(2a)}-e^{(4a+2c)}}}{b}$$

```
[In] integrate(-coth(b*x-c)*coth(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*x + (e^(2*a + 2*c) + 1)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) -
1) + (e^(2*a) + e^(4*a + 2*c))*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(
4*a + 2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \coth(c-bx)\coth(a+bx)\,dx = \frac{\coth(a+c)\,\ln\left(4\,e^{2a}\,e^{2c}+4\,e^{4a}\,e^{4c}-4\,e^{4a}\,e^{2c}\,e^{2bx}-4\,e^{6a}\,e^{4c}\,e^{2bx}\right)}{b} - \frac{\coth(a+c)\,\ln\left(4\,e^{2a}\,e^{2bx}-4\,e^{2a}\,e^{2c}-4\,e^{4a}\,e^{4c}+4\,e^{4a}\,e^{2c}\,e^{2bx}\right)}{b} - x$$

```
[In] int(coth(a + b*x)*coth(c - b*x),x)
```

```
[Out] (coth(a + c)*log(4*exp(2*a)*exp(2*c) + 4*exp(4*a)*exp(4*c) - 4*exp(4*a)*exp
(2*c)*exp(2*b*x) - 4*exp(6*a)*exp(4*c)*exp(2*b*x)))/b - (coth(a + c)*log(4*
exp(2*a)*exp(2*b*x) - 4*exp(2*a)*exp(2*c) - 4*exp(4*a)*exp(4*c) + 4*exp(4*a
)*exp(2*c)*exp(2*b*x)))/b - x
```

3.139 $\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx=\frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a-c)\log(\cosh(c+bx))}{b}$$

```
[Out] csch(a-c)*ln(cosh(b*x+a))/b-csch(a-c)*ln(cosh(b*x+c))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5763, 3556}

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx=\frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a-c)\log(\cosh(bx+c))}{b}$$

```
[In] Int[Sech[a + b*x]*Sech[c + b*x],x]
[Out] (Csch[a - c]*Log[Cosh[a + b*x]])/b - (Csch[a - c]*Log[Cosh[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5763

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
```

```
eQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \operatorname{csch}(a-c)\int \tanh(a+bx)\,dx - \operatorname{csch}(a-c)\int \tanh(c+bx)\,dx \\ &= \frac{\operatorname{csch}(a-c)\log(\cosh(a+bx))}{b} - \frac{\operatorname{csch}(a-c)\log(\cosh(c+bx))}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{\operatorname{csch}(a-c)(\log(\cosh(a+bx)) - \log(\cosh(c+bx)))}{b}$$

```
[In] Integrate[Sech[a + b*x]*Sech[c + b*x],x]
[Out] (Csch[a - c]*(Log[Cosh[a + b*x]] - Log[Cosh[c + b*x]]))/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.14

method	result	size
risch	$-\frac{2\ln\left(e^{2bx+2a}+e^{2a-2c}\right)e^{a+c}}{\left(e^{2a}-e^{2c}\right)b}+\frac{2\ln\left(1+e^{2bx+2a}\right)e^{a+c}}{\left(e^{2a}-e^{2c}\right)b}$	77

```
[In] int(sech(b*x+a)*sech(b*x+c),x,method=_RETURNVERBOSE)
[Out] -2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/(exp(2*a)-exp(2*c))/b*exp(a+c)+2*ln(1+ex
p(2*b*x+2*a))/(exp(2*a)-exp(2*c))/b*exp(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.11

$$\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx$$
$$= \frac{2\left(\left(\cosh\left(-a+c\right)-\sinh\left(-a+c\right)\right)\log\left(\frac{2\left(\cosh(bx+c)\cosh(-a+c)-\sinh(bx+c)\sinh(-a+c)\right)}{\cosh(bx+c)\cosh(-a+c)-\left(\cosh(-a+c)+\sinh(-a+c)\right)\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right.\right.}{b\cosh\left(-a+c\right)^2-2\,b\cosh\left(-a+c\right)\sinh\left(-a+c\right)+b\sinh}$$

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

```
[Out] 2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(b*x + c)*cosh(-a + c) - sinh(b
*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-a
 + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - sinh(
-a + c))*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a +
 c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F]

$$\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx = \int \text{sech}\left(a+bx\right)\text{sech}\left(bx+c\right)dx$$

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x)
```

```
[Out] Integral(sech(a + b*x)*sech(b*x + c), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \text{sech}(a+bx)\text{sech}(c+bx)\,dx = \frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b(e^{(2\,a)}-e^{(2\,c)})} - \frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx)}+e^{(2\,c)}\right)}{b(e^{(2\,a)}-e^{(2\,c)})}$$

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)
*log(e^(-2*b*x) + e^(2*c))/(b*(e^(2*a) - e^(2*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{2\left(\frac{e^{(3a+c)}\log\left(e^{(2bx+2a)}+1\right)}{e^{(4a)}-e^{(2a+2c)}} - \frac{e^{(a+3c)}\log\left(e^{(2bx+2c)}+1\right)}{e^{(2a+2c)}-e^{(4c)}}\right)}{b}$$

```
[In] integrate(sech(b*x+a)*sech(b*x+c),x, algorithm="giac")
```

```
[Out] 2*(e^(3*a + c)*log(e^(2*b*x + 2*a) + 1)/(e^(4*a) - e^(2*a + 2*c)) - e^(a +
3*c)*log(e^(2*b*x + 2*c) + 1)/(e^(2*a + 2*c) - e^(4*c)))/b
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 7.39

$$\int \operatorname{sech}(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{4\sqrt{e^{2a-2c}}\,\operatorname{atan}\left(\frac{b\,(e^{-a}e^{c}+e^{-3a}e^{3c})\,(e^{2a}e^{-2c})^{3/2}}{\sqrt{-b^2(e^{2a}e^{-2c}-1)^2}} + \frac{e^{2a}e^{2bx}\left(\frac{2e^{-c}e^{a}}{b\left(e^{2a}e^{-2c}\right)^{3/2}} + \frac{2\left(e^{-a}e^{c}+e^{-3a}e^{3c}\right)\left(b\sqrt{e^{2a}e^{-2c}}+b\left(e^{2a}e^{-2c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2a}e^{-2c}-1\right)^2}\sqrt{2b^2e^{2a}e^{-2c}-b^2-b^2e^{4a}e^{-4c}}}\right)\sqrt{2b^2e^{2a}e^{-2c}-b^2-b^2e^{4a}e^{-4c}}}{4}\right)}{\sqrt{2b^2e^{2a-2c}-b^2e^{4a-4c}-b^2}}$$

```
[In] int(1/(cosh(a + b*x)*cosh(c + b*x)),x)
```

```
[Out] (4*exp(2*a - 2*c)^(1/2)*atan((b*(exp(-a)*exp(c) + exp(-3*a)*exp(3*c))*(exp(
2*a)*exp(-2*c))^(3/2))/(-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2) + (exp(2*a)*
exp(2*b*x)*((2*exp(-c)*exp(a))/(b*(exp(2*a)*exp(-2*c))^(3/2)) + (2*(exp(-a)
*exp(c) + exp(-3*a)*exp(3*c))*(b*(exp(2*a)*exp(-2*c))^(1/2) + b*(exp(2*a)*e
xp(-2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2)*(2*b^2*exp(2*a)*
exp(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2)))*(2*b^2*exp(2*a)*exp(-2*c)
 - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2))/4))/(2*b^2*exp(2*a - 2*c) - b^2*exp
(4*a - 4*c) - b^2)^(1/2)
```

3.140 $\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}$$

```
[Out] -csch(a+c)*ln(cosh(b*x-c))/b+csch(a+c)*ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5763, 3556}

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=\frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b}$$

```
[In] Int[Sech[c - b*x]*Sech[a + b*x],x]
[Out] -((Csch[a + c]*Log[Cosh[c - b*x]])/b) + (Csch[a + c]*Log[Cosh[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5763

```
Int[Sech[(a_.) + (b_.)*(x_)]*Sech[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Cs
ch[(b*c - a*d)/d], Int[Tanh[a + b*x], x], x] + Dist[Csch[(b*c - a*d)/b], In
```

```
t[Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && N
eQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \operatorname{csch}(a+c)\int \tanh(c-bx)\,dx + \operatorname{csch}(a+c)\int \tanh(a+bx)\,dx \\ &= -\frac{\operatorname{csch}(a+c)\log(\cosh(c-bx))}{b} + \frac{\operatorname{csch}(a+c)\log(\cosh(a+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{csch}(a+c)(\log(\cosh(c-bx)) - \log(\cosh(a+bx)))}{b}$$

```
[In] Integrate[Sech[c - b*x]*Sech[a + b*x],x]
[Out] -((Csch[a + c]*(Log[Cosh[c - b*x]] - Log[Cosh[a + b*x]]))/b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.

Time = 0.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.27

method	result	size
risch	$-\frac{2\ln\left(\mathrm{e}^{2a+2c}+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{\left(\mathrm{e}^{2a+2c}-1\right)b}+\frac{2\ln\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{a+c}}{b\left(\mathrm{e}^{2a+2c}-1\right)}$	75

```
[In] int(sech(b*x-c)*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2*ln(exp(2*a+2*c)+exp(2*b*x+2*a))/(exp(2*a+2*c)-1)/b*exp(a+c)+2/b/(exp(2*a
+2*c)-1)*ln(1+exp(2*b*x+2*a))*exp(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.73

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$$
$$=\frac{2\left((\cosh(a+c)-\sinh(a+c))\log\left(\frac{2(\cosh(bx+a)\cosh(a+c)-\sinh(bx+a)\sinh(a+c))}{\cosh(bx+a)\cosh(a+c)-(\cosh(a+c)+\sinh(a+c))\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-(\cos\right.}{b\cosh(a+c)^2-2\,b\cosh(a+c)\sinh(a+c)+b\sinh(a+c)}$$

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(b*x + a)*cosh(a + c) - sinh(b*x
+ a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c))*
sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c))*l
og(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b
*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F]

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=\int \operatorname{sech}(a+bx)\operatorname{sech}(bx-c)\,dx$$

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x)
```

```
[Out] Integral(sech(a + b*x)*sech(b*x - c), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx=\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}-\frac{2\,e^{(a+c)}\log\left(e^{(-2\,bx+2\,c)}+1\right)}{b(e^{(2\,a+2\,c)}-1)}$$

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-2*b*x - 2*a) + 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)
*log(e^(-2*b*x + 2*c) + 1)/(b*(e^(2*a + 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\left(\frac{e^{(a+c)}\log\left(e^{(2\,bx)}+e^{(2\,c)}\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

```
[In] integrate(sech(b*x-c)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(e^(a + c)*log(e^(2*b*x) + e^(2*c))/(e^(2*a + 2*c) - 1) + e^(3*a + c)*lo
g(e^(2*b*x + 2*a) + 1)/(e^(2*a) - e^(4*a + 2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 268, normalized size of antiderivative = 8.12

$$\int \operatorname{sech}(c-bx)\operatorname{sech}(a+bx)\,dx$$

$$= \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\left(\frac{2\,\mathrm{e}^{a}\,\mathrm{e}^{c}}{b\,{\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}\right)}^{3/2}}+\frac{2\,\mathrm{e}^{-3\,a}\,\mathrm{e}^{-3\,c}\,\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}+1\right)\,\left(b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}}+b\,{\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}\right)}^{3/2}\right)}{\sqrt{-b^2\,{\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}-1\right)}^2}\,\sqrt{2\,b^2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}-b^2-b^2\,\mathrm{e}^{4\,a}\,\mathrm{e}^{4\,c}}}\right)\,\sqrt{2\,b^2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,c}-b^2-b^2\,\mathrm{e}^{4\,a}\,\mathrm{e}^{4\,c}}}{4}+\frac{b\,\mathrm{e}^{-3\,a}\,\mathrm{e}^{-3\,c}\,(\mathrm{e}\ldots}{\sqrt{-b\ldots}}\right.}{\sqrt{2\,b^2\,\mathrm{e}^{2\,a+2\,c}-b^2\,\mathrm{e}^{4\,a+4\,c}-b^2}}$$

```
[In] int(1/(cosh(a + b*x)*cosh(c - b*x)),x)
```

```
[Out] (4*atan((exp(2*a)*exp(2*b*x)*((2*exp(a)*exp(c))/(b*(exp(2*a)*exp(2*c))^(3/2
)) + (2*exp(-3*a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(b*(exp(2*a)*exp(2*c))^
(1/2) + b*(exp(2*a)*exp(2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(2*c) - 1)^2)^(1/
2)*(2*b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2)))*(2*b^2*e
xp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2))/4 + (b*exp(-3*a)*exp
(-3*c)*(exp(2*a)*exp(2*c) + 1)*(exp(2*a)*exp(2*c))^(3/2))/(-b^2*(exp(2*a)*e
xp(2*c) - 1)^2)^(1/2))*exp(2*a + 2*c)^(1/2))/(2*b^2*exp(2*a + 2*c) - b^2*ex
p(4*a + 4*c) - b^2)^(1/2)
```

3.141 $\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{\operatorname{csch}(a-c)\log(\sinh(a+bx))}{b} + \frac{\operatorname{csch}(a-c)\log(\sinh(c+bx))}{b}$$

```
[Out] -csch(a-c)*ln(sinh(b*x+a))/b+csch(a-c)*ln(sinh(b*x+c))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5764, 3556}

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = \frac{\operatorname{csch}(a-c)\log(\sinh(bx+c))}{b} - \frac{\operatorname{csch}(a-c)\log(\sinh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]*Csch[c + b*x],x]
```

```
[Out] -((Csch[a - c]*Log[Sinh[a + b*x]])/b) + (Csch[a - c]*Log[Sinh[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5764

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
```

```
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\operatorname{csch}(a-c)\int \coth(a+bx)\,dx\right)+\operatorname{csch}(a-c)\int \coth(c+bx)\,dx\\ &= -\frac{\operatorname{csch}(a-c)\log(\sinh(a+bx))}{b}+\frac{\operatorname{csch}(a-c)\log(\sinh(c+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{\operatorname{csch}(a-c)(\log(\sinh(a+bx))-\log(\sinh(c+bx)))}{b}$$

```
[In] Integrate[Csch[a + b*x]*Csch[c + b*x],x]
[Out] -((Csch[a - c]*(Log[Sinh[a + b*x]] - Log[Sinh[c + b*x]]))/b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

method	result	size
risch	$-\frac{2\ln\left(e^{2bx+2a}-1\right)e^{a+c}}{\left(e^{2a}-e^{2c}\right)b}+\frac{2\ln\left(e^{2bx+2a}-e^{2a-2c}\right)e^{a+c}}{\left(e^{2a}-e^{2c}\right)b}$	79

```
[In] int(csch(b*x+a)*csch(b*x+c),x,method=_RETURNVERBOSE)
[Out] -2*ln(exp(2*b*x+2*a)-1)/(exp(2*a)-exp(2*c))/b*exp(a+c)+2*ln(exp(2*b*x+2*a)-
exp(2*a-2*c))/(exp(2*a)-exp(2*c))/b*exp(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.11

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\left(\left(\cosh\left(-a+c\right)-\sinh\left(-a+c\right)\right)\log\left(\frac{2\left(\cosh(-a+c)\sinh(bx+c)-\cosh(bx+c)\sinh(-a+c)\right)}{\cosh(bx+c)\cosh(-a+c)-\left(\cosh(-a+c)+\sinh(-a+c)\right)\sinh(bx+c)+\cosh(bx+c)\sinh(-a+c)}\right)\right.}{b\cosh\left(-a+c\right)^{2}-2\,b\cosh\left(-a+c\right)\sinh\left(-a+c\right)+b\sinh}$$

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="fricas")
```

```
[Out] -2*((cosh(-a + c) - sinh(-a + c))*log(2*(cosh(-a + c)*sinh(b*x + c) - cosh(
b*x + c)*sinh(-a + c))/(cosh(b*x + c)*cosh(-a + c) - (cosh(-a + c) + sinh(-
a + c))*sinh(b*x + c) + cosh(b*x + c)*sinh(-a + c))) - (cosh(-a + c) - sinh
(-a + c))*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a
+ c)^2 - 2*b*cosh(-a + c)*sinh(-a + c) + b*sinh(-a + c)^2 - b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{csch}\left(bx+c\right)dx$$

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x)
```

```
[Out] Integral(csch(a + b*x)*csch(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.69

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b(e^{(2\,a)}-e^{(2\,c)})}-\frac{2\,e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b(e^{(2\,a)}-e^{(2\,c)})}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx)}+e^{c}\right)}{b(e^{(2\,a)}-e^{(2\,c)})}+\frac{2\,e^{(a+c)}\log\left(e^{(-bx)}-e^{c}\right)}{b(e^{(2\,a)}-e^{(2\,c)})}$$

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="maxima")
```

```
[Out] -2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a) - e^(2*c))) - 2*e^(a + c)*lo
g(e^(-b*x - a) - 1)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) + e^
c)/(b*(e^(2*a) - e^(2*c))) + 2*e^(a + c)*log(e^(-b*x) - e^c)/(b*(e^(2*a) -
e^(2*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\left(\frac{e^{(3a+c)}\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{e^{(4a)}-e^{(2a+2c)}}-\frac{e^{(a+3c)}\log\left(\left|e^{(2bx+2c)}-1\right|\right)}{e^{(2a+2c)}-e^{(4c)}}\right)}{b}$$

```
[In] integrate(csch(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

```
[Out] -2*(e^(3*a + c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(4*a) - e^(2*a + 2*c)) - e
^(a + 3*c)*log(abs(e^(2*b*x + 2*c) - 1))/(e^(2*a + 2*c) - e^(4*c)))/b
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 266, normalized size of antiderivative = 7.39

$$\int \operatorname{csch}(a+bx)\operatorname{csch}(c+bx)\,dx =$$

$$-\frac{4\sqrt{e^{2a-2c}}\operatorname{atan}\left(\frac{b\left(e^{-a}e^{c}+e^{-3a}e^{3c}\right)\left(e^{2a}e^{-2c}\right)^{3/2}}{\sqrt{-b^2\left(e^{2a}e^{-2c}-1\right)^2}}-\frac{e^{2a}e^{2bx}\left(\frac{2e^{-c}e^{a}}{b\left(e^{2a}e^{-2c}\right)^{3/2}}+\frac{2\left(e^{-a}e^{c}+e^{-3a}e^{3c}\right)\left(b\sqrt{e^{2a}e^{-2c}}+b\left(e^{2a}e^{-2c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2a}e^{-2c}-1\right)^2}\sqrt{2b^2e^{2a}e^{-2c}-b^2-b^2e^{4a}e^{-4c}}}\right)\sqrt{2b^2e^{2a}e^{-2c}-b^2-b^2e^{4a}e^{-4c}}}{4}\right)}{\sqrt{2b^2e^{2a-2c}-b^2e^{4a-4c}-b^2}}$$

```
[In] int(1/(sinh(a + b*x)*sinh(c + b*x)),x)
```

```
[Out] -(4*exp(2*a - 2*c)^(1/2)*atan((b*(exp(-a)*exp(c) + exp(-3*a)*exp(3*c))*(exp
(2*a)*exp(-2*c))^(3/2))/(-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2) - (exp(2*a)
*exp(2*b*x)*((2*exp(-c)*exp(a))/(b*(exp(2*a)*exp(-2*c))^(3/2)) + (2*(exp(-a
)*exp(c) + exp(-3*a)*exp(3*c))*(b*(exp(2*a)*exp(-2*c))^(1/2) + b*(exp(2*a)*
exp(-2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(-2*c) - 1)^2)^(1/2)*(2*b^2*exp(2*a)
*exp(-2*c) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2)))*(2*b^2*exp(2*a)*exp(-2*c
) - b^2 - b^2*exp(4*a)*exp(-4*c))^(1/2))/4))/(2*b^2*exp(2*a - 2*c) - b^2*ex
p(4*a - 4*c) - b^2)^(1/2)
```

3.142 $\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}$$

```
[Out] -csch(a+c)*ln(-sinh(b*x-c))/b+csch(a+c)*ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5764, 3556}

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx=\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}$$

```
[In] Int[Csch[c - b*x]*Csch[a + b*x],x]
```

```
[Out] -((Csch[a + c]*Log[Sinh[c - b*x]])/b) + (Csch[a + c]*Log[Sinh[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5764

```
Int[Csch[(a_.) + (b_.)*(x_)]*Csch[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc
h[(b*c - a*d)/b], Int[Coth[a + b*x], x], x] - Dist[Csch[(b*c - a*d)/d], Int
```

```
[Coth[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && Ne
Q[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \operatorname{csch}(a+c)\int\coth(c-bx)\,dx+\operatorname{csch}(a+c)\int\coth(a+bx)\,dx\\ &= -\frac{\operatorname{csch}(a+c)\log(\sinh(c-bx))}{b}+\frac{\operatorname{csch}(a+c)\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int\operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{csch}(a+c)(\log(\sinh(c-bx))-\log(-\sinh(a+bx)))}{b}$$

```
[In] Integrate[Csch[c - b*x]*Csch[a + b*x],x]
[Out] -((Csch[a + c]*(Log[Sinh[c - b*x]] - Log[-Sinh[a + b*x]]))/b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

method	result	size
risch	$\frac{2\ln(e^{2bx+2a}-1)e^{a+c}}{b(e^{2a+2c}-1)}-\frac{2\ln(-e^{2a+2c}+e^{2bx+2a})e^{a+c}}{b(e^{2a+2c}-1)}$	77

```
[In] int(-csch(b*x-c)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2/b/(exp(2*a+2*c)-1)*ln(exp(2*b*x+2*a)-1)*exp(a+c)-2/b/(exp(2*a+2*c)-1)*ln(
-exp(2*a+2*c)+exp(2*b*x+2*a))*exp(a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.73

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = \frac{2\left((\cosh(a+c)-\sinh(a+c))\log\left(\frac{2(\cosh(a+c)\sinh(bx+a)-\cosh(bx+a)\sinh(a+c))}{\cosh(bx+a)\cosh(a+c)-(\cosh(a+c)+\sinh(a+c))\sinh(bx+a)+\cosh(bx+a)\sinh(a+c)}\right)-(\cos\right.}{b\cosh(a+c)^2-2b\cosh(a+c)\sinh(a+c)+b\sinh(a+c)}$$

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*((cosh(a + c) - sinh(a + c))*log(2*(cosh(a + c)*sinh(b*x + a) - cosh(b*x
+ a)*sinh(a + c))/(cosh(b*x + a)*cosh(a + c) - (cosh(a + c) + sinh(a + c))*
sinh(b*x + a) + cosh(b*x + a)*sinh(a + c))) - (cosh(a + c) - sinh(a + c))*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b*cosh(a + c)^2 - 2*b
*cosh(a + c)*sinh(a + c) + b*sinh(a + c)^2 - b)
```

Sympy [F]

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\int \operatorname{csch}(a+bx)\operatorname{csch}(bx-c)\,dx$$

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x)
```

```
[Out] -Integral(csch(a + b*x)*csch(b*x - c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(36) = 72$.

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.91

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = \frac{2e^{(a+c)}\log\left(e^{(-bx-a)}+1\right)}{b(e^{(2a+2c)}-1)} + \frac{2e^{(a+c)}\log\left(e^{(-bx-a)}-1\right)}{b(e^{(2a+2c)}-1)} - \frac{2e^{(a+c)}\log\left(e^{(-bx+c)}+1\right)}{b(e^{(2a+2c)}-1)} - \frac{2e^{(a+c)}\log\left(e^{(-bx+c)}-1\right)}{b(e^{(2a+2c)}-1)}$$

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(a + c)*log(e^(-b*x - a) + 1)/(b*(e^(2*a + 2*c) - 1)) + 2*e^(a + c)*log
(e^(-b*x - a) - 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) +
 1)/(b*(e^(2*a + 2*c) - 1)) - 2*e^(a + c)*log(e^(-b*x + c) - 1)/(b*(e^(2*a
+ 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\left(\frac{e^{(a+c)}\log\left(\left|e^{(2\,bx)}-e^{(2\,c)}\right|\right)}{e^{(2\,a+2\,c)}-1}+\frac{e^{(3\,a+c)}\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{e^{(2\,a)}-e^{(4\,a+2\,c)}}\right)}{b}$$

```
[In] integrate(-csch(b*x-c)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(e^(a + c)*log(abs(e^(2*b*x) - e^(2*c)))/(e^(2*a + 2*c) - 1) + e^(3*a +
c)*log(abs(e^(2*b*x + 2*a) - 1))/(e^(2*a) - e^(4*a + 2*c)))/b
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 269, normalized size of antiderivative = 8.15

$$\int \operatorname{csch}(c-bx)\operatorname{csch}(a+bx)\,dx =$$

$$-\frac{4\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\left(\frac{2\,e^{a}\,e^{c}}{b\left(e^{2\,a}\,e^{2\,c}\right)^{3/2}}+\frac{2\,e^{-3\,a}\,e^{-3\,c}\left(e^{2\,a}\,e^{2\,c}+1\right)\left(b\,\sqrt{e^{2\,a}\,e^{2\,c}}+b\left(e^{2\,a}\,e^{2\,c}\right)^{3/2}\right)}{\sqrt{-b^2\left(e^{2\,a}\,e^{2\,c}-1\right)^2}\,\sqrt{2\,b^2\,e^{2\,a}\,e^{2\,c}-b^2-b^2\,e^{4\,a}\,e^{4\,c}}}\right)\sqrt{2\,b^2\,e^{2\,a}\,e^{2\,c}-b^2-b^2\,e^{4\,a}\,e^{4\,c}}}{4}-\frac{b\,e^{-3\,a}\,e^{-3\cdots}}{\sqrt{\cdots}}\right)}{\sqrt{2\,b^2\,e^{2\,a+2\,c}-b^2\,e^{4\,a+4\,c}-b^2}}$$

```
[In] int(1/(sinh(a + b*x)*sinh(c - b*x)),x)
```

```
[Out] -(4*atan((exp(2*a)*exp(2*b*x)*((2*exp(a)*exp(c))/(b*(exp(2*a)*exp(2*c))^(3/
2)) + (2*exp(-3*a)*exp(-3*c)*(exp(2*a)*exp(2*c) + 1)*(b*(exp(2*a)*exp(2*c))
^(1/2) + b*(exp(2*a)*exp(2*c))^(3/2)))/((-b^2*(exp(2*a)*exp(2*c) - 1)^2)^(1
/2)*(2*b^2*exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2)))*(2*b^2*
exp(2*a)*exp(2*c) - b^2 - b^2*exp(4*a)*exp(4*c))^(1/2))/4 - (b*exp(-3*a)*ex
p(-3*c)*(exp(2*a)*exp(2*c) + 1)*(exp(2*a)*exp(2*c))^(3/2))/(-b^2*(exp(2*a)*
exp(2*c) - 1)^2)^(1/2))*exp(2*a + 2*c)^(1/2))/(2*b^2*exp(2*a + 2*c) - b^2*e
xp(4*a + 4*c) - b^2)^(1/2)
```

3.143 $\int \sinh(a+bx)\tanh(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = -\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+c))*cosh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5739, 2717, 3855}

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \frac{\sinh(a+bx)}{b}-\frac{\cosh(a-c)\arctan(\sinh(bx+c))}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x],x]
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + Sinh[a + b*x]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5739

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\cosh(a-c)\int \operatorname{sech}(c+bx)\,dx\right)+\int \cosh(a+bx)\,dx\\ &= -\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = -\frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}+\frac{\cosh(bx)\sinh(a)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] (-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (Co
sh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.76

method	result
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln(e^{bx+a}}{}$

```
[In] int(sinh(b*x+a)*tanh(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a
-c)*exp(2*a)+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*I*ln(
```

```
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 11.28

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)}{}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh(-a
 + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*co
sh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh(b*x +
 c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x +
 c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*co
sh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \int \sinh(a+bx)\tanh(bx+c)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x)
[Out] Integral(sinh(a + b*x)*tanh(b*x + c), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(bx+a)}}{2b} - \frac{e^{(-bx-a)}}{2b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
[Out] (e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b -
 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = -\frac{2\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}+e^{(-bx-a)}}{2b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
[Out] -1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) +
e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \sinh(a+bx)\tanh(c+bx)\,dx = \frac{e^{a+bx}}{2b}-\frac{e^{-a-bx}}{2b}-\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}+e^{2a}\,e^{-2c}\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{b^2}}$$

```
[In] int(sinh(a + b*x)*tanh(c + b*x),x)
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2
*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*e
xp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(b^2)^(1/2)
```

3.144 $\int \sinh(a+bx)\tanh^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\cosh(a-c)\text{sech}(c+bx)}{b}-\frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}$$

```
[Out] cosh(b*x+a)/b+cosh(a-c)*sech(b*x+c)/b-arctan(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5739, 5742, 2718, 3855, 2686, 8}

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx=-\frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\cosh(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (ArcTan[Sinh[c + b*x]]*Si
nh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5739

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5742

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -(\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx) + \int \cosh(a+bx)\tanh(c+bx)\,dx \\
&= \frac{\cosh(a-c)\text{Subst}(\int 1\,dx, x, \operatorname{sech}(c+bx))}{b} - \sinh(a-c)\int \operatorname{sech}(c+bx)\,dx + \int \sinh(a+bx)\,dx \\
&= \frac{\cosh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} - \frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx$$
$$= \frac{\cosh(a)\cosh(bx)}{b} + \frac{\cosh(a-c)\text{sech}(c+bx)}{b} - \frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b + (Cosh[a - c]*Sech[c + b*x])/b - (2*ArcTan[((Cosh[c]
 - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[
(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.56

method	result
risch	$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}(e^{2a}+e^{2c})}{b(e^{2bx+2a+2c}+e^{2a})} + \frac{i\ln(e^{bx+a}-ie^{a-c})e^{-a-c}e^{2a}}{2b} - \frac{i\ln(e^{bx+a}-ie^{a-c})e^{-a-c}e^{2c}}{2b} - \frac{i\ln(e^{bx+a}+ie^{a-c})e^{-}}{2b}$

```
[In] int(sinh(b*x+a)*tanh(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(
2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)
^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-1/2*I*ln(exp(b*x+a)
+I*exp(a-c))/b*exp(-a-c)*exp(a)^2+1/2*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-
c)*exp(c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 902, normalized size of antiderivative = 20.04

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - 2
*((cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)
*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x + c))*sinh(-a
 + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh
(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(
3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x +
 c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*x + c)^3*cosh(-a
 + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a
 + c)^2 + 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x
 + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-
a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)
^3*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)^3 + b*cos
h(b*x + c)*cosh(-a + c) + 3*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)
*sinh(-a + c))*sinh(b*x + c)^2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh
(-a + c) - (3*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(
b*x + c)^3 + b*cosh(b*x + c))*sinh(-a + c))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \int \sinh\left(a+bx\right)\tanh^2\left(bx+c\right)dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.33

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} + \frac{e^{(-bx-a)}}{2b} + \frac{\left(3e^{(2a)}+2e^{(2c)}\right)e^{(-2bx-2a)}+e^{(2c)}}{2b\left(e^{(-bx-a+2c)}+e^{(-3bx-a)}\right)}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] (e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a)/b
+ 1/2*((3*e^(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x - a
+ 2*c) + e^(-3*b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = -\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{2e^{(2bx+4a)}+3e^{(2bx+2a+2c)}+e^{(2a)}}{e^{(3bx+3a+2c)}+e^{(bx+3a)}} - e^{(bx+a)}}{2b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
4*a) + 3*e^(2*b*x + 2*a + 2*c) + e^(2*a))/(e^(3*b*x + 3*a + 2*c) + e^(b*x +
 3*a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.84

$$\int \sinh(a+bx)\tanh^2(c+bx)\,dx = \frac{e^{a+bx}}{2b} + \frac{e^{-a-bx}}{2b} + \frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}-e^{2a}\,e^{-2c}\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\left(e^{4a}\,e^{-4c}-2e^{2a}\,e^{-2c}+1\right)}}\right)\sqrt{e^{2c-2a}\left(e^{4a-4c}-2e^{2a-2c}+1\right)}}{\sqrt{b^2}} + \frac{e^{a+bx}\left(e^{2a-2c}+1\right)}{b\left(e^{2a-2c}+e^{2a+2bx}\right)}$$

```
[In] int(sinh(a + b*x)*tanh(c + b*x)^2,x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(e
xp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(exp
(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(b^2)^(1/2) + (exp(a + b*x)*(ex
p(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) + exp(2*a + 2*b*x)))
```

3.145 $\int \sinh(a+bx)\tanh^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\arctan(\sinh(c+bx))\cosh(a-c)}{2b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)\tanh(c+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+c))*cosh(a-c)/b+sech(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b+
1/2*cosh(a-c)*sech(b*x+c)*tanh(b*x+c)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5739, 5742, 2717, 3855, 2686, 8, 2691}

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\cosh(a-c)\arctan(\sinh(bx+c))}{2b} + \frac{\sinh(a-c)\operatorname{sech}(bx+c)}{b} + \frac{\cosh(a-c)\tanh(bx+c)\operatorname{sech}(bx+c)}{2b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[c + b*x]]*Cosh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c])/
b + Sinh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x]*Tanh[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5739

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5742

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh^2(c+bx)\,dx\right)+\int \cosh(a+bx)\tanh^2(c+bx)\,dx\\ &= \frac{\cosh(a-c)\operatorname{sech}(c+bx)\tanh(c+bx)}{2b}-\frac{1}{2}\cosh(a-c)\int \operatorname{sech}(c+bx)\,dx\\ &\quad -\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx+\int \sinh(a+bx)\tanh(c+bx)\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{2b} \\
&\quad + \frac{\cosh(a-c)\operatorname{sech}(c+bx)\tanh(c+bx)}{2b} - \cosh(a-c)\int \operatorname{sech}(c+bx)\,dx \\
&\quad + \frac{\sinh(a-c)\text{Subst}(\int 1\,dx, x, \operatorname{sech}(c+bx))}{b} + \int \cosh(a+bx)\,dx \\
&= -\frac{3\arctan(\sinh(c+bx))\cosh(a-c)}{2b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b} \\
&\quad + \frac{\sinh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)\tanh(c+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$
\int \sinh(a+bx)\tanh^3(c+bx)\,dx
$$
$$
= \frac{-12\arctan\left(\sinh(c)+\cosh(c)\tanh\left(\frac{bx}{2}\right)\right)\cosh(a-c)+\operatorname{sech}^2(c+bx)(2\sinh(a-2c-bx)+5\sinh(a+bx)}{4b}
$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (-12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + Sech[c + b*x]^2*
(2*Sinh[a - 2*c - b*x] + 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.33

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(3\,e^{2bx+4a+2c} - e^{2bx+2a+4c} + e^{4a} - 3\,e^{2a+2c}\right)}{2b\left(e^{2bx+2a+2c}+e^{2a}\right)^2} + \frac{3i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{4b} + \frac{3i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-}}{4b}$

```
[In] int(sinh(b*x+a)*tanh(b*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)-exp
(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2+
3/4*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*a)+3/4*I*ln(exp(b*x+a)-I*
exp(a-c))/b*exp(-a-c)*exp(2*c)-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*
exp(2*a)-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1737 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 1737, normalized size of antiderivative = 24.12

$$\int \sinh(a + bx) \tanh^3(c + bx) \, dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + (15*cosh(b*x
+ c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*cosh(-
a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c)
- 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh(b*x +
c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 - 2)*cosh(b*x + c)
 - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 + (15*cosh(b*
x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^2 + (15*co
sh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh(-a + c)^2 -
 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(-a + c) + 2*c
osh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + (cosh(b*x + c)^6 + 5*cosh(
b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a + c)^2 + 1)*co
sh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c
)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - c
osh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x
+ c)^4 + 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)
*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2
- 2*(5*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)*sinh(
b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^3
 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) -
 2*(5*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
+ c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 + 2*cosh(b*x + c)^3 + cosh(b*x + c
))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*(cosh(-a + c)^2
 + 1)*cosh(b*x + c)^4 + 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*
x + c)^4 + 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*co
sh(b*x + c)^4*cosh(-a + c) + 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))
*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) + 2*cosh
(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*arctan
(cosh(b*x + c) + sinh(b*x + c)) + 2*(3*cosh(b*x + c)^5*cosh(-a + c)^2 + 2*(
5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 + 10*cosh(b*x +
c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 - 5)*cosh(b*x +
c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) + 10*cosh(b*x + c)^3*cosh(-a + c) +
```

```
2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c
)^6*cosh(-a + c) + 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cosh(
-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a + c
) - b*sinh(-a + c))*sinh(b*x + c)^5 + 2*b*cosh(b*x + c)^3*cosh(-a + c) + 5*
(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)
^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a + c) - (5*b*cosh(b*x +
 c)^2 + b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) + 2
*(5*b*cosh(b*x + c)^3*cosh(-a + c) + 3*b*cosh(b*x + c)*cosh(-a + c) - (5*b*
cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b*c
osh(b*x + c)^4*cosh(-a + c) + 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a
+ c) - (5*b*cosh(b*x + c)^4 + 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b
*x + c) - (b*cosh(b*x + c)^5 + 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sinh(
-a + c))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx=\int \sinh{(a+bx)}\tanh^3{(bx+c)}\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)*tanh(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = \frac{3\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b}-\frac{e^{(-bx-a)}}{2b}+\frac{\left(5e^{(2a+2c)}-e^{(4c)}\right)e^{(-2bx-2a)}+\left(2e^{(4a)}-3e^{(2a+2c)}\right)e^{(-4bx-4a)}+e^{(4c)}}{2b\left(e^{(-bx-a+4c)}+2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/2*(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a
)/b + 1/2*((5*e^(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) - 3*e^
(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-3*b
*x - a + 2*c) + e^(-5*b*x - a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{3e^{(3bx+5a+2c)}-e^{(3bx+3a+4c)}+e^{(bx+5a)}-3e^{(bx+3a+2c)}}{\left(e^{(2bx+2a+2c)}+e^{(2a)}\right)^2} - e^{(bx+a)} + e^{(-bx-}}{2b}$$

```
[In] integrate(sinh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
5*a + 2*c) - e^(3*b*x + 3*a + 4*c) + e^(b*x + 5*a) - 3*e^(b*x + 3*a + 2*c))
/(e^(2*b*x + 2*a + 2*c) + e^(2*a))^2 - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(a+bx)\tanh^3(c+bx)\,dx = \int \sinh(a+b\,x)\,\tanh(c+b\,x)^3\,dx$$

```
[In] int(sinh(a + b*x)*tanh(c + b*x)^3,x)
```

```
[Out] int(sinh(a + b*x)*tanh(c + b*x)^3, x)
```

3.146 $\int \coth(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \coth(c+bx)\sinh(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5741, 2717, 3855}

$$\int \coth(c+bx)\sinh(a+bx)\,dx = \frac{\sinh(a+bx)}{b} - \frac{\sinh(a-c)\operatorname{arctanh}(\cosh(bx+c))}{b}$$

```
[In] Int[Coth[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b) + Sinh[a + b*x]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5741

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sinh(a-c)\int \operatorname{csch}(c+bx)\,dx + \int \cosh(a+bx)\,dx \\ &= -\frac{\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \coth(c+bx)\sinh(a+bx)\,dx$$

$$= \frac{\cosh(bx)\sinh(a)}{b} - \frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Coth[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b
*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]
*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(29) = 58.

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 5.34

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{\ln\left(e^{bx+a}+e^{a-c}\right.}{2b}$

```
[In] int(coth(b*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*
exp(2*a)-1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)
```

```
+exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp
(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 439, normalized size of antiderivative = 15.14

$$\int \coth(c+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)}{}$$

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh
(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
- 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(b*x +
c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)
^2 - 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sin
h(-a + c)^2 - 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) - 1) + 2*
(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) +
 cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) - 1)/(b*cosh(b*x + c)*cosh(-a
+ c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sin
h(b*x + c))
```

Sympy [F]

$$\int \coth(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\coth(bx+c)\,dx$$

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*coth(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(29) = 58$.

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.24

$$\int \coth(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2\,b} + \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x
 - a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \coth(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(bx+a+c)} + e^{a}\right) - \left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(bx+a+c)} - e^{a}\right|\right) - e^{(bx+a)}}{2\,b}$$

```
[In] integrate(coth(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e^
(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) - e^(b*x
+ a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.79

$$\int \coth(c+bx)\sinh(a+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{2\,b} - \frac{\mathrm{e}^{-a-b\,x}}{2\,b} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\mathrm{e}^{2\,c}\,\mathrm{e}^{b\,x}\,\left(\sqrt{-b^2}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{\mathrm{e}^{-2\,a}\,\mathrm{e}^{2\,c}\,\left(\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,c}-2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}+1\right)}}\right)\,\sqrt{\mathrm{e}^{2\,c-2\,a}\,\left(\mathrm{e}^{4\,a-4\,c}-2\,\mathrm{e}^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}}$$

```
[In] int(coth(c + b*x)*sinh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*x
)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(e
xp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(-b^2)^(1/2)
```

3.147 $\int \coth^2(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b} - \frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b-csch(b*x+c)*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5741, 5740, 2718, 3855, 2686, 8}

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\operatorname{arctanh}(\cosh(bx+c))}{b} - \frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b} + \frac{\cosh(a+bx)}{b}$$

```
[In] Int[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b - (Csch[c + b*x
]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5740

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5741

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sinh(a-c)\int \coth(c+bx)\text{csch}(c+bx)\,dx + \int \cosh(a+bx)\coth(c+bx)\,dx \\
&= \cosh(a-c)\int \text{csch}(c+bx)\,dx \\
&\quad - \frac{(i\sinh(a-c))\text{Subst}(\int 1\,dx, x, -i\text{csch}(c+bx))}{b} + \int \sinh(a+bx)\,dx \\
&= -\frac{\text{arctanh}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b} - \frac{\text{csch}(c+bx)\sinh(a-c)}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx$$
$$= -\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b} + \frac{\cosh(a)\cosh(bx)}{b} - \frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Coth[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b + (Cosh[a]*Cosh[b*x])/b - (Csch[c + b*x]*Sinh[a - c])/b + (Sinh[a]*Sinh[
b*x])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(46) = 92.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.28

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a}}{2b}$

```
[In] int(coth(b*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+
1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a-c)
)/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. 2(46) = 92.

Time = 0.27 (sec) , antiderivative size = 1237, normalized size of antiderivative = 26.89

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - (
(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*c
osh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a +
 c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh(b
*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*
cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c
) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*
x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b
*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c)^
2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)
*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)
^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c)
 - cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(
-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(cosh(b*x + c) + sin
h(b*x + c) - 1) + 2*(2*cosh(b*x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3
- 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) -
2*(2*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-a + c) - 3*cosh(b*x + c)^2*co
sh(-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)^3*cosh(-a + c) + (b*cosh(-a
+ c) - b*sinh(-a + c))*sinh(b*x + c)^3 - b*cosh(b*x + c)*cosh(-a + c) + 3*(
b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)^
2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (3*b*cosh(b*x + c)
^2 - b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^3 - b*cosh(b*x + c))
*sinh(-a + c))
```

Sympy [F]

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\coth^2(bx+c)\,dx$$

```
[In] integrate(coth(b*x+c)**2*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*coth(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.04

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{e^{(-bx-a)}}{2b} - \frac{\left(3e^{(2a)}-2e^{(2c)}\right)e^{(-2bx-2a)}-e^{(2c)}}{2b\left(e^{(-bx-a+2c)}-e^{(-3bx-a)}\right)}$$

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((3*e^
(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) - e^(
-3*b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right) - \left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right) + \frac{2e^{(2bx+4\cdots}}{e^{(3b\cdots}}}{2b}$$

```
[In] integrate(coth(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e^
(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + (2*e^(2
*b*x + 4*a) - 3*e^(2*b*x + 2*a + 2*c) + e^(2*a))/(e^(3*b*x + 3*a + 2*c) - e
^(b*x + 3*a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.93

$$\int \coth^2(c+bx)\sinh(a+bx)\,dx$$
$$=\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}-\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\,\left(\sqrt{-b^2}+e^{2a}\,e^{-2c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2a}\,e^{2c}\,(2\,e^{2a}\,e^{-2c}+e^{4a}\,e^{-4c}+1)}}\right)\,\sqrt{e^{2c-2a}\,\left(2\,e^{2a-2c}+e^{4a-4c}+1\right)}}{\sqrt{-b^2}}$$
$$+\frac{e^{a+bx}\,\left(e^{2a-2c}-1\right)}{b\,\left(e^{2a-2c}-e^{2a+2bx}\right)}$$

```
[In] int(coth(c + b*x)^2*sinh(a + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2
*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(-b^2)^(1/2) + (exp(a + b*x)*
(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
```

3.148 $\int \coth^3(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{3\text{arctanh}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\text{csch}(c+bx)\sinh(a-c)}{2b} + \frac{\sinh(a+bx)}{b}$$

```
[Out] -cosh(a-c)*csch(b*x+c)/b-3/2*arctanh(cosh(b*x+c))*sinh(a-c)/b-1/2*coth(b*x+
c)*csch(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5741, 5740, 2717, 3855, 2686, 8, 2691}

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = -\frac{3\sinh(a-c)\text{arctanh}(\cosh(bx+c))}{2b} - \frac{\cosh(a-c)\text{csch}(bx+c)}{b} - \frac{\sinh(a-c)\coth(bx+c)\text{csch}(bx+c)}{2b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Int[Coth[c + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (3*ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/(
2*b) - (Coth[c + b*x]*Csch[c + b*x]*Sinh[a - c])/(2*b) + Sinh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5740

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5741

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \sinh(a-c)\int \coth^2(c+bx)\operatorname{csch}(c+bx)\,dx+\int \cosh(a+bx)\coth^2(c+bx)\,dx\\
&= -\frac{\coth(c+bx)\operatorname{csch}(c+bx)\sinh(a-c)}{2b}+\cosh(a-c)\int \coth(c+bx)\operatorname{csch}(c+bx)\,dx\\
&\quad+\frac{1}{2}\sinh(a-c)\int \operatorname{csch}(c+bx)\,dx+\int \coth(c+bx)\sinh(a+bx)\,dx
\end{aligned}$$

$$= -\frac{\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\operatorname{csch}(c+bx)\sinh(a-c)}{2b} - \frac{(i\cosh(a-c))\operatorname{Subst}(\int 1\,dx, x, -i\operatorname{csch}(c+bx))}{b} + \sinh(a-c)\int \operatorname{csch}(c+bx)\,dx + \int \cosh(a+bx)\,dx$$

$$= -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} - \frac{3\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{2b} - \frac{\coth(c+bx)\operatorname{csch}(c+bx)\sinh(a-c)}{2b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx$$

$$= \frac{-12\operatorname{arctanh}\left(\cosh(c)+\sinh(c)\tanh\left(\frac{bx}{2}\right)\right)\sinh(a-c)+\operatorname{csch}^2(c+bx)(2\sinh(a-2c-bx)-5\sinh(a+bx)}{4b}$$

```
[In] Integrate[Coth[c + b*x]^3*Sinh[a + b*x],x]
[Out] (-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Sinh[a - c] + Csch[c + b*x]^2
*(2*Sinh[a - 2*c - b*x] - 5*Sinh[a + b*x] + Sinh[a + 2*c + 3*b*x]))/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(69) = 138$.

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.15

method	result
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}-e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^2}+\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{4b}-\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{}}{4b}$

```
[In] int(coth(b*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)-ex
p(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2+3/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-3/4*ln(exp(b*x+a)-exp(a-
c))/b*exp(-a-c)*exp(2*c)-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)+3
/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 2372, normalized size of antiderivative = 32.49

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-a
+ c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a
+ c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + 2*(15*
cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh
(-a + c) - 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(co
sh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 + 2)*cosh
(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 +
2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x +
c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cos
h(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + 2*(cosh(b*x +
 c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 - 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c
) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*s
inh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x
+ c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh
(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
cosh(-a + c)^2 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a
+ c) + 1)*sinh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*
cosh(b*x + c)^3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*
cosh(b*x + c) - 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a
 + c))*sinh(-a + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3
 + cosh(b*x + c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*
(cosh(-a + c)^2 - 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)
^2 + (5*cosh(b*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(5*cosh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c)
+ cosh(-a + c))*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-
a + c) - 2*cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(
-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + 3*((cosh(-a + c)^2 - 1)*c
osh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) -
cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x
```

```
 + c)^4 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1
)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*sinh
(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^
3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)
- 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*(cosh(-a + c)^
2 - 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b
*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*c
osh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c)
)*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) - 2*cos
h(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(c
osh(b*x + c) + sinh(b*x + c) - 1) + 4*(3*cosh(b*x + c)^5*cosh(-a + c)^2 - 2
*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 - 10*cosh(b*x
+ c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 + 5)*cosh(b*x
+ c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) - 10*cosh(b*x + c)^3*cosh(-a + c)
+ 2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 4*(cosh(b*x +
 c)^6*cosh(-a + c) - 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cos
h(-a + c))*sinh(-a + c) - 2)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a +
 c) - b*sinh(-a + c))*sinh(b*x + c)^5 - 2*b*cosh(b*x + c)^3*cosh(-a + c) +
5*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x +
c)^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (5*b*cosh(b*x
 + c)^2 - b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) +
 2*(5*b*cosh(b*x + c)^3*cosh(-a + c) - 3*b*cosh(b*x + c)*cosh(-a + c) - (5*
b*cosh(b*x + c)^3 - 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b
*cosh(b*x + c)^4*cosh(-a + c) - 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-
a + c) - (5*b*cosh(b*x + c)^4 - 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh
(b*x + c) - (b*cosh(b*x + c)^5 - 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sin
h(-a + c))
```

Sympy [F]

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\coth^3\left(bx+c\right)dx$$

```
[In] integrate(coth(b*x+c)**3*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*coth(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(69) = 138$.

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.55

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx$$
$$= -\frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b} + \frac{3\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b} - \frac{e^{(-bx-a)}}{2b} - \frac{\left(5e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a)}-\left(2e^{(4a)}+3e^{(2a+2c)}\right)e^{(-4bx-4a)}-e^{(4c)}}{2b\left(e^{(-bx-a+4c)}-2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -3/4*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((5*e^
(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) + 3*e^(2*a + 2*c))*e^(
-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c) +
e^(-5*b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.32

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx =$$
$$-\frac{3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-3\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2(3e^{(3}}{}}{4b}$$

```
[In] integrate(coth(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(3*(e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - 3
*(e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + 2*(
3*e^(3*b*x + 5*a + 2*c) + e^(3*b*x + 3*a + 4*c) - e^(b*x + 5*a) - 3*e^(b*x
+ 3*a + 2*c))/(e^(2*b*x + 2*a + 2*c) - e^(2*a))^2 - 2*e^(b*x + a) + 2*e^(-b
*x - a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(c+bx)\sinh(a+bx)\,dx=\int \coth(c+b\,x)^3\sinh(a+b\,x)\;dx$$

```
[In] int(coth(c + b*x)^3*sinh(a + b*x),x)
[Out] int(coth(c + b*x)^3*sinh(a + b*x), x)
```

3.149 $\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx=\frac{\cosh(a-c)\log(\cosh(c+bx))}{b}+x\sinh(a-c)$$

```
[Out] cosh(a-c)*ln(cosh(b*x+c))/b+x*sinh(a-c)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5743, 3556, 8}

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx=\frac{\cosh(a-c)\log(\cosh(bx+c))}{b}+x\sinh(a-c)$$

```
[In] Int[Sech[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5743

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a-c)\int \tanh(c+bx)\,dx + \sinh(a-c)\int 1\,dx \\ &= \frac{\cosh(a-c)\log(\cosh(c+bx))}{b} + x\sinh(a-c) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \frac{\cosh(a-c)\log(\cosh(c+bx))}{b} + x\sinh(a-c)$$

```
[In] Integrate[Sech[c + b*x]*Sinh[a + b*x],x]
[Out] (Cosh[a - c]*Log[Cosh[c + b*x]])/b + x*Sinh[a - c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(26) = 52.

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.69

method	result
risch	$x\,e^{a-c} - e^{-a-c}e^{2a}x - e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} - \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln(e^{2bx+2a}+e^{2a-2c})e^{-a-c}e^{2a}}{2b} + \frac{\ln(e^{2bx+2a}+e^{2a-2}}{2b}$

```
[In] int(sech(b*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a-1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = -\frac{2\,bx - \left(\cosh\left(-a+c\right)^{2} - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^{2} + 1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
[Out] -1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c
) - b*sinh(-a + c))
```

Sympy [F]

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*sech(b*x + c), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-2\,bx)} + e^{(2\,c)}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)*
e^(a - c)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = -\frac{2\,bxe^{(-a+c)} - \left(e^{(2\,a+c)} + e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)} + 1\right)}{2\,b}$$

```
[In] integrate(sech(b*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x
+ 2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \operatorname{sech}(c+bx)\sinh(a+bx)\,dx = \frac{\mathrm{e}^{2\,c-2\,a}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} + \mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\right)\,\left(2\,b\,\mathrm{e}^{3\,a-3\,c} + 2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2} - x\,\mathrm{e}^{c-a}$$

```
[In] int(sinh(a + b*x)/cosh(c + b*x),x)
```

```
[Out] (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*c))*(2*b*exp(3*a
- 3*c) + 2*b*exp(a - c)))/(4*b^2) - x*exp(c - a)
```

3.150 $\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b}+\frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}$$

```
[Out] -cosh(a-c)*sech(b*x+c)/b+arctan(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5743, 2686, 8, 3855}

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = \frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}-\frac{\cosh(a-c)\operatorname{sech}(bx+c)}{b}$$

```
[In] Int[Sech[c + b*x]^2*Sinh[a + b*x],x]
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5743

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx+\sinh(a-c)\int \operatorname{sech}(c+bx)\,dx\\ &= \frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}-\frac{\cosh(a-c)\text{Subst}(\int 1\,dx,x,\operatorname{sech}(c+bx))}{b}\\ &= -\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b}+\frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} + \frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b}$$

```
[In] Integrate[Sech[c + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a - c]*Sech[c + b*x])/b) + (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*
x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)
/2]*Sinh[c])]*Sinh[a - c])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.17

method	result
risch	$-\frac{e^{bx+a}(e^{2a}+e^{2c})}{b(e^{2bx+2a+2c}+e^{2a})}+\frac{i\ln(e^{bx+a}+ie^{a-c})e^{-a-c}e^{2a}}{2b}-\frac{i\ln(e^{bx+a}+ie^{a-c})e^{-a-c}e^{2c}}{2b}-\frac{i\ln(e^{bx+a}-ie^{a-c})e^{-a-c}e^{2a}}{2b}+\frac{i\ln(e^{bx}}{}$

```
[In] int(sech(b*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2-1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(c)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)^2+
1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 405, normalized size of antiderivative = 11.57

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$$
$$= \frac{2\cosh(bx+c)\cosh(-a+c)\sinh(-a+c)-\cosh(bx+c)\sinh(-a+c)^2+\left(\left(\cosh(-a+c)^2-1\right)\cosh(b\right.}{}$$

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 +
 ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1)*s
inh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a +
c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sin
h(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) -
 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 + 1)*cosh(b*x +
 c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*s
inh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a
 + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) -
 b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 + b)*sinh
(-a + c))
```

Sympy [F]

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx=\int \sinh(a+bx)\operatorname{sech}^2(bx+c)\,dx$$

```
[In] integrate(sech(b*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*sech(b*x + c)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx=-\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b}-\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) + e^(2*c)
)*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx=\frac{\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{\left(e^{(bx+2a)}+e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

```
[In] integrate(sech(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] ((e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) + e^(b
*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.29

$$\int \operatorname{sech}^2(c+bx)\sinh(a+bx)\,dx$$
$$= -\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{b^2}} - \frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}+1\right)}{b\,\left(e^{2\,a-2\,c}+e^{2\,a+2\,b\,x}\right)}$$

```
[In] int(sinh(a + b*x)/cosh(c + b*x)^2,x)
```

```
[Out] - (atan((exp(-a)*exp(2*c)*exp(b*x)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^
(1/2)))/(b*(exp(-2*a)*exp(2*c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) +
 1))^(1/2)))*(exp(2*c - 2*a)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2)
)/(b^2)^(1/2) - (exp(a + b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) + ex
p(2*a + 2*b*x)))
```

3.151 $\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\cosh(a-c)\operatorname{sech}^2(c+bx)}{2b}+\frac{\sinh(a-c)\tanh(c+bx)}{b}$$

```
[Out] -1/2*cosh(a-c)*sech(b*x+c)^2/b+sinh(a-c)*tanh(b*x+c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5743, 2686, 30, 3852, 8}

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx=\frac{\sinh(a-c)\tanh(bx+c)}{b}-\frac{\cosh(a-c)\operatorname{sech}^2(bx+c)}{2b}$$

```
[In] Int[Sech[c + b*x]^3*Sinh[a + b*x],x]
[Out] -1/2*(Cosh[a - c]*Sech[c + b*x]^2)/b + (Sinh[a - c]*Tanh[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5743

```
Int[Sech[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Cosh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \operatorname{sech}^2(c+bx)\tanh(c+bx)\,dx+\sinh(a-c)\int \operatorname{sech}^2(c+bx)\,dx\\ &= -\frac{\cosh(a-c)\text{Subst}\left(\int x\,dx,x,\operatorname{sech}(c+bx)\right)}{b}+\frac{(i\sinh(a-c))\text{Subst}\left(\int 1\,dx,x,-i\tanh(c+bx)\right)}{b}\\ &= -\frac{\cosh(a-c)\operatorname{sech}^2(c+bx)}{2b}+\frac{\sinh(a-c)\tanh(c+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{sech}(c)\operatorname{sech}^2(c+bx)(\cosh(a)-\sinh(a-c)\sinh(c+2bx))}{2b}$$

```
[In] Integrate[Sech[c + b*x]^3*Sinh[a + b*x],x]
[Out] -1/2*(Sech[c]*Sech[c + b*x]^2*(Cosh[a] - Sinh[a - c]*Sinh[c + 2*b*x]))/b
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{-1-\cosh(2bx+2c)+2\cosh(2bx+a+c)}{2b(1+\cosh(2bx+2c))}$	42
risch	$-\frac{\left(2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}-\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$	58

```
[In] int(sech(b*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(-1-cosh(2*b*x+2*c)+2*cosh(2*b*x+a+c))/(1+cosh(2*b*x+2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.47

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx =$$

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2+3\,b\cosh(bx+c)\cosh(-a+c)^2+\left(b\cosh(-a+c)^2-b\sinh(-a+c)\right.}$$

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
[Out] -2*(cosh(b*x + c)*cosh(-a + c) + cosh(-a + c)*sinh(b*x + c) - 2*cosh(b*x +
c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*cosh
(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*c
osh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)
^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cosh(b*x
 + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)*sinh(
-a + c)^2)*sinh(b*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^3(bx+c)\,dx$$

```
[In] integrate(sech(b*x+c)**3*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*sech(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.16

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = -\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)} + \frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)} - \frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)}\right)}$$

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4
*c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}+1\right)^2}$$

```
[In] integrate(sech(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(c+bx)\sinh(a+bx)\,dx = \int \frac{\sinh(a+b\,x)}{\cosh\left(c+b\,x\right)^3}\,dx$$

```
[In] int(sinh(a + b*x)/cosh(c + b*x)^3,x)
```

```
[Out] int(sinh(a + b*x)/cosh(c + b*x)^3, x)
```

3.152 $\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = x\cosh(a-c)+\frac{\log(\sinh(c+bx))\sinh(a-c)}{b}$$

```
[Out] x*cosh(a-c)+ln(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5745, 3556, 8}

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{\sinh(a-c)\log(\sinh(bx+c))}{b}+x\cosh(a-c)$$

```
[In] Int[Csch[c + b*x]*Sinh[a + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5745

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int 1\,dx + \sinh(a-c)\int \coth(c+bx)\,dx\\ &= x\cosh(a-c) + \frac{\log(\sinh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = x\cosh(a-c) + \frac{\log(\sinh(c+bx))\sinh(a-c)}{b}$$

```
[In] Integrate[Csch[c + b*x]*Sinh[a + b*x],x]
[Out] x*Cosh[a - c] + (Log[Sinh[c + b*x]]*Sinh[a - c])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(26) = 52.

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.77

method	result
risch	$x\,e^{a-c} - e^{-a-c}e^{2a}x + e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} + \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln(e^{2bx+2a}-e^{2a-2c})e^{-a-c}e^{2a}}{2b} - \frac{\ln(e^{2bx+2a}-e^{2a-2c})}{2b}$

```
[In] int(csch(b*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a+1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)-1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.31

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{2\,bx + \left(\cosh\left(-a+c\right)^2 - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^2 - 1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
 - 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c)
 - b*sinh(-a + c))
```

Sympy [F]

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\operatorname{csch}\left(bx+c\right)\,dx$$

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*csch(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) - e
^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = \frac{2\,bxe^{(-a+c)} + \left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)} - 1\right|\right)}{2\,b}$$

```
[In] integrate(csch(b*x+c)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*b
*x + 2*c) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}(c+bx)\sinh(a+bx)\,dx = x\,\mathrm{e}^{c-a} + \frac{\mathrm{e}^{2\,c-2\,a}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} - \mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,c}\right)\,\left(2\,b\,\mathrm{e}^{3\,a-3\,c} - 2\,b\,\mathrm{e}^{a-c}\right)}{4\,b^2}$$

```
[In] int(sinh(a + b*x)/sinh(c + b*x),x)
[Out] x*exp(c - a) + (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*c)
)*(2*b*exp(3*a - 3*c) - 2*b*exp(a - c)))/(4*b^2)
```

3.153 $\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b}-\frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*cosh(a-c)/b-csch(b*x+c)*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5745, 2686, 8, 3855}

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx=-\frac{\cosh(a-c)\operatorname{arctanh}(\cosh(bx+c))}{b}-\frac{\sinh(a-c)\operatorname{csch}(bx+c)}{b}$$

```
[In] Int[Csch[c + b*x]^2*Sinh[a + b*x],x]
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5745

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \operatorname{csch}(c+bx)\,dx+\sinh(a-c)\int \coth(c+bx)\operatorname{csch}(c+bx)\,dx\\ &= -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b}-\frac{(i\sinh(a-c))\operatorname{Subst}\left(\int 1\,dx,x,-i\operatorname{csch}(c+bx)\right)}{b}\\ &= -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b}-\frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\begin{aligned}&\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx\\ &= -\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}\\ &\quad-\frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}\end{aligned}$$

```
[In] Integrate[Csch[c + b*x]^2*Sinh[a + b*x],x]
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b - (Csch[c + b*x]*Sinh[a - c])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.78

method	result
risch	$\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2c}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{\cdots}\right.}{}$

```
[In] int(csch(b*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))-1/2*ln(ex
p(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-
a-c)*exp(2*c)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b
*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 617, normalized size of antiderivative = 17.14

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$$
$$= \frac{4\cosh(bx+c)\cosh(-a+c)\sinh(-a+c)-2\cosh(bx+c)\sinh(-a+c)^2-2\left(\cosh(-a+c)^2-1\right)\cos\cdots}{}$$

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
c)^2 - 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c) - ((cosh(-a + c)^2 + 1)*cosh(b*
x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 +
 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c
)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^
2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b
*x + c) + 1) + ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*
cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x
 + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh
(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*
sinh(-a + c) - 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(cosh(-a + c)^
2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c))/(b*cos
h(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)
^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sin
h(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{csch}^2(bx+c)\,dx$$

```
[In] integrate(csch(b*x+c)**2*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*csch(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) - e^(2*c))*e^(-b*x - a
)/(b*(e^(-2*b*x) - e^(2*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx = -\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right) - \left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right) + \frac{2\left(e^{(bx+2a)}-e^{(bx+2c)}\right)}{e^{(2bx+2c)}-1}}{2b}$$

```
[In] integrate(csch(b*x+c)^2*sinh(b*x+a),x, algorithm="giac")
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a +
 c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a) -
e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \operatorname{csch}^2(c+bx)\sinh(a+bx)\,dx$$
$$=\frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}-1\right)}{b\,\left(e^{2\,a-2\,c}-e^{2\,a+2\,b\,x}\right)}-\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{-b^2}}$$

```
[In] int(sinh(a + b*x)/sinh(c + b*x)^2,x)
```

```
[Out] (exp(a + b*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
 - (atan((exp(-a)*exp(2*c)*exp(b*x)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^
2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c
) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1
/2))/(-b^2)^(1/2)
```

3.154 $\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\cosh(a-c)\coth(c+bx)}{b}-\frac{\operatorname{csch}^2(c+bx)\sinh(a-c)}{2b}$$

```
[Out] -cosh(a-c)*coth(b*x+c)/b-1/2*csch(b*x+c)^2*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5745, 2686, 30, 3852, 8}

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx=-\frac{\cosh(a-c)\coth(bx+c)}{b}-\frac{\sinh(a-c)\operatorname{csch}^2(bx+c)}{2b}$$

```
[In] Int[Csch[c + b*x]^3*Sinh[a + b*x],x]
[Out] -((Cosh[a - c]*Coth[c + b*x])/b) - (Csch[c + b*x]^2*Sinh[a - c])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5745

```
Int[Csch[w_]^(n_.)*Sinh[v_], x_Symbol] :> Dist[Sinh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cosh(a-c)\int \operatorname{csch}^2(c+bx)\,dx+\sinh(a-c)\int \coth(c+bx)\operatorname{csch}^2(c+bx)\,dx\\
&= -\frac{(i\cosh(a-c))\operatorname{Subst}\left(\int 1\,dx,x,-i\coth(c+bx)\right)}{b}\\
&\quad+\frac{\sinh(a-c)\operatorname{Subst}\left(\int x\,dx,x,-i\operatorname{csch}(c+bx)\right)}{b}\\
&= -\frac{\cosh(a-c)\coth(c+bx)}{b}-\frac{\operatorname{csch}^2(c+bx)\sinh(a-c)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx=-\frac{(\cosh(a)-\cosh(a-c)\cosh(c+2bx))\operatorname{csch}(c)\operatorname{csch}^2(c+bx)}{2b}$$

```
[In] Integrate[Csch[c + b*x]^3*Sinh[a + b*x],x]
[Out] -1/2*((Cosh[a] - Cosh[a - c]*Cosh[c + 2*b*x])*Csch[c]*Csch[c + b*x]^2)/b
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

method	result	size
risch	$\frac{\left(-2\,\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}+\mathrm{e}^{2c}\right)\mathrm{e}^{3a-c}}{\left(-\mathrm{e}^{2bx+2a+2c}+\mathrm{e}^{2a}\right)^{2}b}$	57
parallelrisch	$-\frac{\left(\sinh(bx+a)\left(-\frac{\operatorname{sech}\left(\frac{bx}{2}+\frac{c}{2}\right)^{2}}{2}+1\right)\operatorname{csch}\left(\frac{bx}{2}+\frac{c}{2}\right)+\operatorname{sech}\left(\frac{bx}{2}+\frac{c}{2}\right)\cosh(bx+a)\right)\operatorname{csch}\left(\frac{bx}{2}+\frac{c}{2}\right)}{4b}$	63

```
[In] int(csch(b*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-exp(2*b*x+2*a+2*c)+exp(2*a))^2/b*(-2*exp(2*b*x+2*a+2*c)+exp(2*a)+exp(2*
c))*exp(3*a-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 6.31

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx =$$

$$-\frac{}{b\cosh\left(bx+c\right)^{3}\cosh\left(-a+c\right)^{2}-b\cosh\left(bx+c\right)\cosh\left(-a+c\right)^{2}+\left(b\cosh\left(-a+c\right)^{2}-b\sinh\left(-a+c\right)^{2}\right)\mathrm{s}}$$

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*((2*cosh(-a + c) - sinh(-a + c))*sinh(b*x + c) - cosh(b*x + c)*sinh(-a +
 c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 + (
b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c)*c
osh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*cosh(b
*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cosh(-a
+ c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*sinh(b*
x + c))
```

Sympy [F]

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = \int \sinh\left(a+bx\right)\operatorname{csch}^{3}\left(bx+c\right)dx$$

```
[In] integrate(csch(b*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Integral(sinh(a + b*x)*csch(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(37) = 74$.

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.36

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = -\frac{2\,e^{(-2\,bx+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)} + \frac{e^{(2\,a+3\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)} + \frac{e^{(5\,c)}}{b\left(2\,e^{(-2\,bx+a+2\,c)}-e^{(-4\,bx+a)}-e^{(a+4\,c)}\right)}$$

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] -2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4
*c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}-e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}-1\right)^2}$$

```
[In] integrate(csch(b*x+c)^3*sinh(b*x+a),x, algorithm="giac")
[Out] -(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) - e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+bx)\sinh(a+bx)\,dx = \int \frac{\sinh(a+b\,x)}{\sinh\left(c+b\,x\right)^3}\,dx$$

```
[In] int(sinh(a + b*x)/sinh(c + b*x)^3,x)
[Out] int(sinh(a + b*x)/sinh(c + b*x)^3, x)
```

3.155 $\int \cosh(a+bx)\tanh(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}$$

```
[Out] cosh(b*x+a)/b-arctan(sinh(b*x+c))*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5742, 2718, 3855}

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\sinh(a-c)\arctan(\sinh(bx+c))}{b}$$

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] Cosh[a + b*x]/b - (ArcTan[Sinh[c + b*x]]*Sinh[a - c])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5742

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -(\sinh(a-c)\int \operatorname{sech}(c+bx)\,dx) + \int \sinh(a+bx)\,dx\\ &= \frac{\cosh(a+bx)}{b} - \frac{\arctan(\sinh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = \frac{\cosh(a)\cosh(bx)}{b} - \frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x])/b - (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[
c] + Cosh[c]*Sinh[(b*x)/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c]
)]*Sinh[a - c])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.76

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}\right.}{}$

```
[In] int(cosh(b*x+a)*tanh(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a
-c)*exp(a)^2-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(c)^2-1/2*I*ln(
```

```
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(a)^2+1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 11.28

$$\int \cosh(a + bx) \tanh(c + bx) \, dx = \frac{\cosh{(bx+c)}^2 \cosh{(-a+c)}^2 - 2 \cosh{(bx+c)}^2 \cosh{(-a+c)} \sinh{(-a+c)} + \cosh{(bx+c)}^2 \sinh{(-a+c)}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-
a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + 2*(2*cosh(b*x + c)*cosh(-a
 + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*co
sh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 - 1)*sinh(b*x + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(cosh(b*x +
 c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x +
 c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*co
sh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))
```

Sympy [F]

$$\int \cosh(a + bx) \tanh(c + bx) \, dx = \int \cosh{(a + bx)} \tanh{(bx + c)} \, dx$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \cosh(a+bx) \tanh(c+bx) \, dx = \frac{\left(e^{(2a)} - e^{(2c)}\right) \arctan\left(e^{(-bx-c)}\right) e^{(-a-c)}}{b} + \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="maxima")
```

```
[Out] (e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(b*x + a)/b +
 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = -\frac{2\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - e^(b*x + a) -
e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \cosh(a+bx)\tanh(c+bx)\,dx = \frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}}{2b}+\frac{\operatorname{atan}\left(\frac{e^{-a}\,e^{2c}\,e^{bx}\left(\sqrt{b^2}-e^{2a}\,e^{-2c}\,\sqrt{b^2}\right)}{b\sqrt{e^{-2a}\,e^{2c}\,\left(e^{4a}\,e^{-4c}-2\,e^{2a}\,e^{-2c}+1\right)}}\right)\sqrt{e^{2c-2a}\,\left(e^{4a-4c}-2\,e^{2a-2c}+1\right)}}{\sqrt{b^2}}$$

```
[In] int(cosh(a + b*x)*tanh(c + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) + (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) - exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(e
xp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(exp
(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(b^2)^(1/2)
```

3.156 $\int \cosh(a + bx) \tanh^2(c + bx)\, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \cosh(a + bx) \tanh^2(c + bx)\, dx = -\frac{\arctan(\sinh(c + bx)) \cosh(a - c)}{b} + \frac{\operatorname{sech}(c + bx) \sinh(a - c)}{b} + \frac{\sinh(a + bx)}{b}$$

```
[Out] -arctan(sinh(b*x+c))*cosh(a-c)/b+sech(b*x+c)*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5742, 5739, 2717, 3855, 2686, 8}

$$\int \cosh(a + bx) \tanh^2(c + bx)\, dx = -\frac{\cosh(a - c) \arctan(\sinh(bx + c))}{b} + \frac{\sinh(a - c)\operatorname{sech}(bx + c)}{b} + \frac{\sinh(a + bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] -((ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b) + (Sech[c + b*x]*Sinh[a - c])/b +
Sinh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5739

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5742

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx\right)+\int \sinh(a+bx)\tanh(c+bx)\,dx\\
&= -\left(\cosh(a-c)\int \operatorname{sech}(c+bx)\,dx\right)\\
&\quad+\frac{\sinh(a-c)\operatorname{Subst}\left(\int 1\,dx,x,\operatorname{sech}(c+bx)\right)}{b}+\int \cosh(a+bx)\,dx\\
&= -\frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}+\frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b}+\frac{\sinh(a+bx)}{b}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = -\frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b} + \frac{\cosh(bx)\sinh(a)}{b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x]^2,x]
```

```
[Out] (-2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)
/2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b + (Co
sh[b*x]*Sinh[a])/b + (Sech[c + b*x]*Sinh[a - c])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.60

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)} + \frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b} + \frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2c}}{2b} - \frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-}}{2b}$

```
[In] int(cosh(b*x+a)*tanh(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(
2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*
a)+1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*I*ln(exp(b*x+a)
+I*exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-
c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 902, normalized size of antiderivative = 20.04

$$\int \cosh(a + bx) \tanh^2(c + bx) \, dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 + 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - 2
*((cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh
(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)
*cosh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 + cosh(b*x + c))*sinh(-a
 + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 + 1)*cosh
(b*x + c)^2 + (3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(
3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x +
 c) - 2*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*arctan(cosh(b*x + c) + sinh(b*x + c)) + 2*(2*cosh(b*x + c)^3*cosh(-a
 + c)^2 + (2*cosh(b*x + c)^3 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a
 + c)^2 - 1)*cosh(b*x + c) - 2*(2*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x
 + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-
a + c) + 3*cosh(b*x + c)^2*cosh(-a + c))*sinh(-a + c) - 1)/(b*cosh(b*x + c)
^3*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)^3 + b*cos
h(b*x + c)*cosh(-a + c) + 3*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)
*sinh(-a + c))*sinh(b*x + c)^2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh
(-a + c) - (3*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(
b*x + c)^3 + b*cosh(b*x + c))*sinh(-a + c))
```

Sympy [F]

$$\int \cosh(a + bx) \tanh^2(c + bx) \, dx = \int \cosh (a + bx) \tanh^2 (bx + c) \, dx$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = \frac{\left(e^{(2\,a)}+e^{(2\,c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{e^{(-bx-a)}}{2\,b} + \frac{\left(3\,e^{(2\,a)}-2\,e^{(2\,c)}\right)e^{(-2\,bx-2\,a)}+e^{(2\,c)}}{2\,b\left(e^{(-bx-a+2\,c)}+e^{(-3\,bx-a)}\right)}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="maxima")
[Out] (e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - 1/2*e^(-b*x - a)/b
+ 1/2*((3*e^(2*a) - 2*e^(2*c))*e^(-2*b*x - 2*a) + e^(2*c))/(b*(e^(-b*x - a
+ 2*c) + e^(-3*b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = -\frac{2\left(e^{(2\,a)}+e^{(2\,c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{2\,e^{(2\,bx+4\,a)}-3\,e^{(2\,bx+2\,a+2\,c)}-e^{(2\,a)}}{e^{(3\,bx+3\,a+2\,c)}+e^{(bx+3\,a)}} - e^{(bx+a)}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^2,x, algorithm="giac")
[Out] -1/2*(2*(e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (2*e^(2*b*x +
4*a) - 3*e^(2*b*x + 2*a + 2*c) - e^(2*a))/(e^(3*b*x + 3*a + 2*c) + e^(b*x +
 3*a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.84

$$\int \cosh(a+bx)\tanh^2(c+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{e^{-a-b\,x}}{2\,b} - \frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{b^2}} + \frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}-1\right)}{b\,\left(e^{2\,a-2\,c}+e^{2\,a+2\,b\,x}\right)}$$

```
[In] int(cosh(a + b*x)*tanh(c + b*x)^2,x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*(2
*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2*e
xp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(b^2)^(1/2) + (exp(a + b*x)*(ex
p(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) + exp(2*a + 2*b*x)))
```

3.157 $\int \cosh(a+bx)\tanh^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \frac{\cosh(a+bx)}{b}+\frac{\cosh(a-c)\text{sech}(c+bx)}{b}-\frac{3\arctan(\sinh(c+bx))\sinh(a-c)}{2b}+\frac{\text{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}$$

```
[Out] cosh(b*x+a)/b+cosh(a-c)*sech(b*x+c)/b-3/2*arctan(sinh(b*x+c))*sinh(a-c)/b+1
/2*sech(b*x+c)*sinh(a-c)*tanh(b*x+c)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5742, 5739, 2718, 3855, 2686, 8, 2691}

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = -\frac{3\sinh(a-c)\arctan(\sinh(bx+c))}{2b}+\frac{\cosh(a-c)\text{sech}(bx+c)}{b}+\frac{\sinh(a-c)\tanh(bx+c)\text{sech}(bx+c)}{2b}+\frac{\cosh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] Cosh[a + b*x]/b + (Cosh[a - c]*Sech[c + b*x])/b - (3*ArcTan[Sinh[c + b*x]]*
Sinh[a - c])/(2*b) + (Sech[c + b*x]*Sinh[a - c]*Tanh[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5739

```
Int[Sinh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Cosh[v]*Tanh[w]^(n - 1), x] -
 Dist[Cosh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5742

```
Int[Cosh[v_]*Tanh[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Tanh[w]^(n - 1), x] -
 Dist[Sinh[v - w], Int[Sech[w]*Tanh[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh^2(c+bx)\,dx\right)+\int \sinh(a+bx)\tanh^2(c+bx)\,dx\\
&= \frac{\operatorname{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}-\cosh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx\\
&\quad -\frac{1}{2}\sinh(a-c)\int \operatorname{sech}(c+bx)\,dx+\int \cosh(a+bx)\tanh(c+bx)\,dx
\end{aligned}$$

$$= -\frac{\arctan(\sinh(c+bx))\sinh(a-c)}{2b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b} + \frac{\cosh(a-c)\text{Subst}(\int 1\,dx, x, \operatorname{sech}(c+bx))}{b} - \sinh(a-c)\int \operatorname{sech}(c+bx)\,dx + \int \sinh(a+bx)\,dx$$

$$= \frac{\cosh(a+bx)}{b} + \frac{\cosh(a-c)\operatorname{sech}(c+bx)}{b} - \frac{3\arctan(\sinh(c+bx))\sinh(a-c)}{2b} + \frac{\operatorname{sech}(c+bx)\sinh(a-c)\tanh(c+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.60

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx$$

$$= \frac{\cosh(a-2c)\operatorname{sech}(c)\operatorname{sech}(c+bx) - \cosh(a-c-bx)\operatorname{sech}(c)\operatorname{sech}^2(c+bx) + \cosh(a-c+bx)\operatorname{sech}(c)\operatorname{sech}^2(c}{}$$

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + b*x]^3,x]
```

```
[Out] (Cosh[a - 2*c]*Sech[c]*Sech[c + b*x] - Cosh[a - c - b*x]*Sech[c]*Sech[c + b
*x]^2 + Cosh[a - c + b*x]*Sech[c]*Sech[c + b*x]^2 + Cosh[a]*(4*Cosh[b*x] +
3*Sech[c]*Sech[c + b*x]) - 12*ArcTan[Sinh[c] + Cosh[c]*Tanh[(b*x)/2]]*Sinh[
a - c] + 4*Sinh[a]*Sinh[b*x])/(4*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.31

method	result
risch	$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}(3\,e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}+3\,e^{2a+2c})}{2b(e^{2bx+2a+2c}+e^{2a})^2} + \frac{3i\ln(e^{bx+a}-ie^{a-c})e^{-a-c}e^{2a}}{4b} - \frac{3i\ln(e^{bx+a}-ie^{a-c})e^{-}}{4b}$

```
[In] int(cosh(b*x+a)*tanh(b*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(3*exp(2*b*x+4*a+2*c)+exp
(2*b*x+2*a+4*c)+exp(4*a)+3*exp(2*a+2*c))/b/(exp(2*b*x+2*a+2*c)+exp(2*a))^2+
3/4*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(a)^2-3/4*I*ln(exp(b*x+a)-I*
exp(a-c))/b*exp(-a-c)*exp(c)^2-3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*
exp(a)^2+3/4*I*ln(exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1737 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 1737, normalized size of antiderivative = 24.12

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^6*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 6*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^5 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^4 + (15*cosh(b*x
+ c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + 5*cosh(-
a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c)
+ 2)*sinh(b*x + c)^4 + 4*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(cosh(b*x +
c)^3 + cosh(b*x + c))*sinh(-a + c)^2 + (5*cosh(-a + c)^2 + 2)*cosh(b*x + c)
 - 10*(cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c)^3 + (2*cosh(-a + c)^2 + 5)*cosh(b*x + c)^2 + (15*cosh(b*
x + c)^4*cosh(-a + c)^2 + 6*(5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^2 + (15*co
sh(b*x + c)^4 + 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cosh(-a + c)^2 -
 2*(15*cosh(b*x + c)^4*cosh(-a + c) + 30*cosh(b*x + c)^2*cosh(-a + c) + 2*c
osh(-a + c))*sinh(-a + c) + 5)*sinh(b*x + c)^2 + (cosh(b*x + c)^6 + 5*cosh(
b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a + c)^2 - 1)*co
sh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c
)^2 - 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - c
osh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x
+ c)^4 + 2*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)
*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2
- 2*(5*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) - 1)*sinh(
b*x + c)^3 + 2*(5*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^3
 + 3*cosh(b*x + c))*sinh(-a + c)^2 + 3*(cosh(-a + c)^2 - 1)*cosh(b*x + c) -
 2*(5*cosh(b*x + c)^3*cosh(-a + c) + 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
+ c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 + 2*cosh(b*x + c)^3 + cosh(b*x + c
))*sinh(-a + c)^2 + (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (5*(cosh(-a + c)^2
 - 1)*cosh(b*x + c)^4 + 6*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (5*cosh(b*
x + c)^4 + 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*co
sh(b*x + c)^4*cosh(-a + c) + 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))
*sinh(-a + c) - 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) + 2*cosh
(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*arctan
(cosh(b*x + c) + sinh(b*x + c)) + 2*(3*cosh(b*x + c)^5*cosh(-a + c)^2 + 2*(
5*cosh(-a + c)^2 + 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 + 10*cosh(b*x +
c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 + 5)*cosh(b*x +
c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) + 10*cosh(b*x + c)^3*cosh(-a + c) +
```

```
2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 2*(cosh(b*x + c
)^6*cosh(-a + c) + 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cosh(
-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a + c
) - b*sinh(-a + c))*sinh(b*x + c)^5 + 2*b*cosh(b*x + c)^3*cosh(-a + c) + 5*
(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)
^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a + c) - (5*b*cosh(b*x +
 c)^2 + b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) + 2
*(5*b*cosh(b*x + c)^3*cosh(-a + c) + 3*b*cosh(b*x + c)*cosh(-a + c) - (5*b*
cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b*c
osh(b*x + c)^4*cosh(-a + c) + 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-a
+ c) - (5*b*cosh(b*x + c)^4 + 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh(b
*x + c) - (b*cosh(b*x + c)^5 + 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sinh(
-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \int \cosh\left(a+bx\right)\tanh^3\left(bx+c\right)dx$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*tanh(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\begin{aligned}\int \cosh(a+bx)\tanh^3(c+bx)\,dx &= \frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{2b}+\frac{e^{(-bx-a)}}{2b}\\&\quad+\frac{\left(5e^{(2a+2c)}+e^{(4c)}\right)e^{(-2bx-2a)}+\left(2e^{(4a)}+3e^{(2a+2c)}\right)e^{(-4bx-4a)}+e^{(4c)}}{2b\left(e^{(-bx-a+4c)}+2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}\end{aligned}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/2*(e^(2*a) - e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b + 1/2*e^(-b*x - a
)/b + 1/2*((5*e^(2*a + 2*c) + e^(4*c))*e^(-2*b*x - 2*a) + (2*e^(4*a) + 3*e^
(2*a + 2*c))*e^(-4*b*x - 4*a) + e^(4*c))/(b*(e^(-b*x - a + 4*c) + 2*e^(-3*b
*x - a + 2*c) + e^(-5*b*x - a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx =$$

$$-\frac{3\left(e^{(2a)}-e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)}-\frac{3e^{(3bx+5a+2c)}+e^{(3bx+3a+4c)}+e^{(bx+5a)}+3e^{(bx+3a+2c)}}{\left(e^{(2bx+2a+2c)}+e^{(2a)}\right)^2}-e^{(bx+a)}-e^{(-bx-}}{2b}$$

```
[In] integrate(cosh(b*x+a)*tanh(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(e^(2*a) - e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (3*e^(3*b*x +
5*a + 2*c) + e^(3*b*x + 3*a + 4*c) + e^(b*x + 5*a) + 3*e^(b*x + 3*a + 2*c))
/(e^(2*b*x + 2*a + 2*c) + e^(2*a))^2 - e^(b*x + a) - e^(-b*x - a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\tanh^3(c+bx)\,dx = \int \cosh(a+b\,x)\,\tanh(c+b\,x)^3\,dx$$

```
[In] int(cosh(a + b*x)*tanh(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)*tanh(c + b*x)^3, x)
```

3.158 $\int \cosh(a+bx)\coth(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \cosh(a+bx)\coth(c+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5740, 2718, 3855}

$$\int \cosh(a+bx)\coth(c+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\cosh(a-c)\operatorname{arctanh}(\cosh(bx+c))}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x],x]
[Out] -((ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/b) + Cosh[a + b*x]/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5740

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \operatorname{csch}(c+bx)\,dx + \int \sinh(a+bx)\,dx\\ &= -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{b} + \frac{\cosh(a+bx)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\begin{aligned}&\int \cosh(a+bx)\coth(c+bx)\,dx\\ &= -\frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}\\ &\quad+\frac{\cosh(a)\cosh(bx)}{b}+\frac{\sinh(a)\sinh(bx)}{b}\end{aligned}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(
b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])
/b + (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 5.34

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right.}{2b}$

```
[In] int(cosh(b*x+a)*coth(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*
exp(2*a)+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)
```

+exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp
(2*c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 15.14

$$\int \cosh(a+bx)\coth(c+bx)\,dx = \frac{\cosh(bx+c)^2\cosh(-a+c)^2 - 2\cosh(bx+c)^2\cosh(-a+c)\sinh(-a+c) + \cosh(bx+c)^2\sinh(-a+c)}{}$$

[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="fricas")

[Out] 1/2*(cosh(b*x + c)^2*cosh(-a + c)^2 - 2*cosh(b*x + c)^2*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)^2*sinh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^2 + (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) - (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c))*log(cosh(b*x + c) + sinh(b*x + c) - 1) + 2*(cosh(b*x + c)*cosh(-a + c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c) + 1)/(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c))

Sympy [F]

$$\int \cosh(a+bx)\coth(c+bx)\,dx = \int \cosh(a+bx)\coth(bx+c)\,dx$$

[In] integrate(cosh(b*x+a)*coth(b*x+c),x)

[Out] Integral(cosh(a + b*x)*coth(b*x + c), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(29) = 58$.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.10

$$\int \cosh(a+bx)\coth(c+bx)\,dx = -\frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(b*x + a)/b + 1/2*e^(-b*x
 - a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.14

$$\int \cosh(a+bx)\coth(c+bx)\,dx = -\frac{\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)-e^{(bx+a)}}{2b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*((e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e^
(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) - e^(b*x
+ a) - e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.79

$$\int \cosh(a+bx)\coth(c+bx)\,dx = \frac{e^{a+b\,x}}{2\,b}+\frac{e^{-a-b\,x}}{2\,b}-\frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{-b^2}}$$

```
[In] int(cosh(a + b*x)*coth(c + b*x),x)
```

```
[Out] exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (atan((exp(-a)*exp(2*c)*exp(b*x
)*((-b^2)^(1/2) + exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)*
(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(2
*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/(-b^2)^(1/2)
```

3.159 $\int \cosh(a+bx)\coth^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} - \frac{\operatorname{arctanh}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}$$

```
[Out] -cosh(a-c)*csch(b*x+c)/b-arctanh(cosh(b*x+c))*sinh(a-c)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5740, 5741, 2717, 3855, 2686, 8}

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\sinh(a-c)\operatorname{arctanh}(\cosh(bx+c))}{b} - \frac{\cosh(a-c)\operatorname{csch}(bx+c)}{b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b +
 Sinh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5740

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5741

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \coth(c+bx)\text{csch}(c+bx)\,dx + \int \coth(c+bx)\sinh(a+bx)\,dx \\ &= -\frac{(i\cosh(a-c))\text{Subst}(\int 1\,dx, x, -i\text{csch}(c+bx))}{b} \\ &\quad + \sinh(a-c)\int \text{csch}(c+bx)\,dx + \int \cosh(a+bx)\,dx \\ &= -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{\text{arctanh}(\cosh(c+bx))\sinh(a-c)}{b} + \frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx$$
$$= -\frac{\cosh(a-c)\operatorname{csch}(c+bx)}{b} + \frac{\cosh(bx)\sinh(a)}{b} - \frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) + (Cosh[b*x]*Sinh[a])/b - ((2*I)*ArcTan[((
Cosh[c] - Sinh[c])*(Cosh[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh
[c]*Cosh[(b*x)/2] - I*Cosh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b + (Cosh[a]*Sin
h[b*x])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 4.24

method	result
risch	$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)} + \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b} - \frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b} - \frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a}}{2b}$

```
[In] int(cosh(b*x+a)*coth(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b-1/2*exp(-b*x-a)/b+1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp
(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-
1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a-c)
)/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 1237, normalized size of antiderivative = 26.89

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(b*x + c)^4*cosh(-a + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh
(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^4 + 4*(cosh(b*x + c)*cosh(-a + c)^
2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a + c)^
2)*sinh(b*x + c)^3 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + 3*(2*cosh(b*x
 + c)^2*cosh(-a + c)^2 + (2*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a +
 c)^2 - 2*(2*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)
*sinh(b*x + c)^2 + (cosh(b*x + c)^4 - 3*cosh(b*x + c)^2)*sinh(-a + c)^2 - (
(cosh(-a + c)^2 - 1)*cosh(b*x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sin
h(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-
a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*c
osh(b*x + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a +
 c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh(b
*x + c)^2 + (3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*
cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c
) - 2*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*
x + c)^3 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 -
 1)*sinh(b*x + c)^3 - 3*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b
*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c)^
2 + (cosh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)
*cosh(b*x + c) + (3*(cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (3*cosh(b*x + c)
^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(3*cosh(b*x + c)^2*cosh(-a + c)
 - cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^3*cosh(
-a + c) - cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(cosh(b*x + c) + sin
h(b*x + c) - 1) + 2*(2*cosh(b*x + c)^3*cosh(-a + c)^2 + (2*cosh(b*x + c)^3
- 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c) -
2*(2*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a +
 c))*sinh(b*x + c) - 2*(cosh(b*x + c)^4*cosh(-a + c) - 3*cosh(b*x + c)^2*co
sh(-a + c))*sinh(-a + c) + 1)/(b*cosh(b*x + c)^3*cosh(-a + c) + (b*cosh(-a
+ c) - b*sinh(-a + c))*sinh(b*x + c)^3 - b*cosh(b*x + c)*cosh(-a + c) + 3*(
b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c)^
2 + (3*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (3*b*cosh(b*x + c)
^2 - b)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^3 - b*cosh(b*x + c))
*sinh(-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = \int \cosh\left(a+bx\right)\coth^2\left(bx+c\right)dx$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)**2,x)
[Out] Integral(cosh(a + b*x)*coth(b*x + c)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(46) = 92$.

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.13

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\left(e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)}-e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2\,b} - \frac{e^{(-bx-a)}}{2\,b} - \frac{\left(3\,e^{(2\,a)}+2\,e^{(2\,c)}\right)e^{(-2\,bx-2\,a)}-e^{(2\,c)}}{2\,b\left(e^{(-bx-a+2\,c)}-e^{(-3\,bx-a)}\right)}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="maxima")
[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) -
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b - 1/2*e^(-b*x - a)/b - 1/2*((3*e^
(2*a) + 2*e^(2*c))*e^(-2*b*x - 2*a) - e^(2*c))/(b*(e^(-b*x - a + 2*c) - e^(
-3*b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx = -\frac{\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-\left(e^{(2\,a+c)}-e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2\,e^{(2\,bx+4}}{e^{(3\,b}}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^2,x, algorithm="giac")
[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - (e^
(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + (2*e^(2
*b*x + 4*a) + 3*e^(2*b*x + 2*a + 2*c) - e^(2*a))/(e^(3*b*x + 3*a + 2*c) - e
^(b*x + 3*a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \cosh(a+bx)\coth^2(c+bx)\,dx$$
$$= \frac{e^{a+b\,x}}{2\,b} - \frac{e^{-a-b\,x}}{2\,b} - \frac{\mathrm{atan}\left(-\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}} + \frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}+1\right)}{b\,\left(e^{2\,a-2\,c}-e^{2\,a+2\,b\,x}\right)}$$

```
[In] int(cosh(a + b*x)*coth(c + b*x)^2,x)
```

```
[Out] exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (atan(-(exp(-a)*exp(2*c)*exp(b*
x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^(1/2)))/(b*(exp(-2*a)*exp(2*c)
*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) + 1))^(1/2)))*(exp(2*c - 2*a)*(
exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2))/(-b^2)^(1/2) + (exp(a + b*x)
*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) - exp(2*a + 2*b*x)))
```

3.160 $\int \cosh(a+bx)\coth^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\text{arctanh}(\cosh(c+bx))\cosh(a-c)}{2b} + \frac{\cosh(a+bx)}{b} - \frac{\cosh(a-c)\coth(c+bx)\text{csch}(c+bx)}{2b} - \frac{\text{csch}(c+bx)\sinh(a-c)}{b}$$

```
[Out] -3/2*arctanh(cosh(b*x+c))*cosh(a-c)/b+cosh(b*x+a)/b-1/2*cosh(a-c)*coth(b*x+
c)*csch(b*x+c)/b-csch(b*x+c)*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5740, 5741, 2718, 3855, 2686, 8, 2691}

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\cosh(a-c)\text{arctanh}(\cosh(bx+c))}{2b} - \frac{\sinh(a-c)\text{csch}(bx+c)}{b} - \frac{\cosh(a-c)\coth(bx+c)\text{csch}(bx+c)}{2b} + \frac{\cosh(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cosh[c + b*x]]*Cosh[a - c])/(2*b) + Cosh[a + b*x]/b - (Cosh[a -
 c]*Coth[c + b*x]*Csch[c + b*x])/(2*b) - (Csch[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5740

```
Int[Cosh[v_]*Coth[w_]^(n_.), x_Symbol] :> Int[Sinh[v]*Coth[w]^(n - 1), x] +
 Dist[Cosh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rule 5741

```
Int[Coth[w_]^(n_.)*Sinh[v_], x_Symbol] :> Int[Cosh[v]*Coth[w]^(n - 1), x] +
 Dist[Sinh[v - w], Int[Csch[w]*Coth[w]^(n - 1), x], x] /; GtQ[n, 0] && NeQ[
w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cosh(a-c)\int \coth^2(c+bx)\operatorname{csch}(c+bx)\,dx + \int \coth^2(c+bx)\sinh(a+bx)\,dx \\
&= -\frac{\cosh(a-c)\coth(c+bx)\operatorname{csch}(c+bx)}{2b} + \frac{1}{2}\cosh(a-c)\int \operatorname{csch}(c+bx)\,dx \\
&\quad + \sinh(a-c)\int \coth(c+bx)\operatorname{csch}(c+bx)\,dx + \int \cosh(a+bx)\coth(c+bx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{2b} \\
&\quad -\frac{\cosh(a-c)\coth(c+bx)\operatorname{csch}(c+bx)}{2b}+\cosh(a-c)\int \operatorname{csch}(c+bx)\,dx \\
&\quad -\frac{(i\sinh(a-c))\text{Subst}(\int 1\,dx,x,-i\operatorname{csch}(c+bx))}{b}+\int \sinh(a+bx)\,dx \\
&= -\frac{3\operatorname{arctanh}(\cosh(c+bx))\cosh(a-c)}{2b}+\frac{\cosh(a+bx)}{b} \\
&\quad -\frac{\cosh(a-c)\coth(c+bx)\operatorname{csch}(c+bx)}{2b}-\frac{\operatorname{csch}(c+bx)\sinh(a-c)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$
\int \cosh(a+bx)\coth^3(c+bx)\,dx
$$

$$
= \frac{-12\operatorname{arctanh}\left(\cosh(c)+\sinh(c)\tanh\left(\frac{bx}{2}\right)\right)\cosh(a-c)+\left(2\cosh(a-2c-bx)-5\cosh(a+bx)+\cosh(a-\right.}{4b}
$$

```
[In] Integrate[Cosh[a + b*x]*Coth[c + b*x]^3,x]
```

```
[Out] (-12*ArcTanh[Cosh[c] + Sinh[c]*Tanh[(b*x)/2]]*Cosh[a - c] + (2*Cosh[a - 2*c
 - b*x] - 5*Cosh[a + b*x] + Cosh[a + 2*c + 3*b*x])*Csch[c + b*x]^2)/(4*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(69) = 138.

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.12

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{e^{bx+a}\left(-3\,e^{2bx+4a+2c}+e^{2bx+2a+4c}+e^{4a}-3\,e^{2a+2c}\right)}{2b\left(-e^{2bx+2a+2c}+e^{2a}\right)^{2}}+\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{4b}+\frac{3\ln\left(e^{bx+a}-e^{a-c}\right)e^{-}}{4b}$

```
[In] int(cosh(b*x+a)*coth(b*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(b*x+a)/b+1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)*(-3*exp(2*b*x+4*a+2*c)+ex
p(2*b*x+2*a+4*c)+exp(4*a)-3*exp(2*a+2*c))/b/(-exp(2*b*x+2*a+2*c)+exp(2*a))^
2+3/4*ln(exp(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)+3/4*ln(exp(b*x+a)-exp(a-
c))/b*exp(-a-c)*exp(2*c)-3/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)-3
/4*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(69) = 138$.

Time = 0.28 (sec) , antiderivative size = 2372, normalized size of antiderivative = 32.49

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cosh(b*x + c)^6*cosh(-a + c)^2 + 2*(cosh(-a + c)^2 - 2*cosh(-a + c)*
sinh(-a + c) + sinh(-a + c)^2)*sinh(b*x + c)^6 + 12*(cosh(b*x + c)*cosh(-a
+ c)^2 - 2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) + cosh(b*x + c)*sinh(-a
+ c)^2)*sinh(b*x + c)^5 - 2*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^4 + 2*(15*
cosh(b*x + c)^2*cosh(-a + c)^2 + 5*(3*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
 5*cosh(-a + c)^2 - 10*(3*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh
(-a + c) + 2)*sinh(b*x + c)^4 + 8*(5*cosh(b*x + c)^3*cosh(-a + c)^2 + 5*(co
sh(b*x + c)^3 - cosh(b*x + c))*sinh(-a + c)^2 - (5*cosh(-a + c)^2 - 2)*cosh
(b*x + c) - 10*(cosh(b*x + c)^3*cosh(-a + c) - cosh(b*x + c)*cosh(-a + c))*
sinh(-a + c))*sinh(b*x + c)^3 + 2*(2*cosh(-a + c)^2 - 5)*cosh(b*x + c)^2 +
2*(15*cosh(b*x + c)^4*cosh(-a + c)^2 - 6*(5*cosh(-a + c)^2 - 2)*cosh(b*x +
c)^2 + (15*cosh(b*x + c)^4 - 30*cosh(b*x + c)^2 + 2)*sinh(-a + c)^2 + 2*cos
h(-a + c)^2 - 2*(15*cosh(b*x + c)^4*cosh(-a + c) - 30*cosh(b*x + c)^2*cosh(
-a + c) + 2*cosh(-a + c))*sinh(-a + c) - 5)*sinh(b*x + c)^2 + 2*(cosh(b*x +
 c)^6 - 5*cosh(b*x + c)^4 + 2*cosh(b*x + c)^2)*sinh(-a + c)^2 - 3*((cosh(-a
 + c)^2 + 1)*cosh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c
) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*s
inh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x
+ c))*sinh(b*x + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh
(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 -
cosh(-a + c)^2 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a
+ c) - 1)*sinh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*
cosh(b*x + c)^3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*
cosh(b*x + c) - 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a
 + c))*sinh(-a + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3
 + cosh(b*x + c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*
(cosh(-a + c)^2 + 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)
^2 + (5*cosh(b*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a +
 c)^2 - 2*(5*cosh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c)
+ cosh(-a + c))*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-
a + c) - 2*cosh(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(
-a + c))*log(cosh(b*x + c) + sinh(b*x + c) + 1) + 3*((cosh(-a + c)^2 + 1)*c
osh(b*x + c)^5 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a +
c)^2 + 1)*sinh(b*x + c)^5 - 5*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) -
cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sinh(b*x
```

```
 + c)^4 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1
)*cosh(b*x + c)^2 + (5*cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(5*cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) - 1)*sinh
(b*x + c)^3 + 2*(5*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^3 + (5*cosh(b*x + c)^
3 - 3*cosh(b*x + c))*sinh(-a + c)^2 - 3*(cosh(-a + c)^2 + 1)*cosh(b*x + c)
- 2*(5*cosh(b*x + c)^3*cosh(-a + c) - 3*cosh(b*x + c)*cosh(-a + c))*sinh(-a
 + c))*sinh(b*x + c)^2 + (cosh(b*x + c)^5 - 2*cosh(b*x + c)^3 + cosh(b*x +
c))*sinh(-a + c)^2 + (cosh(-a + c)^2 + 1)*cosh(b*x + c) + (5*(cosh(-a + c)^
2 + 1)*cosh(b*x + c)^4 - 6*(cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (5*cosh(b
*x + c)^4 - 6*cosh(b*x + c)^2 + 1)*sinh(-a + c)^2 + cosh(-a + c)^2 - 2*(5*c
osh(b*x + c)^4*cosh(-a + c) - 6*cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c)
)*sinh(-a + c) + 1)*sinh(b*x + c) - 2*(cosh(b*x + c)^5*cosh(-a + c) - 2*cos
h(b*x + c)^3*cosh(-a + c) + cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*log(c
osh(b*x + c) + sinh(b*x + c) - 1) + 4*(3*cosh(b*x + c)^5*cosh(-a + c)^2 - 2
*(5*cosh(-a + c)^2 - 2)*cosh(b*x + c)^3 + (3*cosh(b*x + c)^5 - 10*cosh(b*x
+ c)^3 + 2*cosh(b*x + c))*sinh(-a + c)^2 + (2*cosh(-a + c)^2 - 5)*cosh(b*x
+ c) - 2*(3*cosh(b*x + c)^5*cosh(-a + c) - 10*cosh(b*x + c)^3*cosh(-a + c)
+ 2*cosh(b*x + c)*cosh(-a + c))*sinh(-a + c))*sinh(b*x + c) - 4*(cosh(b*x +
 c)^6*cosh(-a + c) - 5*cosh(b*x + c)^4*cosh(-a + c) + 2*cosh(b*x + c)^2*cos
h(-a + c))*sinh(-a + c) + 2)/(b*cosh(b*x + c)^5*cosh(-a + c) + (b*cosh(-a +
 c) - b*sinh(-a + c))*sinh(b*x + c)^5 - 2*b*cosh(b*x + c)^3*cosh(-a + c) +
5*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x +
c)^4 + 2*(5*b*cosh(b*x + c)^2*cosh(-a + c) - b*cosh(-a + c) - (5*b*cosh(b*x
 + c)^2 - b)*sinh(-a + c))*sinh(b*x + c)^3 + b*cosh(b*x + c)*cosh(-a + c) +
 2*(5*b*cosh(b*x + c)^3*cosh(-a + c) - 3*b*cosh(b*x + c)*cosh(-a + c) - (5*
b*cosh(b*x + c)^3 - 3*b*cosh(b*x + c))*sinh(-a + c))*sinh(b*x + c)^2 + (5*b
*cosh(b*x + c)^4*cosh(-a + c) - 6*b*cosh(b*x + c)^2*cosh(-a + c) + b*cosh(-
a + c) - (5*b*cosh(b*x + c)^4 - 6*b*cosh(b*x + c)^2 + b)*sinh(-a + c))*sinh
(b*x + c) - (b*cosh(b*x + c)^5 - 2*b*cosh(b*x + c)^3 + b*cosh(b*x + c))*sin
h(-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx=\int \cosh\left(a+bx\right)\coth^3\left(bx+c\right)dx$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)*coth(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(69) = 138$.

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.52

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{4b} + \frac{3\left(e^{(2a)}+e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{4b} + \frac{e^{(-bx-a)}}{2b} - \frac{\left(5e^{(2a+2c)}-e^{(4c)}\right)e^{(-2bx-2a)}-\left(2e^{(4a)}-3e^{(2a+2c)}\right)e^{(-4bx-4a)}-e^{(4c)}}{2b\left(e^{(-bx-a+4c)}-2e^{(-3bx-a+2c)}+e^{(-5bx-a)}\right)}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3/4*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 3/4*(e^(2*a) +
e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + 1/2*e^(-b*x - a)/b - 1/2*((5*e^
(2*a + 2*c) - e^(4*c))*e^(-2*b*x - 2*a) - (2*e^(4*a) - 3*e^(2*a + 2*c))*e^(
-4*b*x - 4*a) - e^(4*c))/(b*(e^(-b*x - a + 4*c) - 2*e^(-3*b*x - a + 2*c) +
e^(-5*b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(69) = 138$.

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.29

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = -\frac{3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+a+c)}+e^{a}\right)-3\left(e^{(2a+c)}+e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+a+c)}-e^{a}\right|\right)+\frac{2(3e^{(3}}{}}{4b}$$

```
[In] integrate(cosh(b*x+a)*coth(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*(e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(e^(b*x + a + c) + e^a) - 3
*(e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + a + c) - e^a)) + 2*(
3*e^(3*b*x + 5*a + 2*c) - e^(3*b*x + 3*a + 4*c) - e^(b*x + 5*a) + 3*e^(b*x
+ 3*a + 2*c))/(e^(2*b*x + 2*a + 2*c) - e^(2*a))^2 - 2*e^(b*x + a) - 2*e^(-b
*x - a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\coth^3(c+bx)\,dx = \int \cosh(a+b\,x)\,\coth(c+b\,x)^3\,dx$$

```
[In] int(cosh(a + b*x)*coth(c + b*x)^3,x)
[Out] int(cosh(a + b*x)*coth(c + b*x)^3, x)
```

3.161 $\int \cosh(a+bx)\textbf{sech}(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = x\cosh(a-c)+\frac{\log(\cosh(c+bx))\sinh(a-c)}{b}$$

```
[Out] x*cosh(a-c)+ln(cosh(b*x+c))*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5746, 3556, 8}

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = \frac{\sinh(a-c)\log(\cosh(bx+c))}{b}+x\cosh(a-c)$$

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x],x]
```

```
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5746

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int 1\,dx + \sinh(a-c)\int \tanh(c+bx)\,dx \\ &= x\cosh(a-c) + \frac{\log(\cosh(c+bx))\sinh(a-c)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = x\cosh(a-c) + \frac{\log(\cosh(c+bx))\sinh(a-c)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x],x]
[Out] x*Cosh[a - c] + (Log[Cosh[c + b*x]]*Sinh[a - c])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(26) = 52$.

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.62

method	result
risch	$x\,\mathrm{e}^{a-c} - \mathrm{e}^{-a-c}\mathrm{e}^{2a}x + \mathrm{e}^{-a-c}\mathrm{e}^{2c}x - \frac{\mathrm{e}^{-a-c}\mathrm{e}^{2a}a}{b} + \frac{\mathrm{e}^{-a-c}\mathrm{e}^{2c}a}{b} + \frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2c}\right)\mathrm{e}^{-a-c}\mathrm{e}^{2a}}{2b} - \frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2}\right.}{2b}$

```
[In] int(cosh(b*x+a)*sech(b*x+c),x,method=_RETURNVERBOSE)
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x+exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a+1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)-1/2*ln(exp(2*b*x+2*a)+exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.31

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{2\,bx + \left(\cosh\left(-a+c\right)^2 - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^2 - 1\right)\log\left(\frac{2\,\cosh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2
 - 1)*log(2*cosh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c)
 - b*sinh(-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = \int \cosh\left(a+bx\right)\operatorname{sech}\left(bx+c\right)dx$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)*sech(b*x + c), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \cosh(a+bx)\operatorname{sech}(c+bx)\,dx = \frac{\left(e^{(2\,a)} - e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-2\,bx)} + e^{(2\,c)}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-2*b*x) + e^(2*c))/b + (b*x + a)*
e^(a - c)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = \frac{2\,bxe^{(-a+c)} + \left(e^{(2\,a+c)} - e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(e^{(2\,bx+2\,c)} + 1\right)}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c),x, algorithm="giac")
[Out] 1/2*(2*b*x*e^(-a + c) + (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(2*b*x +
 2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \cosh(a+bx)\text{sech}(c+bx)\,dx = x\,e^{c-a} + \frac{e^{2\,c-2\,a}\,\ln\left(e^{2\,a}\,e^{2\,b\,x} + e^{2\,a}\,e^{-2\,c}\right)\,\left(2\,b\,e^{3\,a-3\,c} - 2\,b\,e^{a-c}\right)}{4\,b^2}$$

```
[In] int(cosh(a + b*x)/cosh(c + b*x),x)
[Out] x*exp(c - a) + (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*c)
)*(2*b*exp(3*a - 3*c) - 2*b*exp(a - c)))/(4*b^2)
```

3.162 $\int \cosh(a+bx)\mathbf{sech}^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = \frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b} - \frac{\text{sech}(c+bx)\sinh(a-c)}{b}$$

```
[Out] arctan(sinh(b*x+c))*cosh(a-c)/b-sech(b*x+c)*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5746, 2686, 8, 3855}

$$\int \cosh(a+bx)\text{sech}^2(c+bx)\,dx = \frac{\cosh(a-c)\arctan(\sinh(bx+c))}{b} - \frac{\sinh(a-c)\text{sech}(bx+c)}{b}$$

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

```
[Out] (ArcTan[Sinh[c + b*x]]*Cosh[a - c])/b - (Sech[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5746

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \operatorname{sech}(c+bx)\,dx+\sinh(a-c)\int \operatorname{sech}(c+bx)\tanh(c+bx)\,dx\\ &= \frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}-\frac{\sinh(a-c)\text{Subst}(\int 1\,dx,x,\operatorname{sech}(c+bx))}{b}\\ &= \frac{\arctan(\sinh(c+bx))\cosh(a-c)}{b}-\frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\begin{aligned}&\int \cosh(a+bx)\operatorname{sech}^2(c+bx)\,dx\\ &=\frac{2\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh\left(\frac{bx}{2}\right)\sinh(c)+\cosh(c)\sinh\left(\frac{bx}{2}\right)\right)}{\cosh(c)\cosh\left(\frac{bx}{2}\right)-\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\cosh(a-c)}{b}\\ &\quad-\frac{\operatorname{sech}(c+bx)\sinh(a-c)}{b}\end{aligned}$$

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x]^2,x]
```

```
[Out] (2*ArcTan[((Cosh[c] - Sinh[c])*(Cosh[(b*x)/2]*Sinh[c] + Cosh[c]*Sinh[(b*x)/
2]))/(Cosh[c]*Cosh[(b*x)/2] - Cosh[(b*x)/2]*Sinh[c])]*Cosh[a - c])/b - (Sec
h[c + b*x]*Sinh[a - c])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 5.23

method	result
risch	$-\frac{e^{bx+a}\left(e^{2a}-e^{2c}\right)}{b\left(e^{2bx+2a+2c}+e^{2a}\right)}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{i\ln\left(e^{bx+a}+ie^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{i\ln\left(e^{bx+a}-ie^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{i\ln\left(e^{bx}\right.}{}$

```
[In] int(cosh(b*x+a)*sech(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*exp(b*x+a)*(exp(2*a)-exp(2*c))/(exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*I*ln(
exp(b*x+a)+I*exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*I*ln(exp(b*x+a)+I*exp(a-c))
/b*exp(-a-c)*exp(2*c)-1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*a)-
1/2*I*ln(exp(b*x+a)-I*exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(35) = 70$.

Time = 0.25 (sec) , antiderivative size = 405, normalized size of antiderivative = 11.57

$$\int \cosh(a+bx)\mathrm{sech}^2(c+bx)\,dx$$
$$=\frac{2\cosh\left(bx+c\right)\cosh\left(-a+c\right)\sinh\left(-a+c\right)-\cosh\left(bx+c\right)\sinh\left(-a+c\right)^2+\left(\left(\cosh\left(-a+c\right)^2+1\right)\cosh\left(b\right.\right.}{}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 +
 ((cosh(-a + c)^2 + 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*s
inh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 + 1)*s
inh(-a + c)^2 + cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a +
c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 + 1)*cosh(b*x + c))*sin
h(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) + cosh(-a + c))*sinh(-a + c) +
 1)*arctan(cosh(b*x + c) + sinh(b*x + c)) - (cosh(-a + c)^2 - 1)*cosh(b*x +
 c) - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*s
inh(b*x + c))/(b*cosh(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a
 + c))*sinh(b*x + c)^2 + b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) -
 b*cosh(b*x + c)*sinh(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 + b)*sinh
(-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\mathrm{sech}^2(c+bx)\,dx = \int \cosh\left(a+bx\right)\mathrm{sech}^2\left(bx+c\right)dx$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)**2,x)
[Out] Integral(cosh(a + b*x)*sech(b*x + c)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \cosh(a+bx)\mathrm{sech}^2(c+bx)\,dx = -\frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(-bx-c)}\right)e^{(-a-c)}}{b} - \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}+e^{(2c)}\right)}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="maxima")
[Out] -(e^(2*a) + e^(2*c))*arctan(e^(-b*x - c))*e^(-a - c)/b - (e^(2*a) - e^(2*c)
)*e^(-b*x - a)/(b*(e^(-2*b*x) + e^(2*c)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \cosh(a+bx)\mathrm{sech}^2(c+bx)\,dx = \frac{\left(e^{(2a)}+e^{(2c)}\right)\arctan\left(e^{(bx+c)}\right)e^{(-a-c)} - \frac{\left(e^{(bx+2a)}-e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}+1}}{b}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^2,x, algorithm="giac")
[Out] ((e^(2*a) + e^(2*c))*arctan(e^(b*x + c))*e^(-a - c) - (e^(b*x + 2*a) - e^(b
*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.23

$$\int \cosh(a+bx)\mathrm{sech}^2(c+bx)\,dx = \frac{\mathrm{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{b^2}+e^{2\,a}\,e^{-2\,c}\,\sqrt{b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(2\,e^{2\,a}\,e^{-2\,c}+e^{4\,a}\,e^{-4\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(2\,e^{2\,a-2\,c}+e^{4\,a-4\,c}+1\right)}}{\sqrt{b^2}} - \frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}-1\right)}{b\,\left(e^{2\,a-2\,c}+e^{2\,a+2\,b\,x}\right)}$$

```
[In] int(cosh(a + b*x)/cosh(c + b*x)^2,x)
```

```
[Out] (atan((exp(-a)*exp(2*c)*exp(b*x)*((b^2)^(1/2) + exp(2*a)*exp(-2*c)*(b^2)^(1
/2)))/(b*(exp(-2*a)*exp(2*c)*(2*exp(2*a)*exp(-2*c) + exp(4*a)*exp(-4*c) + 1
))^(1/2)))*(exp(2*c - 2*a)*(2*exp(2*a - 2*c) + exp(4*a - 4*c) + 1))^(1/2))/
(b^2)^(1/2) - (exp(a + b*x)*(exp(2*a - 2*c) - 1))/(b*(exp(2*a - 2*c) + exp(
2*a + 2*b*x)))
```

3.163 $\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx=-\frac{\operatorname{sech}^2(c+bx)\sinh(a-c)}{2b}+\frac{\cosh(a-c)\tanh(c+bx)}{b}$$

```
[Out] -1/2*sech(b*x+c)^2*sinh(a-c)/b+cosh(a-c)*tanh(b*x+c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5746, 2686, 30, 3852, 8}

$$\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx=\frac{\cosh(a-c)\tanh(bx+c)}{b}-\frac{\sinh(a-c)\operatorname{sech}^2(bx+c)}{2b}$$

```
[In] Int[Cosh[a + b*x]*Sech[c + b*x]^3,x]
[Out] -1/2*(Sech[c + b*x]^2*Sinh[a - c])/b + (Cosh[a - c]*Tanh[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5746

```
Int[Cosh[v_]*Sech[w_]^(n_.), x_Symbol] :> Dist[Sinh[v - w], Int[Tanh[w]*Sec
h[w]^(n - 1), x], x] + Dist[Cosh[v - w], Int[Sech[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cosh(a-c)\int \operatorname{sech}^2(c+bx)\,dx+\sinh(a-c)\int \operatorname{sech}^2(c+bx)\tanh(c+bx)\,dx\\
&= \frac{(i\cosh(a-c))\text{Subst}(\int 1\,dx,x,-i\tanh(c+bx))}{b}-\frac{\sinh(a-c)\text{Subst}(\int x\,dx,x,\operatorname{sech}(c+bx))}{b}\\
&= -\frac{\operatorname{sech}^2(c+bx)\sinh(a-c)}{2b}+\frac{\cosh(a-c)\tanh(c+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx=-\frac{\operatorname{sech}(c)\operatorname{sech}^2(c+bx)(\sinh(a)-\cosh(a-c)\sinh(c+2bx))}{2b}$$

```
[In] Integrate[Cosh[a + b*x]*Sech[c + b*x]^3,x]
[Out] -1/2*(Sech[c]*Sech[c + b*x]^2*(Sinh[a] - Cosh[a - c]*Sinh[c + 2*b*x]))/b
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\sinh(2bx+a+c)}{b(1+\cosh(2bx+2c))}$	26
risch	$-\frac{\left(2\,e^{2bx+2a+2c}+e^{2a}+e^{2c}\right)e^{3a-c}}{\left(e^{2bx+2a+2c}+e^{2a}\right)^{2}b}$	56

```
[In] int(cosh(b*x+a)*sech(b*x+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*sinh(2*b*x+a+c)/(1+cosh(2*b*x+2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(36) = 72.

Time = 0.25 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.53

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx =$$

$$-\frac{}{b\cosh{(bx+c)}^3\cosh{(-a+c)}^2+3\,b\cosh{(bx+c)}\cosh{(-a+c)}^2+\left(b\cosh{(-a+c)}^2-b\sinh{(-a+c)}\right.}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="fricas")
[Out] -2*(2*cosh(b*x + c)*cosh(-a + c) - cosh(b*x + c)*sinh(-a + c) - sinh(b*x +
c)*sinh(-a + c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 + 3*b*cosh(b*x + c)*cosh
(-a + c)^2 + (b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*c
osh(b*x + c)*cosh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)
^2 - (b*cosh(b*x + c)^3 + 3*b*cosh(b*x + c))*sinh(-a + c)^2 + (3*b*cosh(b*x
 + c)^2*cosh(-a + c)^2 + b*cosh(-a + c)^2 - (3*b*cosh(b*x + c)^2 + b)*sinh(
-a + c)^2)*sinh(b*x + c))
```

Sympy [F]

$$\int \cosh(a+bx)\text{sech}^3(c+bx)\,dx = \int \cosh{(a+bx)}\,\text{sech}^3{(bx+c)}\,dx$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)**3,x)
[Out] Integral(cosh(a + b*x)*sech(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.13

$$\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx = \frac{2\,e^{(-2\,bx+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})} + \frac{e^{(2\,a+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})} + \frac{e^{(5\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)}+e^{(-4\,bx+a)}+e^{(a+4\,c)})}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c
))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*
c))) + e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) + e^(-4*b*x + a) + e^(a + 4*c)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)}+e^{(2\,a)}+e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)}+1\right)^2}$$

```
[In] integrate(cosh(b*x+a)*sech(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) + e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\operatorname{sech}^3(c+bx)\,dx = \int \frac{\cosh(a+b\,x)}{{\cosh\left(c+b\,x\right)}^3}\,dx$$

```
[In] int(cosh(a + b*x)/cosh(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)/cosh(c + b*x)^3, x)
```

3.164 $\int \cosh(a+bx)\mathbf{csch}(c+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cosh(a+bx)\text{csch}(c+bx)\,dx = \frac{\cosh(a-c)\log(\sinh(c+bx))}{b} + x\sinh(a-c)$$

```
[Out] cosh(a-c)*ln(sinh(b*x+c))/b+x*sinh(a-c)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5744, 3556, 8}

$$\int \cosh(a+bx)\text{csch}(c+bx)\,dx = \frac{\cosh(a-c)\log(\sinh(bx+c))}{b} + x\sinh(a-c)$$

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x],x]
```

```
[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5744

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(a-c)\int \coth(c+bx)\,dx + \sinh(a-c)\int 1\,dx \\ &= \frac{\cosh(a-c)\log(\sinh(c+bx))}{b} + x\sinh(a-c)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx = \frac{\cosh(a-c)\log(\sinh(c+bx))}{b} + x\sinh(a-c)$$

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x],x]
[Out] (Cosh[a - c]*Log[Sinh[c + b*x]])/b + x*Sinh[a - c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 5.85

method	result
risch	$x\,e^{a-c} - e^{-a-c}e^{2a}x - e^{-a-c}e^{2c}x - \frac{e^{-a-c}e^{2a}a}{b} - \frac{e^{-a-c}e^{2c}a}{b} + \frac{\ln(e^{2bx+2a}-e^{2a-2c})e^{-a-c}e^{2a}}{2b} + \frac{\ln(e^{2bx+2a}-e^{2a-2c})}{2b}$

```
[In] int(cosh(b*x+a)*csch(b*x+c),x,method=_RETURNVERBOSE)
[Out] x*exp(a-c)-exp(-a-c)*exp(2*a)*x-exp(-a-c)*exp(2*c)*x-1/b*exp(-a-c)*exp(2*a)
*a-1/b*exp(-a-c)*exp(2*c)*a+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)
*exp(2*a)+1/2*ln(exp(2*b*x+2*a)-exp(2*a-2*c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\,bx - \left(\cosh\left(-a+c\right)^{2} - 2\,\cosh\left(-a+c\right)\sinh\left(-a+c\right) + \sinh\left(-a+c\right)^{2} + 1\right)\log\left(\frac{2\,\sinh(bx+c)}{\cosh(bx+c)-\sinh(bx+c)}\right)}{2\left(b\cosh\left(-a+c\right) - b\sinh\left(-a+c\right)\right)}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="fricas")
[Out] -1/2*(2*b*x - (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^
2 + 1)*log(2*sinh(b*x + c)/(cosh(b*x + c) - sinh(b*x + c))))/(b*cosh(-a + c
) - b*sinh(-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx = \int \cosh\left(a+bx\right)\operatorname{csch}\left(bx+c\right)dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x)
[Out] Integral(cosh(a + b*x)*csch(b*x + c), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx = \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} + e^{c}\right)}{2\,b} + \frac{\left(e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}\log\left(e^{(-bx)} - e^{c}\right)}{2\,b} + \frac{(bx+a)e^{(a-c)}}{b}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="maxima")
[Out] 1/2*(e^(2*a) + e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) + e
^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (b*x + a)*e^(a - c)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx = -\frac{2\,bxe^{(-a+c)} - \left(e^{(2\,a+c)} + e^{(3\,c)}\right)e^{(-a-2\,c)}\log\left(\left|e^{(2\,bx+2\,c)} - 1\right|\right)}{2\,b}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*e^(-a + c) - (e^(2*a + c) + e^(3*c))*e^(-a - 2*c)*log(abs(e^(2*
b*x + 2*c) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \cosh(a+bx)\operatorname{csch}(c+bx)\,dx = \frac{e^{2\,c-2\,a}\,\ln\left(e^{2\,a}\,e^{2\,b\,x} - e^{2\,a}\,e^{-2\,c}\right)\,\left(2\,b\,e^{3\,a-3\,c} + 2\,b\,e^{a-c}\right)}{4\,b^2} - x\,e^{c-a}$$

```
[In] int(cosh(a + b*x)/sinh(c + b*x),x)
```

```
[Out] (exp(2*c - 2*a)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*c))*(2*b*exp(3*a
- 3*c) + 2*b*exp(a - c)))/(4*b^2) - x*exp(c - a)
```

3.165 $\int \cosh(a+bx)\mathbf{csch}^2(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{\text{arctanh}(\cosh(c+bx))\sinh(a-c)}{b}$$

```
[Out] -cosh(a-c)*csch(b*x+c)/b-arctanh(cosh(b*x+c))*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5744, 2686, 8, 3855}

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = -\frac{\sinh(a-c)\text{arctanh}(\cosh(bx+c))}{b} - \frac{\cosh(a-c)\text{csch}(bx+c)}{b}$$

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - (ArcTanh[Cosh[c + b*x]]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5744

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cosh(a-c)\int \coth(c+bx)\text{csch}(c+bx)\,dx + \sinh(a-c)\int \text{csch}(c+bx)\,dx \\
&= -\frac{\text{arctanh}(\cosh(c+bx))\sinh(a-c)}{b} - \frac{(i\cosh(a-c))\text{Subst}(\int 1\,dx, x, -i\text{csch}(c+bx))}{b} \\
&= -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} - \frac{\text{arctanh}(\cosh(c+bx))\sinh(a-c)}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.50

$$\begin{aligned}
&\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx \\
&= -\frac{\cosh(a-c)\text{csch}(c+bx)}{b} \\
&\quad - \frac{2i\arctan\left(\frac{(\cosh(c)-\sinh(c))\left(\cosh(c)\cosh\left(\frac{bx}{2}\right)+\sinh(c)\sinh\left(\frac{bx}{2}\right)\right)}{i\cosh(c)\cosh\left(\frac{bx}{2}\right)-i\cosh\left(\frac{bx}{2}\right)\sinh(c)}\right)\sinh(a-c)}{b}
\end{aligned}$$

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x]^2,x]
```

```
[Out] -((Cosh[a - c]*Csch[c + b*x])/b) - ((2*I)*ArcTan[((Cosh[c] - Sinh[c])*(Cosh
[c]*Cosh[(b*x)/2] + Sinh[c]*Sinh[(b*x)/2]))/(I*Cosh[c]*Cosh[(b*x)/2] - I*Co
sh[(b*x)/2]*Sinh[c])]*Sinh[a - c])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(36) = 72$.

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.72

method	result
risch	$\frac{e^{bx+a}\left(e^{2a}+e^{2c}\right)}{b\left(-e^{2bx+2a+2c}+e^{2a}\right)}+\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2a}}{2b}-\frac{\ln\left(e^{bx+a}-e^{a-c}\right)e^{-a-c}e^{2c}}{2b}-\frac{\ln\left(e^{bx+a}+e^{a-c}\right)e^{-a-c}e^{2a}}{2b}+\frac{\ln\left(e^{bx+a}+e^{a-c}\right)\ldots}{}$

```
[In] int(cosh(b*x+a)*csch(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*exp(b*x+a)*(exp(2*a)+exp(2*c))/(-exp(2*b*x+2*a+2*c)+exp(2*a))+1/2*ln(ex
p(b*x+a)-exp(a-c))/b*exp(-a-c)*exp(2*a)-1/2*ln(exp(b*x+a)-exp(a-c))/b*exp(-
a-c)*exp(2*c)-1/2*ln(exp(b*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*a)+1/2*ln(exp(b
*x+a)+exp(a-c))/b*exp(-a-c)*exp(2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 617, normalized size of antiderivative = 17.14

$$\int \cosh(a+bx)\operatorname{csch}^2(c+bx)\,dx = \frac{4\cosh(bx+c)\cosh(-a+c)\sinh(-a+c)-2\cosh(bx+c)\sinh(-a+c)^2-2\left(\cosh(-a+c)^2+1\right)\cos\ldots}{}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - 2*cosh(b*x + c)*sinh(-a +
c)^2 - 2*(cosh(-a + c)^2 + 1)*cosh(b*x + c) - ((cosh(-a + c)^2 - 1)*cosh(b*
x + c)^2 + (cosh(-a + c)^2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 -
 1)*sinh(b*x + c)^2 + (cosh(b*x + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2
 - 2*(2*cosh(b*x + c)*cosh(-a + c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c
)^2 - (cosh(-a + c)^2 - 1)*cosh(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^
2*cosh(-a + c) - cosh(-a + c))*sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b
*x + c) + 1) + ((cosh(-a + c)^2 - 1)*cosh(b*x + c)^2 + (cosh(-a + c)^2 - 2*
cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 - 1)*sinh(b*x + c)^2 + (cosh(b*x
 + c)^2 - 1)*sinh(-a + c)^2 - cosh(-a + c)^2 - 2*(2*cosh(b*x + c)*cosh(-a +
 c)*sinh(-a + c) - cosh(b*x + c)*sinh(-a + c)^2 - (cosh(-a + c)^2 - 1)*cosh
(b*x + c))*sinh(b*x + c) - 2*(cosh(b*x + c)^2*cosh(-a + c) - cosh(-a + c))*
sinh(-a + c) + 1)*log(cosh(b*x + c) + sinh(b*x + c) - 1) - 2*(cosh(-a + c)^
2 - 2*cosh(-a + c)*sinh(-a + c) + sinh(-a + c)^2 + 1)*sinh(b*x + c))/(b*cos
h(b*x + c)^2*cosh(-a + c) + (b*cosh(-a + c) - b*sinh(-a + c))*sinh(b*x + c)
^2 - b*cosh(-a + c) + 2*(b*cosh(b*x + c)*cosh(-a + c) - b*cosh(b*x + c)*sin
h(-a + c))*sinh(b*x + c) - (b*cosh(b*x + c)^2 - b)*sinh(-a + c))
```

Sympy [F]

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = \int \cosh(a+bx)\,\text{csch}^2(bx+c)\,dx$$

[In] integrate(cosh(b*x+a)*csch(b*x+c)**2,x)

[Out] Integral(cosh(a + b*x)*csch(b*x + c)**2, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(36) = 72$.

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = -\frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}+e^{c}\right)}{2b} + \frac{\left(e^{(2a)}-e^{(2c)}\right)e^{(-a-c)}\log\left(e^{(-bx)}-e^{c}\right)}{2b} + \frac{\left(e^{(2a)}+e^{(2c)}\right)e^{(-bx-a)}}{b\left(e^{(-2bx)}-e^{(2c)}\right)}$$

[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="maxima")

[Out] -1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) + e^c)/b + 1/2*(e^(2*a) - e^(2*c))*e^(-a - c)*log(e^(-b*x) - e^c)/b + (e^(2*a) + e^(2*c))*e^(-b*x - a)/(b*(e^(-2*b*x) - e^(2*c)))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.94

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = -\frac{\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(e^{(bx+c)}+1\right)-\left(e^{(2a+c)}-e^{(3c)}\right)e^{(-a-2c)}\log\left(\left|e^{(bx+c)}-1\right|\right)+\frac{2\left(e^{(bx+2a)}+e^{(bx+2c)}\right)e^{(-a)}}{e^{(2bx+2c)}-1}}{2b}$$

[In] integrate(cosh(b*x+a)*csch(b*x+c)^2,x, algorithm="giac")

[Out] -1/2*((e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(e^(b*x + c) + 1) - (e^(2*a + c) - e^(3*c))*e^(-a - 2*c)*log(abs(e^(b*x + c) - 1)) + 2*(e^(b*x + 2*a) + e^(b*x + 2*c))*e^(-a)/(e^(2*b*x + 2*c) - 1))/b

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \cosh(a+bx)\text{csch}^2(c+bx)\,dx = \frac{\text{atan}\left(\frac{e^{-a}\,e^{2\,c}\,e^{b\,x}\,\left(\sqrt{-b^2}-e^{2\,a}\,e^{-2\,c}\,\sqrt{-b^2}\right)}{b\,\sqrt{e^{-2\,a}\,e^{2\,c}\,\left(e^{4\,a}\,e^{-4\,c}-2\,e^{2\,a}\,e^{-2\,c}+1\right)}}\right)\,\sqrt{e^{2\,c-2\,a}\,\left(e^{4\,a-4\,c}-2\,e^{2\,a-2\,c}+1\right)}}{\sqrt{-b^2}} + \frac{e^{a+b\,x}\,\left(e^{2\,a-2\,c}+1\right)}{b\,\left(e^{2\,a-2\,c}-e^{2\,a+2\,b\,x}\right)}$$

```
[In] int(cosh(a + b*x)/sinh(c + b*x)^2,x)
```

```
[Out] (atan((exp(-a)*exp(2*c)*exp(b*x)*((-b^2)^(1/2) - exp(2*a)*exp(-2*c)*(-b^2)^
(1/2)))/(b*(exp(-2*a)*exp(2*c)*(exp(4*a)*exp(-4*c) - 2*exp(2*a)*exp(-2*c) +
 1))^(1/2)))*(exp(2*c - 2*a)*(exp(4*a - 4*c) - 2*exp(2*a - 2*c) + 1))^(1/2)
)/(-b^2)^(1/2) + (exp(a + b*x)*(exp(2*a - 2*c) + 1))/(b*(exp(2*a - 2*c) - e
xp(2*a + 2*b*x)))
```

3.166 $\int \cosh(a+bx)\mathbf{csch}^3(c+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \cosh(a+bx)\text{csch}^3(c+bx)\,dx = -\frac{\cosh(a-c)\text{csch}^2(c+bx)}{2b} - \frac{\coth(c+bx)\sinh(a-c)}{b}$$

```
[Out] -1/2*cosh(a-c)*csch(b*x+c)^2/b-coth(b*x+c)*sinh(a-c)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5744, 2686, 30, 3852, 8}

$$\int \cosh(a+bx)\text{csch}^3(c+bx)\,dx = -\frac{\cosh(a-c)\text{csch}^2(bx+c)}{2b} - \frac{\sinh(a-c)\coth(bx+c)}{b}$$

```
[In] Int[Cosh[a + b*x]*Csch[c + b*x]^3,x]
```

```
[Out] -1/2*(Cosh[a - c]*Csch[c + b*x]^2)/b - (Coth[c + b*x]*Sinh[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5744

```
Int[Cosh[v_]*Csch[w_]^(n_.), x_Symbol] :> Dist[Cosh[v - w], Int[Coth[w]*Csc
h[w]^(n - 1), x], x] + Dist[Sinh[v - w], Int[Csch[w]^(n - 1), x], x] /; GtQ
[n, 0] && NeQ[w, v] && FreeQ[v - w, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \cosh(a-c)\int \coth(c+bx)\text{csch}^2(c+bx)\,dx+\sinh(a-c)\int \text{csch}^2(c+bx)\,dx\\
&= \frac{\cosh(a-c)\text{Subst}\left(\int x\,dx,x,-i\text{csch}(c+bx)\right)}{b}-\frac{(i\sinh(a-c))\text{Subst}\left(\int 1\,dx,x,-i\coth(c+bx)\right)}{b}\\
&= -\frac{\cosh(a-c)\text{csch}^2(c+bx)}{2b}-\frac{\coth(c+bx)\sinh(a-c)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cosh(a+bx)\text{csch}^3(c+bx)\,dx=-\frac{\text{csch}(c)\text{csch}^2(c+bx)(\sinh(a)-\cosh(c+2bx)\sinh(a-c))}{2b}$$

```
[In] Integrate[Cosh[a + b*x]*Csch[c + b*x]^3,x]
[Out] -1/2*(Csch[c]*Csch[c + b*x]^2*(Sinh[a] - Cosh[c + 2*b*x]*Sinh[a - c]))/b
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$-\frac{\operatorname{sech}\left(\frac{bx}{2}+\frac{c}{2}\right)^{2} \operatorname{csch}\left(\frac{bx}{2}+\frac{c}{2}\right)^{2} \cosh(2bx+a+c)}{8b}$	36
risch	$\frac{\left(-2\,{\mathrm e}^{2bx+2a+2c}+{\mathrm e}^{2a}-{\mathrm e}^{2c}\right){\mathrm e}^{3a-c}}{\left(-{\mathrm e}^{2bx+2a+2c}+{\mathrm e}^{2a}\right)^{2} b}$	59

```
[In] int(cosh(b*x+a)*csch(b*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/8/b*sech(1/2*b*x+1/2*c)^2*csch(1/2*b*x+1/2*c)^2*cosh(2*b*x+a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 6.23

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx =$$

$$-\frac{}{b\cosh(bx+c)^3\cosh(-a+c)^2 - b\cosh(bx+c)\cosh(-a+c)^2 + \left(b\cosh(-a+c)^2 - b\sinh(-a+c)^2\right) s}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="fricas")
[Out] -2*(cosh(b*x + c)*cosh(-a + c) + (cosh(-a + c) - 2*sinh(-a + c))*sinh(b*x +
 c))/(b*cosh(b*x + c)^3*cosh(-a + c)^2 - b*cosh(b*x + c)*cosh(-a + c)^2 + (
b*cosh(-a + c)^2 - b*sinh(-a + c)^2)*sinh(b*x + c)^3 + 3*(b*cosh(b*x + c)*c
osh(-a + c)^2 - b*cosh(b*x + c)*sinh(-a + c)^2)*sinh(b*x + c)^2 - (b*cosh(b
*x + c)^3 - b*cosh(b*x + c))*sinh(-a + c)^2 + 3*(b*cosh(b*x + c)^2*cosh(-a
+ c)^2 - b*cosh(-a + c)^2 - (b*cosh(b*x + c)^2 - b)*sinh(-a + c)^2)*sinh(b*
x + c))
```

Sympy [F]

$$\int \cosh(a+bx)\operatorname{csch}^3(c+bx)\,dx = \int \cosh(a+bx)\operatorname{csch}^3(bx+c)\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)**3,x)
[Out] Integral(cosh(a + b*x)*csch(b*x + c)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(37) = 74.

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.38

$$\int \cosh(a+bx)\mathrm{csch}^3(c+bx)\,dx = \frac{2\,e^{(-2\,bx+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)} - e^{(-4\,bx+a)} - e^{(a+4\,c)})} + \frac{e^{(2\,a+3\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)} - e^{(-4\,bx+a)} - e^{(a+4\,c)})} - \frac{e^{(5\,c)}}{b(2\,e^{(-2\,bx+a+2\,c)} - e^{(-4\,bx+a)} - e^{(a+4\,c)})}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*b*x + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c
))) + e^(2*a + 3*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*
c))) - e^(5*c)/(b*(2*e^(-2*b*x + a + 2*c) - e^(-4*b*x + a) - e^(a + 4*c)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \cosh(a+bx)\mathrm{csch}^3(c+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a+2\,c)} - e^{(2\,a)} + e^{(2\,c)}\right)e^{(-a-c)}}{b\left(e^{(2\,bx+2\,c)} - 1\right)^2}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -(2*e^(2*b*x + 2*a + 2*c) - e^(2*a) + e^(2*c))*e^(-a - c)/(b*(e^(2*b*x + 2*
c) - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\mathrm{csch}^3(c+bx)\,dx = \int \frac{\cosh(a+b\,x)}{{\sinh\left(c+b\,x\right)}^3}\,dx$$

```
[In] int(cosh(a + b*x)/sinh(c + b*x)^3,x)
```

```
[Out] int(cosh(a + b*x)/sinh(c + b*x)^3, x)
```

3.167 $\int \sinh(a+bx)\sinh(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \sinh(a+bx)\sinh(c+dx)\,dx=-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

```
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5732, 2717}

$$\int \sinh(a+bx)\sinh(c+dx)\,dx=\frac{\sinh(a+x(b+d)+c)}{2(b+d)}-\frac{\sinh(a+x(b-d)-c)}{2(b-d)}$$

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x],x]
```

```
[Out] -1/2*Sinh[a - c + (b - d)*x]/(b - d) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
```

```
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{2}\cosh(a-c+(b-d)x)+\frac{1}{2}\cosh(a+c+(b+d)x)\right)dx\\ &= -\left(\frac{1}{2}\int \cosh(a-c+(b-d)x)\,dx\right)+\frac{1}{2}\int \cosh(a+c+(b+d)x)\,dx\\ &= -\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = -\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x],x]
[Out] -1/2*Sinh[a - c + (b - d)*x]/(b - d) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$	40
parallelrisch	$\frac{(-b-d)\sinh(a-c+(b-d)x)+\sinh(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$	52
risch	$\frac{\left(b\,e^{2bx+2a}-e^{2bx+2a}d+b+d\right)e^{-bx+dx-a+c}}{4(b+d)(b-d)}-\frac{\left(b\,e^{2bx+2a}+e^{2bx+2a}d+b-d\right)e^{-bx-dx-a-c}}{4(b+d)(b-d)}$	112

```
[In] int(sinh(b*x+a)*sinh(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = -\frac{d\cosh(dx+c)\sinh(bx+a) - b\cosh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2 - (b^2-d^2)\sinh(bx+a)^2}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(d*cosh(d*x + c)*sinh(b*x + a) - b*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^
2)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(32) = 64.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \begin{cases} x\sinh(a)\sinh(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sinh(a-dx)\sinh(c+dx)}{2} + \frac{x\cosh(a-dx)\cosh(c+dx)}{2} + \frac{\sinh(a-dx)\cosh(c+dx)}{2d} & \text{for } b=-d \\ \frac{x\sinh(a+dx)\sinh(c+dx)}{2} - \frac{x\cosh(a+dx)\cosh(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{b\sinh(c+dx)\cosh(a+bx)}{b^2-d^2} - \frac{d\sinh(a+bx)\cosh(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(c
 + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 + sinh(a - d*x)*cosh(c + d*x)/(
2*d), Eq(b, -d)), (x*sinh(a + d*x)*sinh(c + d*x)/2 - x*cosh(a + d*x)*cosh(c
 + d*x)/2 + sinh(c + d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*sinh(c + d*x)*
cosh(a + b*x)/(b**2 - d**2) - d*sinh(a + b*x)*cosh(c + d*x)/(b**2 - d**2),
True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{4(b+d)} - \frac{e^{(bx-dx+a-c)}}{4(b-d)} + \frac{e^{(-bx+dx-a+c)}}{4(b-d)} - \frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) - 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/4
*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sinh(a+bx)\sinh(c+dx)\,dx = \frac{b\cosh(a+b\,x)\,\sinh(c+d\,x) - d\cosh(c+d\,x)\,\sinh(a+b\,x)}{b^2-d^2}$$

```
[In] int(sinh(a + b*x)*sinh(c + d*x),x)
```

```
[Out] (b*cosh(a + b*x)*sinh(c + d*x) - d*cosh(c + d*x)*sinh(a + b*x))/(b^2 - d^2)
```

3.168 $\int \sinh(a+bx)\sinh^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = -\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

```
[Out] -1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5732, 2718}

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cosh(a+bx)}{2b}$$

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
```

```
[Out] -1/2*Cosh[a + b*x]/b + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
 2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(-\frac{1}{2}\sinh(a+bx)+\frac{1}{4}\sinh(a-2c+(b-2d)x)+\frac{1}{4}\sinh(a+2c+(b+2d)x)\right) dx$$
$$= \frac{1}{4}\int \sinh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sinh(a+2c+(b+2d)x)\,dx-\frac{1}{2}\int \sinh(a+bx)\,dx$$
$$= -\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{1}{4}\left(-\frac{2\cosh(a)\cosh(bx)}{b}+\frac{\cosh(a-2c+bx-2dx)}{b-2d}+\frac{\cosh(a+2c+bx+2dx)}{b+2d}-\frac{2\sinh(a)\sinh(bx)}{b}\right)$$

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x]^2,x]
[Out] ((-2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a
+ 2*c + b*x + 2*d*x]/(b + 2*d) - (2*Sinh[a]*Sinh[b*x])/b)/4
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)x)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\cosh(a-2c+(b-2d)x)+b(b-2d)\cosh(a+2c+(b+2d)x)+(-2b^2+8d^2)\cosh(bx+a)-8d^2}{4b^3-16b\,d^2}$
risch	$-\frac{e^{bx+a}}{4b}-\frac{e^{-bx-a}}{4b}+\frac{(b\,e^{2bx+2a}-2\,e^{2bx+2a}d+b+2d)\,e^{-bx+2dx-a+2c}}{8(b+2d)(b-2d)}+\frac{(b\,e^{2bx+2a}+2\,e^{2bx+2a}d+b-2d)\,e^{-bx-2dx-a-2c}}{8(b+2d)(b-2d)}$

```
[In] int(sinh(b*x+a)*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)
*x)/(b+2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{b^2\cosh(bx+a)\cosh(dx+c)^2 - 4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c) + b^2\cosh(bx+a)\sinh(dx+}{2\left((b^3-4\,bd^2)\cosh(bx+a)^2 - (b^3-4\,bd^2)\sinh(bx+a)^2\right)}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)*
sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x
+ a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(49) = 98$.

Time = 0.72 (sec) , antiderivative size = 405, normalized size of antiderivative = 6.53

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \begin{cases} x\sinh(a)\sinh^2(c) \\ \left(\frac{x\sinh^2(c+dx)}{2} - \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4} + \frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2} - \frac{\sinh(a-2dx)\sinh(c+dx)\cosh(c}{4d} \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4} - \frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2} - \frac{\sinh(a+2dx)\sinh(c+dx)\cosh(c}{4d} \\ \frac{b^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} - \frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} - \frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} + \frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2
/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), Eq(
b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c +
d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 - sinh(a - 2*d*
x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)**2*cosh(a - 2*d*x)/(2*
d), Eq(b, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a + 2*d*x)
*cosh(c + d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/2 - sin
```

```
h(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*cosh(a +
2*d*x)/(2*d), Eq(b, 2*d)), (b**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b
*d**2) - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d**2)
- 2*d**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cosh(a +
 b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} - \frac{e^{(bx+a)}}{4\,b} + \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} + \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} - \frac{e^{(-bx-a)}}{4\,b}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) - 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \sinh(a+bx)\sinh^2(c+dx)\,dx$$
$$=\frac{b^2\left(\cosh(a+b\,x)-\cosh(a+b\,x)\,\cosh(c+d\,x)^2\right)-2\,d^2\cosh(a+b\,x)+2\,b\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\mathrm{si}}{4\,b\,d^2-b^3}$$

```
[In] int(sinh(a + b*x)*sinh(c + d*x)^2,x)
```

```
[Out] (b^2*(cosh(a + b*x) - cosh(a + b*x)*cosh(c + d*x)^2) - 2*d^2*cosh(a + b*x)
+ 2*b*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.169 $\int \sinh(a+bx)\sinh^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = -\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)} -\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8(b+3d)}$$

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5732, 2717}

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = -\frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sinh(a+x(b-d)-c)}{8(b-d)} -\frac{3\sinh(a+x(b+d)+c)}{8(b+d)}+\frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[In] Int[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

```
[Out] -1/8*Sinh[a - 3*c + (b - 3*d)*x]/(b - 3*d) + (3*Sinh[a - c + (b - d)*x])/(8
*(b - d)) - (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3
*d)*x]/(8*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{8}\cosh(a-3c+(b-3d)x)+\frac{3}{8}\cosh(a-c+(b-d)x)\right.\\ &\qquad\left.-\frac{3}{8}\cosh(a+c+(b+d)x)+\frac{1}{8}\cosh(a+3c+(b+3d)x)\right)\,dx\\ &= -\left(\frac{1}{8}\int \cosh(a-3c+(b-3d)x)\,dx\right)+\frac{1}{8}\int \cosh(a+3c+(b+3d)x)\,dx\\ &\quad+\frac{3}{8}\int \cosh(a-c+(b-d)x)\,dx-\frac{3}{8}\int \cosh(a+c+(b+d)x)\,dx\\ &= -\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}\\ &\quad-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8(b+3d)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \frac{1}{8}\left(-\frac{\sinh(a-3c+bx-3dx)}{b-3d}+\frac{3\sinh(a-c+bx-dx)}{b-d}\right.$$
$$\left.+\frac{\sinh(a+3c+bx+3dx)}{b+3d}-\frac{3\sinh(a+c+(b+d)x)}{b+d}\right)$$

```
[In] Integrate[Sinh[a + b*x]*Sinh[c + d*x]^3,x]
```

```
[Out] (-(Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sinh[a - c + b*x - d*x])/(b
- d) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sinh[a + c + (b + d)*x])/
(b + d))/8
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{(b\,\mathrm{e}^{2bx+2a}-3\,\mathrm{e}^{2bx+2a}d+b+3d)\mathrm{e}^{-bx+3dx-a+3c}}{16(b+3d)(b-3d)}-\frac{3(b\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{2bx+2a}d+b+d)\mathrm{e}^{-bx+dx-a+c}}{16(b+d)(b-d)}+\frac{3(b\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}d+b}{16(b+d)(b-}$
parallelrisch	$\frac{-12d^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+12d^2b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+(-24b^2d+36d^3)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{(b-d)(b+3d)(b-3d)(b+d)\left(\tanh\right.}$

```
[In] int(sinh(b*x+a)*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)-3/8*sinh(a+c
+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(83) = 166.

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.40

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = -\frac{9\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-\left(b^3-bd^2\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^3+3\left(\left(b^2\right.\right.}{4\left(\left(b^4-10\,b^2\right.\right.}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*(b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3 -
b*d^2)*cosh(b*x + a)*sinh(d*x + c)^3 + 3*((b^2*d - d^3)*cosh(d*x + c)^3 - (
b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b^3 - b*d^2)*cosh(b*x + a
)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a))*sinh(d*x + c))/((b^4 - 1
0*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x +
a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(76) = 152$.

Time = 1.91 (sec) , antiderivative size = 918, normalized size of antiderivative = 10.09

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sinh(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*s
inh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 +
3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)*co
sh(c + d*x)**3/8 - sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + s
inh(a - 3*d*x)*cosh(c + d*x)**3/(24*d) - 3*sinh(c + d*x)**3*cosh(a - 3*d*x)
/(8*d), Eq(b, -3*d)), (3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a -
d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)*
cosh(c + d*x)/8 - 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a - d*x)*si
nh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*sinh(a - d*x)*cosh(c + d*x)**3/(8*d)
 + sinh(c + d*x)**3*cosh(a - d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)*sin
h(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*
sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d*x)*cosh(c +
 d*x)**3/8 + 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*sinh(
a + d*x)*cosh(c + d*x)**3/(8*d) - sinh(c + d*x)**3*cosh(a + d*x)/(8*d), Eq(
b, d)), (x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a + 3*d*x)*sinh(c
+ d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c + d
*x)/8 - x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 - sinh(a + 3*d*x)*sinh(c + d*x
)**2*cosh(c + d*x)/(4*d) + sinh(a + 3*d*x)*cosh(c + d*x)**3/(24*d) + 3*sinh
(c + d*x)**3*cosh(a + 3*d*x)/(8*d), Eq(b, 3*d)), (b**3*sinh(c + d*x)**3*cos
h(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh(a + b*x)*sinh(c +
 d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*sinh(c + d
*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh(c + d*x
)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*si
nh(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) -
 6*d**3*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4), True
))
```

Maxima [F(-2)]

Exception generated.

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} - \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} - \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} + \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} + \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} - \frac{e^{(-bx-3\,dx-a-3\,c)}}{16\,(b+3\,d)}$$

```
[In] integrate(sinh(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) - 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x -
a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.00

$$\int \sinh(a+bx)\sinh^3(c+dx)\,dx = \frac{6\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(c+d\,x)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{6\,d^3\cosh(c+d\,x)^3\sinh(a+b\,x)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{3\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2\,(b^2-3\,d^2)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{\cosh(a+b\,x)\,\sinh(c+d\,x)^3\,(7\,b\,d^2-b^3)}{b^4-10\,b^2\,d^2+9\,d^4}$$

```
[In] int(sinh(a + b*x)*sinh(c + d*x)^3,x)
```

```
[Out] (6*b*d^2*cosh(a + b*x)*cosh(c + d*x)^2*sinh(c + d*x))/(b^4 + 9*d^4 - 10*b^2
*d^2) - (6*d^3*cosh(c + d*x)^3*sinh(a + b*x))/(b^4 + 9*d^4 - 10*b^2*d^2) -
(3*d*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2*(b^2 - 3*d^2))/(b^4 + 9*d^
4 - 10*b^2*d^2) - (cosh(a + b*x)*sinh(c + d*x)^3*(7*b*d^2 - b^3))/(b^4 + 9*
d^4 - 10*b^2*d^2)
```

3.170 $\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx=\frac{x}{4}-\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}$$

```
[Out] 1/4*x-1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sinh(2*d
*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5732, 2717}

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx=\frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

```
[In] Int[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

```
[Out] x/4 - Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
- Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}-\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)-\frac{1}{4}\cosh(2c+2dx)\right.\\
&\qquad \left.+\frac{1}{8}\cosh(2(a+c)+2(b+d)x)\right)\,dx\\
&= \frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx\\
&\quad -\frac{1}{4}\int \cosh(2a+2bx)\,dx-\frac{1}{4}\int \cosh(2c+2dx)\,dx\\
&= \frac{x}{4}-\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}\\
&\quad -\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$$
$$= \frac{(-2b^2d+2d^3)\sinh(2(a+bx))+bd(b+d)\sinh(2(a-c+(b-d)x))+b(b-d)(-2(b+d)\sinh(2(c+dx))}{16b(b-d)d(b+d)}$$

```
[In] Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)
*x)] + b*(b - d)*(-2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a
 + c + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}-\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sinh((2b-2d)x+2a-2c)+4\left(\frac{bd\sinh((2b+2d)x+2a+2c)}{4}+\left(-\frac{d\sinh(2bx+2a)}{2}+b\left(dx-\frac{\sinh(2dx+2c)}{2}\right)\right)(b+d)\right)(b-d)}{16b^3d-16d^3b}$
risch	$\frac{x}{4}-\frac{e^{2bx+2a}}{16b}+\frac{e^{-2bx-2a}}{16b}-\frac{\left(-d\,e^{4bx+4a}b+d^2e^{4bx+4a}+2b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2+bd+d^2\right)e^{-2bx+2dx-2a+2c}}{32(b+d)(b-d)d}+\frac{(d\,e^{4bx+}}{}$

```
[In] int(sinh(b*x+a)^2*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x-1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/8/(2*b-2*d)*sinh((2*b-2
*d)*x+2*a-2*c)+1/8/(2*b+2*d)*sinh((2*b+2*d)*x+2*a+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx$$
$$=\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2+(b^3d-bd^3)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2-(b^2d-\right.}{4\left(\left(b^3d-bd^3\right)\cos\right.}$$

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x
+ (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 - (b^2*d - d^3)*cosh(b*x + a))*sinh(
b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 +
b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2
 - (b^3*d - b*d^3)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(76) = 152.

Time = 1.56 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)**2*sinh(d*x+c)**2,x)
```

[Out] Piecewise((x*sinh(a)**2*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)**2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)**2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*cosh(c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) - sinh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) - 3*sinh(c + d*x)*cosh(a - d*x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d*x)*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)**2/8 + 3*x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + 5*sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)/(8*d) + sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d) - sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d)), ((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*sinh(c)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a + b*x)/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a + b*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3), True))

Maxima [F(-2)]

Exception generated.

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for more deta

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = \frac{1}{4}\,x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} - \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} + \frac{e^{(-2\,bx-2\,a)}}{16\,b} - \frac{e^{(2\,dx+2\,c)}}{16\,d} + \frac{e^{(-2\,dx-2\,c)}}{16\,d}$$

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
+ 2*a - 2*c)/(b - d) - 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2*
a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) + 1/16*e^(-2
*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

$$\int \sinh^2(a+bx)\sinh^2(c+dx)\,dx = -\frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x) - b^3\cosh(c+d\,x)\,\sinh(c+d\,x) - b\,d^3\,x + b^3\,d\,x - 2\,b^2\,d\cosh(a+b\,x)\,\mathrm{si}$$

```
[In] int(sinh(a + b*x)^2*sinh(c + d*x)^2,x)
```

```
[Out] -(d^3*cosh(a + b*x)*sinh(a + b*x) - b^3*cosh(c + d*x)*sinh(c + d*x) - b*d^3
*x + b^3*d*x - 2*b^2*d*cosh(a + b*x)*sinh(a + b*x) + 2*b*d^2*cosh(c + d*x)*
sinh(c + d*x) + 2*b^2*d*cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x) - 2*b*d
^2*cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^3 - 4*b^3*d)
```

3.171 $\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\begin{aligned}\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = &-\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)} \\ &+\frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\cosh(c+dx)}{8d} \\ &-\frac{\cosh(3c+3dx)}{24d}-\frac{3\cosh(2a+c+(2b+d)x)}{16(2b+d)} \\ &+\frac{\cosh(2a+3c+(2b+3d)x)}{16(2b+3d)}\end{aligned}$$

```
[Out] -1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*b+
d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5732, 2718}

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = -\frac{\cosh(2a+x(2b-3d)-3c)}{16(2b-3d)} + \frac{3\cosh(2a+x(2b-d)-c)}{16(2b-d)} - \frac{3\cosh(2a+x(2b+d)+c)}{16(2b+d)} + \frac{\cosh(2a+x(2b+3d)+3c)}{16(2b+3d)} + \frac{3\cosh(c+dx)}{8d} - \frac{\cosh(3c+3dx)}{24d}$$

```
[In] Int[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

```
[Out] -1/16*Cosh[2*a - 3*c + (2*b - 3*d)*x]/(2*b - 3*d) + (3*Cosh[2*a - c + (2*b
- d)*x])/(16*(2*b - d)) + (3*Cosh[c + d*x])/(8*d) - Cosh[3*c + 3*d*x]/(24*d
) - (3*Cosh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Cosh[2*a + 3*c + (2*b
+ 3*d)*x]/(16*(2*b + 3*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(-\frac{1}{16}\sinh(2a-3c+(2b-3d)x) + \frac{3}{16}\sinh(2a-c+(2b-d)x) + \frac{3}{8}\sinh(c+dx) - \frac{1}{8}\sinh(3c+3dx) - \frac{3}{16}\sinh(2a+c+(2b+d)x) + \frac{1}{16}\sinh(2a+3c+(2b+3d)x)\right) dx$$

$$= -\left(\frac{1}{16}\int \sinh(2a-3c+(2b-3d)x)\,dx\right) + \frac{1}{16}\int \sinh(2a+3c+(2b+3d)x)\,dx - \frac{1}{8}\int \sinh(3c+3dx)\,dx + \frac{3}{16}\int \sinh(2a-c+(2b-d)x)\,dx - \frac{3}{16}\int \sinh(2a+c+(2b+d)x)\,dx + \frac{3}{8}\int \sinh(c+dx)\,dx$$

$$= -\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\cosh(c+dx)}{8d} - \frac{\cosh(3c+3dx)}{24d} - \frac{3\cosh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\cosh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \frac{1}{48}\left(\frac{18\cosh(c)\cosh(dx)}{d} - \frac{2\cosh(3c)\cosh(3dx)}{d} - \frac{3\cosh(2a-3c+2bx-3dx)}{2b-3d} + \frac{9\cosh(2a-c+2bx-dx)}{2b-d} - \frac{9\cosh(2a+c+2bx+dx)}{2b+d} + \frac{3\cosh(2a+3c+2bx+3dx)}{2b+3d} + \frac{18\sinh(c)\sinh(dx)}{d} - \frac{2\sinh(3c)\sinh(3dx)}{d}\right)$$

```
[In] Integrate[Sinh[a + b*x]^2*Sinh[c + d*x]^3,x]
```

```
[Out] ((18*Cosh[c]*Cosh[d*x])/d - (2*Cosh[3*c]*Cosh[3*d*x])/d - (3*Cosh[2*a - 3*c
 + 2*b*x - 3*d*x])/(2*b - 3*d) + (9*Cosh[2*a - c + 2*b*x - d*x])/(2*b - d)
- (9*Cosh[2*a + c + 2*b*x + d*x])/(2*b + d) + (3*Cosh[2*a + 3*c + 2*b*x + 3
*d*x])/(2*b + 3*d) + (18*Sinh[c]*Sinh[d*x])/d - (2*Sinh[3*c]*Sinh[3*d*x])/d
)/48
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\cosh(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\cosh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\cosh(dx+c)}{8d}-\frac{\cosh(3dx+3c)}{24d}-\frac{3\cosh(2a+c+(2b+d)x)}{16(2b+d)}+$
parallelrisch	$(-24b^3d-36d^2b^2+6d^3b+9d^4)\cosh(2a-3c+(2b-3d)x)+(72b^3d+36d^2b^2-162d^3b-81d^4)\cosh(2a-c+(2b-d)x)+(24b^3d-36d^2$
risch	$-\frac{(-6d\,e^{4bx+4a}b+9d^2e^{4bx+4a}+8b^2e^{2bx+2a}-18\,e^{2bx+2a}d^2+6bd+9d^2)e^{-2bx+3dx-2a+3c}}{96(2b+3d)(2b-3d)d}+\frac{3(-2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2}{3}$

```
[In] int(sinh(b*x+a)^2*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*cosh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*cosh(2*a-c+(2*b-d)*x)/(2*b-d
)+3/8*cosh(d*x+c)/d-1/24*cosh(3*d*x+3*c)/d-3/16*cosh(2*a+c+(2*b+d)*x)/(2*b+
d)+1/16*cosh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(132) = 264$.

Time = 0.25 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.88

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx$$
$$=\frac{12\,(4\,b^3d-bd^3)\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^3-\left(16\,b^4-40\,b^2d^2+9\,d^4+9\,(4\,b^2d^2-d^4)\cosh(\right.}{}$$

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(4*b^3*d - b*d^3)*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^3 - (1
6*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x
+ c)^3 - 9*((4*b^2*d^2 - d^4)*cosh(d*x + c)^3 - (4*b^2*d^2 - 9*d^4)*cosh(d*
x + c))*sinh(b*x + a)^2 + 36*((4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)
^2 - (4*b^3*d - 9*b*d^3)*cosh(b*x + a))*sinh(b*x + a)*sinh(d*x + c) - 3*(9*
(4*b^2*d^2 - d^4)*cosh(d*x + c)*sinh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*
d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c))*sinh(d*x + c)^2 +
 9*(16*b^4 - 40*b^2*d^2 + 9*d^4 + (4*b^2*d^2 - 9*d^4)*cosh(b*x + a)^2)*cosh
(d*x + c))/((16*b^4*d - 40*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 4
0*b^2*d^3 + 9*d^5)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(116) = 232$.

Time = 5.48 (sec) , antiderivative size = 2001, normalized size of antiderivative = 13.90

$$\int \sinh^2(a + bx) \sinh^3(c + dx)\, dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)**2*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**2*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x
/2)**2*sinh(c + d*x)**3/16 + 3*x*sinh(a - 3*d*x/2)**2*sinh(c + d*x)*cosh(c
+ d*x)**2/16 + 3*x*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cos
h(c + d*x)/8 + x*sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/8 + x
*sinh(c + d*x)**3*cosh(a - 3*d*x/2)**2/16 + 3*x*sinh(c + d*x)*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**2/16 - 7*sinh(a - 3*d*x/2)**2*cosh(c + d*x)**3/(16*d
) - 5*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/(8*d) - 3*sinh(a
 - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/(4*d) - sinh(c
 + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/d + 11*cosh(a - 3*d*x/2)**2*c
osh(c + d*x)**3/(48*d), Eq(b, -3*d/2)), (3*x*sinh(a - d*x/2)**2*sinh(c + d*
x)**3/16 - 3*x*sinh(a - d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/16 + 3*x*s
inh(a - d*x/2)*sinh(c + d*x)**2*cosh(a - d*x/2)*cosh(c + d*x)/8 - 3*x*sinh(
a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)**3/8 + 3*x*sinh(c + d*x)**3*cosh(a
 - d*x/2)**2/16 - 3*x*sinh(c + d*x)*cosh(a - d*x/2)**2*cosh(c + d*x)**2/16
+ sinh(a - d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 31*sinh(a - d*x/2)*
*2*cosh(c + d*x)**3/(48*d) + 3*sinh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*
x/2)/(8*d) - sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2
/(4*d) + cosh(a - d*x/2)**2*cosh(c + d*x)**3/(48*d), Eq(b, -d/2)), (3*x*sin
h(a + d*x/2)**2*sinh(c + d*x)**3/16 - 3*x*sinh(a + d*x/2)**2*sinh(c + d*x)*
cosh(c + d*x)**2/16 - 3*x*sinh(a + d*x/2)*sinh(c + d*x)**2*cosh(a + d*x/2)*
cosh(c + d*x)/8 + 3*x*sinh(a + d*x/2)*cosh(a + d*x/2)*cosh(c + d*x)**3/8 +
3*x*sinh(c + d*x)**3*cosh(a + d*x/2)**2/16 - 3*x*sinh(c + d*x)*cosh(a + d*x
/2)**2*cosh(c + d*x)**2/16 + sinh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d
*x)/d - 31*sinh(a + d*x/2)**2*cosh(c + d*x)**3/(48*d) - 3*sinh(a + d*x/2)*s
inh(c + d*x)**3*cosh(a + d*x/2)/(8*d) + sinh(a + d*x/2)*sinh(c + d*x)*cosh(
a + d*x/2)*cosh(c + d*x)**2/(4*d) + cosh(a + d*x/2)**2*cosh(c + d*x)**3/(48
*d), Eq(b, d/2)), (x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**3/16 + 3*x*sinh(a
+ 3*d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/16 - 3*x*sinh(a + 3*d*x/2)*sin
h(c + d*x)**2*cosh(a + 3*d*x/2)*cosh(c + d*x)/8 - x*sinh(a + 3*d*x/2)*cosh(
a + 3*d*x/2)*cosh(c + d*x)**3/8 + x*sinh(c + d*x)**3*cosh(a + 3*d*x/2)**2/1
6 + 3*x*sinh(c + d*x)*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**2/16 + sinh(a + 3
*d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 5*sinh(a + 3*d*x/2)**2*cosh(c
 + d*x)**3/(48*d) - sinh(a + 3*d*x/2)*sinh(c + d*x)**3*cosh(a + 3*d*x/2)/(2
4*d) - 5*sinh(a + 3*d*x/2)*sinh(c + d*x)*cosh(a + 3*d*x/2)*cosh(c + d*x)**2
/(4*d) + 9*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**3/(16*d), Eq(b, 3*d/2)), ((x
```

```
*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2
*b))*sinh(c)**3, Eq(d, 0)), (24*b**4*sinh(a + b*x)**2*sinh(c + d*x)**2*cosh
(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 16*b**4*sinh(a + b*x)**2*
cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 24*b**4*sinh(c + d
*x)**2*cosh(a + b*x)**2*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5)
 + 16*b**4*cosh(a + b*x)**2*cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 2
7*d**5) + 24*b**3*d*sinh(a + b*x)*sinh(c + d*x)**3*cosh(a + b*x)/(48*b**4*d
 - 120*b**2*d**3 + 27*d**5) - 78*b**2*d**2*sinh(a + b*x)**2*sinh(c + d*x)**
2*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 40*b**2*d**2*sinh(a
 + b*x)**2*cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b**2
*d**2*sinh(c + d*x)**2*cosh(a + b*x)**2*cosh(c + d*x)/(48*b**4*d - 120*b**2
*d**3 + 27*d**5) - 40*b**2*d**2*cosh(a + b*x)**2*cosh(c + d*x)**3/(48*b**4*
d - 120*b**2*d**3 + 27*d**5) - 42*b*d**3*sinh(a + b*x)*sinh(c + d*x)**3*cos
h(a + b*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sinh(a + b*x)*
sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 2
7*d**5) + 27*d**4*sinh(a + b*x)**2*sinh(c + d*x)**2*cosh(c + d*x)/(48*b**4*
d - 120*b**2*d**3 + 27*d**5) - 18*d**4*sinh(a + b*x)**2*cosh(c + d*x)**3/(4
8*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)} - \frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)} + \frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)} - \frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)} - \frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)} + \frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)} - \frac{3\,e^{(-2\,bx-dx-2\,a-c)}}{32\,(2\,b+d)} + \frac{e^{(-2\,bx-3\,dx-2\,a-3\,c)}}{32\,(2\,b+3\,d)} - \frac{e^{(3\,dx+3\,c)}}{48\,d} + \frac{3\,e^{(dx+c)}}{16\,d} + \frac{3\,e^{(-dx-c)}}{16\,d} - \frac{e^{(-3\,dx-3\,c)}}{48\,d}$$

```
[In] integrate(sinh(b*x+a)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) + 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) + 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) - 1/4
8*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d - 1/48*e^(-3
*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.34

$$\int \sinh^2(a+bx)\sinh^3(c+dx)\,dx = \frac{\cosh(c+d\,x)\,\sinh(a+b\,x)^2\,\sinh(c+d\,x)^2\,(8\,b^4-26\,b^2\,d^2+9\,d^4)}{d\,(16\,b^4-40\,b^2\,d^2+9\,d^4)} - \cosh(c+d\,x)^3\,\sinh(a+b\,x)^2\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}+\frac{1}{3\,d}\right) - \frac{2\cosh(a+b\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^3\,(7\,b\,d^2-4\,b^3)}{16\,b^4-40\,b^2\,d^2+9\,d^4} - \frac{\cosh(a+b\,x)^2\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(8\,b^4-14\,b^2\,d^2)}{d\,(16\,b^4-40\,b^2\,d^2+9\,d^4)} - \cosh(a+b\,x)^2\cosh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{1}{3\,d}\right) + \frac{12\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(a+b\,x)\,\sinh(c+d\,x)}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$

```
[In] int(sinh(a + b*x)^2*sinh(c + d*x)^3,x)
```

```
[Out] (cosh(c + d*x)*sinh(a + b*x)^2*sinh(c + d*x)^2*(8*b^4 + 9*d^4 - 26*b^2*d^2)
)/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - cosh(c + d*x)^3*sinh(a + b*x)^2*((3*d
^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)) - (2*cosh(a + b*x)*sinh(a + b*
x)*sinh(c + d*x)^3*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (cosh
(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x)^2*(8*b^4 - 14*b^2*d^2))/(d*(16*b^4
+ 9*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^2*cosh(c + d*x)^3*((3*d^3)/(16*b^4 +
 9*d^4 - 40*b^2*d^2) - 1/(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)^2*s
inh(a + b*x)*sinh(c + d*x))/(16*b^4 + 9*d^4 - 40*b^2*d^2)
```

3.172 $\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} - \frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\sinh(3a-c+(3b-d)x)}{32(3b-d)} + \frac{9\sinh(a+c+(b+d)x)}{32(b+d)} + \frac{\sinh(3(a+c)+3(b+d)x)}{96(b+d)} - \frac{3\sinh(3a+c+(3b+d)x)}{32(3b+d)} - \frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}$$

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)-1/96*sinh(3
*a-3*c+3*(b-d)*x)/(b-d)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sinh(a+c+(b
+d)*x)/(b+d)+1/96*sinh(3*a+3*c+3*(b+d)*x)/(b+d)-3/32*sinh(3*a+c+(3*b+d)*x)/
(3*b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5732, 2717}

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)} - \frac{9\sinh(a+x(b-d)-c)}{32(b-d)} - \frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)} + \frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)} + \frac{9\sinh(a+x(b+d)+c)}{32(b+d)} + \frac{\sinh(3(a+c)+3x(b+d))}{96(b+d)} - \frac{3\sinh(3a+x(3b+d)+c)}{32(3b+d)} - \frac{3\sinh(a+x(b+3d)+3c)}{32(b+3d)}$$

```
[In] Int[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]
```

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) - Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) - (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) - (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{32}\cosh(a-3c+(b-3d)x) - \frac{9}{32}\cosh(a-c+(b-d)x) - \frac{1}{32}\cosh(3(a-c)+3(b-d)x) + \frac{3}{32}\cosh(3a-c+(3b-d)x) + \frac{9}{32}\cosh(a+c+(b+d)x) + \frac{1}{32}\cosh(3(a+c)+3(b+d)x) - \frac{3}{32}\cosh(3a+c+(3b+d)x) - \frac{3}{32}\cosh(a+3c+(b+3d)x)\right)dx \\
&= -\left(\frac{1}{32}\int \cosh(3(a-c)+3(b-d)x)\,dx\right) + \frac{1}{32}\int \cosh(3(a+c)+3(b+d)x)\,dx \\
&\quad + \frac{3}{32}\int \cosh(a-3c+(b-3d)x)\,dx + \frac{3}{32}\int \cosh(3a-c+(3b-d)x)\,dx \\
&\quad - \frac{3}{32}\int \cosh(3a+c+(3b+d)x)\,dx - \frac{3}{32}\int \cosh(a+3c+(b+3d)x)\,dx \\
&\quad - \frac{9}{32}\int \cosh(a-c+(b-d)x)\,dx + \frac{9}{32}\int \cosh(a+c+(b+d)x)\,dx \\
&= \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\sinh(a-c+(b-d)x)}{32(b-d)} \\
&\quad - \frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\sinh(3a-c+(3b-d)x)}{32(3b-d)} \\
&\quad + \frac{9\sinh(a+c+(b+d)x)}{32(b+d)} + \frac{\sinh(3(a+c)+3(b+d)x)}{96(b+d)} \\
&\quad - \frac{3\sinh(3a+c+(3b+d)x)}{32(3b+d)} - \frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{1}{96}\left(\frac{9\sinh(a-3c+bx-3dx)}{b-3d} - \frac{27\sinh(a-c+bx-dx)}{b-d} - \frac{\sinh(3(a-c+bx-dx))}{b-d} + \frac{9\sinh(3a-c+3bx-dx)}{3b-d} - \frac{9\sinh(3a+c+3bx+dx)}{3b+d} - \frac{9\sinh(a+3c+bx+3dx)}{b+3d} + \frac{27\sinh(a+c+(b+d)x)}{b+d} + \frac{\sinh(3(a+c+(b+d)x))}{b+d}\right)$$

[In] Integrate[Sinh[a + b*x]^3*Sinh[c + d*x]^3,x]

[Out] ((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Sinh[a - c + b*x - d*x])/(b - d) - Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x - d*x])/(3*b - d) - (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Sinh[a + 3*c + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sinh[3*(a + c + (b + d)*x)]/(b + d))/96

Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$\frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}-\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}-\frac{\sinh((3b-3d)x+3a}{32(3b-3d)}$
parallelrisch	$\frac{9(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d)(b+d)\sinh(3a-c+(3b-d)x)}{32}-\frac{9\left(\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b+d)\sinh((3b-3d)x+3a-3c)}{3}-\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d}{3}\right.}{}$
risch	$\frac{\left(b^{3}e^{6bx+6a}-b^{2}d\,e^{6bx+6a}-9b\,d^{2}e^{6bx+6a}+9d^{3}e^{6bx+6a}-9b^{3}e^{4bx+4a}+27b^{2}d\,e^{4bx+4a}+9b\,d^{2}e^{4bx+4a}-27d^{3}e^{4bx+4a}-9b^{3}e^{2bx+2a}-\right.}{192(b+d)(b+3d)(b-d)(b-3d)}$

[In] int(sinh(b*x+a)^3*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(a+c+(b+d)*x)/(b+d)-3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)-1/32/(3*b-3*d)*sinh((3*b-3*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)-3/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/32/(3*b+3*d)*sinh((3*b+3*d)*x+3*a+3*c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.75

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = -\frac{\left(\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cosh\left(dx+c\right)^3-9\left(b^4d-10\,b^2d^3+9\,d^5\right)\cosh\left(dx+c\right)\right)\sinh\left(bx+a\right)^3-\left(\left(9\,b^5\right.\right.}{}$$

[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="fricas")

[Out] -1/48*(((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)^3 - 9*(b^4*d - 10*b^2*d^3 + 9*d^5)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 + 3*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*sinh(b*x + a)^2 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(d*x + c)^3 + 3*((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(81*b^4*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*c

```
osh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 - 3*((81*b^4*d - 90*b^2*d^3 + 9
*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c)^3 - 9*
(9*b^4*d - 82*b^2*d^3 + 9*d^5 - (b^4*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^
2)*cosh(d*x + c))*sinh(b*x + a) + 3*(9*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cosh(b*
x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*
b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^2 - 3*((9*b^5 - 82*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^2 - 9*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cos
h(b*x + a))*sinh(b*x + a)^2 - 9*(9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a
))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^
4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*x +
a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.

Time = 16.69 (sec) , antiderivative size = 3580, normalized size of antiderivative = 18.36

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(b*x+a)**3*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d
*x)**3*sinh(c + d*x)**3/32 + 9*x*sinh(a - 3*d*x)**3*sinh(c + d*x)*cosh(c +
d*x)**2/32 + 9*x*sinh(a - 3*d*x)**2*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c
 + d*x)/32 + 3*x*sinh(a - 3*d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)**3/32 - 3
*x*sinh(a - 3*d*x)*sinh(c + d*x)**3*cosh(a - 3*d*x)**2/32 - 9*x*sinh(a - 3*
d*x)*sinh(c + d*x)*cosh(a - 3*d*x)**2*cosh(c + d*x)**2/32 - 9*x*sinh(c + d*
x)**2*cosh(a - 3*d*x)**3*cosh(c + d*x)/32 - 3*x*cosh(a - 3*d*x)**3*cosh(c +
 d*x)**3/32 - 13*sinh(a - 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(320*d)
+ sinh(a - 3*d*x)**3*cosh(c + d*x)**3/(12*d) - 101*sinh(a - 3*d*x)**2*sinh(
c + d*x)**3*cosh(a - 3*d*x)/(320*d) + 3*sinh(a - 3*d*x)**2*sinh(c + d*x)*co
sh(a - 3*d*x)*cosh(c + d*x)**2/(20*d) - 27*sinh(a - 3*d*x)*cosh(a - 3*d*x)*
*2*cosh(c + d*x)**3/(320*d) + sinh(c + d*x)**3*cosh(a - 3*d*x)**3/(5*d) - 5
1*sinh(c + d*x)*cosh(a - 3*d*x)**3*cosh(c + d*x)**2/(320*d), Eq(b, -3*d)),
(5*x*sinh(a - d*x)**3*sinh(c + d*x)**3/16 - 3*x*sinh(a - d*x)**3*sinh(c + d
*x)*cosh(c + d*x)**2/16 + 9*x*sinh(a - d*x)**2*sinh(c + d*x)**2*cosh(a - d*
x)*cosh(c + d*x)/16 - 3*x*sinh(a - d*x)**2*cosh(a - d*x)*cosh(c + d*x)**3/1
6 - 3*x*sinh(a - d*x)*sinh(c + d*x)**3*cosh(a - d*x)**2/16 + 9*x*sinh(a - d
*x)*sinh(c + d*x)*cosh(a - d*x)**2*cosh(c + d*x)**2/16 - 3*x*sinh(c + d*x)*
*2*cosh(a - d*x)**3*cosh(c + d*x)/16 + 5*x*cosh(a - d*x)**3*cosh(c + d*x)**
3/16 + sinh(a - d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(2*d) - sinh(a - d*x
)**3*cosh(c + d*x)**3/(48*d) - 3*sinh(a - d*x)**2*sinh(c + d*x)**3*cosh(a -
 d*x)/(16*d) + 3*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)
**2/(4*d) + 5*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)**3/(16*d) + sinh
```

(c + d*x)**3*cosh(a - d*x)**3/(16*d), Eq(b, -d)), (3*x*sinh(a - d*x/3)**3*sinh(c + d*x)**3/32 - 3*x*sinh(a - d*x/3)**3*sinh(c + d*x)*cosh(c + d*x)**2/32 + 9*x*sinh(a - d*x/3)**2*sinh(c + d*x)**2*cosh(a - d*x/3)*cosh(c + d*x)/32 - 9*x*sinh(a - d*x/3)**2*cosh(a - d*x/3)*cosh(c + d*x)**3/32 + 9*x*sinh(a - d*x/3)*sinh(c + d*x)**3*cosh(a - d*x/3)**2/32 - 9*x*sinh(a - d*x/3)*sinh(c + d*x)*cosh(a - d*x/3)**2*cosh(c + d*x)**2/32 + 3*x*sinh(c + d*x)**2*cosh(a - d*x/3)**3*cosh(c + d*x)/32 - 3*x*cosh(a - d*x/3)**3*cosh(c + d*x)**3/32 + 351*sinh(a - d*x/3)**3*sinh(c + d*x)**2*cosh(c + d*x)/(320*d) - 3*sinh(a - d*x/3)**3*cosh(c + d*x)**3/(4*d) + 183*sinh(a - d*x/3)**2*sinh(c + d*x)**3*cosh(a - d*x/3)/(320*d) - 9*sinh(a - d*x/3)**2*sinh(c + d*x)*cosh(a - d*x/3)*cosh(c + d*x)**2/(20*d) + 9*sinh(a - d*x/3)*cosh(a - d*x/3)**2*cosh(c + d*x)**3/(320*d) - sinh(c + d*x)**3*cosh(a - d*x/3)**3/(10*d) + 33*sinh(c + d*x)*cosh(a - d*x/3)**3*cosh(c + d*x)**2/(320*d), Eq(b, -d/3)), (3*x*sinh(a + d*x/3)**3*sinh(c + d*x)**3/32 - 3*x*sinh(a + d*x/3)**3*sinh(c + d*x)*cosh(c + d*x)**2/32 - 9*x*sinh(a + d*x/3)**2*sinh(c + d*x)**2*cosh(a + d*x/3)*cosh(c + d*x)/32 + 9*x*sinh(a + d*x/3)**2*cosh(a + d*x/3)*cosh(c + d*x)**3/32 + 9*x*sinh(a + d*x/3)*sinh(c + d*x)**3*cosh(a + d*x/3)**2/32 - 9*x*sinh(a + d*x/3)*sinh(c + d*x)*cosh(a + d*x/3)**2*cosh(c + d*x)**2/32 - 3*x*sinh(c + d*x)**2*cosh(a + d*x/3)**3*cosh(c + d*x)/32 + 3*x*cosh(a + d*x/3)**3*cosh(c + d*x)**3/32 + 303*sinh(a + d*x/3)**3*sinh(c + d*x)**2*cosh(c + d*x)/(320*d) - 3*sinh(a + d*x/3)**3*cosh(c + d*x)**3/(5*d) - 39*sinh(a + d*x/3)**2*sinh(c + d*x)**3*cosh(a + d*x/3)/(320*d) - 9*sinh(a + d*x/3)*sinh(c + d*x)**2*cosh(a + d*x/3)**2*cosh(c + d*x)/(20*d) + 153*sinh(a + d*x/3)*cosh(a + d*x/3)**2*cosh(c + d*x)**3/(320*d) + sinh(c + d*x)**3*cosh(a + d*x/3)**3/(4*d) - 81*sinh(c + d*x)*cosh(a + d*x/3)**3*cosh(c + d*x)**2/(320*d), Eq(b, d/3)), (5*x*sinh(a + d*x)**3*sinh(c + d*x)**3/16 - 3*x*sinh(a + d*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/16 - 9*x*sinh(a + d*x)**2*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/16 + 3*x*sinh(a + d*x)**2*cosh(a + d*x)*cosh(c + d*x)**3/16 - 3*x*sinh(a + d*x)*sinh(c + d*x)**3*cosh(a + d*x)**2/16 + 9*x*sinh(a + d*x)*sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)**2/16 + 3*x*sinh(c + d*x)**2*cosh(a + d*x)**3*cosh(c + d*x)/16 - 5*x*cosh(a + d*x)**3*cosh(c + d*x)**3/16 + 11*sinh(a + d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(16*d) - 19*sinh(a + d*x)**3*cosh(c + d*x)**3/(48*d) - 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)**2*cosh(c + d*x)/(4*d) + sinh(a + d*x)*cosh(a + d*x)**2*cosh(c + d*x)**3/(2*d) + 5*sinh(c + d*x)**3*cosh(a + d*x)**3/(16*d) - 3*sinh(c + d*x)*cosh(a + d*x)**3*cosh(c + d*x)**2/(16*d), Eq(b, d)), (3*x*sinh(a + 3*d*x)**3*sinh(c + d*x)**3/32 + 9*x*sinh(a + 3*d*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/32 - 9*x*sinh(a + 3*d*x)**2*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c + d*x)/32 - 3*x*sinh(a + 3*d*x)**2*cosh(a + 3*d*x)*cosh(c + d*x)**3/32 - 3*x*sinh(a + 3*d*x)*sinh(c + d*x)**3*cosh(a + 3*d*x)**2/32 - 9*x*sinh(a + 3*d*x)*sinh(c + d*x)*cosh(a + 3*d*x)**2*cosh(c + d*x)**2/32 + 9*x*sinh(c + d*x)**2*cosh(a + 3*d*x)**3*cosh(c + d*x)/32 + 3*x*cosh(a + 3*d*x)**3*cosh(c + d*x)**3/32 - 61*sinh(a + 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(320*d) + sinh(a + 3*d*x)**3*cosh(c + d*x)**3/(30*d) + 117*sinh(a + 3*d*x)**2*sinh(c + d*x)**3*cosh(a + 3*d*x)/(320*d) + 3*sinh(a + 3*d*x)*sinh(c + d

```
*x)**2*cosh(a + 3*d*x)**2*cosh(c + d*x)/(20*d) - 11*sinh(a + 3*d*x)*cosh(a
+ 3*d*x)**2*cosh(c + d*x)**3/(320*d) - sinh(c + d*x)**3*cosh(a + 3*d*x)**3/
(4*d) + 3*sinh(c + d*x)*cosh(a + 3*d*x)**3*cosh(c + d*x)**2/(320*d), Eq(b,
3*d)), (27*b**5*sinh(a + b*x)**2*sinh(c + d*x)**3*cosh(a + b*x)/(27*b**6 -
273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 18*b**5*sinh(c + d*x)**3*cosh(a
+ b*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 63*b**4*d*s
inh(a + b*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(27*b**6 - 273*b**4*d**2 + 2
73*b**2*d**4 - 27*d**6) + 54*b**4*d*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a +
 b*x)**2*cosh(c + d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6)
- 210*b**3*d**2*sinh(a + b*x)**2*sinh(c + d*x)**3*cosh(a + b*x)/(27*b**6 -
273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 126*b**3*d**2*sinh(a + b*x)**2*s
inh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2 + 273*
b**2*d**4 - 27*d**6) + 122*b**3*d**2*sinh(c + d*x)**3*cosh(a + b*x)**3/(27*
b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 120*b**3*d**2*sinh(c + d*
x)*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2 + 273*b**2*d*
*4 - 27*d**6) + 210*b**2*d**3*sinh(a + b*x)**3*sinh(c + d*x)**2*cosh(c + d*
x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 122*b**2*d**3*sinh
(a + b*x)**3*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27
*d**6) - 126*b**2*d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a + b*x)**2*cosh
(c + d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 120*b**2*d*
*3*sinh(a + b*x)*cosh(a + b*x)**2*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2
 + 273*b**2*d**4 - 27*d**6) + 63*b*d**4*sinh(a + b*x)**2*sinh(c + d*x)**3*c
osh(a + b*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 54*b*d**
4*sinh(a + b*x)**2*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**6 -
273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 27*d**5*sinh(a + b*x)**3*sinh(c
+ d*x)**2*cosh(c + d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6)
 + 18*d**5*sinh(a + b*x)**3*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273
*b**2*d**4 - 27*d**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = \frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} - \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} - \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} - \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} + \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} - \frac{9\,e^{(bx-dx+a-c)}}{64\,(b-d)} + \frac{3\,e^{(bx-3\,dx+a-3\,c)}}{64\,(b-3\,d)} - \frac{3\,e^{(-bx+3\,dx-a+3\,c)}}{64\,(b-3\,d)} + \frac{9\,e^{(-bx+dx-a+c)}}{64\,(b-d)} - \frac{9\,e^{(-bx-dx-a-c)}}{64\,(b+d)} + \frac{3\,e^{(-bx-3\,dx-a-3\,c)}}{64\,(b+3\,d)} + \frac{e^{(-3\,bx+3\,dx-3\,a+3\,c)}}{192\,(b-d)} - \frac{3\,e^{(-3\,bx+dx-3\,a+c)}}{64\,(3\,b-d)} + \frac{3\,e^{(-3\,bx-dx-3\,a-c)}}{64\,(3\,b+d)} - \frac{e^{(-3\,bx-3\,dx-3\,a-3\,c)}}{192\,(b+d)}$$

```
[In] integrate(sinh(b*x+a)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) - 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) - 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9/64
*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) + 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) + 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) + 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 906, normalized size of antiderivative = 4.65

$$\int \sinh^3(a+bx)\sinh^3(c+dx)\,dx = e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4} + \frac{e^{-6a-6bx}(-9b^3-3b^2d+9bd^2+3d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-2a-2bx}(-81b^3+81b^2d+9bd^2-9d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}(-81b^3-81b^2d+9bd^2+9d^3)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-c+3bx-dx}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{576b^4-640b^2d^2+64d^4} + \frac{e^{-6a-6bx}(-9b^3+3b^2d+9bd^2-3d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-2a-2bx}(-81b^3-81b^2d+9bd^2+9d^3)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}(-81b^3+81b^2d+9bd^2-9d^3)}{576b^4-640b^2d^2+64d^4}\right) + e^{3a-3c+3bx-3dx}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-6a-6bx}(-b^3+b^2d+9bd^2-9d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-2a-2bx}(-9b^3-27b^2d+9bd^2+27d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}(-9b^3+27b^2d+9bd^2-27d^3)}{192b^4-1920b^2d^2+1728d^4}\right) - e^{3a+3c+3bx+3dx}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-6a-6bx}(-b^3-b^2d+9bd^2+9d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-2a-2bx}(-9b^3+27b^2d+9bd^2-27d^3)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}(-9b^3-27b^2d+9bd^2+27d^3)}{192b^4-1920b^2d^2+1728d^4}\right)$$

```
[In] int(sinh(a + b*x)^3*sinh(c + d*x)^3,x)
```

```
[Out] exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4 +
64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*
d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 + 81*
b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a - 4*
```

```
b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2
)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(576*b
^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d - 9*b^3
 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2
- 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a
 - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^
2*d^2)) + exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + b^2*d -
 b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9
*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) -
(exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*
d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b*d^2 + b^2*d - b
^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*
d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*
a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 192
0*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(19
2*b^4 + 1728*d^4 - 1920*b^2*d^2))
```

3.173 $\int \cosh(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cosh(a+bx)\cosh(c+dx)\,dx=\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

```
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5733, 2717}

$$\int \cosh(a+bx)\cosh(c+dx)\,dx=\frac{\sinh(a+x(b-d)-c)}{2(b-d)}+\frac{\sinh(a+x(b+d)+c)}{2(b+d)}$$

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x],x]
```

```
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
```

```
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}\cosh(a-c+(b-d)x)+\frac{1}{2}\cosh(a+c+(b+d)x)\right)\,dx\\ &= \frac{1}{2}\int \cosh(a-c+(b-d)x)\,dx+\frac{1}{2}\int \cosh(a+c+(b+d)x)\,dx\\ &= \frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\cosh(c+dx)\,dx=\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2(b+d)}$$

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x],x]
[Out] Sinh[a - c + (b - d)*x]/(2*(b - d)) + Sinh[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}$	40
parallelrisch	$\frac{(b+d)\sinh(a-c+(b-d)x)+\sinh(a+c+(b+d)x)(b-d)}{2b^2-2d^2}$	48
risch	$\frac{\left(b\,e^{2bx+2a}-e^{2bx+2a}d-b-d\right)e^{-bx+dx-a+c}}{4(b+d)(b-d)}+\frac{\left(b\,e^{2bx+2a}+e^{2bx+2a}d-b+d\right)e^{-bx-dx-a-c}}{4(b+d)(b-d)}$	116

```
[In] int(cosh(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/2*sinh(a-c+(b-d)*x)/(b-d)+1/2*sinh(a+c+(b+d)*x)/(b+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \frac{b\cosh(dx+c)\sinh(bx+a) - d\cosh(bx+a)\sinh(dx+c)}{(b^2-d^2)\cosh(bx+a)^2 - (b^2-d^2)\sinh(bx+a)^2}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cosh(d*x + c)*sinh(b*x + a) - d*cosh(b*x + a)*sinh(d*x + c))/((b^2 - d^2
)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(32) = 64.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \begin{cases} x\cosh(a)\cosh(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sinh(a-dx)\sinh(c+dx)}{2} + \frac{x\cosh(a-dx)\cosh(c+dx)}{2} - \frac{\sinh(a-dx)\cosh(c+dx)}{2d} & \text{for } b=-d \\ -\frac{x\sinh(a+dx)\sinh(c+dx)}{2} + \frac{x\cosh(a+dx)\cosh(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{b\sinh(a+bx)\cosh(c+dx)}{b^2-d^2} - \frac{d\sinh(c+dx)\cosh(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*sinh(c
 + d*x)/2 + x*cosh(a - d*x)*cosh(c + d*x)/2 - sinh(a - d*x)*cosh(c + d*x)/(
2*d), Eq(b, -d)), (-x*sinh(a + d*x)*sinh(c + d*x)/2 + x*cosh(a + d*x)*cosh(
c + d*x)/2 + sinh(c + d*x)*cosh(a + d*x)/(2*d), Eq(b, d)), (b*sinh(a + b*x)
*cosh(c + d*x)/(b**2 - d**2) - d*sinh(c + d*x)*cosh(a + b*x)/(b**2 - d**2),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{4\,(b+d)} + \frac{e^{(bx-dx+a-c)}}{4\,(b-d)} - \frac{e^{(-bx+dx-a+c)}}{4\,(b-d)} - \frac{e^{(-bx-dx-a-c)}}{4\,(b+d)}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) - 1/4
*e^(-b*x + d*x - a + c)/(b - d) - 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cosh(a+bx)\cosh(c+dx)\,dx = \frac{b\cosh(c+d\,x)\,\sinh(a+b\,x) - d\cosh(a+b\,x)\,\sinh(c+d\,x)}{b^2-d^2}$$

```
[In] int(cosh(a + b*x)*cosh(c + d*x),x)
```

```
[Out] (b*cosh(c + d*x)*sinh(a + b*x) - d*cosh(a + b*x)*sinh(c + d*x))/(b^2 - d^2)
```

3.174 $\int \cosh(a+bx)\cosh^2(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \frac{\sinh(a+bx)}{2b} + \frac{\sinh(a-2c+(b-2d)x)}{4(b-2d)} + \frac{\sinh(a+2c+(b+2d)x)}{4(b+2d)}$$

```
[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5733, 2717}

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \frac{\sinh(a+x(b-2d)-2c)}{4(b-2d)} + \frac{\sinh(a+x(b+2d)+2c)}{4(b+2d)} + \frac{\sinh(a+bx)}{2b}$$

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
```

```
[Out] Sinh[a + b*x]/(2*b) + Sinh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sinh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{2}\cosh(a+bx)+\frac{1}{4}\cosh(a-2c+(b-2d)x)+\frac{1}{4}\cosh(a+2c+(b+2d)x)\right)\,dx$$
$$= \frac{1}{4}\int \cosh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \cosh(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \cosh(a+bx)\,dx$$
$$= \frac{\sinh(a+bx)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sinh(a+2c+(b+2d)x)}{4(b+2d)}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \frac{1}{4}\left(\frac{2\cosh(bx)\sinh(a)}{b}+\frac{2\cosh(a)\sinh(bx)}{b}+\frac{\sinh(a-2c+bx-2dx)}{b-2d}+\frac{\sinh(a+2c+bx+2dx)}{b+2d}\right)$$

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x]^2,x]
[Out] ((2*Cosh[b*x]*Sinh[a])/b + (2*Cosh[a]*Sinh[b*x])/b + Sinh[a - 2*c + b*x - 2
*d*x]/(b - 2*d) + Sinh[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}$
parallelrisch	$\frac{b(b+2d)\sinh(a-2c+(b-2d)x)+2(b-2d)\left(\frac{b\sinh(a+2c+(b+2d)x)}{2}+\sinh(bx+a)(b+2d)\right)}{4b^3-16b\,d^2}$
risch	$\frac{e^{bx+a}}{4b}-\frac{e^{-bx-a}}{4b}+\frac{\left(b\,e^{2bx+2a}-2\,e^{2bx+2a}d-b-2d\right)e^{-bx+2dx-a+2c}}{8(b+2d)(b-2d)}+\frac{\left(b\,e^{2bx+2a}+2\,e^{2bx+2a}d-b+2d\right)e^{-bx-2dx-a-2c}}{8(b+2d)(b-2d)}$

```
[In] int(cosh(b*x+a)*cosh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/2*sinh(b*x+a)/b+1/4*sinh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sinh(a+2*c+(b+2*d)*x)/(b+2*d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.85

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = -\frac{4\,bd\cosh(bx+a)\cosh(dx+c)\sinh(dx+c)-b^2\sinh(bx+a)\sinh(dx+c)^2-\left(b^2\cosh(dx+c)^2+b^2-\right.}{2\left((b^3-4\,bd^2)\cosh(bx+a)^2-(b^3-4\,bd^2)\sinh(bx+a)^2\right)}$$

[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")

[Out] -1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(d*x + c) - b^2*sinh(b*x + a)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^2 + b^2 - 4*d^2)*sinh(b*x + a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(49) = 98.

Time = 0.70 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \begin{cases} x\cosh(a)\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\cosh(a) \\ \frac{x\sinh(a-2dx)\sinh(c+dx)\cosh(c+dx)}{2}+\frac{x\sinh^2(c+dx)\cosh(a-2dx)}{4}+\frac{x\cosh(a-2dx)\cosh^2(c+dx)}{4}+\frac{\sinh(a-2dx)\sinh^2(c+dx)}{2d}+3 \\ -\frac{x\sinh(a+2dx)\sinh(c+dx)\cosh(c+dx)}{2}+\frac{x\sinh^2(c+dx)\cosh(a+2dx)}{4}+\frac{x\cosh(a+2dx)\cosh^2(c+dx)}{4}-\frac{\sinh(a+2dx)\sinh^2(c+dx)}{2d}+ \\ \frac{b^2\sinh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sinh(c+dx)\cosh(a+bx)\cosh(c+dx)}{b^3-4bd^2}+\frac{2d^2\sinh(a+bx)\sinh^2(c+dx)}{b^3-4bd^2}-\frac{2d^2\sinh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

[In] integrate(cosh(b*x+a)*cosh(d*x+c)**2,x)

[Out] Piecewise((x*cosh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a), Eq(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*sinh(c + d*x)**2*cosh(a - 2*d*x)/4 + x*cosh(a - 2*d*x)*cosh(c + d*x)**2/4 + sinh(a - 2*d*x)*sinh(c + d*x)**2/(2*d) + 3*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/(4*d), Eq(b, -2*d)), (-x*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*sinh(c + d*x)**2*cosh(a + 2*d*x)/4 + x*cosh(a + 2*d*x)*cosh(c + d*x)**2/4 -

```
 sinh(a + 2*d*x)*sinh(c + d*x)**2/(2*d) + 3*sinh(c + d*x)*cosh(a + 2*d*x)*c
osh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*sinh(a + b*x)*cosh(c + d*x)**2/(b**3
 - 4*b*d**2) - 2*b*d*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)/(b**3 - 4*b*
d**2) + 2*d**2*sinh(a + b*x)*sinh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*d**2*si
nh(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} + \frac{e^{(bx+a)}}{4\,b} - \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} - \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} - \frac{e^{(-bx-a)}}{4\,b}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/4*e^(b*x + a)/b - 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) - 1/4*e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \cosh(a+bx)\cosh^2(c+dx)\,dx$$
$$=\frac{2\,d^2\sinh(a+b\,x)-b^2\cosh(c+d\,x)^2\sinh(a+b\,x)+2\,b\,d\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(c+d\,x)}{4\,b\,d^2-b^3}$$

```
[In] int(cosh(a + b*x)*cosh(c + d*x)^2,x)
```

```
[Out] (2*d^2*sinh(a + b*x) - b^2*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d*cosh(a + b
*x)*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.175 $\int \cosh(a+bx)\cosh^3(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)} + \frac{3\sinh(a-c+(b-d)x)}{8(b-d)} + \frac{3\sinh(a+c+(b+d)x)}{8(b+d)} + \frac{\sinh(a+3c+(b+3d)x)}{8(b+3d)}$$

```
[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+
(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5733, 2717}

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{\sinh(a+x(b-3d)-3c)}{8(b-3d)} + \frac{3\sinh(a+x(b-d)-c)}{8(b-d)} + \frac{3\sinh(a+x(b+d)+c)}{8(b+d)} + \frac{\sinh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[In] Int[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

```
[Out] Sinh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sinh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Sinh[a + c + (b + d)*x])/(8*(b + d)) + Sinh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8}\cosh(a-3c+(b-3d)x)+\frac{3}{8}\cosh(a-c+(b-d)x)\right.\\
&\quad \left.+\frac{3}{8}\cosh(a+c+(b+d)x)+\frac{1}{8}\cosh(a+3c+(b+3d)x)\right)\,dx\\
&= \frac{1}{8}\int \cosh(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \cosh(a+3c+(b+3d)x)\,dx\\
&\quad +\frac{3}{8}\int \cosh(a-c+(b-d)x)\,dx+\frac{3}{8}\int \cosh(a+c+(b+d)x)\,dx\\
&= \frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}\\
&\quad +\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8(b+3d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{1}{8}\left(\frac{\sinh(a-3c+bx-3dx)}{b-3d}+\frac{3\sinh(a-c+bx-dx)}{b-d}\right.\\ \left.+\frac{\sinh(a+3c+bx+3dx)}{b+3d}+\frac{3\sinh(a+c+(b+d)x)}{b+d}\right)$$

```
[In] Integrate[Cosh[a + b*x]*Cosh[c + d*x]^3,x]
```

```
[Out] (Sinh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sinh[a - c + b*x - d*x])/(b - d
) + Sinh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sinh[a + c + (b + d)*x])/(b
+ d))/8
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{(b\,e^{2bx+2a}-3\,e^{2bx+2a}d-b-3d)e^{-bx+3dx-a+3c}}{16(b+3d)(b-3d)}+\frac{3(b\,e^{2bx+2a}-e^{2bx+2a}d-b-d)e^{-bx+dx-a+c}}{16(b+d)(b-d)}+\frac{3(b\,e^{2bx+2a}+e^{2bx+2a}d-b}{16(b+d)(b-}$
parallelrisch	$\frac{2b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(b^2-7d^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6-6d\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)(b^2-3d^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+6b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(b^2+d^2)\tanh(}{(b-d)(b+3d)(b-3d)(b+d)}$

[In] int(cosh(b*x+a)*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/8*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sinh(a-c+(b-d)*x)/(b-d)+3/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(a+3*c+(b+3*d)*x)/(b+3*d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(83) = 166$.

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.38

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx$$
$$=\frac{3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)\sinh\left(dx+c\right)^2-3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\sinh\left(dx+c\right)^3+\left(\left(b^3-\right.\right.}{4\left(\left(b^4-10\,b^2d^2\right.\right.}$$

[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")

[Out] 1/4*(3*(b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)*sinh(d*x + c)^2 - 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(d*x + c)^3 + ((b^3 - b*d^2)*cosh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a))*sinh(d*x + c))/((b^4 - 10*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x + a)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.

Time = 1.88 (sec) , antiderivative size = 921, normalized size of antiderivative = 10.12

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cosh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sinh(a - 3*d*x)*s
inh(c + d*x)**3/8 + 3*x*sinh(a - 3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 +
3*x*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/8 + x*cosh(a - 3*d*x)*co
sh(c + d*x)**3/8 + sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 7
*sinh(a - 3*d*x)*cosh(c + d*x)**3/(24*d) + sinh(c + d*x)**3*cosh(a - 3*d*x)
/(8*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**3/8 + 3*x*sinh(a -
 d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a - d*x)
*cosh(c + d*x)/8 + 3*x*cosh(a - d*x)*cosh(c + d*x)**3/8 + 3*sinh(a - d*x)*s
inh(c + d*x)**2*cosh(c + d*x)/(4*d) - 5*sinh(a - d*x)*cosh(c + d*x)**3/(8*d
) + 3*sinh(c + d*x)**3*cosh(a - d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)*
sinh(c + d*x)**3/8 - 3*x*sinh(a + d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3
*x*sinh(c + d*x)**2*cosh(a + d*x)*cosh(c + d*x)/8 + 3*x*cosh(a + d*x)*cosh(
c + d*x)**3/8 - 3*sinh(a + d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + 5*si
nh(a + d*x)*cosh(c + d*x)**3/(8*d) + 3*sinh(c + d*x)**3*cosh(a + d*x)/(8*d)
, Eq(b, d)), (-x*sinh(a + 3*d*x)*sinh(c + d*x)**3/8 - 3*x*sinh(a + 3*d*x)*s
inh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh
(c + d*x)/8 + x*cosh(a + 3*d*x)*cosh(c + d*x)**3/8 - sinh(a + 3*d*x)*sinh(c
 + d*x)**2*cosh(c + d*x)/(4*d) + 7*sinh(a + 3*d*x)*cosh(c + d*x)**3/(24*d)
+ sinh(c + d*x)**3*cosh(a + 3*d*x)/(8*d), Eq(b, 3*d)), (b**3*sinh(a + b*x)*
cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh(c + d*x)*co
sh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh
(a + b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7
*b*d**2*sinh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d
**3*sinh(c + d*x)**3*cosh(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*
sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4)
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} + \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} + \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} - \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} - \frac{e^{(-bx-3\,dx-a-3\,c)}}{16\,(b+3\,d)}$$

```
[In] integrate(cosh(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) - 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) - 3/16*e^(-b*x + d*x -
a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)/(b + d) - 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \cosh(a+bx)\cosh^3(c+dx)\,dx = \frac{b\cosh(c+d\,x)^3\,\sinh(a+b\,x)\,\left(b^2-7\,d^2\right)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{3\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(b^2\,d-3\,d^3\right)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{6\,d^3\cosh(a+b\,x)\,\sinh(c+d\,x)^3}{b^4-10\,b^2\,d^2+9\,d^4} + \frac{6\,b\,d^2\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2}{b^4-10\,b^2\,d^2+9\,d^4}$$

```
[In] int(cosh(a + b*x)*cosh(c + d*x)^3,x)
```

```
[Out] (b*cosh(c + d*x)^3*sinh(a + b*x)*(b^2 - 7*d^2))/(b^4 + 9*d^4 - 10*b^2*d^2)
- (3*cosh(a + b*x)*cosh(c + d*x)^2*sinh(c + d*x)*(b^2*d - 3*d^3))/(b^4 + 9*
d^4 - 10*b^2*d^2) - (6*d^3*cosh(a + b*x)*sinh(c + d*x)^3)/(b^4 + 9*d^4 - 10
*b^2*d^2) + (6*b*d^2*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2)/(b^4 + 9*
d^4 - 10*b^2*d^2)
```

3.176 $\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}$$

```
[Out] 1/4*x+1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sinh(2*d
*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5733, 2717}

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2c+2dx)}{8d}+\frac{x}{4}$$

```
[In] Int[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

```
[Out] x/4 + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d))
+ Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)+\frac{1}{4}\cosh(2c+2dx)\right. \\
&\qquad \left.+\frac{1}{8}\cosh(2(a+c)+2(b+d)x)\right)\,dx \\
&= \frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx \\
&\quad +\frac{1}{4}\int \cosh(2a+2bx)\,dx+\frac{1}{4}\int \cosh(2c+2dx)\,dx \\
&= \frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)} \\
&\quad +\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$$
$$= \frac{2d(b^2-d^2)\sinh(2(a+bx))+bd(b+d)\sinh(2(a-c+(b-d)x))+b(b-d)(2(b+d)\sinh(2(c+dx))+d(}{16b(b-d)d(b+d)}$$

```
[In] Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x)
] + b*(b - d)*(2*(b + d)*Sinh[2*(c + d*x)] + d*(4*(b + d)*x + Sinh[2*(a + c
 + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}+\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sinh((2b-2d)x+2a-2c)+4\left(\frac{bd\sinh((2b+2d)x+2a+2c)}{4}+\left(\frac{d\sinh(2bx+2a)}{2}+b\left(dx+\frac{\sinh(2dx+2c)}{2}\right)\right)(b+d)\right)(b-d)}{16b^3d-16d^3b}$
risch	$\frac{x}{4}+\frac{e^{2bx+2a}}{16b}-\frac{e^{-2bx-2a}}{16b}+\frac{(d\,e^{4bx+4a}b-d^2e^{4bx+4a}+2b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2-bd-d^2)e^{-2bx+2dx-2a+2c}}{32(b+d)(b-d)d}-\frac{(-d\,e^{4bx+}}{}$

[In] int(cosh(b*x+a)^2*cosh(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/4*x+1/8*sinh(2*b*x+2*a)/b+1/8*sinh(2*d*x+2*c)/d+1/8/(2*b-2*d)*sinh((2*b-2*d)*x+2*a-2*c)+1/8/(2*b+2*d)*sinh((2*b+2*d)*x+2*a+2*c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx$$
$$=\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2+(b^3d-bd^3)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2+(b^2d-\right.}{4\left((b^3d-bd^3)\cos\right.}$$

[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 + (b^3*d - b*d^3)*x + (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sinh(b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 - b^3 + b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2 - (b^3*d - b*d^3)*sinh(b*x + a)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(76) = 152.

Time = 1.58 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \text{Too large to display}$$

[In] integrate(cosh(b*x+a)**2*cosh(d*x+c)**2,x)

```
[Out] Piecewise((x*cosh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*cosh(a)*
*2, Eq(b, 0)), (3*x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 - x*sinh(a - d*x)**
2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c +
 d*x)/2 - x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 + 3*x*cosh(a - d*x)**2*cosh
(c + d*x)**2/8 + sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) + sinh(
a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) + 5*sinh(c + d*x)*cosh(a - d*
x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (3*x*sinh(a + d*x)**2*sinh(c + d*x)*
*2/8 - x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d*x
)*cosh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)**2*cosh(a + d*x)**2/8 + 3
*x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)**2*sinh(c + d*x)*cos
h(c + d*x)/(8*d) - sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)/(2*d) + 5*s
inh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(8*d), Eq(b, d)), ((-x*sinh(a +
 b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*cosh
(c)**2, Eq(d, 0)), (b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d -
4*b*d**3) - b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3
) - b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b**3
*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b**3*sinh(a
+ b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*sinh(c +
 d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sinh(
a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*si
nh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(a + b
*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*sinh(c + d*x)**2*c
osh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cosh(a + b*x)**2*cosh(c +
d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cos
h(c + d*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*sinh(c + d*x)**2*cosh
(a + b*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c +
 d*x)**2/(4*b**3*d - 4*b*d**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{1}{4}\,x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} + \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} - \frac{e^{(-2\,bx-2\,a)}}{16\,b} + \frac{e^{(2\,dx+2\,c)}}{16\,d} - \frac{e^{(-2\,dx-2\,c)}}{16\,d}$$

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
+ 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2*
a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^(-2
*b*x - 2*a)/b + 1/16*e^(2*d*x + 2*c)/d - 1/16*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \cosh^2(a+bx)\cosh^2(c+dx)\,dx = \frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x) - b^3\cosh(c+d\,x)\,\sinh(c+d\,x) + b\,d^3\,x - b^3\,d\,x - 2\,b^2\,d\cosh(a+b\,x)\,\cosh}{4\,b\,d^3-4\,b^3\,d}$$

```
[In] int(cosh(a + b*x)^2*cosh(c + d*x)^2,x)
```

```
[Out] (d^3*cosh(a + b*x)*sinh(a + b*x) - b^3*cosh(c + d*x)*sinh(c + d*x) + b*d^3*
x - b^3*d*x - 2*b^2*d*cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d^2
*cosh(a + b*x)^2*cosh(c + d*x)*sinh(c + d*x))/(4*b*d^3 - 4*b^3*d)
```

3.177 $\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sinh(c+dx)}{8d} + \frac{\sinh(3c+3dx)}{24d} + \frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\sinh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5733, 2717}

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)} + \frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)} + \frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)} + \frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)} + \frac{3\sinh(c+dx)}{8d} + \frac{\sinh(3c+3dx)}{24d}$$

```
[In] Int[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) + (3*Sinh[c + d*x])/(8*d) + Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{16}\cosh(2a-3c+(2b-3d)x) + \frac{3}{16}\cosh(2a-c+(2b-d)x) + \frac{3}{8}\cosh(c+dx) + \frac{1}{8}\cosh(3c+3dx) + \frac{3}{16}\cosh(2a+c+(2b+d)x) + \frac{1}{16}\cosh(2a+3c+(2b+3d)x)\right)dx$$

$$= \frac{1}{16}\int \cosh(2a-3c+(2b-3d)x)\,dx + \frac{1}{16}\int \cosh(2a+3c+(2b+3d)x)\,dx + \frac{1}{8}\int \cosh(3c+3dx)\,dx + \frac{3}{16}\int \cosh(2a-c+(2b-d)x)\,dx + \frac{3}{16}\int \cosh(2a+c+(2b+d)x)\,dx + \frac{3}{8}\int \cosh(c+dx)\,dx$$

$$= \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} + \frac{3\sinh(c+dx)}{8d} + \frac{\sinh(3c+3dx)}{24d} + \frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\sinh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{1}{48}\left(\frac{18\cosh(dx)\sinh(c)}{d} + \frac{2\cosh(3dx)\sinh(3c)}{d} + \frac{18\cosh(c)\sinh(dx)}{d} + \frac{2\cosh(3c)\sinh(3dx)}{d} + \frac{3\sinh(2a-3c+2bx-3dx)}{2b-3d} + \frac{9\sinh(2a-c+2bx-dx)}{2b-d} + \frac{9\sinh(2a+c+2bx+dx)}{2b+d} + \frac{3\sinh(2a+3c+2bx+3dx)}{2b+3d}\right)$$

```
[In] Integrate[Cosh[a + b*x]^2*Cosh[c + d*x]^3,x]
```

```
[Out] ((18*Cosh[d*x]*Sinh[c])/d + (2*Cosh[3*d*x]*Sinh[3*c])/d + (18*Cosh[c]*Sinh[
d*x])/d + (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x])
/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a + c
+ 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d)
)/48
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sinh(dx+c)}{8d}+\frac{\sinh(3dx+3c)}{24d}+\frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)}+\frac{\sinh(}{}$
parallelrisch	$\frac{(24b^3d+36d^2b^2-6d^3b-9d^4)\sinh(2a-3c+(2b-3d)x)+72\left(d\left(b+\frac{3d}{2}\right)\left(b+\frac{d}{2}\right)\sinh(2a-c+(2b-d)x)+\left(\frac{d\left(b+\frac{d}{2}\right)\sinh(2a+3c+(2b}{3}\right.\right.}{768d\,b^4-1920b^2d^3+432d^5}$
risch	$\frac{(6d\,e^{4bx+4a}b-9d^2e^{4bx+4a}+8b^2e^{2bx+2a}-18\,e^{2bx+2a}d^2-6bd-9d^2)\,e^{-2bx+3dx-2a+3c}}{96(2b+3d)(2b-3d)d}+\frac{3(2d\,e^{4bx+4a}b-d^2e^{4bx+4a}+8b^2e^{2bx+}}{32(2b-}$

```
[In] int(cosh(b*x+a)^2*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
+3/8*sinh(d*x+c)/d+1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(132) = 264.

Time = 0.25 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.76

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx$$
$$=\frac{36\,(4\,b^3d-bd^3)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)^2+\left(16\,b^4-40\,b^2d^2+9\,d^4-9\,(4\,b^2d\right.}{}$$

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(d
*x + c)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2 - d^4)*cosh(b*x + a
)^2 - 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*d -
 b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x + a)
*cosh(d*x + c))*sinh(b*x + a) + 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 - 3*(4*b^2
*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 - 9*(4*b^2*d^2
 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 - 3*(4*b^2*d^2 - 9*d^4 + 3*(4*b^2*
d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^4*d - 40
*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*
x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(116) = 232$.

Time = 5.29 (sec) , antiderivative size = 2008, normalized size of antiderivative = 13.94

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)**2*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cosh(a)**2*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d
*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + x*sinh(a - 3*d*x/2)**2*cosh(c
+ d*x)**3/16 + x*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/8 + 3
*x*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/8 + 3
*x*sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**3/16 + 9*sinh(a - 3*d*x/2)**2*sinh(c + d*x)**3/(16*d
) + 5*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cosh(c + d*x)/(4
*d) + sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/(24*d) - 5*sinh(
c + d*x)**3*cosh(a - 3*d*x/2)**2/(48*d) + sinh(c + d*x)*cosh(a - 3*d*x/2)**
2*cosh(c + d*x)**2/d, Eq(b, -3*d/2)), (-3*x*sinh(a - d*x/2)**2*sinh(c + d*x
)**2*cosh(c + d*x)/16 + 3*x*sinh(a - d*x/2)**2*cosh(c + d*x)**3/16 - 3*x*si
nh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*x/2)/8 + 3*x*sinh(a - d*x/2)*sinh
(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(a
- d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a - d*x/2)**2*cosh(c + d*x)**3/16 +
 49*sinh(a - d*x/2)**2*sinh(c + d*x)**3/(48*d) - sinh(a - d*x/2)**2*sinh(c
+ d*x)*cosh(c + d*x)**2/d + 7*sinh(a - d*x/2)*sinh(c + d*x)**2*cosh(a - d*x
/2)*cosh(c + d*x)/(4*d) - 13*sinh(a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)*
*3/(8*d) + 17*sinh(c + d*x)**3*cosh(a - d*x/2)**2/(48*d), Eq(b, -d/2)), (-3
*x*sinh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a + d*x/
2)**2*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x/2)*sinh(c + d*x)**3*cosh(a + d
*x/2)/8 - 3*x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*x/2)*cosh(c + d*x)**
2/8 - 3*x*sinh(c + d*x)**2*cosh(a + d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a
 + d*x/2)**2*cosh(c + d*x)**3/16 + 49*sinh(a + d*x/2)**2*sinh(c + d*x)**3/(
48*d) - sinh(a + d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/d - 7*sinh(a + d*
x/2)*sinh(c + d*x)**2*cosh(a + d*x/2)*cosh(c + d*x)/(4*d) + 13*sinh(a + d*x
/2)*cosh(a + d*x/2)*cosh(c + d*x)**3/(8*d) + 17*sinh(c + d*x)**3*cosh(a + d
*x/2)**2/(48*d), Eq(b, d/2)), (3*x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**2*co
sh(c + d*x)/16 + x*sinh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16 - x*sinh(a + 3*
d*x/2)*sinh(c + d*x)**3*cosh(a + 3*d*x/2)/8 - 3*x*sinh(a + 3*d*x/2)*sinh(c
+ d*x)*cosh(a + 3*d*x/2)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a +
 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16
+ 11*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**3/(48*d) - sinh(a + 3*d*x/2)**2*si
nh(c + d*x)*cosh(c + d*x)**2/d + 3*sinh(a + 3*d*x/2)*sinh(c + d*x)**2*cosh(
a + 3*d*x/2)*cosh(c + d*x)/(4*d) + 5*sinh(a + 3*d*x/2)*cosh(a + 3*d*x/2)*co
sh(c + d*x)**3/(8*d) - 7*sinh(c + d*x)**3*cosh(a + 3*d*x/2)**2/(16*d), Eq(b
```

```
, 3*d/2)), ((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*c
osh(a + b*x)/(2*b))*cosh(c)**3, Eq(d, 0)), (16*b**4*sinh(a + b*x)**2*sinh(c
 + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 24*b**4*sinh(a + b*x)**2
*sinh(c + d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 16*
b**4*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5
) + 24*b**4*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(48*b**4*d - 12
0*b**2*d**3 + 27*d**5) + 24*b**3*d*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x
)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*sinh(a + b*x)**2*
sinh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b**2*d**2*sinh(
a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*
d**5) + 40*b**2*d**2*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**
2*d**3 + 27*d**5) - 78*b**2*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*
x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sinh(a + b*x)*sinh(
c + d*x)**2*cosh(a + b*x)*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**
5) - 42*b*d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**3/(48*b**4*d - 12
0*b**2*d**3 + 27*d**5) - 18*d**4*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4
*d - 120*b**2*d**3 + 27*d**5) + 27*d**4*sinh(c + d*x)*cosh(a + b*x)**2*cosh
(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)} + \frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)} + \frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)} + \frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)} - \frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)} - \frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)} - \frac{3\,e^{(-2\,bx-dx-2\,a-c)}}{32\,(2\,b+d)} - \frac{e^{(-2\,bx-3\,dx-2\,a-3\,c)}}{32\,(2\,b+3\,d)} + \frac{e^{(3\,dx+3\,c)}}{48\,d} + \frac{3\,e^{(dx+c)}}{16\,d} - \frac{3\,e^{(-dx-c)}}{16\,d} - \frac{e^{(-3\,dx-3\,c)}}{48\,d}$$

```
[In] integrate(cosh(b*x+a)^2*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) + 1/4
8*e^(3*d*x + 3*c)/d + 3/16*e^(d*x + c)/d - 3/16*e^(-d*x - c)/d - 1/48*e^(-3
*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.34

$$\int \cosh^2(a+bx)\cosh^3(c+dx)\,dx = \frac{\cosh(a+b\,x)^2\cosh(c+d\,x)^2\sinh(c+d\,x)\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)} - \sinh(a+b\,x)^2\sinh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{1}{3\,d}\right) - \frac{2\cosh(a+b\,x)\,\cosh(c+d\,x)^3\sinh(a+b\,x)\,\left(7\,b\,d^2-4\,b^3\right)}{16\,b^4-40\,b^2\,d^2+9\,d^4} - \cosh(a+b\,x)^2\sinh(c+d\,x)^3\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}+\frac{1}{3\,d}\right) + \frac{12\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)^2}{16\,b^4-40\,b^2\,d^2+9\,d^4} - \frac{2\,b^2\cosh(c+d\,x)^2\sinh(a+b\,x)^2\sinh(c+d\,x)\,\left(4\,b^2-7\,d^2\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}$$

```
[In] int(cosh(a + b*x)^2*cosh(c + d*x)^3,x)
```

```
[Out] (cosh(a + b*x)^2*cosh(c + d*x)^2*sinh(c + d*x)*(8*b^4 + 9*d^4 - 26*b^2*d^2)
)/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - sinh(a + b*x)^2*sinh(c + d*x)^3*((3*d
^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) - 1/(3*d)) - (2*cosh(a + b*x)*cosh(c + d*
x)^3*sinh(a + b*x)*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - cosh(
a + b*x)^2*sinh(c + d*x)^3*((3*d^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)
) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x)^2)/(1
6*b^4 + 9*d^4 - 40*b^2*d^2) - (2*b^2*cosh(c + d*x)^2*sinh(a + b*x)^2*sinh(c
 + d*x)*(4*b^2 - 7*d^2))/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2))
```

3.178 $\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sinh(a-c+(b-d)x)}{32(b-d)} + \frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\sinh(3a-c+(3b-d)x)}{32(3b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)} + \frac{\sinh(3(a+c)+3(b+d)x)}{96(b+d)} + \frac{3\sinh(3a+c+(3b+d)x)}{32(3b+d)} + \frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}$$

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+1/96*sinh(3
*a-3*c+3*(b-d)*x)/(b-d)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sinh(a+c+(b
+d)*x)/(b+d)+1/96*sinh(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sinh(3*a+c+(3*b+d)*x)/
(3*b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5733, 2717}

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \frac{3\sinh(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sinh(a+x(b-d)-c)}{32(b-d)} + \frac{\sinh(3(a-c)+3x(b-d))}{96(b-d)} + \frac{3\sinh(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sinh(a+x(b+d)+c)}{32(b+d)} + \frac{\sinh(3(a+c)+3x(b+d))}{96(b+d)} + \frac{3\sinh(3a+x(3b+d)+c)}{32(3b+d)} + \frac{3\sinh(a+x(b+3d)+3c)}{32(b+3d)}$$

```
[In] Int[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

```
[Out] (3*Sinh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sinh[a - c + (b - d)*x]
)/(32*(b - d)) + Sinh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sinh[3*a -
 c + (3*b - d)*x])/(32*(3*b - d)) + (9*Sinh[a + c + (b + d)*x])/(32*(b + d)
) + Sinh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sinh[3*a + c + (3*b + d
)*x])/(32*(3*b + d)) + (3*Sinh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{32}\cosh(a-3c+(b-3d)x)+\frac{9}{32}\cosh(a-c+(b-d)x)+\frac{1}{32}\cosh(3(a-c)+3(b-d)x)+\frac{3}{32}\cosh(3a-c+(3b-d)x)+\frac{9}{32}\cosh(a+c+(b+d)x)+\frac{1}{32}\cosh(3(a+c)+3(b+d)x)+\frac{3}{32}\cosh(3a+c+(3b+d)x)+\frac{3}{32}\cosh(a+3c+(b+3d)x)\right)dx$$

$$=\frac{1}{32}\int\cosh(3(a-c)+3(b-d)x)\,dx+\frac{1}{32}\int\cosh(3(a+c)+3(b+d)x)\,dx+\frac{3}{32}\int\cosh(a-3c+(b-3d)x)\,dx+\frac{3}{32}\int\cosh(3a-c+(3b-d)x)\,dx+\frac{3}{32}\int\cosh(3a+c+(3b+d)x)\,dx+\frac{3}{32}\int\cosh(a+3c+(b+3d)x)\,dx+\frac{9}{32}\int\cosh(a-c+(b-d)x)\,dx+\frac{9}{32}\int\cosh(a+c+(b+d)x)\,dx$$

$$=\frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{\sinh(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\sinh(3a-c+(3b-d)x)}{32(3b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}+\frac{\sinh(3(a+c)+3(b+d)x)}{96(b+d)}+\frac{3\sinh(3a+c+(3b+d)x)}{32(3b+d)}+\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int\cosh^3(a+bx)\cosh^3(c+dx)\,dx=\frac{1}{96}\left(\frac{9\sinh(a-3c+bx-3dx)}{b-3d}+\frac{27\sinh(a-c+bx-dx)}{b-d}+\frac{\sinh(3(a-c+bx-dx))}{b-d}+\frac{9\sinh(3a-c+3bx-dx)}{3b-d}+\frac{9\sinh(3a+c+3bx+dx)}{3b+d}+\frac{9\sinh(a+3c+bx+3dx)}{b+3d}+\frac{27\sinh(a+c+(b+d)x)}{b+d}+\frac{\sinh(3(a+c+(b+d)x))}{b+d}\right)$$

```
[In] Integrate[Cosh[a + b*x]^3*Cosh[c + d*x]^3,x]
```

```
[Out] ((9*Sinh[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sinh[a - c + b*x - d*x])/(
b - d) + Sinh[3*(a - c + b*x - d*x)]/(b - d) + (9*Sinh[3*a - c + 3*b*x - d*
x])/(3*b - d) + (9*Sinh[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sinh[a + 3*c
 + b*x + 3*d*x])/(b + 3*d) + (27*Sinh[a + c + (b + d)*x])/(b + d) + Sinh[3*
(a + c + (b + d)*x)]/(b + d))/96
```

Maple [A] (verified)

Time = 11.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$\frac{3\sinh(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sinh(a-c+(b-d)x)}{32(b-d)}+\frac{9\sinh(a+c+(b+d)x)}{32(b+d)}+\frac{3\sinh(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sinh((3b-3d)x+3a}{96b-96d}$
parallelrisch	$\frac{\frac{9(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d)(b+d)\sinh(3a-c+(3b-d)x)}{32}+\frac{9\left(\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b+d)\sinh((3b-3d)x+3a-3c)}{3}+\frac{(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d}{3}\right.}{}}{}$
risch	$\frac{\left(b^{3}\mathrm{e}^{6bx+6a}-b^{2}d\,\mathrm{e}^{6bx+6a}-9b\,d^{2}\mathrm{e}^{6bx+6a}+9d^{3}\mathrm{e}^{6bx+6a}+9b^{3}\mathrm{e}^{4bx+4a}-27b^{2}d\,\mathrm{e}^{4bx+4a}-9b\,d^{2}\mathrm{e}^{4bx+4a}+27d^{3}\mathrm{e}^{4bx+4a}-9b^{3}\mathrm{e}^{2bx+2a}-\right.}{192(b+d)(b+3d)(b-d)(b-3d)}$

```
[In] int(cosh(b*x+a)^3*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/32*sinh(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sinh(a-c+(b-d)*x)/(b-d)+9/32*sinh(a
+c+(b+d)*x)/(b+d)+3/32*sinh(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*sinh((3
*b-3*d)*x+3*a-3*c)+3/32*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sinh(3*a+c+(3*b+
d)*x)/(3*b+d)+1/32/(3*b+3*d)*sinh((3*b+3*d)*x+3*a+3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.72

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx$$
$$=\frac{\left(\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cosh\left(dx+c\right)^3+27\left(b^5-10\,b^3d^2+9\,bd^4\right)\cosh\left(dx+c\right)\right)\sinh\left(bx+a\right)^3-\left(\left(9\,b^4d\right.\right.}{}$$

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d^
2 + 9*b*d^4)*cosh(d*x + c))*sinh(b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^
5)*cosh(b*x + a)^3 + 3*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)*sinh(b*
x + a)^2 + 27*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(d*x + c)^3 +
 3*((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(d*x + c)*sinh(b*x + a)^3 + 3*(27*b^
5 - 30*b^3*d^2 + 3*b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*
```

```
cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((27*b^5 - 30*b^3*d^2 + 3*
b*d^4 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^3 + 9
*(9*b^5 - 82*b^3*d^2 + 9*b*d^4 + 3*(b^5 - 10*b^3*d^2 + 9*b*d^4)*cosh(b*x +
a)^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(b^4*d - 10*b^2*d^3 + 9*d^5)*cosh
(b*x + a)^3 + ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^3 + 27*(9*b^4*d
 - 10*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*((9*b^4*d - 82*b^2*
d^3 + 9*d^5)*cosh(b*x + a)*cosh(d*x + c)^2 + 3*(b^4*d - 10*b^2*d^3 + 9*d^5)
*cosh(b*x + a))*sinh(b*x + a)^2 + 9*(9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x
 + a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
 a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*
x + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3582 vs. $2(172) = 344$.

Time = 16.83 (sec) , antiderivative size = 3582, normalized size of antiderivative = 18.37

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(b*x+a)**3*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cosh(a)**3*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sinh(a - 3*
d*x)**3*sinh(c + d*x)**3/32 - 9*x*sinh(a - 3*d*x)**3*sinh(c + d*x)*cosh(c +
 d*x)**2/32 - 9*x*sinh(a - 3*d*x)**2*sinh(c + d*x)**2*cosh(a - 3*d*x)*cosh(
c + d*x)/32 - 3*x*sinh(a - 3*d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)**3/32 +
3*x*sinh(a - 3*d*x)*sinh(c + d*x)**3*cosh(a - 3*d*x)**2/32 + 9*x*sinh(a - 3
*d*x)*sinh(c + d*x)*cosh(a - 3*d*x)**2*cosh(c + d*x)**2/32 + 9*x*sinh(c + d
*x)**2*cosh(a - 3*d*x)**3*cosh(c + d*x)/32 + 3*x*cosh(a - 3*d*x)**3*cosh(c
+ d*x)**3/32 - 3*sinh(a - 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(320*d)
+ sinh(a - 3*d*x)**3*cosh(c + d*x)**3/(4*d) - 11*sinh(a - 3*d*x)**2*sinh(c
+ d*x)**3*cosh(a - 3*d*x)/(320*d) + 3*sinh(a - 3*d*x)**2*sinh(c + d*x)*cosh
(a - 3*d*x)*cosh(c + d*x)**2/(20*d) - 117*sinh(a - 3*d*x)*cosh(a - 3*d*x)**
2*cosh(c + d*x)**3/(320*d) + sinh(c + d*x)**3*cosh(a - 3*d*x)**3/(30*d) - 6
1*sinh(c + d*x)*cosh(a - 3*d*x)**3*cosh(c + d*x)**2/(320*d), Eq(b, -3*d)),
(5*x*sinh(a - d*x)**3*sinh(c + d*x)**3/16 - 3*x*sinh(a - d*x)**3*sinh(c + d
*x)*cosh(c + d*x)**2/16 + 9*x*sinh(a - d*x)**2*sinh(c + d*x)**2*cosh(a - d*
x)*cosh(c + d*x)/16 - 3*x*sinh(a - d*x)**2*cosh(a - d*x)*cosh(c + d*x)**3/1
6 - 3*x*sinh(a - d*x)*sinh(c + d*x)**3*cosh(a - d*x)**2/16 + 9*x*sinh(a - d
*x)*sinh(c + d*x)*cosh(a - d*x)**2*cosh(c + d*x)**2/16 - 3*x*sinh(c + d*x)*
*2*cosh(a - d*x)**3*cosh(c + d*x)/16 + 5*x*cosh(a - d*x)**3*cosh(c + d*x)**
3/16 - sinh(a - d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(2*d) + 7*sinh(a - d
*x)**3*cosh(c + d*x)**3/(48*d) - 3*sinh(a - d*x)**2*sinh(c + d*x)**3*cosh(a
 - d*x)/(16*d) - 3*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*
x)**2/(4*d) - 11*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)**3/(16*d) + s
```

inh(c + d*x)**3*cosh(a - d*x)**3/(16*d), Eq(b, -d)), (-3*x*sinh(a - d*x/3)*
*3*sinh(c + d*x)**3/32 + 3*x*sinh(a - d*x/3)**3*sinh(c + d*x)*cosh(c + d*x)
2/32 - 9*x*sinh(a - d*x/3)2*sinh(c + d*x)**2*cosh(a - d*x/3)*cosh(c + d
*x)/32 + 9*x*sinh(a - d*x/3)**2*cosh(a - d*x/3)*cosh(c + d*x)**3/32 - 9*x*s
inh(a - d*x/3)*sinh(c + d*x)**3*cosh(a - d*x/3)**2/32 + 9*x*sinh(a - d*x/3)
*sinh(c + d*x)*cosh(a - d*x/3)**2*cosh(c + d*x)**2/32 - 3*x*sinh(c + d*x)**
2*cosh(a - d*x/3)**3*cosh(c + d*x)/32 + 3*x*cosh(a - d*x/3)**3*cosh(c + d*x
)**3/32 + 81*sinh(a - d*x/3)**3*sinh(c + d*x)**2*cosh(c + d*x)/(320*d) - si
nh(a - d*x/3)**3*cosh(c + d*x)**3/(4*d) + 153*sinh(a - d*x/3)**2*sinh(c + d
*x)**3*cosh(a - d*x/3)/(320*d) - 9*sinh(a - d*x/3)**2*sinh(c + d*x)*cosh(a
- d*x/3)*cosh(c + d*x)**2/(20*d) + 39*sinh(a - d*x/3)*cosh(a - d*x/3)**2*co
sh(c + d*x)**3/(320*d) - 3*sinh(c + d*x)**3*cosh(a - d*x/3)**3/(5*d) + 303*
sinh(c + d*x)*cosh(a - d*x/3)**3*cosh(c + d*x)**2/(320*d), Eq(b, -d/3)), (3
*x*sinh(a + d*x/3)**3*sinh(c + d*x)**3/32 - 3*x*sinh(a + d*x/3)**3*sinh(c +
d*x)*cosh(c + d*x)**2/32 - 9*x*sinh(a + d*x/3)**2*sinh(c + d*x)**2*cosh(a
+ d*x/3)*cosh(c + d*x)/32 + 9*x*sinh(a + d*x/3)**2*cosh(a + d*x/3)*cosh(c +
d*x)**3/32 + 9*x*sinh(a + d*x/3)*sinh(c + d*x)**3*cosh(a + d*x/3)**2/32 -
9*x*sinh(a + d*x/3)*sinh(c + d*x)*cosh(a + d*x/3)**2*cosh(c + d*x)**2/32 -
3*x*sinh(c + d*x)**2*cosh(a + d*x/3)**3*cosh(c + d*x)/32 + 3*x*cosh(a + d*x
/3)**3*cosh(c + d*x)**3/32 - 33*sinh(a + d*x/3)**3*sinh(c + d*x)**2*cosh(c
+ d*x)/(320*d) + sinh(a + d*x/3)**3*cosh(c + d*x)**3/(10*d) + 9*sinh(a + d*
x/3)**2*sinh(c + d*x)**3*cosh(a + d*x/3)/(320*d) + 9*sinh(a + d*x/3)*sinh(c
+ d*x)**2*cosh(a + d*x/3)**2*cosh(c + d*x)/(20*d) - 183*sinh(a + d*x/3)*co
sh(a + d*x/3)**2*cosh(c + d*x)**3/(320*d) - 3*sinh(c + d*x)**3*cosh(a + d*x
/3)**3/(4*d) + 351*sinh(c + d*x)*cosh(a + d*x/3)**3*cosh(c + d*x)**2/(320*d
), Eq(b, d/3)), (-5*x*sinh(a + d*x)**3*sinh(c + d*x)**3/16 + 3*x*sinh(a + d
*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/16 + 9*x*sinh(a + d*x)**2*sinh(c + d*
x)**2*cosh(a + d*x)*cosh(c + d*x)/16 - 3*x*sinh(a + d*x)**2*cosh(a + d*x)*c
osh(c + d*x)**3/16 + 3*x*sinh(a + d*x)*sinh(c + d*x)**3*cosh(a + d*x)**2/16
- 9*x*sinh(a + d*x)*sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)**2/16 - 3
*x*sinh(c + d*x)**2*cosh(a + d*x)**3*cosh(c + d*x)/16 + 5*x*cosh(a + d*x)**
3*cosh(c + d*x)**3/16 + 5*sinh(a + d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(
16*d) - 13*sinh(a + d*x)**3*cosh(c + d*x)**3/(48*d) - 3*sinh(a + d*x)*sinh(
c + d*x)**2*cosh(a + d*x)**2*cosh(c + d*x)/(4*d) + sinh(a + d*x)*cosh(a + d
*x)**2*cosh(c + d*x)**3/(2*d) + 3*sinh(c + d*x)**3*cosh(a + d*x)**3/(16*d)
+ 3*sinh(c + d*x)*cosh(a + d*x)**3*cosh(c + d*x)**2/(16*d), Eq(b, d)), (3*x
*sinh(a + 3*d*x)**3*sinh(c + d*x)**3/32 + 9*x*sinh(a + 3*d*x)**3*sinh(c + d
*x)*cosh(c + d*x)**2/32 - 9*x*sinh(a + 3*d*x)**2*sinh(c + d*x)**2*cosh(a +
3*d*x)*cosh(c + d*x)/32 - 3*x*sinh(a + 3*d*x)**2*cosh(a + 3*d*x)*cosh(c + d
*x)**3/32 - 3*x*sinh(a + 3*d*x)*sinh(c + d*x)**3*cosh(a + 3*d*x)**2/32 - 9*
x*sinh(a + 3*d*x)*sinh(c + d*x)*cosh(a + 3*d*x)**2*cosh(c + d*x)**2/32 + 9*
x*sinh(c + d*x)**2*cosh(a + 3*d*x)**3*cosh(c + d*x)/32 + 3*x*cosh(a + 3*d*x
)**3*cosh(c + d*x)**3/32 + 3*sinh(a + 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d
*x)/(320*d) - sinh(a + 3*d*x)**3*cosh(c + d*x)**3/(4*d) - 11*sinh(a + 3*d*x
)**2*sinh(c + d*x)**3*cosh(a + 3*d*x)/(320*d) + 3*sinh(a + 3*d*x)**2*sinh(c

```
 + d*x)*cosh(a + 3*d*x)*cosh(c + d*x)**2/(20*d) + 117*sinh(a + 3*d*x)*cosh(
a + 3*d*x)**2*cosh(c + d*x)**3/(320*d) + sinh(c + d*x)**3*cosh(a + 3*d*x)**
3/(30*d) - 61*sinh(c + d*x)*cosh(a + 3*d*x)**3*cosh(c + d*x)**2/(320*d), Eq
(b, 3*d)), (-18*b**5*sinh(a + b*x)**3*cosh(c + d*x)**3/(27*b**6 - 273*b**4*
d**2 + 273*b**2*d**4 - 27*d**6) + 27*b**5*sinh(a + b*x)*cosh(a + b*x)**2*co
sh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 54*b**
4*d*sinh(a + b*x)**2*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**6
- 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 63*b**4*d*sinh(c + d*x)*cosh(a
 + b*x)**3*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d
**6) - 120*b**3*d**2*sinh(a + b*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/(27*b*
*6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 122*b**3*d**2*sinh(a + b*x)
**3*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) +
126*b**3*d**2*sinh(a + b*x)*sinh(c + d*x)**2*cosh(a + b*x)**2*cosh(c + d*x)
/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 210*b**3*d**2*sinh(a
 + b*x)*cosh(a + b*x)**2*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b*
*2*d**4 - 27*d**6) + 120*b**2*d**3*sinh(a + b*x)**2*sinh(c + d*x)**3*cosh(a
 + b*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 126*b**2*d**3
*sinh(a + b*x)**2*sinh(c + d*x)*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**6 - 2
73*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 122*b**2*d**3*sinh(c + d*x)**3*co
sh(a + b*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 210*b*
*2*d**3*sinh(c + d*x)*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**6 - 273*b**4
*d**2 + 273*b**2*d**4 - 27*d**6) - 54*b*d**4*sinh(a + b*x)*sinh(c + d*x)**2
*cosh(a + b*x)**2*cosh(c + d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 -
27*d**6) + 63*b*d**4*sinh(a + b*x)*cosh(a + b*x)**2*cosh(c + d*x)**3/(27*b*
*6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 18*d**5*sinh(c + d*x)**3*co
sh(a + b*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 27*d**
5*sinh(c + d*x)*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2
+ 273*b**2*d**4 - 27*d**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = \frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} + \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} + \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} + \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} + \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} + \frac{9\,e^{(bx-dx+a-c)}}{64\,(b-d)} + \frac{3\,e^{(bx-3\,dx+a-3\,c)}}{64\,(b-3\,d)} - \frac{3\,e^{(-bx+3\,dx-a+3\,c)}}{64\,(b-3\,d)} - \frac{9\,e^{(-bx+dx-a+c)}}{64\,(b-d)} - \frac{9\,e^{(-bx-dx-a-c)}}{64\,(b+d)} - \frac{3\,e^{(-bx-3\,dx-a-3\,c)}}{64\,(b+3\,d)} - \frac{e^{(-3\,bx+3\,dx-3\,a+3\,c)}}{192\,(b-d)} - \frac{3\,e^{(-3\,bx+dx-3\,a+c)}}{64\,(3\,b-d)} - \frac{3\,e^{(-3\,bx-dx-3\,a-c)}}{64\,(3\,b+d)} - \frac{e^{(-3\,bx-3\,dx-3\,a-3\,c)}}{192\,(b+d)}$$

```
[In] integrate(cosh(b*x+a)^3*cosh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) + 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9/64
*e^(b*x + d*x + a + c)/(b + d) + 9/64*e^(b*x - d*x + a - c)/(b - d) + 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) - 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) - 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) - 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 908, normalized size of antiderivative = 4.66

$$\int \cosh^3(a+bx)\cosh^3(c+dx)\,dx = -e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4} - \frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4} + \frac{e^{-2a-2bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-c+3bx-dx}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{576b^4-640b^2d^2+64d^4} - \frac{e^{-6a-6bx}\left(-9b^3+3b^2d+9bd^2-3d^3\right)}{576b^4-640b^2d^2+64d^4} + \frac{e^{-2a-2bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4} - \frac{e^{-4a-4bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4}\right) - e^{3a-3c+3bx-3dx}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-6a-6bx}\left(-b^3+b^2d+9bd^2-9d^3\right)}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-2a-2bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right) - e^{3a+3c+3bx+3dx}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-6a-6bx}\left(-b^3-b^2d+9bd^2+9d^3\right)}{192b^4-1920b^2d^2+1728d^4} + \frac{e^{-2a-2bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4} - \frac{e^{-4a-4bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right)$$

```
[In] int(cosh(a + b*x)^3*cosh(c + d*x)^3,x)
```

```
[Out] - exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4
+ 64*d^4 - 640*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 +
3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 2*a - 2*b*x)*(9*b*d^2 + 8
1*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a -
```

```
4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d
^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(576
*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d - 9*b
^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 2*a - 2*b*x)*(9*b*d^
2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4
*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*
b^2*d^2)) - exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)
/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*b*d^2 + b^2*d
 - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 2*a - 2*b*x)*
(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)
- (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 172
8*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b*d^2 + b^2*d -
 b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 6*a - 6*b*x)*(9*
b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(-
2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1
920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2))
```

3.179 $\int \cosh(c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a-c+(b-d)x)}{2(b-d)} + \frac{\cosh(a+c+(b+d)x)}{2(b+d)}$$

```
[Out] 1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5737, 2718}

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a+x(b-d)-c)}{2(b-d)} + \frac{\cosh(a+x(b+d)+c)}{2(b+d)}$$

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
```

```
]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}\sinh(a-c+(b-d)x)+\frac{1}{2}\sinh(a+c+(b+d)x)\right)\,dx\\
&= \frac{1}{2}\int \sinh(a-c+(b-d)x)\,dx+\frac{1}{2}\int \sinh(a+c+(b+d)x)\,dx\\
&= \frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2(b+d)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cosh(c+dx)\sinh(a+bx)\,dx=\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2(b+d)}$$

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x],x]
[Out] Cosh[a - c + (b - d)*x]/(2*(b - d)) + Cosh[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	40
risch	$\frac{\left(b\,{e}^{2bx+2a}-{e}^{2bx+2a}d+b+d\right){e}^{-bx+dx-a+c}}{4(b+d)(b-d)}+\frac{\left(b\,{e}^{2bx+2a}+{e}^{2bx+2a}d+b-d\right){e}^{-bx-dx-a-c}}{4(b+d)(b-d)}$	112
parallelrisch	$\frac{2b-4d\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}{(b^{2}-d^{2})\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}$	116

```
[In] int(cosh(d*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*cosh(a-c+(b-d)*x)/(b-d)+1/2*cosh(a+c+(b+d)*x)/(b+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{b\cosh\left(bx+a\right)\cosh\left(dx+c\right)-d\sinh\left(bx+a\right)\sinh\left(dx+c\right)}{\left(b^2-d^2\right)\cosh\left(bx+a\right)^2-\left(b^2-d^2\right)\sinh\left(bx+a\right)^2}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(d*x + c) - d*sinh(b*x + a)*sinh(d*x + c))/((b^2 - d^2
)*cosh(b*x + a)^2 - (b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(32) = 64.

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.56

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \begin{cases} x\sinh\left(a\right)\cosh\left(c\right) & \text{for}\ b=0\wedge d=0 \\ \frac{x\sinh\left(a-dx\right)\cosh\left(c+dx\right)}{2}+\frac{x\sinh\left(c+dx\right)\cosh\left(a-dx\right)}{2}+\frac{\sinh\left(a-dx\right)\sinh\left(c+dx\right)}{2d} & \text{for}\ b=-d \\ \frac{x\sinh\left(a+dx\right)\cosh\left(c+dx\right)}{2}-\frac{x\sinh\left(c+dx\right)\cosh\left(a+dx\right)}{2}+\frac{\cosh\left(a+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for}\ b=d \\ \frac{b\cosh\left(a+bx\right)\cosh\left(c+dx\right)}{b^2-d^2}-\frac{d\sinh\left(a+bx\right)\sinh\left(c+dx\right)}{b^2-d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x)*cosh(c
 + d*x)/2 + x*sinh(c + d*x)*cosh(a - d*x)/2 + sinh(a - d*x)*sinh(c + d*x)/(
2*d), Eq(b, -d)), (x*sinh(a + d*x)*cosh(c + d*x)/2 - x*sinh(c + d*x)*cosh(a
 + d*x)/2 + cosh(a + d*x)*cosh(c + d*x)/(2*d), Eq(b, d)), (b*cosh(a + b*x)*
cosh(c + d*x)/(b**2 - d**2) - d*sinh(a + b*x)*sinh(c + d*x)/(b**2 - d**2),
True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{e^{(bx+dx+a+c)}}{4(b+d)} + \frac{e^{(bx-dx+a-c)}}{4(b-d)} + \frac{e^{(-bx+dx-a+c)}}{4(b-d)} + \frac{e^{(-bx-dx-a-c)}}{4(b+d)}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + d*x + a + c)/(b + d) + 1/4*e^(b*x - d*x + a - c)/(b - d) + 1/4
*e^(-b*x + d*x - a + c)/(b - d) + 1/4*e^(-b*x - d*x - a - c)/(b + d)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cosh(c+dx)\sinh(a+bx)\,dx = \frac{b\cosh(a+b\,x)\,\cosh(c+d\,x) - d\sinh(a+b\,x)\,\sinh(c+d\,x)}{b^2-d^2}$$

```
[In] int(cosh(c + d*x)*sinh(a + b*x),x)
```

```
[Out] (b*cosh(a + b*x)*cosh(c + d*x) - d*sinh(a + b*x)*sinh(c + d*x))/(b^2 - d^2)
```

3.180 $\int \cosh^2(c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx=\frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)}$$

```
[Out] 1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5737, 2718}

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx=\frac{\cosh(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cosh(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\cosh(a+bx)}{2b}$$

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/(2*b) + Cosh[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cosh[a +
2*c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{2}\sinh(a+bx)+\frac{1}{4}\sinh(a-2c+(b-2d)x)+\frac{1}{4}\sinh(a+2c+(b+2d)x)\right)dx \\ &= \frac{1}{4}\int \sinh(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sinh(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \sinh(a+bx)\,dx \\ &= \frac{\cosh(a+bx)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cosh(a+2c+(b+2d)x)}{4(b+2d)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{1}{4}\left(\frac{2\cosh(a)\cosh(bx)}{b}+\frac{\cosh(a-2c+bx-2dx)}{b-2d}+\frac{\cosh(a+2c+bx+2dx)}{b+2d}+\frac{2\sinh(a)\sinh(bx)}{b}\right)$$

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x],x]
[Out] ((2*Cosh[a]*Cosh[b*x])/b + Cosh[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cosh[a +
 2*c + b*x + 2*d*x]/(b + 2*d) + (2*Sinh[a]*Sinh[b*x])/b)/4
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result
default	$\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b+2d)x)}{4b+8d}$
risch	$\frac{e^{bx+a}}{4b}+\frac{e^{-bx-a}}{4b}+\frac{\left(b\,e^{2bx+2a}-2\,e^{2bx+2a}d+b+2d\right)e^{-bx+2dx-a+2c}}{8(b+2d)(b-2d)}+\frac{\left(b\,e^{2bx+2a}+2\,e^{2bx+2a}d+b-2d\right)e^{-bx-2dx-a-2c}}{8(b+2d)(b-2d)}$
parallelrisch	$\frac{(-2b^2+4d^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+8d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)b+\left(-4\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2b^2-8d^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+8d\tanh\left(\frac{bx}{2}+\right.}{b(b-2d)(b+2d)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cosh(b*x+a)/b+1/4*cosh(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cosh(a+2*c+(b+2*d)*
x)/(b+2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{b^2\cosh(bx+a)\cosh(dx+c)^2 - 4\,bd\cosh(dx+c)\sinh(bx+a)\sinh(dx+c) + b^2\cosh(bx+a)\sinh(dx+}{2\left((b^3-4\,bd^2)\cosh(bx+a)^2 - (b^3-4\,bd^2)\sinh(bx+a)^2\right)}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 - 4*b*d*cosh(d*x + c)*sinh(b*x + a)*
sinh(d*x + c) + b^2*cosh(b*x + a)*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x
+ a))/((b^3 - 4*b*d^2)*cosh(b*x + a)^2 - (b^3 - 4*b*d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(49) = 98.

Time = 0.71 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \begin{cases} x\sinh(a)\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2} + \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\sinh(a) \\ \frac{x\sinh(a-2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a-2dx)\cosh^2(c+dx)}{4} + \frac{x\sinh(c+dx)\cosh(a-2dx)\cosh(c+dx)}{2} + \frac{3\sinh(a-2dx)\sinh(c+dx)\cosh(}{4d} \\ \frac{x\sinh(a+2dx)\sinh^2(c+dx)}{4} + \frac{x\sinh(a+2dx)\cosh^2(c+dx)}{4} - \frac{x\sinh(c+dx)\cosh(a+2dx)\cosh(c+dx)}{2} + \frac{3\sinh(a+2dx)\sinh(c+dx)\cosh(}{4d} \\ \frac{b^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} - \frac{2bd\sinh(a+bx)\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} + \frac{2d^2\sinh^2(c+dx)\cosh(a+bx)}{b^3-4bd^2} - \frac{2d^2\cosh(a+bx)\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**
2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a), Eq
(b, 0)), (x*sinh(a - 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a - 2*d*x)*cosh(c +
 d*x)**2/4 + x*sinh(c + d*x)*cosh(a - 2*d*x)*cosh(c + d*x)/2 + 3*sinh(a - 2
*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) + sinh(c + d*x)**2*cosh(a - 2*d*x)/
(2*d), Eq(b, -2*d)), (x*sinh(a + 2*d*x)*sinh(c + d*x)**2/4 + x*sinh(a + 2*d
*x)*cosh(c + d*x)**2/4 - x*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/2 +
```

```
3*sinh(a + 2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d) - sinh(c + d*x)**2*cosh
(a + 2*d*x)/(2*d), Eq(b, 2*d)), (b**2*cosh(a + b*x)*cosh(c + d*x)**2/(b**3
- 4*b*d**2) - 2*b*d*sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)/(b**3 - 4*b*d
**2) + 2*d**2*sinh(c + d*x)**2*cosh(a + b*x)/(b**3 - 4*b*d**2) - 2*d**2*cos
h(a + b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{8\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{8\,(b-2\,d)} + \frac{e^{(bx+a)}}{4\,b} + \frac{e^{(-bx+2\,dx-a+2\,c)}}{8\,(b-2\,d)} + \frac{e^{(-bx-2\,dx-a-2\,c)}}{8\,(b+2\,d)} + \frac{e^{(-bx-a)}}{4\,b}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/8*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/4*e^(b*x + a)/b + 1/8*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) + 1/8
*e^(-b*x - 2*d*x - a - 2*c)/(b + 2*d) + 1/4*e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \cosh^2(c+dx)\sinh(a+bx)\,dx$$
$$= \frac{2\,d^2\cosh(a+b\,x)-b^2\cosh(a+b\,x)\,\cosh(c+d\,x)^2+2\,b\,d\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)}{4\,b\,d^2-b^3}$$

```
[In] int(cosh(c + d*x)^2*sinh(a + b*x),x)
[Out] (2*d^2*cosh(a + b*x) - b^2*cosh(a + b*x)*cosh(c + d*x)^2 + 2*b*d*cosh(c + d
*x)*sinh(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.181 $\int \cosh^3(c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(a+3c+(b+3d)x)}{8(b+3d)}$$

```
[Out] 1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+
(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5737, 2718}

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(a+x(b+3d)+3c)}{8(b+3d)}$$

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Cosh[a - c + (b - d)*x])/(8*
(b - d)) + (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[a + 3*c + (b + 3*
d)*x]/(8*(b + 3*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{8}\sinh(a-3c+(b-3d)x)+\frac{3}{8}\sinh(a-c+(b-d)x)\right.\\&\qquad\left.+\frac{3}{8}\sinh(a+c+(b+d)x)+\frac{1}{8}\sinh(a+3c+(b+3d)x)\right)\,dx\\&=\frac{1}{8}\int \sinh(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \sinh(a+3c+(b+3d)x)\,dx\\&\quad+\frac{3}{8}\int \sinh(a-c+(b-d)x)\,dx+\frac{3}{8}\int \sinh(a+c+(b+d)x)\,dx\\&=\frac{\cosh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}\\&\quad+\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(a+3c+(b+3d)x)}{8(b+3d)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx=\frac{1}{8}\left(\frac{\cosh(a-3c+bx-3dx)}{b-3d}+\frac{3\cosh(a-c+bx-dx)}{b-d}\right.\\\left.+\frac{\cosh(a+3c+bx+3dx)}{b+3d}+\frac{3\cosh(a+c+(b+d)x)}{b+d}\right)$$

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x],x]
[Out] (Cosh[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Cosh[a - c + b*x - d*x])/(b - d
) + Cosh[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Cosh[a + c + (b + d)*x])/(b
+ d))/8
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

method	result
default	$\frac{\cosh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{\left(b\,\mathrm{e}^{2bx+2a}-3\,\mathrm{e}^{2bx+2a}d+b+3d\right)\mathrm{e}^{-bx+3dx-a+3c}}{16(b+3d)(b-3d)}+\frac{3\left(b\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{2bx+2a}d+b+d\right)\mathrm{e}^{-bx+dx-a+c}}{16(b+d)(b-d)}+\frac{3\left(b\,\mathrm{e}^{2bx+2a}+\mathrm{e}^{2bx+2a}d+b\right.}{16(b+d)(b-}$
parallelrisch	$\frac{2b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}(b^{2}-7d^{2})\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12d\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(b^{2}-3d^{2})\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+6b\left(4\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d^{2}+b^{2}-3d^{2}\right)\mathrm{ta}}{(b-d)(b+3d)(b-3d)(b+d)\left(\tanh\right.}$

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*cosh(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*cosh(a-c+(b-d)*x)/(b-d)+3/8*cosh(a+c+
(b+d)*x)/(b+d)+1/8*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(83) = 166.

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.34

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx$$
$$=\frac{(b^3-bd^2)\cosh(bx+a)\cosh(dx+c)^3+3(b^3-bd^2)\cosh(bx+a)\cosh(dx+c)\sinh(dx+c)^2-3(b^2d-}{4\left((b^4-10b^2d^2+\right.}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(b^3 - b*d^2)*cosh(b*x
 + a)*cosh(d*x + c)*sinh(d*x + c)^2 - 3*(b^2*d - d^3)*sinh(b*x + a)*sinh(d*
x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(b*x + a)*cosh(d*x + c) - 3*(b^2*d - 9*d^3
 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a)*sinh(d*x + c))/((b^4 - 10
*b^2*d^2 + 9*d^4)*cosh(b*x + a)^2 - (b^4 - 10*b^2*d^2 + 9*d^4)*sinh(b*x + a
)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.

Time = 1.96 (sec) , antiderivative size = 921, normalized size of antiderivative = 10.12

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d*x)
*sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sinh(a - 3*d*x)*cosh(c + d*x)**3/8 +
x*sinh(c + d*x)**3*cosh(a - 3*d*x)/8 + 3*x*sinh(c + d*x)*cosh(a - 3*d*x)*co
sh(c + d*x)**2/8 + sinh(a - 3*d*x)*sinh(c + d*x)**3/(8*d) + sinh(c + d*x)**
2*cosh(a - 3*d*x)*cosh(c + d*x)/(4*d) - 7*cosh(a - 3*d*x)*cosh(c + d*x)**3/
(24*d), Eq(b, -3*d)), (-3*x*sinh(a - d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8
+ 3*x*sinh(a - d*x)*cosh(c + d*x)**3/8 - 3*x*sinh(c + d*x)**3*cosh(a - d*x)
/8 + 3*x*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)**2/8 + 3*sinh(a - d*x)*s
inh(c + d*x)**3/(8*d) + 3*sinh(c + d*x)**2*cosh(a - d*x)*cosh(c + d*x)/(4*d
) - 5*cosh(a - d*x)*cosh(c + d*x)**3/(8*d), Eq(b, -d)), (-3*x*sinh(a + d*x)
*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(a + d*x)*cosh(c + d*x)**3/8 +
3*x*sinh(c + d*x)**3*cosh(a + d*x)/8 - 3*x*sinh(c + d*x)*cosh(a + d*x)*cosh
(c + d*x)**2/8 + 3*sinh(a + d*x)*sinh(c + d*x)**3/(8*d) - 3*sinh(c + d*x)**
2*cosh(a + d*x)*cosh(c + d*x)/(4*d) + 5*cosh(a + d*x)*cosh(c + d*x)**3/(8*d
), Eq(b, d)), (3*x*sinh(a + 3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + x*sin
h(a + 3*d*x)*cosh(c + d*x)**3/8 - x*sinh(c + d*x)**3*cosh(a + 3*d*x)/8 - 3*
x*sinh(c + d*x)*cosh(a + 3*d*x)*cosh(c + d*x)**2/8 + sinh(a + 3*d*x)*sinh(c
 + d*x)**3/(8*d) - sinh(c + d*x)**2*cosh(a + 3*d*x)*cosh(c + d*x)/(4*d) + 7
*cosh(a + 3*d*x)*cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*cosh(a + b*x)*
cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sinh(a + b*x)*si
nh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sinh
(c + d*x)**2*cosh(a + b*x)*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7
*b*d**2*cosh(a + b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d
**3*sinh(a + b*x)*sinh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*
sinh(a + b*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4)
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{16\,(b+3\,d)} + \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} + \frac{e^{(bx-3\,dx+a-3\,c)}}{16\,(b-3\,d)} + \frac{e^{(-bx+3\,dx-a+3\,c)}}{16\,(b-3\,d)} + \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} + \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} + \frac{e^{(-bx-3\,dx-a-3\,c)}}{16\,(b+3\,d)}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/16*e^(b*x + d*x + a + c)/(b +
d) + 3/16*e^(b*x - d*x + a - c)/(b - d) + 1/16*e^(b*x - 3*d*x + a - 3*c)/(b
 - 3*d) + 1/16*e^(-b*x + 3*d*x - a + 3*c)/(b - 3*d) + 3/16*e^(-b*x + d*x -
a + c)/(b - d) + 3/16*e^(-b*x - d*x - a - c)/(b + d) + 1/16*e^(-b*x - 3*d*x
 - a - 3*c)/(b + 3*d)
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.00

$$\int \cosh^3(c+dx)\sinh(a+bx)\,dx = \frac{6\,b\,d^2\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(c+d\,x)^2}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{6\,d^3\sinh(a+b\,x)\,\sinh(c+d\,x)^3}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{3\,d\cosh(c+d\,x)^2\sinh(a+b\,x)\,\sinh(c+d\,x)\,\left(b^2-3\,d^2\right)}{b^4-10\,b^2\,d^2+9\,d^4} - \frac{\cosh(a+b\,x)\,\cosh(c+d\,x)^3\,\left(7\,b\,d^2-b^3\right)}{b^4-10\,b^2\,d^2+9\,d^4}$$

```
[In] int(cosh(c + d*x)^3*sinh(a + b*x),x)
```

```
[Out] (6*b*d^2*cosh(a + b*x)*cosh(c + d*x)*sinh(c + d*x)^2)/(b^4 + 9*d^4 - 10*b^2
*d^2) - (6*d^3*sinh(a + b*x)*sinh(c + d*x)^3)/(b^4 + 9*d^4 - 10*b^2*d^2) -
(3*d*cosh(c + d*x)^2*sinh(a + b*x)*sinh(c + d*x)*(b^2 - 3*d^2))/(b^4 + 9*d^
4 - 10*b^2*d^2) - (cosh(a + b*x)*cosh(c + d*x)^3*(7*b*d^2 - b^3))/(b^4 + 9*
d^4 - 10*b^2*d^2)
```

3.182 $\int \cosh(c+dx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)} - \frac{\sinh(c+dx)}{2d} + \frac{\sinh(2a+c+(2b+d)x)}{4(2b+d)}$$

```
[Out] 1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)*
x)/(2*b+d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5737, 2717}

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2a+x(2b-d)-c)}{4(2b-d)} + \frac{\sinh(2a+x(2b+d)+c)}{4(2b+d)} - \frac{\sinh(c+dx)}{2d}$$

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[2*a - c + (2*b - d)*x]/(4*(2*b - d)) - Sinh[c + d*x]/(2*d) + Sinh[2*a
+ c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{4}\cosh(2a-c+(2b-d)x)-\frac{1}{2}\cosh(c+dx)+\frac{1}{4}\cosh(2a+c+(2b+d)x)\right)dx\\ &= \frac{1}{4}\int \cosh(2a-c+(2b-d)x)\,dx+\frac{1}{4}\int \cosh(2a+c+(2b+d)x)\,dx-\frac{1}{2}\int \cosh(c+dx)\,dx\\ &= \frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\sinh(c+dx)}{2d}+\frac{\sinh(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{1}{4}\left(-\frac{2\cosh(dx)\sinh(c)}{d}-\frac{2\cosh(c)\sinh(dx)}{d}+\frac{\sinh(2a-c+2bx-dx)}{2b-d}+\frac{\sinh(2a+c+2bx+dx)}{2b+d}\right)$$

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x]^2,x]
[Out] ((-2*Cosh[d*x]*Sinh[c])/d - (2*Cosh[c]*Sinh[d*x])/d + Sinh[2*a - c + 2*b*x
- d*x]/(2*b - d) + Sinh[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result
default	$\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}-\frac{\sinh(dx+c)}{2d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}$
parallelrisch	$\frac{(2bd+d^2)\sinh(2a-c+(2b-d)x)+(2bd-d^2)\sinh(2a+c+(2b+d)x)+(-8b^2+2d^2)\sinh(dx+c)}{16b^2d-4d^3}$
risch	$-\frac{(-2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2+2bd+d^2)e^{-2bx+dx-2a+c}}{8(2b+d)(2b-d)d}+\frac{(2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2e^{2bx+2a}-}{8(2b+d)(}$

```
[In] int(cosh(d*x+c)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/4*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*sinh(d*x+c)/d+1/4*sinh(2*a+c+(2*b+d)*x)/(2*b+d)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx$$
$$= \frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)-\left(d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\,b^2-d^2\right)\sinh\left(dx+c\right)}{2\left(\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)^2-\left(4\,b^2d-d^3\right)\sinh\left(bx+a\right)^2\right)}$$

[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - (d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*b^2 - d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(b*x + a)^2 - (4*b^2*d - d^3)*sinh(b*x + a)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(49) = 98.

Time = 0.74 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.00

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} x\sinh^2\left(a\right)\cosh\left(c\right) \\ \frac{x\sinh^2\left(a-\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}+\frac{x\sinh\left(a-\frac{dx}{2}\right)\sinh\left(c+dx\right)\cosh\left(a-\frac{dx}{2}\right)}{2}+\frac{x\cosh^2\left(a-\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}+\frac{\sinh^2\left(a-\frac{dx}{2}\right)\sinh\left(c+dx\right)}{d} \\ \frac{x\sinh^2\left(a+\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}-\frac{x\sinh\left(a+\frac{dx}{2}\right)\sinh\left(c+dx\right)\cosh\left(a+\frac{dx}{2}\right)}{2}+\frac{x\cosh^2\left(a+\frac{dx}{2}\right)\cosh\left(c+dx\right)}{4}+\frac{\sinh^2\left(a+\frac{dx}{2}\right)\sinh\left(c+dx\right)}{d} \\ \left(\frac{x\sinh^2\left(a+bx\right)}{2}-\frac{x\cosh^2\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}\right)\cosh\left(c\right) \\ \frac{2b^2\sinh^2\left(a+bx\right)\sinh\left(c+dx\right)}{4b^2d-d^3}-\frac{2b^2\sinh\left(c+dx\right)\cosh^2\left(a+bx\right)}{4b^2d-d^3}+\frac{2bd\sinh\left(a+bx\right)\cosh\left(a+bx\right)\cosh\left(c+dx\right)}{4b^2d-d^3}-\frac{d^2\sinh^2\left(a+bx\right)\sinh\left(c+dx\right)}{4b^2d-d^3} \end{cases}$$

[In] integrate(cosh(d*x+c)*sinh(b*x+a)**2,x)

[Out] Piecewise((x*sinh(a)**2*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*sinh(a - d*x/2)**2*cosh(c + d*x)/4 + x*sinh(a - d*x/2)*sinh(c + d*x)*cosh(a - d*x/2)/2 + x*cosh(a - d*x/2)**2*cosh(c + d*x)/4 + sinh(a - d*x/2)**2*sinh(c + d*x)/d + sinh(a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)/(2*d), Eq(b, -d/2)), (x*sinh(a + d*x/2)**2*cosh(c + d*x)/4 - x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*x/2)/2 + x*cosh(a + d*x/2)**2*cosh(c + d*x)/4 + sinh(a + d*x/2)**2*sinh(c + d

```
*x)/d - sinh(a + d*x/2)*cosh(a + d*x/2)*cosh(c + d*x)/(2*d), Eq(b, d/2)), (
(x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/
(2*b))*cosh(c), Eq(d, 0)), (2*b**2*sinh(a + b*x)**2*sinh(c + d*x)/(4*b**2*d
 - d**3) - 2*b**2*sinh(c + d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3) + 2*b*d*
sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)/(4*b**2*d - d**3) - d**2*sinh(a +
 b*x)**2*sinh(c + d*x)/(4*b**2*d - d**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.82

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx = \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} + \frac{e^{(2\,bx-dx+2\,a-c)}}{8\,(2\,b-d)} - \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} - \frac{e^{(-2\,bx-dx-2\,a-c)}}{8\,(2\,b+d)} - \frac{e^{(dx+c)}}{4\,d} + \frac{e^{(-dx-c)}}{4\,d}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/8*e^(2*b*x - d*x + 2*a - c)/(2*
b - d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 1/8*e^(-2*b*x - d*x - 2
*a - c)/(2*b + d) - 1/4*e^(d*x + c)/d + 1/4*e^(-d*x - c)/d
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \cosh(c+dx)\sinh^2(a+bx)\,dx$$
$$=\frac{d^2\left(\sinh(c+d\,x)-\cosh(a+b\,x)^2\sinh(c+d\,x)\right)-2\,b^2\sinh(c+d\,x)+2\,b\,d\cosh(a+b\,x)\,\cosh(c+d\,x)\,\mathrm{si}}{4\,b^2\,d-d^3}$$

```
[In] int(cosh(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] (d^2*(sinh(c + d*x) - cosh(a + b*x)^2*sinh(c + d*x)) - 2*b^2*sinh(c + d*x)
+ 2*b*d*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x))/(4*b^2*d - d^3)
```

3.183 $\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = -\frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)} - \frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}$$

```
[Out] -1/4*x+1/8*sinh(2*b*x+2*a)/b+1/16*sinh(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sinh(2*
d*x+2*c)/d+1/16*sinh(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5737, 2717}

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sinh(2(a+c)+2x(b+d))}{16(b+d)} + \frac{\sinh(2a+2bx)}{8b}-\frac{\sinh(2c+2dx)}{8d}-\frac{x}{4}$$

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*x + Sinh[2*a + 2*b*x]/(8*b) + Sinh[2*(a - c) + 2*(b - d)*x]/(16*(b - d
)) - Sinh[2*c + 2*d*x]/(8*d) + Sinh[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{4}+\frac{1}{4}\cosh(2a+2bx)+\frac{1}{8}\cosh(2(a-c)+2(b-d)x)-\frac{1}{4}\cosh(2c+2dx)\right.\\
&\qquad\left.+\frac{1}{8}\cosh(2(a+c)+2(b+d)x)\right)\,dx\\
&= -\frac{x}{4}+\frac{1}{8}\int \cosh(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cosh(2(a+c)+2(b+d)x)\,dx\\
&\quad+\frac{1}{4}\int \cosh(2a+2bx)\,dx-\frac{1}{4}\int \cosh(2c+2dx)\,dx\\
&= -\frac{x}{4}+\frac{\sinh(2a+2bx)}{8b}+\frac{\sinh(2(a-c)+2(b-d)x)}{16(b-d)}\\
&\quad-\frac{\sinh(2c+2dx)}{8d}+\frac{\sinh(2(a+c)+2(b+d)x)}{16(b+d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$$
$$=\frac{2d(b^2-d^2)\sinh(2(a+bx))+bd(b+d)\sinh(2(a-c+(b-d)x))-b(b-d)(4d(b+d)x+2(b+d)\sinh(2}{16b(b-d)d(b+d)}$$

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sinh[2*(a + b*x)] + b*d*(b + d)*Sinh[2*(a - c + (b - d)*x)
] - b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sinh[2*(c + d*x)] - d*Sinh[2*(a +
c + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$-\frac{x}{4}+\frac{\sinh(2bx+2a)}{8b}-\frac{\sinh(2dx+2c)}{8d}+\frac{\sinh((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sinh((2b+2d)x+2a+2c)}{16b+16d}$
parallelrisch	$\frac{bd(b+d)\sinh((2b-2d)x+2a-2c)-4\left(-\frac{bd\sinh((2b+2d)x+2a+2c)}{4}+\left(-\frac{d\sinh(2bx+2a)}{2}+b\left(dx+\frac{\sinh(2dx+2c)}{2}\right)\right)(b+d)\right)(b-d)}{16b^3d-16d^3b}$
risch	$-\frac{x}{4}+\frac{e^{2bx+2a}}{16b}-\frac{e^{-2bx-2a}}{16b}-\frac{\left(-d\,e^{4bx+4a}b+d^2e^{4bx+4a}+2b^2e^{2bx+2a}-2\,e^{2bx+2a}d^2+bd+d^2\right)e^{-2bx+2dx-2a+2c}}{32(b+d)(b-d)d}+\frac{(d\,e^{4bx-}}{}$

[In] int(cosh(d*x+c)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/4*x+1/8*sinh(2*b*x+2*a)/b-1/8*sinh(2*d*x+2*c)/d+1/8/(2*b-2*d)*sinh((2*b-2*d)*x+2*a-2*c)+1/8/(2*b+2*d)*sinh((2*b+2*d)*x+2*a+2*c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx$$
$$=\frac{b^2d\cosh(bx+a)\sinh(bx+a)\sinh(dx+c)^2-(b^3d-bd^3)x+\left(b^2d\cosh(bx+a)\cosh(dx+c)^2+(b^2d-d\right.}{4\left((b^3d-bd^3)\cosh\right.}$$

[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")

[Out] 1/4*(b^2*d*cosh(b*x + a)*sinh(b*x + a)*sinh(d*x + c)^2 - (b^3*d - b*d^3)*x + (b^2*d*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - d^3)*cosh(b*x + a))*sinh(b*x + a) - (b*d^2*cosh(d*x + c)*sinh(b*x + a)^2 + (b*d^2*cosh(b*x + a)^2 + b^3 - b*d^2)*cosh(d*x + c))*sinh(d*x + c))/((b^3*d - b*d^3)*cosh(b*x + a)^2 - (b^3*d - b*d^3)*sinh(b*x + a)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(76) = 152.

Time = 1.56 (sec) , antiderivative size = 1027, normalized size of antiderivative = 11.67

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(cosh(d*x+c)**2*sinh(b*x+a)**2,x)

```
[Out] Piecewise((x*sinh(a)**2*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)*
*2, Eq(b, 0)), (-x*sinh(a - d*x)**2*sinh(c + d*x)**2/8 + 3*x*sinh(a - d*x)*
*2*cosh(c + d*x)**2/8 + x*sinh(a - d*x)*sinh(c + d*x)*cosh(a - d*x)*cosh(c
+ d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a - d*x)**2/8 - x*cosh(a - d*x)**2*cos
h(c + d*x)**2/8 + 5*sinh(a - d*x)**2*sinh(c + d*x)*cosh(c + d*x)/(8*d) + si
nh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)/(2*d) + sinh(c + d*x)*cosh(a - d
*x)**2*cosh(c + d*x)/(8*d), Eq(b, -d)), (-x*sinh(a + d*x)**2*sinh(c + d*x)*
*2/8 + 3*x*sinh(a + d*x)**2*cosh(c + d*x)**2/8 - x*sinh(a + d*x)*sinh(c + d
*x)*cosh(a + d*x)*cosh(c + d*x)/2 + 3*x*sinh(c + d*x)**2*cosh(a + d*x)**2/8
 - x*cosh(a + d*x)**2*cosh(c + d*x)**2/8 + sinh(a + d*x)*sinh(c + d*x)**2*c
osh(a + d*x)/(8*d) + 5*sinh(a + d*x)*cosh(a + d*x)*cosh(c + d*x)**2/(8*d) -
 sinh(c + d*x)*cosh(a + d*x)**2*cosh(c + d*x)/(2*d), Eq(b, d)), ((x*sinh(a
+ b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/(2*b))*cos
h(c)**2, Eq(d, 0)), (-b**3*d*x*sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d
- 4*b*d**3) + b**3*d*x*sinh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d*
*3) + b**3*d*x*sinh(c + d*x)**2*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*
*3*d*x*cosh(a + b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b**3*sinh(
a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*sinh(c
 + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*sin
h(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*
sinh(a + b*x)**2*sinh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(a +
 b*x)**2*cosh(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sinh(c + d*x)**2
*cosh(a + b*x)**2/(4*b**3*d - 4*b*d**3) + b*d**3*x*cosh(a + b*x)**2*cosh(c
+ d*x)**2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sinh(a + b*x)**2*sinh(c + d*x)*c
osh(c + d*x)/(4*b**3*d - 4*b*d**3) + d**3*sinh(a + b*x)*sinh(c + d*x)**2*co
sh(a + b*x)/(4*b**3*d - 4*b*d**3) - d**3*sinh(a + b*x)*cosh(a + b*x)*cosh(c
 + d*x)**2/(4*b**3*d - 4*b*d**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(2*d)/b>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = -\frac{1}{4}\,x + \frac{e^{(2\,bx+2\,dx+2\,a+2\,c)}}{32\,(b+d)} + \frac{e^{(2\,bx-2\,dx+2\,a-2\,c)}}{32\,(b-d)} + \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx+2\,dx-2\,a+2\,c)}}{32\,(b-d)} - \frac{e^{(-2\,bx-2\,dx-2\,a-2\,c)}}{32\,(b+d)} - \frac{e^{(-2\,bx-2\,a)}}{16\,b} - \frac{e^{(2\,dx+2\,c)}}{16\,d} + \frac{e^{(-2\,dx-2\,c)}}{16\,d}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x + 1/32*e^(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/32*e^(2*b*x - 2*d*x
 + 2*a - 2*c)/(b - d) + 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x + 2*d*x - 2
*a + 2*c)/(b - d) - 1/32*e^(-2*b*x - 2*d*x - 2*a - 2*c)/(b + d) - 1/16*e^(-
2*b*x - 2*a)/b - 1/16*e^(2*d*x + 2*c)/d + 1/16*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int \cosh^2(c+dx)\sinh^2(a+bx)\,dx = -\frac{d^3\cosh(a+b\,x)\,\sinh(a+b\,x)+b^3\cosh(c+d\,x)\,\sinh(c+d\,x)-b\,d^3\,x+b^3\,d\,x-2\,b\,d^2\cosh(c+d\,x)\,\mathrm{sin}}{4\,b\,d}$$

```
[In] int(cosh(c + d*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] -(d^3*cosh(a + b*x)*sinh(a + b*x) + b^3*cosh(c + d*x)*sinh(c + d*x) - b*d^3
*x + b^3*d*x - 2*b*d^2*cosh(c + d*x)*sinh(c + d*x) - 2*b^2*d*cosh(a + b*x)*
cosh(c + d*x)^2*sinh(a + b*x) + 2*b*d^2*cosh(a + b*x)^2*cosh(c + d*x)*sinh(
c + d*x))/(4*b*d*(b^2 - d^2))
```

3.184 $\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} - \frac{3\sinh(c+dx)}{8d} - \frac{\sinh(3c+3dx)}{24d} + \frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\sinh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5737, 2717}

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{\sinh(2a+x(2b-3d)-3c)}{16(2b-3d)} + \frac{3\sinh(2a+x(2b-d)-c)}{16(2b-d)} + \frac{3\sinh(2a+x(2b+d)+c)}{16(2b+d)} + \frac{\sinh(2a+x(2b+3d)+3c)}{16(2b+3d)} - \frac{3\sinh(c+dx)}{8d} - \frac{\sinh(3c+3dx)}{24d}$$

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sinh[2*a - c + (2*b -
 d)*x])/(16*(2*b - d)) - (3*Sinh[c + d*x])/(8*d) - Sinh[3*c + 3*d*x]/(24*d)
 + (3*Sinh[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sinh[2*a + 3*c + (2*b +
 3*d)*x]/(16*(2*b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{16}\cosh(2a-3c+(2b-3d)x) + \frac{3}{16}\cosh(2a-c+(2b-d)x) - \frac{3}{8}\cosh(c+dx) - \frac{1}{8}\cosh(3c+3dx) + \frac{3}{16}\cosh(2a+c+(2b+d)x) + \frac{1}{16}\cosh(2a+3c+(2b+3d)x)\right) dx$$

$$= \frac{1}{16}\int \cosh(2a-3c+(2b-3d)x)\,dx + \frac{1}{16}\int \cosh(2a+3c+(2b+3d)x)\,dx - \frac{1}{8}\int \cosh(3c+3dx)\,dx + \frac{3}{16}\int \cosh(2a-c+(2b-d)x)\,dx + \frac{3}{16}\int \cosh(2a+c+(2b+d)x)\,dx - \frac{3}{8}\int \cosh(c+dx)\,dx$$

$$= \frac{\sinh(2a-3c+(2b-3d)x)}{16(2b-3d)} + \frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)} - \frac{3\sinh(c+dx)}{8d} - \frac{\sinh(3c+3dx)}{24d} + \frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)} + \frac{\sinh(2a+3c+(2b+3d)x)}{16(2b+3d)}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{1}{48}\left(-\frac{18\cosh(dx)\sinh(c)}{d} - \frac{2\cosh(3dx)\sinh(3c)}{d} - \frac{18\cosh(c)\sinh(dx)}{d} - \frac{2\cosh(3c)\sinh(3dx)}{d} + \frac{3\sinh(2a-3c+2bx-3dx)}{2b-3d} + \frac{9\sinh(2a-c+2bx-dx)}{2b-d} + \frac{9\sinh(2a+c+2bx+dx)}{2b+d} + \frac{3\sinh(2a+3c+2bx+3dx)}{2b+3d}\right)$$

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] ((-18*Cosh[d*x]*Sinh[c])/d - (2*Cosh[3*d*x]*Sinh[3*c])/d - (18*Cosh[c]*Sinh
[d*x])/d - (2*Cosh[3*c]*Sinh[3*d*x])/d + (3*Sinh[2*a - 3*c + 2*b*x - 3*d*x]
)/(2*b - 3*d) + (9*Sinh[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sinh[2*a + c
 + 2*b*x + d*x])/(2*b + d) + (3*Sinh[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d
))/48
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sinh(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sinh(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\sinh(dx+c)}{8d}-\frac{\sinh(3dx+3c)}{24d}+\frac{3\sinh(2a+c+(2b+d)x)}{16(2b+d)}+\frac{\sinh(2a$
parallelrisch	$\frac{(24b^3d+36d^2b^2-6d^3b-9d^4)\sinh(2a-3c+(2b-3d)x)+72\left(d\left(b+\frac{3d}{2}\right)\left(b+\frac{d}{2}\right)\sinh(2a-c+(2b-d)x)+\left(\frac{d\left(b+\frac{d}{2}\right)\sinh(2a+3c+(2b+3d}{3}\right.\right.}{768d\,b^4-1920b^2d^3+432d^5}$
risch	$-\frac{\left(-6d\,e^{4bx+4a}b+9d^2e^{4bx+4a}+8b^2e^{2bx+2a}-18\,e^{2bx+2a}d^2+6bd+9d^2\right)e^{-2bx+3dx-2a+3c}}{96(2b+3d)(2b-3d)d}-\frac{3\left(-2d\,e^{4bx+4a}b+d^2e^{4bx+4a}+8b^2e^{}\right.}{32(}$

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*sinh(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sinh(2*a-c+(2*b-d)*x)/(2*b-d)
-3/8*sinh(d*x+c)/d-1/24*sinh(3*d*x+3*c)/d+3/16*sinh(2*a+c+(2*b+d)*x)/(2*b+d
)+1/16*sinh(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(132) = 264.

Time = 0.26 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.76

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx$$
$$=\frac{36\,(4\,b^3d-bd^3)\cosh(bx+a)\cosh(dx+c)\sinh(bx+a)\sinh(dx+c)^2-\left(16\,b^4-40\,b^2d^2+9\,d^4+9\,(4\,b^2d^2\right.}{}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(36*(4*b^3*d - b*d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)*sinh(d
*x + c)^2 - (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cosh(b*x + a
)^2 + 9*(4*b^2*d^2 - d^4)*sinh(b*x + a)^2)*sinh(d*x + c)^3 + 12*((4*b^3*d -
 b*d^3)*cosh(b*x + a)*cosh(d*x + c)^3 + 3*(4*b^3*d - 9*b*d^3)*cosh(b*x + a)
*cosh(d*x + c))*sinh(b*x + a) - 3*(48*b^4 - 120*b^2*d^2 + 27*d^4 + 3*(4*b^2
*d^2 - 9*d^4)*cosh(b*x + a)^2 + (16*b^4 - 40*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2
 - d^4)*cosh(b*x + a)^2)*cosh(d*x + c)^2 + 3*(4*b^2*d^2 - 9*d^4 + 3*(4*b^2*
d^2 - d^4)*cosh(d*x + c)^2)*sinh(b*x + a)^2)*sinh(d*x + c))/((16*b^4*d - 40
*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 - (16*b^4*d - 40*b^2*d^3 + 9*d^5)*sinh(b*
x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(116) = 232.

Time = 5.55 (sec) , antiderivative size = 2001, normalized size of antiderivative = 13.90

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a)**2*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - 3*d
*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + x*sinh(a - 3*d*x/2)**2*cosh(c
+ d*x)**3/16 + x*sinh(a - 3*d*x/2)*sinh(c + d*x)**3*cosh(a - 3*d*x/2)/8 + 3
*x*sinh(a - 3*d*x/2)*sinh(c + d*x)*cosh(a - 3*d*x/2)*cosh(c + d*x)**2/8 + 3
*x*sinh(c + d*x)**2*cosh(a - 3*d*x/2)**2*cosh(c + d*x)/16 + x*cosh(a - 3*d*
x/2)**2*cosh(c + d*x)**3/16 - 7*sinh(a - 3*d*x/2)**2*sinh(c + d*x)**3/(16*d
) - 3*sinh(a - 3*d*x/2)*sinh(c + d*x)**2*cosh(a - 3*d*x/2)*cosh(c + d*x)/(4
*d) - 5*sinh(a - 3*d*x/2)*cosh(a - 3*d*x/2)*cosh(c + d*x)**3/(8*d) + 11*sin
h(c + d*x)**3*cosh(a - 3*d*x/2)**2/(48*d) - sinh(c + d*x)*cosh(a - 3*d*x/2)
**2*cosh(c + d*x)**2/d, Eq(b, -3*d/2)), (-3*x*sinh(a - d*x/2)**2*sinh(c + d
*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a - d*x/2)**2*cosh(c + d*x)**3/16 - 3*x*
sinh(a - d*x/2)*sinh(c + d*x)**3*cosh(a - d*x/2)/8 + 3*x*sinh(a - d*x/2)*si
nh(c + d*x)*cosh(a - d*x/2)*cosh(c + d*x)**2/8 - 3*x*sinh(c + d*x)**2*cosh(
a - d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a - d*x/2)**2*cosh(c + d*x)**3/16
 - 31*sinh(a - d*x/2)**2*sinh(c + d*x)**3/(48*d) + sinh(a - d*x/2)**2*sinh(
c + d*x)*cosh(c + d*x)**2/d - sinh(a - d*x/2)*sinh(c + d*x)**2*cosh(a - d*x
/2)*cosh(c + d*x)/(4*d) + 3*sinh(a - d*x/2)*cosh(a - d*x/2)*cosh(c + d*x)**
3/(8*d) + sinh(c + d*x)**3*cosh(a - d*x/2)**2/(48*d), Eq(b, -d/2)), (-3*x*s
inh(a + d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x)/16 + 3*x*sinh(a + d*x/2)**
2*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x/2)*sinh(c + d*x)**3*cosh(a + d*x/2
)/8 - 3*x*sinh(a + d*x/2)*sinh(c + d*x)*cosh(a + d*x/2)*cosh(c + d*x)**2/8
 - 3*x*sinh(c + d*x)**2*cosh(a + d*x/2)**2*cosh(c + d*x)/16 + 3*x*cosh(a + d
*x/2)**2*cosh(c + d*x)**3/16 - 31*sinh(a + d*x/2)**2*sinh(c + d*x)**3/(48*d
) + sinh(a + d*x/2)**2*sinh(c + d*x)*cosh(c + d*x)**2/d + sinh(a + d*x/2)*s
inh(c + d*x)**2*cosh(a + d*x/2)*cosh(c + d*x)/(4*d) - 3*sinh(a + d*x/2)*cos
h(a + d*x/2)*cosh(c + d*x)**3/(8*d) + sinh(c + d*x)**3*cosh(a + d*x/2)**2/(
48*d), Eq(b, d/2)), (3*x*sinh(a + 3*d*x/2)**2*sinh(c + d*x)**2*cosh(c + d*x
)/16 + x*sinh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16 - x*sinh(a + 3*d*x/2)*sin
h(c + d*x)**3*cosh(a + 3*d*x/2)/8 - 3*x*sinh(a + 3*d*x/2)*sinh(c + d*x)*cos
h(a + 3*d*x/2)*cosh(c + d*x)**2/8 + 3*x*sinh(c + d*x)**2*cosh(a + 3*d*x/2)*
*2*cosh(c + d*x)/16 + x*cosh(a + 3*d*x/2)**2*cosh(c + d*x)**3/16 - 5*sinh(a
 + 3*d*x/2)**2*sinh(c + d*x)**3/(48*d) + sinh(a + 3*d*x/2)**2*sinh(c + d*x)
*cosh(c + d*x)**2/d - 5*sinh(a + 3*d*x/2)*sinh(c + d*x)**2*cosh(a + 3*d*x/2
)*cosh(c + d*x)/(4*d) - sinh(a + 3*d*x/2)*cosh(a + 3*d*x/2)*cosh(c + d*x)**
3/(24*d) + 9*sinh(c + d*x)**3*cosh(a + 3*d*x/2)**2/(16*d), Eq(b, 3*d/2)), (
```

```
(x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh(a + b*x)/
(2*b))*cosh(c)**3, Eq(d, 0)), (-16*b**4*sinh(a + b*x)**2*sinh(c + d*x)**3/(
48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**4*sinh(a + b*x)**2*sinh(c + d*
x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 16*b**4*sinh(c
+ d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 24*b**4*
sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3
+ 27*d**5) + 24*b**3*d*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**3/(48*b**
4*d - 120*b**2*d**3 + 27*d**5) + 40*b**2*d**2*sinh(a + b*x)**2*sinh(c + d*x
)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 78*b**2*d**2*sinh(a + b*x)**2*
sinh(c + d*x)*cosh(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b
**2*d**2*sinh(c + d*x)**3*cosh(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*
d**5) + 42*b**2*d**2*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(48*b*
*4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sinh(a + b*x)*sinh(c + d*x)**2*
cosh(a + b*x)*cosh(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42*b*d*
*3*sinh(a + b*x)*cosh(a + b*x)*cosh(c + d*x)**3/(48*b**4*d - 120*b**2*d**3
+ 27*d**5) - 18*d**4*sinh(a + b*x)**2*sinh(c + d*x)**3/(48*b**4*d - 120*b**
2*d**3 + 27*d**5) + 27*d**4*sinh(a + b*x)**2*sinh(c + d*x)*cosh(c + d*x)**2
/(48*b**4*d - 120*b**2*d**3 + 27*d**5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*d)/b>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{e^{(2\,bx+3\,dx+2\,a+3\,c)}}{32\,(2\,b+3\,d)}+\frac{3\,e^{(2\,bx+dx+2\,a+c)}}{32\,(2\,b+d)}+\frac{3\,e^{(2\,bx-dx+2\,a-c)}}{32\,(2\,b-d)}+\frac{e^{(2\,bx-3\,dx+2\,a-3\,c)}}{32\,(2\,b-3\,d)}-\frac{e^{(-2\,bx+3\,dx-2\,a+3\,c)}}{32\,(2\,b-3\,d)}-\frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{32\,(2\,b-d)}-\frac{3\,e^{(-2\,bx-dx-2\,a-c)}}{32\,(2\,b+d)}-\frac{e^{(-2\,bx-3\,dx-2\,a-3\,c)}}{32\,(2\,b+3\,d)}-\frac{e^{(3\,dx+3\,c)}}{48\,d}-\frac{3\,e^{(dx+c)}}{16\,d}+\frac{3\,e^{(-dx-c)}}{16\,d}+\frac{e^{(-3\,dx-3\,c)}}{48\,d}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*e^(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/32*e^(2*b*x + d*x + 2*a
+ c)/(2*b + d) + 3/32*e^(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/32*e^(2*b*x -
 3*d*x + 2*a - 3*c)/(2*b - 3*d) - 1/32*e^(-2*b*x + 3*d*x - 2*a + 3*c)/(2*b
- 3*d) - 3/32*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) - 3/32*e^(-2*b*x - d*x -
 2*a - c)/(2*b + d) - 1/32*e^(-2*b*x - 3*d*x - 2*a - 3*c)/(2*b + 3*d) - 1/4
8*e^(3*d*x + 3*c)/d - 3/16*e^(d*x + c)/d + 3/16*e^(-d*x - c)/d + 1/48*e^(-3
*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.34

$$\int \cosh^3(c+dx)\sinh^2(a+bx)\,dx = \frac{{\cosh\left(c+d\,x\right)}^2\,{\sinh\left(a+b\,x\right)}^2\,\sinh\left(c+d\,x\right)\,\left(8\,b^4-26\,b^2\,d^2+9\,d^4\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}-{\sinh\left(a+b\,x\right)}^2\,{\sinh\left(c+d\,x\right)}^3\,\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}+\frac{1}{3\,d}\right)-\frac{2\,\cosh\left(a+b\,x\right)\,{\cosh\left(c+d\,x\right)}^3\,\sinh\left(a+b\,x\right)\,\left(7\,b\,d^2-4\,b^3\right)}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{2\,{\cosh\left(a+b\,x\right)}^2\,{\cosh\left(c+d\,x\right)}^2\,\sinh\left(c+d\,x\right)\,\left(4\,b^4-7\,b^2\,d^2\right)}{d\,\left(16\,b^4-40\,b^2\,d^2+9\,d^4\right)}-{\cosh\left(a+b\,x\right)}^2\,{\sinh\left(c+d\,x\right)}^3\,\left(\frac{3\,d^3}{16\,b^4-40\,b^2\,d^2+9\,d^4}-\frac{1}{3\,d}\right)+\frac{12\,b\,d^2\,\cosh\left(a+b\,x\right)\,\cosh\left(c+d\,x\right)\,\sinh\left(a+b\,x\right)\,{\sinh\left(c+d\,x\right)}^2}{16\,b^4-40\,b^2\,d^2+9\,d^4}$$

```
[In] int(cosh(c + d*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] (cosh(c + d*x)^2*sinh(a + b*x)^2*sinh(c + d*x)*(8*b^4 + 9*d^4 - 26*b^2*d^2)
)/(d*(16*b^4 + 9*d^4 - 40*b^2*d^2)) - sinh(a + b*x)^2*sinh(c + d*x)^3*((3*d
^3)/(16*b^4 + 9*d^4 - 40*b^2*d^2) + 1/(3*d)) - (2*cosh(a + b*x)*cosh(c + d*
x)^3*sinh(a + b*x)*(7*b*d^2 - 4*b^3))/(16*b^4 + 9*d^4 - 40*b^2*d^2) - (2*co
sh(a + b*x)^2*cosh(c + d*x)^2*sinh(c + d*x)*(4*b^4 - 7*b^2*d^2))/(d*(16*b^4
 + 9*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^2*sinh(c + d*x)^3*((3*d^3)/(16*b^4
+ 9*d^4 - 40*b^2*d^2) - 1/(3*d)) + (12*b*d^2*cosh(a + b*x)*cosh(c + d*x)*si
nh(a + b*x)*sinh(c + d*x)^2)/(16*b^4 + 9*d^4 - 40*b^2*d^2)
```

3.185 $\int \cosh(c+dx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx=-\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(3a+c+(3b+d)x)}{8(3b+d)}$$

```
[Out] -3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5737, 2718}

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx=-\frac{3\cosh(a+x(b-d)-c)}{8(b-d)}+\frac{\cosh(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cosh(a+x(b+d)+c)}{8(b+d)}+\frac{\cosh(3a+x(3b+d)+c)}{8(3b+d)}$$

```
[In] Int[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a - c + (b - d)*x])/(8*(b - d)) + Cosh[3*a - c + (3*b - d)*x]/(8*(
3*b - d)) - (3*Cosh[a + c + (b + d)*x])/(8*(b + d)) + Cosh[3*a + c + (3*b +
 d)*x]/(8*(3*b + d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{8}\sinh(a-c+(b-d)x)+\frac{1}{8}\sinh(3a-c+(3b-d)x) \right.\\
&\qquad \left. -\frac{3}{8}\sinh(a+c+(b+d)x)+\frac{1}{8}\sinh(3a+c+(3b+d)x)\right) dx\\
&= \frac{1}{8}\int \sinh(3a-c+(3b-d)x)\,dx+\frac{1}{8}\int \sinh(3a+c+(3b+d)x)\,dx\\
&\quad -\frac{3}{8}\int \sinh(a-c+(b-d)x)\,dx-\frac{3}{8}\int \sinh(a+c+(b+d)x)\,dx\\
&= -\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{8(3b-d)}\\
&\quad -\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(3a+c+(3b+d)x)}{8(3b+d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \frac{1}{8}\left(-\frac{3\cosh(a-c+bx-dx)}{b-d}+\frac{\cosh(3a-c+3bx-dx)}{3b-d} +\frac{\cosh(3a+c+3bx+dx)}{3b+d}-\frac{3\cosh(a+c+(b+d)x)}{b+d}\right)$$

```
[In] Integrate[Cosh[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] ((-3*Cosh[a - c + b*x - d*x])/(b - d) + Cosh[3*a - c + 3*b*x - d*x]/(3*b -
d) + Cosh[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cosh[a + c + (b + d)*x])/(b
 + d))/8
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$-\frac{3\cosh(a-c+(b-d)x)}{8(b-d)}+\frac{\cosh(3a-c+(3b-d)x)}{24b-8d}-\frac{3\cosh(a+c+(b+d)x)}{8(b+d)}+\frac{\cosh(3a+c+(3b+d)x)}{24b+8d}$
parallelrisch	$\frac{-12\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}b^{3}+24d\,b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+\left(-12\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b\,d^{2}+36b^{3}-12b\,d^{2}\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+(-64b}{9\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(b-\frac{d}{3}\right)\left(1+\tanh(\right.}$
risch	$\frac{(3b^{3}e^{6bx+6a}-b^{2}d\,e^{6bx+6a}-3b\,d^{2}e^{6bx+6a}+d^{3}e^{6bx+6a}-27b^{3}e^{4bx+4a}+27b^{2}d\,e^{4bx+4a}+3b\,d^{2}e^{4bx+4a}-3d^{3}e^{4bx+4a}-27b^{3}e^{2bx+2a}}{16(3b+d)(b+d)(3b-d)(b-d)}$

```
[In] int(cosh(d*x+c)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cosh(a+c
+(b+d)*x)/(b+d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(89) = 178.

Time = 0.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.51

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx$$
$$=\frac{9\left(b^{3}-bd^{2}\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{2}+3\left(\left(b^{3}-bd^{2}\right)\cosh\left(bx+a\right)^{3}-\left(9\,b^{3}-bd^{2}\right)\cosh\left(b\right.\right.}{4\left(\left(9\,b^{4}-10\,b^{2}d^{2}+d^{4}\right)\cosh\left(bx+a\right)^{4}-2\left(9\,b^{4}-10\,b^{2}d^{2}\right.\right.}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((b^3
- b*d^2)*cosh(b*x + a)^3 - (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) - (
(b^2*d - d^3)*sinh(b*x + a)^3 - 3*(9*b^2*d - d^3 - (b^2*d - d^3)*cosh(b*x +
 a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x +
 a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b
^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(76) = 152$.

Time = 1.94 (sec) , antiderivative size = 935, normalized size of antiderivative = 9.64

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(c), Eq(b, 0) & Eq(d, 0)), (3*x*sinh(a - d*x)**
3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)/8 - 3*
x*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)/8 - 3*x*sinh(c + d*x)*cosh(a
 - d*x)**3/8 - sinh(a - d*x)**3*sinh(c + d*x)/(8*d) - 3*sinh(a - d*x)**2*co
sh(a - d*x)*cosh(c + d*x)/(4*d) + 3*cosh(a - d*x)**3*cosh(c + d*x)/(8*d), E
q(b, -d)), (x*sinh(a - d*x/3)**3*cosh(c + d*x)/8 + 3*x*sinh(a - d*x/3)**2*s
inh(c + d*x)*cosh(a - d*x/3)/8 + 3*x*sinh(a - d*x/3)*cosh(a - d*x/3)**2*cos
h(c + d*x)/8 + x*sinh(c + d*x)*cosh(a - d*x/3)**3/8 + 9*sinh(a - d*x/3)**3*
sinh(c + d*x)/(8*d) + 3*sinh(a - d*x/3)**2*cosh(a - d*x/3)*cosh(c + d*x)/(4
*d) - cosh(a - d*x/3)**3*cosh(c + d*x)/(8*d), Eq(b, -d/3)), (x*sinh(a + d*x
/3)**3*cosh(c + d*x)/8 - 3*x*sinh(a + d*x/3)**2*sinh(c + d*x)*cosh(a + d*x/
3)/8 + 3*x*sinh(a + d*x/3)*cosh(a + d*x/3)**2*cosh(c + d*x)/8 - x*sinh(c +
d*x)*cosh(a + d*x/3)**3/8 + 7*sinh(a + d*x/3)**3*sinh(c + d*x)/(8*d) - 3*si
nh(a + d*x/3)*sinh(c + d*x)*cosh(a + d*x/3)**2/(4*d) + 3*cosh(a + d*x/3)**3
*cosh(c + d*x)/(8*d), Eq(b, d/3)), (3*x*sinh(a + d*x)**3*cosh(c + d*x)/8 -
3*x*sinh(a + d*x)**2*sinh(c + d*x)*cosh(a + d*x)/8 - 3*x*sinh(a + d*x)*cosh
(a + d*x)**2*cosh(c + d*x)/8 + 3*x*sinh(c + d*x)*cosh(a + d*x)**3/8 + 5*sin
h(a + d*x)**3*sinh(c + d*x)/(8*d) - 3*sinh(a + d*x)*sinh(c + d*x)*cosh(a +
d*x)**2/(4*d) + 3*cosh(a + d*x)**3*cosh(c + d*x)/(8*d), Eq(b, d)), (9*b**3*
sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4)
 - 6*b**3*cosh(a + b*x)**3*cosh(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 7
*b**2*d*sinh(a + b*x)**3*sinh(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 6*b
**2*d*sinh(a + b*x)*sinh(c + d*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 +
 d**4) - 3*b*d**2*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)/(9*b**4 - 10
*b**2*d**2 + d**4) + d**3*sinh(a + b*x)**3*sinh(c + d*x)/(9*b**4 - 10*b**2*
d**2 + d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(89) = 178$.

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \cosh(c+dx)\sinh^3(a+bx)\,dx &= \frac{e^{(3\,bx+dx+3\,a+c)}}{16\,(3\,b+d)} + \frac{e^{(3\,bx-dx+3\,a-c)}}{16\,(3\,b-d)} - \frac{3\,e^{(bx+dx+a+c)}}{16\,(b+d)} \\ &\quad - \frac{3\,e^{(bx-dx+a-c)}}{16\,(b-d)} - \frac{3\,e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{3\,e^{(-bx-dx-a-c)}}{16\,(b+d)} \\ &\quad + \frac{e^{(-3\,bx+dx-3\,a+c)}}{16\,(3\,b-d)} + \frac{e^{(-3\,bx-dx-3\,a-c)}}{16\,(3\,b+d)}\end{aligned}$$

```
[In] integrate(cosh(d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/16*e^(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/16*e^(b*x + d*x + a + c)/(b + d) - 3/16*e^(b*x - d*x + a - c)/
(b - d) - 3/16*e^(-b*x + d*x - a + c)/(b - d) - 3/16*e^(-b*x - d*x - a - c)
/(b + d) + 1/16*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/16*e^(-3*b*x - d*x
 - 3*a - c)/(3*b + d)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.89

$$\int \cosh(c+dx)\sinh^3(a+bx)\,dx$$
$$= \frac{6\,b^2\,d\cosh(a+b\,x)^2\sinh(a+b\,x)\,\sinh(c+d\,x)}{9\,b^4-10\,b^2\,d^2+d^4} - \frac{d\sinh(a+b\,x)^3\sinh(c+d\,x)\,\left(7\,b^2-d^2\right)}{9\,b^4-10\,b^2\,d^2+d^4} - \frac{3\cosh(a+b\,x)\,\cosh(c+d\,x)\,\sinh(a+b\,x)^2\,\left(b\,d^2-3\,b^3\right)}{9\,b^4-10\,b^2\,d^2+d^4} - \frac{6\,b^3\cosh(a+b\,x)^3\cosh(c+d\,x)}{9\,b^4-10\,b^2\,d^2+d^4}$$

```
[In] int(cosh(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] (6*b^2*d*cosh(a + b*x)^2*sinh(a + b*x)*sinh(c + d*x))/(9*b^4 + d^4 - 10*b^2
*d^2) - (d*sinh(a + b*x)^3*sinh(c + d*x)*(7*b^2 - d^2))/(9*b^4 + d^4 - 10*b
^2*d^2) - (3*cosh(a + b*x)*cosh(c + d*x)*sinh(a + b*x)^2*(b*d^2 - 3*b^3))/(
9*b^4 + d^4 - 10*b^2*d^2) - (6*b^3*cosh(a + b*x)^3*cosh(c + d*x))/(9*b^4 +
d^4 - 10*b^2*d^2)
```

3.186 $\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a+bx)}{8b}+\frac{\cosh(3a+3bx)}{24b}-\frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cosh(3a-2c+(3b-2d)x)}{16(3b-2d)}-\frac{3\cosh(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\cosh(3a+2c+(3b+2d)x)}{16(3b+2d)}$$

```
[Out] -3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*d
)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+2*
d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used

$= \{5737, 2718\}$

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a+x(b-2d)-2c)}{16(b-2d)} + \frac{\cosh(3a+x(3b-2d)-2c)}{16(3b-2d)} - \frac{3\cosh(a+x(b+2d)+2c)}{16(b+2d)} + \frac{\cosh(3a+x(3b+2d)+2c)}{16(3b+2d)} - \frac{3\cosh(a+bx)}{8b} + \frac{\cosh(3a+3bx)}{24b}$$

```
[In] Int[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a + b*x])/(8*b) + Cosh[3*a + 3*b*x]/(24*b) - (3*Cosh[a - 2*c + (b
- 2*d)*x])/(16*(b - 2*d)) + Cosh[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)
) - (3*Cosh[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cosh[3*a + 2*c + (3*b
+ 2*d)*x]/(16*(3*b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\text{integral} = \int \left(-\frac{3}{8}\sinh(a+bx) + \frac{1}{8}\sinh(3a+3bx) - \frac{3}{16}\sinh(a-2c+(b-2d)x) + \frac{1}{16}\sinh(3a-2c+(3b-2d)x) - \frac{3}{16}\sinh(a+2c+(b+2d)x) + \frac{1}{16}\sinh(3a+2c+(3b+2d)x)\right) dx$$

$$= \frac{1}{16}\int \sinh(3a-2c+(3b-2d)x)\,dx + \frac{1}{16}\int \sinh(3a+2c+(3b+2d)x)\,dx + \frac{1}{8}\int \sinh(3a+3bx)\,dx - \frac{3}{16}\int \sinh(a-2c+(b-2d)x)\,dx - \frac{3}{16}\int \sinh(a+2c+(b+2d)x)\,dx - \frac{3}{8}\int \sinh(a+bx)\,dx$$

$$
\begin{aligned}
&= -\frac{3\cosh(a+bx)}{8b} + \frac{\cosh(3a+3bx)}{24b} \\
&\quad - \frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)} + \frac{\cosh(3a-2c+(3b-2d)x)}{16(3b-2d)} \\
&\quad - \frac{3\cosh(a+2c+(b+2d)x)}{16(b+2d)} + \frac{\cosh(3a+2c+(3b+2d)x)}{16(3b+2d)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$
\begin{aligned}
\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \frac{1}{48}\Bigg(& -\frac{18\cosh(a)\cosh(bx)}{b} + \frac{2\cosh(3a)\cosh(3bx)}{b} \\
& - \frac{9\cosh(a-2c+bx-2dx)}{b-2d} \\
& + \frac{3\cosh(3a-2c+3bx-2dx)}{3b-2d} \\
& - \frac{9\cosh(a+2c+bx+2dx)}{b+2d} \\
& + \frac{3\cosh(3a+2c+3bx+2dx)}{3b+2d} - \frac{18\sinh(a)\sinh(bx)}{b} \\
& + \frac{2\sinh(3a)\sinh(3bx)}{b}\Bigg)
\end{aligned}
$$

```
[In] Integrate[Cosh[c + d*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] ((-18*Cosh[a]*Cosh[b*x])/b + (2*Cosh[3*a]*Cosh[3*b*x])/b - (9*Cosh[a - 2*c
+ b*x - 2*d*x])/(b - 2*d) + (3*Cosh[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d)
 - (9*Cosh[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cosh[3*a + 2*c + 3*b*x +
2*d*x])/(3*b + 2*d) - (18*Sinh[a]*Sinh[b*x])/b + (2*Sinh[3*a]*Sinh[3*b*x])/
b)/48
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

method	result
default	$-\frac{3\cosh(bx+a)}{8b}+\frac{\cosh(3bx+3a)}{24b}-\frac{3\cosh(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cosh(3a-2c+(3b-2d)x)}{48b-32d}-\frac{3\cosh(a+2c+(b+2d)x)}{16(b+2d)}+$ c
parallelrisch	$\frac{9(b+2d)\left(b+\frac{2d}{3}\right)(b-2d)b\cosh(3a-2c+(3b-2d)x)+9(b+2d)\left(b-\frac{2d}{3}\right)(b-2d)b\cosh(3a+2c+(3b+2d)x)-81(b+2d)\left(b+\frac{2d}{3}\right)\left(b-\frac{2}{3}\right.}{}$
risch	$\frac{e^{3bx+3a}}{48b}-\frac{3\,e^{bx+a}}{16b}-\frac{3\,e^{-bx-a}}{16b}+\frac{e^{-3bx-3a}}{48b}+\frac{(3b^3e^{6bx+6a}-2b^2d\,e^{6bx+6a}-12b\,d^2e^{6bx+6a}+8d^3e^{6bx+6a}-27b^3e^{4bx+4a}+}{}$

```
[In] int(cosh(d*x+c)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cosh(b*x+a)/b+1/24*cosh(3*b*x+3*a)/b-3/16*cosh(a-2*c+(b-2*d)*x)/(b-2*d
)+1/16*cosh(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cosh(a+2*c+(b+2*d)*x)/(b+2*
d)+1/16*cosh(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.21

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \frac{(9\,b^4-40\,b^2d^2+16\,d^4)\cosh{(bx+a)}^3+9\,\left((b^4-4\,b^2d^2)\cosh{(bx+a)}^3-(9\,b^4-4\,b^2d^2)\cosh{(bx+a)}\right)\cosh}{}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*((9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x + a)^3 + 9*((b^4 - 4*b^2*d^2)*
cosh(b*x + a)^3 - (9*b^4 - 4*b^2*d^2)*cosh(b*x + a))*cosh(d*x + c)^2 + 3*(9
*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 + (9*b^4 - 40*b^2*d^2 + 16
*d^4)*cosh(b*x + a))*sinh(b*x + a)^2 + 9*((b^4 - 4*b^2*d^2)*cosh(b*x + a)^3
 + 3*(b^4 - 4*b^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b^4 - 4*b^2*d^2)*
cosh(b*x + a))*sinh(d*x + c)^2 - 9*(9*b^4 - 40*b^2*d^2 + 16*d^4)*cosh(b*x +
 a) - 12*((b^3*d - 4*b*d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3*d - 4*
b*d^3 - (b^3*d - 4*b*d^3)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a))*sin
h(d*x + c))/((9*b^5 - 40*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^4 - 2*(9*b^5 - 4
0*b^3*d^2 + 16*b*d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b^5 - 40*b^3*d^2
 + 16*b*d^4)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(116) = 232$.

Time = 5.34 (sec) , antiderivative size = 2030, normalized size of antiderivative = 14.71

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x
)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*sinh(a)*
*3, Eq(b, 0)), (3*x*sinh(a - 2*d*x)**3*sinh(c + d*x)**2/16 + 3*x*sinh(a - 2
*d*x)**3*cosh(c + d*x)**2/16 + 3*x*sinh(a - 2*d*x)**2*sinh(c + d*x)*cosh(a
- 2*d*x)*cosh(c + d*x)/8 - 3*x*sinh(a - 2*d*x)*sinh(c + d*x)**2*cosh(a - 2*
d*x)**2/16 - 3*x*sinh(a - 2*d*x)*cosh(a - 2*d*x)**2*cosh(c + d*x)**2/16 - 3
```

*x*sinh(c + d*x)*cosh(a - 2*d*x)**3*cosh(c + d*x)/8 + 13*sinh(a - 2*d*x)**3
*sinh(c + d*x)*cosh(c + d*x)/(16*d) + sinh(a - 2*d*x)**2*sinh(c + d*x)**2*c
osh(a - 2*d*x)/(2*d) - 7*sinh(a - 2*d*x)*sinh(c + d*x)*cosh(a - 2*d*x)**2*c
osh(c + d*x)/(8*d) - 49*sinh(c + d*x)**2*cosh(a - 2*d*x)**3/(96*d) - 17*cos
h(a - 2*d*x)**3*cosh(c + d*x)**2/(96*d), Eq(b, -2*d)), (x*sinh(a - 2*d*x/3)
3*sinh(c + d*x)2/16 + x*sinh(a - 2*d*x/3)**3*cosh(c + d*x)**2/16 + 3*x*
sinh(a - 2*d*x/3)**2*sinh(c + d*x)*cosh(a - 2*d*x/3)*cosh(c + d*x)/8 + 3*x*
sinh(a - 2*d*x/3)*sinh(c + d*x)**2*cosh(a - 2*d*x/3)**2/16 + 3*x*sinh(a - 2
*d*x/3)*cosh(a - 2*d*x/3)**2*cosh(c + d*x)**2/16 + x*sinh(c + d*x)*cosh(a -
 2*d*x/3)**3*cosh(c + d*x)/8 + 15*sinh(a - 2*d*x/3)**3*sinh(c + d*x)*cosh(c
 + d*x)/(16*d) + 3*sinh(a - 2*d*x/3)**2*sinh(c + d*x)**2*cosh(a - 2*d*x/3)/
(2*d) + 9*sinh(a - 2*d*x/3)*sinh(c + d*x)*cosh(a - 2*d*x/3)**2*cosh(c + d*x
)/(8*d) - 11*sinh(c + d*x)**2*cosh(a - 2*d*x/3)**3/(32*d) + 21*cosh(a - 2*d
*x/3)**3*cosh(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (x*sinh(a + 2*d*x/3)**3*s
inh(c + d*x)**2/16 + x*sinh(a + 2*d*x/3)**3*cosh(c + d*x)**2/16 - 3*x*sinh(
a + 2*d*x/3)**2*sinh(c + d*x)*cosh(a + 2*d*x/3)*cosh(c + d*x)/8 + 3*x*sinh(
a + 2*d*x/3)*sinh(c + d*x)**2*cosh(a + 2*d*x/3)**2/16 + 3*x*sinh(a + 2*d*x/
3)*cosh(a + 2*d*x/3)**2*cosh(c + d*x)**2/16 - x*sinh(c + d*x)*cosh(a + 2*d*
x/3)**3*cosh(c + d*x)/8 + 15*sinh(a + 2*d*x/3)**3*sinh(c + d*x)*cosh(c + d*
x)/(16*d) - 3*sinh(a + 2*d*x/3)**2*sinh(c + d*x)**2*cosh(a + 2*d*x/3)/(2*d)
 + 9*sinh(a + 2*d*x/3)*sinh(c + d*x)*cosh(a + 2*d*x/3)**2*cosh(c + d*x)/(8*
d) + 11*sinh(c + d*x)**2*cosh(a + 2*d*x/3)**3/(32*d) - 21*cosh(a + 2*d*x/3)
3*cosh(c + d*x)2/(32*d), Eq(b, 2*d/3)), (3*x*sinh(a + 2*d*x)**3*sinh(c
 + d*x)**2/16 + 3*x*sinh(a + 2*d*x)**3*cosh(c + d*x)**2/16 - 3*x*sinh(a + 2*
d*x)**2*sinh(c + d*x)*cosh(a + 2*d*x)*cosh(c + d*x)/8 - 3*x*sinh(a + 2*d*x)
*sinh(c + d*x)**2*cosh(a + 2*d*x)**2/16 - 3*x*sinh(a + 2*d*x)*cosh(a + 2*d*
x)**2*cosh(c + d*x)**2/16 + 3*x*sinh(c + d*x)*cosh(a + 2*d*x)**3*cosh(c + d
*x)/8 + 13*sinh(a + 2*d*x)**3*sinh(c + d*x)*cosh(c + d*x)/(16*d) - sinh(a +
 2*d*x)**2*sinh(c + d*x)**2*cosh(a + 2*d*x)/(2*d) - 7*sinh(a + 2*d*x)*sinh(
c + d*x)*cosh(a + 2*d*x)**2*cosh(c + d*x)/(8*d) + 49*sinh(c + d*x)**2*cosh(
a + 2*d*x)**3/(96*d) + 17*cosh(a + 2*d*x)**3*cosh(c + d*x)**2/(96*d), Eq(b,
 2*d)), (27*b**4*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**5 -
 120*b**3*d**2 + 48*b*d**4) - 18*b**4*cosh(a + b*x)**3*cosh(c + d*x)**2/(27
*b**5 - 120*b**3*d**2 + 48*b*d**4) - 42*b**3*d*sinh(a + b*x)**3*sinh(c + d*
x)*cosh(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 36*b**3*d*sinh(a +
 b*x)*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)/(27*b**5 - 120*b**3*d**2
 + 48*b*d**4) + 42*b**2*d**2*sinh(a + b*x)**2*sinh(c + d*x)**2*cosh(a + b*x
)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 78*b**2*d**2*sinh(a + b*x)**2*cos
h(a + b*x)*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 40*b**2
*d**2*sinh(c + d*x)**2*cosh(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**
4) + 40*b**2*d**2*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d**
2 + 48*b*d**4) + 24*b*d**3*sinh(a + b*x)**3*sinh(c + d*x)*cosh(c + d*x)/(27
*b**5 - 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sinh(a + b*x)**2*sinh(c + d*x)
2*cosh(a + b*x)/(27*b5 - 120*b**3*d**2 + 48*b*d**4) + 24*d**4*sinh(a +
b*x)**2*cosh(a + b*x)*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4

```
) + 16*d**4*sinh(c + d*x)**2*cosh(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48
*b*d**4) - 16*d**4*cosh(a + b*x)**3*cosh(c + d*x)**2/(27*b**5 - 120*b**3*d*
*2 + 48*b*d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(126) = 252$.

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.86

$$\begin{aligned}\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx &= \frac{e^{(3\,bx+2\,dx+3\,a+2\,c)}}{32\,(3\,b+2\,d)} + \frac{e^{(3\,bx-2\,dx+3\,a-2\,c)}}{32\,(3\,b-2\,d)} + \frac{e^{(3\,bx+3\,a)}}{48\,b} \\ &\quad - \frac{3\,e^{(bx+2\,dx+a+2\,c)}}{32\,(b+2\,d)} - \frac{3\,e^{(bx-2\,dx+a-2\,c)}}{32\,(b-2\,d)} - \frac{3\,e^{(bx+a)}}{16\,b} \\ &\quad - \frac{3\,e^{(-bx+2\,dx-a+2\,c)}}{32\,(b-2\,d)} - \frac{3\,e^{(-bx-2\,dx-a-2\,c)}}{32\,(b+2\,d)} - \frac{3\,e^{(-bx-a)}}{16\,b} \\ &\quad + \frac{e^{(-3\,bx+2\,dx-3\,a+2\,c)}}{32\,(3\,b-2\,d)} + \frac{e^{(-3\,bx-2\,dx-3\,a-2\,c)}}{32\,(3\,b+2\,d)} + \frac{e^{(-3\,bx-3\,a)}}{48\,b}\end{aligned}$$

```
[In] integrate(cosh(d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/32*e^(3*b*x - 2*d*x + 3*
a - 2*c)/(3*b - 2*d) + 1/48*e^(3*b*x + 3*a)/b - 3/32*e^(b*x + 2*d*x + a + 2
*c)/(b + 2*d) - 3/32*e^(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/16*e^(b*x + a)
/b - 3/32*e^(-b*x + 2*d*x - a + 2*c)/(b - 2*d) - 3/32*e^(-b*x - 2*d*x - a -
 2*c)/(b + 2*d) - 3/16*e^(-b*x - a)/b + 1/32*e^(-3*b*x + 2*d*x - 3*a + 2*c)
/(3*b - 2*d) + 1/32*e^(-3*b*x - 2*d*x - 3*a - 2*c)/(3*b + 2*d) + 1/48*e^(-3
*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.44

$$\int \cosh^2(c+dx)\sinh^3(a+bx)\,dx$$
$$= \frac{\cosh(a+b\,x)\,\cosh(c+d\,x)^2\sinh(a+b\,x)^2\,(9\,b^4-26\,b^2\,d^2+8\,d^4)}{b\,(9\,b^4-40\,b^2\,d^2+16\,d^4)}$$
$$-\cosh(a+b\,x)^3\sinh(c+d\,x)^2\left(\frac{3\,b^3}{9\,b^4-40\,b^2\,d^2+16\,d^4}-\frac{1}{3\,b}\right)$$
$$-\cosh(a+b\,x)^3\cosh(c+d\,x)^2\left(\frac{3\,b^3}{9\,b^4-40\,b^2\,d^2+16\,d^4}+\frac{1}{3\,b}\right)$$
$$-\frac{2\,d\cosh(c+d\,x)\,\sinh(a+b\,x)^3\sinh(c+d\,x)\,(7\,b^2-4\,d^2)}{9\,b^4-40\,b^2\,d^2+16\,d^4}$$
$$+\frac{12\,b^2\,d\cosh(a+b\,x)^2\cosh(c+d\,x)\,\sinh(a+b\,x)\,\sinh(c+d\,x)}{9\,b^4-40\,b^2\,d^2+16\,d^4}$$
$$+\frac{2\,d^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2\sinh(c+d\,x)^2\,(7\,b^2-4\,d^2)}{b\,(9\,b^4-40\,b^2\,d^2+16\,d^4)}$$

```
[In] int(cosh(c + d*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] (cosh(a + b*x)*cosh(c + d*x)^2*sinh(a + b*x)^2*(9*b^4 + 8*d^4 - 26*b^2*d^2)
)/(b*(9*b^4 + 16*d^4 - 40*b^2*d^2)) - cosh(a + b*x)^3*sinh(c + d*x)^2*((3*b
^3)/(9*b^4 + 16*d^4 - 40*b^2*d^2) - 1/(3*b)) - cosh(a + b*x)^3*cosh(c + d*x
)^2*((3*b^3)/(9*b^4 + 16*d^4 - 40*b^2*d^2) + 1/(3*b)) - (2*d*cosh(c + d*x)*
sinh(a + b*x)^3*sinh(c + d*x)*(7*b^2 - 4*d^2))/(9*b^4 + 16*d^4 - 40*b^2*d^2
) + (12*b^2*d*cosh(a + b*x)^2*cosh(c + d*x)*sinh(a + b*x)*sinh(c + d*x))/(9
*b^4 + 16*d^4 - 40*b^2*d^2) + (2*d^2*cosh(a + b*x)*sinh(a + b*x)^2*sinh(c +
 d*x)^2*(7*b^2 - 4*d^2))/(b*(9*b^4 + 16*d^4 - 40*b^2*d^2))
```

3.187 $\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)} - \frac{9\cosh(a-c+(b-d)x)}{32(b-d)} + \frac{\cosh(3(a-c)+3(b-d)x)}{96(b-d)} + \frac{3\cosh(3a-c+(3b-d)x)}{32(3b-d)} - \frac{9\cosh(a+c+(b+d)x)}{32(b+d)} + \frac{\cosh(3(a+c)+3(b+d)x)}{96(b+d)} + \frac{3\cosh(3a+c+(3b+d)x)}{32(3b+d)} - \frac{3\cosh(a+3c+(b+3d)x)}{32(b+3d)}$$

```
[Out] -3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)+1/96*cosh(
3*a-3*c+3*(b-d)*x)/(b-d)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*cosh(a+c+(
b+d)*x)/(b+d)+1/96*cosh(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*cosh(3*a+c+(3*b+d)*x)
/(3*b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5737, 2718}

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = -\frac{3\cosh(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cosh(a+x(b-d)-c)}{32(b-d)}+\frac{\cosh(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cosh(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cosh(a+x(b+d)+c)}{32(b+d)}+\frac{\cosh(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\cosh(3a+x(3b+d)+c)}{32(3b+d)}-\frac{3\cosh(a+x(b+3d)+3c)}{32(b+3d)}$$

```
[In] Int[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
[Out] (-3*Cosh[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cosh[a - c + (b - d)*x
])/(32*(b - d)) + Cosh[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cosh[3*a
- c + (3*b - d)*x])/(32*(3*b - d)) - (9*Cosh[a + c + (b + d)*x])/(32*(b + d
)) + Cosh[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cosh[3*a + c + (3*b +
d)*x])/(32*(3*b + d)) - (3*Cosh[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{32}\sinh(a-3c+(b-3d)x)-\frac{9}{32}\sinh(a-c+(b-d)x)\right.\\
&\quad+\frac{1}{32}\sinh(3(a-c)+3(b-d)x)+\frac{3}{32}\sinh(3a-c+(3b-d)x)\\
&\quad-\frac{9}{32}\sinh(a+c+(b+d)x)+\frac{1}{32}\sinh(3(a+c)+3(b+d)x)\\
&\quad\left.+\frac{3}{32}\sinh(3a+c+(3b+d)x)-\frac{3}{32}\sinh(a+3c+(b+3d)x)\right)\,dx\\
&=\frac{1}{32}\int\sinh(3(a-c)+3(b-d)x)\,dx+\frac{1}{32}\int\sinh(3(a+c)+3(b+d)x)\,dx\\
&\quad-\frac{3}{32}\int\sinh(a-3c+(b-3d)x)\,dx+\frac{3}{32}\int\sinh(3a-c+(3b-d)x)\,dx\\
&\quad+\frac{3}{32}\int\sinh(3a+c+(3b+d)x)\,dx-\frac{3}{32}\int\sinh(a+3c+(b+3d)x)\,dx\\
&\quad-\frac{9}{32}\int\sinh(a-c+(b-d)x)\,dx-\frac{9}{32}\int\sinh(a+c+(b+d)x)\,dx\\
&=-\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cosh(a-c+(b-d)x)}{32(b-d)}\\
&\quad+\frac{\cosh(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\cosh(3a-c+(3b-d)x)}{32(3b-d)}\\
&\quad-\frac{9\cosh(a+c+(b+d)x)}{32(b+d)}+\frac{\cosh(3(a+c)+3(b+d)x)}{96(b+d)}\\
&\quad+\frac{3\cosh(3a+c+(3b+d)x)}{32(3b+d)}-\frac{3\cosh(a+3c+(b+3d)x)}{32(b+3d)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\begin{aligned}
\int\cosh^3(c+dx)\sinh^3(a+bx)\,dx &= \frac{1}{96}\left(-\frac{9\cosh(a-3c+bx-3dx)}{b-3d}\right.\\
&\quad-\frac{27\cosh(a-c+bx-dx)}{b-d}+\frac{\cosh(3(a-c+bx-dx))}{b-d}\\
&\quad+\frac{9\cosh(3a-c+3bx-dx)}{3b-d}\\
&\quad+\frac{9\cosh(3a+c+3bx+dx)}{3b+d}\\
&\quad-\frac{9\cosh(a+3c+bx+3dx)}{b+3d}\\
&\quad-\frac{27\cosh(a+c+(b+d)x)}{b+d}\\
&\quad\left.+\frac{\cosh(3(a+c+(b+d)x))}{b+d}\right)
\end{aligned}$$

```
[In] Integrate[Cosh[c + d*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] ((-9*Cosh[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cosh[a - c + b*x - d*x])/
(b - d) + Cosh[3*(a - c + b*x - d*x)]/(b - d) + (9*Cosh[3*a - c + 3*b*x - d
*x])/(3*b - d) + (9*Cosh[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cosh[a + 3*
c + b*x + 3*d*x])/(b + 3*d) - (27*Cosh[a + c + (b + d)*x])/(b + d) + Cosh[3
*(a + c + (b + d)*x)]/(b + d))/96
```

Maple [A] (verified)

Time = 11.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.97

method	result
default	$-\frac{3\cosh(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cosh(a-c+(b-d)x)}{32(b-d)}-\frac{9\cosh(a+c+(b+d)x)}{32(b+d)}-\frac{3\cosh(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\cosh((3b-3d)x\cdots}{96b-96\cdots}$
parallelrisch	$\frac{9(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b+d)\left(b-\frac{d}{3}\right)\cosh((3b-3d)x+3a-3c)+27(b+3d)\left(b+\frac{d}{3}\right)(b-3d)(b-d)(b+d)\cosh(3a-c+(3b-d)x)+9(b\cdots}{\cdots}$
risch	$\frac{(b^3e^{6bx+6a}-b^2d\,e^{6bx+6a}-9b\,d^2e^{6bx+6a}+9d^3e^{6bx+6a}-9b^3e^{4bx+4a}+27b^2d\,e^{4bx+4a}+9b\,d^2e^{4bx+4a}-27d^3e^{4bx+4a}-9b^3e^{2bx+2a}\cdots}{192(b+d)(b+3d)(b-d)(b-3d)}$

```
[In] int(cosh(d*x+c)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/32*cosh(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cosh(a-c+(b-d)*x)/(b-d)-9/32*cosh(
a+c+(b+d)*x)/(b+d)-3/32*cosh(a+3*c+(b+3*d)*x)/(b+3*d)+1/32/(3*b-3*d)*cosh((
3*b-3*d)*x+3*a-3*c)+3/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cosh(3*a+c+(3*b
+d)*x)/(3*b+d)+1/32/(3*b+3*d)*cosh((3*b+3*d)*x+3*a+3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.74

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx$$
$$= \frac{\left((9\,b^5-82\,b^3d^2+9\,bd^4)\cosh\left(bx+a\right)^3-9\left(9\,b^5-10\,b^3d^2+bd^4\right)\cosh\left(bx+a\right)\right)\cosh\left(dx+c\right)^3-\left((9\,b^4d\cdots\right.}{\cdots}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d
^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5
)*sinh(b*x + a)^3 - 3*(81*b^4*d - 90*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^
3 + 9*d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*((9*b^5 - 82
*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)^3 + 27*(b^5 - 10*b^3*d^2 +
9*b*d^4)*cosh(b*x + a)*cosh(d*x + c))*sinh(b*x + a)^2 + 3*(3*(9*b^5 - 82*b^
3*d^2 + 9*b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 + ((9*b^5 - 82
```

```
*b^3*d^2 + 9*b*d^4)*cosh(b*x + a)^3 - 9*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b
*x + a))*cosh(d*x + c))*sinh(d*x + c)^2 + 27*((b^5 - 10*b^3*d^2 + 9*b*d^4)*
cosh(b*x + a)^3 - (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cosh(b*x + a))*cosh(d*x +
c) - 3*((3*b^4*d - 30*b^2*d^3 + 27*d^5 + (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos
h(d*x + c)^2)*sinh(b*x + a)^3 - 3*(27*b^4*d - 246*b^2*d^3 + 27*d^5 - 3*(b^4
*d - 10*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2 + (81*b^4*d - 90*b^2*d^3 + 9*d^5 -
 (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cosh(b*x + a)^2)*cosh(d*x + c)^2)*sinh(b*x
+ a))*sinh(d*x + c))/((9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x +
a)^4 - 2*(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + (9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.

Time = 16.87 (sec) , antiderivative size = 3580, normalized size of antiderivative = 18.36

$$\int \cosh^3(c + dx) \sinh^3(a + bx) \, dx = \text{Too large to display}$$

```
[In] integrate(cosh(d*x+c)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a)**3*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (9*x*sinh(a - 3*d
*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/32 + 3*x*sinh(a - 3*d*x)**3*cosh(c +
d*x)**3/32 + 3*x*sinh(a - 3*d*x)**2*sinh(c + d*x)**3*cosh(a - 3*d*x)/32 + 9
*x*sinh(a - 3*d*x)**2*sinh(c + d*x)*cosh(a - 3*d*x)*cosh(c + d*x)**2/32 - 9
*x*sinh(a - 3*d*x)*sinh(c + d*x)**2*cosh(a - 3*d*x)**2*cosh(c + d*x)/32 - 3
*x*sinh(a - 3*d*x)*cosh(a - 3*d*x)**2*cosh(c + d*x)**3/32 - 3*x*sinh(c + d*
x)**3*cosh(a - 3*d*x)**3/32 - 9*x*sinh(c + d*x)*cosh(a - 3*d*x)**3*cosh(c +
 d*x)**2/32 + sinh(a - 3*d*x)**3*sinh(c + d*x)**3/(12*d) - 13*sinh(a - 3*d*
x)**3*sinh(c + d*x)*cosh(c + d*x)**2/(320*d) + 3*sinh(a - 3*d*x)**2*sinh(c
+ d*x)**2*cosh(a - 3*d*x)*cosh(c + d*x)/(20*d) - 101*sinh(a - 3*d*x)**2*cos
h(a - 3*d*x)*cosh(c + d*x)**3/(320*d) - 27*sinh(a - 3*d*x)*sinh(c + d*x)**3
*cosh(a - 3*d*x)**2/(320*d) - 51*sinh(c + d*x)**2*cosh(a - 3*d*x)**3*cosh(c
 + d*x)/(320*d) + cosh(a - 3*d*x)**3*cosh(c + d*x)**3/(5*d), Eq(b, -3*d)),
(-3*x*sinh(a - d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/16 + 5*x*sinh(a - d*x
)**3*cosh(c + d*x)**3/16 - 3*x*sinh(a - d*x)**2*sinh(c + d*x)**3*cosh(a - d
*x)/16 + 9*x*sinh(a - d*x)**2*sinh(c + d*x)*cosh(a - d*x)*cosh(c + d*x)**2/
16 + 9*x*sinh(a - d*x)*sinh(c + d*x)**2*cosh(a - d*x)**2*cosh(c + d*x)/16 -
 3*x*sinh(a - d*x)*cosh(a - d*x)**2*cosh(c + d*x)**3/16 + 5*x*sinh(c + d*x)
**3*cosh(a - d*x)**3/16 - 3*x*sinh(c + d*x)*cosh(a - d*x)**3*cosh(c + d*x)*
*2/16 - sinh(a - d*x)**3*sinh(c + d*x)**3/(48*d) + sinh(a - d*x)**3*sinh(c
+ d*x)*cosh(c + d*x)**2/(2*d) + 3*sinh(a - d*x)**2*sinh(c + d*x)**2*cosh(a
- d*x)*cosh(c + d*x)/(4*d) - 3*sinh(a - d*x)**2*cosh(a - d*x)*cosh(c + d*x)
**3/(16*d) + 5*sinh(a - d*x)*sinh(c + d*x)**3*cosh(a - d*x)**2/(16*d) + cos
h(a - d*x)**3*cosh(c + d*x)**3/(16*d), Eq(b, -d)), (-3*x*sinh(a - d*x/3)**3
```

```
*sinh(c + d*x)**2*cosh(c + d*x)/32 + 3*x*sinh(a - d*x/3)**3*cosh(c + d*x)**
3/32 - 9*x*sinh(a - d*x/3)**2*sinh(c + d*x)**3*cosh(a - d*x/3)/32 + 9*x*sin
h(a - d*x/3)**2*sinh(c + d*x)*cosh(a - d*x/3)*cosh(c + d*x)**2/32 - 9*x*sin
h(a - d*x/3)*sinh(c + d*x)**2*cosh(a - d*x/3)**2*cosh(c + d*x)/32 + 9*x*sin
h(a - d*x/3)*cosh(a - d*x/3)**2*cosh(c + d*x)**3/32 - 3*x*sinh(c + d*x)**3*
cosh(a - d*x/3)**3/32 + 3*x*sinh(c + d*x)*cosh(a - d*x/3)**3*cosh(c + d*x)*
*2/32 - 3*sinh(a - d*x/3)**3*sinh(c + d*x)**3/(4*d) + 351*sinh(a - d*x/3)**
3*sinh(c + d*x)*cosh(c + d*x)**2/(320*d) - 9*sinh(a - d*x/3)**2*sinh(c + d*
x)**2*cosh(a - d*x/3)*cosh(c + d*x)/(20*d) + 183*sinh(a - d*x/3)**2*cosh(a
- d*x/3)*cosh(c + d*x)**3/(320*d) + 9*sinh(a - d*x/3)*sinh(c + d*x)**3*cosh
(a - d*x/3)**2/(320*d) + 33*sinh(c + d*x)**2*cosh(a - d*x/3)**3*cosh(c + d*
x)/(320*d) - cosh(a - d*x/3)**3*cosh(c + d*x)**3/(10*d), Eq(b, -d/3)), (-3*
x*sinh(a + d*x/3)**3*sinh(c + d*x)**2*cosh(c + d*x)/32 + 3*x*sinh(a + d*x/3
)**3*cosh(c + d*x)**3/32 + 9*x*sinh(a + d*x/3)**2*sinh(c + d*x)**3*cosh(a +
 d*x/3)/32 - 9*x*sinh(a + d*x/3)**2*sinh(c + d*x)*cosh(a + d*x/3)*cosh(c +
d*x)**2/32 - 9*x*sinh(a + d*x/3)*sinh(c + d*x)**2*cosh(a + d*x/3)**2*cosh(c
 + d*x)/32 + 9*x*sinh(a + d*x/3)*cosh(a + d*x/3)**2*cosh(c + d*x)**3/32 + 3
*x*sinh(c + d*x)**3*cosh(a + d*x/3)**3/32 - 3*x*sinh(c + d*x)*cosh(a + d*x/
3)**3*cosh(c + d*x)**2/32 - 3*sinh(a + d*x/3)**3*sinh(c + d*x)**3/(5*d) + 3
03*sinh(a + d*x/3)**3*sinh(c + d*x)*cosh(c + d*x)**2/(320*d) - 39*sinh(a +
d*x/3)**2*cosh(a + d*x/3)*cosh(c + d*x)**3/(320*d) + 153*sinh(a + d*x/3)*si
nh(c + d*x)**3*cosh(a + d*x/3)**2/(320*d) - 9*sinh(a + d*x/3)*sinh(c + d*x)
*cosh(a + d*x/3)**2*cosh(c + d*x)**2/(20*d) - 81*sinh(c + d*x)**2*cosh(a +
d*x/3)**3*cosh(c + d*x)/(320*d) + cosh(a + d*x/3)**3*cosh(c + d*x)**3/(4*d)
, Eq(b, d/3)), (-3*x*sinh(a + d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/16 + 5
*x*sinh(a + d*x)**3*cosh(c + d*x)**3/16 + 3*x*sinh(a + d*x)**2*sinh(c + d*x
)**3*cosh(a + d*x)/16 - 9*x*sinh(a + d*x)**2*sinh(c + d*x)*cosh(a + d*x)*co
sh(c + d*x)**2/16 + 9*x*sinh(a + d*x)*sinh(c + d*x)**2*cosh(a + d*x)**2*cos
h(c + d*x)/16 - 3*x*sinh(a + d*x)*cosh(a + d*x)**2*cosh(c + d*x)**3/16 - 5*
x*sinh(c + d*x)**3*cosh(a + d*x)**3/16 + 3*x*sinh(c + d*x)*cosh(a + d*x)**3
*cosh(c + d*x)**2/16 - 19*sinh(a + d*x)**3*sinh(c + d*x)**3/(48*d) + 11*sin
h(a + d*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/(16*d) + sinh(a + d*x)*sinh(c
+ d*x)**3*cosh(a + d*x)**2/(2*d) - 3*sinh(a + d*x)*sinh(c + d*x)*cosh(a + d
*x)**2*cosh(c + d*x)**2/(4*d) - 3*sinh(c + d*x)**2*cosh(a + d*x)**3*cosh(c
+ d*x)/(16*d) + 5*cosh(a + d*x)**3*cosh(c + d*x)**3/(16*d), Eq(b, d)), (9*x
*sinh(a + 3*d*x)**3*sinh(c + d*x)**2*cosh(c + d*x)/32 + 3*x*sinh(a + 3*d*x)
**3*cosh(c + d*x)**3/32 - 3*x*sinh(a + 3*d*x)**2*sinh(c + d*x)**3*cosh(a +
3*d*x)/32 - 9*x*sinh(a + 3*d*x)**2*sinh(c + d*x)*cosh(a + 3*d*x)*cosh(c + d
*x)**2/32 - 9*x*sinh(a + 3*d*x)*sinh(c + d*x)**2*cosh(a + 3*d*x)**2*cosh(c
+ d*x)/32 - 3*x*sinh(a + 3*d*x)*cosh(a + 3*d*x)**2*cosh(c + d*x)**3/32 + 3*
x*sinh(c + d*x)**3*cosh(a + 3*d*x)**3/32 + 9*x*sinh(c + d*x)*cosh(a + 3*d*x
)**3*cosh(c + d*x)**2/32 + sinh(a + 3*d*x)**3*sinh(c + d*x)**3/(30*d) - 61*
sinh(a + 3*d*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/(320*d) + 117*sinh(a + 3*
d*x)**2*cosh(a + 3*d*x)*cosh(c + d*x)**3/(320*d) - 11*sinh(a + 3*d*x)*sinh(
c + d*x)**3*cosh(a + 3*d*x)**2/(320*d) + 3*sinh(a + 3*d*x)*sinh(c + d*x)*co
```

```
sh(a + 3*d*x)**2*cosh(c + d*x)**2/(20*d) + 3*sinh(c + d*x)**2*cosh(a + 3*d*
x)**3*cosh(c + d*x)/(320*d) - cosh(a + 3*d*x)**3*cosh(c + d*x)**3/(4*d), Eq
(b, 3*d)), (27*b**5*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)**3/(27*b**
6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 18*b**5*cosh(a + b*x)**3*cos
h(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 63*b**4
*d*sinh(a + b*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2
 + 273*b**2*d**4 - 27*d**6) + 54*b**4*d*sinh(a + b*x)*sinh(c + d*x)*cosh(a
+ b*x)**2*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d*
*6) + 126*b**3*d**2*sinh(a + b*x)**2*sinh(c + d*x)**2*cosh(a + b*x)*cosh(c
+ d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 210*b**3*d**2*
sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 +
273*b**2*d**4 - 27*d**6) - 120*b**3*d**2*sinh(c + d*x)**2*cosh(a + b*x)**3*
cosh(c + d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 122*b**
3*d**2*cosh(a + b*x)**3*cosh(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**
2*d**4 - 27*d**6) - 122*b**2*d**3*sinh(a + b*x)**3*sinh(c + d*x)**3/(27*b**
6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 210*b**2*d**3*sinh(a + b*x)*
*3*sinh(c + d*x)*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4
- 27*d**6) + 120*b**2*d**3*sinh(a + b*x)*sinh(c + d*x)**3*cosh(a + b*x)**2/
(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 126*b**2*d**3*sinh(a
+ b*x)*sinh(c + d*x)*cosh(a + b*x)**2*cosh(c + d*x)**2/(27*b**6 - 273*b**4*
d**2 + 273*b**2*d**4 - 27*d**6) - 54*b*d**4*sinh(a + b*x)**2*sinh(c + d*x)*
*2*cosh(a + b*x)*cosh(c + d*x)/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 2
7*d**6) + 63*b*d**4*sinh(a + b*x)**2*cosh(a + b*x)*cosh(c + d*x)**3/(27*b**
6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) + 18*d**5*sinh(a + b*x)**3*sin
h(c + d*x)**3/(27*b**6 - 273*b**4*d**2 + 273*b**2*d**4 - 27*d**6) - 27*d**5
*sinh(a + b*x)**3*sinh(c + d*x)*cosh(c + d*x)**2/(27*b**6 - 273*b**4*d**2 +
 273*b**2*d**4 - 27*d**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.

Time = 0.30 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,dx+3\,a+3\,c)}}{192\,(b+d)} + \frac{3\,e^{(3\,bx+dx+3\,a+c)}}{64\,(3\,b+d)} + \frac{3\,e^{(3\,bx-dx+3\,a-c)}}{64\,(3\,b-d)} + \frac{e^{(3\,bx-3\,dx+3\,a-3\,c)}}{192\,(b-d)} - \frac{3\,e^{(bx+3\,dx+a+3\,c)}}{64\,(b+3\,d)} - \frac{9\,e^{(bx+dx+a+c)}}{64\,(b+d)} - \frac{9\,e^{(bx-dx+a-c)}}{64\,(b-d)} - \frac{3\,e^{(bx-3\,dx+a-3\,c)}}{64\,(b-3\,d)} - \frac{3\,e^{(-bx+3\,dx-a+3\,c)}}{64\,(b-3\,d)} - \frac{9\,e^{(-bx+dx-a+c)}}{64\,(b-d)} - \frac{9\,e^{(-bx-dx-a-c)}}{64\,(b+d)} - \frac{3\,e^{(-bx-3\,dx-a-3\,c)}}{64\,(b+3\,d)} + \frac{e^{(-3\,bx+3\,dx-3\,a+3\,c)}}{192\,(b-d)} + \frac{3\,e^{(-3\,bx+dx-3\,a+c)}}{64\,(3\,b-d)} + \frac{3\,e^{(-3\,bx-dx-3\,a-c)}}{64\,(3\,b+d)} + \frac{e^{(-3\,bx-3\,dx-3\,a-3\,c)}}{192\,(b+d)}$$

```
[In] integrate(cosh(d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*e^(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/64*e^(3*b*x + d*x + 3*a + c
)/(3*b + d) + 3/64*e^(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/192*e^(3*b*x - 3
*d*x + 3*a - 3*c)/(b - d) - 3/64*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9/64
*e^(b*x + d*x + a + c)/(b + d) - 9/64*e^(b*x - d*x + a - c)/(b - d) - 3/64*
e^(b*x - 3*d*x + a - 3*c)/(b - 3*d) - 3/64*e^(-b*x + 3*d*x - a + 3*c)/(b -
3*d) - 9/64*e^(-b*x + d*x - a + c)/(b - d) - 9/64*e^(-b*x - d*x - a - c)/(b
 + d) - 3/64*e^(-b*x - 3*d*x - a - 3*c)/(b + 3*d) + 1/192*e^(-3*b*x + 3*d*x
 - 3*a + 3*c)/(b - d) + 3/64*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 3/64*e^
(-3*b*x - d*x - 3*a - c)/(3*b + d) + 1/192*e^(-3*b*x - 3*d*x - 3*a - 3*c)/(
b + d)
```

Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 908, normalized size of antiderivative = 4.66

$$\int \cosh^3(c+dx)\sinh^3(a+bx)\,dx = -e^{3a+c+3bx+dx}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{576b^4-640b^2d^2+64d^4}+\frac{e^{-6a-6bx}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{576b^4-640b^2d^2+64d^4}-\frac{e^{-2a-2bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4}-\frac{e^{-4a-4bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4}\right)-e^{3a-c+3bx-dx}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{576b^4-640b^2d^2+64d^4}+\frac{e^{-6a-6bx}\left(-9b^3+3b^2d+9bd^2-3d^3\right)}{576b^4-640b^2d^2+64d^4}-\frac{e^{-2a-2bx}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{576b^4-640b^2d^2+64d^4}-\frac{e^{-4a-4bx}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{576b^4-640b^2d^2+64d^4}\right)-e^{3a-3c+3bx-3dx}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{192b^4-1920b^2d^2+1728d^4}+\frac{e^{-6a-6bx}\left(-b^3+b^2d+9bd^2-9d^3\right)}{192b^4-1920b^2d^2+1728d^4}-\frac{e^{-2a-2bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4}-\frac{e^{-4a-4bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right)-e^{3a+3c+3bx+3dx}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{192b^4-1920b^2d^2+1728d^4}+\frac{e^{-6a-6bx}\left(-b^3-b^2d+9bd^2+9d^3\right)}{192b^4-1920b^2d^2+1728d^4}-\frac{e^{-2a-2bx}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{192b^4-1920b^2d^2+1728d^4}-\frac{e^{-4a-4bx}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{192b^4-1920b^2d^2+1728d^4}\right)$$

```
[In] int(cosh(c + d*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] - exp(3*a + c + 3*b*x + d*x)*((9*b*d^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(576*b^4
+ 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 - 3*b^2*d - 9*b^3 +
3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^2 + 8
1*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4*a -
```

```
4*b*x)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d
^2)) - exp(3*a - c + 3*b*x - d*x)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(576
*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + 3*b^2*d - 9*b
^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 2*a - 2*b*x)*(9*b*d^
2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- 4
*a - 4*b*x)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4 - 640*
b^2*d^2)) - exp(3*a - 3*c + 3*b*x - 3*d*x)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)
/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*b*d^2 + b^2*d
 - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- 2*a - 2*b*x)*
(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)
- (exp(- 4*a - 4*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 172
8*d^4 - 1920*b^2*d^2)) - exp(3*a + 3*c + 3*b*x + 3*d*x)*((9*b*d^2 + b^2*d -
 b^3 - 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp(- 6*a - 6*b*x)*(9*
b*d^2 - b^2*d - b^3 + 9*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(-
2*a - 2*b*x)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1
920*b^2*d^2) - (exp(- 4*a - 4*b*x)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2))
```

3.188 $\int \sinh(a+bx)\tanh(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 121

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2(c+dx)}\right)}{b}$$

```
[Out] 1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b-exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-1
/2*b/d],-exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],-e
xp(2*d*x+2*c))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5720, 2225, 2283}

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = -\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2(c+dx)}\right)}{b} + \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -1/2*b/d, 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5720

```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-E^
(-(a + b*x))/2 + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}+\frac{e^{-a-bx}}{1+e^{2(c+dx)}}-\frac{e^{a+bx}}{1+e^{2(c+dx)}}\right)\,dx \\
&= -\left(\frac{1}{2}\int e^{-a-bx}\,dx\right)+\frac{1}{2}\int e^{a+bx}\,dx+\int \frac{e^{-a-bx}}{1+e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1+e^{2(c+dx)}}\,dx \\
&= \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}-\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b} \\
&\quad -\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2(c+dx)}\right)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$
\int \sinh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}\left(1+e^{2(a+bx)}-2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)-2e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(1\right.\right.}{2b}
$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] (E^(-a - b*x)*(1 + E^(2*(a + b*x)) - 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b
/(2*d), -E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), -E^(2*(c + d*x))]))/(2*b)
```

Maple [F]

$$\int \sinh(bx+a)\tanh(dx+c)\,dx$$

```
[In] int(sinh(b*x+a)*tanh(d*x+c),x)
[Out] int(sinh(b*x+a)*tanh(d*x+c),x)
```

Fricas [F]

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh(bx+a)\tanh(dx+c)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
[Out] integral(sinh(b*x + a)*tanh(d*x + c), x)
```

Sympy [F]

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh(a+bx)\tanh(c+dx)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x)
[Out] Integral(sinh(a + b*x)*tanh(c + d*x), x)
```

Maxima [F]

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh(bx+a)\tanh(dx+c)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
[Out] 1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a)
 - 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

Giac [F]

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

```
[In] integrate(sinh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)*tanh(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(a+bx)\tanh(c+dx)\,dx = \int \sinh(a+b\,x)\,\tanh(c+d\,x)\,dx$$

```
[In] int(sinh(a + b*x)*tanh(c + d*x),x)
[Out] int(sinh(a + b*x)*tanh(c + d*x), x)
```

3.189 $\int \coth(c+dx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 117

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b} - \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}$$

```
[Out] 1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b-exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-1
/2*b/d],exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],exp
(2*d*x+2*c))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5722, 2225, 2283}

$$\int \coth(c+dx)\sinh(a+bx)\,dx = -\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2(c+dx)}\right)}{b} + \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[In] Int[Coth[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(2*b) + E^(a + b*x)/(2*b) - (E^(-a - b*x)*Hypergeometric2F1[1,
 -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2F
1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5722

```
Int[Coth[(c_.) + (d_.)*(x_)]*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-E^
(-(a + b*x))/2 + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}+\frac{e^{-a-bx}}{1-e^{2(c+dx)}}-\frac{e^{a+bx}}{1-e^{2(c+dx)}}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-a-bx}\,dx\right)+\frac{1}{2}\int e^{a+bx}\,dx+\int \frac{e^{-a-bx}}{1-e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1-e^{2(c+dx)}}\,dx\\
&= \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}-\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b}\\
&\quad-\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 240 vs. $2(117) = 234$.

Time = 2.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.05

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \frac{\cosh(a)\cosh(bx)\coth(c)}{b} + \frac{e^{-a+2c-bx}\left(be^{2dx}\,\text{Hypergeometric2F1}\left(1,1-\frac{b}{2d},2-\frac{b}{2d},e^{2(c+dx)}\right)-(b-2d)\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1\right.\right.}{b(b-2d)\left(-1+e^{2c}\right)} - \frac{e^{a+2c}\left(-\frac{e^{(b+2d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{b}{2d},2+\frac{b}{2d},e^{2(c+dx)}\right)}{b+2d}+\frac{e^{bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}\right)}{-1+e^{2c}} + \frac{\coth(c)\sinh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Coth[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (Cosh[a]*Cosh[b*x]*Coth[c])/b + (E^(-a + 2*c - b*x)*(b*E^(2*d*x)*Hypergeome
tric2F1[1, 1 - b/(2*d), 2 - b/(2*d), E^(2*(c + d*x))] - (b - 2*d)*Hypergeom
etric2F1[1, -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))]))/(b*(b - 2*d)*(-1 + E^
(2*c))) - (E^(a + 2*c)*(-((E^((b + 2*d)*x)*Hypergeometric2F1[1, 1 + b/(2*d)
, 2 + b/(2*d), E^(2*(c + d*x))])/(b + 2*d)) + (E^(b*x)*Hypergeometric2F1[1,
 b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b))/(-1 + E^(2*c)) + (Coth[c]*Sinh
[a]*Sinh[b*x])/b
```

Maple [F]

$$\int \coth(dx+c)\sinh(bx+a)\,dx$$

```
[In] int(coth(d*x+c)*sinh(b*x+a),x)
```

```
[Out] int(coth(d*x+c)*sinh(b*x+a),x)
```

Fricas [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth(dx+c)\sinh(bx+a)\,dx$$

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)*sinh(b*x + a), x)
```

Sympy [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\coth(c+dx)\,dx$$

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*coth(c + d*x), x)
```

Maxima [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth(dx+c)\sinh(bx+a)\,dx$$

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a) -
 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x + 2
*a) - 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

Giac [F]

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth(dx+c)\sinh(bx+a)\,dx$$

```
[In] integrate(coth(d*x+c)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(coth(d*x + c)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(c+dx)\sinh(a+bx)\,dx = \int \coth(c+d\,x)\,\sinh(a+b\,x)\,dx$$

```
[In] int(coth(c + d*x)*sinh(a + b*x),x)
[Out] int(coth(c + d*x)*sinh(a + b*x), x)
```

3.190 $\int \cosh(a+bx)\coth(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \cosh(a+bx)\coth(c+dx)\,dx = -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b} + \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}$$

```
[Out] -1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],ex
p(2*d*x+2*c))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5721, 2225, 2283}

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b} - \frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,e^{2(c+dx)}\right)}{b} - \frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[In] Int[Cosh[a + b*x]*Coth[c + d*x],x]
```

```
[Out] -1/2*E^(-a - b*x)/b + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -1/2*b/d, 1 - b/(2*d), E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric2
F1[1, b/(2*d), 1 + b/(2*d), E^(2*(c + d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5721

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}-\frac{e^{-a-bx}}{1-e^{2(c+dx)}}-\frac{e^{a+bx}}{1-e^{2(c+dx)}}\right)dx\\
&= \frac{1}{2}\int e^{-a-bx}\,dx+\frac{1}{2}\int e^{a+bx}\,dx-\int \frac{e^{-a-bx}}{1-e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1-e^{2(c+dx)}}\,dx\\
&= -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)}{b}\\
&\quad -\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},e^{2(c+dx)}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \frac{e^{-a-bx}\left(-1+e^{2(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},e^{2(c+dx)}\right)-2e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(1\right.\right.}{2b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[c + d*x],x]
```

```
[Out] (E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d),
 1 + b/(2*d), E^(2*(c + d*x))]))/(2*b)
```

Maple [F]

$$\int \cosh (bx + a) \coth (dx + c) \, dx$$

```
[In] int(cosh(b*x+a)*coth(d*x+c),x)
[Out] int(cosh(b*x+a)*coth(d*x+c),x)
```

Fricas [F]

$$\int \cosh(a + bx) \coth(c + dx) \, dx = \int \cosh (bx + a) \coth (dx + c) \, dx$$

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*coth(d*x + c), x)
```

Sympy [F]

$$\int \cosh(a + bx) \coth(c + dx) \, dx = \int \cosh (a + bx) \coth (c + dx) \, dx$$

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x)
[Out] Integral(cosh(a + b*x)*coth(c + d*x), x)
```

Maxima [F]

$$\int \cosh(a + bx) \coth(c + dx) \, dx = \int \cosh (bx + a) \coth (dx + c) \, dx$$

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="maxima")
[Out] 1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate((e^(2*b*x + 2*a) +
 1)/(e^(b*x + d*x + a + c) + e^(b*x + a)), x) + 1/2*integrate((e^(2*b*x + 2
*a) + 1)/(e^(b*x + d*x + a + c) - e^(b*x + a)), x)
```

Giac [F]

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh\left(bx+a\right)\coth\left(dx+c\right)\,dx$$

```
[In] integrate(cosh(b*x+a)*coth(d*x+c),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*coth(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\coth(c+dx)\,dx = \int \cosh(a+b\,x)\,\coth(c+d\,x)\,dx$$

```
[In] int(cosh(a + b*x)*coth(c + d*x),x)
[Out] int(cosh(a + b*x)*coth(c + d*x), x)
```

3.191 $\int \cosh(a+bx)\tanh(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 120

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2(c+dx)}\right)}{b}$$

```
[Out] -1/2*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+exp(-b*x-a)*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],-exp(2*d*x+2*c))/b-exp(b*x+a)*hypergeom([1, 1/2*b/d],[1+1/2*b/d],-
exp(2*d*x+2*c))/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5723, 2225, 2283}

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b}-\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},\frac{b}{2d}+1,-e^{2(c+dx)}\right)}{b}-\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}$$

```
[In] Int[Cosh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] -1/2*E^(-a - b*x)/b + E^(a + b*x)/(2*b) + (E^(-a - b*x)*Hypergeometric2F1[1
, -1/2*b/d, 1 - b/(2*d), -E^(2*(c + d*x))])/b - (E^(a + b*x)*Hypergeometric
2F1[1, b/(2*d), 1 + b/(2*d), -E^(2*(c + d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5723

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-a-bx}+\frac{1}{2}e^{a+bx}-\frac{e^{-a-bx}}{1+e^{2(c+dx)}}-\frac{e^{a+bx}}{1+e^{2(c+dx)}}\right)dx\\
&= \frac{1}{2}\int e^{-a-bx}\,dx+\frac{1}{2}\int e^{a+bx}\,dx-\int \frac{e^{-a-bx}}{1+e^{2(c+dx)}}\,dx-\int \frac{e^{a+bx}}{1+e^{2(c+dx)}}\,dx\\
&= -\frac{e^{-a-bx}}{2b}+\frac{e^{a+bx}}{2b}+\frac{e^{-a-bx}\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)}{b}\\
&\quad -\frac{e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b}{2d},1+\frac{b}{2d},-e^{2(c+dx)}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \frac{e^{-a-bx}\left(-1+e^{2(a+bx)}+2\,\text{Hypergeometric2F1}\left(1,-\frac{b}{2d},1-\frac{b}{2d},-e^{2(c+dx)}\right)-2e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(\right.\right.}{2b}$$

```
[In] Integrate[Cosh[a + b*x]*Tanh[c + d*x],x]
```

```
[Out] (E^(-a - b*x)*(-1 + E^(2*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), -E^(2*(c + d*x))] - 2*E^(2*(a + b*x))*Hypergeometric2F1[1, b/(2*d)
, 1 + b/(2*d), -E^(2*(c + d*x))]))/(2*b)
```

Maple [F]

$$\int \cosh(bx+a)\tanh(dx+c)\,dx$$

```
[In] int(cosh(b*x+a)*tanh(d*x+c),x)
[Out] int(cosh(b*x+a)*tanh(d*x+c),x)
```

Fricas [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx=\int \cosh(bx+a)\tanh(dx+c)\,dx$$

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*tanh(d*x + c), x)
```

Sympy [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx=\int \cosh(a+bx)\tanh(c+dx)\,dx$$

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x)
[Out] Integral(cosh(a + b*x)*tanh(c + d*x), x)
```

Maxima [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx=\int \cosh(bx+a)\tanh(dx+c)\,dx$$

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="maxima")
[Out] 1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)/b - 1/2*integrate(2*(e^(2*b*x + 2*a)
 + 1)/(e^(b*x + 2*d*x + a + 2*c) + e^(b*x + a)), x)
```

Giac [F]

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh\left(bx+a\right)\tanh\left(dx+c\right)\,dx$$

```
[In] integrate(cosh(b*x+a)*tanh(d*x+c),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*tanh(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(a+bx)\tanh(c+dx)\,dx = \int \cosh(a+b\,x)\,\tanh(c+d\,x)\,dx$$

```
[In] int(cosh(a + b*x)*tanh(c + d*x),x)
[Out] int(cosh(a + b*x)*tanh(c + d*x), x)
```

3.192 $\int \sinh(x)\sinh(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sinh(x)\sinh(2x)\,dx = \frac{2\sinh^3(x)}{3}$$

```
[Out] 2/3*sinh(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4367}

$$\int \sinh(x)\sinh(2x)\,dx = \frac{1}{6}\sinh(3x) - \frac{\sinh(x)}{2}$$

```
[In] Int[Sinh[x]*Sinh[2*x],x]
```

```
[Out] -1/2*Sinh[x] + Sinh[3*x]/6
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \sinh(x)\sinh(2x)\,dx = -\frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

```
[In] Integrate[Sinh[x]*Sinh[2*x],x]
[Out] -1/2*Sinh[x] + Sinh[3*x]/6
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
parallelrisch	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
risch	$\frac{e^{3x}}{12}-\frac{e^{x}}{4}+\frac{e^{-x}}{4}-\frac{e^{-3x}}{12}$	24

```
[In] int(sinh(x)*sinh(2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*sinh(x)+1/6*sinh(3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \sinh(x)\sinh(2x)\,dx = \frac{1}{6}\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^2 - 1\right)\sinh(x)$$

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="fricas")
[Out] 1/6*sinh(x)^3 + 1/2*(cosh(x)^2 - 1)*sinh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \sinh(x)\sinh(2x)\,dx = \frac{2\sinh\left(x\right)\cosh\left(2x\right)}{3} - \frac{\sinh\left(2x\right)\cosh\left(x\right)}{3}$$

```
[In] integrate(sinh(x)*sinh(2*x),x)
[Out] 2*sinh(x)*cosh(2*x)/3 - sinh(2*x)*cosh(x)/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \sinh(x)\sinh(2x)\,dx = -\frac{1}{12}\left(3\,e^{(-2\,x)} - 1\right)e^{(3\,x)} + \frac{1}{4}\,e^{(-x)} - \frac{1}{12}\,e^{(-3\,x)}$$

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="maxima")
[Out] -1/12*(3*e^(-2*x) - 1)*e^(3*x) + 1/4*e^(-x) - 1/12*e^(-3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \sinh(x)\sinh(2x)\,dx = \frac{1}{12}\left(3\,e^{(2\,x)} - 1\right)e^{(-3\,x)} + \frac{1}{12}\,e^{(3\,x)} - \frac{1}{4}\,e^{x}$$

```
[In] integrate(sinh(x)*sinh(2*x),x, algorithm="giac")
[Out] 1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sinh(x)\sinh(2x)\,dx = \frac{2\sinh(x)^3}{3}$$

```
[In] int(sinh(2*x)*sinh(x),x)
[Out] (2*sinh(x)^3)/3
```

3.193 $\int \sinh(x)\sinh(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

```
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4367}

$$\int \sinh(x)\sinh(3x)\,dx = \frac{1}{8}\sinh(4x) - \frac{1}{4}\sinh(2x)$$

```
[In] Int[Sinh[x]*Sinh[3*x],x]
```

```
[Out] -1/4*Sinh[2*x] + Sinh[4*x]/8
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{4}\sinh(2x)+\frac{1}{8}\sinh(4x)$$

```
[In] Integrate[Sinh[x]*Sinh[3*x],x]
[Out] -1/4*Sinh[2*x] + Sinh[4*x]/8
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$	14
risch	$\frac{{\rm e}^{4x}}{16}-\frac{{\rm e}^{2x}}{8}+\frac{{\rm e}^{-2x}}{8}-\frac{{\rm e}^{-4x}}{16}$	26
parallelrisch	$\frac{\sinh(5x)-2\sinh(x)-\sinh(3x)+4\sinh(2x)-2\sinh(4x)}{16\cosh(x)-16}$	37

```
[In] int(sinh(x)*sinh(3*x),x,method=_RETURNVERBOSE)
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \sinh(x)\sinh(3x)\,dx = \frac{1}{2}\cosh(x)\sinh(x)^3+\frac{1}{2}\left(\cosh(x)^3-\cosh(x)\right)\sinh(x)$$

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="fricas")
[Out] 1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 - cosh(x))*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sinh(x)\sinh(3x)\,dx = \frac{3\sinh{(x)}\cosh{(3x)}}{8} - \frac{\sinh{(3x)}\cosh{(x)}}{8}$$

```
[In] integrate(sinh(x)*sinh(3*x),x)
[Out] 3*sinh(x)*cosh(3*x)/8 - sinh(3*x)*cosh(x)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(3x)\,dx = -\frac{1}{16}\left(2e^{(-2x)} - 1\right)e^{(4x)} + \frac{1}{8}e^{(-2x)} - \frac{1}{16}e^{(-4x)}$$

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="maxima")
[Out] -1/16*(2*e^(-2*x) - 1)*e^(4*x) + 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(3x)\,dx = \frac{1}{16}\left(2e^{(2x)} - 1\right)e^{(-4x)} + \frac{1}{16}e^{(4x)} - \frac{1}{8}e^{(2x)}$$

```
[In] integrate(sinh(x)*sinh(3*x),x, algorithm="giac")
[Out] 1/16*(2*e^(2*x) - 1)*e^(-4*x) + 1/16*e^(4*x) - 1/8*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sinh(x)\sinh(3x)\,dx = \frac{\sinh(4x)}{8} - \frac{\sinh(2x)}{4}$$

```
[In] int(sinh(3*x)*sinh(x),x)
[Out] sinh(4*x)/8 - sinh(2*x)/4
```

3.194 $\int \sinh(x)\sinh(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

```
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4367\}$

$$\int \sinh(x)\sinh(4x)\,dx = \frac{1}{10}\sinh(5x) - \frac{1}{6}\sinh(3x)$$

```
[In] Int[Sinh[x]*Sinh[4*x],x]
[Out] -1/6*Sinh[3*x] + Sinh[5*x]/10
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

```
[In] Integrate[Sinh[x]*Sinh[4*x],x]
[Out] -1/6*Sinh[3*x] + Sinh[5*x]/10
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
parallelrisch	$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
risch	$\frac{e^{5x}}{20}-\frac{e^{3x}}{12}+\frac{e^{-3x}}{12}-\frac{e^{-5x}}{20}$	26

```
[In] int(sinh(x)*sinh(4*x),x,method=_RETURNVERBOSE)
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \sinh(x)\sinh(4x)\,dx = \frac{1}{10}\sinh(x)^5 + \frac{1}{6}\left(6\cosh(x)^2 - 1\right)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^4 - \cosh(x)^2\right)\sinh(x)$$

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="fricas")
[Out] 1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 - 1)*sinh(x)^3 + 1/2*(cosh(x)^4 - cosh(x)
^2)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sinh(x)\sinh(4x)\,dx = \frac{4\sinh{(x)}\cosh{(4x)}}{15} - \frac{\sinh{(4x)}\cosh{(x)}}{15}$$

```
[In] integrate(sinh(x)*sinh(4*x),x)
[Out] 4*sinh(x)*cosh(4*x)/15 - sinh(4*x)*cosh(x)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(4x)\,dx = -\frac{1}{60}\left(5e^{(-2x)} - 3\right)e^{(5x)} + \frac{1}{12}e^{(-3x)} - \frac{1}{20}e^{(-5x)}$$

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="maxima")
[Out] -1/60*(5*e^(-2*x) - 3)*e^(5*x) + 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sinh(x)\sinh(4x)\,dx = \frac{1}{60}\left(5e^{(2x)} - 3\right)e^{(-5x)} + \frac{1}{20}e^{(5x)} - \frac{1}{12}e^{(3x)}$$

```
[In] integrate(sinh(x)*sinh(4*x),x, algorithm="giac")
[Out] 1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sinh(x)\sinh(4x)\,dx = \frac{4\sinh(x)^3\left(6\sinh(x)^2 + 5\right)}{15}$$

```
[In] int(sinh(4*x)*sinh(x),x)
[Out] (4*sinh(x)^3*(6*sinh(x)^2 + 5))/15
```

3.195 $\int \sinh(x) \sinh(mx) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \sinh(x) \sinh(mx) \, dx = -\frac{\sinh((1-m)x)}{2(1-m)} + \frac{\sinh((1+m)x)}{2(1+m)}$$

```
[Out] -1/2*sinh((1-m)*x)/(1-m)+1/2*sinh((1+m)*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5732, 2717}

$$\int \sinh(x) \sinh(mx) \, dx = \frac{\sinh((m+1)x)}{2(m+1)} - \frac{\sinh((1-m)x)}{2(1-m)}$$

```
[In] Int[Sinh[x]*Sinh[m*x],x]
[Out] -1/2*Sinh[(1 - m)*x]/(1 - m) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5732

```
Int[Sinh[v_]^(p_.)*Sinh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Sinh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((1+m)x)\right)\,dx \\
&= -\left(\frac{1}{2}\int \cosh((1-m)x)\,dx\right)+\frac{1}{2}\int \cosh((1+m)x)\,dx \\
&= -\frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((1+m)x)}{2(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sinh(x)\sinh(mx)\,dx=\frac{m\cosh(mx)\sinh(x)-\cosh(x)\sinh(mx)}{-1+m^2}$$

```
[In] Integrate[Sinh[x]*Sinh[m*x],x]
[Out] (m*Cosh[m*x]*Sinh[x] - Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\sinh(x(-1+m))}{2(-1+m)}+\frac{\sinh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(-1-m)\sinh(x(-1+m))+\sinh((1+m)x)(-1+m)}{2m^2-2}$	34
risch	$\frac{(m\,e^{2x}-e^{2x}-m-1)\,e^{x(-1+m)}}{4(1+m)(-1+m)}+\frac{(m\,e^{2x}+e^{2x}-m+1)\,e^{-(1+m)x}}{4(1+m)(-1+m)}$	71

```
[In] int(sinh(x)*sinh(m*x),x,method=_RETURNVERBOSE)
[Out] -1/2/(-1+m)*sinh(x*(-1+m))+1/2*sinh((1+m)*x)/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \sinh(x)\sinh(mx)\,dx = \frac{m\cosh(mx)\sinh(x) - \cosh(x)\sinh(mx)}{(m^2-1)\cosh(x)^2 - (m^2-1)\sinh(x)^2}$$

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(m*x)*sinh(x) - cosh(x)*sinh(m*x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(22) = 44.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \sinh(x)\sinh(mx)\,dx = \begin{cases} -\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} - \frac{\sinh(x)\cosh(x)}{2} & \text{for } m = -1 \\ \frac{x\sinh^2(x)}{2} - \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} & \text{for } m = 1 \\ \frac{m\sinh(x)\cosh(mx)}{m^2-1} - \frac{\sinh(mx)\cosh(x)}{m^2-1} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(x)*sinh(m*x),x)
```

```
[Out] Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 - sinh(x)*cosh(x)/2, Eq(m, -1))
, (x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, 1)), (m*sinh(
x)*cosh(m*x)/(m**2 - 1) - sinh(m*x)*cosh(x)/(m**2 - 1), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sinh(x)\sinh(mx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \sinh(x)\sinh(mx)\,dx = \frac{e^{(mx+x)}}{4\,(m+1)} - \frac{e^{(mx-x)}}{4\,(m-1)} + \frac{e^{(-mx+x)}}{4\,(m-1)} - \frac{e^{(-mx-x)}}{4\,(m+1)}$$

```
[In] integrate(sinh(x)*sinh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m - 1
) - 1/4*e^(-m*x - x)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sinh(x)\sinh(mx)\,dx = -\frac{\sinh(m\,x)\,\cosh(x) - m\cosh(m\,x)\,\sinh(x)}{m^2-1}$$

```
[In] int(sinh(m*x)*sinh(x),x)
```

```
[Out] -(sinh(m*x)*cosh(x) - m*cosh(m*x)*sinh(x))/(m^2 - 1)
```

3.196 $\int \cosh(2x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

```
[Out] -1/2*cosh(x)+1/6*cosh(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4369}

$$\int \cosh(2x)\sinh(x)\,dx = \frac{1}{6}\cosh(3x) - \frac{\cosh(x)}{2}$$

```
[In] Int[Cosh[2*x]*Sinh[x],x]
```

```
[Out] -1/2*Cosh[x] + Cosh[3*x]/6
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

```
[In] Integrate[Cosh[2*x]*Sinh[x],x]
[Out] -1/2*Cosh[x] + Cosh[3*x]/6
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	12
parallelrisch	$\frac{1}{3}-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	13
risch	$\frac{e^{3x}}{12}-\frac{e^{x}}{4}-\frac{e^{-x}}{4}+\frac{e^{-3x}}{12}$	24

```
[In] int(sinh(x)*cosh(2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*cosh(x)+1/6*cosh(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(2x)\sinh(x)\,dx = \frac{1}{6}\cosh(x)^3 + \frac{1}{2}\cosh(x)\sinh(x)^2 - \frac{1}{2}\cosh(x)$$

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="fricas")
[Out] 1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 - 1/2*cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(2x)\sinh(x)\,dx = \frac{2\sinh\left(x\right)\sinh\left(2x\right)}{3} - \frac{\cosh\left(x\right)\cosh\left(2x\right)}{3}$$

```
[In] integrate(cosh(2*x)*sinh(x),x)
[Out] 2*sinh(x)*sinh(2*x)/3 - cosh(x)*cosh(2*x)/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{1}{12}\left(3e^{(-2x)} - 1\right)e^{(3x)} - \frac{1}{4}e^{(-x)} + \frac{1}{12}e^{(-3x)}$$

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="maxima")
[Out] -1/12*(3*e^(-2*x) - 1)*e^(3*x) - 1/4*e^(-x) + 1/12*e^(-3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cosh(2x)\sinh(x)\,dx = -\frac{1}{12}\left(3e^{(2x)} - 1\right)e^{(-3x)} + \frac{1}{12}e^{(3x)} - \frac{1}{4}e^{x}$$

```
[In] integrate(cosh(2*x)*sinh(x),x, algorithm="giac")
[Out] -1/12*(3*e^(2*x) - 1)*e^(-3*x) + 1/12*e^(3*x) - 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cosh(2x)\sinh(x)\,dx = \frac{2\cosh(x)^3}{3} - \cosh(x)$$

```
[In] int(cosh(2*x)*sinh(x),x)
[Out] (2*cosh(x)^3)/3 - cosh(x)
```

3.197 $\int \cosh(3x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(3x)\sinh(x)\,dx = -\frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

```
[Out] -1/4*cosh(2*x)+1/8*cosh(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4369}

$$\int \cosh(3x)\sinh(x)\,dx = \frac{1}{8}\cosh(4x) - \frac{1}{4}\cosh(2x)$$

```
[In] Int[Cosh[3*x]*Sinh[x],x]
```

```
[Out] -1/4*Cosh[2*x] + Cosh[4*x]/8
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(3x)\sinh(x)\,dx = -\frac{1}{2}\cosh^2(x) + \frac{1}{8}\cosh(4x)$$

```
[In] Integrate[Cosh[3*x]*Sinh[x],x]
[Out] -1/2*Cosh[x]^2 + Cosh[4*x]/8
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}-\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}+\frac{e^{-4x}}{16}$	26
parallelrisch	$\frac{\cosh(5x)-\cosh(3x)+4\cosh(2x)-2-2\cosh(4x)}{16\cosh(x)-16}$	34

```
[In] int(sinh(x)*cosh(3*x),x,method=_RETURNVERBOSE)
[Out] -1/4*cosh(2*x)+1/8*cosh(4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cosh(3x)\sinh(x)\,dx = \frac{1}{8}\cosh(x)^4 + \frac{1}{8}\sinh(x)^4 + \frac{1}{4}\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - \frac{1}{4}\cosh(x)^2$$

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="fricas")
[Out] 1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 - 1)*sinh(x)^2 - 1/4*cosh(
x)^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(3x)\sinh(x)\,dx = \frac{3\sinh{(x)}\sinh{(3x)}}{8} - \frac{\cosh{(x)}\cosh{(3x)}}{8}$$

```
[In] integrate(cosh(3*x)*sinh(x),x)
[Out] 3*sinh(x)*sinh(3*x)/8 - cosh(x)*cosh(3*x)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(3x)\sinh(x)\,dx = -\frac{1}{16}\left(2\,e^{(-2\,x)} - 1\right)e^{(4\,x)} - \frac{1}{8}\,e^{(-2\,x)} + \frac{1}{16}\,e^{(-4\,x)}$$

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="maxima")
[Out] -1/16*(2*e^(-2*x) - 1)*e^(4*x) - 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \cosh(3x)\sinh(x)\,dx = \frac{1}{16}\left(e^{(2\,x)} + e^{(-2\,x)}\right)^2 - \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(cosh(3*x)*sinh(x),x, algorithm="giac")
[Out] 1/16*(e^(2*x) + e^(-2*x))^2 - 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \cosh(3x)\sinh(x)\,dx = \sinh(x)^4 + \frac{\sinh(x)^2}{2}$$

```
[In] int(cosh(3*x)*sinh(x),x)
[Out] sinh(x)^2/2 + sinh(x)^4
```

3.198 $\int \cosh(4x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

```
[Out] -1/6*cosh(3*x)+1/10*cosh(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4369}

$$\int \cosh(4x)\sinh(x)\,dx = \frac{1}{10}\cosh(5x) - \frac{1}{6}\cosh(3x)$$

```
[In] Int[Cosh[4*x]*Sinh[x],x]
[Out] -1/6*Cosh[3*x] + Cosh[5*x]/10
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

```
[In] Integrate[Cosh[4*x]*Sinh[x],x]
[Out] -1/6*Cosh[3*x] + Cosh[5*x]/10
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	14
parallelrisch	$\frac{1}{15}-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	15
risch	$\frac{e^{5x}}{20}-\frac{e^{3x}}{12}-\frac{e^{-3x}}{12}+\frac{e^{-5x}}{20}$	26

```
[In] int(cosh(4*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] -1/6*cosh(3*x)+1/10*cosh(5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \cosh(4x)\sinh(x)\,dx = \frac{1}{10}\cosh(x)^5 + \frac{1}{2}\cosh(x)\sinh(x)^4 - \frac{1}{6}\cosh(x)^3 + \frac{1}{2}\left(2\cosh(x)^3 - \cosh(x)\right)\sinh(x)^2$$

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="fricas")
[Out] 1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 - 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3 -
 cosh(x))*sinh(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(4x)\sinh(x)\,dx = \frac{4\sinh{(x)}\sinh{(4x)}}{15} - \frac{\cosh{(x)}\cosh{(4x)}}{15}$$

```
[In] integrate(cosh(4*x)*sinh(x),x)
[Out] 4*sinh(x)*sinh(4*x)/15 - cosh(x)*cosh(4*x)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{60}\left(5\,e^{(-2\,x)} - 3\right)e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{20}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="maxima")
[Out] -1/60*(5*e^(-2*x) - 3)*e^(5*x) - 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(4x)\sinh(x)\,dx = -\frac{1}{60}\left(5\,e^{(2\,x)} - 3\right)e^{(-5\,x)} + \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(3\,x)}$$

```
[In] integrate(cosh(4*x)*sinh(x),x, algorithm="giac")
[Out] -1/60*(5*e^(2*x) - 3)*e^(-5*x) + 1/20*e^(5*x) - 1/12*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cosh(4x)\sinh(x)\,dx = \frac{8\cosh(x)^5}{5} - \frac{8\cosh(x)^3}{3} + \cosh(x)$$

```
[In] int(cosh(4*x)*sinh(x),x)
[Out] cosh(x) - (8*cosh(x)^3)/3 + (8*cosh(x)^5)/5
```

3.199 $\int \cosh(mx)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cosh(mx)\sinh(x)\,dx = \frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((1+m)x)}{2(1+m)}$$

```
[Out] 1/2*cosh((1-m)*x)/(1-m)+1/2*cosh((1+m)*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5737, 2718}

$$\int \cosh(mx)\sinh(x)\,dx = \frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((m+1)x)}{2(m+1)}$$

```
[In] Int[Cosh[m*x]*Sinh[x],x]
```

```
[Out] Cosh[(1 - m)*x]/(2*(1 - m)) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}\sinh((1-m)x)+\frac{1}{2}\sinh((1+m)x)\right)\,dx\\ &= \frac{1}{2}\int \sinh((1-m)x)\,dx+\frac{1}{2}\int \sinh((1+m)x)\,dx\\ &= \frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cosh(mx)\sinh(x)\,dx = \frac{-\cosh(x)\cosh(mx)+m\sinh(x)\sinh(mx)}{-1+m^2}$$

```
[In] Integrate[Cosh[m*x]*Sinh[x],x]
[Out] (-(Cosh[x]*Cosh[m*x]) + m*Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cosh(x(-1+m))}{2(-1+m)}+\frac{\cosh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(-1-m)\cosh(x(-1+m))+2+\cosh((1+m)x)(-1+m)}{2m^2-2}$	35
risch	$\frac{(m\,e^{2x}-e^{2x}-m-1)e^{x(-1+m)}}{4(1+m)(-1+m)}-\frac{(m\,e^{2x}+e^{2x}-m+1)e^{-(1+m)x}}{4(1+m)(-1+m)}$	71

```
[In] int(cosh(m*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] -1/2/(-1+m)*cosh(x*(-1+m))+1/2*cosh((1+m)*x)/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(mx)\sinh(x)\,dx = \frac{m\sinh(mx)\sinh(x) - \cosh(mx)\cosh(x)}{(m^2-1)\cosh(x)^2 - (m^2-1)\sinh(x)^2}$$

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] (m*sinh(m*x)*sinh(x) - cosh(m*x)*cosh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \cosh(mx)\sinh(x)\,dx = \begin{cases} \frac{\cosh^2(x)}{2} & \text{for } m=-1 \vee m=1 \\ \frac{m\sinh(x)\sinh(mx)}{m^2-1} - \frac{\cosh(x)\cosh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(m*x)*sinh(x),x)
```

```
[Out] Piecewise((cosh(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sinh(x)*sinh(m*x)/(m**2
- 1) - cosh(x)*cosh(m*x)/(m**2 - 1), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(mx)\sinh(x)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cosh(mx)\sinh(x)\,dx = \frac{e^{(mx+x)}}{4\,(m+1)} - \frac{e^{(mx-x)}}{4\,(m-1)} - \frac{e^{(-mx+x)}}{4\,(m-1)} + \frac{e^{(-mx-x)}}{4\,(m+1)}$$

```
[In] integrate(cosh(m*x)*sinh(x),x, algorithm="giac")
[Out] 1/4*e^(m*x + x)/(m + 1) - 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m - 1
) + 1/4*e^(-m*x - x)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cosh(mx)\sinh(x)\,dx = -\frac{\cosh(m\,x)\,\cosh(x) - m\sinh(m\,x)\,\sinh(x)}{m^2-1}$$

```
[In] int(cosh(m*x)*sinh(x),x)
[Out] -(cosh(m*x)*cosh(x) - m*sinh(m*x)*sinh(x))/(m^2 - 1)
```

3.200 $\int \sinh(x) \tanh(2x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \sinh(x) \tanh(2x) \, dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)$$

```
[Out] sinh(x)-1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 327, 209}

$$\int \sinh(x) \tanh(2x) \, dx = \sinh(x) - \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[In] Int[Sinh[x]*Tanh[2*x],x]
[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int -\frac{2x^2}{1+2x^2}\,dx, x, \sinh(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x^2}{1+2x^2}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(2x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} + \sinh(x)$$

```
[In] Integrate[Sinh[x]*Tanh[2*x],x]
[Out] -(ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]) + Sinh[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\sinh(x) - \frac{\arctan\left(\sinh(x)\sqrt{2}\right)\sqrt{2}}{2}$	16
default	$\sinh(x) - \frac{\arctan\left(\sinh(x)\sqrt{2}\right)\sqrt{2}}{2}$	16
risch	$\frac{e^{x}}{2} - \frac{e^{-x}}{2} + \frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{4} - \frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{4}$	54

```
[In] int(sinh(x)*tanh(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(x)-1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 6.05

$$\int \sinh(x)\tanh(2x)\,dx = -\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\mathrm{arcta}}{2\left(\cosh\left(x\right)+\mathrm{sir}}$$

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2*
sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(cosh
(x) - sinh(x))) - cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) +
 sinh(x))
```

Sympy [F]

$$\int \sinh(x)\tanh(2x)\,dx = \int \sinh\left(x\right)\tanh\left(2x\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(2*x),x)
```

```
[Out] Integral(sinh(x)*tanh(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \sinh(x)\tanh(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) - 1/2*e^(-x) + 1/2*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \sinh(x)\tanh(2x)\,dx = -\frac{1}{4}\,\sqrt{2}\left(\pi + 2\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \sinh(x)\tanh(2x)\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2} + \frac{\sqrt{2}\,e^{3\,x}}{2}\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}\right)}{2}$$

```
[In] int(tanh(2*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x)
)/2))/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2))/2
```

3.201 $\int \sinh(x)\tanh(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\arctan(\sinh(x)) - \frac{1}{3}\arctan(2\sinh(x)) + \sinh(x)$$

```
[Out] -1/3*arctan(sinh(x))-1/3*arctan(2*sinh(x))+sinh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1293, 1177, 209}

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\arctan(\sinh(x)) - \frac{1}{3}\arctan(2\sinh(x)) + \sinh(x)$$

```
[In] Int[Sinh[x]*Tanh[3*x],x]
```

```
[Out] -1/3*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2(-3-4x^2)}{1+5x^2+4x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) + \frac{1}{4}\text{Subst}\left(\int \frac{-4-8x^2}{1+5x^2+4x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right) - \frac{4}{3}\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{3}\arctan(\sinh(x)) - \frac{1}{3}\arctan(2\sinh(x)) + \sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\arctan(\sinh(x)) - \frac{1}{3}\arctan(2\sinh(x)) + \sinh(x)$$

```
[In] Integrate[Sinh[x]*Tanh[3*x],x]
[Out] -1/3*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/3 + Sinh[x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln\left(e^{x}-i\right)}{3}-\frac{i\ln\left(e^{x}+i\right)}{3}+\frac{i\ln\left(e^{2x}-ie^{x}-1\right)}{6}-\frac{i\ln\left(e^{2x}+ie^{x}-1\right)}{6}$	60

```
[In] int(sinh(x)*tanh(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/3*I*ln(exp(x)-I)-1/3*I*ln(exp(x)+I)+1/6*I*ln(exp(2
*x)-I*exp(x)-1)-1/6*I*ln(exp(2*x)+I*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.00

$$\int \sinh(x)\tanh(3x)\,dx$$
$$=\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(-\frac{\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}}{\cosh(x)-\sinh(x)}\right)-6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\right.}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)/(cosh(x) - sinh(x))) - 6*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x))
+ 3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 3)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sinh(x)\tanh(3x)\,dx=\int \sinh\left(x\right)\tanh\left(3x\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(3*x),x)
```

```
[Out] Integral(sinh(x)*tanh(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \sinh(x)\tanh(3x)\,dx = \frac{1}{3}\arctan\left(\sqrt{3} + 2\,e^{(-x)}\right) + \frac{1}{3}\arctan\left(-\sqrt{3} + 2\,e^{(-x)}\right) + \frac{2}{3}\arctan\left(e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(sqrt(3) + 2*e^(-x)) + 1/3*arctan(-sqrt(3) + 2*e^(-x)) + 2/3*arct
an(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \sinh(x)\tanh(3x)\,dx = -\frac{1}{3}\,\pi - \frac{1}{3}\arctan\left(\left(e^{(2\,x)} - 1\right)e^{(-x)}\right) - \frac{1}{3}\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] -1/3*pi - 1/3*arctan((e^(2*x) - 1)*e^(-x)) - 1/3*arctan(1/2*(e^(2*x) - 1)*e
^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \sinh(x)\tanh(3x)\,dx = \frac{\mathrm{e}^{x}}{2} - \mathrm{atan}\left(\mathrm{e}^{x}\right) - \frac{\mathrm{atan}\left(\mathrm{e}^{3\,x}\right)}{3} - \frac{\mathrm{e}^{-x}}{2}$$

```
[In] int(tanh(3*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x)) - atan(exp(3*x))/3 - exp(-x)/2
```

3.202 $\int \sinh(x)\tanh(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 69

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right) + \sinh(x)$$

```
[Out] sinh(x)-1/4*arctan(2*sinh(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arcta
n(2*sinh(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1293, 1180, 209}

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right) + \sinh(x)$$

```
[In] Int[Sinh[x]*Tanh[4*x],x]
```

```
[Out] -1/4*(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]) - (Sqrt[2 +
Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{4x^2(-1-2x^2)}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\\
&= -\left(4\text{Subst}\left(\int \frac{x^2(-1-2x^2)}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\right)\\
&= \sinh(x) + \frac{1}{2}\text{Subst}\left(\int \frac{-2-8x^2}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) + \left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{4-2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right)\\
&\quad - \left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{4+2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right) + \sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right) + \sinh(x)$$

```
[In] Integrate[Sinh[x]*Tanh[4*x],x]
[Out] -1/4*(Sqrt[2 - Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]) - (Sqrt[2 +
Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[2]]])/4 + Sinh[x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^{4}+128\_Z^{2}+1\right)}\_R\ln\left(-8\_R\,e^{x}+e^{2x}-1\right)\right)$	42

```
[In] int(sinh(x)*tanh(4*x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-8*_R*exp(x)+exp(2*x)-1),_R=RootOf(2048*_Z
^4+128*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.09

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{\sqrt{\sqrt{2}-2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{2}-2}(\cosh(x)+\sin$$

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="fricas")
[Out] -1/8*(sqrt(sqrt(2) - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + sqrt(sqrt(2) - 2)*(cosh(x) + sinh(x)) - 1) - sqrt(sqrt(2)
```

```
- 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sq
rt(sqrt(2) - 2)*(cosh(x) + sinh(x)) - 1) + sqrt(-sqrt(2) - 2)*(cosh(x) + si
nh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(2) - 2)*(
cosh(x) + sinh(x)) - 1) - sqrt(-sqrt(2) - 2)*(cosh(x) + sinh(x))*log(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(2) - 2)*(cosh(x) + sinh(x)
) - 1) - 4*cosh(x)^2 - 8*cosh(x)*sinh(x) - 4*sinh(x)^2 + 4)/(cosh(x) + sinh
(x))
```

Sympy [F]

$$\int \sinh(x)\tanh(4x)\,dx = \int \sinh{\left(x \right)}\tanh{\left(4x \right)}\,dx$$

```
[In] integrate(sinh(x)*tanh(4*x),x)
[Out] Integral(sinh(x)*tanh(4*x), x)
```

Maxima [F]

$$\int \sinh(x)\tanh(4x)\,dx = \int \sinh\left(x\right)\tanh\left(4\,x\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate(2*(e^(7*x) + e^x)/(e^(8*x) + 1), x
)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sinh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(sinh(x)*tanh(4*x),x, algorithm="giac")
[Out] -1/4*sqrt(sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(sqrt(2) + 2)) - 1/4*sqrt
(-sqrt(2) + 2)*arctan(-(e^(-x) - e^x)/sqrt(-sqrt(2) + 2)) - 1/2*e^(-x) + 1/
2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sinh(x)\tanh(4x)\,dx = \frac{e^x}{2} - \frac{e^{-x}}{2} - \frac{\operatorname{atan}\left(\frac{e^{-x}\,(e^{2\,x}-1)}{\sqrt{\sqrt{2}+2}}\right)\,\sqrt{\sqrt{2}+2}}{4} - \frac{\operatorname{atan}\left(\frac{e^{-x}\,(e^{2\,x}-1)}{\sqrt{2-\sqrt{2}}}\right)\,\sqrt{2-\sqrt{2}}}{4}$$

```
[In] int(tanh(4*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (atan((exp(-x)*(exp(2*x) - 1))/(2^(1/2) + 2)^(1/2))*
(2^(1/2) + 2)^(1/2))/4 - (atan((exp(-x)*(exp(2*x) - 1))/(2 - 2^(1/2))^(1/2)
)*(2 - 2^(1/2))^(1/2))/4
```

3.203 $\int \sinh(x) \tanh(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 87

$$\int \sinh(x)\tanh(5x)\,dx = -\frac{1}{5}\arctan(\sinh(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

```
[Out] -1/5*arctan(sinh(x))+sinh(x)-1/5*arctan(sinh(x)*(5^(1/2)+1))*(1/2*5^(1/2)-1
/2)-1/5*arctan(2*sinh(x)*2^(1/2)/(3+5^(1/2))^(1/2))*(1/2+1/2*5^(1/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6874, 2098, 209, 1180}

$$\int \sinh(x)\tanh(5x)\,dx = -\frac{1}{5}\arctan(\sinh(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

```
[In] Int[Sinh[x]*Tanh[5*x],x]
```

```
[Out] -1/5*ArcTan[Sinh[x]] - (Sqrt[(3 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])
]*Sinh[x]])/5 - (Sqrt[(3 - Sqrt[5])/2]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]
])/5 + Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2(-5-20x^2-16x^4)}{1+13x^2+28x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-1+\frac{1+8x^2+8x^4}{1+13x^2+28x^4+16x^6}\right)\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \text{Subst}\left(\int \frac{1+8x^2+8x^4}{1+13x^2+28x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \text{Subst}\left(\int \left(\frac{1}{5\,(1+x^2)}+\frac{4(1+6x^2)}{5\,(1+12x^2+16x^4)}\right)\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \frac{1}{5}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right) - \frac{4}{5}\text{Subst}\left(\int \frac{1+6x^2}{1+12x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{5}\arctan(\sinh(x)) + \sinh(x)\\
&\quad - \frac{1}{5}\left(4\left(3-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{6-2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right)\\
&\quad - \frac{1}{5}\left(4\left(3+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{6+2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right)
\end{aligned}$$

$$= -\frac{1}{5}\arctan(\sinh(x)) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sinh(x)\tanh(5x)\,dx = \frac{1}{10}\left(-2\arctan(\sinh(x)) - \sqrt{2\left(3+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{3+\sqrt{5}}}\sinh(x)\right) - \sqrt{6-2\sqrt{5}}\arctan\left(\sqrt{2\left(3+\sqrt{5}\right)}\sinh(x)\right) + 10\sinh(x)\right)$$

```
[In] Integrate[Sinh[x]*Tanh[5*x],x]
```

```
[Out] (-2*ArcTan[Sinh[x]] - Sqrt[2*(3 + Sqrt[5])]*ArcTan[2*Sqrt[2/(3 + Sqrt[5])]*
Sinh[x]] - Sqrt[6 - 2*Sqrt[5]]*ArcTan[Sqrt[2*(3 + Sqrt[5])]*Sinh[x]] + 10*S
inh[x])/10
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result	s
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln(e^{x}-i)}{5}-\frac{i\ln(e^{x}+i)}{5}+\left(\sum_{\_R=\mathrm{RootOf}(10000\_Z^{4}+300\_Z^{2}+1)}\_R\ln\left(-10\_R\,e^{x}+e^{2x}-1\right)\right)$	6

```
[In] int(sinh(x)*tanh(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/5*I*ln(exp(x)-I)-1/5*I*ln(exp(x)+I)+sum(_R*ln(-10*
_R*exp(x)+exp(2*x)-1),_R=RootOf(10000*_Z^4+300*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.52

$$\int \sinh(x)\tanh(5x)\,dx = -\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}-3}\log\left(2\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+2\,\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(x\right)\right.\right.}{}$$

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="fricas")
```

```
[Out] -1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) - 3)*log(2*cosh(x)^
2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*s
qrt(sqrt(5) - 3) - 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) -
3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + s
qrt(2)*sinh(x))*sqrt(sqrt(5) - 3) - 2) + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x)
)*sqrt(-sqrt(5) - 3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (s
qrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 3) - 2) - (sqrt(2)*cosh(x
) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x)
 + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 3) - 2
) + 8*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - 10*cosh(x)^2 - 20*cos
h(x)*sinh(x) - 10*sinh(x)^2 + 10)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sinh(x)\tanh(5x)\,dx = \int \sinh\left(x\right)\tanh\left(5x\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(5*x),x)
```

```
[Out] Integral(sinh(x)*tanh(5*x), x)
```

Maxima [F]

$$\int \sinh(x)\tanh(5x)\,dx = \int \sinh\left(x\right)\tanh\left(5\,x\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 2/5*arctan(e^x) - 1/2*integrate(2/5*(3*e^(7*x) -
 e^(5*x) - e^(3*x) + 3*e^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sinh(x)\tanh(5x)\,dx = -\frac{1}{10}\,\pi - \frac{1}{10}\left(\sqrt{5}+1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}+1}\right) - \frac{1}{10}\left(\sqrt{5}-1\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{5}-1}\right) - \frac{1}{5}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)*tanh(5*x),x, algorithm="giac")
```

```
[Out] -1/10*pi - 1/10*(sqrt(5) + 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) + 1)) - 1/1
0*(sqrt(5) - 1)*arctan(-2*(e^(-x) - e^x)/(sqrt(5) - 1)) - 1/5*arctan(1/2*(e
^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \sinh(x)\tanh(5x)\,dx = \frac{e^{x}}{2} - \frac{2\,\mathrm{atan}(e^{x})}{5} - \frac{e^{-x}}{2} - 2\,\mathrm{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{10\sqrt{\frac{3}{200}-\frac{\sqrt{5}}{200}}}\right)\sqrt{\frac{3}{200}-\frac{\sqrt{5}}{200}} - 2\,\mathrm{atan}\left(\frac{e^{-x}\left(e^{2x}-1\right)}{10\sqrt{\frac{\sqrt{5}}{200}+\frac{3}{200}}}\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{3}{200}}$$

```
[In] int(tanh(5*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - (2*atan(exp(x)))/5 - exp(-x)/2 - 2*atan((exp(-x)*(exp(2*x) - 1))
/(10*(3/200 - 5^(1/2)/200)^(1/2)))*(3/200 - 5^(1/2)/200)^(1/2) - 2*atan((ex
p(-x)*(exp(2*x) - 1))/(10*(5^(1/2)/200 + 3/200)^(1/2)))*(5^(1/2)/200 + 3/20
0)^(1/2)
```

3.204 $\int \sinh(x)\tanh(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 87

$$\int \sinh(x)\tanh(6x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right) + \sinh(x)$$

```
[Out] sinh(x)-1/6*arctan(sinh(x)*2^(1/2))*2^(1/2)-1/6*arctan(2*sinh(x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctan(2*sinh(x)/(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6874, 2098, 209, 1180}

$$\int \sinh(x)\tanh(6x)\,dx = -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right) + \sinh(x)$$

```
[In] Int[Sinh[x]*Tanh[6*x],x]
```

```
[Out] -1/3*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{2x^2(-3-16x^2-16x^4)}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{x^2(-3-16x^2-16x^4)}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\right)\\
&= -\left(2\text{Subst}\left(\int \left(-\frac{1}{2}+\frac{1+12x^2+16x^4}{2\left(1+18x^2+48x^4+32x^6\right)}\right)\,dx, x, \sinh(x)\right)\right)\\
&= \sinh(x)-\text{Subst}\left(\int \frac{1+12x^2+16x^4}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= \sinh(x)-\text{Subst}\left(\int \left(\frac{1}{3\left(1+2x^2\right)}+\frac{2(1+8x^2)}{3\left(1+16x^2+16x^4\right)}\right)\,dx, x, \sinh(x)\right)
\end{aligned}
$$

$$=\sinh(x)-\frac{1}{3}\text{Subst}\left(\int\frac{1}{1+2x^2}\,dx,x,\sinh(x)\right)-\frac{2}{3}\text{Subst}\left(\int\frac{1+8x^2}{1+16x^2+16x^4}\,dx,x,\sinh(x)\right)$$

$$=-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\sinh(x)$$
$$-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{8-4\sqrt{3}+16x^2}\,dx,x,\sinh(x)\right)$$
$$-\frac{1}{3}\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int\frac{1}{8+4\sqrt{3}+16x^2}\,dx,x,\sinh(x)\right)$$

$$=-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)$$
$$-\frac{1}{6}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)+\sinh(x)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int\sinh(x)\tanh(6x)\,dx=-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)+\sinh(x)$$

```
[In] Integrate[Sinh[x]*Tanh[6*x],x]
[Out] -1/3*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] - (Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x]
)/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sq
rt[3]]])/6 + Sinh[x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{12}+\left(\sum_{\_R=\text{RootOf}(20736\_Z^4+576\_Z^2+1)}\_R\ln(-12\_$

```
[In] int(sinh(x)*tanh(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*
I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)+sum(_R*ln(-12*_R*exp(x)+exp(2*x)-
1),_R=RootOf(20736*_Z^4+576*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(65) = 130.

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.45

$$\int \sinh(x)\tanh(6x)\,dx =$$
$$-\frac{\sqrt{\sqrt{3}-2}(\cosh{(x)}+\sinh{(x)})\log\left(\cosh{(x)}^2+2\cosh{(x)}\sinh{(x)}+\sinh{(x)}^2+\sqrt{\sqrt{3}-2}(\cosh{(x)}+\text{s}\right.}{}$$

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(sqrt(3) - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(sqrt(3) - 2)*(cosh(x) + sinh(x)) - 1) - sqrt(sqrt(3)
 - 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - s
qrt(sqrt(3) - 2)*(cosh(x) + sinh(x)) - 1) + sqrt(-sqrt(3) - 2)*(cosh(x) + s
inh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(3) - 2)*
(cosh(x) + sinh(x)) - 1) - sqrt(-sqrt(3) - 2)*(cosh(x) + sinh(x))*log(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(3) - 2)*(cosh(x) + sinh(x
)) - 1) + 2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x)
+ 1/2*sqrt(2)*sinh(x)) - 2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*
(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2
))/(cosh(x) - sinh(x))) - 6*cosh(x)^2 - 12*cosh(x)*sinh(x) - 6*sinh(x)^2 +
6)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sinh(x)\tanh(6x)\,dx = \int \sinh{(x)}\tanh{(6x)}\,dx$$

```
[In] integrate(sinh(x)*tanh(6*x),x)
```

```
[Out] Integral(sinh(x)*tanh(6*x), x)
```

Maxima [F]

$$\int \sinh(x)\tanh(6x)\,dx = \int \sinh\left(x\right)\tanh\left(6\,x\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)
) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/2*integrate(2/3*
(2*e^(7*x) - e^(5*x) - e^(3*x) + 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \sinh(x)\tanh(6x)\,dx = &-\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}+\sqrt{2}}\right)\\&-\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{2\left(e^{(-x)}-e^{x}\right)}{\sqrt{6}-\sqrt{2}}\right)\\&-\frac{1}{12}\sqrt{2}\left(\pi+2\arctan\left(\frac{1}{2}\sqrt{2}(e^{(2x)}-1)e^{(-x)}\right)\right)-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\end{aligned}$$

```
[In] integrate(sinh(x)*tanh(6*x),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(6) + sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) + sqrt(2))) - 1
/12*(sqrt(6) - sqrt(2))*arctan(-2*(e^(-x) - e^x)/(sqrt(6) - sqrt(2))) - 1/1
2*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x) +
1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \sinh(x)\tanh(6x)\,dx = &\frac{e^{x}}{2}-\frac{e^{-x}}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{-x}\,(e^{2x}-1)}{2}\right)}{6}\\&-2\,\mathrm{atan}\left(\frac{e^{-x}\,(e^{2x}-1)}{12\,\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}}\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}\\&-2\,\mathrm{atan}\left(\frac{e^{-x}\,(e^{2x}-1)}{12\,\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}}\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}\end{aligned}$$

```
[In] int(tanh(6*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(-x)*(exp(2*x) - 1))/2))/6
 - 2*atan((exp(-x)*(exp(2*x) - 1))/(12*(1/72 - 3^(1/2)/144)^(1/2)))*(1/72 -
 3^(1/2)/144)^(1/2) - 2*atan((exp(-x)*(exp(2*x) - 1))/(12*(3^(1/2)/144 + 1/
72)^(1/2)))*(3^(1/2)/144 + 1/72)^(1/2)
```

3.205 $\int \sinh(x) \tanh(nx) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 81

$$\int \sinh(x) \tanh(nx) \, dx = \frac{e^{-x}}{2} + \frac{e^x}{2} - e^{-x} \text{Hypergeometric2F1}\left(1, -\frac{1}{2n}, 1 - \frac{1}{2n}, -e^{2nx}\right) - e^x \text{Hypergeometric2F1}\left(1, \frac{1}{2n}, \frac{1}{2}\left(2 + \frac{1}{n}\right), -e^{2nx}\right)$$

```
[Out] 1/2/exp(x)+1/2*exp(x)-hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*n*x))/exp(x)-e
xp(x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*n*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5720, 2225, 2283}

$$\int \sinh(x) \tanh(nx) \, dx = -e^{-x} \text{Hypergeometric2F1}\left(1, -\frac{1}{2n}, 1 - \frac{1}{2n}, -e^{2nx}\right) - e^x \text{Hypergeometric2F1}\left(1, \frac{1}{2n}, \frac{1}{2}\left(2 + \frac{1}{n}\right), -e^{2nx}\right) + \frac{e^{-x}}{2} + \frac{e^x}{2}$$

```
[In] Int[Sinh[x]*Tanh[n*x],x]
```

```
[Out] 1/(2*E^x) + E^x/2 - Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -E^(2*n*x)]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5720

```
Int[Sinh[(a_.) + (b_.)*(x_)]*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[-E^
(-(a + b*x))/2 + E^(a + b*x)/2 + 1/(E^(a + b*x)*(1 + E^(2*(c + d*x)))) - E^
(a + b*x)/(1 + E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{e^{-x}}{2}+\frac{e^x}{2}+\frac{e^{-x}}{1+e^{2nx}}-\frac{e^x}{1+e^{2nx}}\right) dx \\
&= -\left(\frac{1}{2}\int e^{-x}\,dx\right)+\frac{\int e^x\,dx}{2}+\int \frac{e^{-x}}{1+e^{2nx}}\,dx-\int \frac{e^x}{1+e^{2nx}}\,dx \\
&= \frac{e^{-x}}{2}+\frac{e^x}{2}-e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2nx}\right) \\
&\quad -e^x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2nx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \sinh(x)\tanh(nx)\,dx = \frac{1}{2}\Bigg(e^{-x}+e^x-2e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2nx}\right) - 2e^x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},-e^{2nx}\right)\Bigg)$$

```
[In] Integrate[Sinh[x]*Tanh[n*x],x]
```

```
[Out] (E^(-x) + E^x - (2*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^(2*n*x)])
/E^x - 2*E^x*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^(2*n*x)])/2
```

Maple [F]

$$\int \sinh(x) \tanh(nx) \, dx$$

```
[In] int(sinh(x)*tanh(n*x),x)
[Out] int(sinh(x)*tanh(n*x),x)
```

Fricas [F]

$$\int \sinh(x) \tanh(nx) \, dx = \int \sinh(x) \tanh(nx) \, dx$$

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="fricas")
[Out] integral(sinh(x)*tanh(n*x), x)
```

Sympy [F]

$$\int \sinh(x) \tanh(nx) \, dx = \int \sinh(x) \tanh(nx) \, dx$$

```
[In] integrate(sinh(x)*tanh(n*x),x)
[Out] Integral(sinh(x)*tanh(n*x), x)
```

Maxima [F]

$$\int \sinh(x) \tanh(nx) \, dx = \int \sinh(x) \tanh(nx) \, dx$$

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/2*integrate(2*(e^(2*x) - 1)/(e^(2*n*x + x) + e
^x), x)
```

Giac [F]

$$\int \sinh(x)\tanh(nx)\,dx = \int \sinh\left(x\right)\tanh\left(nx\right)\,dx$$

```
[In] integrate(sinh(x)*tanh(n*x),x, algorithm="giac")
[Out] integrate(sinh(x)*tanh(n*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(x)\tanh(nx)\,dx = \int \tanh(n\,x)\,\sinh(x)\,dx$$

```
[In] int(tanh(n*x)*sinh(x),x)
[Out] int(tanh(n*x)*sinh(x), x)
```

3.206 $\int \coth(2x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \coth(2x)\sinh(x)\,dx=-\frac{1}{2}\arctan(\sinh(x))+\sinh(x)$$

```
[Out] -1/2*arctan(sinh(x))+sinh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {396, 209}

$$\int \coth(2x)\sinh(x)\,dx=\sinh(x)-\frac{1}{2}\arctan(\sinh(x))$$

```
[In] Int[Coth[2*x]*Sinh[x],x]
```

```
[Out] -1/2*ArcTan[Sinh[x]] + Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+2x^2}{2+2x^2}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{2}\arctan(\sinh(x)) + \sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \coth(2x)\sinh(x)\,dx = -\frac{1}{2}\arctan(\sinh(x)) + \sinh(x)$$

```
[In] Integrate[Coth[2*x]*Sinh[x],x]
[Out] -1/2*ArcTan[Sinh[x]] + Sinh[x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\arctan(\sinh(x))}{2}+\sinh(x)$	9
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln(e^{x}-i)}{2}-\frac{i\ln(e^{x}+i)}{2}$	30

```
[In] int(coth(2*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(sinh(x))+sinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 4.20

$$\int \coth(2x)\sinh(x)\,dx = -\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-\cosh\left(x\right)^{2}-2\cosh\left(x\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+1}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="fricas")
[Out] -1/2*(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(
x)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \coth(2x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(2x\right)\,dx$$

```
[In] integrate(coth(2*x)*sinh(x),x)
[Out] Integral(sinh(x)*coth(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \coth(2x)\sinh(x)\,dx = \arctan\left(e^{(-x)}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="maxima")
[Out] arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \coth(2x)\sinh(x)\,dx = -\arctan\left(e^{x}\right) - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(coth(2*x)*sinh(x),x, algorithm="giac")
[Out] -arctan(e^x) - 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \coth(2x)\sinh(x)\,dx = \frac{e^{x}}{2} - \mathrm{atan}(e^{x}) - \frac{e^{-x}}{2}$$

```
[In] int(coth(2*x)*sinh(x),x)
[Out] exp(x)/2 - atan(exp(x)) - exp(-x)/2
```

3.207 $\int \coth(3x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \coth(3x)\sinh(x)\,dx = -\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)$$

```
[Out] sinh(x)-1/3*arctan(2/3*sinh(x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {396, 209}

$$\int \coth(3x)\sinh(x)\,dx = \sinh(x) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[Coth[3*x]*Sinh[x],x]
```

```
[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1+4x^2}{3+4x^2}\,dx, x, \sinh(x)\right)\\ &= \sinh(x) - 2\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth(3x)\sinh(x)\,dx = -\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sinh(x)$$

```
[In] Integrate[Coth[3*x]*Sinh[x],x]
[Out] -(ArcTan[(2*Sinh[x])/Sqrt[3]]/Sqrt[3]) + Sinh[x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\sinh(x) - \frac{\arctan\left(\frac{2\sinh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
default	$\sinh(x) - \frac{\arctan\left(\frac{2\sinh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$\frac{e^{x}}{2} - \frac{e^{-x}}{2} + \frac{i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\,e^{x}-1\right)}{6} - \frac{i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\,e^{x}-1\right)}{6}$	54

```
[In] int(coth(3*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] sinh(x)-1/3*arctan(2/3*sinh(x)*3^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 5.90

$$\int \coth(3x)\sinh(x)\,dx = -\frac{2\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh(x)+\frac{1}{3}\sqrt{3}\sinh(x)\right)-2\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\mathrm{ar}}{6\left(\cosh(x)\right.}$$

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/
3*sqrt(3)*sinh(x)) - 2*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqr
t(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))
/(cosh(x) - sinh(x))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/
(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \coth(3x)\sinh(x)\,dx = \int \sinh(x)\coth(3x)\,dx$$

```
[In] integrate(coth(3*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

$$\int \coth(3x)\sinh(x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}+1\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}-1\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) + 1/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*e^(-x) - 1)) - 1/2*e^(-x) + 1/2*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \coth(3x)\sinh(x)\,dx = -\frac{1}{6}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(coth(3*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) - 1/2*e^(-x)
 + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \coth(3x)\sinh(x)\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,e^{x}}{3} + \frac{\sqrt{3}\,e^{3x}}{3}\right)}{3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{x}}{3}\right)}{3}$$

```
[In] int(coth(3*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 - (3^(1/2)*atan((2*3^(1/2)*exp(x))/3 + (3^(1/2)*exp(3*
x))/3))/3 - (3^(1/2)*atan((3^(1/2)*exp(x))/3))/3
```

3.208 $\int \coth(4x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} + \sinh(x)$$

```
[Out] -1/4*arctan(sinh(x))+sinh(x)-1/4*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1690, 1180, 209}

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} + \sinh(x)$$

```
[In] Int[Coth[4*x]*Sinh[x],x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+8x^2+8x^4}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right) \\
&= \text{Subst}\left(\int \left(1-\frac{3+4x^2}{4+12x^2+8x^4}\right)\,dx, x, \sinh(x)\right) \\
&= \sinh(x)-\text{Subst}\left(\int \frac{3+4x^2}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right) \\
&= \sinh(x)-2\text{Subst}\left(\int \frac{1}{4+8x^2}\,dx, x, \sinh(x)\right)-2\text{Subst}\left(\int \frac{1}{8+8x^2}\,dx, x, \sinh(x)\right) \\
&= -\frac{1}{4}\arctan(\sinh(x))-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}+\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x))-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}+\sinh(x)$$

```
[In] Integrate[Coth[4*x]*Sinh[x],x]
[Out] -1/4*ArcTan[Sinh[x]] - ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2]) + Sinh[x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\frac{i\ln\left(e^{x}-i\right)}{4}-\frac{i\ln\left(e^{x}+i\right)}{4}+\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{8}$	72

```
[In] int(coth(4*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/8*I*2^(1/2)*
ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/8*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-
1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.57

$$\int \coth(4x)\sinh(x)\,dx =$$

$$-\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\arctan\left(\frac{1}{2}\sqrt{2}\cosh\left(x\right)+\frac{1}{2}\sqrt{2}\sinh\left(x\right)\right)-\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\text{arcta}}{}$$

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(1/2*sqrt(2)*cosh(x) + 1/2*
sqrt(2)*sinh(x)) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*arctan(-1/2*(sqrt(2)
*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))/(cosh
(x) - sinh(x))) + 2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - 2*cosh(
x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \coth(4x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(4x\right)\,dx$$

```
[In] integrate(coth(4*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(4*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \coth(4x)\sinh(x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right) + \frac{1}{2}\arctan\left(e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) + 1/4*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x)) - 1/2*e^(-x) + 1/2*e
^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \coth(4x)\sinh(x)\,dx = -\frac{1}{8}\pi - \frac{1}{8}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(coth(4*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1
/4*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \coth(4x)\sinh(x)\,dx = \frac{e^{x}}{2} - \frac{\operatorname{atan}(e^{x})}{2} - \frac{e^{-x}}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,e^{x}}{2} + \frac{\sqrt{2}\,e^{3x}}{2}\right)}{4} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,e^{x}}{2}\right)}{4}$$

```
[In] int(coth(4*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x))/2 - exp(-x)/2 - (2^(1/2)*atan((2^(1/2)*exp(x))/2 +
(2^(1/2)*exp(3*x))/2))/4 - (2^(1/2)*atan((2^(1/2)*exp(x))/2))/4
```

3.209 $\int \coth(5x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 82

$$\int \coth(5x)\sinh(x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

```
[Out] sinh(x)-1/10*arctan(1/5*sinh(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)
-1/10*arctan(2*sinh(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1690, 1180, 209}

$$\int \coth(5x)\sinh(x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)$$

```
[In] Int[Coth[5*x]*Sinh[x],x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]]) - (Sqr
t[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(2*(5 + Sqrt[5]))/5]*Sinh[x]])/5 + Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+12x^2+16x^4}{5+20x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{4(1+2x^2)}{5+20x^2+16x^4}\right)\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - 4\text{Subst}\left(\int \frac{1+2x^2}{5+20x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= \sinh(x) - \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10-2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right)\\
&\quad - \frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10+2\sqrt{5}+16x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)\\
&\quad - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sinh(x)\right) + \sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \coth(5x)\sinh(x)\,dx = \frac{1}{10}\left(-\sqrt{10-2\sqrt{5}}\arctan\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sinh(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sinh(x)\right)+10\sinh(x)\right)$$

```
[In] Integrate[Coth[5*x]*Sinh[x],x]
```

```
[Out] (-(Sqrt[10 - 2*Sqrt[5]]*ArcTan[Sqrt[2 + 2/Sqrt[5]]*Sinh[x]]) - Sqrt[2*(5 +
Sqrt[5])]*ArcTan[2*Sqrt[2/(5 + Sqrt[5])]*Sinh[x]] + 10*Sinh[x])/10
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{e^{x}}{2}-\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(2000\_Z^{4}+100\_Z^{2}+1\right)}\_R\ln\left(-10\_R\,e^{x}+e^{2x}-1\right)\right)$	42

```
[In] int(coth(5*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+sum(_R*ln(-10*_R*exp(x)+exp(2*x)-1),_R=RootOf(2000*_
Z^4+100*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(54) = 108.

Time = 0.25 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.57

$$\int \coth(5x)\sinh(x)\,dx = -\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}-5}\log\left(2\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+2\,\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(x\right)\right.\right.}{}$$

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) - 5)*log(2*cosh(x)^
2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*s
```

```
qrt(sqrt(5) - 5) - 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) -
5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + s
qrt(2)*sinh(x))*sqrt(sqrt(5) - 5) - 2) + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x)
)*sqrt(-sqrt(5) - 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (s
qrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 5) - 2) - (sqrt(2)*cosh(x
) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x)
 + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) - 5) - 2
) - 10*cosh(x)^2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 + 10)/(cosh(x) + sinh(
x))
```

Sympy [F]

$$\int \coth(5x)\sinh(x)\,dx = \int \sinh(x)\coth(5x)\,dx$$

```
[In] integrate(coth(5*x)*sinh(x),x)
[Out] Integral(sinh(x)*coth(5*x), x)
```

Maxima [F]

$$\int \coth(5x)\sinh(x)\,dx = \int \coth(5x)\sinh(x)\,dx$$

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(3*x) + e^(2*x) + e^x)/(e^(4*x)
 + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/2*integrate((e^(3*x) - e^(2*x) + e^
x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \coth(5x)\sinh(x)\,dx = -\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right) - \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{e^{(-x)}-e^{x}}{\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(coth(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(1/2*sqrt(5) + 5/2))
- 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(e^(-x) - e^x)/sqrt(-1/2*sqrt(5) + 5/2
)) - 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \coth(5x)\sinh(x)\,dx = \frac{e^x}{2} - \frac{e^{-x}}{2} + \ln\left(40\,e^x\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\,e^{2\,x} + 4\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} + \ln\left(40\,e^x\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\,e^{2\,x} + 4\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}} - \ln\left(4\,e^{2\,x} + 40\,e^x\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}} - \ln\left(4\,e^{2\,x} + 40\,e^x\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}} - 4\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}$$

```
[In] int(coth(5*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - exp(-x)/2 + log(40*exp(x)*(- 5^(1/2)/200 - 1/40)^(1/2) - 4*exp(2
*x) + 4)*(- 5^(1/2)/200 - 1/40)^(1/2) + log(40*exp(x)*(5^(1/2)/200 - 1/40)^
(1/2) - 4*exp(2*x) + 4)*(5^(1/2)/200 - 1/40)^(1/2) - log(4*exp(2*x) + 40*ex
p(x)*(- 5^(1/2)/200 - 1/40)^(1/2) - 4)*(- 5^(1/2)/200 - 1/40)^(1/2) - log(4
*exp(2*x) + 40*exp(x)*(5^(1/2)/200 - 1/40)^(1/2) - 4)*(5^(1/2)/200 - 1/40)^
(1/2)
```

3.210 $\int \coth(6x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\arctan(\sinh(x)) - \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sinh(x)$$

```
[Out] -1/6*arctan(sinh(x))-1/6*arctan(2*sinh(x))+sinh(x)-1/6*arctan(2/3*sinh(x)*3
^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2098, 209}

$$\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\arctan(\sinh(x)) - \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sinh(x)$$

```
[In] Int[Coth[6*x]*Sinh[x],x]
```

```
[Out] -1/6*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2
*Sqrt[3]) + Sinh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+18x^2+48x^4+32x^6}{2\left(3+19x^2+32x^4+16x^6\right)}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+18x^2+48x^4+32x^6}{3+19x^2+32x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2-\frac{1}{3\left(1+x^2\right)}-\frac{2}{3\left(1+4x^2\right)}-\frac{2}{3+4x^2}\right)dx, x, \sinh(x)\right)\\
&= \sinh(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&\quad -\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)-\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{6}\arctan(\sinh(x))-\frac{1}{6}\arctan(2\sinh(x))-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$
\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\arctan(\sinh(x))-\frac{1}{6}\arctan(2\sinh(x))-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sinh(x)
$$

```
[In] Integrate[Coth[6*x]*Sinh[x],x]
[Out] -1/6*ArcTan[Sinh[x]] - ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2
*Sqrt[3]) + Sinh[x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.68

method	result
risch	$\frac{e^x}{2}-\frac{e^{-x}}{2}+\frac{i\ln(e^x-i)}{6}-\frac{i\ln(e^x+i)}{6}+\frac{i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\,e^x-1\right)}{12}-\frac{i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\,e^x-1\right)}{12}+\frac{i\ln(e^{2x}-ie^x-1)}{12}-\frac{i\ln(e^{2x}$

```
[In] int(coth(6*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)-1/2*exp(-x)+1/6*I*ln(exp(x)-I)-1/6*I*ln(exp(x)+I)+1/12*I*3^(1/2)
*ln(exp(2*x)-I*3^(1/2)*exp(x)-1)-1/12*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2)*exp(x
)-1)+1/12*I*ln(exp(2*x)-I*exp(x)-1)-1/12*I*ln(exp(2*x)+I*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\int \coth(6x)\sinh(x)\,dx =$$
$$-\frac{\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\arctan\left(\frac{1}{3}\sqrt{3}\cosh\left(x\right)+\frac{1}{3}\sqrt{3}\sinh\left(x\right)\right)-\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)\right)\mathrm{arct}}{}$$

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*((sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(1/3*sqrt(3)*cosh(x) + 1/3*
sqrt(3)*sinh(x)) - (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*arctan(-1/3*(sqrt(3)
*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2 + 2*sqrt(3))/(co
sh(x) - sinh(x))) - (cosh(x) + sinh(x))*arctan(-(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 3*(cosh(x) + sinh(x))*arctan(cosh(x
) + sinh(x)) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 3)/(cosh(x)
+ sinh(x))
```

Sympy [F]

$$\int \coth(6x)\sinh(x)\,dx = \int \sinh\left(x\right)\coth\left(6x\right)\,dx$$

```
[In] integrate(coth(6*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*coth(6*x), x)
```

Maxima [F]

$$\int \coth(6x)\sinh(x)\,dx = \int \coth(6x)\sinh(x)\,dx$$

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/
6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x - 1)) - 1/3*arctan(e^x) - 1/2*integrate
(1/3*(e^(3*x) + e^x)/(e^(4*x) - e^(2*x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \coth(6x)\sinh(x)\,dx = -\frac{1}{6}\pi - \frac{1}{12}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right) - \frac{1}{6}\arctan\left(\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{6}\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right) - \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(coth(6*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) -
1/6*arctan((e^(2*x) - 1)*e^(-x)) - 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 1
/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \coth(6x)\sinh(x)\,dx = \frac{e^{x}}{2} - \frac{\mathrm{atan}(e^{x})}{3} - \frac{e^{-x}}{2} - \frac{\mathrm{atan}\left(36\,e^{-x}\left(\frac{e^{2x}}{36}-\frac{1}{36}\right)\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(4\sqrt{3}\,e^{-x}\left(\frac{e^{2x}}{12}-\frac{1}{12}\right)\right)}{6}$$

```
[In] int(coth(6*x)*sinh(x),x)
```

```
[Out] exp(x)/2 - atan(exp(x))/3 - exp(-x)/2 - atan(36*exp(-x)*(exp(2*x)/36 - 1/36
))/6 - (3^(1/2)*atan(4*3^(1/2)*exp(-x)*(exp(2*x)/12 - 1/12)))/6
```

3.211 $\int \operatorname{sech}(2x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \operatorname{sech}(2x)\sinh(x)\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4442, 213}

$$\int \operatorname{sech}(2x)\sinh(x)\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[In] Int[Sech[2*x]*Sinh[x],x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
```

```
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)\\ &= -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

```
[In] Integrate[Sech[2*x]*Sinh[x],x]
[Out] -((ArcTanh[Sqrt[2] - I*Tanh[x/2]] + ArcTanh[Sqrt[2] + I*Tanh[x/2]])/Sqrt[2]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 1.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

method	result	size
risch	$\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{4}$	39

```
[In] int(sech(2*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2))
*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\cosh(x)^2 + \sinh(x)^2 - 2\sqrt{2}\cosh(x) + 2}{\cosh(x)^2 + \sinh(x)^2}\right)$$

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
+ sinh(x)^2))
```

Sympy [F]

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(2x)\,dx$$

```
[In] integrate(sech(2*x)*sinh(x),x)
[Out] Integral(sinh(x)*sech(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right)$$

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x} + e^{(2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x} + e^{(2x)} + 1\right)$$

```
[In] integrate(sech(2*x)*sinh(x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*e^x
+ e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}(2x)\sinh(x)\,dx = -\frac{\sqrt{2}\left(\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right) - \ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)\right)}{4}$$

```
[In] int(sinh(x)/cosh(2*x),x)
```

```
[Out] -(2^(1/2)*(log(exp(2*x) + 2^(1/2)*exp(x) + 1) - log(exp(2*x) - 2^(1/2)*exp(
x) + 1)))/4
```

3.212 $\int \mathrm{sech}(3x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \mathrm{sech}(3x)\sinh(x)\,dx = -\frac{1}{3}\log(\cosh(x)) + \frac{1}{6}\log\left(3-4\cosh^2(x)\right)$$

```
[Out] -1/3*ln(cosh(x))+1/6*ln(3-4*cosh(x)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4442, 272, 36, 29, 31}

$$\int \mathrm{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(3-4\cosh^2(x)\right) - \frac{1}{3}\log(\cosh(x))$$

```
[In] Int[Sech[3*x]*Sinh[x],x]
[Out] -1/3*Log[Cosh[x]] + Log[3 - 4*Cosh[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\left(-3+4x^2\right)}\,dx, x, \cosh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-3+4x)}\,dx, x, \cosh^2(x)\right)\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cosh^2(x)\right)\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3+4x}\,dx, x, \cosh^2(x)\right)\\
&= -\frac{1}{3}\log(\cosh(x))+\frac{1}{6}\log\left(3-4\cosh^2(x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\left(5+8\sinh^2(x)\right)\right)$$

```
[In] Integrate[Sech[3*x]*Sinh[x],x]
[Out] -1/3*ArcTanh[(5 + 8*Sinh[x]^2)/3]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{\ln(1+e^{2x})}{3}+\frac{\ln(1-e^{2x}+e^{4x})}{6}$	26

```
[In] int(sech(3*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(1+exp(2*x))+1/6*ln(1-exp(2*x)+exp(4*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) - \frac{1}{3}\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="fricas")
[Out] 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(3x)\,dx$$

```
[In] integrate(sech(3*x)*sinh(x),x)
[Out] Integral(sinh(x)*sech(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{6}\log\left(-\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right) - \frac{1}{3}\log\left(e^{(-2x)} + 1\right)$$

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="maxima")
[Out] 1/6*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/6*log(-sqrt(3)*e^(-x) + e^(-2*x)
 + 1) - 1/3*log(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{1}{6}\log\left(\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{6}\log\left(-\sqrt{3}e^{x} + e^{(2x)} + 1\right) - \frac{1}{3}\log\left(e^{(2x)} + 1\right)$$

```
[In] integrate(sech(3*x)*sinh(x),x, algorithm="giac")
[Out] 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1) -
1/3*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \operatorname{sech}(3x)\sinh(x)\,dx = \frac{\ln\left(\mathrm{e}^{2x} - \mathrm{e}^{4x} - 1\right)}{6} - \frac{\ln\left(3\,\mathrm{e}^{2x} + 3\right)}{3}$$

```
[In] int(sinh(x)/cosh(3*x),x)
[Out] log(exp(2*x) - exp(4*x) - 1)/6 - log(3*exp(2*x) + 3)/3
```

3.213 $\int \operatorname{sech}(4x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \operatorname{sech}(4x)\sinh(x)\,dx=\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/2*arctanh(2*cosh(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2*
cosh(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4442, 1107, 213}

$$\int \operatorname{sech}(4x)\sinh(x)\,dx=\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[In] Int[Sech[4*x]*Sinh[x],x]
```

```
[Out] ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[
(2*Cosh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\ &= \sqrt{2}\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)\\ &\quad - \sqrt{2}\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \frac{1}{16}\text{RootSum}\Big[1 + \#1^8\&, \frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+x\#1^2+2\log\left(-\cosh\left(\frac{x}{2}\right)-\text{s}\right.}{\#1^5}$$

```
[In] Integrate[Sech[4*x]*Sinh[x],x]
[Out] RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Si
nh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x
/2]*#1]*#1^2)/#1^5 & ]/16
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(32768\_Z^4-512\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(4096\_R^3-48\_R\right)\mathrm{e}^x+1\right)\right)$	40

```
[In] int(sech(4*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(4096*_R^3-48*_R)*exp(x)+1),_R=RootOf(32768*_Z^4-512*_
Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\begin{aligned}\int \operatorname{sech}(4x)\sinh(x)\,dx = {} & \frac{1}{8}\sqrt{\sqrt{2}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2 \\ & +\left(\left(\sqrt{2}-1\right)\cosh(x)+\left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{\sqrt{2}+2}+1\Big) \\ & -\frac{1}{8}\sqrt{\sqrt{2}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2 \\ & -\left(\left(\sqrt{2}-1\right)\cosh(x)+\left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{\sqrt{2}+2}+1\Big) \\ & -\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2 \\ & +\left(\left(\sqrt{2}+1\right)\cosh(x)+\left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{-\sqrt{2}+2}+1\Big) \\ & +\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2 \\ & -\left(\left(\sqrt{2}+1\right)\cosh(x)+\left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{-\sqrt{2}+2}+1\Big)\end{aligned}$$

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqr
t(2) - 1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqr
t(sqrt(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) -
1)*cosh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(sqrt(2) + 2) + 1) - 1/8*sqrt(-sqrt
```

```
(2) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(2) + 1)*cos
h(x) + (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1) + 1/8*sqrt(-sqrt(2) +
 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) + 1)*cosh(x)
+ (sqrt(2) + 1)*sinh(x))*sqrt(-sqrt(2) + 2) + 1)
```

Sympy [F]

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(4x)\,dx$$

```
[In] integrate(sech(4*x)*sinh(x),x)
[Out] Integral(sinh(x)*sech(4*x), x)
```

Maxima [F]

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \int \operatorname{sech}(4x)\sinh(x)\,dx$$

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="maxima")
[Out] integrate(sech(4*x)*sinh(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(49) = 98.

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \operatorname{sech}(4x)\sinh(x)\,dx = &-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)\\&-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)\end{aligned}$$

```
[In] integrate(sech(4*x)*sinh(x),x, algorithm="giac")
[Out] -1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt
(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sqrt(sqrt(2) + 2)*log
(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.54

$$\int \operatorname{sech}(4x)\sinh(x)\,dx = \ln\left(3\,e^{2\,x}-2\,\sqrt{2}+8\,e^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-2\,\sqrt{2}\,e^{2\,x}-8\,\sqrt{2}\,e^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+3\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-\ln\left(3\,e^{2\,x}-2\,\sqrt{2}-8\,e^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-2\,\sqrt{2}\,e^{2\,x}+8\,\sqrt{2}\,e^{x}\,\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}+3\right)\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{64}}-\ln\left(3\,e^{2\,x}+2\,\sqrt{2}-8\,e^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+2\,\sqrt{2}\,e^{2\,x}-8\,\sqrt{2}\,e^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+3\right)\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+\ln\left(3\,e^{2\,x}+2\,\sqrt{2}+8\,e^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+2\,\sqrt{2}\,e^{2\,x}+8\,\sqrt{2}\,e^{x}\,\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}+3\right)\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{32}}$$

```
[In] int(sinh(x)/cosh(4*x),x)
```

```
[Out] log(3*exp(2*x) - 2*2^(1/2) + 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) - 2*2^(1/2)
*exp(2*x) - 8*2^(1/2)*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) + 3)*(1/32 - 2^(1/2)
/64)^(1/2) - log(3*exp(2*x) - 2*2^(1/2) - 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2
) - 2*2^(1/2)*exp(2*x) + 8*2^(1/2)*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) + 3)*(1
/32 - 2^(1/2)/64)^(1/2) - log(3*exp(2*x) + 2*2^(1/2) - 8*exp(x)*(2^(1/2)/64
 + 1/32)^(1/2) + 2*2^(1/2)*exp(2*x) - 8*2^(1/2)*exp(x)*(2^(1/2)/64 + 1/32)^
(1/2) + 3)*(2^(1/2)/64 + 1/32)^(1/2) + log(3*exp(2*x) + 2*2^(1/2) + 8*exp(x
)*(2^(1/2)/64 + 1/32)^(1/2) + 2*2^(1/2)*exp(2*x) + 8*2^(1/2)*exp(x)*(2^(1/2
)/64 + 1/32)^(1/2) + 3)*(2^(1/2)/64 + 1/32)^(1/2)
```

3.214 $\int \operatorname{sech}(5x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 62

$$\int \operatorname{sech}(5x)\sinh(x)\,dx=\frac{1}{5}\log(\cosh(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cosh^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cosh^2(x)\right)$$

```
[Out] 1/5*ln(cosh(x))-1/20*ln(5-8*cosh(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*cos
h(x)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4442, 1128, 719, 29, 646, 31}

$$\int \operatorname{sech}(5x)\sinh(x)\,dx=-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cosh^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cosh^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\cosh(x))$$

```
[In] Int[Sech[5*x]*Sinh[x],x]
```

```
[Out] Log[Cosh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cosh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] - 8*Cosh[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \cosh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \cosh^2(x)\right)\\
&= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cosh^2(x)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \cosh^2(x)\right)\\
&= \frac{1}{5}\log(\cosh(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \cosh^2(x)\right)\\
&\quad -\frac{1}{5}\left(4\left(1+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x}\,dx, x, \cosh^2(x)\right)
\end{aligned}
$$

$$= \frac{1}{5}\log(\cosh(x)) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cosh^2(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cosh^2(x)\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \frac{1}{20}\left(4\log(\cosh(x)) + \left(-1+\sqrt{5}\right)\log\left(3-\sqrt{5}+8\sinh^2(x)\right) - \left(1+\sqrt{5}\right)\log\left(3+\sqrt{5}+8\sinh^2(x)\right)\right)$$

```
[In] Integrate[Sech[5*x]*Sinh[x],x]
```

```
[Out] (4*Log[Cosh[x]] + (-1 + Sqrt[5])*Log[3 - Sqrt[5] + 8*Sinh[x]^2] - (1 + Sqrt
[5])*Log[3 + Sqrt[5] + 8*Sinh[x]^2])/20
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 1.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result
risch	$\frac{\ln(1+e^{2x})}{5} - \frac{\ln\left(e^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20} + \frac{\ln\left(e^{4x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20} - \frac{\ln\left(e^{4x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2x}+1\right)}{20} - \frac{\ln\left(e^{4x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e\right.}{20}$

```
[In] int(sech(5*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*ln(1+exp(2*x))-1/20*ln(exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(
exp(4*x)+(-1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2*5^(1/
2)-1/2)*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2*5^(1/2)-1/2)*exp(2*x)+1)*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.94

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4 + 4\sinh(x)^4 - 4\left(\sqrt{5}+1\right)\cosh(x)^2 + 4\left(6\cosh(x)^2 - \sqrt{5} - 1\right)\sinh(x)^2 + \sqrt{5} + 7}{2\cosh(x)^4 + 2\sinh(x)^4 + 2\left(6\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 1}\right) - \frac{1}{20}\log\left(\frac{2\cosh(x)^4 + 2\sinh(x)^4 + 2\left(6\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 1}{\cosh(x)^4 - 4\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 4\cosh(x)\sinh(x)^3 + \sinh(x)^4}\right) + \frac{1}{5}\log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 - 4*(sqrt(5) + 1)*cosh(x)^2 + 4
*(6*cosh(x)^2 - sqrt(5) - 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*sinh
(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)) - 1/20*log((2*cos
h(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 1)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
 + sinh(x)^4)) + 1/5*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(5x)\,dx$$

```
[In] integrate(sech(5*x)*sinh(x),x)
```

```
[Out] Integral(sinh(x)*sech(5*x), x)
```

Maxima [F]

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \int \operatorname{sech}(5x)\sinh(x)\,dx$$

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="maxima")
```

```
[Out] -2/5*integrate((e^(6*x) - e^(4*x) + e^(2*x) - 1)*e^(2*x)/(e^(8*x) - e^(6*x)
 + e^(4*x) - e^(2*x) + 1), x) + 2/5*integrate(e^(6*x)/(e^(8*x) - e^(6*x) +
e^(4*x) - e^(2*x) + 1), x) + 1/5*integrate(e^(4*x)/(e^(8*x) - e^(6*x) + e^(
4*x) - e^(2*x) + 1), x) - 4/5*integrate(e^(2*x)/(e^(8*x) - e^(6*x) + e^(4*x
) - e^(2*x) + 1), x) + 1/5*log(e^(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.90

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right) + \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(-\frac{1}{2}\sqrt{2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right) - \frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\sqrt{-2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right) - \frac{1}{20}\left(\sqrt{5}+1\right)\log\left(-\frac{1}{2}\sqrt{-2\sqrt{5}+10}e^{x}+e^{(2x)}+1\right) + \frac{1}{5}\log\left(e^{(2x)}+1\right)$$

```
[In] integrate(sech(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/20*(
sqrt(5) - 1)*log(-1/2*sqrt(2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(
5) + 1)*log(1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5) +
1)*log(-1/2*sqrt(-2*sqrt(5) + 10)*e^x + e^(2*x) + 1) + 1/5*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \operatorname{sech}(5x)\sinh(x)\,dx = \frac{\ln\left(5\,\mathrm{e}^{2x}+5\right)}{5} - \ln\left(\mathrm{e}^{2x}+2\,\mathrm{e}^{4x}+\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\left(20\,\mathrm{e}^{2x}+30\,\mathrm{e}^{4x}+30\right)+2\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(\mathrm{e}^{2x}+2\,\mathrm{e}^{4x}-\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)\left(20\,\mathrm{e}^{2x}+30\,\mathrm{e}^{4x}+30\right)+2\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)$$

```
[In] int(sinh(x)/cosh(5*x),x)
```

```
[Out] log(5*exp(2*x) + 5)/5 - log(exp(2*x) + 2*exp(4*x) + (5^(1/2)/20 + 1/20)*(20
*exp(2*x) + 30*exp(4*x) + 30) + 2)*(5^(1/2)/20 + 1/20) + log(exp(2*x) + 2*e
xp(4*x) - (5^(1/2)/20 - 1/20)*(20*exp(2*x) + 30*exp(4*x) + 30) + 2)*(5^(1/2
)/20 - 1/20)
```

3.215 $\int \operatorname{sech}(6x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] 1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)-1/6*arctanh(2*cosh(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctanh(2*cosh(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4442, 2082, 213, 1180}

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[In] Int[Sech[6*x]*Sinh[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cosh[x]]/(3*Sqrt[2]) - ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) - ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{-1+18x^2-48x^4+32x^6}\,dx, x, \cosh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{3\,(-1+2x^2)}+\frac{4(-1+2x^2)}{3\,(1-16x^2+16x^4)}\right)\,dx, x, \cosh(x)\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right)\right)+\frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \cosh(x)\right)\\
&= \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}+\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \cosh(x)\right)\\
&\quad +\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x, \cosh(x)\right)\\
&= \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.16

$$\int \operatorname{sech}(6x)\sinh(x)\,dx$$
$$= \frac{1}{24}\left(4\sqrt{2}\left(\operatorname{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)\right)+\operatorname{RootSum}\left[1-\#1^4\right.\right.$$
$$+\#1^8\&,\frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+x\#1^2+2\log\left(-\cosh\left(\frac{x}{2}\right)-\right.}{}$$

```
[In] Integrate[Sech[6*x]*Sinh[x],x]
```

```
[Out] (4*Sqrt[2]*(ArcTanh[Sqrt[2] - I*Tanh[x/2]] + ArcTanh[Sqrt[2] + I*Tanh[x/2]]
) + RootSum[1 - #1^4 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*
#1 - Sinh[x/2]*#1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]
*#1 - Sinh[x/2]*#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2
]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/24
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{12}-\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{12}+2\left(\sum_{\_R=\operatorname{RootOf}(331776\_Z^4-2304\_Z^2+1)}\_R\ln\left(e^{2x}+\left(13824\_R^3\right.\right.\right.$

```
[In] int(sech(6*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)-1/12*ln(1+exp(2*x)-exp(x)*2^(1/2
))*2^(1/2)+2*sum(_R*ln(exp(2*x)+(13824*_R^3-96*_R)*exp(x)+1),_R=RootOf(3317
76*_Z^4-2304*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.94

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{3}-2\right)\cosh(x) + \left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{\sqrt{3}+2} + 1\right) - \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{3}-2\right)\cosh(x) + \left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{\sqrt{3}+2} + 1\right) - \frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \left(\left(\sqrt{3}+2\right)\cosh(x) + \left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{-\sqrt{3}+2} + 1\right) + \frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \left(\left(\sqrt{3}+2\right)\cosh(x) + \left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{-\sqrt{3}+2} + 1\right) + \frac{1}{12}\sqrt{2}\log\left(\frac{\cosh(x)^2 + \sinh(x)^2 + 2\sqrt{2}\cosh(x) + 2}{\cosh(x)^2 + \sinh(x)^2}\right)$$

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sq
rt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*s
qrt(sqrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3)
- 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(sqrt(3) + 2) + 1) - 1/12*sqrt(-s
qrt(3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3) + 2)*
cosh(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(-sqrt(
3) + 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) + 2)*cosh
(x) + (sqrt(3) + 2)*sinh(x))*sqrt(-sqrt(3) + 2) + 1) + 1/12*sqrt(2)*log((co
sh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2))
```

Sympy [F]

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \int \sinh(x)\operatorname{sech}(6x)\,dx$$

```
[In] integrate(sech(6*x)*sinh(x),x)
[Out] Integral(sinh(x)*sech(6*x), x)
```

Maxima [F]

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \int \operatorname{sech}(6\,x)\sinh(x)\,dx$$

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^x
 + e^(2*x) + 1) + integrate(1/3*(e^(7*x) - e^(5*x) + e^(3*x) - e^x)/(e^(8*x
) - e^(4*x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \operatorname{sech}(6x)\sinh(x)\,dx = &-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^{x}+e^{(2\,x)}+1\right)\\&+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)e^{x}+e^{(2\,x)}+1\right)\\&-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\frac{1}{2}\left(\sqrt{6}-\sqrt{2}\right)e^{x}+e^{(2\,x)}+1\right)\\&+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(-\frac{1}{2}\left(\sqrt{6}-\sqrt{2}\right)e^{x}+e^{(2\,x)}+1\right)\\&+\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2\,x)}+1\right)-\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2\,x)}+1\right)\end{aligned}$$

```
[In] integrate(sech(6*x)*sinh(x),x, algorithm="giac")
[Out] -1/24*(sqrt(6) - sqrt(2))*log(1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) +
1/24*(sqrt(6) - sqrt(2))*log(-1/2*(sqrt(6) + sqrt(2))*e^x + e^(2*x) + 1) -
1/24*(sqrt(6) + sqrt(2))*log(1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1) + 1
/24*(sqrt(6) + sqrt(2))*log(-1/2*(sqrt(6) - sqrt(2))*e^x + e^(2*x) + 1) + 1
/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) - 1/12*sqrt(2)*log(-sqrt(2)*e^x
+ e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.39

$$\int \operatorname{sech}(6x)\sinh(x)\,dx = \frac{\sqrt{2}\ln\left(e^{2x}+\sqrt{2}\,e^{x}+1\right)}{12} - \frac{\sqrt{2}\ln\left(e^{2x}-\sqrt{2}\,e^{x}+1\right)}{12} + \ln\left(7\,e^{2x}-4\sqrt{3}-24\,e^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-4\sqrt{3}\,e^{2x} + 12\sqrt{3}\,e^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}+7\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}} - \ln\left(7\,e^{2x}-4\sqrt{3}+24\,e^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}-4\sqrt{3}\,e^{2x} - 12\sqrt{3}\,e^{x}\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}}+7\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}}{144}} + \ln\left(7\,e^{2x}+4\sqrt{3}-24\,e^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+4\sqrt{3}\,e^{2x} - 12\sqrt{3}\,e^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+7\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}} - \ln\left(7\,e^{2x}+4\sqrt{3}+24\,e^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+4\sqrt{3}\,e^{2x} + 12\sqrt{3}\,e^{x}\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}+7\right)\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}$$

```
[In] int(sinh(x)/cosh(6*x),x)
```

```
[Out] (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/12 - (2^(1/2)*log(exp(2*x) - 2
^(1/2)*exp(x) + 1))/12 + log(7*exp(2*x) - 4*3^(1/2) - 24*exp(x)*(1/72 - 3^(
1/2)/144)^(1/2) - 4*3^(1/2)*exp(2*x) + 12*3^(1/2)*exp(x)*(1/72 - 3^(1/2)/14
4)^(1/2) + 7)*(1/72 - 3^(1/2)/144)^(1/2) - log(7*exp(2*x) - 4*3^(1/2) + 24*
exp(x)*(1/72 - 3^(1/2)/144)^(1/2) - 4*3^(1/2)*exp(2*x) - 12*3^(1/2)*exp(x)*
(1/72 - 3^(1/2)/144)^(1/2) + 7)*(1/72 - 3^(1/2)/144)^(1/2) + log(7*exp(2*x)
 + 4*3^(1/2) - 24*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 4*3^(1/2)*exp(2*x) -
12*3^(1/2)*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 7)*(3^(1/2)/144 + 1/72)^(1/2
) - log(7*exp(2*x) + 4*3^(1/2) + 24*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 4*3
^(1/2)*exp(2*x) + 12*3^(1/2)*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 7)*(3^(1/2
)/144 + 1/72)^(1/2)
```

3.216 $\int \operatorname{csch}(2x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \frac{1}{2}\arctan(\sinh(x))$$

```
[Out] 1/2*arctan(sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4373, 3855}

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \frac{1}{2}\arctan(\sinh(x))$$

```
[In] Int[Csch[2*x]*Sinh[x],x]
```

```
[Out] ArcTan[Sinh[x]]/2
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\arctan(\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \frac{1}{2}\arctan(\sinh(x))$$

```
[In] Integrate[Csch[2*x]*Sinh[x],x]
[Out] ArcTan[Sinh[x]]/2
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\arctan(\sinh(x))}{2}$	6
risch	$\frac{i\ln(e^x+i)}{2}-\frac{i\ln(e^x-i)}{2}$	20

```
[In] int(csch(2*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(sinh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \arctan(\cosh(x)+\sinh(x))$$

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="fricas")
[Out] arctan(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(2x)\,dx$$

```
[In] integrate(csch(2*x)*sinh(x),x)
[Out] Integral(sinh(x)*csch(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = -\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="maxima")
[Out] -arctan(e^(-x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \arctan\left(e^{x}\right)$$

```
[In] integrate(csch(2*x)*sinh(x),x, algorithm="giac")
[Out] arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.43

$$\int \operatorname{csch}(2x)\sinh(x)\,dx = \operatorname{atan}(e^{x})$$

```
[In] int(sinh(x)/sinh(2*x),x)
[Out] atan(exp(x))
```

3.217 $\int \operatorname{csch}(3x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*tanh(x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {209}

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[Csch[3*x]*Sinh[x],x]
[Out] ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\text{Subst}\left(\int \frac{1}{3+x^2}\,dx,x,\tanh(x)\right)\\&=\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Integrate[Csch[3*x]*Sinh[x],x]
[Out] ArcTan[Tanh[x]/Sqrt[3]]/Sqrt[3]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

method	result	size
risch	$\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

```
[In] int(csch(3*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] 1/6*I*3^(1/2)*ln(exp(2*x)+1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)+1/2-
1/2*I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(-\frac{3\,\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right)$$

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x)))
```

Sympy [F]

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(3x)\,dx$$

```
[In] integrate(csch(3*x)*sinh(x),x)
[Out] Integral(sinh(x)*csch(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}+1\right)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(-x)}-1\right)\right)$$

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-x) + 1)) - 1/3*sqrt(3)*arctan(1/3*sqr
t(3)*(2*e^(-x) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \operatorname{csch}(3x)\sinh(x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right)$$

```
[In] integrate(csch(3*x)*sinh(x),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \operatorname{csch}(3x)\sinh(x)\,dx=\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\mathrm{e}^{2\,x}+1\right)}{3}\right)}{3}$$

```
[In] int(sinh(x)/sinh(3*x),x)
[Out] (3^(1/2)*atan((3^(1/2)*(2*exp(2*x) + 1))/3))/3
```

3.218 $\int \mathbf{csch}(4x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctan(sinh(x))+1/4*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1107, 209}

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}} - \frac{1}{4}\arctan(\sinh(x))$$

```
[In] Int[Csch[4*x]*Sinh[x],x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{4+12x^2+8x^4}\,dx, x, \sinh(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{4+8x^2}\,dx, x, \sinh(x)\right) - 2\text{Subst}\left(\int \frac{1}{8+8x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{4}\arctan(\sinh(x)) + \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{2\sqrt{2}}$$

```
[In] Integrate[Csch[4*x]*Sinh[x],x]
[Out] -1/4*ArcTan[Sinh[x]] + ArcTan[Sqrt[2]*Sinh[x]]/(2*Sqrt[2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

method	result	size
risch	$\frac{i\ln(e^x-i)}{4}-\frac{i\ln(e^x+i)}{4}+\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^x-1\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^x-1\right)}{8}$	62

```
[In] int(csch(4*x)*sinh(x),x,method=_RETURNVERBOSE)
[Out] 1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/8*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*e
xp(x)-1)-1/8*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x) + \frac{1}{2}\sqrt{2}\sinh(x)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 + \sqrt{2}}{2(\cosh(x) - \sinh(x))}\right) - \frac{1}{2}\arctan(\cosh(x) + \sinh(x))$$

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/4*sqrt(2)
*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))/(cosh(x) - sinh(x))) - 1/2*arctan(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(4x)\,dx$$

```
[In] integrate(csch(4*x)*sinh(x),x)
[Out] Integral(sinh(x)*csch(4*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2e^{(-x)}\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2e^{(-x)}\right)\right) + \frac{1}{2}\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/4*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x))) + 1/2*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = -\frac{1}{8}\pi + \frac{1}{8}\sqrt{2}\left(\pi + 2\arctan\left(\frac{1}{2}\sqrt{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right) - \frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

```
[In] integrate(csch(4*x)*sinh(x),x, algorithm="giac")
[Out] -1/8*pi + 1/8*sqrt(2)*(pi + 2*arctan(1/2*sqrt(2)*(e^(2*x) - 1)*e^(-x))) - 1
/4*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}(4x)\sinh(x)\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^x}{2} + \frac{\sqrt{2}\,e^{3x}}{2}\right) + 2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^x}{2}\right)\right)}{8} - \frac{\mathrm{atan}(e^x)}{2}$$

```
[In] int(sinh(x)/sinh(4*x),x)
[Out] (2^(1/2)*(2*atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x))/2) + 2*atan((2^(1/
2)*exp(x))/2)))/8 - atan(exp(x))/2
```

3.219 $\int \operatorname{csch}(5x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 75

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

```
[Out] 1/10*arctan(tanh(x)/(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(t
anh(x)/(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1180, 209}

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)$$

```
[In] Int[Csch[5*x]*Sinh[x],x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 - 2*Sqrt[5]]])/5 - (Sqrt[(5 +
Sqrt[5])/2]*ArcTan[Tanh[x]/Sqrt[5 + 2*Sqrt[5]]])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-x^2}{5+10x^2+x^4}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{10}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5-2\sqrt{5}+x^2}\,dx, x, \tanh(x)\right)\\
&\quad -\frac{1}{10}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5+2\sqrt{5}+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5-2\sqrt{5}}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\tanh(x)}{\sqrt{5+2\sqrt{5}}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{\sqrt{5+\sqrt{5}}\arctan\left(\frac{\left(-3+\sqrt{5}\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\arctan\left(\frac{\left(3+\sqrt{5}\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

```
[In] Integrate[Csch[5*x]*Sinh[x],x]
[Out] (Sqrt[5 + Sqrt[5]]*ArcTan[((-3 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTan[((3 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}\left(32000\_Z^{4}+400\_Z^{2}+1\right)}\_R\ln\left(4000\_R^{3}-200\_R^{2}+\mathrm{e}^{2x}+30\_R-1\right)\right)$	41

```
[In] int(csch(5*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(4000*_R^3-200*_R^2+exp(2*x)+30*_R-1),_R=RootOf(32000*_Z^4+400*_
Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.24

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}-2\sqrt{5}+2\Big)-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}-2\sqrt{5}+2\Big)-\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}+2\sqrt{5}+2\Big)+\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\Big(8\cosh(x)^2+16\cosh(x)\sinh(x)+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}+2\sqrt{5}+2\Big)$$

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*sin
h(x)^2 + (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) - 2*sqrt(5) + 2) - 1
/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*sinh
(x)^2 - (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) - 2*sqrt(5) + 2) - 1/
```

```
20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*sinh
(x)^2 + (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) + 2*sqrt(5) + 2) + 1
/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*sin
h(x)^2 - (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) + 2*sqrt(5) + 2)
```

Sympy [F]

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(5x)\,dx$$

```
[In] integrate(csch(5*x)*sinh(x),x)
[Out] Integral(sinh(x)*csch(5*x), x)
```

Maxima [F]

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \int \operatorname{csch}(5\,x)\sinh(x)\,dx$$

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="maxima")
[Out] 1/10*(-1)^(3/5)*log((-1)^(1/5) + e^(-2*x)) + 1/10*sqrt(5)*(-1)^(3/5)*log((s
qrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*e^(-2*
x))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*
e^(-2*x)))/sqrt(2*sqrt(5) - 10) - 1/10*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)
^(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))/(sqrt(
5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*e^(-2*x))
)/sqrt(-2*sqrt(5) - 10) - 1/10*log(-(sqrt(5)*(-1)^(1/5) + (-1)^(1/5))*e^(-2
*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) + 1/10*l
og((sqrt(5)*(-1)^(1/5) - (-1)^(1/5))*e^(-2*x) + 2*(-1)^(2/5) + 2*e^(-4*x))/
(sqrt(5)*(-1)^(2/5) - (-1)^(2/5)) - 1/10*integrate((e^(3*x) + 2*e^(2*x) + 3
*e^x + 4)*e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(
(e^(3*x) - 2*e^(2*x) + 3*e^x - 4)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x +
1), x) + 1/10*log(e^x + 1) + 1/10*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = \frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4e^{(2x)}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4e^{(2x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

```
[In] integrate(csch(5*x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) - 1)/sqrt(2*sqrt(5)
 + 10)) - 1/10*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) + 1)/sqrt(-
2*sqrt(5) + 10))
```

Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.76

$$\int \operatorname{csch}(5x)\sinh(x)\,dx = 2\,\mathrm{atan}\left(\frac{\frac{e^{2x}}{5}+\frac{9\sqrt{5}}{25}+\frac{6\sqrt{5}\,e^{2x}}{25}+\frac{4}{5}}{5\,e^{2x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\frac{9\sqrt{5}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}}{5}+\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+\frac{9\sqrt{5}\,e^{2x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}}{5}}\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}} + \sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\left(\ln\left(\frac{9\sqrt{5}}{25}-\frac{e^{2x}}{5}+\frac{6\sqrt{5}\,e^{2x}}{25}-\frac{4}{5}-e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,5\mathrm{i}+\frac{\sqrt{5}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5} - \sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,1\mathrm{i}+\frac{\sqrt{5}\,e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5}\right)1\mathrm{i}-\ln\left(\frac{9\sqrt{5}}{25}-\frac{e^{2x}}{5}+\frac{6\sqrt{5}\,e^{2x}}{25}-\frac{4}{5} + e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,5\mathrm{i}-\frac{\sqrt{5}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5}+\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,1\mathrm{i}-\frac{\sqrt{5}\,e^{2x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}\,9\mathrm{i}}{5}\right)1\mathrm{i}\right)$$

```
[In] int(sinh(x)/sinh(5*x),x)
```

```
[Out] 2*atan((exp(2*x)/5 + (9*5^(1/2))/25 + (6*5^(1/2)*exp(2*x))/25 + 4/5)/(5*exp
(2*x)*(5^(1/2)/200 + 1/40)^(1/2) + (9*5^(1/2)*(5^(1/2)/200 + 1/40)^(1/2))/5
 + (5^(1/2)/200 + 1/40)^(1/2) + (9*5^(1/2)*exp(2*x)*(5^(1/2)/200 + 1/40)^(1
/2))/5))*(5^(1/2)/200 + 1/40)^(1/2) + (1/40 - 5^(1/2)/200)^(1/2)*(log((5^(1
/2)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5 - exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*
```

```
5i - exp(2*x)/5 + (9*5^(1/2))/25 - (1/40 - 5^(1/2)/200)^(1/2)*1i + (6*5^(1/
2)*exp(2*x))/25 + (5^(1/2)*exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*9i)/5 - 4/5)
*1i - log(exp(2*x)*(1/40 - 5^(1/2)/200)^(1/2)*5i - exp(2*x)/5 - (5^(1/2)*(1
/40 - 5^(1/2)/200)^(1/2)*9i)/5 + (9*5^(1/2))/25 + (1/40 - 5^(1/2)/200)^(1/2
)*1i + (6*5^(1/2)*exp(2*x))/25 - (5^(1/2)*exp(2*x)*(1/40 - 5^(1/2)/200)^(1/
2)*9i)/5 - 4/5)*1i)
```

3.220 $\int \operatorname{csch}(6x)\sinh(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \frac{1}{6}\arctan(\sinh(x)) + \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(sinh(x))+1/6*arctan(2*sinh(x))-1/6*arctan(2/3*sinh(x)*3^(1/2))*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2082, 209}

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \frac{1}{6}\arctan(\sinh(x)) + \frac{1}{6}\arctan(2\sinh(x)) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[Csch[6*x]*Sinh[x],x]
```

```
[Out] ArcTan[Sinh[x]]/6 + ArcTan[2*Sinh[x]]/6 - ArcTan[(2*Sinh[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{2\left(3+19x^2+32x^4+16x^6\right)}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+19x^2+32x^4+16x^6}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{3\left(1+x^2\right)}+\frac{2}{3\left(1+4x^2\right)}-\frac{2}{3+4x^2}\right)\,dx, x, \sinh(x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sinh(x)\right)\\
&\quad -\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{6}\arctan(\sinh(x))+\frac{1}{6}\arctan(2\sinh(x))-\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \operatorname{csch}(6x)\sinh(x)\,dx=\frac{1}{6}\left(\arctan(\sinh(x))+\arctan(2\sinh(x))-\sqrt{3}\arctan\left(\frac{2\sinh(x)}{\sqrt{3}}\right)\right)$$

```
[In] Integrate[Csch[6*x]*Sinh[x],x]
[Out] (ArcTan[Sinh[x]] + ArcTan[2*Sinh[x]] - Sqrt[3]*ArcTan[(2*Sinh[x])/Sqrt[3]])
/6
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

method	result	size
risch	$\frac{i\ln(\mathrm{e}^{x}+i)}{6}-\frac{i\ln(\mathrm{e}^{x}-i)}{6}+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{3}\,\mathrm{e}^{x}-1\right)}{12}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2x}+i\sqrt{3}\,\mathrm{e}^{x}-1\right)}{12}+\frac{i\ln(\mathrm{e}^{2x}+i\mathrm{e}^{x}-1)}{12}-\frac{i\ln(\mathrm{e}^{2x}-i\mathrm{e}^{x}-1)}{12}$	92

```
[In] int(csch(6*x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*ln(exp(x)+I)-1/6*I*ln(exp(x)-I)+1/12*I*3^(1/2)*ln(exp(2*x)-I*3^(1/2)*
exp(x)-1)-1/12*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2)*exp(x)-1)+1/12*I*ln(exp(2*x)
+I*exp(x)-1)-1/12*I*ln(exp(2*x)-I*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.97

$$\begin{aligned}
&\int \operatorname{csch}(6x)\sinh(x)\,dx \\
&= -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\cosh\left(x\right)+\frac{1}{3}\sqrt{3}\sinh\left(x\right)\right) \\
&\quad+\frac{1}{6}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh\left(x\right)^{2}+2\sqrt{3}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{3}\sinh\left(x\right)^{2}+2\sqrt{3}}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right) \\
&\quad-\frac{1}{6}\arctan\left(-\frac{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}{\cosh\left(x\right)-\sinh\left(x\right)}\right) \\
&\quad+\frac{1}{2}\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)
\end{aligned}$$

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*cosh(x) + 1/3*sqrt(3)*sinh(x)) + 1/6*sqrt(3
)*arctan(-1/3*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh
(x)^2 + 2*sqrt(3))/(cosh(x) - sinh(x))) - 1/6*arctan(-(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))) + 1/2*arctan(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \int \sinh(x)\operatorname{csch}(6x)\,dx$$

```
[In] integrate(csch(6*x)*sinh(x),x)
[Out] Integral(sinh(x)*csch(6*x), x)
```

Maxima [F]

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \int \operatorname{csch}(6x)\sinh(x)\,dx$$

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + 1/3*arctan(e^x) + integrate(1/6*(e^(3*x) + e^x)/(e^(4*x)
- e^(2*x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \frac{1}{6}\pi - \frac{1}{12}\sqrt{3}\left(\pi + 2\arctan\left(\frac{1}{3}\sqrt{3}(e^{(2x)} - 1)e^{(-x)}\right)\right) + \frac{1}{6}\arctan\left((e^{(2x)} - 1)e^{(-x)}\right) + \frac{1}{6}\arctan\left(\frac{1}{2}(e^{(2x)} - 1)e^{(-x)}\right)$$

```
[In] integrate(csch(6*x)*sinh(x),x, algorithm="giac")
[Out] 1/6*pi - 1/12*sqrt(3)*(pi + 2*arctan(1/3*sqrt(3)*(e^(2*x) - 1)*e^(-x))) + 1
/6*arctan((e^(2*x) - 1)*e^(-x)) + 1/6*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \operatorname{csch}(6x)\sinh(x)\,dx = \frac{\operatorname{atan}(e^x)}{3} - \frac{\operatorname{atan}(e^{-x} - e^x)}{6} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,e^x}{3} - \frac{\sqrt{3}\,e^{-x}}{3}\right)}{6}$$

```
[In] int(sinh(x)/sinh(6*x),x)
```

```
[Out] atan(exp(x))/3 - atan(exp(-x) - exp(x))/6 - (3^(1/2)*atan((3^(1/2)*exp(x))/
3 - (3^(1/2)*exp(-x))/3))/6
```

3.221 $\int \cosh(x)\sinh(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cosh(x)\sinh(2x)\,dx = \frac{2\cosh^3(x)}{3}$$

```
[Out] 2/3*cosh(x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4369}

$$\int \cosh(x)\sinh(2x)\,dx = \frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

```
[In] Int[Cosh[x]*Sinh[2*x],x]
```

```
[Out] Cosh[x]/2 + Cosh[3*x]/6
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \cosh(x)\sinh(2x)\,dx = \frac{\cosh(x)}{2} + \frac{1}{6}\cosh(3x)$$

```
[In] Integrate[Cosh[x]*Sinh[2*x],x]
[Out] Cosh[x]/2 + Cosh[3*x]/6
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	12
parallelrisch	$-\frac{2}{3}+\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	13
risch	$\frac{e^{3x}}{12}+\frac{e^{x}}{4}+\frac{e^{-x}}{4}+\frac{e^{-3x}}{12}$	24

```
[In] int(cosh(x)*sinh(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*cosh(x)+1/6*cosh(3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cosh(x)\sinh(2x)\,dx = \frac{1}{6}\cosh(x)^3 + \frac{1}{2}\cosh(x)\sinh(x)^2 + \frac{1}{2}\cosh(x)$$

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="fricas")
[Out] 1/6*cosh(x)^3 + 1/2*cosh(x)*sinh(x)^2 + 1/2*cosh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \cosh(x)\sinh(2x)\,dx = -\frac{\sinh(x)\sinh(2x)}{3} + \frac{2\cosh(x)\cosh(2x)}{3}$$

```
[In] integrate(cosh(x)*sinh(2*x),x)
[Out] -sinh(x)*sinh(2*x)/3 + 2*cosh(x)*cosh(2*x)/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \cosh(x)\sinh(2x)\,dx = \frac{1}{12}\left(3e^{(-2x)} + 1\right)e^{(3x)} + \frac{1}{4}e^{(-x)} + \frac{1}{12}e^{(-3x)}$$

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="maxima")
[Out] 1/12*(3*e^(-2*x) + 1)*e^(3*x) + 1/4*e^(-x) + 1/12*e^(-3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \cosh(x)\sinh(2x)\,dx = \frac{1}{12}\left(3e^{(2x)} + 1\right)e^{(-3x)} + \frac{1}{12}e^{(3x)} + \frac{1}{4}e^{x}$$

```
[In] integrate(cosh(x)*sinh(2*x),x, algorithm="giac")
[Out] 1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cosh(x)\sinh(2x)\,dx = \frac{2\cosh(x)^3}{3}$$

```
[In] int(sinh(2*x)*cosh(x),x)
[Out] (2*cosh(x)^3)/3
```

3.222 $\int \cosh(x)\sinh(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

```
[Out] 1/4*cosh(2*x)+1/8*cosh(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4369\}$

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

```
[In] Int[Cosh[x]*Sinh[3*x],x]
```

```
[Out] Cosh[2*x]/4 + Cosh[4*x]/8
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\cosh(2x) + \frac{1}{8}\cosh(4x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\sinh(3x)\,dx = \frac{\cosh^2(x)}{2} + \frac{1}{8}\cosh(4x)$$

```
[In] Integrate[Cosh[x]*Sinh[3*x],x]
[Out] Cosh[x]^2/2 + Cosh[4*x]/8
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}+\frac{e^{2x}}{8}+\frac{e^{-2x}}{8}+\frac{e^{-4x}}{16}$	26
parallelrisch	$\frac{\cosh(5x)-4\cosh(x)+3\cosh(3x)-4\cosh(2x)+6-2\cosh(4x)}{16\cosh(x)-16}$	38

```
[In] int(cosh(x)*sinh(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*cosh(2*x)+1/8*cosh(4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{8}\cosh(x)^4 + \frac{1}{8}\sinh(x)^4 + \frac{1}{4}\left(3\cosh(x)^2+1\right)\sinh(x)^2 + \frac{1}{4}\cosh(x)^2$$

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="fricas")
[Out] 1/8*cosh(x)^4 + 1/8*sinh(x)^4 + 1/4*(3*cosh(x)^2 + 1)*sinh(x)^2 + 1/4*cosh(
x)^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\sinh(3x)\,dx = -\frac{\sinh{(x)}\sinh{(3x)}}{8} + \frac{3\cosh{(x)}\cosh{(3x)}}{8}$$

```
[In] integrate(cosh(x)*sinh(3*x),x)
[Out] -sinh(x)*sinh(3*x)/8 + 3*cosh(x)*cosh(3*x)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{16}\left(2\,e^{(-2\,x)} + 1\right)e^{(4\,x)} + \frac{1}{8}\,e^{(-2\,x)} + \frac{1}{16}\,e^{(-4\,x)}$$

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="maxima")
[Out] 1/16*(2*e^(-2*x) + 1)*e^(4*x) + 1/8*e^(-2*x) + 1/16*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \cosh(x)\sinh(3x)\,dx = \frac{1}{16}\left(e^{(2\,x)} + e^{(-2\,x)}\right)^2 + \frac{1}{8}\,e^{(2\,x)} + \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(cosh(x)*sinh(3*x),x, algorithm="giac")
[Out] 1/16*(e^(2*x) + e^(-2*x))^2 + 1/8*e^(2*x) + 1/8*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \cosh(x)\sinh(3x)\,dx = \cosh(x)^4 - \frac{\cosh(x)^2}{2}$$

```
[In] int(sinh(3*x)*cosh(x),x)
[Out] cosh(x)^4 - cosh(x)^2/2
```

3.223 $\int \cosh(x)\sinh(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

```
[Out] 1/6*cosh(3*x)+1/10*cosh(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4369}

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

```
[In] Int[Cosh[x]*Sinh[4*x],x]
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{6}\cosh(3x) + \frac{1}{10}\cosh(5x)$$

```
[In] Integrate[Cosh[x]*Sinh[4*x],x]
[Out] Cosh[3*x]/6 + Cosh[5*x]/10
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	14
parallelrisch	$-\frac{4}{15}+\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	15
risch	$\frac{e^{5x}}{20}+\frac{e^{3x}}{12}+\frac{e^{-3x}}{12}+\frac{e^{-5x}}{20}$	26

```
[In] int(cosh(x)*sinh(4*x),x,method=_RETURNVERBOSE)
[Out] 1/6*cosh(3*x)+1/10*cosh(5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{10}\cosh(x)^5 + \frac{1}{2}\cosh(x)\sinh(x)^4 + \frac{1}{6}\cosh(x)^3 + \frac{1}{2}\left(2\cosh(x)^3 + \cosh(x)\right)\sinh(x)^2$$

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="fricas")
[Out] 1/10*cosh(x)^5 + 1/2*cosh(x)*sinh(x)^4 + 1/6*cosh(x)^3 + 1/2*(2*cosh(x)^3 +
 cosh(x))*sinh(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\sinh(4x)\,dx = -\frac{\sinh(x)\sinh(4x)}{15} + \frac{4\cosh(x)\cosh(4x)}{15}$$

```
[In] integrate(cosh(x)*sinh(4*x),x)
[Out] -sinh(x)*sinh(4*x)/15 + 4*cosh(x)*cosh(4*x)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{60}\left(5\,e^{(-2\,x)} + 3\right)e^{(5\,x)} + \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{20}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="maxima")
[Out] 1/60*(5*e^(-2*x) + 3)*e^(5*x) + 1/12*e^(-3*x) + 1/20*e^(-5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\sinh(4x)\,dx = \frac{1}{60}\left(5\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{20}\,e^{(5\,x)} + \frac{1}{12}\,e^{(3\,x)}$$

```
[In] integrate(cosh(x)*sinh(4*x),x, algorithm="giac")
[Out] 1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cosh(x)\sinh(4x)\,dx = \frac{4\cosh(x)^3\left(6\cosh(x)^2 - 5\right)}{15}$$

```
[In] int(sinh(4*x)*cosh(x),x)
[Out] (4*cosh(x)^3*(6*cosh(x)^2 - 5))/15
```

3.224 $\int \cosh(x)\sinh(mx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cosh(x)\sinh(mx)\,dx = -\frac{\cosh((1-m)x)}{2(1-m)} + \frac{\cosh((1+m)x)}{2(1+m)}$$

```
[Out] -1/2*cosh((1-m)*x)/(1-m)+1/2*cosh((1+m)*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5737, 2718}

$$\int \cosh(x)\sinh(mx)\,dx = \frac{\cosh((m+1)x)}{2(m+1)} - \frac{\cosh((1-m)x)}{2(1-m)}$$

```
[In] Int[Cosh[x]*Sinh[m*x],x]
```

```
[Out] -1/2*Cosh[(1 - m)*x]/(1 - m) + Cosh[(1 + m)*x]/(2*(1 + m))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}\sinh((1-m)x)+\frac{1}{2}\sinh((1+m)x)\right)\,dx \\
&= -\left(\frac{1}{2}\int \sinh((1-m)x)\,dx\right)+\frac{1}{2}\int \sinh((1+m)x)\,dx \\
&= -\frac{\cosh((1-m)x)}{2(1-m)}+\frac{\cosh((1+m)x)}{2(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cosh(x)\sinh(mx)\,dx = \frac{m\cosh(x)\cosh(mx)-\sinh(x)\sinh(mx)}{-1+m^2}$$

```
[In] Integrate[Cosh[x]*Sinh[m*x],x]
[Out] (m*Cosh[x]*Cosh[m*x] - Sinh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cosh(x(-1+m))}{-2+2m}+\frac{\cosh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(1+m)\cosh(x(-1+m))+\cosh((1+m)x)(-1+m)-2m}{2m^2-2}$	35
risch	$\frac{(m\,\mathrm{e}^{2x}-\mathrm{e}^{2x}+m+1)\mathrm{e}^{x(-1+m)}}{4(1+m)(-1+m)}+\frac{(m\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+m-1)\mathrm{e}^{-(1+m)x}}{4(1+m)(-1+m)}$	67

```
[In] int(cosh(x)*sinh(m*x),x,method=_RETURNVERBOSE)
[Out] 1/2/(-1+m)*cosh(x*(-1+m))+1/2*cosh((1+m)*x)/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(x)\sinh(mx)\,dx = \frac{m\cosh(mx)\cosh(x) - \sinh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2 - (m^2-1)\sinh(x)^2}$$

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(m*x)*cosh(x) - sinh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(x)\sinh(mx)\,dx = \begin{cases} -\frac{\cosh^2(x)}{2} & \text{for } m = -1 \\ \frac{\cosh^2(x)}{2} & \text{for } m = 1 \\ \frac{m\cosh(x)\cosh(mx)}{m^2-1} - \frac{\sinh(x)\sinh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)*sinh(m*x),x)
```

```
[Out] Piecewise((-cosh(x)**2/2, Eq(m, -1)), (cosh(x)**2/2, Eq(m, 1)), (m*cosh(x)*
cosh(m*x)/(m**2 - 1) - sinh(x)*sinh(m*x)/(m**2 - 1), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(x)\sinh(mx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cosh(x)\sinh(mx)\,dx = \frac{e^{(mx+x)}}{4\,(m+1)} + \frac{e^{(mx-x)}}{4\,(m-1)} + \frac{e^{(-mx+x)}}{4\,(m-1)} + \frac{e^{(-mx-x)}}{4\,(m+1)}$$

```
[In] integrate(cosh(x)*sinh(m*x),x, algorithm="giac")
```

```
[Out] 1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) + 1/4*e^(-m*x + x)/(m - 1
) + 1/4*e^(-m*x - x)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cosh(x)\sinh(mx)\,dx = -\frac{\sinh(m\,x)\,\sinh(x) - m\cosh(m\,x)\,\cosh(x)}{m^2-1}$$

```
[In] int(sinh(m*x)*cosh(x),x)
```

```
[Out] -(sinh(m*x)*sinh(x) - m*cosh(m*x)*cosh(x))/(m^2 - 1)
```

3.225 $\int \cosh(x)\cosh(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(x)\cosh(2x)\,dx = \frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

```
[Out] 1/2*sinh(x)+1/6*sinh(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4368\}$

$$\int \cosh(x)\cosh(2x)\,dx = \frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

```
[In] Int[Cosh[x]*Cosh[2*x],x]
```

```
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(2x)\,dx = \frac{\sinh(x)}{2} + \frac{1}{6}\sinh(3x)$$

```
[In] Integrate[Cosh[x]*Cosh[2*x],x]
[Out] Sinh[x]/2 + Sinh[3*x]/6
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
parallelrisch	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}$	12
risch	$\frac{e^{3x}}{12}+\frac{e^{x}}{4}-\frac{e^{-x}}{4}-\frac{e^{-3x}}{12}$	24

```
[In] int(cosh(x)*cosh(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sinh(x)+1/6*sinh(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cosh(x)\cosh(2x)\,dx = \frac{1}{6}\sinh\left(x\right)^{3} + \frac{1}{2}\left(\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)$$

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="fricas")
[Out] 1/6*sinh(x)^3 + 1/2*(cosh(x)^2 + 1)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(x)\cosh(2x)\,dx = -\frac{\sinh{(x)}\cosh{(2x)}}{3} + \frac{2\sinh{(2x)}\cosh{(x)}}{3}$$

```
[In] integrate(cosh(x)*cosh(2*x),x)
[Out] -sinh(x)*cosh(2*x)/3 + 2*sinh(2*x)*cosh(x)/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \cosh(x)\cosh(2x)\,dx = \frac{1}{12}\left(3e^{(-2x)} + 1\right)e^{(3x)} - \frac{1}{4}e^{(-x)} - \frac{1}{12}e^{(-3x)}$$

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="maxima")
[Out] 1/12*(3*e^(-2*x) + 1)*e^(3*x) - 1/4*e^(-x) - 1/12*e^(-3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cosh(x)\cosh(2x)\,dx = -\frac{1}{12}\left(3e^{(2x)} + 1\right)e^{(-3x)} + \frac{1}{12}e^{(3x)} + \frac{1}{4}e^{x}$$

```
[In] integrate(cosh(x)*cosh(2*x),x, algorithm="giac")
[Out] -1/12*(3*e^(2*x) + 1)*e^(-3*x) + 1/12*e^(3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cosh(x)\cosh(2x)\,dx = \frac{2\sinh(x)^3}{3} + \sinh(x)$$

```
[In] int(cosh(2*x)*cosh(x),x)
[Out] sinh(x) + (2*sinh(x)^3)/3
```

3.226 $\int \cosh(x)\cosh(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

```
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4368\}$

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

```
[In] Int[Cosh[x]*Cosh[3*x],x]
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{4}\sinh(2x) + \frac{1}{8}\sinh(4x)$$

```
[In] Integrate[Cosh[x]*Cosh[3*x],x]
[Out] Sinh[2*x]/4 + Sinh[4*x]/8
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$	14
parallelrisch	$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}$	14
risch	$\frac{e^{4x}}{16}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}-\frac{e^{-4x}}{16}$	26

```
[In] int(cosh(x)*cosh(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{2}\cosh(x)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^3 + \cosh(x)\right)\sinh(x)$$

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="fricas")
[Out] 1/2*cosh(x)*sinh(x)^3 + 1/2*(cosh(x)^3 + cosh(x))*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(3x)\,dx = -\frac{\sinh(x)\cosh(3x)}{8} + \frac{3\sinh(3x)\cosh(x)}{8}$$

```
[In] integrate(cosh(x)*cosh(3*x),x)
[Out] -sinh(x)*cosh(3*x)/8 + 3*sinh(3*x)*cosh(x)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(3x)\,dx = \frac{1}{16}\left(2e^{(-2x)}+1\right)e^{(4x)} - \frac{1}{8}e^{(-2x)} - \frac{1}{16}e^{(-4x)}$$

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="maxima")
[Out] 1/16*(2*e^(-2*x) + 1)*e^(4*x) - 1/8*e^(-2*x) - 1/16*e^(-4*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(3x)\,dx = -\frac{1}{16}\left(2e^{(2x)}+1\right)e^{(-4x)} + \frac{1}{16}e^{(4x)} + \frac{1}{8}e^{(2x)}$$

```
[In] integrate(cosh(x)*cosh(3*x),x, algorithm="giac")
[Out] -1/16*(2*e^(2*x) + 1)*e^(-4*x) + 1/16*e^(4*x) + 1/8*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(3x)\,dx = \frac{e^{-4x}\left(e^{2x}-1\right)\left(e^{2x}+1\right)^3}{16}$$

```
[In] int(cosh(3*x)*cosh(x),x)
[Out] (exp(-4*x)*(exp(2*x) - 1)*(exp(2*x) + 1)^3)/16
```

3.227 $\int \cosh(x)\cosh(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

```
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4368\}$

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

```
[In] Int[Cosh[x]*Cosh[4*x],x]
```

```
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{6}\sinh(3x) + \frac{1}{10}\sinh(5x)$$

```
[In] Integrate[Cosh[x]*Cosh[4*x],x]
[Out] Sinh[3*x]/6 + Sinh[5*x]/10
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
parallelrisch	$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}$	14
risch	$\frac{e^{5x}}{20}+\frac{e^{3x}}{12}-\frac{e^{-3x}}{12}-\frac{e^{-5x}}{20}$	26

```
[In] int(cosh(x)*cosh(4*x),x,method=_RETURNVERBOSE)
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{10}\sinh(x)^5 + \frac{1}{6}\left(6\cosh(x)^2+1\right)\sinh(x)^3 + \frac{1}{2}\left(\cosh(x)^4+\cosh(x)^2\right)\sinh(x)$$

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="fricas")
[Out] 1/10*sinh(x)^5 + 1/6*(6*cosh(x)^2 + 1)*sinh(x)^3 + 1/2*(cosh(x)^4 + cosh(x)
^2)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cosh(x)\cosh(4x)\,dx = -\frac{\sinh(x)\cosh(4x)}{15} + \frac{4\sinh(4x)\cosh(x)}{15}$$

```
[In] integrate(cosh(x)*cosh(4*x),x)
[Out] -sinh(x)*cosh(4*x)/15 + 4*sinh(4*x)*cosh(x)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(4x)\,dx = \frac{1}{60}\left(5\,e^{(-2\,x)} + 3\right)e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{20}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="maxima")
[Out] 1/60*(5*e^(-2*x) + 3)*e^(5*x) - 1/12*e^(-3*x) - 1/20*e^(-5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cosh(x)\cosh(4x)\,dx = -\frac{1}{60}\left(5\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{20}\,e^{(5\,x)} + \frac{1}{12}\,e^{(3\,x)}$$

```
[In] integrate(cosh(x)*cosh(4*x),x, algorithm="giac")
[Out] -1/60*(5*e^(2*x) + 3)*e^(-5*x) + 1/20*e^(5*x) + 1/12*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cosh(x)\cosh(4x)\,dx = \frac{8\sinh(x)^5}{5} + \frac{8\sinh(x)^3}{3} + \sinh(x)$$

```
[In] int(cosh(4*x)*cosh(x),x)
[Out] sinh(x) + (8*sinh(x)^3)/3 + (8*sinh(x)^5)/5
```

3.228 $\int \cosh(x) \cosh(mx) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cosh(x) \cosh(mx) \, dx = \frac{\sinh((1-m)x)}{2(1-m)} + \frac{\sinh((1+m)x)}{2(1+m)}$$

```
[Out] 1/2*sinh((1-m)*x)/(1-m)+1/2*sinh((1+m)*x)/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5733, 2717}

$$\int \cosh(x) \cosh(mx) \, dx = \frac{\sinh((1-m)x)}{2(1-m)} + \frac{\sinh((m+1)x)}{2(m+1)}$$

```
[In] Int[Cosh[x]*Cosh[m*x],x]
[Out] Sinh[(1 - m)*x]/(2*(1 - m)) + Sinh[(1 + m)*x]/(2*(1 + m))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5733

```
Int[Cosh[v_]^(p_.)*Cosh[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cosh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}\cosh((1-m)x)+\frac{1}{2}\cosh((1+m)x)\right) dx\\ &= \frac{1}{2}\int \cosh((1-m)x)\,dx+\frac{1}{2}\int \cosh((1+m)x)\,dx\\ &= \frac{\sinh((1-m)x)}{2(1-m)}+\frac{\sinh((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cosh(x)\cosh(mx)\,dx=\frac{-\cosh(mx)\sinh(x)+m\cosh(x)\sinh(mx)}{-1+m^2}$$

```
[In] Integrate[Cosh[x]*Cosh[m*x],x]
[Out] (-(Cosh[m*x]*Sinh[x]) + m*Cosh[x]*Sinh[m*x])/(-1 + m^2)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sinh(x(-1+m))}{-2+2m}+\frac{\sinh((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(1+m)\sinh(x(-1+m))+\sinh((1+m)x)(-1+m)}{2m^2-2}$	32
risch	$\frac{(m\,\mathrm{e}^{2x}-\mathrm{e}^{2x}+m+1)\,\mathrm{e}^{x(-1+m)}}{4(1+m)(-1+m)}-\frac{(m\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+m-1)\,\mathrm{e}^{-(1+m)x}}{4(1+m)(-1+m)}$	67

```
[In] int(cosh(x)*cosh(m*x),x,method=_RETURNVERBOSE)
[Out] 1/2/(-1+m)*sinh(x*(-1+m))+1/2*sinh((1+m)*x)/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cosh(x)\cosh(mx)\,dx = \frac{m\cosh(x)\sinh(mx) - \cosh(mx)\sinh(x)}{(m^2-1)\cosh(x)^2 - (m^2-1)\sinh(x)^2}$$

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="fricas")
```

```
[Out] (m*cosh(x)*sinh(m*x) - cosh(m*x)*sinh(x))/((m^2 - 1)*cosh(x)^2 - (m^2 - 1)*
sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \cosh(x)\cosh(mx)\,dx = \begin{cases} -\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m\sinh(mx)\cosh(x)}{m^2-1} - \frac{\sinh(x)\cosh(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)*cosh(m*x),x)
```

```
[Out] Piecewise((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2, Eq(m, -1)
| Eq(m, 1)), (m*sinh(m*x)*cosh(x)/(m**2 - 1) - sinh(x)*cosh(m*x)/(m**2 - 1)
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \cosh(x)\cosh(mx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cosh(x)\cosh(mx)\,dx = \frac{e^{(mx+x)}}{4\,(m+1)} + \frac{e^{(mx-x)}}{4\,(m-1)} - \frac{e^{(-mx+x)}}{4\,(m-1)} - \frac{e^{(-mx-x)}}{4\,(m+1)}$$

```
[In] integrate(cosh(x)*cosh(m*x),x, algorithm="giac")
[Out] 1/4*e^(m*x + x)/(m + 1) + 1/4*e^(m*x - x)/(m - 1) - 1/4*e^(-m*x + x)/(m - 1
) - 1/4*e^(-m*x - x)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cosh(x)\cosh(mx)\,dx = -\frac{\cosh(m\,x)\,\sinh(x) - m\sinh(m\,x)\,\cosh(x)}{m^2-1}$$

```
[In] int(cosh(m*x)*cosh(x),x)
[Out] -(cosh(m*x)*sinh(x) - m*sinh(m*x)*cosh(x))/(m^2 - 1)
```

3.229 $\int \cosh(x)\tanh(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \cosh(x)\tanh(2x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}} + \cosh(x)$$

```
[Out] cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 327, 213}

$$\int \cosh(x)\tanh(2x)\,dx = \cosh(x) - \frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}}$$

```
[In] Int[Cosh[x]*Tanh[2*x],x]
```

```
[Out] -(ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2]) + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^2}{-1+2x^2}\,dx, x, \cosh(x)\right) \\
&= 2\text{Subst}\left(\int \frac{x^2}{-1+2x^2}\,dx, x, \cosh(x)\right) \\
&= \cosh(x) + \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cosh(x)\right) \\
&= -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{\sqrt{2}} + \cosh(x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$
\int \cosh(x)\tanh(2x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{2}} + \cosh(x)
$$

```
[In] Integrate[Cosh[x]*Tanh[2*x],x]
[Out] -(ArcTanh[Sqrt[2] - I*Tanh[x/2]]/Sqrt[2]) - ArcTanh[Sqrt[2] + I*Tanh[x/2]]/
Sqrt[2] + Cosh[x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\cosh\left(x\right)-\frac{\operatorname{arctanh}\left(\cosh\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	16
default	$\cosh\left(x\right)-\frac{\operatorname{arctanh}\left(\cosh\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	16
risch	$\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}+\frac{\ln\left(1+{\mathrm e}^{2 x}-{\mathrm e}^{x} \sqrt{2}\right) \sqrt{2}}{4}-\frac{\ln\left(1+{\mathrm e}^{2 x}+{\mathrm e}^{x} \sqrt{2}\right) \sqrt{2}}{4}$	49

```
[In] int(cosh(x)*tanh(2*x),x,method=_RETURNVERBOSE)
[Out] cosh(x)-1/2*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \cosh(x)\tanh(2x)\,dx = \frac{2\,\cosh\left(x\right)^{2} + \left(\sqrt{2}\cosh\left(x\right) + \sqrt{2}\sinh\left(x\right)\right)\log\left(\frac{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}-2\,\sqrt{2}\cosh\left(x\right)+2}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}}\right) + 4\,\cosh\left(x\right)\sinh\left(x\right) + 2\,\sin}{4\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}$$

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="fricas")
[Out] 1/4*(2*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + sin
h(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) + 4*cosh(x)*sinh(x
) + 2*sinh(x)^2 + 2)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \cosh(x)\tanh(2x)\,dx = \int \cosh\left(x\right)\tanh\left(2x\right)\,dx$$

```
[In] integrate(cosh(x)*tanh(2*x),x)
[Out] Integral(cosh(x)*tanh(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \cosh(x)\tanh(2x)\,dx = -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="maxima")
[Out] -1/4*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \cosh(x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - e^{x}}{\sqrt{2} + e^{(-x)} + e^{x}}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(cosh(x)*tanh(2*x),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \cosh(x)\tanh(2x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} - \frac{\sqrt{2}\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right)}{4} + \frac{\sqrt{2}\ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)}{4}$$

```
[In] int(tanh(2*x)*cosh(x),x)
[Out] exp(-x)/2 + exp(x)/2 - (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/4 + (2^
(1/2)*log(exp(2*x) - 2^(1/2)*exp(x) + 1))/4
```

3.230 $\int \cosh(x)\tanh(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cosh(x)\tanh(3x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x)$$

```
[Out] cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {396, 212}

$$\int \cosh(x)\tanh(3x)\,dx = \cosh(x) - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[Cosh[x]*Tanh[3*x],x]
```

```
[Out] -(ArcTanh[(2*Cosh[x])/Sqrt[3]]/Sqrt[3]) + Cosh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \cosh(x)\right) \\ &= \cosh(x) - 2\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \cosh(x)\right) \\ &= -\frac{\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x) \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \cosh(x)\tanh(3x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} + \cosh(x)$$

```
[In] Integrate[Cosh[x]*Tanh[3*x],x]
[Out] -(ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]]/Sqrt[3]) - ArcTanh[(2 + I*Tanh[x/2])/S
qrt[3]]/Sqrt[3] + Cosh[x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\cosh(x) - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
default	$\cosh(x) - \frac{\operatorname{arctanh}\left(\frac{2\cosh(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$\frac{e^{x}}{2} + \frac{e^{-x}}{2} + \frac{\ln\left(1+e^{2x}-e^{x}\sqrt{3}\right)\sqrt{3}}{6} - \frac{\ln\left(1+e^{2x}+e^{x}\sqrt{3}\right)\sqrt{3}}{6}$	49

```
[In] int(cosh(x)*tanh(3*x),x,method=_RETURNVERBOSE)
[Out] cosh(x)-1/3*arctanh(2/3*cosh(x)*3^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \cosh(x)\tanh(3x)\,dx = \frac{3\cosh(x)^2 + \left(\sqrt{3}\cosh(x) + \sqrt{3}\sinh(x)\right)\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right) + 6\cosh(x)\sinh(x) + 3\,s}{6\left(\cosh(x)+\sinh(x)\right)}$$

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 + 2
*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) + 6*co
sh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \cosh(x)\tanh(3x)\,dx = \int \cosh(x)\tanh(3x)\,dx$$

```
[In] integrate(cosh(x)*tanh(3*x),x)
```

```
[Out] Integral(cosh(x)*tanh(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\begin{aligned}\int \cosh(x)\tanh(3x)\,dx = &-\frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right)\\ &+ \frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}e^{(-x)} + e^{(-2x)} + 1\right)\\ &- \frac{1}{12}\sqrt{3}\log\left(\sqrt{3}e^{x} + e^{(2x)} + 1\right) + \frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}e^{x} + e^{(2x)} + 1\right)\\ &+ \frac{1}{6}\arctan\left(\sqrt{3} + 2e^{(-x)}\right) + \frac{1}{6}\arctan\left(\sqrt{3} + 2e^{x}\right)\\ &+ \frac{1}{6}\arctan\left(-\sqrt{3} + 2e^{(-x)}\right) + \frac{1}{6}\arctan\left(-\sqrt{3} + 2e^{x}\right)\\ &+ \frac{1}{3}\arctan\left(e^{(-x)}\right) + \frac{1}{3}\arctan\left(e^{x}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}\end{aligned}$$

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(sqrt(3)*e^(-x) + e^(-2*x) + 1) + 1/12*sqrt(3)*log(-sqrt(3
)*e^(-x) + e^(-2*x) + 1) - 1/12*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/
12*sqrt(3)*log(-sqrt(3)*e^x + e^(2*x) + 1) + 1/6*arctan(sqrt(3) + 2*e^(-x))
 + 1/6*arctan(sqrt(3) + 2*e^x) + 1/6*arctan(-sqrt(3) + 2*e^(-x)) + 1/6*arct
an(-sqrt(3) + 2*e^x) + 1/3*arctan(e^(-x)) + 1/3*arctan(e^x) + 1/2*e^(-x) +
1/2*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \cosh(x)\tanh(3x)\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

```
[In] integrate(cosh(x)*tanh(3*x),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \cosh(x)\tanh(3x)\,dx = \frac{e^{-x}}{2}+\frac{e^{x}}{2}+\frac{\sqrt{3}\,\ln\left(\frac{e^{2x}}{3}-\frac{\sqrt{3}\,e^{x}}{3}+\frac{1}{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\frac{e^{2x}}{3}+\frac{\sqrt{3}\,e^{x}}{3}+\frac{1}{3}\right)}{6}$$

```
[In] int(tanh(3*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 + (3^(1/2)*log(exp(2*x)/3 - (3^(1/2)*exp(x))/3 + 1/3))
/6 - (3^(1/2)*log(exp(2*x)/3 + (3^(1/2)*exp(x))/3 + 1/3))/6
```

3.231 $\int \cosh(x)\tanh(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 69

$$\int \cosh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right) + \cosh(x)$$

```
[Out] cosh(x)-1/4*arctanh(2*cosh(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arct
anh(2*cosh(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {12, 1293, 1180, 213}

$$\int \cosh(x)\tanh(4x)\,dx = -\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right) + \cosh(x)$$

```
[In] Int[Cosh[x]*Tanh[4*x],x]
```

```
[Out] -1/4*(Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 - Sqrt[2]]]) - (Sqrt[2 +
 Sqrt[2]]*ArcTanh[(2*Cosh[x])/Sqrt[2 + Sqrt[2]]])/4 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{4x^2(-1+2x^2)}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\
&= 4\text{Subst}\left(\int \frac{x^2(-1+2x^2)}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\
&= \cosh(x) - \frac{1}{2}\text{Subst}\left(\int \frac{2-8x^2}{1-8x^2+8x^4}\,dx, x, \cosh(x)\right)\\
&= \cosh(x) - \left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)\\
&\quad + \left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{4}\sqrt{2-\sqrt{2}}\,\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\,\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{2}}}\right) + \cosh(x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \cosh(x)\tanh(4x)\,dx = \cosh(x) + \frac{1}{16}\text{RootSum}\Bigg[1 + \#1^8\&, \frac{-x - 2\log\left(-\cosh\left(\frac{x}{2}\right) - \sinh\left(\frac{x}{2}\right) + \cosh\left(\frac{x}{2}\right)\#1 - \sinh\left(\frac{x}{2}\right)\#1\right) + x\#1^6 + 2\log\left(-\cosh\left(\frac{x}{2}\right) - \right.}{\#1^7}$$

```
[In] Integrate[Cosh[x]*Tanh[4*x],x]
```

```
[Out] Cosh[x] + RootSum[1 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/
2]*#1 - Sinh[x/2]*#1] + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#
1 - Sinh[x/2]*#1]*#1^6)/#1^7 & ]/16
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\left(\sum_{\_R=\text{RootOf}\left(2048\_Z^{4}-128\_Z^{2}+1\right)}\_R\ln\left(-8\_R\,e^{x}+e^{2x}+1\right)\right)$	42

```
[In] int(cosh(x)*tanh(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+sum(_R*ln(-8*_R*exp(x)+exp(2*x)+1),_R=RootOf(2048*_Z
^4-128*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.09

$$\int \cosh(x)\tanh(4x)\,dx = -\frac{\sqrt{\sqrt{2}+2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + \sqrt{\sqrt{2}+2}(\cosh(x)+\text{s}\right.}{}$$

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + sqrt(sqrt(2) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(2)
+ 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sq
rt(sqrt(2) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(2) + 2)*(cosh(x) + si
nh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(2) + 2)*(
cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(2) + 2)*(cosh(x) + sinh(x))*log(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(2) + 2)*(cosh(x) + sinh(x)
) + 1) - 4*cosh(x)^2 - 8*cosh(x)*sinh(x) - 4*sinh(x)^2 - 4)/(cosh(x) + sinh
(x))
```

Sympy [F]

$$\int \cosh(x)\tanh(4x)\,dx = \int \cosh{(x)}\tanh{(4x)}\,dx$$

```
[In] integrate(cosh(x)*tanh(4*x),x)
[Out] Integral(cosh(x)*tanh(4*x), x)
```

Maxima [F]

$$\int \cosh(x)\tanh(4x)\,dx = \int \cosh{(x)}\tanh{(4\,x)}\,dx$$

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^x)/(e^(8*x) + 1), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(49) = 98.

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \cosh(x)\tanh(4x)\,dx = &-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}+e^{(-x)}+e^{x}\right)\\&-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+e^{(-x)}+e^{x}\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}+e^{(-x)}+e^{x}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\end{aligned}$$

```
[In] integrate(cosh(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(sqr
t(2) + 2)*log(-sqrt(sqrt(2) + 2) + e^(-x) + e^x) - 1/8*sqrt(-sqrt(2) + 2)*l
og(sqrt(-sqrt(2) + 2) + e^(-x) + e^x) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(-s
qrt(2) + 2) + e^(-x) + e^x) + 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.93

$$\int \cosh(x)\tanh(4x)\,dx = \frac{e^{-x}}{2} + \frac{e^x}{2} + \ln\left(e^{2x} - 8\,e^x\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} + 1\right)\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} - \ln\left(e^{2x} + 8\,e^x\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} + 1\right)\sqrt{\frac{1}{32} - \frac{\sqrt{2}}{64}} + \ln\left(e^{2x} - 8\,e^x\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}} + 1\right)\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}} - \ln\left(e^{2x} + 8\,e^x\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}} + 1\right)\sqrt{\frac{\sqrt{2}}{64} + \frac{1}{32}}$$

```
[In] int(tanh(4*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 + log(exp(2*x) - 8*exp(x)*(1/32 - 2^(1/2)/64)^(1/2) +
1)*(1/32 - 2^(1/2)/64)^(1/2) - log(exp(2*x) + 8*exp(x)*(1/32 - 2^(1/2)/64)^
(1/2) + 1)*(1/32 - 2^(1/2)/64)^(1/2) + log(exp(2*x) - 8*exp(x)*(2^(1/2)/64
+ 1/32)^(1/2) + 1)*(2^(1/2)/64 + 1/32)^(1/2) - log(exp(2*x) + 8*exp(x)*(2^(
1/2)/64 + 1/32)^(1/2) + 1)*(2^(1/2)/64 + 1/32)^(1/2)
```

3.232 $\int \cosh(x)\tanh(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 82

$$\int \cosh(x)\tanh(5x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right) + \cosh(x)$$

```
[Out] cosh(x)-1/10*arctanh(1/5*cosh(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-1/10*arctanh(2*cosh(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1690, 1180, 213}

$$\int \cosh(x)\tanh(5x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right) + \cosh(x)$$

```
[In] Int[Cosh[x]*Tanh[5*x],x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cosh[x]]) - (Sq
rt[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cosh[x]])/5 + Cosh[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \cosh(x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{4(1-2x^2)}{5-20x^2+16x^4}\right)\,dx, x, \cosh(x)\right)\\
&= \cosh(x) - 4\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \cosh(x)\right)\\
&= \cosh(x) + \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \cosh(x)\right)\\
&\quad + \frac{1}{5}\left(4\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\text{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cosh(x)\right)\\
&\quad - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cosh(x)\right) + \cosh(x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.04

$$\int \cosh(x)\tanh(5x)\,dx = \cosh(x) + \frac{1}{4}\text{RootSum}\Big[1 - \#1^2 + \#1^4 - \#1^6 + \#1^8\&, \frac{-x - 2\log\left(-\cosh\left(\frac{x}{2}\right) - \sinh\left(\frac{x}{2}\right) + \cosh\left(\frac{x}{2}\right)\#1 - \sinh\left(\frac{x}{2}\right)\#1\right) + x\#1^2 + 2\log\left(-\cosh\left(\frac{x}{2}\right) - s\right.}{}$$

```
[In] Integrate[Cosh[x]*Tanh[5*x],x]
```

```
[Out] Cosh[x] + RootSum[1 - #1^2 + #1^4 - #1^6 + #1^8 & , (-x - 2*Log[-Cosh[x/2]
- Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] + x*#1^2 + 2*Log[-Cosh[x/2] - Si
nh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - x*#1^4 - 2*Log[-Cosh[x/2] - S
inh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + x*#1^6 + 2*Log[-Cosh[x/2] -
Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1 + 2*#1^3 - 3*#1^5 + 4*#
1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{e^x}{2} + \frac{e^{-x}}{2} + \left(\sum_{\_R=\text{RootOf}(2000\_Z^4-100\_Z^2+1)} \_R\ln\left(-10\_R\,e^x + e^{2x} + 1\right)\right)$	42

```
[In] int(cosh(x)*tanh(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+sum(_R*ln(-10*_R*exp(x)+exp(2*x)+1),_R=RootOf(2000*_
Z^4-100*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.57

$$\int \cosh(x)\tanh(5x)\,dx = -\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\sqrt{5}+5}\log\left(2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}+\left(\sqrt{2}\cosh\left(\right.\right.\right.}{}$$

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="fricas")
```

```
[Out] -1/20*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) + 5)*log(2*cosh(x)^
2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*s
qrt(sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sqrt(5) +
5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + s
qrt(2)*sinh(x))*sqrt(sqrt(5) + 5) + 2) + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x)
)*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + (s
qrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2) - (sqrt(2)*cosh(x
) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x)
 + 2*sinh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(-sqrt(5) + 5) + 2
) - 10*cosh(x)^2 - 20*cosh(x)*sinh(x) - 10*sinh(x)^2 - 10)/(cosh(x) + sinh(
x))
```

Sympy [F]

$$\int \cosh(x)\tanh(5x)\,dx = \int \cosh\left(x\right)\tanh\left(5x\right)\,dx$$

```
[In] integrate(cosh(x)*tanh(5*x),x)
```

```
[Out] Integral(cosh(x)*tanh(5*x), x)
```

Maxima [F]

$$\int \cosh(x)\tanh(5x)\,dx = \int \cosh\left(x\right)\tanh\left(5\,x\right)\,dx$$

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate(2*(e^(7*x) - e^(5*x) + e^(3*x) - e
^x)/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(54) = 108$.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

$$\int \cosh(x)\tanh(5x)\,dx = -\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right) + \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(-\sqrt{\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right) - \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right) + \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(-\sqrt{-\frac{1}{2}\sqrt{5}+\frac{5}{2}}+e^{(-x)}+e^{x}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

```
[In] integrate(cosh(x)*tanh(5*x),x, algorithm="giac")
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/
20*sqrt(2*sqrt(5) + 10)*log(-sqrt(1/2*sqrt(5) + 5/2) + e^(-x) + e^x) - 1/20
*sqrt(-2*sqrt(5) + 10)*log(sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/20*
sqrt(-2*sqrt(5) + 10)*log(-sqrt(-1/2*sqrt(5) + 5/2) + e^(-x) + e^x) + 1/2*e
^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \cosh(x)\tanh(5x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} + \ln\left(4\,e^{2\,x} - 40\,e^{x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}+4\right)\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}} - \ln\left(4\,e^{2\,x} + 40\,e^{x}\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}}+4\right)\sqrt{\frac{1}{40}-\frac{\sqrt{5}}{200}} + \ln\left(4\,e^{2\,x} - 40\,e^{x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+4\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}} - \ln\left(4\,e^{2\,x} + 40\,e^{x}\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}+4\right)\sqrt{\frac{\sqrt{5}}{200}+\frac{1}{40}}$$

```
[In] int(tanh(5*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 + log(4*exp(2*x) - 40*exp(x)*(1/40 - 5^(1/2)/200)^(1/2
) + 4)*(1/40 - 5^(1/2)/200)^(1/2) - log(4*exp(2*x) + 40*exp(x)*(1/40 - 5^(1
/2)/200)^(1/2) + 4)*(1/40 - 5^(1/2)/200)^(1/2) + log(4*exp(2*x) - 40*exp(x)
*(5^(1/2)/200 + 1/40)^(1/2) + 4)*(5^(1/2)/200 + 1/40)^(1/2) - log(4*exp(2*x
) + 40*exp(x)*(5^(1/2)/200 + 1/40)^(1/2) + 4)*(5^(1/2)/200 + 1/40)^(1/2)
```

3.233 $\int \cosh(x)\tanh(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 87

$$\int \cosh(x)\tanh(6x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right) + \cosh(x)$$

```
[Out] cosh(x)-1/6*arctanh(cosh(x)*2^(1/2))*2^(1/2)-1/6*arctanh(2*cosh(x)/(1/2*6^(
1/2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))-1/6*arctanh(2*cosh(x)/(1/2*6^(
1/2)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {12, 6874, 2098, 213, 1180}

$$\int \cosh(x)\tanh(6x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}} - \frac{1}{6}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{6}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right) + \cosh(x)$$

```
[In] Int[Cosh[x]*Tanh[6*x],x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Cosh[x]]/Sqrt[2] - (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cosh[
x])/Sqrt[2 - Sqrt[3]]])/6 - (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cosh[x])/Sqrt[2 +
 Sqrt[3]]])/6 + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^2(-3+16x^2-16x^4)}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right) \\
&= 2\text{Subst}\left(\int \frac{x^2(-3+16x^2-16x^4)}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right) \\
&= 2\text{Subst}\left(\int \left(\frac{1}{2}-\frac{1-12x^2+16x^4}{2\,(1-18x^2+48x^4-32x^6)}\right)\,dx, x, \cosh(x)\right) \\
&= \cosh(x)-\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \cosh(x)\right) \\
&= \cosh(x)-\text{Subst}\left(\int \left(-\frac{1}{3\,(-1+2x^2)}-\frac{2(-1+8x^2)}{3\,(1-16x^2+16x^4)}\right)\,dx, x, \cosh(x)\right)
\end{aligned}$$

$$= \cosh(x)+\frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\cosh(x)\right)+\frac{2}{3}\mathrm{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx,x,\cosh(x)\right)$$

$$= -\frac{\mathrm{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}+\cosh(x)+\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx,x,\cosh(x)\right)+\frac{1}{3}\left(4\left(2+\sqrt{3}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx,x,\cosh(x)\right)$$

$$= -\frac{\mathrm{arctanh}\left(\sqrt{2}\cosh(x)\right)}{3\sqrt{2}}-\frac{1}{6}\sqrt{2-\sqrt{3}}\mathrm{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{6}\sqrt{2+\sqrt{3}}\mathrm{arctanh}\left(\frac{2\cosh(x)}{\sqrt{2+\sqrt{3}}}\right)+\cosh(x)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.23

$$\int \cosh(x)\tanh(6x)\,dx$$

$$= \frac{1}{24}\Bigg(-4\left(\sqrt{2}\mathrm{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right)+\sqrt{2}\mathrm{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)-6\cosh(x)\right)+\mathrm{RootSum}\Bigg[1-\#1^4+\#1^8\&,\frac{-2x-4\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)-x\#1^2-2\log\left(-\cosh\left(\frac{x}{2}\right)-\right.}{}$$

```
[In] Integrate[Cosh[x]*Tanh[6*x],x]
```

```
[Out] (-4*(Sqrt[2]*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + Sqrt[2]*ArcTanh[Sqrt[2] + I*T
anh[x/2]] - 6*Cosh[x]) + RootSum[1 - #1^4 + #1^8 & , (-2*x - 4*Log[-Cosh[x/
2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] - x*#1^2 - 2*Log[-Cosh[x/2] -
 Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + x*#1^4 + 2*Log[-Cosh[x/2]
- Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 2*x*#1^6 + 4*Log[-Cosh[x/
2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-#1^3 + 2*#1^7) & ])/2
4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

method	result
risch	$\frac{e^x}{2}+\frac{e^{-x}}{2}+\left(\sum_{\_R=\mathrm{RootOf}(20736\_Z^4-576\_Z^2+1)}\_R\ln\left(-12\_R\,e^x+e^{2x}+1\right)\right)+\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{12}-\frac{\ln(}{}$

```
[In] int(cosh(x)*tanh(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+sum(_R*ln(-12*_R*exp(x)+exp(2*x)+1),_R=RootOf(20736*
_Z^4-576*_Z^2+1))+1/12*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/12*ln(1+exp(
2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.97

$$\int \cosh(x)\tanh(6x)\,dx =$$

$$-\frac{\sqrt{\sqrt{3}+2}(\cosh(x)+\sinh(x))\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+\sqrt{\sqrt{3}+2}(\cosh(x)+\mathrm{s}\right.}{}$$

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) - sqrt(sqrt(3)
 + 2)*(cosh(x) + sinh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - s
qrt(sqrt(3) + 2)*(cosh(x) + sinh(x)) + 1) + sqrt(-sqrt(3) + 2)*(cosh(x) + s
inh(x))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(-sqrt(3) + 2)*
(cosh(x) + sinh(x)) + 1) - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x))*log(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - sqrt(-sqrt(3) + 2)*(cosh(x) + sinh(x
)) + 1) - 6*cosh(x)^2 - (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2
+ sinh(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 12*cosh(x)*
sinh(x) - 6*sinh(x)^2 - 6)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \cosh(x)\tanh(6x)\,dx = \int \cosh(x)\tanh(6x)\,dx$$

```
[In] integrate(cosh(x)*tanh(6*x),x)
[Out] Integral(cosh(x)*tanh(6*x), x)
```

Maxima [F]

$$\int \cosh(x)\tanh(6x)\,dx = \int \cosh(x)\tanh(6x)\,dx$$

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/12*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) + 1/
12*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + 1/2*integrate(2/3*(2*e^(7*x) +
 e^(5*x) - e^(3*x) - 2*e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \cosh(x)\tanh(6x)\,dx = &-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &+\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}+e^{(-x)}+e^{x}\right)\\ &+\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}\end{aligned}$$

```
[In] integrate(cosh(x)*tanh(6*x),x, algorithm="giac")
[Out] -1/24*(sqrt(6) + sqrt(2))*log(1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) - 1
/24*(sqrt(6) - sqrt(2))*log(1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1/2
4*(sqrt(6) - sqrt(2))*log(-1/2*sqrt(6) + 1/2*sqrt(2) + e^(-x) + e^x) + 1/24
*(sqrt(6) + sqrt(2))*log(-1/2*sqrt(6) - 1/2*sqrt(2) + e^(-x) + e^x) + 1/12*
sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e^(-x
) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.95

$$\int \cosh(x)\tanh(6x)\,dx = \frac{e^{-x}}{2} + \frac{e^{x}}{2} - \frac{\sqrt{2}\,\ln\left(e^{2x} + \sqrt{2}\,e^{x} + 1\right)}{12} + \frac{\sqrt{2}\,\ln\left(e^{2x} - \sqrt{2}\,e^{x} + 1\right)}{12} + \ln\left(e^{2x} - 12\,e^{x}\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} + 1\right)\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} - \ln\left(e^{2x} + 12\,e^{x}\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} + 1\right)\sqrt{\frac{1}{72} - \frac{\sqrt{3}}{144}} + \ln\left(e^{2x} - 12\,e^{x}\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}} + 1\right)\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}} - \ln\left(e^{2x} + 12\,e^{x}\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}} + 1\right)\sqrt{\frac{\sqrt{3}}{144} + \frac{1}{72}}$$

```
[In] int(tanh(6*x)*cosh(x),x)
```

```
[Out] exp(-x)/2 + exp(x)/2 - (2^(1/2)*log(exp(2*x) + 2^(1/2)*exp(x) + 1))/12 + (2
^(1/2)*log(exp(2*x) - 2^(1/2)*exp(x) + 1))/12 + log(exp(2*x) - 12*exp(x)*(1
/72 - 3^(1/2)/144)^(1/2) + 1)*(1/72 - 3^(1/2)/144)^(1/2) - log(exp(2*x) + 1
2*exp(x)*(1/72 - 3^(1/2)/144)^(1/2) + 1)*(1/72 - 3^(1/2)/144)^(1/2) + log(e
xp(2*x) - 12*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 1)*(3^(1/2)/144 + 1/72)^(1
/2) - log(exp(2*x) + 12*exp(x)*(3^(1/2)/144 + 1/72)^(1/2) + 1)*(3^(1/2)/144
 + 1/72)^(1/2)
```

3.234 $\int \cosh(x)\coth(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cosh(x)\coth(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x)) + \cosh(x)$$

```
[Out] -1/2*arctanh(cosh(x))+cosh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 396, 212}

$$\int \cosh(x)\coth(2x)\,dx = \cosh(x) - \frac{1}{2}\text{arctanh}(\cosh(x))$$

```
[In] Int[Cosh[x]*Coth[2*x],x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]] + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{-1+2x^2}{2(1-x^2)}\,dx, x, \cosh(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x^2}{1-x^2}\,dx, x, \cosh(x)\right)\right) \\
&= \cosh(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{2}\text{arctanh}(\cosh(x)) + \cosh(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \cosh(x)\coth(2x)\,dx = \cosh(x) - \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Cosh[x]*Coth[2*x],x]
[Out] Cosh[x] - Log[Cosh[x/2]]/2 + Log[Sinh[x/2]]/2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

method	result	size
default	$\cosh(x) - 2\,\text{arctanh}(e^x) - \frac{\ln(\tanh(\frac{x}{2}))}{2}$	16
risch	$\frac{e^x}{2} + \frac{e^{-x}}{2} + \frac{\ln(e^x-1)}{2} - \frac{\ln(e^x+1)}{2}$	26

```
[In] int(cosh(x)*coth(2*x),x,method=_RETURNVERBOSE)
[Out] cosh(x)-2*arctanh(exp(x))-1/2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 5.20

$$\int \cosh(x)\coth(2x)\,dx$$
$$= \frac{\cosh(x)^2 - (\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x) + 1) + (\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x}{2(\cosh(x) + \sinh(x))}$$

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="fricas")
[Out] 1/2*(cosh(x)^2 - (cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + (cosh(x)
+ sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/
(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \cosh(x)\coth(2x)\,dx = \int \cosh(x)\coth(2x)\,dx$$

```
[In] integrate(cosh(x)*coth(2*x),x)
[Out] Integral(cosh(x)*coth(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \cosh(x)\coth(2x)\,dx = \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x} - \frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="maxima")
[Out] 1/2*e^(-x) + 1/2*e^x - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \cosh(x)\coth(2x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{2}\,\log\left(e^{x}+1\right) + \frac{1}{2}\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(cosh(x)*coth(2*x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \cosh(x)\coth(2x)\,dx = \frac{\ln\left(1-\mathrm{e}^{x}\right)}{2} - \frac{\ln\left(-\mathrm{e}^{x}-1\right)}{2} + \frac{\mathrm{e}^{-x}}{2} + \frac{\mathrm{e}^{x}}{2}$$

```
[In] int(coth(2*x)*cosh(x),x)
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + exp(-x)/2 + exp(x)/2
```

3.235 $\int \cosh(x)\coth(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \cosh(x)\coth(3x)\,dx = \cosh(x) + \frac{1}{6}\log(1-2\cosh(x)) + \frac{1}{6}\log(1-\cosh(x)) - \frac{1}{6}\log(1+\cosh(x)) - \frac{1}{6}\log(1+2\cosh(x))$$

```
[Out] cosh(x)+1/6*ln(1-2*cosh(x))+1/6*ln(1-cosh(x))-1/6*ln(1+cosh(x))-1/6*ln(1+2*
cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1293, 1175, 630, 31}

$$\int \cosh(x)\coth(3x)\,dx = \cosh(x) + \frac{1}{6}\log(1-2\cosh(x)) + \frac{1}{6}\log(1-\cosh(x)) - \frac{1}{6}\log(\cosh(x)+1) - \frac{1}{6}\log(2\cosh(x)+1)$$

```
[In] Int[Cosh[x]*Coth[3*x],x]
```

```
[Out] Cosh[x] + Log[1 - 2*Cosh[x]]/6 + Log[1 - Cosh[x]]/6 - Log[1 + Cosh[x]]/6 -
Log[1 + 2*Cosh[x]]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2(3-4x^2)}{1-5x^2+4x^4}\,dx, x, \cosh(x)\right)\\
&= \cosh(x)+\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx, x, \cosh(x)\right)\\
&= \cosh(x)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx, x, \cosh(x)\right)\\
&\quad+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx, x, \cosh(x)\right)\\
&= \cosh(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \cosh(x)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx, x, \cosh(x)\right)\\
&\quad-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{1}{2}+x}\,dx, x, \cosh(x)\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cosh(x)\right)\\
&= \cosh(x)+\frac{1}{6}\log(1-2\cosh(x))+\frac{1}{6}\log(1-\cosh(x))-\frac{1}{6}\log(1+\cosh(x))-\frac{1}{6}\log(1\\
&\quad+2\cosh(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \cosh(x)\coth(3x)\,dx = \cosh(x) - \frac{1}{3}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{6}\log(1-2\cosh(x)) - \frac{1}{6}\log(1+2\cosh(x)) + \frac{1}{3}\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Cosh[x]*Coth[3*x],x]
```

```
[Out] Cosh[x] - Log[Cosh[x/2]]/3 + Log[1 - 2*Cosh[x]]/6 - Log[1 + 2*Cosh[x]]/6 +
Log[Sinh[x/2]]/3
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^{x}+1)}{3}+\frac{\ln(e^{x}-1)}{3}+\frac{\ln(e^{2x}-e^{x}+1)}{6}-\frac{\ln(1+e^{x}+e^{2x})}{6}$	50

```
[In] int(cosh(x)*coth(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)-1/3*ln(exp(x)+1)+1/3*ln(exp(x)-1)+1/6*ln(exp(2*x)-ex
p(x)+1)-1/6*ln(1+exp(x)+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(37) = 74.

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.31

$$\int \cosh(x)\coth(3x)\,dx = \frac{3\cosh(x)^2 - (\cosh(x)+\sinh(x))\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right) + (\cosh(x)+\sinh(x))\log\left(\frac{2\cosh(x)-1}{\cosh(x)-\sinh(x)}\right) - 2(\cosh$$

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cosh(x)^2 - (cosh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(
x))) + (cosh(x) + sinh(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(co
sh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cos
h(x) + sinh(x) - 1) + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 3)/(cosh(x) + sinh(
x))
```

Sympy [F]

$$\int \cosh(x)\coth(3x)\,dx = \int \cosh{(x)}\coth{(3x)}\,dx$$

```
[In] integrate(cosh(x)*coth(3*x),x)
[Out] Integral(cosh(x)*coth(3*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \cosh(x)\coth(3x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{6}\,\log\left(e^{(-x)} + e^{(-2\,x)} + 1\right) - \frac{1}{3}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{3}\,\log\left(e^{(-x)} - 1\right) + \frac{1}{6}\,\log\left(-e^{(-x)} + e^{(-2\,x)} + 1\right)$$

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="maxima")
[Out] 1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^(-2*x) + 1) - 1/3*log(e^(-x) + 1)
 + 1/3*log(e^(-x) - 1) + 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \cosh(x)\coth(3x)\,dx = \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} + 1\right) + \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} - 1\right) + \frac{1}{6}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

```
[In] integrate(cosh(x)*coth(3*x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x - 1/6*log(e^(-x) + e^x + 2) - 1/6*log(e^(-x) + e^x + 1
) + 1/6*log(e^(-x) + e^x - 1) + 1/6*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \cosh(x)\coth(3x)\,dx = \frac{\ln(6-6\,\mathrm{e}^{x})}{3} - \frac{\ln(-6\,\mathrm{e}^{x}-6)}{3} + \frac{\mathrm{e}^{-x}}{2} + \frac{\ln(\mathrm{e}^{x}-\mathrm{e}^{2\,x}-1)}{6} - \frac{\ln(-\mathrm{e}^{2\,x}-\mathrm{e}^{x}-1)}{6} + \frac{\mathrm{e}^{x}}{2}$$

```
[In] int(coth(3*x)*cosh(x),x)
[Out] log(6 - 6*exp(x))/3 - log(- 6*exp(x) - 6)/3 + exp(-x)/2 + log(exp(x) - exp(
2*x) - 1)/6 - log(- exp(2*x) - exp(x) - 1)/6 + exp(x)/2
```

3.236 $\int \cosh(x)\coth(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \cosh(x)\coth(4x)\,dx = -\frac{1}{4}\text{arctanh}(\cosh(x)) - \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}} + \cosh(x)$$

```
[Out] -1/4*arctanh(cosh(x))+cosh(x)-1/4*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1690, 1180, 213}

$$\int \cosh(x)\coth(4x)\,dx = -\frac{1}{4}\text{arctanh}(\cosh(x)) - \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}} + \cosh(x)$$

```
[In] Int[Cosh[x]*Coth[4*x],x]
[Out] -1/4*ArcTanh[Cosh[x]] - ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2]) + Cosh[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{-1+8x^2-8x^4}{4-12x^2+8x^4}\,dx, x, \cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(-1+\frac{3-4x^2}{4-12x^2+8x^4}\right)\,dx, x, \cosh(x)\right)\\
&= \cosh(x)-\text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \cosh(x)\right)\\
&= \cosh(x)+2\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \cosh(x)\right)+2\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{4}\text{arctanh}(\cosh(x))-\frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}+\cosh(x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \cosh(x)\coth(4x)\,dx = \frac{1}{4}\left(-\sqrt{2}\text{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right) - \sqrt{2}\text{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right)+4\cosh(x)-\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

```
[In] Integrate[Cosh[x]*Coth[4*x],x]
[Out] (-(Sqrt[2]*ArcTanh[Sqrt[2] - I*Tanh[x/2]]) - Sqrt[2]*ArcTanh[Sqrt[2] + I*Ta
nh[x/2]] + 4*Cosh[x] - Log[Cosh[x/2]] + Log[Sinh[x/2]])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

method	result	size
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{\ln(e^{x}-1)}{4}-\frac{\ln(e^{x}+1)}{4}+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{2}\right)\sqrt{2}}{8}$	63

```
[In] int(cosh(x)*coth(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)+1/4*ln(exp(x)-1)-1/4*ln(exp(x)+1)+1/8*ln(1+exp(2*x)-
exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.61

$$\int \cosh(x)\coth(4x)\,dx = \frac{4\cosh(x)^2+\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(\frac{\cosh(x)^2+\sinh(x)^2-2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right)-2\left(\cosh(x)+\sinh(x)\right)\log}{8\,(\cos}$$

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)^2 + (sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log((cosh(x)^2 + sin
h(x)^2 - 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2 + sinh(x)^2)) - 2*(cosh(x) + sin
h(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh
(x) - 1) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 + 4)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \cosh(x)\coth(4x)\,dx = \int \cosh(x)\coth(4x)\,dx$$

```
[In] integrate(cosh(x)*coth(4*x),x)
```

```
[Out] Integral(cosh(x)*coth(4*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int \cosh(x)\coth(4x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x} - \frac{1}{4}\log\left(e^{(-x)} + 1\right) + \frac{1}{4}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="maxima")
[Out] -1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*
e^(-x) + e^(-2*x) + 1) + 1/2*e^(-x) + 1/2*e^x - 1/4*log(e^(-x) + 1) + 1/4*l
og(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \cosh(x)\coth(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - e^{x}}{\sqrt{2} + e^{(-x)} + e^{x}}\right) + \frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x} - \frac{1}{8}\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{8}\log\left(e^{(-x)} + e^{x} - 2\right)$$

```
[In] integrate(cosh(x)*coth(4*x),x, algorithm="giac")
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) + 1/2*e
^(-x) + 1/2*e^x - 1/8*log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \cosh(x)\coth(4x)\,dx = \frac{\ln\left(\frac{1}{2} - \frac{e^{x}}{2}\right)}{4} - \frac{\ln\left(-\frac{e^{x}}{2} - \frac{1}{2}\right)}{4} + \frac{e^{-x}}{2} + \frac{e^{x}}{2} - \frac{\sqrt{2}\,\ln\left(-\frac{e^{2x}}{8} - \frac{\sqrt{2}\,e^{x}}{8} - \frac{1}{8}\right)}{8} + \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,e^{x}}{8} - \frac{e^{2x}}{8} - \frac{1}{8}\right)}{8}$$

```
[In] int(coth(4*x)*cosh(x),x)
[Out] log(1/2 - exp(x)/2)/4 - log(- exp(x)/2 - 1/2)/4 + exp(-x)/2 + exp(x)/2 - (2
^(1/2)*log(- exp(2*x)/8 - (2^(1/2)*exp(x))/8 - 1/8))/8 + (2^(1/2)*log((2^(1
/2)*exp(x))/8 - exp(2*x)/8 - 1/8))/8
```

3.237 $\int \cosh(x)\coth(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 110

$$\int \cosh(x)\coth(5x)\,dx = -\frac{1}{5}\text{arctanh}(\cosh(x)) + \cosh(x) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cosh(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(x)\right) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(x)\right)$$

```
[Out] -1/5*arctanh(cosh(x))+cosh(x)+1/20*ln(1-4*cosh(x)-5^(1/2))*(-5^(1/2)+1)-1/2
0*ln(1+4*cosh(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1-4*cosh(x)+5^(1/2))*(5^(1/2
)+1)-1/20*ln(1+4*cosh(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used

$= \{2100, 213, 646, 31\}$

$$\int \cosh(x)\coth(5x)\,dx = -\frac{1}{5}\text{arctanh}(\cosh(x)) + \cosh(x) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cosh(x)-\sqrt{5}+1\right) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cosh(x)+\sqrt{5}+1\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cosh(x)-\sqrt{5}+1\right) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\cosh(x)+\sqrt{5}+1\right)$$

```
[In] Int[Cosh[x]*Coth[5*x],x]
```

```
[Out] -1/5*ArcTanh[Cosh[x]] + Cosh[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cosh[x
]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]])/20 - ((1 - Sqrt[5])*L
og[1 - Sqrt[5] + 4*Cosh[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cosh[x
]])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2(5-20x^2+16x^4)}{1-13x^2+28x^4-16x^6}\,dx, x, \cosh(x)\right) \\
&= -\text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)\,dx, x, \cosh(x)\right) \\
&= \cosh(x)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right) \\
&\quad +\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \cosh(x)\right) \\
&\quad -\frac{2}{5}\text{Subst}\left(\int \frac{-1+x}{-1+2x+4x^2}\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{5}\text{arctanh}(\cosh(x))+\cosh(x)-\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \cosh(x)\right) \\
&\quad +\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-1+\sqrt{5}+4x}\,dx, x, \cosh(x)\right) \\
&\quad +\frac{1}{5}\left(1+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-1-\sqrt{5}+4x}\,dx, x, \cosh(x)\right) \\
&\quad -\frac{1}{5}\left(1+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{5}+4x}\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{5}\text{arctanh}(\cosh(x))+\cosh(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right) \\
&\quad +\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cosh(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(x)\right) \\
&\quad -\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$
\begin{aligned}
\int \cosh(x)\coth(5x)\,dx = \frac{1}{100}\Bigg(&100\cosh(x)-20\log\left(\cosh\left(\frac{x}{2}\right)\right) \\
&+\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cosh(x)\right) \\
&+\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cosh(x)\right) \\
&-\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(x)\right) \\
&-\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(x)\right)+20\log\left(\sinh\left(\frac{x}{2}\right)\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[Cosh[x]*Coth[5*x],x]
```

```
[Out] (100*Cosh[x] - 20*Log[Cosh[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] -
 4*Cosh[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cosh[x]] - Sqrt[5]*
(-5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 4*Cosh[x]] + 20*Log[Sinh[x/2]])/100
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.73

method	result
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^{x}+1)}{5}+\frac{\ln(e^{x}-1)}{5}+\frac{\ln\left(e^{2x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{x}+1\right)}{20}+\frac{\ln\left(e^{2x}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{x}+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{2x}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{x}+1\right)}{20}$

```
[In] int(cosh(x)*coth(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)+1/2*exp(-x)-1/5*ln(exp(x)+1)+1/5*ln(exp(x)-1)+1/20*ln(exp(2*x)+(
-1/2-1/2*5^(1/2))*exp(x)+1)+1/20*ln(exp(2*x)+(-1/2-1/2*5^(1/2))*exp(x)+1)*5
^(1/2)+1/20*ln(exp(2*x)+(1/2*5^(1/2)-1/2)*exp(x)+1)-1/20*ln(exp(2*x)+(1/2*5
^(1/2)-1/2)*exp(x)+1)*5^(1/2)-1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)+1)+
1/20*ln(exp(2*x)+(1/2-1/2*5^(1/2))*exp(x)+1)*5^(1/2)-1/20*ln(exp(2*x)+(1/2+
1/2*5^(1/2))*exp(x)+1)-1/20*ln(exp(2*x)+(1/2+1/2*5^(1/2))*exp(x)+1)*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(80) = 160$.

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.47

$$\int \cosh(x)\coth(5x)\,dx$$

$$= \frac{10\cosh(x)^2+\left(\sqrt{5}\cosh(x)+\sqrt{5}\sinh(x)\right)\log\left(-\frac{4\left(\sqrt{5}-1\right)\cosh(x)-4\cosh(x)^2-4\sinh(x)^2+\sqrt{5}-7}{2\cosh(x)^2+2\sinh(x)^2+2\cosh(x)+1}\right)+\left(\sqrt{5}\cosh(x)\right.}{}$$

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*(10*cosh(x)^2 + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqrt(5) -
 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 + sqrt(5) - 7)/(2*cosh(x)^2 + 2*sin
h(x)^2 + 2*cosh(x) + 1)) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x))*log(-(4*(sqr
t(5) + 1)*cosh(x) - 4*cosh(x)^2 - 4*sinh(x)^2 - sqrt(5) - 7)/(2*cosh(x)^2 +
 2*sinh(x)^2 - 2*cosh(x) + 1)) - (cosh(x) + sinh(x))*log((2*cosh(x)^2 + 2*s
inh(x)^2 + 2*cosh(x) + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + (c
osh(x) + sinh(x))*log((2*cosh(x)^2 + 2*sinh(x)^2 - 2*cosh(x) + 1)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x) + sinh(x))*log(cosh(x) + s
inh(x) + 1) + 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 20*cosh(x)
*sinh(x) + 10*sinh(x)^2 + 10)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \cosh(x)\coth(5x)\,dx = \int \cosh{(x)}\coth{(5x)}\,dx$$

```
[In] integrate(cosh(x)*coth(5*x),x)
[Out] Integral(cosh(x)*coth(5*x), x)
```

Maxima [F]

$$\int \cosh(x)\coth(5x)\,dx = \int \cosh{(x)}\coth{(5\,x)}\;dx$$

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) + 1)*e^(-x) - 1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/
(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 1/5*integrate((e^(3*x) - e^(2
*x) + e^x - 1)*e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integ
rate(e^(3*x)/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) + 3/10*integrate(e
^(3*x)/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)
/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^(2*x)/(e^(4
*x) - e^(3*x) + e^(2*x) - e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3
*x) + e^(2*x) + e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2
*x) - e^x + 1), x) - 1/5*log(e^x + 1) + 1/5*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.43

$$\begin{aligned}\int \cosh(x)\coth(5x)\,dx =& \frac{1}{20}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-2\,e^{(-x)}-2\,e^{x}+1}{\sqrt{5}+2\,e^{(-x)}+2\,e^{x}-1}\right)\\&+\frac{1}{20}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-2\,e^{(-x)}-2\,e^{x}-1}{\sqrt{5}+2\,e^{(-x)}+2\,e^{x}+1}\right)+\frac{1}{2}\,e^{(-x)}\\&+\frac{1}{2}\,e^{x}-\frac{1}{20}\,\log\left(\left(e^{(-x)}+e^{x}\right)^{2}+e^{(-x)}+e^{x}-1\right)\\&+\frac{1}{20}\,\log\left(\left(e^{(-x)}+e^{x}\right)^{2}-e^{(-x)}-e^{x}-1\right)\\&-\frac{1}{10}\,\log\left(e^{(-x)}+e^{x}+2\right)+\frac{1}{10}\,\log\left(e^{(-x)}+e^{x}-2\right)\end{aligned}$$

```
[In] integrate(cosh(x)*coth(5*x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x + 1)/(sqrt(5) + 2*e^(-x) + 2*
e^x - 1)) + 1/20*sqrt(5)*log(-(sqrt(5) - 2*e^(-x) - 2*e^x - 1)/(sqrt(5) + 2
*e^(-x) + 2*e^x + 1)) + 1/2*e^(-x) + 1/2*e^x - 1/20*log((e^(-x) + e^x)^2 +
e^(-x) + e^x - 1) + 1/20*log((e^(-x) + e^x)^2 - e^(-x) - e^x - 1) - 1/10*lo
g(e^(-x) + e^x + 2) + 1/10*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \cosh(x)\coth(5x)\,dx &= \frac{\ln(10-10\,e^x)}{5}-\frac{\ln(-10\,e^x-10)}{5}+\frac{e^{-x}}{2}+\frac{e^x}{2}\\ &\quad-\ln\left(-e^{2x}-10\,e^x\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-1\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)\\ &\quad+\ln\left(10\,e^x\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-e^{2x}-1\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)\\ &\quad-\ln\left(-e^{2x}-10\,e^x\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-1\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\\ &\quad+\ln\left(10\,e^x\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-e^{2x}-1\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\end{aligned}$$

```
[In] int(coth(5*x)*cosh(x),x)
```

```
[Out] log(10 - 10*exp(x))/5 - log(- 10*exp(x) - 10)/5 + exp(-x)/2 + exp(x)/2 - lo
g(- exp(2*x) - 10*exp(x)*(5^(1/2)/20 - 1/20) - 1)*(5^(1/2)/20 - 1/20) + log
(10*exp(x)*(5^(1/2)/20 - 1/20) - exp(2*x) - 1)*(5^(1/2)/20 - 1/20) - log(-
exp(2*x) - 10*exp(x)*(5^(1/2)/20 + 1/20) - 1)*(5^(1/2)/20 + 1/20) + log(10*
exp(x)*(5^(1/2)/20 + 1/20) - exp(2*x) - 1)*(5^(1/2)/20 + 1/20)
```

3.238 $\int \cosh(x) \coth(6x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \cosh(x)\coth(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cosh(x)) - \frac{1}{6}\text{arctanh}(2\cosh(x)) - \frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \cosh(x)$$

```
[Out] -1/6*arctanh(cosh(x))-1/6*arctanh(2*cosh(x))+cosh(x)-1/6*arctanh(2/3*cosh(x
)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2098, 213}

$$\int \cosh(x)\coth(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cosh(x)) - \frac{1}{6}\text{arctanh}(2\cosh(x)) - \frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \cosh(x)$$

```
[In] Int[Cosh[x]*Coth[6*x],x]
```

```
[Out] -1/6*ArcTanh[Cosh[x]] - ArcTanh[2*Cosh[x]]/6 - ArcTanh[(2*Cosh[x])/Sqrt[3]]
/(2*Sqrt[3]) + Cosh[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{2(3-19x^2+32x^4-16x^6)}\,dx, x, \cosh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \cosh(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-2-\frac{1}{3(-1+x^2)}-\frac{2}{-3+4x^2}-\frac{2}{3(-1+4x^2)}\right)dx, x, \cosh(x)\right)\right)\\
&= \cosh(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)\\
&\quad+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cosh(x)\right)+\text{Subst}\left(\int \frac{1}{-3+4x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{6}\text{arctanh}(\cosh(x))-\frac{1}{6}\text{arctanh}(2\cosh(x))-\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cosh(x)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\begin{aligned}
\int \cosh(x)\coth(6x)\,dx &= \frac{1}{12}\left(-2\sqrt{3}\text{arctanh}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)\right.\\
&\quad-2\sqrt{3}\text{arctanh}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+12\cosh(x)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)\\
&\quad\left.+\log(1-2\cosh(x))-\log(1+2\cosh(x))+2\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)
\end{aligned}$$

```
[In] Integrate[Cosh[x]*Coth[6*x],x]
```

[Out] (-2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + I*T
anh[x/2])/Sqrt[3]] + 12*Cosh[x] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - L
og[1 + 2*Cosh[x]] + 2*Log[Sinh[x/2]])/12

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

method	result
risch	$\frac{e^{x}}{2}+\frac{e^{-x}}{2}-\frac{\ln(e^{x}+1)}{6}+\frac{\ln(e^{x}-1)}{6}+\frac{\ln\left(1+e^{2x}-e^{x}\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{2x}+e^{x}\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln(e^{2x}-e^{x}+1)}{12}-\frac{\ln(1+e^{x}+e^{2x})}{12}$

[In] int(cosh(x)*coth(6*x),x,method=_RETURNVERBOSE)

[Out] 1/2*exp(x)+1/2*exp(-x)-1/6*ln(exp(x)+1)+1/6*ln(exp(x)-1)+1/12*ln(1+exp(2*x)
-exp(x)*3^(1/2))*3^(1/2)-1/12*ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2)+1/12*ln
(exp(2*x)-exp(x)+1)-1/12*ln(1+exp(x)+exp(2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 4.13

$$\int \cosh(x)\coth(6x)\,dx = \frac{6\cosh(x)^2+\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)\right)\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right)-(\cosh(x)+\sinh(x))\log}{}$$

[In] integrate(cosh(x)*coth(6*x),x, algorithm="fricas")

[Out] 1/12*(6*cosh(x)^2 + (sqrt(3)*cosh(x) + sqrt(3)*sinh(x))*log((2*cosh(x)^2 +
2*sinh(x)^2 - 4*sqrt(3)*cosh(x) + 5)/(2*cosh(x)^2 + 2*sinh(x)^2 - 1)) - (co
sh(x) + sinh(x))*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) + (cosh(x) + sinh
(x))*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(cosh(x) + sinh(x))*log(c
osh(x) + sinh(x) + 1) + 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) +
12*cosh(x)*sinh(x) + 6*sinh(x)^2 + 6)/(cosh(x) + sinh(x))

Sympy [F]

$$\int \cosh(x)\coth(6x)\,dx = \int \cosh\left(x\right)\coth\left(6x\right)\,dx$$

```
[In] integrate(cosh(x)*coth(6*x),x)
[Out] Integral(cosh(x)*coth(6*x), x)
```

Maxima [F]

$$\int \cosh(x)\coth(6x)\,dx = \int \cosh\left(x\right)\coth\left(6\,x\right)\,dx$$

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) + 1)*e^(-x) + 1/2*integrate((e^(3*x) - e^x)/(e^(4*x) - e^(2*x)
 + 1), x) - 1/12*log(e^(2*x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6
*log(e^x + 1) + 1/6*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.34

$$\begin{aligned}\int \cosh(x)\coth(6x)\,dx = {} & \frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right)+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x} \\ & -\frac{1}{12}\,\log\left(e^{(-x)}+e^{x}+2\right)-\frac{1}{12}\,\log\left(e^{(-x)}+e^{x}+1\right) \\ & +\frac{1}{12}\,\log\left(e^{(-x)}+e^{x}-1\right)+\frac{1}{12}\,\log\left(e^{(-x)}+e^{x}-2\right)\end{aligned}$$

```
[In] integrate(cosh(x)*coth(6*x),x, algorithm="giac")
[Out] 1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) + 1/2*
e^(-x) + 1/2*e^x - 1/12*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1)
+ 1/12*log(e^(-x) + e^x - 1) + 1/12*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

$$\int \cosh(x)\coth(6x)\,dx = \frac{\ln\left(\frac{1}{3}-\frac{e^x}{3}\right)}{6} - \frac{\ln\left(-\frac{e^x}{3}-\frac{1}{3}\right)}{6} + \frac{e^{-x}}{2} - \frac{\ln\left(-\frac{e^{2x}}{36}-\frac{e^x}{36}-\frac{1}{36}\right)}{12} + \frac{\ln\left(\frac{e^x}{36}-\frac{e^{2x}}{36}-\frac{1}{36}\right)}{12} + \frac{e^x}{2} - \frac{\sqrt{3}\ln\left(-\frac{e^{2x}}{12}-\frac{\sqrt{3}\,e^x}{12}-\frac{1}{12}\right)}{12} + \frac{\sqrt{3}\ln\left(\frac{\sqrt{3}\,e^x}{12}-\frac{e^{2x}}{12}-\frac{1}{12}\right)}{12}$$

```
[In] int(coth(6*x)*cosh(x),x)
```

```
[Out] log(1/3 - exp(x)/3)/6 - log(- exp(x)/3 - 1/3)/6 + exp(-x)/2 - log(- exp(2*x
)/36 - exp(x)/36 - 1/36)/12 + log(exp(x)/36 - exp(2*x)/36 - 1/36)/12 + exp(
x)/2 - (3^(1/2)*log(- exp(2*x)/12 - (3^(1/2)*exp(x))/12 - 1/12))/12 + (3^(1
/2)*log((3^(1/2)*exp(x))/12 - exp(2*x)/12 - 1/12))/12
```

3.239 $\int \cosh(x)\coth(nx)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 76

$$\int \cosh(x)\coth(nx)\,dx = -\frac{e^{-x}}{2}+\frac{e^{x}}{2}+e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right) - e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2nx}\right)$$

```
[Out] -1/2/exp(x)+1/2*exp(x)+hypergeom([1, -1/2/n],[1-1/2/n],exp(2*n*x))/exp(x)-e
xp(x)*hypergeom([1, 1/2/n],[1+1/2/n],exp(2*n*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5721, 2225, 2283}

$$\int \cosh(x)\coth(nx)\,dx = e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right) - e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2nx}\right)-\frac{e^{-x}}{2}+\frac{e^{x}}{2}$$

```
[In] Int[Cosh[x]*Coth[n*x],x]
```

```
[Out] -1/2*1/E^x + E^x/2 + Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E^(2*n*x)]
/E^x - E^x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, E^(2*n*x)]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5721

```
Int[Cosh[(a_.) + (b_.)*(x_)]*Coth[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(
E^(a + b*x)*2) + E^(a + b*x)/2 - 1/(E^(a + b*x)*(1 - E^(2*(c + d*x)))) - E^
(a + b*x)/(1 - E^(2*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 -
d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}-\frac{e^{-x}}{1-e^{2nx}}-\frac{e^{x}}{1-e^{2nx}}\right)dx\\
&= \frac{1}{2}\int e^{-x}\,dx+\frac{\int e^{x}\,dx}{2}-\int \frac{e^{-x}}{1-e^{2nx}}\,dx-\int \frac{e^{x}}{1-e^{2nx}}\,dx\\
&= -\frac{e^{-x}}{2}+\frac{e^{x}}{2}+e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right)\\
&\quad -e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2nx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\begin{aligned}
\int \cosh(x)\coth(nx)\,dx &= \frac{1}{2}\Bigg(-e^{-x}+e^{x}+2e^{-x}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2nx}\right)\\
&\quad -2e^{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},e^{2nx}\right)\Bigg)
\end{aligned}$$

```
[In] Integrate[Cosh[x]*Coth[n*x],x]
```

```
[Out] (-E^(-x) + E^x + (2*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E^(2*n*x)])
/E^x - 2*E^x*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), E^(2*n*x)])/2
```

Maple [F]

$$\int \cosh(x)\coth(nx)\,dx$$

```
[In] int(cosh(x)*coth(n*x),x)
[Out] int(cosh(x)*coth(n*x),x)
```

Fricas [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh(x)\coth(nx)\,dx$$

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="fricas")
[Out] integral(cosh(x)*coth(n*x), x)
```

Sympy [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh(x)\coth(nx)\,dx$$

```
[In] integrate(cosh(x)*coth(n*x),x)
[Out] Integral(cosh(x)*coth(n*x), x)
```

Maxima [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh(x)\coth(nx)\,dx$$

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="maxima")
[Out] 1/2*(e^(2*x) - 1)*e^(-x) - 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) + e^x),
 x) + 1/2*integrate((e^(2*x) + 1)/(e^(n*x + x) - e^x), x)
```

Giac [F]

$$\int \cosh(x)\coth(nx)\,dx = \int \cosh\left(x\right)\coth\left(nx\right)\,dx$$

```
[In] integrate(cosh(x)*coth(n*x),x, algorithm="giac")
[Out] integrate(cosh(x)*coth(n*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh(x)\coth(nx)\,dx = \int \coth(n\,x)\,\cosh(x)\,dx$$

```
[In] int(coth(n*x)*cosh(x),x)
[Out] int(coth(n*x)*cosh(x), x)
```

3.240 $\int \cosh(x)\mathbf{sech}(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(x)\text{sech}(2x)\,dx = \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(sinh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4441, 209}

$$\int \cosh(x)\text{sech}(2x)\,dx = \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[In] Int[Cosh[x]*Sech[2*x],x]
```

```
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
```

```
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)$$
$$= \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{sech}(2x)\,dx = \frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[In] Integrate[Cosh[x]*Sech[2*x],x]
[Out] ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

method	result	size
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2x}+i\sqrt{2}\,e^{x}-1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2x}-i\sqrt{2}\,e^{x}-1\right)}{4}$	44

```
[In] int(cosh(x)*sech(2*x),x,method=_RETURNVERBOSE)
[Out] 1/4*I*2^(1/2)*ln(exp(2*x)+I*2^(1/2)*exp(x)-1)-1/4*I*2^(1/2)*ln(exp(2*x)-I*2
^(1/2)*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.53

$$\int \cosh(x)\operatorname{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+\sqrt{2}}{2(\cosh(x)-\sinh(x))}\right)$$

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) - 1/2*sqrt(2)
*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \cosh(x)\operatorname{sech}(2x)\,dx = \int \cosh(x)\operatorname{sech}(2x)\,dx$$

```
[In] integrate(cosh(x)*sech(2*x),x)
[Out] Integral(cosh(x)*sech(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \cosh(x)\operatorname{sech}(2x)\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cosh(x)\mathrm{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)$$

```
[In] integrate(cosh(x)*sech(2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x))
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \cosh(x)\mathrm{sech}(2x)\,dx = \frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{x}}{2}+\frac{\sqrt{2}\,\mathrm{e}^{3\,x}}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{e}^{x}}{2}\right)\right)}{2}$$

```
[In] int(cosh(x)/cosh(2*x),x)
```

```
[Out] (2^(1/2)*(atan((2^(1/2)*exp(x))/2 + (2^(1/2)*exp(3*x))/2) + atan((2^(1/2)*e
xp(x))/2)))/2
```

3.241 $\int \cosh(x)\mathbf{sech}(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(tanh(x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {209}

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

```
[In] Int[Cosh[x]*Sech[3*x],x]
```

```
[Out] ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \tanh(x)\right) \\ &= \frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\arctan\left(\sqrt{3}\tanh(x)\right)}{\sqrt{3}}$$

```
[In] Integrate[Cosh[x]*Sech[3*x],x]
[Out] ArcTan[Sqrt[3]*Tanh[x]]/Sqrt[3]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

method	result	size
risch	$\frac{i\sqrt{3}\,\ln\left(e^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\sqrt{3}\,\ln\left(e^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

```
[In] int(cosh(x)*sech(3*x),x,method=_RETURNVERBOSE)
[Out] 1/6*I*3^(1/2)*ln(exp(2*x)-1/2+1/2*I*3^(1/2))-1/6*I*3^(1/2)*ln(exp(2*x)-1/2-
1/2*I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \cosh(x)\text{sech}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)$$

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) - s
inh(x)))
```

Sympy [F]

$$\int \cosh(x)\text{sech}(3x)\,dx = \int \cosh(x)\,\text{sech}(3x)\,dx$$

```
[In] integrate(cosh(x)*sech(3*x),x)
[Out] Integral(cosh(x)*sech(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 7.60

$$\begin{aligned}\int \cosh(x)\text{sech}(3x)\,dx = &-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{(-2x)}-1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)\\&+\frac{1}{6}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{12}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)\\&+\frac{1}{12}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{6}\log\left(e^{(2x)}+1\right)\\&+\frac{1}{6}\log\left(e^{(-2x)}+1\right)-\frac{1}{12}\log\left(-e^{(-2x)}+e^{(-4x)}+1\right)\end{aligned}$$

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-2*x) - 1)) - 1/6*sqrt(3)*arctan(sqrt
(3) + 2*e^x) + 1/6*sqrt(3)*arctan(-sqrt(3) + 2*e^x) + 1/12*log(sqrt(3)*e^x
+ e^(2*x) + 1) + 1/12*log(-sqrt(3)*e^x + e^(2*x) + 1) - 1/6*log(e^(2*x) + 1
) + 1/6*log(e^(-2*x) + 1) - 1/12*log(-e^(-2*x) + e^(-4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{(2x)}-1)\right)$$

```
[In] integrate(cosh(x)*sech(3*x),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(x)\text{sech}(3x)\,dx = \frac{\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}\,(2\,e^{2\,x}-1)}{3}\right)}{3}$$

```
[In] int(cosh(x)/cosh(3*x),x)
[Out] (3^(1/2)*atan((3^(1/2)*(2*exp(2*x) - 1))/3))/3
```

3.242 $\int \cosh(x)\mathbf{sech}(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \cosh(x)\text{sech}(4x)\,dx = \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/2*arctan(2*sinh(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctan(2*si
nh(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4441, 1107, 209}

$$\int \cosh(x)\text{sech}(4x)\,dx = \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[In] Int[Cosh[x]*Sech[4*x],x]
```

```
[Out] ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(2
*Sinh[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1+8x^2+8x^4}\,dx, x, \sinh(x)\right)\\
&= \sqrt{2}\text{Subst}\left(\int \frac{1}{4-2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right) - \sqrt{2}\text{Subst}\left(\int \frac{1}{4+2\sqrt{2}+8x^2}\,dx, x, \sinh(x)\right)\\
&= \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$
\int \cosh(x)\text{sech}(4x)\,dx = \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}
$$

```
[In] Integrate[Cosh[x]*Sech[4*x],x]
[Out] (Sqrt[2 + Sqrt[2]]*ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTan[(2*Sin
h[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}(32768\_Z^4+512\_Z^2+1)}\_R\ln\left(\mathrm{e}^{2x}+\left(-4096\_R^3-48\_R\right)\mathrm{e}^{x}-1\right)\right)$	40

```
[In] int(cosh(x)*sech(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(-4096*_R^3-48*_R)*exp(x)-1),_R=RootOf(32768*_Z^4+512*
_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\begin{aligned}\int \cosh(x)\mathrm{sech}(4x)\,dx = &-\frac{1}{8}\sqrt{\sqrt{2}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&+\left(\left(\sqrt{2}+1\right)\cosh(x)+\left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{\sqrt{2}-2}-1\Big)\\&+\frac{1}{8}\sqrt{\sqrt{2}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&-\left(\left(\sqrt{2}+1\right)\cosh(x)+\left(\sqrt{2}+1\right)\sinh(x)\right)\sqrt{\sqrt{2}-2}-1\Big)\\&+\frac{1}{8}\sqrt{-\sqrt{2}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&+\left(\left(\sqrt{2}-1\right)\cosh(x)+\left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{-\sqrt{2}-2}-1\Big)\\&-\frac{1}{8}\sqrt{-\sqrt{2}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\\&-\left(\left(\sqrt{2}-1\right)\cosh(x)+\left(\sqrt{2}-1\right)\sinh(x)\right)\sqrt{-\sqrt{2}-2}-1\Big)\end{aligned}$$

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sq
rt(2) + 1)*cosh(x) + (sqrt(2) + 1)*sinh(x))*sqrt(sqrt(2) - 2) - 1) + 1/8*sq
rt(sqrt(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) +
 1)*cosh(x) + (sqrt(2) + 1)*sinh(x))*sqrt(sqrt(2) - 2) - 1) + 1/8*sqrt(-sqr
```

```
t(2) - 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(2) - 1)*co
sh(x) + (sqrt(2) - 1)*sinh(x))*sqrt(-sqrt(2) - 2) - 1) - 1/8*sqrt(-sqrt(2)
- 2)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(2) - 1)*cosh(x)
 + (sqrt(2) - 1)*sinh(x))*sqrt(-sqrt(2) - 2) - 1)
```

Sympy [F]

$$\int \cosh(x)\text{sech}(4x)\,dx = \int \cosh{(x)}\,\text{sech}\,(4x)\,dx$$

```
[In] integrate(cosh(x)*sech(4*x),x)
[Out] Integral(cosh(x)*sech(4*x), x)
```

Maxima [F]

$$\int \cosh(x)\text{sech}(4x)\,dx = \int \cosh{(x)}\,\text{sech}\,(4\,x)\,\,dx$$

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="maxima")
[Out] integrate(cosh(x)*sech(4*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(49) = 98$.

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \cosh(x)\text{sech}(4x)\,dx = {} & \frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^{x}}{\sqrt{-\sqrt{2}+2}}\right) \\ & +\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^{x}}{\sqrt{-\sqrt{2}+2}}\right) \\ & -\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^{x}}{\sqrt{\sqrt{2}+2}}\right) \\ & -\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^{x}}{\sqrt{\sqrt{2}+2}}\right)\end{aligned}$$

```
[In] integrate(cosh(x)*sech(4*x),x, algorithm="giac")
[Out] 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2)
) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2)
 + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(sq
rt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/s
qrt(sqrt(2) + 2))
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$\int \cosh(x)\operatorname{sech}(4x)\,dx = \frac{\operatorname{atan}\left(\frac{3\,e^{2\,x}-2\,\sqrt{2}+2\,\sqrt{2}\,e^{2\,x}-3}{e^x\,\sqrt{\sqrt{2}+2}+\sqrt{2}\,e^x\,\sqrt{\sqrt{2}+2}}\right)\,\sqrt{\sqrt{2}+2}}{4} + \frac{\operatorname{atan}\left(\frac{3\,e^{2\,x}+2\,\sqrt{2}-2\,\sqrt{2}\,e^{2\,x}-3}{e^x\,\sqrt{2-\sqrt{2}}-\sqrt{2}\,e^x\,\sqrt{2-\sqrt{2}}}\right)\,\sqrt{2-\sqrt{2}}}{4}$$

```
[In] int(cosh(x)/cosh(4*x),x)
```

```
[Out] (atan((3*exp(2*x) - 2*2^(1/2) + 2*2^(1/2)*exp(2*x) - 3)/(exp(x)*(2^(1/2) +
2)^(1/2) + 2^(1/2)*exp(x)*(2^(1/2) + 2)^(1/2)))*(2^(1/2) + 2)^(1/2))/4 + (a
tan((3*exp(2*x) + 2*2^(1/2) - 2*2^(1/2)*exp(2*x) - 3)/(exp(x)*(2 - 2^(1/2))
^(1/2) - 2^(1/2)*exp(x)*(2 - 2^(1/2))^(1/2)))*(2 - 2^(1/2))^(1/2))/4
```

3.243 $\int \cosh(x)\mathbf{sech}(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 75

$$\int \cosh(x)\text{sech}(5x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)$$

```
[Out] -1/10*arctan((5-2*5^(1/2))^(1/2)*tanh(x))*(10-2*5^(1/2))^(1/2)+1/10*arctan(
(5+2*5^(1/2))^(1/2)*tanh(x))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1180, 209}

$$\int \cosh(x)\text{sech}(5x)\,dx = \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)$$

```
[In] Int[Cosh[x]*Sech[5*x],x]
```

```
[Out] -1/5*(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[5 - 2*Sqrt[5]]*Tanh[x]]) + (Sqrt[(5
 + Sqrt[5])/2]*ArcTan[Sqrt[5 + 2*Sqrt[5]]*Tanh[x]])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-x^2}{1+10x^2+5x^4}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\left(-1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5+2\sqrt{5}+5x^2}\,dx, x, \tanh(x)\right)\\
&\quad+\frac{1}{2}\left(-1+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{5-2\sqrt{5}+5x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{5-2\sqrt{5}}\tanh(x)\right)\\
&\quad+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{5+2\sqrt{5}}\tanh(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \cosh(x)\operatorname{sech}(5x)\,dx = \frac{\sqrt{5+\sqrt{5}}\arctan\left(\frac{\left(5+\sqrt{5}\right)\tanh(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\arctan\left(\frac{\left(-5+\sqrt{5}\right)\tanh(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

```
[In] Integrate[Cosh[x]*Sech[5*x],x]
[Out] (Sqrt[5 + Sqrt[5]]*ArcTan[((5 + Sqrt[5])*Tanh[x])/Sqrt[10 - 2*Sqrt[5]]] + S
qrt[5 - Sqrt[5]]*ArcTan[((-5 + Sqrt[5])*Tanh[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}\left(32000\_Z^4+400\_Z^2+1\right)}\_R\ln\left(-4000\_R^3+200\_R^2+\mathrm{e}^{2x}-30\_R+1\right)\right)$	41

```
[In] int(cosh(x)*sech(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(-4000*_R^3+200*_R^2+exp(2*x)-30*_R+1),_R=RootOf(32000*_Z^4+400*
_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(49) = 98$.

Time = 0.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.24

$$\begin{aligned}\int \cosh(x)\operatorname{sech}(5x)\,dx = &-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)\right.\\&\left.+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}+2\sqrt{5}-2\right)\\&+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)\right.\\&\left.+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-5}+2\sqrt{5}-2\right)\\&+\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)\right.\\&\left.+8\sinh(x)^2+\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}-2\sqrt{5}-2\right)\\&-\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}-5}\log\left(8\cosh(x)^2+16\cosh(x)\sinh(x)\right.\\&\left.+8\sinh(x)^2-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-5}-2\sqrt{5}-2\right)\end{aligned}$$

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*si
nh(x)^2 + (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) + 2*sqrt(5) - 2) +
1/20*sqrt(2)*sqrt(sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*sin
h(x)^2 - (sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 5) + 2*sqrt(5) - 2) + 1
```

```
/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*sin
h(x)^2 + (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) - 2*sqrt(5) - 2) -
1/20*sqrt(2)*sqrt(-sqrt(5) - 5)*log(8*cosh(x)^2 + 16*cosh(x)*sinh(x) + 8*si
nh(x)^2 - (sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 5) - 2*sqrt(5) - 2)
```

Sympy [F]

$$\int \cosh(x)\text{sech}(5x)\,dx = \int \cosh(x)\,\text{sech}(5x)\,dx$$

```
[In] integrate(cosh(x)*sech(5*x),x)
[Out] Integral(cosh(x)*sech(5*x), x)
```

Maxima [F]

$$\int \cosh(x)\text{sech}(5x)\,dx = \int \cosh(x)\,\text{sech}(5\,x)\,\,dx$$

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="maxima")
[Out] 1/5*sqrt(5)*arctan((sqrt(5) + 4*e^(-2*x) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*
sqrt(5) + 10) - 1/5*sqrt(5)*arctan(-(sqrt(5) - 4*e^(-2*x) + 1)/sqrt(-2*sqrt
(5) + 10))/sqrt(-2*sqrt(5) + 10) - 1/10*log(-(sqrt(5) + 1)*e^(-2*x) + 2*e^(
-4*x) + 2)/(sqrt(5) + 1) + 1/10*log((sqrt(5) - 1)*e^(-2*x) + 2*e^(-4*x) + 2
)/(sqrt(5) - 1) - 1/5*integrate((e^(7*x) - 2*e^(5*x) - 2*e^(3*x) + e^x)*e^x
/(e^(8*x) - e^(6*x) + e^(4*x) - e^(2*x) + 1), x) + 1/10*log(e^(2*x) + 1) -
1/10*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \cosh(x)\text{sech}(5x)\,dx = -\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,e^{(2\,x)}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,e^{(2\,x)}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$

```
[In] integrate(cosh(x)*sech(5*x),x, algorithm="giac")
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*e^(2*x) - 1)/sqrt(2*sqrt(5)
 + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*e^(2*x) + 1)/sqrt(
-2*sqrt(5) + 10))
```

Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.96

$$\int \cosh(x)\text{sech}(5x)\,dx = \ln\left(1-\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,e^{2\,x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(48\,e^{2\,x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\,(360\,e^{2\,x}-360)-72\right)-8\right)-e^{2\,x}\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}-\ln\left(\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,e^{2\,x}+\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\,(360\,e^{2\,x}-360)-48\,e^{2\,x}+72\right)-8\right)-e^{2\,x}+1\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}-\ln\left(\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,e^{2\,x}+\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}\,(360\,e^{2\,x}-360)-48\,e^{2\,x}+72\right)-8\right)-e^{2\,x}+1\right)\sqrt{-\frac{\sqrt{5}}{200}-\frac{1}{40}}+\ln\left(1-\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(4\,e^{2\,x}+\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\left(48\,e^{2\,x}+\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}\,(360\,e^{2\,x}-360)-72\right)-8\right)-e^{2\,x}\right)\sqrt{\frac{\sqrt{5}}{200}-\frac{1}{40}}$$

```
[In] int(cosh(x)/cosh(5*x),x)
```

```
[Out] log(1 - (- 5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (- 5^(1/2)/200 - 1/40)^(
1/2)*(48*exp(2*x) + (- 5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 72)
 - 8) - exp(2*x))*(- 5^(1/2)/200 - 1/40)^(1/2) - log((5^(1/2)/200 - 1/40)^(
1/2)*(4*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*((5^(1/2)/200 - 1/40)^(1/2)*(
360*exp(2*x) - 360) - 48*exp(2*x) + 72) - 8) - exp(2*x) + 1)*(5^(1/2)/200 -
 1/40)^(1/2) - log((- 5^(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (- 5^(1/2)/20
0 - 1/40)^(1/2)*((- 5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 48*exp
(2*x) + 72) - 8) - exp(2*x) + 1)*(- 5^(1/2)/200 - 1/40)^(1/2) + log(1 - (5^
(1/2)/200 - 1/40)^(1/2)*(4*exp(2*x) + (5^(1/2)/200 - 1/40)^(1/2)*(48*exp(2*
```

```
x) + (5^(1/2)/200 - 1/40)^(1/2)*(360*exp(2*x) - 360) - 72) - 8) - exp(2*x))
*(5^(1/2)/200 - 1/40)^(1/2)
```

3.244 $\int \cosh(x)\mathbf{sech}(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \cosh(x)\text{sech}(6x)\,dx=-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -1/6*arctan(sinh(x)*2^(1/2))*2^(1/2)+1/6*arctan(2*sinh(x)/(1/2*6^(1/2)-1/2*
2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctan(2*sinh(x)/(1/2*6^(1/2)+1/2*2
^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4441, 2082, 209, 1180}

$$\int \cosh(x)\text{sech}(6x)\,dx=-\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[In] Int[Cosh[x]*Sech[6*x],x]
```

```
[Out] -1/3*ArcTan[Sqrt[2]*Sinh[x]]/Sqrt[2] + ArcTan[(2*Sinh[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1+18x^2+48x^4+32x^6}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{3\left(1+2x^2\right)}+\frac{4(1+2x^2)}{3\left(1+16x^2+16x^4\right)}\right)\,dx, x, \sinh(x)\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \sinh(x)\right)\right)+\frac{4}{3}\text{Subst}\left(\int \frac{1+2x^2}{1+16x^2+16x^4}\,dx, x, \sinh(x)\right)\\
&= -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{4}{3}\text{Subst}\left(\int \frac{1}{8-4\sqrt{3}+16x^2}\,dx, x, \sinh(x)\right)\\
&\quad+\frac{4}{3}\text{Subst}\left(\int \frac{1}{8+4\sqrt{3}+16x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{\arctan\left(\sqrt{2}\sinh(x)\right)}{3\sqrt{2}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \cosh(x)\mathrm{sech}(6x)\,dx = \frac{1}{6}\left(-\sqrt{2}\arctan\left(\sqrt{2}\sinh(x)\right)+\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sinh(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

```
[In] Integrate[Cosh[x]*Sech[6*x],x]
```

```
[Out] (-(Sqrt[2]*ArcTan[Sqrt[2]*Sinh[x]]) + Sqrt[2 + Sqrt[3]]*ArcTan[(2*Sinh[x])/
Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTan[(2*Sinh[x])/Sqrt[2 + Sqrt[3]]
])/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

method	result
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(331776\_Z^4+2304\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2x}+\left(13824\_R^3+96\_R\right)\mathrm{e}^{x}-1\right)\right)+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-i\sqrt{2}\,\mathrm{e}^{x}-\right.}{12}$

```
[In] int(cosh(x)*sech(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(exp(2*x)+(13824*_R^3+96*_R)*exp(x)-1),_R=RootOf(331776*_Z^4+230
4*_Z^2+1))+1/12*I*2^(1/2)*ln(exp(2*x)-I*2^(1/2)*exp(x)-1)-1/12*I*2^(1/2)*ln
(exp(2*x)+I*2^(1/2)*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.32

$$\int \cosh(x)\text{sech}(6x)\,dx$$
$$= -\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\cosh(x)+\frac{1}{2}\sqrt{2}\sinh(x)\right)$$
$$+\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+\sqrt{2}}{2(\cosh(x)-\sinh(x))}\right)$$
$$+\frac{1}{12}\sqrt{\sqrt{3}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2$$
$$+\left(\left(\sqrt{3}+2\right)\cosh(x)+\left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{\sqrt{3}-2}-1\Big)$$
$$-\frac{1}{12}\sqrt{\sqrt{3}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2$$
$$-\left(\left(\sqrt{3}+2\right)\cosh(x)+\left(\sqrt{3}+2\right)\sinh(x)\right)\sqrt{\sqrt{3}-2}-1\Big)$$
$$-\frac{1}{12}\sqrt{-\sqrt{3}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2$$
$$+\left(\left(\sqrt{3}-2\right)\cosh(x)+\left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{-\sqrt{3}-2}-1\Big)$$
$$+\frac{1}{12}\sqrt{-\sqrt{3}-2}\log\Big(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2$$
$$-\left(\left(\sqrt{3}-2\right)\cosh(x)+\left(\sqrt{3}-2\right)\sinh(x)\right)\sqrt{-\sqrt{3}-2}-1\Big)$$

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*cosh(x) + 1/2*sqrt(2)*sinh(x)) + 1/6*sqrt(2
)*arctan(-1/2*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh
(x)^2 + sqrt(2))/(cosh(x) - sinh(x))) + 1/12*sqrt(sqrt(3) - 2)*log(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + ((sqrt(3) + 2)*cosh(x) + (sqrt(3) + 2)*
sinh(x))*sqrt(sqrt(3) - 2) - 1) - 1/12*sqrt(sqrt(3) - 2)*log(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - ((sqrt(3) + 2)*cosh(x) + (sqrt(3) + 2)*sinh(x
))*sqrt(sqrt(3) - 2) - 1) - 1/12*sqrt(-sqrt(3) - 2)*log(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + ((sqrt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sq
rt(-sqrt(3) - 2) - 1) + 1/12*sqrt(-sqrt(3) - 2)*log(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - ((sqrt(3) - 2)*cosh(x) + (sqrt(3) - 2)*sinh(x))*sqrt(-
sqrt(3) - 2) - 1)
```

Sympy [F]

$$\int \cosh(x)\text{sech}(6x)\,dx = \int \cosh(x)\,\text{sech}(6x)\,dx$$

```
[In] integrate(cosh(x)*sech(6*x),x)
[Out] Integral(cosh(x)*sech(6*x), x)
```

Maxima [F]

$$\int \cosh(x)\text{sech}(6x)\,dx = \int \cosh(x)\,\text{sech}(6\,x)\,\,dx$$

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/6*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + integrate(1/3*(e^(7*x) + e^(5*x) + e^(3*x) +
 e^x)/(e^(8*x) - e^(4*x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\begin{aligned}\int \cosh(x)\text{sech}(6x)\,dx &= \frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}-\sqrt{2}+4\,e^{x}}{\sqrt{6}+\sqrt{2}}\right)\\ &\quad+\frac{1}{12}\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(-\frac{\sqrt{6}-\sqrt{2}-4\,e^{x}}{\sqrt{6}+\sqrt{2}}\right)\\ &\quad+\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{\sqrt{6}+\sqrt{2}+4\,e^{x}}{\sqrt{6}-\sqrt{2}}\right)\\ &\quad+\frac{1}{12}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(-\frac{\sqrt{6}+\sqrt{2}-4\,e^{x}}{\sqrt{6}-\sqrt{2}}\right)\\ &\quad-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{x}\right)\right)\\ &\quad-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{x}\right)\right)\end{aligned}$$

```
[In] integrate(cosh(x)*sech(6*x),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - sqrt(2))*arctan((sqrt(6) - sqrt(2) + 4*e^x)/(sqrt(6) + sqrt
(2))) + 1/12*(sqrt(6) - sqrt(2))*arctan(-(sqrt(6) - sqrt(2) - 4*e^x)/(sqrt(
6) + sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan((sqrt(6) + sqrt(2) + 4*e^x
)/(sqrt(6) - sqrt(2))) + 1/12*(sqrt(6) + sqrt(2))*arctan(-(sqrt(6) + sqrt(2
) - 4*e^x)/(sqrt(6) - sqrt(2))) - 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*e^x)) - 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^x))
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.42

$$\int \cosh(x)\mathrm{sech}(6x)\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}+4\,\sqrt{3}-4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^x}{2}-\frac{3\,\sqrt{6}\,e^x}{2}}\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}-4\,\sqrt{3}+4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^x}{2}+\frac{3\,\sqrt{6}\,e^x}{2}}\right)}{12} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}+4\,\sqrt{3}-4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^x}{2}-\frac{3\,\sqrt{6}\,e^x}{2}}\right)}{12} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{7\,e^{2\,x}-4\,\sqrt{3}+4\,\sqrt{3}\,e^{2\,x}-7}{\frac{5\,\sqrt{2}\,e^x}{2}+\frac{3\,\sqrt{6}\,e^x}{2}}\right)}{12} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{-x}\,\left(e^{2\,x}-1\right)}{2}\right)}{6}$$

```
[In] int(cosh(x)/cosh(6*x),x)
```

```
[Out] (2^(1/2)*atan((7*exp(2*x) + 4*3^(1/2) - 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)
*exp(x))/2 - (3*6^(1/2)*exp(x))/2)))/12 + (2^(1/2)*atan((7*exp(2*x) - 4*3^(
1/2) + 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)*exp(x))/2 + (3*6^(1/2)*exp(x))/2
)))/12 - (6^(1/2)*atan((7*exp(2*x) + 4*3^(1/2) - 4*3^(1/2)*exp(2*x) - 7)/((
5*2^(1/2)*exp(x))/2 - (3*6^(1/2)*exp(x))/2)))/12 + (6^(1/2)*atan((7*exp(2*x
) - 4*3^(1/2) + 4*3^(1/2)*exp(2*x) - 7)/((5*2^(1/2)*exp(x))/2 + (3*6^(1/2)*
exp(x))/2)))/12 - (2^(1/2)*atan((2^(1/2)*exp(-x)*(exp(2*x) - 1))/2))/6
```

3.245 $\int \cosh(x)\mathbf{csch}(2x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \cosh(x)\text{csch}(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x))$$

```
[Out] -1/2*arctanh(cosh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4372, 3855}

$$\int \cosh(x)\text{csch}(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x))$$

```
[In] Int[Cosh[x]*Csch[2*x],x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \operatorname{csch}(x)\,dx$$
$$= -\frac{1}{2}\operatorname{arctanh}(\cosh(x))$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.00

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = \frac{1}{2}\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

```
[In] Integrate[Cosh[x]*Csch[2*x],x]
[Out] (-Log[Cosh[x/2]] + Log[Sinh[x/2]])/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$	8
risch	$\frac{\ln(e^x-1)}{2} - \frac{\ln(e^x+1)}{2}$	16

```
[In] int(cosh(x)*csch(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cosh(x)\operatorname{csch}(2x)\,dx = -\frac{1}{2}\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + \frac{1}{2}\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="fricas")
[Out] -1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int \cosh(x)\text{csch}(2x)\,dx = \int \cosh(x)\,\text{csch}(2x)\,dx$$

```
[In] integrate(cosh(x)*csch(2*x),x)
[Out] Integral(cosh(x)*csch(2*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cosh(x)\text{csch}(2x)\,dx = -\frac{1}{2}\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="maxima")
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \cosh(x)\text{csch}(2x)\,dx = -\frac{1}{2}\log\left(e^{x} + 1\right) + \frac{1}{2}\log\left(|e^{x} - 1|\right)$$

```
[In] integrate(cosh(x)*csch(2*x),x, algorithm="giac")
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cosh(x)\text{csch}(2x)\,dx = \frac{\ln\left(1 - e^{x}\right)}{2} - \frac{\ln\left(-e^{x} - 1\right)}{2}$$

```
[In] int(cosh(x)/sinh(2*x),x)
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2
```

3.246 $\int \cosh(x)\mathbf{csch}(3x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(3+4\sinh^2(x)\right)$$

```
[Out] 1/3*ln(sinh(x))-1/6*ln(3+4*sinh(x)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {4441, 272, 36, 29, 31}

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(4\sinh^2(x)+3\right)$$

```
[In] Int[Cosh[x]*Csch[3*x],x]
```

```
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\,(3+4x^2)}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(3+4x)}\,dx, x, \sinh^2(x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{3+4x}\,dx, x, \sinh^2(x)\right)\\
&= \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(3+4\sinh^2(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{csch}(3x)\,dx = \frac{1}{3}\log(\sinh(x)) - \frac{1}{6}\log\left(3+4\sinh^2(x)\right)$$

```
[In] Integrate[Cosh[x]*Csch[3*x],x]
[Out] Log[Sinh[x]]/3 - Log[3 + 4*Sinh[x]^2]/6
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\ln(e^{2x}-1)}{3}-\frac{\ln(e^{4x}+e^{2x}+1)}{6}$	24

```
[In] int(cosh(x)*csch(3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(exp(2*x)-1)-1/6*ln(exp(4*x)+exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = -\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\frac{1}{3}\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="fricas")
[Out] -1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = \int \cosh(x)\operatorname{csch}(3x)\,dx$$

```
[In] integrate(cosh(x)*csch(3*x),x)
[Out] Integral(cosh(x)*csch(3*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = -\frac{1}{6}\log\left(e^{(-x)} + e^{(-2x)} + 1\right) + \frac{1}{3}\log\left(e^{(-x)} + 1\right) + \frac{1}{3}\log\left(e^{(-x)} - 1\right) - \frac{1}{6}\log\left(-e^{(-x)} + e^{(-2x)} + 1\right)$$

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="maxima")
[Out] -1/6*log(e^(-x) + e^(-2*x) + 1) + 1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 1)
 - 1/6*log(-e^(-x) + e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = -\frac{1}{6}\log\left(e^{(2x)} + e^{x} + 1\right) - \frac{1}{6}\log\left(e^{(2x)} - e^{x} + 1\right) + \frac{1}{3}\log\left(e^{x} + 1\right) + \frac{1}{3}\log\left(|e^{x} - 1|\right)$$

```
[In] integrate(cosh(x)*csch(3*x),x, algorithm="giac")
[Out] -1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) + 1/3*log(e^x + 1)
 + 1/3*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \cosh(x)\operatorname{csch}(3x)\,dx = \frac{\ln\left(3\,e^{2\,x} - 3\right)}{3} - \frac{\ln\left(-e^{2\,x} - e^{4\,x} - 1\right)}{6}$$

```
[In] int(cosh(x)/sinh(3*x),x)
[Out] log(3*exp(2*x) - 3)/3 - log(- exp(2*x) - exp(4*x) - 1)/6
```

3.247 $\int \cosh(x)\mathbf{csch}(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \cosh(x)\text{csch}(4x)\,dx = -\frac{1}{4}\text{arctanh}(\cosh(x)) + \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(cosh(x))+1/4*arctanh(cosh(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1107, 212}

$$\int \cosh(x)\text{csch}(4x)\,dx = \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}} - \frac{1}{4}\text{arctanh}(\cosh(x))$$

```
[In] Int[Cosh[x]*Csch[4*x],x]
```

```
[Out] -1/4*ArcTanh[Cosh[x]] + ArcTanh[Sqrt[2]*Cosh[x]]/(2*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{-4+12x^2-8x^4}\,dx, x, \cosh(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{4-8x^2}\,dx, x, \cosh(x)\right) - 2\text{Subst}\left(\int \frac{1}{8-8x^2}\,dx, x, \cosh(x)\right)\\ &= -\frac{1}{4}\text{arctanh}(\cosh(x)) + \frac{\text{arctanh}\left(\sqrt{2}\cosh(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int \cosh(x)\text{csch}(4x)\,dx = \frac{1}{4}\left(\sqrt{2}\text{arctanh}\left(\sqrt{2}-i\tanh\left(\frac{x}{2}\right)\right) + \sqrt{2}\text{arctanh}\left(\sqrt{2}+i\tanh\left(\frac{x}{2}\right)\right) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

```
[In] Integrate[Cosh[x]*Csch[4*x],x]
[Out] (Sqrt[2]*ArcTanh[Sqrt[2] - I*Tanh[x/2]] + Sqrt[2]*ArcTanh[Sqrt[2] + I*Tanh[
x/2]] - Log[Cosh[x/2]] + Log[Sinh[x/2]])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

method	result	size
risch	$\frac{\ln(e^x-1)}{4}-\frac{\ln(e^x+1)}{4}+\frac{\ln\left(1+e^{2x}+e^x\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln\left(1+e^{2x}-e^x\sqrt{2}\right)\sqrt{2}}{8}$	53

```
[In] int(cosh(x)*csch(4*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(exp(x)-1)-1/4*ln(exp(x)+1)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cosh(x)\text{csch}(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\cosh(x)^2+\sinh(x)^2+2\sqrt{2}\cosh(x)+2}{\cosh(x)^2+\sinh(x)^2}\right) - \frac{1}{4}\log(\cosh(x)+\sinh(x)+1) + \frac{1}{4}\log(\cosh(x)+\sinh(x)-1)$$

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log((cosh(x)^2 + sinh(x)^2 + 2*sqrt(2)*cosh(x) + 2)/(cosh(x)^2
+ sinh(x)^2)) - 1/4*log(cosh(x) + sinh(x) + 1) + 1/4*log(cosh(x) + sinh(x)
- 1)
```

Sympy [F]

$$\int \cosh(x)\text{csch}(4x)\,dx = \int \cosh(x)\,\text{csch}(4x)\,dx$$

```
[In] integrate(cosh(x)*csch(4*x),x)
[Out] Integral(cosh(x)*csch(4*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \cosh(x)\text{csch}(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right) - \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{(-x)}+e^{(-2x)}+1\right) - \frac{1}{4}\log\left(e^{(-x)}+1\right) + \frac{1}{4}\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*log(sqrt(2)*e^(-x) + e^(-2*x) + 1) - 1/8*sqrt(2)*log(-sqrt(2)*e
^(-x) + e^(-2*x) + 1) - 1/4*log(e^(-x) + 1) + 1/4*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-e^{x}}{\sqrt{2}+e^{(-x)}+e^{x}}\right) - \frac{1}{8}\log\left(e^{(-x)}+e^{x}+2\right) + \frac{1}{8}\log\left(e^{(-x)}+e^{x}-2\right)$$

```
[In] integrate(cosh(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - e^x)/(sqrt(2) + e^(-x) + e^x)) - 1/8*
log(e^(-x) + e^x + 2) + 1/8*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \cosh(x)\operatorname{csch}(4x)\,dx = \frac{\ln\left(\frac{1}{2}-\frac{e^{x}}{2}\right)}{4} - \frac{\ln\left(-\frac{e^{x}}{2}-\frac{1}{2}\right)}{4} + \frac{\sqrt{2}\,\ln\left(-\frac{e^{2\,x}}{8}-\frac{\sqrt{2}\,e^{x}}{8}-\frac{1}{8}\right)}{8} - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,e^{x}}{8}-\frac{e^{2\,x}}{8}-\frac{1}{8}\right)}{8}$$

```
[In] int(cosh(x)/sinh(4*x),x)
```

```
[Out] log(1/2 - exp(x)/2)/4 - log(- exp(x)/2 - 1/2)/4 + (2^(1/2)*log(- exp(2*x)/8
 - (2^(1/2)*exp(x))/8 - 1/8))/8 - (2^(1/2)*log((2^(1/2)*exp(x))/8 - exp(2*x
)/8 - 1/8))/8
```

3.248 $\int \cosh(x)\mathbf{csch}(5x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 62

$$\int \cosh(x)\text{csch}(5x)\,dx = \frac{1}{5}\log(\sinh(x)) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}+8\sinh^2(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}+8\sinh^2(x)\right)$$

```
[Out] 1/5*ln(sinh(x))-1/20*ln(5+8*sinh(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5+8*sin
h(x)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4441, 1128, 719, 29, 646, 31}

$$\int \cosh(x)\text{csch}(5x)\,dx = -\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(8\sinh^2(x)-\sqrt{5}+5\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(8\sinh^2(x)+\sqrt{5}+5\right) + \frac{1}{5}\log(\sinh(x))$$

```
[In] Int[Cosh[x]*Csch[5*x],x]
```

```
[Out] Log[Sinh[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] + 8*Sinh[x]^2])/20 - ((1 -
Sqrt[5])*Log[5 + Sqrt[5] + 8*Sinh[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x\,(5+20x^2+16x^4)}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\,(5+20x+16x^2)}\,dx, x, \sinh^2(x)\right)\\
&= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right) + \frac{1}{10}\text{Subst}\left(\int \frac{-20-16x}{5+20x+16x^2}\,dx, x, \sinh^2(x)\right)\\
&= \frac{1}{5}\log(\sinh(x)) - \frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10+2\sqrt{5}+16x}\,dx, x, \sinh^2(x)\right)\\
&\quad - \frac{1}{5}\left(4\left(1+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{10-2\sqrt{5}+16x}\,dx, x, \sinh^2(x)\right)
\end{aligned}
$$

$$= \frac{1}{5}\log(\sinh(x)) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}+8\sinh^2(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}+8\sinh^2(x)\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \cosh(x)\operatorname{csch}(5x)\,dx = \frac{1}{20}\left(-\left(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cosh(2x)\right)\right) + \left(-1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cosh(2x)\right) + 4\log(\sinh(x))\right)$$

```
[In] Integrate[Cosh[x]*Csch[5*x],x]
```

```
[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cosh[2*x]]) + (-1 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cosh[2*x]] + 4*Log[Sinh[x]])/20
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result
risch	$\frac{\ln\left(e^{2x}-1\right)}{5} - \frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20} + \frac{\ln\left(e^{4x}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)\sqrt{5}}{20} - \frac{\ln\left(e^{4x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+1\right)}{20} - \frac{\ln\left(e^{4x}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2x}+\right.}{20}$

```
[In] int(cosh(x)*csch(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*ln(exp(2*x)-1)-1/20*ln(exp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)+1/20*ln(e
xp(4*x)+(1/2+1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)-1/20*ln(exp(4*x)+(1/2-1/2*5^(
1/2))*exp(2*x)+1)-1/20*ln(exp(4*x)+(1/2-1/2*5^(1/2))*exp(2*x)+1)*5^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.90

$$\int \cosh(x)\operatorname{csch}(5x)\,dx$$
$$= \frac{1}{20}\sqrt{5}\log\left(\frac{4\cosh(x)^4 + 4\sinh(x)^4 + 4\left(\sqrt{5}+1\right)\cosh(x)^2 + 4\left(6\cosh(x)^2 + \sqrt{5}+1\right)\sinh(x)^2 + \sqrt{5}+7}{2\cosh(x)^4 + 2\sinh(x)^4 + 2\left(6\cosh(x)^2+1\right)\sinh(x)^2 + 2\cosh(x)^2 + 1}\right)$$
$$-\frac{1}{20}\log\left(\frac{2\cosh(x)^4 + 2\sinh(x)^4 + 2\left(6\cosh(x)^2+1\right)\sinh(x)^2 + 2\cosh(x)^2 + 1}{\cosh(x)^4 - 4\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 4\cosh(x)\sinh(x)^3 + \sinh(x)^4}\right)$$
$$+\frac{1}{5}\log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((4*cosh(x)^4 + 4*sinh(x)^4 + 4*(sqrt(5) + 1)*cosh(x)^2 + 4
*(6*cosh(x)^2 + sqrt(5) + 1)*sinh(x)^2 + sqrt(5) + 7)/(2*cosh(x)^4 + 2*sinh
(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)) - 1/20*log((2*cos
h(x)^4 + 2*sinh(x)^4 + 2*(6*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 1)/(co
sh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3
 + sinh(x)^4)) + 1/5*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \cosh(x)\operatorname{csch}(5x)\,dx = \int \cosh(x)\operatorname{csch}(5x)\,dx$$

```
[In] integrate(cosh(x)*csch(5*x),x)
```

```
[Out] Integral(cosh(x)*csch(5*x), x)
```

Maxima [F]

$$\int \cosh(x)\operatorname{csch}(5x)\,dx = \int \cosh(x)\operatorname{csch}(5x)\,dx$$

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="maxima")
```

```
[Out] -1/5*integrate((e^(3*x) + e^(2*x) + e^x + 1)*e^x/(e^(4*x) + e^(3*x) + e^(2*
x) + e^x + 1), x) - 1/5*integrate((e^(3*x) - e^(2*x) + e^x - 1)*e^x/(e^(4*x
) - e^(3*x) + e^(2*x) - e^x + 1), x) + 3/10*integrate(e^(3*x)/(e^(4*x) + e^
(3*x) + e^(2*x) + e^x + 1), x) - 3/10*integrate(e^(3*x)/(e^(4*x) - e^(3*x)
+ e^(2*x) - e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) + e^(3*x) + e^(2
*x) + e^x + 1), x) + 1/10*integrate(e^(2*x)/(e^(4*x) - e^(3*x) + e^(2*x) -
e^x + 1), x) - 1/10*integrate(e^x/(e^(4*x) + e^(3*x) + e^(2*x) + e^x + 1),
x) + 1/10*integrate(e^x/(e^(4*x) - e^(3*x) + e^(2*x) - e^x + 1), x) + 1/5*l
og(e^x + 1) + 1/5*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.74

$$\int \cosh(x)\text{csch}(5x)\,dx = -\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right) - \frac{1}{20}\left(\sqrt{5}+1\right)\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(-\frac{1}{2}\left(\sqrt{5}-1\right)e^{x}+e^{(2x)}+1\right) + \frac{1}{5}\log\left(e^{x}+1\right) + \frac{1}{5}\log\left(|e^{x}-1|\right)$$

```
[In] integrate(cosh(x)*csch(5*x),x, algorithm="giac")
```

```
[Out] -1/20*(sqrt(5) + 1)*log(1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) - 1/20*(sqrt(5
) + 1)*log(-1/2*(sqrt(5) + 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(1
/2*(sqrt(5) - 1)*e^x + e^(2*x) + 1) + 1/20*(sqrt(5) - 1)*log(-1/2*(sqrt(5)
- 1)*e^x + e^(2*x) + 1) + 1/5*log(e^x + 1) + 1/5*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

$$\int \cosh(x)\text{csch}(5x)\,dx = \frac{\ln\left(5-5\,e^{2x}\right)}{5} - \ln\left(2\,e^{4x}-e^{2x}+\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)\left(30\,e^{4x}-20\,e^{2x}+30\right)+2\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right) + \ln\left(2\,e^{4x}-e^{2x}-\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)\left(30\,e^{4x}-20\,e^{2x}+30\right)+2\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)$$

```
[In] int(cosh(x)/sinh(5*x),x)
```

```
[Out] log(5 - 5*exp(2*x))/5 - log(2*exp(4*x) - exp(2*x) + (5^(1/2)/20 + 1/20)*(30
*exp(4*x) - 20*exp(2*x) + 30) + 2)*(5^(1/2)/20 + 1/20) + log(2*exp(4*x) - e
xp(2*x) - (5^(1/2)/20 - 1/20)*(30*exp(4*x) - 20*exp(2*x) + 30) + 2)*(5^(1/2
)/20 - 1/20)
```

3.249 $\int \cosh(x)\mathbf{csch}(6x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \cosh(x)\text{csch}(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cosh(x)) - \frac{1}{6}\text{arctanh}(2\cosh(x)) + \frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(cosh(x))-1/6*arctanh(2*cosh(x))+1/6*arctanh(2/3*cosh(x)*3^(1/2
))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2082, 213}

$$\int \cosh(x)\text{csch}(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cosh(x)) - \frac{1}{6}\text{arctanh}(2\cosh(x)) + \frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[In] Int[Cosh[x]*Csch[6*x],x]
```

```
[Out] -1/6*ArcTanh[Cosh[x]] - ArcTanh[2*Cosh[x]]/6 + ArcTanh[(2*Cosh[x])/Sqrt[3]]
/(2*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{2\,(3-19x^2+32x^4-16x^6)}\,dx, x, \cosh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \cosh(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3\,(-1+x^2)}+\frac{2}{-3+4x^2}-\frac{2}{3\,(-1+4x^2)}\right)dx, x, \cosh(x)\right)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cosh(x)\right)\\
&\quad -\text{Subst}\left(\int \frac{1}{-3+4x^2}\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{6}\text{arctanh}(\cosh(x))-\frac{1}{6}\text{arctanh}(2\cosh(x))+\frac{\text{arctanh}\left(\frac{2\cosh(x)}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\begin{aligned}
\int \cosh(x)\text{csch}(6x)\,dx &= \frac{1}{12}\left(2\sqrt{3}\text{arctanh}\left(\frac{2-i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)\right.\\
&\quad +2\sqrt{3}\text{arctanh}\left(\frac{2+i\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)\\
&\quad \left.+\log(1-2\cosh(x))-\log(1+2\cosh(x))+2\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)
\end{aligned}$$

```
[In] Integrate[Cosh[x]*Csch[6*x],x]
[Out] (2*Sqrt[3]*ArcTanh[(2 - I*Tanh[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + I*Ta
nh[x/2])/Sqrt[3]] - 2*Log[Cosh[x/2]] + Log[1 - 2*Cosh[x]] - Log[1 + 2*Cosh[
x]] + 2*Log[Sinh[x/2]])/12
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.14

method	result	size
risch	$-\frac{\ln(e^x+1)}{6}+\frac{\ln(e^x-1)}{6}-\frac{\ln(1+e^x+e^{2x})}{12}+\frac{\ln\left(1+e^{2x}+e^x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+e^{2x}-e^x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln(e^{2x}-e^x+1)}{12}$	77

```
[In] int(cosh(x)*csch(6*x),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(exp(x)+1)+1/6*ln(exp(x)-1)-1/12*ln(1+exp(x)+exp(2*x))+1/12*ln(1+exp
(2*x)+exp(x)*3^(1/2))*3^(1/2)-1/12*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)+1/
12*ln(exp(2*x)-exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\begin{aligned}\int \cosh(x)\operatorname{csch}(6x)\,dx &= \frac{1}{12}\sqrt{3}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+4\sqrt{3}\cosh(x)+5}{2\cosh(x)^2+2\sinh(x)^2-1}\right)\\ &\quad-\frac{1}{12}\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right)+\frac{1}{12}\log\left(\frac{2\cosh(x)-1}{\cosh(x)-\sinh(x)}\right)\\ &\quad-\frac{1}{6}\log\left(\cosh(x)+\sinh(x)+1\right)+\frac{1}{6}\log\left(\cosh(x)+\sinh(x)-1\right)\end{aligned}$$

```
[In] integrate(cosh(x)*csch(6*x),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*log((2*cosh(x)^2 + 2*sinh(x)^2 + 4*sqrt(3)*cosh(x) + 5)/(2*cos
h(x)^2 + 2*sinh(x)^2 - 1)) - 1/12*log((2*cosh(x) + 1)/(cosh(x) - sinh(x)))
+ 1/12*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 1/6*log(cosh(x) + sinh(x)
 + 1) + 1/6*log(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int \cosh(x)\operatorname{csch}(6x)\,dx = \int \cosh(x)\operatorname{csch}(6x)\,dx$$

```
[In] integrate(cosh(x)*csch(6*x),x)
[Out] Integral(cosh(x)*csch(6*x), x)
```

Maxima [F]

$$\int \cosh(x)\text{csch}(6x)\,dx = \int \cosh(x)\,\text{csch}(6\,x)\,dx$$

[In] integrate(cosh(x)*csch(6*x),x, algorithm="maxima")

[Out] -integrate(1/2*(e^(3*x) - e^x)/(e^(4*x) - e^(2*x) + 1), x) - 1/12*log(e^(2*x) + e^x + 1) + 1/12*log(e^(2*x) - e^x + 1) - 1/6*log(e^x + 1) + 1/6*log(e^x - 1)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

$$\int \cosh(x)\text{csch}(6x)\,dx = -\frac{1}{12}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-e^{x}}{\sqrt{3}+e^{(-x)}+e^{x}}\right) - \frac{1}{12}\log\left(e^{(-x)}+e^{x}+2\right) - \frac{1}{12}\log\left(e^{(-x)}+e^{x}+1\right) + \frac{1}{12}\log\left(e^{(-x)}+e^{x}-1\right) + \frac{1}{12}\log\left(e^{(-x)}+e^{x}-2\right)$$

[In] integrate(cosh(x)*csch(6*x),x, algorithm="giac")

[Out] -1/12*sqrt(3)*log(-(sqrt(3) - e^(-x) - e^x)/(sqrt(3) + e^(-x) + e^x)) - 1/12*log(e^(-x) + e^x + 2) - 1/12*log(e^(-x) + e^x + 1) + 1/12*log(e^(-x) + e^x - 1) + 1/12*log(e^(-x) + e^x - 2)

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \cosh(x)\text{csch}(6x)\,dx = \frac{\ln\left(\frac{1}{3}-\frac{e^{x}}{3}\right)}{6} - \frac{\ln\left(-\frac{e^{x}}{3}-\frac{1}{3}\right)}{6} - \frac{\ln\left(-\frac{e^{2\,x}}{36}-\frac{e^{x}}{36}-\frac{1}{36}\right)}{12} + \frac{\ln\left(\frac{e^{x}}{36}-\frac{e^{2\,x}}{36}-\frac{1}{36}\right)}{12} + \frac{\sqrt{3}\,\ln\left(-\frac{e^{2\,x}}{12}-\frac{\sqrt{3}\,e^{x}}{12}-\frac{1}{12}\right)}{12} - \frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,e^{x}}{12}-\frac{e^{2\,x}}{12}-\frac{1}{12}\right)}{12}$$

[In] int(cosh(x)/sinh(6*x),x)

[Out] log(1/3 - exp(x)/3)/6 - log(- exp(x)/3 - 1/3)/6 - log(- exp(2*x)/36 - exp(x)/36 - 1/36)/12 + log(exp(x)/36 - exp(2*x)/36 - 1/36)/12 + (3^(1/2)*log(- exp(2*x)/12 - (3^(1/2)*exp(x))/12 - 1/12))/12 - (3^(1/2)*log((3^(1/2)*exp(x))/12 - exp(2*x)/12 - 1/12))/12

3.250 $\int x^m \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

```
[Out] 2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+m
,2*b*x)/b/exp(2*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 12, 3389, 2212}

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} + \frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}$$

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{2}x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{2}\int x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx - \frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx \\
&= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{2^{-3-m}e^{-2a}x^m(-b^2x^2)^{-m}\left(e^{4a}(bx)^m\Gamma(1+m,-2bx)+(-bx)^m\Gamma(1+m,2bx)\right)}{b}$$

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] (2^(-3 - m)*x^m*(E^(4*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*Gamma[1
+ m, 2*b*x]))/(b*E^(2*a)*(-(b^2*x^2))^m)
```

Maple [F]

$$\int x^m \cosh (bx+a) \sinh (bx+a) \, dx$$

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{\cosh (m \log (2\,b) + 2\,a)\,\Gamma(m+1, 2\,bx) + \cosh (m \log (-2\,b) - 2\,a)\,\Gamma(m+1, -2\,bx) - \Gamma(m+1, 2\,bx) \sinh (}{8\,b}$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/8*(cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) + cosh(m*log(-2*b) - 2*a)*g
amma(m + 1, -2*b*x) - gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - gamma(m
+ 1, -2*b*x)*sinh(m*log(-2*b) - 2*a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \frac{1}{4}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1, 2\,bx) - \frac{1}{4}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+1, -2\,bx)$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x)^
(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x)
```

Giac [F]

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(a+b\,x)\,\sinh(a+b\,x)\,dx$$

```
[In] int(x^m*cosh(a + b*x)*sinh(a + b*x),x)
[Out] int(x^m*cosh(a + b*x)*sinh(a + b*x), x)
```

3.251 $\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}$$

```
[Out] 3/8*x/b^3+1/4*x^3/b-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sin
h(b*x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5480, 3392, 30, 2715, 8}

$$\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx = -\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}+\frac{3x\sinh^2(a+bx)}{4b^3}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3x}{8b^3}+\frac{x^3}{4b}$$

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b^4) - (3*x^
2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh[a + b*x]^2)/(4*b^3) + (x
^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3 \sinh^2(a+bx)}{2b} - \frac{3\int x^2 \sinh^2(a+bx)\,dx}{2b} \\
&= -\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3} \\
&\quad + \frac{x^3\sinh^2(a+bx)}{2b} - \frac{3\int \sinh^2(a+bx)\,dx}{4b^3} + \frac{3\int x^2\,dx}{4b} \\
&= \frac{x^3}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} \\
&\quad + \frac{3x\sinh^2(a+bx)}{4b^3} + \frac{x^3\sinh^2(a+bx)}{2b} + \frac{3\int 1\,dx}{8b^3}
\end{aligned}$$

$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3} + \frac{x^3\sinh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(6bx+4b^3x^3)\cosh(2(a+bx))-3(1+2b^2x^2)\sinh(2(a+bx))}{16b^4}$$

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] ((6*b*x + 4*b^3*x^3)*Cosh[2*(a + b*x)] - 3*(1 + 2*b^2*x^2)*Sinh[2*(a + b*x)
])/(16*b^4)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

method	result
risch	$\frac{(4x^3b^3-6x^2b^2+6bx-3)e^{2bx+2a}}{32b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)e^{-2bx-2a}}{32b^4}$
derivativedivides	$\frac{-\frac{a^3\cosh(bx+a)^2}{2}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a}{2}\right.}{}$
default	$\frac{-\frac{a^3\cosh(bx+a)^2}{2}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a}{2}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*x^3+6*b^2
*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{\left(2\,b^3x^3+3\,bx\right)\cosh\left(bx+a\right)^2-3\left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\left(2\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^2}{8\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/8*((2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 3*(2*b^2*x^2 + 1)*cosh(b*x + a)*
sinh(b*x + a) + (2*b^3*x^3 + 3*b*x)*sinh(b*x + a)^2)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases}\frac{x^3\sinh^2\left(a+bx\right)}{4b}+\frac{x^3\cosh^2\left(a+bx\right)}{4b}-\frac{3x^2\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^2}+\frac{3x\sinh^2\left(a+bx\right)}{8b^3}+\frac{3x\cosh^2\left(a+bx\right)}{8b^3}-\frac{3\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{8b^4}\\ \frac{x^4\sinh\left(a\right)\cosh\left(a\right)}{4}\end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((x**3*sinh(a + b*x)**2/(4*b) + x**3*cosh(a + b*x)**2/(4*b) - 3*x*
*2*sinh(a + b*x)*cosh(a + b*x)/(4*b**2) + 3*x*sinh(a + b*x)**2/(8*b**3) + 3
*x*cosh(a + b*x)**2/(8*b**3) - 3*sinh(a + b*x)*cosh(a + b*x)/(8*b**4), Ne(b
, 0)), (x**4*sinh(a)*cosh(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int x^3\cosh(a+bx)\sinh(a+bx)\,dx=\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^4}+\frac{\left(4\,b^3x^3+6\,b^2x^2+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 3*e^(2*a))*e^
(2*b*x)/b^4 + 1/32*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3)e^{(2\,bx+2\,a)}}{32\,b^4} + \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/32*(4*b^3*
x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int x^3 \cosh(a+bx)\sinh(a+bx)\,dx = -\frac{\frac{3\sinh(2\,a+2\,b\,x)}{2} - 2\,b^3\,x^3\cosh(2\,a+2\,b\,x) + 3\,b^2\,x^2\sinh(2\,a+2\,b\,x) - 3\,b\,x\cosh(2\,a+2\,b\,x)}{8\,b^4}$$

```
[In] int(x^3*cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] -((3*sinh(2*a + 2*b*x))/2 - 2*b^3*x^3*cosh(2*a + 2*b*x) + 3*b^2*x^2*sinh(2*
a + 2*b*x) - 3*b*x*cosh(2*a + 2*b*x))/(8*b^4)
```

3.252 $\int x^2 \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int x^2 \cosh(a+bx) \sinh(a+bx) \, dx = \frac{x^2}{4b} - \frac{x \cosh(a+bx) \sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2 \sinh^2(a+bx)}{2b}$$

```
[Out] 1/4*x^2/b-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4*sinh(b*x+a)^2/b^3+1/2*x^2*s
inh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5480, 3391, 30}

$$\int x^2 \cosh(a+bx) \sinh(a+bx) \, dx = \frac{\sinh^2(a+bx)}{4b^3} - \frac{x \sinh(a+bx) \cosh(a+bx)}{2b^2} + \frac{x^2 \sinh^2(a+bx)}{2b} + \frac{x^2}{4b}$$

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] x^2/(4*b) - (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^
3) + (x^2*Sinh[a + b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sinh^2(a+bx)}{2b} - \frac{\int x\sinh^2(a+bx)\,dx}{b} \\
&= -\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2\sinh^2(a+bx)}{2b} + \frac{\int x\,dx}{2b} \\
&= \frac{x^2}{4b} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2\sinh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(1+2b^2x^2)\cosh(2(a+bx)) - 2bx\sinh(2(a+bx))}{8b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] ((1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 2*b*x*Sinh[2*(a + b*x)])/(8*b^3)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(2x^2b^2-2bx+1)\mathrm{e}^{2bx+2a}}{16b^3}+\frac{(2x^2b^2+2bx+1)\mathrm{e}^{-2bx-2a}}{16b^3}$
derivativedivides	$\frac{\frac{a^2\cosh(bx+a)^2}{2}-2a\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)+\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+}{2}}{b^3}$
default	$\frac{\frac{a^2\cosh(bx+a)^2}{2}-2a\left(\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}\right)+\frac{(bx+a)^2\cosh(bx+a)^2}{2}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+}{2}}{b^3}$

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+1)/b^3*ex
p(-2*b*x-2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{4\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)^2-\left(2\,b^2x^2+1\right)\sinh\left(bx+a\right)^2}{8\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*x^2 + 1)*cosh(b*x + a)^2 -
 (2*b^2*x^2 + 1)*sinh(b*x + a)^2)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int x^2\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x^2\sinh^2\left(a+bx\right)}{4b}+\frac{x^2\cosh^2\left(a+bx\right)}{4b}-\frac{x\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b^2}+\frac{\sinh^2\left(a+bx\right)}{4b^3} & \text{for}\ b\neq 0 \\ \frac{x^3\sinh\left(a\right)\cosh\left(a\right)}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**2/(4*b) + x**2*cosh(a + b*x)**2/(4*b) - x*si
nh(a + b*x)*cosh(a + b*x)/(2*b**2) + sinh(a + b*x)**2/(4*b**3), Ne(b, 0)),
(x**3*sinh(a)*cosh(a)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^3} + \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/16*(2*
b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{(2\,b^2x^2 - 2\,bx + 1)e^{(2\,bx+2\,a)}}{16\,b^3} + \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(2*b^2*x^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 + 1/16*(2*b^2*x^2 + 2*b*x
+ 1)*e^(-2*b*x - 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int x^2 \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\frac{\cosh(2\,a+2\,b\,x)}{2} - b\,x\sinh(2\,a+2\,b\,x) + b^2\,x^2\cosh(2\,a+2\,b\,x)}{4\,b^3}$$

```
[In] int(x^2*cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] (cosh(2*a + 2*b*x)/2 - b*x*sinh(2*a + 2*b*x) + b^2*x^2*cosh(2*a + 2*b*x))/(
4*b^3)
```

3.253 $\int x \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx=\frac{x}{4b}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}$$

```
[Out] 1/4*x/b-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5480, 2715, 8}

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx=-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}$$

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] x/(4*b) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sinh^2(a+bx)}{2b} - \frac{\int \sinh^2(a+bx)\,dx}{2b}\\ &= -\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b} + \frac{\int 1\,dx}{4b}\\ &= \frac{x}{4b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{-2bx\cosh(2(a+bx))+\sinh(2(a+bx))}{8b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] -1/8*(-2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/b^2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{(2bx-1)\mathrm{e}^{2bx+2a}}{16b^2}+\frac{(2bx+1)\mathrm{e}^{-2bx-2a}}{16b^2}$	42
derivativedivides	$\frac{\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{a\cosh(bx+a)^2}{2}}{b^2}$	53
default	$\frac{\frac{(bx+a)\cosh(bx+a)^2}{2}-\frac{\cosh(bx+a)\sinh(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{a\cosh(bx+a)^2}{2}}{b^2}$	53

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{bx\cosh\left(bx+a\right)^2+bx\sinh\left(bx+a\right)^2-\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{4\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/4*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a
))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x\sinh^2\left(a+bx\right)}{4b}+\frac{x\cosh^2\left(a+bx\right)}{4b}-\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^2} & \text{for}\ b\neq 0\\ \frac{x^2\sinh\left(a\right)\cosh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((x*sinh(a + b*x)**2/(4*b) + x*cosh(a + b*x)**2/(4*b) - sinh(a + b
*x)*cosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/16*(2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x -
 2*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} + \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(2*b*x - 1)*e^(2*b*x + 2*a)/b^2 + 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^
2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int x\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{\sinh(2\,a+2\,b\,x)-2\,b\,x\cosh(2\,a+2\,b\,x)}{8\,b^2}$$

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x),x)
```

```
[Out] -(sinh(2*a + 2*b*x) - 2*b*x*cosh(2*a + 2*b*x))/(8*b^2)
```

3.254 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\sinh^2(a+bx)}{2b}$$

```
[Out] 1/2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2644, 30}

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\sinh^2(a+bx)}{2b}$$

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}(\int x\,dx, x, i\sinh(a+bx))}{b}\\ &= \frac{\sinh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{1}{2}\left(\frac{\cosh(2a)\cosh(2bx)}{2b} + \frac{\sinh(2a)\sinh(2bx)}{2b}\right)$$

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] ((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^2}{2b}$	14
default	$\frac{\cosh(bx+a)^2}{2b}$	14
risch	$\frac{e^{2bx+2a}}{8b} + \frac{e^{-2bx-2a}}{8b}$	30

```
[In] int(cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*cosh(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2 + \sinh(bx+a)^2}{4\,b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((sinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2}{2b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/2*cosh(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(2bx+2a)}}{8b} + \frac{e^{(-2bx-2a)}}{8b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^2}{2\,b}$$

```
[In] int(cosh(a + b*x)*sinh(a + b*x),x)
[Out] cosh(a + b*x)^2/(2*b)
```

3.255 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x} dx = \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[Out] 1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5556, 12, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x} dx = \frac{1}{2}\sinh(2a)\text{Chi}(2bx) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sinh(2a+2bx)}{2x}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx \\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx \\
&= \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$
\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{2}(\text{Chi}(2bx)\sinh(2a) + \cosh(2a)\text{Shi}(2bx))
$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x,x]
[Out] (CoshIntegral[2*b*x]*Sinh[2*a] + Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)}{4}-\frac{\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)}{4}$	26

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/4*exp(-2*a)*Ei(1,2*b*x)-1/4*exp(2*a)*Ei(1,-2*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\left(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \frac{1}{4}\left(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx)\right)\sinh\left(2\,a\right)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] 1/4*(Ei(2*b*x) - Ei(-2*b*x))*cosh(2*a) + 1/4*(Ei(2*b*x) + Ei(-2*b*x))*sinh(
2*a)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\operatorname{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{4}\operatorname{Ei}(-2\,bx)\,e^{(-2\,a)}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\operatorname{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{4}\operatorname{Ei}(-2\,bx)\,e^{(-2\,a)}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x, x)
```

3.256 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2} dx = b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx)$$

```
[Out] b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5556, 12, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2} dx = b\cosh(2a)\text{Chi}(2bx) + b\sinh(2a)\text{Shi}(2bx) - \frac{\sinh(2a+2bx)}{2x}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sinh(2a+2bx)}{2x^2}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= -\frac{\sinh(2a+2bx)}{2x} + b\int \frac{\cosh(2a+2bx)}{x}\,dx \\
&= -\frac{\sinh(2a+2bx)}{2x} + (b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx + (b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx \\
&= b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}\left(2b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2(a+bx))}{x} + 2b\sinh(2a)\text{Shi}(2bx)\right)$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^2,x]
```

```
[Out] (2*b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*(a + b*x)]/x + 2*b*Sinh[2*a]*Si
nhIntegral[2*b*x])/2
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{2\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)bx+2\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)bx+\mathrm{e}^{2bx+2a}-\mathrm{e}^{-2bx-2a}}{4x}$	55

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*exp(2*a)*Ei(1,-2*b*x)*b*x+2*exp(-2*a)*Ei(1,2*b*x)*b*x+exp(2*b*x+2*a
)-exp(-2*b*x-2*a))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{(bx\text{Ei}(2\,bx) + bx\text{Ei}(-2\,bx))\cosh(2\,a) - 2\,\cosh(bx+a)\sinh(bx+a) + (bx\text{Ei}(2\,bx) - bx\text{Ei}(-2\,bx))\sinh(}{2\,x}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) - 2*cosh(b*x + a)*sinh(b*x
+ a) + (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\cosh(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**2,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}be^{(-2a)}\Gamma(-1,2bx) + \frac{1}{2}be^{(2a)}\Gamma(-1,-2bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{2\,bx\text{Ei}(2\,bx)\,e^{(2\,a)} + 2\,bx\text{Ei}(-2\,bx)\,e^{(-2\,a)} - e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}}{4\,x}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
[Out] 1/4*(2*b*x*Ei(2*b*x)*e^(2*a) + 2*b*x*Ei(-2*b*x)*e^(-2*a) - e^(2*b*x + 2*a)
+ e^(-2*b*x - 2*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x^2,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x^2, x)
```

3.257 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{b\cosh(2a+2bx)}{2x} + b^2\text{Chi}(2bx)\sinh(2a) - \frac{\sinh(2a+2bx)}{4x^2} + b^2\cosh(2a)\text{Shi}(2bx)$$

```
[Out] -1/2*b*cosh(2*b*x+2*a)/x+b^2*cosh(2*a)*Shi(2*b*x)+b^2*Chi(2*b*x)*sinh(2*a)-
1/4*sinh(2*b*x+2*a)/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5556, 12, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2\sinh(2a)\text{Chi}(2bx) + b^2\cosh(2a)\text{Shi}(2bx) - \frac{\sinh(2a+2bx)}{4x^2} - \frac{b\cosh(2a+2bx)}{2x}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

```
[Out] -1/2*(b*Cosh[2*a + 2*b*x])/x + b^2*CoshIntegral[2*b*x]*Sinh[2*a] - Sinh[2*a
 + 2*b*x]/(4*x^2) + b^2*Cosh[2*a]*SinhIntegral[2*b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sinh(2a+2bx)}{2x^3}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^3}\,dx \\
&= -\frac{\sinh(2a+2bx)}{4x^2} + \frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx \\
&= -\frac{b\cosh(2a+2bx)}{2x} - \frac{\sinh(2a+2bx)}{4x^2} + b^2\int \frac{\sinh(2a+2bx)}{x}\,dx
\end{aligned}
$$

$$= -\frac{b\cosh(2a+2bx)}{2x} - \frac{\sinh(2a+2bx)}{4x^2} + \left(b^2\cosh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx + \left(b^2\sinh(2a)\right)\int \frac{\cosh(2bx)}{x}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{2x} + b^2\text{Chi}(2bx)\sinh(2a) - \frac{\sinh(2a+2bx)}{4x^2} + b^2\cosh(2a)\text{Shi}(2bx)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{1}{2}\left(2b^2\text{Chi}(2bx)\sinh(2a) - \frac{2bx\cosh(2(a+bx)) + \sinh(2(a+bx))}{2x^2} + 2b^2\cosh(2a)\text{Shi}(2bx)\right)$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^3,x]
```

```
[Out] (2*b^2*CoshIntegral[2*b*x]*Sinh[2*a] - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a
 + b*x)])/(2*x^2) + 2*b^2*Cosh[2*a]*SinhIntegral[2*b*x])/2
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

method	result	size
risch	$-\frac{-4\,{\rm e}^{-2a}\,\text{Ei}_1(2bx)x^2b^2+4\,{\rm e}^{2a}\,\text{Ei}_1(-2bx)x^2b^2+2\,{\rm e}^{-2bx-2a}bx+2\,{\rm e}^{2bx+2a}bx-{\rm e}^{-2bx-2a}+{\rm e}^{2bx+2a}}{8x^2}$	89

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-4*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2+4*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2+2*ex
p(-2*b*x-2*a)*b*x+2*exp(2*b*x+2*a)*b*x-exp(-2*b*x-2*a)+exp(2*b*x+2*a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.73

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{bx\cosh\left(bx+a\right)^2 + bx\sinh\left(bx+a\right)^2 - \left(b^2x^2\mathrm{Ei}(2\,bx) - b^2x^2\mathrm{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \cosh\left(bx+a\right)\sinh\left(\right.}{2\,x^2}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 - (b^2*x^2*Ei(2*b*x) - b^2*
x^2*Ei(-2*b*x))*cosh(2*a) + cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2*Ei(2*b*x
) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2e^{(-2\,a)}\Gamma(-2,2\,bx) - b^2e^{(2\,a)}\Gamma(-2,-2\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-2*a)*gamma(-2, 2*b*x) - b^2*e^(2*a)*gamma(-2, -2*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx$$
$$= \frac{4\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 4\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 2\,bxe^{(2\,bx+2\,a)} - 2\,bxe^{(-2\,bx-2\,a)} - e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)}}{8\,x^2}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^3,x, algorithm="giac")
[Out] 1/8*(4*b^2*x^2*Ei(2*b*x)*e^(2*a) - 4*b^2*x^2*Ei(-2*b*x)*e^(-2*a) - 2*b*x*e^
(2*b*x + 2*a) - 2*b*x*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x^3,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x^3, x)
```

3.258 $\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4} dx = -\frac{b\cosh(2a+2bx)}{6x^2} + \frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{2}{3}b^3\sinh(2a)\text{Shi}(2bx)$$

```
[Out] 2/3*b^3*Chi(2*b*x)*cosh(2*a)-1/6*b*cosh(2*b*x+2*a)/x^2+2/3*b^3*Shi(2*b*x)*s
inh(2*a)-1/6*sinh(2*b*x+2*a)/x^3-1/3*b^2*sinh(2*b*x+2*a)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5556, 12, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4} dx = \frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx) + \frac{2}{3}b^3\sinh(2a)\text{Shi}(2bx) - \frac{b^2\sinh(2a+2bx)}{3x} - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b\cosh(2a+2bx)}{6x^2}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/6*(b*Cosh[2*a + 2*b*x])/x^2 + (2*b^3*Cosh[2*a]*CoshIntegral[2*b*x])/3 -
Sinh[2*a + 2*b*x]/(6*x^3) - (b^2*Sinh[2*a + 2*b*x])/(3*x) + (2*b^3*Sinh[2*a
]*SinhIntegral[2*b*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sinh(2a+2bx)}{2x^4}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= -\frac{\sinh(2a+2bx)}{6x^3} + \frac{1}{3}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx
\end{aligned}
$$

$$= -\frac{b\cosh(2a+2bx)}{6x^2} - \frac{\sinh(2a+2bx)}{6x^3} + \frac{1}{3}b^2\int \frac{\sinh(2a+2bx)}{x^2}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{6x^2} - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{1}{3}(2b^3)\int \frac{\cosh(2a+2bx)}{x}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{6x^2} - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{1}{3}(2b^3\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx + \frac{1}{3}(2b^3\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{6x^2} + \frac{2}{3}b^3\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{6x^3} - \frac{b^2\sinh(2a+2bx)}{3x} + \frac{2}{3}b^3\sinh(2a)\text{Shi}(2bx)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{bx\cosh(2(a+bx)) - 4b^3x^3\cosh(2a)\text{Chi}(2bx) + \sinh(2(a+bx)) + 2b^2x^2\sinh(2(a+bx)) - 4b^3x^3\sinh(2}{6x^3}$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/6*(b*x*Cosh[2*(a + b*x)] - 4*b^3*x^3*Cosh[2*a]*CoshIntegral[2*b*x] + Sin
h[2*(a + b*x)] + 2*b^2*x^2*Sinh[2*(a + b*x)] - 4*b^3*x^3*Sinh[2*a]*SinhInte
gral[2*b*x])/x^3
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.42

method	result	si
risch	$-\frac{4\,e^{2a}\,\text{Ei}_1(-2bx)x^3b^3+4\,e^{-2a}\,\text{Ei}_1(2bx)x^3b^3+2\,e^{2bx+2a}b^2x^2-2\,e^{-2bx-2a}b^2x^2+e^{2bx+2a}bx+e^{-2bx-2a}bx+e^{2bx+2a}-e^{-2bx-2a}}{12x^3}$	12

```
[In] int(cosh(b*x+a)*sinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(4*exp(2*a)*Ei(1,-2*b*x)*x^3*b^3+4*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3+2*ex
p(2*b*x+2*a)*b^2*x^2-2*exp(-2*b*x-2*a)*b^2*x^2+exp(2*b*x+2*a)*b*x+exp(-2*b*
x-2*a)*b*x+exp(2*b*x+2*a)-exp(-2*b*x-2*a))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx =$$
$$-\frac{bx\cosh{(bx+a)}^2+bx\sinh{(bx+a)}^2+2\,(2\,b^2x^2+1)\cosh{(bx+a)}\sinh{(bx+a)}-2\,(b^3x^3\mathrm{Ei}(2\,bx)+b^3x^3}{6\,x^3}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*x*cosh(b*x + a)^2 + b*x*sinh(b*x + a)^2 + 2*(2*b^2*x^2 + 1)*cosh(b*
x + a)*sinh(b*x + a) - 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a)
 - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\sinh{(a+bx)}\cosh{(a+bx)}}{x^4}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.36

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = 2\,b^3e^{(-2\,a)}\Gamma(-3,2\,bx)+2\,b^3e^{(2\,a)}\Gamma(-3,-2\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 2*b^3*e^(-2*a)*gamma(-3, 2*b*x) + 2*b^3*e^(2*a)*gamma(-3, -2*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = \frac{4\,b^3x^3\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 4\,b^3x^3\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 2\,b^2x^2e^{(2\,bx+2\,a)} + 2\,b^2x^2e^{(-2\,bx-2\,a)} - bxe^{(2\,bx+2\,a)} - bxe^{(-2\,bx}}{12\,x^3}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*x^3*Ei(2*b*x)*e^(2*a) + 4*b^3*x^3*Ei(-2*b*x)*e^(-2*a) - 2*b^2*x
^2*e^(2*b*x + 2*a) + 2*b^2*x^2*e^(-2*b*x - 2*a) - b*x*e^(2*b*x + 2*a) - b*x
*e^(-2*b*x - 2*a) - e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x))/x^4,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x))/x^4, x)
```

3.259 $\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b} + \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{8b} + \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{8b}$$

```
[Out] 1/8*3^(-1-m)*exp(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)+1/8*exp(a)*x^m*GAM
MA(1+m,-b*x)/b/((-b*x)^m)+1/8*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)+1/8*3^(
-1-m)*x^m*GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 3389, 2212}

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{8b} + \frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{8b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{8b} + \frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{8b}$$

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) + (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) + (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}x^m\sinh(a+bx)+\frac{1}{4}x^m\sinh(3a+3bx)\right)\,dx\\
&= \frac{1}{4}\int x^m\sinh(a+bx)\,dx+\frac{1}{4}\int x^m\sinh(3a+3bx)\,dx\\
&= \frac{1}{8}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{8}\int e^{i(ia+ibx)}x^m\,dx+\frac{1}{8}\int e^{-i(3ia+3ibx)}x^m\,dx-\frac{1}{8}\int e^{i(3ia+3ibx)}x^m\,dx\\
&= \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b}+\frac{e^ax^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b}\\
&\quad+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{8b}+\frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{-3a}x^m\Big(3e^{2a}(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)+(bx)^{-m}\Gamma(1+m,bx))+3^{-m}(-b^2x^2)^{-m}\,(e^{6a}(bx)^m\Gamma(1+m,-3bx$$
$$24b$$

[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x],x]

[Out] (x^m*(3*E^(2*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b
*x]/(b*x)^m) + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x))^m*Gamma[1 +
 m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))

Maple [F]

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)\,dx$$

[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)

[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a),x)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.21

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(m\log(3b)+3a)\Gamma(m+1,3bx)+3\cosh(m\log(b)+a)\Gamma(m+1,bx)+3\cosh(m\log(-b)-a)\Gamma(m$$

[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")

[Out] 1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) + 3*cosh(m*log(b) + a)*gam
ma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) + cosh(m*log(-3*b
) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a)
- 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, -3*b*x)*sinh(m*lo
g(-3*b) - 3*a) - 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b

Sympy [F]

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int x^m \sinh(a+bx)\cosh^2(a+bx)\,dx$$

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a),x)
[Out] Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx =& \frac{1}{8}(3\,bx)^{-m-1}x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx)\\ &+\frac{1}{8}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx)\\ &-\frac{1}{8}(-bx)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx)\\ &-\frac{1}{8}(-3\,bx)^{-m-1}x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx)\end{aligned}$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-m
 - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/8*(-b*x)^(-m - 1)*x^(m + 1)*e^
a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1,
 -3*b*x)
```

Giac [F]

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \sinh(bx+a)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)\ dx$$

```
[In] int(x^m*cosh(a + b*x)^2*sinh(a + b*x),x)
[Out] int(x^m*cosh(a + b*x)^2*sinh(a + b*x), x)
```

3.260 $\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{4x\cosh(a+bx)}{3b^3}+\frac{2x\cosh^3(a+bx)}{9b^3}+\frac{x^3\cosh^3(a+bx)}{3b} - \frac{14\sinh(a+bx)}{9b^4}-\frac{2x^2\sinh(a+bx)}{3b^2} - \frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2}-\frac{2\sinh^3(a+bx)}{27b^4}$$

```
[Out] 4/3*x*cosh(b*x+a)/b^3+2/9*x*cosh(b*x+a)^3/b^3+1/3*x^3*cosh(b*x+a)^3/b-14/9*
sinh(b*x+a)/b^4-2/3*x^2*sinh(b*x+a)/b^2-1/3*x^2*cosh(b*x+a)^2*sinh(b*x+a)/b
^2-2/27*sinh(b*x+a)^3/b^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5481, 3392, 3377, 2717, 2713}

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{2\sinh^3(a+bx)}{27b^4}-\frac{14\sinh(a+bx)}{9b^4}+\frac{2x\cosh^3(a+bx)}{9b^3} + \frac{4x\cosh(a+bx)}{3b^3}-\frac{2x^2\sinh(a+bx)}{3b^2} - \frac{x^2\sinh(a+bx)\cosh^2(a+bx)}{3b^2}+\frac{x^3\cosh^3(a+bx)}{3b}$$

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (4*x*Cosh[a + b*x])/(3*b^3) + (2*x*Cosh[a + b*x]^3)/(9*b^3) + (x^3*Cosh[a +
 b*x]^3)/(3*b) - (14*Sinh[a + b*x])/(9*b^4) - (2*x^2*Sinh[a + b*x])/(3*b^2)
```

```
- (x^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b^2) - (2*Sinh[a + b*x]^3)/(27*b^
4)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3\cosh^3(a+bx)}{3b} - \frac{\int x^2\cosh^3(a+bx)\,dx}{b} \\
&= \frac{2x\cosh^3(a+bx)}{9b^3} + \frac{x^3\cosh^3(a+bx)}{3b} - \frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2} \\
&\quad - \frac{2\int\cosh^3(a+bx)\,dx}{9b^3} - \frac{2\int x^2\cosh(a+bx)\,dx}{3b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x\cosh^3(a+bx)}{9b^3} + \frac{x^3\cosh^3(a+bx)}{3b} - \frac{2x^2\sinh(a+bx)}{3b^2} \\
&\quad - \frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2} - \frac{(2i)\text{Subst}\left(\int (1-x^2)\ dx, x, -i\sinh(a+bx)\right)}{9b^4} \\
&\quad + \frac{4\int x\sinh(a+bx)\,dx}{3b^2} \\
&= \frac{4x\cosh(a+bx)}{3b^3} + \frac{2x\cosh^3(a+bx)}{9b^3} + \frac{x^3\cosh^3(a+bx)}{3b} - \frac{2\sinh(a+bx)}{9b^4} \\
&\quad - \frac{2x^2\sinh(a+bx)}{3b^2} - \frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2} - \frac{2\sinh^3(a+bx)}{27b^4} \\
&\quad - \frac{4\int\cosh(a+bx)\,dx}{3b^3} \\
&= \frac{4x\cosh(a+bx)}{3b^3} + \frac{2x\cosh^3(a+bx)}{9b^3} + \frac{x^3\cosh^3(a+bx)}{3b} - \frac{14\sinh(a+bx)}{9b^4} \\
&\quad - \frac{2x^2\sinh(a+bx)}{3b^2} - \frac{x^2\cosh^2(a+bx)\sinh(a+bx)}{3b^2} - \frac{2\sinh^3(a+bx)}{27b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$
\int x^3\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{27bx(6+b^2x^2)\cosh(a+bx) + (6bx+9b^3x^3)\cosh(3(a+bx)) - 2(82+45b^2x^2+(2+9b^2x^2)\cosh(2(a+b
}{108b^4}
$$

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (27*b*x*(6 + b^2*x^2)*Cosh[a + b*x] + (6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)]
 - 2*(82 + 45*b^2*x^2 + (2 + 9*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(
108*b^4)
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(9x^3b^3-9x^2b^2+6bx-2)e^{3bx+3a}}{216b^4} + \frac{(x^3b^3-3x^2b^2+6bx-6)e^{bx+a}}{8b^4} + \frac{(x^3b^3+3x^2b^2+6bx+6)e^{-bx-a}}{8b^4} + \frac{(9x^3b^3+9x^2b^2}{}$
derivativedivides	$\frac{-\frac{a^3\cosh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\,\mathrm{si}}{9}\right.}{}$
default	$\frac{-\frac{a^3\cosh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)-3a\left(\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\,\mathrm{si}}{9}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(9*b^3*x^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)+1/8*(b^3*x^3-3*b^2*x
^2+6*b*x-6)/b^4*exp(b*x+a)+1/8*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a)+
1/216*(9*b^3*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{3\,(3\,b^3x^3+2\,bx)\cosh{(bx+a)}^3+9\,(3\,b^3x^3+2\,bx)\cosh{(bx+a)}\sinh{(bx+a)}^2-(9\,b^2x^2+2)\sinh{(bx+a)}^3}{108\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/108*(3*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 9*(3*b^3*x^3 + 2*b*x)*cosh(b
*x + a)*sinh(b*x + a)^2 - (9*b^2*x^2 + 2)*sinh(b*x + a)^3 + 27*(b^3*x^3 + 6
*b*x)*cosh(b*x + a) - 3*(27*b^2*x^2 + (9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 54)
*sinh(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x^3\cosh^3{(a+bx)}}{3b}+\frac{2x^2\sinh^3{(a+bx)}}{3b^2}-\frac{x^2\sinh{(a+bx)}\cosh^2{(a+bx)}}{b^2}-\frac{4x\sinh^2{(a+bx)}\cosh{(a+bx)}}{3b^3}+\frac{14x\cosh^3{(a+bx)}}{9b^3}+\frac{40\sinh^3{(a+b}}{27b^4} \\ \frac{x^4\sinh{(a)}\cosh^2{(a)}}{4} \end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**3*cosh(a + b*x)**3/(3*b) + 2*x**2*sinh(a + b*x)**3/(3*b**2) -
 x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 4*x*sinh(a + b*x)**2*cosh(a + b
*x)/(3*b**3) + 14*x*cosh(a + b*x)**3/(9*b**3) + 40*sinh(a + b*x)**3/(27*b**
4) - 14*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), (x**4*sinh(a)*c
osh(a)**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\left(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4} + \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)e^{(bx)}}{8\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{8\,b^4} + \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*e
^(3*b*x)/b^4 + 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b*x
)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b
^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int x^3 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2)e^{(3\,bx+3\,a)}}{216\,b^4} + \frac{(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)e^{(bx+a)}}{8\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{8\,b^4} + \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 + 1/8*(b^3*x^
3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*
b*x - 3*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int x^3 \cosh^2(a+bx) \sinh(a+bx)\,dx = \frac{\frac{2\,x\cosh(a+b\,x)^3}{9}+\frac{4\,x\cosh(a+b\,x)}{3}}{b^3} - \frac{\frac{2\,x^2\sinh(a+b\,x)}{3}+\frac{x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{3}}{b^2} - \frac{40\sinh(a+b\,x)}{27\,b^4} - \frac{2\cosh(a+b\,x)^2\sinh(a+b\,x)}{27\,b^4} + \frac{x^3\cosh(a+b\,x)^3}{3\,b}$$

```
[In] int(x^3*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] ((4*x*cosh(a + b*x))/3 + (2*x*cosh(a + b*x)^3)/9)/b^3 - ((2*x^2*sinh(a + b*
x))/3 + (x^2*cosh(a + b*x)^2*sinh(a + b*x))/3)/b^2 - (40*sinh(a + b*x))/(27
*b^4) - (2*cosh(a + b*x)^2*sinh(a + b*x))/(27*b^4) + (x^3*cosh(a + b*x)^3)/
(3*b)
```

3.261 $\int x^2 \cosh^2(a+bx) \sinh(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^2 \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{4\cosh(a+bx)}{9b^3} + \frac{2\cosh^3(a+bx)}{27b^3} + \frac{x^2\cosh^3(a+bx)}{3b} - \frac{4x\sinh(a+bx)}{9b^2} - \frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2}$$

```
[Out] 4/9*cosh(b*x+a)/b^3+2/27*cosh(b*x+a)^3/b^3+1/3*x^2*cosh(b*x+a)^3/b-4/9*x*si
nh(b*x+a)/b^2-2/9*x*cosh(b*x+a)^2*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5481, 3391, 3377, 2718}

$$\int x^2 \cosh^2(a+bx) \sinh(a+bx) \, dx = \frac{2\cosh^3(a+bx)}{27b^3} + \frac{4\cosh(a+bx)}{9b^3} - \frac{4x\sinh(a+bx)}{9b^2} - \frac{2x\sinh(a+bx)\cosh^2(a+bx)}{9b^2} + \frac{x^2\cosh^3(a+bx)}{3b}$$

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (4*Cosh[a + b*x])/(9*b^3) + (2*Cosh[a + b*x]^3)/(27*b^3) + (x^2*Cosh[a + b*
x]^3)/(3*b) - (4*x*Sinh[a + b*x])/(9*b^2) - (2*x*Cosh[a + b*x]^2*Sinh[a + b
*x])/(9*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2\cosh^3(a+bx)}{3b} - \frac{2\int x\cosh^3(a+bx)\,dx}{3b}\\
&= \frac{2\cosh^3(a+bx)}{27b^3} + \frac{x^2\cosh^3(a+bx)}{3b} - \frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2} - \frac{4\int x\cosh(a+bx)\,dx}{9b}\\
&= \frac{2\cosh^3(a+bx)}{27b^3} + \frac{x^2\cosh^3(a+bx)}{3b} - \frac{4x\sinh(a+bx)}{9b^2}\\
&\quad - \frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2} + \frac{4\int \sinh(a+bx)\,dx}{9b^2}\\
&= \frac{4\cosh(a+bx)}{9b^3} + \frac{2\cosh^3(a+bx)}{27b^3} + \frac{x^2\cosh^3(a+bx)}{3b}\\
&\quad - \frac{4x\sinh(a+bx)}{9b^2} - \frac{2x\cosh^2(a+bx)\sinh(a+bx)}{9b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int x^2 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{27(2+b^2x^2)\cosh(a+bx)+(2+9b^2x^2)\cosh(3(a+bx))-6bx(9\sinh(a+bx)+\sinh(3(a+bx)))}{108b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (27*(2 + b^2*x^2)*Cosh[a + b*x] + (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] - 6*b*x
*(9*Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(108*b^3)
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(9x^2b^2-6bx+2)\mathrm{e}^{3bx+3a}}{216b^3}+\frac{(x^2b^2-2bx+2)\mathrm{e}^{bx+a}}{8b^3}+\frac{(x^2b^2+2bx+2)\mathrm{e}^{-bx-a}}{8b^3}+\frac{(9x^2b^2+6bx+2)\mathrm{e}^{-3bx-3a}}{216b^3}$
derivativedivides	$\frac{\frac{a^2\cosh(bx+a)^3}{3}-2a\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)+\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\sinh(bx+\cdots}{9}\cdots}{b^3}$
default	$\frac{\frac{a^2\cosh(bx+a)^3}{3}-2a\left(\frac{(bx+a)\cosh(bx+a)^3}{3}-\frac{2\sinh(bx+a)}{9}-\frac{\cosh(bx+a)^2\sinh(bx+a)}{9}\right)+\frac{(bx+a)^2\cosh(bx+a)^3}{3}-\frac{4(bx+a)\sinh(bx+\cdots}{9}\cdots}{b^3}$

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(9*b^2*x^2-6*b*x+2)/b^3*exp(3*b*x+3*a)+1/8*(b^2*x^2-2*b*x+2)/b^3*exp(
b*x+a)+1/8*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)+1/216*(9*b^2*x^2+6*b*x+2)/b^3*
exp(-3*b*x-3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int x^2 \cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{6\,bx\sinh\left(bx+a\right)^3-\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)^3-3\left(9\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-27\left(b^2x^2\cdots\right.}{108\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/108*(6*b*x*sinh(b*x + a)^3 - (9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 3*(9*b^2*
x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 27*(b^2*x^2 + 2)*cosh(b*x + a) + 1
8*(b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$=\begin{cases}\frac{x^2\cosh^3(a+bx)}{3b}+\frac{4x\sinh^3(a+bx)}{9b^2}-\frac{2x\sinh(a+bx)\cosh^2(a+bx)}{3b^2}-\frac{4\sinh^2(a+bx)\cosh(a+bx)}{9b^3}+\frac{14\cosh^3(a+bx)}{27b^3} & \text{for } b\neq 0\\ \frac{x^3\sinh(a)\cosh^2(a)}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x**2*cosh(a + b*x)**3/(3*b) + 4*x*sinh(a + b*x)**3/(9*b**2) - 2*
x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 4*sinh(a + b*x)**2*cosh(a + b*x
)/(9*b**3) + 14*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), (x**3*sinh(a)*cosh(a
)**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int x^2\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)})e^{(3\,bx)}}{216\,b^3}+\frac{(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{8\,b^3}+\frac{(b^2x^2+2\,bx+2)e^{(-bx-a)}}{8\,b^3}+\frac{(9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 + 1/8*(
b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 + 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int x^2 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{(9\,b^2x^2 - 6\,bx + 2)e^{(3\,bx+3\,a)}}{216\,b^3} + \frac{(b^2x^2 - 2\,bx + 2)e^{(bx+a)}}{8\,b^3} + \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{8\,b^3} + \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 + 1/8*(b^2*x^2 - 2*b*x +
2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + 1/216*(9*
b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int x^2 \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\frac{4\cosh(a+b\,x)}{9} - b\left(\frac{2\,x\sinh(a+b\,x)\cosh(a+b\,x)^2}{9} + \frac{4\,x\sinh(a+b\,x)}{9}\right) + \frac{2\cosh(a+b\,x)^3}{27} + \frac{b^2\,x^2\cosh(a+b\,x)^3}{3}}{b^3}$$

```
[In] int(x^2*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] ((4*cosh(a + b*x))/9 - b*((4*x*sinh(a + b*x))/9 + (2*x*cosh(a + b*x)^2*sinh
(a + b*x))/9) + (2*cosh(a + b*x)^3)/27 + (b^2*x^2*cosh(a + b*x)^3)/3)/b^3
```

3.262 $\int x \cosh^2(a + bx) \sinh(a + bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x \cosh^2(a + bx) \sinh(a + bx) \, dx = \frac{x \cosh^3(a + bx)}{3b} - \frac{\sinh(a + bx)}{3b^2} - \frac{\sinh^3(a + bx)}{9b^2}$$

```
[Out] 1/3*x*cosh(b*x+a)^3/b-1/3*sinh(b*x+a)/b^2-1/9*sinh(b*x+a)^3/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5481, 2713}

$$\int x \cosh^2(a + bx) \sinh(a + bx) \, dx = -\frac{\sinh^3(a + bx)}{9b^2} - \frac{\sinh(a + bx)}{3b^2} + \frac{x \cosh^3(a + bx)}{3b}$$

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (x*Cosh[a + b*x]^3)/(3*b) - Sinh[a + b*x]/(3*b^2) - Sinh[a + b*x]^3/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\cosh^3(a+bx)}{3b} - \frac{\int \cosh^3(a+bx)\,dx}{3b}\\ &= \frac{x\cosh^3(a+bx)}{3b} - \frac{i\text{Subst}\left(\int (1-x^2)\,dx, x, -i\sinh(a+bx)\right)}{3b^2}\\ &= \frac{x\cosh^3(a+bx)}{3b} - \frac{\sinh(a+bx)}{3b^2} - \frac{\sinh^3(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{-9bx\cosh(a+bx) - 3bx\cosh(3(a+bx)) + 9\sinh(a+bx) + \sinh(3(a+bx))}{36b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x],x]
[Out] -1/36*(-9*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + 9*Sinh[a + b*x] + S
inh[3*(a + b*x)])/b^2
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\cosh(bx+a)^3}{3} - \frac{2\sinh(bx+a)}{9} - \frac{\cosh(bx+a)^2\sinh(bx+a)}{9} - \frac{a\cosh(bx+a)^3}{3}}{b^2}$	56
default	$\frac{\frac{(bx+a)\cosh(bx+a)^3}{3} - \frac{2\sinh(bx+a)}{9} - \frac{\cosh(bx+a)^2\sinh(bx+a)}{9} - \frac{a\cosh(bx+a)^3}{3}}{b^2}$	56
risch	$\frac{(3bx-1)e^{3bx+3a}}{72b^2} + \frac{(bx-1)e^{bx+a}}{8b^2} + \frac{(bx+1)e^{-bx-a}}{8b^2} + \frac{(3bx+1)e^{-3bx-3a}}{72b^2}$	77

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b^2*(1/3*(b*x+a)*cosh(b*x+a)^3-2/9*sinh(b*x+a)-1/9*cosh(b*x+a)^2*sinh(b*x
+a)-1/3*a*cosh(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{3\,bx\cosh\left(bx+a\right)^3+9\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+9\,bx\cosh\left(bx+a\right)-\sinh\left(bx+a\right)^3-3\,\left(\cosh\left(bx+\right.\right.}{36\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/36*(3*b*x*cosh(b*x + a)^3 + 9*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 9*b*x*c
osh(b*x + a) - sinh(b*x + a)^3 - 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x\cosh^3(a+bx)}{3b}+\frac{2\sinh^3(a+bx)}{9b^2}-\frac{\sinh(a+bx)\cosh^2(a+bx)}{3b^2} & \text{for } b\neq 0 \\ \frac{x^2\sinh(a)\cosh^2(a)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*cosh(a + b*x)**3/(3*b) + 2*sinh(a + b*x)**3/(9*b**2) - sinh(a
+ b*x)*cosh(a + b*x)**2/(3*b**2), Ne(b, 0)), (x**2*sinh(a)*cosh(a)**2/2, Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}+\frac{\left(bxe^a-e^a\right)e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 1/8*(b*x*e^a - e^a)*e^(b*x)/
b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^2} + \frac{(bx-1)e^{(bx+a)}}{8\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{8\,b^2} + \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 + 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8*
(b*x + 1)*e^(-b*x - a)/b^2 + 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int x\cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{-3\,b\,x\cosh(a+b\,x)^3+\sinh(a+b\,x)\,\cosh(a+b\,x)^2+2\sinh(a+b\,x)}{9\,b^2}$$

```
[In] int(x*cosh(a + b*x)^2*sinh(a + b*x),x)
```

```
[Out] -(2*sinh(a + b*x) + cosh(a + b*x)^2*sinh(a + b*x) - 3*b*x*cosh(a + b*x)^3)/
(9*b^2)
```

3.263 $\int \cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh^3(a+bx)}{3b}$$

```
[Out] 1/3*cosh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2645, 30}

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh^3(a+bx)}{3b}$$

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^2\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh^3(a+bx)}{3b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x],x]
[Out] Cosh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^3}{3b}$	14
default	$\frac{\cosh(bx+a)^3}{3b}$	14
risch	$\frac{e^{3bx+3a}}{24b}+\frac{e^{bx+a}}{8b}+\frac{e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}$	55

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/3*cosh(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.53

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + 3\cosh(bx+a)}{12\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 3*cosh(b*x + a))/
b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\cosh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a),x)
[Out] Piecewise((cosh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^3}{3b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/3*cosh(b*x + a)^3/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3bx+3a)}}{24b} + \frac{e^{(bx+a)}}{8b} + \frac{e^{(-bx-a)}}{8b} + \frac{e^{(-3bx-3a)}}{24b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
[Out] 1/24*e^(3*b*x + 3*a)/b + 1/8*e^(b*x + a)/b + 1/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^3}{3\,b}$$

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x),x)
[Out] cosh(a + b*x)^3/(3*b)
```

3.264 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\text{Chi}(bx)\sinh(a) + \frac{1}{4}\text{Chi}(3bx)\sinh(3a) + \frac{1}{4}\cosh(a)\text{Shi}(bx) + \frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

```
[Out] 1/4*cosh(a)*Shi(b*x)+1/4*cosh(3*a)*Shi(3*b*x)+1/4*Chi(b*x)*sinh(a)+1/4*Chi(
3*b*x)*sinh(3*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\sinh(a)\text{Chi}(bx) + \frac{1}{4}\sinh(3a)\text{Chi}(3bx) + \frac{1}{4}\cosh(a)\text{Shi}(bx) + \frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (Cosh[a
]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sinh(a+bx)}{4x}+\frac{\sinh(3a+3bx)}{4x}\right) dx\\
&= \frac{1}{4}\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x}\,dx\\
&= \frac{1}{4}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx\\
&\quad+\frac{1}{4}\sinh(a)\int \frac{\cosh(bx)}{x}\,dx+\frac{1}{4}\sinh(3a)\int \frac{\cosh(3bx)}{x}\,dx\\
&= \frac{1}{4}\text{Chi}(bx)\sinh(a)+\frac{1}{4}\text{Chi}(3bx)\sinh(3a)+\frac{1}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}(\mathrm{Chi}(bx)\sinh(a)+\mathrm{Chi}(3bx)\sinh(3a)+\cosh(a)\mathrm{Shi}(bx)+\cosh(3a)\mathrm{Shi}(3bx))$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x,x]
[Out] (CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + Cosh[a]*SinhIn
tegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)}{8}+\frac{\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)}{8}-\frac{\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)}{8}-\frac{\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)}{8}$	47

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)-1/8*exp(a)*Ei(1,-b*x)-1/8*e
xp(3*a)*Ei(1,-3*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}\left(\mathrm{Ei}(3\,bx)-\mathrm{Ei}(-3\,bx)\right)\cosh\left(3\,a\right)+\frac{1}{8}\left(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx)\right)\cosh\left(a\right)+\frac{1}{8}\left(\mathrm{Ei}(3\,bx)+\mathrm{Ei}(-3\,bx)\right)\sinh\left(3\,a\right)+\frac{1}{8}\left(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx)\right)\sinh\left(a\right)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] 1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 1/8*(Ei(b*x) - Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 1/8*(Ei(b*x) + Ei(-b*x))*sinh(a)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\cosh^2(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/8
*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 1/8
*Ei(b*x)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x, x)
```

3.265 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{4}b\cosh(a)\text{Chi}(bx)+\frac{3}{4}b\cosh(3a)\text{Chi}(3bx) - \frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x} + \frac{1}{4}b\sinh(a)\text{Shi}(bx)+\frac{3}{4}b\sinh(3a)\text{Shi}(3bx)$$

```
[Out] 1/4*b*Chi(b*x)*cosh(a)+3/4*b*Chi(3*b*x)*cosh(3*a)+1/4*b*Shi(b*x)*sinh(a)+3/
4*b*Shi(3*b*x)*sinh(3*a)-1/4*sinh(b*x+a)/x-1/4*sinh(3*b*x+3*a)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{4}b\cosh(a)\text{Chi}(bx)+\frac{3}{4}b\cosh(3a)\text{Chi}(3bx) + \frac{1}{4}b\sinh(a)\text{Shi}(bx)+\frac{3}{4}b\sinh(3a)\text{Shi}(3bx) - \frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x])/4 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/4 - S
inh[a + b*x]/(4*x) - Sinh[3*a + 3*b*x]/(4*x) + (b*Sinh[a]*SinhIntegral[b*x]
)/4 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sinh(a+bx)}{4x^2}+\frac{\sinh(3a+3bx)}{4x^2}\right)dx \\
&= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^2}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^2}\,dx \\
&= -\frac{\sinh(a+bx)}{4x}-\frac{\sinh(3a+3bx)}{4x}+\frac{1}{4}b\int \frac{\cosh(a+bx)}{x}\,dx+\frac{1}{4}(3b)\int \frac{\cosh(3a+3bx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sinh(a+bx)}{4x} - \frac{\sinh(3a+3bx)}{4x} + \frac{1}{4}(b\cosh(a))\int\frac{\cosh(bx)}{x}\,dx \\
&\quad + \frac{1}{4}(3b\cosh(3a))\int\frac{\cosh(3bx)}{x}\,dx \\
&\quad + \frac{1}{4}(b\sinh(a))\int\frac{\sinh(bx)}{x}\,dx + \frac{1}{4}(3b\sinh(3a))\int\frac{\sinh(3bx)}{x}\,dx \\
&= \frac{1}{4}b\cosh(a)\text{Chi}(bx) + \frac{3}{4}b\cosh(3a)\text{Chi}(3bx) - \frac{\sinh(a+bx)}{4x} \\
&\quad - \frac{\sinh(3a+3bx)}{4x} + \frac{1}{4}b\sinh(a)\text{Shi}(bx) + \frac{3}{4}b\sinh(3a)\text{Shi}(3bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$
\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx
= \frac{bx\cosh(a)\text{Chi}(bx) + 3bx\cosh(3a)\text{Chi}(3bx) - \sinh(a+bx) - \sinh(3(a+bx)) + bx\sinh(a)\text{Shi}(bx) + 3bx\,\mathrm{s}}{4x}
$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*x*Cosh[a]*CoshIntegral[b*x] + 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] - Sinh
[a + b*x] - Sinh[3*(a + b*x)] + b*x*Sinh[a]*SinhIntegral[b*x] + 3*b*x*Sinh[
3*a]*SinhIntegral[3*b*x])/(4*x)
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{e^{-a}\,\text{Ei}_1(bx)bx+3\,e^{3a}\,\text{Ei}_1(-3bx)bx+3\,e^{-3a}\,\text{Ei}_1(3bx)bx+e^{a}\,\text{Ei}_1(-bx)bx+e^{bx+a}-e^{-bx-a}+e^{3bx+3a}-e^{-3bx-3a}}{8x}$	95

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(exp(-a)*Ei(1,b*x)*b*x+3*exp(3*a)*Ei(1,-3*b*x)*b*x+3*exp(-3*a)*Ei(1,3*
b*x)*b*x+exp(a)*Ei(1,-b*x)*b*x+exp(b*x+a)-exp(-b*x-a)+exp(3*b*x+3*a)-exp(-3
*b*x-3*a))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx =$$
$$-\frac{2\sinh{(bx+a)}^3 - 3\,(bx\mathrm{Ei}(3\,bx) + bx\mathrm{Ei}(-3\,bx))\cosh{(3\,a)} - (bx\mathrm{Ei}(bx) + bx\mathrm{Ei}(-bx))\cosh{(a)} + 2\,(3\cos}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sinh(b*x + a)^3 - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*cosh(3*a) - (b
*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) - (b*x*Ei(b*x) - b*x*Ei(-b
*x))*sinh(a))/x
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\sinh{(a+bx)}\cosh^2{(a+bx)}}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{3}{8}\,be^{(-3\,a)}\Gamma(-1,3\,bx) + \frac{1}{8}\,be^{(-a)}\Gamma(-1,bx) + \frac{1}{8}\,be^{a}\Gamma(-1,-bx) + \frac{3}{8}\,be^{(3\,a)}\Gamma(-1,-3\,bx)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) + 1/8*b*e^a*g
amma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{3\,bx\text{Ei}(3\,bx)\,e^{(3\,a)} + bx\text{Ei}(-bx)\,e^{(-a)} + 3\,bx\text{Ei}(-3\,bx)\,e^{(-3\,a)} + bx\text{Ei}(bx)\,e^{a} - e^{(3\,bx+3\,a)} - e^{(bx+a)} + e^{(-bx-a)}}{8\,x}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) + 3*b*x*Ei(-3*b*x)*e^(-3
*a) + b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) - e^(b*x + a) + e^(-b*x - a) + e^(-
3*b*x - 3*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x^2,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x^2, x)
```

3.266 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}+\frac{1}{8}b^2\text{Chi}(bx)\sinh(a)+\frac{9}{8}b^2\text{Chi}(3bx)\sinh(3a)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}+\frac{1}{8}b^2\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Shi}(3bx)$$

```
[Out] -1/8*b*cosh(b*x+a)/x-3/8*b*cosh(3*b*x+3*a)/x+1/8*b^2*cosh(a)*Shi(b*x)+9/8*b
^2*cosh(3*a)*Shi(3*b*x)+1/8*b^2*Chi(b*x)*sinh(a)+9/8*b^2*Chi(3*b*x)*sinh(3*
a)-1/8*sinh(b*x+a)/x^2-1/8*sinh(3*b*x+3*a)/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{1}{8}b^2\sinh(a)\text{Chi}(bx)+\frac{9}{8}b^2\sinh(3a)\text{Chi}(3bx)+\frac{1}{8}b^2\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^2\cosh(3a)\text{Shi}(3bx)-\frac{\sinh(a+bx)}{8x^2}-\frac{\sinh(3a+3bx)}{8x^2}-\frac{b\cosh(a+bx)}{8x}-\frac{3b\cosh(3a+3bx)}{8x}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

```
[Out] -1/8*(b*Cosh[a + b*x])/x - (3*b*Cosh[3*a + 3*b*x])/(8*x) + (b^2*CoshIntegra
l[b*x]*Sinh[a])/8 + (9*b^2*CoshIntegral[3*b*x]*Sinh[3*a])/8 - Sinh[a + b*x]
/(8*x^2) - Sinh[3*a + 3*b*x]/(8*x^2) + (b^2*Cosh[a]*SinhIntegral[b*x])/8 +
(9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\sinh(a+bx)}{4x^3}+\frac{\sinh(3a+3bx)}{4x^3}\right) dx\\ &= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^3}\,dx+\frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^3}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sinh(a+bx)}{8x^2} - \frac{\sinh(3a+3bx)}{8x^2} + \frac{1}{8}b\int \frac{\cosh(a+bx)}{x^2}\,dx + \frac{1}{8}(3b)\int \frac{\cosh(3a+3bx)}{x^2}\,dx \\
&= -\frac{b\cosh(a+bx)}{8x} - \frac{3b\cosh(3a+3bx)}{8x} - \frac{\sinh(a+bx)}{8x^2} - \frac{\sinh(3a+3bx)}{8x^2} \\
&\quad + \frac{1}{8}b^2\int \frac{\sinh(a+bx)}{x}\,dx + \frac{1}{8}(9b^2)\int \frac{\sinh(3a+3bx)}{x}\,dx \\
&= -\frac{b\cosh(a+bx)}{8x} - \frac{3b\cosh(3a+3bx)}{8x} - \frac{\sinh(a+bx)}{8x^2} - \frac{\sinh(3a+3bx)}{8x^2} \\
&\quad + \frac{1}{8}\left(b^2\cosh(a)\right)\int \frac{\sinh(bx)}{x}\,dx + \frac{1}{8}\left(9b^2\cosh(3a)\right)\int \frac{\sinh(3bx)}{x}\,dx \\
&\quad + \frac{1}{8}\left(b^2\sinh(a)\right)\int \frac{\cosh(bx)}{x}\,dx + \frac{1}{8}\left(9b^2\sinh(3a)\right)\int \frac{\cosh(3bx)}{x}\,dx \\
&= -\frac{b\cosh(a+bx)}{8x} - \frac{3b\cosh(3a+3bx)}{8x} + \frac{1}{8}b^2\text{Chi}(bx)\sinh(a) + \frac{9}{8}b^2\text{Chi}(3bx)\sinh(3a) \\
&\quad - \frac{\sinh(a+bx)}{8x^2} - \frac{\sinh(3a+3bx)}{8x^2} + \frac{1}{8}b^2\cosh(a)\text{Shi}(bx) + \frac{9}{8}b^2\cosh(3a)\text{Shi}(3bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$
\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{bx\cosh(a+bx) + 3bx\cosh(3(a+bx)) - b^2x^2\text{Chi}(bx)\sinh(a) - 9b^2x^2\text{Chi}(3bx)\sinh(3a) + \sinh(a+bx) -}{8x^2}
$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^3,x]
```

```
[Out] -1/8*(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^2*x^2*CoshIntegral[b*
x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*a] + Sinh[a + b*x] + Sinh
[3*(a + b*x)] - b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*Sin
hIntegral[3*b*x])/x^2
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{-9\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^2b^2+9\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^2b^2-\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^2b^2+\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^2b^2+3\,\mathrm{e}^{-3bx-3a}bx+3\,\mathrm{e}^{3bx+3a}bx+\mathrm{e}^{-bx-a}bx+\mathrm{e}^{bx+a}b}{16x^2}$

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-9*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2+9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2-exp
(-a)*Ei(1,b*x)*x^2*b^2+exp(a)*Ei(1,-b*x)*x^2*b^2+3*exp(-3*b*x-3*a)*b*x+3*ex
p(3*b*x+3*a)*b*x+exp(-b*x-a)*b*x+exp(b*x+a)*b*x-exp(-3*b*x-3*a)+exp(3*b*x+3
*a)-exp(-b*x-a)+exp(b*x+a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+2\,bx\cosh\left(bx+a\right)+2\,\sinh\left(bx+a\right)^3-9\,(b^2x$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*b*x
*cosh(b*x + a) + 2*sinh(b*x + a)^3 - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b
*x))*cosh(3*a) - (b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(3*cosh(b
*x + a)^2 + 1)*sinh(b*x + a) - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*s
inh(3*a) - (b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{9}{8}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx)+\frac{1}{8}\,b^2e^{(-a)}\Gamma(-2,bx) - \frac{1}{8}\,b^2e^a\Gamma(-2,-bx)-\frac{9}{8}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) - 1/8*b^2
*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx$$
$$= \frac{9\,b^2x^2\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - b^2x^2\mathrm{Ei}(-bx)\,e^{(-a)} - 9\,b^2x^2\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + b^2x^2\mathrm{Ei}(bx)\,e^{a} - 3\,bxe^{(3\,bx+3\,a)} - bxe^{(bx+}}{16\,x^2}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) - 9*b^2*x^2*Ei(
-3*b*x)*e^(-3*a) + b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) - b*x*e^(b*x
 + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - e^(b*
x + a) + e^(-b*x - a) + e^(-3*b*x - 3*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x^3,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x^3, x)
```

3.267 $\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 154

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx &= -\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}+\frac{1}{24}b^3\cosh(a)\text{Chi}(bx)\\ &+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx)-\frac{\sinh(a+bx)}{12x^3}-\frac{b^2\sinh(a+bx)}{24x}-\frac{\sinh(3a+3bx)}{12x^3}\\ &-\frac{3b^2\sinh(3a+3bx)}{8x}+\frac{1}{24}b^3\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)\end{aligned}$$

```
[Out] 1/24*b^3*Chi(b*x)*cosh(a)+9/8*b^3*Chi(3*b*x)*cosh(3*a)-1/24*b*cosh(b*x+a)/x
^2-1/8*b*cosh(3*b*x+3*a)/x^2+1/24*b^3*Shi(b*x)*sinh(a)+9/8*b^3*Shi(3*b*x)*s
inh(3*a)-1/12*sinh(b*x+a)/x^3-1/24*b^2*sinh(b*x+a)/x-1/12*sinh(3*b*x+3*a)/x
^3-3/8*b^2*sinh(3*b*x+3*a)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx &= \frac{1}{24}b^3\cosh(a)\text{Chi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx)\\ &+\frac{1}{24}b^3\sinh(a)\text{Shi}(bx)+\frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)\\ &-\frac{b^2\sinh(a+bx)}{24x}-\frac{3b^2\sinh(3a+3bx)}{8x}-\frac{\sinh(a+bx)}{12x^3}\\ &-\frac{\sinh(3a+3bx)}{12x^3}-\frac{b\cosh(a+bx)}{24x^2}-\frac{b\cosh(3a+3bx)}{8x^2}\end{aligned}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/24*(b*Cosh[a + b*x])/x^2 - (b*Cosh[3*a + 3*b*x])/(8*x^2) + (b^3*Cosh[a]*
CoshIntegral[b*x])/24 + (9*b^3*Cosh[3*a]*CoshIntegral[3*b*x])/8 - Sinh[a +
b*x]/(12*x^3) - (b^2*Sinh[a + b*x])/(24*x) - Sinh[3*a + 3*b*x]/(12*x^3) - (
3*b^2*Sinh[3*a + 3*b*x])/(8*x) + (b^3*Sinh[a]*SinhIntegral[b*x])/24 + (9*b^
3*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\sinh(a+bx)}{4x^4} + \frac{\sinh(3a+3bx)}{4x^4}\right) dx \\ &= \frac{1}{4}\int \frac{\sinh(a+bx)}{x^4}\,dx + \frac{1}{4}\int \frac{\sinh(3a+3bx)}{x^4}\,dx\end{aligned}$$

$$= -\frac{\sinh(a+bx)}{12x^3} - \frac{\sinh(3a+3bx)}{12x^3} + \frac{1}{12}b\int \frac{\cosh(a+bx)}{x^3}\,dx + \frac{1}{4}b\int \frac{\cosh(3a+3bx)}{x^3}\,dx$$

$$= -\frac{b\cosh(a+bx)}{24x^2} - \frac{b\cosh(3a+3bx)}{8x^2} - \frac{\sinh(a+bx)}{12x^3} - \frac{\sinh(3a+3bx)}{12x^3} + \frac{1}{24}b^2\int \frac{\sinh(a+bx)}{x^2}\,dx + \frac{1}{8}(3b^2)\int \frac{\sinh(3a+3bx)}{x^2}\,dx$$

$$= -\frac{b\cosh(a+bx)}{24x^2} - \frac{b\cosh(3a+3bx)}{8x^2} - \frac{\sinh(a+bx)}{12x^3} - \frac{b^2\sinh(a+bx)}{24x} - \frac{\sinh(3a+3bx)}{12x^3} - \frac{3b^2\sinh(3a+3bx)}{8x} + \frac{1}{24}b^3\int \frac{\cosh(a+bx)}{x}\,dx + \frac{1}{8}(9b^3)\int \frac{\cosh(3a+3bx)}{x}\,dx$$

$$= -\frac{b\cosh(a+bx)}{24x^2} - \frac{b\cosh(3a+3bx)}{8x^2} - \frac{\sinh(a+bx)}{12x^3} - \frac{b^2\sinh(a+bx)}{24x} - \frac{\sinh(3a+3bx)}{12x^3} - \frac{3b^2\sinh(3a+3bx)}{8x} + \frac{1}{24}(b^3\cosh(a))\int \frac{\cosh(bx)}{x}\,dx + \frac{1}{8}(9b^3\cosh(3a))\int \frac{\cosh(3bx)}{x}\,dx + \frac{1}{24}(b^3\sinh(a))\int \frac{\sinh(bx)}{x}\,dx + \frac{1}{8}(9b^3\sinh(3a))\int \frac{\sinh(3bx)}{x}\,dx$$

$$= -\frac{b\cosh(a+bx)}{24x^2} - \frac{b\cosh(3a+3bx)}{8x^2} + \frac{1}{24}b^3\cosh(a)\text{Chi}(bx) + \frac{9}{8}b^3\cosh(3a)\text{Chi}(3bx) - \frac{\sinh(a+bx)}{12x^3} - \frac{b^2\sinh(a+bx)}{24x} - \frac{\sinh(3a+3bx)}{12x^3} - \frac{3b^2\sinh(3a+3bx)}{8x} + \frac{1}{24}b^3\sinh(a)\text{Shi}(bx) + \frac{9}{8}b^3\sinh(3a)\text{Shi}(3bx)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{bx\cosh(a+bx) + 3bx\cosh(3(a+bx)) - b^3x^3\cosh(a)\text{Chi}(bx) - 27b^3x^3\cosh(3a)\text{Chi}(3bx) + 2\sinh(a+\dots}{\dots}$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/24*(b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - b^3*x^3*Cosh[a]*CoshIn
tegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b*x] + 2*Sinh[a + b*x] +
b^2*x^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] -
 b^3*x^3*Sinh[a]*SinhIntegral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*
x])/x^3
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3b^3+27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3+\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3+9\,\mathrm{e}^{3bx+3a}b^2x^2-9\,\mathrm{e}^{-3bx-3a}b^2x^2-\mathrm{e}^{-bx-a}b^2x^2+}{48x^3}$

[In] int(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/48*(27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3+27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3+ex
p(-a)*Ei(1,b*x)*x^3*b^3+exp(a)*Ei(1,-b*x)*x^3*b^3+9*exp(3*b*x+3*a)*b^2*x^2-
9*exp(-3*b*x-3*a)*b^2*x^2-exp(-b*x-a)*b^2*x^2+exp(b*x+a)*b^2*x^2+3*exp(3*b*
x+3*a)*b*x+3*exp(-3*b*x-3*a)*b*x+exp(-b*x-a)*b*x+exp(b*x+a)*b*x+2*exp(3*b*x
+3*a)-2*exp(-3*b*x-3*a)-2*exp(-b*x-a)+2*exp(b*x+a))/x^3

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx =$$
$$-\frac{6\,bx\cosh\left(bx+a\right)^3+18\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+2\left(9\,b^2x^2+2\right)\sinh\left(bx+a\right)^3+2\,bx\cosh\left(bx+a\right.}{}$$

[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="fricas")

[Out] -1/48*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(9*
b^2*x^2 + 2)*sinh(b*x + a)^3 + 2*b*x*cosh(b*x + a) - 27*(b^3*x^3*Ei(3*b*x)
+ b^3*x^3*Ei(-3*b*x))*cosh(3*a) - (b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*cosh
(a) + 2*(b^2*x^2 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a) - 2
7*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a) - (b^3*x^3*Ei(b*x) - b
^3*x^3*Ei(-b*x))*sinh(a))/x^3

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{x^4}\,dx$$

[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)/x**4,x)

[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**2/x**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \frac{27}{8}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx) + \frac{1}{8}\,b^3e^{(-a)}\Gamma(-3,bx) + \frac{1}{8}\,b^3e^{a}\Gamma(-3,-bx) + \frac{27}{8}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="maxima")
[Out] 27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) + 1/8*b^
3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \frac{27\,b^3x^3\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + b^3x^3\mathrm{Ei}(-bx)\,e^{(-a)} + 27\,b^3x^3\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + b^3x^3\mathrm{Ei}(bx)\,e^{a} - 9\,b^2x^2e^{(3\,bx+3\,a)} - b^2}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)/x^4,x, algorithm="giac")
[Out] 1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) + 27*b^3*x^3*E
i(-3*b*x)*e^(-3*a) + b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) - b^2*
x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) + 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*b*x
*e^(3*b*x + 3*a) - b*x*e^(b*x + a) - b*x*e^(-b*x - a) - 3*b*x*e^(-3*b*x - 3
*a) - 2*e^(3*b*x + 3*a) - 2*e^(b*x + a) + 2*e^(-b*x - a) + 2*e^(-3*b*x - 3*
a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x))/x^4,x)
[Out] int((cosh(a + b*x)^2*sinh(a + b*x))/x^4, x)
```

3.268 $\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{2^{-2(3+m)}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} + \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{2^{-2(3+m)}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}$$

```
[Out] exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)+2^(-4-m)*exp(2*a)*x
^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-4-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/
((b*x)^m)+x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 3389, 2212}

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b} + \frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} + \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b} + \frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b}$$

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) + (2^(-4 -
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 +
m))*b*E^(4*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}x^m \sinh(2a+2bx) + \frac{1}{8}x^m \sinh(4a+4bx)\right) dx \\
&= \frac{1}{8}\int x^m \sinh(4a+4bx)\,dx + \frac{1}{4}\int x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx - \frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx \\
&\quad + \frac{1}{8}\int e^{-i(2ia+2ibx)}x^m\,dx - \frac{1}{8}\int e^{i(2ia+2ibx)}x^m\,dx \\
&= \frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} + \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} \\
&\quad + \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{4^{-3-m}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{4^{-3-m}e^{-4a}x^m(-b^2x^2)^{-m}\left(e^{8a}(bx)^m\Gamma(1+m,-4bx)+2^{2+m}e^{6a}(bx)^m\Gamma(1+m,-2bx)+(-bx)^m\left(2^{2+m}e^{2a}\Gamma(1\right.\right.}{b}$$

[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x],x]

[Out] (4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] + 2^(2 + m)*E^(6*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(2^(2 + m)*E^(2*a)*Gamma[1 + m, 2*b*x] + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)

Maple [F]

$$\int x^m \cosh(bx+a)^3 \sinh(bx+a)\,dx$$

[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)

[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a),x)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{\cosh(m\log(4b)+4a)\Gamma(m+1,4bx)+4\cosh(m\log(2b)+2a)\Gamma(m+1,2bx)+4\cosh(m\log(-2b)-2}{}$$

[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")

[Out] 1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) + 4*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) + 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cosh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*log(4*b) + 4*a) - 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - 4*gamma(m + 1, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*b) - 4*a))/b

Sympy [F]

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int x^m \sinh(a+bx)\cosh^3(a+bx)\,dx$$

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a),x)
[Out] Integral(x**m*sinh(a + b*x)*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx =& \frac{1}{16}(4\,bx)^{-m-1}x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx) \\ &+ \frac{1}{8}(2\,bx)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) \\ &- \frac{1}{8}(-2\,bx)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) \\ &- \frac{1}{16}(-4\,bx)^{-m-1}x^{m+1}e^{(4\,a)}\Gamma(m+1,-4\,bx)\end{aligned}$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) + 1/8*(2*b*x)^
(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/8*(-2*b*x)^(-m - 1)*x^(
m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e^(4
*a)*gamma(m + 1, -4*b*x)
```

Giac [F]

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \sinh(bx+a)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a+bx)\sinh(a+bx)\,dx = \int x^m\cosh(a+b\,x)^3\sinh(a+b\,x)\,dx$$

```
[In] int(x^m*cosh(a + b*x)^3*sinh(a + b*x),x)
[Out] int(x^m*cosh(a + b*x)^3*sinh(a + b*x), x)
```

3.269 $\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} - \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{3\cosh^3(a+bx)\sinh(a+bx)}{128b^4} - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2}$$

```
[Out] -45/256*x/b^3-3/32*x^3/b+9/32*x*cosh(b*x+a)^2/b^3+3/32*x*cosh(b*x+a)^4/b^3+
1/4*x^3*cosh(b*x+a)^4/b-45/256*cosh(b*x+a)*sinh(b*x+a)/b^4-9/32*x^2*cosh(b*
x+a)*sinh(b*x+a)/b^2-3/128*cosh(b*x+a)^3*sinh(b*x+a)/b^4-3/16*x^2*cosh(b*x+
a)^3*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{5481, 3392, 30, 2715, 8\}$

$$\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{3\sinh(a+bx)\cosh^3(a+bx)}{128b^4} - \frac{45\sinh(a+bx)\cosh(a+bx)}{256b^4} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{9x\cosh^2(a+bx)}{32b^3} - \frac{3x^2\sinh(a+bx)\cosh^3(a+bx)}{16b^2} - \frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{45x}{256b^3} - \frac{3x^3}{32b}$$

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (9*x*Cosh[a + b*x]^2)/(32*b^3) + (3*x*
Cosh[a + b*x]^4)/(32*b^3) + (x^3*Cosh[a + b*x]^4)/(4*b) - (45*Cosh[a + b*x]
*Sinh[a + b*x])/(256*b^4) - (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) -
(3*Cosh[a + b*x]^3*Sinh[a + b*x])/(128*b^4) - (3*x^2*Cosh[a + b*x]^3*Sinh[a
 + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3\cosh^4(a+bx)}{4b} - \frac{3\int x^2\cosh^4(a+bx)\,dx}{4b}\\
&= \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2}\\
&\quad - \frac{3\int \cosh^4(a+bx)\,dx}{32b^3} - \frac{9\int x^2\cosh^2(a+bx)\,dx}{16b}\\
&= \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3}\\
&\quad + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2}\\
&\quad - \frac{3\cosh^3(a+bx)\sinh(a+bx)}{128b^4} - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2}\\
&\quad - \frac{9\int \cosh^2(a+bx)\,dx}{128b^3} - \frac{9\int \cosh^2(a+bx)\,dx}{32b^3} - \frac{9\int x^2\,dx}{32b}\\
&= -\frac{3x^3}{32b} + \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3}\\
&\quad + \frac{x^3\cosh^4(a+bx)}{4b} - \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4}\\
&\quad - \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{3\cosh^3(a+bx)\sinh(a+bx)}{128b^4}\\
&\quad - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - \frac{9\int 1\,dx}{256b^3} - \frac{9\int 1\,dx}{64b^3}\\
&= -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{9x\cosh^2(a+bx)}{32b^3} + \frac{3x\cosh^4(a+bx)}{32b^3} + \frac{x^3\cosh^4(a+bx)}{4b}\\
&\quad - \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} - \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2}\\
&\quad - \frac{3\cosh^3(a+bx)\sinh(a+bx)}{128b^4} - \frac{3x^2\cosh^3(a+bx)\sinh(a+bx)}{16b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx)\,dx$$
$$= \frac{32bx(3+2b^2x^2)\cosh(2(a+bx))+2bx(3+8b^2x^2)\cosh(4(a+bx))-3(16+32b^2x^2+(1+8b^2x^2)\cosh(2(a}{512b^4}$$

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (32*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + 2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a
 + b*x)] - 3*(16 + 32*b^2*x^2 + (1 + 8*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[2*(
a + b*x)])/(512*b^4)
```

Maple [A] (verified)

Time = 7.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(32x^3b^3-24x^2b^2+12bx-3)e^{4bx+4a}}{2048b^4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)e^{2bx+2a}}{64b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)e^{-2bx-2a}}{64b^4}+\frac{(32x^3}{}$
derivativedivides	$\frac{-\frac{a^3\cosh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\cosh}{4}\right.}{}$
default	$\frac{-\frac{a^3\cosh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\cosh}{4}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2048*(32*b^3*x^3-24*b^2*x^2+12*b*x-3)/b^4*exp(4*b*x+4*a)+1/64*(4*b^3*x^3-
6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/64*(4*b^3*x^3+6*b^2*x^2+6*b*x+3)/b^
4*exp(-2*b*x-2*a)+1/2048*(32*b^3*x^3+24*b^2*x^2+12*b*x+3)/b^4*exp(-4*b*x-4*
a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.23

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx)\,dx$$
$$= \frac{(8\,b^3x^3+3\,bx)\cosh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^4+}{}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 + 16*(2*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^2 + 2*(16*b^3*x^3 + 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)
^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 + 16*(2*b
^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.46

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx)\,dx$$
$$= \begin{cases} -\frac{3x^3\sinh^4(a+bx)}{32b} + \frac{3x^3\sinh^2(a+bx)\cosh^2(a+bx)}{16b} + \frac{5x^3\cosh^4(a+bx)}{32b} + \frac{9x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2} - \frac{15x^2\sinh(a+bx)\cosh^3(a}{32b^2} \\ \frac{x^4\sinh(a)\cosh^3(a)}{4} \end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) + 5*x**3*cosh(a + b*x)**4/(32*b) + 9*x**2*sinh(a + b*x)**
3*cosh(a + b*x)/(32*b**2) - 15*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2
) - 45*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x)**
2/(128*b**3) + 51*x*cosh(a + b*x)**4/(256*b**3) + 45*sinh(a + b*x)**3*cosh(
a + b*x)/(256*b**4) - 51*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne(b, 0
)), (x**4*sinh(a)*cosh(a)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10

$$\int x^3 \cosh^3(a+bx) \sinh(a+bx)\,dx$$
$$= \frac{\left(32\,b^3x^3e^{(4\,a)} - 24\,b^2x^2e^{(4\,a)} + 12\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4} + \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4} + \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*a
))*e^(4*b*x)/b^4 + 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 + 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b
*x - 4*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{(32\,b^3x^3 - 24\,b^2x^2 + 12\,bx - 3)e^{(4\,bx+4\,a)}}{2048\,b^4} + \frac{(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3)e^{(2\,bx+2\,a)}}{64\,b^4} + \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 + 1/64*(4
*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 + 1/64*(4*b^3*x^3 + 6
*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^
2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

$$\int x^3 \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\frac{x^3\cosh(2\,a+2\,b\,x)}{8} + \frac{x^3\cosh(4\,a+4\,b\,x)}{32}}{b} - \frac{\frac{3\,x^2\sinh(2\,a+2\,b\,x)}{16} + \frac{3\,x^2\sinh(4\,a+4\,b\,x)}{128}}{b^2} + \frac{\frac{3\,x\cosh(2\,a+2\,b\,x)}{16} + \frac{3\,x\cosh(4\,a+4\,b\,x)}{256}}{b^3} - \frac{3\sinh(2\,a+2\,b\,x)}{32\,b^4} - \frac{3\sinh(4\,a+4\,b\,x)}{1024\,b^4}$$

```
[In] int(x^3*cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] ((x^3*cosh(2*a + 2*b*x))/8 + (x^3*cosh(4*a + 4*b*x))/32)/b - ((3*x^2*sinh(2
*a + 2*b*x))/16 + (3*x^2*sinh(4*a + 4*b*x))/128)/b^2 + ((3*x*cosh(2*a + 2*b
*x))/16 + (3*x*cosh(4*a + 4*b*x))/256)/b^3 - (3*sinh(2*a + 2*b*x))/(32*b^4)
 - (3*sinh(4*a + 4*b*x))/(1024*b^4)
```

3.270 $\int x^2 \cosh^3(a+bx) \sinh(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int x^2 \cosh^3(a+bx) \sinh(a+bx) \, dx = -\frac{3x^2}{32b} + \frac{3\cosh^2(a+bx)}{32b^3} + \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b} - \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2}$$

```
[Out] -3/32*x^2/b+3/32*cosh(b*x+a)^2/b^3+1/32*cosh(b*x+a)^4/b^3+1/4*x^2*cosh(b*x+
a)^4/b-3/16*x*cosh(b*x+a)*sinh(b*x+a)/b^2-1/8*x*cosh(b*x+a)^3*sinh(b*x+a)/b
^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5481, 3391, 30}

$$\int x^2 \cosh^3(a+bx) \sinh(a+bx) \, dx = \frac{\cosh^4(a+bx)}{32b^3} + \frac{3\cosh^2(a+bx)}{32b^3} - \frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b^2} - \frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2} + \frac{x^2\cosh^4(a+bx)}{4b} - \frac{3x^2}{32b}$$

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-3*x^2)/(32*b) + (3*Cosh[a + b*x]^2)/(32*b^3) + Cosh[a + b*x]^4/(32*b^3) +
 (x^2*Cosh[a + b*x]^4)/(4*b) - (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) -
 (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(8*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\cosh^4(a+bx)}{4b} - \frac{\int x\cosh^4(a+bx)\,dx}{2b}\\
&= \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b} - \frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2} - \frac{3\int x\cosh^2(a+bx)\,dx}{8b}\\
&= \frac{3\cosh^2(a+bx)}{32b^3} + \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b}\\
&\quad - \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2} - \frac{3\int x\,dx}{16b}\\
&= -\frac{3x^2}{32b} + \frac{3\cosh^2(a+bx)}{32b^3} + \frac{\cosh^4(a+bx)}{32b^3} + \frac{x^2\cosh^4(a+bx)}{4b}\\
&\quad - \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{x\cosh^3(a+bx)\sinh(a+bx)}{8b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{16(1+2b^2x^2)\cosh(2(a+bx))+(1+8b^2x^2)\cosh(4(a+bx))-4bx(8\sinh(2(a+bx))+\sinh(4(a+bx)))}{256b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x],x]
[Out] (16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] -
 4*b*x*(8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)]))/(256*b^3)
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(8x^2b^2-4bx+1)\mathrm{e}^{4bx+4a}}{512b^3}+\frac{(2x^2b^2-2bx+1)\mathrm{e}^{2bx+2a}}{32b^3}+\frac{(2x^2b^2+2bx+1)\mathrm{e}^{-2bx-2a}}{32b^3}+\frac{(8x^2b^2+4bx+1)\mathrm{e}^{-4bx-4a}}{512b^3}$
derivativedivides	$\frac{\frac{a^2\cosh(bx+a)^4}{4}-2a\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)+\frac{(bx+a)^2\cosh(bx+a}{4}}{b^3}$
default	$\frac{\frac{a^2\cosh(bx+a)^4}{4}-2a\left(\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)+\frac{(bx+a)^2\cosh(bx+a}{4}}{b^3}$

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/512*(8*b^2*x^2-4*b*x+1)/b^3*exp(4*b*x+4*a)+1/32*(2*b^2*x^2-2*b*x+1)/b^3*e
xp(2*b*x+2*a)+1/32*(2*b^2*x^2+2*b*x+1)/b^3*exp(-2*b*x-2*a)+1/512*(8*b^2*x^2
+4*b*x+1)/b^3*exp(-4*b*x-4*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int x^2 \cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{16\,bx\cosh(bx+a)\sinh(bx+a)^3-(8\,b^2x^2+1)\cosh(bx+a)^4-(8\,b^2x^2+1)\sinh(bx+a)^4-16\,(2\,b^2x}{}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x + a
)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^2
- 2*(16*b^2*x^2 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 +
16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{3x^2\sinh^4{(a+bx)}}{32b}+\frac{3x^2\sinh^2{(a+bx)}\cosh^2{(a+bx)}}{16b}+\frac{5x^2\cosh^4{(a+bx)}}{32b}+\frac{3x\sinh^3{(a+bx)}\cosh{(a+bx)}}{16b^2}-\frac{5x\sinh{(a+bx)}\cosh^3{(a+bx)}}{16b^2} \\ \frac{x^3\sinh{(a)}\cosh^3{(a)}}{3} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(16*b) + 5*x**2*cosh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**3*c
osh(a + b*x)/(16*b**2) - 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) - 3*s
inh(a + b*x)**4/(64*b**3) + 5*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x**3*
sinh(a)*cosh(a)**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3} + \frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^3} + \frac{(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2+4\,bx+1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 + 1/32*(2
*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 + 1/32*(2*b^2*x^2
 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*
x - 4*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{(8\,b^2x^2-4\,bx+1)e^{(4\,bx+4\,a)}}{512\,b^3} + \frac{(2\,b^2x^2-2\,bx+1)e^{(2\,bx+2\,a)}}{32\,b^3} + \frac{(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2+4\,bx+1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/32*(2*b^2*x^2 - 2*b*x
 + 1)*e^(2*b*x + 2*a)/b^3 + 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b
^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int x^2\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{3\cosh(a+b\,x)^2}{32\,b^3} - \frac{\frac{3\,x^2}{32}-\frac{x^2\cosh(a+b\,x)^4}{4}}{b} - \frac{\frac{x\sinh(a+b\,x)\cosh(a+b\,x)^3}{8}+\frac{3\,x\sinh(a+b\,x)\cosh(a+b\,x)}{16}}{b^2} + \frac{\cosh(a+b\,x)^4}{32\,b^3}$$

```
[In] int(x^2*cosh(a + b*x)^3*sinh(a + b*x),x)
```

```
[Out] (3*cosh(a + b*x)^2)/(32*b^3) - ((3*x^2)/32 - (x^2*cosh(a + b*x)^4)/4)/b - (
(x*cosh(a + b*x)^3*sinh(a + b*x))/8 + (3*x*cosh(a + b*x)*sinh(a + b*x))/16)
/b^2 + cosh(a + b*x)^4/(32*b^3)
```

3.271 $\int x\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx=-\frac{3x}{32b}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2}$$

```
[Out] -3/32*x/b+1/4*x*cosh(b*x+a)^4/b-3/32*cosh(b*x+a)*sinh(b*x+a)/b^2-1/16*cosh(
b*x+a)^3*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5481, 2715, 8}

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx=-\frac{\sinh(a+bx)\cosh^3(a+bx)}{16b^2}-\frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2}+\frac{x\cosh^4(a+bx)}{4b}-\frac{3x}{32b}$$

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (-3*x)/(32*b) + (x*Cosh[a + b*x]^4)/(4*b) - (3*Cosh[a + b*x]*Sinh[a + b*x])
/(32*b^2) - (Cosh[a + b*x]^3*Sinh[a + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\cosh^4(a+bx)}{4b} - \frac{\int \cosh^4(a+bx)\,dx}{4b}\\ &= \frac{x\cosh^4(a+bx)}{4b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\int \cosh^2(a+bx)\,dx}{16b}\\ &= \frac{x\cosh^4(a+bx)}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\int 1\,dx}{32b}\\ &= -\frac{3x}{32b} + \frac{x\cosh^4(a+bx)}{4b} - \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{16b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{-16bx\cosh(2(a+bx)) - 4bx\cosh(4(a+bx)) + 8\sinh(2(a+bx)) + \sinh(4(a+bx))}{128b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x],x]
[Out] -1/128*(-16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 8*Sinh[2*(a +
 b*x)] + Sinh[4*(a + b*x)])/b^2
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\cosh(bx+a)^4}{4}}{b^2}$	69
default	$\frac{\frac{(bx+a)\cosh(bx+a)^4}{4}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{16}-\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\cosh(bx+a)^4}{4}}{b^2}$	69
risch	$\frac{(4bx-1)e^{4bx+4a}}{256b^2}+\frac{(2bx-1)e^{2bx+2a}}{32b^2}+\frac{(2bx+1)e^{-2bx-2a}}{32b^2}+\frac{(4bx+1)e^{-4bx-4a}}{256b^2}$	82

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*cosh(b*x+a)^4-1/16*cosh(b*x+a)^3*sinh(b*x+a)-3/32*cosh(b
*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-1/4*a*cosh(b*x+a)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{bx\cosh{(bx+a)}^4+bx\sinh{(bx+a)}^4+4\,bx\cosh{(bx+a)}^2-\cosh{(bx+a)}\sinh{(bx+a)}^3+2\,(3\,bx\cosh{(bx}}{32\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^2 - c
osh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 + 2*b*x)*sinh(b*x +
 a)^2 - (cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{3x\sinh^4{(a+bx)}}{32b}+\frac{3x\sinh^2{(a+bx)}\cosh^2{(a+bx)}}{16b}+\frac{5x\cosh^4{(a+bx)}}{32b}+\frac{3\sinh^3{(a+bx)}\cosh{(a+bx)}}{32b^2}-\frac{5\sinh{(a+bx)}\cosh^3{(a+bx)}}{32b^2} & \text{fo} \\ \frac{x^2\sinh{(a)}\cosh^3{(a)}}{2} & \text{ot} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-3*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x
)**2/(16*b) + 5*x*cosh(a + b*x)**4/(32*b) + 3*sinh(a + b*x)**3*cosh(a + b*x
)/(32*b**2) - 5*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**2*
sinh(a)*cosh(a)**3/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/32*(2*b*x*e^(2*a) - e^(2*
a))*e^(2*b*x)/b^2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}+\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b^
2 + 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x - 4
*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{\frac{3\,x}{32}-\frac{x\cosh(a+b\,x)^4}{4}}{b}-\frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{16\,b^2}-\frac{3\cosh(a+b\,x)\,\sinh(a+b\,x)}{32\,b^2}$$

```
[In] int(x*cosh(a + b*x)^3*sinh(a + b*x),x)
[Out] - ((3*x)/32 - (x*cosh(a + b*x)^4)/4)/b - (cosh(a + b*x)^3*sinh(a + b*x))/(1
6*b^2) - (3*cosh(a + b*x)*sinh(a + b*x))/(32*b^2)
```

3.272 $\int \cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^4(a+bx)}{4b}$$

```
[Out] 1/4*cosh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2645, 30}

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{\cosh^4(a+bx)}{4b}$$

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^3\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\cosh^4(a+bx)}{4b}$$

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x],x]
[Out] Cosh[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^4}{4b}$	14
default	$\frac{\cosh(bx+a)^4}{4b}$	14
risch	$\frac{e^{4bx+4a}}{64b}+\frac{e^{2bx+2a}}{16b}+\frac{e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}$	58

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/4*cosh(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^4 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2+2\right)\sinh(bx+a)^2 + 4\cosh(bx+a)^2}{32\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 2)*sinh(b*
x + a)^2 + 4*cosh(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\cosh^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sinh(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a),x)
[Out] Piecewise((cosh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)*cosh(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^4}{4\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/4*cosh(b*x + a)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{e^{(4\,bx+4\,a)}}{64\,b} + \frac{e^{(2\,bx+2\,a)}}{16\,b} + \frac{e^{(-2\,bx-2\,a)}}{16\,b} + \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] 1/64*e^(4*b*x + 4*a)/b + 1/16*e^(2*b*x + 2*a)/b + 1/16*e^(-2*b*x - 2*a)/b +
 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)^4}{4\,b}$$

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x),x)
[Out] cosh(a + b*x)^4/(4*b)
```

3.273 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\text{Chi}(2bx)\sinh(2a) + \frac{1}{8}\text{Chi}(4bx)\sinh(4a) + \frac{1}{4}\cosh(2a)\text{Shi}(2bx) + \frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[Out] 1/4*cosh(2*a)*Shi(2*b*x)+1/8*cosh(4*a)*Shi(4*b*x)+1/4*Chi(2*b*x)*sinh(2*a)+
1/8*Chi(4*b*x)*sinh(4*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{4}\sinh(2a)\text{Chi}(2bx) + \frac{1}{8}\sinh(4a)\text{Chi}(4bx) + \frac{1}{4}\cosh(2a)\text{Shi}(2bx) + \frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/4 + (CoshIntegral[4*b*x]*Sinh[4*a])/8 + (Co
sh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right) dx \\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x}\,dx \\
&= \frac{1}{4}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{8}\cosh(4a)\int \frac{\sinh(4bx)}{x}\,dx \\
&\quad +\frac{1}{4}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{8}\sinh(4a)\int \frac{\cosh(4bx)}{x}\,dx \\
&= \frac{1}{4}\text{Chi}(2bx)\sinh(2a)+\frac{1}{8}\text{Chi}(4bx)\sinh(4a)+\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{8}(2\text{Chi}(2bx)\sinh(2a)+\text{Chi}(4bx)\sinh(4a) + 2\cosh(2a)\text{Shi}(2bx)+\cosh(4a)\text{Shi}(4bx))$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x,x]
[Out] (2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] + 2*Cosh[2
*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{e^{-4a}\,\text{Ei}_1(4bx)}{16}+\frac{e^{-2a}\,\text{Ei}_1(2bx)}{8}-\frac{e^{2a}\,\text{Ei}_1(-2bx)}{8}-\frac{e^{4a}\,\text{Ei}_1(-4bx)}{16}$	50

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/16*exp(-4*a)*Ei(1,4*b*x)+1/8*exp(-2*a)*Ei(1,2*b*x)-1/8*exp(2*a)*Ei(1,-2*b
*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{16}\left(\text{Ei}(4\,bx)-\text{Ei}(-4\,bx)\right)\cosh\left(4\,a\right) + \frac{1}{8}\left(\text{Ei}(2\,bx)-\text{Ei}(-2\,bx)\right)\cosh\left(2\,a\right) + \frac{1}{16}\left(\text{Ei}(4\,bx)+\text{Ei}(-4\,bx)\right)\sinh\left(4\,a\right) + \frac{1}{8}\left(\text{Ei}(2\,bx)+\text{Ei}(-2\,bx)\right)\sinh\left(2\,a\right)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] 1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) + 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) + 1/8*(Ei(2*b*x) + Ei(-2*b*
x))*sinh(2*a)
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \int \frac{\sinh{(a+bx)}\cosh^3{(a+bx)}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/8*Ei(2*b*x)*e^(2*a) - 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x}\,dx=\int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x,x)
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x, x)
```

3.274 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx) - \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x} + \frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)$$

```
[Out] 1/2*b*Chi(2*b*x)*cosh(2*a)+1/2*b*Chi(4*b*x)*cosh(4*a)+1/2*b*Shi(2*b*x)*sinh
(2*a)+1/2*b*Shi(4*b*x)*sinh(4*a)-1/4*sinh(2*b*x+2*a)/x-1/8*sinh(4*b*x+4*a)/
x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx) + \frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx) - \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

```
[Out] (b*Cosh[2*a]*CoshIntegral[2*b*x])/2 + (b*Cosh[4*a]*CoshIntegral[4*b*x])/2 -
 Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) + (b*Sinh[2*a]*SinhInteg
ral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^2}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^2}\,dx\\ &= -\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}+\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{2}b\int \frac{\cosh(4a+4bx)}{x}\,dx\end{aligned}$$

$$= -\frac{\sinh(2a+2bx)}{4x} - \frac{\sinh(4a+4bx)}{8x} + \frac{1}{2}(b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx + \frac{1}{2}(b\cosh(4a))\int \frac{\cosh(4bx)}{x}\,dx + \frac{1}{2}(b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}(b\sinh(4a))\int \frac{\sinh(4bx)}{x}\,dx$$

$$= \frac{1}{2}b\cosh(2a)\text{Chi}(2bx) + \frac{1}{2}b\cosh(4a)\text{Chi}(4bx) - \frac{\sinh(2a+2bx)}{4x} - \frac{\sinh(4a+4bx)}{8x} + \frac{1}{2}b\sinh(2a)\text{Shi}(2bx) + \frac{1}{2}b\sinh(4a)\text{Shi}(4bx)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{4bx\cosh(2a)\text{Chi}(2bx) + 4bx\cosh(4a)\text{Chi}(4bx) - 2\sinh(2(a+bx)) - \sinh(4(a+bx)) + 4bx\sinh(2a)\text{Shi}(2b}{8x}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^2,x]
```

```
[Out] (4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] + 4*b*x*Cosh[4*a]*CoshIntegral[4*b*x]
- 2*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegral[2*
b*x] + 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/(8*x)
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{4\,e^{2a}\,\text{Ei}_1(-2bx)bx+4\,e^{4a}\,\text{Ei}_1(-4bx)bx+4\,e^{-2a}\,\text{Ei}_1(2bx)bx+4\,e^{-4a}\,\text{Ei}_1(4bx)bx+2\,e^{2bx+2a}+e^{4bx+4a}-2\,e^{-2bx-2a}-e^{-4bx-4a}}{16x}$	105

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(4*exp(2*a)*Ei(1,-2*b*x)*b*x+4*exp(4*a)*Ei(1,-4*b*x)*b*x+4*exp(-2*a)*
Ei(1,2*b*x)*b*x+4*exp(-4*a)*Ei(1,4*b*x)*b*x+2*exp(2*b*x+2*a)+exp(4*b*x+4*a)
-2*exp(-2*b*x-2*a)-exp(-4*b*x-4*a))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = -\frac{2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\left(bx\mathrm{Ei}(4\,bx)+bx\mathrm{Ei}(-4\,bx)\right)\cosh\left(4\,a\right)-\left(bx\mathrm{Ei}(2\,bx)+bx\mathrm{Ei}(-2\,bx)\right)\mathrm{co}}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*co
sh(4*a) - (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*a)
- (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \frac{1}{4}\,be^{(-4\,a)}\Gamma(-1,4\,bx)+\frac{1}{4}\,be^{(-2\,a)}\Gamma(-1,2\,bx) + \frac{1}{4}\,be^{(2\,a)}\Gamma(-1,-2\,bx)+\frac{1}{4}\,be^{(4\,a)}\Gamma(-1,-4\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/4*b*e
^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx$$
$$= \frac{4\,bx\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + 4\,bx\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 4\,bx\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 4\,bx\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2}}{16\,x}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) + 4*b*x*Ei(2*b*x)*e^(2*a) + 4*b*x*Ei(-2*b*x)*
e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a)
+ 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^2}\,dx = \int \frac{{\cosh\left(a+b\,x\right)}^3\,\sinh\left(a+b\,x\right)}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x^2,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x^2, x)
```

3.275 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = & -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}\\ & +\frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a)+b^2\text{Chi}(4bx)\sinh(4a)\\ & -\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}\\ & +\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)\end{aligned}$$

```
[Out] -1/4*b*cosh(2*b*x+2*a)/x-1/4*b*cosh(4*b*x+4*a)/x+1/2*b^2*cosh(2*a)*Shi(2*b*
x)+b^2*cosh(4*a)*Shi(4*b*x)+1/2*b^2*Chi(2*b*x)*sinh(2*a)+b^2*Chi(4*b*x)*sin
h(4*a)-1/8*sinh(2*b*x+2*a)/x^2-1/16*sinh(4*b*x+4*a)/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = & \frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx)+b^2\sinh(4a)\text{Chi}(4bx)\\ & +\frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx)+b^2\cosh(4a)\text{Shi}(4bx)\\ & -\frac{\sinh(2a+2bx)}{8x^2}-\frac{\sinh(4a+4bx)}{16x^2}\\ & -\frac{b\cosh(2a+2bx)}{4x}-\frac{b\cosh(4a+4bx)}{4x}\end{aligned}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

```
[Out] -1/4*(b*Cosh[2*a + 2*b*x])/x - (b*Cosh[4*a + 4*b*x])/(4*x) + (b^2*CoshInteg
ral[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] - Sinh[2*a + 2*
b*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right) dx$$
$$= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^3}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^3}\,dx$$

$$= -\frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} + \frac{1}{4}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx + \frac{1}{4}b\int \frac{\cosh(4a+4bx)}{x^2}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} - \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} + \frac{1}{2}b^2\int \frac{\sinh(2a+2bx)}{x}\,dx + b^2\int \frac{\sinh(4a+4bx)}{x}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} - \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} + \frac{1}{2}\left(b^2\cosh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx + \left(b^2\cosh(4a)\right)\int \frac{\sinh(4bx)}{x}\,dx + \frac{1}{2}\left(b^2\sinh(2a)\right)\int \frac{\cosh(2bx)}{x}\,dx + \left(b^2\sinh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx$$

$$= -\frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a) + b^2\text{Chi}(4bx)\sinh(4a) - \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} + \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2\cosh(a)\text{Chi}(2bx)\sinh(a) + b^2\text{Chi}(4bx)\sinh(4a) - \frac{2bx\cosh(2(a+bx)) + \sinh(2(a+bx))}{8x^2} - \frac{4bx\cosh(4(a+bx)) + \sinh(4(a+bx))}{16x^2} + \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^3,x]
```

```
[Out] b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + b^2*CoshIntegral[4*b*x]*Sinh[4*a]
 - (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4*(a
 + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) + (b^2*Cosh[2*a]*SinhIntegral[2*b*x]
)/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{-16\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^2b^2+8\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2-8\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2+16\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^2b^2+4\,\mathrm{e}^{-4bx-4a}bx+4\,\mathrm{e}^{2bx+2a}bx+4\,\mathrm{e}^{-2bx-}}{32x^2}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-16*exp(-4*a)*Ei(1,4*b*x)*x^2*b^2+8*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2-8*
exp(-2*a)*Ei(1,2*b*x)*x^2*b^2+16*exp(4*a)*Ei(1,-4*b*x)*x^2*b^2+4*exp(-4*b*x
-4*a)*b*x+4*exp(2*b*x+2*a)*b*x+4*exp(-2*b*x-2*a)*b*x+4*exp(4*b*x+4*a)*b*x-e
xp(-4*b*x-4*a)+2*exp(2*b*x+2*a)-2*exp(-2*b*x-2*a)+exp(4*b*x+4*a))/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(113) = 226.

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = -\frac{bx\cosh\left(bx+a\right)^4+bx\sinh\left(bx+a\right)^4+bx\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(6\,bx\cosh\left(bx\right.\right.}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + b*x*cosh(b*x + a)^2 + cos
h(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2
- 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) - (b^2*x^2*Ei(2*b*x)
 - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b
*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) - (b^2*x^2*E
i(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = b^2e^{(-4\,a)}\Gamma(-2,4\,bx) + \frac{1}{2}\,b^2e^{(-2\,a)}\Gamma(-2,2\,bx) - \frac{1}{2}\,b^2e^{(2\,a)}\Gamma(-2,-2\,bx) - b^2e^{(4\,a)}\Gamma(-2,-4\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-4*a)*gamma(-2, 4*b*x) + 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) - 1/2*b^2
*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = \frac{16\,b^2x^2\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + 8\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 8\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 16\,b^2x^2\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+}}{32}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 8*b^2*x^2*Ei(2*b*x)*e^(2*a) - 8*b^2*x^
2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x + 4
*a) - 4*b*x*e^(2*b*x + 2*a) - 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x - 4*
a) - e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x -
 4*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^3}\,dx = \int \frac{{\cosh\left(a+b\,x\right)}^3\,\sinh\left(a+b\,x\right)}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x^3,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x^3, x)
```

3.276 $\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = &-\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}\\ &+\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)+\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)\\ &-\frac{\sinh(2a+2bx)}{12x^3}-\frac{b^2\sinh(2a+2bx)}{6x}\\ &-\frac{\sinh(4a+4bx)}{24x^3}-\frac{b^2\sinh(4a+4bx)}{3x}\\ &+\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)\end{aligned}$$

```
[Out] 1/3*b^3*Chi(2*b*x)*cosh(2*a)+4/3*b^3*Chi(4*b*x)*cosh(4*a)-1/12*b*cosh(2*b*x
+2*a)/x^2-1/12*b*cosh(4*b*x+4*a)/x^2+1/3*b^3*Shi(2*b*x)*sinh(2*a)+4/3*b^3*S
hi(4*b*x)*sinh(4*a)-1/12*sinh(2*b*x+2*a)/x^3-1/6*b^2*sinh(2*b*x+2*a)/x-1/24
*sinh(4*b*x+4*a)/x^3-1/3*b^2*sinh(4*b*x+4*a)/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{5556, 3378, 3384, 3379, 3382\}$

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \frac{1}{3}b^3\cosh(2a)\mathrm{Chi}(2bx) + \frac{4}{3}b^3\cosh(4a)\mathrm{Chi}(4bx) + \frac{1}{3}b^3\sinh(2a)\mathrm{Shi}(2bx) + \frac{4}{3}b^3\sinh(4a)\mathrm{Shi}(4bx) - \frac{b^2\sinh(2a+2bx)}{6x} - \frac{b^2\sinh(4a+4bx)}{3x} - \frac{\sinh(2a+2bx)}{12x^3} - \frac{\sinh(4a+4bx)}{24x^3} - \frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/12*(b*Cosh[2*a + 2*b*x])/x^2 - (b*Cosh[4*a + 4*b*x])/(12*x^2) + (b^3*Cos
h[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 - S
inh[2*a + 2*b*x]/(12*x^3) - (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*
x]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) + (b^3*Sinh[2*a]*SinhIntegral[2
*b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx\\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^4}\,dx+\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^4}\,dx\\
&= -\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}+\frac{1}{6}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{6}b\int \frac{\cosh(4a+4bx)}{x^3}\,dx\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{\sinh(2a+2bx)}{12x^3}\\
&\quad -\frac{\sinh(4a+4bx)}{24x^3}+\frac{1}{6}b^2\int \frac{\sinh(2a+2bx)}{x^2}\,dx+\frac{1}{3}b^2\int \frac{\sinh(4a+4bx)}{x^2}\,dx\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{\sinh(2a+2bx)}{12x^3}\\
&\quad -\frac{b^2\sinh(2a+2bx)}{6x}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{b^2\sinh(4a+4bx)}{3x}\\
&\quad +\frac{1}{3}b^3\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{3}\left(4b^3\right)\int \frac{\cosh(4a+4bx)}{x}\,dx\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{\sinh(2a+2bx)}{12x^3}\\
&\quad -\frac{b^2\sinh(2a+2bx)}{6x}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{b^2\sinh(4a+4bx)}{3x}\\
&\quad +\frac{1}{3}\left(b^3\cosh(2a)\right)\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{3}\left(4b^3\cosh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx\\
&\quad +\frac{1}{3}\left(b^3\sinh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{3}\left(4b^3\sinh(4a)\right)\int \frac{\sinh(4bx)}{x}\,dx\\
&= -\frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{1}{3}b^3\cosh(2a)\mathrm{Chi}(2bx)\\
&\quad +\frac{4}{3}b^3\cosh(4a)\mathrm{Chi}(4bx)-\frac{\sinh(2a+2bx)}{12x^3}-\frac{b^2\sinh(2a+2bx)}{6x}-\frac{\sinh(4a+4bx)}{24x^3}\\
&\quad -\frac{b^2\sinh(4a+4bx)}{3x}+\frac{1}{3}b^3\sinh(2a)\mathrm{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\mathrm{Shi}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{2bx\cosh(2(a+bx))+2bx\cosh(4(a+bx))-8b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-32b^3x^3\cosh(4a)\mathrm{Chi}(4bx)+2\,\mathrm{si}}{}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x])/x^4,x]
```

```
[Out] -1/24*(2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] - 8*b^3*x^3*Cosh[2
*a]*CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] + 2*Sinh
[2*(a + b*x)] + 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2
*Sinh[4*(a + b*x)] - 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x^3*S
inh[4*a]*SinhIntegral[4*b*x])/x^3
```

Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{32\,e^{-4a}\,\mathrm{Ei}_1(4bx)x^3b^3+8\,e^{2a}\,\mathrm{Ei}_1(-2bx)x^3b^3+32\,e^{4a}\,\mathrm{Ei}_1(-4bx)x^3b^3+8\,e^{-2a}\,\mathrm{Ei}_1(2bx)x^3b^3+8\,e^{4bx+4a}b^2x^2-8\,e^{-4bx-4a}b^2x^2+4\,e^{2b}}{48}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(32*exp(-4*a)*Ei(1,4*b*x)*x^3*b^3+8*exp(2*a)*Ei(1,-2*b*x)*x^3*b^3+32*
exp(4*a)*Ei(1,-4*b*x)*x^3*b^3+8*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3+8*exp(4*b*x+4
*a)*b^2*x^2-8*exp(-4*b*x-4*a)*b^2*x^2+4*exp(2*b*x+2*a)*b^2*x^2-4*exp(-2*b*x
-2*a)*b^2*x^2+2*exp(4*b*x+4*a)*b*x+2*exp(-4*b*x-4*a)*b*x+2*exp(2*b*x+2*a)*b
*x+2*exp(-2*b*x-2*a)*b*x+exp(4*b*x+4*a)-exp(-4*b*x-4*a)+2*exp(2*b*x+2*a)-2*
exp(-2*b*x-2*a))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = -\frac{bx\cosh(bx+a)^4+bx\sinh(bx+a)^4+2\,(8\,b^2x^2+1)\cosh(bx+a)\sinh(bx+a)^3+bx\cosh(bx+a)^2+}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a)^3 + b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 + b*
x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a) -
 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 + 1)*
cosh(b*x + a)^3 + (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^3*x^3
*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) - 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3
*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\sinh(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)*cosh(a + b*x)**3/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.35

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = 4\,b^3e^{(-4\,a)}\Gamma(-3,4\,bx) + b^3e^{(-2\,a)}\Gamma(-3,2\,bx) + b^3e^{(2\,a)}\Gamma(-3,-2\,bx) + 4\,b^3e^{(4\,a)}\Gamma(-3,-4\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + b^3*e^(-2*a)*gamma(-3, 2*b*x) + b^3*e^(2*
a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \frac{32\,b^3x^3\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + 8\,b^3x^3\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 8\,b^3x^3\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 32\,b^3x^3\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 8\,b^2x^2e^{(4\,bx}}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) + 8*b^3*x^3*Ei(2*b*x)*e^(2*a) + 8*b^3*x^
3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b*x
 + 4*a) - 4*b^2*x^2*e^(2*b*x + 2*a) + 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2*x^
2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a) - 2*b*x*
e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - 2*e^(2*b*x +
2*a) + 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x))/x^4,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x))/x^4, x)
```

3.277 $\int \frac{\cosh(x)\sinh(x)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\cosh(x)\sinh(x)}{x} \, dx = \frac{\text{Shi}(2x)}{2}$$

```
[Out] 1/2*Shi(2*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5556, 12, 3379}

$$\int \frac{\cosh(x)\sinh(x)}{x} \, dx = \frac{\text{Shi}(2x)}{2}$$

```
[In] Int[(Cosh[x]*Sinh[x])/x,x]
```

```
[Out] SinhIntegral[2*x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sinh(2x)}{2x}\,dx\\ &= \frac{1}{2}\int \frac{\sinh(2x)}{x}\,dx\\ &= \frac{\mathrm{Shi}(2x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{\mathrm{Shi}(2x)}{2}$$

```
[In] Integrate[(Cosh[x]*Sinh[x])/x,x]
[Out] SinhIntegral[2*x]/2
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\mathrm{Shi}(2x)}{2}$	7
meijerg	$-\frac{i\,\mathrm{Si}(2ix)}{2}$	9
risch	$\frac{\mathrm{Ei}_1(2x)}{4}-\frac{\mathrm{Ei}_1(-2x)}{4}$	16

```
[In] int(cosh(x)*sinh(x)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*Shi(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}(2\,x) - \frac{1}{4}\,\mathrm{Ei}(-2\,x)$$

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="fricas")
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

Sympy [F]

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \int \frac{\sinh(x)\cosh(x)}{x}\,dx$$

```
[In] integrate(cosh(x)*sinh(x)/x,x)
[Out] Integral(sinh(x)*cosh(x)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}(2\,x) - \frac{1}{4}\,\mathrm{Ei}(-2\,x)$$

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \frac{1}{4}\,\mathrm{Ei}(2\,x) - \frac{1}{4}\,\mathrm{Ei}(-2\,x)$$

```
[In] integrate(cosh(x)*sinh(x)/x,x, algorithm="giac")
[Out] 1/4*Ei(2*x) - 1/4*Ei(-2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh(x)}{x}\,dx = \int \frac{\cosh(x)\,\sinh(x)}{x}\,dx$$

```
[In] int((cosh(x)*sinh(x))/x,x)
[Out] int((cosh(x)*sinh(x))/x, x)
```

3.278 $\int \frac{\cosh(x)\sinh(x)}{x^2} dx$

Optimal result

Integrand size = 8, antiderivative size = 16

$$\int \frac{\cosh(x)\sinh(x)}{x^2} dx = \text{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

```
[Out] Chi(2*x)-1/2*sinh(2*x)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5556, 12, 3378, 3382}

$$\int \frac{\cosh(x)\sinh(x)}{x^2} dx = \text{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

```
[In] Int[(Cosh[x]*Sinh[x])/x^2,x]
```

```
[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sinh(2x)}{2x^2}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2x)}{x^2}\,dx \\
&= -\frac{\sinh(2x)}{2x} + \int \frac{\cosh(2x)}{x}\,dx \\
&= \text{Chi}(2x) - \frac{\sinh(2x)}{2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \text{Chi}(2x) - \frac{\sinh(2x)}{2x}$$

```
[In] Integrate[(Cosh[x]*Sinh[x])/x^2,x]
[Out] CoshIntegral[2*x] - Sinh[2*x]/(2*x)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\mathrm{Chi}\left(2x\right)-\frac{\sinh(2x)}{2x}$	15
risch	$\frac{-2\ \mathrm{Ei}_1(2x)x-2\ \mathrm{Ei}_1(-2x)x+\mathrm{e}^{-2x}-\mathrm{e}^{2x}}{4x}$	33
meijerg	$\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(2)+4\ln(x)+2i\pi}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{2\sinh(2x)}{\sqrt{\pi}\,x}+\frac{4\ \mathrm{Chi}(2x)-4\ln(2x)-4\gamma}{\sqrt{\pi}}\right)}{4}$	65

```
[In] int(cosh(x)*sinh(x)/x^2,x,method=_RETURNVERBOSE)
[Out] Chi(2*x)-1/2*sinh(2*x)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \frac{x\mathrm{Ei}(2\,x) + x\mathrm{Ei}(-2\,x) - 2\,\cosh\left(x\right)\sinh\left(x\right)}{2\,x}$$

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="fricas")
[Out] 1/2*(x*Ei(2*x) + x*Ei(-2*x) - 2*cosh(x)*sinh(x))/x
```

Sympy [F]

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \int \frac{\sinh\left(x\right)\cosh\left(x\right)}{x^2}\,dx$$

```
[In] integrate(cosh(x)*sinh(x)/x**2,x)
[Out] Integral(sinh(x)*cosh(x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \frac{1}{2}\,\Gamma(-1, 2\,x) + \frac{1}{2}\,\Gamma(-1, -2\,x)$$

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="maxima")
[Out] 1/2*gamma(-1, 2*x) + 1/2*gamma(-1, -2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \frac{2\,x\text{Ei}(2\,x) + 2\,x\text{Ei}(-2\,x) - e^{(2\,x)} + e^{(-2\,x)}}{4\,x}$$

```
[In] integrate(cosh(x)*sinh(x)/x^2,x, algorithm="giac")
[Out] 1/4*(2*x*Ei(2*x) + 2*x*Ei(-2*x) - e^(2*x) + e^(-2*x))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh(x)}{x^2}\,dx = \int \frac{\cosh(x)\,\sinh(x)}{x^2}\,dx$$

```
[In] int((cosh(x)*sinh(x))/x^2,x)
[Out] int((cosh(x)*sinh(x))/x^2, x)
```

3.279 $\int \frac{\cosh(x)\sinh(x)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\cosh(x)\sinh(x)}{x^3} dx = -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \text{Shi}(2x)$$

```
[Out] -1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5556, 12, 3378, 3379}

$$\int \frac{\cosh(x)\sinh(x)}{x^3} dx = \text{Shi}(2x) - \frac{\sinh(2x)}{4x^2} - \frac{\cosh(2x)}{2x}$$

```
[In] Int[(Cosh[x]*Sinh[x])/x^3,x]
```

```
[Out] -1/2*Cosh[2*x]/x - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sinh(2x)}{2x^3}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2x)}{x^3}\,dx\\
&= -\frac{\sinh(2x)}{4x^2} + \frac{1}{2}\int \frac{\cosh(2x)}{x^2}\,dx\\
&= -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \int \frac{\sinh(2x)}{x}\,dx\\
&= -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \text{Shi}(2x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = -\frac{\cosh(2x)}{2x} - \frac{\sinh(2x)}{4x^2} + \text{Shi}(2x)$$

```
[In] Integrate[(Cosh[x]*Sinh[x])/x^3,x]
[Out] -1/2*Cosh[2*x]/x - Sinh[2*x]/(4*x^2) + SinhIntegral[2*x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\cosh(2x)}{2x}+\operatorname{Shi}(2x)-\frac{\sinh(2x)}{4x^2}$	24
meijerg	$\frac{i\sqrt{\pi}\left(\frac{2i\cosh(2x)}{x\sqrt{\pi}}+\frac{i\sinh(2x)}{x^2\sqrt{\pi}}-\frac{4\operatorname{Si}(2ix)}{\sqrt{\pi}}\right)}{4}$	44
risch	$-\frac{-4\operatorname{Ei}_1(2x)x^2+4\operatorname{Ei}_1(-2x)x^2+2\,\mathrm{e}^{-2x}x+2\,\mathrm{e}^{2x}x-\mathrm{e}^{-2x}+\mathrm{e}^{2x}}{8x^2}$	51

```
[In] int(cosh(x)*sinh(x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*cosh(2*x)/x+Shi(2*x)-1/4*sinh(2*x)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \frac{x^2\mathrm{Ei}(2\,x)-x^2\mathrm{Ei}(-2\,x)-x\cosh(x)^2-x\sinh(x)^2-\cosh(x)\sinh(x)}{2\,x^2}$$

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="fricas")
[Out] 1/2*(x^2*Ei(2*x) - x^2*Ei(-2*x) - x*cosh(x)^2 - x*sinh(x)^2 - cosh(x)*sinh(
x))/x^2
```

Sympy [F]

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \int \frac{\sinh(x)\cosh(x)}{x^3}\,dx$$

```
[In] integrate(cosh(x)*sinh(x)/x**3,x)
[Out] Integral(sinh(x)*cosh(x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \Gamma(-2,2\,x) - \Gamma(-2,-2\,x)$$

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="maxima")
[Out] gamma(-2, 2*x) - gamma(-2, -2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \frac{4\,x^2\mathrm{Ei}(2\,x) - 4\,x^2\mathrm{Ei}(-2\,x) - 2\,xe^{(2\,x)} - 2\,xe^{(-2\,x)} - e^{(2\,x)} + e^{(-2\,x)}}{8\,x^2}$$

```
[In] integrate(cosh(x)*sinh(x)/x^3,x, algorithm="giac")
[Out] 1/8*(4*x^2*Ei(2*x) - 4*x^2*Ei(-2*x) - 2*x*e^(2*x) - 2*x*e^(-2*x) - e^(2*x)
+ e^(-2*x))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh(x)}{x^3}\,dx = \int \frac{\cosh(x)\,\sinh(x)}{x^3}\,dx$$

```
[In] int((cosh(x)*sinh(x))/x^3,x)
[Out] int((cosh(x)*sinh(x))/x^3, x)
```

3.280 $\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b} - \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{8b} - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{8b}$$

```
[Out] 1/8*3^(-1-m)*exp(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/8*exp(a)*x^m*GAM
MA(1+m,-b*x)/b/((-b*x)^m)+1/8*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/8*3^(
-1-m)*x^m*GAMMA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 3388, 2212}

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{8b} - \frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{8b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{8b} - \frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{8b}$$

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (3^(-1 - m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(8*b*(-(b*x))^m) - (E^a*x^m*G
amma[1 + m, -(b*x)])/(8*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(8*b*E^a*(b
*x)^m) - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(8*b*E^(3*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{4}x^m \cosh(a+bx) + \frac{1}{4}x^m \cosh(3a+3bx)\right) dx \\
&= -\left(\frac{1}{4}\int x^m \cosh(a+bx)\,dx\right) + \frac{1}{4}\int x^m \cosh(3a+3bx)\,dx \\
&= -\left(\frac{1}{8}\int e^{-i(ia+ibx)}x^m\,dx\right) - \frac{1}{8}\int e^{i(ia+ibx)}x^m\,dx \\
&\quad + \frac{1}{8}\int e^{-i(3ia+3ibx)}x^m\,dx + \frac{1}{8}\int e^{i(3ia+3ibx)}x^m\,dx \\
&= \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{8b} - \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{8b} \\
&\quad + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{8b} - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{e^{-3a}x^m\Big(-3e^{4a}(-bx)^{-m}\Gamma(1+m,-bx)+3e^{2a}(bx)^{-m}\Gamma(1+m,bx)+3^{-m}(-b^2x^2)^{-m}\left(e^{6a}(bx)^m\Gamma(1+m,-3\right.}{24b}$$

```
[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-3*E^(4*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (3*E^(2*a)*Gamma[1 + m
, b*x])/(b*x)^m + (E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b*x))^m*Gamma[
1 + m, 3*b*x])/(3^m*(-(b^2*x^2))^m)))/(24*b*E^(3*a))
```

Maple [F]

$$\int x^m \cosh(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.21

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{\cosh(m\log(3b)+3a)\,\Gamma(m+1,3bx)-3\cosh(m\log(b)+a)\,\Gamma(m+1,bx)+3\cosh(m\log(-b)-a)\,\Gamma}{}$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) - 3*cosh(m*log(b) + a)*ga
mma(m + 1, b*x) + 3*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) - cosh(m*log(-3*
b) - 3*a)*gamma(m + 1, -3*b*x) - gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a)
 - 3*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + gamma(m + 1, -3*b*x)*sinh(m*l
og(-3*b) - 3*a) + 3*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

Sympy [F]

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int x^m \sinh^2{(a+bx)}\cosh{(a+bx)}\,dx$$

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = &-\frac{1}{8}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx) \\ &+\frac{1}{8}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx) \\ &+\frac{1}{8}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx) \\ &-\frac{1}{8}\,(-3\,bx)^{-m-1}\,x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx)\end{aligned}$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*x)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/8*(b*x)^(-
m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/8*(-b*x)^(-m - 1)*x^(m + 1)*e
^a*gamma(m + 1, -b*x) - 1/8*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1
, -3*b*x)
```

Giac [F]

$$\int x^m \cosh(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh{(bx+a)}\sinh{(bx+a)}^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx) \sinh^2(a+bx) \, dx = \int x^m \cosh(a+b\,x) \, \sinh(a+b\,x)^2 \, dx$$

```
[In] int(x^m*cosh(a + b*x)*sinh(a + b*x)^2,x)
[Out] int(x^m*cosh(a + b*x)*sinh(a + b*x)^2, x)
```

3.281 $\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{14\cosh(a+bx)}{9b^4}+\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{2\cosh^3(a+bx)}{27b^4}-\frac{4x\sinh(a+bx)}{3b^3}-\frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2}+\frac{2x\sinh^3(a+bx)}{9b^3}+\frac{x^3\sinh^3(a+bx)}{3b}$$

```
[Out] 14/9*cosh(b*x+a)/b^4+2/3*x^2*cosh(b*x+a)/b^2-2/27*cosh(b*x+a)^3/b^4-4/3*x*s
inh(b*x+a)/b^3-1/3*x^2*cosh(b*x+a)*sinh(b*x+a)^2/b^2+2/9*x*sinh(b*x+a)^3/b^
3+1/3*x^3*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5480, 3392, 3377, 2718, 2713}

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx=-\frac{2\cosh^3(a+bx)}{27b^4}+\frac{14\cosh(a+bx)}{9b^4}+\frac{2x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)}{3b^3}+\frac{2x^2\cosh(a+bx)}{3b^2}-\frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{3b^2}+\frac{x^3\sinh^3(a+bx)}{3b}$$

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (14*Cosh[a + b*x])/(9*b^4) + (2*x^2*Cosh[a + b*x])/(3*b^2) - (2*Cosh[a + b*
x]^3)/(27*b^4) - (4*x*Sinh[a + b*x])/(3*b^3) - (x^2*Cosh[a + b*x]*Sinh[a +
```

```
b*x]^2)/(3*b^2) + (2*x*Sinh[a + b*x]^3)/(9*b^3) + (x^3*Sinh[a + b*x]^3)/(3*
b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3 \sinh^3(a+bx)}{3b} - \frac{\int x^2 \sinh^3(a+bx)\,dx}{b} \\ &= -\frac{x^2 \cosh(a+bx)\sinh^2(a+bx)}{3b^2} + \frac{2x\sinh^3(a+bx)}{9b^3} \\ &\quad + \frac{x^3\sinh^3(a+bx)}{3b} - \frac{2\int \sinh^3(a+bx)\,dx}{9b^3} + \frac{2\int x^2\sinh(a+bx)\,dx}{3b}\end{aligned}$$

$$
\begin{aligned}
&= \frac{2x^2\cosh(a+bx)}{3b^2} - \frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2} \\
&\quad + \frac{2x\sinh^3(a+bx)}{9b^3} + \frac{x^3\sinh^3(a+bx)}{3b} \\
&\quad + \frac{2\text{Subst}\left(\int (1-x^2)\,dx, x, \cosh(a+bx)\right)}{9b^4} - \frac{4\int x\cosh(a+bx)\,dx}{3b^2} \\
&= \frac{2\cosh(a+bx)}{9b^4} + \frac{2x^2\cosh(a+bx)}{3b^2} - \frac{2\cosh^3(a+bx)}{27b^4} \\
&\quad - \frac{4x\sinh(a+bx)}{3b^3} - \frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2} \\
&\quad + \frac{2x\sinh^3(a+bx)}{9b^3} + \frac{x^3\sinh^3(a+bx)}{3b} + \frac{4\int \sinh(a+bx)\,dx}{3b^3} \\
&= \frac{14\cosh(a+bx)}{9b^4} + \frac{2x^2\cosh(a+bx)}{3b^2} - \frac{2\cosh^3(a+bx)}{27b^4} - \frac{4x\sinh(a+bx)}{3b^3} \\
&\quad - \frac{x^2\cosh(a+bx)\sinh^2(a+bx)}{3b^2} + \frac{2x\sinh^3(a+bx)}{9b^3} + \frac{x^3\sinh^3(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$
\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx
= \frac{81(2+b^2x^2)\cosh(a+bx) - (2+9b^2x^2)\cosh(3(a+bx)) + 6bx(-26-3b^2x^2+(2+3b^2x^2)\cosh(2(a+bx
}{108b^4}
$$

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (81*(2 + b^2*x^2)*Cosh[a + b*x] - (2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] + 6*b*x
*(-26 - 3*b^2*x^2 + (2 + 3*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(108*
b^4)
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(9x^3b^3-9x^2b^2+6bx-2)\mathrm{e}^{3bx+3a}}{216b^4} - \frac{(x^3b^3-3x^2b^2+6bx-6)\mathrm{e}^{bx+a}}{8b^4} + \frac{(x^3b^3+3x^2b^2+6bx+6)\mathrm{e}^{-bx-a}}{8b^4} - \frac{(9x^3b^3+9x^2b^2}{}$
derivativedivides	$\frac{-\frac{a^3\sinh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right)-3a\left(\frac{(bx+a)^2\sinh(bx+a)^3}{3}+\frac{4(bx+a)\,\mathrm{co}}{9}\right.}{}$
default	$\frac{-\frac{a^3\sinh(bx+a)^3}{3}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right)-3a\left(\frac{(bx+a)^2\sinh(bx+a)^3}{3}+\frac{4(bx+a)\,\mathrm{co}}{9}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(9*b^3*x^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)-1/8*(b^3*x^3-3*b^2*x
^2+6*b*x-6)/b^4*exp(b*x+a)+1/8*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a)-
1/216*(9*b^3*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{(9\,b^2x^2+2)\cosh\left(bx+a\right)^3+3\,(9\,b^2x^2+2)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-3\,(3\,b^3x^3+2\,bx)\sinh\left(bx+a\right)^3}{108\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*((9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a)*s
inh(b*x + a)^2 - 3*(3*b^3*x^3 + 2*b*x)*sinh(b*x + a)^3 - 81*(b^2*x^2 + 2)*c
osh(b*x + a) + 9*(3*b^3*x^3 - (3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 18*b*x)
*sinh(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{x^3\sinh^3(a+bx)}{3b}-\frac{x^2\sinh^2(a+bx)\cosh(a+bx)}{b^2}+\frac{2x^2\cosh^3(a+bx)}{3b^2}+\frac{14x\sinh^3(a+bx)}{9b^3}-\frac{4x\sinh(a+bx)\cosh^2(a+bx)}{3b^3}-\frac{14\sinh^2(a+b}{9} \\ \frac{x^4\sinh^2(a)\cosh(a)}{4} \end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**3/(3*b) - x**2*sinh(a + b*x)**2*cosh(a + b*x
)/b**2 + 2*x**2*cosh(a + b*x)**3/(3*b**2) + 14*x*sinh(a + b*x)**3/(9*b**3)
- 4*x*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) - 14*sinh(a + b*x)**2*cosh(a
+ b*x)/(9*b**4) + 40*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), (x**4*sinh(a)**
2*cosh(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^4} - \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)e^{(bx)}}{8\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{8\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3*a) - 2*e^(3*a))*e
^(3*b*x)/b^4 - 1/8*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^(b*x
)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b
^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int x^3\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{(9\,b^3x^3 - 9\,b^2x^2 + 6\,bx - 2)e^{(3\,bx+3\,a)}}{216\,b^4} - \frac{(b^3x^3 - 3\,b^2x^2 + 6\,bx - 6)e^{(bx+a)}}{8\,b^4} + \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{8\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/8*(b^3*x^
3 - 3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/8*(b^3*x^3 + 3*b^2*x^2 + 6*b
*x + 6)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*
b*x - 3*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3 \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{14\,x\,\sinh(a+b\,x)^3}{9}-\frac{4\,x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{3}}{b^3} + \frac{\frac{2\,x^2\,\cosh(a+b\,x)^3}{3}-x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2} + \frac{40\cosh(a+b\,x)^3}{27\,b^4} - \frac{14\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{9\,b^4} + \frac{x^3\sinh(a+b\,x)^3}{3\,b}$$

```
[In] int(x^3*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] ((14*x*sinh(a + b*x)^3)/9 - (4*x*cosh(a + b*x)^2*sinh(a + b*x))/3)/b^3 + ((
2*x^2*cosh(a + b*x)^3)/3 - x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2 + (40*cos
h(a + b*x)^3)/(27*b^4) - (14*cosh(a + b*x)*sinh(a + b*x)^2)/(9*b^4) + (x^3*
sinh(a + b*x)^3)/(3*b)
```

3.282 $\int x^2 \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^2 \cosh(a+bx) \sinh^2(a+bx) \, dx = \frac{4x\cosh(a+bx)}{9b^2} - \frac{4\sinh(a+bx)}{9b^3} - \frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2} + \frac{2\sinh^3(a+bx)}{27b^3} + \frac{x^2\sinh^3(a+bx)}{3b}$$

```
[Out] 4/9*x*cosh(b*x+a)/b^2-4/9*sinh(b*x+a)/b^3-2/9*x*cosh(b*x+a)*sinh(b*x+a)^2/b
^2+2/27*sinh(b*x+a)^3/b^3+1/3*x^2*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5480, 3391, 3377, 2717}

$$\int x^2 \cosh(a+bx) \sinh^2(a+bx) \, dx = \frac{2\sinh^3(a+bx)}{27b^3} - \frac{4\sinh(a+bx)}{9b^3} + \frac{4x\cosh(a+bx)}{9b^2} - \frac{2x\sinh^2(a+bx)\cosh(a+bx)}{9b^2} + \frac{x^2\sinh^3(a+bx)}{3b}$$

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (4*x*Cosh[a + b*x])/(9*b^2) - (4*Sinh[a + b*x])/(9*b^3) - (2*x*Cosh[a + b*x
]*Sinh[a + b*x]^2)/(9*b^2) + (2*Sinh[a + b*x]^3)/(27*b^3) + (x^2*Sinh[a + b
*x]^3)/(3*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sinh^3(a+bx)}{3b} - \frac{2\int x\sinh^3(a+bx)\,dx}{3b}\\
&= -\frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2} + \frac{2\sinh^3(a+bx)}{27b^3} + \frac{x^2\sinh^3(a+bx)}{3b} + \frac{4\int x\sinh(a+bx)\,dx}{9b}\\
&= \frac{4x\cosh(a+bx)}{9b^2} - \frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2}\\
&\quad + \frac{2\sinh^3(a+bx)}{27b^3} + \frac{x^2\sinh^3(a+bx)}{3b} - \frac{4\int\cosh(a+bx)\,dx}{9b^2}\\
&= \frac{4x\cosh(a+bx)}{9b^2} - \frac{4\sinh(a+bx)}{9b^3} - \frac{2x\cosh(a+bx)\sinh^2(a+bx)}{9b^2}\\
&\quad + \frac{2\sinh^3(a+bx)}{27b^3} + \frac{x^2\sinh^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int x^2 \cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{27bx\cosh(a+bx) - 3bx\cosh(3(a+bx)) + (-26 - 9b^2x^2 + (2+9b^2x^2)\cosh(2(a+bx)))\sinh(a+bx)}{54b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (27*b*x*Cosh[a + b*x] - 3*b*x*Cosh[3*(a + b*x)] + (-26 - 9*b^2*x^2 + (2 + 9
*b^2*x^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(54*b^3)
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(9x^2b^2-6bx+2)e^{3bx+3a}}{216b^3} - \frac{(x^2b^2-2bx+2)e^{bx+a}}{8b^3} + \frac{(x^2b^2+2bx+2)e^{-bx-a}}{8b^3} - \frac{(9x^2b^2+6bx+2)e^{-3bx-3a}}{216b^3}$
derivativedivides	$\frac{\frac{a^2\sinh(bx+a)^3}{3} - 2a\left(\frac{(bx+a)\sinh(bx+a)^3}{3} + \frac{2\cosh(bx+a)}{9} - \frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right) + \frac{(bx+a)^2\sinh(bx+a)^3}{3} + \frac{4(bx+a)\cosh(bx+\cdots}{9}}{b^3}$
default	$\frac{\frac{a^2\sinh(bx+a)^3}{3} - 2a\left(\frac{(bx+a)\sinh(bx+a)^3}{3} + \frac{2\cosh(bx+a)}{9} - \frac{\cosh(bx+a)\sinh(bx+a)^2}{9}\right) + \frac{(bx+a)^2\sinh(bx+a)^3}{3} + \frac{4(bx+a)\cosh(bx+\cdots}{9}}{b^3}$

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(9*b^2*x^2-6*b*x+2)/b^3*exp(3*b*x+3*a)-1/8*(b^2*x^2-2*b*x+2)/b^3*exp(
b*x+a)+1/8*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-1/216*(9*b^2*x^2+6*b*x+2)/b^3*
exp(-3*b*x-3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int x^2 \cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{6\,bx\cosh(bx+a)^3 + 18\,bx\cosh(bx+a)\sinh(bx+a)^2 - (9\,b^2x^2+2)\sinh(bx+a)^3 - 54\,bx\cosh(bx+\cdots}{108\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(6*b*x*cosh(b*x + a)^3 + 18*b*x*cosh(b*x + a)*sinh(b*x + a)^2 - (9*b
^2*x^2 + 2)*sinh(b*x + a)^3 - 54*b*x*cosh(b*x + a) + 3*(9*b^2*x^2 - (9*b^2*
x^2 + 2)*cosh(b*x + a)^2 + 18)*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{x^2\sinh^3(a+bx)}{3b}-\frac{2x\sinh^2(a+bx)\cosh(a+bx)}{3b^2}+\frac{4x\cosh^3(a+bx)}{9b^2}+\frac{14\sinh^3(a+bx)}{27b^3}-\frac{4\sinh(a+bx)\cosh^2(a+bx)}{9b^3} & \text{for } b\neq 0\\ \frac{x^3\sinh^2(a)\cosh(a)}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**3/(3*b) - 2*x*sinh(a + b*x)**2*cosh(a + b*x)
/(3*b**2) + 4*x*cosh(a + b*x)**3/(9*b**2) + 14*sinh(a + b*x)**3/(27*b**3) -
 4*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cos
h(a)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{\left(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)}\right)e^{(3\,bx)}}{216\,b^3}-\frac{(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{8\,b^3}+\frac{(b^2x^2+2\,bx+2)e^{(-bx-a)}}{8\,b^3}-\frac{(9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/8*(
b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/216*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{(9\,b^2x^2-6\,bx+2)e^{(3\,bx+3\,a)}}{216\,b^3}-\frac{(b^2x^2-2\,bx+2)e^{(bx+a)}}{8\,b^3}+\frac{(b^2x^2+2\,bx+2)e^{(-bx-a)}}{8\,b^3}-\frac{(9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*x^2 - 6*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/8*(b^2*x^2 - 2*b*x +
2)*e^(b*x + a)/b^3 + 1/8*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/216*(9*
b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int x^2\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{\frac{4\,x\cosh(a+b\,x)^3}{9}-\frac{2\,x\cosh(a+b\,x)\sinh(a+b\,x)^2}{3}}{b^2}+\frac{14\sinh(a+b\,x)^3}{27\,b^3}-\frac{4\cosh(a+b\,x)^2\sinh(a+b\,x)}{9\,b^3}+\frac{x^2\sinh(a+b\,x)^3}{3\,b}$$

```
[In] int(x^2*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] ((4*x*cosh(a + b*x)^3)/9 - (2*x*cosh(a + b*x)*sinh(a + b*x)^2)/3)/b^2 + (14
*sinh(a + b*x)^3)/(27*b^3) - (4*cosh(a + b*x)^2*sinh(a + b*x))/(9*b^3) + (x
^2*sinh(a + b*x)^3)/(3*b)
```

3.283 $\int x \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x \cosh(a+bx) \sinh^2(a+bx) \, dx = \frac{\cosh(a+bx)}{3b^2} - \frac{\cosh^3(a+bx)}{9b^2} + \frac{x \sinh^3(a+bx)}{3b}$$

```
[Out] 1/3*cosh(b*x+a)/b^2-1/9*cosh(b*x+a)^3/b^2+1/3*x*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5480, 2713}

$$\int x \cosh(a+bx) \sinh^2(a+bx) \, dx = -\frac{\cosh^3(a+bx)}{9b^2} + \frac{\cosh(a+bx)}{3b^2} + \frac{x \sinh^3(a+bx)}{3b}$$

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/(3*b^2) - Cosh[a + b*x]^3/(9*b^2) + (x*Sinh[a + b*x]^3)/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
```

```
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sinh^3(a+bx)}{3b} - \frac{\int \sinh^3(a+bx)\,dx}{3b} \\
&= \frac{x\sinh^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int (1-x^2)\,dx, x, \cosh(a+bx)\right)}{3b^2} \\
&= \frac{\cosh(a+bx)}{3b^2} - \frac{\cosh^3(a+bx)}{9b^2} + \frac{x\sinh^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{9\cosh(a+bx) - \cosh(3(a+bx)) + 12bx\sinh^3(a+bx)}{36b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
[Out] (9*Cosh[a + b*x] - Cosh[3*(a + b*x)] + 12*b*x*Sinh[a + b*x]^3)/(36*b^2)
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}-\frac{a\sinh(bx+a)^3}{3}}{b^2}$	56
default	$\frac{\frac{(bx+a)\sinh(bx+a)^3}{3}+\frac{2\cosh(bx+a)}{9}-\frac{\cosh(bx+a)\sinh(bx+a)^2}{9}-\frac{a\sinh(bx+a)^3}{3}}{b^2}$	56
risch	$\frac{(3bx-1)e^{3bx+3a}}{72b^2}-\frac{(bx-1)e^{bx+a}}{8b^2}+\frac{(bx+1)e^{-bx-a}}{8b^2}-\frac{(3bx+1)e^{-3bx-3a}}{72b^2}$	77

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b^2*(1/3*(b*x+a)*sinh(b*x+a)^3+2/9*cosh(b*x+a)-1/9*cosh(b*x+a)*sinh(b*x+a
)^2-1/3*a*sinh(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{3\,bx\sinh\left(bx+a\right)^3-\cosh\left(bx+a\right)^3-3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+9\left(bx\cosh\left(bx+a\right)^2-bx\right)\sinh\left(bx+\right.}{36\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/36*(3*b*x*sinh(b*x + a)^3 - cosh(b*x + a)^3 - 3*cosh(b*x + a)*sinh(b*x +
a)^2 + 9*(b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) + 9*cosh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{x\sinh^3(a+bx)}{3b}-\frac{\sinh^2(a+bx)\cosh(a+bx)}{3b^2}+\frac{2\cosh^3(a+bx)}{9b^2} & \text{for } b\neq 0 \\ \frac{x^2\sinh^2(a)\cosh(a)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**2,x)
[Out] Piecewise((x*sinh(a + b*x)**3/(3*b) - sinh(a + b*x)**2*cosh(a + b*x)/(3*b**
2) + 2*cosh(a + b*x)**3/(9*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)/2, Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{72\,b^2}-\frac{\left(bxe^{a}-e^{a}\right)e^{(bx)}}{8\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{8\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/72*(3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 1/8*(b*x*e^a - e^a)*e^(b*x)/
b^2 + 1/8*(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{72\,b^2} - \frac{(bx-1)e^{(bx+a)}}{8\,b^2} + \frac{(bx+1)e^{(-bx-a)}}{8\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x - 1)*e^(3*b*x + 3*a)/b^2 - 1/8*(b*x - 1)*e^(b*x + a)/b^2 + 1/8*
(b*x + 1)*e^(-b*x - a)/b^2 - 1/72*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int x\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{2\cosh(a+b\,x)^3 - 3\cosh(a+b\,x)\,\sinh(a+b\,x)^2 + 3\,b\,x\sinh(a+b\,x)^3}{9\,b^2}$$

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] (2*cosh(a + b*x)^3 - 3*cosh(a + b*x)*sinh(a + b*x)^2 + 3*b*x*sinh(a + b*x)^
3)/(9*b^2)
```

3.284 $\int \cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh^3(a+bx)}{3b}$$

```
[Out] 1/3*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2644, 30}

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh^3(a+bx)}{3b}$$

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{i\text{Subst}\left(\int x^2\,dx, x, i\sinh(a+bx)\right)}{b} \\ &= \frac{\sinh^3(a+bx)}{3b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh^3(a+bx)}{3b}$$

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^2,x]
[Out] Sinh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^3}{3b}$	14
default	$\frac{\sinh(bx+a)^3}{3b}$	14
risch	$\frac{e^{3bx+3a}}{24b} - \frac{e^{bx+a}}{8b} + \frac{e^{-bx-a}}{8b} - \frac{e^{-3bx-3a}}{24b}$	55

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*sinh(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(bx+a)^3 + 3\left(\cosh(bx+a)^2 - 1\right)\sinh(bx+a)}{12\,b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 - 1)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{\sinh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^2(a)\cosh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2,x)
[Out] Piecewise((sinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(bx+a)^3}{3b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*sinh(b*x + a)^3/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(3bx+3a)}}{24b} - \frac{e^{(bx+a)}}{8b} + \frac{e^{(-bx-a)}}{8b} - \frac{e^{(-3bx-3a)}}{24b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b + 1/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(a+b\,x)^3}{3\,b}$$

```
[In] int(cosh(a + b*x)*sinh(a + b*x)^2,x)
[Out] sinh(a + b*x)^3/(3*b)
```

3.285 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x} dx = -\frac{1}{4}\cosh(a)\text{Chi}(bx) + \frac{1}{4}\cosh(3a)\text{Chi}(3bx) - \frac{1}{4}\sinh(a)\text{Shi}(bx) + \frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[Out] -1/4*Chi(b*x)*cosh(a)+1/4*Chi(3*b*x)*cosh(3*a)-1/4*Shi(b*x)*sinh(a)+1/4*Shi
(3*b*x)*sinh(3*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x} dx = -\frac{1}{4}\cosh(a)\text{Chi}(bx) + \frac{1}{4}\cosh(3a)\text{Chi}(3bx) - \frac{1}{4}\sinh(a)\text{Shi}(bx) + \frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
```

```
[Out] -1/4*(Cosh[a]*CoshIntegral[b*x]) + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (Sin
h[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{4x} + \frac{\cosh(3a+3bx)}{4x}\right) dx \\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx\right) + \frac{1}{4}\int \frac{\cosh(3a+3bx)}{x}\,dx \\
&= -\left(\frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\right) + \frac{1}{4}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx \\
&\quad - \frac{1}{4}\sinh(a)\int \frac{\sinh(bx)}{x}\,dx + \frac{1}{4}\sinh(3a)\int \frac{\sinh(3bx)}{x}\,dx \\
&= -\frac{1}{4}\cosh(a)\text{Chi}(bx) + \frac{1}{4}\cosh(3a)\text{Chi}(3bx) - \frac{1}{4}\sinh(a)\text{Shi}(bx) + \frac{1}{4}\sinh(3a)\text{Shi}(3bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{4}(-\cosh(a)\mathrm{Chi}(bx)+\cosh(3a)\mathrm{Chi}(3bx) - \sinh(a)\mathrm{Shi}(bx)+\sinh(3a)\mathrm{Shi}(3bx))$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x,x]
[Out] (-(Cosh[a]*CoshIntegral[b*x]) + Cosh[3*a]*CoshIntegral[3*b*x] - Sinh[a]*Sin
hIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{-3a}\,\mathrm{Ei}_1(3bx)}{8}+\frac{e^{-a}\,\mathrm{Ei}_1(bx)}{8}+\frac{e^{a}\,\mathrm{Ei}_1(-bx)}{8}-\frac{e^{3a}\,\mathrm{Ei}_1(-3bx)}{8}$	47

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+1/8*exp(-a)*Ei(1,b*x)+1/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\left(\mathrm{Ei}(3\,bx)+\mathrm{Ei}(-3\,bx)\right)\cosh(3\,a) - \frac{1}{8}\left(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx)\right)\cosh(a) + \frac{1}{8}\left(\mathrm{Ei}(3\,bx)-\mathrm{Ei}(-3\,bx)\right)\sinh(3\,a) - \frac{1}{8}\left(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx)\right)\sinh(a)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 1/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/8
*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{1}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 1/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 1/8
*Ei(b*x)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x, x)
```

3.286 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{\cosh(a+bx)}{4x} - \frac{\cosh(3a+3bx)}{4x} - \frac{1}{4}b\text{Chi}(bx)\sinh(a) + \frac{3}{4}b\text{Chi}(3bx)\sinh(3a) - \frac{1}{4}b\cosh(a)\text{Shi}(bx) + \frac{3}{4}b\cosh(3a)\text{Shi}(3bx)$$

```
[Out] 1/4*cosh(b*x+a)/x-1/4*cosh(3*b*x+3*a)/x-1/4*b*cosh(a)*Shi(b*x)+3/4*b*cosh(3
*a)*Shi(3*b*x)-1/4*b*Chi(b*x)*sinh(a)+3/4*b*Chi(3*b*x)*sinh(3*a)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{1}{4}b\sinh(a)\text{Chi}(bx) + \frac{3}{4}b\sinh(3a)\text{Chi}(3bx) - \frac{1}{4}b\cosh(a)\text{Shi}(bx) + \frac{3}{4}b\cosh(3a)\text{Shi}(3bx) + \frac{\cosh(a+bx)}{4x} - \frac{\cosh(3a+3bx)}{4x}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] Cosh[a + b*x]/(4*x) - Cosh[3*a + 3*b*x]/(4*x) - (b*CoshIntegral[b*x]*Sinh[a
])/4 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/4 - (b*Cosh[a]*SinhIntegral[b*x]
)/4 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{4x^2} + \frac{\cosh(3a+3bx)}{4x^2}\right) dx \\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^2}\, dx\right) + \frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^2}\, dx \\
&= \frac{\cosh(a+bx)}{4x} - \frac{\cosh(3a+3bx)}{4x} - \frac{1}{4}b\int \frac{\sinh(a+bx)}{x}\, dx + \frac{1}{4}(3b)\int \frac{\sinh(3a+3bx)}{x}\, dx
\end{aligned}
$$

$$= \frac{\cosh(a+bx)}{4x} - \frac{\cosh(3a+3bx)}{4x} - \frac{1}{4}(b\cosh(a))\int\frac{\sinh(bx)}{x}\,dx + \frac{1}{4}(3b\cosh(3a))\int\frac{\sinh(3bx)}{x}\,dx - \frac{1}{4}(b\sinh(a))\int\frac{\cosh(bx)}{x}\,dx + \frac{1}{4}(3b\sinh(3a))\int\frac{\cosh(3bx)}{x}\,dx$$

$$= \frac{\cosh(a+bx)}{4x} - \frac{\cosh(3a+3bx)}{4x} - \frac{1}{4}b\text{Chi}(bx)\sinh(a) + \frac{3}{4}b\text{Chi}(3bx)\sinh(3a) - \frac{1}{4}b\cosh(a)\text{Shi}(bx) + \frac{3}{4}b\cosh(3a)\text{Shi}(3bx)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int\frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{-\cosh(a+bx)+\cosh(3(a+bx))+bx\text{Chi}(bx)\sinh(a)-3bx\text{Chi}(3bx)\sinh(3a)+bx\cosh(a)\text{Shi}(bx)-}{4x}$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] -1/4*(-Cosh[a + b*x] + Cosh[3*(a + b*x)] + b*x*CoshIntegral[b*x]*Sinh[a] -
3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] + b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x
*Cosh[3*a]*SinhIntegral[3*b*x])/x
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{-3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)bx+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)bx-\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)bx+3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)bx+\mathrm{e}^{-3bx-3a}-\mathrm{e}^{-bx-a}+\mathrm{e}^{3bx+3a}-\mathrm{e}^{bx+a}}{8x}$	96

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-3*exp(-3*a)*Ei(1,3*b*x)*b*x+exp(-a)*Ei(1,b*x)*b*x-exp(a)*Ei(1,-b*x)*
b*x+3*exp(3*a)*Ei(1,-3*b*x)*b*x+exp(-3*b*x-3*a)-exp(-b*x-a)+exp(3*b*x+3*a)-
exp(b*x+a))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{2\,\cosh\left(bx+a\right)^3+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-3\left(bx\mathrm{Ei}(3\,bx)-bx\mathrm{Ei}(-3\,bx)\right)\cosh\left(3\,a\right)+\left(bx\mathrm{Ei}(bx)\right.}{}$$

[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] -1/8*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*cosh(3*a) + (b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + (b*x*Ei(b*x) + b*x*Ei(-b*x))*sinh(a) - 2*cosh(b*x + a))/x

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{x^2}\,dx$$

[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**2,x)

[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{3}{8}\,be^{(-3\,a)}\Gamma(-1,3\,bx)+\frac{1}{8}\,be^{(-a)}\Gamma(-1,bx) - \frac{1}{8}\,be^{a}\Gamma(-1,-bx)+\frac{3}{8}\,be^{(3\,a)}\Gamma(-1,-3\,bx)$$

[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")

[Out] -3/8*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/8*b*e^(-a)*gamma(-1, b*x) - 1/8*b*e^a*gamma(-1, -b*x) + 3/8*b*e^(3*a)*gamma(-1, -3*b*x)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{3\,bx\text{Ei}(3\,bx)\,e^{(3\,a)} + bx\text{Ei}(-bx)\,e^{(-a)} - 3\,bx\text{Ei}(-3\,bx)\,e^{(-3\,a)} - bx\text{Ei}(bx)\,e^{a} - e^{(3\,bx+3\,a)} + e^{(bx+a)} + e^{(-bx-a)}}{8\,x}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
[Out] 1/8*(3*b*x*Ei(3*b*x)*e^(3*a) + b*x*Ei(-b*x)*e^(-a) - 3*b*x*Ei(-3*b*x)*e^(-3
*a) - b*x*Ei(b*x)*e^a - e^(3*b*x + 3*a) + e^(b*x + a) + e^(-b*x - a) - e^(-
3*b*x - 3*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^2,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^2, x)
```

3.287 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{\cosh(a+bx)}{8x^2} - \frac{\cosh(3a+3bx)}{8x^2} - \frac{1}{8}b^2\cosh(a)\text{Chi}(bx) + \frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx) + \frac{b\sinh(a+bx)}{8x} - \frac{3b\sinh(3a+3bx)}{8x} - \frac{1}{8}b^2\sinh(a)\text{Shi}(bx) + \frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)$$

```
[Out] -1/8*b^2*Chi(b*x)*cosh(a)+9/8*b^2*Chi(3*b*x)*cosh(3*a)+1/8*cosh(b*x+a)/x^2-
1/8*cosh(3*b*x+3*a)/x^2-1/8*b^2*Shi(b*x)*sinh(a)+9/8*b^2*Shi(3*b*x)*sinh(3*
a)+1/8*b*sinh(b*x+a)/x-3/8*b*sinh(3*b*x+3*a)/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{1}{8}b^2\cosh(a)\text{Chi}(bx) + \frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx) - \frac{1}{8}b^2\sinh(a)\text{Shi}(bx) + \frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx) + \frac{\cosh(a+bx)}{8x^2} - \frac{\cosh(3a+3bx)}{8x^2} + \frac{b\sinh(a+bx)}{8x} - \frac{3b\sinh(3a+3bx)}{8x}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] Cosh[a + b*x]/(8*x^2) - Cosh[3*a + 3*b*x]/(8*x^2) - (b^2*Cosh[a]*CoshIntegr
al[b*x])/8 + (9*b^2*Cosh[3*a]*CoshIntegral[3*b*x])/8 + (b*Sinh[a + b*x])/(8
*x) - (3*b*Sinh[3*a + 3*b*x])/(8*x) - (b^2*Sinh[a]*SinhIntegral[b*x])/8 + (
9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{4x^3}+\frac{\cosh(3a+3bx)}{4x^3}\right)\,dx\\ &= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^3}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^3}\,dx\end{aligned}$$

$$= \frac{\cosh(a+bx)}{8x^2} - \frac{\cosh(3a+3bx)}{8x^2} - \frac{1}{8}b\int \frac{\sinh(a+bx)}{x^2}\,dx + \frac{1}{8}(3b)\int \frac{\sinh(3a+3bx)}{x^2}\,dx$$

$$= \frac{\cosh(a+bx)}{8x^2} - \frac{\cosh(3a+3bx)}{8x^2} + \frac{b\sinh(a+bx)}{8x} - \frac{3b\sinh(3a+3bx)}{8x} - \frac{1}{8}b^2\int \frac{\cosh(a+bx)}{x}\,dx + \frac{1}{8}(9b^2)\int \frac{\cosh(3a+3bx)}{x}\,dx$$

$$= \frac{\cosh(a+bx)}{8x^2} - \frac{\cosh(3a+3bx)}{8x^2} + \frac{b\sinh(a+bx)}{8x} - \frac{3b\sinh(3a+3bx)}{8x} - \frac{1}{8}\left(b^2\cosh(a)\right)\int \frac{\cosh(bx)}{x}\,dx + \frac{1}{8}\left(9b^2\cosh(3a)\right)\int \frac{\cosh(3bx)}{x}\,dx - \frac{1}{8}\left(b^2\sinh(a)\right)\int \frac{\sinh(bx)}{x}\,dx + \frac{1}{8}\left(9b^2\sinh(3a)\right)\int \frac{\sinh(3bx)}{x}\,dx$$

$$= \frac{\cosh(a+bx)}{8x^2} - \frac{\cosh(3a+3bx)}{8x^2} - \frac{1}{8}b^2\cosh(a)\text{Chi}(bx) + \frac{9}{8}b^2\cosh(3a)\text{Chi}(3bx) + \frac{b\sinh(a+bx)}{8x} - \frac{3b\sinh(3a+3bx)}{8x} - \frac{1}{8}b^2\sinh(a)\text{Shi}(bx) + \frac{9}{8}b^2\sinh(3a)\text{Shi}(3bx)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{\cosh(a+bx) - \cosh(3(a+bx)) - b^2x^2\cosh(a)\text{Chi}(bx) + 9b^2x^2\cosh(3a)\text{Chi}(3bx) + bx\sinh(a+bx) - 3bx\,s}{8x^2}$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] (Cosh[a + b*x] - Cosh[3*(a + b*x)] - b^2*x^2*Cosh[a]*CoshIntegral[b*x] + 9*
b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a
 + b*x)] - b^2*x^2*Sinh[a]*SinhIntegral[b*x] + 9*b^2*x^2*Sinh[3*a]*SinhInte
gral[3*b*x])/(8*x^2)
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.34

method	result
risch	$\frac{-9\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^2b^2-9\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^2b^2+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^2b^2+\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^2b^2+\mathrm{e}^{bx+a}bx-3\,\mathrm{e}^{3bx+3a}bx+3\,\mathrm{e}^{-3bx-3a}bx-\mathrm{e}^{-bx-a}bx+}{16x^2}$

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2-9*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2+exp(
-a)*Ei(1,b*x)*x^2*b^2+exp(a)*Ei(1,-b*x)*x^2*b^2+exp(b*x+a)*b*x-3*exp(3*b*x+
3*a)*b*x+3*exp(-3*b*x-3*a)*b*x-exp(-b*x-a)*b*x+exp(b*x+a)-exp(3*b*x+3*a)-ex
p(-3*b*x-3*a)+exp(-b*x-a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.64

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{6\,bx\sinh\left(bx+a\right)^3+2\,\cosh\left(bx+a\right)^3+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-9\left(b^2x^2\mathrm{Ei}(3\,bx)+b^2x^2\mathrm{Ei}(-3\,b\right.}{}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x
 + a)^2 - 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*cosh(3*a) + (b^2*x^2*E
i(b*x) + b^2*x^2*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b
*x + a) - 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + (b^2*x^2*E
i(b*x) - b^2*x^2*Ei(-b*x))*sinh(a) - 2*cosh(b*x + a))/x^2
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{9}{8}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx)+\frac{1}{8}\,b^2e^{(-a)}\Gamma(-2,bx) + \frac{1}{8}\,b^2e^{a}\Gamma(-2,-bx)-\frac{9}{8}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -9/8*b^2*e^(-3*a)*gamma(-2, 3*b*x) + 1/8*b^2*e^(-a)*gamma(-2, b*x) + 1/8*b^
2*e^a*gamma(-2, -b*x) - 9/8*b^2*e^(3*a)*gamma(-2, -3*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{9\,b^2x^2\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - b^2x^2\mathrm{Ei}(-bx)\,e^{(-a)} + 9\,b^2x^2\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - b^2x^2\mathrm{Ei}(bx)\,e^{a} - 3\,bxe^{(3\,bx+3\,a)} + bxe^{(bx+}}{16\,x^2}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*x^2*Ei(3*b*x)*e^(3*a) - b^2*x^2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(
-3*b*x)*e^(-3*a) - b^2*x^2*Ei(b*x)*e^a - 3*b*x*e^(3*b*x + 3*a) + b*x*e^(b*x
 + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) + e^(b*
x + a) + e^(-b*x - a) - e^(-3*b*x - 3*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^3,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^3, x)
```

3.288 $\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 154

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = {} & \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x} \\ & -\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x} \\ & -\frac{1}{24}b^3\mathrm{Chi}(bx)\sinh(a)+\frac{9}{8}b^3\mathrm{Chi}(3bx)\sinh(3a) \\ & +\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2} \\ & -\frac{1}{24}b^3\cosh(a)\mathrm{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\mathrm{Shi}(3bx)\end{aligned}$$

```
[Out] 1/12*cosh(b*x+a)/x^3+1/24*b^2*cosh(b*x+a)/x-1/12*cosh(3*b*x+3*a)/x^3-3/8*b^
2*cosh(3*b*x+3*a)/x-1/24*b^3*cosh(a)*Shi(b*x)+9/8*b^3*cosh(3*a)*Shi(3*b*x)-
1/24*b^3*Chi(b*x)*sinh(a)+9/8*b^3*Chi(3*b*x)*sinh(3*a)+1/24*b*sinh(b*x+a)/x
^2-1/8*b*sinh(3*b*x+3*a)/x^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used

= {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{1}{24}b^3\sinh(a)\mathrm{Chi}(bx) + \frac{9}{8}b^3\sinh(3a)\mathrm{Chi}(3bx) - \frac{1}{24}b^3\cosh(a)\mathrm{Shi}(bx) + \frac{9}{8}b^3\cosh(3a)\mathrm{Shi}(3bx) + \frac{b^2\cosh(a+bx)}{24x} - \frac{3b^2\cosh(3a+3bx)}{8x} + \frac{\cosh(a+bx)}{12x^3} - \frac{\cosh(3a+3bx)}{12x^3} + \frac{b\sinh(a+bx)}{24x^2} - \frac{b\sinh(3a+3bx)}{8x^2}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] Cosh[a + b*x]/(12*x^3) + (b^2*Cosh[a + b*x])/(24*x) - Cosh[3*a + 3*b*x]/(12
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(8*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/2
4 + (9*b^3*CoshIntegral[3*b*x]*Sinh[3*a])/8 + (b*Sinh[a + b*x])/(24*x^2) -
(b*Sinh[3*a + 3*b*x])/(8*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/24 + (9*b^3
*Cosh[3*a]*SinhIntegral[3*b*x])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{4x^4}+\frac{\cosh(3a+3bx)}{4x^4}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\cosh(a+bx)}{x^4}\,dx\right)+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x^4}\,dx\\
&= \frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{1}{12}b\int \frac{\sinh(a+bx)}{x^3}\,dx+\frac{1}{4}b\int \frac{\sinh(3a+3bx)}{x^3}\,dx\\
&= \frac{\cosh(a+bx)}{12x^3}-\frac{\cosh(3a+3bx)}{12x^3}+\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2}\\
&\quad -\frac{1}{24}b^2\int \frac{\cosh(a+bx)}{x^2}\,dx+\frac{1}{8}\left(3b^2\right)\int \frac{\cosh(3a+3bx)}{x^2}\,dx\\
&= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}\\
&\quad +\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2}-\frac{1}{24}b^3\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{8}\left(9b^3\right)\int \frac{\sinh(3a+3bx)}{x}\,dx\\
&= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}\\
&\quad -\frac{3b^2\cosh(3a+3bx)}{8x}+\frac{b\sinh(a+bx)}{24x^2}-\frac{b\sinh(3a+3bx)}{8x^2}\\
&\quad -\frac{1}{24}\left(b^3\cosh(a)\right)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{8}\left(9b^3\cosh(3a)\right)\int \frac{\sinh(3bx)}{x}\,dx\\
&\quad -\frac{1}{24}\left(b^3\sinh(a)\right)\int \frac{\cosh(bx)}{x}\,dx+\frac{1}{8}\left(9b^3\sinh(3a)\right)\int \frac{\cosh(3bx)}{x}\,dx\\
&= \frac{\cosh(a+bx)}{12x^3}+\frac{b^2\cosh(a+bx)}{24x}-\frac{\cosh(3a+3bx)}{12x^3}-\frac{3b^2\cosh(3a+3bx)}{8x}\\
&\quad -\frac{1}{24}b^3\text{Chi}(bx)\sinh(a)+\frac{9}{8}b^3\text{Chi}(3bx)\sinh(3a)+\frac{b\sinh(a+bx)}{24x^2}\\
&\quad -\frac{b\sinh(3a+3bx)}{8x^2}-\frac{1}{24}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{8}b^3\cosh(3a)\text{Shi}(3bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx$$
$$= \frac{2\cosh(a+bx)+b^2x^2\cosh(a+bx)-2\cosh(3(a+bx))-9b^2x^2\cosh(3(a+bx))-b^3x^3\text{Chi}(bx)\sinh(a)+2}{}$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] (2*Cosh[a + b*x] + b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^2*
Cosh[3*(a + b*x)] - b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshInte
gral[3*b*x]*Sinh[3*a] + b*x*Sinh[a + b*x] - 3*b*x*Sinh[3*(a + b*x)] - b^3*x
^3*Cosh[a]*SinhIntegral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x])/(2
4*x^3)
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{-27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3+\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3-\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3+27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3b^3+9\,\mathrm{e}^{-3bx-3a}b^2x^2-\mathrm{e}^{-bx-a}b^2x^2-\mathrm{e}^{bx+a}b^2x^2+9}{48x^3}$

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3+exp(-a)*Ei(1,b*x)*x^3*b^3-exp(a)*E
i(1,-b*x)*x^3*b^3+27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3+9*exp(-3*b*x-3*a)*b^2*x^
2-exp(-b*x-a)*b^2*x^2-exp(b*x+a)*b^2*x^2+9*exp(3*b*x+3*a)*b^2*x^2-3*exp(-3*
b*x-3*a)*b*x+exp(-b*x-a)*b*x-exp(b*x+a)*b*x+3*exp(3*b*x+3*a)*b*x+2*exp(-3*b
*x-3*a)-2*exp(-b*x-a)-2*exp(b*x+a)+2*exp(3*b*x+3*a))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.45

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx =$$
$$-\frac{6\,bx\sinh(bx+a)^3+2\,(9\,b^2x^2+2)\cosh(bx+a)^3+6\,(9\,b^2x^2+2)\cosh(bx+a)\sinh(bx+a)^2-2\,(b^2x^2}{}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/48*(6*b*x*sinh(b*x + a)^3 + 2*(9*b^2*x^2 + 2)*cosh(b*x + a)^3 + 6*(9*b^2
*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a) - 2
7*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + (b^3*x^3*Ei(b*x) - b
^3*x^3*Ei(-b*x))*cosh(a) + 2*(9*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a) -
27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*sinh(3*a) + (b^3*x^3*Ei(b*x) +
b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\sinh^2(a+bx)\cosh(a+bx)}{x^4}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**2/x**4,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{27}{8}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx)+\frac{1}{8}\,b^3e^{(-a)}\Gamma(-3,bx) - \frac{1}{8}\,b^3e^{a}\Gamma(-3,-bx)+\frac{27}{8}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
[Out] -27/8*b^3*e^(-3*a)*gamma(-3, 3*b*x) + 1/8*b^3*e^(-a)*gamma(-3, b*x) - 1/8*b
^3*e^a*gamma(-3, -b*x) + 27/8*b^3*e^(3*a)*gamma(-3, -3*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.44

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{27\,b^3x^3\mathrm{Ei}(3\,bx)\,e^{(3\,a)}+b^3x^3\mathrm{Ei}(-bx)\,e^{(-a)}-27\,b^3x^3\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-b^3x^3\mathrm{Ei}(bx)\,e^{a}-9\,b^2x^2e^{(3\,bx+3\,a)}+b^2}{}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(27*b^3*x^3*Ei(3*b*x)*e^(3*a) + b^3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*E
i(-3*b*x)*e^(-3*a) - b^3*x^3*Ei(b*x)*e^a - 9*b^2*x^2*e^(3*b*x + 3*a) + b^2*
x^2*e^(b*x + a) + b^2*x^2*e^(-b*x - a) - 9*b^2*x^2*e^(-3*b*x - 3*a) - 3*b*x
*e^(3*b*x + 3*a) + b*x*e^(b*x + a) - b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3
*a) - 2*e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a) - 2*e^(-3*b*x - 3*
a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^4,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x)^2)/x^4, x)
```

3.289 $\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x^{1+m}}{8(1+m)} + \frac{2^{-2(3+m)}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} - \frac{2^{-2(3+m)}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}$$

```
[Out] -1/8*x^(1+m)/(1+m)+exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)-
x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3388, 2212}

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b} - \frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b} - \frac{x^{m+1}}{8(m+1)}$$

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/8*x^(1 + m)/(1 + m) + (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*
b*(-(b*x))^m) - (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 + m))*b*E^(4*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x^m}{8}+\frac{1}{8}x^m\cosh(4a+4bx)\right)dx\\
&= -\frac{x^{1+m}}{8(1+m)}+\frac{1}{8}\int x^m\cosh(4a+4bx)\,dx\\
&= -\frac{x^{1+m}}{8(1+m)}+\frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx+\frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx\\
&= -\frac{x^{1+m}}{8(1+m)}+\frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b}-\frac{4^{-3-m}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int x^m\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{2^{-2-2(2+m)}e^{-4a}x^m\left(-b^2x^2\right)^{-m}\left(2^{3+2m}be^{4a}x\left(-b^2x^2\right)^m-e^{8a}(1+m)(bx)^m\Gamma(1+m,-4bx)+(1+m)(-bx)^{m}\cdots\right.}{b(1+m)}$$

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -((2^(-2 - 2*(2 + m))*x^m*(2^(3 + 2*m)*b*E^(4*a)*x*(-(b^2*x^2))^m - E^(8*a)
*(1 + m)*(b*x)^m*Gamma[1 + m, -4*b*x] + (1 + m)*(-(b*x))^m*Gamma[1 + m, 4*b
*x]))/(b*E^(4*a)*(1 + m)*(-(b^2*x^2))^m))
```

Maple [F]

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^2\, dx$$

```
[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\, dx = -\frac{8\,bx\cosh(m\log(x)) + (m+1)\cosh(m\log(4\,b)+4\,a)\,\Gamma(m+1,4\,bx) - (m+1)\cosh(m\log(-4\,b)-4$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(8*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4
*b*x) - (m + 1)*cosh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - (m + 1)*gamm
a(m + 1, 4*b*x)*sinh(m*log(4*b) + 4*a) + (m + 1)*gamma(m + 1, -4*b*x)*sinh(
m*log(-4*b) - 4*a) + 8*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F]

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\, dx = \int x^m \sinh^2(a+bx)\cosh^2(a+bx)\, dx$$

```
[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\, dx = &-\frac{1}{16}\,(4\,bx)^{-m-1}\,x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx) \\ &-\frac{1}{16}\,(-4\,bx)^{-m-1}\,x^{m+1}e^{(4\,a)}\Gamma(m+1,-4\,bx) \\ &-\frac{x^{m+1}}{8\,(m+1)}\end{aligned}$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/16*(-4*b*
x)^(-m - 1)*x^(m + 1)*e^(4*a)*gamma(m + 1, -4*b*x) - 1/8*x^(m + 1)/(m + 1)
```

Giac [F]

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \sinh\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(a+b\,x)^2 \sinh(a+b\,x)^2\,dx$$

```
[In] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^2, x)
```

3.290 $\int x^3 \cosh^2(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int x^3 \cosh^2(a+bx) \sinh^2(a+bx) \, dx = -\frac{x^4}{32} - \frac{3\cosh(4a+4bx)}{1024b^4} - \frac{3x^2\cosh(4a+4bx)}{128b^2} + \frac{3x\sinh(4a+4bx)}{256b^3} + \frac{x^3\sinh(4a+4bx)}{32b}$$

```
[Out] -1/32*x^4-3/1024*cosh(4*b*x+4*a)/b^4-3/128*x^2*cosh(4*b*x+4*a)/b^2+3/256*x*
sinh(4*b*x+4*a)/b^3+1/32*x^3*sinh(4*b*x+4*a)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5556, 3377, 2718}

$$\int x^3 \cosh^2(a+bx) \sinh^2(a+bx) \, dx = -\frac{3\cosh(4a+4bx)}{1024b^4} + \frac{3x\sinh(4a+4bx)}{256b^3} - \frac{3x^2\cosh(4a+4bx)}{128b^2} + \frac{x^3\sinh(4a+4bx)}{32b} - \frac{x^4}{32}$$

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/32*x^4 - (3*Cosh[4*a + 4*b*x])/(1024*b^4) - (3*x^2*Cosh[4*a + 4*b*x])/(1
28*b^2) + (3*x*Sinh[4*a + 4*b*x])/(256*b^3) + (x^3*Sinh[4*a + 4*b*x])/(32*b
)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x^3}{8}+\frac{1}{8}x^3\cosh(4a+4bx)\right)dx\\
&= -\frac{x^4}{32}+\frac{1}{8}\int x^3\cosh(4a+4bx)\,dx\\
&= -\frac{x^4}{32}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{3\int x^2\sinh(4a+4bx)\,dx}{32b}\\
&= -\frac{x^4}{32}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{x^3\sinh(4a+4bx)}{32b}+\frac{3\int x\cosh(4a+4bx)\,dx}{64b^2}\\
&= -\frac{x^4}{32}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}+\frac{x^3\sinh(4a+4bx)}{32b}-\frac{3\int\sinh(4a+4bx)\,dx}{256b^3}\\
&= -\frac{x^4}{32}-\frac{3\cosh(4a+4bx)}{1024b^4}-\frac{3x^2\cosh(4a+4bx)}{128b^2}+\frac{3x\sinh(4a+4bx)}{256b^3}+\frac{x^3\sinh(4a+4bx)}{32b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$
\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{-32b^4x^4-3(1+8b^2x^2)\cosh(4(a+bx))+4bx(3+8b^2x^2)\sinh(4(a+bx))}{1024b^4}
$$

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^4*x^4 - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 4*b*x*(3 + 8*b^2*x^2)*
Sinh[4*(a + b*x)])/(1024*b^4)
```

Maple [A] (verified)

Time = 11.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x^4}{32}+\frac{(32x^3b^3-24x^2b^2+12bx-3)e^{4bx+4a}}{2048b^4}-\frac{(32x^3b^3+24x^2b^2+12bx+3)e^{-4bx-4a}}{2048b^4}$
derivativedivides	$\frac{-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sin}{8}\right.}{}$
default	$\frac{-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sin}{8}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*x^4+1/2048*(32*b^3*x^3-24*b^2*x^2+12*b*x-3)/b^4*exp(4*b*x+4*a)-1/2048
*(32*b^3*x^3+24*b^2*x^2+12*b*x+3)/b^4*exp(-4*b*x-4*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(69) = 138.

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.77

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{32\,b^4x^4+3\,(8\,b^2x^2+1)\cosh{(bx+a)}^4-16\,(8\,b^3x^3+3\,bx)\cosh{(bx+a)}^3\sinh{(bx+a)}+18\,(8\,b^2x^2+1}{1}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(32*b^4*x^4 + 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 - 16*(8*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^3*sinh(b*x + a) + 18*(8*b^2*x^2 + 1)*cosh(b*x + a)^2*si
nh(b*x + a)^2 - 16*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(8
*b^2*x^2 + 1)*sinh(b*x + a)^4)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(76) = 152.

Time = 0.55 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.16

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$

$$=\begin{cases}-\frac{x^4\sinh^4(a+bx)}{32}+\frac{x^4\sinh^2(a+bx)\cosh^2(a+bx)}{16}-\frac{x^4\cosh^4(a+bx)}{32}+\frac{x^3\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{8b}-\\ \frac{x^4\sinh^2(a)\cosh^2(a)}{4}\end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**4*sinh(a + b*x)**4/32 + x**4*sinh(a + b*x)**2*cosh(a + b*x)*
*2/16 - x**4*cosh(a + b*x)**4/32 + x**3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - 3*x**2*sinh(a + b*x)**4/(12
8*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 3*x**2*cosh(
a + b*x)**4/(128*b**2) + 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 3*x
*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) - 3*sinh(a + b*x)**4/(256*b**4) -
 3*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*sinh(a)**2*cosh(a)**2/4, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{32}\,x^4 + \frac{\left(32\,b^3x^3e^{(4\,a)} - 24\,b^2x^2e^{(4\,a)} + 12\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4} - \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a
) - 3*e^(4*a))*e^(4*b*x)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3
)*e^(-4*b*x - 4*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int x^3\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{32}\,x^4 + \frac{(32\,b^3x^3 - 24\,b^2x^2 + 12\,bx - 3)e^{(4\,bx+4\,a)}}{2048\,b^4} - \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b
^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^3 \cosh^2(a+bx) \sinh^2(a+bx) \, dx$$
$$= -\frac{\frac{3\cosh(4\,a+4\,b\,x)}{1024} - \frac{3\,b\,x\sinh(4\,a+4\,b\,x)}{256} + \frac{3\,b^2\,x^2\cosh(4\,a+4\,b\,x)}{128} - \frac{b^3\,x^3\sinh(4\,a+4\,b\,x)}{32}}{b^4} - \frac{x^4}{32}$$

```
[In] int(x^3*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] - ((3*cosh(4*a + 4*b*x))/1024 - (3*b*x*sinh(4*a + 4*b*x))/256 + (3*b^2*x^2*
cosh(4*a + 4*b*x))/128 - (b^3*x^3*sinh(4*a + 4*b*x))/32)/b^4 - x^4/32
```

3.291 $\int x^2 \cosh^2(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int x^2 \cosh^2(a+bx) \sinh^2(a+bx) \, dx = -\frac{x^3}{24} - \frac{x \cosh(4a+4bx)}{64b^2} + \frac{\sinh(4a+4bx)}{256b^3} + \frac{x^2 \sinh(4a+4bx)}{32b}$$

```
[Out] -1/24*x^3-1/64*x*cosh(4*b*x+4*a)/b^2+1/256*sinh(4*b*x+4*a)/b^3+1/32*x^2*sin
h(4*b*x+4*a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5556, 3377, 2717}

$$\int x^2 \cosh^2(a+bx) \sinh^2(a+bx) \, dx = \frac{\sinh(4a+4bx)}{256b^3} - \frac{x \cosh(4a+4bx)}{64b^2} + \frac{x^2 \sinh(4a+4bx)}{32b} - \frac{x^3}{24}$$

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/24*x^3 - (x*Cosh[4*a + 4*b*x])/(64*b^2) + Sinh[4*a + 4*b*x]/(256*b^3) +
(x^2*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x^2}{8}+\frac{1}{8}x^2\cosh(4a+4bx)\right)\,dx\\
&= -\frac{x^3}{24}+\frac{1}{8}\int x^2\cosh(4a+4bx)\,dx\\
&= -\frac{x^3}{24}+\frac{x^2\sinh(4a+4bx)}{32b}-\frac{\int x\sinh(4a+4bx)\,dx}{16b}\\
&= -\frac{x^3}{24}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{x^2\sinh(4a+4bx)}{32b}+\frac{\int\cosh(4a+4bx)\,dx}{64b^2}\\
&= -\frac{x^3}{24}-\frac{x\cosh(4a+4bx)}{64b^2}+\frac{\sinh(4a+4bx)}{256b^3}+\frac{x^2\sinh(4a+4bx)}{32b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{-32b^3x^3-12bx\cosh(4(a+bx))+3(1+8b^2x^2)\sinh(4(a+bx))}{768b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-32*b^3*x^3 - 12*b*x*Cosh[4*(a + b*x)] + 3*(1 + 8*b^2*x^2)*Sinh[4*(a + b*x
)])/(768*b^3)
```

Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{x^3}{24}+\frac{(8x^2b^2-4bx+1)\,\mathrm{e}^{4bx+4a}}{512b^3}-\frac{(8x^2b^2+4bx+1)\,\mathrm{e}^{-4bx-4a}}{512b^3}$
derivativedivides	$\frac{a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx}{8}\right.}{}$
default	$\frac{a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx}{8}\right.}{}$

```
[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*x^3+1/512*(8*b^2*x^2-4*b*x+1)/b^3*exp(4*b*x+4*a)-1/512*(8*b^2*x^2+4*b
*x+1)/b^3*exp(-4*b*x-4*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{8\,b^3x^3+3\,bx\cosh{(bx+a)}^4+18\,bx\cosh{(bx+a)}^2\sinh{(bx+a)}^2+3\,bx\sinh{(bx+a)}^4-3\,(8\,b^2x^2+1)\,\mathrm{co}}{192\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(8*b^3*x^3 + 3*b*x*cosh(b*x + a)^4 + 18*b*x*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + 3*b*x*sinh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*
x + a) - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(53) = 106.

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.40

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x^3\sinh^4(a+bx)}{24}+\frac{x^3\sinh^2(a+bx)\cosh^2(a+bx)}{12}-\frac{x^3\cosh^4(a+bx)}{24}+\frac{x^2\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{8b}- \\ \frac{x^3\sinh^2(a)\cosh^2(a)}{3} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**3*sinh(a + b*x)**4/24 + x**3*sinh(a + b*x)**2*cosh(a + b*x)*
*2/12 - x**3*cosh(a + b*x)**4/24 + x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - x*sinh(a + b*x)**4/(64*b**2
) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - x*cosh(a + b*x)**4/(6
4*b**2) + sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + sinh(a + b*x)*cosh(a +
 b*x)**3/(64*b**3), Ne(b, 0)), (x**3*sinh(a)**2*cosh(a)**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{24}\,x^3 + \frac{\left(8\,b^2x^2e^{(4\,a)} - 4\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3} - \frac{\left(8\,b^2x^2 + 4\,bx + 1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/24*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b
^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int x^2\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{24}\,x^3 + \frac{\left(8\,b^2x^2 - 4\,bx + 1\right)e^{(4\,bx+4\,a)}}{512\,b^3} - \frac{\left(8\,b^2x^2 + 4\,bx + 1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/24*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/512*(8*b^
2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2 \cosh^2(a+bx) \sinh^2(a+bx) \, dx = \frac{\frac{\sinh(4a+4bx)}{256} + \frac{b^2 x^2 \sinh(4a+4bx)}{32} - \frac{b x \cosh(4a+4bx)}{64}}{b^3} - \frac{x^3}{24}$$

```
[In] int(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] (sinh(4*a + 4*b*x)/256 + (b^2*x^2*sinh(4*a + 4*b*x))/32 - (b*x*cosh(4*a + 4
*b*x))/64)/b^3 - x^3/24
```

3.292 $\int x \cosh^2(a + bx) \sinh^2(a + bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int x \cosh^2(a + bx) \sinh^2(a + bx) \, dx = -\frac{x^2}{16} - \frac{\cosh(4a + 4bx)}{128b^2} + \frac{x \sinh(4a + 4bx)}{32b}$$

```
[Out] -1/16*x^2-1/128*cosh(4*b*x+4*a)/b^2+1/32*x*sinh(4*b*x+4*a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 3377, 2718}

$$\int x \cosh^2(a + bx) \sinh^2(a + bx) \, dx = -\frac{\cosh(4a + 4bx)}{128b^2} + \frac{x \sinh(4a + 4bx)}{32b} - \frac{x^2}{16}$$

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/16*x^2 - Cosh[4*a + 4*b*x]/(128*b^2) + (x*Sinh[4*a + 4*b*x])/(32*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{8}+\frac{1}{8}x\cosh(4a+4bx)\right)dx \\
&= -\frac{x^2}{16}+\frac{1}{8}\int x\cosh(4a+4bx)\,dx \\
&= -\frac{x^2}{16}+\frac{x\sinh(4a+4bx)}{32b}-\frac{\int \sinh(4a+4bx)\,dx}{32b} \\
&= -\frac{x^2}{16}-\frac{\cosh(4a+4bx)}{128b^2}+\frac{x\sinh(4a+4bx)}{32b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$
\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{-8a^2+8b^2x^2+\cosh(4(a+bx))-4bx\sinh(4(a+bx))}{128b^2}
$$

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] -1/128*(-8*a^2 + 8*b^2*x^2 + Cosh[4*(a + b*x)] - 4*b*x*Sinh[4*(a + b*x)])/b
^2
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x^2}{16}+\frac{(4bx-1)e^{4bx+4a}}{256b^2}-\frac{(4bx+1)e^{-4bx-4a}}{256b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^2}{16}-\frac{\cosh(bx+a)^4}{16}+\frac{\cosh(bx+a)^2}{16}-a\Bigl(\frac{\cosh(bx+a)^3\sinh(bx+}{4}}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}-\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}-\frac{(bx+a)^2}{16}-\frac{\cosh(bx+a)^4}{16}+\frac{\cosh(bx+a)^2}{16}-a\Bigl(\frac{\cosh(bx+a)^3\sinh(bx+}{4}}{b^2}$

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*x^2+1/256*(4*b*x-1)/b^2*exp(4*b*x+4*a)-1/256*(4*b*x+1)/b^2*exp(-4*b*x
-4*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{16\,bx\cosh(bx+a)^3\sinh(bx+a)+16\,bx\cosh(bx+a)\sinh(bx+a)^3-8\,b^2x^2-\cosh(bx+a)^4-6\,\cosh}{128\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*x
 + a)^3 - 8*b^2*x^2 - cosh(b*x + a)^4 - 6*cosh(b*x + a)^2*sinh(b*x + a)^2 -
 sinh(b*x + a)^4)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(34) = 68$.

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x^2\sinh^4(a+bx)}{16}+\frac{x^2\sinh^2(a+bx)\cosh^2(a+bx)}{8}-\frac{x^2\cosh^4(a+bx)}{16}+\frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{8b}- \\ \frac{x^2\sinh^2(a)\cosh^2(a)}{2} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x)**4/16 + x**2*sinh(a + b*x)**2*cosh(a + b*x)*
*2/8 - x**2*cosh(a + b*x)**4/16 + x*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) +
x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) - sinh(a + b*x)**4/(32*b**2) - cosh(
a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*sinh(a)**2*cosh(a)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}\,x^2 + \frac{\left(4\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}\,x^2 + \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/256*(4*b*x + 1)*e^(-4
*b*x - 4*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int x\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\frac{\cosh(4\,a+4\,b\,x)}{128} - \frac{b\,x\sinh(4\,a+4\,b\,x)}{32}}{b^2} - \frac{x^2}{16}$$

```
[In] int(x*cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

```
[Out] - (cosh(4*a + 4*b*x)/128 - (b*x*sinh(4*a + 4*b*x))/32)/b^2 - x^2/16
```

3.293 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{8}-\frac{\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

```
[Out] -1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{\sinh(a+bx)\cosh(a+bx)}{8b}-\frac{x}{8}$$

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/8*x - (Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*
x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{1}{4}\int \cosh^2(a+bx)\,dx \\ &= -\frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b} - \frac{\int 1\,dx}{8} \\ &= -\frac{x}{8} - \frac{\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{-4(a+bx)+\sinh(4(a+bx))}{32b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{x}{8}+\frac{e^{4bx+4a}}{64b}-\frac{e^{-4bx-4a}}{64b}$	33
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}}{b}$	43
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}}{b}$	43

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/8*x+1/64*exp(4*b*x+4*a)/b-1/64*exp(-4*b*x-4*a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \neq \\ x\sinh^2(a)\cosh^2(a) & \text{otherw} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)
*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{bx+a}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(4\,a+4\,b\,x)}{32\,b} - \frac{x}{8}$$

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
[Out] sinh(4*a + 4*b*x)/(32*b) - x/8
```

3.294 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\cosh(4a)\text{Chi}(4bx) - \frac{\log(x)}{8} + \frac{1}{8}\sinh(4a)\text{Shi}(4bx)$$

```
[Out] 1/8*Chi(4*b*x)*cosh(4*a)-1/8*ln(x)+1/8*Shi(4*b*x)*sinh(4*a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}\cosh(4a)\text{Chi}(4bx) + \frac{1}{8}\sinh(4a)\text{Shi}(4bx) - \frac{\log(x)}{8}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (Cosh[4*a]*CoshIntegral[4*b*x])/8 - Log[x]/8 + (Sinh[4*a]*SinhIntegral[4*b*
x])/8
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8x} + \frac{\cosh(4a+4bx)}{8x}\right) dx \\
&= -\frac{\log(x)}{8} + \frac{1}{8}\int \frac{\cosh(4a+4bx)}{x}\,dx \\
&= -\frac{\log(x)}{8} + \frac{1}{8}\cosh(4a)\int \frac{\cosh(4bx)}{x}\,dx + \frac{1}{8}\sinh(4a)\int \frac{\sinh(4bx)}{x}\,dx \\
&= \frac{1}{8}\cosh(4a)\text{Chi}(4bx) - \frac{\log(x)}{8} + \frac{1}{8}\sinh(4a)\text{Shi}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{8}(\cosh(4a)\text{Chi}(4bx) - \log(2bx) + \sinh(4a)\text{Shi}(4bx))$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x,x]
[Out] (Cosh[4*a]*CoshIntegral[4*b*x] - Log[2*b*x] + Sinh[4*a]*SinhIntegral[4*b*x]
)/8
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\ln(x)}{8}-\frac{\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)}{16}-\frac{\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)}{16}$	30

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/8*ln(x)-1/16*exp(-4*a)*Ei(1,4*b*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}\left(\mathrm{Ei}(4\,bx)+\mathrm{Ei}(-4\,bx)\right)\cosh\left(4\,a\right) + \frac{1}{16}\left(\mathrm{Ei}(4\,bx)-\mathrm{Ei}(-4\,bx)\right)\sinh\left(4\,a\right) - \frac{1}{8}\log\left(x\right)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/16*(Ei(4*b*x) + Ei(-4*b*x))*cosh(4*a) + 1/16*(Ei(4*b*x) - Ei(-4*b*x))*sin
h(4*a) - 1/8*log(x)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh^2\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - \frac{1}{8}\,\log{(x)}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} + \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - \frac{1}{8}\,\log{(x)}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
[Out] 1/16*Ei(4*b*x)*e^(4*a) + 1/16*Ei(-4*b*x)*e^(-4*a) - 1/8*log(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^2}{x}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x,x)
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x, x)
```

3.295 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2} dx = \frac{1}{8x} - \frac{\cosh(4a+4bx)}{8x} + \frac{1}{2}b\text{Chi}(4bx)\sinh(4a) + \frac{1}{2}b\cosh(4a)\text{Shi}(4bx)$$

```
[Out] 1/8/x-1/8*cosh(4*b*x+4*a)/x+1/2*b*cosh(4*a)*Shi(4*b*x)+1/2*b*Chi(4*b*x)*sin
h(4*a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2} dx = \frac{1}{2}b\sinh(4a)\text{Chi}(4bx) + \frac{1}{2}b\cosh(4a)\text{Shi}(4bx) - \frac{\cosh(4a+4bx)}{8x} + \frac{1}{8x}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] 1/(8*x) - Cosh[4*a + 4*b*x]/(8*x) + (b*CoshIntegral[4*b*x]*Sinh[4*a])/2 + (
b*Cosh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8x^2} + \frac{\cosh(4a+4bx)}{8x^2}\right) dx \\
&= \frac{1}{8x} + \frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^2}\,dx \\
&= \frac{1}{8x} - \frac{\cosh(4a+4bx)}{8x} + \frac{1}{2}b\int \frac{\sinh(4a+4bx)}{x}\,dx \\
&= \frac{1}{8x} - \frac{\cosh(4a+4bx)}{8x} + \frac{1}{2}(b\cosh(4a))\int \frac{\sinh(4bx)}{x}\,dx + \frac{1}{2}(b\sinh(4a))\int \frac{\cosh(4bx)}{x}\,dx \\
&= \frac{1}{8x} - \frac{\cosh(4a+4bx)}{8x} + \frac{1}{2}b\text{Chi}(4bx)\sinh(4a) + \frac{1}{2}b\cosh(4a)\text{Shi}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{1-\cosh(4(a+bx))+4bx\text{Chi}(4bx)\sinh(4a)+4bx\cosh(4a)\text{Shi}(4bx)}{8x}$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^2,x]
[Out] (1 - Cosh[4*(a + b*x)] + 4*b*x*CoshIntegral[4*b*x]*Sinh[4*a] + 4*b*x*Cosh[4
*a]*SinhIntegral[4*b*x])/(8*x)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{-4\,\mathrm{e}^{-4a}\,\text{Ei}_1(4bx)bx+4\,\mathrm{e}^{4a}\,\text{Ei}_1(-4bx)bx+\mathrm{e}^{-4bx-4a}+\mathrm{e}^{4bx+4a}-2}{16x}$	54

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/16*(-4*exp(-4*a)*Ei(1,4*b*x)*b*x+4*exp(4*a)*Ei(1,-4*b*x)*b*x+exp(-4*b*x-
4*a)+exp(4*b*x+4*a)-2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.69

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{\cosh(bx+a)^4+6\,\cosh(bx+a)^2\sinh(bx+a)^2+\sinh(bx+a)^4-2\,(bx\text{Ei}(4\,bx)-bx\text{Ei}(-4\,bx))\cosh(}{8\,x}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] -1/8*(cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x + a)^4
 - 2*(b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*cosh(4*a) - 2*(b*x*Ei(4*b*x) + b*x*Ei
(-4*b*x))*sinh(4*a) - 1)/x
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{1}{4}\,be^{(-4\,a)}\Gamma(-1,4\,bx) + \frac{1}{4}\,be^{(4\,a)}\Gamma(-1,-4\,bx) + \frac{1}{8\,x}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] -1/4*b*e^(-4*a)*gamma(-1, 4*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x) + 1/8/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{4\,bx\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 4\,bx\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} - e^{(-4\,bx-4\,a)} + 2}{16\,x}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a)
 - e^(-4*b*x - 4*a) + 2)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^2}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^2,x)
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^2, x)
```

3.296 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{1}{16x^2} - \frac{\cosh(4a+4bx)}{16x^2} + b^2\cosh(4a)\mathrm{Chi}(4bx) - \frac{b\sinh(4a+4bx)}{4x} + b^2\sinh(4a)\mathrm{Shi}(4bx)$$

```
[Out] 1/16/x^2+b^2*Chi(4*b*x)*cosh(4*a)-1/16*cosh(4*b*x+4*a)/x^2+b^2*Shi(4*b*x)*s
inh(4*a)-1/4*b*sinh(4*b*x+4*a)/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = b^2\cosh(4a)\mathrm{Chi}(4bx) + b^2\sinh(4a)\mathrm{Shi}(4bx) - \frac{\cosh(4a+4bx)}{16x^2} - \frac{b\sinh(4a+4bx)}{4x} + \frac{1}{16x^2}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] 1/(16*x^2) - Cosh[4*a + 4*b*x]/(16*x^2) + b^2*Cosh[4*a]*CoshIntegral[4*b*x]
 - (b*Sinh[4*a + 4*b*x])/(4*x) + b^2*Sinh[4*a]*SinhIntegral[4*b*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8x^3}+\frac{\cosh(4a+4bx)}{8x^3}\right)dx \\
&= \frac{1}{16x^2}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^3}\,dx \\
&= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+\frac{1}{4}b\int \frac{\sinh(4a+4bx)}{x^2}\,dx \\
&= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x}+b^2\int \frac{\cosh(4a+4bx)}{x}\,dx \\
&= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}-\frac{b\sinh(4a+4bx)}{4x} \\
&\quad +\left(b^2\cosh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx+\left(b^2\sinh(4a)\right)\int \frac{\sinh(4bx)}{x}\,dx \\
&= \frac{1}{16x^2}-\frac{\cosh(4a+4bx)}{16x^2}+b^2\cosh(4a)\text{Chi}(4bx)-\frac{b\sinh(4a+4bx)}{4x}+b^2\sinh(4a)\text{Shi}(4bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{1-\cosh(4(a+bx))+16b^2x^2\cosh(4a)\text{Chi}(4bx)-4bx\sinh(4(a+bx))+16b^2x^2\sinh(4a)\text{Shi}(4bx)}{16x^2}$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] (1 - Cosh[4*(a + b*x)] + 16*b^2*x^2*Cosh[4*a]*CoshIntegral[4*b*x] - 4*b*x*S
inh[4*(a + b*x)] + 16*b^2*x^2*Sinh[4*a]*SinhIntegral[4*b*x])/(16*x^2)
```

Maple [A] (verified)

Time = 6.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{-16\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^2b^2-16\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^2b^2+4\,\mathrm{e}^{-4bx-4a}bx-4\,\mathrm{e}^{4bx+4a}bx-\mathrm{e}^{-4bx-4a}-\mathrm{e}^{4bx+4a}+2}{32x^2}$	92

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(-16*exp(-4*a)*Ei(1,4*b*x)*x^2*b^2-16*exp(4*a)*Ei(1,-4*b*x)*x^2*b^2+4*
exp(-4*b*x-4*a)*b*x-4*exp(4*b*x+4*a)*b*x-exp(-4*b*x-4*a)-exp(4*b*x+4*a)+2)/
x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.09

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{16\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\cosh\left(bx+a\right)^4+6\,\cosh\left(bx+a\right)}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 16*b*x*cosh(b*x + a)*sinh(b*x
 + a)^3 + cosh(b*x + a)^4 + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + sinh(b*x +
a)^4 - 8*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*cosh(4*a) - 8*(b^2*x^2*Ei
(4*b*x) - b^2*x^2*Ei(-4*b*x))*sinh(4*a) - 1)/x^2
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\sinh^2{(a+bx)}\cosh^2{(a+bx)}}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**3,x)
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.54

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -b^2e^{(-4\,a)}\Gamma(-2,4\,bx) - b^2e^{(4\,a)}\Gamma(-2,-4\,bx) + \frac{1}{16\,x^2}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
[Out] -b^2*e^(-4*a)*gamma(-2, 4*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x) + 1/16/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{16\,b^2x^2\text{Ei}(4\,bx)\,e^{(4\,a)} + 16\,b^2x^2\text{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+4\,a)} + 4\,bxe^{(-4\,bx-4\,a)} - e^{(4\,bx+4\,a)} - e^{(-4\,bx-4\,a)} +}{32\,x^2}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^3,x, algorithm="giac")
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) + 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x
*e^(4*b*x + 4*a) + 4*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4
*a) + 2)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^2}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^3,x)
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^3, x)
```

3.297 $\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b^2\cosh(4a+4bx)}{3x}+\frac{4}{3}b^3\text{Chi}(4bx)\sinh(4a)-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)$$

```
[Out] 1/24/x^3-1/24*cosh(4*b*x+4*a)/x^3-1/3*b^2*cosh(4*b*x+4*a)/x+4/3*b^3*cosh(4*
a)*Shi(4*b*x)+4/3*b^3*Chi(4*b*x)*sinh(4*a)-1/12*b*sinh(4*b*x+4*a)/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{4}{3}b^3\sinh(4a)\text{Chi}(4bx)+\frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)-\frac{b^2\cosh(4a+4bx)}{3x}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{24x^3}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] 1/(24*x^3) - Cosh[4*a + 4*b*x]/(24*x^3) - (b^2*Cosh[4*a + 4*b*x])/(3*x) + (
4*b^3*CoshIntegral[4*b*x]*Sinh[4*a])/3 - (b*Sinh[4*a + 4*b*x])/(12*x^2) + (
4*b^3*Cosh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8x^4}+\frac{\cosh(4a+4bx)}{8x^4}\right)dx\\
&= \frac{1}{24x^3}+\frac{1}{8}\int \frac{\cosh(4a+4bx)}{x^4}\,dx\\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}+\frac{1}{6}b\int \frac{\sinh(4a+4bx)}{x^3}\,dx\\
&= \frac{1}{24x^3}-\frac{\cosh(4a+4bx)}{24x^3}-\frac{b\sinh(4a+4bx)}{12x^2}+\frac{1}{3}b^2\int \frac{\cosh(4a+4bx)}{x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{24x^3} - \frac{\cosh(4a+4bx)}{24x^3} - \frac{b^2\cosh(4a+4bx)}{3x} \\
&\quad - \frac{b\sinh(4a+4bx)}{12x^2} + \frac{1}{3}\left(4b^3\right)\int \frac{\sinh(4a+4bx)}{x}\,dx \\
&= \frac{1}{24x^3} - \frac{\cosh(4a+4bx)}{24x^3} - \frac{b^2\cosh(4a+4bx)}{3x} - \frac{b\sinh(4a+4bx)}{12x^2} \\
&\quad + \frac{1}{3}\left(4b^3\cosh(4a)\right)\int \frac{\sinh(4bx)}{x}\,dx + \frac{1}{3}\left(4b^3\sinh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx \\
&= \frac{1}{24x^3} - \frac{\cosh(4a+4bx)}{24x^3} - \frac{b^2\cosh(4a+4bx)}{3x} \\
&\quad + \frac{4}{3}b^3\text{Chi}(4bx)\sinh(4a) - \frac{b\sinh(4a+4bx)}{12x^2} + \frac{4}{3}b^3\cosh(4a)\text{Shi}(4bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$
\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{-1+\cosh(4(a+bx))+8b^2x^2\cosh(4(a+bx))-32b^3x^3\text{Chi}(4bx)\sinh(4a)+2bx\sinh(4(a+bx))-32b}{24x^3}
$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] -1/24*(-1 + Cosh[4*(a + b*x)] + 8*b^2*x^2*Cosh[4*(a + b*x)] - 32*b^3*x^3*Co
shIntegral[4*b*x]*Sinh[4*a] + 2*b*x*Sinh[4*(a + b*x)] - 32*b^3*x^3*Cosh[4*a
]*SinhIntegral[4*b*x])/x^3
```

Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{-32\,e^{-4a}\,\text{Ei}_1(4bx)x^3b^3+32\,e^{4a}\,\text{Ei}_1(-4bx)x^3b^3+8\,e^{-4bx-4a}b^2x^2+8\,e^{4bx+4a}b^2x^2-2\,e^{-4bx-4a}bx+2\,e^{4bx+4a}bx+e^{-4bx-4a}+e^{4bx+4}}{48x^3}$

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-32*exp(-4*a)*Ei(1,4*b*x)*x^3*b^3+32*exp(4*a)*Ei(1,-4*b*x)*x^3*b^3+8
*exp(-4*b*x-4*a)*b^2*x^2+8*exp(4*b*x+4*a)*b^2*x^2-2*exp(-4*b*x-4*a)*b*x+2*e
xp(4*b*x+4*a)*b*x+exp(-4*b*x-4*a)+exp(4*b*x+4*a)-2)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(80) = 160.

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{8\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^4+6\,(8\,b}{}$$

[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="fricas")

[Out] -1/24*(8*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + (8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 6*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 *sinh(b*x + a)^2 + (8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 16*(b^3*x^3*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*cosh(4*a) - 16*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*sinh(4*a) - 1)/x^3

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\sinh^2(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**2/x**4,x)

[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**2/x**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -4\,b^3e^{(-4\,a)}\Gamma(-3,4\,bx)+4\,b^3e^{(4\,a)}\Gamma(-3,-4\,bx)+\frac{1}{24\,x^3}$$

[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="maxima")

[Out] -4*b^3*e^(-4*a)*gamma(-3, 4*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x) + 1/24/x ^3

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx$$
$$= \frac{32\,b^3x^3\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 32\,b^3x^3\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 8\,b^2x^2e^{(4\,bx+4\,a)} - 8\,b^2x^2e^{(-4\,bx-4\,a)} - 2\,bxe^{(4\,bx+4\,a)} + 2\,bx}{48\,x^3}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2
*x^2*e^(4*b*x + 4*a) - 8*b^2*x^2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) +
 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) - e^(-4*b*x - 4*a) + 2)/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)^2}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^4,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^2)/x^4, x)
```

3.298 $\int x^m \cosh^3(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int x^m \cosh^3(a+bx) \sinh^2(a+bx) \, dx = \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b} + \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b} - \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{16b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{16b} - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{32b} - \frac{5^{-1-m}e^{-5a}x^m(bx)^{-m}\Gamma(1+m,5bx)}{32b}$$

```
[Out] 1/32*5^(-1-m)*exp(5*a)*x^m*GAMMA(1+m,-5*b*x)/b/((-b*x)^m)+1/32*3^(-1-m)*exp
(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/16*exp(a)*x^m*GAMMA(1+m,-b*x)/b/
((-b*x)^m)+1/16*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/32*3^(-1-m)*x^m*GAM
MA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)-1/32*5^(-1-m)*x^m*GAMMA(1+m,5*b*x)/b/exp
(5*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3388, 2212}

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-5bx)}{32b} + \frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{32b} - \frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{16b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{16b} - \frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{32b} - \frac{e^{-5a}5^{-m-1}x^m(bx)^{-m}\Gamma(m+1,5bx)}{32b}$$

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) + (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) - (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x^m\cosh(a+bx)+\frac{1}{16}x^m\cosh(3a+3bx)+\frac{1}{16}x^m\cosh(5a+5bx)\right)dx\\
&= \frac{1}{16}\int x^m\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^m\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^m\cosh(a+bx)\,dx\\
&= \frac{1}{32}\int e^{-i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{i(3ia+3ibx)}x^m\,dx+\frac{1}{32}\int e^{-i(5ia+5ibx)}x^m\,dx\\
&\quad+\frac{1}{32}\int e^{i(5ia+5ibx)}x^m\,dx-\frac{1}{16}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{16}\int e^{i(ia+ibx)}x^m\,dx\\
&= \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b}+\frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b}\\
&\quad-\frac{e^ax^m(-bx)^{-m}\Gamma(1+m,-bx)}{16b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{16b}\\
&\quad-\frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{32b}-\frac{5^{-1-m}e^{-5a}x^m(bx)^{-m}\Gamma(1+m,5bx)}{32b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.84

$$\int x^m\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{e^{-5a}x^m\Big(-30e^{6a}(-bx)^{-m}\Gamma(1+m,-bx)+30e^{4a}(bx)^{-m}\Gamma(1+m,bx)+5\ 3^{-m}e^{2a}\left(-b^2x^2\right)^{-m}\left(e^{6a}(bx)^m\Gamma(1+\right.}{4}$$

```
[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-30*E^(6*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + (30*E^(4*a)*Gamma[1 +
 m, b*x])/(b*x)^m + (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] - (-(b
*x))^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Ga
mma[1 + m, -5*b*x] - (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m))
)/(480*b*E^(5*a))
```

Maple [F]

$$\int x^{m} \cosh \left(b x+a\right)^{3} \sinh \left(b x+a\right)^{2}d x$$

```
[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{3\,\cosh\left(m\log\left(5\,b\right)+5\,a\right)\Gamma(m+1,5\,bx)+5\,\cosh\left(m\log\left(3\,b\right)+3\,a\right)\Gamma(m+1,3\,bx)-30\,\cosh\left(m\log\left(b\right)\right.}{}$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) + 5*cosh(m*log(3*b) +
3*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) + 30*cos
h(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m + 1
, -3*b*x) - 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m + 1,
 5*b*x)*sinh(m*log(5*b) + 5*a) - 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*
a) - 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x)*sin
h(m*log(-3*b) - 3*a) + 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a) + 30*
gamma(m + 1, b*x)*sinh(m*log(b) + a))/b
```

Sympy [F]

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int x^{m}\sinh^{2}\left(a+bx\right)\cosh^{3}\left(a+bx\right)\,dx$$

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Integral(x**m*sinh(a + b*x)**2*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{32}\,(5\,bx)^{-m-1}\,x^{m+1}e^{(-5\,a)}\Gamma(m+1,5\,bx) - \frac{1}{32}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx) + \frac{1}{16}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx) + \frac{1}{16}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx) - \frac{1}{32}\,(-3\,bx)^{-m-1}\,x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx) - \frac{1}{32}\,(-5\,bx)^{-m-1}\,x^{m+1}e^{(5\,a)}\Gamma(m+1,-5\,bx)$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x
)^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) + 1/16*(b*x)^(-m - 1)*x^(
m + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamma(
m + 1, -b*x) - 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*b*x
) - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

Giac [F]

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^3 \sinh\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)^2\,dx$$

```
[In] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^2, x)
```

3.299 $\int x^3 \cosh^3(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 202

$$\int x^3 \cosh^3(a+bx) \sinh^2(a+bx) \, dx = \frac{3\cosh(a+bx)}{4b^4} + \frac{3x^2\cosh(a+bx)}{8b^2} - \frac{\cosh(3a+3bx)}{216b^4} - \frac{x^2\cosh(3a+3bx)}{48b^2} - \frac{3\cosh(5a+5bx)}{5000b^4} - \frac{3x^2\cosh(5a+5bx)}{400b^2} - \frac{3x\sinh(a+bx)}{4b^3} - \frac{x^3\sinh(a+bx)}{8b} + \frac{x\sinh(3a+3bx)}{72b^3} + \frac{x^3\sinh(3a+3bx)}{48b} + \frac{3x\sinh(5a+5bx)}{1000b^3} + \frac{x^3\sinh(5a+5bx)}{80b}$$

```
[Out] 3/4*cosh(b*x+a)/b^4+3/8*x^2*cosh(b*x+a)/b^2-1/216*cosh(3*b*x+3*a)/b^4-1/48*
x^2*cosh(3*b*x+3*a)/b^2-3/5000*cosh(5*b*x+5*a)/b^4-3/400*x^2*cosh(5*b*x+5*a
)/b^2-3/4*x*sinh(b*x+a)/b^3-1/8*x^3*sinh(b*x+a)/b+1/72*x*sinh(3*b*x+3*a)/b^
3+1/48*x^3*sinh(3*b*x+3*a)/b+3/1000*x*sinh(5*b*x+5*a)/b^3+1/80*x^3*sinh(5*b
*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3377, 2718}

$$\int x^3 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{3\cosh(a+bx)}{4b^4} - \frac{\cosh(3a+3bx)}{216b^4} - \frac{3\cosh(5a+5bx)}{5000b^4} - \frac{3x\sinh(a+bx)}{4b^3} + \frac{x\sinh(3a+3bx)}{72b^3} + \frac{3x\sinh(5a+5bx)}{1000b^3} + \frac{3x^2\cosh(a+bx)}{8b^2} - \frac{x^2\cosh(3a+3bx)}{48b^2} - \frac{3x^2\cosh(5a+5bx)}{400b^2} - \frac{x^3\sinh(a+bx)}{8b} + \frac{x^3\sinh(3a+3bx)}{48b} + \frac{x^3\sinh(5a+5bx)}{80b}$$

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (3*Cosh[a + b*x])/(4*b^4) + (3*x^2*Cosh[a + b*x])/(8*b^2) - Cosh[3*a + 3*b*
x]/(216*b^4) - (x^2*Cosh[3*a + 3*b*x])/(48*b^2) - (3*Cosh[5*a + 5*b*x])/(50
00*b^4) - (3*x^2*Cosh[5*a + 5*b*x])/(400*b^2) - (3*x*Sinh[a + b*x])/(4*b^3)
 - (x^3*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(72*b^3) + (x^3*Sinh[3
*a + 3*b*x])/(48*b) + (3*x*Sinh[5*a + 5*b*x])/(1000*b^3) + (x^3*Sinh[5*a +
5*b*x])/(80*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x^3\cosh(a+bx)+\frac{1}{16}x^3\cosh(3a+3bx)+\frac{1}{16}x^3\cosh(5a+5bx)\right)\,dx \\
&= \frac{1}{16}\int x^3\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^3\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^3\cosh(a+bx)\,dx \\
&= -\frac{x^3\sinh(a+bx)}{8b}+\frac{x^3\sinh(3a+3bx)}{48b}+\frac{x^3\sinh(5a+5bx)}{80b} \\
&\quad -\frac{3\int x^2\sinh(5a+5bx)\,dx}{80b}-\frac{\int x^2\sinh(3a+3bx)\,dx}{16b}+\frac{3\int x^2\sinh(a+bx)\,dx}{8b} \\
&= \frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2} \\
&\quad -\frac{x^3\sinh(a+bx)}{8b}+\frac{x^3\sinh(3a+3bx)}{48b}+\frac{x^3\sinh(5a+5bx)}{80b} \\
&\quad +\frac{3\int x\cosh(5a+5bx)\,dx}{200b^2}+\frac{\int x\cosh(3a+3bx)\,dx}{24b^2}-\frac{3\int x\cosh(a+bx)\,dx}{4b^2} \\
&= \frac{3x^2\cosh(a+bx)}{8b^2}-\frac{x^2\cosh(3a+3bx)}{48b^2}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh(a+bx)}{4b^3} \\
&\quad -\frac{x^3\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{72b^3}+\frac{x^3\sinh(3a+3bx)}{48b}+\frac{3x\sinh(5a+5bx)}{1000b^3} \\
&\quad +\frac{x^3\sinh(5a+5bx)}{80b}-\frac{3\int\sinh(5a+5bx)\,dx}{1000b^3}-\frac{\int\sinh(3a+3bx)\,dx}{72b^3}+\frac{3\int\sinh(a+bx)\,dx}{4b^3} \\
&= \frac{3\cosh(a+bx)}{4b^4}+\frac{3x^2\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{216b^4}-\frac{x^2\cosh(3a+3bx)}{48b^2} \\
&\quad -\frac{3\cosh(5a+5bx)}{5000b^4}-\frac{3x^2\cosh(5a+5bx)}{400b^2}-\frac{3x\sinh(a+bx)}{4b^3}-\frac{x^3\sinh(a+bx)}{8b} \\
&\quad +\frac{x\sinh(3a+3bx)}{72b^3}+\frac{x^3\sinh(3a+3bx)}{48b}+\frac{3x\sinh(5a+5bx)}{1000b^3}+\frac{x^3\sinh(5a+5bx)}{80b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.62

$$
\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx
= \frac{101250(2+b^2x^2)\cosh(a+bx)-625(2+9b^2x^2)\cosh(3(a+bx))-81(2+25b^2x^2)\cosh(5(a+bx))+30b}{270000}
$$

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (101250*(2 + b^2*x^2)*Cosh[a + b*x] - 625*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)]
 - 81*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(-6598 - 825*b^2*x^2 + 8*
(38 + 75*b^2*x^2)*Cosh[2*(a + b*x)] + 9*(6 + 25*b^2*x^2)*Cosh[4*(a + b*x)])
*Sinh[a + b*x])/(270000*b^4)
```

Maple [A] (verified)

Time = 24.68 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

method	result
risch	$\frac{(125x^3b^3-75x^2b^2+30bx-6)e^{5bx+5a}}{20000b^4}+\frac{(9x^3b^3-9x^2b^2+6bx-2)e^{3bx+3a}}{864b^4}-\frac{(x^3b^3-3x^2b^2+6bx-6)e^{bx+a}}{16b^4}+\frac{(x^3b^3+3x$
derivativedivides	$\frac{-a^3\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\right.}{}$
default	$\frac{-a^3\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\right.}{}$

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20000*(125*b^3*x^3-75*b^2*x^2+30*b*x-6)/b^4*exp(5*b*x+5*a)+1/864*(9*b^3*x
^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)-1/16*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b
^4*exp(b*x+a)+1/16*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a)-1/864*(9*b^3
*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)-1/20000*(125*b^3*x^3+75*b^2*x^2
+30*b*x+6)/b^4*exp(-5*b*x-5*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.36

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx =$$

$$-\frac{81\,(25\,b^2x^2+2)\cosh\left(bx+a\right)^5+405\,(25\,b^2x^2+2)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-135\,(25\,b^3x^3+6\,bx)\sin}{}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/270000*(81*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 405*(25*b^2*x^2 + 2)*cosh(
b*x + a)*sinh(b*x + a)^4 - 135*(25*b^3*x^3 + 6*b*x)*sinh(b*x + a)^5 + 625*(
9*b^2*x^2 + 2)*cosh(b*x + a)^3 - 75*(75*b^3*x^3 + 18*(25*b^3*x^3 + 6*b*x)*c
osh(b*x + a)^2 + 50*b*x)*sinh(b*x + a)^3 + 15*(54*(25*b^2*x^2 + 2)*cosh(b*x
 + a)^3 + 125*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2 - 101250*(b^2*
x^2 + 2)*cosh(b*x + a) + 225*(150*b^3*x^3 - 3*(25*b^3*x^3 + 6*b*x)*cosh(b*x
 + a)^4 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^2 + 900*b*x)*sinh(b*x + a))/
b^4
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases}-\frac{2x^3\sinh^5(a+bx)}{15b}+\frac{x^3\sinh^3(a+bx)\cosh^2(a+bx)}{3b}+\frac{2x^2\sinh^4(a+bx)\cosh(a+bx)}{5b^2}-\frac{13x^2\sinh^2(a+bx)\cosh^3(a+bx)}{15b^2}+\frac{26x^2\cosh^5(a+bx)}{75b^2} \\ \frac{x^4\sinh^2(a)\cosh^3(a)}{4}\end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x**3*sinh(a + b*x)**5/(15*b) + x**3*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(3*b) + 2*x**2*sinh(a + b*x)**4*cosh(a + b*x)/(5*b**2) - 13*x**2*s
inh(a + b*x)**2*cosh(a + b*x)**3/(15*b**2) + 26*x**2*cosh(a + b*x)**5/(75*b
**2) - 856*x*sinh(a + b*x)**5/(1125*b**3) + 338*x*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(225*b**3) - 52*x*sinh(a + b*x)*cosh(a + b*x)**4/(75*b**3) + 856*s
inh(a + b*x)**4*cosh(a + b*x)/(1125*b**4) - 5114*sinh(a + b*x)**2*cosh(a +
b*x)**3/(3375*b**4) + 12568*cosh(a + b*x)**5/(16875*b**4), Ne(b, 0)), (x**4
*sinh(a)**2*cosh(a)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{(125\,b^3x^3e^{(5\,a)}-75\,b^2x^2e^{(5\,a)}+30\,bxe^{(5\,a)}-6\,e^{(5\,a)})e^{(5\,bx)}}{20000\,b^4}+\frac{(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)})e^{(3\,bx)}}{864\,b^4}-\frac{(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)e^{(bx)}}{16\,b^4}+\frac{(b^3x^3+3\,b^2x^2+6\,bx+6)e^{(-bx-a)}}{16\,b^4}-\frac{(9\,b^3x^3+9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{864\,b^4}-\frac{(125\,b^3x^3+75\,b^2x^2+30\,bx+6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(5
*a))*e^(5*b*x)/b^4 + 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b
*x*e^a - 6*e^a)*e^(b*x)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*
x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{(125\,b^3x^3-75\,b^2x^2+30\,bx-6)e^{(5\,bx+5\,a)}}{20000\,b^4} + \frac{(9\,b^3x^3-9\,b^2x^2+6\,bx-2)e^{(3\,bx+3\,a)}}{864\,b^4} - \frac{(b^3x^3-3\,b^2x^2+6\,bx-6)e^{(bx+a)}}{16\,b^4} + \frac{(b^3x^3+3\,b^2x^2+6\,bx+6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3+9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{864\,b^4} - \frac{(125\,b^3x^3+75\,b^2x^2+30\,bx+6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 + 1/864
*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3 -
3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 + 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x
+ 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^4 - 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5
*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int x^3\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{x\sinh(3\,a+3\,b\,x)}{72}-\frac{3\,x\sinh(a+b\,x)}{4}+\frac{3\,x\sinh(5\,a+5\,b\,x)}{1000}}{b^3} + \frac{\frac{x^3\sinh(3\,a+3\,b\,x)}{48}+\frac{x^3\sinh(5\,a+5\,b\,x)}{80}-\frac{x^3\sinh(a+b\,x)}{8}}{b} + \frac{3\cosh(a+b\,x)}{4\,b^4} - \frac{\cosh(3\,a+3\,b\,x)}{216\,b^4} - \frac{3\cosh(5\,a+5\,b\,x)}{5000\,b^4} - \frac{\frac{x^2\cosh(3\,a+3\,b\,x)}{48}-\frac{3\,x^2\cosh(a+b\,x)}{8}+\frac{3\,x^2\cosh(5\,a+5\,b\,x)}{400}}{b^2}$$

```
[In] int(x^3*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] ((x*sinh(3*a + 3*b*x))/72 - (3*x*sinh(a + b*x))/4 + (3*x*sinh(5*a + 5*b*x))
/1000)/b^3 + ((x^3*sinh(3*a + 3*b*x))/48 + (x^3*sinh(5*a + 5*b*x))/80 - (x^
3*sinh(a + b*x))/8)/b + (3*cosh(a + b*x))/(4*b^4) - cosh(3*a + 3*b*x)/(216*
b^4) - (3*cosh(5*a + 5*b*x))/(5000*b^4) - ((x^2*cosh(3*a + 3*b*x))/48 - (3*
x^2*cosh(a + b*x))/8 + (3*x^2*cosh(5*a + 5*b*x))/400)/b^2
```

3.300 $\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{x\cosh(a+bx)}{4b^2} - \frac{x\cosh(3a+3bx)}{72b^2} - \frac{x\cosh(5a+5bx)}{200b^2} - \frac{\sinh(a+bx)}{4b^3} - \frac{x^2\sinh(a+bx)}{8b} + \frac{\sinh(3a+3bx)}{216b^3} + \frac{x^2\sinh(3a+3bx)}{48b} + \frac{\sinh(5a+5bx)}{1000b^3} + \frac{x^2\sinh(5a+5bx)}{80b}$$

```
[Out] 1/4*x*cosh(b*x+a)/b^2-1/72*x*cosh(3*b*x+3*a)/b^2-1/200*x*cosh(5*b*x+5*a)/b^
2-1/4*sinh(b*x+a)/b^3-1/8*x^2*sinh(b*x+a)/b+1/216*sinh(3*b*x+3*a)/b^3+1/48*
x^2*sinh(3*b*x+3*a)/b+1/1000*sinh(5*b*x+5*a)/b^3+1/80*x^2*sinh(5*b*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5556, 3377, 2717}

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{\sinh(a+bx)}{4b^3} + \frac{\sinh(3a+3bx)}{216b^3} + \frac{\sinh(5a+5bx)}{1000b^3} + \frac{x\cosh(a+bx)}{4b^2} - \frac{x\cosh(3a+3bx)}{72b^2} - \frac{x\cosh(5a+5bx)}{200b^2} - \frac{x^2\sinh(a+bx)}{8b} + \frac{x^2\sinh(3a+3bx)}{48b} + \frac{x^2\sinh(5a+5bx)}{80b}$$

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (x*Cosh[a + b*x])/(4*b^2) - (x*Cosh[3*a + 3*b*x])/(72*b^2) - (x*Cosh[5*a +
5*b*x])/(200*b^2) - Sinh[a + b*x]/(4*b^3) - (x^2*Sinh[a + b*x])/(8*b) + Sin
h[3*a + 3*b*x]/(216*b^3) + (x^2*Sinh[3*a + 3*b*x])/(48*b) + Sinh[5*a + 5*b*
x]/(1000*b^3) + (x^2*Sinh[5*a + 5*b*x])/(80*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x^2\cosh(a+bx)+\frac{1}{16}x^2\cosh(3a+3bx)+\frac{1}{16}x^2\cosh(5a+5bx)\right)\,dx\\
&= \frac{1}{16}\int x^2\cosh(3a+3bx)\,dx+\frac{1}{16}\int x^2\cosh(5a+5bx)\,dx-\frac{1}{8}\int x^2\cosh(a+bx)\,dx\\
&= -\frac{x^2\sinh(a+bx)}{8b}+\frac{x^2\sinh(3a+3bx)}{48b}+\frac{x^2\sinh(5a+5bx)}{80b}\\
&\quad-\frac{\int x\sinh(5a+5bx)\,dx}{40b}-\frac{\int x\sinh(3a+3bx)\,dx}{24b}+\frac{\int x\sinh(a+bx)\,dx}{4b}\\
&= \frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}\\
&\quad-\frac{x^2\sinh(a+bx)}{8b}+\frac{x^2\sinh(3a+3bx)}{48b}+\frac{x^2\sinh(5a+5bx)}{80b}\\
&\quad+\frac{\int\cosh(5a+5bx)\,dx}{200b^2}+\frac{\int\cosh(3a+3bx)\,dx}{72b^2}-\frac{\int\cosh(a+bx)\,dx}{4b^2}\\
&= \frac{x\cosh(a+bx)}{4b^2}-\frac{x\cosh(3a+3bx)}{72b^2}-\frac{x\cosh(5a+5bx)}{200b^2}-\frac{\sinh(a+bx)}{4b^3}\\
&\quad-\frac{x^2\sinh(a+bx)}{8b}+\frac{\sinh(3a+3bx)}{216b^3}+\frac{x^2\sinh(3a+3bx)}{48b}+\frac{\sinh(5a+5bx)}{1000b^3}\\
&\quad+\frac{x^2\sinh(5a+5bx)}{80b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int x^2\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$= \frac{-6750(-2bx\cosh(a+bx)+(2+b^2x^2)\sinh(a+bx))+125(-6bx\cosh(3(a+bx))+(2+9b^2x^2)\sinh(3(a}{54000b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-6750*(-2*b*x*Cosh[a + b*x] + (2 + b^2*x^2)*Sinh[a + b*x]) + 125*(-6*b*x*C
osh[3*(a + b*x)] + (2 + 9*b^2*x^2)*Sinh[3*(a + b*x)]) + 27*(-10*b*x*Cosh[5*
(a + b*x)] + (2 + 25*b^2*x^2)*Sinh[5*(a + b*x)]))/(54000*b^3)
```

Maple [A] (verified)

Time = 18.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(25x^2b^2-10bx+2)e^{5bx+5a}}{4000b^3}+\frac{(9x^2b^2-6bx+2)e^{3bx+3a}}{864b^3}-\frac{(x^2b^2-2bx+2)e^{bx+a}}{16b^3}+\frac{(x^2b^2+2bx+2)e^{-bx-a}}{16b^3}-\frac{(9x^2b^2}{}$
derivativedivides	$\frac{a^2\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx}{}\right.}{}$
default	$\frac{a^2\left(\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx}{}\right.}{}$

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4000*(25*b^2*x^2-10*b*x+2)/b^3*exp(5*b*x+5*a)+1/864*(9*b^2*x^2-6*b*x+2)/b
^3*exp(3*b*x+3*a)-1/16*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/16*(b^2*x^2+2*b*x
+2)/b^3*exp(-b*x-a)-1/864*(9*b^2*x^2+6*b*x+2)/b^3*exp(-3*b*x-3*a)-1/4000*(2
5*b^2*x^2+10*b*x+2)/b^3*exp(-5*b*x-5*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.41

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{270\,bx\cosh\left(bx+a\right)^5+1350\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-27\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5+750\,bx\,\mathrm{c}}{}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/54000*(270*b*x*cosh(b*x + a)^5 + 1350*b*x*cosh(b*x + a)*sinh(b*x + a)^4
- 27*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 + 750*b*x*cosh(b*x + a)^3 - 5*(225*b^
2*x^2 + 54*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 50)*sinh(b*x + a)^3 - 13500*b
*x*cosh(b*x + a) + 450*(6*b*x*cosh(b*x + a)^3 + 5*b*x*cosh(b*x + a))*sinh(b
*x + a)^2 - 15*(9*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 450*b^2*x^2 + 25*(9*b^
2*x^2 + 2)*cosh(b*x + a)^2 - 900)*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.23

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases} -\frac{2x^2\sinh^5(a+bx)}{15b}+\frac{x^2\sinh^3(a+bx)\cosh^2(a+bx)}{3b}+\frac{4x\sinh^4(a+bx)\cosh(a+bx)}{15b^2}-\frac{26x\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2}+\frac{52x\cosh^5(a+b}{225b^2} \\ \frac{x^3\sinh^2(a)\cosh^3(a)}{3} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x**2*sinh(a + b*x)**5/(15*b) + x**2*sinh(a + b*x)**3*cosh(a +
 b*x)**2/(3*b) + 4*x*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 26*x*sinh(a
 + b*x)**2*cosh(a + b*x)**3/(45*b**2) + 52*x*cosh(a + b*x)**5/(225*b**2) -
856*sinh(a + b*x)**5/(3375*b**3) + 338*sinh(a + b*x)**3*cosh(a + b*x)**2/(6
75*b**3) - 52*sinh(a + b*x)*cosh(a + b*x)**4/(225*b**3), Ne(b, 0)), (x**3*s
inh(a)**2*cosh(a)**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(25\,b^2x^2e^{(5\,a)} - 10\,bxe^{(5\,a)} + 2\,e^{(5\,a)}\right)e^{(5\,bx)}}{4000\,b^3} + \frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{864\,b^3} - \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)e^{(bx)}}{16\,b^3} + \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{16\,b^3} - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^3} - \frac{(25\,b^2x^2 + 10\,bx + 2)e^{(-5\,bx-5\,a)}}{4000\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 + 1/
864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*(b
^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 + 1/16*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 - 1/400
0*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{(25\,b^2x^2 - 10\,bx + 2)e^{(5\,bx+5\,a)}}{4000\,b^3} + \frac{(9\,b^2x^2 - 6\,bx + 2)e^{(3\,bx+3\,a)}}{864\,b^3} - \frac{(b^2x^2 - 2\,bx + 2)e^{(bx+a)}}{16\,b^3} + \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{16\,b^3} - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^3} - \frac{(25\,b^2x^2 + 10\,bx + 2)e^{(-5\,bx-5\,a)}}{4000\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 + 1/864*(9*b^2*x^2 - 6
*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^3
+ 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x +
2)*e^(-3*b*x - 3*a)/b^3 - 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)
/b^3
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int x^2 \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{x^2\sinh(3a+3bx)}{48}+\frac{x^2\sinh(5a+5bx)}{80}-\frac{x^2\sinh(a+bx)}{8}}{b} - \frac{\sinh(a+bx)}{4b^3} - \frac{\frac{x\cosh(3a+3bx)}{72}-\frac{x\cosh(a+bx)}{4}+\frac{x\cosh(5a+5bx)}{200}}{b^2} + \frac{\sinh(3a+3bx)}{216b^3} + \frac{\sinh(5a+5bx)}{1000b^3}$$

```
[In] int(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] ((x^2*sinh(3*a + 3*b*x))/48 + (x^2*sinh(5*a + 5*b*x))/80 - (x^2*sinh(a + b*
x))/8)/b - sinh(a + b*x)/(4*b^3) - ((x*cosh(3*a + 3*b*x))/72 - (x*cosh(a +
b*x))/4 + (x*cosh(5*a + 5*b*x))/200)/b^2 + sinh(3*a + 3*b*x)/(216*b^3) + si
nh(5*a + 5*b*x)/(1000*b^3)
```

3.301 $\int x \cosh^3(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int x \cosh^3(a+bx) \sinh^2(a+bx) \, dx = \frac{\cosh(a+bx)}{8b^2} - \frac{\cosh(3a+3bx)}{144b^2} - \frac{\cosh(5a+5bx)}{400b^2} - \frac{x \sinh(a+bx)}{8b} + \frac{x \sinh(3a+3bx)}{48b} + \frac{x \sinh(5a+5bx)}{80b}$$

```
[Out] 1/8*cosh(b*x+a)/b^2-1/144*cosh(3*b*x+3*a)/b^2-1/400*cosh(5*b*x+5*a)/b^2-1/8
*x*sinh(b*x+a)/b+1/48*x*sinh(3*b*x+3*a)/b+1/80*x*sinh(5*b*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 3377, 2718}

$$\int x \cosh^3(a+bx) \sinh^2(a+bx) \, dx = \frac{\cosh(a+bx)}{8b^2} - \frac{\cosh(3a+3bx)}{144b^2} - \frac{\cosh(5a+5bx)}{400b^2} - \frac{x \sinh(a+bx)}{8b} + \frac{x \sinh(3a+3bx)}{48b} + \frac{x \sinh(5a+5bx)}{80b}$$

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Cosh[a + b*x]/(8*b^2) - Cosh[3*a + 3*b*x]/(144*b^2) - Cosh[5*a + 5*b*x]/(40
0*b^2) - (x*Sinh[a + b*x])/(8*b) + (x*Sinh[3*a + 3*b*x])/(48*b) + (x*Sinh[5
*a + 5*b*x])/(80*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x\cosh(a+bx)+\frac{1}{16}x\cosh(3a+3bx)+\frac{1}{16}x\cosh(5a+5bx)\right)dx\\
&= \frac{1}{16}\int x\cosh(3a+3bx)\,dx+\frac{1}{16}\int x\cosh(5a+5bx)\,dx-\frac{1}{8}\int x\cosh(a+bx)\,dx\\
&= -\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}\\
&\quad-\frac{\int\sinh(5a+5bx)\,dx}{80b}-\frac{\int\sinh(3a+3bx)\,dx}{48b}+\frac{\int\sinh(a+bx)\,dx}{8b}\\
&= \frac{\cosh(a+bx)}{8b^2}-\frac{\cosh(3a+3bx)}{144b^2}-\frac{\cosh(5a+5bx)}{400b^2}\\
&\quad-\frac{x\sinh(a+bx)}{8b}+\frac{x\sinh(3a+3bx)}{48b}+\frac{x\sinh(5a+5bx)}{80b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{450\cosh(a+bx)-25\cosh(3(a+bx))-9\cosh(5(a+bx))-450bx\sinh(a+bx)+75bx\sinh(3(a+bx))+}{3600b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (450*Cosh[a + b*x] - 25*Cosh[3*(a + b*x)] - 9*Cosh[5*(a + b*x)] - 450*b*x*S
inh[a + b*x] + 75*b*x*Sinh[3*(a + b*x)] + 45*b*x*Sinh[5*(a + b*x)])/(3600*b
^2)
```

Maple [A] (verified)

Time = 13.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(5bx-1)\mathrm{e}^{5bx+5a}}{800b^2}+\frac{(3bx-1)\mathrm{e}^{3bx+3a}}{288b^2}-\frac{(bx-1)\mathrm{e}^{bx+a}}{16b^2}+\frac{(bx+1)\mathrm{e}^{-bx-a}}{16b^2}-\frac{(3bx+1)\mathrm{e}^{-3bx-3a}}{288b^2}-\frac{(5bx+1)\mathrm{e}^{-5bx-5a}}{800b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{15}-\frac{\cosh(bx+a)^5}{25}+\frac{2\cosh(bx+a)}{15}+\frac{\cosh(bx+a)^3}{45}}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{2(bx+a)\sinh(bx+a)}{15}-\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{15}-\frac{\cosh(bx+a)^5}{25}+\frac{2\cosh(bx+a)}{15}+\frac{\cosh(bx+a)^3}{45}}{b^2}$

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/800*(5*b*x-1)/b^2*exp(5*b*x+5*a)+1/288*(3*b*x-1)/b^2*exp(3*b*x+3*a)-1/16*
(b*x-1)/b^2*exp(b*x+a)+1/16*(b*x+1)/b^2*exp(-b*x-a)-1/288*(3*b*x+1)/b^2*exp
(-3*b*x-3*a)-1/800*(5*b*x+1)/b^2*exp(-5*b*x-5*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.62

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{45\,bx\sinh\left(bx+a\right)^5-9\,\cosh\left(bx+a\right)^5-45\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+75\,\left(6\,bx\cosh\left(bx+a\right)^2+bx\right)\sin}{}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3600*(45*b*x*sinh(b*x + a)^5 - 9*cosh(b*x + a)^5 - 45*cosh(b*x + a)*sinh(
b*x + a)^4 + 75*(6*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 - 25*cosh(b*x
 + a)^3 - 15*(6*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^2 + 225*(b
*x*cosh(b*x + a)^4 + b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) + 450*cosh(
b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\begin{cases} -\frac{2x\sinh^5(a+bx)}{15b}+\frac{x\sinh^3(a+bx)\cosh^2(a+bx)}{3b}+\frac{2\sinh^4(a+bx)\cosh(a+bx)}{15b^2}-\frac{13\sinh^2(a+bx)\cosh^3(a+bx)}{45b^2}+\frac{26\cosh^5(a+bx)}{225b^2} \\ \frac{x^2\sinh^2(a)\cosh^3(a)}{2} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*x*sinh(a + b*x)**5/(15*b) + x*sinh(a + b*x)**3*cosh(a + b*x)*
*2/(3*b) + 2*sinh(a + b*x)**4*cosh(a + b*x)/(15*b**2) - 13*sinh(a + b*x)**2
*cosh(a + b*x)**3/(45*b**2) + 26*cosh(a + b*x)**5/(225*b**2), Ne(b, 0)), (x
**2*sinh(a)**2*cosh(a)**3/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(5\,bxe^{(5\,a)}-e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2}+\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2} - \frac{(bxe^a-e^a)e^{(bx)}}{16\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}-\frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 + 1/288*(3*b*x*e^(3*a) - e^(3
*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 + 1/16*(b*x + 1)*e^(-
b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e
^(-5*b*x - 5*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2}+\frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2}-\frac{(bx-1)e^{(bx+a)}}{16\,b^2}+\frac{(bx+1)e^{(-bx-a)}}{16\,b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2}-\frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 + 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/b
^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 + 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1/2
88*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 - 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)/b^
2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{b\left(\frac{2\,x\,\sinh(a+b\,x)^5}{15}-\frac{x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{3}\right)-\frac{2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^4}{15}-\frac{26\,\cosh(a+b\,x)^5}{225}+\frac{13\,\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{45}}{b^2}$$

```
[In] int(x*cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] -(b*((2*x*sinh(a + b*x)^5)/15 - (x*cosh(a + b*x)^2*sinh(a + b*x)^3)/3) - (2
*cosh(a + b*x)*sinh(a + b*x)^4)/15 - (26*cosh(a + b*x)^5)/225 + (13*cosh(a
+ b*x)^3*sinh(a + b*x)^2)/45)/b^2
```

3.302 $\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh^3(a+bx)}{3b} + \frac{\sinh^5(a+bx)}{5b}$$

```
[Out] 1/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2644, 14}

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh^5(a+bx)}{5b} + \frac{\sinh^3(a+bx)}{3b}$$

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] Sinh[a + b*x]^3/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{i\text{Subst}\left(\int x^2(1-x^2)\ dx, x, i\sinh(a+bx)\right)}{b} \\ &= \frac{i\text{Subst}\left(\int (x^2-x^4)\ dx, x, i\sinh(a+bx)\right)}{b} \\ &= \frac{\sinh^3(a+bx)}{3b} + \frac{\sinh^5(a+bx)}{5b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{(7+3\cosh(2(a+bx)))\sinh^3(a+bx)}{30b}$$

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
[Out] ((7 + 3*Cosh[2*(a + b*x)])*Sinh[a + b*x]^3)/(30*b)
```

Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\sinh(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\sinh(bx+a)^3}{3}}{b}$	26
risch	$\frac{e^{5bx+5a}}{160b}+\frac{e^{3bx+3a}}{96b}-\frac{e^{bx+a}}{16b}+\frac{e^{-bx-a}}{16b}-\frac{e^{-3bx-3a}}{96b}-\frac{e^{-5bx-5a}}{160b}$	83

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*sinh(b*x+a)^5+1/3*sinh(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{3\,\sinh(bx+a)^5 + 5\,\left(6\,\cosh(bx+a)^2+1\right)\sinh(bx+a)^3 + 15\,\left(\cosh(bx+a)^4+\cosh(bx+a)^2-2\right)\sinh}{240\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + cosh(b*x + a)^2 - 2)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{2\sinh^5(a+bx)}{15b} + \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^2(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*sinh(a + b*x)**5/(15*b) + sinh(a + b*x)**3*cosh(a + b*x)**2/(
3*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(27) = 54$.

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,a)} - 30\,e^{(-4\,bx-4\,a)} + 3\right)e^{(5\,bx+5\,a)}}{480\,b} + \frac{30\,e^{(-bx-a)} - 5\,e^{(-3\,bx-3\,a)} - 3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*e^(-2*b*x - 2*a) - 30*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1/
480*(30*e^(-b*x - a) - 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{e^{(3\,bx+3\,a)}}{96\,b} - \frac{e^{(bx+a)}}{16\,b} + \frac{e^{(-bx-a)}}{16\,b} - \frac{e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 1/96*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b + 1/1
6*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{3\sinh(a+b\,x)^5 + 5\sinh(a+b\,x)^3}{15\,b}$$

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x)^2,x)
```

```
[Out] (5*sinh(a + b*x)^3 + 3*sinh(a + b*x)^5)/(15*b)
```

3.303 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x} dx = -\frac{1}{8}\cosh(a)\text{Chi}(bx) + \frac{1}{16}\cosh(3a)\text{Chi}(3bx) + \frac{1}{16}\cosh(5a)\text{Chi}(5bx) - \frac{1}{8}\sinh(a)\text{Shi}(bx) + \frac{1}{16}\sinh(3a)\text{Shi}(3bx) + \frac{1}{16}\sinh(5a)\text{Shi}(5bx)$$

```
[Out] -1/8*Chi(b*x)*cosh(a)+1/16*Chi(3*b*x)*cosh(3*a)+1/16*Chi(5*b*x)*cosh(5*a)-1
/8*Shi(b*x)*sinh(a)+1/16*Shi(3*b*x)*sinh(3*a)+1/16*Shi(5*b*x)*sinh(5*a)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x} dx = -\frac{1}{8}\cosh(a)\text{Chi}(bx) + \frac{1}{16}\cosh(3a)\text{Chi}(3bx) + \frac{1}{16}\cosh(5a)\text{Chi}(5bx) - \frac{1}{8}\sinh(a)\text{Shi}(bx) + \frac{1}{16}\sinh(3a)\text{Shi}(3bx) + \frac{1}{16}\sinh(5a)\text{Shi}(5bx)$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

```
[Out] -1/8*(Cosh[a]*CoshIntegral[b*x]) + (Cosh[3*a]*CoshIntegral[3*b*x])/16 + (Co
sh[5*a]*CoshIntegral[5*b*x])/16 - (Sinh[a]*SinhIntegral[b*x])/8 + (Sinh[3*a
]*SinhIntegral[3*b*x])/16 + (Sinh[5*a]*SinhIntegral[5*b*x])/16
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{8x}+\frac{\cosh(3a+3bx)}{16x}+\frac{\cosh(5a+5bx)}{16x}\right)dx\\
&= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x}\,dx\\
&= -\left(\frac{1}{8}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\right)+\frac{1}{16}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx+\frac{1}{16}\cosh(5a)\int \frac{\cosh(5bx)}{x}\,dx\\
&\quad -\frac{1}{8}\sinh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{16}\sinh(3a)\int \frac{\sinh(3bx)}{x}\,dx+\frac{1}{16}\sinh(5a)\int \frac{\sinh(5bx)}{x}\,dx\\
&= -\frac{1}{8}\cosh(a)\text{Chi}(bx)+\frac{1}{16}\cosh(3a)\text{Chi}(3bx)+\frac{1}{16}\cosh(5a)\text{Chi}(5bx)\\
&\quad -\frac{1}{8}\sinh(a)\text{Shi}(bx)+\frac{1}{16}\sinh(3a)\text{Shi}(3bx)+\frac{1}{16}\sinh(5a)\text{Shi}(5bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{16}(-2\cosh(a)\text{Chi}(bx)+\cosh(3a)\text{Chi}(3bx) + \cosh(5a)\text{Chi}(5bx)-2\sinh(a)\text{Shi}(bx) + \sinh(3a)\text{Shi}(3bx)+\sinh(5a)\text{Shi}(5bx))$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x,x]
```

```
[Out] (-2*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] + Cosh[5*a]*C
oshIntegral[5*b*x] - 2*Sinh[a]*SinhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3
*b*x] + Sinh[5*a]*SinhIntegral[5*b*x])/16
```

Maple [A] (verified)

Time = 8.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{e^{-5a}\text{Ei}_1(5bx)}{32}-\frac{e^{-3a}\text{Ei}_1(3bx)}{32}+\frac{e^{-a}\text{Ei}_1(bx)}{16}+\frac{e^{a}\text{Ei}_1(-bx)}{16}-\frac{e^{3a}\text{Ei}_1(-3bx)}{32}-\frac{e^{5a}\text{Ei}_1(-5bx)}{32}$	71

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)+1/16*exp(-a)*Ei(1,b*
x)+1/16*exp(a)*Ei(1,-b*x)-1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5*
b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{32}\left(\text{Ei}(5\,bx)+\text{Ei}(-5\,bx)\right)\cosh(5\,a) + \frac{1}{32}\left(\text{Ei}(3\,bx)+\text{Ei}(-3\,bx)\right)\cosh(3\,a) - \frac{1}{16}\left(\text{Ei}(bx)+\text{Ei}(-bx)\right)\cosh(a) + \frac{1}{32}\left(\text{Ei}(5\,bx)-\text{Ei}(-5\,bx)\right)\sinh(5\,a) + \frac{1}{32}\left(\text{Ei}(3\,bx)-\text{Ei}(-3\,bx)\right)\sinh(3\,a) - \frac{1}{16}\left(\text{Ei}(bx)-\text{Ei}(-bx)\right)\sinh(a)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/32*(Ei(5*b*x) + Ei(-5*b*x))*cosh(5*a) + 1/32*(Ei(3*b*x) + Ei(-3*b*x))*cos
h(3*a) - 1/16*(Ei(b*x) + Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) - Ei(-5*b*x))*
sinh(5*a) + 1/32*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) - Ei(-b
*x))*sinh(a)
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) + 1/32*Ei(3*b*x)*e^(3*a) - 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) + 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{x}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x, x)
```

3.304 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x} - \frac{1}{8}b\text{Chi}(bx)\sinh(a)+\frac{3}{16}b\text{Chi}(3bx)\sinh(3a) + \frac{5}{16}b\text{Chi}(5bx)\sinh(5a)-\frac{1}{8}b\cosh(a)\text{Shi}(bx) + \frac{3}{16}b\cosh(3a)\text{Shi}(3bx)+\frac{5}{16}b\cosh(5a)\text{Shi}(5bx)$$

```
[Out] 1/8*cosh(b*x+a)/x-1/16*cosh(3*b*x+3*a)/x-1/16*cosh(5*b*x+5*a)/x-1/8*b*cosh(
a)*Shi(b*x)+3/16*b*cosh(3*a)*Shi(3*b*x)+5/16*b*cosh(5*a)*Shi(5*b*x)-1/8*b*C
hi(b*x)*sinh(a)+3/16*b*Chi(3*b*x)*sinh(3*a)+5/16*b*Chi(5*b*x)*sinh(5*a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{1}{8}b\sinh(a)\text{Chi}(bx)+\frac{3}{16}b\sinh(3a)\text{Chi}(3bx) + \frac{5}{16}b\sinh(5a)\text{Chi}(5bx)-\frac{1}{8}b\cosh(a)\text{Shi}(bx) + \frac{3}{16}b\cosh(3a)\text{Shi}(3bx)+\frac{5}{16}b\cosh(5a)\text{Shi}(5bx) + \frac{\cosh(a+bx)}{8x}-\frac{\cosh(3a+3bx)}{16x}-\frac{\cosh(5a+5bx)}{16x}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] Cosh[a + b*x]/(8*x) - Cosh[3*a + 3*b*x]/(16*x) - Cosh[5*a + 5*b*x]/(16*x) -
 (b*CoshIntegral[b*x]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x]*Sinh[3*a])/16 +
 (5*b*CoshIntegral[5*b*x]*Sinh[5*a])/16 - (b*Cosh[a]*SinhIntegral[b*x])/8 +
 (3*b*Cosh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Cosh[5*a]*SinhIntegral[5*b*x
])/16
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{8x^2} + \frac{\cosh(3a+3bx)}{16x^2} + \frac{\cosh(5a+5bx)}{16x^2}\right) dx \\ &= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^2}\,dx + \frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^2}\,dx - \frac{1}{8}\int \frac{\cosh(a+bx)}{x^2}\,dx\end{aligned}$$

$$= \frac{\cosh(a+bx)}{8x} - \frac{\cosh(3a+3bx)}{16x} - \frac{\cosh(5a+5bx)}{16x} - \frac{1}{8}b\int \frac{\sinh(a+bx)}{x}\,dx + \frac{1}{16}(3b)\int \frac{\sinh(3a+3bx)}{x}\,dx + \frac{1}{16}(5b)\int \frac{\sinh(5a+5bx)}{x}\,dx$$

$$= \frac{\cosh(a+bx)}{8x} - \frac{\cosh(3a+3bx)}{16x} - \frac{\cosh(5a+5bx)}{16x} - \frac{1}{8}(b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx + \frac{1}{16}(3b\cosh(3a))\int \frac{\sinh(3bx)}{x}\,dx + \frac{1}{16}(5b\cosh(5a))\int \frac{\sinh(5bx)}{x}\,dx - \frac{1}{8}(b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx + \frac{1}{16}(3b\sinh(3a))\int \frac{\cosh(3bx)}{x}\,dx + \frac{1}{16}(5b\sinh(5a))\int \frac{\cosh(5bx)}{x}\,dx$$

$$= \frac{\cosh(a+bx)}{8x} - \frac{\cosh(3a+3bx)}{16x} - \frac{\cosh(5a+5bx)}{16x} - \frac{1}{8}b\text{Chi}(bx)\sinh(a) + \frac{3}{16}b\text{Chi}(3bx)\sinh(3a) + \frac{5}{16}b\text{Chi}(5bx)\sinh(5a) - \frac{1}{8}b\cosh(a)\text{Shi}(bx) + \frac{3}{16}b\cosh(3a)\text{Shi}(3bx) + \frac{5}{16}b\cosh(5a)\text{Shi}(5bx)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{-2\cosh(a+bx) + \cosh(3(a+bx)) + \cosh(5(a+bx)) + 2bx\text{Chi}(bx)\sinh(a) - 3bx\text{Chi}(3bx)\sinh(3a) - 5}{16x}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^2,x]
```

```
[Out] -1/16*(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b*x*Cos
hIntegral[b*x]*Sinh[a] - 3*b*x*CoshIntegral[3*b*x]*Sinh[3*a] - 5*b*x*CoshIn
tegral[5*b*x]*Sinh[5*a] + 2*b*x*Cosh[a]*SinhIntegral[b*x] - 3*b*x*Cosh[3*a]
*SinhIntegral[3*b*x] - 5*b*x*Cosh[5*a]*SinhIntegral[5*b*x])/x
```

Maple [A] (verified)

Time = 9.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
risch	$\frac{3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)bx-2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)bx+2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)bx-3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)bx+5\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)bx-5\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)bx+2\,\mathrm{e}^{bx+a}-\mathrm{e}^{-3bx-}}{32x}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(3*exp(-3*a)*Ei(1,3*b*x)*b*x-2*exp(-a)*Ei(1,b*x)*b*x+2*exp(a)*Ei(1,-b*
x)*b*x-3*exp(3*a)*Ei(1,-3*b*x)*b*x+5*exp(-5*a)*Ei(1,5*b*x)*b*x-5*exp(5*a)*E
i(1,-5*b*x)*b*x+2*exp(b*x+a)-exp(-3*b*x-3*a)+2*exp(-b*x-a)-exp(3*b*x+3*a)-e
xp(-5*b*x-5*a)-exp(5*b*x+5*a))/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(106) = 212.

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{2\,\cosh\left(bx+a\right)^5+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\,\cosh\left(bx+a\right)^3+2\left(10\,\cosh\left(bx+a\right)^3+3\,\cosh\left(\right.\right.}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*cosh(b*x +
a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(b*x*Ei
(5*b*x) - b*x*Ei(-5*b*x))*cosh(5*a) - 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*co
sh(3*a) + 2*(b*x*Ei(b*x) - b*x*Ei(-b*x))*cosh(a) - 5*(b*x*Ei(5*b*x) + b*x*E
i(-5*b*x))*sinh(5*a) - 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*
x*Ei(b*x) + b*x*Ei(-b*x))*sinh(a) - 4*cosh(b*x + a))/x
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh^3\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = -\frac{5}{32}\,be^{(-5\,a)}\Gamma(-1,5\,bx) - \frac{3}{32}\,be^{(-3\,a)}\Gamma(-1,3\,bx) + \frac{1}{16}\,be^{(-a)}\Gamma(-1,bx) - \frac{1}{16}\,be^{a}\Gamma(-1,-bx) + \frac{3}{32}\,be^{(3\,a)}\Gamma(-1,-3\,bx) + \frac{5}{32}\,be^{(5\,a)}\Gamma(-1,-5\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) + 1/16
*b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) + 3/32*b*e^(3*a)*gamm
a(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \frac{5\,bx\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + 3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + 2\,bx\mathrm{Ei}(-bx)\,e^{(-a)} - 3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - 5\,bx\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \cdots}{32\,x}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x*Ei(5*b*x)*e^(5*a) + 3*b*x*Ei(3*b*x)*e^(3*a) + 2*b*x*Ei(-b*x)*e^
(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) - 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b*x
)*e^a - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) + 2*e^(-b*x - a)
- e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^2,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^2, x)
```

3.305 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx =& \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}\\&-\frac{\cosh(5a+5bx)}{32x^2}-\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)\\&+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)\\&+\frac{b\sinh(a+bx)}{16x}-\frac{3b\sinh(3a+3bx)}{32x}\\&-\frac{5b\sinh(5a+5bx)}{32x}-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)\\&+\frac{9}{32}b^2\sinh(3a)\text{Shi}(3bx)+\frac{25}{32}b^2\sinh(5a)\text{Shi}(5bx)\end{aligned}$$

```
[Out] -1/16*b^2*Chi(b*x)*cosh(a)+9/32*b^2*Chi(3*b*x)*cosh(3*a)+25/32*b^2*Chi(5*b*
x)*cosh(5*a)+1/16*cosh(b*x+a)/x^2-1/32*cosh(3*b*x+3*a)/x^2-1/32*cosh(5*b*x+
5*a)/x^2-1/16*b^2*Shi(b*x)*sinh(a)+9/32*b^2*Shi(3*b*x)*sinh(3*a)+25/32*b^2*
Shi(5*b*x)*sinh(5*a)+1/16*b*sinh(b*x+a)/x-3/32*b*sinh(3*b*x+3*a)/x-5/32*b*s
inh(5*b*x+5*a)/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{1}{16}b^2\cosh(a)\text{Chi}(bx)+\frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\sinh(a)\text{Shi}(bx)+\frac{9}{32}b^2\sinh(3a)\text{Shi}(3bx)+\frac{25}{32}b^2\sinh(5a)\text{Shi}(5bx)+\frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x}-\frac{3b\sinh(3a+3bx)}{32x}-\frac{5b\sinh(5a+5bx)}{32x}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] Cosh[a + b*x]/(16*x^2) - Cosh[3*a + 3*b*x]/(32*x^2) - Cosh[5*a + 5*b*x]/(32
*x^2) - (b^2*Cosh[a]*CoshIntegral[b*x])/16 + (9*b^2*Cosh[3*a]*CoshIntegral[
3*b*x])/32 + (25*b^2*Cosh[5*a]*CoshIntegral[5*b*x])/32 + (b*Sinh[a + b*x])/
(16*x) - (3*b*Sinh[3*a + 3*b*x])/(32*x) - (5*b*Sinh[5*a + 5*b*x])/(32*x) -
(b^2*Sinh[a]*SinhIntegral[b*x])/16 + (9*b^2*Sinh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Sinh[5*a]*SinhIntegral[5*b*x])/32
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{8x^3}+\frac{\cosh(3a+3bx)}{16x^3}+\frac{\cosh(5a+5bx)}{16x^3}\right) dx \\
&= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^3}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^3}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x^3}\,dx \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}-\frac{1}{16}b\int \frac{\sinh(a+bx)}{x^2}\,dx \\
&\quad +\frac{1}{32}(3b)\int \frac{\sinh(3a+3bx)}{x^2}\,dx+\frac{1}{32}(5b)\int \frac{\sinh(5a+5bx)}{x^2}\,dx \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2}+\frac{b\sinh(a+bx)}{16x} \\
&\quad -\frac{3b\sinh(3a+3bx)}{32x}-\frac{5b\sinh(5a+5bx)}{32x}-\frac{1}{16}b^2\int \frac{\cosh(a+bx)}{x}\,dx \\
&\quad +\frac{1}{32}\left(9b^2\right)\int \frac{\cosh(3a+3bx)}{x}\,dx+\frac{1}{32}\left(25b^2\right)\int \frac{\cosh(5a+5bx)}{x}\,dx \\
&= \frac{\cosh(a+bx)}{16x^2}-\frac{\cosh(3a+3bx)}{32x^2}-\frac{\cosh(5a+5bx)}{32x^2} \\
&\quad +\frac{b\sinh(a+bx)}{16x}-\frac{3b\sinh(3a+3bx)}{32x}-\frac{5b\sinh(5a+5bx)}{32x} \\
&\quad -\frac{1}{16}\left(b^2\cosh(a)\right)\int \frac{\cosh(bx)}{x}\,dx+\frac{1}{32}\left(9b^2\cosh(3a)\right)\int \frac{\cosh(3bx)}{x}\,dx \\
&\quad +\frac{1}{32}\left(25b^2\cosh(5a)\right)\int \frac{\cosh(5bx)}{x}\,dx-\frac{1}{16}\left(b^2\sinh(a)\right)\int \frac{\sinh(bx)}{x}\,dx \\
&\quad +\frac{1}{32}\left(9b^2\sinh(3a)\right)\int \frac{\sinh(3bx)}{x}\,dx+\frac{1}{32}\left(25b^2\sinh(5a)\right)\int \frac{\sinh(5bx)}{x}\,dx
\end{aligned}$$

$$= \frac{\cosh(a+bx)}{16x^2} - \frac{\cosh(3a+3bx)}{32x^2} - \frac{\cosh(5a+5bx)}{32x^2} - \frac{1}{16}b^2\cosh(a)\text{Chi}(bx) + \frac{9}{32}b^2\cosh(3a)\text{Chi}(3bx) + \frac{25}{32}b^2\cosh(5a)\text{Chi}(5bx) + \frac{b\sinh(a+bx)}{16x} - \frac{3b\sinh(3a+3bx)}{32x} - \frac{5b\sinh(5a+5bx)}{32x} - \frac{1}{16}b^2\sinh(a)\text{Shi}(bx) + \frac{9}{32}b^2\sinh(3a)\text{Shi}(3bx) + \frac{25}{32}b^2\sinh(5a)\text{Shi}(5bx)$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{-2\cosh(a+bx)+\cosh(3(a+bx))+\cosh(5(a+bx))+2b^2x^2\cosh(a)\text{Chi}(bx)-9b^2x^2\cosh(3a)\text{Chi}(3bx)}{}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^3,x]
```

```
[Out] -1/32*(-2*Cosh[a + b*x] + Cosh[3*(a + b*x)] + Cosh[5*(a + b*x)] + 2*b^2*x^2
*Cosh[a]*CoshIntegral[b*x] - 9*b^2*x^2*Cosh[3*a]*CoshIntegral[3*b*x] - 25*b
^2*x^2*Cosh[5*a]*CoshIntegral[5*b*x] - 2*b*x*Sinh[a + b*x] + 3*b*x*Sinh[3*(
a + b*x)] + 5*b*x*Sinh[5*(a + b*x)] + 2*b^2*x^2*Sinh[a]*SinhIntegral[b*x] -
 9*b^2*x^2*Sinh[3*a]*SinhIntegral[3*b*x] - 25*b^2*x^2*Sinh[5*a]*SinhIntegra
l[5*b*x])/x^2
```

Maple [A] (verified)

Time = 14.24 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.36

method	result
risch	$\frac{-9\,e^{-3a}\,\text{Ei}_1(3bx)x^2b^2+2\,e^{-a}\,\text{Ei}_1(bx)x^2b^2+2\,e^{a}\,\text{Ei}_1(-bx)x^2b^2-25\,e^{-5a}\,\text{Ei}_1(5bx)x^2b^2-25\,e^{5a}\,\text{Ei}_1(-5bx)x^2b^2-9\,e^{3a}\,\text{Ei}_1(-3bx)x^2b^2+2}{}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-9*exp(-3*a)*Ei(1,3*b*x)*x^2*b^2+2*exp(-a)*Ei(1,b*x)*x^2*b^2+2*exp(a)
*Ei(1,-b*x)*x^2*b^2-25*exp(-5*a)*Ei(1,5*b*x)*x^2*b^2-25*exp(5*a)*Ei(1,-5*b*
x)*x^2*b^2-9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2+2*exp(b*x+a)*b*x-2*exp(-b*x-a)*b
*x+3*exp(-3*b*x-3*a)*b*x+5*exp(-5*b*x-5*a)*b*x-5*exp(5*b*x+5*a)*b*x-3*exp(3
*b*x+3*a)*b*x+2*exp(b*x+a)+2*exp(-b*x-a)-exp(-3*b*x-3*a)-exp(-5*b*x-5*a)-ex
p(5*b*x+5*a)-exp(3*b*x+3*a))/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(160) = 320.

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.84

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = -\frac{10\,bx\sinh\left(bx+a\right)^5+2\,\cosh\left(bx+a\right)^5+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\,\left(50\,bx\cosh\left(bx+a\right)^2+3\,b\right.}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*b*x*sinh(b*x + a)^5 + 2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b
*x + a)^4 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*cosh(b*x
 + a)^3 + 2*(10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 25*(b^
2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*cosh(5*a) - 9*(b^2*x^2*Ei(3*b*x) + b^
2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b*x) + b^2*x^2*Ei(-b*x))*cosh(a
) + 2*(25*b*x*cosh(b*x + a)^4 + 9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a
) - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*sinh(5*a) - 9*(b^2*x^2*Ei(3
*b*x) - b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x) - b^2*x^2*Ei(-b*
x))*sinh(a) - 4*cosh(b*x + a))/x^2
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\sinh^2\left(a+bx\right)\cosh^3\left(a+bx\right)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = &-\frac{25}{32}\,b^2e^{(-5\,a)}\Gamma(-2,5\,bx)-\frac{9}{32}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx)\\&+\frac{1}{16}\,b^2e^{(-a)}\Gamma(-2,bx)+\frac{1}{16}\,b^2e^{a}\Gamma(-2,-bx)\\&-\frac{9}{32}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx)-\frac{25}{32}\,b^2e^{(5\,a)}\Gamma(-2,-5\,bx)\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) +
 1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) - 9/32*b^2*e
^(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.32

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \frac{25\,b^2x^2\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + 9\,b^2x^2\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - 2\,b^2x^2\mathrm{Ei}(-bx)\,e^{(-a)} + 9\,b^2x^2\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + 25\,b^2x^2\mathrm{Ei}(-5\,b$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) + 9*b^2*x^2*Ei(3*b*x)*e^(3*a) - 2*b^2*x^
2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) + 25*b^2*x^2*Ei(-5*b*x)*e
^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) - 3*b*x*e^(3*b*x +
3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) + 5*
b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) - e^(3*b*x + 3*a) + 2*e^(b*x + a) +
2*e^(-b*x - a) - e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^3, x)
```

3.306 $\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 238

$$\begin{aligned}\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx =& \frac{\cosh(a+bx)}{24x^3}+\frac{b^2\cosh(a+bx)}{48x}-\frac{\cosh(3a+3bx)}{48x^3}\\&-\frac{3b^2\cosh(3a+3bx)}{32x}-\frac{\cosh(5a+5bx)}{48x^3}\\&-\frac{25b^2\cosh(5a+5bx)}{96x}-\frac{1}{48}b^3\text{Chi}(bx)\sinh(a)\\&+\frac{9}{32}b^3\text{Chi}(3bx)\sinh(3a)+\frac{125}{96}b^3\text{Chi}(5bx)\sinh(5a)\\&+\frac{b\sinh(a+bx)}{48x^2}-\frac{b\sinh(3a+3bx)}{32x^2}\\&-\frac{5b\sinh(5a+5bx)}{96x^2}-\frac{1}{48}b^3\cosh(a)\text{Shi}(bx)\\&+\frac{9}{32}b^3\cosh(3a)\text{Shi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Shi}(5bx)\end{aligned}$$

```
[Out] 1/24*cosh(b*x+a)/x^3+1/48*b^2*cosh(b*x+a)/x-1/48*cosh(3*b*x+3*a)/x^3-3/32*b
^2*cosh(3*b*x+3*a)/x-1/48*cosh(5*b*x+5*a)/x^3-25/96*b^2*cosh(5*b*x+5*a)/x-1
/48*b^3*cosh(a)*Shi(b*x)+9/32*b^3*cosh(3*a)*Shi(3*b*x)+125/96*b^3*cosh(5*a)
*Shi(5*b*x)-1/48*b^3*Chi(b*x)*sinh(a)+9/32*b^3*Chi(3*b*x)*sinh(3*a)+125/96*
b^3*Chi(5*b*x)*sinh(5*a)+1/48*b*sinh(b*x+a)/x^2-1/32*b*sinh(3*b*x+3*a)/x^2-
5/96*b*sinh(5*b*x+5*a)/x^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{1}{48}b^3\sinh(a)\text{Chi}(bx)+\frac{9}{32}b^3\sinh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\sinh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\cosh(a)\text{Shi}(bx)+\frac{9}{32}b^3\cosh(3a)\text{Shi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Shi}(5bx)+\frac{b^2\cosh(a+bx)}{48x}-\frac{3b^2\cosh(3a+3bx)}{32x}-\frac{25b^2\cosh(5a+5bx)}{96x}+\frac{\cosh(a+bx)}{24x^3}-\frac{\cosh(3a+3bx)}{48x^3}-\frac{\cosh(5a+5bx)}{48x^3}+\frac{b\sinh(a+bx)}{48x^2}-\frac{b\sinh(3a+3bx)}{32x^2}-\frac{5b\sinh(5a+5bx)}{96x^2}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] Cosh[a + b*x]/(24*x^3) + (b^2*Cosh[a + b*x])/(48*x) - Cosh[3*a + 3*b*x]/(48
*x^3) - (3*b^2*Cosh[3*a + 3*b*x])/(32*x) - Cosh[5*a + 5*b*x]/(48*x^3) - (25
*b^2*Cosh[5*a + 5*b*x])/(96*x) - (b^3*CoshIntegral[b*x]*Sinh[a])/48 + (9*b^
3*CoshIntegral[3*b*x]*Sinh[3*a])/32 + (125*b^3*CoshIntegral[5*b*x]*Sinh[5*a
])/96 + (b*Sinh[a + b*x])/(48*x^2) - (b*Sinh[3*a + 3*b*x])/(32*x^2) - (5*b*
Sinh[5*a + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*SinhIntegral[b*x])/48 + (9*b^3*C
osh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*SinhIntegral[5*b*x])/
96
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{\cosh(a+bx)}{8x^4}+\frac{\cosh(3a+3bx)}{16x^4}+\frac{\cosh(5a+5bx)}{16x^4}\right)dx \\
&= \frac{1}{16}\int \frac{\cosh(3a+3bx)}{x^4}\,dx+\frac{1}{16}\int \frac{\cosh(5a+5bx)}{x^4}\,dx-\frac{1}{8}\int \frac{\cosh(a+bx)}{x^4}\,dx \\
&= \frac{\cosh(a+bx)}{24x^3}-\frac{\cosh(3a+3bx)}{48x^3}-\frac{\cosh(5a+5bx)}{48x^3}-\frac{1}{24}b\int \frac{\sinh(a+bx)}{x^3}\,dx \\
&\quad +\frac{1}{16}b\int \frac{\sinh(3a+3bx)}{x^3}\,dx+\frac{1}{48}(5b)\int \frac{\sinh(5a+5bx)}{x^3}\,dx \\
&= \frac{\cosh(a+bx)}{24x^3}-\frac{\cosh(3a+3bx)}{48x^3}-\frac{\cosh(5a+5bx)}{48x^3}+\frac{b\sinh(a+bx)}{48x^2} \\
&\quad -\frac{b\sinh(3a+3bx)}{32x^2}-\frac{5b\sinh(5a+5bx)}{96x^2}-\frac{1}{48}b^2\int \frac{\cosh(a+bx)}{x^2}\,dx \\
&\quad +\frac{1}{32}(3b^2)\int \frac{\cosh(3a+3bx)}{x^2}\,dx+\frac{1}{96}(25b^2)\int \frac{\cosh(5a+5bx)}{x^2}\,dx \\
&= \frac{\cosh(a+bx)}{24x^3}+\frac{b^2\cosh(a+bx)}{48x}-\frac{\cosh(3a+3bx)}{48x^3}-\frac{3b^2\cosh(3a+3bx)}{32x} \\
&\quad -\frac{\cosh(5a+5bx)}{48x^3}-\frac{25b^2\cosh(5a+5bx)}{96x}+\frac{b\sinh(a+bx)}{48x^2} \\
&\quad -\frac{b\sinh(3a+3bx)}{32x^2}-\frac{5b\sinh(5a+5bx)}{96x^2}-\frac{1}{48}b^3\int \frac{\sinh(a+bx)}{x}\,dx \\
&\quad +\frac{1}{32}(9b^3)\int \frac{\sinh(3a+3bx)}{x}\,dx+\frac{1}{96}(125b^3)\int \frac{\sinh(5a+5bx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} \\
&\quad - \frac{3b^2\cosh(3a+3bx)}{32x} - \frac{\cosh(5a+5bx)}{48x^3} - \frac{25b^2\cosh(5a+5bx)}{96x} \\
&\quad + \frac{b\sinh(a+bx)}{48x^2} - \frac{b\sinh(3a+3bx)}{32x^2} - \frac{5b\sinh(5a+5bx)}{96x^2} \\
&\quad - \frac{1}{48}\left(b^3\cosh(a)\right)\int \frac{\sinh(bx)}{x}\,dx + \frac{1}{32}\left(9b^3\cosh(3a)\right)\int \frac{\sinh(3bx)}{x}\,dx \\
&\quad + \frac{1}{96}\left(125b^3\cosh(5a)\right)\int \frac{\sinh(5bx)}{x}\,dx - \frac{1}{48}\left(b^3\sinh(a)\right)\int \frac{\cosh(bx)}{x}\,dx \\
&\quad + \frac{1}{32}\left(9b^3\sinh(3a)\right)\int \frac{\cosh(3bx)}{x}\,dx + \frac{1}{96}\left(125b^3\sinh(5a)\right)\int \frac{\cosh(5bx)}{x}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cosh(a+bx)}{24x^3} + \frac{b^2\cosh(a+bx)}{48x} - \frac{\cosh(3a+3bx)}{48x^3} - \frac{3b^2\cosh(3a+3bx)}{32x} \\
&\quad - \frac{\cosh(5a+5bx)}{48x^3} - \frac{25b^2\cosh(5a+5bx)}{96x} - \frac{1}{48}b^3\mathrm{Chi}(bx)\sinh(a) \\
&\quad + \frac{9}{32}b^3\mathrm{Chi}(3bx)\sinh(3a) + \frac{125}{96}b^3\mathrm{Chi}(5bx)\sinh(5a) + \frac{b\sinh(a+bx)}{48x^2} \\
&\quad - \frac{b\sinh(3a+3bx)}{32x^2} - \frac{5b\sinh(5a+5bx)}{96x^2} - \frac{1}{48}b^3\cosh(a)\mathrm{Shi}(bx) \\
&\quad + \frac{9}{32}b^3\cosh(3a)\mathrm{Shi}(3bx) + \frac{125}{96}b^3\cosh(5a)\mathrm{Shi}(5bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89

$$
\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx
= \frac{4\cosh(a+bx) + 2b^2x^2\cosh(a+bx) - 2\cosh(3(a+bx)) - 9b^2x^2\cosh(3(a+bx)) - 2\cosh(5(a+bx)) - 25}{}
$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^2)/x^4,x]
```

```
[Out] (4*Cosh[a + b*x] + 2*b^2*x^2*Cosh[a + b*x] - 2*Cosh[3*(a + b*x)] - 9*b^2*x^
2*Cosh[3*(a + b*x)] - 2*Cosh[5*(a + b*x)] - 25*b^2*x^2*Cosh[5*(a + b*x)] -
2*b^3*x^3*CoshIntegral[b*x]*Sinh[a] + 27*b^3*x^3*CoshIntegral[3*b*x]*Sinh[3
*a] + 125*b^3*x^3*CoshIntegral[5*b*x]*Sinh[5*a] + 2*b*x*Sinh[a + b*x] - 3*b
*x*Sinh[3*(a + b*x)] - 5*b*x*Sinh[5*(a + b*x)] - 2*b^3*x^3*Cosh[a]*SinhInte
gral[b*x] + 27*b^3*x^3*Cosh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Cosh[5*a
]*SinhIntegral[5*b*x])/(96*x^3)
```

Maple [A] (verified)

Time = 21.04 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{-27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3+2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3-2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3+27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3b^3-125\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)x^3b^3+125\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5b}{}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(-27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3+2*exp(-a)*Ei(1,b*x)*x^3*b^3-2*exp
(a)*Ei(1,-b*x)*x^3*b^3+27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3-125*exp(-5*a)*Ei(1,
5*b*x)*x^3*b^3+125*exp(5*a)*Ei(1,-5*b*x)*x^3*b^3+9*exp(-3*b*x-3*a)*b^2*x^2-
2*exp(-b*x-a)*b^2*x^2+9*exp(3*b*x+3*a)*b^2*x^2-2*exp(b*x+a)*b^2*x^2+25*exp(
-5*b*x-5*a)*b^2*x^2+25*exp(5*b*x+5*a)*b^2*x^2-3*exp(-3*b*x-3*a)*b*x+2*exp(-
b*x-a)*b*x+3*exp(3*b*x+3*a)*b*x-2*exp(b*x+a)*b*x-5*exp(-5*b*x-5*a)*b*x+5*ex
p(5*b*x+5*a)*b*x+2*exp(-3*b*x-3*a)-4*exp(-b*x-a)+2*exp(3*b*x+3*a)-4*exp(b*x
+a)+2*exp(-5*b*x-5*a)+2*exp(5*b*x+5*a))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.67

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx =$$
$$-\frac{10\,bx\sinh\left(bx+a\right)^5+2\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)^5+10\left(25\,b^2x^2+2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/192*(10*b*x*sinh(b*x + a)^5 + 2*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 + 10*(2
5*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(9*b^2*x^2 + 2)*cosh(b*x +
 a)^3 + 2*(50*b*x*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^3 + 2*(10*(25*b^2*
x^2 + 2)*cosh(b*x + a)^3 + 3*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a)^2
 - 4*(b^2*x^2 + 2)*cosh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei(-5*b
*x))*cosh(5*a) - 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*
(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*cosh(a) + 2*(25*b*x*cosh(b*x + a)^4 +
9*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) + b^3
*x^3*Ei(-5*b*x))*sinh(5*a) - 27*(b^3*x^3*Ei(3*b*x) + b^3*x^3*Ei(-3*b*x))*si
nh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{x^4}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**2/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2*cosh(a + b*x)**3/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.37

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = -\frac{125}{32}\,b^3e^{(-5\,a)}\Gamma(-3,5\,bx) - \frac{27}{32}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx) + \frac{1}{16}\,b^3e^{(-a)}\Gamma(-3,bx) - \frac{1}{16}\,b^3e^{a}\Gamma(-3,-bx) + \frac{27}{32}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx) + \frac{125}{32}\,b^3e^{(5\,a)}\Gamma(-3,-5\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x)
 + 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) + 27/32*b^
3*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \frac{125\,b^3x^3\mathrm{Ei}(5\,bx)\,e^{(5\,a)} + 27\,b^3x^3\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + 2\,b^3x^3\mathrm{Ei}(-bx)\,e^{(-a)} - 27\,b^3x^3\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - 125\,b^3x^3\mathrm{Ei}(}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) + 27*b^3*x^3*Ei(3*b*x)*e^(3*a) + 2*b^3
*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) - 125*b^3*x^3*Ei(-5*b
*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) - 9*b^2*x
^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) + 2*b^2*x^2*e^(-b*x - a) - 9*b^2
```

```
*x^2*e^(-3*b*x - 3*a) - 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x + 5*a)
 - 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) - 2*b*x*e^(-b*x - a) + 3*b*x*e
^(-3*b*x - 3*a) + 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) - 2*e^(3*b*x +
 3*a) + 4*e^(b*x + a) + 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) - 2*e^(-5*b*x -
 5*a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^2(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^2}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^4,x)
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^2)/x^4, x)
```

3.307 $\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{2^{-2(3+m)}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} - \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{2^{-2(3+m)}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}$$

```
[Out] exp(4*a)*x^m*GAMMA(1+m,-4*b*x)/(2^(6+2*m))/b/((-b*x)^m)-2^(-4-m)*exp(2*a)*x
^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^(-4-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/
((b*x)^m)+x^m*GAMMA(1+m,4*b*x)/(2^(6+2*m))/b/exp(4*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 3389, 2212}

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{4a}2^{-2(m+3)}x^m(-bx)^{-m}\Gamma(m+1,-4bx)}{b} - \frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} - \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b} + \frac{e^{-4a}2^{-2(m+3)}x^m(bx)^{-m}\Gamma(m+1,4bx)}{b}$$

```
[In] Int[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(4*a)*x^m*Gamma[1 + m, -4*b*x])/(2^(2*(3 + m))*b*(-(b*x))^m) - (2^(-4 - 
m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma
[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (x^m*Gamma[1 + m, 4*b*x])/(2^(2*(3 + 
m))*b*E^(4*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{4}x^m \sinh(2a+2bx) + \frac{1}{8}x^m \sinh(4a+4bx)\right) dx \\
&= \frac{1}{8}\int x^m \sinh(4a+4bx)\,dx - \frac{1}{4}\int x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{16}\int e^{-i(4ia+4ibx)}x^m\,dx - \frac{1}{16}\int e^{i(4ia+4ibx)}x^m\,dx \\
&\quad - \frac{1}{8}\int e^{-i(2ia+2ibx)}x^m\,dx + \frac{1}{8}\int e^{i(2ia+2ibx)}x^m\,dx \\
&= \frac{4^{-3-m}e^{4a}x^m(-bx)^{-m}\Gamma(1+m,-4bx)}{b} - \frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} \\
&\quad - \frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{4^{-3-m}e^{-4a}x^m(bx)^{-m}\Gamma(1+m,4bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{4^{-3-m}e^{-4a}x^m(-b^2x^2)^{-m}\left(e^{8a}(bx)^m\Gamma(1+m,-4bx)-2^{2+m}e^{6a}(bx)^m\Gamma(1+m,-2bx)+(-bx)^m\left(-2^{2+m}e^{2a}\Gamma\right.\right.}{b}$$

[In] Integrate[x^m*Cosh[a + b*x]*Sinh[a + b*x]^3,x]

[Out] (4^(-3 - m)*x^m*(E^(8*a)*(b*x)^m*Gamma[1 + m, -4*b*x] - 2^(2 + m)*E^(6*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(2^(2 + m)*E^(2*a)*Gamma[1 + m, 2*b*x]) + Gamma[1 + m, 4*b*x])))/(b*E^(4*a)*(-(b^2*x^2))^m)

Maple [F]

$$\int x^m \cosh(bx+a)\sinh(bx+a)^3\,dx$$

[In] int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)

[Out] int(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\cosh(m\log(4b)+4a)\Gamma(m+1,4bx)-4\cosh(m\log(2b)+2a)\Gamma(m+1,2bx)-4\cosh(m\log(-2b)-2}{}$$

[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/64*(cosh(m*log(4*b) + 4*a)*gamma(m + 1, 4*b*x) - 4*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) - 4*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cosh(m*log(-4*b) - 4*a)*gamma(m + 1, -4*b*x) - gamma(m + 1, 4*b*x)*sinh(m*log(4*b) + 4*a) + 4*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 4*gamma(m + 1, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -4*b*x)*sinh(m*log(-4*b) - 4*a))/b

Sympy [F]

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \int x^m \sinh^3{(a+bx)}\cosh{(a+bx)}\,dx$$

```
[In] integrate(x**m*cosh(b*x+a)*sinh(b*x+a)**3,x)
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

$$\begin{aligned}\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = {} & \frac{1}{16}\,(4\,bx)^{-m-1}\,x^{m+1}e^{(-4\,a)}\Gamma(m+1,4\,bx) \\ & - \frac{1}{8}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) \\ & + \frac{1}{8}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) \\ & - \frac{1}{16}\,(-4\,bx)^{-m-1}\,x^{m+1}e^{(4\,a)}\Gamma(m+1,-4\,bx)\end{aligned}$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/16*(4*b*x)^(-m - 1)*x^(m + 1)*e^(-4*a)*gamma(m + 1, 4*b*x) - 1/8*(2*b*x)^
(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 1/8*(-2*b*x)^(-m - 1)*x^(
m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/16*(-4*b*x)^(-m - 1)*x^(m + 1)*e^(4
*a)*gamma(m + 1, -4*b*x)
```

Giac [F]

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh{(bx+a)}\sinh{(bx+a)}^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh(a+b\,x)\,\sinh(a+b\,x)^3\,dx$$

```
[In] int(x^m*cosh(a + b*x)*sinh(a + b*x)^3,x)
[Out] int(x^m*cosh(a + b*x)*sinh(a + b*x)^3, x)
```

3.308 $\int x^3 \cosh(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int x^3 \cosh(a+bx) \sinh^3(a+bx) \, dx = -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3(a+bx)}{128b^4} - \frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b}$$

```
[Out] -45/256*x/b^3-3/32*x^3/b+45/256*cosh(b*x+a)*sinh(b*x+a)/b^4+9/32*x^2*cosh(b
*x+a)*sinh(b*x+a)/b^2-9/32*x*sinh(b*x+a)^2/b^3-3/128*cosh(b*x+a)*sinh(b*x+a
)^3/b^4-3/16*x^2*cosh(b*x+a)*sinh(b*x+a)^3/b^2+3/32*x*sinh(b*x+a)^4/b^3+1/4
*x^3*sinh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

$= \{5480, 3392, 30, 2715, 8\}$

$$\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{3\sinh^3(a+bx)\cosh(a+bx)}{128b^4} + \frac{45\sinh(a+bx)\cosh(a+bx)}{256b^4} + \frac{3x\sinh^4(a+bx)}{32b^3} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3x^2\sinh^3(a+bx)\cosh(a+bx)}{16b^2} + \frac{9x^2\sinh(a+bx)\cosh(a+bx)}{32b^2} + \frac{x^3\sinh^4(a+bx)}{4b} - \frac{45x}{256b^3} - \frac{3x^3}{32b}$$

```
[In] Int[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-45*x)/(256*b^3) - (3*x^3)/(32*b) + (45*Cosh[a + b*x]*Sinh[a + b*x])/(256*
b^4) + (9*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (9*x*Sinh[a + b*x]^2)
/(32*b^3) - (3*Cosh[a + b*x]*Sinh[a + b*x]^3)/(128*b^4) - (3*x^2*Cosh[a + b
*x]*Sinh[a + b*x]^3)/(16*b^2) + (3*x*Sinh[a + b*x]^4)/(32*b^3) + (x^3*Sinh[
a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x^3\sinh^4(a+bx)}{4b} - \frac{3\int x^2\sinh^4(a+bx)\,dx}{4b}$$

$$= -\frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b} - \frac{3\int\sinh^4(a+bx)\,dx}{32b^3} + \frac{9\int x^2\sinh^2(a+bx)\,dx}{16b}$$

$$= \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3(a+bx)}{128b^4} - \frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b} + \frac{9\int\sinh^2(a+bx)\,dx}{128b^3} + \frac{9\int\sinh^2(a+bx)\,dx}{32b^3} - \frac{9\int x^2\,dx}{32b}$$

$$= -\frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3(a+bx)}{128b^4} - \frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b} - \frac{9\int 1\,dx}{256b^3} - \frac{9\int 1\,dx}{64b^3}$$

$$= -\frac{45x}{256b^3} - \frac{3x^3}{32b} + \frac{45\cosh(a+bx)\sinh(a+bx)}{256b^4} + \frac{9x^2\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{9x\sinh^2(a+bx)}{32b^3} - \frac{3\cosh(a+bx)\sinh^3(a+bx)}{128b^4} - \frac{3x^2\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{3x\sinh^4(a+bx)}{32b^3} + \frac{x^3\sinh^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.61

$$\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{2bx(3+8b^2x^2)\cosh(4(a+bx))+48(1+2b^2x^2)\sinh(2(a+bx))-\cosh(2(a+bx))\,(32bx(3+2b^2x^2)+3(1}{512b^4}$$

```
[In] Integrate[x^3*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (2*b*x*(3 + 8*b^2*x^2)*Cosh[4*(a + b*x)] + 48*(1 + 2*b^2*x^2)*Sinh[2*(a + b
*x)] - Cosh[2*(a + b*x)]*(32*b*x*(3 + 2*b^2*x^2) + 3*(1 + 8*b^2*x^2)*Sinh[2
*(a + b*x)]))/(512*b^4)
```

Maple [A] (verified)

Time = 13.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(32x^3b^3-24x^2b^2+12bx-3)e^{4bx+4a}}{2048b^4}-\frac{(4x^3b^3-6x^2b^2+6bx-3)e^{2bx+2a}}{64b^4}-\frac{(4x^3b^3+6x^2b^2+6bx+3)e^{-2bx-2a}}{64b^4}+\frac{(32x^3}{}$
derivativedivides	$\frac{-\frac{a^3\sinh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\sinh(}{4}\right.}{}$
default	$\frac{-\frac{a^3\sinh(bx+a)^4}{4}+3a^2\left(\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}\right)-3a\left(\frac{(bx+a)^2\sinh(}{4}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2048*(32*b^3*x^3-24*b^2*x^2+12*b*x-3)/b^4*exp(4*b*x+4*a)-1/64*(4*b^3*x^3-
6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)-1/64*(4*b^3*x^3+6*b^2*x^2+6*b*x+3)/b^
4*exp(-2*b*x-2*a)+1/2048*(32*b^3*x^3+24*b^2*x^2+12*b*x+3)/b^4*exp(-4*b*x-4*
a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.23

$$\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{(8\,b^3x^3+3\,bx)\cosh\left(bx+a\right)^4-3\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\left(8\,b^3x^3+3\,bx\right)\sinh\left(bx+a\right)^4-}{}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a
)*sinh(b*x + a)^3 + (8*b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 16*(2*b^3*x^3 + 3
*b*x)*cosh(b*x + a)^2 - 2*(16*b^3*x^3 - 3*(8*b^3*x^3 + 3*b*x)*cosh(b*x + a)
^2 + 24*b*x)*sinh(b*x + a)^2 - 3*((8*b^2*x^2 + 1)*cosh(b*x + a)^3 - 16*(2*b
^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.46

$$\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{5x^3\sinh^4(a+bx)}{32b} + \frac{3x^3\sinh^2(a+bx)\cosh^2(a+bx)}{16b} - \frac{3x^3\cosh^4(a+bx)}{32b} - \frac{15x^2\sinh^3(a+bx)\cosh(a+bx)}{32b^2} + \frac{9x^2\sinh(a+bx)\cosh^3(a+b}{32b^2} \\ \frac{x^4\sinh^3(a)\cosh(a)}{4} \end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x**3*sinh(a + b*x)**4/(32*b) + 3*x**3*sinh(a + b*x)**2*cosh(a
+ b*x)**2/(16*b) - 3*x**3*cosh(a + b*x)**4/(32*b) - 15*x**2*sinh(a + b*x)**
3*cosh(a + b*x)/(32*b**2) + 9*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2)
 + 51*x*sinh(a + b*x)**4/(256*b**3) + 9*x*sinh(a + b*x)**2*cosh(a + b*x)**2
/(128*b**3) - 45*x*cosh(a + b*x)**4/(256*b**3) - 51*sinh(a + b*x)**3*cosh(a
 + b*x)/(256*b**4) + 45*sinh(a + b*x)*cosh(a + b*x)**3/(256*b**4), Ne(b, 0)
), (x**4*sinh(a)**3*cosh(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10

$$\int x^3 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\left(32\,b^3x^3e^{(4\,a)} - 24\,b^2x^2e^{(4\,a)} + 12\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4} - \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{64\,b^4} - \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a) - 3*e^(4*a
))*e^(4*b*x)/b^4 - 1/64*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/64*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b
*x - 4*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(32\,b^3x^3-24\,b^2x^2+12\,bx-3)e^{(4\,bx+4\,a)}}{2048\,b^4} - \frac{(4\,b^3x^3-6\,b^2x^2+6\,bx-3)e^{(2\,bx+2\,a)}}{64\,b^4} - \frac{(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{64\,b^4} + \frac{(32\,b^3x^3+24\,b^2x^2+12\,bx+3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^4 - 1/64*(4
*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/64*(4*b^3*x^3 + 6
*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/2048*(32*b^3*x^3 + 24*b^2*x^
2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int x^3\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\frac{3\,x^2\sinh(2\,a+2\,b\,x)}{16}-\frac{3\,x^2\sinh(4\,a+4\,b\,x)}{128}}{b^2} - \frac{\frac{x^3\cosh(2\,a+2\,b\,x)}{8}-\frac{x^3\cosh(4\,a+4\,b\,x)}{32}}{b} - \frac{\frac{3\,x\cosh(2\,a+2\,b\,x)}{16}-\frac{3\,x\cosh(4\,a+4\,b\,x)}{256}}{b^3} + \frac{3\sinh(2\,a+2\,b\,x)}{32\,b^4} - \frac{3\sinh(4\,a+4\,b\,x)}{1024\,b^4}$$

```
[In] int(x^3*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] ((3*x^2*sinh(2*a + 2*b*x))/16 - (3*x^2*sinh(4*a + 4*b*x))/128)/b^2 - ((x^3*
cosh(2*a + 2*b*x))/8 - (x^3*cosh(4*a + 4*b*x))/32)/b - ((3*x*cosh(2*a + 2*b
*x))/16 - (3*x*cosh(4*a + 4*b*x))/256)/b^3 + (3*sinh(2*a + 2*b*x))/(32*b^4)
 - (3*sinh(4*a + 4*b*x))/(1024*b^4)
```

3.309 $\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{3x^2}{32b}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2}+\frac{\sinh^4(a+bx)}{32b^3}+\frac{x^2\sinh^4(a+bx)}{4b}$$

```
[Out] -3/32*x^2/b+3/16*x*cosh(b*x+a)*sinh(b*x+a)/b^2-3/32*sinh(b*x+a)^2/b^3-1/8*x
*cosh(b*x+a)*sinh(b*x+a)^3/b^2+1/32*sinh(b*x+a)^4/b^3+1/4*x^2*sinh(b*x+a)^4
/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5480, 3391, 30}

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh^4(a+bx)}{32b^3}-\frac{3\sinh^2(a+bx)}{32b^3}-\frac{x\sinh^3(a+bx)\cosh(a+bx)}{8b^2}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{16b^2}+\frac{x^2\sinh^4(a+bx)}{4b}-\frac{3x^2}{32b}$$

```
[In] Int[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(32*b) + (3*x*Cosh[a + b*x]*Sinh[a + b*x])/(16*b^2) - (3*Sinh[a +
b*x]^2)/(32*b^3) - (x*Cosh[a + b*x]*Sinh[a + b*x]^3)/(8*b^2) + Sinh[a + b*x
]^4/(32*b^3) + (x^2*Sinh[a + b*x]^4)/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sinh^4(a+bx)}{4b} - \frac{\int x\sinh^4(a+bx)\,dx}{2b} \\
&= -\frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2} + \frac{\sinh^4(a+bx)}{32b^3} + \frac{x^2\sinh^4(a+bx)}{4b} + \frac{3\int x\sinh^2(a+bx)\,dx}{8b} \\
&= \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\sinh^2(a+bx)}{32b^3} - \frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2} \\
&\quad + \frac{\sinh^4(a+bx)}{32b^3} + \frac{x^2\sinh^4(a+bx)}{4b} - \frac{3\int x\,dx}{16b} \\
&= -\frac{3x^2}{32b} + \frac{3x\cosh(a+bx)\sinh(a+bx)}{16b^2} - \frac{3\sinh^2(a+bx)}{32b^3} \\
&\quad - \frac{x\cosh(a+bx)\sinh^3(a+bx)}{8b^2} + \frac{\sinh^4(a+bx)}{32b^3} + \frac{x^2\sinh^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{-16(1+2b^2x^2)\cosh(2(a+bx)) + (1+8b^2x^2)\cosh(4(a+bx)) + 4bx(8\sinh(2(a+bx)) - \sinh(4(a+bx))}{256b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
[Out] (-16*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 8*b^2*x^2)*Cosh[4*(a + b*x)]
+ 4*b*x*(8*Sinh[2*(a + b*x)] - Sinh[4*(a + b*x)]))/(256*b^3)
```

Maple [A] (verified)

Time = 10.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(8x^2b^2-4bx+1)e^{4bx+4a}}{512b^3} - \frac{(2x^2b^2-2bx+1)e^{2bx+2a}}{32b^3} - \frac{(2x^2b^2+2bx+1)e^{-2bx-2a}}{32b^3} + \frac{(8x^2b^2+4bx+1)e^{-4bx-4a}}{512b^3}$
derivativedivides	$\frac{\frac{a^2\sinh(bx+a)^4}{4} - 2a\left(\frac{(bx+a)\sinh(bx+a)^4}{4} - \frac{\cosh(bx+a)\sinh(bx+a)^3}{16} + \frac{3\cosh(bx+a)\sinh(bx+a)}{32} - \frac{3bx}{32} - \frac{3a}{32}\right) + \frac{(bx+a)^2\sinh(bx+a}{4}}{b^3}$
default	$\frac{\frac{a^2\sinh(bx+a)^4}{4} - 2a\left(\frac{(bx+a)\sinh(bx+a)^4}{4} - \frac{\cosh(bx+a)\sinh(bx+a)^3}{16} + \frac{3\cosh(bx+a)\sinh(bx+a)}{32} - \frac{3bx}{32} - \frac{3a}{32}\right) + \frac{(bx+a)^2\sinh(bx+a}{4}}{b^3}$

```
[In] int(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/512*(8*b^2*x^2-4*b*x+1)/b^3*exp(4*b*x+4*a)-1/32*(2*b^2*x^2-2*b*x+1)/b^3*e
xp(2*b*x+2*a)-1/32*(2*b^2*x^2+2*b*x+1)/b^3*exp(-2*b*x-2*a)+1/512*(8*b^2*x^2
+4*b*x+1)/b^3*exp(-4*b*x-4*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{16\,bx\cosh(bx+a)\sinh(bx+a)^3 - (8\,b^2x^2+1)\cosh(bx+a)^4 - (8\,b^2x^2+1)\sinh(bx+a)^4 + 16\,(2\,b^2x}{}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - (8*b^2*x^2 + 1)*cosh(b*x + a
)^4 - (8*b^2*x^2 + 1)*sinh(b*x + a)^4 + 16*(2*b^2*x^2 + 1)*cosh(b*x + a)^2
+ 2*(16*b^2*x^2 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 +
16*(b*x*cosh(b*x + a)^3 - 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{5x^2\sinh^4(a+bx)}{32b} + \frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{16b} - \frac{3x^2\cosh^4(a+bx)}{32b} - \frac{5x\sinh^3(a+bx)\cosh(a+bx)}{16b^2} + \frac{3x\sinh(a+bx)\cosh^3(a+bx)}{16b^2} + \\ \frac{x^3\sinh^3(a)\cosh(a)}{3} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x**2*sinh(a + b*x)**4/(32*b) + 3*x**2*sinh(a + b*x)**2*cosh(a
+ b*x)**2/(16*b) - 3*x**2*cosh(a + b*x)**4/(32*b) - 5*x*sinh(a + b*x)**3*co
sh(a + b*x)/(16*b**2) + 3*x*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2) + 5*si
nh(a + b*x)**4/(64*b**3) - 3*cosh(a + b*x)**4/(64*b**3), Ne(b, 0)), (x**3*s
inh(a)**3*cosh(a)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int x^2 \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(8\,b^2x^2e^{(4\,a)} - 4\,bxe^{(4\,a)} + e^{(4\,a)})e^{(4\,bx)}}{512\,b^3} - \frac{(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)})e^{(2\,bx)}}{32\,b^3} - \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2 + 4\,bx + 1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3 - 1/32*(2
*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/32*(2*b^2*x^2
 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*
x - 4*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int x^2 \cosh(a+bx) \sinh^3(a+bx)\,dx = \frac{(8\,b^2x^2 - 4\,bx + 1)e^{(4\,bx+4\,a)}}{512\,b^3} - \frac{(2\,b^2x^2 - 2\,bx + 1)e^{(2\,bx+2\,a)}}{32\,b^3} - \frac{(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{32\,b^3} + \frac{(8\,b^2x^2 + 4\,bx + 1)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 - 1/32*(2*b^2*x^2 - 2*b*x
 + 1)*e^(2*b*x + 2*a)/b^3 - 1/32*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b
^3 + 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int x^2 \cosh(a+bx) \sinh^3(a+bx)\,dx = -\frac{\frac{\cosh(2\,a+2\,b\,x)}{16} - \frac{\cosh(4\,a+4\,b\,x)}{256} + b^2\left(\frac{x^2\cosh(2\,a+2\,b\,x)}{8} - \frac{x^2\cosh(4\,a+4\,b\,x)}{32}\right) - b\left(\frac{x\sinh(2\,a+2\,b\,x)}{8} - \frac{x\sinh(4\,a+4\,b\,x)}{64}\right)}{b^3}$$

```
[In] int(x^2*cosh(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(cosh(2*a + 2*b*x)/16 - cosh(4*a + 4*b*x)/256 + b^2*((x^2*cosh(2*a + 2*b*x
))/8 - (x^2*cosh(4*a + 4*b*x))/32) - b*((x*sinh(2*a + 2*b*x))/8 - (x*sinh(4
*a + 4*b*x))/64))/b^3
```

3.310 $\int x \cosh(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int x \cosh(a+bx) \sinh^3(a+bx) \, dx = -\frac{3x}{32b} + \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b}$$

```
[Out] -3/32*x/b+3/32*cosh(b*x+a)*sinh(b*x+a)/b^2-1/16*cosh(b*x+a)*sinh(b*x+a)^3/b
^2+1/4*x*sinh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5480, 2715, 8}

$$\int x \cosh(a+bx) \sinh^3(a+bx) \, dx = -\frac{\sinh^3(a+bx)\cosh(a+bx)}{16b^2} + \frac{3\sinh(a+bx)\cosh(a+bx)}{32b^2} + \frac{x\sinh^4(a+bx)}{4b} - \frac{3x}{32b}$$

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x)/(32*b) + (3*Cosh[a + b*x]*Sinh[a + b*x])/(32*b^2) - (Cosh[a + b*x]*S
inh[a + b*x]^3)/(16*b^2) + (x*Sinh[a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sinh^4(a+bx)}{4b} - \frac{\int \sinh^4(a+bx)\,dx}{4b} \\
&= -\frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b} + \frac{3\int \sinh^2(a+bx)\,dx}{16b} \\
&= \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b} - \frac{3\int 1\,dx}{32b} \\
&= -\frac{3x}{32b} + \frac{3\cosh(a+bx)\sinh(a+bx)}{32b^2} - \frac{\cosh(a+bx)\sinh^3(a+bx)}{16b^2} + \frac{x\sinh^4(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$
\begin{aligned}
&\int x\cosh(a+bx)\sinh^3(a+bx)\,dx \\
&= -\frac{16bx\cosh(2(a+bx)) - 4bx\cosh(4(a+bx)) - 8\sinh(2(a+bx)) + \sinh(4(a+bx))}{128b^2}
\end{aligned}
$$

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] -1/128*(16*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] - 8*Sinh[2*(a +
b*x)] + Sinh[4*(a + b*x)])/b^2
```

Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\sinh(bx+a)^4}{4}}{b^2}$	69
default	$\frac{\frac{(bx+a)\sinh(bx+a)^4}{4}-\frac{\cosh(bx+a)\sinh(bx+a)^3}{16}+\frac{3\cosh(bx+a)\sinh(bx+a)}{32}-\frac{3bx}{32}-\frac{3a}{32}-\frac{a\sinh(bx+a)^4}{4}}{b^2}$	69
risch	$\frac{(4bx-1)e^{4bx+4a}}{256b^2}-\frac{(2bx-1)e^{2bx+2a}}{32b^2}-\frac{(2bx+1)e^{-2bx-2a}}{32b^2}+\frac{(4bx+1)e^{-4bx-4a}}{256b^2}$	82

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/4*(b*x+a)*sinh(b*x+a)^4-1/16*cosh(b*x+a)*sinh(b*x+a)^3+3/32*cosh(b
*x+a)*sinh(b*x+a)-3/32*b*x-3/32*a-1/4*a*sinh(b*x+a)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{bx\cosh(bx+a)^4+bx\sinh(bx+a)^4-4\,bx\cosh(bx+a)^2-\cosh(bx+a)\sinh(bx+a)^3+2\,(3\,bx\cosh(bx}{32\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - 4*b*x*cosh(b*x + a)^2 - c
osh(b*x + a)*sinh(b*x + a)^3 + 2*(3*b*x*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x +
 a)^2 - (cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{5x\sinh^4(a+bx)}{32b}+\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{16b}-\frac{3x\cosh^4(a+bx)}{32b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{32b^2}+\frac{3\sinh(a+bx)\cosh^3(a+bx)}{32b^2} & \text{for } b \\ \frac{x^2\sinh^3(a)\cosh(a)}{2} & \text{othe}\end{cases}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**4/(32*b) + 3*x*sinh(a + b*x)**2*cosh(a + b*x)
**2/(16*b) - 3*x*cosh(a + b*x)**4/(32*b) - 5*sinh(a + b*x)**3*cosh(a + b*x)
/(32*b**2) + 3*sinh(a + b*x)*cosh(a + b*x)**3/(32*b**2), Ne(b, 0)), (x**2*s
inh(a)**3*cosh(a)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 - 1/32*(2*b*x*e^(2*a) - e^(2*
a))*e^(2*b*x)/b^2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x +
1)*e^(-4*b*x - 4*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2}-\frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{32\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{32\,b^2}+\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 - 1/32*(2*b*x - 1)*e^(2*b*x + 2*a)/b^
2 - 1/32*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/256*(4*b*x + 1)*e^(-4*b*x - 4
*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int x\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=-\frac{\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(2\,a+2\,b\,x)}{16}+b\left(\frac{x\cosh(2\,a+2\,b\,x)}{8}-\frac{x\cosh(4\,a+4\,b\,x)}{32}\right)}{b^2}$$

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^3,x)
[Out] -(sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/16 + b*((x*cosh(2*a + 2*b*x))/8
 - (x*cosh(4*a + 4*b*x))/32))/b^2
```

3.311 $\int \cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh^4(a+bx)}{4b}$$

```
[Out] 1/4*sinh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2644, 30}

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh^4(a+bx)}{4b}$$

```
[In] Int[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^3\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh^4(a+bx)}{4b}$$

```
[In] Integrate[Cosh[a + b*x]*Sinh[a + b*x]^3,x]
[Out] Sinh[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^4}{4b}$	14
default	$\frac{\sinh(bx+a)^4}{4b}$	14
risch	$\frac{e^{4bx+4a}}{64b} - \frac{e^{2bx+2a}}{16b} - \frac{e^{-2bx-2a}}{16b} + \frac{e^{-4bx-4a}}{64b}$	58

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*sinh(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.60

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^4 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 - 2\right)\sinh(bx+a)^2 - 4\cosh(bx+a)^2}{32\,b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/32*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 2)*sinh(b*
x + a)^2 - 4*cosh(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^4(a+bx)}{4b} & \text{for } b\neq 0 \\ x\sinh^3(a)\cosh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3,x)
[Out] Piecewise((sinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh(bx+a)^4}{4b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*sinh(b*x + a)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(4bx+4a)}}{64b} - \frac{e^{(2bx+2a)}}{16b} - \frac{e^{(-2bx-2a)}}{16b} + \frac{e^{(-4bx-4a)}}{64b}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] 1/64*e^(4*b*x + 4*a)/b - 1/16*e^(2*b*x + 2*a)/b - 1/16*e^(-2*b*x - 2*a)/b +
 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{\sinh(a+b\,x)^4}{4\,b}$$

```
[In] int(cosh(a + b*x)*sinh(a + b*x)^3,x)
[Out] sinh(a + b*x)^4/(4*b)
```

3.312 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{1}{4}\text{Chi}(2bx)\sinh(2a)+\frac{1}{8}\text{Chi}(4bx)\sinh(4a)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[Out] -1/4*cosh(2*a)*Shi(2*b*x)+1/8*cosh(4*a)*Shi(4*b*x)-1/4*Chi(2*b*x)*sinh(2*a)
+1/8*Chi(4*b*x)*sinh(4*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{1}{4}\sinh(2a)\text{Chi}(2bx)+\frac{1}{8}\sinh(4a)\text{Chi}(4bx)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

```
[Out] -1/4*(CoshIntegral[2*b*x]*Sinh[2*a]) + (CoshIntegral[4*b*x]*Sinh[4*a])/8 -
(Cosh[2*a]*SinhIntegral[2*b*x])/4 + (Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{\sinh(2a+2bx)}{4x}+\frac{\sinh(4a+4bx)}{8x}\right)dx\\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x}\,dx\\
&= -\left(\frac{1}{4}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx\right)+\frac{1}{8}\cosh(4a)\int \frac{\sinh(4bx)}{x}\,dx\\
&\quad -\frac{1}{4}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{8}\sinh(4a)\int \frac{\cosh(4bx)}{x}\,dx\\
&= -\frac{1}{4}\text{Chi}(2bx)\sinh(2a)+\frac{1}{8}\text{Chi}(4bx)\sinh(4a)-\frac{1}{4}\cosh(2a)\text{Shi}(2bx)+\frac{1}{8}\cosh(4a)\text{Shi}(4bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{8}(-2\mathrm{Chi}(2bx)\sinh(2a)+\mathrm{Chi}(4bx)\sinh(4a) - 2\cosh(2a)\mathrm{Shi}(2bx)+\cosh(4a)\mathrm{Shi}(4bx))$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-2*CoshIntegral[2*b*x]*Sinh[2*a] + CoshIntegral[4*b*x]*Sinh[4*a] - 2*Cosh[
2*a]*SinhIntegral[2*b*x] + Cosh[4*a]*SinhIntegral[4*b*x])/8
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{e^{-4a}\,\mathrm{Ei}_1(4bx)}{16}-\frac{e^{-2a}\,\mathrm{Ei}_1(2bx)}{8}+\frac{e^{2a}\,\mathrm{Ei}_1(-2bx)}{8}-\frac{e^{4a}\,\mathrm{Ei}_1(-4bx)}{16}$	50

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*exp(-4*a)*Ei(1,4*b*x)-1/8*exp(-2*a)*Ei(1,2*b*x)+1/8*exp(2*a)*Ei(1,-2*b
*x)-1/16*exp(4*a)*Ei(1,-4*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}\left(\mathrm{Ei}(4\,bx)-\mathrm{Ei}(-4\,bx)\right)\cosh(4\,a) - \frac{1}{8}\left(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx)\right)\cosh(2\,a) + \frac{1}{16}\left(\mathrm{Ei}(4\,bx)+\mathrm{Ei}(-4\,bx)\right)\sinh(4\,a) - \frac{1}{8}\left(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx)\right)\sinh(2\,a)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(4*b*x) - Ei(-4*b*x))*cosh(4*a) - 1/8*(Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) + 1/16*(Ei(4*b*x) + Ei(-4*b*x))*sinh(4*a) - 1/8*(Ei(2*b*x) + Ei(-2*b*
x))*sinh(2*a)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}\,\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - \frac{1}{8}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{16}\,\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(4*b*x)*e^(4*a) - 1/8*Ei(2*b*x)*e^(2*a) + 1/8*Ei(-2*b*x)*e^(-2*a) -
1/16*Ei(-4*b*x)*e^(-4*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x,x)
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x, x)
```

3.313 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}-\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)$$

```
[Out] -1/2*b*Chi(2*b*x)*cosh(2*a)+1/2*b*Chi(4*b*x)*cosh(4*a)-1/2*b*Shi(2*b*x)*sin
h(2*a)+1/2*b*Shi(4*b*x)*sinh(4*a)+1/4*sinh(2*b*x+2*a)/x-1/8*sinh(4*b*x+4*a)
/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{1}{2}b\cosh(2a)\text{Chi}(2bx)+\frac{1}{2}b\cosh(4a)\text{Chi}(4bx)-\frac{1}{2}b\sinh(2a)\text{Shi}(2bx)+\frac{1}{2}b\sinh(4a)\text{Shi}(4bx)+\frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -1/2*(b*Cosh[2*a]*CoshIntegral[2*b*x]) + (b*Cosh[4*a]*CoshIntegral[4*b*x])/
2 + Sinh[2*a + 2*b*x]/(4*x) - Sinh[4*a + 4*b*x]/(8*x) - (b*Sinh[2*a]*SinhIn
tegral[2*b*x])/2 + (b*Sinh[4*a]*SinhIntegral[4*b*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\sinh(2a+2bx)}{4x^2}+\frac{\sinh(4a+4bx)}{8x^2}\right) dx \\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^2}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= \frac{\sinh(2a+2bx)}{4x}-\frac{\sinh(4a+4bx)}{8x}-\frac{1}{2}b\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{2}b\int \frac{\cosh(4a+4bx)}{x}\,dx
\end{aligned}$$

$$= \frac{\sinh(2a+2bx)}{4x} - \frac{\sinh(4a+4bx)}{8x} - \frac{1}{2}(b\cosh(2a))\int\frac{\cosh(2bx)}{x}\,dx + \frac{1}{2}(b\cosh(4a))\int\frac{\cosh(4bx)}{x}\,dx - \frac{1}{2}(b\sinh(2a))\int\frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}(b\sinh(4a))\int\frac{\sinh(4bx)}{x}\,dx$$

$$= -\frac{1}{2}b\cosh(2a)\mathrm{Chi}(2bx) + \frac{1}{2}b\cosh(4a)\mathrm{Chi}(4bx) + \frac{\sinh(2a+2bx)}{4x} - \frac{\sinh(4a+4bx)}{8x} - \frac{1}{2}b\sinh(2a)\mathrm{Shi}(2bx) + \frac{1}{2}b\sinh(4a)\mathrm{Shi}(4bx)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int\frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{4bx\cosh(2a)\mathrm{Chi}(2bx) - 4bx\cosh(4a)\mathrm{Chi}(4bx) - 2\sinh(2(a+bx)) + \sinh(4(a+bx)) + 4bx\sinh(2a)\mathrm{Shi}(}{8x}$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -1/8*(4*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 4*b*x*Cosh[4*a]*CoshIntegral[4*
b*x] - 2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 4*b*x*Sinh[2*a]*SinhIntegr
al[2*b*x] - 4*b*x*Sinh[4*a]*SinhIntegral[4*b*x])/x
```

Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{-4\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)bx+4\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)bx-4\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)bx+4\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)bx+\mathrm{e}^{-4bx-4a}-2\,\mathrm{e}^{-2bx-2a}-\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}}{16x}$	105

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-4*exp(-4*a)*Ei(1,4*b*x)*b*x+4*exp(-2*a)*Ei(1,2*b*x)*b*x-4*exp(4*a)*E
i(1,-4*b*x)*b*x+4*exp(2*a)*Ei(1,-2*b*x)*b*x+exp(-4*b*x-4*a)-2*exp(-2*b*x-2*
a)-exp(4*b*x+4*a)+2*exp(2*b*x+2*a))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{2\cosh(bx+a)\sinh(bx+a)^3 - (bx\mathrm{Ei}(4\,bx) + bx\mathrm{Ei}(-4\,bx))\cosh(4\,a) + (bx\mathrm{Ei}(2\,bx) + bx\mathrm{Ei}(-2\,bx))\,\mathrm{co}}{}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cosh(b*x + a)*sinh(b*x + a)^3 - (b*x*Ei(4*b*x) + b*x*Ei(-4*b*x))*co
sh(4*a) + (b*x*Ei(2*b*x) + b*x*Ei(-2*b*x))*cosh(2*a) + 2*(cosh(b*x + a)^3 -
 cosh(b*x + a))*sinh(b*x + a) - (b*x*Ei(4*b*x) - b*x*Ei(-4*b*x))*sinh(4*a)
+ (b*x*Ei(2*b*x) - b*x*Ei(-2*b*x))*sinh(2*a))/x
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\cosh(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{1}{4}\,be^{(-4\,a)}\Gamma(-1,4\,bx) - \frac{1}{4}\,be^{(-2\,a)}\Gamma(-1,2\,bx) - \frac{1}{4}\,be^{(2\,a)}\Gamma(-1,-2\,bx) + \frac{1}{4}\,be^{(4\,a)}\Gamma(-1,-4\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*e^(-4*a)*gamma(-1, 4*b*x) - 1/4*b*e^(-2*a)*gamma(-1, 2*b*x) - 1/4*b*e
^(2*a)*gamma(-1, -2*b*x) + 1/4*b*e^(4*a)*gamma(-1, -4*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx$$
$$= \frac{4\,bx\text{Ei}(4\,bx)\,e^{(4\,a)} - 4\,bx\text{Ei}(2\,bx)\,e^{(2\,a)} - 4\,bx\text{Ei}(-2\,bx)\,e^{(-2\,a)} + 4\,bx\text{Ei}(-4\,bx)\,e^{(-4\,a)} - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)} - 2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)}}{16\,x}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x*Ei(4*b*x)*e^(4*a) - 4*b*x*Ei(2*b*x)*e^(2*a) - 4*b*x*Ei(-2*b*x)*
e^(-2*a) + 4*b*x*Ei(-4*b*x)*e^(-4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a)
- 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^2,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^2, x)
```

3.314 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} - \frac{1}{2}b^2\text{Chi}(2bx)\sinh(2a) + b^2\text{Chi}(4bx)\sinh(4a) + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx)$$

```
[Out] 1/4*b*cosh(2*b*x+2*a)/x-1/4*b*cosh(4*b*x+4*a)/x-1/2*b^2*cosh(2*a)*Shi(2*b*x
)+b^2*cosh(4*a)*Shi(4*b*x)-1/2*b^2*Chi(2*b*x)*sinh(2*a)+b^2*Chi(4*b*x)*sinh
(4*a)+1/8*sinh(2*b*x+2*a)/x^2-1/16*sinh(4*b*x+4*a)/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{1}{2}b^2\sinh(2a)\text{Chi}(2bx) + b^2\sinh(4a)\text{Chi}(4bx) - \frac{1}{2}b^2\cosh(2a)\text{Shi}(2bx) + b^2\cosh(4a)\text{Shi}(4bx) + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} + \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(4*x) - (b*Cosh[4*a + 4*b*x])/(4*x) - (b^2*CoshIntegr
al[2*b*x]*Sinh[2*a])/2 + b^2*CoshIntegral[4*b*x]*Sinh[4*a] + Sinh[2*a + 2*b
*x]/(8*x^2) - Sinh[4*a + 4*b*x]/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b*
x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{\sinh(2a+2bx)}{4x^3}+\frac{\sinh(4a+4bx)}{8x^3}\right)dx\\ &= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^3}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^3}\,dx\end{aligned}$$

$$= \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{4}b\int \frac{\cosh(2a+2bx)}{x^2}\,dx + \frac{1}{4}b\int \frac{\cosh(4a+4bx)}{x^2}\,dx$$

$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{2}b^2\int \frac{\sinh(2a+2bx)}{x}\,dx + b^2\int \frac{\sinh(4a+4bx)}{x}\,dx$$

$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{2}\left(b^2\cosh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx + \left(b^2\cosh(4a)\right)\int \frac{\sinh(4bx)}{x}\,dx - \frac{1}{2}\left(b^2\sinh(2a)\right)\int \frac{\cosh(2bx)}{x}\,dx + \left(b^2\sinh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx$$

$$= \frac{b\cosh(2a+2bx)}{4x} - \frac{b\cosh(4a+4bx)}{4x} - \frac{1}{2}b^2\mathrm{Chi}(2bx)\sinh(2a) + b^2\mathrm{Chi}(4bx)\sinh(4a) + \frac{\sinh(2a+2bx)}{8x^2} - \frac{\sinh(4a+4bx)}{16x^2} - \frac{1}{2}b^2\cosh(2a)\mathrm{Shi}(2bx) + b^2\cosh(4a)\mathrm{Shi}(4bx)$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -b^2\cosh(a)\mathrm{Chi}(2bx)\sinh(a) + b^2\mathrm{Chi}(4bx)\sinh(4a) + \frac{2bx\cosh(2(a+bx)) + \sinh(2(a+bx))}{8x^2} - \frac{4bx\cosh(4(a+bx)) + \sinh(4(a+bx))}{16x^2} - \frac{1}{2}b^2\cosh(2a)\mathrm{Shi}(2bx) + b^2\cosh(4a)\mathrm{Shi}(4bx)$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] -(b^2*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a]) + b^2*CoshIntegral[4*b*x]*Sinh[4
*a] + (2*b*x*Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])/(8*x^2) - (4*b*x*Cosh[4
*(a + b*x)] + Sinh[4*(a + b*x)])/(16*x^2) - (b^2*Cosh[2*a]*SinhIntegral[2*b
*x])/2 + b^2*Cosh[4*a]*SinhIntegral[4*b*x]
```

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

method	result
risch	$\frac{8\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2-16\,\mathrm{e}^{4a}\,\mathrm{Ei}_1(-4bx)x^2b^2+16\,\mathrm{e}^{-4a}\,\mathrm{Ei}_1(4bx)x^2b^2-8\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2+4\,\mathrm{e}^{2bx+2a}bx-4\,\mathrm{e}^{4bx+4a}bx-4\,\mathrm{e}^{-4bx-4a}bx+}{32x^2}$

[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)

[Out] 1/32*(8*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2-16*exp(4*a)*Ei(1,-4*b*x)*x^2*b^2+16*exp(-4*a)*Ei(1,4*b*x)*x^2*b^2-8*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2+4*exp(2*b*x+2*a)*b*x-4*exp(4*b*x+4*a)*b*x-4*exp(-4*b*x-4*a)*b*x+4*exp(-2*b*x-2*a)*b*x+2*exp(2*b*x+2*a)-exp(4*b*x+4*a)+exp(-4*b*x-4*a)-2*exp(-2*b*x-2*a))/x^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(113) = 226.

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.83

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{bx\cosh{(bx+a)}^4+bx\sinh{(bx+a)}^4-bx\cosh{(bx+a)}^2+\cosh{(bx+a)}\sinh{(bx+a)}^3+\left(6\,bx\cosh{(bx}\right.}{}$$

[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="fricas")

[Out] -1/4*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a)^3 + (6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 - 2*(b^2*x^2*Ei(4*b*x) - b^2*x^2*Ei(-4*b*x))*cosh(4*a) + (b^2*x^2*Ei(2*b*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + (cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 2*(b^2*x^2*Ei(4*b*x) + b^2*x^2*Ei(-4*b*x))*sinh(4*a) + (b^2*x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\sinh^3{(a+bx)}\cosh{(a+bx)}}{x^3}\,dx$$

[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**3,x)

[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = b^2e^{(-4\,a)}\Gamma(-2,4\,bx) - \frac{1}{2}\,b^2e^{(-2\,a)}\Gamma(-2,2\,bx) + \frac{1}{2}\,b^2e^{(2\,a)}\Gamma(-2,-2\,bx) - b^2e^{(4\,a)}\Gamma(-2,-4\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] b^2*e^(-4*a)*gamma(-2, 4*b*x) - 1/2*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 1/2*b^2
*e^(2*a)*gamma(-2, -2*b*x) - b^2*e^(4*a)*gamma(-2, -4*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{16\,b^2x^2\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 8\,b^2x^2\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + 8\,b^2x^2\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - 16\,b^2x^2\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 4\,bxe^{(4\,bx+}}{32}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/32*(16*b^2*x^2*Ei(4*b*x)*e^(4*a) - 8*b^2*x^2*Ei(2*b*x)*e^(2*a) + 8*b^2*x^
2*Ei(-2*b*x)*e^(-2*a) - 16*b^2*x^2*Ei(-4*b*x)*e^(-4*a) - 4*b*x*e^(4*b*x + 4
*a) + 4*b*x*e^(2*b*x + 2*a) + 4*b*x*e^(-2*b*x - 2*a) - 4*b*x*e^(-4*b*x - 4*
a) - e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x -
 4*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^3,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^3, x)
```

3.315 $\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 169

$$\begin{aligned}\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4} dx = &\frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2} \\ &- \frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx) + \frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx) \\ &+ \frac{\sinh(2a+2bx)}{12x^3} + \frac{b^2\sinh(2a+2bx)}{6x} \\ &- \frac{\sinh(4a+4bx)}{24x^3} - \frac{b^2\sinh(4a+4bx)}{3x} \\ &- \frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx) + \frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)\end{aligned}$$

```
[Out] -1/3*b^3*Chi(2*b*x)*cosh(2*a)+4/3*b^3*Chi(4*b*x)*cosh(4*a)+1/12*b*cosh(2*b*
x+2*a)/x^2-1/12*b*cosh(4*b*x+4*a)/x^2-1/3*b^3*Shi(2*b*x)*sinh(2*a)+4/3*b^3*
Shi(4*b*x)*sinh(4*a)+1/12*sinh(2*b*x+2*a)/x^3+1/6*b^2*sinh(2*b*x+2*a)/x-1/2
4*sinh(4*b*x+4*a)/x^3-1/3*b^2*sinh(4*b*x+4*a)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used

$= \{5556, 3378, 3384, 3379, 3382\}$

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx) + \frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx) - \frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx) + \frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx) + \frac{b^2\sinh(2a+2bx)}{6x} - \frac{b^2\sinh(4a+4bx)}{3x} + \frac{\sinh(2a+2bx)}{12x^3} - \frac{\sinh(4a+4bx)}{24x^3} + \frac{b\cosh(2a+2bx)}{12x^2} - \frac{b\cosh(4a+4bx)}{12x^2}$$

```
[In] Int[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(12*x^2) - (b*Cosh[4*a + 4*b*x])/(12*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/3 + (4*b^3*Cosh[4*a]*CoshIntegral[4*b*x])/3 + Si
nh[2*a + 2*b*x]/(12*x^3) + (b^2*Sinh[2*a + 2*b*x])/(6*x) - Sinh[4*a + 4*b*x
]/(24*x^3) - (b^2*Sinh[4*a + 4*b*x])/(3*x) - (b^3*Sinh[2*a]*SinhIntegral[2*
b*x])/3 + (4*b^3*Sinh[4*a]*SinhIntegral[4*b*x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\sinh(2a+2bx)}{4x^4}+\frac{\sinh(4a+4bx)}{8x^4}\right)dx\\
&= \frac{1}{8}\int \frac{\sinh(4a+4bx)}{x^4}\,dx-\frac{1}{4}\int \frac{\sinh(2a+2bx)}{x^4}\,dx\\
&= \frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{1}{6}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{6}b\int \frac{\cosh(4a+4bx)}{x^3}\,dx\\
&= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}-\frac{\sinh(4a+4bx)}{24x^3}\\
&\quad -\frac{1}{6}b^2\int \frac{\sinh(2a+2bx)}{x^2}\,dx+\frac{1}{3}b^2\int \frac{\sinh(4a+4bx)}{x^2}\,dx\\
&= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}+\frac{b^2\sinh(2a+2bx)}{6x}\\
&\quad -\frac{\sinh(4a+4bx)}{24x^3}-\frac{b^2\sinh(4a+4bx)}{3x}-\frac{1}{3}b^3\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{3}\left(4b^3\right)\int \frac{\cosh(4a+4bx)}{x}\,dx\\
&= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}+\frac{\sinh(2a+2bx)}{12x^3}\\
&\quad +\frac{b^2\sinh(2a+2bx)}{6x}-\frac{\sinh(4a+4bx)}{24x^3}-\frac{b^2\sinh(4a+4bx)}{3x}\\
&\quad -\frac{1}{3}\left(b^3\cosh(2a)\right)\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{3}\left(4b^3\cosh(4a)\right)\int \frac{\cosh(4bx)}{x}\,dx\\
&\quad -\frac{1}{3}\left(b^3\sinh(2a)\right)\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{3}\left(4b^3\sinh(4a)\right)\int \frac{\sinh(4bx)}{x}\,dx\\
&= \frac{b\cosh(2a+2bx)}{12x^2}-\frac{b\cosh(4a+4bx)}{12x^2}-\frac{1}{3}b^3\cosh(2a)\text{Chi}(2bx)\\
&\quad +\frac{4}{3}b^3\cosh(4a)\text{Chi}(4bx)+\frac{\sinh(2a+2bx)}{12x^3}+\frac{b^2\sinh(2a+2bx)}{6x}-\frac{\sinh(4a+4bx)}{24x^3}\\
&\quad -\frac{b^2\sinh(4a+4bx)}{3x}-\frac{1}{3}b^3\sinh(2a)\text{Shi}(2bx)+\frac{4}{3}b^3\sinh(4a)\text{Shi}(4bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{-2bx\cosh(2(a+bx))+2bx\cosh(4(a+bx))+8b^3x^3\cosh(2a)\mathrm{Chi}(2bx)-32b^3x^3\cosh(4a)\mathrm{Chi}(4bx)-2}{}$$

```
[In] Integrate[(Cosh[a + b*x]*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] -1/24*(-2*b*x*Cosh[2*(a + b*x)] + 2*b*x*Cosh[4*(a + b*x)] + 8*b^3*x^3*Cosh[
2*a]*CoshIntegral[2*b*x] - 32*b^3*x^3*Cosh[4*a]*CoshIntegral[4*b*x] - 2*Sin
h[2*(a + b*x)] - 4*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^
2*Sinh[4*(a + b*x)] + 8*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 32*b^3*x^3*
Sinh[4*a]*SinhIntegral[4*b*x])/x^3
```

Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

method	result
risch	$\frac{-32\,e^{-4a}\,\mathrm{Ei}_1(4bx)x^3b^3+8\,e^{-2a}\,\mathrm{Ei}_1(2bx)x^3b^3+8\,e^{2a}\,\mathrm{Ei}_1(-2bx)x^3b^3-32\,e^{4a}\,\mathrm{Ei}_1(-4bx)x^3b^3+8\,e^{-4bx-4a}b^2x^2-4\,e^{-2bx-2a}b^2x^2+4\,e^{2}}{48x}$

```
[In] int(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(-32*exp(-4*a)*Ei(1,4*b*x)*x^3*b^3+8*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3+8*e
xp(2*a)*Ei(1,-2*b*x)*x^3*b^3-32*exp(4*a)*Ei(1,-4*b*x)*x^3*b^3+8*exp(-4*b*x-
4*a)*b^2*x^2-4*exp(-2*b*x-2*a)*b^2*x^2+4*exp(2*b*x+2*a)*b^2*x^2-8*exp(4*b*x
+4*a)*b^2*x^2-2*exp(-4*b*x-4*a)*b*x+2*exp(-2*b*x-2*a)*b*x+2*exp(2*b*x+2*a)*
b*x-2*exp(4*b*x+4*a)*b*x+exp(-4*b*x-4*a)-2*exp(-2*b*x-2*a)+2*exp(2*b*x+2*a)
-exp(4*b*x+4*a))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{bx\cosh(bx+a)^4+bx\sinh(bx+a)^4+2\left(8b^2x^2+1\right)\cosh(bx+a)\sinh(bx+a)^3-bx\cosh(bx+a)^2+}{}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x*cosh(b*x + a)^4 + b*x*sinh(b*x + a)^4 + 2*(8*b^2*x^2 + 1)*cosh(b
*x + a)*sinh(b*x + a)^3 - b*x*cosh(b*x + a)^2 + (6*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 - 8*(b^3*x^3*Ei(4*b*x) + b^3*x^3*Ei(-4*b*x))*cosh(4*a) +
 2*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*cosh(2*a) + 2*((8*b^2*x^2 + 1)*
cosh(b*x + a)^3 - (2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) - 8*(b^3*x^3
*Ei(4*b*x) - b^3*x^3*Ei(-4*b*x))*sinh(4*a) + 2*(b^3*x^3*Ei(2*b*x) - b^3*x^3
*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\sinh^3{(a+bx)}\cosh{(a+bx)}}{x^4}\,dx$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = 4\,b^3e^{(-4\,a)}\Gamma(-3,4\,bx) - b^3e^{(-2\,a)}\Gamma(-3,2\,bx) - b^3e^{(2\,a)}\Gamma(-3,-2\,bx) + 4\,b^3e^{(4\,a)}\Gamma(-3,-4\,bx)$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 4*b^3*e^(-4*a)*gamma(-3, 4*b*x) - b^3*e^(-2*a)*gamma(-3, 2*b*x) - b^3*e^(2*
a)*gamma(-3, -2*b*x) + 4*b^3*e^(4*a)*gamma(-3, -4*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{32\,b^3x^3\mathrm{Ei}(4\,bx)\,e^{(4\,a)} - 8\,b^3x^3\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 8\,b^3x^3\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 32\,b^3x^3\mathrm{Ei}(-4\,bx)\,e^{(-4\,a)} - 8\,b^2x^2e^{(4\,bx}}{}$$

```
[In] integrate(cosh(b*x+a)*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(32*b^3*x^3*Ei(4*b*x)*e^(4*a) - 8*b^3*x^3*Ei(2*b*x)*e^(2*a) - 8*b^3*x^
3*Ei(-2*b*x)*e^(-2*a) + 32*b^3*x^3*Ei(-4*b*x)*e^(-4*a) - 8*b^2*x^2*e^(4*b*x
 + 4*a) + 4*b^2*x^2*e^(2*b*x + 2*a) - 4*b^2*x^2*e^(-2*b*x - 2*a) + 8*b^2*x^
2*e^(-4*b*x - 4*a) - 2*b*x*e^(4*b*x + 4*a) + 2*b*x*e^(2*b*x + 2*a) + 2*b*x*
e^(-2*b*x - 2*a) - 2*b*x*e^(-4*b*x - 4*a) - e^(4*b*x + 4*a) + 2*e^(2*b*x +
2*a) - 2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^4,x)
```

```
[Out] int((cosh(a + b*x)*sinh(a + b*x)^3)/x^4, x)
```

3.316 $\int x^m \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\begin{aligned}\int x^m \cosh^2(a+bx) \sinh^3(a+bx) \, dx = &\frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b} \\ &- \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b} \\ &- \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{16b} \\ &- \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{16b} \\ &- \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{32b} \\ &+ \frac{5^{-1-m}e^{-5a}x^m(bx)^{-m}\Gamma(1+m,5bx)}{32b}\end{aligned}$$

```
[Out] 1/32*5^(-1-m)*exp(5*a)*x^m*GAMMA(1+m,-5*b*x)/b/((-b*x)^m)-1/32*3^(-1-m)*exp
(3*a)*x^m*GAMMA(1+m,-3*b*x)/b/((-b*x)^m)-1/16*exp(a)*x^m*GAMMA(1+m,-b*x)/b/
((-b*x)^m)-1/16*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-1/32*3^(-1-m)*x^m*GAM
MA(1+m,3*b*x)/b/exp(3*a)/((b*x)^m)+1/32*5^(-1-m)*x^m*GAMMA(1+m,5*b*x)/b/exp
(5*a)/((b*x)^m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3389, 2212}

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{5a}5^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-5bx)}{32b} - \frac{e^{3a}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-3bx)}{32b} - \frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{16b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{16b} - \frac{e^{-3a}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,3bx)}{32b} + \frac{e^{-5a}5^{-m-1}x^m(bx)^{-m}\Gamma(m+1,5bx)}{32b}$$

```
[In] Int[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (5^(-1 - m)*E^(5*a)*x^m*Gamma[1 + m, -5*b*x])/(32*b*(-(b*x))^m) - (3^(-1 -
m)*E^(3*a)*x^m*Gamma[1 + m, -3*b*x])/(32*b*(-(b*x))^m) - (E^a*x^m*Gamma[1 +
 m, -(b*x)])/(16*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(16*b*E^a*(b*x)^m)
 - (3^(-1 - m)*x^m*Gamma[1 + m, 3*b*x])/(32*b*E^(3*a)*(b*x)^m) + (5^(-1 - m
)*x^m*Gamma[1 + m, 5*b*x])/(32*b*E^(5*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x^m \sinh(a+bx) - \frac{1}{16}x^m \sinh(3a+3bx) + \frac{1}{16}x^m \sinh(5a+5bx)\right) dx \\
&= -\left(\frac{1}{16}\int x^m \sinh(3a+3bx)\,dx\right) + \frac{1}{16}\int x^m \sinh(5a+5bx)\,dx - \frac{1}{8}\int x^m \sinh(a+bx)\,dx \\
&= -\left(\frac{1}{32}\int e^{-i(3ia+3ibx)}x^m\,dx\right) + \frac{1}{32}\int e^{i(3ia+3ibx)}x^m\,dx + \frac{1}{32}\int e^{-i(5ia+5ibx)}x^m\,dx \\
&\quad - \frac{1}{32}\int e^{i(5ia+5ibx)}x^m\,dx - \frac{1}{16}\int e^{-i(ia+ibx)}x^m\,dx + \frac{1}{16}\int e^{i(ia+ibx)}x^m\,dx \\
&= \frac{5^{-1-m}e^{5a}x^m(-bx)^{-m}\Gamma(1+m,-5bx)}{32b} - \frac{3^{-1-m}e^{3a}x^m(-bx)^{-m}\Gamma(1+m,-3bx)}{32b} \\
&\quad - \frac{e^{a}x^m(-bx)^{-m}\Gamma(1+m,-bx)}{16b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{16b} \\
&\quad - \frac{3^{-1-m}e^{-3a}x^m(bx)^{-m}\Gamma(1+m,3bx)}{32b} + \frac{5^{-1-m}e^{-5a}x^m(bx)^{-m}\Gamma(1+m,5bx)}{32b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.83

$$
\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx
= \frac{e^{-5a}x^m\left(-30e^{4a}(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)+(bx)^{-m}\Gamma(1+m,bx)) - 5\ 3^{-m}e^{2a}(-b^2x^2)^{-m}\left(e^{6a}(bx)^m\Gamma(1+\right.\right.}{4}
$$

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (x^m*(-30*E^(4*a)*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m,
 b*x]/(b*x)^m) - (5*E^(2*a)*(E^(6*a)*(b*x)^m*Gamma[1 + m, -3*b*x] + (-(b*x)
)^m*Gamma[1 + m, 3*b*x]))/(3^m*(-(b^2*x^2))^m) + (3*(E^(10*a)*(b*x)^m*Gamma
[1 + m, -5*b*x] + (-(b*x))^m*Gamma[1 + m, 5*b*x]))/(5^m*(-(b^2*x^2))^m)))/(
480*b*E^(5*a))
```

Maple [F]

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^3\,dx$$

[In] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)

[Out] int(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{3\cosh(m\log(5b)+5a)\Gamma(m+1,5bx) - 5\cosh(m\log(3b)+3a)\Gamma(m+1,3bx) - 30\cosh(m\log(b)+$$

[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/480*(3*cosh(m*log(5*b) + 5*a)*gamma(m + 1, 5*b*x) - 5*cosh(m*log(3*b) + 3*a)*gamma(m + 1, 3*b*x) - 30*cosh(m*log(b) + a)*gamma(m + 1, b*x) - 30*cosh(m*log(-b) - a)*gamma(m + 1, -b*x) - 5*cosh(m*log(-3*b) - 3*a)*gamma(m + 1, -3*b*x) + 3*cosh(m*log(-5*b) - 5*a)*gamma(m + 1, -5*b*x) - 3*gamma(m + 1, 5*b*x)*sinh(m*log(5*b) + 5*a) + 5*gamma(m + 1, 3*b*x)*sinh(m*log(3*b) + 3*a) + 30*gamma(m + 1, -b*x)*sinh(m*log(-b) - a) + 5*gamma(m + 1, -3*b*x)*sinh(m*log(-3*b) - 3*a) - 3*gamma(m + 1, -5*b*x)*sinh(m*log(-5*b) - 5*a) + 30*gamma(m + 1, b*x)*sinh(m*log(b) + a))/b

Sympy [F]

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int x^m \sinh^3(a+bx)\cosh^2(a+bx)\,dx$$

[In] integrate(x**m*cosh(b*x+a)**2*sinh(b*x+a)**3,x)

[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{1}{32}\,(5\,bx)^{-m-1}\,x^{m+1}e^{(-5\,a)}\Gamma(m+1,5\,bx) - \frac{1}{32}\,(3\,bx)^{-m-1}\,x^{m+1}e^{(-3\,a)}\Gamma(m+1,3\,bx) - \frac{1}{16}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx) + \frac{1}{16}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx) + \frac{1}{32}\,(-3\,bx)^{-m-1}\,x^{m+1}e^{(3\,a)}\Gamma(m+1,-3\,bx) - \frac{1}{32}\,(-5\,bx)^{-m-1}\,x^{m+1}e^{(5\,a)}\Gamma(m+1,-5\,bx)$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/32*(5*b*x)^(-m - 1)*x^(m + 1)*e^(-5*a)*gamma(m + 1, 5*b*x) - 1/32*(3*b*x)
^(-m - 1)*x^(m + 1)*e^(-3*a)*gamma(m + 1, 3*b*x) - 1/16*(b*x)^(-m - 1)*x^(m
 + 1)*e^(-a)*gamma(m + 1, b*x) + 1/16*(-b*x)^(-m - 1)*x^(m + 1)*e^a*gamma(m
 + 1, -b*x) + 1/32*(-3*b*x)^(-m - 1)*x^(m + 1)*e^(3*a)*gamma(m + 1, -3*b*x)
 - 1/32*(-5*b*x)^(-m - 1)*x^(m + 1)*e^(5*a)*gamma(m + 1, -5*b*x)
```

Giac [F]

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \sinh\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^2*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \int x^m\,\mathrm{cosh}(a+b\,x)^2\,\mathrm{sinh}(a+b\,x)^3\,dx$$

```
[In] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
[Out] int(x^m*cosh(a + b*x)^2*sinh(a + b*x)^3, x)
```

3.317 $\int x^3 \cosh^2(a+bx) \sinh^3(a+bx)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 202

$$\int x^3 \cosh^2(a+bx) \sinh^3(a+bx)\, dx = -\frac{3x\cosh(a+bx)}{4b^3} - \frac{x^3\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{72b^3} - \frac{x^3\cosh(3a+3bx)}{48b} + \frac{3x\cosh(5a+5bx)}{1000b^3} + \frac{x^3\cosh(5a+5bx)}{80b} + \frac{3\sinh(a+bx)}{4b^4} + \frac{3x^2\sinh(a+bx)}{8b^2} + \frac{\sinh(3a+3bx)}{216b^4} + \frac{x^2\sinh(3a+3bx)}{48b^2} - \frac{3\sinh(5a+5bx)}{5000b^4} - \frac{3x^2\sinh(5a+5bx)}{400b^2}$$

```
[Out] -3/4*x*cosh(b*x+a)/b^3-1/8*x^3*cosh(b*x+a)/b-1/72*x*cosh(3*b*x+3*a)/b^3-1/4
8*x^3*cosh(3*b*x+3*a)/b+3/1000*x*cosh(5*b*x+5*a)/b^3+1/80*x^3*cosh(5*b*x+5*
a)/b+3/4*sinh(b*x+a)/b^4+3/8*x^2*sinh(b*x+a)/b^2+1/216*sinh(3*b*x+3*a)/b^4+
1/48*x^2*sinh(3*b*x+3*a)/b^2-3/5000*sinh(5*b*x+5*a)/b^4-3/400*x^2*sinh(5*b*
x+5*a)/b^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3377, 2717}

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{3\sinh(a+bx)}{4b^4}+\frac{\sinh(3a+3bx)}{216b^4} -\frac{3\sinh(5a+5bx)}{5000b^4}-\frac{3x\cosh(a+bx)}{4b^3} -\frac{x\cosh(3a+3bx)}{72b^3}+\frac{3x\cosh(5a+5bx)}{1000b^3} +\frac{3x^2\sinh(a+bx)}{8b^2}+\frac{x^2\sinh(3a+3bx)}{48b^2} -\frac{3x^2\sinh(5a+5bx)}{400b^2}-\frac{x^3\cosh(a+bx)}{8b} -\frac{x^3\cosh(3a+3bx)}{48b}+\frac{x^3\cosh(5a+5bx)}{80b}$$

```
[In] Int[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x*Cosh[a + b*x])/(4*b^3) - (x^3*Cosh[a + b*x])/(8*b) - (x*Cosh[3*a + 3*
b*x])/(72*b^3) - (x^3*Cosh[3*a + 3*b*x])/(48*b) + (3*x*Cosh[5*a + 5*b*x])/(
1000*b^3) + (x^3*Cosh[5*a + 5*b*x])/(80*b) + (3*Sinh[a + b*x])/(4*b^4) + (3
*x^2*Sinh[a + b*x])/(8*b^2) + Sinh[3*a + 3*b*x]/(216*b^4) + (x^2*Sinh[3*a +
 3*b*x])/(48*b^2) - (3*Sinh[5*a + 5*b*x])/(5000*b^4) - (3*x^2*Sinh[5*a + 5*
b*x])/(400*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{1}{8}x^3\sinh(a+bx) - \frac{1}{16}x^3\sinh(3a+3bx) + \frac{1}{16}x^3\sinh(5a+5bx)\right)dx$$

$$= -\left(\frac{1}{16}\int x^3\sinh(3a+3bx)\,dx\right) + \frac{1}{16}\int x^3\sinh(5a+5bx)\,dx - \frac{1}{8}\int x^3\sinh(a+bx)\,dx$$

$$= -\frac{x^3\cosh(a+bx)}{8b} - \frac{x^3\cosh(3a+3bx)}{48b} + \frac{x^3\cosh(5a+5bx)}{80b} - \frac{3\int x^2\cosh(5a+5bx)\,dx}{80b} + \frac{\int x^2\cosh(3a+3bx)\,dx}{16b} + \frac{3\int x^2\cosh(a+bx)\,dx}{8b}$$

$$= -\frac{x^3\cosh(a+bx)}{8b} - \frac{x^3\cosh(3a+3bx)}{48b} + \frac{x^3\cosh(5a+5bx)}{80b} + \frac{3x^2\sinh(a+bx)}{8b^2} + \frac{x^2\sinh(3a+3bx)}{48b^2} - \frac{3x^2\sinh(5a+5bx)}{400b^2} + \frac{3\int x\sinh(5a+5bx)\,dx}{200b^2} - \frac{\int x\sinh(3a+3bx)\,dx}{24b^2} - \frac{3\int x\sinh(a+bx)\,dx}{4b^2}$$

$$= -\frac{3x\cosh(a+bx)}{4b^3} - \frac{x^3\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{72b^3} - \frac{x^3\cosh(3a+3bx)}{48b} + \frac{3x\cosh(5a+5bx)}{1000b^3} + \frac{x^3\cosh(5a+5bx)}{80b} + \frac{3x^2\sinh(a+bx)}{8b^2} + \frac{x^2\sinh(3a+3bx)}{48b^2} - \frac{3x^2\sinh(5a+5bx)}{400b^2} - \frac{3\int\cosh(5a+5bx)\,dx}{1000b^3} + \frac{\int\cosh(3a+3bx)\,dx}{72b^3} + \frac{3\int\cosh(a+bx)\,dx}{4b^3}$$

$$= -\frac{3x\cosh(a+bx)}{4b^3} - \frac{x^3\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{72b^3} - \frac{x^3\cosh(3a+3bx)}{48b} + \frac{3x\cosh(5a+5bx)}{1000b^3} + \frac{x^3\cosh(5a+5bx)}{80b} + \frac{3\sinh(a+bx)}{4b^4} + \frac{3x^2\sinh(a+bx)}{8b^2} + \frac{\sinh(3a+3bx)}{216b^4} + \frac{x^2\sinh(3a+3bx)}{48b^2} - \frac{3\sinh(5a+5bx)}{5000b^4} - \frac{3x^2\sinh(5a+5bx)}{400b^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{-33750(bx(6+b^2x^2)\cosh(a+bx) - 3(2+b^2x^2)\sinh(a+bx)) - 625((6bx+9b^3x^3)\cosh(3(a+bx)) - (2\ldots}{270000b\ldots}$$

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-33750*(b*x*(6 + b^2*x^2)*Cosh[a + b*x] - 3*(2 + b^2*x^2)*Sinh[a + b*x]) -
 625*((6*b*x + 9*b^3*x^3)*Cosh[3*(a + b*x)] - (2 + 9*b^2*x^2)*Sinh[3*(a + b
*x)]) + 27*(5*b*x*(6 + 25*b^2*x^2)*Cosh[5*(a + b*x)] - 3*(2 + 25*b^2*x^2)*S
inh[5*(a + b*x)]))/(270000*b^4)
```

Maple [A] (verified)

Time = 39.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

method	result
risch	$\frac{(125x^3b^3-75x^2b^2+30bx-6)\,\mathrm{e}^{5bx+5a}}{20000b^4}-\frac{(9x^3b^3-9x^2b^2+6bx-2)\,\mathrm{e}^{3bx+3a}}{864b^4}-\frac{(x^3b^3-3x^2b^2+6bx-6)\,\mathrm{e}^{bx+a}}{16b^4}-\frac{(x^3b^3+3x$
derivativedivides	$\frac{-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh}{25}\right.}{}$
default	$\frac{-a^3\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh}{25}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20000*(125*b^3*x^3-75*b^2*x^2+30*b*x-6)/b^4*exp(5*b*x+5*a)-1/864*(9*b^3*x
^3-9*b^2*x^2+6*b*x-2)/b^4*exp(3*b*x+3*a)-1/16*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b
^4*exp(b*x+a)-1/16*(b^3*x^3+3*b^2*x^2+6*b*x+6)/b^4*exp(-b*x-a)-1/864*(9*b^3
*x^3+9*b^2*x^2+6*b*x+2)/b^4*exp(-3*b*x-3*a)+1/20000*(125*b^3*x^3+75*b^2*x^2
+30*b*x+6)/b^4*exp(-5*b*x-5*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.36

$$\int x^3\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{135\,(25\,b^3x^3+6\,bx)\cosh{(bx+a)}^5+675\,(25\,b^3x^3+6\,bx)\cosh{(bx+a)}\sinh{(bx+a)}^4-81\,(25\,b^2x^2+2)\sin}{}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270000*(135*(25*b^3*x^3 + 6*b*x)*cosh(b*x + a)^5 + 675*(25*b^3*x^3 + 6*b*
x)*cosh(b*x + a)*sinh(b*x + a)^4 - 81*(25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 18
75*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a)^3 + 5*(1125*b^2*x^2 - 162*(25*b^2*x^2
+ 2)*cosh(b*x + a)^2 + 250)*sinh(b*x + a)^3 + 225*(6*(25*b^3*x^3 + 6*b*x)*c
osh(b*x + a)^3 - 25*(3*b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x + a)^2 - 33
750*(b^3*x^3 + 6*b*x)*cosh(b*x + a) - 15*(27*(25*b^2*x^2 + 2)*cosh(b*x + a)
^4 - 6750*b^2*x^2 - 125*(9*b^2*x^2 + 2)*cosh(b*x + a)^2 - 13500)*sinh(b*x +
 a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} \frac{x^3\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2x^3\cosh^5(a+bx)}{15b} + \frac{26x^2\sinh^5(a+bx)}{75b^2} - \frac{13x^2\sinh^3(a+bx)\cosh^2(a+bx)}{15b^2} + \frac{2x^2\sinh(a+bx)\cosh^4(a+ \\ \frac{x^4\sinh^3(a)\cosh^2(a)}{4} \end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**3*cosh(a + b
*x)**5/(15*b) + 26*x**2*sinh(a + b*x)**5/(75*b**2) - 13*x**2*sinh(a + b*x)*
*3*cosh(a + b*x)**2/(15*b**2) + 2*x**2*sinh(a + b*x)*cosh(a + b*x)**4/(5*b*
*2) - 52*x*sinh(a + b*x)**4*cosh(a + b*x)/(75*b**3) + 338*x*sinh(a + b*x)**
2*cosh(a + b*x)**3/(225*b**3) - 856*x*cosh(a + b*x)**5/(1125*b**3) + 12568*
sinh(a + b*x)**5/(16875*b**4) - 5114*sinh(a + b*x)**3*cosh(a + b*x)**2/(337
5*b**4) + 856*sinh(a + b*x)*cosh(a + b*x)**4/(1125*b**4), Ne(b, 0)), (x**4*
sinh(a)**3*cosh(a)**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{(125\,b^3x^3e^{(5\,a)} - 75\,b^2x^2e^{(5\,a)} + 30\,bxe^{(5\,a)} - 6\,e^{(5\,a)})e^{(5\,bx)}}{20000\,b^4} - \frac{(9\,b^3x^3e^{(3\,a)} - 9\,b^2x^2e^{(3\,a)} + 6\,bxe^{(3\,a)} - 2\,e^{(3\,a)})e^{(3\,bx)}}{864\,b^4} - \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)e^{(bx)}}{16\,b^4} - \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3 + 9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^4} + \frac{(125\,b^3x^3 + 75\,b^2x^2 + 30\,bx + 6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/20000*(125*b^3*x^3*e^(5*a) - 75*b^2*x^2*e^(5*a) + 30*b*x*e^(5*a) - 6*e^(5
*a))*e^(5*b*x)/b^4 - 1/864*(9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 1/16*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b
*x*e^a - 6*e^a)*e^(b*x)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*
x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{(125\,b^3x^3-75\,b^2x^2+30\,bx-6)e^{(5\,bx+5\,a)}}{20000\,b^4} - \frac{(9\,b^3x^3-9\,b^2x^2+6\,bx-2)e^{(3\,bx+3\,a)}}{864\,b^4} - \frac{(b^3x^3-3\,b^2x^2+6\,bx-6)e^{(bx+a)}}{16\,b^4} - \frac{(b^3x^3+3\,b^2x^2+6\,bx+6)e^{(-bx-a)}}{16\,b^4} - \frac{(9\,b^3x^3+9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{864\,b^4} + \frac{(125\,b^3x^3+75\,b^2x^2+30\,bx+6)e^{(-5\,bx-5\,a)}}{20000\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/20000*(125*b^3*x^3 - 75*b^2*x^2 + 30*b*x - 6)*e^(5*b*x + 5*a)/b^4 - 1/864
*(9*b^3*x^3 - 9*b^2*x^2 + 6*b*x - 2)*e^(3*b*x + 3*a)/b^4 - 1/16*(b^3*x^3 -
3*b^2*x^2 + 6*b*x - 6)*e^(b*x + a)/b^4 - 1/16*(b^3*x^3 + 3*b^2*x^2 + 6*b*x
+ 6)*e^(-b*x - a)/b^4 - 1/864*(9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^4 + 1/20000*(125*b^3*x^3 + 75*b^2*x^2 + 30*b*x + 6)*e^(-5*b*x - 5
*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int x^3 \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{12568\sinh(a+b\,x)}{16875\,b^4} - \frac{\frac{x^3\cosh(a+b\,x)^3}{3} - \frac{x^3\cosh(a+b\,x)^5}{5}}{b} - \frac{-\frac{6\,x\cosh(a+b\,x)^5}{125} + \frac{26\,x\cosh(a+b\,x)^3}{225} + \frac{52\,x\cosh(a+b\,x)}{75}}{b^3} + \frac{\frac{26\,x^2\sinh(a+b\,x)}{75} + \frac{13\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{75} - \frac{3\,x^2\cosh(a+b\,x)^4\sinh(a+b\,x)}{25}}{b^2} + \frac{434\cosh(a+b\,x)^2\sinh(a+b\,x)}{16875\,b^4} - \frac{6\cosh(a+b\,x)^4\sinh(a+b\,x)}{625\,b^4}$$

```
[In] int(x^3*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] (12568*sinh(a + b*x))/(16875*b^4) - ((x^3*cosh(a + b*x)^3)/3 - (x^3*cosh(a
+ b*x)^5)/5)/b - ((52*x*cosh(a + b*x))/75 + (26*x*cosh(a + b*x)^3)/225 - (6
*x*cosh(a + b*x)^5)/125)/b^3 + ((26*x^2*sinh(a + b*x))/75 + (13*x^2*cosh(a
+ b*x)^2*sinh(a + b*x))/75 - (3*x^2*cosh(a + b*x)^4*sinh(a + b*x))/25)/b^2
+ (434*cosh(a + b*x)^2*sinh(a + b*x))/(16875*b^4) - (6*cosh(a + b*x)^4*sinh
(a + b*x))/(625*b^4)
```

3.318 $\int x^2 \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx) \, dx = -\frac{\cosh(a+bx)}{4b^3} - \frac{x^2 \cosh(a+bx)}{8b} - \frac{\cosh(3a+3bx)}{216b^3} - \frac{x^2 \cosh(3a+3bx)}{48b} + \frac{\cosh(5a+5bx)}{1000b^3} + \frac{x^2 \cosh(5a+5bx)}{80b} + \frac{x \sinh(a+bx)}{4b^2} + \frac{x \sinh(3a+3bx)}{72b^2} - \frac{x \sinh(5a+5bx)}{200b^2}$$

```
[Out] -1/4*cosh(b*x+a)/b^3-1/8*x^2*cosh(b*x+a)/b-1/216*cosh(3*b*x+3*a)/b^3-1/48*x
^2*cosh(3*b*x+3*a)/b+1/1000*cosh(5*b*x+5*a)/b^3+1/80*x^2*cosh(5*b*x+5*a)/b+
1/4*x*sinh(b*x+a)/b^2+1/72*x*sinh(3*b*x+3*a)/b^2-1/200*x*sinh(5*b*x+5*a)/b^
2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3377, 2718}

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx) \, dx = -\frac{\cosh(a+bx)}{4b^3} - \frac{\cosh(3a+3bx)}{216b^3} + \frac{\cosh(5a+5bx)}{1000b^3} + \frac{x \sinh(a+bx)}{4b^2} + \frac{x \sinh(3a+3bx)}{72b^2} - \frac{x \sinh(5a+5bx)}{200b^2} - \frac{x^2 \cosh(a+bx)}{8b} - \frac{x^2 \cosh(3a+3bx)}{48b} + \frac{x^2 \cosh(5a+5bx)}{80b}$$

```
[In] Int[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -1/4*Cosh[a + b*x]/b^3 - (x^2*Cosh[a + b*x])/(8*b) - Cosh[3*a + 3*b*x]/(216
*b^3) - (x^2*Cosh[3*a + 3*b*x])/(48*b) + Cosh[5*a + 5*b*x]/(1000*b^3) + (x^
2*Cosh[5*a + 5*b*x])/(80*b) + (x*Sinh[a + b*x])/(4*b^2) + (x*Sinh[3*a + 3*b
*x])/(72*b^2) - (x*Sinh[5*a + 5*b*x])/(200*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x^2\sinh(a+bx) - \frac{1}{16}x^2\sinh(3a+3bx) + \frac{1}{16}x^2\sinh(5a+5bx)\right) dx \\
&= -\left(\frac{1}{16}\int x^2\sinh(3a+3bx)\,dx\right) + \frac{1}{16}\int x^2\sinh(5a+5bx)\,dx - \frac{1}{8}\int x^2\sinh(a+bx)\,dx \\
&= -\frac{x^2\cosh(a+bx)}{8b} - \frac{x^2\cosh(3a+3bx)}{48b} + \frac{x^2\cosh(5a+5bx)}{80b} \\
&\quad - \frac{\int x\cosh(5a+5bx)\,dx}{40b} + \frac{\int x\cosh(3a+3bx)\,dx}{24b} + \frac{\int x\cosh(a+bx)\,dx}{4b} \\
&= -\frac{x^2\cosh(a+bx)}{8b} - \frac{x^2\cosh(3a+3bx)}{48b} + \frac{x^2\cosh(5a+5bx)}{80b} \\
&\quad + \frac{x\sinh(a+bx)}{4b^2} + \frac{x\sinh(3a+3bx)}{72b^2} - \frac{x\sinh(5a+5bx)}{200b^2} \\
&\quad + \frac{\int \sinh(5a+5bx)\,dx}{200b^2} - \frac{\int \sinh(3a+3bx)\,dx}{72b^2} - \frac{\int \sinh(a+bx)\,dx}{4b^2} \\
&= -\frac{\cosh(a+bx)}{4b^3} - \frac{x^2\cosh(a+bx)}{8b} - \frac{\cosh(3a+3bx)}{216b^3} - \frac{x^2\cosh(3a+3bx)}{48b} \\
&\quad + \frac{\cosh(5a+5bx)}{1000b^3} + \frac{x^2\cosh(5a+5bx)}{80b} + \frac{x\sinh(a+bx)}{4b^2} + \frac{x\sinh(3a+3bx)}{72b^2} \\
&\quad - \frac{x\sinh(5a+5bx)}{200b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.66

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\,dx = \frac{-6750(2+b^2x^2)\cosh(a+bx)-125(2+9b^2x^2)\cosh(3(a+bx))+27(2+25b^2x^2)\cosh(5(a+bx))+30bx(}{54000b^3}$$

[In] Integrate[x^2*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]

[Out] (-6750*(2 + b^2*x^2)*Cosh[a + b*x] - 125*(2 + 9*b^2*x^2)*Cosh[3*(a + b*x)] + 27*(2 + 25*b^2*x^2)*Cosh[5*(a + b*x)] + 30*b*x*(450*Sinh[a + b*x] + 25*Sinh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)]))/(54000*b^3)

Maple [A] (verified)

Time = 26.87 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(25x^2b^2-10bx+2)\mathrm{e}^{5bx+5a}}{4000b^3}-\frac{(9x^2b^2-6bx+2)\mathrm{e}^{3bx+3a}}{864b^3}-\frac{(x^2b^2-2bx+2)\mathrm{e}^{bx+a}}{16b^3}-\frac{(x^2b^2+2bx+2)\mathrm{e}^{-bx-a}}{16b^3}-\frac{(9x^2b^2}{}$
derivativedivides	$\frac{a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh(b}{25}\right.}{}$
default	$\frac{a^2\left(\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh(b}{25}\right.}{}$

[In] int(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4000*(25*b^2*x^2-10*b*x+2)/b^3*exp(5*b*x+5*a)-1/864*(9*b^2*x^2-6*b*x+2)/b^3*exp(3*b*x+3*a)-1/16*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)-1/16*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-1/864*(9*b^2*x^2+6*b*x+2)/b^3*exp(-3*b*x-3*a)+1/4000*(25*b^2*x^2+10*b*x+2)/b^3*exp(-5*b*x-5*a)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.45

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\,dx = -\frac{270\,bx\sinh(bx+a)^5-27\,(25\,b^2x^2+2)\cosh(bx+a)^5-135\,(25\,b^2x^2+2)\cosh(bx+a)\sinh(bx+a)^4+}{}$$

[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")

```
[Out] -1/54000*(270*b*x*sinh(b*x + a)^5 - 27*(25*b^2*x^2 + 2)*cosh(b*x + a)^5 - 1
35*(25*b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^4 + 125*(9*b^2*x^2 + 2)*cos
h(b*x + a)^3 + 150*(18*b*x*cosh(b*x + a)^2 - 5*b*x)*sinh(b*x + a)^3 - 15*(1
8*(25*b^2*x^2 + 2)*cosh(b*x + a)^3 - 25*(9*b^2*x^2 + 2)*cosh(b*x + a))*sinh
(b*x + a)^2 + 6750*(b^2*x^2 + 2)*cosh(b*x + a) + 450*(3*b*x*cosh(b*x + a)^4
 - 5*b*x*cosh(b*x + a)^2 - 30*b*x)*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.23

$$\int x^2\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases} \frac{x^2\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2x^2\cosh^5(a+bx)}{15b} + \frac{52x\sinh^5(a+bx)}{225b^2} - \frac{26x\sinh^3(a+bx)\cosh^2(a+bx)}{45b^2} + \frac{4x\sinh(a+bx)\cosh^4(a+bx)}{15b^2} \\ \frac{x^3\sinh^3(a)\cosh^2(a)}{3} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x**2*cosh(a + b
*x)**5/(15*b) + 52*x*sinh(a + b*x)**5/(225*b**2) - 26*x*sinh(a + b*x)**3*co
sh(a + b*x)**2/(45*b**2) + 4*x*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2) - 5
2*sinh(a + b*x)**4*cosh(a + b*x)/(225*b**3) + 338*sinh(a + b*x)**2*cosh(a +
 b*x)**3/(675*b**3) - 856*cosh(a + b*x)**5/(3375*b**3), Ne(b, 0)), (x**3*si
nh(a)**3*cosh(a)**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\begin{aligned}\int x^2\cosh^2(a+bx)\sinh^3(a+bx)\,dx = {} & \frac{(25\,b^2x^2e^{(5\,a)} - 10\,bxe^{(5\,a)} + 2\,e^{(5\,a)})e^{(5\,bx)}}{4000\,b^3} \\ & - \frac{(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)})e^{(3\,bx)}}{864\,b^3} \\ & - \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)e^{(bx)}}{16\,b^3} \\ & - \frac{(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{16\,b^3} \\ & - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{864\,b^3} \\ & + \frac{(25\,b^2x^2 + 10\,bx + 2)e^{(-5\,bx-5\,a)}}{4000\,b^3}\end{aligned}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4000*(25*b^2*x^2*e^(5*a) - 10*b*x*e^(5*a) + 2*e^(5*a))*e^(5*b*x)/b^3 - 1/
864*(9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))*e^(3*b*x)/b^3 - 1/16*(b
^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 1/16*(b^2*x^2 + 2*b*x + 2)*e^
(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^3 + 1/400
0*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\, dx = \frac{(25\, b^2x^2 - 10\, bx + 2)e^{(5\, bx+5\, a)}}{4000\, b^3} - \frac{(9\, b^2x^2 - 6\, bx + 2)e^{(3\, bx+3\, a)}}{864\, b^3} - \frac{(b^2x^2 - 2\, bx + 2)e^{(bx+a)}}{16\, b^3} - \frac{(b^2x^2 + 2\, bx + 2)e^{(-bx-a)}}{16\, b^3} - \frac{(9\, b^2x^2 + 6\, bx + 2)e^{(-3\, bx-3\, a)}}{864\, b^3} + \frac{(25\, b^2x^2 + 10\, bx + 2)e^{(-5\, bx-5\, a)}}{4000\, b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4000*(25*b^2*x^2 - 10*b*x + 2)*e^(5*b*x + 5*a)/b^3 - 1/864*(9*b^2*x^2 - 6
*b*x + 2)*e^(3*b*x + 3*a)/b^3 - 1/16*(b^2*x^2 - 2*b*x + 2)*e^(b*x + a)/b^3
- 1/16*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - 1/864*(9*b^2*x^2 + 6*b*x +
2)*e^(-3*b*x - 3*a)/b^3 + 1/4000*(25*b^2*x^2 + 10*b*x + 2)*e^(-5*b*x - 5*a)
/b^3
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int x^2 \cosh^2(a+bx) \sinh^3(a+bx)\, dx = -\frac{780\cosh(a+b\,x) + 130\cosh(a+b\,x)^3 - 54\cosh(a+b\,x)^5 - 780\,b\,x\sinh(a+b\,x) + 1125\,b^2\,x^2\cosh(a+b}{337}$$

```
[In] int(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
```

```
[Out] -(780*cosh(a + b*x) + 130*cosh(a + b*x)^3 - 54*cosh(a + b*x)^5 - 780*b*x*si
nh(a + b*x) + 1125*b^2*x^2*cosh(a + b*x)^3 - 675*b^2*x^2*cosh(a + b*x)^5 -
390*b*x*cosh(a + b*x)^2*sinh(a + b*x) + 270*b*x*cosh(a + b*x)^4*sinh(a + b*
x))/(3375*b^3)
```

3.319 $\int x \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int x \cosh^2(a+bx) \sinh^3(a+bx) \, dx = -\frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b} + \frac{\sinh(a+bx)}{8b^2} + \frac{\sinh(3a+3bx)}{144b^2} - \frac{\sinh(5a+5bx)}{400b^2}$$

```
[Out] -1/8*x*cosh(b*x+a)/b-1/48*x*cosh(3*b*x+3*a)/b+1/80*x*cosh(5*b*x+5*a)/b+1/8*
sinh(b*x+a)/b^2+1/144*sinh(3*b*x+3*a)/b^2-1/400*sinh(5*b*x+5*a)/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5556, 3377, 2717}

$$\int x \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{\sinh(a+bx)}{8b^2} + \frac{\sinh(3a+3bx)}{144b^2} - \frac{\sinh(5a+5bx)}{400b^2} - \frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b}$$

```
[In] Int[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -1/8*(x*Cosh[a + b*x])/b - (x*Cosh[3*a + 3*b*x])/(48*b) + (x*Cosh[5*a + 5*b
*x])/(80*b) + Sinh[a + b*x]/(8*b^2) + Sinh[3*a + 3*b*x]/(144*b^2) - Sinh[5*
a + 5*b*x]/(400*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}x\sinh(a+bx) - \frac{1}{16}x\sinh(3a+3bx) + \frac{1}{16}x\sinh(5a+5bx)\right)\,dx \\
&= -\left(\frac{1}{16}\int x\sinh(3a+3bx)\,dx\right) + \frac{1}{16}\int x\sinh(5a+5bx)\,dx - \frac{1}{8}\int x\sinh(a+bx)\,dx \\
&= -\frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b} \\
&\quad - \frac{\int\cosh(5a+5bx)\,dx}{80b} + \frac{\int\cosh(3a+3bx)\,dx}{48b} + \frac{\int\cosh(a+bx)\,dx}{8b} \\
&= -\frac{x\cosh(a+bx)}{8b} - \frac{x\cosh(3a+3bx)}{48b} + \frac{x\cosh(5a+5bx)}{80b} \\
&\quad + \frac{\sinh(a+bx)}{8b^2} + \frac{\sinh(3a+3bx)}{144b^2} - \frac{\sinh(5a+5bx)}{400b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{-450bx\cosh(a+bx) - 75bx\cosh(3(a+bx)) + 45bx\cosh(5(a+bx)) + 450\sinh(a+bx) + 25\sinh(3(a+b}{3600b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-450*b*x*Cosh[a + b*x] - 75*b*x*Cosh[3*(a + b*x)] + 45*b*x*Cosh[5*(a + b*x
)] + 450*Sinh[a + b*x] + 25*Sinh[3*(a + b*x)] - 9*Sinh[5*(a + b*x)])/(3600*
b^2)
```

Maple [A] (verified)

Time = 19.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh(bx+a)^4}{25}+\frac{26\sinh(bx+a)}{225}+\frac{13\cosh(bx+a)^2\sinh(bx+a)}{225}-}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{5}-\frac{2(bx+a)\cosh(bx+a)^3}{15}-\frac{\sinh(bx+a)\cosh(bx+a)^4}{25}+\frac{26\sinh(bx+a)}{225}+\frac{13\cosh(bx+a)^2\sinh(bx+a)}{225}-}{b^2}$
risch	$\frac{(5bx-1)e^{5bx+5a}}{800b^2}-\frac{(3bx-1)e^{3bx+3a}}{288b^2}-\frac{(bx-1)e^{bx+a}}{16b^2}-\frac{(bx+1)e^{-bx-a}}{16b^2}-\frac{(3bx+1)e^{-3bx-3a}}{288b^2}+\frac{(5bx+1)e^{-5bx-5}}{800b^2}$

```
[In] int(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/5*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)^3-2/15*(b*x+a)*cosh(b*x+a)^3-1
/25*sinh(b*x+a)*cosh(b*x+a)^4+26/225*sinh(b*x+a)+13/225*cosh(b*x+a)^2*sinh(
b*x+a)-a*(1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{45\,bx\cosh{(bx+a)}^5+225\,bx\cosh{(bx+a)}\sinh{(bx+a)}^4-75\,bx\cosh{(bx+a)}^3-9\,\sinh{(bx+a)}^5-5\,(1}{}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3600*(45*b*x*cosh(b*x + a)^5 + 225*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 75
*b*x*cosh(b*x + a)^3 - 9*sinh(b*x + a)^5 - 5*(18*cosh(b*x + a)^2 - 5)*sinh(
b*x + a)^3 - 450*b*x*cosh(b*x + a) + 225*(2*b*x*cosh(b*x + a)^3 - b*x*cosh(
b*x + a))*sinh(b*x + a)^2 - 15*(3*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 30)
*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{x\sinh^2{(a+bx)}\cosh^3{(a+bx)}}{3b}-\frac{2x\cosh^5{(a+bx)}}{15b}+\frac{26\sinh^5{(a+bx)}}{225b^2}-\frac{13\sinh^3{(a+bx)}\cosh^2{(a+bx)}}{45b^2}+\frac{2\sinh{(a+bx)}\cosh^4{(a+bx)}}{15b^2} & \text{fo}\\ \frac{x^2\sinh^3{(a)}\cosh^2{(a)}}{2} & \text{ot}\end{cases}$$

```
[In] integrate(x*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*x*cosh(a + b*x)**5
/(15*b) + 26*sinh(a + b*x)**5/(225*b**2) - 13*sinh(a + b*x)**3*cosh(a + b*x
)**2/(45*b**2) + 2*sinh(a + b*x)*cosh(a + b*x)**4/(15*b**2), Ne(b, 0)), (x*
*2*sinh(a)**3*cosh(a)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(5\,bxe^{(5\,a)}-e^{(5\,a)}\right)e^{(5\,bx)}}{800\,b^2} - \frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{288\,b^2} - \frac{(bxe^{a}-e^{a})e^{(bx)}}{16\,b^2} - \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} + \frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/800*(5*b*x*e^(5*a) - e^(5*a))*e^(5*b*x)/b^2 - 1/288*(3*b*x*e^(3*a) - e^(3
*a))*e^(3*b*x)/b^2 - 1/16*(b*x*e^a - e^a)*e^(b*x)/b^2 - 1/16*(b*x + 1)*e^(-
b*x - a)/b^2 - 1/288*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e
^(-5*b*x - 5*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{(5\,bx-1)e^{(5\,bx+5\,a)}}{800\,b^2} - \frac{(3\,bx-1)e^{(3\,bx+3\,a)}}{288\,b^2} - \frac{(bx-1)e^{(bx+a)}}{16\,b^2} - \frac{(bx+1)e^{(-bx-a)}}{16\,b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{288\,b^2} + \frac{(5\,bx+1)e^{(-5\,bx-5\,a)}}{800\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/800*(5*b*x - 1)*e^(5*b*x + 5*a)/b^2 - 1/288*(3*b*x - 1)*e^(3*b*x + 3*a)/b
^2 - 1/16*(b*x - 1)*e^(b*x + a)/b^2 - 1/16*(b*x + 1)*e^(-b*x - a)/b^2 - 1/2
88*(3*b*x + 1)*e^(-3*b*x - 3*a)/b^2 + 1/800*(5*b*x + 1)*e^(-5*b*x - 5*a)/b^
2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int x\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{\frac{26\sinh(a+b\,x)}{225}-b\left(\frac{x\cosh(a+b\,x)^3}{3}-\frac{x\cosh(a+b\,x)^5}{5}\right)+\frac{13\cosh(a+b\,x)^2\sinh(a+b\,x)}{225}-\frac{\cosh(a+b\,x)^4\sinh(a+b\,x)}{25}}{b^2}$$

```
[In] int(x*cosh(a + b*x)^2*sinh(a + b*x)^3,x)
[Out] ((26*sinh(a + b*x))/225 - b*((x*cosh(a + b*x)^3)/3 - (x*cosh(a + b*x)^5)/5)
 + (13*cosh(a + b*x)^2*sinh(a + b*x))/225 - (cosh(a + b*x)^4*sinh(a + b*x))
/25)/b^2
```

3.320 $\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx=-\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}$$

```
[Out] -1/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 14}

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx=\frac{\cosh^5(a+bx)}{5b}-\frac{\cosh^3(a+bx)}{3b}$$

```
[In] Int[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -1/3*Cosh[a + b*x]^3/b + Cosh[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int (x^2-x^4)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh^3(a+bx)(-7+3\cosh(2(a+bx)))}{30b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
[Out] (Cosh[a + b*x]^3*(-7 + 3*Cosh[2*(a + b*x)]))/(30*b)
```

Maple [A] (verified)

Time = 12.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{5}-\frac{\cosh(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\cosh(bx+a)^5}{5}-\frac{\cosh(bx+a)^3}{3}}{b}$	26
risch	$\frac{e^{5bx+5a}}{160b}-\frac{e^{3bx+3a}}{96b}-\frac{e^{bx+a}}{16b}-\frac{e^{-bx-a}}{16b}-\frac{e^{-3bx-3a}}{96b}+\frac{e^{-5bx-5a}}{160b}$	83

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*cosh(b*x+a)^5-1/3*cosh(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.55

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{3\cosh\left(bx+a\right)^5 + 15\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4 - 5\cosh\left(bx+a\right)^3 + 15\left(2\cosh\left(bx+a\right)^3 - \cosh\left(bx+a\right.\right.}{240\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 5*cosh(b*x +
a)^3 + 15*(2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 30*cosh(b*x
 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)\cosh^3(a+bx)}{3b} - \frac{2\cosh^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sinh^3(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) - 2*cosh(a + b*x)**5/(15
*b), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(27) = 54$.

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,a)} + 30\,e^{(-4\,bx-4\,a)} - 3\right)e^{(5\,bx+5\,a)}}{480\,b} - \frac{30\,e^{(-bx-a)} + 5\,e^{(-3\,bx-3\,a)} - 3\,e^{(-5\,bx-5\,a)}}{480\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/480*(5*e^(-2*b*x - 2*a) + 30*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b - 1
/480*(30*e^(-b*x - a) + 5*e^(-3*b*x - 3*a) - 3*e^(-5*b*x - 5*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{e^{(3\,bx+3\,a)}}{96\,b} - \frac{e^{(bx+a)}}{16\,b} - \frac{e^{(-bx-a)}}{16\,b} - \frac{e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] 1/160*e^(5*b*x + 5*a)/b - 1/96*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b - 1/1
6*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{5\cosh(a+b\,x)^3 - 3\cosh(a+b\,x)^5}{15\,b}$$

```
[In] int(cosh(a + b*x)^2*sinh(a + b*x)^3,x)
[Out] -(5*cosh(a + b*x)^3 - 3*cosh(a + b*x)^5)/(15*b)
```

3.321 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{1}{8}\text{Chi}(bx)\sinh(a) - \frac{1}{16}\text{Chi}(3bx)\sinh(3a) + \frac{1}{16}\text{Chi}(5bx)\sinh(5a) - \frac{1}{8}\cosh(a)\text{Shi}(bx) - \frac{1}{16}\cosh(3a)\text{Shi}(3bx) + \frac{1}{16}\cosh(5a)\text{Shi}(5bx)$$

```
[Out] -1/8*cosh(a)*Shi(b*x)-1/16*cosh(3*a)*Shi(3*b*x)+1/16*cosh(5*a)*Shi(5*b*x)-1
/8*Chi(b*x)*sinh(a)-1/16*Chi(3*b*x)*sinh(3*a)+1/16*Chi(5*b*x)*sinh(5*a)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{1}{8}\sinh(a)\text{Chi}(bx) - \frac{1}{16}\sinh(3a)\text{Chi}(3bx) + \frac{1}{16}\sinh(5a)\text{Chi}(5bx) - \frac{1}{8}\cosh(a)\text{Shi}(bx) - \frac{1}{16}\cosh(3a)\text{Shi}(3bx) + \frac{1}{16}\cosh(5a)\text{Shi}(5bx)$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]
```

```
[Out] -1/8*(CoshIntegral[b*x]*Sinh[a]) - (CoshIntegral[3*b*x]*Sinh[3*a])/16 + (Co
shIntegral[5*b*x]*Sinh[5*a])/16 - (Cosh[a]*SinhIntegral[b*x])/8 - (Cosh[3*a
]*SinhIntegral[3*b*x])/16 + (Cosh[5*a]*SinhIntegral[5*b*x])/16
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\sinh(a+bx)}{8x} - \frac{\sinh(3a+3bx)}{16x} + \frac{\sinh(5a+5bx)}{16x}\right) dx \\
&= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x}\,dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x}\,dx - \frac{1}{8}\int \frac{\sinh(a+bx)}{x}\,dx \\
&= -\left(\frac{1}{8}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx\right) - \frac{1}{16}\cosh(3a)\int \frac{\sinh(3bx)}{x}\,dx + \frac{1}{16}\cosh(5a)\int \frac{\sinh(5bx)}{x}\,dx \\
&\quad - \frac{1}{8}\sinh(a)\int \frac{\cosh(bx)}{x}\,dx - \frac{1}{16}\sinh(3a)\int \frac{\cosh(3bx)}{x}\,dx + \frac{1}{16}\sinh(5a)\int \frac{\cosh(5bx)}{x}\,dx \\
&= -\frac{1}{8}\text{Chi}(bx)\sinh(a) - \frac{1}{16}\text{Chi}(3bx)\sinh(3a) + \frac{1}{16}\text{Chi}(5bx)\sinh(5a) \\
&\quad - \frac{1}{8}\cosh(a)\text{Shi}(bx) - \frac{1}{16}\cosh(3a)\text{Shi}(3bx) + \frac{1}{16}\cosh(5a)\text{Shi}(5bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{16}(-2\text{Chi}(bx)\sinh(a)-\text{Chi}(3bx)\sinh(3a) + \text{Chi}(5bx)\sinh(5a)-2\cosh(a)\text{Shi}(bx) - \cosh(3a)\text{Shi}(3bx)+\cosh(5a)\text{Shi}(5bx))$$

[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x,x]

[Out] (-2*CoshIntegral[b*x]*Sinh[a] - CoshIntegral[3*b*x]*Sinh[3*a] + CoshIntegra
l[5*b*x]*Sinh[5*a] - 2*Cosh[a]*SinhIntegral[b*x] - Cosh[3*a]*SinhIntegral[3
*b*x] + Cosh[5*a]*SinhIntegral[5*b*x])/16

Maple [A] (verified)

Time = 11.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{e^{-5a}\,\text{Ei}_1(5bx)}{32}-\frac{e^{-3a}\,\text{Ei}_1(3bx)}{32}-\frac{e^{-a}\,\text{Ei}_1(bx)}{16}+\frac{e^{a}\,\text{Ei}_1(-bx)}{16}+\frac{e^{3a}\,\text{Ei}_1(-3bx)}{32}-\frac{e^{5a}\,\text{Ei}_1(-5bx)}{32}$	71

[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x,method=_RETURNVERBOSE)

[Out] 1/32*exp(-5*a)*Ei(1,5*b*x)-1/32*exp(-3*a)*Ei(1,3*b*x)-1/16*exp(-a)*Ei(1,b*x
)+1/16*exp(a)*Ei(1,-b*x)+1/32*exp(3*a)*Ei(1,-3*b*x)-1/32*exp(5*a)*Ei(1,-5*b
*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{32}\left(\text{Ei}(5\,bx)-\text{Ei}(-5\,bx)\right)\cosh\left(5\,a\right) - \frac{1}{32}\left(\text{Ei}(3\,bx)-\text{Ei}(-3\,bx)\right)\cosh\left(3\,a\right) - \frac{1}{16}\left(\text{Ei}(bx)-\text{Ei}(-bx)\right)\cosh\left(a\right) + \frac{1}{32}\left(\text{Ei}(5\,bx)+\text{Ei}(-5\,bx)\right)\sinh\left(5\,a\right) - \frac{1}{32}\left(\text{Ei}(3\,bx)+\text{Ei}(-3\,bx)\right)\sinh\left(3\,a\right) - \frac{1}{16}\left(\text{Ei}(bx)+\text{Ei}(-bx)\right)\sinh\left(a\right)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/32*(Ei(5*b*x) - Ei(-5*b*x))*cosh(5*a) - 1/32*(Ei(3*b*x) - Ei(-3*b*x))*cos
h(3*a) - 1/16*(Ei(b*x) - Ei(-b*x))*cosh(a) + 1/32*(Ei(5*b*x) + Ei(-5*b*x))*
sinh(5*a) - 1/32*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) - 1/16*(Ei(b*x) + Ei(-b
*x))*sinh(a)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3{(a+bx)}\cosh^2{(a+bx)}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = {} & \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} \\ & + \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} \\ & - \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = {} & \frac{1}{32}\,\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - \frac{1}{32}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} \\ & + \frac{1}{16}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{32}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} \\ & - \frac{1}{32}\,\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} - \frac{1}{16}\,\mathrm{Ei}(bx)\,e^{a}\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/32*Ei(5*b*x)*e^(5*a) - 1/32*Ei(3*b*x)*e^(3*a) + 1/16*Ei(-b*x)*e^(-a) + 1/
32*Ei(-3*b*x)*e^(-3*a) - 1/32*Ei(-5*b*x)*e^(-5*a) - 1/16*Ei(b*x)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x, x)
```

3.322 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = &-\frac{1}{8}b\cosh(a)\text{Chi}(bx)-\frac{3}{16}b\cosh(3a)\text{Chi}(3bx)\\&+\frac{5}{16}b\cosh(5a)\text{Chi}(5bx)+\frac{\sinh(a+bx)}{8x}\\&+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}-\frac{1}{8}b\sinh(a)\text{Shi}(bx)\\&-\frac{3}{16}b\sinh(3a)\text{Shi}(3bx)+\frac{5}{16}b\sinh(5a)\text{Shi}(5bx)\end{aligned}$$

```
[Out] -1/8*b*Chi(b*x)*cosh(a)-3/16*b*Chi(3*b*x)*cosh(3*a)+5/16*b*Chi(5*b*x)*cosh(
5*a)-1/8*b*Shi(b*x)*sinh(a)-3/16*b*Shi(3*b*x)*sinh(3*a)+5/16*b*Shi(5*b*x)*s
inh(5*a)+1/8*sinh(b*x+a)/x+1/16*sinh(3*b*x+3*a)/x-1/16*sinh(5*b*x+5*a)/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = &-\frac{1}{8}b\cosh(a)\text{Chi}(bx)-\frac{3}{16}b\cosh(3a)\text{Chi}(3bx)\\&+\frac{5}{16}b\cosh(5a)\text{Chi}(5bx)-\frac{1}{8}b\sinh(a)\text{Shi}(bx)\\&-\frac{3}{16}b\sinh(3a)\text{Shi}(3bx)+\frac{5}{16}b\sinh(5a)\text{Shi}(5bx)\\&+\frac{\sinh(a+bx)}{8x}+\frac{\sinh(3a+3bx)}{16x}-\frac{\sinh(5a+5bx)}{16x}\end{aligned}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -1/8*(b*Cosh[a]*CoshIntegral[b*x]) - (3*b*Cosh[3*a]*CoshIntegral[3*b*x])/16
 + (5*b*Cosh[5*a]*CoshIntegral[5*b*x])/16 + Sinh[a + b*x]/(8*x) + Sinh[3*a
+ 3*b*x]/(16*x) - Sinh[5*a + 5*b*x]/(16*x) - (b*Sinh[a]*SinhIntegral[b*x])/
8 - (3*b*Sinh[3*a]*SinhIntegral[3*b*x])/16 + (5*b*Sinh[5*a]*SinhIntegral[5*
b*x])/16
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{\sinh(a+bx)}{8x^2} - \frac{\sinh(3a+3bx)}{16x^2} + \frac{\sinh(5a+5bx)}{16x^2}\right) dx \\ &= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^2}\, dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^2}\, dx - \frac{1}{8}\int \frac{\sinh(a+bx)}{x^2}\, dx \end{aligned}$$

$$= \frac{\sinh(a+bx)}{8x} + \frac{\sinh(3a+3bx)}{16x} - \frac{\sinh(5a+5bx)}{16x} - \frac{1}{8}b\int \frac{\cosh(a+bx)}{x}\,dx - \frac{1}{16}(3b)\int \frac{\cosh(3a+3bx)}{x}\,dx + \frac{1}{16}(5b)\int \frac{\cosh(5a+5bx)}{x}\,dx$$

$$= \frac{\sinh(a+bx)}{8x} + \frac{\sinh(3a+3bx)}{16x} - \frac{\sinh(5a+5bx)}{16x} - \frac{1}{8}(b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx - \frac{1}{16}(3b\cosh(3a))\int \frac{\cosh(3bx)}{x}\,dx + \frac{1}{16}(5b\cosh(5a))\int \frac{\cosh(5bx)}{x}\,dx - \frac{1}{8}(b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx - \frac{1}{16}(3b\sinh(3a))\int \frac{\sinh(3bx)}{x}\,dx + \frac{1}{16}(5b\sinh(5a))\int \frac{\sinh(5bx)}{x}\,dx$$

$$= -\frac{1}{8}b\cosh(a)\mathrm{Chi}(bx) - \frac{3}{16}b\cosh(3a)\mathrm{Chi}(3bx) + \frac{5}{16}b\cosh(5a)\mathrm{Chi}(5bx) + \frac{\sinh(a+bx)}{8x} + \frac{\sinh(3a+3bx)}{16x} - \frac{\sinh(5a+5bx)}{16x} - \frac{1}{8}b\sinh(a)\mathrm{Shi}(bx) - \frac{3}{16}b\sinh(3a)\mathrm{Shi}(3bx) + \frac{5}{16}b\sinh(5a)\mathrm{Shi}(5bx)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{-2bx\cosh(a)\mathrm{Chi}(bx) - 3bx\cosh(3a)\mathrm{Chi}(3bx) + 5bx\cosh(5a)\mathrm{Chi}(5bx) + 2\sinh(a+bx) + \sinh(3(a+bx)}{16x}$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] (-2*b*x*Cosh[a]*CoshIntegral[b*x] - 3*b*x*Cosh[3*a]*CoshIntegral[3*b*x] + 5
*b*x*Cosh[5*a]*CoshIntegral[5*b*x] + 2*Sinh[a + b*x] + Sinh[3*(a + b*x)] -
Sinh[5*(a + b*x)] - 2*b*x*Sinh[a]*SinhIntegral[b*x] - 3*b*x*Sinh[3*a]*SinhI
ntegral[3*b*x] + 5*b*x*Sinh[5*a]*SinhIntegral[5*b*x])/(16*x)
```

Maple [A] (verified)

Time = 12.74 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{5\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)bx-3\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)bx+5\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)bx-3\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)bx-2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)bx-2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)bx+\mathrm{e}^{5bx+5a}-2\,\mathrm{e}^{bx+}}{32x}$

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(5*exp(5*a)*Ei(1,-5*b*x)*b*x-3*exp(3*a)*Ei(1,-3*b*x)*b*x+5*exp(-5*a)*
Ei(1,5*b*x)*b*x-3*exp(-3*a)*Ei(1,3*b*x)*b*x-2*exp(-a)*Ei(1,b*x)*b*x-2*exp(a
)*Ei(1,-b*x)*b*x+exp(5*b*x+5*a)-2*exp(b*x+a)-exp(3*b*x+3*a)-exp(-5*b*x-5*a)
+exp(-3*b*x-3*a)+2*exp(-b*x-a))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx =$$
$$-\frac{2\sinh\left(bx+a\right)^5+2\left(10\cosh\left(bx+a\right)^2-1\right)\sinh\left(bx+a\right)^3-5\left(bx\mathrm{Ei}(5\,bx)+bx\mathrm{Ei}(-5\,bx)\right)\cosh\left(5\,a\right)+3}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 5*(
b*x*Ei(5*b*x) + b*x*Ei(-5*b*x))*cosh(5*a) + 3*(b*x*Ei(3*b*x) + b*x*Ei(-3*b*
x))*cosh(3*a) + 2*(b*x*Ei(b*x) + b*x*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)
^4 - 3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 5*(b*x*Ei(5*b*x) - b*x*Ei(-5*b*
x))*sinh(5*a) + 3*(b*x*Ei(3*b*x) - b*x*Ei(-3*b*x))*sinh(3*a) + 2*(b*x*Ei(b*
x) - b*x*Ei(-b*x))*sinh(a))/x
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3\left(a+bx\right)\cosh^2\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{5}{32}\,be^{(-5\,a)}\Gamma(-1,5\,bx) - \frac{3}{32}\,be^{(-3\,a)}\Gamma(-1,3\,bx) - \frac{1}{16}\,be^{(-a)}\Gamma(-1,bx) - \frac{1}{16}\,be^{a}\Gamma(-1,-bx) - \frac{3}{32}\,be^{(3\,a)}\Gamma(-1,-3\,bx) + \frac{5}{32}\,be^{(5\,a)}\Gamma(-1,-5\,bx)$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 5/32*b*e^(-5*a)*gamma(-1, 5*b*x) - 3/32*b*e^(-3*a)*gamma(-1, 3*b*x) - 1/16*
b*e^(-a)*gamma(-1, b*x) - 1/16*b*e^a*gamma(-1, -b*x) - 3/32*b*e^(3*a)*gamma
(-1, -3*b*x) + 5/32*b*e^(5*a)*gamma(-1, -5*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{5\,bx\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - 3\,bx\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - 2\,bx\mathrm{Ei}(-bx)\,e^{(-a)} - 3\,bx\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + 5\,bx\mathrm{Ei}(-5\,bx)\,e^{(-5\,a)} -}{32\,x}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/32*(5*b*x*Ei(5*b*x)*e^(5*a) - 3*b*x*Ei(3*b*x)*e^(3*a) - 2*b*x*Ei(-b*x)*e^
(-a) - 3*b*x*Ei(-3*b*x)*e^(-3*a) + 5*b*x*Ei(-5*b*x)*e^(-5*a) - 2*b*x*Ei(b*x
)*e^a - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) - 2*e^(-b*x - a)
- e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^2,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^2, x)
```

3.323 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} - \frac{1}{16}b^2\text{Chi}(bx)\sinh(a) - \frac{9}{32}b^2\text{Chi}(3bx)\sinh(3a) + \frac{25}{32}b^2\text{Chi}(5bx)\sinh(5a) + \frac{\sinh(a+bx)}{16x^2} + \frac{\sinh(3a+3bx)}{32x^2} - \frac{\sinh(5a+5bx)}{32x^2} - \frac{1}{16}b^2\cosh(a)\text{Shi}(bx) - \frac{9}{32}b^2\cosh(3a)\text{Shi}(3bx) + \frac{25}{32}b^2\cosh(5a)\text{Shi}(5bx)$$

```
[Out] 1/16*b*cosh(b*x+a)/x+3/32*b*cosh(3*b*x+3*a)/x-5/32*b*cosh(5*b*x+5*a)/x-1/16
*b^2*cosh(a)*Shi(b*x)-9/32*b^2*cosh(3*a)*Shi(3*b*x)+25/32*b^2*cosh(5*a)*Shi
(5*b*x)-1/16*b^2*Chi(b*x)*sinh(a)-9/32*b^2*Chi(3*b*x)*sinh(3*a)+25/32*b^2*C
hi(5*b*x)*sinh(5*a)+1/16*sinh(b*x+a)/x^2+1/32*sinh(3*b*x+3*a)/x^2-1/32*sinh
(5*b*x+5*a)/x^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = &-\frac{1}{16}b^2\sinh(a)\text{Chi}(bx)-\frac{9}{32}b^2\sinh(3a)\text{Chi}(3bx)\\&+\frac{25}{32}b^2\sinh(5a)\text{Chi}(5bx)-\frac{1}{16}b^2\cosh(a)\text{Shi}(bx)\\&-\frac{9}{32}b^2\cosh(3a)\text{Shi}(3bx)+\frac{25}{32}b^2\cosh(5a)\text{Shi}(5bx)\\&+\frac{\sinh(a+bx)}{16x^2}+\frac{\sinh(3a+3bx)}{32x^2}\\&-\frac{\sinh(5a+5bx)}{32x^2}+\frac{b\cosh(a+bx)}{16x}\\&+\frac{3b\cosh(3a+3bx)}{32x}-\frac{5b\cosh(5a+5bx)}{32x}\end{aligned}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (b*Cosh[a + b*x])/(16*x) + (3*b*Cosh[3*a + 3*b*x])/(32*x) - (5*b*Cosh[5*a +
 5*b*x])/(32*x) - (b^2*CoshIntegral[b*x]*Sinh[a])/16 - (9*b^2*CoshIntegral[
3*b*x]*Sinh[3*a])/32 + (25*b^2*CoshIntegral[5*b*x]*Sinh[5*a])/32 + Sinh[a +
 b*x]/(16*x^2) + Sinh[3*a + 3*b*x]/(32*x^2) - Sinh[5*a + 5*b*x]/(32*x^2) -
(b^2*Cosh[a]*SinhIntegral[b*x])/16 - (9*b^2*Cosh[3*a]*SinhIntegral[3*b*x])/
32 + (25*b^2*Cosh[5*a]*SinhIntegral[5*b*x])/32
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\sinh(a+bx)}{8x^3} - \frac{\sinh(3a+3bx)}{16x^3} + \frac{\sinh(5a+5bx)}{16x^3}\right) dx \\
&= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^3}\, dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^3}\, dx - \frac{1}{8}\int \frac{\sinh(a+bx)}{x^3}\, dx \\
&= \frac{\sinh(a+bx)}{16x^2} + \frac{\sinh(3a+3bx)}{32x^2} - \frac{\sinh(5a+5bx)}{32x^2} - \frac{1}{16}b\int \frac{\cosh(a+bx)}{x^2}\, dx \\
&\quad - \frac{1}{32}(3b)\int \frac{\cosh(3a+3bx)}{x^2}\, dx + \frac{1}{32}(5b)\int \frac{\cosh(5a+5bx)}{x^2}\, dx \\
&= \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} + \frac{\sinh(a+bx)}{16x^2} \\
&\quad + \frac{\sinh(3a+3bx)}{32x^2} - \frac{\sinh(5a+5bx)}{32x^2} - \frac{1}{16}b^2\int \frac{\sinh(a+bx)}{x}\, dx \\
&\quad - \frac{1}{32}\left(9b^2\right)\int \frac{\sinh(3a+3bx)}{x}\, dx + \frac{1}{32}\left(25b^2\right)\int \frac{\sinh(5a+5bx)}{x}\, dx \\
&= \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} \\
&\quad + \frac{\sinh(a+bx)}{16x^2} + \frac{\sinh(3a+3bx)}{32x^2} - \frac{\sinh(5a+5bx)}{32x^2} \\
&\quad - \frac{1}{16}\left(b^2\cosh(a)\right)\int \frac{\sinh(bx)}{x}\, dx - \frac{1}{32}\left(9b^2\cosh(3a)\right)\int \frac{\sinh(3bx)}{x}\, dx \\
&\quad + \frac{1}{32}\left(25b^2\cosh(5a)\right)\int \frac{\sinh(5bx)}{x}\, dx - \frac{1}{16}\left(b^2\sinh(a)\right)\int \frac{\cosh(bx)}{x}\, dx \\
&\quad - \frac{1}{32}\left(9b^2\sinh(3a)\right)\int \frac{\cosh(3bx)}{x}\, dx + \frac{1}{32}\left(25b^2\sinh(5a)\right)\int \frac{\cosh(5bx)}{x}\, dx
\end{aligned}$$

$$= \frac{b\cosh(a+bx)}{16x} + \frac{3b\cosh(3a+3bx)}{32x} - \frac{5b\cosh(5a+5bx)}{32x} - \frac{1}{16}b^2\text{Chi}(bx)\sinh(a) - \frac{9}{32}b^2\text{Chi}(3bx)\sinh(3a) + \frac{25}{32}b^2\text{Chi}(5bx)\sinh(5a) + \frac{\sinh(a+bx)}{16x^2} + \frac{\sinh(3a+3bx)}{32x^2} - \frac{\sinh(5a+5bx)}{32x^2} - \frac{1}{16}b^2\cosh(a)\text{Shi}(bx) - \frac{9}{32}b^2\cosh(3a)\text{Shi}(3bx) + \frac{25}{32}b^2\cosh(5a)\text{Shi}(5bx)$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{2bx\cosh(a+bx) + 3bx\cosh(3(a+bx)) - 5bx\cosh(5(a+bx)) - 2b^2x^2\text{Chi}(bx)\sinh(a) - 9b^2x^2\text{Chi}(3bx)\,\text{s}}{}$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
2*b^2*x^2*CoshIntegral[b*x]*Sinh[a] - 9*b^2*x^2*CoshIntegral[3*b*x]*Sinh[3*
a] + 25*b^2*x^2*CoshIntegral[5*b*x]*Sinh[5*a] + 2*Sinh[a + b*x] + Sinh[3*(a
 + b*x)] - Sinh[5*(a + b*x)] - 2*b^2*x^2*Cosh[a]*SinhIntegral[b*x] - 9*b^2*
x^2*Cosh[3*a]*SinhIntegral[3*b*x] + 25*b^2*x^2*Cosh[5*a]*SinhIntegral[5*b*x
])/(32*x^2)
```

Maple [A] (verified)

Time = 13.45 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.34

method	result
risch	$\frac{-25\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)x^2b^2+25\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)x^2b^2-9\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^2b^2-2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^2b^2+2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^2b^2+9\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^2b^2-}{}$

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-25*exp(5*a)*Ei(1,-5*b*x)*x^2*b^2+25*exp(-5*a)*Ei(1,5*b*x)*x^2*b^2-9*
exp(-3*a)*Ei(1,3*b*x)*x^2*b^2-2*exp(-a)*Ei(1,b*x)*x^2*b^2+2*exp(a)*Ei(1,-b*
x)*x^2*b^2+9*exp(3*a)*Ei(1,-3*b*x)*x^2*b^2+2*exp(b*x+a)*b*x-5*exp(5*b*x+5*a
)*b*x-5*exp(-5*b*x-5*a)*b*x+3*exp(-3*b*x-3*a)*b*x+2*exp(-b*x-a)*b*x+3*exp(3
*b*x+3*a)*b*x+2*exp(b*x+a)-exp(5*b*x+5*a)+exp(-5*b*x-5*a)-exp(-3*b*x-3*a)-2
*exp(-b*x-a)+exp(3*b*x+3*a))/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(160) = 320.

Time = 0.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.83

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{10\,bx\cosh\left(bx+a\right)^5+50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-6\,bx\cosh\left(bx+a\right)^3+2\,\sinh\left(bx+a\right)^5+2\,(10}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 - 6*b*
x*cosh(b*x + a)^3 + 2*sinh(b*x + a)^5 + 2*(10*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a)^3 - 4*b*x*cosh(b*x + a) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x
+ a))*sinh(b*x + a)^2 - 25*(b^2*x^2*Ei(5*b*x) - b^2*x^2*Ei(-5*b*x))*cosh(5*
a) + 9*(b^2*x^2*Ei(3*b*x) - b^2*x^2*Ei(-3*b*x))*cosh(3*a) + 2*(b^2*x^2*Ei(b
*x) - b^2*x^2*Ei(-b*x))*cosh(a) + 2*(5*cosh(b*x + a)^4 - 3*cosh(b*x + a)^2
- 2)*sinh(b*x + a) - 25*(b^2*x^2*Ei(5*b*x) + b^2*x^2*Ei(-5*b*x))*sinh(5*a)
+ 9*(b^2*x^2*Ei(3*b*x) + b^2*x^2*Ei(-3*b*x))*sinh(3*a) + 2*(b^2*x^2*Ei(b*x)
 + b^2*x^2*Ei(-b*x))*sinh(a))/x^2
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx &= \frac{25}{32}\,b^2e^{(-5\,a)}\Gamma(-2,5\,bx)-\frac{9}{32}\,b^2e^{(-3\,a)}\Gamma(-2,3\,bx)\\&\quad-\frac{1}{16}\,b^2e^{(-a)}\Gamma(-2,bx)+\frac{1}{16}\,b^2e^{a}\Gamma(-2,-bx)\\&\quad+\frac{9}{32}\,b^2e^{(3\,a)}\Gamma(-2,-3\,bx)-\frac{25}{32}\,b^2e^{(5\,a)}\Gamma(-2,-5\,bx)\end{aligned}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 25/32*b^2*e^(-5*a)*gamma(-2, 5*b*x) - 9/32*b^2*e^(-3*a)*gamma(-2, 3*b*x) -
1/16*b^2*e^(-a)*gamma(-2, b*x) + 1/16*b^2*e^a*gamma(-2, -b*x) + 9/32*b^2*e^
(3*a)*gamma(-2, -3*b*x) - 25/32*b^2*e^(5*a)*gamma(-2, -5*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx$$
$$= \frac{25\,b^2x^2\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - 9\,b^2x^2\mathrm{Ei}(3\,bx)\,e^{(3\,a)} + 2\,b^2x^2\mathrm{Ei}(-bx)\,e^{(-a)} + 9\,b^2x^2\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - 25\,b^2x^2\mathrm{Ei}(-5}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(25*b^2*x^2*Ei(5*b*x)*e^(5*a) - 9*b^2*x^2*Ei(3*b*x)*e^(3*a) + 2*b^2*x^
2*Ei(-b*x)*e^(-a) + 9*b^2*x^2*Ei(-3*b*x)*e^(-3*a) - 25*b^2*x^2*Ei(-5*b*x)*e
^(-5*a) - 2*b^2*x^2*Ei(b*x)*e^a - 5*b*x*e^(5*b*x + 5*a) + 3*b*x*e^(3*b*x +
3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e^(-3*b*x - 3*a) - 5*
b*x*e^(-5*b*x - 5*a) - e^(5*b*x + 5*a) + e^(3*b*x + 3*a) + 2*e^(b*x + a) -
2*e^(-b*x - a) - e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^3, x)
```

3.324 $\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 238

$$\begin{aligned}\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx &= \frac{b\cosh(a+bx)}{48x^2}+\frac{b\cosh(3a+3bx)}{32x^2}-\frac{5b\cosh(5a+5bx)}{96x^2}\\ &\quad-\frac{1}{48}b^3\cosh(a)\mathrm{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\mathrm{Chi}(3bx)\\ &\quad+\frac{125}{96}b^3\cosh(5a)\mathrm{Chi}(5bx)+\frac{\sinh(a+bx)}{24x^3}\\ &\quad+\frac{b^2\sinh(a+bx)}{48x}+\frac{\sinh(3a+3bx)}{48x^3}\\ &\quad+\frac{3b^2\sinh(3a+3bx)}{32x}-\frac{\sinh(5a+5bx)}{48x^3}\\ &\quad-\frac{25b^2\sinh(5a+5bx)}{96x}-\frac{1}{48}b^3\sinh(a)\mathrm{Shi}(bx)\\ &\quad-\frac{9}{32}b^3\sinh(3a)\mathrm{Shi}(3bx)+\frac{125}{96}b^3\sinh(5a)\mathrm{Shi}(5bx)\end{aligned}$$

```
[Out] -1/48*b^3*Chi(b*x)*cosh(a)-9/32*b^3*Chi(3*b*x)*cosh(3*a)+125/96*b^3*Chi(5*b
*x)*cosh(5*a)+1/48*b*cosh(b*x+a)/x^2+1/32*b*cosh(3*b*x+3*a)/x^2-5/96*b*cosh
(5*b*x+5*a)/x^2-1/48*b^3*Shi(b*x)*sinh(a)-9/32*b^3*Shi(3*b*x)*sinh(3*a)+125
/96*b^3*Shi(5*b*x)*sinh(5*a)+1/24*sinh(b*x+a)/x^3+1/48*b^2*sinh(b*x+a)/x+1/
48*sinh(3*b*x+3*a)/x^3+3/32*b^2*sinh(3*b*x+3*a)/x-1/48*sinh(5*b*x+5*a)/x^3-
25/96*b^2*sinh(5*b*x+5*a)/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{1}{48}b^3\cosh(a)\text{Chi}(bx)-\frac{9}{32}b^3\cosh(3a)\text{Chi}(3bx)+\frac{125}{96}b^3\cosh(5a)\text{Chi}(5bx)-\frac{1}{48}b^3\sinh(a)\text{Shi}(bx)-\frac{9}{32}b^3\sinh(3a)\text{Shi}(3bx)+\frac{125}{96}b^3\sinh(5a)\text{Shi}(5bx)+\frac{b^2\sinh(a+bx)}{48x}+\frac{3b^2\sinh(3a+3bx)}{32x}-\frac{25b^2\sinh(5a+5bx)}{96x}+\frac{\sinh(a+bx)}{24x^3}+\frac{\sinh(3a+3bx)}{48x^3}-\frac{\sinh(5a+5bx)}{48x^3}+\frac{b\cosh(a+bx)}{48x^2}+\frac{b\cosh(3a+3bx)}{32x^2}-\frac{5b\cosh(5a+5bx)}{96x^2}$$

```
[In] Int[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[a + b*x])/(48*x^2) + (b*Cosh[3*a + 3*b*x])/(32*x^2) - (5*b*Cosh[5*a
 + 5*b*x])/(96*x^2) - (b^3*Cosh[a]*CoshIntegral[b*x])/48 - (9*b^3*Cosh[3*a]
*CoshIntegral[3*b*x])/32 + (125*b^3*Cosh[5*a]*CoshIntegral[5*b*x])/96 + Sin
h[a + b*x]/(24*x^3) + (b^2*Sinh[a + b*x])/(48*x) + Sinh[3*a + 3*b*x]/(48*x^
3) + (3*b^2*Sinh[3*a + 3*b*x])/(32*x) - Sinh[5*a + 5*b*x]/(48*x^3) - (25*b^
2*Sinh[5*a + 5*b*x])/(96*x) - (b^3*Sinh[a]*SinhIntegral[b*x])/48 - (9*b^3*S
inh[3*a]*SinhIntegral[3*b*x])/32 + (125*b^3*Sinh[5*a]*SinhIntegral[5*b*x])/
96
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\sinh(a+bx)}{8x^4} - \frac{\sinh(3a+3bx)}{16x^4} + \frac{\sinh(5a+5bx)}{16x^4}\right) dx \\
&= -\left(\frac{1}{16}\int \frac{\sinh(3a+3bx)}{x^4}\,dx\right) + \frac{1}{16}\int \frac{\sinh(5a+5bx)}{x^4}\,dx - \frac{1}{8}\int \frac{\sinh(a+bx)}{x^4}\,dx \\
&= \frac{\sinh(a+bx)}{24x^3} + \frac{\sinh(3a+3bx)}{48x^3} - \frac{\sinh(5a+5bx)}{48x^3} - \frac{1}{24}b\int \frac{\cosh(a+bx)}{x^3}\,dx \\
&\quad - \frac{1}{16}b\int \frac{\cosh(3a+3bx)}{x^3}\,dx + \frac{1}{48}(5b)\int \frac{\cosh(5a+5bx)}{x^3}\,dx \\
&= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} \\
&\quad + \frac{\sinh(3a+3bx)}{48x^3} - \frac{\sinh(5a+5bx)}{48x^3} - \frac{1}{48}b^2\int \frac{\sinh(a+bx)}{x^2}\,dx \\
&\quad - \frac{1}{32}(3b^2)\int \frac{\sinh(3a+3bx)}{x^2}\,dx + \frac{1}{96}(25b^2)\int \frac{\sinh(5a+5bx)}{x^2}\,dx \\
&= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} \\
&\quad + \frac{b^2\sinh(a+bx)}{48x} + \frac{\sinh(3a+3bx)}{48x^3} + \frac{3b^2\sinh(3a+3bx)}{32x} \\
&\quad - \frac{\sinh(5a+5bx)}{48x^3} - \frac{25b^2\sinh(5a+5bx)}{96x} - \frac{1}{48}b^3\int \frac{\cosh(a+bx)}{x}\,dx \\
&\quad - \frac{1}{32}(9b^3)\int \frac{\cosh(3a+3bx)}{x}\,dx + \frac{1}{96}(125b^3)\int \frac{\cosh(5a+5bx)}{x}\,dx
\end{aligned}$$

$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} + \frac{\sinh(a+bx)}{24x^3} + \frac{b^2\sinh(a+bx)}{48x} + \frac{\sinh(3a+3bx)}{48x^3} + \frac{3b^2\sinh(3a+3bx)}{32x} - \frac{\sinh(5a+5bx)}{48x^3} - \frac{25b^2\sinh(5a+5bx)}{96x} - \frac{1}{48}\left(b^3\cosh(a)\right)\int\frac{\cosh(bx)}{x}\,dx - \frac{1}{32}\left(9b^3\cosh(3a)\right)\int\frac{\cosh(3bx)}{x}\,dx + \frac{1}{96}\left(125b^3\cosh(5a)\right)\int\frac{\cosh(5bx)}{x}\,dx - \frac{1}{48}\left(b^3\sinh(a)\right)\int\frac{\sinh(bx)}{x}\,dx - \frac{1}{32}\left(9b^3\sinh(3a)\right)\int\frac{\sinh(3bx)}{x}\,dx + \frac{1}{96}\left(125b^3\sinh(5a)\right)\int\frac{\sinh(5bx)}{x}\,dx$$

$$= \frac{b\cosh(a+bx)}{48x^2} + \frac{b\cosh(3a+3bx)}{32x^2} - \frac{5b\cosh(5a+5bx)}{96x^2} - \frac{1}{48}b^3\cosh(a)\text{Chi}(bx) - \frac{9}{32}b^3\cosh(3a)\text{Chi}(3bx) + \frac{125}{96}b^3\cosh(5a)\text{Chi}(5bx) + \frac{\sinh(a+bx)}{24x^3} + \frac{b^2\sinh(a+bx)}{48x} + \frac{\sinh(3a+3bx)}{48x^3} + \frac{3b^2\sinh(3a+3bx)}{32x} - \frac{\sinh(5a+5bx)}{48x^3} - \frac{25b^2\sinh(5a+5bx)}{96x} - \frac{1}{48}b^3\sinh(a)\text{Shi}(bx) - \frac{9}{32}b^3\sinh(3a)\text{Shi}(3bx) + \frac{125}{96}b^3\sinh(5a)\text{Shi}(5bx)$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{2bx\cosh(a+bx) + 3bx\cosh(3(a+bx)) - 5bx\cosh(5(a+bx)) - 2b^3x^3\cosh(a)\text{Chi}(bx) - 27b^3x^3\cosh(3a)}{}$$

```
[In] Integrate[(Cosh[a + b*x]^2*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (2*b*x*Cosh[a + b*x] + 3*b*x*Cosh[3*(a + b*x)] - 5*b*x*Cosh[5*(a + b*x)] -
2*b^3*x^3*Cosh[a]*CoshIntegral[b*x] - 27*b^3*x^3*Cosh[3*a]*CoshIntegral[3*b
*x] + 125*b^3*x^3*Cosh[5*a]*CoshIntegral[5*b*x] + 4*Sinh[a + b*x] + 2*b^2*x
^2*Sinh[a + b*x] + 2*Sinh[3*(a + b*x)] + 9*b^2*x^2*Sinh[3*(a + b*x)] - 2*Si
nh[5*(a + b*x)] - 25*b^2*x^2*Sinh[5*(a + b*x)] - 2*b^3*x^3*Sinh[a]*SinhInte
gral[b*x] - 27*b^3*x^3*Sinh[3*a]*SinhIntegral[3*b*x] + 125*b^3*x^3*Sinh[5*a
]*SinhIntegral[5*b*x])/(96*x^3)
```

Maple [A] (verified)

Time = 21.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{125\,\mathrm{e}^{5a}\,\mathrm{Ei}_1(-5bx)x^3b^3+125\,\mathrm{e}^{-5a}\,\mathrm{Ei}_1(5bx)x^3b^3-27\,\mathrm{e}^{-3a}\,\mathrm{Ei}_1(3bx)x^3b^3-2\,\mathrm{e}^{-a}\,\mathrm{Ei}_1(bx)x^3b^3-2\,\mathrm{e}^{a}\,\mathrm{Ei}_1(-bx)x^3b^3-27\,\mathrm{e}^{3a}\,\mathrm{Ei}_1(-3bx)x^3}{}$

```
[In] int(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(125*exp(5*a)*Ei(1,-5*b*x)*x^3*b^3+125*exp(-5*a)*Ei(1,5*b*x)*x^3*b^3
-27*exp(-3*a)*Ei(1,3*b*x)*x^3*b^3-2*exp(-a)*Ei(1,b*x)*x^3*b^3-2*exp(a)*Ei(1
,-b*x)*x^3*b^3-27*exp(3*a)*Ei(1,-3*b*x)*x^3*b^3+25*exp(5*b*x+5*a)*b^2*x^2-2
*exp(b*x+a)*b^2*x^2-25*exp(-5*b*x-5*a)*b^2*x^2+9*exp(-3*b*x-3*a)*b^2*x^2+2*
exp(-b*x-a)*b^2*x^2-9*exp(3*b*x+3*a)*b^2*x^2+5*exp(5*b*x+5*a)*b*x-2*exp(b*x
+a)*b*x+5*exp(-5*b*x-5*a)*b*x-3*exp(-3*b*x-3*a)*b*x-2*exp(-b*x-a)*b*x-3*exp
(3*b*x+3*a)*b*x+2*exp(5*b*x+5*a)-4*exp(b*x+a)-2*exp(-5*b*x-5*a)+2*exp(-3*b*
x-3*a)+4*exp(-b*x-a)-2*exp(3*b*x+3*a))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx =$$
$$-\frac{10\,bx\cosh\left(bx+a\right)^5+50\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+2\left(25\,b^2x^2+2\right)\sinh\left(bx+a\right)^5-6\,bx\cosh\left(bx\right.}{}$$

```
[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/192*(10*b*x*cosh(b*x + a)^5 + 50*b*x*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(
25*b^2*x^2 + 2)*sinh(b*x + a)^5 - 6*b*x*cosh(b*x + a)^3 - 2*(9*b^2*x^2 - 10
*(25*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^3 - 4*b*x*cosh(b*x + a
) + 2*(50*b*x*cosh(b*x + a)^3 - 9*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 125*
(b^3*x^3*Ei(5*b*x) + b^3*x^3*Ei(-5*b*x))*cosh(5*a) + 27*(b^3*x^3*Ei(3*b*x)
+ b^3*x^3*Ei(-3*b*x))*cosh(3*a) + 2*(b^3*x^3*Ei(b*x) + b^3*x^3*Ei(-b*x))*co
sh(a) + 2*(5*(25*b^2*x^2 + 2)*cosh(b*x + a)^4 - 2*b^2*x^2 - 3*(9*b^2*x^2 +
2)*cosh(b*x + a)^2 - 4)*sinh(b*x + a) - 125*(b^3*x^3*Ei(5*b*x) - b^3*x^3*Ei
(-5*b*x))*sinh(5*a) + 27*(b^3*x^3*Ei(3*b*x) - b^3*x^3*Ei(-3*b*x))*sinh(3*a)
 + 2*(b^3*x^3*Ei(b*x) - b^3*x^3*Ei(-b*x))*sinh(a))/x^3
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\sinh^3(a+bx)\cosh^2(a+bx)}{x^4}\,dx$$

[In] integrate(cosh(b*x+a)**2*sinh(b*x+a)**3/x**4,x)

[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**2/x**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.37

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{125}{32}\,b^3e^{(-5\,a)}\Gamma(-3,5\,bx) - \frac{27}{32}\,b^3e^{(-3\,a)}\Gamma(-3,3\,bx) - \frac{1}{16}\,b^3e^{(-a)}\Gamma(-3,bx) - \frac{1}{16}\,b^3e^{a}\Gamma(-3,-bx) - \frac{27}{32}\,b^3e^{(3\,a)}\Gamma(-3,-3\,bx) + \frac{125}{32}\,b^3e^{(5\,a)}\Gamma(-3,-5\,bx)$$

[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="maxima")

[Out] 125/32*b^3*e^(-5*a)*gamma(-3, 5*b*x) - 27/32*b^3*e^(-3*a)*gamma(-3, 3*b*x) - 1/16*b^3*e^(-a)*gamma(-3, b*x) - 1/16*b^3*e^a*gamma(-3, -b*x) - 27/32*b^3*e^(3*a)*gamma(-3, -3*b*x) + 125/32*b^3*e^(5*a)*gamma(-3, -5*b*x)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{125\,b^3x^3\mathrm{Ei}(5\,bx)\,e^{(5\,a)} - 27\,b^3x^3\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - 2\,b^3x^3\mathrm{Ei}(-bx)\,e^{(-a)} - 27\,b^3x^3\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + 125\,b^3x^3\mathrm{E}}{}$$

[In] integrate(cosh(b*x+a)^2*sinh(b*x+a)^3/x^4,x, algorithm="giac")

[Out] 1/192*(125*b^3*x^3*Ei(5*b*x)*e^(5*a) - 27*b^3*x^3*Ei(3*b*x)*e^(3*a) - 2*b^3*x^3*Ei(-b*x)*e^(-a) - 27*b^3*x^3*Ei(-3*b*x)*e^(-3*a) + 125*b^3*x^3*Ei(-5*b*x)*e^(-5*a) - 2*b^3*x^3*Ei(b*x)*e^a - 25*b^2*x^2*e^(5*b*x + 5*a) + 9*b^2*x^2*e^(3*b*x + 3*a) + 2*b^2*x^2*e^(b*x + a) - 2*b^2*x^2*e^(-b*x - a) - 9*b^2

```
*x^2*e^(-3*b*x - 3*a) + 25*b^2*x^2*e^(-5*b*x - 5*a) - 5*b*x*e^(5*b*x + 5*a)
 + 3*b*x*e^(3*b*x + 3*a) + 2*b*x*e^(b*x + a) + 2*b*x*e^(-b*x - a) + 3*b*x*e
^(-3*b*x - 3*a) - 5*b*x*e^(-5*b*x - 5*a) - 2*e^(5*b*x + 5*a) + 2*e^(3*b*x +
 3*a) + 4*e^(b*x + a) - 4*e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + 2*e^(-5*b*x -
 5*a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)^3}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^4,x)
[Out] int((cosh(a + b*x)^2*sinh(a + b*x)^3)/x^4, x)
```

3.325 $\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{2^{-7-m}3^{-1-m}e^{6a}x^m(-bx)^{-m}\Gamma(1+m,-6bx)}{b} - \frac{3\ 2^{-7-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{3\ 2^{-7-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \frac{2^{-7-m}3^{-1-m}e^{-6a}x^m(bx)^{-m}\Gamma(1+m,6bx)}{b}$$

```
[Out] 2^(-7-m)*3^(-1-m)*exp(6*a)*x^m*GAMMA(1+m,-6*b*x)/b/((-b*x)^m)-3*2^(-7-m)*ex
p(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-3*2^(-7-m)*x^m*GAMMA(1+m,2*b*x)/b
/exp(2*a)/((b*x)^m)+2^(-7-m)*3^(-1-m)*x^m*GAMMA(1+m,6*b*x)/b/exp(6*a)/((b*x
)^m)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5556, 3389, 2212}

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{6a}2^{-m-7}3^{-m-1}x^m(-bx)^{-m}\Gamma(m+1,-6bx)}{b} - \frac{3e^{2a}2^{-m-7}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} - \frac{3e^{-2a}2^{-m-7}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b} + \frac{e^{-6a}2^{-m-7}3^{-m-1}x^m(bx)^{-m}\Gamma(m+1,6bx)}{b}$$

```
[In] Int[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*E^(6*a)*x^m*Gamma[1 + m, -6*b*x])/(b*(-(b*x))^m) - (
3*2^(-7 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) - (3*2^(-7 -
m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + (2^(-7 - m)*3^(-1 - m)*x^
m*Gamma[1 + m, 6*b*x])/(b*E^(6*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{32}x^m\sinh(2a+2bx)+\frac{1}{32}x^m\sinh(6a+6bx)\right)dx\\
&= \frac{1}{32}\int x^m\sinh(6a+6bx)\,dx-\frac{3}{32}\int x^m\sinh(2a+2bx)\,dx\\
&= \frac{1}{64}\int e^{-i(6ia+6ibx)}x^m\,dx-\frac{1}{64}\int e^{i(6ia+6ibx)}x^m\,dx\\
&\quad-\frac{3}{64}\int e^{-i(2ia+2ibx)}x^m\,dx+\frac{3}{64}\int e^{i(2ia+2ibx)}x^m\,dx\\
&= \frac{2^{-7-m}3^{-1-m}e^{6a}x^m(-bx)^{-m}\Gamma(1+m,-6bx)}{b}-\frac{3\ 2^{-7-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}\\
&\quad-\frac{3\ 2^{-7-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}+\frac{2^{-7-m}3^{-1-m}e^{-6a}x^m(bx)^{-m}\Gamma(1+m,6bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{2^{-7-m}3^{-1-m}e^{-6a}x^m(-b^2x^2)^{-m}\left(e^{12a}(bx)^m\Gamma(1+m,-6bx)-3^{2+m}e^{8a}(bx)^m\Gamma(1+m,-2bx)+(-bx)^m\left(-3\right.\right.}{b}$$

[In] Integrate[x^m*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]

[Out] (2^(-7 - m)*3^(-1 - m)*x^m*(E^(12*a)*(b*x)^m*Gamma[1 + m, -6*b*x] - 3^(2 + m)*E^(8*a)*(b*x)^m*Gamma[1 + m, -2*b*x] + (-(b*x))^m*(-(3^(2 + m)*E^(4*a)*Gamma[1 + m, 2*b*x]) + Gamma[1 + m, 6*b*x])))/(b*E^(6*a)*(-(b^2*x^2))^m)

Maple [F]

$$\int x^m \cosh\left(bx+a\right)^3 \sinh\left(bx+a\right)^3 dx$$

[In] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)

[Out] int(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.11

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh\left(m\log\left(6\,b\right)+6\,a\right)\Gamma(m+1,6\,bx)-9\,\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma(m+1,2\,bx)-9\,\cosh\left(m\log\left(-2\,b\right)-\right.}{}$$

[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/384*(cosh(m*log(6*b) + 6*a)*gamma(m + 1, 6*b*x) - 9*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b*x) - 9*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + cosh(m*log(-6*b) - 6*a)*gamma(m + 1, -6*b*x) - gamma(m + 1, 6*b*x)*sinh(m*log(6*b) + 6*a) + 9*gamma(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + 9*gamma(m + 1, -2*b*x)*sinh(m*log(-2*b) - 2*a) - gamma(m + 1, -6*b*x)*sinh(m*log(-6*b) - 6*a))/b

Sympy [F]

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int x^m \sinh^3{(a+bx)}\cosh^3{(a+bx)}\,dx$$

```
[In] integrate(x**m*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**3*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\begin{aligned}\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = {} & \frac{1}{64}\,(6\,bx)^{-m-1}\,x^{m+1}e^{(-6\,a)}\Gamma(m+1,6\,bx) \\ & -\frac{3}{64}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) \\ & +\frac{3}{64}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) \\ & -\frac{1}{64}\,(-6\,bx)^{-m-1}\,x^{m+1}e^{(6\,a)}\Gamma(m+1,-6\,bx)\end{aligned}$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(6*b*x)^(-m - 1)*x^(m + 1)*e^(-6*a)*gamma(m + 1, 6*b*x) - 3/64*(2*b*x)
^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) + 3/64*(-2*b*x)^(-m - 1)*x
^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/64*(-6*b*x)^(-m - 1)*x^(m + 1)*e^
(6*a)*gamma(m + 1, -6*b*x)
```

Giac [F]

$$\int x^m \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \int x^m \cosh{(bx+a)}^3 \sinh{(bx+a)}^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a+bx) \sinh^3(a+bx) \, dx = \int x^m \cosh(a+b\,x)^3 \sinh(a+b\,x)^3 \, dx$$

```
[In] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
[Out] int(x^m*cosh(a + b*x)^3*sinh(a + b*x)^3, x)
```

3.326 $\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\,dx = -\frac{9x\cosh(2a+2bx)}{128b^3} - \frac{3x^3\cosh(2a+2bx)}{64b} + \frac{x\cosh(6a+6bx)}{1152b^3} + \frac{x^3\cosh(6a+6bx)}{192b} + \frac{9\sinh(2a+2bx)}{256b^4} + \frac{9x^2\sinh(2a+2bx)}{128b^2} - \frac{\sinh(6a+6bx)}{6912b^4} - \frac{x^2\sinh(6a+6bx)}{384b^2}$$

```
[Out] -9/128*x*cosh(2*b*x+2*a)/b^3-3/64*x^3*cosh(2*b*x+2*a)/b+1/1152*x*cosh(6*b*x
+6*a)/b^3+1/192*x^3*cosh(6*b*x+6*a)/b+9/256*sinh(2*b*x+2*a)/b^4+9/128*x^2*s
inh(2*b*x+2*a)/b^2-1/6912*sinh(6*b*x+6*a)/b^4-1/384*x^2*sinh(6*b*x+6*a)/b^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5556, 3377, 2717}

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\,dx = \frac{9\sinh(2a+2bx)}{256b^4} - \frac{\sinh(6a+6bx)}{6912b^4} - \frac{9x\cosh(2a+2bx)}{128b^3} + \frac{x\cosh(6a+6bx)}{1152b^3} + \frac{9x^2\sinh(2a+2bx)}{128b^2} - \frac{x^2\sinh(6a+6bx)}{384b^2} - \frac{3x^3\cosh(2a+2bx)}{64b} + \frac{x^3\cosh(6a+6bx)}{192b}$$

```
[In] Int[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-9*x*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^3*Cosh[2*a + 2*b*x])/(64*b) + (x*
Cosh[6*a + 6*b*x])/(1152*b^3) + (x^3*Cosh[6*a + 6*b*x])/(192*b) + (9*Sinh[2
*a + 2*b*x])/(256*b^4) + (9*x^2*Sinh[2*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6
*b*x]/(6912*b^4) - (x^2*Sinh[6*a + 6*b*x])/(384*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{32}x^3\sinh(2a+2bx)+\frac{1}{32}x^3\sinh(6a+6bx)\right)\,dx \\
&= \frac{1}{32}\int x^3\sinh(6a+6bx)\,dx-\frac{3}{32}\int x^3\sinh(2a+2bx)\,dx \\
&= -\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b} \\
&\quad -\frac{\int x^2\cosh(6a+6bx)\,dx}{64b}+\frac{9\int x^2\cosh(2a+2bx)\,dx}{64b} \\
&= -\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x^3\cosh(6a+6bx)}{192b}+\frac{9x^2\sinh(2a+2bx)}{128b^2} \\
&\quad -\frac{x^2\sinh(6a+6bx)}{384b^2}+\frac{\int x\sinh(6a+6bx)\,dx}{192b^2}-\frac{9\int x\sinh(2a+2bx)\,dx}{64b^2} \\
&= -\frac{9x\cosh(2a+2bx)}{128b^3}-\frac{3x^3\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{1152b^3}+\frac{x^3\cosh(6a+6bx)}{192b} \\
&\quad +\frac{9x^2\sinh(2a+2bx)}{128b^2}-\frac{x^2\sinh(6a+6bx)}{384b^2}-\frac{\int\cosh(6a+6bx)\,dx}{1152b^3}+\frac{9\int\cosh(2a+2bx)\,dx}{128b^3}
\end{aligned}
$$

$$= -\frac{9x\cosh(2a+2bx)}{128b^3} - \frac{3x^3\cosh(2a+2bx)}{64b} + \frac{x\cosh(6a+6bx)}{1152b^3} + \frac{x^3\cosh(6a+6bx)}{192b} + \frac{9\sinh(2a+2bx)}{256b^4} + \frac{9x^2\sinh(2a+2bx)}{128b^2} - \frac{\sinh(6a+6bx)}{6912b^4} - \frac{x^2\sinh(6a+6bx)}{384b^2}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.63

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{81bx(3+2b^2x^2)\cosh(2(a+bx)) - 3(bx+6b^3x^3)\cosh(6(a+bx)) + (-121-234b^2x^2+(1+18b^2x^2)\cos}{3456b^4}$$

```
[In] Integrate[x^3*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] -1/3456*(81*b*x*(3 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(b*x + 6*b^3*x^3)*Cos
h[6*(a + b*x)] + (-121 - 234*b^2*x^2 + (1 + 18*b^2*x^2)*Cosh[4*(a + b*x)])*
Sinh[2*(a + b*x)])/b^4
```

Maple [A] (verified)

Time = 88.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(36x^3b^3-18x^2b^2+6bx-1)e^{6bx+6a}}{13824b^4} - \frac{3(4x^3b^3-6x^2b^2+6bx-3)e^{2bx+2a}}{512b^4} - \frac{3(4x^3b^3+6x^2b^2+6bx+3)e^{-2bx-2a}}{512b^4} + \frac{(36x$
derivativedivides	$\frac{-a^3\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh(b}{36}\right.}{}$
default	$\frac{-a^3\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6}-\frac{(bx+a)\cosh(bx+a)^4}{12}-\frac{\sinh(bx+a)\cosh(b}{36}\right.}{}$

```
[In] int(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13824*(36*b^3*x^3-18*b^2*x^2+6*b*x-1)/b^4*exp(6*b*x+6*a)-3/512*(4*b^3*x^3
-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)-3/512*(4*b^3*x^3+6*b^2*x^2+6*b*x+3)/
b^4*exp(-2*b*x-2*a)+1/13824*(36*b^3*x^3+18*b^2*x^2+6*b*x+1)/b^4*exp(-6*b*x-
6*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.73

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\, dx$$
$$= \frac{3\left(6\,b^3x^3+bx\right)\cosh\left(bx+a\right)^6-10\left(18\,b^2x^2+1\right)\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3+45\left(6\,b^3x^3+bx\right)\cosh\left(bx-\right.}{}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3456*(3*(6*b^3*x^3 + b*x)*cosh(b*x + a)^6 - 10*(18*b^2*x^2 + 1)*cosh(b*x
+ a)^3*sinh(b*x + a)^3 + 45*(6*b^3*x^3 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)
^4 - 3*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(6*b^3*x^3 + b*x)
*sinh(b*x + a)^6 - 81*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 9*(18*b^3*x^3 -
 5*(6*b^3*x^3 + b*x)*cosh(b*x + a)^4 + 27*b*x)*sinh(b*x + a)^2 - 3*((18*b^2
*x^2 + 1)*cosh(b*x + a)^5 - 81*(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a)
)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(141) = 282.

Time = 1.02 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.20

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\, dx$$
$$= \begin{cases} -\frac{x^3\sinh^6(a+bx)}{24b}+\frac{x^3\sinh^4(a+bx)\cosh^2(a+bx)}{8b}+\frac{x^3\sinh^2(a+bx)\cosh^4(a+bx)}{8b}-\frac{x^3\cosh^6(a+bx)}{24b}+\frac{x^2\sinh^5(a+bx)\cosh(a+bx)}{8b^2} \\ \frac{x^4\sinh^3(a)\cosh^3(a)}{4} \end{cases}$$

```
[In] integrate(x**3*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x**3*sinh(a + b*x)**6/(24*b) + x**3*sinh(a + b*x)**4*cosh(a + b
*x)**2/(8*b) + x**3*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**3*cosh(a +
 b*x)**6/(24*b) + x**2*sinh(a + b*x)**5*cosh(a + b*x)/(8*b**2) - x**2*sinh(
a + b*x)**3*cosh(a + b*x)**3/(3*b**2) + x**2*sinh(a + b*x)*cosh(a + b*x)**5
/(8*b**2) - 5*x*sinh(a + b*x)**6/(72*b**3) + x*sinh(a + b*x)**4*cosh(a + b*
x)**2/(12*b**3) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(12*b**3) - 5*x*cosh(
a + b*x)**6/(72*b**3) + 5*sinh(a + b*x)**5*cosh(a + b*x)/(72*b**4) - 31*sin
h(a + b*x)**3*cosh(a + b*x)**3/(216*b**4) + 5*sinh(a + b*x)*cosh(a + b*x)**
5/(72*b**4), Ne(b, 0)), (x**4*sinh(a)**3*cosh(a)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(36\,b^3x^3e^{(6\,a)}-18\,b^2x^2e^{(6\,a)}+6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{13824\,b^4} - \frac{3\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{512\,b^4} - \frac{3\,(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{512\,b^4} + \frac{(36\,b^3x^3+18\,b^2x^2+6\,bx+1)e^{(-6\,bx-6\,a)}}{13824\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13824*(36*b^3*x^3*e^(6*a) - 18*b^2*x^2*e^(6*a) + 6*b*x*e^(6*a) - e^(6*a))
*e^(6*b*x)/b^4 - 3/512*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*
a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 3/512*(4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e
^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2*x^2 + 6*b*x + 1)*e^(-6*b
*x - 6*a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int x^3\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(36\,b^3x^3-18\,b^2x^2+6\,bx-1)e^{(6\,bx+6\,a)}}{13824\,b^4} - \frac{3\,(4\,b^3x^3-6\,b^2x^2+6\,bx-3)e^{(2\,bx+2\,a)}}{512\,b^4} - \frac{3\,(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{512\,b^4} + \frac{(36\,b^3x^3+18\,b^2x^2+6\,bx+1)e^{(-6\,bx-6\,a)}}{13824\,b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/13824*(36*b^3*x^3 - 18*b^2*x^2 + 6*b*x - 1)*e^(6*b*x + 6*a)/b^4 - 3/512*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 3/512*(4*b^3*x^3 +
 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 + 1/13824*(36*b^3*x^3 + 18*b^2
*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int x^3 \cosh^3(a+bx) \sinh^3(a+bx)\, dx = \frac{\frac{9\,x^2\sinh(2\,a+2\,b\,x)}{128}-\frac{x^2\sinh(6\,a+6\,b\,x)}{384}}{b^2} - \frac{\frac{3\,x^3\cosh(2\,a+2\,b\,x)}{64}-\frac{x^3\cosh(6\,a+6\,b\,x)}{192}}{b} - \frac{\frac{9\,x\cosh(2\,a+2\,b\,x)}{128}-\frac{x\cosh(6\,a+6\,b\,x)}{1152}}{b^3} + \frac{9\sinh(2\,a+2\,b\,x)}{256\,b^4} - \frac{\sinh(6\,a+6\,b\,x)}{6912\,b^4}$$

```
[In] int(x^3*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] ((9*x^2*sinh(2*a + 2*b*x))/128 - (x^2*sinh(6*a + 6*b*x))/384)/b^2 - ((3*x^3
*cosh(2*a + 2*b*x))/64 - (x^3*cosh(6*a + 6*b*x))/192)/b - ((9*x*cosh(2*a +
2*b*x))/128 - (x*cosh(6*a + 6*b*x))/1152)/b^3 + (9*sinh(2*a + 2*b*x))/(256*
b^4) - sinh(6*a + 6*b*x)/(6912*b^4)
```

3.327 $\int x^2 \cosh^3(a+bx) \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int x^2 \cosh^3(a+bx) \sinh^3(a+bx)\,dx = -\frac{3\cosh(2a+2bx)}{128b^3} - \frac{3x^2\cosh(2a+2bx)}{64b} + \frac{\cosh(6a+6bx)}{3456b^3} + \frac{x^2\cosh(6a+6bx)}{192b} + \frac{3x\sinh(2a+2bx)}{64b^2} - \frac{x\sinh(6a+6bx)}{576b^2}$$

```
[Out] -3/128*cosh(2*b*x+2*a)/b^3-3/64*x^2*cosh(2*b*x+2*a)/b+1/3456*cosh(6*b*x+6*a
)/b^3+1/192*x^2*cosh(6*b*x+6*a)/b+3/64*x*sinh(2*b*x+2*a)/b^2-1/576*x*sinh(6
*b*x+6*a)/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5556, 3377, 2718}

$$\int x^2 \cosh^3(a+bx) \sinh^3(a+bx)\,dx = -\frac{3\cosh(2a+2bx)}{128b^3} + \frac{\cosh(6a+6bx)}{3456b^3} + \frac{3x\sinh(2a+2bx)}{64b^2} - \frac{x\sinh(6a+6bx)}{576b^2} - \frac{3x^2\cosh(2a+2bx)}{64b} + \frac{x^2\cosh(6a+6bx)}{192b}$$

```
[In] Int[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[2*a + 2*b*x])/(128*b^3) - (3*x^2*Cosh[2*a + 2*b*x])/(64*b) + Cosh[
6*a + 6*b*x]/(3456*b^3) + (x^2*Cosh[6*a + 6*b*x])/(192*b) + (3*x*Sinh[2*a +
 2*b*x])/(64*b^2) - (x*Sinh[6*a + 6*b*x])/(576*b^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{32}x^2\sinh(2a+2bx)+\frac{1}{32}x^2\sinh(6a+6bx)\right)dx \\
&= \frac{1}{32}\int x^2\sinh(6a+6bx)\,dx-\frac{3}{32}\int x^2\sinh(2a+2bx)\,dx \\
&= -\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}-\frac{\int x\cosh(6a+6bx)\,dx}{96b}+\frac{3\int x\cosh(2a+2bx)\,dx}{32b} \\
&= -\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{x^2\cosh(6a+6bx)}{192b}+\frac{3x\sinh(2a+2bx)}{64b^2} \\
&\quad -\frac{x\sinh(6a+6bx)}{576b^2}+\frac{\int\sinh(6a+6bx)\,dx}{576b^2}-\frac{3\int\sinh(2a+2bx)\,dx}{64b^2} \\
&= -\frac{3\cosh(2a+2bx)}{128b^3}-\frac{3x^2\cosh(2a+2bx)}{64b}+\frac{\cosh(6a+6bx)}{3456b^3} \\
&\quad +\frac{x^2\cosh(6a+6bx)}{192b}+\frac{3x\sinh(2a+2bx)}{64b^2}-\frac{x\sinh(6a+6bx)}{576b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int x^2 \cosh^3(a+bx) \sinh^3(a+bx)\,dx$$
$$= \frac{-81(1+2b^2x^2)\cosh(2(a+bx)) + (1+18b^2x^2)\cosh(6(a+bx)) + 6bx(27\sinh(2(a+bx)) - \sinh(6(a+bx)))}{3456b^3}$$

```
[In] Integrate[x^2*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-81*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] + (1 + 18*b^2*x^2)*Cosh[6*(a + b*x)]
 + 6*b*x*(27*Sinh[2*(a + b*x)] - Sinh[6*(a + b*x)]))/(3456*b^3)
```

Maple [A] (verified)

Time = 63.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
risch	$\frac{(18x^2b^2-6bx+1)\mathrm{e}^{6bx+6a}}{6912b^3} - \frac{3(2x^2b^2-2bx+1)\mathrm{e}^{2bx+2a}}{256b^3} - \frac{3(2x^2b^2+2bx+1)\mathrm{e}^{-2bx-2a}}{256b^3} + \frac{(18x^2b^2+6bx+1)\mathrm{e}^{-6bx-6a}}{6912b^3}$
derivativedivides	$a^2\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6} - \frac{\cosh(bx+a)^4}{12}\right) - 2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6} - \frac{(bx+a)\cosh(bx+a)^4}{12} - \frac{\sinh(bx+a)\cosh(bx+$
default	$a^2\left(\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6} - \frac{\cosh(bx+a)^4}{12}\right) - 2a\left(\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6} - \frac{(bx+a)\cosh(bx+a)^4}{12} - \frac{\sinh(bx+a)\cosh(bx+$

```
[In] int(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6912*(18*b^2*x^2-6*b*x+1)/b^3*exp(6*b*x+6*a)-3/256*(2*b^2*x^2-2*b*x+1)/b^
3*exp(2*b*x+2*a)-3/256*(2*b^2*x^2+2*b*x+1)/b^3*exp(-2*b*x-2*a)+1/6912*(18*b
^2*x^2+6*b*x+1)/b^3*exp(-6*b*x-6*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.92

$$\int x^2 \cosh^3(a+bx) \sinh^3(a+bx)\,dx =$$
$$-\frac{120\,bx\cosh(bx+a)^3\sinh(bx+a)^3 + 36\,bx\cosh(bx+a)\sinh(bx+a)^5 - (18\,b^2x^2+1)\cosh(bx+a)^6 -}{}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3456*(120*b*x*cosh(b*x + a)^3*sinh(b*x + a)^3 + 36*b*x*cosh(b*x + a)*sin
h(b*x + a)^5 - (18*b^2*x^2 + 1)*cosh(b*x + a)^6 - 15*(18*b^2*x^2 + 1)*cosh(
b*x + a)^2*sinh(b*x + a)^4 - (18*b^2*x^2 + 1)*sinh(b*x + a)^6 + 81*(2*b^2*x
^2 + 1)*cosh(b*x + a)^2 - 3*(5*(18*b^2*x^2 + 1)*cosh(b*x + a)^4 - 54*b^2*x^
2 - 27)*sinh(b*x + a)^2 + 36*(b*x*cosh(b*x + a)^5 - 9*b*x*cosh(b*x + a))*si
nh(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(102) = 204$.

Time = 0.77 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

$$\int x^2 \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} -\frac{x^2\sinh^6(a+bx)}{24b} + \frac{x^2\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x^2\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x^2\cosh^6(a+bx)}{24b} + \frac{x\sinh^5(a+bx)\cosh(a+bx)}{12b^2} - \\ \frac{x^3\sinh^3(a)\cosh^3(a)}{3} \end{cases}$$

```
[In] integrate(x**2*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x)**6/(24*b) + x**2*sinh(a + b*x)**4*cosh(a + b
*x)**2/(8*b) + x**2*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x**2*cosh(a +
 b*x)**6/(24*b) + x*sinh(a + b*x)**5*cosh(a + b*x)/(12*b**2) - 2*x*sinh(a +
 b*x)**3*cosh(a + b*x)**3/(9*b**2) + x*sinh(a + b*x)*cosh(a + b*x)**5/(12*b
**2) - sinh(a + b*x)**6/(72*b**3) + sinh(a + b*x)**2*cosh(a + b*x)**4/(18*b
**3) - 7*cosh(a + b*x)**6/(216*b**3), Ne(b, 0)), (x**3*sinh(a)**3*cosh(a)**
3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int x^2 \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(18\,b^2x^2e^{(6\,a)} - 6\,bxe^{(6\,a)} + e^{(6\,a)}\right)e^{(6\,bx)}}{6912\,b^3} - \frac{3\,\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^3} - \frac{3\,(2\,b^2x^2 + 2\,bx + 1)e^{(-2\,bx-2\,a)}}{256\,b^3} + \frac{(18\,b^2x^2 + 6\,bx + 1)e^{(-6\,bx-6\,a)}}{6912\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6912*(18*b^2*x^2*e^(6*a) - 6*b*x*e^(6*a) + e^(6*a))*e^(6*b*x)/b^3 - 3/256
*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 3/256*(2*b^2
*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^
(-6*b*x - 6*a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(18\,b^2x^2-6\,bx+1)e^{(6\,bx+6\,a)}}{6912\,b^3} - \frac{3\,(2\,b^2x^2-2\,bx+1)e^{(2\,bx+2\,a)}}{256\,b^3} - \frac{3\,(2\,b^2x^2+2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^3} + \frac{(18\,b^2x^2+6\,bx+1)e^{(-6\,bx-6\,a)}}{6912\,b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/6912*(18*b^2*x^2 - 6*b*x + 1)*e^(6*b*x + 6*a)/b^3 - 3/256*(2*b^2*x^2 - 2*
b*x + 1)*e^(2*b*x + 2*a)/b^3 - 3/256*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*
a)/b^3 + 1/6912*(18*b^2*x^2 + 6*b*x + 1)*e^(-6*b*x - 6*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int x^2\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\frac{3\cosh(2\,a+2\,b\,x)}{128}-\frac{\cosh(6\,a+6\,b\,x)}{3456}+b^2\left(\frac{3\,x^2\cosh(2\,a+2\,b\,x)}{64}-\frac{x^2\cosh(6\,a+6\,b\,x)}{192}\right)-b\left(\frac{3\,x\sinh(2\,a+2\,b\,x)}{64}-\frac{x\sinh(6\,a+6\,b\,x)}{576}\right)}{b^3}$$

```
[In] int(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] -((3*cosh(2*a + 2*b*x))/128 - cosh(6*a + 6*b*x)/3456 + b^2*((3*x^2*cosh(2*a
 + 2*b*x))/64 - (x^2*cosh(6*a + 6*b*x))/192) - b*((3*x*sinh(2*a + 2*b*x))/6
4 - (x*sinh(6*a + 6*b*x))/576))/b^3
```

3.328 $\int x \cosh^3(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int x \cosh^3(a+bx) \sinh^3(a+bx) \, dx = -\frac{3x\cosh(2a+2bx)}{64b} + \frac{x\cosh(6a+6bx)}{192b} + \frac{3\sinh(2a+2bx)}{128b^2} - \frac{\sinh(6a+6bx)}{1152b^2}$$

```
[Out] -3/64*x*cosh(2*b*x+2*a)/b+1/192*x*cosh(6*b*x+6*a)/b+3/128*sinh(2*b*x+2*a)/b
^2-1/1152*sinh(6*b*x+6*a)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5556, 3377, 2717}

$$\int x \cosh^3(a+bx) \sinh^3(a+bx) \, dx = \frac{3\sinh(2a+2bx)}{128b^2} - \frac{\sinh(6a+6bx)}{1152b^2} - \frac{3x\cosh(2a+2bx)}{64b} + \frac{x\cosh(6a+6bx)}{192b}$$

```
[In] Int[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*x*Cosh[2*a + 2*b*x])/(64*b) + (x*Cosh[6*a + 6*b*x])/(192*b) + (3*Sinh[2
*a + 2*b*x])/(128*b^2) - Sinh[6*a + 6*b*x]/(1152*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3}{32}x\sinh(2a+2bx)+\frac{1}{32}x\sinh(6a+6bx)\right)\,dx\\
&= \frac{1}{32}\int x\sinh(6a+6bx)\,dx-\frac{3}{32}\int x\sinh(2a+2bx)\,dx\\
&= -\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}-\frac{\int\cosh(6a+6bx)\,dx}{192b}+\frac{3\int\cosh(2a+2bx)\,dx}{64b}\\
&= -\frac{3x\cosh(2a+2bx)}{64b}+\frac{x\cosh(6a+6bx)}{192b}+\frac{3\sinh(2a+2bx)}{128b^2}-\frac{\sinh(6a+6bx)}{1152b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{54bx\cosh(2(a+bx))-6bx\cosh(6(a+bx))-27\sinh(2(a+bx))+\sinh(6(a+bx))}{1152b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
[Out] -1/1152*(54*b*x*Cosh[2*(a + b*x)] - 6*b*x*Cosh[6*(a + b*x)] - 27*Sinh[2*(a
+ b*x)] + Sinh[6*(a + b*x)])/b^2
```

Maple [A] (verified)

Time = 44.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(6bx-1)e^{6bx+6a}}{2304b^2} - \frac{3(2bx-1)e^{2bx+2a}}{256b^2} - \frac{3(2bx+1)e^{-2bx-2a}}{256b^2} + \frac{(6bx+1)e^{-6bx-6a}}{2304b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6} - \frac{(bx+a)\cosh(bx+a)^4}{12} - \frac{\sinh(bx+a)\cosh(bx+a)^5}{36} + \frac{\cosh(bx+a)^3\sinh(bx+a)}{36} + \frac{\cosh(bx+a)\sinh(bx}{24}}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^4}{6} - \frac{(bx+a)\cosh(bx+a)^4}{12} - \frac{\sinh(bx+a)\cosh(bx+a)^5}{36} + \frac{\cosh(bx+a)^3\sinh(bx+a)}{36} + \frac{\cosh(bx+a)\sinh(bx}{24}}{b^2}$

```
[In] int(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2304*(6*b*x-1)/b^2*exp(6*b*x+6*a)-3/256*(2*b*x-1)/b^2*exp(2*b*x+2*a)-3/25
6*(2*b*x+1)/b^2*exp(-2*b*x-2*a)+1/2304*(6*b*x+1)/b^2*exp(-6*b*x-6*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(59) = 118$.

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.21

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{3\,bx\cosh\left(bx+a\right)^6 + 45\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4 + 3\,bx\sinh\left(bx+a\right)^6 - 10\,\cosh\left(bx+a\right)^3\sinh\left(bx\right.}{}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/576*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 - 3*cosh(b*x + a)*s
inh(b*x + a)^5 - 27*b*x*cosh(b*x + a)^2 + 9*(5*b*x*cosh(b*x + a)^4 - 3*b*x)
*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^5 - 9*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(63) = 126$.

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.21

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{x\sinh^6(a+bx)}{24b} + \frac{x\sinh^4(a+bx)\cosh^2(a+bx)}{8b} + \frac{x\sinh^2(a+bx)\cosh^4(a+bx)}{8b} - \frac{x\cosh^6(a+bx)}{24b} + \frac{\sinh^5(a+bx)\cosh(a+bx)}{24b^2} - \sinh \\ \frac{x^2\sinh^3(a)\cosh^3(a)}{2} \end{cases}$$

```
[In] integrate(x*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**6/(24*b) + x*sinh(a + b*x)**4*cosh(a + b*x)**2
/(8*b) + x*sinh(a + b*x)**2*cosh(a + b*x)**4/(8*b) - x*cosh(a + b*x)**6/(24
*b) + sinh(a + b*x)**5*cosh(a + b*x)/(24*b**2) - sinh(a + b*x)**3*cosh(a +
b*x)**3/(9*b**2) + sinh(a + b*x)*cosh(a + b*x)**5/(24*b**2), Ne(b, 0)), (x*
*2*sinh(a)**3*cosh(a)**3/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(6\,bxe^{(6\,a)}-e^{(6\,a)}\right)e^{(6\,bx)}}{2304\,b^2}-\frac{3\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2304*(6*b*x*e^(6*a) - e^(6*a))*e^(6*b*x)/b^2 - 3/256*(2*b*x*e^(2*a) - e^(
2*a))*e^(2*b*x)/b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*
x + 1)*e^(-6*b*x - 6*a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{(6\,bx-1)e^{(6\,bx+6\,a)}}{2304\,b^2}-\frac{3\,(2\,bx-1)e^{(2\,bx+2\,a)}}{256\,b^2}-\frac{3\,(2\,bx+1)e^{(-2\,bx-2\,a)}}{256\,b^2}+\frac{(6\,bx+1)e^{(-6\,bx-6\,a)}}{2304\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2304*(6*b*x - 1)*e^(6*b*x + 6*a)/b^2 - 3/256*(2*b*x - 1)*e^(2*b*x + 2*a)/
b^2 - 3/256*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 + 1/2304*(6*b*x + 1)*e^(-6*b*x
 - 6*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= -\frac{\frac{\sinh(6\,a+6\,b\,x)}{1152}-\frac{3\sinh(2\,a+2\,b\,x)}{128}+b\left(\frac{3\,x\cosh(2\,a+2\,b\,x)}{64}-\frac{x\cosh(6\,a+6\,b\,x)}{192}\right)}{b^2}$$

```
[In] int(x*cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] -(sinh(6*a + 6*b*x)/1152 - (3*sinh(2*a + 2*b*x))/128 + b*((3*x*cosh(2*a + 2
*b*x))/64 - (x*cosh(6*a + 6*b*x))/192))/b^2
```

3.329 $\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{\sinh^4(a+bx)}{4b}+\frac{\sinh^6(a+bx)}{6b}$$

```
[Out] 1/4*sinh(b*x+a)^4/b+1/6*sinh(b*x+a)^6/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 14}

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{\sinh^6(a+bx)}{6b}+\frac{\sinh^4(a+bx)}{4b}$$

```
[In] Int[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]^4/(4*b) + Sinh[a + b*x]^6/(6*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^3(1-x^2)\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int (x^3-x^5)\,dx, x, i\sinh(a+bx)\right)}{b}$$
$$= \frac{\sinh^4(a+bx)}{4b} + \frac{\sinh^6(a+bx)}{6b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{1}{8}\left(-\frac{3\cosh(2(a+bx))}{8b} + \frac{\cosh(6(a+bx))}{24b}\right)$$

```
[In] Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
[Out] ((-3*Cosh[2*(a + b*x)])/(8*b) + Cosh[6*(a + b*x)]/(24*b))/8
```

Maple [A] (verified)

Time = 31.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{\left(\cosh(bx+a)^2-1\right)^3}{6}+\frac{\left(\cosh(bx+a)^2-1\right)^2}{4}}{b}$	34
default	$\frac{\frac{\left(\cosh(bx+a)^2-1\right)^3}{6}+\frac{\left(\cosh(bx+a)^2-1\right)^2}{4}}{b}$	34
risch	$\frac{e^{6bx+6a}}{384b}-\frac{3\,e^{2bx+2a}}{128b}-\frac{3\,e^{-2bx-2a}}{128b}+\frac{e^{-6bx-6a}}{384b}$	58

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/6*(cosh(b*x+a)^2-1)^3+1/4*(cosh(b*x+a)^2-1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(27) = 54.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.32

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh\left(bx+a\right)^6+15\,\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4+\sinh\left(bx+a\right)^6+3\left(5\,\cosh\left(bx+a\right)^4-3\right)\sinh\left(bx+a\right)^2}{192\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(cosh(b*x + a)^6 + 15*cosh(b*x + a)^2*sinh(b*x + a)^4 + sinh(b*x + a)
^6 + 3*(5*cosh(b*x + a)^4 - 3)*sinh(b*x + a)^2 - 9*cosh(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)\cosh^4(a+bx)}{4b} - \frac{\cosh^6(a+bx)}{12b} & \text{for } b \neq 0 \\ x\sinh^3(a)\cosh^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)**4/(4*b) - cosh(a + b*x)**6/(12*b
), Ne(b, 0)), (x*sinh(a)**3*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(9\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{9\,e^{(-2\,bx-2\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/384*(9*e^(-2*b*x - 2*
a) - e^(-6*b*x - 6*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{3\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{3\,e^{(-2\,bx-2\,a)}}{128\,b} + \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/384*e^(6*b*x + 6*a)/b - 3/128*e^(2*b*x + 2*a)/b - 3/128*e^(-2*b*x - 2*a)/
b + 1/384*e^(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{2\sinh(a+b\,x)^6 + 3\sinh(a+b\,x)^4}{12\,b}$$

```
[In] int(cosh(a + b*x)^3*sinh(a + b*x)^3,x)
```

```
[Out] (3*sinh(a + b*x)^4 + 2*sinh(a + b*x)^6)/(12*b)
```

3.330 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{3}{32}\text{Chi}(2bx)\sinh(2a) + \frac{1}{32}\text{Chi}(6bx)\sinh(6a) - \frac{3}{32}\cosh(2a)\text{Shi}(2bx) + \frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

```
[Out] -3/32*cosh(2*a)*Shi(2*b*x)+1/32*cosh(6*a)*Shi(6*b*x)-3/32*Chi(2*b*x)*sinh(2
*a)+1/32*Chi(6*b*x)*sinh(6*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5556, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = -\frac{3}{32}\sinh(2a)\text{Chi}(2bx) + \frac{1}{32}\sinh(6a)\text{Chi}(6bx) - \frac{3}{32}\cosh(2a)\text{Shi}(2bx) + \frac{1}{32}\cosh(6a)\text{Shi}(6bx)$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
```

```
[Out] (-3*CoshIntegral[2*b*x]*Sinh[2*a])/32 + (CoshIntegral[6*b*x]*Sinh[6*a])/32
- (3*Cosh[2*a]*SinhIntegral[2*b*x])/32 + (Cosh[6*a]*SinhIntegral[6*b*x])/32
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3\sinh(2a+2bx)}{32x}+\frac{\sinh(6a+6bx)}{32x}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x}\,dx \\
&= -\left(\frac{1}{32}(3\cosh(2a))\int \frac{\sinh(2bx)}{x}\,dx\right)+\frac{1}{32}\cosh(6a)\int \frac{\sinh(6bx)}{x}\,dx \\
&\quad -\frac{1}{32}(3\sinh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{32}\sinh(6a)\int \frac{\cosh(6bx)}{x}\,dx \\
&= -\frac{3}{32}\text{Chi}(2bx)\sinh(2a)+\frac{1}{32}\text{Chi}(6bx)\sinh(6a)-\frac{3}{32}\cosh(2a)\text{Shi}(2bx)+\frac{1}{32}\cosh(6a)\text{Shi}(6bx)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{32}(-6\cosh(a)\text{Chi}(2bx)\sinh(a) + \text{Chi}(6bx)\sinh(6a) - 3\cosh(2a)\text{Shi}(2bx) + \cosh(6a)\text{Shi}(6bx))$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x,x]
[Out] (-6*Cosh[a]*CoshIntegral[2*b*x]*Sinh[a] + CoshIntegral[6*b*x]*Sinh[6*a] - 3
*Cosh[2*a]*SinhIntegral[2*b*x] + Cosh[6*a]*SinhIntegral[6*b*x])/32
```

Maple [A] (verified)

Time = 28.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{e^{-6a}\,\text{Ei}_1(6bx)}{64}-\frac{3\,e^{-2a}\,\text{Ei}_1(2bx)}{64}+\frac{3\,e^{2a}\,\text{Ei}_1(-2bx)}{64}-\frac{e^{6a}\,\text{Ei}_1(-6bx)}{64}$	50

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x,method=_RETURNVERBOSE)
[Out] 1/64*exp(-6*a)*Ei(1,6*b*x)-3/64*exp(-2*a)*Ei(1,2*b*x)+3/64*exp(2*a)*Ei(1,-2
*b*x)-1/64*exp(6*a)*Ei(1,-6*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{64}\,(\text{Ei}(6\,bx)-\text{Ei}(-6\,bx))\cosh(6\,a) - \frac{3}{64}\,(\text{Ei}(2\,bx)-\text{Ei}(-2\,bx))\cosh(2\,a) + \frac{1}{64}\,(\text{Ei}(6\,bx)+\text{Ei}(-6\,bx))\sinh(6\,a) - \frac{3}{64}\,(\text{Ei}(2\,bx)+\text{Ei}(-2\,bx))\sinh(2\,a)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
[Out] 1/64*(Ei(6*b*x) - Ei(-6*b*x))*cosh(6*a) - 3/64*(Ei(2*b*x) - Ei(-2*b*x))*cos
h(2*a) + 1/64*(Ei(6*b*x) + Ei(-6*b*x))*sinh(6*a) - 3/64*(Ei(2*b*x) + Ei(-2*
b*x))*sinh(2*a)
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{64}\,\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - \frac{3}{64}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + \frac{3}{64}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{64}\,\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
- 1/64*Ei(-6*b*x)*e^(-6*a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \frac{1}{64}\,\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - \frac{3}{64}\,\mathrm{Ei}(2\,bx)\,e^{(2\,a)} + \frac{3}{64}\,\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} - \frac{1}{64}\,\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/64*Ei(6*b*x)*e^(6*a) - 3/64*Ei(2*b*x)*e^(2*a) + 3/64*Ei(-2*b*x)*e^(-2*a)
- 1/64*Ei(-6*b*x)*e^(-6*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3\sinh(a+b\,x)^3}{x}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x,x)
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x, x)
```

3.331 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)$$

```
[Out] -3/16*b*Chi(2*b*x)*cosh(2*a)+3/16*b*Chi(6*b*x)*cosh(6*a)-3/16*b*Shi(2*b*x)*
sinh(2*a)+3/16*b*Shi(6*b*x)*sinh(6*a)+3/32*sinh(2*b*x+2*a)/x-1/32*sinh(6*b*
x+6*a)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{3}{16}b\cosh(2a)\text{Chi}(2bx)+\frac{3}{16}b\cosh(6a)\text{Chi}(6bx)-\frac{3}{16}b\sinh(2a)\text{Shi}(2bx)+\frac{3}{16}b\sinh(6a)\text{Shi}(6bx)+\frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] (-3*b*Cosh[2*a]*CoshIntegral[2*b*x])/16 + (3*b*Cosh[6*a]*CoshIntegral[6*b*x
])/16 + (3*Sinh[2*a + 2*b*x])/(32*x) - Sinh[6*a + 6*b*x]/(32*x) - (3*b*Sinh
[2*a]*SinhIntegral[2*b*x])/16 + (3*b*Sinh[6*a]*SinhIntegral[6*b*x])/16
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^2}+\frac{\sinh(6a+6bx)}{32x^2}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^2}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^2}\,dx \\
&= \frac{3\sinh(2a+2bx)}{32x}-\frac{\sinh(6a+6bx)}{32x} \\
&\quad -\frac{1}{16}(3b)\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{16}(3b)\int \frac{\cosh(6a+6bx)}{x}\,dx
\end{aligned}$$

$$= \frac{3\sinh(2a+2bx)}{32x} - \frac{\sinh(6a+6bx)}{32x} - \frac{1}{16}(3b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx + \frac{1}{16}(3b\cosh(6a))\int \frac{\cosh(6bx)}{x}\,dx - \frac{1}{16}(3b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{16}(3b\sinh(6a))\int \frac{\sinh(6bx)}{x}\,dx$$

$$= -\frac{3}{16}b\cosh(2a)\mathrm{Chi}(2bx) + \frac{3}{16}b\cosh(6a)\mathrm{Chi}(6bx) + \frac{3\sinh(2a+2bx)}{32x} - \frac{\sinh(6a+6bx)}{32x} - \frac{3}{16}b\sinh(2a)\mathrm{Shi}(2bx) + \frac{3}{16}b\sinh(6a)\mathrm{Shi}(6bx)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{6bx\cosh(2a)\mathrm{Chi}(2bx) - 6bx\cosh(6a)\mathrm{Chi}(6bx) - 3\sinh(2(a+bx)) + \sinh(6(a+bx)) + 6bx\sinh(2a)\mathrm{Sh}}{32x}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^2,x]
```

```
[Out] -1/32*(6*b*x*Cosh[2*a]*CoshIntegral[2*b*x] - 6*b*x*Cosh[6*a]*CoshIntegral[6
*b*x] - 3*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 6*b*x*Sinh[2*a]*SinhInteg
ral[2*b*x] - 6*b*x*Sinh[6*a]*SinhIntegral[6*b*x])/x
```

Maple [A] (verified)

Time = 30.96 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{6\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)bx-6\,\mathrm{e}^{6a}\,\mathrm{Ei}_1(-6bx)bx-6\,\mathrm{e}^{-6a}\,\mathrm{Ei}_1(6bx)bx+6\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)bx+3\,\mathrm{e}^{2bx+2a}-\mathrm{e}^{6bx+6a}+\mathrm{e}^{-6bx-6a}-3\,\mathrm{e}^{-2bx-2a}}{64x}$	105

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(6*exp(2*a)*Ei(1,-2*b*x)*b*x-6*exp(6*a)*Ei(1,-6*b*x)*b*x-6*exp(-6*a)*E
i(1,6*b*x)*b*x+6*exp(-2*a)*Ei(1,2*b*x)*b*x+3*exp(2*b*x+2*a)-exp(6*b*x+6*a)+
exp(-6*b*x-6*a)-3*exp(-2*b*x-2*a))/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(77) = 154$.

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = -\frac{20\cosh(bx+a)^3\sinh(bx+a)^3 + 6\cosh(bx+a)\sinh(bx+a)^5 - 3\,(bx\mathrm{Ei}(6\,bx) + bx\mathrm{Ei}(-6\,bx))\cosh(6\,a)}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^5
 - 3*(b*x*Ei(6*b*x) + b*x*Ei(-6*b*x))*cosh(6*a) + 3*(b*x*Ei(2*b*x) + b*x*Ei
(-2*b*x))*cosh(2*a) + 6*(cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a) - 3
*(b*x*Ei(6*b*x) - b*x*Ei(-6*b*x))*sinh(6*a) + 3*(b*x*Ei(2*b*x) - b*x*Ei(-2*
b*x))*sinh(2*a))/x
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{3}{32}\,be^{(-6\,a)}\Gamma(-1,6\,bx) - \frac{3}{32}\,be^{(-2\,a)}\Gamma(-1,2\,bx) - \frac{3}{32}\,be^{(2\,a)}\Gamma(-1,-2\,bx) + \frac{3}{32}\,be^{(6\,a)}\Gamma(-1,-6\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 3/32*b*e^(-6*a)*gamma(-1, 6*b*x) - 3/32*b*e^(-2*a)*gamma(-1, 2*b*x) - 3/32*
b*e^(2*a)*gamma(-1, -2*b*x) + 3/32*b*e^(6*a)*gamma(-1, -6*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \frac{6\,bx\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - 6\,bx\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 6\,bx\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 6\,bx\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)} - e^{(6\,bx+6\,a)} + 3\,e^{(2\,bx+}}{64\,x}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
[Out] 1/64*(6*b*x*Ei(6*b*x)*e^(6*a) - 6*b*x*Ei(2*b*x)*e^(2*a) - 6*b*x*Ei(-2*b*x)*
e^(-2*a) + 6*b*x*Ei(-6*b*x)*e^(-6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a)
- 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^3}{x^2}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^2,x)
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^2, x)
```

3.332 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 131

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} - \frac{3}{16}b^2\text{Chi}(2bx)\sinh(2a) + \frac{9}{16}b^2\text{Chi}(6bx)\sinh(6a) + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} - \frac{3}{16}b^2\cosh(2a)\text{Shi}(2bx) + \frac{9}{16}b^2\cosh(6a)\text{Shi}(6bx)$$

```
[Out] 3/32*b*cosh(2*b*x+2*a)/x-3/32*b*cosh(6*b*x+6*a)/x-3/16*b^2*cosh(2*a)*Shi(2*
b*x)+9/16*b^2*cosh(6*a)*Shi(6*b*x)-3/16*b^2*Chi(2*b*x)*sinh(2*a)+9/16*b^2*C
hi(6*b*x)*sinh(6*a)+3/64*sinh(2*b*x+2*a)/x^2-1/64*sinh(6*b*x+6*a)/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5556, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{3}{16}b^2\sinh(2a)\text{Chi}(2bx) + \frac{9}{16}b^2\sinh(6a)\text{Chi}(6bx) - \frac{3}{16}b^2\cosh(2a)\text{Shi}(2bx) + \frac{9}{16}b^2\cosh(6a)\text{Shi}(6bx) + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} + \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] (3*b*Cosh[2*a + 2*b*x])/(32*x) - (3*b*Cosh[6*a + 6*b*x])/(32*x) - (3*b^2*Co
shIntegral[2*b*x]*Sinh[2*a])/16 + (9*b^2*CoshIntegral[6*b*x]*Sinh[6*a])/16
+ (3*Sinh[2*a + 2*b*x])/(64*x^2) - Sinh[6*a + 6*b*x]/(64*x^2) - (3*b^2*Cosh
[2*a]*SinhIntegral[2*b*x])/16 + (9*b^2*Cosh[6*a]*SinhIntegral[6*b*x])/16
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^3}+\frac{\sinh(6a+6bx)}{32x^3}\right)\,dx\\ &= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^3}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^3}\,dx\end{aligned}$$

$$= \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} - \frac{1}{32}(3b)\int \frac{\cosh(2a+2bx)}{x^2}\,dx + \frac{1}{32}(3b)\int \frac{\cosh(6a+6bx)}{x^2}\,dx$$

$$= \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} - \frac{1}{16}(3b^2)\int \frac{\sinh(2a+2bx)}{x}\,dx + \frac{1}{16}(9b^2)\int \frac{\sinh(6a+6bx)}{x}\,dx$$

$$= \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} - \frac{1}{16}(3b^2\cosh(2a))\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{16}(9b^2\cosh(6a))\int \frac{\sinh(6bx)}{x}\,dx - \frac{1}{16}(3b^2\sinh(2a))\int \frac{\cosh(2bx)}{x}\,dx + \frac{1}{16}(9b^2\sinh(6a))\int \frac{\cosh(6bx)}{x}\,dx$$

$$= \frac{3b\cosh(2a+2bx)}{32x} - \frac{3b\cosh(6a+6bx)}{32x} - \frac{3}{16}b^2\text{Chi}(2bx)\sinh(2a) + \frac{9}{16}b^2\text{Chi}(6bx)\sinh(6a) + \frac{3\sinh(2a+2bx)}{64x^2} - \frac{\sinh(6a+6bx)}{64x^2} - \frac{3}{16}b^2\cosh(2a)\text{Shi}(2bx) + \frac{9}{16}b^2\cosh(6a)\text{Shi}(6bx)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{-6bx\cosh(2(a+bx)) + 6bx\cosh(6(a+bx)) + 12b^2x^2\text{Chi}(2bx)\sinh(2a) - 36b^2x^2\text{Chi}(6bx)\sinh(6a) - 3\,\text{s}}{64x^2}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^3,x]
```

```
[Out] -1/64*(-6*b*x*Cosh[2*(a + b*x)] + 6*b*x*Cosh[6*(a + b*x)] + 12*b^2*x^2*Cosh
Integral[2*b*x]*Sinh[2*a] - 36*b^2*x^2*CoshIntegral[6*b*x]*Sinh[6*a] - 3*Si
nh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 12*b^2*x^2*Cosh[2*a]*SinhIntegral[2*b
*x] - 36*b^2*x^2*Cosh[6*a]*SinhIntegral[6*b*x])/x^2
```

Maple [A] (verified)

Time = 32.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{-36\,\mathrm{e}^{-6a}\,\mathrm{Ei}_1(6bx)x^2b^2+12\,\mathrm{e}^{-2a}\,\mathrm{Ei}_1(2bx)x^2b^2-12\,\mathrm{e}^{2a}\,\mathrm{Ei}_1(-2bx)x^2b^2+36\,\mathrm{e}^{6a}\,\mathrm{Ei}_1(-6bx)x^2b^2+6\,\mathrm{e}^{-6bx-6a}bx-6\,\mathrm{e}^{-2bx-2a}bx-6\,\mathrm{e}^{2}}{128x^2}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
[Out] -1/128*(-36*exp(-6*a)*Ei(1,6*b*x)*x^2*b^2+12*exp(-2*a)*Ei(1,2*b*x)*x^2*b^2-
12*exp(2*a)*Ei(1,-2*b*x)*x^2*b^2+36*exp(6*a)*Ei(1,-6*b*x)*x^2*b^2+6*exp(-6*
b*x-6*a)*b*x-6*exp(-2*b*x-2*a)*b*x-6*exp(2*b*x+2*a)*b*x+6*exp(6*b*x+6*a)*b*
x-exp(-6*b*x-6*a)+3*exp(-2*b*x-2*a)-3*exp(2*b*x+2*a)+exp(6*b*x+6*a))/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(115) = 230$.

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.09

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = -\frac{3\,bx\cosh\left(bx+a\right)^6+45\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4+3\,bx\sinh\left(bx+a\right)^6+10\,\cosh\left(bx+a\right)^3\sinh\left(\right.}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="fricas")
[Out] -1/32*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 + 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 3*cosh(b*x + a)*s
inh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 + 3*(15*b*x*cosh(b*x + a)^4 - b*x)*s
inh(b*x + a)^2 - 9*(b^2*x^2*Ei(6*b*x) - b^2*x^2*Ei(-6*b*x))*cosh(6*a) + 3*(
b^2*x^2*Ei(2*b*x) - b^2*x^2*Ei(-2*b*x))*cosh(2*a) + 3*(cosh(b*x + a)^5 - co
sh(b*x + a))*sinh(b*x + a) - 9*(b^2*x^2*Ei(6*b*x) + b^2*x^2*Ei(-6*b*x))*sin
h(6*a) + 3*(b^2*x^2*Ei(2*b*x) + b^2*x^2*Ei(-2*b*x))*sinh(2*a))/x^2
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\sinh^3(a+bx)\cosh^3(a+bx)}{x^3}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**3,x)
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{9}{16}\,b^2e^{(-6\,a)}\Gamma(-2,6\,bx) - \frac{3}{16}\,b^2e^{(-2\,a)}\Gamma(-2,2\,bx) + \frac{3}{16}\,b^2e^{(2\,a)}\Gamma(-2,-2\,bx) - \frac{9}{16}\,b^2e^{(6\,a)}\Gamma(-2,-6\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 9/16*b^2*e^(-6*a)*gamma(-2, 6*b*x) - 3/16*b^2*e^(-2*a)*gamma(-2, 2*b*x) + 3
/16*b^2*e^(2*a)*gamma(-2, -2*b*x) - 9/16*b^2*e^(6*a)*gamma(-2, -6*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \frac{36\,b^2x^2\text{Ei}(6\,bx)\,e^{(6\,a)} - 12\,b^2x^2\text{Ei}(2\,bx)\,e^{(2\,a)} + 12\,b^2x^2\text{Ei}(-2\,bx)\,e^{(-2\,a)} - 36\,b^2x^2\text{Ei}(-6\,bx)\,e^{(-6\,a)} - 6\,bxe^{(6\,bx}}{128}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/128*(36*b^2*x^2*Ei(6*b*x)*e^(6*a) - 12*b^2*x^2*Ei(2*b*x)*e^(2*a) + 12*b^2
*x^2*Ei(-2*b*x)*e^(-2*a) - 36*b^2*x^2*Ei(-6*b*x)*e^(-6*a) - 6*b*x*e^(6*b*x
+ 6*a) + 6*b*x*e^(2*b*x + 2*a) + 6*b*x*e^(-2*b*x - 2*a) - 6*b*x*e^(-6*b*x -
 6*a) - e^(6*b*x + 6*a) + 3*e^(2*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*
x - 6*a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^3}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^3}{x^3}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^3,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^3, x)
```

3.333 $\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{b\cosh(2a+2bx)}{32x^2} - \frac{b\cosh(6a+6bx)}{32x^2} - \frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx) + \frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx) + \frac{\sinh(2a+2bx)}{32x^3} + \frac{b^2\sinh(2a+2bx)}{16x} - \frac{\sinh(6a+6bx)}{96x^3} - \frac{3b^2\sinh(6a+6bx)}{16x} - \frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx) + \frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)$$

```
[Out] -1/8*b^3*Chi(2*b*x)*cosh(2*a)+9/8*b^3*Chi(6*b*x)*cosh(6*a)+1/32*b*cosh(2*b*
x+2*a)/x^2-1/32*b*cosh(6*b*x+6*a)/x^2-1/8*b^3*Shi(2*b*x)*sinh(2*a)+9/8*b^3*
Shi(6*b*x)*sinh(6*a)+1/32*sinh(2*b*x+2*a)/x^3+1/16*b^2*sinh(2*b*x+2*a)/x-1/
96*sinh(6*b*x+6*a)/x^3-3/16*b^2*sinh(6*b*x+6*a)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{5556, 3378, 3384, 3379, 3382\}$

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx) + \frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx) - \frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx) + \frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx) + \frac{b^2\sinh(2a+2bx)}{16x} - \frac{3b^2\sinh(6a+6bx)}{16x} + \frac{\sinh(2a+2bx)}{32x^3} - \frac{\sinh(6a+6bx)}{96x^3} + \frac{b\cosh(2a+2bx)}{32x^2} - \frac{b\cosh(6a+6bx)}{32x^2}$$

```
[In] Int[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] (b*Cosh[2*a + 2*b*x])/(32*x^2) - (b*Cosh[6*a + 6*b*x])/(32*x^2) - (b^3*Cosh
[2*a]*CoshIntegral[2*b*x])/8 + (9*b^3*Cosh[6*a]*CoshIntegral[6*b*x])/8 + Si
nh[2*a + 2*b*x]/(32*x^3) + (b^2*Sinh[2*a + 2*b*x])/(16*x) - Sinh[6*a + 6*b*
x]/(96*x^3) - (3*b^2*Sinh[6*a + 6*b*x])/(16*x) - (b^3*Sinh[2*a]*SinhIntegra
l[2*b*x])/8 + (9*b^3*Sinh[6*a]*SinhIntegral[6*b*x])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{3\sinh(2a+2bx)}{32x^4}+\frac{\sinh(6a+6bx)}{32x^4}\right)dx \\
&= \frac{1}{32}\int \frac{\sinh(6a+6bx)}{x^4}\,dx-\frac{3}{32}\int \frac{\sinh(2a+2bx)}{x^4}\,dx \\
&= \frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3}-\frac{1}{16}b\int \frac{\cosh(2a+2bx)}{x^3}\,dx+\frac{1}{16}b\int \frac{\cosh(6a+6bx)}{x^3}\,dx \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}-\frac{\sinh(6a+6bx)}{96x^3} \\
&\quad -\frac{1}{16}b^2\int \frac{\sinh(2a+2bx)}{x^2}\,dx+\frac{1}{16}(3b^2)\int \frac{\sinh(6a+6bx)}{x^2}\,dx \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3}+\frac{b^2\sinh(2a+2bx)}{16x}-\frac{\sinh(6a+6bx)}{96x^3} \\
&\quad -\frac{3b^2\sinh(6a+6bx)}{16x}-\frac{1}{8}b^3\int \frac{\cosh(2a+2bx)}{x}\,dx+\frac{1}{8}(9b^3)\int \frac{\cosh(6a+6bx)}{x}\,dx \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}+\frac{\sinh(2a+2bx)}{32x^3} \\
&\quad +\frac{b^2\sinh(2a+2bx)}{16x}-\frac{\sinh(6a+6bx)}{96x^3}-\frac{3b^2\sinh(6a+6bx)}{16x} \\
&\quad -\frac{1}{8}(b^3\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx+\frac{1}{8}(9b^3\cosh(6a))\int \frac{\cosh(6bx)}{x}\,dx \\
&\quad -\frac{1}{8}(b^3\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx+\frac{1}{8}(9b^3\sinh(6a))\int \frac{\sinh(6bx)}{x}\,dx \\
&= \frac{b\cosh(2a+2bx)}{32x^2}-\frac{b\cosh(6a+6bx)}{32x^2}-\frac{1}{8}b^3\cosh(2a)\text{Chi}(2bx) \\
&\quad +\frac{9}{8}b^3\cosh(6a)\text{Chi}(6bx)+\frac{\sinh(2a+2bx)}{32x^3}+\frac{b^2\sinh(2a+2bx)}{16x}-\frac{\sinh(6a+6bx)}{96x^3} \\
&\quad -\frac{3b^2\sinh(6a+6bx)}{16x}-\frac{1}{8}b^3\sinh(2a)\text{Shi}(2bx)+\frac{9}{8}b^3\sinh(6a)\text{Shi}(6bx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{-3bx\cosh(2(a+bx))+3bx\cosh(6(a+bx))+12b^3x^3\cosh(2a)\text{Chi}(2bx)-108b^3x^3\cosh(6a)\text{Chi}(6bx)-}{}$$

```
[In] Integrate[(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/x^4,x]
```

```
[Out] -1/96*(-3*b*x*Cosh[2*(a + b*x)] + 3*b*x*Cosh[6*(a + b*x)] + 12*b^3*x^3*Cosh
[2*a]*CoshIntegral[2*b*x] - 108*b^3*x^3*Cosh[6*a]*CoshIntegral[6*b*x] - 3*S
inh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[6*(a + b*x)] + 18*b^2
*x^2*Sinh[6*(a + b*x)] + 12*b^3*x^3*Sinh[2*a]*SinhIntegral[2*b*x] - 108*b^3
*x^3*Sinh[6*a]*SinhIntegral[6*b*x])/x^3
```

Maple [A] (verified)

Time = 49.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

method	result
risch	$\frac{12\,e^{2a}\,\text{Ei}_1(-2bx)x^3b^3-108\,e^{-6a}\,\text{Ei}_1(6bx)x^3b^3+12\,e^{-2a}\,\text{Ei}_1(2bx)x^3b^3-108\,e^{6a}\,\text{Ei}_1(-6bx)x^3b^3+6\,e^{2bx+2a}b^2x^2+18\,e^{-6bx-6a}b^2x^2-6\,e}{192}$

```
[In] int(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(12*exp(2*a)*Ei(1,-2*b*x)*x^3*b^3-108*exp(-6*a)*Ei(1,6*b*x)*x^3*b^3+1
2*exp(-2*a)*Ei(1,2*b*x)*x^3*b^3-108*exp(6*a)*Ei(1,-6*b*x)*x^3*b^3+6*exp(2*b
*x+2*a)*b^2*x^2+18*exp(-6*b*x-6*a)*b^2*x^2-6*exp(-2*b*x-2*a)*b^2*x^2-18*exp
(6*b*x+6*a)*b^2*x^2+3*exp(2*b*x+2*a)*b*x-3*exp(-6*b*x-6*a)*b*x+3*exp(-2*b*x
-2*a)*b*x-3*exp(6*b*x+6*a)*b*x+3*exp(2*b*x+2*a)+exp(-6*b*x-6*a)-3*exp(-2*b*
x-2*a)-exp(6*b*x+6*a))/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(149) = 298$.

Time = 0.25 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = -\frac{3\,bx\cosh(bx+a)^6+45\,bx\cosh(bx+a)^2\sinh(bx+a)^4+3\,bx\sinh(bx+a)^6+20\,(18\,b^2x^2+1)\cosh(bx}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(3*b*x*cosh(b*x + a)^6 + 45*b*x*cosh(b*x + a)^2*sinh(b*x + a)^4 + 3*b
*x*sinh(b*x + a)^6 + 20*(18*b^2*x^2 + 1)*cosh(b*x + a)^3*sinh(b*x + a)^3 +
6*(18*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^5 - 3*b*x*cosh(b*x + a)^2 +
3*(15*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^2 - 54*(b^3*x^3*Ei(6*b*x) +
b^3*x^3*Ei(-6*b*x))*cosh(6*a) + 6*(b^3*x^3*Ei(2*b*x) + b^3*x^3*Ei(-2*b*x))*
cosh(2*a) + 6*((18*b^2*x^2 + 1)*cosh(b*x + a)^5 - (2*b^2*x^2 + 1)*cosh(b*x
+ a))*sinh(b*x + a) - 54*(b^3*x^3*Ei(6*b*x) - b^3*x^3*Ei(-6*b*x))*sinh(6*a)
 + 6*(b^3*x^3*Ei(2*b*x) - b^3*x^3*Ei(-2*b*x))*sinh(2*a))/x^3
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\sinh^3{(a+bx)}\cosh^3{(a+bx)}}{x^4}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*sinh(b*x+a)**3/x**4,x)
```

```
[Out] Integral(sinh(a + b*x)**3*cosh(a + b*x)**3/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{27}{8}\,b^3e^{(-6\,a)}\Gamma(-3,6\,bx) - \frac{3}{8}\,b^3e^{(-2\,a)}\Gamma(-3,2\,bx) - \frac{3}{8}\,b^3e^{(2\,a)}\Gamma(-3,-2\,bx) + \frac{27}{8}\,b^3e^{(6\,a)}\Gamma(-3,-6\,bx)$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 27/8*b^3*e^(-6*a)*gamma(-3, 6*b*x) - 3/8*b^3*e^(-2*a)*gamma(-3, 2*b*x) - 3/
8*b^3*e^(2*a)*gamma(-3, -2*b*x) + 27/8*b^3*e^(6*a)*gamma(-3, -6*b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \frac{108\,b^3x^3\mathrm{Ei}(6\,bx)\,e^{(6\,a)} - 12\,b^3x^3\mathrm{Ei}(2\,bx)\,e^{(2\,a)} - 12\,b^3x^3\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)} + 108\,b^3x^3\mathrm{Ei}(-6\,bx)\,e^{(-6\,a)} - 18\,b^2}{}$$

```
[In] integrate(cosh(b*x+a)^3*sinh(b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/192*(108*b^3*x^3*Ei(6*b*x)*e^(6*a) - 12*b^3*x^3*Ei(2*b*x)*e^(2*a) - 12*b^
3*x^3*Ei(-2*b*x)*e^(-2*a) + 108*b^3*x^3*Ei(-6*b*x)*e^(-6*a) - 18*b^2*x^2*e^
(6*b*x + 6*a) + 6*b^2*x^2*e^(2*b*x + 2*a) - 6*b^2*x^2*e^(-2*b*x - 2*a) + 18
*b^2*x^2*e^(-6*b*x - 6*a) - 3*b*x*e^(6*b*x + 6*a) + 3*b*x*e^(2*b*x + 2*a) +
 3*b*x*e^(-2*b*x - 2*a) - 3*b*x*e^(-6*b*x - 6*a) - e^(6*b*x + 6*a) + 3*e^(2
*b*x + 2*a) - 3*e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{x^4}\,dx = \int \frac{\cosh(a+b\,x)^3\,\sinh(a+b\,x)^3}{x^4}\,dx$$

```
[In] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^4,x)
```

```
[Out] int((cosh(a + b*x)^3*sinh(a + b*x)^3)/x^4, x)
```

3.334 $\int x^m \tanh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \tanh(a+bx)\,dx = \text{Int}(x^m \tanh(a+bx), x)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

```
[In] Int[x^m*Tanh[a + b*x],x]
[Out] Defer[Int][x^m*Tanh[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \tanh(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

[In] Integrate[x^m*Tanh[a + b*x],x]

[Out] Integrate[x^m*Tanh[a + b*x], x]

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

[In] int(x^m*sech(b*x+a)*sinh(b*x+a),x)

[Out] int(x^m*sech(b*x+a)*sinh(b*x+a),x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")

[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a), x)

Sympy [N/A]

Not integrable

Time = 10.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int x^m \tanh(a+bx)\,dx = \int x^m \sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a),x)

[Out] Integral(x**m*sinh(a + b*x)*sech(a + b*x), x)

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 10.00

$$\int x^m \tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int x^m \tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)}{\cosh(a+b\,x)}\,dx$$

```
[In] int((x^m*sinh(a + b*x))/cosh(a + b*x),x)
```

```
[Out] int((x^m*sinh(a + b*x))/cosh(a + b*x), x)
```

3.335 $\int x^3 \tanh(a + bx)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 91

$$\int x^3 \tanh(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}$$

```
[Out] -1/4*x^4+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-
3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3799, 2221, 2611, 6744, 2320, 6724}

$$\int x^3 \tanh(a+bx)\,dx = \frac{3 \operatorname{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4} - \frac{3x \operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{3x^2 \operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} + \frac{x^3 \log\left(e^{2(a+bx)}+1\right)}{b} - \frac{x^4}{4}$$

```
[In] Int[x^3*Tanh[a + b*x],x]
```

```
[Out] -1/4*x^4 + (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, -E^(2*(a +
b*x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4
, -E^(2*(a + b*x))])/(4*b^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^4}{4} + 2\int \frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 + e^{2(a+bx)}\right)}{b} - \frac{3\int x^2 \log\left(1 + e^{2(a+bx)}\right) dx}{b}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 + e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} - \frac{3\int x \operatorname{PolyLog}\left(2, -e^{2(a+bx)}\right) dx}{b^2}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 + e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, -e^{2(a+bx)}\right)}{2b^3} + \frac{3\int \operatorname{PolyLog}\left(3, -e^{2(a+bx)}\right) dx}{2b^3}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 + e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, -e^{2(a+bx)}\right)}{2b^3} + \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-x)}{x} dx, x, e^{2(a+bx)}\right)}{4b^4}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 + e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, -e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4, -e^{2(a+bx)}\right)}{4b^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int x^3 \tanh(a + bx)\, dx$$
$$= \frac{b^4x^4 + 4b^3x^3 \log\left(1 + e^{-2(a+bx)}\right) - 6b^2x^2 \operatorname{PolyLog}\left(2, -e^{-2(a+bx)}\right) - 6bx \operatorname{PolyLog}\left(3, -e^{-2(a+bx)}\right) - 3 \operatorname{PolyLo}}{4b^4}$$

```
[In] Integrate[x^3*Tanh[a + b*x],x]
```

```
[Out] (b^4*x^4 + 4*b^3*x^3*Log[1 + E^(-2*(a + b*x))] - 6*b^2*x^2*PolyLog[2, -E^(-
2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(-2*(a + b*x))] - 3*PolyLog[4, -E^(-2*(
a + b*x))])/(4*b^4)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{x^4}{4} - \frac{3a^4}{2b^4} - \frac{2a^3x}{b^3} + \frac{x^3 \ln(1+e^{2bx+2a})}{b} + \frac{3x^2 \operatorname{polylog}(2,-e^{2bx+2a})}{2b^2} - \frac{3x \operatorname{polylog}(3,-e^{2bx+2a})}{2b^3} + \frac{2a^3 \ln(e^{bx+a})}{b^4} + \frac{3 \operatorname{poly}}{}$

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] -1/4*x^4-3/2/b^4*a^4-2/b^3*a^3*x+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog (2,-exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+2/b^4*a^3*ln(e xp(b*x+a))+3/4*polylog(4,-exp(2*b*x+2*a))/b^4

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.82

$$\int x^3 \tanh(a+bx)\,dx = -\frac{b^4x^4 - 12\,b^2x^2\mathrm{Li}_2(i\,\cosh(bx+a) + i\,\sinh(bx+a)) - 12\,b^2x^2\mathrm{Li}_2(-i\,\cosh(bx+a) - i\,\sinh(bx+a)) +$$

[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")

[Out] -1/4*(b^4*x^4 - 12*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*b^2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) + I) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) + 24*b*x*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 24*b*x*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 4*(b^3*x^3 + a^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 24*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^4

Sympy [F]

$$\int x^3 \tanh(a+bx)\,dx = \int x^3 \sinh(a+bx)\,\mathrm{sech}(a+bx)\,dx$$

[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a),x)

[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int x^3 \tanh(a+bx)\,dx = -\frac{1}{4}\,x^4 + \frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right) + 6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - 6\,bx\mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right) + 3\,\mathrm{Li}_4\left(-e^{(2\,bx+2\,a)}\right)}{3\,b^4}$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*
b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x +
 2*a)))/b^4
```

Giac [F]

$$\int x^3 \tanh(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)\ dx$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)}{\cosh(a+b\,x)}\,dx$$

```
[In] int((x^3*sinh(a + b*x))/cosh(a + b*x),x)
```

```
[Out] int((x^3*sinh(a + b*x))/cosh(a + b*x), x)
```

3.336 $\int x^2 \tanh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int x^2 \tanh(a+bx)\,dx = -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}$$

```
[Out] -1/3*x^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*po
lylog(3,-exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3799, 2221, 2611, 2320, 6724}

$$\int x^2 \tanh(a+bx)\,dx = -\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}+\frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^3}{3}$$

```
[In] Int[x^2*Tanh[a + b*x],x]
```

```
[Out] -1/3*x^3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + (x*PolyLog[2, -E^(2*(a + b*x)
)])/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3}{3}+2\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx \\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{2\int x\log\left(1+e^{2(a+bx)}\right)\,dx}{b} \\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\int \text{PolyLog}\left(2,-e^{2(a+bx)}\right)\,dx}{b^2} \\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3} \\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int x^2 \tanh(a+bx)\,dx = \frac{2b^2x^2\left(bx+3\log\left(1+e^{-2(a+bx)}\right)\right)-6bx\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)-3\,\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{6b^3}$$

```
[In] Integrate[x^2*Tanh[a + b*x],x]
[Out] (2*b^2*x^2*(b*x + 3*Log[1 + E^(-2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^(-2*(a
 + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/(6*b^3)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

method	result	size
risch	$-\frac{x^3}{3}-\frac{2a^2\ln(e^{bx+a})}{b^3}+\frac{2a^2x}{b^2}+\frac{4a^3}{3b^3}+\frac{x^2\ln(1+e^{2bx+2a})}{b}+\frac{x\,\text{polylog}(2,-e^{2bx+2a})}{b^2}-\frac{\text{polylog}(3,-e^{2bx+2a})}{2b^3}$	94

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*
x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b
^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.18

$$\int x^2 \tanh(a+bx)\,dx = -\frac{b^3x^3-6\,bx\text{Li}_2(i\cosh(bx+a)+i\sinh(bx+a))-6\,bx\text{Li}_2(-i\cosh(bx+a)-i\sinh(bx+a))-3\,a^2\log\ldots}{}$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] -1/3*(b^3*x^3 - 6*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*b*x*dilo
g(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(b^2*x^2 - a^2
)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*(b^2*x^2 - a^2)*log(-I*cos
h(b*x + a) - I*sinh(b*x + a) + 1) + 6*polylog(3, I*cosh(b*x + a) + I*sinh(b
*x + a)) + 6*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^3
```

Sympy [F]

$$\int x^2 \tanh(a+bx)\,dx = \int x^2 \sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a),x)
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int x^2 \tanh(a+bx)\,dx = -\frac{1}{3}x^3 + \frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right) + 2bx\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - \mathrm{Li}_3\left(-e^{(2bx+2a)}\right)}{2b^3}$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/3*x^3 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x
+ 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3
```

Giac [F]

$$\int x^2 \tanh(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh(a+bx)\,dx = \int \frac{x^2\sinh(a+bx)}{\cosh(a+bx)}\,dx$$

```
[In] int((x^2*sinh(a + b*x))/cosh(a + b*x),x)
[Out] int((x^2*sinh(a + b*x))/cosh(a + b*x), x)
```

3.337 $\int x \tanh(a+bx) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int x \tanh(a+bx) \, dx = -\frac{x^2}{2} + \frac{x \log\left(1+e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}$$

```
[Out] -1/2*x^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3799, 2221, 2317, 2438}

$$\int x \tanh(a+bx) \, dx = \frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} + \frac{x \log\left(e^{2(a+bx)}+1\right)}{b} - \frac{x^2}{2}$$

```
[In] Int[x*Tanh[a + b*x],x]
```

```
[Out] -1/2*x^2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*x))]/(2
*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2}{2} + 2\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx \\
&= -\frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\int \log\left(1+e^{2(a+bx)}\right)\,dx}{b} \\
&= -\frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^2} \\
&= -\frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$
\int x\tanh(a+bx)\,dx = \frac{bx\left(bx+2\log\left(1+e^{-2(a+bx)}\right)\right) - \operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right)}{2b^2}
$$

```
[In] Integrate[x*Tanh[a + b*x],x]
[Out] (b*x*(b*x + 2*Log[1 + E^(-2*(a + b*x))]) - PolyLog[2, -E^(-2*(a + b*x))])/(
2*b^2)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

method	result	size
risch	$-\frac{x^2}{2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{x\ln(1+e^{2bx+2a})}{b}+\frac{\operatorname{polylog}(2,-e^{2bx+2a})}{2b^2}+\frac{2a\ln(e^{bx+a})}{b^2}$	70

```
[In] int(x*sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2-2/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+
2*a))/b^2+2/b^2*a*ln(exp(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.13

$$\int x\tanh(a+bx)\,dx = -\frac{b^2x^2+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-i\right)-2\,(bx+$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 2*a*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*a*log(cosh(b
*x + a) + sinh(b*x + a) - I) - 2*(b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x
 + a) + 1) - 2*(b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*di
log(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*dilog(-I*cosh(b*x + a) - I*sinh(
b*x + a)))/b^2
```

Sympy [F]

$$\int x\tanh(a+bx)\,dx = \int x\sinh\left(a+bx\right)\operatorname{sech}\left(a+bx\right)\,dx$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x)
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x\tanh(a+bx)\,dx = -\frac{1}{2}\,x^2 + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/2*x^2 + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b
^2
```

Giac [F]

$$\int x\tanh(a+bx)\,dx = \int x\,\mathrm{sech}\,(bx+a)\sinh\,(bx+a)\,dx$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\tanh(a+bx)\,dx = \int \frac{x\sinh(a+b\,x)}{\cosh\,(a+b\,x)}\,dx$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x), x)
```

3.338 $\int \tanh(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

```
[Out] ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

```
[In] Int[Tanh[a + b*x],x]
```

```
[Out] Log[Cosh[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \tanh(a+bx)\,dx=\frac{\log(\cosh(a+bx))}{b}$$

```
[In] Integrate[Tanh[a + b*x],x]
[Out] Log[Cosh[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$-\frac{\ln(\operatorname{sech}(bx+a))}{b}$	13
default	$-\frac{\ln(\operatorname{sech}(bx+a))}{b}$	13
parallelrisch	$\frac{-bx-\ln(1-\tanh(bx+a))}{b}$	23
risch	$-x-\frac{2a}{b}+\frac{\ln(1+e^{2bx+2a})}{b}$	27

```
[In] int(sech(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/b*ln(sech(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \tanh(a+bx)\,dx=-\frac{bx-\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] -(b*x - log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int \tanh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \tanh(a+bx)\,dx = \frac{\log\left(e^{(bx+a)} + e^{(-bx-a)}\right)}{b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] log(e^(b*x + a) + e^(-b*x - a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \tanh(a+bx)\,dx = -\frac{bx + a - \log\left(e^{(2\,bx+2\,a)} + 1\right)}{b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] -(b*x + a - log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \tanh(a+bx)\,dx = \frac{\ln\left(\cosh(a+b\,x)\right)}{b}$$

```
[In] int(sinh(a + b*x)/cosh(a + b*x),x)
[Out] log(cosh(a + b*x))/b
```

3.339 $\int \frac{\tanh(a+bx)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\tanh(a+bx)}{x} dx = \text{Int}\left(\frac{\tanh(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh(a+bx)}{x} dx = \int \frac{\tanh(a+bx)}{x} dx$$

```
[In] Int[Tanh[a + b*x]/x,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\tanh(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 9.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(a + bx)}{x} dx = \int \frac{\tanh(a + bx)}{x} dx$$

```
[In] Integrate[Tanh[a + b*x]/x,x]
[Out] Integrate[Tanh[a + b*x]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{sech}(bx + a) \sinh(bx + a)}{x} dx$$

```
[In] int(sech(b*x+a)*sinh(b*x+a)/x,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a + bx)}{x} dx = \int \frac{\operatorname{sech}(bx + a) \sinh(bx + a)}{x} dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\tanh(a + bx)}{x} dx = \int \frac{\sinh(a + bx) \operatorname{sech}(a + bx)}{x} dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] -2*integrate(1/(x*e^(2*b*x + 2*a) + x), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)}{x\cosh(a+b\,x)}\,dx$$

```
[In] int(sinh(a + b*x)/(x*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)/(x*cosh(a + b*x)), x)
```

3.340 $\int \frac{\tanh(a+bx)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\tanh(a+bx)}{x^2} dx = \text{Int}\left(\frac{\tanh(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh(a+bx)}{x^2} dx = \int \frac{\tanh(a+bx)}{x^2} dx$$

```
[In] Int[Tanh[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\tanh(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 16.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\tanh(a+bx)}{x^2}\,dx$$

```
[In] Integrate[Tanh[a + b*x]/x^2,x]
[Out] Integrate[Tanh[a + b*x]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}dx$$

```
[In] int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x**2,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="maxima")
[Out] -1/x - 2*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)}{x^2\cosh(a+b\,x)}\,dx$$

```
[In] int(sinh(a + b*x)/(x^2*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)/(x^2*cosh(a + b*x)), x)
```

3.341 $\int x^m \mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \operatorname{Int}(x^m \operatorname{sech}(a+bx)\tanh(a+bx),x)$$

```
[Out] CannotIntegrate(x^m*sech(b*x+a)*tanh(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$$

```
[In] Int[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] Defer[Int][x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 48.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx$$

```
[In] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\,dx$$

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 35.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^m \text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^m \sinh(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a),x)
[Out] Integral(x**m*sinh(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \mathrm{sech}(a+bx)\tanh(a+bx)\,dx=\int x^m\,\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \mathrm{sech}(a+bx)\tanh(a+bx)\,dx=\int x^m\,\mathrm{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \mathrm{sech}(a+bx)\tanh(a+bx)\,dx=\int \frac{x^m\,\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^m*sinh(a + b*x))/cosh(a + b*x)^2,x)
[Out] int((x^m*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

3.342 $\int x^3 \text{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx=\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}-\frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}-\frac{x^3\text{sech}(a+bx)}{b}$$

```
[Out] 6*x^2*arctan(exp(b*x+a))/b^2-6*I*x*polylog(2,-I*exp(b*x+a))/b^3+6*I*x*polyl
og(2,I*exp(b*x+a))/b^3+6*I*polylog(3,-I*exp(b*x+a))/b^4-6*I*polylog(3,I*exp
(b*x+a))/b^4-x^3*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5526, 4265, 2611, 2320, 6724}

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx=\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}+\frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}-\frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}-\frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{x^3\text{sech}(a+bx)}{b}$$

```
[In] Int[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^
3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a +
 b*x)])/b^4 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (x^3*Sech[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{x^3\text{sech}(a+bx)}{b}+\frac{3\int x^2\text{sech}(a+bx)\,dx}{b}$$

$$= \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{x^3 \operatorname{sech}(a+bx)}{b} - \frac{(6i)\int x \log\left(1-ie^{a+bx}\right)\,dx}{b^2} + \frac{(6i)\int x \log\left(1+ie^{a+bx}\right)\,dx}{b^2}$$

$$= \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{6ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{x^3 \operatorname{sech}(a+bx)}{b} + \frac{(6i)\int \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^3} - \frac{(6i)\int \operatorname{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^3}$$

$$= \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{6ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{x^3 \operatorname{sech}(a+bx)}{b} + \frac{(6i)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^4} - \frac{(6i)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx, x, e^{a+bx}\right)}{b^4}$$

$$= \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{6ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{6i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} - \frac{6i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} - \frac{x^3 \operatorname{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int x^3 \operatorname{sech}(a+bx) \tanh(a+bx)\,dx = \frac{3i\left(b^2x^2 \log\left(1-ie^{a+bx}\right) - b^2x^2 \log\left(1+ie^{a+bx}\right) - 2bx \operatorname{PolyLog}\left(2,-ie^{a+bx}\right) + 2bx \operatorname{PolyLog}\left(2,ie^{a+bx}\right) + 2\right.}{b^4} - \frac{x^3 \operatorname{sech}(a+bx)}{b}$$

```
[In] Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2
*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*Pol
yLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (x^3*Sech[a
 + b*x])/b
```

Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)\, dx$$

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

```
[Out] int(x^3*sech(b*x+a)^2*sinh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(90) = 180.

Time = 0.27 (sec) , antiderivative size = 672, normalized size of antiderivative = 5.95

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,b^3x^3\cosh(bx+a)+2\,b^3x^3\sinh(bx+a)+6\left(-i\,bx\cosh(bx+a)^2-2i\,bx\cosh(bx+a)\sinh(bx+a)-\right.}{}$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) + 6*(-I*b*x*cosh(b*x +
a)^2 - 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) - I*b*x*sinh(b*x + a)^2 - I*b*x)
*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(I*b*x*cosh(b*x + a)^2 + 2*I*
b*x*cosh(b*x + a)*sinh(b*x + a) + I*b*x*sinh(b*x + a)^2 + I*b*x)*dilog(-I*c
osh(b*x + a) - I*sinh(b*x + a)) + 3*(-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(
b*x + a)*sinh(b*x + a) - I*a^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x + a) +
 sinh(b*x + a) + I) + 3*(I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*sinh
(b*x + a) + I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x + a
) - I) + 3*(I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 + 2*(I*b^2*x^2
- I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b^2*x^2 - I*a^2)*sinh(b*x + a)^2
- I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 3*(-I*b^2*x^2 + (-I*b
^2*x^2 + I*a^2)*cosh(b*x + a)^2 + 2*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh
(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(-I*cosh(b*x +
 a) - I*sinh(b*x + a) + 1) + 6*(I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(
b*x + a) + I*sinh(b*x + a)^2 + I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) + 6*(-I*cosh(b*x + a)^2 - 2*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x
 + a)^2 - I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x
+ a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [F]

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^3\sinh{(a+bx)}\,\text{sech}^2\,(a+bx)\,dx$$

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a),x)
[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^3\,\text{sech}\,(bx+a)^2\sinh{(bx+a)}\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 6*integrate(x^2*e^(b*x + a)/(b
*e^(2*b*x + 2*a) + b), x)
```

Giac [F]

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^3\,\text{sech}\,(bx+a)^2\sinh{(bx+a)}\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{sech}(a+bx)\tanh(a+bx)\,dx=\int\frac{x^3\sinh(a+b\,x)}{\cosh{(a+b\,x)}^2}\,dx$$

```
[In] int((x^3*sinh(a + b*x))/cosh(a + b*x)^2,x)
[Out] int((x^3*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

3.343 $\int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{x^2\operatorname{sech}(a+bx)}{b}$$

```
[Out] 4*x*arctan(exp(b*x+a))/b^2-2*I*polylog(2,-I*exp(b*x+a))/b^3+2*I*polylog(2,I
*exp(b*x+a))/b^3-x^2*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5526, 4265, 2317, 2438}

$$\int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{x^2\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 +
((2*I)*PolyLog[2, I*E^(a + b*x)])/b^3 - (x^2*Sech[a + b*x])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\text{sech}(a+bx)}{b}+\frac{2\int x\text{sech}(a+bx)\,dx}{b}\\
&= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}-\frac{(2i)\int\log\left(1-ie^{a+bx}\right)\,dx}{b^2}+\frac{(2i)\int\log\left(1+ie^{a+bx}\right)\,dx}{b^2}\\
&= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{sech}(a+bx)}{b}-\frac{(2i)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&\quad+\frac{(2i)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{x^2\text{sech}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx$$
$$=\frac{2i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{b^3}$$
$$-\frac{x^2\text{sech}(a+bx)}{b}$$

```
[In] Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] ((2*I)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x)]) - PolyLog[2,
(-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b^3 - (x^2*Sech[a + b*x])/b
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(62) = 124.

Time = 0.75 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{2x^2 e^{bx+a}}{b(1+e^{2bx+2a})}-\frac{2i\ln(1+ie^{bx+a})x}{b^2}-\frac{2i\ln(1+ie^{bx+a})a}{b^3}+\frac{2i\ln(1-ie^{bx+a})x}{b^2}+\frac{2i\ln(1-ie^{bx+a})a}{b^3}-\frac{2i\,\text{dilog}(1+ie^{bx+a})}{b^3}+$

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2*I/b^2*ln(1+I*exp(b*x+a))*x-2*I/b^3
*ln(1+I*exp(b*x+a))*a+2*I/b^2*ln(1-I*exp(b*x+a))*x+2*I/b^3*ln(1-I*exp(b*x+a
))*a-2*I/b^3*dilog(1+I*exp(b*x+a))+2*I/b^3*dilog(1-I*exp(b*x+a))-4/b^3*a*ar
ctan(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 468, normalized size of antiderivative = 6.78

$$\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\left(b^2x^2\cosh(bx+a)+b^2x^2\sinh(bx+a)+\left(-i\cosh(bx+a)^2-2i\cosh(bx+a)\sinh(bx+a)-i\sinh\right.\right.}{}$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a) + (-I*cosh(b*x + a)^2 - 2
*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 - I)*dilog(I*cosh(b*x +
a) + I*sinh(b*x + a)) + (I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x + a
) + I*sinh(b*x + a)^2 + I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (I*a
*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh(b*x + a) + I*a*sinh(b*x + a)^2
+ I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-I*a*cosh(b*x + a)^2 - 2*I
*a*cosh(b*x + a)*sinh(b*x + a) - I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x +
a) + sinh(b*x + a) - I) + ((I*b*x + I*a)*cosh(b*x + a)^2 + 2*(I*b*x + I*a)*
cosh(b*x + a)*sinh(b*x + a) + (I*b*x + I*a)*sinh(b*x + a)^2 + I*b*x + I*a)*
log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((-I*b*x - I*a)*cosh(b*x + a)^
2 + 2*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*sinh(b*x
+ a)^2 - I*b*x - I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*cos
h(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b^
3)
```

Sympy [F]

$$\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^2\sinh{(a+bx)}\,\text{sech}^2\,(a+bx)\,dx$$

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^2\,\text{sech}\,(bx+a)^2\sinh{(bx+a)}\,dx$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 4*integrate(x*e^(b*x + a)/(b*e
^(2*b*x + 2*a) + b), x)
```

Giac [F]

$$\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx=\int x^2\,\text{sech}\,(bx+a)^2\sinh{(bx+a)}\,dx$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^2*sinh(a + b*x))/cosh(a + b*x)^2,x)
[Out] int((x^2*sinh(a + b*x))/cosh(a + b*x)^2, x)
```

3.344 $\int x\mathbf{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int x\operatorname{sech}(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{x\operatorname{sech}(a+bx)}{b}$$

```
[Out] arctan(sinh(b*x+a))/b^2-x*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5526, 3855}

$$\int x\operatorname{sech}(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{x\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[x*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] ArcTan[Sinh[a + b*x]]/b^2 - (x*Sech[a + b*x])/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
```

```
qQ[q, 1]
```

Rubi steps

$$\text{integral} = -\frac{x\text{sech}(a+bx)}{b} + \frac{\int \text{sech}(a+bx)\,dx}{b}$$
$$= \frac{\arctan(\sinh(a+bx))}{b^2} - \frac{x\text{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int x\text{sech}(a+bx)\tanh(a+bx)\,dx = \frac{2\arctan\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} - \frac{x\text{sech}(a+bx)}{b}$$

```
[In] Integrate[x*Sech[a + b*x]*Tanh[a + b*x],x]
[Out] (2*ArcTan[Tanh[a/2 + (b*x)/2]])/b^2 - (x*Sech[a + b*x])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)} + \frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{b^2} - \frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{b^2}$	59

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)+I)-I/b^2*ln(exp(b*
x+a)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.83

$$\int x\mathrm{sech}(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\,(bx\cosh(bx+a)+bx\sinh(bx+a)-(\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)}{b^2\cosh(bx+a)^2+2\,b^2\cosh(bx+a)\sinh(bx+a)+b^2\sinh(bx+}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x +
a)))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*
x + a)^2 + b^2)
```

Sympy [F]

$$\int x\mathrm{sech}(a+bx)\tanh(a+bx)\,dx = \int x\sinh(a+bx)\,\mathrm{sech}^2(a+bx)\,dx$$

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x\mathrm{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}+\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) + 2*arctan(e^(b*x + a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int x\mathrm{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\left(\pi + bxe^{(bx+a)} + \pi e^{(2\,bx+2\,a)} - \arctan\left(e^{(bx+a)}\right)e^{(2\,bx+2\,a)} - \arctan\left(e^{(bx+a)}\right)\right)}{b^2e^{(2\,bx+2\,a)} + b^2}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -2*(pi + b*x*e^(b*x + a) + pi*e^(2*b*x + 2*a) - arctan(e^(b*x + a))*e^(2*b*
x + 2*a) - arctan(e^(b*x + a)))/(b^2*e^(2*b*x + 2*a) + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int x\mathrm{sech}(a+bx)\tanh(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^4}}{b^2}\right)}{\sqrt{b^4}} - \frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^2,x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^4)^(1/2))/b^2))/(b^4)^(1/2) - (2*x*exp(a + b*x)
)/(b*(exp(2*a + 2*b*x) + 1))
```

3.345 $\int \text{sech}(a + bx) \tanh(a + bx)\, dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \text{sech}(a + bx) \tanh(a + bx)\, dx = -\frac{\text{sech}(a + bx)}{b}$$

```
[Out] -sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2686, 8}

$$\int \text{sech}(a + bx) \tanh(a + bx)\, dx = -\frac{\text{sech}(a + bx)}{b}$$

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(Sech[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}(\int 1\,dx, x, \text{sech}(a+bx))}{b}$$
$$= -\frac{\text{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
[Out] -(Sech[a + b*x]/b)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\text{sech}(bx+a)}{b}$	12
default	$-\frac{\text{sech}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}$	25

```
[In] int(sech(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -sech(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \text{sech}(a+bx)\tanh(a+bx)\,dx$$
$$= -\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+b}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \int \sinh{(a+bx)}\operatorname{sech}^{2}{(a+bx)}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} + e^{(-bx-a)})}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} + e^{(-bx-a)})}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{1}{b\cosh{(a+b\,x)}}$$

```
[In] int(sinh(a + b*x)/cosh(a + b*x)^2,x)
[Out] -1/(b*cosh(a + b*x))
```

3.346 $\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \text{Int}\left(\frac{\text{sech}(a+bx)\tanh(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x,x]
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\mathrm{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)}{x}dx$$

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\mathrm{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\sinh\left(a+bx\right)\mathrm{sech}^{2}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.75

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) - 2*integrate(e^(b*x + a)/(b*x^2
*e^(2*b*x + 2*a) + b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)}{x\cosh\left(a+b\,x\right)^{2}}\,dx$$

```
[In] int(sinh(a + b*x)/(x*cosh(a + b*x)^2),x)
[Out] int(sinh(a + b*x)/(x*cosh(a + b*x)^2), x)
```

3.347 $\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\text{Int}\left(\frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]
[Out] Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\text{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2,x]
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)}{x^{2}}dx$$

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)}{x^{2}}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\sinh\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)/x**2,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="maxima")
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) - 4*integrate(e^(b*x + a)/(b
*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)}{x^2\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^2),x)
[Out] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^2), x)
```

3.348 $\int x^m \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{sech}^2(a+bx)\tanh(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*sech(b*x+a)^2*tanh(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$$

```
[In] Int[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] Defer[Int][x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 39.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{sech}^2(a+bx) \tanh(a+bx) \, dx = \int x^m \text{sech}^2(a+bx) \tanh(a+bx) \, dx$$

```
[In] Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sech[a + b*x]^2*Tanh[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a) \, dx$$

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{sech}^2(a+bx) \tanh(a+bx) \, dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a) \, dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 110.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \text{sech}^2(a+bx) \tanh(a+bx) \, dx = \int x^m \sinh(a+bx) \operatorname{sech}^3(a+bx) \, dx$$

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a),x)
[Out] Integral(x**m*sinh(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^3 \sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^3 \sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^m*sinh(a + b*x))/cosh(a + b*x)^3,x)
[Out] int((x^m*sinh(a + b*x))/cosh(a + b*x)^3, x)
```

3.349 $\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx=\frac{3x^2}{2b^2}-\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}$$

```
[Out] 3/2*x^2/b^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-exp(2*b*x+2*a))/b^4
-1/2*x^3*sech(b*x+a)^2/b+3/2*x^2*tanh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5526, 4269, 3799, 2221, 2317, 2438}

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{3\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}-\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\text{sech}^2(a+bx)}{2b}+\frac{3x^2}{2b^2}$$

```
[In] Int[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (3*x^2)/(2*b^2) - (3*x*Log[1 + E^(2*(a + b*x))])/b^3 - (3*PolyLog[2, -E^(2*
(a + b*x))])/(2*b^4) - (x^3*Sech[a + b*x]^2)/(2*b) + (3*x^2*Tanh[a + b*x])/
(2*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3\int x^2\operatorname{sech}^2(a+bx)\,dx}{2b}\\ &= -\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{3\int x\tanh(a+bx)\,dx}{b^2}\\ &= \frac{3x^2}{2b^2}-\frac{x^3\operatorname{sech}^2(a+bx)}{2b}+\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{6\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b^2}\end{aligned}$$

$$= \frac{3x^2}{2b^2} - \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{x^3\text{sech}^2(a+bx)}{2b} + \frac{3x^2\tanh(a+bx)}{2b^2} + \frac{3\int\log\left(1+e^{2(a+bx)}\right)\,dx}{b^3}$$

$$= \frac{3x^2}{2b^2} - \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{x^3\text{sech}^2(a+bx)}{2b} + \frac{3x^2\tanh(a+bx)}{2b^2} + \frac{3\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^4}$$

$$= \frac{3x^2}{2b^2} - \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4} - \frac{x^3\text{sech}^2(a+bx)}{2b} + \frac{3x^2\tanh(a+bx)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int x^3\text{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{3\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right) + bx\left(-\frac{6bx}{1+e^{2a}} - 6\log\left(1+e^{-2(a+bx)}\right) - b^2x^2\text{sech}^2(a+bx) + 3bx\text{sech}(a)\text{sech}(a+\right.}{2b^4}$$

```
[In] Integrate[x^3*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (3*PolyLog[2, -E^(-2*(a + b*x))] + b*x*((-6*b*x)/(1 + E^(2*a)) - 6*Log[1 +
E^(-2*(a + b*x))] - b^2*x^2*Sech[a + b*x]^2 + 3*b*x*Sech[a]*Sech[a + b*x]*S
inh[b*x]))/(2*b^4)
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

method	result	si
risch	$-\frac{x^2\left(2\,\mathrm{e}^{2bx+2a}bx+3\,\mathrm{e}^{2bx+2a}+3\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2} + \frac{3x^2}{b^2} + \frac{6ax}{b^3} + \frac{3a^2}{b^4} - \frac{3x\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^3} - \frac{3\,\text{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^4} - \frac{6a\ln\left(\mathrm{e}^{bx+a}\right)}{b^4}$	1

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2+3/b
^2*x^2+6/b^3*a*x+3/b^4*a^2-3*x*ln(1+exp(2*b*x+2*a))/b^3-3/2*polylog(2,-exp(
2*b*x+2*a))/b^4-6/b^4*a*ln(exp(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1113, normalized size of antiderivative = 13.41

$$\int x^3 \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(
b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 + 6
*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh(b*
x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
 + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(cosh(b*x + a)^4 +
4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(a*cosh(
b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*co
sh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x
 + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*si
nh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x
+ a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cos
h(b*x + a) + sinh(b*x + a) - I) - 3*((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*c
osh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2
+ b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x +
a) + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b*x + a)*cosh(b*x
+ a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a
)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x +
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*
x + a))*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*
(6*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b
*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x
 + a)^3 + b^4*sinh(b*x + a)^4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh
(b*x + a)^2 + b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x
+ a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^3 \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^3 \sinh(a+bx)\,\text{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a),x)
[Out] Integral(x**3*sinh(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int x^3 \text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{3\,x^2 + \left(2\,bx^3 e^{(2\,a)} + 3\,x^2 e^{(2\,a)}\right)e^{(2\,bx)}}{b^2 e^{(4\,bx+4\,a)} + 2\,b^2 e^{(2\,bx+2\,a)} + b^2} + \frac{3\,x^2}{b^2} - \frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right) + \text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] -(3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
 + 2*b^2*e^(2*b*x + 2*a) + b^2) + 3*x^2/b^2 - 3/2*(2*b*x*log(e^(2*b*x + 2*a
) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

Giac [F]

$$\int x^3 \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^3\,\text{sech}\left(bx+a\right)^3 \sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^3*sinh(a + b*x))/cosh(a + b*x)^3,x)
[Out] int((x^3*sinh(a + b*x))/cosh(a + b*x)^3, x)
```

3.350 $\int x^2 \mathbf{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\tanh(a+bx)}{b^2}$$

```
[Out] -ln(cosh(b*x+a))/b^3-1/2*x^2*sech(b*x+a)^2/b+x*tanh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5526, 4269, 3556}

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\text{sech}^2(a+bx)}{2b}$$

```
[In] Int[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Tanh[a + b*x])
/b^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{\int x\text{sech}^2(a+bx)\,dx}{b}\\ &= -\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\tanh(a+bx)}{b^2}-\frac{\int\tanh(a+bx)\,dx}{b^2}\\ &= -\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\tanh(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int x^2\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{\log(\cosh(a+bx))}{b^3}-\frac{x^2\text{sech}^2(a+bx)}{2b}+\frac{x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b^2}+\frac{x\tanh(a)}{b^2}$$

```
[In] Integrate[x^2*Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] -(Log[Cosh[a + b*x]]/b^3) - (x^2*Sech[a + b*x]^2)/(2*b) + (x*Sech[a]*Sech[a
 + b*x]*Sinh[b*x])/b^2 + (x*Tanh[a])/b^2
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{2x}{b^2}+\frac{2a}{b^3}-\frac{2x\left(\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}+1\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}-\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b^3}$	73

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x/b^2+2/b^3*a-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+
2*a))^2-1/b^3*ln(1+exp(2*b*x+2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(40) = 80.

Time = 0.26 (sec) , antiderivative size = 378, normalized size of antiderivative = 9.00

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{2\,bx\cosh(bx+a)^4 + 8\,bx\cosh(bx+a)\sinh(bx+a)^3 + 2\,bx\sinh(bx+a)^4 - 2\,(b^2x^2-bx)\cosh(bx+a)^2 -}{}$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(b
*x + a)^4 - 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 6*b*x*cosh(b*x
 + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*lo
g(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*x + a)
^3 - (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x^2\sinh(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x**2*sinh(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\left(\left(bx^2e^{(2\,a)} - xe^{(2\,a)}\right)e^{(2\,bx)} - xe^{(4\,bx+4\,a)}\right)}{b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2} - \frac{\log\left(\left(e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,a)}\right)}{b^3}$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) - x*e^(4*b*x + 4*a))/(b^2*e^(4*b*
x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - log((e^(2*b*x + 2*a) + 1)*e^(-2*a
))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(40) = 80$.

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.38

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,b^2x^2e^{(2\,bx+2\,a)} - 2\,bxe^{(4\,bx+4\,a)} - 2\,bxe^{(2\,bx+2\,a)} + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)} - 1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(-e^{(2\,bx}\right.}{b^3e^{(4\,bx+4\,a)} + 2\,b^3e^{(2\,bx+2\,a)} + b^3}$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -(2*b^2*x^2*e^(2*b*x + 2*a) - 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a)
 + e^(4*b*x + 4*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*
b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) + 2*b^3*e
^(2*b*x + 2*a) + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int x^2 \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{\frac{x^2}{b} - \frac{x^2\,\mathrm{e}^{2\,a+2\,b\,x}}{b}}{2\,\mathrm{e}^{2\,a+2\,b\,x} + \mathrm{e}^{4\,a+4\,b\,x} + 1} - \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} + 1\right)}{b^3} + \frac{2\,x}{b^2} - \frac{b\,x^2 + 2\,x}{b^2\,\left(\mathrm{e}^{2\,a+2\,b\,x} + 1\right)}$$

```
[In] int((x^2*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

```
[Out] (x^2/b - (x^2*exp(2*a + 2*b*x))/b)/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) +
 1) - log(exp(2*a)*exp(2*b*x) + 1)/b^3 + (2*x)/b^2 - (2*x + b*x^2)/(b^2*(ex
p(2*a + 2*b*x) + 1))
```

3.351 $\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\tanh(a+bx)}{2b^2}$$

```
[Out] -1/2*x*sech(b*x+a)^2/b+1/2*tanh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5526, 3852, 8}

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx=\frac{\tanh(a+bx)}{2b^2}-\frac{x\text{sech}^2(a+bx)}{2b}$$

```
[In] Int[x*Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*(x*Sech[a + b*x]^2)/b + Tanh[a + b*x]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\int \text{sech}^2(a+bx)\,dx}{2b}\\ &= -\frac{x\text{sech}^2(a+bx)}{2b}+\frac{i\text{Subst}(\int 1\,dx,x,-i\tanh(a+bx))}{2b^2}\\ &= -\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\tanh(a+bx)}{2b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{x\text{sech}^2(a+bx)}{2b}+\frac{\tanh(a+bx)}{2b^2}$$

[In] Integrate[x*Sech[a + b*x]^2*Tanh[a + b*x],x]

[Out] -1/2*(x*Sech[a + b*x]^2)/b + Tanh[a + b*x]/(2*b^2)

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{2\,e^{2bx+2a}bx+e^{2bx+2a}+1}{b^2\left(1+e^{2bx+2a}\right)^2}$	43

[In] int(x*sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)

[Out] -(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.50

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\left(bx\sinh\left(bx+a\right)+\left(bx+1\right)\cosh\left(bx+a\right)\right)}{b^2\cosh\left(bx+a\right)^3+3\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b^2\sinh\left(bx+a\right)^3+3\,b^2\cosh\left(bx+a\right)+\left(3\,b^2\cosh\left(\right.\right.}$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(b*x*sinh(b*x + a) + (b*x + 1)*cosh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*b
^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + 3*b^2*cosh(b*x + a
) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = \int x\sinh\left(a+bx\right)\text{sech}^3\left(a+bx\right)\,dx$$

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(26) = 52$.

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.37

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,bxe^{(4\,bx+4\,a)}+\left(4\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{2\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)} + \frac{2\,bxe^{(4\,bx+4\,a)}-e^{(2\,bx+2\,a)}-1}{2\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^(4*b*x + 4*a) + (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2
*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(2*b*x*e^(4*b*x + 4*a
) - e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 6.13

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{4\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) - 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}\right.}{2\left(b^2e^{(4\,bx+4\,a)}+2\right.}$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) + 1) - 2*
e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x +
2*a) - 1) + 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)
 - log(e^(2*b*x + 2*a) + 1) + log(-e^(2*b*x + 2*a) - 1) + 2)/(b^2*e^(4*b*x
+ 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x\text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{\mathrm{e}^{2\,a+2\,b\,x}\,\left(2\,b\,x+1\right)+1}{b^2\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^2}$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^3,x)
```

```
[Out] -(exp(2*a + 2*b*x)*(2*b*x + 1) + 1)/(b^2*(exp(2*a + 2*b*x) + 1)^2)
```

3.352 $\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -1/2*sech(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

```
[In] Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/2*Sech[a + b*x]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}(\int x\,dx, x, \text{sech}(a+bx))}{b}$$
$$= -\frac{\text{sech}^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{\text{sech}^2(a+bx)}{2b}$$

```
[In] Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
[Out] -1/2*Sech[a + b*x]^2/b
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\text{sech}(bx+a)^2}{2b}$	14
default	$-\frac{\text{sech}(bx+a)^2}{2b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b(1+\mathrm{e}^{2bx+2a})^2}$	28

```
[In] int(sech(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2*sech(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \text{sech}^2(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{2\left(\cosh(bx+a)+\sinh(bx+a)\right)}{b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3+3\,b\cosh(bx+a)+\left(3\,b\cosh(bx+\right.}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x + a)
^2 + b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a),x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) + e^(-b*x - a))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)^2}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{1}{2\,b\cosh\left(a+b\,x\right)^2}$$

```
[In] int(sinh(a + b*x)/cosh(a + b*x)^3,x)
[Out] -1/(2*b*cosh(a + b*x)^2)
```

3.353 $\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)^2*tanh(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x,x]
[Out] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x}dx$$

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
[Out] int(sech(b*x+a)^3*sinh(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\sinh(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)/x,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.61

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="maxima")
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 4*integrate(1/2/(b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)}{x\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(sinh(a + b*x)/(x*cosh(a + b*x)^3),x)
[Out] int(sinh(a + b*x)/(x*cosh(a + b*x)^3), x)
```

3.354 $\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(sech(b*x+a)^2*tanh(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Int[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]
[Out] Defer[Int][(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2,x]
[Out] Integrate[(Sech[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}dx$$

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
[Out] int(sech(b*x+a)^3*sinh(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 4.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx=\int \frac{\sinh(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)/x**2,x)
[Out] Integral(sinh(a + b*x)*sech(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.56

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 12*integrate(1/2/(b^2*x^4*e^(2*b*x + 2*a)
 + b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)}{x^2\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)/(x^2*cosh(a + b*x)^3), x)
```

3.355 $\int x^m \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} - \mathrm{Int}(x^m\operatorname{sech}(a+bx),x)$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)
/((b*x)^m)-Unintegrable(x^m*sech(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

```
[In] Int[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Defer[Int][x^m*Sech[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^m \cosh(a+bx)\,dx - \int x^m\operatorname{sech}(a+bx)\,dx \\
&= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx + \frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx - \int x^m\operatorname{sech}(a+bx)\,dx \\
&= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} - \int x^m\operatorname{sech}(a+bx)\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 14.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh(a+bx)\tanh(a+bx)\,dx$$

```
[In] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 38.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a)**2,x)
[Out] Integral(x**m*sinh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x^m \sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^2}{\cosh(a+b\,x)}\,dx$$

```
[In] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x),x)
[Out] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.356 $\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 195

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} - \frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b}$$

```
[Out] -2*x^3*arctan(exp(b*x+a))/b-6*cosh(b*x+a)/b^4-3*x^2*cosh(b*x+a)/b^2+3*I*x^2
*polylog(2,-I*exp(b*x+a))/b^2-3*I*x^2*polylog(2,I*exp(b*x+a))/b^2-6*I*x*pol
ylog(3,-I*exp(b*x+a))/b^3+6*I*x*polylog(3,I*exp(b*x+a))/b^3+6*I*polylog(4,-
I*exp(b*x+a))/b^4-6*I*polylog(4,I*exp(b*x+a))/b^4+6*x*sinh(b*x+a)/b^3+x^3*s
inh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5557, 3377, 2718, 4265, 2611, 6744, 2320, 6724}

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} -\frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}-\frac{6\cosh(a+bx)}{b^4} -\frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} +\frac{6x\sinh(a+bx)}{b^3}+\frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} -\frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} -\frac{3x^2\cosh(a+bx)}{b^2}+\frac{x^3\sinh(a+bx)}{b}$$

```
[In] Int[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a + b*
x])/b^2 + ((3*I)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog
[2, I*E^(a + b*x)])/b^2 - ((6*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*
I)*x*PolyLog[3, I*E^(a + b*x)])/b^3 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/
b^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^
3*Sinh[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3\cosh(a+bx)\,dx-\int x^3\operatorname{sech}(a+bx)\,dx\\ &= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}+\frac{x^3\sinh(a+bx)}{b}+\frac{(3i)\int x^2\log\left(1-ie^{a+bx}\right)\,dx}{b}\\ &\quad-\frac{(3i)\int x^2\log\left(1+ie^{a+bx}\right)\,dx}{b}-\frac{3\int x^2\sinh(a+bx)\,dx}{b}\end{aligned}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{x^3\sinh(a+bx)}{b} - \frac{(6i)\int x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} + \frac{(6i)\int x\,\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2} + \frac{6\int x\cosh(a+bx)\,dx}{b^2}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b} + \frac{(6i)\int \text{PolyLog}\left(3,-ie^{a+bx}\right)\,dx}{b^3} - \frac{(6i)\int \text{PolyLog}\left(3,ie^{a+bx}\right)\,dx}{b^3} - \frac{6\int \sinh(a+bx)\,dx}{b^3}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b} + \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} - \frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int x^3\sinh(a+bx)\tanh(a+bx)\,dx = -\frac{i\left(-6i\cosh(a+bx) - 3ib^2x^2\cosh(a+bx) + b^3x^3\log\left(1-ie^{a+bx}\right) - b^3x^3\log\left(1+ie^{a+bx}\right) - 3b^2x^2\,\text{PolyLo}\right.}{}$$

```
[In] Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-6*I)*Cosh[a + b*x] - (3*I)*b^2*x^2*Cosh[a + b*x] + b^3*x^3*Log[1 -
 I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, (-I
```

```
)*E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 6*b*x*PolyLog[3, (-I
)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)] - 6*PolyLog[4, (-I)*E^(a +
 b*x)] + 6*PolyLog[4, I*E^(a + b*x)] + (6*I)*b*x*Sinh[a + b*x] + I*b^3*x^3*
Sinh[a + b*x]))/b^4
```

Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

[In] int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)

[Out] int(x^3*sech(b*x+a)*sinh(b*x+a)^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(162) = 324.

Time = 0.27 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.12

$$\int x^3\sinh(a+bx)\tanh(a+bx)\,dx = -\frac{b^3x^3+3\,b^2x^2-(b^3x^3-3\,b^2x^2+6\,bx-6)\cosh(bx+a)^2-2\,(b^3x^3-3\,b^2x^2+6\,bx-6)\cosh(bx+a)\,\mathrm{si}}{}$$

[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")

```
[Out] -1/2*(b^3*x^3 + 3*b^2*x^2 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)
^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) - (b^3
*x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x + 6*(I*b^2*x^2*cosh(b
*x + a) + I*b^2*x^2*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
 + 6*(-I*b^2*x^2*cosh(b*x + a) - I*b^2*x^2*sinh(b*x + a))*dilog(-I*cosh(b*x
 + a) - I*sinh(b*x + a)) + 2*(-I*a^3*cosh(b*x + a) - I*a^3*sinh(b*x + a))*l
og(cosh(b*x + a) + sinh(b*x + a) + I) + 2*(I*a^3*cosh(b*x + a) + I*a^3*sinh
(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*((-I*b^3*x^3 - I*a^3)
*cosh(b*x + a) + (-I*b^3*x^3 - I*a^3)*sinh(b*x + a))*log(I*cosh(b*x + a) +
I*sinh(b*x + a) + 1) + 2*((I*b^3*x^3 + I*a^3)*cosh(b*x + a) + (I*b^3*x^3 +
I*a^3)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 12*(I*c
osh(b*x + a) + I*sinh(b*x + a))*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a
)) + 12*(-I*cosh(b*x + a) - I*sinh(b*x + a))*polylog(4, -I*cosh(b*x + a) -
I*sinh(b*x + a)) + 12*(-I*b*x*cosh(b*x + a) - I*b*x*sinh(b*x + a))*polylog(
3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 12*(I*b*x*cosh(b*x + a) + I*b*x*sin
h(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6)/(b^4*cosh(b
*x + a) + b^4*sinh(b*x + a))
```

Sympy [F]

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^3 \sinh^2{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a)**2,x)
[Out] Integral(x**3*sinh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^3 \operatorname{sech}{(bx+a)}\sinh{(bx+a)}^2\,dx$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(b
*x) - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - 2*integrate(
x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^3 \operatorname{sech}{(bx+a)}\sinh{(bx+a)}^2\,dx$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x^3\sinh(a+b\,x)^2}{\cosh{(a+b\,x)}}\,dx$$

```
[In] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x),x)
[Out] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.357 $\int x^2 \sinh(a+bx) \tanh(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int x^2 \sinh(a+bx) \tanh(a+bx) \, dx = -\frac{2x^2 \arctan\left(e^{a+bx}\right)}{b} - \frac{2x\cosh(a+bx)}{b^2} + \frac{2ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{2ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{2i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{2i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{2\sinh(a+bx)}{b^3} + \frac{x^2\sinh(a+bx)}{b}$$

```
[Out] -2*x^2*arctan(exp(b*x+a))/b-2*x*cosh(b*x+a)/b^2+2*I*x*polylog(2,-I*exp(b*x+
a))/b^2-2*I*x*polylog(2,I*exp(b*x+a))/b^2-2*I*polylog(3,-I*exp(b*x+a))/b^3+
2*I*polylog(3,I*exp(b*x+a))/b^3+2*sinh(b*x+a)/b^3+x^2*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5557, 3377, 2717, 4265, 2611, 2320, 6724}

$$\int x^2 \sinh(a+bx) \tanh(a+bx) \, dx = -\frac{2x^2 \arctan\left(e^{a+bx}\right)}{b} - \frac{2i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{2i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{2\sinh(a+bx)}{b^3} + \frac{2ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{2ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} + \frac{x^2\sinh(a+bx)}{b}$$

```
[In] Int[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (2*x*Cosh[a + b*x])/b^2 + ((2*I)*x*PolyLog
[2, (-I)*E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 - ((2*
I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[3, I*E^(a + b*x)])/b^
3 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
```

```
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\cosh(a+bx)\,dx-\int x^2\text{sech}(a+bx)\,dx\\
&= -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{x^2\sinh(a+bx)}{b}+\frac{(2i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b}\\
&\quad-\frac{(2i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b}-\frac{2\int x\sinh(a+bx)\,dx}{b}\\
&= -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{2x\cosh(a+bx)}{b^2}+\frac{2ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}\\
&\quad-\frac{2ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{x^2\sinh(a+bx)}{b}-\frac{(2i)\int\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2}\\
&\quad+\frac{(2i)\int\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2}+\frac{2\int\cosh(a+bx)\,dx}{b^2}\\
&= -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{2x\cosh(a+bx)}{b^2}+\frac{2ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}\\
&\quad-\frac{2ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2\sinh(a+bx)}{b^3}+\frac{x^2\sinh(a+bx)}{b}\\
&\quad-\frac{(2i)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}+\frac{(2i)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&= -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{2x\cosh(a+bx)}{b^2}+\frac{2ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}\\
&\quad-\frac{2ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{2i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}\\
&\quad+\frac{2i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}+\frac{x^2\sinh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{i\left(-2ibx\cosh(a+bx)+b^2x^2\log\left(1-ie^{a+bx}\right)-b^2x^2\log\left(1+ie^{a+bx}\right)-2bx\,\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bx\,\text{P}\right.}{}$$

```
[In] Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-2*I)*b*x*Cosh[a + b*x] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*
Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog
[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a +
 b*x)] + (2*I)*Sinh[a + b*x] + I*b^2*x^2*Sinh[a + b*x]))/b^3
```

Maple [F]

$$\int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^2*sech(b*x+a)*sinh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(112) = 224.

Time = 0.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.53

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{b^2x^2-(b^2x^2-2\,bx+2)\cosh(bx+a)^2-2\,(b^2x^2-2\,bx+2)\cosh(bx+a)\sinh(bx+a)-(b^2x^2-2\,bx+}{}$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 2*b*x
+ 2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 +
2*b*x + 4*(I*b*x*cosh(b*x + a) + I*b*x*sinh(b*x + a))*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) + 4*(-I*b*x*cosh(b*x + a) - I*b*x*sinh(b*x + a))*dilog(
-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(I*a^2*cosh(b*x + a) + I*a^2*sinh(b
*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + 2*(-I*a^2*cosh(b*x + a) -
 I*a^2*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*((-I*b^2*x
```

```
^2 + I*a^2)*cosh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a))*log(I*cosh(
b*x + a) + I*sinh(b*x + a) + 1) + 2*((I*b^2*x^2 - I*a^2)*cosh(b*x + a) + (I
*b^2*x^2 - I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1
) + 4*(-I*cosh(b*x + a) - I*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*s
inh(b*x + a)) + 4*(I*cosh(b*x + a) + I*sinh(b*x + a))*polylog(3, -I*cosh(b*
x + a) - I*sinh(b*x + a)) + 2)/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F]

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2 \sinh^2{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a)**2,x)
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) - (b^2*x^2 + 2*b
*x + 2)*e^(-b*x))*e^(-a)/b^3 - 2*integrate(x^2*e^(b*x + a)/(e^(2*b*x + 2*a)
 + 1), x)
```

Giac [F]

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(a+bx)\tanh(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^2}{\cosh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x),x)
[Out] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.358 $\int x \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx=-\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\cosh(a+bx)}{b^2}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{x\sinh(a+bx)}{b}$$

```
[Out] -2*x*arctan(exp(b*x+a))/b-cosh(b*x+a)/b^2+I*polylog(2,-I*exp(b*x+a))/b^2-I*
polylog(2,I*exp(b*x+a))/b^2+x*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5557, 3377, 2718, 4265, 2317, 2438}

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx=-\frac{2x\arctan\left(e^{a+bx}\right)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}$$

```
[In] Int[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - Cosh[a + b*x]/b^2 + (I*PolyLog[2, (-I)*E^(a
+ b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])/b^2 + (x*Sinh[a + b*x])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x\cosh(a+bx)\,dx - \int x\text{sech}(a+bx)\,dx \\
&= -\frac{2x\arctan\left(e^{a+bx}\right)}{b} + \frac{x\sinh(a+bx)}{b} + \frac{i\int\log\left(1-ie^{a+bx}\right)\,dx}{b} \\
&\quad - \frac{i\int\log\left(1+ie^{a+bx}\right)\,dx}{b} - \frac{\int\sinh(a+bx)\,dx}{b}
\end{aligned}
$$

$$= -\frac{2x\arctan\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{x\sinh(a+bx)}{b} + \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^2} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{b^2}$$

$$= -\frac{2x\arctan\left(e^{a+bx}\right)}{b} - \frac{\cosh(a+bx)}{b^2} + \frac{i\,\text{PolyLog}\left(2, -ie^{a+bx}\right)}{b^2} - \frac{i\,\text{PolyLog}\left(2, ie^{a+bx}\right)}{b^2} + \frac{x\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = -\frac{i\left(-i\cosh(a+bx) + bx\log\left(1-ie^{a+bx}\right) - bx\log\left(1+ie^{a+bx}\right) - \text{PolyLog}\left(2, -ie^{a+bx}\right) + \text{PolyLog}\left(2, ie^{a}\right.}{b^2}$$

```
[In] Integrate[x*Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] ((-I)*((-I)*Cosh[a + b*x] + b*x*Log[1 - I*E^(a + b*x)] - b*x*Log[1 + I*E^(a
 + b*x)] - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)] + I*b*x
*Sinh[a + b*x]))/b^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(70) = 140.

Time = 0.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.10

method	result
risch	$\frac{(bx-1)e^{bx+a}}{2b^2} - \frac{(bx+1)e^{-bx-a}}{2b^2} + \frac{i\ln(1+ie^{bx+a})x}{b} + \frac{i\ln(1+ie^{bx+a})a}{b^2} - \frac{i\ln(1-ie^{bx+a})x}{b} - \frac{i\ln(1-ie^{bx+a})a}{b^2} + \frac{i\,\text{dilog}(1+}{b^2}$

```
[In] int(x*sech(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)+I/b*ln(1+I*exp(b*x+a
))*x+I/b^2*ln(1+I*exp(b*x+a))*a-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b
*x+a))*a+I/b^2*dilog(1+I*exp(b*x+a))-I/b^2*dilog(1-I*exp(b*x+a))+2/b^2*a*ar
ctan(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.26

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{(bx-1)\cosh{(bx+a)}^2 + 2\,(bx-1)\cosh{(bx+a)}\sinh{(bx+a)} + (bx-1)\sinh{(bx+a)}^2 - bx - 2\,(i\,\cosh{(}$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
(b*x - 1)*sinh(b*x + a)^2 - b*x - 2*(I*cosh(b*x + a) + I*sinh(b*x + a))*dil
og(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*(-I*cosh(b*x + a) - I*sinh(b*x +
a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(-I*a*cosh(b*x + a) - I*a
*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - 2*(I*a*cosh(b*x +
a) + I*a*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - 2*((-I*b*x
 - I*a)*cosh(b*x + a) + (-I*b*x - I*a)*sinh(b*x + a))*log(I*cosh(b*x + a) +
 I*sinh(b*x + a) + 1) - 2*((I*b*x + I*a)*cosh(b*x + a) + (I*b*x + I*a)*sinh
(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 1)/(b^2*cosh(b*x +
 a) + b^2*sinh(b*x + a))
```

Sympy [F]

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\sinh^2{(a+bx)}\,\mathrm{sech}\,{(a+bx)}\,dx$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx = \int x\,\mathrm{sech}\,{(bx+a)}\sinh{(bx+a)}^2\,dx$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) - (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - 2*i
ntegrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx=\int x\,\mathrm{sech}\,(bx+a)\sinh{(bx+a)}^2\,dx$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sinh(a+bx)\tanh(a+bx)\,dx=\int \frac{x\,{\sinh(a+b\,x)}^2}{\cosh\left(a+b\,x\right)}\,dx$$

```
[In] int((x*sinh(a + b*x)^2)/cosh(a + b*x),x)
[Out] int((x*sinh(a + b*x)^2)/cosh(a + b*x), x)
```

3.359 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 327, 209}

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{\sinh(a+bx)}{b}-\frac{\arctan(\sinh(a+bx))}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b} \\ &= \frac{\sinh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(a+bx)\right)}{b} \\ &= -\frac{\arctan(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{\arctan(\sinh(a+bx))}{b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
[Out] -(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	21
default	$\frac{\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	21
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	59

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\cosh\left(bx+\right.}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a
)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 + 1)
/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \int \sinh^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(bx+a)}}{2b}-\frac{e^{(-bx-a)}}{2b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\arctan\left(e^{(bx+a)}\right) - e^{(bx+a)} + e^{(-bx-a)}}{2b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^{a+bx}}{2b} - \frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-bx}}{2b}$$

```
[In] int(sinh(a + b*x)^2/cosh(a + b*x),x)
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
- exp(- a - b*x)/(2*b)
```

3.360 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx)-\text{Int}\left(\frac{\text{sech}(a+bx)}{x},x\right)$$

```
[Out] Chi(b*x)*cosh(a)+Shi(b*x)*sinh(a)-Unintegrable(sech(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
```

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] - Defer[Int][Sech[a +
 b*x]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh(a+bx)}{x}\,dx - \int \frac{\text{sech}(a+bx)}{x}\,dx \\ &= \cosh(a)\int \frac{\cosh(bx)}{x}\,dx + \sinh(a)\int \frac{\sinh(bx)}{x}\,dx - \int \frac{\text{sech}(a+bx)}{x}\,dx \\ &= \cosh(a)\text{Chi}(bx)+\sinh(a)\text{Shi}(bx) - \int \frac{\text{sech}(a+bx)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 5.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x,x]
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^{2}}{x}dx$$

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^{2}}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx=\int \frac{\sinh^{2}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2/x,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="maxima")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^2}{x\cosh(a+b\,x)}\,dx$$

```
[In] int(sinh(a + b*x)^2/(x*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)^2/(x*cosh(a + b*x)), x)
```

3.361 $\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = -\frac{\cosh(a+bx)}{x} + b\text{Chi}(bx)\sinh(a) + b\cosh(a)\text{Shi}(bx) - \text{Int}\left(\frac{\text{sech}(a+bx)}{x^2},x\right)$$

```
[Out] -cosh(b*x+a)/x+b*cosh(a)*Shi(b*x)+b*Chi(b*x)*sinh(a)-Unintegrable(sech(b*x+
a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
```

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] - Defer[Int][Sech[a + b*x]/x^2, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(a+bx)}{x^2}\,dx - \int \frac{\operatorname{sech}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x} + b\int \frac{\sinh(a+bx)}{x}\,dx - \int \frac{\operatorname{sech}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x} + (b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx + (b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx - \int \frac{\operatorname{sech}(a+bx)}{x^2}\,dx\\
&= -\frac{\cosh(a+bx)}{x} + b\operatorname{Chi}(bx)\sinh(a) + b\cosh(a)\operatorname{Shi}(bx) - \int \frac{\operatorname{sech}(a+bx)}{x^2}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2,x]
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}dx$$

```
[In] int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sinh(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^2}{x^2\cosh\left(a+b\,x\right)}\,dx$$

```
[In] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)), x)
```

3.362 $\int x^m \tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^2(a+bx)\,dx = \operatorname{Int}\left(x^m \tanh^2(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2(a+bx)\,dx$$

```
[In] Int[x^m*Tanh[a + b*x]^2,x]
[Out] Defer[Int][x^m*Tanh[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \tanh^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Tanh[a + b*x]^2,x]
[Out] Integrate[x^m*Tanh[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2\,dx$$

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 120.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \sinh^2(a+bx) \operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Integral(x**m*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
+ (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^2(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^2}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
[Out] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

3.363 $\int x^3 \tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{x^3}{b}+\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\tanh(a+bx)}{b}$$

```
[Out] -x^3/b+1/4*x^4+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a)
)/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4-x^3*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3801, 3799, 2221, 2611, 2320, 6724, 30}

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{3\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^3\tanh(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

```
[In] Int[x^3*Tanh[a + b*x]^2,x]
```

```
[Out] -(x^3/b) + x^4/4 + (3*x^2*Log[1 + E^(2*(a + b*x))])/b^2 + (3*x*PolyLog[2, -
E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, -E^(2*(a + b*x))])/(2*b^4) - (x^3*Tan
h[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\tanh(a+bx)}{b}+\frac{3\int x^2\tanh(a+bx)\,dx}{b}+\int x^3\,dx\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\tanh(a+bx)}{b}+\frac{6\int \frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^3\tanh(a+bx)}{b}-\frac{6\int x\log\left(1+e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}\\
&\quad -\frac{x^3\tanh(a+bx)}{b}-\frac{3\int \text{PolyLog}\left(2,-e^{2(a+bx)}\right)\,dx}{b^3}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}\\
&\quad -\frac{x^3\tanh(a+bx)}{b}-\frac{3\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^4}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}+\frac{3x^2\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}\\
&\quad -\frac{3\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}-\frac{x^3\tanh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int x^3\tanh^2(a+bx)\,dx=\frac{1}{4}\left(-\frac{12x\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)}{b^3}-\frac{6\,\text{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{b^4}+x^2\left(\frac{8x}{b+be^{2a}}+x^2+\frac{12\log\left(1+e^{-2(a+bx)}\right)}{b^2}-\frac{4x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b}\right)\right)$$

```
[In] Integrate[x^3*Tanh[a + b*x]^2,x]
```

```
[Out] ((-12*x*PolyLog[2, -E^(-2*(a + b*x))])/b^3 - (6*PolyLog[3, -E^(-2*(a + b*x)
)])/b^4 + x^2*((8*x)/(b + b*E^(2*a)) + x^2 + (12*Log[1 + E^(-2*(a + b*x))])
/b^2 - (4*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b))/4
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

method	result
risch	$\frac{x^4}{4}+\frac{2x^3}{b(1+e^{2bx+2a})}-\frac{6a^2\ln(e^{bx+a})}{b^4}-\frac{2x^3}{b}+\frac{6x\,a^2}{b^3}+\frac{4a^3}{b^4}+\frac{3x^2\ln(1+e^{2bx+2a})}{b^2}+\frac{3x\,\mathrm{polylog}(2,-e^{2bx+2a})}{b^3}-\frac{3\,\mathrm{polylo}}{}$

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+2*x^3/b/(1+exp(2*b*x+2*a))-6/b^4*a^2*ln(exp(b*x+a))-2*x^3/b+6*x/b^3
*a^2+4/b^4*a^3+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(2*b*x+2*a)
)/b^3-3/2*polylog(3,-exp(2*b*x+2*a))/b^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 721, normalized size of antiderivative = 8.10

$$\int x^3\tanh^2(a+bx)\,dx$$
$$=\frac{b^4x^4-8a^3+(b^4x^4-8b^3x^3-8a^3)\cosh(bx+a)^2+2(b^4x^4-8b^3x^3-8a^3)\cosh(bx+a)\sinh(bx+a)+}{}$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^4 - 8*a^3 + (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 + 2*(b
^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^4*x^4 - 8*b^3*
x^3 - 8*a^3)*sinh(b*x + a)^2 + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a
)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(I*cosh(b*x + a) + I*sinh
(b*x + a)) + 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) +
b*x*sinh(b*x + a)^2 + b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 12*(
a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)
^2 + a^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 12*(a^2*cosh(b*x + a)^2
+ 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 + a^2)*log(cosh(b
*x + a) + sinh(b*x + a) - I) + 12*(b^2*x^2 + (b^2*x^2 - a^2)*cosh(b*x + a)^
2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*sinh(b*
x + a)^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 12*(b^2*x^2 +
(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x
+ a) + (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-I*cosh(b*x + a) - I*sinh
(b*x + a) + 1) - 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(3,
 -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x
 + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [F]

$$\int x^3 \tanh^2(a+bx)\,dx = \int x^3 \sinh^2{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Integral(x**3*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int x^3 \tanh^2(a+bx)\,dx = -\frac{2\,x^3}{b} + \frac{bx^4e^{(2\,bx+2\,a)} + bx^4 + 8\,x^3}{4\,(be^{(2\,bx+2\,a)} + b)} + \frac{3\,\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)} + 1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4}$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) + b*x^4 + 8*x^3)/(b*e^(2*b*x + 2*a) +
 b) + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a
)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
```

Giac [F]

$$\int x^3 \tanh^2(a+bx)\,dx = \int x^3 \operatorname{sech}{(bx+a)}^2 \sinh{(bx+a)}^2\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh^2(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^2}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
[Out] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

3.364 $\int x^2 \tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^2 \tanh^2(a+bx)\,dx = -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}-\frac{x^2\tanh(a+bx)}{b}$$

```
[Out] -x^2/b+1/3*x^3+2*x*ln(1+exp(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3-
x^2*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3801, 3799, 2221, 2317, 2438, 30}

$$\int x^2 \tanh^2(a+bx)\,dx = \frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(e^{2(a+bx)}+1\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

```
[In] Int[x^2*Tanh[a + b*x]^2,x]
```

```
[Out] -(x^2/b) + x^3/3 + (2*x*Log[1 + E^(2*(a + b*x))])/b^2 + PolyLog[2, -E^(2*(a
 + b*x))]/b^3 - (x^2*Tanh[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\tanh(a+bx)}{b}+\frac{2\int x\tanh(a+bx)\,dx}{b}+\int x^2\,dx\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\tanh(a+bx)}{b}+\frac{4\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{2\int\log\left(1+e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}+\frac{2x\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{x^2\tanh(a+bx)}{b}-\frac{\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2(a+bx)}\right)}{b^3}
\end{aligned}$$

$$= -\frac{x^2}{b} + \frac{x^3}{3} + \frac{2x\log\left(1 + e^{2(a+bx)}\right)}{b^2} + \frac{\text{PolyLog}\left(2, -e^{2(a+bx)}\right)}{b^3} - \frac{x^2\tanh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int x^2\tanh^2(a+bx)\,dx = -\frac{\text{PolyLog}\left(2, -e^{-2(a+bx)}\right)}{b^3} + \frac{1}{3}x\left(\frac{6x}{b+be^{2a}} + x^2 + \frac{6\log\left(1 + e^{-2(a+bx)}\right)}{b^2} - \frac{3x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b}\right)$$

```
[In] Integrate[x^2*Tanh[a + b*x]^2,x]
```

```
[Out] -(PolyLog[2, -E^(-2*(a + b*x))]/b^3) + (x*((6*x)/(b + b*E^(2*a)) + x^2 + (6
*Log[1 + E^(-2*(a + b*x))])/b^2 - (3*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b))
/3
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{x^3}{3} + \frac{2x^2}{b\left(1+e^{2bx+2a}\right)} - \frac{2x^2}{b} - \frac{4ax}{b^2} - \frac{2a^2}{b^3} + \frac{2x\ln\left(1+e^{2bx+2a}\right)}{b^2} + \frac{\text{polylog}\left(2,-e^{2bx+2a}\right)}{b^3} + \frac{4a\ln\left(e^{bx+a}\right)}{b^3}$	99

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+2*x^2/b/(1+exp(2*b*x+2*a))-2*x^2/b-4*a*x/b^2-2/b^3*a^2+2*x*ln(1+exp
(2*b*x+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+4/b^3*a*ln(exp(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 515, normalized size of antiderivative = 7.92

$$\int x^2\tanh^2(a+bx)\,dx$$

$$= \frac{b^3x^3 + (b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)^2 + 2\,(b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)\sinh(bx+a) + (b^3x^3 -}{}$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^3 + (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 + 2*(b^3*x^3 -
 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 - 6*b^2*x^2 + 6*
a^2)*sinh(b*x + a)^2 + 6*a^2 + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*
x + a) + sinh(b*x + a)^2 + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dil
og(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*
x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x
+ a) + I) - 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh
(b*x + a)^2 + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 6*((b*x + a)*cosh
(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x
+ a)^2 + b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 6*((b*x + a)
*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh
(b*x + a)^2 + b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*co
sh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b
^3)
```

Sympy [F]

$$\int x^2 \tanh^2(a+bx)\,dx = \int x^2 \sinh^2{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int x^2 \tanh^2(a+bx)\,dx = -\frac{2\,x^2}{b} + \frac{bx^3e^{(2\,bx+2\,a)} + bx^3 + 6\,x^2}{3\,(be^{(2\,bx+2\,a)} + b)} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^3}$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) + b*x^3 + 6*x^2)/(b*e^(2*b*x + 2*a) +
 b) + (2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^3
```

Giac [F]

$$\int x^2 \tanh^2(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^2 \,dx$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh^2(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^2}{\cosh(a+b\,x)^2}\,dx$$

```
[In] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
[Out] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^2, x)
```

3.365 $\int x \tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int x\tanh^2(a+bx)\,dx=\frac{x^2}{2}+\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}$$

```
[Out] 1/2*x^2+ln(cosh(b*x+a))/b^2-x*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3801, 3556, 30}

$$\int x\tanh^2(a+bx)\,dx=\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}+\frac{x^2}{2}$$

```
[In] Int[x*Tanh[a + b*x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[a + b*x]]/b^2 - (x*Tanh[a + b*x])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\tanh(a+bx)}{b}+\frac{\int \tanh(a+bx)\,dx}{b}+\int x\,dx\\ &= \frac{x^2}{2}+\frac{\log(\cosh(a+bx))}{b^2}-\frac{x\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int x\tanh^2(a+bx)\,dx = \frac{b^2x^2+2\log(\cosh(a+bx))-2bx\text{sech}(a)\text{sech}(a+bx)\sinh(bx)-2bx\tanh(a)}{2b^2}$$

```
[In] Integrate[x*Tanh[a + b*x]^2,x]
[Out] (b^2*x^2 + 2*Log[Cosh[a + b*x]] - 2*b*x*Sech[a]*Sech[a + b*x]*Sinh[b*x] - 2
*b*x*Tanh[a])/(2*b^2)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result	size
parallelrisch	$\frac{x^2b^2-2\tanh(bx+a)xb-2bx-2\ln(1-\tanh(bx+a))}{2b^2}$	41
risch	$\frac{x^2}{2}-\frac{2x}{b}-\frac{2a}{b^2}+\frac{2x}{b(1+e^{2bx+2a})}+\frac{\ln(1+e^{2bx+2a})}{b^2}$	54

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(x^2*b^2-2*tanh(b*x+a)*x*b-2*b*x-2*ln(1-tanh(b*x+a)))/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.97

$$\int x\tanh^2(a+bx)\,dx = \frac{b^2x^2 + (b^2x^2 - 4\,bx)\cosh\left(bx+a\right)^2 + 2\,(b^2x^2 - 4\,bx)\cosh\left(bx+a\right)\sinh\left(bx+a\right) + (b^2x^2 - 4\,bx)\sinh\left(bx+\right.}{2\,\left(b^2\cosh\left(bx+a\right)^2 + 2\,b^2\cosh\left(bx+a\right)\right.}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 4*b*x)*cosh
(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 + 2*(cosh(b*x +
 a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b
*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

Sympy [F]

$$\int x\tanh^2(a+bx)\,dx = \int x\sinh^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)\,dx$$

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int x\tanh^2(a+bx)\,dx = -\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b} + \frac{bx^2 + \left(bx^2e^{(2\,a)} - 2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\,\left(be^{(2\,bx+2\,a)}+b\right)} + \frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) + 1/2*(b*x^2 + (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) + b) + log((e^(2*b*x + 2*a) + 1)
*e^(-2*a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int x\tanh^2(a+bx)\,dx$$
$$= \frac{b^2x^2e^{(2\,bx+2\,a)} + b^2x^2 - 4\,bxe^{(2\,bx+2\,a)} + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} + 1\right) + 2\,\log\left(e^{(2\,bx+2\,a)} + 1\right)}{2\,(b^2e^{(2\,bx+2\,a)} + b^2)}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x
 + 2*a)*log(e^(2*b*x + 2*a) + 1) + 2*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*b*
x + 2*a) + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int x\tanh^2(a+bx)\,dx = \frac{\frac{x^2\cosh(a+b\,x)}{2} - \frac{x\sinh(a+b\,x)}{b}}{\cosh\left(a+b\,x\right)} + \frac{\ln\left(\cosh(a+b\,x)\right)}{b^2}$$

```
[In] int((x*sinh(a + b*x)^2)/cosh(a + b*x)^2,x)
```

```
[Out] ((x^2*cosh(a + b*x))/2 - (x*sinh(a + b*x))/b)/cosh(a + b*x) + log(cosh(a +
b*x))/b^2
```

3.366 $\int \tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \tanh^2(a+bx)\,dx = x - \frac{\tanh(a+bx)}{b}$$

```
[Out] x-tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tanh^2(a+bx)\,dx = x - \frac{\tanh(a+bx)}{b}$$

```
[In] Int[Tanh[a + b*x]^2,x]
```

```
[Out] x - Tanh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{\tanh(a+bx)}{b} + \int 1\,dx$$
$$= x - \frac{\tanh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \tanh^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))}{b} - \frac{\tanh(a+bx)}{b}$$

```
[In] Integrate[Tanh[a + b*x]^2,x]
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
parallelrisch	$\frac{bx-\tanh(bx+a)}{b}$	17
derivativedivides	$\frac{bx+a-\tanh(bx+a)}{b}$	18
default	$\frac{bx+a-\tanh(bx+a)}{b}$	18
risch	$x+\frac{2}{b\left(1+e^{2bx+2a}\right)}$	21

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] (b*x-tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \tanh^2(a+bx)\,dx = \frac{(bx+1)\cosh(bx+a) - \sinh(bx+a)}{b\cosh(bx+a)}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + 1)*cosh(b*x + a) - sinh(b*x + a))/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \tanh^2(a+bx)\,dx = \int \sinh^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \tanh^2(a+bx)\,dx = x + \frac{a}{b} - \frac{2}{b(e^{(-2\,bx-2\,a)}+1)}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] x + a/b - 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \tanh^2(a+bx)\,dx = \frac{bx + a + \frac{2}{e^{(2\,bx+2\,a)}+1}}{b}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] (b*x + a + 2/(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \tanh^2(a+bx)\,dx = x + \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(sinh(a + b*x)^2/cosh(a + b*x)^2,x)
[Out] x + 2/(b*(exp(2*a + 2*b*x) + 1))
```

3.367 $\int \frac{\tanh^2(a+bx)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^2(a+bx)}{x} dx = \text{Int}\left(\frac{\tanh^2(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^2(a+bx)}{x} dx = \int \frac{\tanh^2(a+bx)}{x} dx$$

```
[In] Int[Tanh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Tanh[a + b*x]^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\tanh^2(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 17.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(a+bx)}{x}\,dx=\int \frac{\tanh^2(a+bx)}{x}\,dx$$

```
[In] Integrate[Tanh[a + b*x]^2/x,x]
[Out] Integrate[Tanh[a + b*x]^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}}{x}dx$$

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\tanh^2(a+bx)}{x}\,dx=\int \frac{\sinh^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="maxima")
[Out] 2/(b*x*e^(2*b*x + 2*a) + b*x) + 2*integrate(1/(b*x^2*e^(2*b*x + 2*a) + b*x^
2), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^2}{x\cosh(a+b\,x)^2}\,dx$$

```
[In] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^2),x)
[Out] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^2), x)
```

3.368 $\int \frac{\tanh^2(a+bx)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \mathrm{Int}\left(\frac{\tanh^2(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^2(a+bx)}{x^2} dx = \int \frac{\tanh^2(a+bx)}{x^2} dx$$

```
[In] Int[Tanh[a + b*x]^2/x^2,x]
[Out] Defer[Int][Tanh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\tanh^2(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 10.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\tanh^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[Tanh[a + b*x]^2/x^2,x]
[Out] Integrate[Tanh[a + b*x]^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}}{x^{2}}dx$$

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\sinh^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 5.67

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*x*e^(2*b*x + 2*a) + b*x - 2)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integr
ate(1/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^2(a+bx)}{x^2}\,dx=\int \frac{\sinh(a+b\,x)^2}{x^2\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^2),x)
```

```
[Out] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^2), x)
```

3.369 $\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \operatorname{Int}(x^m \operatorname{sech}(a+bx),x) - \operatorname{Int}\left(x^m \operatorname{sech}^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*sech(b*x+a),x)-Unintegrable(x^m*sech(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$

```
[In] Int[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Defer[Int][x^m*Sech[a + b*x], x] - Defer[Int][x^m*Sech[a + b*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{sech}(a+bx)\,dx - \int x^m \operatorname{sech}^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 52.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Integrate[x^m*Sech[a + b*x]*Tanh[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\,dx$$

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3\sinh(bx+a)^2\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^2}{\cosh(a+b\,x)^3}\,dx$$

```
[In] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
[Out] int((x^m*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.370 $\int x^3 \text{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 240

$$\int x^3 \text{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{6x\arctan\left(e^{a+bx}\right)}{b^3}+\frac{x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4}-\frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^4}+\frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{3ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{3i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4}+\frac{3i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}-\frac{3x^2\text{sech}(a+bx)}{2b^2}-\frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] 6*x*arctan(exp(b*x+a))/b^3+x^3*arctan(exp(b*x+a))/b-3*I*polylog(2,-I*exp(b*
x+a))/b^4-3/2*I*x^2*polylog(2,-I*exp(b*x+a))/b^2+3*I*polylog(2,I*exp(b*x+a)
)/b^4+3/2*I*x^2*polylog(2,I*exp(b*x+a))/b^2+3*I*x*polylog(3,-I*exp(b*x+a))/
b^3-3*I*x*polylog(3,I*exp(b*x+a))/b^3-3*I*polylog(4,-I*exp(b*x+a))/b^4+3*I*
polylog(4,I*exp(b*x+a))/b^4-3/2*x^2*sech(b*x+a)/b^2-1/2*x^3*sech(b*x+a)*tan
h(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5563, 4265, 2611, 6744, 2320, 6724, 4271, 2317, 2438}

$$\begin{aligned}\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx =& \frac{6x\arctan\left(e^{a+bx}\right)}{b^3}+\frac{x^3\arctan\left(e^{a+bx}\right)}{b}\\ &-\frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4}+\frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^4}\\ &-\frac{3i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4}+\frac{3i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}\\ &+\frac{3ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}-\frac{3ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}\\ &-\frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}\\ &+\frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}-\frac{3x^2\text{sech}(a+bx)}{2b^2}\\ &-\frac{x^3\tanh(a+bx)\text{sech}(a+bx)}{2b}\end{aligned}$$

```
[In] Int[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (6*x*ArcTan[E^(a + b*x)])/b^3 + (x^3*ArcTan[E^(a + b*x)])/b - ((3*I)*PolyLo
g[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((3*I)*PolyLog[2, I*E^(a + b*x)])/b^4 + (((3*I)/2)*x^2*PolyLog[2, I*E
^(a + b*x)])/b^2 + ((3*I)*x*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*x*Po
lyLog[3, I*E^(a + b*x)])/b^3 - ((3*I)*PolyLog[4, (-I)*E^(a + b*x)])/b^4 + (
(3*I)*PolyLog[4, I*E^(a + b*x)])/b^4 - (3*x^2*Sech[a + b*x])/(2*b^2) - (x^3
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5563

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3\text{sech}(a+bx)\,dx - \int x^3\text{sech}^3(a+bx)\,dx \\
&= \frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b} \\
&\quad - \frac{1}{2}\int x^3\text{sech}(a+bx)\,dx + \frac{3\int x\text{sech}(a+bx)\,dx}{b^2} \\
&\quad - \frac{(3i)\int x^2\log\left(1-ie^{a+bx}\right)\,dx}{b} + \frac{(3i)\int x^2\log\left(1+ie^{a+bx}\right)\,dx}{b} \\
&= \frac{6x\arctan\left(e^{a+bx}\right)}{b^3} + \frac{x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} \\
&\quad + \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b} \\
&\quad - \frac{(3i)\int\log\left(1-ie^{a+bx}\right)\,dx}{b^3} + \frac{(3i)\int\log\left(1+ie^{a+bx}\right)\,dx}{b^3} \\
&\quad + \frac{(6i)\int x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} - \frac{(6i)\int x\,\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2} \\
&\quad + \frac{(3i)\int x^2\log\left(1-ie^{a+bx}\right)\,dx}{2b} - \frac{(3i)\int x^2\log\left(1+ie^{a+bx}\right)\,dx}{2b} \\
&= \frac{6x\arctan\left(e^{a+bx}\right)}{b^3} + \frac{x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} \\
&\quad + \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} + \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} \\
&\quad - \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b} \\
&\quad - \frac{(3i)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{(3i)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} \\
&\quad - \frac{(6i)\int\text{PolyLog}\left(3,-ie^{a+bx}\right)\,dx}{b^3} + \frac{(6i)\int\text{PolyLog}\left(3,ie^{a+bx}\right)\,dx}{b^3} \\
&\quad - \frac{(3i)\int x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} + \frac{(3i)\int x\,\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2}
\end{aligned}
$$

$$= \frac{6x\arctan\left(e^{a+bx}\right)}{b^3} + \frac{x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} + \frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^4} + \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} + \frac{3ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{3ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{(3i)\int \text{PolyLog}\left(3,-ie^{a+bx}\right)\,dx}{b^3} - \frac{(3i)\int \text{PolyLog}\left(3,ie^{a+bx}\right)\,dx}{b^3}$$

$$= \frac{6x\arctan\left(e^{a+bx}\right)}{b^3} + \frac{x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} + \frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^4} + \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} + \frac{3ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{3ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} + \frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4}$$

$$= \frac{6x\arctan\left(e^{a+bx}\right)}{b^3} + \frac{x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4} - \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} + \frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^4} + \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} + \frac{3ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{3ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{3i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} + \frac{3i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} - \frac{3x^2\text{sech}(a+bx)}{2b^2} - \frac{x^3\text{sech}(a+bx)\tanh(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.02

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{-i\left(6bx\log\left(1-ie^{a+bx}\right)+b^3x^3\log\left(1-ie^{a+bx}\right)-6bx\log\left(1+ie^{a+bx}\right)-b^3x^3\log\left(1+ie^{a+bx}\right)-3(2+b\right.}{}$$

```
[In] Integrate[x^3*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -1/2*((-I)*(6*b*x*Log[1 - I*E^(a + b*x)] + b^3*x^3*Log[1 - I*E^(a + b*x)] -
 6*b*x*Log[1 + I*E^(a + b*x)] - b^3*x^3*Log[1 + I*E^(a + b*x)] - 3*(2 + b^2
*x^2)*PolyLog[2, (-I)*E^(a + b*x)] + 3*(2 + b^2*x^2)*PolyLog[2, I*E^(a + b*
x)] + 6*b*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*x*PolyLog[3, I*E^(a + b*x)]
- 6*PolyLog[4, (-I)*E^(a + b*x)] + 6*PolyLog[4, I*E^(a + b*x)]) + b^3*x^3*S
ech[a]*Sech[a + b*x]^2*Sinh[b*x] + b^2*x^2*Sech[a + b*x]*(3 + b*x*Tanh[a]))
/b^4
```

Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2 \, dx$$

```
[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2163 vs. 2(186) = 372.

Time = 0.29 (sec) , antiderivative size = 2163, normalized size of antiderivative = 9.01

$$\int x^3 \operatorname{sech}(a+bx) \tanh^2(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*cos
h(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 - 2*(b
^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) + 3*((-I*b^2*x^2 - 2*I)*cosh(b*x + a)^4 +
 4*(-I*b^2*x^2 - 2*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*x^2 - 2*I)*si
nh(b*x + a)^4 - I*b^2*x^2 + 2*(-I*b^2*x^2 - 2*I)*cosh(b*x + a)^2 + 2*(-I*b^
2*x^2 + 3*(-I*b^2*x^2 - 2*I)*cosh(b*x + a)^2 - 2*I)*sinh(b*x + a)^2 + 4*((-
I*b^2*x^2 - 2*I)*cosh(b*x + a)^3 + (-I*b^2*x^2 - 2*I)*cosh(b*x + a))*sinh(b
*x + a) - 2*I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 3*((I*b^2*x^2 + 2
*I)*cosh(b*x + a)^4 + 4*(I*b^2*x^2 + 2*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (
I*b^2*x^2 + 2*I)*sinh(b*x + a)^4 + I*b^2*x^2 + 2*(I*b^2*x^2 + 2*I)*cosh(b*x
 + a)^2 + 2*(I*b^2*x^2 + 3*(I*b^2*x^2 + 2*I)*cosh(b*x + a)^2 + 2*I)*sinh(b*
x + a)^2 + 4*((I*b^2*x^2 + 2*I)*cosh(b*x + a)^3 + (I*b^2*x^2 + 2*I)*cosh(b*
x + a))*sinh(b*x + a) + 2*I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - ((
-I*a^3 - 6*I*a)*cosh(b*x + a)^4 - 4*(I*a^3 + 6*I*a)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (-I*a^3 - 6*I*a)*sinh(b*x + a)^4 - I*a^3 - 2*(I*a^3 + 6*I*a)*cosh(
b*x + a)^2 - 2*(I*a^3 + 3*(I*a^3 + 6*I*a)*cosh(b*x + a)^2 + 6*I*a)*sinh(b*x
 + a)^2 - 4*((I*a^3 + 6*I*a)*cosh(b*x + a)^3 + (I*a^3 + 6*I*a)*cosh(b*x + a
```

```
))*sinh(b*x + a) - 6*I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((I*a^3
+ 6*I*a)*cosh(b*x + a)^4 - 4*(-I*a^3 - 6*I*a)*cosh(b*x + a)*sinh(b*x + a)^3
 + (I*a^3 + 6*I*a)*sinh(b*x + a)^4 + I*a^3 - 2*(-I*a^3 - 6*I*a)*cosh(b*x +
a)^2 - 2*(-I*a^3 + 3*(-I*a^3 - 6*I*a)*cosh(b*x + a)^2 - 6*I*a)*sinh(b*x + a
)^2 - 4*((-I*a^3 - 6*I*a)*cosh(b*x + a)^3 + (-I*a^3 - 6*I*a)*cosh(b*x + a))
*sinh(b*x + a) + 6*I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (-I*b^3*x^
3 + (-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I*a)*cosh(b*x + a)^4 - 4*(I*b^3*x^3 +
 I*a^3 + 6*I*b*x + 6*I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^3*x^3 - I*a
^3 - 6*I*b*x - 6*I*a)*sinh(b*x + a)^4 - I*a^3 - 2*(I*b^3*x^3 + I*a^3 + 6*I*
b*x + 6*I*a)*cosh(b*x + a)^2 - 2*(I*b^3*x^3 + I*a^3 + 3*(I*b^3*x^3 + I*a^3
+ 6*I*b*x + 6*I*a)*cosh(b*x + a)^2 + 6*I*b*x + 6*I*a)*sinh(b*x + a)^2 - 6*I
*b*x - 4*((I*b^3*x^3 + I*a^3 + 6*I*b*x + 6*I*a)*cosh(b*x + a)^3 + (I*b^3*x^
3 + I*a^3 + 6*I*b*x + 6*I*a)*cosh(b*x + a))*sinh(b*x + a) - 6*I*a)*log(I*co
sh(b*x + a) + I*sinh(b*x + a) + 1) - (I*b^3*x^3 + (I*b^3*x^3 + I*a^3 + 6*I*
b*x + 6*I*a)*cosh(b*x + a)^4 - 4*(-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I*a)*cos
h(b*x + a)*sinh(b*x + a)^3 + (I*b^3*x^3 + I*a^3 + 6*I*b*x + 6*I*a)*sinh(b*x
 + a)^4 + I*a^3 - 2*(-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I*a)*cosh(b*x + a)^2
- 2*(-I*b^3*x^3 - I*a^3 + 3*(-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I*a)*cosh(b*x
 + a)^2 - 6*I*b*x - 6*I*a)*sinh(b*x + a)^2 + 6*I*b*x - 4*((-I*b^3*x^3 - I*a
^3 - 6*I*b*x - 6*I*a)*cosh(b*x + a)^3 + (-I*b^3*x^3 - I*a^3 - 6*I*b*x - 6*I
*a)*cosh(b*x + a))*sinh(b*x + a) + 6*I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) + 6*(-I*cosh(b*x + a)^4 - 4*I*cosh(b*x + a)*sinh(b*x + a)^3 - I*
sinh(b*x + a)^4 + 2*(-3*I*cosh(b*x + a)^2 - I)*sinh(b*x + a)^2 - 2*I*cosh(b
*x + a)^2 + 4*(-I*cosh(b*x + a)^3 - I*cosh(b*x + a))*sinh(b*x + a) - I)*pol
ylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(I*cosh(b*x + a)^4 + 4*I*cos
h(b*x + a)*sinh(b*x + a)^3 + I*sinh(b*x + a)^4 + 2*(3*I*cosh(b*x + a)^2 + I
)*sinh(b*x + a)^2 + 2*I*cosh(b*x + a)^2 + 4*(I*cosh(b*x + a)^3 + I*cosh(b*x
 + a))*sinh(b*x + a) + I)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) +
6*(I*b*x*cosh(b*x + a)^4 + 4*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + I*b*x*si
nh(b*x + a)^4 + 2*I*b*x*cosh(b*x + a)^2 + 2*(3*I*b*x*cosh(b*x + a)^2 + I*b*
x)*sinh(b*x + a)^2 + I*b*x + 4*(I*b*x*cosh(b*x + a)^3 + I*b*x*cosh(b*x + a)
)*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(-I*b*x*
cosh(b*x + a)^4 - 4*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 - I*b*x*sinh(b*x +
a)^4 - 2*I*b*x*cosh(b*x + a)^2 + 2*(-3*I*b*x*cosh(b*x + a)^2 - I*b*x)*sinh(
b*x + a)^2 - I*b*x + 4*(-I*b*x*cosh(b*x + a)^3 - I*b*x*cosh(b*x + a))*sinh(
b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b^3*x^3 - 3*b
^2*x^2 - 3*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^2)*sinh(b*x + a))/(b^4*cosh(
b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 + 2*
b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x + a)^2
 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^3\sinh^2{(a+bx)}\,\text{sech}^3{(a+bx)}\,dx$$

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Integral(x**3*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^3\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) - (b*x^3*e^a - 3*x^2*e^a)*e^(b*
x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*(
b^2*x^3*e^a + 6*x*e^a)*e^(b*x)/(b^2*e^(2*b*x + 2*a) + b^2), x)
```

Giac [F]

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^3\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int\frac{x^3\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)^3}\,dx$$

```
[In] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
[Out] int((x^3*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.371 $\int x^2 \mathrm{sech}(a+bx) \tanh^2(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int x^2 \mathrm{sech}(a+bx) \tanh^2(a+bx) \, dx = \frac{x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{ix \,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{ix \,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{i \,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{i \,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{x \mathrm{sech}(a+bx)}{b^2} - \frac{x^2 \mathrm{sech}(a+bx) \tanh(a+bx)}{2b}$$

```
[Out] x^2*arctan(exp(b*x+a))/b+arctan(sinh(b*x+a))/b^3-I*x*polylog(2,-I*exp(b*x+a
))/b^2+I*x*polylog(2,I*exp(b*x+a))/b^2+I*polylog(3,-I*exp(b*x+a))/b^3-I*pol
ylog(3,I*exp(b*x+a))/b^3-x*sech(b*x+a)/b^2-1/2*x^2*sech(b*x+a)*tanh(b*x+a)/
b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5563, 4265, 2611, 2320, 6724, 4271, 3855}

$$\int x^2 \mathrm{sech}(a+bx) \tanh^2(a+bx) \, dx = \frac{\arctan(\sinh(a+bx))}{b^3} + \frac{x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{i \,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{i \,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{ix \,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{ix \,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{x \mathrm{sech}(a+bx)}{b^2} - \frac{x^2 \tanh(a+bx) \mathrm{sech}(a+bx)}{2b}$$

```
[In] Int[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (I*x*PolyLog[2, (
-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (I*PolyLog[3,
 (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a + b
*x])/b^2 - (x^2*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5563

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\text{sech}(a+bx)\,dx - \int x^2\text{sech}^3(a+bx)\,dx \\
&= \frac{2x^2\arctan\left(e^{a+bx}\right)}{b} - \frac{x\text{sech}(a+bx)}{b^2} - \frac{x^2\text{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{1}{2}\int x^2\text{sech}(a+bx)\,dx \\
&\quad + \frac{\int \text{sech}(a+bx)\,dx}{b^2} - \frac{(2i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b} + \frac{(2i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b} \\
&= \frac{x^2\arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{2ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} \\
&\quad + \frac{2ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{x\text{sech}(a+bx)}{b^2} - \frac{x^2\text{sech}(a+bx)\tanh(a+bx)}{2b} \\
&\quad + \frac{(2i)\int \text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} - \frac{(2i)\int \text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2} \\
&\quad + \frac{i\int x\log\left(1-ie^{a+bx}\right)\,dx}{b} - \frac{i\int x\log\left(1+ie^{a+bx}\right)\,dx}{b} \\
&= \frac{x^2\arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} \\
&\quad + \frac{ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{x\text{sech}(a+bx)}{b^2} - \frac{x^2\text{sech}(a+bx)\tanh(a+bx)}{2b} \\
&\quad + \frac{(2i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{(2i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} \\
&\quad - \frac{i\int \text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} + \frac{i\int \text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2}
\end{aligned}$$

$$= \frac{x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{ix \operatorname{PolyLog}\left(2, -ie^{a+bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(2, ie^{a+bx}\right)}{b^2} + \frac{2i \operatorname{PolyLog}\left(3, -ie^{a+bx}\right)}{b^3} - \frac{2i \operatorname{PolyLog}\left(3, ie^{a+bx}\right)}{b^3} - \frac{x\operatorname{sech}(a+bx)}{b^2} - \frac{x^2\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{i\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \frac{i\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3}$$

$$= \frac{x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{ix \operatorname{PolyLog}\left(2, -ie^{a+bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(2, ie^{a+bx}\right)}{b^2} + \frac{i \operatorname{PolyLog}\left(3, -ie^{a+bx}\right)}{b^3} - \frac{i \operatorname{PolyLog}\left(3, ie^{a+bx}\right)}{b^3} - \frac{x\operatorname{sech}(a+bx)}{b^2} - \frac{x^2\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int x^2\operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$$

$$= \frac{i\left(-4i\arctan\left(e^{a+bx}\right) + b^2x^2\log\left(1-ie^{a+bx}\right) - b^2x^2\log\left(1+ie^{a+bx}\right) - 2bx\operatorname{PolyLog}\left(2,-ie^{a+bx}\right) + 2bx\operatorname{Poly}\right.}{2b^3} - \frac{x\operatorname{sech}(a)\operatorname{sech}(a+bx)(2\cosh(a)+bx\sinh(a))}{2b^2} - \frac{x^2\operatorname{sech}(a)\operatorname{sech}^2(a+bx)\sinh(bx)}{2b}$$

```
[In] Integrate[x^2*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((I/2)*((-4*I)*ArcTan[E^(a + b*x)] + b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x
^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*Poly
Log[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(
a + b*x)]))/b^3 - (x*Sech[a]*Sech[a + b*x]*(2*Cosh[a] + b*x*Sinh[a]))/(2*b^
2) - (x^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x])/(2*b)
```

Maple [F]

$$\int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^2\,dx$$

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1577 vs. $2(118) = 236$.

Time = 0.29 (sec) , antiderivative size = 1577, normalized size of antiderivative = 11.03

$$\int x^2 \text{sech}(a+bx) \tanh^2(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 - 2*(b^2*x^2 - 2*b
*x)*cosh(b*x + a) + 2*(-I*b*x*cosh(b*x + a)^4 - 4*I*b*x*cosh(b*x + a)*sinh(
b*x + a)^3 - I*b*x*sinh(b*x + a)^4 - 2*I*b*x*cosh(b*x + a)^2 + 2*(-3*I*b*x*
cosh(b*x + a)^2 - I*b*x)*sinh(b*x + a)^2 - I*b*x + 4*(-I*b*x*cosh(b*x + a)^
3 - I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x
+ a)) + 2*(I*b*x*cosh(b*x + a)^4 + 4*I*b*x*cosh(b*x + a)*sinh(b*x + a)^3 +
I*b*x*sinh(b*x + a)^4 + 2*I*b*x*cosh(b*x + a)^2 + 2*(3*I*b*x*cosh(b*x + a)^
2 + I*b*x)*sinh(b*x + a)^2 + I*b*x + 4*(I*b*x*cosh(b*x + a)^3 + I*b*x*cosh(
b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - ((I*a^
2 + 2*I)*cosh(b*x + a)^4 - 4*(-I*a^2 - 2*I)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (I*a^2 + 2*I)*sinh(b*x + a)^4 - 2*(-I*a^2 - 2*I)*cosh(b*x + a)^2 - 2*(3*(-
I*a^2 - 2*I)*cosh(b*x + a)^2 - I*a^2 - 2*I)*sinh(b*x + a)^2 + I*a^2 - 4*((-
I*a^2 - 2*I)*cosh(b*x + a)^3 + (-I*a^2 - 2*I)*cosh(b*x + a))*sinh(b*x + a)
+ 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((-I*a^2 - 2*I)*cosh(b*x +
a)^4 - 4*(I*a^2 + 2*I)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*a^2 - 2*I)*sinh(
b*x + a)^4 - 2*(I*a^2 + 2*I)*cosh(b*x + a)^2 - 2*(3*(I*a^2 + 2*I)*cosh(b*x
+ a)^2 + I*a^2 + 2*I)*sinh(b*x + a)^2 - I*a^2 - 4*((I*a^2 + 2*I)*cosh(b*x +
 a)^3 + (I*a^2 + 2*I)*cosh(b*x + a))*sinh(b*x + a) - 2*I)*log(cosh(b*x + a)
 + sinh(b*x + a) - I) - ((-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^4 - 4*(I*b^2*x^
2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*x^2 + I*a^2)*sinh(b*x +
a)^4 - I*b^2*x^2 - 2*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 - 2*(I*b^2*x^2 + 3
*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 - I*a^2)*sinh(b*x + a)^2 + I*a^2 - 4*(
(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^3 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a))*si
nh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((I*b^2*x^2 - I*a
^2)*cosh(b*x + a)^4 - 4*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a)^3
+ (I*b^2*x^2 - I*a^2)*sinh(b*x + a)^4 + I*b^2*x^2 - 2*(-I*b^2*x^2 + I*a^2)*
cosh(b*x + a)^2 - 2*(-I*b^2*x^2 + 3*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 +
I*a^2)*sinh(b*x + a)^2 - I*a^2 - 4*((-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^3 +
(-I*b^2*x^2 + I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I
*sinh(b*x + a) + 1) + 2*(I*cosh(b*x + a)^4 + 4*I*cosh(b*x + a)*sinh(b*x + a
)^3 + I*sinh(b*x + a)^4 + 2*(3*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^2 + 2*I
*cosh(b*x + a)^2 + 4*(I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) +
I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*(-I*cosh(b*x + a)^4 -
```

```
4*I*cosh(b*x + a)*sinh(b*x + a)^3 - I*sinh(b*x + a)^4 + 2*(-3*I*cosh(b*x +
a)^2 - I)*sinh(b*x + a)^2 - 2*I*cosh(b*x + a)^2 + 4*(-I*cosh(b*x + a)^3 - I
*cosh(b*x + a))*sinh(b*x + a) - I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x
 + a)) - 2*(b^2*x^2 - 3*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^2 - 2*b*x)*sinh(b*x
 + a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sin
h(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3
)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x +
a))
```

Sympy [F]

$$\int x^2\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^2\sinh^2{(a+bx)}\,\text{sech}^3{(a+bx)}\,dx$$

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Integral(x**2*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int x^2\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^2\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 2*b^2*integrate(1/2*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - ((b*x
^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) - (b*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e
^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*arctan(e^(b*x + a))/b^3
```

Giac [F]

$$\int x^2\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x^2\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^2\sinh(a+b\,x)^2}{\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
[Out] int((x^2*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.372 $\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 91

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{x\arctan\left(e^{a+bx}\right)}{b} - \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} + \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} - \frac{\text{sech}(a+bx)}{2b^2} - \frac{x\text{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] x*arctan(exp(b*x+a))/b-1/2*I*polylog(2,-I*exp(b*x+a))/b^2+1/2*I*polylog(2,I
*exp(b*x+a))/b^2-1/2*sech(b*x+a)/b^2-1/2*x*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5563, 4265, 2317, 2438, 4270}

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{x\arctan\left(e^{a+bx}\right)}{b} - \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} + \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} - \frac{\text{sech}(a+bx)}{2b^2} - \frac{x\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

```
[In] Int[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (x*ArcTan[E^(a + b*x)])/b - ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + ((I/
2)*PolyLog[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) - (x*Sech[a + b*x
]*Tanh[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5563

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\text{sech}(a+bx)\,dx - \int x\text{sech}^3(a+bx)\,dx \\
&= \frac{2x\arctan\left(e^{a+bx}\right)}{b} - \frac{\text{sech}(a+bx)}{2b^2} - \frac{x\text{sech}(a+bx)\tanh(a+bx)}{2b} \\
&\quad - \frac{1}{2}\int x\text{sech}(a+bx)\,dx - \frac{i\int\log\left(1-ie^{a+bx}\right)\,dx}{b} + \frac{i\int\log\left(1+ie^{a+bx}\right)\,dx}{b}
\end{aligned}$$

$$= \frac{x\arctan(e^{a+bx})}{b} - \frac{\mathrm{sech}(a+bx)}{2b^2} - \frac{x\mathrm{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{i\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{i\mathrm{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{i\int \log(1-ie^{a+bx})\,dx}{2b} - \frac{i\int \log(1+ie^{a+bx})\,dx}{2b}$$

$$= \frac{x\arctan(e^{a+bx})}{b} - \frac{i\,\mathrm{PolyLog}(2,-ie^{a+bx})}{b^2} + \frac{i\,\mathrm{PolyLog}(2,ie^{a+bx})}{b^2} - \frac{\mathrm{sech}(a+bx)}{2b^2} - \frac{x\mathrm{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{i\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{2b^2} - \frac{i\mathrm{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{2b^2}$$

$$= \frac{x\arctan(e^{a+bx})}{b} - \frac{i\,\mathrm{PolyLog}(2,-ie^{a+bx})}{2b^2} + \frac{i\,\mathrm{PolyLog}(2,ie^{a+bx})}{2b^2} - \frac{\mathrm{sech}(a+bx)}{2b^2} - \frac{x\mathrm{sech}(a+bx)\tanh(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int x\mathrm{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{-i(2ia\arctan(e^{a+bx}) + (a+bx)\log(1-ie^{a+bx}) - (a+bx)\log(1+ie^{a+bx}) - \mathrm{PolyLog}(2,-ie^{a+bx}) + \mathrm{I}}{2b^2}$$

```
[In] Integrate[x*Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -1/2*((-I)*((2*I)*a*ArcTan[E^(a + b*x)] + (a + b*x)*Log[1 - I*E^(a + b*x)]
- (a + b*x)*Log[1 + I*E^(a + b*x)] - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog
[2, I*E^(a + b*x)]) + Sech[a + b*x] + b*x*Sech[a + b*x]*Tanh[a + b*x])/b^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(76) = 152.

Time = 1.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{e^{bx+a}(e^{2bx+2a}bx-bx+e^{2bx+2a}+1)}{b^2(1+e^{2bx+2a})^2} - \frac{i\ln(1+ie^{bx+a})x}{2b} - \frac{i\ln(1+ie^{bx+a})a}{2b^2} + \frac{i\ln(1-ie^{bx+a})x}{2b} + \frac{i\ln(1-ie^{bx+a})a}{2b^2} - \frac{i\,\mathrm{dilog}(}{}$

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(b*x+a)*(exp(2*b*x+2*a)*b*x-b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a)
)^2-1/2*I/b*ln(1+I*exp(b*x+a))*x-1/2*I/b^2*ln(1+I*exp(b*x+a))*a+1/2*I/b*ln(
1-I*exp(b*x+a))*x+1/2*I/b^2*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*dilog(1+I*exp(b*
x+a))+1/2*I/b^2*dilog(1-I*exp(b*x+a))-1/b^2*a*arctan(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 1064, normalized size of antiderivative = 11.69

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
^2 + 2*(b*x + 1)*sinh(b*x + a)^3 - 2*(b*x - 1)*cosh(b*x + a) - (I*cosh(b*x
+ a)^4 + 4*I*cosh(b*x + a)*sinh(b*x + a)^3 + I*sinh(b*x + a)^4 - 2*(-3*I*co
sh(b*x + a)^2 - I)*sinh(b*x + a)^2 + 2*I*cosh(b*x + a)^2 - 4*(-I*cosh(b*x +
 a)^3 - I*cosh(b*x + a))*sinh(b*x + a) + I)*dilog(I*cosh(b*x + a) + I*sinh(
b*x + a)) - (-I*cosh(b*x + a)^4 - 4*I*cosh(b*x + a)*sinh(b*x + a)^3 - I*sin
h(b*x + a)^4 - 2*(3*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^2 - 2*I*cosh(b*x +
 a)^2 - 4*(I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) - I)*dilog(-I
*cosh(b*x + a) - I*sinh(b*x + a)) - (-I*a*cosh(b*x + a)^4 - 4*I*a*cosh(b*x
+ a)*sinh(b*x + a)^3 - I*a*sinh(b*x + a)^4 - 2*I*a*cosh(b*x + a)^2 - 2*(3*I
*a*cosh(b*x + a)^2 + I*a)*sinh(b*x + a)^2 - 4*(I*a*cosh(b*x + a)^3 + I*a*co
sh(b*x + a))*sinh(b*x + a) - I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) -
(I*a*cosh(b*x + a)^4 + 4*I*a*cosh(b*x + a)*sinh(b*x + a)^3 + I*a*sinh(b*x +
 a)^4 + 2*I*a*cosh(b*x + a)^2 - 2*(-3*I*a*cosh(b*x + a)^2 - I*a)*sinh(b*x +
 a)^2 - 4*(-I*a*cosh(b*x + a)^3 - I*a*cosh(b*x + a))*sinh(b*x + a) + I*a)*l
og(cosh(b*x + a) + sinh(b*x + a) - I) - ((-I*b*x - I*a)*cosh(b*x + a)^4 - 4
*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*x - I*a)*sinh(b*x + a)
^4 - 2*(I*b*x + I*a)*cosh(b*x + a)^2 - 2*(3*(I*b*x + I*a)*cosh(b*x + a)^2 +
 I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x - 4*((I*b*x + I*a)*cosh(b*x + a)^3 +
(I*b*x + I*a)*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(I*cosh(b*x + a) + I*s
inh(b*x + a) + 1) - ((I*b*x + I*a)*cosh(b*x + a)^4 - 4*(-I*b*x - I*a)*cosh(
b*x + a)*sinh(b*x + a)^3 + (I*b*x + I*a)*sinh(b*x + a)^4 - 2*(-I*b*x - I*a)
*cosh(b*x + a)^2 - 2*(3*(-I*b*x - I*a)*cosh(b*x + a)^2 - I*b*x - I*a)*sinh(
b*x + a)^2 + I*b*x - 4*((-I*b*x - I*a)*cosh(b*x + a)^3 + (-I*b*x - I*a)*cos
h(b*x + a))*sinh(b*x + a) + I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1
) + 2*(3*(b*x + 1)*cosh(b*x + a)^2 - b*x + 1)*sinh(b*x + a))/(b^2*cosh(b*x
+ a)^4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*
```

```
cosh(b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4
*(b^2*cosh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x\sinh^2{(a+bx)}\,\text{sech}^3{(a+bx)}\,dx$$

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a)**2,x)
[Out] Integral(x*sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b*
x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 2*integrate(1/2*x*e^(b*x + a)/(e^
(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^3\sinh\,(bx+a)^2\,dx$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^3*sinh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{sech}(a+bx)\tanh^2(a+bx)\,dx=\int\frac{x\sinh(a+b\,x)^2}{\cosh\,(a+b\,x)^3}\,dx$$

```
[In] int((x*sinh(a + b*x)^2)/cosh(a + b*x)^3,x)
[Out] int((x*sinh(a + b*x)^2)/cosh(a + b*x)^3, x)
```

3.373 $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[In] Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[In] Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
[Out] ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	43
risch	$-\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}$	69

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+
a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 7.91

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - (\cosh(bx+a)^4 + 4\cosh(bx+a)\sin}{b\cosh(bx+a)^4 + 4\,b\cosh}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
+ (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a
)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 +
b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi - \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{4b}$$

[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")

[Out] 1/4*(pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,e^{a+bx}}{b\,\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)} - \frac{e^{a+bx}}{b\,\left(e^{2a+2bx}+1\right)}$$

[In] int(sinh(a + b*x)^2/cosh(a + b*x)^3,x)

[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))

3.374 $\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{x},x\right) - \operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/x,x)-Unintegrable(sech(b*x+a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
[Out] Defer[Int][Sech[a + b*x]/x, x] - Defer[Int][Sech[a + b*x]^3/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{sech}(a+bx)}{x}\,dx - \int \frac{\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x,x]
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x}dx$$

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 8.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 7.28

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) - (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^(
4*b*x + 4*a) + 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/2*(b^2*
x^2*e^a + 2*e^a)*e^(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x}\,dx=\int \frac{\sinh(a+b\,x)^2}{x\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)^2/(x*cosh(a + b*x)^3), x)
```

3.375 $\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{x^2},x\right) - \operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/x^2,x)-Unintegrable(sech(b*x+a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

```
[In] Int[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
[Out] Defer[Int][Sech[a + b*x]/x^2, x] - Defer[Int][Sech[a + b*x]^3/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{sech}(a+bx)}{x^2}\,dx - \int \frac{\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
[Out] Integrate[(Sech[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x^2}dx$$

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 10.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**2/x**2,x)
[Out] Integral(sinh(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 7.39

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] -((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) - (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3
*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/2*(
b^2*x^2*e^a + 6*e^a)*e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^3\sinh\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{sech}(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^2}{x^2\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^3),x)
[Out] int(sinh(a + b*x)^2/(x^2*cosh(a + b*x)^3), x)
```

3.376 $\int x^m \sinh^2(a+bx) \tanh(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \sinh^2(a+bx) \tanh(a+bx) \, dx = \frac{2^{-3-m} e^{2a} x^m (-bx)^{-m} \Gamma(1+m, -2bx)}{b} + \frac{2^{-3-m} e^{-2a} x^m (bx)^{-m} \Gamma(1+m, 2bx)}{b} - \text{Int}(x^m \tanh(a+bx), x)$$

```
[Out] 2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+m
,2*b*x)/b/exp(2*a)/((b*x)^m)-Unintegrable(x^m*tanh(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh^2(a+bx) \tanh(a+bx) \, dx = \int x^m \sinh^2(a+bx) \tanh(a+bx) \, dx$$

```
[In] Int[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) - Defer[Int][x^m*Tanh[a + b*x]
, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^m \cosh(a+bx)\sinh(a+bx)\,dx - \int x^m \tanh(a+bx)\,dx \\
&= \int \frac{1}{2}x^m \sinh(2a+2bx)\,dx - \int x^m \tanh(a+bx)\,dx \\
&= \frac{1}{2}\int x^m \sinh(2a+2bx)\,dx - \int x^m \tanh(a+bx)\,dx \\
&= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx - \frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx - \int x^m \tanh(a+bx)\,dx \\
&= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} \\
&\quad + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} - \int x^m \tanh(a+bx)\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 17.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$
\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx
$$

```
[In] Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x],x]
[Out] Integrate[x^m*Sinh[a + b*x]^2*Tanh[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$
\int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx
$$

```
[In] int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
[Out] int(x^m*sech(b*x+a)*sinh(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 113.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \sinh^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**m*sech(b*x+a)*sinh(b*x+a)**3,x)
[Out] Integral(x**m*sinh(a + b*x)**3*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x),x)
[Out] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.377 $\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 185

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} + \frac{3x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{3\,\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}$$

```
[Out] 3/8*x/b^3+1/4*x^3/b+1/4*x^4-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-e
xp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(
2*b*x+2*a))/b^4-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sinh(b*
x+a)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5557, 5480, 3392, 30, 2715, 8, 3799, 2221, 2611, 6744, 2320, 6724}

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{3\,\text{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4} - \frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4} + \frac{3x\,\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{3x\sinh^2(a+bx)}{4b^3} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2} - \frac{x^3\log\left(e^{2(a+bx)}+1\right)}{b} + \frac{x^3\sinh^2(a+bx)}{2b} + \frac{3x}{8b^3} + \frac{x^3}{4b} + \frac{x^4}{4}$$

```
[In] Int[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) + x^4/4 - (x^3*Log[1 + E^(2*(a + b*x))])/b - (3*x
^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) + (3*x*PolyLog[3, -E^(2*(a + b*x))
])/(2*b^3) - (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Si
nh[a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*S
inh[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\cosh(a+bx)\sinh(a+bx)\,dx-\int x^3\tanh(a+bx)\,dx\\
&=\frac{x^4}{4}+\frac{x^3\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx-\frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}\\
&=\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int\sinh^2(a+bx)\,dx}{4b^3}+\frac{3\int x^2\,dx}{4b}+\frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)\,dx}{b}\\
&=\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}\\
&\quad-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3\int 1\,dx}{8b^3}+\frac{3\int x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)\,dx}{b^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}\\
&\quad+\frac{3x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}\\
&\quad-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)\,dx}{2b^3}\\
&= \frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}\\
&\quad+\frac{3x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}\\
&\quad-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\text{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{4b^4}\\
&= \frac{3x}{8b^3}+\frac{x^3}{4b}+\frac{x^4}{4}-\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}\\
&\quad+\frac{3x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{3\operatorname{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}\\
&\quad-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}\\
&\quad+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$
\int x^3\sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{\cosh(a)(\cosh(a)+\sinh(a))\left(4b^4x^4-6bx\cosh(2(a+bx))-4b^3x^3\cosh(2(a+bx))+16b^3x^3\log\left(1+e^{-\dots}\right.\right.}{}
$$

```
[In] Integrate[x^3*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/8*(Cosh[a]*(Cosh[a] + Sinh[a])*(4*b^4*x^4 - 6*b*x*Cosh[2*(a + b*x)] - 4*
b^3*x^3*Cosh[2*(a + b*x)] + 16*b^3*x^3*Log[1 + E^(-2*(a + b*x))] - 24*b^2*x
^2*PolyLog[2, -E^(-2*(a + b*x))] - 24*b*x*PolyLog[3, -E^(-2*(a + b*x))] - 1
2*PolyLog[4, -E^(-2*(a + b*x))] + 3*Sinh[2*(a + b*x)] + 6*b^2*x^2*Sinh[2*(a
 + b*x)]))/(b^4*(1 + E^(2*a)))
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x^4}{4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)\,\mathrm{e}^{2bx+2a}}{32b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)\,\mathrm{e}^{-2bx-2a}}{32b^4}+\frac{3a^4}{2b^4}-\frac{3\,\mathrm{polylog}(4,-\mathrm{e}^{2bx+2a})}{4b^4}+\frac{2a^3x}{b^3}-\frac{x^3\ln(1}{}$

```
[In] int(x^3*sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*x
^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)+3/2/b^4*a^4-3/4*polylog(4,-exp(2*
b*x+2*a))/b^4+2/b^3*a^3*x-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp
(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-2/b^4*a^3*ln(exp(b*x+
a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 966, normalized size of antiderivative = 5.22

$$\int x^3\sinh^2(a+bx)\tanh(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3*x
^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 + 8*(b^4*x^4 - 2*a^
4)*cosh(b*x + a)^2 + 2*(4*b^4*x^4 - 8*a^4 + 3*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*
x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*(b^2*x^2*cosh(b*x + a)
^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog
(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x
^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-I*cosh(b*x
 + a) - I*sinh(b*x + a)) + 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*si
nh(b*x + a) + a^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) +
 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b*x
 + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 32*((b^3*x^3 + a^3)*cosh(
b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 + a^3
)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 32*((b^3*x^
3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) +
(b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1
) - 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 192*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, -I*cosh(b*x + a) -
 I*sinh(b*x + a)) + 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x
```

```
 + a) + b*x*sinh(b*x + a)^2)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a))
+ 192*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b
*x + a)^2)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((4*b^3*x^3 -
 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^3 + 4*(b^4*x^4 - 2*a^4)*cosh(b*x + a)
)*sinh(b*x + a) + 3)/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x +
a) + b^4*sinh(b*x + a)^2)
```

Sympy [F]

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^3 \sinh^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**3*sech(b*x+a)*sinh(b*x+a)**3,x)
[Out] Integral(x**3*sinh(a + b*x)**3*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{1}{2}x^4 - \frac{(8b^4x^4e^{(2a)} - (4b^3x^3e^{(4a)} - 6b^2x^2e^{(4a)} + 6bxe^{(4a)} - 3e^{(4a)})e^{(2bx)} - (4b^3x^3 + 6b^2x^2 + 6bx + 3)e^{(-2bx)}}{32b^4} - \frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right) + 6b^2x^2\mathrm{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx\mathrm{Li}_3\left(-e^{(2bx+2a)}\right) + 3\mathrm{Li}_4\left(-e^{(2bx+2a)}\right)}{3b^4}$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/2*x^4 - 1/32*(8*b^4*x^4*e^(2*a) - (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
+ 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3
)*e^(-2*b*x))*e^(-2*a)/b^4 - 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^
2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*poly
log(4, -e^(2*b*x + 2*a)))/b^4
```

Giac [F]

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)\sinh(bx+a)^3\,dx$$

```
[In] integrate(x^3*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^3}{\cosh(a+b\,x)}\,dx$$

```
[In] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x),x)
[Out] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.378 $\int x^2 \sinh^2(a+bx) \tanh(a+bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int x^2 \sinh^2(a+bx) \tanh(a+bx)\, dx = \frac{x^2}{4b} + \frac{x^3}{3} - \frac{x^2 \log\left(1+e^{2(a+bx)}\right)}{b} - \frac{x \operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2} + \frac{\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{x \cosh(a+bx) \sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2 \sinh^2(a+bx)}{2b}$$

```
[Out] 1/4*x^2/b+1/3*x^3-x^2*ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b
^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4
*sinh(b*x+a)^2/b^3+1/2*x^2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5557, 5480, 3391, 30, 3799, 2221, 2611, 2320, 6724}

$$\int x^2 \sinh^2(a+bx) \tanh(a+bx)\, dx = \frac{\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{\sinh^2(a+bx)}{4b^3} - \frac{x \operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2} - \frac{x \sinh(a+bx) \cosh(a+bx)}{2b^2} - \frac{x^2 \log\left(e^{2(a+bx)}+1\right)}{b} + \frac{x^2 \sinh^2(a+bx)}{2b} + \frac{x^2}{4b} + \frac{x^3}{3}$$

```
[In] Int[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] x^2/(4*b) + x^3/3 - (x^2*Log[1 + E^(2*(a + b*x))])/b - (x*PolyLog[2, -E^(2*
(a + b*x))])/b^2 + PolyLog[3, -E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2
*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\cosh(a+bx)\sinh(a+bx)\,dx-\int x^2\tanh(a+bx)\,dx\\
&= \frac{x^3}{3}+\frac{x^2\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx-\frac{\int x\sinh^2(a+bx)\,dx}{b}\\
&= \frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{\int x\,dx}{2b}+\frac{2\int x\log\left(1+e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}\\
&\quad-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{\int \text{PolyLog}\left(2,-e^{2(a+bx)}\right)\,dx}{b^2}
\end{aligned}$$

$$=\frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}(2,-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3}$$

$$=\frac{x^2}{4b}+\frac{x^3}{3}-\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}+\frac{\mathrm{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\sinh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

$$\int x^2\sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh(a)(\cosh(a)+\sinh(a))\left(-8b^3x^3+3\cosh(2(a+bx))+6b^2x^2\cosh(2(a+bx))-24b^2x^2\log\left(1+e^{-2(a+b\right.\right.}}{12b^3\left(1+e^{2a}\right)}$$

```
[In] Integrate[x^2*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] (Cosh[a]*(Cosh[a] + Sinh[a])*(-8*b^3*x^3 + 3*Cosh[2*(a + b*x)] + 6*b^2*x^2*
Cosh[2*(a + b*x)] - 24*b^2*x^2*Log[1 + E^(-2*(a + b*x))] + 24*b*x*PolyLog[2
, -E^(-2*(a + b*x))] + 12*PolyLog[3, -E^(-2*(a + b*x))] - 6*b*x*Sinh[2*(a +
 b*x)]))/(12*b^3*(1 + E^(2*a)))
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17

method	result
risch	$\frac{x^3}{3}+\frac{(2x^2b^2-2bx+1)\mathrm{e}^{2bx+2a}}{16b^3}+\frac{(2x^2b^2+2bx+1)\mathrm{e}^{-2bx-2a}}{16b^3}+\frac{2a^2\ln(\mathrm{e}^{bx+a})}{b^3}-\frac{2a^2x}{b^2}-\frac{4a^3}{3b^3}-\frac{x^2\ln(1+\mathrm{e}^{2bx+2a})}{b}-\frac{x\,\mathrm{polylo}}{}$

```
[In] int(x^2*sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+1
)/b^3*exp(-2*b*x-2*a)+2/b^3*a^2*ln(exp(b*x+a))-2/b^2*a^2*x-4/3/b^3*a^3-x^2*
ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(
2*b*x+2*a))/b^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 789, normalized size of antiderivative = 6.07

$$\int x^2 \sinh^2(a+bx) \tanh(a+bx)\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)^4
 + 6*b^2*x^2 + 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 + 2*(8*b^3*x^3 + 16*a^3
 + 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x - 96*
(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a
)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 96*(b*x*cosh(b*x + a)^2 + 2
*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-I*cosh(b*x +
 a) - I*sinh(b*x + a)) - 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh
(b*x + a) + a^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) - 4
8*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x +
 a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 48*((b^2*x^2 - a^2)*cosh(b*
x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*
sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 48*((b^2*x^2
- a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b
^2*x^2 - a^2)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1)
+ 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*po
lylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 96*(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, -I*cosh(b*x + a) - I*s
inh(b*x + a)) + 4*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 + 8*(b^3*x^3 +
 2*a^3)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh
(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2)
```

Sympy [F]

$$\int x^2 \sinh^2(a+bx) \tanh(a+bx)\, dx = \int x^2 \sinh^3{(a+bx)} \operatorname{sech}{(a+bx)}\, dx$$

```
[In] integrate(x**2*sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**3*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{2}{3}x^3 - \frac{\left(16\,b^3x^3e^{(2\,a)} - 3\left(2\,b^2x^2e^{(4\,a)} - 2\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(2\,bx)} - 3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3} - \frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 2/3*x^3 - 1/48*(16*b^3*x^3*e^(2*a) - 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) +
 e^(4*a))*e^(2*b*x) - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3 -
1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - p
olylog(3, -e^(2*b*x + 2*a)))/b^3
```

Giac [F]

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int x^2\,\mathrm{sech}\,(bx+a)\sinh\,(bx+a)^3\,dx$$

```
[In] integrate(x^2*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x^2\,{\sinh(a+b\,x)}^3}{\cosh\,(a+b\,x)}\,dx$$

```
[In] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x),x)
[Out] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.379 $\int x \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{x}{4b}+\frac{x^2}{2}-\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}$$

```
[Out] 1/4*x/b+1/2*x^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2
-1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5557, 5480, 2715, 8, 3799, 2221, 2317, 2438}

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx=-\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}-\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}+\frac{x^2}{2}$$

```
[In] Int[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] x/(4*b) + x^2/2 - (x*Log[1 + E^(2*(a + b*x))])/b - PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/
(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x\cosh(a+bx)\sinh(a+bx)\,dx - \int x\tanh(a+bx)\,dx \\
&= \frac{x^2}{2} + \frac{x\sinh^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx - \frac{\int \sinh^2(a+bx)\,dx}{2b} \\
&= \frac{x^2}{2} - \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} \\
&\quad + \frac{x\sinh^2(a+bx)}{2b} + \frac{\int 1\,dx}{4b} + \frac{\int \log\left(1+e^{2(a+bx)}\right)\,dx}{b} \\
&= \frac{x}{4b} + \frac{x^2}{2} - \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} \\
&\quad + \frac{x\sinh^2(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^2} \\
&= \frac{x}{4b} + \frac{x^2}{2} - \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} \\
&\quad - \frac{\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{x\sinh^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$
\int x\sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{-4a^2+4b^2x^2-2bx\cosh(2(a+bx))+8bx\log\left(1+e^{-2(a+bx)}\right)-4\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)+\sinh(2(a}{8b^2}
$$

```
[In] Integrate[x*Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

```
[Out] -1/8*(-4*a^2 + 4*b^2*x^2 - 2*b*x*Cosh[2*(a + b*x)] + 8*b*x*Log[1 + E^(-2*(a
 + b*x))] - 4*PolyLog[2, -E^(-2*(a + b*x))] + Sinh[2*(a + b*x)])/b^2
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result	siz
risch	$\frac{x^2}{2}+\frac{(2bx-1)e^{2bx+2a}}{16b^2}+\frac{(2bx+1)e^{-2bx-2a}}{16b^2}+\frac{2ax}{b}+\frac{a^2}{b^2}-\frac{x\ln(1+e^{2bx+2a})}{b}-\frac{\text{polylog}(2,-e^{2bx+2a})}{2b^2}-\frac{2a\ln(e^{bx+a})}{b^2}$	11

```
[In] int(x*sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*a
)+2/b*a*x+a^2/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b
^2-2/b^2*a*ln(exp(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 558, normalized size of antiderivative = 6.27

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$=\frac{(2\,bx-1)\cosh\left(bx+a\right)^4+4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(2\,bx-1)\sinh\left(bx+a\right)^4+8\,(b^2x^2-}{}$$

```
[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 + 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2 +
2*(4*b^2*x^2 + 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 + 2*b
*x - 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)
*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 16*(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x
 + a)) + 16*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b
*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 16*(a*cosh(b*x + a)^2 +
 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x + a) + s
inh(b*x + a) - I) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x +
a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(
b*x + a) + 1) - 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*s
inh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x
 + a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x
+ a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*
x + a) + b^2*sinh(b*x + a)^2)
```

Sympy [F]

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx=\int x\sinh^3{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

[In] integrate(x*sech(b*x+a)*sinh(b*x+a)**3,x)

[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\begin{aligned}\int x\sinh^2(a+bx)\tanh(a+bx)\,dx&=x^2-\frac{\left(8\,b^2x^2e^{(2\,a)}-\left(2\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(2\,bx)}-(2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2}\\&\quad-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}\end{aligned}$$

[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")

[Out] x^2 - 1/16*(8*b^2*x^2*e^(2*a) - (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) - (2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2

Giac [F]

$$\int x\sinh^2(a+bx)\tanh(a+bx)\,dx=\int x\operatorname{sech}{(bx+a)}\sinh{(bx+a)}^3\,dx$$

[In] integrate(x*sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")

[Out] integrate(x*sech(b*x + a)*sinh(b*x + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \frac{x\sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)}\,dx$$

```
[In] int((x*sinh(a + b*x)^3)/cosh(a + b*x),x)
[Out] int((x*sinh(a + b*x)^3)/cosh(a + b*x), x)
```

3.380 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[Out] 1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

```
[In] Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
[Out] Cosh[a + b*x]^2/(2*b) - Log[Cosh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh^2(a+bx)}{2b} - \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{-\frac{1}{2}\cosh^2(a+bx)+\log(\cosh(a+bx))}{b}$$

```
[In] Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
[Out] -((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
default	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
risch	$x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{2a}{b}-\frac{\ln(1+e^{2bx+2a})}{b}$	54

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*sinh(b*x+a)^2-ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.04

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{8\,bx\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)^4+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sinh\left(bx+a\right)^4+2\,\left(4\,bx+3\,\cosh\left(\right.\right.}{}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a
)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*co
sh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) + cos
h(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sin
h(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \sinh^3\left(a+bx\right)\operatorname{sech}\left(a+bx\right)\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*b
*x - 2*a) + 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 8\,a + e^{(2\,bx+2\,a)} - 8\,\log\left(e^{(2\,bx+2\,a)} + 1\right)}{8\,b}$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*a + e^(2*b*x + 2*
a) - 8*log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = x - \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} + 1\right)}{b} + \frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b}$$

```
[In] int(sinh(a + b*x)^3/cosh(a + b*x),x)
```

```
[Out] x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

3.381 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) - \text{Int}\left(\frac{\tanh(a+bx)}{x},x\right)$$

```
[Out] 1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)-Unintegrable(tanh(b*x+a)/
x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 - Def
er[Int][Tanh[a + b*x]/x, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \int \frac{\sinh(2a+2bx)}{2x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x}\,dx \\
&= \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) - \int \frac{\tanh(a+bx)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 12.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx$$

```
[In] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x,x]
[Out] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}dx$$

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3/x,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) + 2*integrate(1/(x*e^(2*b*x
 + 2*a) + x), x) - log(x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^3}{x\cosh(a+b\,x)}\,dx$$

```
[In] int(sinh(a + b*x)^3/(x*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)^3/(x*cosh(a + b*x)), x)
```

3.382 $\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx) - \text{Int}\left(\frac{\tanh(a+bx)}{x^2},x\right)$$

```
[Out] b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x-Uninteg
rable(tanh(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Int[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] - Defer[Int][Tanh[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx - \int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= \int \frac{\sinh(2a+2bx)}{2x^2}\,dx - \int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx - \int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x} + b\int \frac{\cosh(2a+2bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= -\frac{\sinh(2a+2bx)}{2x} + (b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx\\
&\quad + (b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx - \int \frac{\tanh(a+bx)}{x^2}\,dx\\
&= b\cosh(2a)\mathrm{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\mathrm{Shi}(2bx) - \int \frac{\tanh(a+bx)}{x^2}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 10.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2,x]
[Out] Integrate[(Sinh[a + b*x]^2*Tanh[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}dx$$

```
[In] int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
[Out] int(sech(b*x+a)*sinh(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)**3/x**2,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) + 1/x + 2
*integrate(1/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)\sinh(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)*sinh(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)*sinh(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(a+bx)\tanh(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{x^2\cosh(a+b\,x)}\,dx$$

```
[In] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)),x)
[Out] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)), x)
```

3.383 $\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} - \text{Int}(x^m\text{sech}(a+bx)\tanh(a+bx),x)$$

```
[Out] -CannotIntegrate(x^m*sech(b*x+a)*tanh(b*x+a),x)+1/2*exp(a)*x^m*GAMMA(1+m,-b
*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)
```

Rubi [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx$$

```
[In] Int[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) - Defer[Int][x^m*Sech[a + b*x]*Tanh[a + b*x], x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^m \sinh(a+bx)\,dx - \int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx\\
&= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx - \frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx - \int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx\\
&= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}\\
&\quad - \int x^m \operatorname{sech}(a+bx)\tanh(a+bx)\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 45.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$
\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx
$$

```
[In] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Integrate[x^m*Sinh[a + b*x]*Tanh[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$
\int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx
$$

```
[In] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
[Out] int(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$
\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx
$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*sech(b*x+a)**2*sinh(b*x+a)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^3}{\cosh(a+b\,x)^2}\,dx$$

```
[In] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
[Out] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

3.384 $\int x^3 \sinh(a+bx) \tanh^2(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int x^3 \sinh(a+bx) \tanh^2(a+bx) \, dx = -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} + \frac{6ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{6ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{6i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} + \frac{6i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} + \frac{x^3 \operatorname{sech}(a+bx)}{b} - \frac{6\sinh(a+bx)}{b^4} - \frac{3x^2\sinh(a+bx)}{b^2}$$

```
[Out] -6*x^2*arctan(exp(b*x+a))/b^2+6*x*cosh(b*x+a)/b^3+x^3*cosh(b*x+a)/b+6*I*x*p
olylog(2,-I*exp(b*x+a))/b^3-6*I*x*polylog(2,I*exp(b*x+a))/b^3-6*I*polylog(3
,-I*exp(b*x+a))/b^4+6*I*polylog(3,I*exp(b*x+a))/b^4+x^3*sech(b*x+a)/b-6*sin
h(b*x+a)/b^4-3*x^2*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

= {5557, 3377, 2717, 5526, 4265, 2611, 2320, 6724}

$$\int x^3 \sinh(a+bx) \tanh^2(a+bx)\, dx = -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6i\,\text{PolyLog}\left(3, -ie^{a+bx}\right)}{b^4} + \frac{6i\,\text{PolyLog}\left(3, ie^{a+bx}\right)}{b^4} - \frac{6\sinh(a+bx)}{b^4} + \frac{6ix\,\text{PolyLog}\left(2, -ie^{a+bx}\right)}{b^3} - \frac{6ix\,\text{PolyLog}\left(2, ie^{a+bx}\right)}{b^3} + \frac{6x\cosh(a+bx)}{b^3} - \frac{3x^2\sinh(a+bx)}{b^2} + \frac{x^3\cosh(a+bx)}{b} + \frac{x^3\text{sech}(a+bx)}{b}$$

```
[In] Int[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a +
b*x])/b + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((6*I)*x*PolyLog[2,
I*E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4 + ((6*I)*Pol
yLog[3, I*E^(a + b*x)])/b^4 + (x^3*Sech[a + b*x])/b - (6*Sinh[a + b*x])/b^4
 - (3*x^2*Sinh[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3\sinh(a+bx)\,dx-\int x^3\operatorname{sech}(a+bx)\tanh(a+bx)\,dx\\
&=\frac{x^3\cosh(a+bx)}{b}+\frac{x^3\operatorname{sech}(a+bx)}{b}-\frac{3\int x^2\cosh(a+bx)\,dx}{b}-\frac{3\int x^2\operatorname{sech}(a+bx)\,dx}{b}\\
&=-\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}+\frac{x^3\cosh(a+bx)}{b}+\frac{x^3\operatorname{sech}(a+bx)}{b}-\frac{3x^2\sinh(a+bx)}{b^2}\\
&\quad+\frac{(6i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b^2}-\frac{(6i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b^2}+\frac{6\int x\sinh(a+bx)\,dx}{b^2}
\end{aligned}
$$

$$= -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} + \frac{6x \cosh(a+bx)}{b^3} + \frac{x^3 \cosh(a+bx)}{b} + \frac{6ix \operatorname{PolyLog}\left(2, -ie^{a+bx}\right)}{b^3} - \frac{6ix \operatorname{PolyLog}\left(2, ie^{a+bx}\right)}{b^3} + \frac{x^3 \operatorname{sech}(a+bx)}{b} - \frac{3x^2 \sinh(a+bx)}{b^2} - \frac{(6i) \int \operatorname{PolyLog}\left(2, -ie^{a+bx}\right) dx}{b^3} + \frac{(6i) \int \operatorname{PolyLog}\left(2, ie^{a+bx}\right) dx}{b^3} - \frac{6 \int \cosh(a+bx) dx}{b^3}$$

$$= -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} + \frac{6x \cosh(a+bx)}{b^3} + \frac{x^3 \cosh(a+bx)}{b} + \frac{6ix \operatorname{PolyLog}\left(2, -ie^{a+bx}\right)}{b^3} - \frac{6ix \operatorname{PolyLog}\left(2, ie^{a+bx}\right)}{b^3} + \frac{x^3 \operatorname{sech}(a+bx)}{b} - \frac{6 \sinh(a+bx)}{b^4} - \frac{3x^2 \sinh(a+bx)}{b^2} - \frac{(6i)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x} dx, x, e^{a+bx}\right)}{b^4} + \frac{(6i)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x} dx, x, e^{a+bx}\right)}{b^4}$$

$$= -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} + \frac{6x \cosh(a+bx)}{b^3} + \frac{x^3 \cosh(a+bx)}{b} + \frac{6ix \operatorname{PolyLog}\left(2, -ie^{a+bx}\right)}{b^3} - \frac{6ix \operatorname{PolyLog}\left(2, ie^{a+bx}\right)}{b^3} - \frac{6i \operatorname{PolyLog}\left(3, -ie^{a+bx}\right)}{b^4} + \frac{6i \operatorname{PolyLog}\left(3, ie^{a+bx}\right)}{b^4} + \frac{x^3 \operatorname{sech}(a+bx)}{b} - \frac{6 \sinh(a+bx)}{b^4} - \frac{3x^2 \sinh(a+bx)}{b^2}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

$$\int x^3 \sinh(a+bx) \tanh^2(a+bx) \, dx = \frac{-3i\left(b^2x^2 \log\left(1-ie^{a+bx}\right) - b^2x^2 \log\left(1+ie^{a+bx}\right) - 2bx \operatorname{PolyLog}\left(2, -ie^{a+bx}\right) + 2bx \operatorname{PolyLog}\left(2, ie^{a+bx}\right) + \right.}{}$$

```
[In] Integrate[x^3*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((-3*I)*(b^2*x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] -
2*b*x*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*Po
lyLog[3, (-I)*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]) + b^3*x^3*Sech[a
+ b*x] + Cosh[b*x]*(b*x*(6 + b^2*x^2)*Cosh[a] - 3*(2 + b^2*x^2)*Sinh[a]) +
(-3*(2 + b^2*x^2)*Cosh[a] + b*x*(6 + b^2*x^2)*Sinh[a])*Sinh[b*x])/b^4
```

Maple [F]

$$\int x^3 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3 \, dx$$

```
[In] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)
[Out] int(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1225 vs. 2(139) = 278.

Time = 0.28 (sec) , antiderivative size = 1225, normalized size of antiderivative = 7.56

$$\int x^3 \sinh(a+bx) \tanh^2(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b^
2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 + 6*(b^3*x^3 + 2*b*x)*cosh(b
*x + a)^2 + 6*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^2
+ 2*b*x)*sinh(b*x + a)^2 + 6*b*x - 12*(I*b*x*cosh(b*x + a)^3 + 3*I*b*x*cosh
(b*x + a)*sinh(b*x + a)^2 + I*b*x*sinh(b*x + a)^3 + I*b*x*cosh(b*x + a) + (
3*I*b*x*cosh(b*x + a)^2 + I*b*x)*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*s
inh(b*x + a)) - 12*(-I*b*x*cosh(b*x + a)^3 - 3*I*b*x*cosh(b*x + a)*sinh(b*x
 + a)^2 - I*b*x*sinh(b*x + a)^3 - I*b*x*cosh(b*x + a) + (-3*I*b*x*cosh(b*x
+ a)^2 - I*b*x)*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) -
6*(I*a^2*cosh(b*x + a)^3 + 3*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2 + I*a^2*si
nh(b*x + a)^3 + I*a^2*cosh(b*x + a) + (3*I*a^2*cosh(b*x + a)^2 + I*a^2)*sin
h(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - 6*(-I*a^2*cosh(b*x + a
)^3 - 3*I*a^2*cosh(b*x + a)*sinh(b*x + a)^2 - I*a^2*sinh(b*x + a)^3 - I*a^2
*cosh(b*x + a) + (-3*I*a^2*cosh(b*x + a)^2 - I*a^2)*sinh(b*x + a))*log(cosh
(b*x + a) + sinh(b*x + a) - I) - 6*((-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^3 +
3*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a)^2 + (-I*b^2*x^2 + I*a^2)
*sinh(b*x + a)^3 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a) + (-I*b^2*x^2 + 3*(-I
*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + I*a^2)*sinh(b*x + a))*log(I*cosh(b*x +
a) + I*sinh(b*x + a) + 1) - 6*((I*b^2*x^2 - I*a^2)*cosh(b*x + a)^3 + 3*(I*b
^2*x^2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a)^2 + (I*b^2*x^2 - I*a^2)*sinh(b*
x + a)^3 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a) + (I*b^2*x^2 + 3*(I*b^2*x^2 -
I*a^2)*cosh(b*x + a)^2 - I*a^2)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sin
h(b*x + a) + 1) - 12*(-I*cosh(b*x + a)^3 - 3*I*cosh(b*x + a)*sinh(b*x + a)^
2 - I*sinh(b*x + a)^3 + (-3*I*cosh(b*x + a)^2 - I)*sinh(b*x + a) - I*cosh(b
*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*(I*cosh(b*x + a
```

```
)^3 + 3*I*cosh(b*x + a)*sinh(b*x + a)^2 + I*sinh(b*x + a)^3 + (3*I*cosh(b*x
 + a)^2 + I)*sinh(b*x + a) + I*cosh(b*x + a))*polylog(3, -I*cosh(b*x + a) -
 I*sinh(b*x + a)) + 4*((b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^3 +
3*(b^3*x^3 + 2*b*x)*cosh(b*x + a))*sinh(b*x + a) + 6)/(b^4*cosh(b*x + a)^3
+ 3*b^4*cosh(b*x + a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3 + b^4*cosh(b*x
+ a) + (3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x + a))
```

Sympy [F]

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3 \sinh^3{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

```
[In] integrate(x**3*sech(b*x+a)**2*sinh(b*x+a)**3,x)
[Out] Integral(x**3*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3 \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(3
*b*x) + 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^2
+ 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) + b^4*e^a) - 6*integrate(x^2*e^
(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Giac [F]

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^3 \operatorname{sech}\left(bx+a\right)^2 \sinh\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^3\,\sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
[Out] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

3.385 $\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx = -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}+\frac{2i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{x^2\operatorname{sech}(a+bx)}{b}-\frac{2x\sinh(a+bx)}{b^2}$$

```
[Out] -4*x*arctan(exp(b*x+a))/b^2+2*cosh(b*x+a)/b^3+x^2*cosh(b*x+a)/b+2*I*polylog
(2,-I*exp(b*x+a))/b^3-2*I*polylog(2,I*exp(b*x+a))/b^3+x^2*sech(b*x+a)/b-2*x
*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5557, 3377, 2718, 5526, 4265, 2317, 2438}

$$\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx = -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{2i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{2i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{2\cosh(a+bx)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}+\frac{x^2\cosh(a+bx)}{b}+\frac{x^2\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x]
)/b + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*I)*PolyLog[2, I*E^(a +
 b*x)])/b^3 + (x^2*Sech[a + b*x])/b - (2*x*Sinh[a + b*x])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
```

```
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2\sinh(a+bx)\,dx-\int x^2\text{sech}(a+bx)\tanh(a+bx)\,dx\\
&= \frac{x^2\cosh(a+bx)}{b}+\frac{x^2\text{sech}(a+bx)}{b}-\frac{2\int x\cosh(a+bx)\,dx}{b}-\frac{2\int x\text{sech}(a+bx)\,dx}{b}\\
&= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{x^2\cosh(a+bx)}{b}+\frac{x^2\text{sech}(a+bx)}{b}-\frac{2x\sinh(a+bx)}{b^2}\\
&\quad+\frac{(2i)\int\log\left(1-ie^{a+bx}\right)\,dx}{b^2}-\frac{(2i)\int\log\left(1+ie^{a+bx}\right)\,dx}{b^2}+\frac{2\int\sinh(a+bx)\,dx}{b^2}\\
&= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}\\
&\quad+\frac{x^2\text{sech}(a+bx)}{b}-\frac{2x\sinh(a+bx)}{b^2}+\frac{(2i)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&\quad-\frac{(2i)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}+\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}\\
&\quad-\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{x^2\text{sech}(a+bx)}{b}-\frac{2x\sinh(a+bx)}{b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

$$
\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx
= \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)-2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)+\frac{1}{2}\text{sech}(a+bx)\left(2+3b^2x^2+(2+b^2x^2)\cosh(2(a+bx))\right.}{b^3}
$$

```
[In] Integrate[x^2*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] ((2*I)*PolyLog[2, (-I)*E^(a + b*x)] - (2*I)*PolyLog[2, I*E^(a + b*x)] + (Se
ch[a + b*x]*(2 + 3*b^2*x^2 + (2 + b^2*x^2)*Cosh[2*(a + b*x)] - (4*I)*b*x*Co
sh[a + b*x]*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x)]) - 2*b*x*Sinh[
2*(a + b*x)]))/2)/b^3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(97) = 194.

Time = 0.89 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.97

method	result
risch	$\frac{(x^2b^2-2bx+2)e^{bx+a}}{2b^3}+\frac{(x^2b^2+2bx+2)e^{-bx-a}}{2b^3}+\frac{2x^2e^{bx+a}}{b(1+e^{2bx+2a})}+\frac{2i\ln(1+ie^{bx+a})x}{b^2}+\frac{2i\ln(1+ie^{bx+a})a}{b^3}-\frac{2i\ln(1-ie^{bx+a})x}{b^2}$

```
[In] int(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)+
2*x^2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*I/b^2*ln(1+I*exp(b*x+a))*x+2*I/b^3*
ln(1+I*exp(b*x+a))*a-2*I/b^2*ln(1-I*exp(b*x+a))*x-2*I/b^3*ln(1-I*exp(b*x+a)
)*a+2*I/b^3*dilog(1+I*exp(b*x+a))-2*I/b^3*dilog(1-I*exp(b*x+a))+4/b^3*a*arc
tan(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(91) = 182.

Time = 0.27 (sec) , antiderivative size = 879, normalized size of antiderivative = 8.45

$$\int x^2\sinh(a+bx)\tanh^2(a+bx)\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2 +
 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 + 2*(3*b^2*x^2 + 3*(b^2*x^2 - 2*b*x + 2)
*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x - 4*(I*cosh(b*x + a)^3 + 3*I*
cosh(b*x + a)*sinh(b*x + a)^2 + I*sinh(b*x + a)^3 + (3*I*cosh(b*x + a)^2 +
I)*sinh(b*x + a) + I*cosh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)
) - 4*(-I*cosh(b*x + a)^3 - 3*I*cosh(b*x + a)*sinh(b*x + a)^2 - I*sinh(b*x
+ a)^3 + (-3*I*cosh(b*x + a)^2 - I)*sinh(b*x + a) - I*cosh(b*x + a))*dilog(
-I*cosh(b*x + a) - I*sinh(b*x + a)) - 4*(-I*a*cosh(b*x + a)^3 - 3*I*a*cosh(
b*x + a)*sinh(b*x + a)^2 - I*a*sinh(b*x + a)^3 - I*a*cosh(b*x + a) + (-3*I*
a*cosh(b*x + a)^2 - I*a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) +
 I) - 4*(I*a*cosh(b*x + a)^3 + 3*I*a*cosh(b*x + a)*sinh(b*x + a)^2 + I*a*si
nh(b*x + a)^3 + I*a*cosh(b*x + a) + (3*I*a*cosh(b*x + a)^2 + I*a)*sinh(b*x
+ a))*log(cosh(b*x + a) + sinh(b*x + a) - I) - 4*((-I*b*x - I*a)*cosh(b*x +
 a)^3 + 3*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a)^2 + (-I*b*x - I*a)*sin
h(b*x + a)^3 + (-I*b*x - I*a)*cosh(b*x + a) + (3*(-I*b*x - I*a)*cosh(b*x +
```

```
a)^2 - I*b*x - I*a)*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) +
1) - 4*((I*b*x + I*a)*cosh(b*x + a)^3 + 3*(I*b*x + I*a)*cosh(b*x + a)*sinh(
b*x + a)^2 + (I*b*x + I*a)*sinh(b*x + a)^3 + (I*b*x + I*a)*cosh(b*x + a) +
(3*(I*b*x + I*a)*cosh(b*x + a)^2 + I*b*x + I*a)*sinh(b*x + a))*log(-I*cosh(
b*x + a) - I*sinh(b*x + a) + 1) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3
+ (3*b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 +
3*b^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3 + b^3*cosh(b*x +
a) + (3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a))
```

Sympy [F]

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2 \sinh^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**2*sech(b*x+a)**2*sinh(b*x+a)**3,x)
[Out] Integral(x**2*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) + 2*(3*b^2*x^2
*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2*b
*x + 3*a) + b^3*e^a) - 4*integrate(x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x
)
```

Giac [F]

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)^2 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^2*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)^2*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
[Out] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^2, x)
```

3.386 $\int x \sinh(a+bx) \tanh^2(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int x \sinh(a+bx) \tanh^2(a+bx) \, dx = -\frac{\arctan(\sinh(a+bx))}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2}$$

```
[Out] -arctan(sinh(b*x+a))/b^2+x*cosh(b*x+a)/b+x*sech(b*x+a)/b-sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5557, 3377, 2717, 5526, 3855}

$$\int x \sinh(a+bx) \tanh^2(a+bx) \, dx = -\frac{\arctan(\sinh(a+bx))}{b^2} - \frac{\sinh(a+bx)}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b}$$

```
[In] Int[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x])/b -
Sinh[a + b*x]/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x\sinh(a+bx)\,dx - \int x\text{sech}(a+bx)\tanh(a+bx)\,dx \\
&= \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b} - \frac{\int \cosh(a+bx)\,dx}{b} - \frac{\int \text{sech}(a+bx)\,dx}{b} \\
&= -\frac{\arctan(\sinh(a+bx))}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$
\int x\sinh(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\arctan\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{x\cosh(a+bx)}{b} + \frac{x\text{sech}(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2}
$$

```
[In] Integrate[x*Sinh[a + b*x]*Tanh[a + b*x]^2,x]
[Out] (-2*ArcTan[Tanh[(a + b*x)/2]])/b^2 + (x*Cosh[a + b*x])/b + (x*Sech[a + b*x]
)/b - Sinh[a + b*x]/b^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

method	result	size
risch	$\frac{(bx-1)\mathrm{e}^{bx+a}}{2b^2}+\frac{(bx+1)\mathrm{e}^{-bx-a}}{2b^2}+\frac{2x\,\mathrm{e}^{bx+a}}{b(1+\mathrm{e}^{2bx+2a})}+\frac{i\ln(\mathrm{e}^{bx+a}-i)}{b^2}-\frac{i\ln(\mathrm{e}^{bx+a}+i)}{b^2}$	94

```
[In] int(x*sech(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)+2*x*exp(b*x+a)/b/(1+
exp(2*b*x+2*a))+I/b^2*ln(exp(b*x+a)-I)-I/b^2*ln(exp(b*x+a)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 283, normalized size of antiderivative = 6.15

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx$$
$$=\frac{(bx-1)\cosh(bx+a)^4+4(bx-1)\cosh(bx+a)\sinh(bx+a)^3+(bx-1)\sinh(bx+a)^4+6\,bx\cosh(bx}{}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x - 1)*sinh(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b*x
 + a)^2 + b*x)*sinh(b*x + a)^2 + b*x - 4*(cosh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
+ cosh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) + 4*((b*x - 1)*cosh(
b*x + a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^3 +
 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 + b^2*cosh(b*x +
 a) + (3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a))
```

Sympy [F]

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx=\int x\sinh^3(a+bx)\,\mathrm{sech}^2(a+bx)\,dx$$

```
[In] integrate(x*sech(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{6\,bxe^{(bx+2\,a)} + \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} + (bx+1)e^{(-bx)}}{2\,\left(b^2e^{(2\,bx+3\,a)} + b^2e^{a}\right)} - \frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/2*(6*b*x*e^(b*x + 2*a) + (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) + (b*x + 1)*e^
(-b*x))/(b^2*e^(2*b*x + 3*a) + b^2*e^a) - 2*arctan(e^(b*x + a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(46) = 92.

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{bxe^{(4\,bx+4\,a)} + 6\,bxe^{(2\,bx+2\,a)} + bx - 4\,\arctan\left(e^{(bx+a)}\right)e^{(3\,bx+3\,a)} - 4\,\arctan\left(e^{(bx+a)}\right)e^{(bx+a)} - e^{(4\,bx+4\,a)} + 1}{2\,\left(b^2e^{(3\,bx+3\,a)} + b^2e^{(bx+a)}\right)}$$

```
[In] integrate(x*sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*(b*x*e^(4*b*x + 4*a) + 6*b*x*e^(2*b*x + 2*a) + b*x - 4*arctan(e^(b*x +
a))*e^(3*b*x + 3*a) - 4*arctan(e^(b*x + a))*e^(b*x + a) - e^(4*b*x + 4*a) +
 1)/(b^2*e^(3*b*x + 3*a) + b^2*e^(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int x\sinh(a+bx)\tanh^2(a+bx)\,dx = \mathrm{e}^{-a-b\,x}\left(\frac{x}{2\,b}+\frac{1}{2\,b^2}\right) - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^4}}{b^2}\right)}{\sqrt{b^4}} + \mathrm{e}^{a+b\,x}\left(\frac{x}{2\,b}-\frac{1}{2\,b^2}\right) + \frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int((x*sinh(a + b*x)^3)/cosh(a + b*x)^2,x)
[Out] exp(- a - b*x)*(x/(2*b) + 1/(2*b^2)) - (2*atan((exp(b*x)*exp(a)*(b^4)^(1/2)
)/b^2))/(b^4)^(1/2) + exp(a + b*x)*(x/(2*b) - 1/(2*b^2)) + (2*x*exp(a + b*x
))/(b*(exp(2*a + 2*b*x) + 1))
```

3.387 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(a+bx)}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b+sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(a+bx)}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(a+bx)}{b} + \frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
[Out] Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}+6\,e^{bx+a}+e^{-bx-a}}{2b\left(1+e^{2bx+2a}\right)}$	46

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh\left(bx+a\right)^2+\sinh\left(bx+a\right)^2+3}{2\,b\cosh\left(bx+a\right)}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b}+\frac{5\,e^{(-2\,bx-2\,a)}+1}{2\,b\left(e^{(-bx-a)}+e^{(-3\,bx-3\,a)}\right)}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3*
b*x - 3*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}}+e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) + e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh(a+b\,x)^2+1}{b\cosh\left(a+b\,x\right)}$$

```
[In] int(sinh(a + b*x)^3/cosh(a + b*x)^2,x)
[Out] (cosh(a + b*x)^2 + 1)/(b*cosh(a + b*x))
```

3.388 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \mathrm{Chi}(bx)\sinh(a)+\cosh(a)\mathrm{Shi}(bx) - \mathrm{Int}\left(\frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{x},x\right)$$

```
[Out] -CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x,x)+cosh(a)*Shi(b*x)+Chi(b*x)*sin
h(a)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
```

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] - Defer[Int][(Sech[a
+ b*x]*Tanh[a + b*x])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sinh(a+bx)}{x}\,dx - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$
$$= \cosh(a)\int \frac{\sinh(bx)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx)}{x}\,dx - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$
$$= \operatorname{Chi}(bx)\sinh(a) + \cosh(a)\operatorname{Shi}(bx) - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x,x]
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}dx$$

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 8.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="maxima")
[Out] -1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a + 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
 + b*x) + 2*integrate(e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^2\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^3}{x\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^2),x)
[Out] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^2), x)
```

3.389 $\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = b\cosh(a)\mathrm{Chi}(bx) - \frac{\sinh(a+bx)}{x} + b\sinh(a)\mathrm{Shi}(bx) - \mathrm{Int}\left(\frac{\mathrm{sech}(a+bx)\tanh(a+bx)}{x^2},x\right)$$

```
[Out] -CannotIntegrate(sech(b*x+a)*tanh(b*x+a)/x^2,x)+b*Chi(b*x)*cosh(a)+b*Shi(b*
x)*sinh(a)-sinh(b*x+a)/x
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx$$

```
[In] Int[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
```

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 - Defer[Int][(Sech[a + b*x]*Tanh[a + b*x])/x^2, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sinh(a+bx)}{x^2}\,dx - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx \\
&= -\frac{\sinh(a+bx)}{x} + b\int \frac{\cosh(a+bx)}{x}\,dx - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx \\
&= -\frac{\sinh(a+bx)}{x} + (b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx \\
&\quad + (b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx \\
&= b\cosh(a)\operatorname{Chi}(bx) - \frac{\sinh(a+bx)}{x} + b\sinh(a)\operatorname{Shi}(bx) - \int \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{x^2}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$
\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx
$$

```
[In] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2,x]
[Out] Integrate[(Sinh[a + b*x]*Tanh[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$
\int \frac{\operatorname{sech}(bx+a)^2 \sinh(bx+a)^3}{x^2}dx
$$

```
[In] int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
[Out] int(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 10.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)**2*sinh(b*x+a)**3/x**2,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.83

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
[Out] 1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) + 2*e^(b*x + a)/(b*
x^2*e^(2*b*x + 2*a) + b*x^2) + 4*integrate(e^(b*x + a)/(b*x^3*e^(2*b*x + 2*
a) + b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2\sinh\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^2*sinh(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^2*sinh(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(a+bx)\tanh^2(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{x^2\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^2),x)
[Out] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^2), x)
```

3.390 $\int x^m \tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^3(a+bx)\,dx = \text{Int}\left(x^m \tanh^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

```
[In] Int[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m \tanh^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

```
[In] Integrate[x^m*Tanh[a + b*x]^3,x]
[Out] Integrate[x^m*Tanh[a + b*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3\,dx$$

```
[In] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
[Out] int(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \tanh^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*sech(b*x+a)**3*sinh(b*x+a)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 14.25

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m +
1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
+ 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^m*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \tanh^3(a+bx)\,dx = \int \frac{x^m \sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
```

```
[Out] int((x^m*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.391 $\int x^3 \tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 183

$$\int x^3 \tanh^3(a+bx)\,dx = -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{3\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x\,\mathrm{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\mathrm{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b-1/4*x^4+3*x*ln(1+exp(2*b*x+2*a))/b^3+x^3*ln(1+exp(2*
b*x+2*a))/b+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,-exp(2*b*x
+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*
a))/b^4-3/2*x^2*tanh(b*x+a)/b^2-1/2*x^3*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3801, 3799, 2221, 2317, 2438, 30, 2611, 6744, 2320, 6724}

$$\int x^3 \tanh^3(a+bx)\,dx = \frac{3\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}+\frac{3\,\mathrm{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4}-\frac{3x\,\mathrm{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}+\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}+\frac{3x^2\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\tanh(a+bx)}{2b^2}+\frac{x^3\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x^3\tanh^2(a+bx)}{2b}-\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}$$

```
[In] Int[x^3*Tanh[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 + (3*x*Log[1 + E^(2*(a + b*x))])/b^3 +
 (x^3*Log[1 + E^(2*(a + b*x))])/b + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4
) + (3*x^2*PolyLog[2, -E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, -E^(2*(a
 + b*x))])/(2*b^3) + (3*PolyLog[4, -E^(2*(a + b*x))])/(4*b^4) - (3*x^2*Tanh
[a + b*x])/(2*b^2) - (x^3*Tanh[a + b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\tanh^2(a+bx)}{2b}+\frac{3\int x^2\tanh^2(a+bx)\,dx}{2b}+\int x^3\tanh(a+bx)\,dx\\
&= -\frac{x^4}{4}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}\\
&\quad+2\int\frac{e^{2(a+bx)}x^3}{1+e^{2(a+bx)}}\,dx+\frac{3\int x\tanh(a+bx)\,dx}{b^2}+\frac{3\int x^2\,dx}{2b}\\
&= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}+\frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b}-\frac{3x^2\tanh(a+bx)}{2b^2}\\
&\quad-\frac{x^3\tanh^2(a+bx)}{2b}+\frac{6\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b^2}-\frac{3\int x^2\log\left(1+e^{2(a+bx)}\right)\,dx}{b}
\end{aligned}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} - \frac{3\int\log\left(1+e^{2(a+bx)}\right)dx}{b^3} - \frac{3\int x\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{b^2}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}dx,x,e^{2(a+bx)}\right)}{2b^4} + \frac{3\int\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)dx}{2b^3}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-x)}{x}dx,x,e^{2(a+bx)}\right)}{4b^4}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{3\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{3\operatorname{PolyLog}\left(4,-e^{2(a+bx)}\right)}{4b^4} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.36

$$\int x^3\tanh^3(a+bx)\,dx$$

$$= \frac{1}{4}\left(\frac{12b^2x^2 + 2b^4x^4 + 12bx\log\left(1+e^{-2(a+bx)}\right) + 12be^{2a}x\log\left(1+e^{-2(a+bx)}\right) + 4b^3x^3\log\left(1+e^{-2(a+bx)}\right) + 4}{} + \frac{2x^3\operatorname{sech}^2(a+bx)}{b} - \frac{6x^2\operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(bx)}{b^2} + x^4\tanh(a)\right)$$

[In] Integrate[x^3*Tanh[a + b*x]^3,x]

[Out] ((12*b^2*x^2 + 2*b^4*x^4 + 12*b*x*Log[1 + E^(-2*(a + b*x))] + 12*b*E^(2*a)*x*Log[1 + E^(-2*(a + b*x))] + 4*b^3*x^3*Log[1 + E^(-2*(a + b*x))] + 4*b^3*E^(2*a)*x^3*Log[1 + E^(-2*(a + b*x))] - 6*(1 + E^(2*a))*(1 + b^2*x^2)*PolyLog[2, -E^(-2*(a + b*x))] - 6*b*(1 + E^(2*a))*x*PolyLog[3, -E^(-2*(a + b*x))] - 3*PolyLog[4, -E^(-2*(a + b*x))] - 3*E^(2*a)*PolyLog[4, -E^(-2*(a + b*x))])/(b^4*(1 + E^(2*a))) + (2*x^3*Sech[a + b*x]^2)/b - (6*x^2*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 + x^4*Tanh[a])/4

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{x^4}{4}+\frac{x^2\left(2\,\mathrm{e}^{2bx+2a}bx+3\,\mathrm{e}^{2bx+2a}+3\right)}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{3\,\mathrm{polylog}\left(2,-\mathrm{e}^{2bx+2a}\right)}{2b^4}+\frac{3\,\mathrm{polylog}\left(4,-\mathrm{e}^{2bx+2a}\right)}{4b^4}-\frac{3a^4}{2b^4}-\frac{3a^2}{b^4}-\frac{3x^2}{b^2}-\frac{6ax}{b^3}-$

[In] int(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/4*x^4+x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3/4*polylog(4,-exp(2*b*x+2*a))/b^4-3/2/b^4*a^4-3/b^4*a^2-3/b^2*x^2-6/b^3*a*x-2/b^3*a^3*x+3*x*ln(1+exp(2*b*x+2*a))/b^3+x^3*ln(1+exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+6/b^4*a*ln(exp(b*x+a))+2/b^4*a^3*ln(exp(b*x+a))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 2207, normalized size of antiderivative = 12.06

$$\int x^3\tanh^3(a+bx)\,dx=\text{Too large to display}$$

[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 + 2*(b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 + 2*(b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 + 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(

b*x + a)^3 + (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 + (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 + 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 + 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 + (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 4*(b^3*x^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 4*(b^3*x^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 + (b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^

```
4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x
+ a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \tanh^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*sech(b*x+a)**3*sinh(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.29

$$\begin{aligned}
&\int x^3 \tanh^3(a+bx)\,dx \\
&= \frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 + 2\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4} \\
&\quad + \frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)} + 1\right) + 6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right) - 6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)}) + 3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})}{3\,b^4} \\
&\quad + \frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}
\end{aligned}$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 + 2*(b^2*x^4*e^(2*a) + 4*b*
x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b
*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + 1/3*(4*b^3*x^3*log(e^(2*
b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)) - 6*b*x*polylog(3, -e^(
2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4 + 3/2*(2*b*x*log(e^(2*b
*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^4
```

Giac [F]

$$\int x^3 \tanh^3(a+bx)\,dx = \int x^3 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3\,dx$$

```
[In] integrate(x^3*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh^3(a+bx)\,dx = \int \frac{x^3 \sinh(a+b\,x)^3}{\cosh(a+b\,x)^3}\,dx$$

```
[In] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
[Out] int((x^3*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.392 $\int x^2 \tanh^3(a + bx) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int x^2 \tanh^3(a+bx)\,dx = \frac{x^2}{2b} - \frac{x^3}{3} + \frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} + \frac{x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2} - \frac{\mathrm{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b}$$

```
[Out] 1/2*x^2/b-1/3*x^3+x^2*ln(1+exp(2*b*x+2*a))/b+ln(cosh(b*x+a))/b^3+x*polylog(
2,-exp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3-x*tanh(b*x+a)/b^2
-1/2*x^2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3801, 3556, 30, 3799, 2221, 2611, 2320, 6724}

$$\int x^2 \tanh^3(a+bx)\,dx = -\frac{\mathrm{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3} + \frac{\log(\cosh(a+bx))}{b^3} + \frac{x\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2} - \frac{x\tanh(a+bx)}{b^2} + \frac{x^2\log\left(e^{2(a+bx)}+1\right)}{b} - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{x^2}{2b} - \frac{x^3}{3}$$

```
[In] Int[x^2*Tanh[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - x^3/3 + (x^2*Log[1 + E^(2*(a + b*x))])/b + Log[Cosh[a + b*x]]/b
^3 + (x*PolyLog[2, -E^(2*(a + b*x))])/b^2 - PolyLog[3, -E^(2*(a + b*x))]/(2
*b^3) - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\tanh^2(a+bx)}{2b}+\frac{\int x\tanh^2(a+bx)\,dx}{b}+\int x^2\tanh(a+bx)\,dx\\
&= -\frac{x^3}{3}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}\\
&\quad+2\int\frac{e^{2(a+bx)}x^2}{1+e^{2(a+bx)}}\,dx+\frac{\int\tanh(a+bx)\,dx}{b^2}+\frac{\int x\,dx}{b}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}\\
&\quad-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}-\frac{2\int x\log\left(1+e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}\\
&\quad-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}-\frac{\int\text{PolyLog}\left(2,-e^{2(a+bx)}\right)\,dx}{b^2}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}\\
&\quad-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}-\frac{\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}+\frac{x^2\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\log(\cosh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^2}\\
&\quad-\frac{\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^3}-\frac{x\tanh(a+bx)}{b^2}-\frac{x^2\tanh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.69

$$\int x^2 \tanh^3(a+bx)\,dx$$
$$= \frac{e^{2a}\left(12e^{-2a}x - 12(1+e^{-2a})\,x + 4b^2e^{-2a}x^3 + 6b(1+e^{-2a})\,x^2\log\left(1+e^{-2(a+bx)}\right) + \frac{6(1+e^{-2a})\log(1+e^{2(a+bx)})}{b} - \right.}{6b^2\left(1+e^{2a}\right)}$$
$$+ \frac{x^2\text{sech}^2(a+bx)}{2b} - \frac{x\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b^2} + \frac{1}{3}x^3\tanh(a)$$

```
[In] Integrate[x^2*Tanh[a + b*x]^3,x]
```

```
[Out] (E^(2*a)*((12*x)/E^(2*a) - 12*(1 + E^(-2*a))*x + (4*b^2*x^3)/E^(2*a) + 6*b*
(1 + E^(-2*a))*x^2*Log[1 + E^(-2*(a + b*x))] + (6*(1 + E^(-2*a))*Log[1 + E^
(2*(a + b*x))])/b - 6*(1 + E^(-2*a))*x*PolyLog[2, -E^(-2*(a + b*x))] - (3*(
1 + E^(-2*a))*PolyLog[3, -E^(-2*(a + b*x))])/b))/(6*b^2*(1 + E^(2*a))) + (x
^2*Sech[a + b*x]^2)/(2*b) - (x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b^2 + (x^3*
Tanh[a])/3
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{x^3}{3} + \frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}+1\right)}{b^2\left(1+e^{2bx+2a}\right)^2} + \frac{\ln\left(1+e^{2bx+2a}\right)}{b^3} - \frac{2\ln\left(e^{bx+a}\right)}{b^3} - \frac{2a^2\ln\left(e^{bx+a}\right)}{b^3} + \frac{2a^2x}{b^2} + \frac{4a^3}{3b^3} + \frac{x^2\ln\left(1+e^{2bx+2a}\right)}{b}$

```
[In] int(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3+2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2
+1/b^3*ln(1+exp(2*b*x+2*a))-2/b^3*ln(exp(b*x+a))-2/b^3*a^2*ln(exp(b*x+a))+2
/b^2*a^2*x+4/3/b^3*a^3+x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*
a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1649, normalized size of antiderivative = 14.22

$$\int x^2 \tanh^3(a + bx) \, dx = \text{Too large to display}$$

[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] -1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*b*x + 6*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*(b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + I) - 3*((a^2 + 1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 + 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 + a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 + (a^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh

```
(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a) + 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*((b^3*x^3
+ 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 + (b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*b
*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b*x + a)^4 + 4*b^3*
cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2
 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x
+ a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2\tanh^3(a+bx)\,dx=\int x^2\sinh^3{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

```
[In] integrate(x**2*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int x^2\tanh^3(a+bx)\,dx$$
$$=-\frac{2}{3}\,x^3+\frac{b^2x^3e^{(4\,bx+4\,a)}+b^2x^3+2\left(b^2x^3e^{(2\,a)}+3\,bx^2e^{(2\,a)}+3\,xe^{(2\,a)}\right)e^{(2\,bx)}+6\,x}{3\left(b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2\right)}-\frac{2\,x}{b^2}+\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}+\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b^3}$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 + 2*(b^2*x^3*e^(2*a) + 3*
b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
+ 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + log(e
^(2*b*x + 2*a) + 1)/b^3
```

Giac [F]

$$\int x^2 \tanh^3(a+bx)\,dx = \int x^2 \operatorname{sech}(bx+a)^3 \sinh(bx+a)^3 \, dx$$

```
[In] integrate(x^2*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh^3(a+bx)\,dx = \int \frac{x^2 \sinh(a+b\,x)^3}{\cosh(a+b\,x)^3}\,dx$$

```
[In] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
[Out] int((x^2*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.393 $\int x\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x\tanh^3(a+bx)\,dx=\frac{x}{2b}-\frac{x^2}{2}+\frac{x\log\left(1+e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}$$

```
[Out] 1/2*x/b-1/2*x^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2
-1/2*tanh(b*x+a)/b^2-1/2*x*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3801, 3554, 8, 3799, 2221, 2317, 2438}

$$\int x\tanh^3(a+bx)\,dx=\frac{\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}+\frac{x\log\left(e^{2(a+bx)}+1\right)}{b}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

```
[In] Int[x*Tanh[a + b*x]^3,x]
```

```
[Out] x/(2*b) - x^2/2 + (x*Log[1 + E^(2*(a + b*x))])/b + PolyLog[2, -E^(2*(a + b*
x))]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*Tanh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x\tanh^2(a+bx)}{2b} + \frac{\int \tanh^2(a+bx)\,dx}{2b} + \int x\tanh(a+bx)\,dx$$

$$= -\frac{x^2}{2} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + 2\int \frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx + \frac{\int 1\,dx}{2b}$$

$$= \frac{x}{2b} - \frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\int \log\left(1+e^{2(a+bx)}\right)\,dx}{b}$$

$$= \frac{x}{2b} - \frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^2}$$

$$= \frac{x}{2b} - \frac{x^2}{2} + \frac{x\log\left(1+e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2, -e^{2(a+bx)}\right)}{2b^2} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int x\tanh^3(a+bx)\,dx$$
$$= \frac{bx\left(bx+2\log\left(1+e^{-2(a+bx)}\right)\right) - \text{PolyLog}\left(2, -e^{-2(a+bx)}\right) + bx\text{sech}^2(a+bx) - \text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{2b^2}$$

```
[In] Integrate[x*Tanh[a + b*x]^3,x]
```

```
[Out] (b*x*(b*x + 2*Log[1 + E^(-2*(a + b*x))]) - PolyLog[2, -E^(-2*(a + b*x))] +
b*x*Sech[a + b*x]^2 - Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.35

method	result	size
risch	$-\frac{x^2}{2} + \frac{2\,e^{2bx+2a}bx+e^{2bx+2a}+1}{b^2\left(1+e^{2bx+2a}\right)^2} - \frac{2ax}{b} - \frac{a^2}{b^2} + \frac{x\ln\left(1+e^{2bx+2a}\right)}{b} + \frac{\text{polylog}\left(2,-e^{2bx+2a}\right)}{2b^2} + \frac{2a\ln\left(e^{bx+a}\right)}{b^2}$	111

```
[In] int(x*sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2+(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2
/b*a*x-a^2/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2+
2/b^2*a*ln(exp(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1106, normalized size of antiderivative = 13.49

$$\int x\tanh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2
 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*(c
osh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x +
 a)) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*
x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^
2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x
 + a) + sinh(b*x + a) + I) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(
b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)
^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x
+ a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 2*((b*x + a)*cosh(b*x +
a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^
4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)
*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x
+ a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((b
*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x
+ a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*
x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 +
(b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*sinh(b
*x + a) + 1) + 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 + (b^2*x^2 - 2*a^2 - 2*
b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cos
h(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 +
2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a
)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\tanh^3(a+bx)\,dx = \int x\sinh^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x*sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(x*sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.60

$$\int x\tanh^3(a+bx)\,dx = -x^2 + \frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 + 2\left(b^2x^2e^{(2\,a)} + 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{2\left(b^2e^{(4\,bx+4\,a)} + 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2
*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))
/b^2
```

Giac [F]

$$\int x\tanh^3(a+bx)\,dx = \int x\operatorname{sech}(bx+a)^3\sinh(bx+a)^3\,dx$$

```
[In] integrate(x*sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sech(b*x + a)^3*sinh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\tanh^3(a+bx)\,dx=\int \frac{x\sinh(a+b\,x)^3}{\cosh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x*sinh(a + b*x)^3)/cosh(a + b*x)^3,x)
[Out] int((x*sinh(a + b*x)^3)/cosh(a + b*x)^3, x)
```

3.394 $\int \tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

```
[In] Int[Tanh[a + b*x]^3,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\tanh^2(a+bx)}{2b} + \int \tanh(a+bx)\,dx$$
$$= \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

```
[In] Integrate[Tanh[a + b*x]^3,x]
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\ln(\cosh(bx+a))-\frac{\tanh(bx+a)^2}{2}}{b}$	23
default	$\frac{\ln(\cosh(bx+a))-\frac{\tanh(bx+a)^2}{2}}{b}$	23
parallelrisch	$\frac{-\tanh(bx+a)^2-2bx-2\ln(1-\tanh(bx+a))}{2b}$	34
risch	$-x-\frac{2a}{b}+\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{\ln\left(1+\mathrm{e}^{2bx+2a}\right)}{b}$	54

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(cosh(b*x+a))-1/2*tanh(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 12.56

$$\int \tanh^3(a+bx)\,dx =$$

$$-\frac{bx\cosh\left(bx+a\right)^4+4\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+bx\sinh\left(bx+a\right)^4+2\left(bx-1\right)\cosh\left(bx+a\right)^2+2}{}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 + 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 + (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [F]

$$\int \tanh^3(a+bx)\,dx = \int \sinh^3\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \tanh^3(a+bx)\,dx = x+\frac{a}{b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}+\frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x
- 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \tanh^3(a+bx)\,dx = -\frac{2\,bx + 2\,a + \frac{3\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+3}{\left(e^{(2\,bx+2\,a)}+1\right)^2} - 2\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{2\,b}$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + 2*a + (3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 3)/(e^(2*b*x +
 2*a) + 1)^2 - 2*log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tanh^3(a+bx)\,dx = \frac{1}{2\,b\cosh\left(a+b\,x\right)^2} + \frac{\ln\left(\cosh(a+b\,x)\right)}{b}$$

```
[In] int(sinh(a + b*x)^3/cosh(a + b*x)^3,x)
```

```
[Out] 1/(2*b*cosh(a + b*x)^2) + log(cosh(a + b*x))/b
```

3.395 $\int \frac{\tanh^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \text{Int}\left(\frac{\tanh^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\tanh^3(a+bx)}{x}\,dx$$

```
[In] Int[Tanh[a + b*x]^3/x,x]
[Out] Defer[Int][Tanh[a + b*x]^3/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\tanh^3(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(a+bx)}{x}\,dx=\int \frac{\tanh^3(a+bx)}{x}\,dx$$

```
[In] Integrate[Tanh[a + b*x]^3/x,x]
[Out] Integrate[Tanh[a + b*x]^3/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x}dx$$

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x}\,dx=\int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 19.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\tanh^3(a+bx)}{x}\,dx=\int \frac{\sinh^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 9.25

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2*
x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate(2*(b^2*x^2 + 1)/(b^2*x^3*e^(2*b*
x + 2*a) + b^2*x^3), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)^3}{x\cosh(a+b\,x)^3}\,dx$$

```
[In] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^3),x)
```

```
[Out] int(sinh(a + b*x)^3/(x*cosh(a + b*x)^3), x)
```

3.396 $\int \frac{\tanh^3(a+bx)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\tanh^3(a+bx)}{x^2} dx = \text{Int}\left(\frac{\tanh^3(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(tanh(b*x+a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^3(a+bx)}{x^2} dx = \int \frac{\tanh^3(a+bx)}{x^2} dx$$

```
[In] Int[Tanh[a + b*x]^3/x^2,x]
[Out] Defer[Int][Tanh[a + b*x]^3/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\tanh^3(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 8.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\tanh^3(a+bx)}{x^2}\,dx$$

```
[In] Integrate[Tanh[a + b*x]^3/x^2,x]
[Out] Integrate[Tanh[a + b*x]^3/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x^2}dx$$

```
[In] int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
[Out] int(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3 \sinh(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 27.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)**3*sinh(b*x+a)**3/x**2,x)
[Out] Integral(sinh(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 11.92

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="maxima")
[Out] -(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 + 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e^
(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3) - integrate(2*(b^2*x^2 + 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4),
 x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3\sinh(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(sech(b*x+a)^3*sinh(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)^3*sinh(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tanh^3(a+bx)}{x^2}\,dx = \int \frac{\sinh(a+b\,x)^3}{x^2\cosh(a+b\,x)^3}\,dx$$

```
[In] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^3),x)
[Out] int(sinh(a + b*x)^3/(x^2*cosh(a + b*x)^3), x)
```

3.397 $\int x^m \coth(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \coth(a+bx)\,dx = \text{Int}(x^m \coth(a+bx), x)$$

```
[Out] Unintegrable(x^m*coth(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \coth(a+bx)\,dx = \int x^m \coth(a+bx)\,dx$$

```
[In] Int[x^m*Coth[a + b*x],x]
[Out] Defer[Int][x^m*Coth[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \coth(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \coth(a+bx)\,dx = \int x^m \coth(a+bx)\,dx$$

```
[In] Integrate[x^m*Coth[a + b*x],x]
[Out] Integrate[x^m*Coth[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int x^m \cosh\left(bx+a\right) \operatorname{csch}\left(bx+a\right) dx$$

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a),x)
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right) \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 108.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh\left(a+bx\right) \operatorname{csch}\left(a+bx\right) dx$$

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a),x)
[Out] Integral(x**m*cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 10.20

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) - m - 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int x^m \coth(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int x^m \coth(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

```
[In] int((x^m*cosh(a + b*x))/sinh(a + b*x),x)
```

```
[Out] int((x^m*cosh(a + b*x))/sinh(a + b*x), x)
```

3.398 $\int x^3 \coth(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int x^3 \coth(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}$$

```
[Out] -1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3
/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3797, 2221, 2611, 6744, 2320, 6724}

$$\int x^3 \coth(a+bx)\,dx = \frac{3 \operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4} - \frac{3x \operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{3x^2 \operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} + \frac{x^3 \log\left(1-e^{2(a+bx)}\right)}{b} - \frac{x^4}{4}$$

```
[In] Int[x^3*Coth[a + b*x],x]
```

```
[Out] -1/4*x^4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x^2*PolyLog[2, E^(2*(a + b
*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^3) + (3*PolyLog[4,
E^(2*(a + b*x))])/(4*b^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^4}{4} - 2\int \frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{3\int x^2 \log\left(1 - e^{2(a+bx)}\right)\,dx}{b}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3\int x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)\,dx}{b^2}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} + \frac{3\int \operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)\,dx}{2b^3}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} + \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx, x, e^{2(a+bx)}\right)}{4b^4}$$

$$= -\frac{x^4}{4} + \frac{x^3 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4, e^{2(a+bx)}\right)}{4b^4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int x^3 \coth(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1 - e^{2a+2bx}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2a+2bx}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, e^{2a+2bx}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4, e^{2a+2bx}\right)}{4b^4}$$

```
[In] Integrate[x^3*Coth[a + b*x],x]
```

```
[Out] -1/4*x^4 + (x^3*Log[1 - E^(2*a + 2*b*x)])/b + (3*x^2*PolyLog[2, E^(2*a + 2*
b*x)])/(2*b^2) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) + (3*PolyLog[4,
E^(2*a + 2*b*x)])/(4*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.30

method	result
risch	$-\frac{x^4}{4} + \frac{3x^2 \operatorname{polylog}(2, e^{bx+a})}{b^2} - \frac{6x \operatorname{polylog}(3, e^{bx+a})}{b^3} + \frac{\ln(e^{bx+a}+1)x^3}{b} + \frac{3x^2 \operatorname{polylog}(2, -e^{bx+a})}{b^2} - \frac{6x \operatorname{polylog}(3, -e^{bx+a})}{b^3} +$

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3+1/b*
ln(exp(b*x+a)+1)*x^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylog(3,-exp(b*
x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^3-3/2/b^4*a^4-2/b^3*a^3*x+6*polylog(4,-exp
(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+1/b^4*ln(1-exp(b*x+a))*a^3+2/b^4*a
^3*ln(exp(b*x+a))-1/b^4*a^3*ln(exp(b*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(78) = 156$.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.48

$$\int x^3 \coth(a+bx)\,dx = -\frac{b^4x^4 - 4\,b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) - 12\,b^2x^2\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)) - 12\,b}{}$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 4*b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*b^2*x
^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*b^2*x^2*dilog(-cosh(b*x + a) -
 sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 24*b*x*pol
ylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*b*x*polylog(3, -cosh(b*x + a) -
 sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
 - 24*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 24*polylog(4, -cosh(b*x +
 a) - sinh(b*x + a)))/b^4
```

Sympy [F]

$$\int x^3 \coth(a+bx)\,dx = \int x^3 \cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x**3*cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int x^3 \coth(a+bx)\,dx$$
$$= -\frac{1}{4}x^4 + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] -1/4*x^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) -
6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3
*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^
(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \coth(a+bx)\,dx = \int x^3 \cosh(bx+a)\,\mathrm{csch}\,(bx+a)\;dx$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)}{\sinh(a+b\,x)}\,dx$$

```
[In] int((x^3*cosh(a + b*x))/sinh(a + b*x),x)
[Out] int((x^3*cosh(a + b*x))/sinh(a + b*x), x)
```

3.399 $\int x^2 \coth(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int x^2 \coth(a+bx)\,dx = -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}$$

```
[Out] -1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*pol
ylog(3,exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3797, 2221, 2611, 2320, 6724}

$$\int x^2 \coth(a+bx)\,dx = -\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^3}{3}$$

```
[In] Int[x^2*Coth[a + b*x],x]
```

```
[Out] -1/3*x^3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(a + b*x))
])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{3}-2\int \frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{2\int x\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\int \text{PolyLog}\left(2,e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3}
\end{aligned}$$

$$= -\frac{x^3}{3} + \frac{x^2 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int x^2 \coth(a+bx)\,dx = -\frac{x^3}{3} + \frac{x^2 \log\left(1 - e^{2a+2bx}\right)}{b} + \frac{x \operatorname{PolyLog}\left(2, e^{2a+2bx}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2a+2bx}\right)}{2b^3}$$

```
[In] Integrate[x^2*Coth[a + b*x],x]
```

```
[Out] -1/3*x^3 + (x^2*Log[1 - E^(2*a + 2*b*x)])/b + (x*PolyLog[2, E^(2*a + 2*b*x)
])/b^2 - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(59) = 118.

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{x^3}{3} + \frac{a^2 \ln(\mathrm{e}^{bx+a}-1)}{b^3} - \frac{2a^2 \ln(\mathrm{e}^{bx+a})}{b^3} + \frac{2a^2 x}{b^2} + \frac{4a^3}{3b^3} + \frac{\ln(\mathrm{e}^{bx+a}+1)x^2}{b} + \frac{2x \operatorname{polylog}(2,-\mathrm{e}^{bx+a})}{b^2} - \frac{2 \operatorname{polylog}(3,-\mathrm{e}^{bx+}}{b^3}$

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))+2/b^2*a^2*x+4/
3/b^3*a^3+1/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-2*polylog
(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+2*x
*polylog(2,exp(b*x+a))/b^2-2*polylog(3,exp(b*x+a))/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.67

$$\int x^2 \coth(a+bx)\,dx = -\frac{b^3x^3 - 3\,b^2x^2 \log\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right) + 1\right) - 6\,bx\mathrm{Li}_2(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)) - 6\,bx\mathrm{Li}_2}{}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 6*b*x*di
log(cosh(b*x + a) + sinh(b*x + a)) - 6*b*x*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(b^2*x^2 - a^2)*lo
g(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 6*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^3
```

Sympy [F]

$$\int x^2 \coth(a+bx)\,dx = \int x^2 \cosh{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int x^2 \coth(a+bx)\,dx = -\frac{1}{3}\,x^3 + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(e^{(bx+a)})}{b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*po
lylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(
e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2 \coth(a+bx)\,dx = \int x^2 \cosh{(bx+a)}\operatorname{csch}{(bx+a)}\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^2*cosh(a + b*x))/sinh(a + b*x),x)
[Out] int((x^2*cosh(a + b*x))/sinh(a + b*x), x)
```

3.400 $\int x \coth(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int x\coth(a+bx)\,dx=-\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

```
[Out] -1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3797, 2221, 2317, 2438}

$$\int x\coth(a+bx)\,dx=\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2}{2}$$

```
[In] Int[x*Coth[a + b*x],x]
```

```
[Out] -1/2*x^2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*
b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{2} - 2\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx \\
&= -\frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\int \log\left(1-e^{2(a+bx)}\right)\,dx}{b} \\
&= -\frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^2} \\
&= -\frac{x^2}{2} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int x\coth(a+bx)\,dx = -\frac{x^2}{2} + \frac{x\log\left(1-e^{2a+2bx}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2a+2bx}\right)}{2b^2}$$

```
[In] Integrate[x*Coth[a + b*x],x]
[Out] -1/2*x^2 + (x*Log[1 - E^(2*a + 2*b*x)])/b + PolyLog[2, E^(2*a + 2*b*x)]/(2*
b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(41) = 82$.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{x^2}{2} - \frac{2ax}{b} - \frac{a^2}{b^2} + \frac{\ln(e^{bx+a}+1)x}{b} + \frac{\mathrm{polylog}(2,-e^{bx+a})}{b^2} + \frac{\ln(1-e^{bx+a})x}{b} + \frac{\ln(1-e^{bx+a})a}{b^2} + \frac{\mathrm{polylog}(2,e^{bx+a})}{b^2} - \frac{a\ln(e$

```
[In] int(x*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2-2/b*a*x-a^2/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2+
1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1
/b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int x\coth(a+bx)\,dx = -\frac{b^2x^2 - 2\,bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right) - 2\,(bx+$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - 2*b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*a*log(cosh
(b*x + a) + sinh(b*x + a) - 1) - 2*(b*x + a)*log(-cosh(b*x + a) - sinh(b*x
+ a) + 1) - 2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*dilog(-cosh(b*x + a)
 - sinh(b*x + a)))/b^2
```

Sympy [F]

$$\int x\coth(a+bx)\,dx = \int x\cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x \coth(a+bx)\,dx = -\frac{1}{2}\,x^2 + \frac{bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] -1/2*x^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(
-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x \coth(a+bx)\,dx = \int x\cosh\left(bx+a\right)\mathrm{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x), x)
```

3.401 $\int \coth(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \coth(a+bx)\,dx = \frac{\log(\sinh(a+bx))}{b}$$

```
[Out] ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$\int \coth(a+bx)\,dx = \frac{\log(\sinh(a+bx))}{b}$$

```
[In] Int[Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\log(\sinh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \coth(a+bx)\,dx = \frac{\log(\cosh(a+bx)) + \log(\tanh(a+bx))}{b}$$

```
[In] Integrate[Coth[a + b*x],x]
[Out] (Log[Cosh[a + b*x]] + Log[Tanh[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$-\frac{\ln(\operatorname{csch}(bx+a))}{b}$	13
default	$-\frac{\ln(\operatorname{csch}(bx+a))}{b}$	13
risch	$-x-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	27
parallelrisch	$\frac{-bx+\ln(\tanh(bx+a))-\ln(1-\tanh(bx+a))}{b}$	30

```
[In] int(cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/b*ln(csch(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \coth(a+bx)\,dx = -\frac{bx - \log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] -(b*x - log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int \coth(a+bx)\,dx = \int \cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \coth(a+bx)\,dx = \frac{\log\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{b}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] log(e^(b*x + a) - e^(-b*x - a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\,dx = -\frac{bx + a - \log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{b}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] -(b*x + a - log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\,dx = \frac{\ln\left(\sinh(a+b\,x)\right)}{b}$$

```
[In] int(cosh(a + b*x)/sinh(a + b*x),x)
[Out] log(sinh(a + b*x))/b
```

3.402 $\int \frac{\coth(a+bx)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\coth(a+bx)}{x} \, dx = \text{Int}\left(\frac{\coth(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(coth(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth(a+bx)}{x} \, dx = \int \frac{\coth(a+bx)}{x} \, dx$$

```
[In] Int[Coth[a + b*x]/x,x]
```

```
[Out] Defer[Int][Coth[a + b*x]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth(a+bx)}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)}{x}\,dx$$

```
[In] Integrate[Coth[a + b*x]/x,x]
[Out] Integrate[Coth[a + b*x]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}dx$$

```
[In] int(cosh(b*x+a)*csch(b*x+a)/x,x)
[Out] int(cosh(b*x+a)*csch(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 3.50

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="maxima")
[Out] -integrate(1/(x*e^(b*x + a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x)
+ log(x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*csch(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)}{x\sinh(a+b\,x)}\,dx$$

```
[In] int(cosh(a + b*x)/(x*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)/(x*sinh(a + b*x)), x)
```

3.403 $\int \frac{\coth(a+bx)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\coth(a+bx)}{x^2} dx = \mathrm{Int}\left(\frac{\coth(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(coth(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)}{x^2} dx$$

```
[In] Int[Coth[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][Coth[a + b*x]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)}{x^2}\,dx$$

```
[In] Integrate[Coth[a + b*x]/x^2,x]
[Out] Integrate[Coth[a + b*x]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}dx$$

```
[In] int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
[Out] int(cosh(b*x+a)*csch(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x**2,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.60

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="maxima")
[Out] -1/x - integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x +
a) - x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*csch(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)}{x^2\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(cosh(a + b*x)/(x^2*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)/(x^2*sinh(a + b*x)), x)
```

3.404 $\int x^m \cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} + \operatorname{Int}(x^m\operatorname{csch}(a+bx),x)$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)
/((b*x)^m)+Unintegrable(x^m*csch(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

```
[In] Int[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Defer[Int][x^m*Csch[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^m\operatorname{csch}(a+bx)\,dx + \int x^m\sinh(a+bx)\,dx \\
&= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx - \frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx + \int x^m\operatorname{csch}(a+bx)\,dx \\
&= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} + \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} + \int x^m\operatorname{csch}(a+bx)\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 17.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(a+bx)\coth(a+bx)\,dx$$

```
[In] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)\,dx$$

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x^m \cosh(a+bx)\coth(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x),x)
[Out] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.405 $\int x^3 \cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{6x\cosh(a+bx)}{b^3}+\frac{x^3\cosh(a+bx)}{b}-\frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{6\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}+\frac{6\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}-\frac{6\sinh(a+bx)}{b^4}-\frac{3x^2\sinh(a+bx)}{b^2}$$

```
[Out] -2*x^3*arctanh(exp(b*x+a))/b+6*x*cosh(b*x+a)/b^3+x^3*cosh(b*x+a)/b-3*x^2*po
lylog(2,-exp(b*x+a))/b^2+3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp
(b*x+a))/b^3-6*x*polylog(3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+6*p
olylog(4,exp(b*x+a))/b^4-6*sinh(b*x+a)/b^4-3*x^2*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5558, 3377, 2717, 4267, 2611, 6744, 2320, 6724}

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx = -\frac{2x^3\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{6\operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} + \frac{6\operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4} - \frac{6\sinh(a+bx)}{b^4} + \frac{6x\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6x\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} + \frac{6x\cosh(a+bx)}{b^3} - \frac{3x^2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3x^2\sinh(a+bx)}{b^2} + \frac{x^3\cosh(a+bx)}{b}$$

```
[In] Int[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(a + b*x)])/b + (6*x*Cosh[a + b*x])/b^3 + (x^3*Cosh[a + b
*x])/b - (3*x^2*PolyLog[2, -E^(a + b*x)])/b^2 + (3*x^2*PolyLog[2, E^(a + b*
x)])/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - (6*x*PolyLog[3, E^(a + b*x)
])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])/b^4
 - (6*Sinh[a + b*x])/b^4 - (3*x^2*Sinh[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3\text{csch}(a+bx)\,dx+\int x^3\sinh(a+bx)\,dx\\ &= -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x^3\cosh(a+bx)}{b}-\frac{3\int x^2\cosh(a+bx)\,dx}{b}\\ &\quad-\frac{3\int x^2\log\left(1-e^{a+bx}\right)\,dx}{b}+\frac{3\int x^2\log\left(1+e^{a+bx}\right)\,dx}{b}\end{aligned}$$

$$= -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3x^2\sinh(a+bx)}{b^2} + \frac{6\int x\,\text{PolyLog}\left(2,-e^{a+bx}\right)dx}{b^2} - \frac{6\int x\,\text{PolyLog}\left(2,e^{a+bx}\right)dx}{b^2} + \frac{6\int x\sinh(a+bx)\,dx}{b^2}$$

$$= -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^2\sinh(a+bx)}{b^2} - \frac{6\int\cosh(a+bx)\,dx}{b^3} - \frac{6\int\text{PolyLog}\left(3,-e^{a+bx}\right)dx}{b^3} + \frac{6\int\text{PolyLog}\left(3,e^{a+bx}\right)dx}{b^3}$$

$$= -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{6\sinh(a+bx)}{b^4} - \frac{3x^2\sinh(a+bx)}{b^2} - \frac{6\text{Subst}\left(\int\frac{\text{PolyLog}(3,-x)}{x}dx,x,e^{a+bx}\right)}{b^4} + \frac{6\text{Subst}\left(\int\frac{\text{PolyLog}(3,x)}{x}dx,x,e^{a+bx}\right)}{b^4}$$

$$= -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{6x\cosh(a+bx)}{b^3} + \frac{x^3\cosh(a+bx)}{b} - \frac{3x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{3x^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{6\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} + \frac{6\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4} - \frac{6\sinh(a+bx)}{b^4} - \frac{3x^2\sinh(a+bx)}{b^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.06

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{6bx\cosh(a+bx) + b^3x^3\cosh(a+bx) + b^3x^3\log\left(1-e^{a+bx}\right) - b^3x^3\log\left(1+e^{a+bx}\right) - 3b^2x^2\,\text{PolyLog}\left(2,-\right.}{}$$

```
[In] Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (6*b*x*Cosh[a + b*x] + b^3*x^3*Cosh[a + b*x] + b^3*x^3*Log[1 - E^(a + b*x)]
 - b^3*x^3*Log[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^
```

```
2*x^2*PolyLog[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*Poly
Log[3, E^(a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)
] - 6*Sinh[a + b*x] - 3*b^2*x^2*Sinh[a + b*x])/b^4
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.49

method	result
risch	$\frac{(x^3b^3-3x^2b^2+6bx-6)e^{bx+a}}{2b^4}+\frac{(x^3b^3+3x^2b^2+6bx+6)e^{-bx-a}}{2b^4}+\frac{6\,\text{polylog}(4,e^{bx+a})}{b^4}-\frac{6\,\text{polylog}(4,-e^{bx+a})}{b^4}+\frac{\ln(1-e^{bx+a})x^3}{b}$

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)+1/2*(b^3*x^3+3*b^2*x^2+6*b*x
+6)/b^4*exp(-b*x-a)+6*polylog(4,exp(b*x+a))/b^4-6*polylog(4,-exp(b*x+a))/b^
4+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,ex
p(b*x+a))/b^3-1/b*ln(exp(b*x+a)+1)*x^3-3*x^2*polylog(2,-exp(b*x+a))/b^2+6*x
*polylog(3,-exp(b*x+a))/b^3+2/b^4*a^3*arctanh(exp(b*x+a))+1/b^4*ln(1-exp(b*
x+a))*a^3-1/b^4*ln(exp(b*x+a)+1)*a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(156) = 312.

Time = 0.27 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.10

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx$$
$$=\frac{b^3x^3+3\,b^2x^2+(b^3x^3-3\,b^2x^2+6\,bx-6)\cosh{(bx+a)}^2+2\,(b^3x^3-3\,b^2x^2+6\,bx-6)\cosh{(bx+a)}\sinh{(}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 3*b^2*x^2 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^
2 + 2*(b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a) + (b^3*
x^3 - 3*b^2*x^2 + 6*b*x - 6)*sinh(b*x + a)^2 + 6*b*x + 6*(b^2*x^2*cosh(b*x
+ a) + b^2*x^2*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b^2
*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 2*(b^3*x^3*cosh(b*x + a) + b^3*x^3*sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) + 1) - 2*(a^3*cosh(b*x + a) + a^3*sinh(b*x + a))*log(cosh
(b*x + a) + sinh(b*x + a) - 1) + 2*((b^3*x^3 + a^3)*cosh(b*x + a) + (b^3*x^
3 + a^3)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 12*(cosh(
b*x + a) + sinh(b*x + a))*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 12*(c
osh(b*x + a) + sinh(b*x + a))*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) -
12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(
```

```
b*x + a)) + 12*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*polylog(3, -cosh(b*x
 + a) - sinh(b*x + a)) + 6)/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F]

$$\int x^3\cosh(a+bx)\coth(a+bx)\,dx=\int x^3\cosh^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a),x)
[Out] Integral(x**3*cosh(a + b*x)**2*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.25

$$\begin{aligned}&\int x^3\cosh(a+bx)\coth(a+bx)\,dx\\&=\frac{\left(\left(b^3x^3e^{(2a)}-3\,b^2x^2e^{(2a)}+6\,bxe^{(2a)}-6\,e^{(2a)}\right)e^{(bx)}+\left(b^3x^3+3\,b^2x^2+6\,bx+6\right)e^{(-bx)}\right)e^{(-a)}}{2\,b^4}\\&\quad-\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}\\&\quad+\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}\end{aligned}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] 1/2*((b^3*x^3*e^(2*a) - 3*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a) - 6*e^(2*a))*e^(b
*x) + (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x))*e^(-a)/b^4 - (b^3*x^3*log
(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*
x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1)
+ 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(
4, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh(a+bx)\coth(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x),x)
[Out] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.406 $\int x^2 \cosh(a+bx) \coth(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 115

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx = -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}$$

```
[Out] -2*x^2*arctanh(exp(b*x+a))/b+2*cosh(b*x+a)/b^3+x^2*cosh(b*x+a)/b-2*x*polylo
g(2,-exp(b*x+a))/b^2+2*x*polylog(2,exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))
/b^3-2*polylog(3,exp(b*x+a))/b^3-2*x*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5558, 3377, 2718, 4267, 2611, 2320, 6724}

$$\int x^2 \cosh(a+bx)\coth(a+bx)\,dx = -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{2\cosh(a+bx)}{b^3}-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{x^2\cosh(a+bx)}{b}$$

```
[In] Int[x^2*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(a + b*x)])/b + (2*Cosh[a + b*x])/b^3 + (x^2*Cosh[a + b*x
])/b - (2*x*PolyLog[2, -E^(a + b*x)])/b^2 + (2*x*PolyLog[2, E^(a + b*x)])/b
^2 + (2*PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 - (
2*x*Sinh[a + b*x])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
```

```
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\text{csch}(a+bx)\,dx+\int x^2\sinh(a+bx)\,dx\\
&= -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x^2\cosh(a+bx)}{b}-\frac{2\int x\cosh(a+bx)\,dx}{b}\\
&\quad-\frac{2\int x\log\left(1-e^{a+bx}\right)\,dx}{b}+\frac{2\int x\log\left(1+e^{a+bx}\right)\,dx}{b}\\
&= -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x^2\cosh(a+bx)}{b}-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\
&\quad+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}+\frac{2\int\text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2}\\
&\quad-\frac{2\int\text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2}+\frac{2\int\sinh(a+bx)\,dx}{b^2}\\
&= -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}\\
&\quad-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2x\sinh(a+bx)}{b^2}\\
&\quad+\frac{2\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}-\frac{2\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&= -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{2\cosh(a+bx)}{b^3}+\frac{x^2\cosh(a+bx)}{b}-\frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\
&\quad+\frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{2x\sinh(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx$$
$$=\frac{2\cosh(a+bx)+b^2x^2\cosh(a+bx)+b^2x^2\log\left(1-e^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}$$

[In] Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x],x]

[Out] (2*Cosh[a + b*x] + b^2*x^2*Cosh[a + b*x] + b^2*x^2*Log[1 - E^(a + b*x)] - b ^2*x^2*Log[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLo g[2, E^(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)] - 2*b*x*Sinh[a + b*x])/b^3

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(x^2b^2-2bx+2)e^{bx+a}}{2b^3}+\frac{(x^2b^2+2bx+2)e^{-bx-a}}{2b^3}-\frac{2a^2\operatorname{arctanh}(e^{bx+a})}{b^3}+\frac{\ln(1-e^{bx+a})x^2}{b}-\frac{\ln(1-e^{bx+a})a^2}{b^3}+\frac{2x\,\mathrm{polylog}(2,e^{b}}{b^2}$

[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)+1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)- 2/b^3*a^2*arctanh(exp(b*x+a))+1/b*ln(1-exp(b*x+a))*x^2-1/b^3*ln(1-exp(b*x+a))*a^2+2*x*polylog(2,exp(b*x+a))/b^2-2*polylog(3,exp(b*x+a))/b^3-1/b*ln(exp (b*x+a)+1)*x^2+1/b^3*ln(exp(b*x+a)+1)*a^2-2*x*polylog(2,-exp(b*x+a))/b^2+2* polylog(3,-exp(b*x+a))/b^3

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(108) = 216.

Time = 0.26 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx$$
$$=\frac{b^2x^2+(b^2x^2-2\,bx+2)\cosh(bx+a)^2+2\,(b^2x^2-2\,bx+2)\cosh(bx+a)\sinh(bx+a)+(b^2x^2-2\,bx+2}{}$$

[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")

[Out] 1/2*(b^2*x^2 + (b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^2 + 2

```
*b*x + 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh
(b*x + a)) - 4*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) - 2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a))*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + 2*(a^2*cosh(b*x + a) + a^2*sinh(b*x + a
))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b^2*x^2 - a^2)*cosh(b*x + a
) + (b^2*x^2 - a^2)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
- 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a
)) + 4*(cosh(b*x + a) + sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x
 + a)) + 2)/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F]

$$\int x^2\cosh(a+bx)\coth(a+bx)\,dx = \int x^2\cosh^2{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a),x)
[Out] Integral(x**2*cosh(a + b*x)**2*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int x^2\cosh(a+bx)\coth(a+bx)\,dx \\
&= \frac{\left((b^2x^2e^{(2a)} - 2bxe^{(2a)} + 2e^{(2a)})e^{(bx)} + (b^2x^2 + 2bx + 2)e^{(-bx)}\right)e^{(-a)}}{2b^3} \\
&\quad - \frac{b^2x^2\log\left(e^{(bx+a)} + 1\right) + 2bx\mathrm{Li}_2(-e^{(bx+a)}) - 2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3} \\
&\quad + \frac{b^2x^2\log\left(-e^{(bx+a)} + 1\right) + 2bx\mathrm{Li}_2(e^{(bx+a)}) - 2\,\mathrm{Li}_3(e^{(bx+a)})}{b^3}
\end{aligned}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] 1/2*((b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b
*x + 2)*e^(-b*x))*e^(-a)/b^3 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(
-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a)
 + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2 \cosh(a+bx) \coth(a+bx) \, dx = \int x^2 \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right) \, dx$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx) \coth(a+bx) \, dx = \int \frac{x^2 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)} \, dx$$

```
[In] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x),x)
[Out] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.407 $\int x \cosh(a+bx) \coth(a+bx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int x \cosh(a+bx) \coth(a+bx) \, dx = -\frac{2x\text{arctanh}(e^{a+bx})}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\text{PolyLog}(2,-e^{a+bx})}{b^2} + \frac{\text{PolyLog}(2,e^{a+bx})}{b^2} - \frac{\sinh(a+bx)}{b^2}$$

```
[Out] -2*x*arctanh(exp(b*x+a))/b+x*cosh(b*x+a)/b-polylog(2,-exp(b*x+a))/b^2+polyl
og(2,exp(b*x+a))/b^2-sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5558, 3377, 2717, 4267, 2317, 2438}

$$\int x \cosh(a+bx) \coth(a+bx) \, dx = -\frac{2x\text{arctanh}(e^{a+bx})}{b} - \frac{\text{PolyLog}(2,-e^{a+bx})}{b^2} + \frac{\text{PolyLog}(2,e^{a+bx})}{b^2} - \frac{\sinh(a+bx)}{b^2} + \frac{x\cosh(a+bx)}{b}$$

```
[In] Int[x*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-2*x*ArcTanh[E^(a + b*x)])/b + (x*Cosh[a + b*x])/b - PolyLog[2, -E^(a + b*
x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 - Sinh[a + b*x]/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x\text{csch}(a+bx)\,dx + \int x\sinh(a+bx)\,dx \\ &= -\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\int\cosh(a+bx)\,dx}{b} \\ &\quad - \frac{\int\log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{\int\log\left(1+e^{a+bx}\right)\,dx}{b}\end{aligned}$$

$$= -\frac{2x\text{arctanh}(e^{a+bx})}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\sinh(a+bx)}{b^2} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{b^2}$$

$$= -\frac{2x\text{arctanh}(e^{a+bx})}{b} + \frac{x\cosh(a+bx)}{b} - \frac{\text{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{\text{PolyLog}(2, e^{a+bx})}{b^2} - \frac{\sinh(a+bx)}{b^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \frac{bx\cosh(a+bx) + bx\log(1-e^{a+bx}) - bx\log(1+e^{a+bx}) - \text{PolyLog}(2, -e^{a+bx}) + \text{PolyLog}(2, e^{a+bx}) - \text{s}}{b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] (b*x*Cosh[a + b*x] + b*x*Log[1 - E^(a + b*x)] - b*x*Log[1 + E^(a + b*x)] -
PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)] - Sinh[a + b*x])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(63) = 126.

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.11

method	result
risch	$\frac{(bx-1)e^{bx+a}}{2b^2} + \frac{(bx+1)e^{-bx-a}}{2b^2} + \frac{\ln(1-e^{bx+a})x}{b} + \frac{\ln(1-e^{bx+a})a}{b^2} + \frac{\text{polylog}(2,e^{bx+a})}{b^2} - \frac{\ln(e^{bx+a}+1)x}{b} - \frac{\ln(e^{bx+a}+1)a}{b^2}$

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)+1/2*(b*x+1)/b^2*exp(-b*x-a)+1/b*ln(1-exp(b*x+a))
*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1/b*ln(exp(b*x+a)+1)*
x-1/b^2*ln(exp(b*x+a)+1)*a-polylog(2,-exp(b*x+a))/b^2+2/b^2*a*arctanh(exp(b
*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(61) = 122$.

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.86

$$\int x\cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{(bx-1)\cosh(bx+a)^2 + 2(bx-1)\cosh(bx+a)\sinh(bx+a) + (bx-1)\sinh(bx+a)^2 + bx + 2(\cosh(bx}{}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^2 + 2*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a) +
(b*x - 1)*sinh(b*x + a)^2 + b*x + 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(c
osh(b*x + a) + sinh(b*x + a)) - 2*(cosh(b*x + a) + sinh(b*x + a))*dilog(-co
sh(b*x + a) - sinh(b*x + a)) - 2*(b*x*cosh(b*x + a) + b*x*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) - 2*(a*cosh(b*x + a) + a*sinh(b*x + a)
)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*((b*x + a)*cosh(b*x + a) + (b*
x + a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(b^2*cos
h(b*x + a) + b^2*sinh(b*x + a))
```

Sympy [F]

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \int x\cosh^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a),x)
[Out] Integral(x*cosh(a + b*x)**2*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int x\cosh(a+bx)\coth(a+bx)\,dx = \frac{\left(\left(bxe^{(2a)} - e^{(2a)}\right)e^{(bx)} + (bx+1)e^{(-bx)}\right)e^{(-a)}}{2b^2} - \frac{bx\log\left(e^{(bx+a)}+1\right) + \operatorname{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right) + \operatorname{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] 1/2*((b*x*e^(2*a) - e^(2*a))*e^(b*x) + (b*x + 1)*e^(-b*x))*e^(-a)/b^2 - (b*
x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) +
 1) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x\cosh(a+bx)\coth(a+bx)\,dx=\int x\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)^2*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\coth(a+bx)\,dx=\int\frac{x\cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x*cosh(a + b*x)^2)/sinh(a + b*x),x)
[Out] int((x*cosh(a + b*x)^2)/sinh(a + b*x), x)
```

3.408 $\int \cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cosh(a+bx)\coth(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 327, 212}

$$\int \cosh(a+bx)\coth(a+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Cosh[a + b*x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
[Out] Cosh[a + b*x]/b - Log[Cosh[(a + b*x)/2]]/b + Log[Sinh[(a + b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	21
default	$\frac{\cosh(bx+a)-2\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	21
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	54

[In] int(cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{\cosh(bx+a)^2 - 2(\cosh(bx+a)+\sinh(bx+a))\log(\cosh(bx+a)+\sinh(bx+a)+1) + 2(\cosh(bx+a)}{2(b\cosh(bx+a)}$$

[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")

[Out] 1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(b*cosh(b*x + a) + b*sinh(b*x + a))

Sympy [F]

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \int \cosh^2(a+bx)\operatorname{csch}(a+bx)\,dx$$

[In] integrate(cosh(b*x+a)**2*csch(b*x+a),x)

[Out] Integral(cosh(a + b*x)**2*csch(a + b*x), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(23) = 46.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{2b} + \frac{e^{(-bx-a)}}{2b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")

[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)} + e^{(-bx-a)} - 2\,\log\left(e^{(bx+a)}+1\right) + 2\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] 1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*x
 + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-b\,x}}{2\,b}$$

```
[In] int(cosh(a + b*x)^2/sinh(a + b*x),x)
[Out] exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) + exp(- a - b*x)/(2*b)
```

3.409 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \mathrm{Chi}(bx)\sinh(a) + \cosh(a)\mathrm{Shi}(bx) + \mathrm{Int}\left(\frac{\mathrm{csch}(a+bx)}{x}, x\right)$$

```
[Out] cosh(a)*Shi(b*x)+Chi(b*x)*sinh(a)+Unintegrable(csch(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
```

```
[Out] CoshIntegral[b*x]*Sinh[a] + Cosh[a]*SinhIntegral[b*x] + Defer[Int][Csch[a +
 b*x]/x, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\mathrm{csch}(a+bx)}{x}\,dx + \int \frac{\sinh(a+bx)}{x}\,dx \\
&= \cosh(a)\int \frac{\sinh(bx)}{x}\,dx + \sinh(a)\int \frac{\cosh(bx)}{x}\,dx + \int \frac{\mathrm{csch}(a+bx)}{x}\,dx \\
&= \mathrm{Chi}(bx)\sinh(a) + \cosh(a)\mathrm{Shi}(bx) + \int \frac{\mathrm{csch}(a+bx)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 8.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx$$

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x,x]
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x}dx$$

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 9.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^{2}\left(a+bx\right)\operatorname{csch}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)/x,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^{2}}{x\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(cosh(a + b*x)^2/(x*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)^2/(x*sinh(a + b*x)), x)
```

3.410 $\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = b\cosh(a)\text{Chi}(bx) - \frac{\sinh(a+bx)}{x} + b\sinh(a)\text{Shi}(bx) + \text{Int}\left(\frac{\text{csch}(a+bx)}{x^2},x\right)$$

```
[Out] b*Chi(b*x)*cosh(a)+b*Shi(b*x)*sinh(a)-sinh(b*x+a)/x+Unintegrable(csch(b*x+a
)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[a]*CoshIntegral[b*x] - Sinh[a + b*x]/x + b*Sinh[a]*SinhIntegral[b*x]
 + Defer[Int][Csch[a + b*x]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\operatorname{csch}(a+bx)}{x^2}\,dx + \int \frac{\sinh(a+bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{x} + b\int \frac{\cosh(a+bx)}{x}\,dx + \int \frac{\operatorname{csch}(a+bx)}{x^2}\,dx\\ &= -\frac{\sinh(a+bx)}{x} + (b\cosh(a))\int \frac{\cosh(bx)}{x}\,dx + (b\sinh(a))\int \frac{\sinh(bx)}{x}\,dx + \int \frac{\operatorname{csch}(a+bx)}{x^2}\,dx\\ &= b\cosh(a)\operatorname{Chi}(bx) - \frac{\sinh(a+bx)}{x} + b\sinh(a)\operatorname{Shi}(bx) + \int \frac{\operatorname{csch}(a+bx)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 24.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2,x]
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)}{x^2}dx$$

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 10.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh^2{(a+bx)}\operatorname{csch}{(a+bx)}}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)/x**2,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}^2\operatorname{csch}{(bx+a)}}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}^2\operatorname{csch}{(bx+a)}}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(a+bx)\coth(a+bx)}{x^2}\,dx=\int \frac{{\mathrm{cosh}\left(a+b\,x\right)}^2}{x^2\,\mathrm{sinh}\left(a+b\,x\right)} \,d x$$

```
[In] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)), x)
```

3.411 $\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \text{Int}(x^m\coth(a+bx),x)$$

```
[Out] 2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)+2^(-3-m)*x^m*GAMMA(1+m
,2*b*x)/b/exp(2*a)/((b*x)^m)+Unintegrable(x^m*coth(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$$

```
[In] Int[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-(b*x))^m) + (2^(-3 - m)*
x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m) + Defer[Int][x^m*Coth[a + b*x]
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^m \coth(a+bx)\,dx + \int x^m \cosh(a+bx)\sinh(a+bx)\,dx \\
&= \int x^m \coth(a+bx)\,dx + \int \frac{1}{2}x^m \sinh(2a+2bx)\,dx \\
&= \frac{1}{2}\int x^m \sinh(2a+2bx)\,dx + \int x^m \coth(a+bx)\,dx \\
&= \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx - \frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx + \int x^m \coth(a+bx)\,dx \\
&= \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} \\
&\quad + \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b} + \int x^m \coth(a+bx)\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 16.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh^2(a+bx)\coth(a+bx)\,dx$$

```
[In] Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] Integrate[x^m*Cosh[a + b*x]^2*Coth[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)dx$$

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x),x)
[Out] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.412 $\int x^3 \cosh^2(a+bx) \coth(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\begin{aligned}\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx &= \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}\\&\quad+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}\\&\quad-\frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}\\&\quad-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}\\&\quad-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}\\&\quad+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}\end{aligned}$$

```
[Out] 3/8*x/b^3+1/4*x^3/b-1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,ex
p(2*b*x+2*a))/b^2-3/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b
*x+2*a))/b^4-3/8*cosh(b*x+a)*sinh(b*x+a)/b^4-3/4*x^2*cosh(b*x+a)*sinh(b*x+a
)/b^2+3/4*x*sinh(b*x+a)^2/b^3+1/2*x^3*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5558, 5480, 3392, 30, 2715, 8, 3797, 2221, 2611, 6744, 2320, 6724}

$$\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{3\,\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b^4}-\frac{3x\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{x^4}{4}$$

```
[In] Int[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (3*x)/(8*b^3) + x^3/(4*b) - x^4/4 + (x^3*Log[1 - E^(2*(a + b*x))])/b + (3*x
^2*PolyLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])
/(2*b^3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4) - (3*Cosh[a + b*x]*Sinh[
a + b*x])/(8*b^4) - (3*x^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (3*x*Sinh
[a + b*x]^2)/(4*b^3) + (x^3*Sinh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\coth(a+bx)\,dx+\int x^3\cosh(a+bx)\sinh(a+bx)\,dx\\
&= -\frac{x^4}{4}+\frac{x^3\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx-\frac{3\int x^2\sinh^2(a+bx)\,dx}{2b}\\
&= -\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^3\sinh^2(a+bx)}{2b}-\frac{3\int\sinh^2(a+bx)\,dx}{4b^3}+\frac{3\int x^2\,dx}{4b}-\frac{3\int x^2\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x^3}{4b}-\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}\\
&\quad-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}\\
&\quad+\frac{3x\sinh^2(a+bx)}{4b^3}+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3\int 1\,dx}{8b^3}-\frac{3\int x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)\,dx}{b^2}\\
&= \frac{3x}{8b^3}+\frac{x^3}{4b}-\frac{x^4}{4}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}\\
&\quad-\frac{3x\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4}\\
&\quad-\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{3x\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^3\sinh^2(a+bx)}{2b}+\frac{3\int\text{PolyLog}\left(3,e^{2(a+bx)}\right)\,dx}{2b^3}
\end{aligned}$$

$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3} + \frac{x^3\sinh^2(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{2(a+bx)}\right)}{4b^4}$$

$$= \frac{3x}{8b^3} + \frac{x^3}{4b} - \frac{x^4}{4} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{3\operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b^4} - \frac{3x^2\cosh(a+bx)\sinh(a+bx)}{4b^2} + \frac{3x\sinh^2(a+bx)}{4b^3} + \frac{x^3\sinh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.31

$$\int x^3\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\sinh(a)(\cosh(a)+\sinh(a))\left(4b^4x^4+6bx\cosh(2(a+bx))+4b^3x^3\cosh(2(a+bx))+16b^3x^3\log\left(1-e^{-a-b}\right.\right.}{}$$

```
[In] Integrate[x^3*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (Sinh[a]*(Cosh[a] + Sinh[a])*(4*b^4*x^4 + 6*b*x*Cosh[2*(a + b*x)] + 4*b^3*x
^3*Cosh[2*(a + b*x)] + 16*b^3*x^3*Log[1 - E^(-a - b*x)] + 16*b^3*x^3*Log[1
+ E^(-a - b*x)] - 48*b^2*x^2*PolyLog[2, -E^(-a - b*x)] - 48*b^2*x^2*PolyLog
[2, E^(-a - b*x)] - 96*b*x*PolyLog[3, -E^(-a - b*x)] - 96*b*x*PolyLog[3, E^
(-a - b*x)] - 96*PolyLog[4, -E^(-a - b*x)] - 96*PolyLog[4, E^(-a - b*x)] -
3*Sinh[2*(a + b*x)] - 6*b^2*x^2*Sinh[2*(a + b*x)]))/(8*b^4*(-1 + E^(2*a)))
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{x^4}{4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)\,\mathrm{e}^{2bx+2a}}{32b^4}+\frac{(4x^3b^3+6x^2b^2+6bx+3)\,\mathrm{e}^{-2bx-2a}}{32b^4}-\frac{a^3\ln(\mathrm{e}^{bx+a}-1)}{b^4}+\frac{2a^3\ln(\mathrm{e}^{bx+a})}{b^4}-\frac{3a^4}{2b^4}+$

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4+1/32*(4*b^3*x^3-6*b^2*x^2+6*b*x-3)/b^4*exp(2*b*x+2*a)+1/32*(4*b^3*
x^3+6*b^2*x^2+6*b*x+3)/b^4*exp(-2*b*x-2*a)-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4
*a^3*ln(exp(b*x+a))-3/2/b^4*a^4+1/b*ln(exp(b*x+a)+1)*x^3+3*x^2*polylog(2,-e
xp(b*x+a))/b^2-6*x*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^3+3*x^
2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3-2/b^3*a^3*x+6*pol
ylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+1/b^4*ln(1-exp(b*x+a))*
a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(159) = 318$.

Time = 0.26 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.87

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^3*x^3 + (4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)^4 + 4*(
4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b^3*x
^3 - 6*b^2*x^2 + 6*b*x - 3)*sinh(b*x + a)^4 + 6*b^2*x^2 - 8*(b^4*x^4 - 2*a^
4)*cosh(b*x + a)^2 - 2*(4*b^4*x^4 - 8*a^4 - 3*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*
x - 3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*(b^2*x^2*cosh(b*x + a)
^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog
(cosh(b*x + a) + sinh(b*x + a)) + 96*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*c
osh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*dilog(-cosh(b*x + a)
- sinh(b*x + a)) + 32*(b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x + a)*si
nh(b*x + a) + b^3*x^3*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) +
1) - 32*(a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh
(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 32*((b^3*x^3 + a^3)*c
osh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 +
 a^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 192*(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(4, co
sh(b*x + a) + sinh(b*x + a)) + 192*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 19
2*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x +
 a)^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 192*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*polylog(3, -cos
h(b*x + a) - sinh(b*x + a)) + 4*((4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*cosh(b
*x + a)^3 - 4*(b^4*x^4 - 2*a^4)*cosh(b*x + a))*sinh(b*x + a) + 3)/(b^4*cosh
(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.25

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{1}{2}\,x^4 + \frac{\left(8\,b^4x^4e^{(2\,a)} + \left(4\,b^3x^3e^{(4\,a)} - 6\,b^2x^2e^{(4\,a)} + 6\,bxe^{(4\,a)} - 3\,e^{(4\,a)}\right)e^{(2\,bx)} + \left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx)}}{32\,b^4} + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3\left(-e^{(bx+a)}\right) + 6\,\mathrm{Li}_4\left(-e^{(bx+a)}\right)}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3\left(e^{(bx+a)}\right) + 6\,\mathrm{Li}_4\left(e^{(bx+a)}\right)}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] -1/2*x^4 + 1/32*(8*b^4*x^4*e^(2*a) + (4*b^3*x^3*e^(4*a) - 6*b^2*x^2*e^(4*a)
 + 6*b*x*e^(4*a) - 3*e^(4*a))*e^(2*b*x) + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x +
3)*e^(-2*b*x))*e^(-2*a)/b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dil
og(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x +
a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) -
6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x),x)
[Out] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.413 $\int x^2 \cosh^2(a+bx) \coth(a+bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 126

$$\int x^2 \cosh^2(a+bx) \coth(a+bx)\, dx = \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x \operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} - \frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2\sinh^2(a+bx)}{2b}$$

```
[Out] 1/4*x^2/b-1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b^
2-1/2*polylog(3,exp(2*b*x+2*a))/b^3-1/2*x*cosh(b*x+a)*sinh(b*x+a)/b^2+1/4*s
inh(b*x+a)^2/b^3+1/2*x^2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5558, 5480, 3391, 30, 3797, 2221, 2611, 2320, 6724}

$$\int x^2 \cosh^2(a+bx) \coth(a+bx)\, dx = -\frac{\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x \operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2} - \frac{x\sinh(a+bx)\cosh(a+bx)}{2b^2} + \frac{x^2 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{x^2\sinh^2(a+bx)}{2b} + \frac{x^2}{4b} - \frac{x^3}{3}$$

```
[In] Int[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] x^2/(4*b) - x^3/3 + (x^2*Log[1 - E^(2*(a + b*x))])/b + (x*PolyLog[2, E^(2*(
a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3) - (x*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b^2) + Sinh[a + b*x]^2/(4*b^3) + (x^2*Sinh[a + b*x]^2)/(2*b
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\coth(a+bx)\,dx+\int x^2\cosh(a+bx)\sinh(a+bx)\,dx\\
&= -\frac{x^3}{3}+\frac{x^2\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx-\frac{\int x\sinh^2(a+bx)\,dx}{b}\\
&= -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}\\
&\quad+\frac{\sinh^2(a+bx)}{4b^3}+\frac{x^2\sinh^2(a+bx)}{2b}+\frac{\int x\,dx}{2b}-\frac{2\int x\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x^2}{4b}-\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}\\
&\quad-\frac{x\cosh(a+bx)\sinh(a+bx)}{2b^2}+\frac{\sinh^2(a+bx)}{4b^3}\\
&\quad+\frac{x^2\sinh^2(a+bx)}{2b}-\frac{\int\text{PolyLog}\left(2,e^{2(a+bx)}\right)\,dx}{b^2}
\end{aligned}$$

$$= \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^2} - \frac{x \cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2 \sinh^2(a+bx)}{2b} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^3}$$

$$= \frac{x^2}{4b} - \frac{x^3}{3} + \frac{x^2 \log\left(1 - e^{2(a+bx)}\right)}{b} + \frac{x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} - \frac{x \cosh(a+bx)\sinh(a+bx)}{2b^2} + \frac{\sinh^2(a+bx)}{4b^3} + \frac{x^2 \sinh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.41

$$\int x^2 \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\sinh(a)(\cosh(a)+\sinh(a))\left(8b^3x^3 + 3\cosh(2(a+bx)) + 6b^2x^2\cosh(2(a+bx)) + 24b^2x^2\log\left(1-e^{-a-bx}\right) + \right.}{}$$

```
[In] Integrate[x^2*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (Sinh[a]*(Cosh[a] + Sinh[a])*(8*b^3*x^3 + 3*Cosh[2*(a + b*x)] + 6*b^2*x^2*C
osh[2*(a + b*x)] + 24*b^2*x^2*Log[1 - E^(-a - b*x)] + 24*b^2*x^2*Log[1 + E^
(-a - b*x)] - 48*b*x*PolyLog[2, -E^(-a - b*x)] - 48*b*x*PolyLog[2, E^(-a -
b*x)] - 48*PolyLog[3, -E^(-a - b*x)] - 48*PolyLog[3, E^(-a - b*x)] - 6*b*x*
Sinh[2*(a + b*x)]))/(12*b^3*(-1 + E^(2*a)))
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.76

method	result
risch	$-\frac{x^3}{3} + \frac{(2x^2b^2-2bx+1)e^{2bx+2a}}{16b^3} + \frac{(2x^2b^2+2bx+1)e^{-2bx-2a}}{16b^3} + \frac{4a^3}{3b^3} + \frac{2a^2x}{b^2} + \frac{\ln(e^{bx+a}+1)x^2}{b} + \frac{2x\operatorname{polylog}(2,-e^{bx+a})}{b^2} +$

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3+1/16*(2*b^2*x^2-2*b*x+1)/b^3*exp(2*b*x+2*a)+1/16*(2*b^2*x^2+2*b*x+
1)/b^3*exp(-2*b*x-2*a)+4/3/b^3*a^3+2/b^2*a^2*x+1/b*ln(exp(b*x+a)+1)*x^2+2*x
*polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,exp(b*x+
a))/b^2+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))-1/b^3*ln(1-exp(
b*x+a))*a^2-2*polylog(3,-exp(b*x+a))/b^3-2*polylog(3,exp(b*x+a))/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(113) = 226$.

Time = 0.27 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.53

$$\int x^2 \cosh^2(a+bx) \coth(a+bx)\,dx$$
$$= \frac{3\,(2\,b^2x^2 - 2\,bx + 1)\cosh\,(bx+a)^4 + 12\,(2\,b^2x^2 - 2\,bx + 1)\cosh\,(bx+a)\sinh\,(bx+a)^3 + 3\,(2\,b^2x^2 - 2\,bx}{}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^4 + 12*(2*b^2*x^2 - 2*b*x + 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + 3*(2*b^2*x^2 - 2*b*x + 1)*sinh(b*x + a)^4
 + 6*b^2*x^2 - 16*(b^3*x^3 + 2*a^3)*cosh(b*x + a)^2 - 2*(8*b^3*x^3 + 16*a^3
 - 9*(2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 6*b*x + 96*
(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a
)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 96*(b*x*cosh(b*x + a)^2 + 2*b*x
*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*dilog(-cosh(b*x + a) -
sinh(b*x + a)) + 48*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh
(b*x + a) + b^2*x^2*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1)
 + 48*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 48*((b^2*x^2 - a^2)*cos
h(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*x^2 - a
^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 96*(cosh(b*x
 + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*polylog(3, cosh(
b*x + a) + sinh(b*x + a)) - 96*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*(3*(
2*b^2*x^2 - 2*b*x + 1)*cosh(b*x + a)^3 - 8*(b^3*x^3 + 2*a^3)*cosh(b*x + a))
*sinh(b*x + a) + 3)/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a
) + b^3*sinh(b*x + a)^2)
```

Sympy [F]

$$\int x^2 \cosh^2(a+bx) \coth(a+bx)\,dx = \int x^2 \cosh^3{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x)**3*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.36

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{2}{3}\,x^3 + \frac{\left(16\,b^3x^3e^{(2\,a)} + 3\left(2\,b^2x^2e^{(4\,a)} - 2\,bxe^{(4\,a)} + e^{(4\,a)}\right)e^{(2\,bx)} + 3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx)}\right)e^{(-2\,a)}}{48\,b^3} + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3} + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/48*(16*b^3*x^3*e^(2*a) + 3*(2*b^2*x^2*e^(4*a) - 2*b*x*e^(4*a)
+ e^(4*a))*e^(2*b*x) + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^3 +
 (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -
e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a
)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx = \int x^2\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{x^2\cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x),x)
```

```
[Out] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.414 $\int x\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{x}{4b}-\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}$$

```
[Out] 1/4*x/b-1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2-
1/4*cosh(b*x+a)*sinh(b*x+a)/b^2+1/2*x*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5558, 5480, 2715, 8, 3797, 2221, 2317, 2438}

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\sinh(a+bx)\cosh(a+bx)}{4b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\sinh^2(a+bx)}{2b}+\frac{x}{4b}-\frac{x^2}{2}$$

```
[In] Int[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] x/(4*b) - x^2/2 + (x*Log[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x
))]/(2*b^2) - (Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2) + (x*Sinh[a + b*x]^2)/(
2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
```

```
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x\coth(a+bx)\,dx+\int x\cosh(a+bx)\sinh(a+bx)\,dx\\
&= -\frac{x^2}{2}+\frac{x\sinh^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx-\frac{\int\sinh^2(a+bx)\,dx}{2b}\\
&= -\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}\\
&\quad+\frac{x\sinh^2(a+bx)}{2b}+\frac{\int 1\,dx}{4b}-\frac{\int\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x}{4b}-\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}\\
&\quad+\frac{x\sinh^2(a+bx)}{2b}-\frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^2}\\
&= \frac{x}{4b}-\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}\\
&\quad-\frac{\cosh(a+bx)\sinh(a+bx)}{4b^2}+\frac{x\sinh^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$
\int x\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{4a^2-4b^2x^2-2bx\cosh(2(a+bx))-8bx\log\left(1-e^{-2(a+bx)}\right)+4\,\text{PolyLog}\left(2,e^{-2(a+bx)}\right)+\sinh(2(a+bx$$
$$8b^2$$

```
[In] Integrate[x*Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] -1/8*(4*a^2 - 4*b^2*x^2 - 2*b*x*Cosh[2*(a + b*x)] - 8*b*x*Log[1 - E^(-2*(a
+ b*x))] + 4*PolyLog[2, E^(-2*(a + b*x))] + Sinh[2*(a + b*x)])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(78) = 156.

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{x^2}{2}+\frac{(2bx-1)\mathrm{e}^{2bx+2a}}{16b^2}+\frac{(2bx+1)\mathrm{e}^{-2bx-2a}}{16b^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln(\mathrm{e}^{bx+a}+1)x}{b}+\frac{\mathrm{polylog}(2,-\mathrm{e}^{bx+a})}{b^2}+\frac{\ln(1-\mathrm{e}^{bx+a})x}{b}+\ln($

[In] int(x*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)

[Out] -1/2*x^2+1/16*(2*b*x-1)/b^2*exp(2*b*x+2*a)+1/16*(2*b*x+1)/b^2*exp(-2*b*x-2*a)-2/b*a*x-a^2/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(77) = 154.

Time = 0.27 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.55

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{(2\,bx-1)\cosh\left(bx+a\right)^4+4\,(2\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+(2\,bx-1)\sinh\left(bx+a\right)^4-8\,(b^2x^2-}{}$$

[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")

[Out] 1/16*((2*b*x - 1)*cosh(b*x + a)^4 + 4*(2*b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (2*b*x - 1)*sinh(b*x + a)^4 - 8*(b^2*x^2 - 2*a^2)*cosh(b*x + a)^2 - 2*(4*b^2*x^2 - 3*(2*b*x - 1)*cosh(b*x + a)^2 - 8*a^2)*sinh(b*x + a)^2 + 2*b*x + 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 16*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 16*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 16*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 16*((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 4*((2*b*x - 1)*cosh(b*x + a)^3 - 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a))*sinh(b*x + a) + 1)/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2)

Sympy [F]

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int x\cosh^3{(a+bx)}\operatorname{csch}{(a+bx)}\,dx$$

[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a),x)

[Out] Integral(x*cosh(a + b*x)**3*csch(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = -x^2 + \frac{\left(8\,b^2x^2e^{(2\,a)} + \left(2\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(2\,bx)} + (2\,bx+1)e^{(-2\,bx)}\right)e^{(-2\,a)}}{16\,b^2} + \frac{bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")

[Out] -x^2 + 1/16*(8*b^2*x^2*e^(2*a) + (2*b*x*e^(4*a) - e^(4*a))*e^(2*b*x) + (2*b*x + 1)*e^(-2*b*x))*e^(-2*a)/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2

Giac [F]

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx = \int x\cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}\,dx$$

[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")

[Out] integrate(x*cosh(b*x + a)^3*csch(b*x + a), x)

Mupad [F(-1)]

Timed out.

$$\int x\cosh^2(a+bx)\coth(a+bx)\,dx=\int \frac{x\cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((x*cosh(a + b*x)^3)/sinh(a + b*x),x)
[Out] int((x*cosh(a + b*x)^3)/sinh(a + b*x), x)
```

3.415 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

```
[Out] ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2670, 14}

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\sinh^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b}$$

```
[In] Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] Log[Sinh[a + b*x]]/b + Sinh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -i\sinh(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, -i\sinh(a+bx)\right)}{b}$$

$$= \frac{\log(\sinh(a+bx))}{b} + \frac{\sinh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{2\log(\sinh(a+bx)) + \sinh^2(a+bx)}{2b}$$

```
[In] Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] (2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
risch	$-x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	55

```
[In] int(cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.52

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{8\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)^4-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\sinh\left(bx+a\right)^4+2\left(4\,bx-3\cos\right.}{}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2*s
inh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) - co
sh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*si
nh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \int \cosh^3\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a),x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(25) = 50.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{bx+a}{b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
- a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$
$$= -\frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 8\,a - e^{(2\,bx+2\,a)} - 8\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*a - e^(2*b*x + 2
*a) - 8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b} - x + \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

```
[In] int(cosh(a + b*x)^3/sinh(a + b*x),x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a + 2
*b*x)/(8*b)
```

3.416 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) + \text{Int}\left(\frac{\coth(a+bx)}{x},x\right)$$

```
[Out] 1/2*cosh(2*a)*Shi(2*b*x)+1/2*Chi(2*b*x)*sinh(2*a)+Unintegrable(coth(b*x+a)/
x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

```
[In] Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
```

```
[Out] (CoshIntegral[2*b*x]*Sinh[2*a])/2 + (Cosh[2*a]*SinhIntegral[2*b*x])/2 + Def
er[Int][Coth[a + b*x]/x, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\coth(a+bx)}{x}\,dx + \int \frac{\cosh(a+bx)\sinh(a+bx)}{x}\,dx \\
&= \int \frac{\coth(a+bx)}{x}\,dx + \int \frac{\sinh(2a+2bx)}{2x}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x}\,dx + \int \frac{\coth(a+bx)}{x}\,dx \\
&= \frac{1}{2}\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx + \frac{1}{2}\sinh(2a)\int \frac{\cosh(2bx)}{x}\,dx + \int \frac{\coth(a+bx)}{x}\,dx \\
&= \frac{1}{2}\text{Chi}(2bx)\sinh(2a) + \frac{1}{2}\cosh(2a)\text{Shi}(2bx) + \int \frac{\coth(a+bx)}{x}\,dx
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx$$

```
[In] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x,x]
[Out] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx+a)^3\,\text{csch}(bx+a)}{x}dx$$

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 21.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh^3\left(a+bx\right)\operatorname{csch}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)/x,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.17

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x)*e^(2*a) - 1/4*Ei(-2*b*x)*e^(-2*a) - integrate(1/(x*e^(b*x + a
) + x), x) + integrate(1/(x*e^(b*x + a) - x), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3}{x\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(cosh(a + b*x)^3/(x*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)^3/(x*sinh(a + b*x)), x)
```

3.417 $\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx) + \text{Int}\left(\frac{\coth(a+bx)}{x^2},x\right)$$

```
[Out] b*Chi(2*b*x)*cosh(2*a)+b*Shi(2*b*x)*sinh(2*a)-1/2*sinh(2*b*x+2*a)/x+Uninteg
rable(coth(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

```
[In] Int[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
```

```
[Out] b*Cosh[2*a]*CoshIntegral[2*b*x] - Sinh[2*a + 2*b*x]/(2*x) + b*Sinh[2*a]*Sin
hIntegral[2*b*x] + Defer[Int][Coth[a + b*x]/x^2, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\coth(a+bx)}{x^2}\,dx + \int \frac{\cosh(a+bx)\sinh(a+bx)}{x^2}\,dx \\
&= \int \frac{\coth(a+bx)}{x^2}\,dx + \int \frac{\sinh(2a+2bx)}{2x^2}\,dx \\
&= \frac{1}{2}\int \frac{\sinh(2a+2bx)}{x^2}\,dx + \int \frac{\coth(a+bx)}{x^2}\,dx \\
&= -\frac{\sinh(2a+2bx)}{2x} + b\int \frac{\cosh(2a+2bx)}{x}\,dx + \int \frac{\coth(a+bx)}{x^2}\,dx \\
&= -\frac{\sinh(2a+2bx)}{2x} + (b\cosh(2a))\int \frac{\cosh(2bx)}{x}\,dx \\
&\quad + (b\sinh(2a))\int \frac{\sinh(2bx)}{x}\,dx + \int \frac{\coth(a+bx)}{x^2}\,dx \\
&= b\cosh(2a)\text{Chi}(2bx) - \frac{\sinh(2a+2bx)}{2x} + b\sinh(2a)\text{Shi}(2bx) + \int \frac{\coth(a+bx)}{x^2}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 7.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2,x]
[Out] Integrate[(Cosh[a + b*x]^2*Coth[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)}{x^{2}}dx$$

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 26.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh^3\left(a+bx\right)\operatorname{csch}\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)/x**2,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.00

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="maxima")
[Out] 1/2*b*e^(-2*a)*gamma(-1, 2*b*x) + 1/2*b*e^(2*a)*gamma(-1, -2*b*x) - 1/x - i
ntegrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x + a) - x^2
), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(a+bx)\coth(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)^3}{x^2\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)),x)
[Out] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)), x)
```

3.418 $\int x\cosh^2(x)\coth^2(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x\cosh^2(x)\coth^2(x)\,dx=\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\log(\sinh(x))+\frac{1}{2}x\cosh(x)\sinh(x)$$

```
[Out] 3/4*x^2-1/4*cosh(x)^2-x*coth(x)+ln(sinh(x))+1/2*x*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5558, 3391, 30, 3801, 3556}

$$\int x\cosh^2(x)\coth^2(x)\,dx=\frac{3x^2}{4}-\frac{\cosh^2(x)}{4}-x\coth(x)+\log(\sinh(x))+\frac{1}{2}x\sinh(x)\cosh(x)$$

```
[In] Int[x*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^2)/4 - Cosh[x]^2/4 - x*Coth[x] + Log[Sinh[x]] + (x*Cosh[x]*Sinh[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\cosh^2(x)\,dx + \int x\coth^2(x)\,dx \\
&= -\frac{1}{4}\cosh^2(x) - x\coth(x) + \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\int x\,dx}{2} + \int x\,dx + \int \coth(x)\,dx \\
&= \frac{3x^2}{4} - \frac{\cosh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) + \frac{1}{2}x\cosh(x)\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cosh^2(x)\coth^2(x)\,dx = \frac{3x^2}{4} - \frac{1}{8}\cosh(2x) - x\coth(x) + \log(\sinh(x)) + \frac{1}{4}x\sinh(2x)$$

```
[In] Integrate[x*Cosh[x]^2*Coth[x]^2,x]
[Out] (3*x^2)/4 - Cosh[2*x]/8 - x*Coth[x] + Log[Sinh[x]] + (x*Sinh[2*x])/4
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
risch	$\frac{3x^2}{4}+\left(-\frac{1}{16}+\frac{x}{8}\right)\mathrm{e}^{2x}+\left(-\frac{1}{16}-\frac{x}{8}\right)\mathrm{e}^{-2x}-2x-\frac{2x}{\mathrm{e}^{2x}-1}+\ln\left(\mathrm{e}^{2x}-1\right)$	48

```
[In] int(x*cosh(x)^2*coth(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^2+(-1/16+1/8*x)*exp(2*x)+(-1/16-1/8*x)*exp(-2*x)-2*x-2*x/(exp(2*x)-1)
+ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 336, normalized size of antiderivative = 10.18

$$\int x\cosh^2(x)\coth^2(x)\,dx$$

$$=\frac{(2x-1)\cosh(x)^6+6(2x-1)\cosh(x)\sinh(x)^5+(2x-1)\sinh(x)^6+(12x^2-34x+1)\cosh(x)^4+(\cdots}{}$$

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x - 1)*cosh(x)^6 + 6*(2*x - 1)*cosh(x)*sinh(x)^5 + (2*x - 1)*sinh(
x)^6 + (12*x^2 - 34*x + 1)*cosh(x)^4 + (15*(2*x - 1)*cosh(x)^2 + 12*x^2 - 3
4*x + 1)*sinh(x)^4 + 4*(5*(2*x - 1)*cosh(x)^3 + (12*x^2 - 34*x + 1)*cosh(x)
)*sinh(x)^3 - (12*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x - 1)*cosh(x)^4 + 6*(1
2*x^2 - 34*x + 1)*cosh(x)^2 - 12*x^2 - 2*x - 1)*sinh(x)^2 + 16*(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2
+ 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2
*(3*(2*x - 1)*cosh(x)^5 + 2*(12*x^2 - 34*x + 1)*cosh(x)^3 - (12*x^2 + 2*x +
 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sin
h(x))
```

Sympy [F]

$$\int x \cosh^2(x) \coth^2(x) \, dx = \int x \cosh^2(x) \coth^2(x) \, dx$$

```
[In] integrate(x*cosh(x)**2*coth(x)**2,x)
[Out] Integral(x*cosh(x)**2*coth(x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \cosh^2(x) \coth^2(x) \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.06

$$\int x \cosh^2(x) \coth^2(x) \, dx = \frac{12 x^2 e^{(4x)} - 12 x^2 e^{(2x)} + 2 x e^{(6x)} - 34 x e^{(4x)} - 2 x e^{(2x)} + 16 e^{(4x)} \log\left(e^{(2x)} - 1\right) - 16 e^{(2x)} \log\left(e^{(2x)} - 1\right) -}{16 \left(e^{(4x)} - e^{(2x)}\right)}$$

```
[In] integrate(x*cosh(x)^2*coth(x)^2,x, algorithm="giac")
[Out] 1/16*(12*x^2*e^(4*x) - 12*x^2*e^(2*x) + 2*x*e^(6*x) - 34*x*e^(4*x) - 2*x*e^
(2*x) + 16*e^(4*x)*log(e^(2*x) - 1) - 16*e^(2*x)*log(e^(2*x) - 1) + 2*x - e
^(6*x) + e^(4*x) - e^(2*x) + 1)/(e^(4*x) - e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int x\cosh^2(x)\coth^2(x)\,dx = \ln\left(e^{2x}-1\right)-2x-e^{-2x}\left(\frac{x}{8}+\frac{1}{16}\right)+e^{2x}\left(\frac{x}{8}-\frac{1}{16}\right)-\frac{2x}{e^{2x}-1}+\frac{3x^2}{4}$$

```
[In] int(x*cosh(x)^2*coth(x)^2,x)
```

```
[Out] log(exp(2*x) - 1) - 2*x - exp(-2*x)*(x/8 + 1/16) + exp(2*x)*(x/8 - 1/16) -
(2*x)/(exp(2*x) - 1) + (3*x^2)/4
```

3.419 $\int x^2 \cosh^2(x) \coth^2(x) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2} x \cosh^2(x) - x^2 \coth(x) + 2x \log\left(1 - e^{2x}\right) + \text{PolyLog}\left(2, e^{2x}\right) + \frac{1}{4} \cosh(x) \sinh(x) + \frac{1}{2} x^2 \cosh(x) \sinh(x)$$

```
[Out] 1/4*x-x^2+1/2*x^3-1/2*x*cosh(x)^2-x^2*coth(x)+2*x*ln(1-exp(2*x))+polylog(2,
exp(2*x))+1/4*cosh(x)*sinh(x)+1/2*x^2*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5558, 3392, 30, 2715, 8, 3801, 3797, 2221, 2317, 2438}

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \text{PolyLog}\left(2, e^{2x}\right) + \frac{x^3}{2} - x^2 - x^2 \coth(x) + \frac{1}{2} x^2 \sinh(x) \cosh(x) + \frac{x}{4} + 2x \log\left(1 - e^{2x}\right) - \frac{1}{2} x \cosh^2(x) + \frac{1}{4} \sinh(x) \cosh(x)$$

```
[In] Int[x^2*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] x/4 - x^2 + x^3/2 - (x*Cosh[x]^2)/2 - x^2*Coth[x] + 2*x*Log[1 - E^(2*x)] +
PolyLog[2, E^(2*x)] + (Cosh[x]*Sinh[x])/4 + (x^2*Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2\cosh^2(x)\,dx + \int x^2\coth^2(x)\,dx \\
&= -\frac{1}{2}x\cosh^2(x) - x^2\coth(x) + \frac{1}{2}x^2\cosh(x)\sinh(x) + \frac{\int x^2\,dx}{2} \\
&\quad + \frac{1}{2}\int \cosh^2(x)\,dx + 2\int x\coth(x)\,dx + \int x^2\,dx \\
&= -x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + \frac{1}{4}\cosh(x)\sinh(x) \\
&\quad + \frac{1}{2}x^2\cosh(x)\sinh(x) + \frac{\int 1\,dx}{4} - 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) \\
&\quad + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cosh(x)\sinh(x) - 2\int \log\left(1-e^{2x}\right)\,dx \\
&= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) \\
&\quad + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cosh(x)\sinh(x) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{x}{4} - x^2 + \frac{x^3}{2} - \frac{1}{2}x\cosh^2(x) - x^2\coth(x) + 2x\log\left(1-e^{2x}\right) \\
&\quad + \text{PolyLog}\left(2,e^{2x}\right) + \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x^2\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \frac{1}{8}\left(8x^2 + 4x^3 - 2x\cosh(2x) - 8x^2\coth(x) + 16x\log\left(1 - e^{-2x}\right) - 8\,\text{PolyLog}\left(2, e^{-2x}\right) + \sinh(2x) + 2x^2\sinh(2x)\right)$$

```
[In] Integrate[x^2*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (8*x^2 + 4*x^3 - 2*x*Cosh[2*x] - 8*x^2*Coth[x] + 16*x*Log[1 - E^(-2*x)] - 8
*PolyLog[2, E^(-2*x)] + Sinh[2*x] + 2*x^2*Sinh[2*x])/8
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x^3}{2} + \left(\frac{1}{16} - \frac{1}{8}x + \frac{1}{8}x^2\right) e^{2x} + \left(-\frac{1}{16} - \frac{1}{8}x - \frac{1}{8}x^2\right) e^{-2x} - \frac{2x^2}{e^{2x}-1} - 2x^2 + 2x\ln\left(1 - e^{x}\right) + 2\,\text{polylog}\left(2\right.$

```
[In] int(x^2*cosh(x)^2*coth(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3+(1/16-1/8*x+1/8*x^2)*exp(x)^2+(-1/16-1/8*x-1/8*x^2)/exp(x)^2-2*x^2/
(exp(x)^2-1)-2*x^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))+2*x*ln(exp(x)+1)+2*
polylog(2,-exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(60) = 120$.

Time = 0.26 (sec) , antiderivative size = 617, normalized size of antiderivative = 8.45

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x^2 - 2*x + 1)*cosh(x)^6 + 6*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^5 +
 (2*x^2 - 2*x + 1)*sinh(x)^6 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^4 + (8*x^
3 + 15*(2*x^2 - 2*x + 1)*cosh(x)^2 - 34*x^2 + 2*x - 1)*sinh(x)^4 + 4*(5*(2*
x^2 - 2*x + 1)*cosh(x)^3 + (8*x^3 - 34*x^2 + 2*x - 1)*cosh(x))*sinh(x)^3 -
(8*x^3 + 2*x^2 + 2*x + 1)*cosh(x)^2 + (15*(2*x^2 - 2*x + 1)*cosh(x)^4 - 8*x
^3 + 6*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^2 - 2*x^2 - 2*x - 1)*sinh(x)^2 +
2*x^2 + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)
*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(cosh(x) +
 sinh(x)) + 32*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2
```

```
- 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*dilog(-cosh
(x) - sinh(x)) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*
cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*s
inh(x))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^
3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2*x*cosh
(x)^3 - x*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*(3*(2*x^2 - 2*x
 + 1)*cosh(x)^5 + 2*(8*x^3 - 34*x^2 + 2*x - 1)*cosh(x)^3 - (8*x^3 + 2*x^2 +
 2*x + 1)*cosh(x))*sinh(x) + 2*x + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x
))*sinh(x))
```

Sympy [F]

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \int x^2 \cosh^2(x) \coth^2(x) \, dx$$

```
[In] integrate(x**2*cosh(x)**2*coth(x)**2,x)
[Out] Integral(x**2*cosh(x)**2*coth(x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^2 \, dx$$

```
[In] integrate(x^2*cosh(x)^2*coth(x)^2,x, algorithm="giac")
[Out] integrate(x^2*cosh(x)^2*coth(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2(x) \coth^2(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^2 \, dx$$

```
[In] int(x^2*cosh(x)^2*coth(x)^2,x)
[Out] int(x^2*cosh(x)^2*coth(x)^2, x)
```

3.420 $\int x^3 \cosh^2(x) \coth^2(x) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1 - e^{2x}\right) + 3x\,\text{PolyLog}\left(2, e^{2x}\right) - \frac{3\,\text{PolyLog}\left(3, e^{2x}\right)}{2} + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3\cosh(x)\sinh(x)$$

```
[Out] 3/8*x^2-x^3+3/8*x^4-3/8*cosh(x)^2-3/4*x^2*cosh(x)^2-x^3*coth(x)+3*x^2*ln(1-
exp(2*x))+3*x*polylog(2,exp(2*x))-3/2*polylog(3,exp(2*x))+3/4*x*cosh(x)*sin
h(x)+1/2*x^3*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5558, 3392, 30, 3391, 3801, 3797, 2221, 2611, 2320, 6724}

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = 3x\,\text{PolyLog}\left(2, e^{2x}\right) - \frac{3\,\text{PolyLog}\left(3, e^{2x}\right)}{2} + \frac{3x^4}{8} - x^3 - x^3\coth(x) + \frac{1}{2}x^3\sinh(x)\cosh(x) + \frac{3x^2}{8} + 3x^2\log\left(1 - e^{2x}\right) - \frac{3}{4}x^2\cosh^2(x) - \frac{3\cosh^2(x)}{8} + \frac{3}{4}x\sinh(x)\cosh(x)$$

```
[In] Int[x^3*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^2)/8 - x^3 + (3*x^4)/8 - (3*Cosh[x]^2)/8 - (3*x^2*Cosh[x]^2)/4 - x^3*C
oth[x] + 3*x^2*Log[1 - E^(2*x)] + 3*x*PolyLog[2, E^(2*x)] - (3*PolyLog[3, E
^(2*x)])/2 + (3*x*Cosh[x]*Sinh[x])/4 + (x^3*Cosh[x]*Sinh[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3\cosh^2(x)\,dx+\int x^3\coth^2(x)\,dx\\
&= -\frac{3}{4}x^2\cosh^2(x)-x^3\coth(x)+\frac{1}{2}x^3\cosh(x)\sinh(x)+\frac{\int x^3\,dx}{2}\\
&\quad+\frac{3}{2}\int x\cosh^2(x)\,dx+3\int x^2\coth(x)\,dx+\int x^3\,dx\\
&= -x^3+\frac{3x^4}{8}-\frac{3\cosh^2(x)}{8}-\frac{3}{4}x^2\cosh^2(x)-x^3\coth(x)+\frac{3}{4}x\cosh(x)\sinh(x)\\
&\quad+\frac{1}{2}x^3\cosh(x)\sinh(x)+\frac{3\int x\,dx}{4}-6\int\frac{e^{2x}x^2}{1-e^{2x}}\,dx\\
&= \frac{3x^2}{8}-x^3+\frac{3x^4}{8}-\frac{3\cosh^2(x)}{8}-\frac{3}{4}x^2\cosh^2(x)-x^3\coth(x)+3x^2\log\left(1-e^{2x}\right)\\
&\quad+\frac{3}{4}x\cosh(x)\sinh(x)+\frac{1}{2}x^3\cosh(x)\sinh(x)-6\int x\log\left(1-e^{2x}\right)\,dx
\end{aligned}
$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1 - e^{2x}\right) + 3x\,\text{PolyLog}\left(2,e^{2x}\right) + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3\cosh(x)\sinh(x) - 3\int \text{PolyLog}\left(2,e^{2x}\right)\,dx$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1 - e^{2x}\right) + 3x\,\text{PolyLog}\left(2,e^{2x}\right) + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3\cosh(x)\sinh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2x}\right)$$

$$= \frac{3x^2}{8} - x^3 + \frac{3x^4}{8} - \frac{3\cosh^2(x)}{8} - \frac{3}{4}x^2\cosh^2(x) - x^3\coth(x) + 3x^2\log\left(1 - e^{2x}\right) + 3x\,\text{PolyLog}\left(2,e^{2x}\right) - \frac{3\,\text{PolyLog}\left(3,e^{2x}\right)}{2} + \frac{3}{4}x\cosh(x)\sinh(x) + \frac{1}{2}x^3\cosh(x)\sinh(x)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int x^3\cosh^2(x)\coth^2(x)\,dx = \frac{3x^4}{8} - \frac{3}{16}\left(1+2x^2\right)\cosh(2x) - x^3\coth(x) + x^2\left(x + 3\log\left(1 - e^{-2x}\right)\right) - 3x\,\text{PolyLog}\left(2,e^{-2x}\right) - \frac{3}{2}\text{PolyLog}\left(3,e^{-2x}\right) + \frac{1}{8}x\left(3+2x^2\right)\sinh(2x)$$

```
[In] Integrate[x^3*Cosh[x]^2*Coth[x]^2,x]
```

```
[Out] (3*x^4)/8 - (3*(1 + 2*x^2)*Cosh[2*x])/16 - x^3*Coth[x] + x^2*(x + 3*Log[1 -
 E^(-2*x)]) - 3*x*PolyLog[2, E^(-2*x)] - (3*PolyLog[3, E^(-2*x)])/2 + (x*(3
 + 2*x^2)*Sinh[2*x])/8
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

method	result
risch	$\frac{3x^4}{8} + \left(-\frac{3}{32} + \frac{3}{16}x - \frac{3}{16}x^2 + \frac{1}{8}x^3\right)\mathrm{e}^{2x} + \left(-\frac{3}{32} - \frac{3}{16}x - \frac{3}{16}x^2 - \frac{1}{8}x^3\right)\mathrm{e}^{-2x} - \frac{2x^3}{\mathrm{e}^{2x}-1} - 2x^3 + 3x^2\ln(1$

```
[In] int(x^3*cosh(x)^2*coth(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(x)^2+(-3/32-3/16*x-3/16*x^2-1/8
*x^3)/exp(x)^2-2*x^3/(exp(x)^2-1)-2*x^3+3*x^2*ln(1-exp(x))+6*x*polylog(2,ex
p(x))-6*polylog(3,exp(x))+3*x^2*ln(exp(x)+1)+6*x*polylog(2,-exp(x))-6*polyl
og(3,-exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 875, normalized size of antiderivative = 8.58

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 + 6*(4*x^3 - 6*x^2 + 6*x - 3)*cos
h(x)*sinh(x)^5 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^6 + (12*x^4 - 68*x^3 + 6
*x^2 - 6*x + 3)*cosh(x)^4 + (12*x^4 - 68*x^3 + 15*(4*x^3 - 6*x^2 + 6*x - 3)
*cosh(x)^2 + 6*x^2 - 6*x + 3)*sinh(x)^4 + 4*(5*(4*x^3 - 6*x^2 + 6*x - 3)*co
sh(x)^3 + (12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x))*sinh(x)^3 + 4*x^3 -
(12*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + (15*(4*x^3 - 6*x^2 + 6*x - 3
)*cosh(x)^4 - 12*x^4 - 4*x^3 + 6*(12*x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x
)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh(x)^4 + 4*x*cosh(x)*s
inh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 - x)*sinh(x)^2 + 2*(2
*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192*(x*cosh(x
)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 - x*cosh(x)^2 + (6*x*cosh(x)^2 -
x)*sinh(x)^2 + 2*(2*x*cosh(x)^3 - x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh
(x)) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*co
sh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 96*(x^2*cosh(x)^4 + 4*x^2*cosh(x
)*sinh(x)^3 + x^2*sinh(x)^4 - x^2*cosh(x)^2 + (6*x^2*cosh(x)^2 - x^2)*sinh(
x)^2 + 2*(2*x^2*cosh(x)^3 - x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) +
1) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*s
inh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*polylog(3, cosh(x
) + sinh(x)) - 192*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x
)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*polylog
(3, -cosh(x) - sinh(x)) + 2*(3*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^5 + 2*(12*
x^4 - 68*x^3 + 6*x^2 - 6*x + 3)*cosh(x)^3 - (12*x^4 + 4*x^3 + 6*x^2 + 6*x +
 3)*cosh(x))*sinh(x) + 6*x + 3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sin
h(x))
```

Sympy [F]

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \int x^3 \cosh^2{(x)} \coth^2{(x)} \, dx$$

```
[In] integrate(x**3*cosh(x)**2*coth(x)**2,x)
[Out] Integral(x**3*cosh(x)**2*coth(x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \int x^3 \cosh{(x)}^2 \coth{(x)}^2 \, dx$$

```
[In] integrate(x^3*cosh(x)^2*coth(x)^2,x, algorithm="giac")
[Out] integrate(x^3*cosh(x)^2*coth(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh^2(x) \coth^2(x) \, dx = \int x^3 \cosh(x)^2 \coth(x)^2 \, dx$$

```
[In] int(x^3*cosh(x)^2*coth(x)^2,x)
[Out] int(x^3*cosh(x)^2*coth(x)^2, x)
```

3.421 $\int x \cosh^2(x) \coth^3(x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int x \cosh^2(x) \coth^3(x) \, dx = \frac{3x}{4} - x^2 - \frac{\coth(x)}{2} - \frac{1}{2}x \coth^2(x) + 2x \log\left(1 - e^{2x}\right) + \text{PolyLog}\left(2, e^{2x}\right) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x \sinh^2(x)$$

```
[Out] 3/4*x-x^2-1/2*coth(x)-1/2*x*coth(x)^2+2*x*ln(1-exp(2*x))+polylog(2,exp(2*x)
)-1/4*cosh(x)*sinh(x)+1/2*x*sinh(x)^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5558, 5480, 2715, 8, 3797, 2221, 2317, 2438, 3801, 3554}

$$\int x \cosh^2(x) \coth^3(x) \, dx = \text{PolyLog}\left(2, e^{2x}\right) - x^2 + \frac{3x}{4} + 2x \log\left(1 - e^{2x}\right) + \frac{1}{2}x \sinh^2(x) - \frac{1}{2}x \coth^2(x) - \frac{\coth(x)}{2} - \frac{1}{4}\sinh(x)\cosh(x)$$

```
[In] Int[x*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x)/4 - x^2 - Coth[x]/2 - (x*Coth[x]^2)/2 + 2*x*Log[1 - E^(2*x)] + PolyLo
g[2, E^(2*x)] - (Cosh[x]*Sinh[x])/4 + (x*Sinh[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\cosh^2(x)\coth(x)\,dx + \int x\coth^3(x)\,dx \\
&= -\frac{1}{2}x\coth^2(x) + \frac{1}{2}\int \coth^2(x)\,dx + 2\int x\coth(x)\,dx + \int x\cosh(x)\sinh(x)\,dx \\
&= -\frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) + \frac{1}{2}x\sinh^2(x) + \frac{\int 1\,dx}{2} \\
&\quad - \frac{1}{2}\int \sinh^2(x)\,dx + 2\left(-\frac{x^2}{2} - 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx\right) \\
&= \frac{x}{2} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x) \\
&\quad + \frac{\int 1\,dx}{4} + 2\left(-\frac{x^2}{2} + x\log\left(1-e^{2x}\right) - \int \log\left(1-e^{2x}\right)\,dx\right) \\
&= \frac{3x}{4} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x) \\
&\quad + 2\left(-\frac{x^2}{2} + x\log\left(1-e^{2x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right)\right) \\
&= \frac{3x}{4} - \frac{\coth(x)}{2} - \frac{1}{2}x\coth^2(x) + 2\left(-\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + \frac{\text{PolyLog}\left(2, e^{2x}\right)}{2}\right) \\
&\quad - \frac{1}{4}\cosh(x)\sinh(x) + \frac{1}{2}x\sinh^2(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x\cosh^2(x)\coth^3(x)\,dx = \frac{1}{8}\big(8x^2+2x\cosh(2x)-4\coth(x)-4x\text{csch}^2(x) + 16x\log\left(1-e^{-2x}\right)-8\,\text{PolyLog}\left(2,e^{-2x}\right)-\sinh(2x)\big)$$

```
[In] Integrate[x*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (8*x^2 + 2*x*Cosh[2*x] - 4*Coth[x] - 4*x*Csch[x]^2 + 16*x*Log[1 - E^(-2*x)]
 - 8*PolyLog[2, E^(-2*x)] - Sinh[2*x])/8
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

method	result
risch	$-x^2+\left(-\frac{1}{16}+\frac{x}{8}\right)e^{2x}+\left(\frac{1}{16}+\frac{x}{8}\right)e^{-2x}-\frac{2\,e^{2x}x+e^{2x}-1}{\left(e^{2x}-1\right)^2}+2x\ln\left(1-e^{x}\right)+2\,\text{polylog}\left(2,e^{x}\right)+2x\ln\left(e\right.$

```
[In] int(x*cosh(x)^2*coth(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x^2+(-1/16+1/8*x)*exp(x)^2+(1/16+1/8*x)/exp(x)^2-(2*x*exp(x)^2+exp(x)^2-1)
/(exp(x)^2-1)^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))+2*x*ln(exp(x)+1)+2*pol
ylog(2,-exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 916, normalized size of antiderivative = 14.54

$$\int x\cosh^2(x)\coth^3(x)\,dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((2*x - 1)*cosh(x)^8 + 8*(2*x - 1)*cosh(x)*sinh(x)^7 + (2*x - 1)*sinh(
x)^8 - 2*(8*x^2 + 2*x - 1)*cosh(x)^6 + 2*(14*(2*x - 1)*cosh(x)^2 - 8*x^2 -
2*x + 1)*sinh(x)^6 + 4*(14*(2*x - 1)*cosh(x)^3 - 3*(8*x^2 + 2*x - 1)*cosh(x
))*sinh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^4 + 2*(35*(2*x - 1)*cosh(x)^4 -
15*(8*x^2 + 2*x - 1)*cosh(x)^2 + 16*x^2 - 14*x - 8)*sinh(x)^4 + 8*(7*(2*x -
 1)*cosh(x)^5 - 5*(8*x^2 + 2*x - 1)*cosh(x)^3 + 2*(8*x^2 - 7*x - 4)*cosh(x)
)*sinh(x)^3 - 2*(8*x^2 + 2*x - 7)*cosh(x)^2 + 2*(14*(2*x - 1)*cosh(x)^6 - 1
5*(8*x^2 + 2*x - 1)*cosh(x)^4 + 12*(8*x^2 - 7*x - 4)*cosh(x)^2 - 8*x^2 - 2*
```

```
x + 7)*sinh(x)^2 + 32*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*co
sh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^
3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^
5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 32*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh
(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^
2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh
(x))*dilog(-cosh(x) - sinh(x)) + 32*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 +
x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cos
h(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh
(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x
))*log(cosh(x) + sinh(x) + 1) + 32*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x
*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh
(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cosh(
x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(x)
)*log(-cosh(x) - sinh(x) + 1) + 4*(2*(2*x - 1)*cosh(x)^7 - 3*(8*x^2 + 2*x -
 1)*cosh(x)^5 + 4*(8*x^2 - 7*x - 4)*cosh(x)^3 - (8*x^2 + 2*x - 7)*cosh(x))*
sinh(x) + 2*x + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(
x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 +
 (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 -
 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int x\cosh^2(x)\coth^3(x)\,dx = \int x\cosh^2{(x)}\coth^3{(x)}\,dx$$

```
[In] integrate(x*cosh(x)**2*coth(x)**3,x)
[Out] Integral(x*cosh(x)**2*coth(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(50) = 100$.

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.32

$$\int x\cosh^2(x)\coth^3(x)\,dx = -2\,x^2 + 2\,x\log\left(e^x+1\right) + 2\,x\log\left(-e^x+1\right) + \frac{5}{8}\,x + \frac{16\,x^2 + (2\,x-1)e^{(6\,x)} + 2\,(8\,x^2 - 2\,x + 1)e^{(4\,x)} - (32\,x^2 + 8\,x + 11)e^{(2\,x)} + (2\,x+1)e^{(-2\,x)} - 14\,x + 9}{16\,(e^{(4\,x)} - 2\,e^{(2\,x)} + 1)} - \frac{5\,\left(2\,xe^{(4\,x)} + e^{(2\,x)} - 1\right)}{16\,(e^{(4\,x)} - 2\,e^{(2\,x)} + 1)} + 2\,\mathrm{Li}_2(-e^x) + 2\,\mathrm{Li}_2(e^x)$$

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
```

```
[Out] -2*x^2 + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 5/8*x + 1/16*(16*x^2 + (2*x
 - 1)*e^(6*x) + 2*(8*x^2 - 2*x + 1)*e^(4*x) - (32*x^2 + 8*x + 11)*e^(2*x) +
 (2*x + 1)*e^(-2*x) - 14*x + 9)/(e^(4*x) - 2*e^(2*x) + 1) - 5/16*(2*x*e^(4*
x) + e^(2*x) - 1)/(e^(4*x) - 2*e^(2*x) + 1) + 2*dilog(-e^x) + 2*dilog(e^x)
```

Giac [F]

$$\int x \cosh^2(x) \coth^3(x) \, dx = \int x \cosh(x)^2 \coth(x)^3 \, dx$$

```
[In] integrate(x*cosh(x)^2*coth(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(x)^2*coth(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cosh^2(x) \coth^3(x) \, dx = \int x \cosh(x)^2 \coth(x)^3 \, dx$$

```
[In] int(x*cosh(x)^2*coth(x)^3,x)
```

```
[Out] int(x*cosh(x)^2*coth(x)^3, x)
```

3.422 $\int x^2 \cosh^2(x) \coth^3(x) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx = \frac{3x^2}{4} - \frac{2x^3}{3} - x\coth(x) - \frac{1}{2}x^2\coth^2(x) + 2x^2\log\left(1-e^{2x}\right) + \log(\sinh(x)) + 2x\,\text{PolyLog}\left(2,e^{2x}\right) - \text{PolyLog}\left(3,e^{2x}\right) - \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\sinh^2(x)}{4} + \frac{1}{2}x^2\sinh^2(x)$$

```
[Out] 3/4*x^2-2/3*x^3-x*coth(x)-1/2*x^2*coth(x)^2+2*x^2*ln(1-exp(2*x))+ln(sinh(x)
)+2*x*polylog(2,exp(2*x))-polylog(3,exp(2*x))-1/2*x*cosh(x)*sinh(x)+1/4*sin
h(x)^2+1/2*x^2*sinh(x)^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5558, 5480, 3391, 30, 3797, 2221, 2611, 2320, 6724, 3801, 3556}

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx = 2x\,\text{PolyLog}\left(2,e^{2x}\right) - \text{PolyLog}\left(3,e^{2x}\right) - \frac{2x^3}{3} + \frac{3x^2}{4} + 2x^2\log\left(1-e^{2x}\right) + \frac{1}{2}x^2\sinh^2(x) - \frac{1}{2}x^2\coth^2(x) + \frac{\sinh^2(x)}{4} - x\coth(x) + \log(\sinh(x)) - \frac{1}{2}x\sinh(x)\cosh(x)$$

```
[In] Int[x^2*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x^2)/4 - (2*x^3)/3 - x*Coth[x] - (x^2*Coth[x]^2)/2 + 2*x^2*Log[1 - E^(2*
x)] + Log[Sinh[x]] + 2*x*PolyLog[2, E^(2*x)] - PolyLog[3, E^(2*x)] - (x*Cos
h[x]*Sinh[x])/2 + Sinh[x]^2/4 + (x^2*Sinh[x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
```

```
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2\cosh^2(x)\coth(x)\,dx+\int x^2\coth^3(x)\,dx\\
&= -\frac{1}{2}x^2\coth^2(x)+2\int x^2\coth(x)\,dx+\int x\coth^2(x)\,dx+\int x^2\cosh(x)\sinh(x)\,dx\\
&= -x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\frac{1}{2}x^2\sinh^2(x)+2\left(-\frac{x^3}{3}-2\int\frac{e^{2x}x^2}{1-e^{2x}}\,dx\right)\\
&\quad+\int x\,dx+\int\coth(x)\,dx-\int x\sinh^2(x)\,dx\\
&= \frac{x^2}{2}-x\coth(x)-\frac{1}{2}x^2\coth^2(x)+\log(\sinh(x))-\frac{1}{2}x\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{4}\\
&\quad+\frac{1}{2}x^2\sinh^2(x)+\frac{\int x\,dx}{2}+2\left(-\frac{x^3}{3}+x^2\log\left(1-e^{2x}\right)-2\int x\log\left(1-e^{2x}\right)\,dx\right)
\end{aligned}
$$

$$= \frac{3x^2}{4} - x\coth(x) - \frac{1}{2}x^2\coth^2(x) + \log(\sinh(x)) - \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\sinh^2(x)}{4} + \frac{1}{2}x^2\sinh^2(x) + 2\left(-\frac{x^3}{3} + x^2\log\left(1-e^{2x}\right) + x\,\text{PolyLog}\left(2,e^{2x}\right) - \int \text{PolyLog}\left(2,e^{2x}\right)\,dx\right)$$

$$= \frac{3x^2}{4} - x\coth(x) - \frac{1}{2}x^2\coth^2(x) + \log(\sinh(x)) - \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\sinh^2(x)}{4} + \frac{1}{2}x^2\sinh^2(x) + 2\left(-\frac{x^3}{3} + x^2\log\left(1-e^{2x}\right) + x\,\text{PolyLog}\left(2,e^{2x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2x}\right)\right)$$

$$= \frac{3x^2}{4} - x\coth(x) - \frac{1}{2}x^2\coth^2(x) + \log(\sinh(x)) + 2\left(-\frac{x^3}{3} + x^2\log\left(1-e^{2x}\right) + x\,\text{PolyLog}\left(2,e^{2x}\right) - \frac{\text{PolyLog}\left(3,e^{2x}\right)}{2}\right) - \frac{1}{2}x\cosh(x)\sinh(x) + \frac{\sinh^2(x)}{4} + \frac{1}{2}x^2\sinh^2(x)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41

$$\int x^2\cosh^2(x)\coth^3(x)\,dx = -x + \frac{2x^3}{3} + \frac{1}{8}\left(1+2x^2\right)\cosh(2x) - x\coth(x) - \frac{1}{2}x^2\text{csch}^2(x) + 2x^2\log\left(1-e^{-x}\right) + 2x^2\log\left(1+e^{-x}\right) + \log\left(1-e^x\right) + \log\left(1+e^x\right) - 4x\,\text{PolyLog}\left(2,-e^{-x}\right) - 4x\,\text{PolyLog}\left(2,e^{-x}\right) - 4\,\text{PolyLog}\left(3,-e^{-x}\right) - 4\,\text{PolyLog}\left(3,e^{-x}\right) - \frac{1}{4}x\sinh(2x)$$

```
[In] Integrate[x^2*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] -x + (2*x^3)/3 + ((1 + 2*x^2)*Cosh[2*x])/8 - x*Coth[x] - (x^2*Csch[x]^2)/2
+ 2*x^2*Log[1 - E^(-x)] + 2*x^2*Log[1 + E^(-x)] + Log[1 - E^x] + Log[1 + E^
x] - 4*x*PolyLog[2, -E^(-x)] - 4*x*PolyLog[2, E^(-x)] - 4*PolyLog[3, -E^(-x
)] - 4*PolyLog[3, E^(-x)] - (x*Sinh[2*x])/4
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{2x^3}{3} + \left(\frac{1}{16} - \frac{1}{8}x + \frac{1}{8}x^2\right)e^{2x} + \left(\frac{1}{16} + \frac{1}{8}x + \frac{1}{8}x^2\right)e^{-2x} - \frac{2x\left(e^{2x}x + e^{2x} - 1\right)}{\left(e^{2x}-1\right)^2} + \ln\left(e^x - 1\right) - 2\ln\left(e^x\right) + \text{l}$

```
[In] int(x^2*cosh(x)^2*coth(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*x^3+(1/16-1/8*x+1/8*x^2)*exp(x)^2+(1/16+1/8*x+1/8*x^2)/exp(x)^2-2*x*(x
*exp(x)^2+exp(x)^2-1)/(exp(x)^2-1)^2+ln(exp(x)-1)-2*ln(exp(x))+ln(exp(x)+1)
+2*x^2*ln(1-exp(x))+4*x*polylog(2,exp(x))-4*polylog(3,exp(x))+2*x^2*ln(exp(
x)+1)+4*x*polylog(2,-exp(x))-4*polylog(3,-exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 1512, normalized size of antiderivative = 15.75

$$\int x^2 \cosh^2(x) \coth^3(x)\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(2*x^2 - 2*x + 1)*cosh(x)^8 + 24*(2*x^2 - 2*x + 1)*cosh(x)*sinh(x)^
7 + 3*(2*x^2 - 2*x + 1)*sinh(x)^8 - 2*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^6
 - 2*(16*x^3 - 42*(2*x^2 - 2*x + 1)*cosh(x)^2 + 6*x^2 + 42*x + 3)*sinh(x)^6
 + 12*(14*(2*x^2 - 2*x + 1)*cosh(x)^3 - (16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)
)*sinh(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^4 + 2*(105*(2*x^2 - 2*
x + 1)*cosh(x)^4 + 32*x^3 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^2 - 42*x
^2 + 48*x + 3)*sinh(x)^4 + 8*(21*(2*x^2 - 2*x + 1)*cosh(x)^5 - 5*(16*x^3 +
6*x^2 + 42*x + 3)*cosh(x)^3 + (32*x^3 - 42*x^2 + 48*x + 3)*cosh(x))*sinh(x)
^3 - 2*(16*x^3 + 6*x^2 + 6*x + 3)*cosh(x)^2 + 2*(42*(2*x^2 - 2*x + 1)*cosh(
x)^6 - 15*(16*x^3 + 6*x^2 + 42*x + 3)*cosh(x)^4 - 16*x^3 + 6*(32*x^3 - 42*x
^2 + 48*x + 3)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 192*(x*cosh
(x)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)
^2 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)
^2 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4
*x*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 192*(x*cosh(x
)^6 + 6*x*cosh(x)*sinh(x)^5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2
 - 2*x)*sinh(x)^4 + 4*(5*x*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2
 + (15*x*cosh(x)^4 - 12*x*cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x
*cosh(x)^3 + x*cosh(x))*sinh(x))*dilog(-cosh(x) - sinh(x)) + 48*((2*x^2 + 1
)*cosh(x)^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(
2*x^2 + 1)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4
*(5*(2*x^2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*
cosh(x)^2 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 +
1)*sinh(x)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^
2 + 1)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 48*(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4
*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*si
nh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(
cosh(x) + sinh(x) - 1) + 96*(x^2*cosh(x)^6 + 6*x^2*cosh(x)*sinh(x)^5 + x^2*
sinh(x)^6 - 2*x^2*cosh(x)^4 + (15*x^2*cosh(x)^2 - 2*x^2)*sinh(x)^4 + x^2*co
```

```
sh(x)^2 + 4*(5*x^2*cosh(x)^3 - 2*x^2*cosh(x))*sinh(x)^3 + (15*x^2*cosh(x)^4
 - 12*x^2*cosh(x)^2 + x^2)*sinh(x)^2 + 2*(3*x^2*cosh(x)^5 - 4*x^2*cosh(x)^3
 + x^2*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) - 192*(cosh(x)^6 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 +
 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*
sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*po
lylog(3, cosh(x) + sinh(x)) - 192*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(
x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2
*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*polylog(3, -cosh(x) - sinh(
x)) + 4*(6*(2*x^2 - 2*x + 1)*cosh(x)^7 - 3*(16*x^3 + 6*x^2 + 42*x + 3)*cosh
(x)^5 + 2*(32*x^3 - 42*x^2 + 48*x + 3)*cosh(x)^3 - (16*x^3 + 6*x^2 + 6*x +
3)*cosh(x))*sinh(x) + 6*x + 3)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))
*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3
*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int x^2 \cosh^2(x) \coth^3(x)\, dx = \int x^2 \cosh^2{\left(x \right)} \coth^3{\left(x \right)}\, dx$$

```
[In] integrate(x**2*cosh(x)**2*coth(x)**3,x)
[Out] Integral(x**2*cosh(x)**2*coth(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.81

$$\begin{aligned} &\int x^2 \cosh^2(x) \coth^3(x)\, dx \\ &= -\frac{4}{3}\, x^3 + 2\, x^2 \log\left(e^x + 1\right) + 2\, x^2 \log\left(-e^x + 1\right) + 4\, x \mathrm{Li}_2(-e^x) + 4\, x \mathrm{Li}_2(e^x) - 2\, x \\ &\quad + \frac{32\, x^3 - 12\, x^2 + 3\,(2\, x^2 - 2\, x + 1)e^{(6\, x)} + 2\,(16\, x^3 - 6\, x^2 + 6\, x - 3)e^{(4\, x)} - 2\,(32\, x^3 + 42\, x^2 + 48\, x - 3)e}{48\,(e^{(4\, x)} - 2\, e^{(2\, x)} + 1)} \\ &\quad + \log\left(e^x + 1\right) + \log\left(e^x - 1\right) - 4\, \mathrm{Li}_3(-e^x) - 4\, \mathrm{Li}_3(e^x) \end{aligned}$$

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
[Out] -4/3*x^3 + 2*x^2*log(e^x + 1) + 2*x^2*log(-e^x + 1) + 4*x*dilog(-e^x) + 4*x
*dilog(e^x) - 2*x + 1/48*(32*x^3 - 12*x^2 + 3*(2*x^2 - 2*x + 1)*e^(6*x) + 2
*(16*x^3 - 6*x^2 + 6*x - 3)*e^(4*x) - 2*(32*x^3 + 42*x^2 + 48*x - 3)*e^(2*x
) + 3*(2*x^2 + 2*x + 1)*e^(-2*x) + 84*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + lo
g(e^x + 1) + log(e^x - 1) - 4*polylog(3, -e^x) - 4*polylog(3, e^x)
```

Giac [F]

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx = \int x^2 \cosh\left(x\right)^2 \coth\left(x\right)^3 \, dx$$

```
[In] integrate(x^2*cosh(x)^2*coth(x)^3,x, algorithm="giac")
[Out] integrate(x^2*cosh(x)^2*coth(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2(x) \coth^3(x) \, dx = \int x^2 \cosh(x)^2 \coth(x)^3 \, dx$$

```
[In] int(x^2*cosh(x)^2*coth(x)^3,x)
[Out] int(x^2*cosh(x)^2*coth(x)^3, x)
```

3.423 $\int x^3 \cosh^2(x) \coth^3(x) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 158

$$\int x^3 \cosh^2(x) \coth^3(x) \, dx = \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{x^4}{2} - \frac{3}{2}x^2 \coth(x) - \frac{1}{2}x^3 \coth^2(x) + 3x \log\left(1-e^{2x}\right) + 2x^3 \log\left(1-e^{2x}\right) + \frac{3 \operatorname{PolyLog}\left(2,e^{2x}\right)}{2} + 3x^2 \operatorname{PolyLog}\left(2,e^{2x}\right) - 3x \operatorname{PolyLog}\left(3,e^{2x}\right) + \frac{3 \operatorname{PolyLog}\left(4,e^{2x}\right)}{2} - \frac{3}{8}\cosh(x)\sinh(x) - \frac{3}{4}x^2 \cosh(x)\sinh(x) + \frac{3}{4}x\sinh^2(x) + \frac{1}{2}x^3\sinh^2(x)$$

```
[Out] 3/8*x-3/2*x^2+3/4*x^3-1/2*x^4-3/2*x^2*coth(x)-1/2*x^3*coth(x)^2+3*x*ln(1-ex
p(2*x))+2*x^3*ln(1-exp(2*x))+3/2*polylog(2,exp(2*x))+3*x^2*polylog(2,exp(2*
x))-3*x*polylog(3,exp(2*x))+3/2*polylog(4,exp(2*x))-3/8*cosh(x)*sinh(x)-3/4
*x^2*cosh(x)*sinh(x)+3/4*x*sinh(x)^2+1/2*x^3*sinh(x)^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {5558, 5480, 3392, 30, 2715, 8, 3797, 2221, 2611, 6744, 2320, 6724, 3801, 2317, 2438}

$$\int x^3 \cosh^2(x) \coth^3(x) \, dx = 3x^2 \operatorname{PolyLog}\left(2,e^{2x}\right) - 3x \operatorname{PolyLog}\left(3,e^{2x}\right) + \frac{3 \operatorname{PolyLog}\left(2,e^{2x}\right)}{2} + \frac{3 \operatorname{PolyLog}\left(4,e^{2x}\right)}{2} - \frac{x^4}{2} + \frac{3x^3}{4} + 2x^3 \log\left(1-e^{2x}\right) + \frac{1}{2}x^3 \sinh^2(x) - \frac{1}{2}x^3 \coth^2(x) - \frac{3x^2}{2} - \frac{3}{2}x^2 \coth(x) - \frac{3}{4}x^2 \sinh(x)\cosh(x) + \frac{3x}{8} + 3x \log\left(1-e^{2x}\right) + \frac{3}{4}x\sinh^2(x) - \frac{3}{8}\sinh(x)\cosh(x)$$

```
[In] Int[x^3*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] (3*x)/8 - (3*x^2)/2 + (3*x^3)/4 - x^4/2 - (3*x^2*Coth[x])/2 - (x^3*Coth[x]^
2)/2 + 3*x*Log[1 - E^(2*x)] + 2*x^3*Log[1 - E^(2*x)] + (3*PolyLog[2, E^(2*x
)])/2 + 3*x^2*PolyLog[2, E^(2*x)] - 3*x*PolyLog[3, E^(2*x)] + (3*PolyLog[4,
 E^(2*x)])/2 - (3*Cosh[x]*Sinh[x])/8 - (3*x^2*Cosh[x]*Sinh[x])/4 + (3*x*Sin
h[x]^2)/4 + (x^3*Sinh[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
```

```
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\cosh^2(x)\coth(x)\,dx+\int x^3\coth^3(x)\,dx\\
&= -\frac{1}{2}x^3\coth^2(x)+\frac{3}{2}\int x^2\coth^2(x)\,dx+2\int x^3\coth(x)\,dx+\int x^3\cosh(x)\sinh(x)\,dx\\
&= -\frac{3}{2}x^2\coth(x)-\frac{1}{2}x^3\coth^2(x)+\frac{1}{2}x^3\sinh^2(x)+\frac{3\int x^2\,dx}{2}\\
&\quad -\frac{3}{2}\int x^2\sinh^2(x)\,dx+2\left(-\frac{x^4}{4}-2\int\frac{e^{2x}x^3}{1-e^{2x}}\,dx\right)+3\int x\coth(x)\,dx\\
&= -\frac{3x^2}{2}+\frac{x^3}{2}-\frac{3}{2}x^2\coth(x)-\frac{1}{2}x^3\coth^2(x)-\frac{3}{4}x^2\cosh(x)\sinh(x)\\
&\quad +\frac{3}{4}x\sinh^2(x)+\frac{1}{2}x^3\sinh^2(x)+\frac{3\int x^2\,dx}{4}-\frac{3}{4}\int\sinh^2(x)\,dx\\
&\quad +2\left(-\frac{x^4}{4}+x^3\log\left(1-e^{2x}\right)-3\int x^2\log\left(1-e^{2x}\right)\,dx\right)-6\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -\frac{3x^2}{2}+\frac{3x^3}{4}-\frac{3}{2}x^2\coth(x)-\frac{1}{2}x^3\coth^2(x)+3x\log\left(1-e^{2x}\right)-\frac{3}{8}\cosh(x)\sinh(x)-\frac{3}{4}x^2\cosh(x)\sinh(x)\\
&\quad +\frac{3}{4}x\sinh^2(x)+\frac{1}{2}x^3\sinh^2(x)+\frac{3\int 1\,dx}{8}-3\int\log\left(1-e^{2x}\right)\,dx+2\left(-\frac{x^4}{4}+x^3\log\left(1\right.\right.\\
&\qquad \left.\left.-e^{2x}\right)+\frac{3}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)-3\int x\,\text{PolyLog}\left(2,e^{2x}\right)\,dx\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) \\
&\quad - \frac{3}{8}\cosh(x)\sinh(x) - \frac{3}{4}x^2\cosh(x)\sinh(x) + \frac{3}{4}x\sinh^2(x) + \frac{1}{2}x^3\sinh^2(x) \\
&\quad + 2\left(-\frac{x^4}{4} + x^3\log\left(1-e^{2x}\right) + \frac{3}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right) - \frac{3}{2}x\,\text{PolyLog}\left(3,e^{2x}\right)\right. \\
&\qquad \left. + \frac{3}{2}\int \text{PolyLog}\left(3,e^{2x}\right)\,dx\right) - \frac{3}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) \\
&\quad + \frac{3\,\text{PolyLog}\left(2,e^{2x}\right)}{2} - \frac{3}{8}\cosh(x)\sinh(x) - \frac{3}{4}x^2\cosh(x)\sinh(x) + \frac{3}{4}x\sinh^2(x) \\
&\quad + \frac{1}{2}x^3\sinh^2(x) + 2\left(-\frac{x^4}{4} + x^3\log\left(1-e^{2x}\right) + \frac{3}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right)\right. \\
&\qquad \left. - \frac{3}{2}x\,\text{PolyLog}\left(3,e^{2x}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx, x, e^{2x}\right)\right) \\
&= \frac{3x}{8} - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{3}{2}x^2\coth(x) - \frac{1}{2}x^3\coth^2(x) + 3x\log\left(1-e^{2x}\right) + \frac{3\,\text{PolyLog}\left(2,e^{2x}\right)}{2} \\
&\quad + 2\left(-\frac{x^4}{4} + x^3\log\left(1-e^{2x}\right) + \frac{3}{2}x^2\,\text{PolyLog}\left(2,e^{2x}\right) - \frac{3}{2}x\,\text{PolyLog}\left(3,e^{2x}\right)\right. \\
&\qquad \left. + \frac{3\,\text{PolyLog}\left(4,e^{2x}\right)}{4}\right) \\
&\quad - \frac{3}{8}\cosh(x)\sinh(x) - \frac{3}{4}x^2\cosh(x)\sinh(x) + \frac{3}{4}x\sinh^2(x) + \frac{1}{2}x^3\sinh^2(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.21

$$
\begin{aligned}
\int x^3\cosh^2(x)\coth^3(x)\,dx &= -3\left(1+2x^2\right)\text{PolyLog}\left(2,-e^{-x}\right) + \frac{1}{8}\left(12x^2 + 4x^4 + 3x\cosh(2x)\right. \\
&\quad + 2x^3\cosh(2x) - 12x^2\coth(x) - 4x^3\text{csch}^2(x) \\
&\quad + 24x\log\left(1-e^{-x}\right) + 16x^3\log\left(1-e^{-x}\right) + 24x\log\left(1+e^{-x}\right) \\
&\quad + 16x^3\log\left(1+e^{-x}\right) - 24\left(1+2x^2\right)\text{PolyLog}\left(2,e^{-x}\right) \\
&\quad - 96x\,\text{PolyLog}\left(3,-e^{-x}\right) - 96x\,\text{PolyLog}\left(3,e^{-x}\right) \\
&\quad - 96\,\text{PolyLog}\left(4,-e^{-x}\right) - 96\,\text{PolyLog}\left(4,e^{-x}\right) \\
&\quad \left. - 3\cosh(x)\sinh(x) - 6x^2\cosh(x)\sinh(x)\right)
\end{aligned}
$$

```
[In] Integrate[x^3*Cosh[x]^2*Coth[x]^3,x]
```

```
[Out] -3*(1 + 2*x^2)*PolyLog[2, -E^(-x)] + (12*x^2 + 4*x^4 + 3*x*Cosh[2*x] + 2*x^
3*Cosh[2*x] - 12*x^2*Coth[x] - 4*x^3*Csch[x]^2 + 24*x*Log[1 - E^(-x)] + 16*
```

```
x^3*Log[1 - E^(-x)] + 24*x*Log[1 + E^(-x)] + 16*x^3*Log[1 + E^(-x)] - 24*(1
 + 2*x^2)*PolyLog[2, E^(-x)] - 96*x*PolyLog[3, -E^(-x)] - 96*x*PolyLog[3, E
^(-x)] - 96*PolyLog[4, -E^(-x)] - 96*PolyLog[4, E^(-x)] - 3*Cosh[x]*Sinh[x]
 - 6*x^2*Cosh[x]*Sinh[x])/8
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{x^4}{2}+\left(-\frac{3}{32}+\frac{3}{16}x-\frac{3}{16}x^2+\frac{1}{8}x^3\right)\mathrm{e}^{2x}+\left(\frac{3}{32}+\frac{3}{16}x+\frac{3}{16}x^2+\frac{1}{8}x^3\right)\mathrm{e}^{-2x}-\frac{x^2\left(2\,\mathrm{e}^{2x}x+3\,\mathrm{e}^{2x}-3\right)}{\left(\mathrm{e}^{2x}-1\right)^2}-3x^2+$

```
[In] int(x^3*cosh(x)^2*coth(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^4+(-3/32+3/16*x-3/16*x^2+1/8*x^3)*exp(x)^2+(3/32+3/16*x+3/16*x^2+1/8
*x^3)/exp(x)^2-x^2*(2*x*exp(x)^2+3*exp(x)^2-3)/(exp(x)^2-1)^2-3*x^2+3*x*ln(
1-exp(x))+3*polylog(2,exp(x))+3*x*ln(exp(x)+1)+3*polylog(2,-exp(x))+2*x^3*l
n(1-exp(x))+6*x^2*polylog(2,exp(x))-12*x*polylog(3,exp(x))+12*polylog(4,exp
(x))+2*x^3*ln(exp(x)+1)+6*x^2*polylog(2,-exp(x))-12*x*polylog(3,-exp(x))+12
*polylog(4,-exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2067 vs. 2(126) = 252.

Time = 0.27 (sec) , antiderivative size = 2067, normalized size of antiderivative = 13.08

$$\int x^3\cosh^2(x)\coth^3(x)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^8 + 8*(4*x^3 - 6*x^2 + 6*x - 3)*cos
h(x)*sinh(x)^7 + (4*x^3 - 6*x^2 + 6*x - 3)*sinh(x)^8 - 2*(8*x^4 + 4*x^3 + 4
2*x^2 + 6*x - 3)*cosh(x)^6 - 2*(8*x^4 + 4*x^3 - 14*(4*x^3 - 6*x^2 + 6*x - 3
)*cosh(x)^2 + 42*x^2 + 6*x - 3)*sinh(x)^6 + 4*(14*(4*x^3 - 6*x^2 + 6*x - 3)
*cosh(x)^3 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x))*sinh(x)^5 + 4*(8
*x^4 - 14*x^3 + 24*x^2 + 3*x)*cosh(x)^4 + 2*(35*(4*x^3 - 6*x^2 + 6*x - 3)*c
osh(x)^4 + 16*x^4 - 28*x^3 - 15*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^
2 + 48*x^2 + 6*x)*sinh(x)^4 + 8*(7*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^5 - 5*
(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^3 + 2*(8*x^4 - 14*x^3 + 24*x^2 +
 3*x)*cosh(x))*sinh(x)^3 + 4*x^3 - 2*(8*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh
(x)^2 + 2*(14*(4*x^3 - 6*x^2 + 6*x - 3)*cosh(x)^6 - 15*(8*x^4 + 4*x^3 + 42*
x^2 + 6*x - 3)*cosh(x)^4 - 8*x^4 - 4*x^3 + 12*(8*x^4 - 14*x^3 + 24*x^2 + 3*
x)*cosh(x)^2 - 6*x^2 - 6*x - 3)*sinh(x)^2 + 6*x^2 + 96*((2*x^2 + 1)*cosh(x)
```

```
^6 + 6*(2*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(2*x^2 + 1
)*cosh(x)^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4*(5*(2*x^
2 + 1)*cosh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*cosh(x)^2
 + (15*(2*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 + 1)*sinh(x
)^2 + 2*(3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^2 + 1)*co
sh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) + 96*((2*x^2 + 1)*cosh(x)^6 + 6*(2
*x^2 + 1)*cosh(x)*sinh(x)^5 + (2*x^2 + 1)*sinh(x)^6 - 2*(2*x^2 + 1)*cosh(x)
^4 + (15*(2*x^2 + 1)*cosh(x)^2 - 4*x^2 - 2)*sinh(x)^4 + 4*(5*(2*x^2 + 1)*co
sh(x)^3 - 2*(2*x^2 + 1)*cosh(x))*sinh(x)^3 + (2*x^2 + 1)*cosh(x)^2 + (15*(2
*x^2 + 1)*cosh(x)^4 - 12*(2*x^2 + 1)*cosh(x)^2 + 2*x^2 + 1)*sinh(x)^2 + 2*(
3*(2*x^2 + 1)*cosh(x)^5 - 4*(2*x^2 + 1)*cosh(x)^3 + (2*x^2 + 1)*cosh(x))*si
nh(x))*dilog(-cosh(x) - sinh(x)) + 32*((2*x^3 + 3*x)*cosh(x)^6 + 6*(2*x^3 +
 3*x)*cosh(x)*sinh(x)^5 + (2*x^3 + 3*x)*sinh(x)^6 - 2*(2*x^3 + 3*x)*cosh(x)
^4 - (4*x^3 - 15*(2*x^3 + 3*x)*cosh(x)^2 + 6*x)*sinh(x)^4 + 4*(5*(2*x^3 + 3
*x)*cosh(x)^3 - 2*(2*x^3 + 3*x)*cosh(x))*sinh(x)^3 + (2*x^3 + 3*x)*cosh(x)^
2 + (15*(2*x^3 + 3*x)*cosh(x)^4 + 2*x^3 - 12*(2*x^3 + 3*x)*cosh(x)^2 + 3*x)
*sinh(x)^2 + 2*(3*(2*x^3 + 3*x)*cosh(x)^5 - 4*(2*x^3 + 3*x)*cosh(x)^3 + (2*
x^3 + 3*x)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 32*((2*x^3 + 3*x)
*cosh(x)^6 + 6*(2*x^3 + 3*x)*cosh(x)*sinh(x)^5 + (2*x^3 + 3*x)*sinh(x)^6 -
2*(2*x^3 + 3*x)*cosh(x)^4 - (4*x^3 - 15*(2*x^3 + 3*x)*cosh(x)^2 + 6*x)*sinh
(x)^4 + 4*(5*(2*x^3 + 3*x)*cosh(x)^3 - 2*(2*x^3 + 3*x)*cosh(x))*sinh(x)^3 +
 (2*x^3 + 3*x)*cosh(x)^2 + (15*(2*x^3 + 3*x)*cosh(x)^4 + 2*x^3 - 12*(2*x^3
+ 3*x)*cosh(x)^2 + 3*x)*sinh(x)^2 + 2*(3*(2*x^3 + 3*x)*cosh(x)^5 - 4*(2*x^3
 + 3*x)*cosh(x)^3 + (2*x^3 + 3*x)*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x)
+ 1) + 384*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2
)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cos
h(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(
x)^3 + cosh(x))*sinh(x))*polylog(4, cosh(x) + sinh(x)) + 384*(cosh(x)^6 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4
 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1
)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*
polylog(4, -cosh(x) - sinh(x)) - 384*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^5 +
 x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x*co
sh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*cos
h(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*sinh(
x))*polylog(3, cosh(x) + sinh(x)) - 384*(x*cosh(x)^6 + 6*x*cosh(x)*sinh(x)^
5 + x*sinh(x)^6 - 2*x*cosh(x)^4 + (15*x*cosh(x)^2 - 2*x)*sinh(x)^4 + 4*(5*x
*cosh(x)^3 - 2*x*cosh(x))*sinh(x)^3 + x*cosh(x)^2 + (15*x*cosh(x)^4 - 12*x*
cosh(x)^2 + x)*sinh(x)^2 + 2*(3*x*cosh(x)^5 - 4*x*cosh(x)^3 + x*cosh(x))*si
nh(x))*polylog(3, -cosh(x) - sinh(x)) + 4*(2*(4*x^3 - 6*x^2 + 6*x - 3)*cosh
(x)^7 - 3*(8*x^4 + 4*x^3 + 42*x^2 + 6*x - 3)*cosh(x)^5 + 4*(8*x^4 - 14*x^3
+ 24*x^2 + 3*x)*cosh(x)^3 - (8*x^4 + 4*x^3 + 6*x^2 + 6*x + 3)*cosh(x))*sinh
(x) + 6*x + 3)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2
 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15
*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*c
```

```
osh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int x^3 \cosh^2(x) \coth^3(x)\, dx = \int x^3 \cosh^2(x) \coth^3(x)\, dx$$

```
[In] integrate(x**3*cosh(x)**2*coth(x)**3,x)
[Out] Integral(x**3*cosh(x)**2*coth(x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.51

$$\begin{aligned}\int x^3 \cosh^2(x) \coth^3(x)\, dx = & -x^4 + 2x^3 \log(e^x + 1) + 2x^3 \log(-e^x + 1) + 6x^2 \mathrm{Li}_2(-e^x) \\ & + 6x^2 \mathrm{Li}_2(e^x) - 3x^2 + 3x \log(e^x + 1) + 3x \log(-e^x + 1) - 12x \mathrm{Li}_3(-e^x) - 12x \mathrm{Li}_3(e^x) \\ & + \frac{16x^4 - 8x^3 + 84x^2 + (4x^3 - 6x^2 + 6x - 3)e^{(6x)} + 2(8x^4 - 4x^3 + 6x^2 - 6x + 3)e^{(4x)} - 4(8x^4 + 14x}{32(e^{(4x)} - 2e^{(2x)} + 1)} \\ & + 3\mathrm{Li}_2(-e^x) + 3\mathrm{Li}_2(e^x) + 12\mathrm{Li}_4(-e^x) + 12\mathrm{Li}_4(e^x)\end{aligned}$$

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="maxima")
[Out] -x^4 + 2*x^3*log(e^x + 1) + 2*x^3*log(-e^x + 1) + 6*x^2*dilog(-e^x) + 6*x^2
*dilog(e^x) - 3*x^2 + 3*x*log(e^x + 1) + 3*x*log(-e^x + 1) - 12*x*polylog(3
, -e^x) - 12*x*polylog(3, e^x) + 1/32*(16*x^4 - 8*x^3 + 84*x^2 + (4*x^3 - 6
*x^2 + 6*x - 3)*e^(6*x) + 2*(8*x^4 - 4*x^3 + 6*x^2 - 6*x + 3)*e^(4*x) - 4*(
8*x^4 + 14*x^3 + 24*x^2 - 3*x)*e^(2*x) + (4*x^3 + 6*x^2 + 6*x + 3)*e^(-2*x)
 - 12*x - 6)/(e^(4*x) - 2*e^(2*x) + 1) + 3*dilog(-e^x) + 3*dilog(e^x) + 12*
polylog(4, -e^x) + 12*polylog(4, e^x)
```

Giac [F]

$$\int x^3 \cosh^2(x) \coth^3(x)\, dx = \int x^3 \cosh(x)^2 \coth(x)^3\, dx$$

```
[In] integrate(x^3*cosh(x)^2*coth(x)^3,x, algorithm="giac")
[Out] integrate(x^3*cosh(x)^2*coth(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh^2(x) \coth^3(x) \, dx = \int x^3 \cosh(x)^2 \coth(x)^3 \, dx$$

```
[In] int(x^3*cosh(x)^2*coth(x)^3,x)
[Out] int(x^3*cosh(x)^2*coth(x)^3, x)
```

3.424 $\int x^m \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \text{Int}(x^m \coth(a+bx)\text{csch}(a+bx),x)$$

```
[Out] CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

```
[In] Int[x^m*Coth[a + b*x]*Csch[a + b*x],x]
[Out] Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx$$

```
[In] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x],x]
[Out] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \cosh(bx+a)\,\text{csch}(bx+a)^2\,dx$$

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)\,\text{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)\,\text{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)\,\text{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^m \coth(a+bx)\text{csch}(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)}{\sinh(a+b\,x)^2}\,dx$$

```
[In] int((x^m*cosh(a + b*x))/sinh(a + b*x)^2,x)
[Out] int((x^m*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

3.425 $\int x^3 \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\operatorname{csch}(a+bx)}{b} - \frac{6x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^4}$$

```
[Out] -6*x^2*arctanh(exp(b*x+a))/b^2-x^3*csch(b*x+a)/b-6*x*polylog(2,-exp(b*x+a))
/b^3+6*x*polylog(2,exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3
,exp(b*x+a))/b^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5527, 4267, 2611, 2320, 6724}

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} + \frac{6\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^4} - \frac{6x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} - \frac{x^3\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2,
 -E^(a + b*x)])/b^3 + (6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^
(a + b*x)])/b^4 - (6*PolyLog[3, E^(a + b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{x^3\text{csch}(a+bx)}{b} + \frac{3\int x^2\text{csch}(a+bx)\,dx}{b}$$

$$= -\frac{6x^2\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6\int x\log(1-e^{a+bx})\,dx}{b^2} + \frac{6\int x\log(1+e^{a+bx})\,dx}{b^2}$$

$$= -\frac{6x^2\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{6x\,\text{PolyLog}(2,e^{a+bx})}{b^3} + \frac{6\int\text{PolyLog}(2,-e^{a+bx})\,dx}{b^3} - \frac{6\int\text{PolyLog}(2,e^{a+bx})\,dx}{b^3}$$

$$= -\frac{6x^2\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{6x\,\text{PolyLog}(2,e^{a+bx})}{b^3} + \frac{6\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{6\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{a+bx}\right)}{b^4}$$

$$= -\frac{6x^2\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{6x\,\text{PolyLog}(2,e^{a+bx})}{b^3} + \frac{6\,\text{PolyLog}(3,-e^{a+bx})}{b^4} - \frac{6\,\text{PolyLog}(3,e^{a+bx})}{b^4}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int x^3\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{-b^3x^3\text{csch}(a+bx) + 3b^2x^2\log(1-e^{a+bx}) - 3b^2x^2\log(1+e^{a+bx}) - 6bx\,\text{PolyLog}(2,-e^{a+bx}) + 6bx\,\text{Poly}}{b^4}$$

```
[In] Integrate[x^3*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-(b^3*x^3*Csch[a + b*x]) + 3*b^2*x^2*Log[1 - E^(a + b*x)] - 3*b^2*x^2*Log[
1 + E^(a + b*x)] - 6*b*x*PolyLog[2, -E^(a + b*x)] + 6*b*x*PolyLog[2, E^(a +
 b*x)] + 6*PolyLog[3, -E^(a + b*x)] - 6*PolyLog[3, E^(a + b*x)])/b^4
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{2x^3 e^{bx+a}}{b(e^{2bx+2a}-1)} - \frac{6a^2\operatorname{arctanh}(e^{bx+a})}{b^4} + \frac{3\ln(1-e^{bx+a})x^2}{b^2} - \frac{3\ln(1-e^{bx+a})a^2}{b^4} + \frac{6x\operatorname{polylog}(2,e^{bx+a})}{b^3} - \frac{6\operatorname{polylog}(3,e^{bx+}}{b^4}$

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh(exp(b*x+a))+3/b^2*
ln(1-exp(b*x+a))*x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*polylog(2,exp(b*x+a))/b
^3-6*polylog(3,exp(b*x+a))/b^4-3/b^2*ln(exp(b*x+a)+1)*x^2+3/b^4*ln(exp(b*x+
a)+1)*a^2-6*x*polylog(2,-exp(b*x+a))/b^3+6*polylog(3,-exp(b*x+a))/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.92

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,b^3x^3\cosh\left(bx+a\right)+2\,b^3x^3\sinh\left(bx+a\right)-6\left(bx\cosh\left(bx+a\right)^2+2\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+bx\,\mathrm{s}\right.}{}$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(co
sh(b*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)
*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)
 + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1
) - 3*(a^2*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b
*x + a)^2 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*x^2 - (b^2
*x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
 - (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a
) + 1) + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 - 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*
cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a
) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a
) + b^4*sinh(b*x + a)^2 - b^4)
```

Sympy [F]

$$\int x^3 \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^{3}\cosh\left(a+bx\right)\text{csch}^{2}\left(a+bx\right)\,dx$$

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cosh(a + b*x)*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,x^3e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b} - \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4} + \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3(e^{(bx+a)})\right)}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) + 1
) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))/b^4
```

Giac [F]

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^3 \cosh(bx+a)\operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)}{\sinh(a+b\,x)^2}\,dx$$

```
[In] int((x^3*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] int((x^3*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

3.426 $\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{4x\operatorname{arctanh}(e^{a+bx})}{b^2} - \frac{x^2\operatorname{csch}(a+bx)}{b} - \frac{2\operatorname{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{2\operatorname{PolyLog}(2,e^{a+bx})}{b^3}$$

```
[Out] -4*x*arctanh(exp(b*x+a))/b^2-x^2*csch(b*x+a)/b-2*polylog(2,-exp(b*x+a))/b^3
+2*polylog(2,exp(b*x+a))/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5527, 4267, 2317, 2438}

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{4x\operatorname{arctanh}(e^{a+bx})}{b^2} - \frac{2\operatorname{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{2\operatorname{PolyLog}(2,e^{a+bx})}{b^3} - \frac{x^2\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch[a + b*x])/b - (2*PolyLog[2, -E^
(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\text{csch}(a+bx)}{b}+\frac{2\int x\text{csch}(a+bx)\,dx}{b}\\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}-\frac{2\int\log\left(1-e^{a+bx}\right)\,dx}{b^2}+\frac{2\int\log\left(1+e^{a+bx}\right)\,dx}{b^2}\\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}\\
&\quad -\frac{2\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}+\frac{2\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}-\frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$
$$= \frac{-bx\left(bx\operatorname{csch}(a+bx)-2\log\left(1-e^{a+bx}\right)+2\log\left(1+e^{a+bx}\right)\right)-2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)+2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3}$$

```
[In] Integrate[x^2*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-(b*x*(b*x*Csch[a + b*x] - 2*Log[1 - E^(a + b*x)] + 2*Log[1 + E^(a + b*x)]
)) - 2*PolyLog[2, -E^(a + b*x)] + 2*PolyLog[2, E^(a + b*x)])/b^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(56) = 112.

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{2x^2 e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{2\ln\left(1-e^{bx+a}\right)x}{b^2}+\frac{2\ln\left(1-e^{bx+a}\right)a}{b^3}+\frac{2\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^3}-\frac{2\ln\left(e^{bx+a}+1\right)x}{b^2}-\frac{2\ln\left(e^{bx+a}+1\right)a}{b^3}-\frac{2\,\mathrm{polyl}}{}$

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*x^2*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln(1-
exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^2*ln(exp(b*x+a)+1)*x-2/b^3*ln
(exp(b*x+a)+1)*a-2*polylog(2,-exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(54) = 108.

Time = 0.25 (sec) , antiderivative size = 367, normalized size of antiderivative = 6.22

$$\int x^2 \coth(a+bx)\operatorname{csch}(a+bx)\,dx =$$
$$-\frac{2\left(b^2x^2\cosh\left(bx+a\right)+b^2x^2\sinh\left(bx+a\right)-\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+\right.\right.}{}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*x^2*cosh(b*x + a) + b^2*x^2*sinh(b*x + a) - (cosh(b*x + a)^2 + 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh
(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^2 + 2*
```

```
b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x +
 a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x
+ a) + a*sinh(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*
x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a
)*sinh(b*x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*
cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 -
 b^3)
```

Sympy [F]

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^2 \cosh(a+bx)\,\text{csch}^2(a+bx)\,dx$$

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cosh(a + b*x)*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,x^2e^{(bx+a)}}{be^{(2\,bx+2\,a)}-b} - \frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^3
```

Giac [F]

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = \int x^2 \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*cosh(b*x + a)*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth(a+bx)\text{csch}(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^2*cosh(a + b*x))/sinh(a + b*x)^2,x)
[Out] int((x^2*cosh(a + b*x))/sinh(a + b*x)^2, x)
```

3.427 $\int x \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b^2-x*csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5527, 3855}

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

```
[In] Int[x*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - (x*Csch[a + b*x])/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
```

```
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\text{csch}(a+bx)}{b}+\frac{\int \text{csch}(a+bx)\,dx}{b}\\ &= -\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{x\text{csch}(a)}{b}-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{x\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b}+\frac{x\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b}$$

```
[In] Integrate[x*Coth[a + b*x]*Csch[a + b*x],x]
[Out] -((x*Csch[a])/b) - Log[Cosh[a/2 + (b*x)/2]]/b^2 + Log[Sinh[a/2 + (b*x)/2]]/
b^2 + (x*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b) + (x*Sech[a/2]*
Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(2*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b^2}$	54

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -2/b*x*exp(b*x+a)/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b*
x+a)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 6.76

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)+\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right.\right.}{b^{2}\cosh\left(bx\right.}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) + (cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a
) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*
cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx = \int x\cosh\left(a+bx\right)\text{csch}^{2}\left(a+bx\right)dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int x\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^{2}} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^{2}}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.72

$$\int x\coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,bxe^{(bx+a)} + e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}+1\right) - e^{(2\,bx+2\,a)}\log\left(e^{(bx+a)}-1\right) - \log\left(e^{(bx+a)}+1\right) + \log\left(e^{(bx+a)}-1\right.}{b^2e^{(2\,bx+2\,a)} - b^2}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*x*e^(b*x + a) + e^(2*b*x + 2*a)*log(e^(b*x + a) + 1) - e^(2*b*x + 2*a
)*log(e^(b*x + a) - 1) - log(e^(b*x + a) + 1) + log(e^(b*x + a) - 1))/(b^2*
e^(2*b*x + 2*a) - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int x\coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^4}}{b^2}\right)}{\sqrt{-b^4}} - \frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(-b^4)^(1/2))/b^2))/(-b^4)^(1/2) - (2*x*exp(a +
b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.428 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2686, 8}

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

```
[In] Int[Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -(Csch[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int 1\,dx, x, -i\text{csch}(a+bx)\right)}{b} \\
&= -\frac{\text{csch}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$
\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}
$$

```
[In] Integrate[Coth[a + b*x]*Csch[a + b*x],x]
[Out] -(Csch[a + b*x]/b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\text{csch}(bx+a)}{b}$	12
default	$-\frac{\text{csch}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	25

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -csch(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$
\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}
$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = \int \cosh(a+bx)\,\text{csch}^2(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{1}{b\sinh(a+b\,x)}$$

```
[In] int(cosh(a + b*x)/sinh(a + b*x)^2,x)
[Out] -1/(b*sinh(a + b*x))
```

3.429 $\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx$$

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x,x]
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}}{x}dx$$

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
[Out] int(cosh(b*x+a)*csch(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 9.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh\left(a+bx\right)\text{csch}^{2}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2/x,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 2*integrate(1/2/(b*x^2*e^(b*x
+ a) + b*x^2), x) - 2*integrate(1/2/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)}{x\sinh\left(a+b\,x\right)^{2}}\,dx$$

```
[In] int(cosh(a + b*x)/(x*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)/(x*sinh(a + b*x)^2), x)
```

3.430 $\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\coth(a+bx)\text{csch}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 32.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2,x]
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^2}{x^2}dx$$

```
[In] int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
[Out] int(cosh(b*x+a)*csch(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 11.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\,\text{csch}^2\,(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**2/x**2,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*
x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)}{x^2\,\sinh(a+b\,x)^2}\,dx$$

```
[In] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^2), x)
```

3.431 $\int x^m \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \coth^2(a+bx)\,dx = \text{Int}\left(x^m \coth^2(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*coth(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \coth^2(a+bx)\,dx$$

```
[In] Int[x^m*Coth[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \coth^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \coth^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Coth[a + b*x]^2,x]
[Out] Integrate[x^m*Coth[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2 dx$$

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 145, normalized size of antiderivative = 12.08

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) - 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) + integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
- (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^2(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^2}{\sinh(a+b\,x)^2}\,dx$$

```
[In] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

3.432 $\int x^3 \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}$$

```
[Out] -x^3/b+1/4*x^4-x^3*coth(b*x+a)/b+3*x^2*ln(1-exp(2*b*x+2*a))/b^2+3*x*polylog
(2,exp(2*b*x+2*a))/b^3-3/2*polylog(3,exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3801, 3797, 2221, 2611, 2320, 6724, 30}

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{3\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}+\frac{3x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^3\coth(a+bx)}{b}-\frac{x^3}{b}+\frac{x^4}{4}$$

```
[In] Int[x^3*Coth[a + b*x]^2,x]
```

```
[Out] -(x^3/b) + x^4/4 - (x^3*Coth[a + b*x])/b + (3*x^2*Log[1 - E^(2*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, E^(2*(a + b*x)
)])/(2*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^3\coth(a+bx)}{b}+\frac{3\int x^2\coth(a+bx)\,dx}{b}+\int x^3\,dx\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}-\frac{6\int\frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{6\int x\log\left(1-e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}\\
&\quad+\frac{3x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\int\text{PolyLog}\left(2,e^{2(a+bx)}\right)\,dx}{b^3}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}\\
&\quad+\frac{3x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^4}\\
&= -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2}\\
&\quad+\frac{3x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 222 vs. $2(87)=174$.

Time = 0.61 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.55

$$\int x^3\coth^2(a+bx)\,dx = \frac{x^4}{4} - \frac{e^{2a}\left(2b^3e^{-2a}x^3-3b^2(1-e^{-2a})\,x^2\log\left(1-e^{-a-bx}\right)-3b^2(1-e^{-2a})\,x^2\log\left(1+e^{-a-bx}\right)+6b(1-e^{-2a})\,x\right.}{} + \frac{x^3\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

```
[In] Integrate[x^3*Coth[a + b*x]^2,x]
```

```
[Out] x^4/4 - (E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^
(-a - b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*b*(1 - E^(
-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, E^(-a
- b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*Pol
yLog[3, E^(-a - b*x)]))/(b^4*(-1 + E^(2*a))) + (x^3*Csch[a]*Csch[a + b*x]*S
inh[b*x])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(83) = 166$.

Time = 0.99 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.28

method	result
risch	$\frac{x^4}{4} - \frac{2x^3}{b(e^{2bx+2a}-1)} + \frac{4a^3}{b^4} + \frac{6x\,a^2}{b^3} - \frac{2x^3}{b} - \frac{6\,\text{polylog}(3,-e^{bx+a})}{b^4} - \frac{6\,\text{polylog}(3,e^{bx+a})}{b^4} + \frac{3\ln(e^{bx+a}+1)x^2}{b^2} + \frac{6x\,\text{polylog}}{}$

```
[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^4-2*x^3/b/(exp(2*b*x+2*a)-1)+4/b^4*a^3+6*x/b^3*a^2-2*x^3/b-6*polylog(
3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4+3/b^2*ln(exp(b*x+a)+1)*x^2+6
*x*polylog(2,-exp(b*x+a))/b^3+3/b^2*ln(1-exp(b*x+a))*x^2+6*x*polylog(2,exp(
b*x+a))/b^3+3/b^4*a^2*ln(exp(b*x+a)-1)-6/b^4*a^2*ln(exp(b*x+a))-3/b^4*ln(1-
exp(b*x+a))*a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 632, normalized size of antiderivative = 7.26

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{b^4x^4 - 8\,a^3 - (b^4x^4 - 8\,b^3x^3 - 8\,a^3)\cosh(bx+a)^2 - 2\,(b^4x^4 - 8\,b^3x^3 - 8\,a^3)\cosh(bx+a)\sinh(bx+a)}{}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
[Out] -1/4*(b^4*x^4 - 8*a^3 - (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 - 2*(
b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) - (b^4*x^4 - 8*b^3
*x^3 - 8*a^3)*sinh(b*x + a)^2 - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(cosh(b*x + a) + sinh(b*
x + a)) - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x
*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 12*(b^2*x^2
*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x
 + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*(a^2*cosh(b*
x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 - a^2)*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) + 12*(b^2*x^2 - (b^2*x^2 - a^2)*cosh(
b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - a^2
)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 24*(cosh
(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*
x + a)^2 - b^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{2\,x^3}{b} + \frac{bx^4e^{(2\,bx+2\,a)} - bx^4 - 8\,x^3}{4\,(be^{(2\,bx+2\,a)} - b)} + \frac{3\,\left(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4} + \frac{3\,\left(b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(e^{(bx+a)})\right)}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) - b*x^4 - 8*x^3)/(b*e^(2*b*x + 2*a) -
 b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polyl
og(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e
^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \coth^2(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
[Out] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

3.433 $\int x^2 \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}$$

```
[Out] -x^2/b+1/3*x^3-x^2*coth(b*x+a)/b+2*x*ln(1-exp(2*b*x+2*a))/b^2+polylog(2,exp
(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3801, 3797, 2221, 2317, 2438, 30}

$$\int x^2 \coth^2(a+bx)\,dx = \frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{x^2\coth(a+bx)}{b}-\frac{x^2}{b}+\frac{x^3}{3}$$

```
[In] Int[x^2*Coth[a + b*x]^2,x]
```

```
[Out] -(x^2/b) + x^3/3 - (x^2*Coth[a + b*x])/b + (2*x*Log[1 - E^(2*(a + b*x))])/b
^2 + PolyLog[2, E^(2*(a + b*x))]/b^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2\coth(a+bx)}{b}+\frac{2\int x\coth(a+bx)\,dx}{b}+\int x^2\,dx\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}-\frac{4\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{2\int \log\left(1-e^{2(a+bx)}\right)\,dx}{b^2}\\
&= -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{b^3}
\end{aligned}
$$

$$= -\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2\coth(a+bx)}{b} + \frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2} + \frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^3}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int x^2\coth^2(a+bx)\,dx = -\frac{2\,\text{PolyLog}\left(2,-e^{-a-bx}\right)}{b^3} - \frac{2\,\text{PolyLog}\left(2,e^{-a-bx}\right)}{b^3} + \frac{1}{3}x\left(\frac{6x}{b-be^{2a}} + x^2 + \frac{6\log\left(1-e^{-a-bx}\right)}{b^2} + \frac{6\log\left(1+e^{-a-bx}\right)}{b^2} + \frac{3x\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}\right)$$

```
[In] Integrate[x^2*Coth[a + b*x]^2,x]
```

```
[Out] (-2*PolyLog[2, -E^(-a - b*x)])/b^3 - (2*PolyLog[2, E^(-a - b*x)])/b^3 + (x*
((6*x)/(b - b*E^(2*a)) + x^2 + (6*Log[1 - E^(-a - b*x)])/b^2 + (6*Log[1 + E
^(-a - b*x)])/b^2 + (3*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b))/3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.

Time = 0.99 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x^3}{3} - \frac{2x^2}{b\left(e^{2bx+2a}-1\right)} - \frac{2x^2}{b} - \frac{4ax}{b^2} - \frac{2a^2}{b^3} + \frac{2\ln\left(e^{bx+a}+1\right)x}{b^2} + \frac{2\,\text{polylog}\left(2,-e^{bx+a}\right)}{b^3} + \frac{2\ln\left(1-e^{bx+a}\right)x}{b^2} + \frac{2\ln\left(1-e^{bx+a}\right.}{b^3}$

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3-2*x^2/b/(exp(2*b*x+2*a)-1)-2*x^2/b-4*a*x/b^2-2/b^3*a^2+2/b^2*ln(exp
(b*x+a)+1)*x+2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln
(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^3*a*ln(exp(b*x+a)-1)+4/b^3
*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 453, normalized size of antiderivative = 6.97

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{b^3x^3 - (b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)^2 - 2\,(b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)\sinh(bx+a) - (b^3x^3}{}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 - 2*(b^3*x^3
- 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^3*x^3 - 6*b^2*x^2 + 6
*a^2)*sinh(b*x + a)^2 + 6*a^2 - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(co
sh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(
-cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
 a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sin
h(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*((b*x + a)*cos
h(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x
 + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x +
 a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

Sympy [F]

$$\int x^2 \coth^2(a+bx)\,dx = \int x^2 \cosh^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{2\,x^2}{b} + \frac{bx^3e^{(2\,bx+2\,a)} - bx^3 - 6\,x^2}{3\,(be^{(2\,bx+2\,a)} - b)} + \frac{2\,\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\,\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) - b*x^3 - 6*x^2)/(b*e^(2*b*x + 2*a) -
 b) + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-
e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2 \coth^2(a+bx)\,dx = \int x^2 \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^2, x)
```

3.434 $\int x \coth^2(a+bx)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int x \coth^2(a+bx)\, dx = \frac{x^2}{2} - \frac{x \coth(a+bx)}{b} + \frac{\log(\sinh(a+bx))}{b^2}$$

```
[Out] 1/2*x^2-x*coth(b*x+a)/b+ln(sinh(b*x+a))/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3801, 3556, 30}

$$\int x \coth^2(a+bx)\, dx = \frac{\log(\sinh(a+bx))}{b^2} - \frac{x \coth(a+bx)}{b} + \frac{x^2}{2}$$

```
[In] Int[x*Coth[a + b*x]^2,x]
```

```
[Out] x^2/2 - (x*Coth[a + b*x])/b + Log[Sinh[a + b*x]]/b^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\coth(a+bx)}{b} + \frac{\int \coth(a+bx)\,dx}{b} + \int x\,dx \\ &= \frac{x^2}{2} - \frac{x\coth(a+bx)}{b} + \frac{\log(\sinh(a+bx))}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int x\coth^2(a+bx)\,dx = \frac{b^2x^2 - 2bx\coth(a) + 2\log(\sinh(a+bx)) + 2bx\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{2b^2}$$

```
[In] Integrate[x*Coth[a + b*x]^2,x]
[Out] (b^2*x^2 - 2*b*x*Coth[a] + 2*Log[Sinh[a + b*x]] + 2*b*x*Csch[a]*Csch[a + b*
x]*Sinh[b*x])/(2*b^2)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{x^2}{2} - \frac{2x}{b} - \frac{2a}{b^2} - \frac{2x}{b\left(e^{2bx+2a}-1\right)} + \frac{\ln\left(e^{2bx+2a}-1\right)}{b^2}$	54
parallelrisch	$\frac{-2\ln(1-\tanh(bx+a))\tanh(bx+a)+2\ln(\tanh(bx+a))\tanh(bx+a)+xb(-2+(bx-2)\tanh(bx+a))}{2\tanh(bx+a)b^2}$	66

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-2*x/b-2*a/b^2-2*x/b/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 6.10

$$\int x\coth^2(a+bx)\,dx = -\frac{b^2x^2 - (b^2x^2 - 4\,bx)\cosh{(bx+a)}^2 - 2\,(b^2x^2 - 4\,bx)\cosh{(bx+a)}\sinh{(bx+a)} - (b^2x^2 - 4\,bx)\sinh{(bx}{2\,\left(b^2\cosh{(bx+a)}^2 + 2\,b^2\cosh{(bx+a}}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 4*b*x)*cos
h(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 - 2*(cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(
b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int x\coth^2(a+bx)\,dx = \int x\cosh^2{(a+bx)}\operatorname{csch}^2{(a+bx)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int x\coth^2(a+bx)\,dx = -\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)} - b} - \frac{bx^2 - \left(bx^2e^{(2\,a)} - 2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\,\left(be^{(2\,bx+2\,a)} - b\right)} + \frac{\log\left(\left(e^{(bx+a)} + 1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)} - 1\right)e^{(-a)}\right)}{b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - 1/2*(b*x^2 - (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) - b) + log((e^(b*x + a) + 1)*e^(
-a))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

$$\int x\coth^2(a+bx)\,dx = \frac{b^2x^2e^{(2\,bx+2\,a)} - b^2x^2 - 4\,bxe^{(2\,bx+2\,a)} + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) - 2\,\log\left(e^{(2\,bx+2\,a)} - 1\right)}{2\left(b^2e^{(2\,bx+2\,a)} - b^2\right)}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) - b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x
 + 2*a)*log(e^(2*b*x + 2*a) - 1) - 2*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*
x + 2*a) - b^2)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int x\coth^2(a+bx)\,dx = \frac{\frac{x^2\sinh(a+b\,x)}{2} - \frac{x\cosh(a+b\,x)}{b}}{\sinh\left(a+b\,x\right)} + \frac{\ln\left(\sinh(a+b\,x)\right)}{b^2}$$

```
[In] int((x*cosh(a + b*x)^2)/sinh(a + b*x)^2,x)
```

```
[Out] ((x^2*sinh(a + b*x))/2 - (x*cosh(a + b*x))/b)/sinh(a + b*x) + log(sinh(a +
b*x))/b^2
```

3.435 $\int \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \coth^2(a+bx)\,dx = x-\frac{\coth(a+bx)}{b}$$

```
[Out] x-coth(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \coth^2(a+bx)\,dx = x-\frac{\coth(a+bx)}{b}$$

```
[In] Int[Coth[a + b*x]^2,x]
```

```
[Out] x - Coth[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{\coth(a+bx)}{b} + \int 1\,dx$$
$$= x - \frac{\coth(a+bx)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \coth^2(a+bx)\,dx = -\frac{\coth(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Coth[a + b*x]^2,x]
[Out] -((Coth[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{bx+a-\coth(bx+a)}{b}$	18
default	$\frac{bx+a-\coth(bx+a)}{b}$	18
risch	$x-\frac{2}{b\left(e^{2bx+2a}-1\right)}$	21
parallelrisch	$\frac{-1+\tanh(bx+a)xb}{b\tanh(bx+a)}$	24

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(b*x+a-coth(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \coth^2(a+bx)\,dx = \frac{(bx+1)\sinh(bx+a) - \cosh(bx+a)}{b\sinh(bx+a)}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + 1)*sinh(b*x + a) - cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^2(a+bx)\,dx = \int \cosh^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \coth^2(a+bx)\,dx = x + \frac{a}{b} + \frac{2}{b(e^{(-2\,bx-2\,a)}-1)}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
[Out] x + a/b + 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \coth^2(a+bx)\,dx = \frac{bx + a - \frac{2}{e^{(2\,bx+2\,a)}-1}}{b}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
[Out] (b*x + a - 2/(e^(2*b*x + 2*a) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \coth^2(a+bx)\,dx = x - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(cosh(a + b*x)^2/sinh(a + b*x)^2,x)
[Out] x - 2/(b*(exp(2*a + 2*b*x) - 1))
```

3.436 $\int \frac{\coth^2(a+bx)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^2(a+bx)}{x} dx = \text{Int}\left(\frac{\coth^2(a+bx)}{x}, x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x} dx = \int \frac{\coth^2(a+bx)}{x} dx$$

```
[In] Int[Coth[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth^2(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\coth^2(a+bx)}{x}\,dx$$

```
[In] Integrate[Coth[a + b*x]^2/x,x]
[Out] Integrate[Coth[a + b*x]^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2}{x}dx$$

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 21.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh^2\left(a+bx\right)\operatorname{csch}^2\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.75

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}^2\operatorname{csch}{(bx+a)}^2}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*x*e^(2*b*x + 2*a) - b*x) + integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x
) - integrate(1/(b*x^2*e^(b*x + a) - b*x^2), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh{(bx+a)}^2\operatorname{csch}{(bx+a)}^2}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2}{x\sinh{(a+b\,x)}^2}\,dx$$

```
[In] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^2), x)
```

3.437 $\int \frac{\coth^2(a+bx)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \text{Int}\left(\frac{\coth^2(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \int \frac{\coth^2(a+bx)}{x^2} dx$$

```
[In] Int[Coth[a + b*x]^2/x^2,x]
[Out] Defer[Int][Coth[a + b*x]^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth^2(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\coth^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[Coth[a + b*x]^2/x^2,x]
[Out] Integrate[Coth[a + b*x]^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}dx$$

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 25.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh^{2}\left(a+bx\right)\operatorname{csch}^{2}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**2/x**2,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 7.58

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}^2\,\mathrm{csch}\,{(bx+a)}^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*x*e^(2*b*x + 2*a) - b*x + 2)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) + 2*integr
ate(1/(b*x^3*e^(b*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) -
b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}^2\,\mathrm{csch}\,{(bx+a)}^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{{\cosh(a+b\,x)}^2}{x^2\,{\sinh{(a+b\,x)}}^2}\,dx$$

```
[In] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^2),x)
```

```
[Out] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^2), x)
```

3.438 $\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} + \mathrm{Int}(x^m\coth(a+bx)\mathrm{csch}(a+bx),x)$$

```
[Out] CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a),x)+1/2*exp(a)*x^m*GAMMA(1+m,-b*
x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

```
[In] Int[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m) + Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x], x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^m \cosh(a+bx)\,dx + \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx \\
&= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx + \frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx + \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx \\
&= \frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b} \\
&\quad + \int x^m \coth(a+bx)\text{csch}(a+bx)\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 26.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh(a+bx)\coth^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2,x]
[Out] Integrate[x^m*Cosh[a + b*x]*Coth[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2\,dx$$

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int x^m \cosh(bx+a)^3 \operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \cosh(a+bx)\coth^2(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^3}{\sinh(a+b\,x)^2}\,dx$$

```
[In] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
[Out] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

3.439 $\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int x^3\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{6\cosh(a+bx)}{b^4}-\frac{3x^2\cosh(a+bx)}{b^2}-\frac{x^3\operatorname{csch}(a+bx)}{b}-\frac{6x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^4}+\frac{6x\sinh(a+bx)}{b^3}+\frac{x^3\sinh(a+bx)}{b}$$

```
[Out] -6*x^2*arctanh(exp(b*x+a))/b^2-6*cosh(b*x+a)/b^4-3*x^2*cosh(b*x+a)/b^2-x^3*
csch(b*x+a)/b-6*x*polylog(2,-exp(b*x+a))/b^3+6*x*polylog(2,exp(b*x+a))/b^3+
6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4+6*x*sinh(b*x+a)/b^
3+x^3*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used

$= \{5558, 3377, 2718, 5527, 4267, 2611, 2320, 6724\}$

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} + \frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4} - \frac{6\cosh(a+bx)}{b^4} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6x\sinh(a+bx)}{b^3} - \frac{3x^2\cosh(a+bx)}{b^2} + \frac{x^3\sinh(a+bx)}{b} - \frac{x^3\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (6*Cosh[a + b*x])/b^4 - (3*x^2*Cosh[a +
 b*x])/b^2 - (x^3*Csch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + (
6*x*PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - (6*Po
lyLog[3, E^(a + b*x)])/b^4 + (6*x*Sinh[a + b*x])/b^3 + (x^3*Sinh[a + b*x])/
b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3\cosh(a+bx)\,dx+\int x^3\coth(a+bx)\text{csch}(a+bx)\,dx\\
&= -\frac{x^3\text{csch}(a+bx)}{b}+\frac{x^3\sinh(a+bx)}{b}+\frac{3\int x^2\text{csch}(a+bx)\,dx}{b}-\frac{3\int x^2\sinh(a+bx)\,dx}{b}\\
&= -\frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{3x^2\cosh(a+bx)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}+\frac{x^3\sinh(a+bx)}{b}\\
&\quad+\frac{6\int x\cosh(a+bx)\,dx}{b^2}-\frac{6\int x\log\left(1-e^{a+bx}\right)\,dx}{b^2}+\frac{6\int x\log\left(1+e^{a+bx}\right)\,dx}{b^2}
\end{aligned}
$$

$$= -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\operatorname{csch}(a+bx)}{b} - \frac{6x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b} + \frac{6\int\operatorname{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^3} - \frac{6\int\operatorname{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^3} - \frac{6\int\sinh(a+bx)\,dx}{b^3}$$

$$= -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\operatorname{csch}(a+bx)}{b} - \frac{6x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b} + \frac{6\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{6\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{a+bx}\right)}{b^4}$$

$$= -\frac{6x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{6\cosh(a+bx)}{b^4} - \frac{3x^2\cosh(a+bx)}{b^2} - \frac{x^3\operatorname{csch}(a+bx)}{b} - \frac{6x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^4} + \frac{6x\sinh(a+bx)}{b^3} + \frac{x^3\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.53

$$\int x^3\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\operatorname{csch}\left(\frac{1}{2}(a+bx)\right)\operatorname{sech}\left(\frac{1}{2}(a+bx)\right)\left(-6bx-3b^3x^3+6bx\cosh(2(a+bx))+b^3x^3\cosh(2(a+bx))+6b^2x^2\log\right.}{}$$

```
[In] Integrate[x^3*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-6*b*x - 3*b^3*x^3 + 6*b*x*Cosh[2*(a
+ b*x)] + b^3*x^3*Cosh[2*(a + b*x)] + 6*b^2*x^2*Log[1 - E^(a + b*x)]*Sinh[a
 + b*x] - 6*b^2*x^2*Log[1 + E^(a + b*x)]*Sinh[a + b*x] - 12*b*x*PolyLog[2,
-E^(a + b*x)]*Sinh[a + b*x] + 12*b*x*PolyLog[2, E^(a + b*x)]*Sinh[a + b*x]
+ 12*PolyLog[3, -E^(a + b*x)]*Sinh[a + b*x] - 12*PolyLog[3, E^(a + b*x)]*Si
nh[a + b*x] - 6*Sinh[2*(a + b*x)] - 3*b^2*x^2*Sinh[2*(a + b*x)]))/(4*b^4)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.69

method	result
risch	$\frac{(x^3b^3-3x^2b^2+6bx-6)e^{bx+a}}{2b^4}-\frac{(x^3b^3+3x^2b^2+6bx+6)e^{-bx-a}}{2b^4}-\frac{2x^3e^{bx+a}}{b(e^{2bx+2a}-1)}-\frac{6a^2\operatorname{arctanh}(e^{bx+a})}{b^4}+\frac{3\ln(1-e^{bx+a})x^2}{b^2}-$

```
[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b^3*x^3-3*b^2*x^2+6*b*x-6)/b^4*exp(b*x+a)-1/2*(b^3*x^3+3*b^2*x^2+6*b*x
+6)/b^4*exp(-b*x-a)-2/b*x^3*exp(b*x+a)/(exp(2*b*x+2*a)-1)-6/b^4*a^2*arctanh
(exp(b*x+a))+3/b^2*ln(1-exp(b*x+a))*x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6*x*poly
log(2,exp(b*x+a))/b^3-6*polylog(3,exp(b*x+a))/b^4-3/b^2*ln(exp(b*x+a)+1)*x^
2+3/b^4*ln(exp(b*x+a)+1)*a^2-6*x*polylog(2,-exp(b*x+a))/b^3+6*polylog(3,-ex
p(b*x+a))/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(136) = 272.

Time = 0.28 (sec) , antiderivative size = 1055, normalized size of antiderivative = 7.38

$$\int x^3\cosh(a+bx)\coth^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^4 + 4*(b^3*x
^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b^
2*x^2 + 6*b*x - 6)*sinh(b*x + a)^4 + 3*b^2*x^2 - 6*(b^3*x^3 + 2*b*x)*cosh(b
*x + a)^2 - 6*(b^3*x^3 - (b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^2
+ 2*b*x)*sinh(b*x + a)^2 + 6*b*x + 12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x
 + a)*sinh(b*x + a)^2 + b*x*sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*co
sh(b*x + a)^2 - b*x)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) -
12*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x*sinh(b*
x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a))
*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 6*(b^2*x^2*cosh(b*x + a)^3 + 3*b^2
*x^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*x^2*sinh(b*x + a)^3 - b^2*x^2*cosh
(b*x + a) + (3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + 6*(a^2*cosh(b*x + a)^3 + 3*a^2*cosh(b*x + a)
*sinh(b*x + a)^2 + a^2*sinh(b*x + a)^3 - a^2*cosh(b*x + a) + (3*a^2*cosh(b*
x + a)^2 - a^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(
(b^2*x^2 - a^2)*cosh(b*x + a)^3 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x
+ a)^2 + (b^2*x^2 - a^2)*sinh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a) -
(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a))*log(-cos
h(b*x + a) - sinh(b*x + a) + 1) - 12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sin
```

```
h(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - co
sh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 12*(cosh(b*x + a)^
3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
- 1)*sinh(b*x + a) - cosh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 - 3*b^2*x^2 + 6*b*x - 6)*cosh(b*x + a)^3 - 3*(b^3*x^3 + 2
*b*x)*cosh(b*x + a))*sinh(b*x + a) + 6)/(b^4*cosh(b*x + a)^3 + 3*b^4*cosh(b
*x + a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3 - b^4*cosh(b*x + a) + (3*b^4*
cosh(b*x + a)^2 - b^4)*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.51

$$\int x^3 \cosh(a+bx)\coth^2(a+bx)\,dx$$
$$= \frac{(b^3x^3e^{(4a)} - 3\,b^2x^2e^{(4a)} + 6\,bxe^{(4a)} - 6\,e^{(4a)})e^{(3\,bx)} - 6\,(b^3x^3e^{(2a)} + 2\,bxe^{(2a)})e^{(bx)} + (b^3x^3 + 3\,b^2x^2 + 6\,bx}{2\,(b^4e^{(2\,bx+3\,a)} - b^4e^{a})}$$
$$- \frac{3\,(b^2x^2\log(e^{(bx+a)}+1) + 2\,bx\mathrm{Li}_2(-e^{(bx+a)}) - 2\,\mathrm{Li}_3(-e^{(bx+a)}))}{b^4}$$
$$+ \frac{3\,(b^2x^2\log(-e^{(bx+a)}+1) + 2\,bx\mathrm{Li}_2(e^{(bx+a)}) - 2\,\mathrm{Li}_3(e^{(bx+a)}))}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*x^3*e^(4*a) - 3*b^2*x^2*e^(4*a) + 6*b*x*e^(4*a) - 6*e^(4*a))*e^(3
*b*x) - 6*(b^3*x^3*e^(2*a) + 2*b*x*e^(2*a))*e^(b*x) + (b^3*x^3 + 3*b^2*x^2
+ 6*b*x + 6)*e^(-b*x))/(b^4*e^(2*b*x + 3*a) - b^4*e^a) - 3*(b^2*x^2*log(e^(
b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4
 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(
3, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \cosh(a+bx) \coth^2(a+bx) \, dx = \int x^3 \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^2 \, dx$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cosh(a+bx) \coth^2(a+bx) \, dx = \int \frac{x^3 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^2} \, dx$$

```
[In] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
[Out] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

3.440 $\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx = -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{2x\cosh(a+bx)}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{2\sinh(a+bx)}{b^3} + \frac{x^2\sinh(a+bx)}{b}$$

```
[Out] -4*x*arctanh(exp(b*x+a))/b^2-2*x*cosh(b*x+a)/b^2-x^2*csch(b*x+a)/b-2*polylo
g(2,-exp(b*x+a))/b^3+2*polylog(2,exp(b*x+a))/b^3+2*sinh(b*x+a)/b^3+x^2*sinh
(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5558, 3377, 2717, 5527, 4267, 2317, 2438}

$$\int x^2 \cosh(a+bx) \coth^2(a+bx) \, dx = -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{2\sinh(a+bx)}{b^3} - \frac{2x\cosh(a+bx)}{b^2} + \frac{x^2\sinh(a+bx)}{b} - \frac{x^2\text{csch}(a+bx)}{b}$$

```
[In] Int[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTanh[E^(a + b*x)])/b^2 - (2*x*Cosh[a + b*x])/b^2 - (x^2*Csch[a + b
*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + (2*PolyLog[2, E^(a + b*x)])/b^3
 + (2*Sinh[a + b*x])/b^3 + (x^2*Sinh[a + b*x])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
```

```
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\cosh(a+bx)\,dx+\int x^2\coth(a+bx)\text{csch}(a+bx)\,dx\\
&= -\frac{x^2\text{csch}(a+bx)}{b}+\frac{x^2\sinh(a+bx)}{b}+\frac{2\int x\text{csch}(a+bx)\,dx}{b}-\frac{2\int x\sinh(a+bx)\,dx}{b}\\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{2x\cosh(a+bx)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}+\frac{x^2\sinh(a+bx)}{b}\\
&\quad+\frac{2\int\cosh(a+bx)\,dx}{b^2}-\frac{2\int\log\left(1-e^{a+bx}\right)\,dx}{b^2}+\frac{2\int\log\left(1+e^{a+bx}\right)\,dx}{b^2}\\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{2x\cosh(a+bx)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}\\
&\quad+\frac{2\sinh(a+bx)}{b^3}+\frac{x^2\sinh(a+bx)}{b}-\frac{2\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&\quad+\frac{2\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{a+bx}\right)}{b^3}\\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{2x\cosh(a+bx)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}-\frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}\\
&\quad+\frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{2\sinh(a+bx)}{b^3}+\frac{x^2\sinh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\text{csch}\left(\frac{1}{2}(a+bx)\right)\text{sech}\left(\frac{1}{2}(a+bx)\right)\left(-2-3b^2x^2+2\cosh(2(a+bx))+b^2x^2\cosh(2(a+bx))+4bx\log\left(1-e^{a+bx}\right)\right.}{}$$

```
[In] Integrate[x^2*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(-2 - 3*b^2*x^2 + 2*Cosh[2*(a + b*x)]
+ b^2*x^2*Cosh[2*(a + b*x)] + 4*b*x*Log[1 - E^(a + b*x)]*Sinh[a + b*x] - 4*
b*x*Log[1 + E^(a + b*x)]*Sinh[a + b*x] - 4*PolyLog[2, -E^(a + b*x)]*Sinh[a
+ b*x] + 4*PolyLog[2, E^(a + b*x)]*Sinh[a + b*x] - 2*b*x*Sinh[2*(a + b*x)])
)/(4*b^3)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.95

method	result
risch	$\frac{(x^2b^2-2bx+2)e^{bx+a}}{2b^3}-\frac{(x^2b^2+2bx+2)e^{-bx-a}}{2b^3}-\frac{2x^2e^{bx+a}}{b(e^{2bx+2a}-1)}+\frac{2\ln(1-e^{bx+a})x}{b^2}+\frac{2\ln(1-e^{bx+a})a}{b^3}+\frac{2\,\mathrm{polylog}(2,e^{bx+a})}{b^3}$

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b^2*x^2-2*b*x+2)/b^3*exp(b*x+a)-1/2*(b^2*x^2+2*b*x+2)/b^3*exp(-b*x-a)-
2/b*x^2*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*ln(1-e
xp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3-2/b^2*ln(exp(b*x+a)+1)*x-2/b^3*ln(
exp(b*x+a)+1)*a-2*polylog(2,-exp(b*x+a))/b^3+4/b^3*a*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(90) = 180$.

Time = 0.27 (sec) , antiderivative size = 731, normalized size of antiderivative = 7.69

$$\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$=\frac{(b^2x^2-2\,bx+2)\cosh{(bx+a)}^4+4\,(b^2x^2-2\,bx+2)\cosh{(bx+a)}\sinh{(bx+a)}^3+(b^2x^2-2\,bx+2)\sinh$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*b*x + 2)*cosh(b
*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 2*b*x + 2)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(3*b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(3*b^2*x^2 - 3*(b^2*x^2 - 2*b*x + 2)
*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 2*b*x + 4*(cosh(b*x + a)^3 + 3*cosh
(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(
b*x + a) - cosh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(cosh(b*
x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) - 4*(b*x*cosh(b*x + a)^3 + 3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 + b*x*
sinh(b*x + a)^3 - b*x*cosh(b*x + a) + (3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*
x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 4*(a*cosh(b*x + a)^3 + 3*a
*cosh(b*x + a)*sinh(b*x + a)^2 + a*sinh(b*x + a)^3 - a*cosh(b*x + a) + (3*a
*cosh(b*x + a)^2 - a)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 4*((b*x + a)*cosh(b*x + a)^3 + 3*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^2
 + (b*x + a)*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a) + (3*(b*x + a)*cosh(
b*x + a)^2 - b*x - a)*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + 4*((b^2*x^2 - 2*b*x + 2)*cosh(b*x + a)^3 - (3*b^2*x^2 + 2)*cosh(b*x + a
))*sinh(b*x + a) + 2)/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)*sinh(b*x +
```

```
a)^2 + b^3*sinh(b*x + a)^3 - b^3*cosh(b*x + a) + (3*b^3*cosh(b*x + a)^2 -
b^3)*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{(b^2x^2e^{(4a)}-2bxe^{(4a)}+2e^{(4a)})e^{(3bx)}-2\left(3b^2x^2e^{(2a)}+2e^{(2a)}\right)e^{(bx)}+(b^2x^2+2bx+2)e^{(-bx)}}{2\left(b^3e^{(2bx+3a)}-b^3e^a\right)} - \frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*((b^2*x^2*e^(4*a) - 2*b*x*e^(4*a) + 2*e^(4*a))*e^(3*b*x) - 2*(3*b^2*x^2
*e^(2*a) + 2*e^(2*a))*e^(b*x) + (b^2*x^2 + 2*b*x + 2)*e^(-b*x))/(b^3*e^(2*b
*x + 3*a) - b^3*e^a) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b
^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2\cosh(a+bx)\coth^2(a+bx)\,dx=\int x^2\cosh(bx+a)^3\operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a+bx)\coth^2(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
[Out] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^2, x)
```

3.441 $\int x \cosh(a+bx) \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b^2}-\frac{\cosh(a+bx)}{b^2}-\frac{x\operatorname{csch}(a+bx)}{b}+\frac{x\sinh(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b^2-cosh(b*x+a)/b^2-x*csch(b*x+a)/b+x*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5558, 3377, 2718, 5527, 3855}

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b^2}-\frac{\cosh(a+bx)}{b^2}+\frac{x\sinh(a+bx)}{b}-\frac{x\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b^2) - Cosh[a + b*x]/b^2 - (x*Csch[a + b*x])/b + (
x*Sinh[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\cosh(a+bx)\,dx + \int x\coth(a+bx)\text{csch}(a+bx)\,dx \\
&= -\frac{x\text{csch}(a+bx)}{b} + \frac{x\sinh(a+bx)}{b} + \frac{\int \text{csch}(a+bx)\,dx}{b} - \frac{\int \sinh(a+bx)\,dx}{b} \\
&= -\frac{\text{arctanh}(\cosh(a+bx))}{b^2} - \frac{\cosh(a+bx)}{b^2} - \frac{x\text{csch}(a+bx)}{b} + \frac{x\sinh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$= \frac{-2\cosh(a+bx) - bx\coth\left(\frac{1}{2}(a+bx)\right) - 2\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right) + 2\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right) + 2bx\sinh(a}{2b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-2*Cosh[a + b*x] - b*x*Coth[(a + b*x)/2] - 2*Log[Cosh[(a + b*x)/2]] + 2*Lo
g[Sinh[(a + b*x)/2]] + 2*b*x*Sinh[a + b*x] + b*x*Tanh[(a + b*x)/2])/(2*b^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{(bx-1)\mathrm{e}^{bx+a}}{2b^2}-\frac{(bx+1)\mathrm{e}^{-bx-a}}{2b^2}-\frac{2x\,\mathrm{e}^{bx+a}}{b(\mathrm{e}^{2bx+2a}-1)}+\frac{\ln(\mathrm{e}^{bx+a}-1)}{b^2}-\frac{\ln(\mathrm{e}^{bx+a}+1)}{b^2}$	89

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x-1)/b^2*exp(b*x+a)-1/2*(b*x+1)/b^2*exp(-b*x-a)-2/b*x*exp(b*x+a)/(ex
p(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b*x+a)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 7.81

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{(bx-1)\cosh(bx+a)^4+4(bx-1)\cosh(bx+a)\sinh(bx+a)^3+(bx-1)\sinh(bx+a)^4-6\,bx\cosh(bx+$$

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x - 1)*cosh(b*x + a)^4 + 4*(b*x - 1)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x - 1)*sinh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 + 6*((b*x - 1)*cosh(b*x
 + a)^2 - b*x)*sinh(b*x + a)^2 + b*x - 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)
*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
- cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(cosh(b*x + a)^
3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
- 1)*sinh(b*x + a) - cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1)
+ 4*((b*x - 1)*cosh(b*x + a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a) + 1)/(b
^2*cosh(b*x + a)^3 + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a
)^3 - b^2*cosh(b*x + a) + (3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

Sympy [F]

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = \int x\cosh^3(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(a + b*x)**3*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{6\,bxe^{(bx+2\,a)} - \left(bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(3\,bx)} - (bx+1)e^{(-bx)}}{2\,\left(b^2e^{(2\,bx+3\,a)} - b^2e^{a}\right)} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b*x*e^(b*x + 2*a) - (b*x*e^(4*a) - e^(4*a))*e^(3*b*x) - (b*x + 1)*e
^(-b*x))/(b^2*e^(2*b*x + 3*a) - b^2*e^a) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(47) = 94$.

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.06

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$= \frac{bxe^{(4\,bx+4\,a)} - 6\,bxe^{(2\,bx+2\,a)} + bx - 2\,e^{(3\,bx+3\,a)}\log\left(e^{(bx+a)}+1\right) + 2\,e^{(bx+a)}\log\left(e^{(bx+a)}+1\right) + 2\,e^{(3\,bx+3\,a)}\mathrm{lo}}{2\,\left(b^2e^{(3\,bx+3\,a)} - b^2e^{(bx+a)}\right)}$$

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x*e^(4*b*x + 4*a) - 6*b*x*e^(2*b*x + 2*a) + b*x - 2*e^(3*b*x + 3*a)*
log(e^(b*x + a) + 1) + 2*e^(b*x + a)*log(e^(b*x + a) + 1) + 2*e^(3*b*x + 3*
a)*log(e^(b*x + a) - 1) - 2*e^(b*x + a)*log(e^(b*x + a) - 1) - e^(4*b*x + 4
*a) + 1)/(b^2*e^(3*b*x + 3*a) - b^2*e^(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int x\cosh(a+bx)\coth^2(a+bx)\,dx = \mathrm{e}^{a+b\,x}\left(\frac{x}{2\,b} - \frac{1}{2\,b^2}\right) - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^4}}{b^2}\right)}{\sqrt{-b^4}} - \mathrm{e}^{-a-b\,x}\left(\frac{x}{2\,b} + \frac{1}{2\,b^2}\right) - \frac{2\,x\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x} - 1\right)}$$

```
[In] int((x*cosh(a + b*x)^3)/sinh(a + b*x)^2,x)
```

```
[Out] exp(a + b*x)*(x/(2*b) - 1/(2*b^2)) - (2*atan((exp(b*x)*exp(a)*(-b^4)^(1/2))
/b^2))/(-b^4)^(1/2) - exp(- a - b*x)*(x/(2*b) + 1/(2*b^2)) - (2*x*exp(a + b
*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.442 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] -csch(b*x+a)/b+sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=\frac{\sinh(a+bx)}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{i\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, -i\sinh(a+bx)\right)}{b}$$
$$= -\frac{\text{csch}(a+bx)}{b} + \frac{\sinh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b} + \frac{\sinh(a+bx)}{b}$$

```
[In] Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
[Out] -(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}-6\,e^{bx+a}+e^{-bx-a}}{2b\left(e^{2bx+2a}-1\right)}$	46

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/sinh(b*x+a)*cosh(b*x+a)^2-2/sinh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh{(bx+a)}^2+\sinh{(bx+a)}^2-3}{2\,b\sinh{(bx+a)}}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh^3{(a+bx)}\operatorname{csch}^2{(a+bx)}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b}-\frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b(e^{(-bx-a)}-e^{(-3\,bx-3\,a)})}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-3
*b*x - 3*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{4}{e^{(bx+a)}-e^{(-bx-a)}}-e^{(bx+a)}+e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(4/(e^(b*x + a) - e^(-b*x - a)) - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=\frac{\sinh(a+b\,x)^2-1}{b\sinh\left(a+b\,x\right)}$$

```
[In] int(cosh(a + b*x)^3/sinh(a + b*x)^2,x)
[Out] (sinh(a + b*x)^2 - 1)/(b*sinh(a + b*x))
```

3.443 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \cosh(a)\mathrm{Chi}(bx)+\sinh(a)\mathrm{Shi}(bx) + \mathrm{Int}\left(\frac{\coth(a+bx)\mathrm{csch}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x,x)+Chi(b*x)*cosh(a)+Shi(b*x)*sinh
(a)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
```

```
[Out] Cosh[a]*CoshIntegral[b*x] + Sinh[a]*SinhIntegral[b*x] + Defer[Int][(Coth[a
+ b*x]*Csch[a + b*x])/x, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(a+bx)}{x}\,dx + \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx \\
&= \cosh(a)\int \frac{\cosh(bx)}{x}\,dx + \sinh(a)\int \frac{\sinh(bx)}{x}\,dx + \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx \\
&= \cosh(a)\operatorname{Chi}(bx) + \sinh(a)\operatorname{Shi}(bx) + \int \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{x}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 18.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x,x]
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x}dx$$

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 52.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh^3(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*Ei(-b*x)*e^(-a) + 1/2*Ei(b*x)*e^a - 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a)
- b*x) - integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x) - integrate(1/(b*x^2*e
^(b*x + a) - b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3}{x\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^2),x)
[Out] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^2), x)
```

3.444 $\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = -\frac{\cosh(a+bx)}{x} + b\mathrm{Chi}(bx)\sinh(a) + b\cosh(a)\mathrm{Shi}(bx) + \mathrm{Int}\left(\frac{\coth(a+bx)\mathrm{csch}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)/x^2,x)-cosh(b*x+a)/x+b*cosh(a)*Shi(
b*x)+b*Chi(b*x)*sinh(a)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

```
[In] Int[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
```

```
[Out] -(Cosh[a + b*x]/x) + b*CoshIntegral[b*x]*Sinh[a] + b*Cosh[a]*SinhIntegral[b
*x] + Defer[Int][(Coth[a + b*x]*Csch[a + b*x])/x^2, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(a+bx)}{x^2}\,dx + \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx \\
&= -\frac{\cosh(a+bx)}{x} + b\int \frac{\sinh(a+bx)}{x}\,dx + \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx \\
&= -\frac{\cosh(a+bx)}{x} + (b\cosh(a))\int \frac{\sinh(bx)}{x}\,dx \\
&\quad + (b\sinh(a))\int \frac{\cosh(bx)}{x}\,dx + \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx \\
&= -\frac{\cosh(a+bx)}{x} + b\text{Chi}(bx)\sinh(a) + b\cosh(a)\text{Shi}(bx) + \int \frac{\coth(a+bx)\text{csch}(a+bx)}{x^2}\,dx
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 14.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2,x]
[Out] Integrate[(Cosh[a + b*x]*Coth[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(bx+a)^3 \operatorname{csch}(bx+a)^2}{x^2}dx$$

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx=\int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 70.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx=\int \frac{\cosh^3(a+bx)\operatorname{csch}^2(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**2/x**2,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.67

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx=\int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] -1/2*b*e^(-a)*gamma(-1, b*x) + 1/2*b*e^a*gamma(-1, -b*x) - 2*e^(b*x + a)/(b
*x^2*e^(2*b*x + 2*a) - b*x^2) - 2*integrate(1/(b*x^3*e^(b*x + a) + b*x^3),
x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx=\int \frac{\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cosh(a+bx)\coth^2(a+bx)}{x^2}\,dx=\int \frac{\cosh(a+b\,x)^3}{x^2\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^2),x)
[Out] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^2), x)
```

3.445 $\int x^m \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Int}\left(x^m \coth(a+bx)\text{csch}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*coth(b*x+a)*csch(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

```
[In] Int[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 35.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] Integrate[x^m*Coth[a + b*x]*Csch[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

```
[In] int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
[Out] int(x^m*cosh(b*x+a)*csch(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int x^m \cosh(bx+a)\operatorname{csch}(bx+a)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)*csch(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^m*cosh(a + b*x))/sinh(a + b*x)^3,x)
[Out] int((x^m*cosh(a + b*x))/sinh(a + b*x)^3, x)
```

3.446 $\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{3x^2}{2b^2} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\text{csch}^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}$$

```
[Out] -3/2*x^2/b^2-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*csch(b*x+a)^2/b+3*x*ln(1-exp(2
*b*x+2*a))/b^3+3/2*polylog(2,exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5527, 4269, 3797, 2221, 2317, 2438}

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\text{csch}^2(a+bx)}{2b} - \frac{3x^2}{2b^2}$$

```
[In] Int[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (-3*x^2)/(2*b^2) - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3*Csch[a + b*x]^2)/(2
*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/
(2*b^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3\int x^2\text{csch}^2(a+bx)\,dx}{2b}\\
&= -\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}+\frac{3\int x\coth(a+bx)\,dx}{b^2}\\
&= -\frac{3x^2}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\text{csch}^2(a+bx)}{2b}-\frac{6\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3x^2}{2b^2} - \frac{3x^2 \coth(a+bx)}{2b^2} - \frac{x^3 \text{csch}^2(a+bx)}{2b} \\
&\quad + \frac{3x \log\left(1-e^{2(a+bx)}\right)}{b^3} - \frac{3\int \log\left(1-e^{2(a+bx)}\right)\,dx}{b^3} \\
&= -\frac{3x^2}{2b^2} - \frac{3x^2 \coth(a+bx)}{2b^2} - \frac{x^3 \text{csch}^2(a+bx)}{2b} \\
&\quad + \frac{3x \log\left(1-e^{2(a+bx)}\right)}{b^3} - \frac{3\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^4} \\
&= -\frac{3x^2}{2b^2} - \frac{3x^2 \coth(a+bx)}{2b^2} - \frac{x^3 \text{csch}^2(a+bx)}{2b} + \frac{3x \log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.42

$$
\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx
= \frac{-6\,\text{PolyLog}\left(2,-e^{-a-bx}\right) - 6\,\text{PolyLog}\left(2,e^{-a-bx}\right) + bx\left(-b^2x^2\text{csch}^2(a+bx) + 6\left(-\frac{bx}{-1+e^{2a}} + \log\left(1-e^{-a-\cdots}\right.\right.\right.}{2b^4}
$$

```
[In] Integrate[x^3*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (-6*PolyLog[2, -E^(-a - b*x)] - 6*PolyLog[2, E^(-a - b*x)] + b*x*(-(b^2*x^2
*Csch[a + b*x]^2) + 6*(-((b*x)/(-1 + E^(2*a))) + Log[1 - E^(-a - b*x)] + Lo
g[1 + E^(-a - b*x)]) + 3*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x]))/(2*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(75) = 150.

Time = 0.54 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.13

method	result
risch	$-\frac{x^2\left(2\,\text{e}^{2bx+2a}bx+3\,\text{e}^{2bx+2a}-3\right)}{b^2\left(\text{e}^{2bx+2a}-1\right)^2} - \frac{3x^2}{b^2} - \frac{6ax}{b^3} - \frac{3a^2}{b^4} + \frac{3\ln\left(\text{e}^{bx+a}+1\right)x}{b^3} + \frac{3\,\text{polylog}\left(2,-\text{e}^{bx+a}\right)}{b^4} + \frac{3\ln\left(1-\text{e}^{bx+a}\right)x}{b^3} + \cdots$

```
[In] int(x^3*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2-3/b
^2*x^2-6/b^3*a*x-3/b^4*a^2+3/b^3*ln(exp(b*x+a)+1)*x+3*polylog(2,-exp(b*x+a)
)/b^4+3/b^3*ln(1-exp(b*x+a))*x+3/b^4*ln(1-exp(b*x+a))*a+3*polylog(2,exp(b*x
+a))/b^4-3/b^4*a*ln(exp(b*x+a)-1)+6/b^4*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 979, normalized size of antiderivative = 11.80

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*
x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*
x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^4 + 4*c
osh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*s
inh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) + 1) + 3*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x +
a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 - a
)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) +
 a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b*x + a)*cosh(b*x + a)^4 +
 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*
(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(
b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*
sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(6*(b^2*x^2
- a^2)*cosh(b*x + a)^3 + (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b*x + a))*sin
h(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^
4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2
- b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 - b^4*cosh(b*x + a))*sinh(b
*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.57

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{3\,x^2 - \left(2\,bx^3e^{(2\,a)} + 3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2} - \frac{3\,x^2}{b^2} + \frac{3\,\left(bx\log\left(e^{(bx+a)}+1\right) + \text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\,\left(bx\log\left(-e^{(bx+a)}+1\right) + \text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a)
- 2*b^2*e^(2*b*x + 2*a) + b^2) - 3*x^2/b^2 + 3*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^4
```

Giac [F]

$$\int x^3 \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^3*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = \int \frac{x^3\cosh(a+b\,x)}{\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^3*cosh(a + b*x))/sinh(a + b*x)^3,x)
[Out] int((x^3*cosh(a + b*x))/sinh(a + b*x)^3, x)
```

3.447 $\int x^2 \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x^2\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\text{csch}^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}$$

```
[Out] -x*coth(b*x+a)/b^2-1/2*x^2*csch(b*x+a)^2/b+ln(sinh(b*x+a))/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5527, 4269, 3556}

$$\int x^2\coth(a+bx)\text{csch}^2(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b^3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\text{csch}^2(a+bx)}{2b}$$

```
[In] Int[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -((x*Coth[a + b*x])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]
/b^3
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\operatorname{csch}^2(a+bx)}{2b}+\frac{\int x\operatorname{csch}^2(a+bx)\,dx}{b}\\
&= -\frac{x\coth(a+bx)}{b^2}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}+\frac{\int \coth(a+bx)\,dx}{b^2}\\
&= -\frac{x\coth(a+bx)}{b^2}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int x^2\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{x\coth(a)}{b^2}-\frac{x^2\operatorname{csch}^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b^2}$$

```
[In] Integrate[x^2*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] -((x*Coth[a])/b^2) - (x^2*Csch[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 +
 (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.71

method	result	size
risch	$-\frac{2x}{b^2}-\frac{2a}{b^3}-\frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b^3}$	72

```
[In] int(x^2*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x/b^2-2/b^3*a-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2
*a)-1)^2+1/b^3*ln(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 9.12

$$\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$

$$-\frac{2\,bx\cosh\left(bx+a\right)^4+8\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+2\,bx\sinh\left(bx+a\right)^4+2\left(b^2x^2-bx\right)\cosh\left(bx+a\right.}{}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a)^4 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 2*b*x*sinh(
b*x + a)^4 + 2*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 6*b*x*cosh(b*
x + a)^2 - b*x)*sinh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*l
og(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*b*x*cosh(b*x + a
)^3 + (b^2*x^2 - b*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 +
4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x
+ a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cos
h(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int x^2\cosh\left(a+bx\right)\operatorname{csch}^3\left(a+bx\right)dx$$

```
[In] integrate(x**2*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cosh(a + b*x)*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{2\left(\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}+xe^{(4bx+4a)}\right)}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2} + \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^3} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) - x*e^(2*a))*e^(2*b*x) + x*e^(4*b*x + 4*a))/(b^2*e^(4*b*
x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + log((e^(b*x + a) + 1)*e^(-a))/b^3
 + log((e^(b*x + a) - 1)*e^(-a))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.31

$$\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{2b^2x^2e^{(2bx+2a)}+2bxe^{(4bx+4a)}-2bxe^{(2bx+2a)}-e^{(4bx+4a)}\log\left(e^{(2bx+2a)}-1\right)+2e^{(2bx+2a)}\log\left(e^{(2bx+2a)}\right.}{b^3e^{(4bx+4a)}-2b^3e^{(2bx+2a)}+b^3}$$

```
[In] integrate(x^2*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(2*b^2*x^2*e^(2*b*x + 2*a) + 2*b*x*e^(4*b*x + 4*a) - 2*b*x*e^(2*b*x + 2*a)
 - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*e^(2*b*x + 2*a)*log(e^(2*b*
x + 2*a) - 1) - log(e^(2*b*x + 2*a) - 1))/(b^3*e^(4*b*x + 4*a) - 2*b^3*e^(2
*b*x + 2*a) + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int x^2 \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}-1\right)}{b^3} - \frac{\frac{x^2}{b}+\frac{x^2\,e^{2a+2bx}}{b}}{e^{4a+4bx}-2\,e^{2a+2bx}+1} - \frac{2x}{b^2} - \frac{b\,x^2+2\,x}{b^2\,\left(e^{2a+2bx}-1\right)}$$

```
[In] int((x^2*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b^3 - (x^2/b + (x^2*exp(2*a + 2*b*x))/b)/(exp(
4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) - (2*x)/b^2 - (2*x + b*x^2)/(b^2*(ex
p(2*a + 2*b*x) - 1))
```

3.448 $\int x \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

```
[Out] -1/2*coth(b*x+a)/b^2-1/2*x*csch(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5527, 3852, 8}

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

```
[In] Int[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] -1/2*Coth[a + b*x]/b^2 - (x*Csch[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\text{csch}^2(a+bx)}{2b}+\frac{\int \text{csch}^2(a+bx)\,dx}{2b}\\ &= -\frac{x\text{csch}^2(a+bx)}{2b}-\frac{i\text{Subst}(\int 1\,dx,x,-i\coth(a+bx))}{2b^2}\\ &= -\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(a+bx)}{2b^2}-\frac{x\text{csch}^2(a+bx)}{2b}$$

```
[In] Integrate[x*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] -1/2*Coth[a + b*x]/b^2 - (x*Csch[a + b*x]^2)/(2*b)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}-1}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}$	43

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.57

$$\int x \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$

$$-\frac{2\,(bx\cosh\left(bx+a\right)+(bx+1)\sinh\left(bx+a\right))}{b^2\cosh\left(bx+a\right)^3+3\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b^2\sinh\left(bx+a\right)^3-b^2\cosh\left(bx+a\right)+3\,\left(b^2\cosh\left(\right.\right.}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(b*x*cosh(b*x + a) + (b*x + 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^3 + 3*b
^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3 - b^2*cosh(b*x + a)
+ 3*(b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a))
```

Sympy [F]

$$\int x \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int x\cosh\left(a+bx\right)\operatorname{csch}^3\left(a+bx\right)dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] Integral(x*cosh(a + b*x)*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.33

$$\int x \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \frac{2\,bxe^{(4\,bx+4\,a)}-\left(4\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}+1}{2\,(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2)} - \frac{2\,bxe^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}-1}{2\,(b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2)}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*e^(4*b*x + 4*a) - (4*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*
e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(2*b*x*e^(4*b*x + 4*a)
 + e^(2*b*x + 2*a) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 6.13

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{4\,bxe^{(2\,bx+2\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right) + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right) + e^{(4\,bx+4\,a)}\log\left(-e^{(2\,bx+2\,a)}+\right.}{2\,(b^2e^{(4\,bx+4\,a)} - 2\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*x*e^(2*b*x + 2*a) - e^(4*b*x + 4*a)*log(e^(2*b*x + 2*a) - 1) + 2*
e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + e^(4*b*x + 4*a)*log(-e^(2*b*x +
2*a) + 1) - 2*e^(2*b*x + 2*a)*log(-e^(2*b*x + 2*a) + 1) + 2*e^(2*b*x + 2*a)
 - log(e^(2*b*x + 2*a) - 1) + log(-e^(2*b*x + 2*a) + 1) - 2)/(b^2*e^(4*b*x
+ 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\mathrm{e}^{2\,a+2\,b\,x}\,(2\,b\,x+1)-1}{b^2\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^2}$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] -(exp(2*a + 2*b*x)*(2*b*x + 1) - 1)/(b^2*(exp(2*a + 2*b*x) - 1)^2)
```

3.449 $\int \coth(a+bx)\text{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

```
[Out] -1/2*csch(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2686, 30}

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

```
[In] Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*Csch[a + b*x]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, -i\text{csch}(a+bx))}{b}$$
$$= -\frac{\text{csch}^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{csch}^2(a+bx)}{2b}$$

[In] Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]

[Out] -1/2*Csch[a + b*x]^2/b

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\text{csch}(bx+a)^2}{2b}$	14
default	$-\frac{\text{csch}(bx+a)^2}{2b}$	14
risch	$-\frac{2\,e^{2bx+2a}}{b(e^{2bx+2a}-1)^2}$	28

[In] int(cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/2*csch(b*x+a)^2/b

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.73

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx =$$
$$-\frac{2(\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3-b\cosh(bx+a)+3\,(b\cosh(bx+a$$

[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")

```
[Out] -2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*s
inh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)^2
 - b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = \int \cosh(a+bx)\,\text{csch}^3(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})^2}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -2/(b*(e^(b*x + a) - e^(-b*x - a))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,a)} - 1)^2}$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] -2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{1}{2\,b\,\sinh{(a+b\,x)}^2}$$

```
[In] int(cosh(a + b*x)/sinh(a + b*x)^3,x)
[Out] -1/(2*b*sinh(a + b*x)^2)
```

3.450 $\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x,x]
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^3}{x}dx$$

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
[Out] int(cosh(b*x+a)*csch(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 21.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+bx)\,\text{csch}^3(a+bx)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3/x,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - 4*integrate(1/4/(b^2*x^3*e^(b*x + a) + b^
2*x^3), x) + 4*integrate(1/4/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)\,\text{csch}\,(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)}{x\sinh(a+b\,x)^3}\,dx$$

```
[In] int(cosh(a + b*x)/(x*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)/(x*sinh(a + b*x)^3), x)
```

3.451 $\int \frac{\coth(a+bx)\mathrm{csch}^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth(a+bx)\mathrm{csch}^2(a+bx)}{x^2}\,dx = \mathrm{Int}\left(\frac{\coth(a+bx)\mathrm{csch}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(coth(b*x+a)*csch(b*x+a)^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+bx)\mathrm{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)\mathrm{csch}^2(a+bx)}{x^2}\,dx$$

```
[In] Int[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
[Out] Defer[Int][(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth(a+bx)\mathrm{csch}^2(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx=\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2,x]
[Out] Integrate[(Coth[a + b*x]*Csch[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{3}}{x^{2}}dx$$

```
[In] int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
[Out] int(cosh(b*x+a)*csch(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx=\int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 27.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx=\int \frac{\cosh\left(a+bx\right)\text{csch}^{3}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)**3/x**2,x)
[Out] Integral(cosh(a + b*x)*csch(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.06

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 12*integrate(1/4/(b^2*x^4*e^(b*x + a) + b
^2*x^4), x) + 12*integrate(1/4/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth(a+bx)\text{csch}^2(a+bx)}{x^2}\,dx = \int \frac{\cosh(a+b\,x)}{x^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)/(x^2*sinh(a + b*x)^3), x)
```

3.452 $\int x^m \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Int}(x^m\text{csch}(a+bx),x) + \text{Int}\left(x^m\text{csch}^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*csch(b*x+a),x)+Unintegrable(x^m*csch(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

```
[In] Int[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] Defer[Int][x^m*Csch[a + b*x], x] + Defer[Int][x^m*Csch[a + b*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m\text{csch}(a+bx)\,dx + \int x^m\text{csch}^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 164.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

```
[In] Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] Integrate[x^m*Coth[a + b*x]^2*Csch[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \cosh{(bx+a)}^2 \operatorname{csch}{(bx+a)}^3\,dx$$

```
[In] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
[Out] int(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^m \cosh{(bx+a)}^2 \operatorname{csch}{(bx+a)}^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \text{csch}(bx+a)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int x^m \cosh(bx+a)^2 \text{csch}(bx+a)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^2}{\sinh(a+b\,x)^3}\,dx$$

```
[In] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
[Out] int((x^m*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.453 $\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 201

$$\int x^3 \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^4} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} + \frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^4} + \frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} + \frac{3x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{3x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} + \frac{3\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}$$

```
[Out] -6*x*arctanh(exp(b*x+a))/b^3-x^3*arctanh(exp(b*x+a))/b-3/2*x^2*csch(b*x+a)/
b^2-1/2*x^3*coth(b*x+a)*csch(b*x+a)/b-3*polylog(2,-exp(b*x+a))/b^4-3/2*x^2*
polylog(2,-exp(b*x+a))/b^2+3*polylog(2,exp(b*x+a))/b^4+3/2*x^2*polylog(2,ex
p(b*x+a))/b^2+3*x*polylog(3,-exp(b*x+a))/b^3-3*x*polylog(3,exp(b*x+a))/b^3-
3*polylog(4,-exp(b*x+a))/b^4+3*polylog(4,exp(b*x+a))/b^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5565, 4267, 2611, 6744, 2320, 6724, 4271, 2317, 2438}

$$\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3}-\frac{x^3\text{arctanh}\left(e^{a+bx}\right)}{b} -\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^4}+\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^4} -\frac{3\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}+\frac{3\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4} +\frac{3x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{3x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} -\frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}+\frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} -\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[In] Int[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (-6*x*ArcTanh[E^(a + b*x)])/b^3 - (x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*Csc
h[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (3*PolyLog[
2, -E^(a + b*x)])/b^4 - (3*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*PolyL
og[2, E^(a + b*x)])/b^4 + (3*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) + (3*x*Po
lyLog[3, -E^(a + b*x)])/b^3 - (3*x*PolyLog[3, E^(a + b*x)])/b^3 - (3*PolyLo
g[4, -E^(a + b*x)])/b^4 + (3*PolyLog[4, E^(a + b*x)])/b^4
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5565

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^3\text{csch}(a+bx)\,dx + \int x^3\text{csch}^3(a+bx)\,dx \\
&= -\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} \\
&\quad - \frac{1}{2}\int x^3\text{csch}(a+bx)\,dx + \frac{3\int x\text{csch}(a+bx)\,dx}{b^2} \\
&\quad - \frac{3\int x^2\log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{3\int x^2\log\left(1+e^{a+bx}\right)\,dx}{b} \\
&= -\frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} \\
&\quad - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} \\
&\quad + \frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3\int\log\left(1-e^{a+bx}\right)\,dx}{b^3} + \frac{3\int\log\left(1+e^{a+bx}\right)\,dx}{b^3} \\
&\quad + \frac{6\int x\,\text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} - \frac{6\int x\,\text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2} \\
&\quad + \frac{3\int x^2\log\left(1-e^{a+bx}\right)\,dx}{2b} - \frac{3\int x^2\log\left(1+e^{a+bx}\right)\,dx}{2b} \\
&= -\frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} - \frac{x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} \\
&\quad - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} \\
&\quad + \frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} + \frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\
&\quad - \frac{3\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{3\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} \\
&\quad - \frac{6\int\text{PolyLog}\left(3,-e^{a+bx}\right)\,dx}{b^3} + \frac{6\int\text{PolyLog}\left(3,e^{a+bx}\right)\,dx}{b^3} \\
&\quad - \frac{3\int x\,\text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} + \frac{3\int x\,\text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2}
\end{aligned}
$$

$$= -\frac{6x\text{arctanh}(e^{a+bx})}{b^3} - \frac{x^3\text{arctanh}(e^{a+bx})}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{3\,\text{PolyLog}(2,-e^{a+bx})}{b^4} - \frac{3x^2\,\text{PolyLog}(2,-e^{a+bx})}{2b^2} + \frac{3\,\text{PolyLog}(2,e^{a+bx})}{b^4} + \frac{3x^2\,\text{PolyLog}(2,e^{a+bx})}{2b^2} + \frac{3x\,\text{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{3x\,\text{PolyLog}(3,e^{a+bx})}{b^3} - \frac{6\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, e^{a+bx}\right)}{b^4} + \frac{6\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx, x, e^{a+bx}\right)}{b^4} + \frac{3\int \text{PolyLog}(3,-e^{a+bx})\,dx}{b^3} - \frac{3\int \text{PolyLog}(3,e^{a+bx})\,dx}{b^3}$$

$$= -\frac{6x\text{arctanh}(e^{a+bx})}{b^3} - \frac{x^3\text{arctanh}(e^{a+bx})}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{3\,\text{PolyLog}(2,-e^{a+bx})}{b^4} - \frac{3x^2\,\text{PolyLog}(2,-e^{a+bx})}{2b^2} + \frac{3\,\text{PolyLog}(2,e^{a+bx})}{b^4} + \frac{3x^2\,\text{PolyLog}(2,e^{a+bx})}{2b^2} + \frac{3x\,\text{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{3x\,\text{PolyLog}(3,e^{a+bx})}{b^3} - \frac{6\,\text{PolyLog}(4,-e^{a+bx})}{b^4} + \frac{6\,\text{PolyLog}(4,e^{a+bx})}{b^4} + \frac{3\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, e^{a+bx}\right)}{b^4} - \frac{3\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx, x, e^{a+bx}\right)}{b^4}$$

$$= -\frac{6x\text{arctanh}(e^{a+bx})}{b^3} - \frac{x^3\text{arctanh}(e^{a+bx})}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{x^3\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{3\,\text{PolyLog}(2,-e^{a+bx})}{b^4} - \frac{3x^2\,\text{PolyLog}(2,-e^{a+bx})}{2b^2} + \frac{3\,\text{PolyLog}(2,e^{a+bx})}{b^4} + \frac{3x^2\,\text{PolyLog}(2,e^{a+bx})}{2b^2} + \frac{3x\,\text{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{3x\,\text{PolyLog}(3,e^{a+bx})}{b^3} - \frac{3\,\text{PolyLog}(4,-e^{a+bx})}{b^4} + \frac{3\,\text{PolyLog}(4,e^{a+bx})}{b^4}$$

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.39

$$\int x^3\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{12b^2x^2\text{csch}(a) + b^3x^3\text{csch}^2\left(\frac{1}{2}(a+bx)\right) - 24bx\log\left(1-e^{a+bx}\right) - 4b^3x^3\log\left(1-e^{a+bx}\right) + 24bx\log\left(1+e^{a\cdots}\right.}{}$$

```
[In] Integrate[x^3*Coth[a + b*x]^2*Csch[a + b*x],x]
```

[Out] -1/8*(12*b^2*x^2*Csch[a] + b^3*x^3*Csch[(a + b*x)/2]^2 - 24*b*x*Log[1 - E^(a + b*x)] - 4*b^3*x^3*Log[1 - E^(a + b*x)] + 24*b*x*Log[1 + E^(a + b*x)] + 4*b^3*x^3*Log[1 + E^(a + b*x)] + 12*(2 + b^2*x^2)*PolyLog[2, -E^(a + b*x)] - 12*(2 + b^2*x^2)*PolyLog[2, E^(a + b*x)] - 24*b*x*PolyLog[3, -E^(a + b*x)] + 24*b*x*PolyLog[3, E^(a + b*x)] + 24*PolyLog[4, -E^(a + b*x)] - 24*PolyLog[4, E^(a + b*x)] + b^3*x^3*Sech[(a + b*x)/2]^2 - 6*b^2*x^2*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] - 6*b^2*x^2*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/b^4

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{x^2 e^{bx+a} \left(e^{2bx+2a} bx+bx+3\, e^{2bx+2a}-3\right)}{b^2 \left(e^{2bx+2a}-1\right)^2}+\frac{3\, \mathrm{polylog}\left(2, e^{bx+a}\right)}{b^4}+\frac{3\, \mathrm{polylog}\left(4, e^{bx+a}\right)}{b^4}-\frac{3\, \mathrm{polylog}\left(2, -e^{bx+a}\right)}{b^4}-\frac{3\, \mathrm{polylog}(4,}{b^4}$

[In] int(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -x^2*exp(b*x+a)*(exp(2*b*x+2*a)*b*x+b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2+3*polylog(2,exp(b*x+a))/b^4+3*polylog(4,exp(b*x+a))/b^4-3*polylog(2,-exp(b*x+a))/b^4-3*polylog(4,-exp(b*x+a))/b^4+3/b^3*ln(1-exp(b*x+a))*x-3/b^3*ln(exp(b*x+a)+1)*x+1/2/b*ln(1-exp(b*x+a))*x^3+3/2*x^2*polylog(2,exp(b*x+a))/b^2-3*x*polylog(3,exp(b*x+a))/b^3-1/2/b*ln(exp(b*x+a)+1)*x^3-3/2*x^2*polylog(2,-exp(b*x+a))/b^2+3*x*polylog(3,-exp(b*x+a))/b^3+3/b^4*ln(1-exp(b*x+a))*a-3/b^4*ln(exp(b*x+a)+1)*a+1/2/b^4*ln(1-exp(b*x+a))*a^3-1/2/b^4*ln(exp(b*x+a)+1)*a^3+6/b^4*a*arctanh(exp(b*x+a))+1/b^4*a^3*arctanh(exp(b*x+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(179) = 358.

Time = 0.28 (sec) , antiderivative size = 1802, normalized size of antiderivative = 8.97

$$\int x^3 \coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")

[Out] -1/2*(2*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)^3 + 6*(b^3*x^3 + 3*b^2*x^2)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*x^3 + 3*b^2*x^2)*sinh(b*x + a)^3 + 2*(b^3*x^3 - 3*b^2*x^2)*cosh(b*x + a) - 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*dilog(cosh(b*x + a) + sinh(b*

```
x + a)) + 3*((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 2)
*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 + 2)*sinh(b
*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh(b*x + a))
*sinh(b*x + a) + 2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^3*x^3 + (b^3
*x^3 + 6*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 6*b*x)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^3*x^3 + 6*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 + 6*b*x)*cosh(b*x +
 a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 6*b*x)*cosh(b*x + a)^2 + 6*b*x)*sinh(b*x
+ a)^2 + 6*b*x + 4*((b^3*x^3 + 6*b*x)*cosh(b*x + a)^3 - (b^3*x^3 + 6*b*x)*c
osh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + ((a^3
 + 6*a)*cosh(b*x + a)^4 + 4*(a^3 + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^
3 + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 6*a)*cosh(b*x + a)^2 - 2*(a^3 - 3
*(a^3 + 6*a)*cosh(b*x + a)^2 + 6*a)*sinh(b*x + a)^2 + 4*((a^3 + 6*a)*cosh(b
*x + a)^3 - (a^3 + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)*log(cosh(b*x +
a) + sinh(b*x + a) - 1) - (b^3*x^3 + (b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x
 + a)^4 + 4*(b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (
b^3*x^3 + a^3 + 6*b*x + 6*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3 + 6*b
*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 + 6*b*x + 6
*a)*cosh(b*x + a)^2 + 6*b*x + 6*a)*sinh(b*x + a)^2 + 6*b*x + 4*((b^3*x^3 +
a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 + 6*b*x + 6*a)*cosh(b*x
 + a))*sinh(b*x + a) + 6*a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*co
sh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x +
a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) -
 sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a
)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^
2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a)
)*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*
x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*
x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x +
 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cos
h(b*x + a) - sinh(b*x + a)) + 2*(b^3*x^3 - 3*b^2*x^2 + 3*(b^3*x^3 + 3*b^2*x
^2)*cosh(b*x + a)^2)*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x +
 a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b^4 + 2
*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 - b
^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.30

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx$$
$$= -\frac{\left(bx^3e^{(3a)}+3x^2e^{(3a)}\right)e^{(3bx)}+\left(bx^3e^a-3x^2e^a\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}$$
$$-\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6bx\mathrm{Li}_3(-e^{(bx+a)})+6\mathrm{Li}_4(-e^{(bx+a)})}{2b^4}$$
$$+\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6bx\mathrm{Li}_3(e^{(bx+a)})+6\mathrm{Li}_4(e^{(bx+a)})}{2b^4}$$
$$-\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -((b*x^3*e^(3*a) + 3*x^2*e^(3*a))*e^(3*b*x) + (b*x^3*e^a - 3*x^2*e^a)*e^(b*
x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^3*x^3*log(
e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x
 + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + 1/2*(b^3*x^3*log(-e^(b*x + a) +
1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polyl
og(4, e^(b*x + a)))/b^4 - 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a))
)/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^3 \cosh{(bx+a)}^2 \operatorname{csch}{(bx+a)}^3 \,dx$$

```
[In] integrate(x^3*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^2}{\sinh{(a+b\,x)}^3}\,dx$$

```
[In] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
[Out] int((x^3*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.454 $\int x^2 \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{x^2\operatorname{arctanh}(e^{a+bx})}{b} - \frac{\operatorname{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b^2} + \frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b^2} + \frac{\operatorname{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{\operatorname{PolyLog}(3,e^{a+bx})}{b^3}$$

```
[Out] -x^2*arctanh(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^3-x*csch(b*x+a)/b^2-1/2*x
^2*coth(b*x+a)*csch(b*x+a)/b-x*polylog(2,-exp(b*x+a))/b^2+x*polylog(2,exp(b
*x+a))/b^2+polylog(3,-exp(b*x+a))/b^3-polylog(3,exp(b*x+a))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5565, 4267, 2611, 2320, 6724, 4271, 3855}

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b^3} - \frac{x^2\operatorname{arctanh}(e^{a+bx})}{b} + \frac{\operatorname{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{\operatorname{PolyLog}(3,e^{a+bx})}{b^3} - \frac{x\operatorname{PolyLog}(2,-e^{a+bx})}{b^2} + \frac{x\operatorname{PolyLog}(2,e^{a+bx})}{b^2} - \frac{x\operatorname{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

[In] Int[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]

[Out] -((x^2*ArcTanh[E^(a + b*x)])/b) - ArcTanh[Cosh[a + b*x]]/b^3 - (x*Csch[a + b*x])/b^2 - (x^2*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (x*PolyLog[2, -E^(a + b*x)])/b^2 + (x*PolyLog[2, E^(a + b*x)])/b^2 + PolyLog[3, -E^(a + b*x)]/b^3 - PolyLog[3, E^(a + b*x)]/b^3

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5565

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^2\text{csch}(a+bx)\,dx + \int x^2\text{csch}^3(a+bx)\,dx \\
&= -\frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b} \\
&\quad - \frac{1}{2}\int x^2\text{csch}(a+bx)\,dx + \frac{\int \text{csch}(a+bx)\,dx}{b^2} \\
&\quad - \frac{2\int x\log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{2\int x\log\left(1+e^{a+bx}\right)\,dx}{b} \\
&= -\frac{x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} \\
&\quad - \frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} \\
&\quad + \frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{2\int \text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} \\
&\quad - \frac{2\int \text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2} + \frac{\int x\log\left(1-e^{a+bx}\right)\,dx}{b} - \frac{\int x\log\left(1+e^{a+bx}\right)\,dx}{b} \\
&= -\frac{x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} \\
&\quad - \frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} \\
&\quad - \frac{\int \text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} + \frac{\int \text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2}
\end{aligned}$$

$$= -\frac{x^2\text{arctanh}(e^{a+bx})}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b^2} + \frac{x\,\text{PolyLog}(2,e^{a+bx})}{b^2} + \frac{2\,\text{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{2\,\text{PolyLog}(3,e^{a+bx})}{b^3} - \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{a+bx}\right)}{b^3}$$

$$= -\frac{x^2\text{arctanh}(e^{a+bx})}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} - \frac{x^2\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{x\,\text{PolyLog}(2,-e^{a+bx})}{b^2} + \frac{x\,\text{PolyLog}(2,e^{a+bx})}{b^2} + \frac{\text{PolyLog}(3,-e^{a+bx})}{b^3} - \frac{\text{PolyLog}(3,e^{a+bx})}{b^3}$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.80

$$\int x^2\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{8bx\text{csch}(a) + b^2x^2\text{csch}^2\left(\frac{1}{2}(a+bx)\right) - 8\log\left(1-e^{a+bx}\right) - 4b^2x^2\log\left(1-e^{a+bx}\right) + 8\log\left(1+e^{a+bx}\right) + 4b^2x^2}{}$$

```
[In] Integrate[x^2*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*(8*b*x*Csch[a] + b^2*x^2*Csch[(a + b*x)/2]^2 - 8*Log[1 - E^(a + b*x)]
- 4*b^2*x^2*Log[1 - E^(a + b*x)] + 8*Log[1 + E^(a + b*x)] + 4*b^2*x^2*Log[1
 + E^(a + b*x)] + 8*b*x*PolyLog[2, -E^(a + b*x)] - 8*b*x*PolyLog[2, E^(a +
b*x)] - 8*PolyLog[3, -E^(a + b*x)] + 8*PolyLog[3, E^(a + b*x)] + b^2*x^2*Se
ch[(a + b*x)/2]^2 - 4*b*x*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] - 4*b*x
*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/b^3
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{x\,\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}bx+bx+2\,\mathrm{e}^{2bx+2a}-2\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2} - \frac{a^2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b^3} + \frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x^2}{2b} - \frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{2b^3} + \frac{x\,\text{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2} -$

```
[In] int(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x*exp(b*x+a)*(exp(2*b*x+2*a)*b*x+b*x+2*exp(2*b*x+2*a)-2)/b^2/(exp(2*b*x+2*
a)-1)^2-1/b^3*a^2*arctanh(exp(b*x+a))+1/2/b*ln(1-exp(b*x+a))*x^2-1/2/b^3*ln
(1-exp(b*x+a))*a^2+x*polylog(2,exp(b*x+a))/b^2-polylog(3,exp(b*x+a))/b^3-1/
2/b*ln(exp(b*x+a)+1)*x^2+1/2/b^3*ln(exp(b*x+a)+1)*a^2-x*polylog(2,-exp(b*x+
a))/b^2+polylog(3,-exp(b*x+a))/b^3-2/b^3*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. $2(114) = 228$.

Time = 0.28 (sec) , antiderivative size = 1311, normalized size of antiderivative = 10.66

$$\int x^2 \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^3 + 6*(b^2*x^2 + 2*b*x)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b^2*x^2 + 2*b*x)*sinh(b*x + a)^3 + 2*(b^2*x^2 - 2*b
*x)*cosh(b*x + a) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x +
 a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a
)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x +
a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x +
 a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*c
osh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*
(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x +
a) - sinh(b*x + a)) + ((b^2*x^2 + 2)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 2)*cosh
(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^
2*x^2 + 2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 2)*cosh(b*x + a)^2 +
 2)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 2)*cosh(b*x + a)^3 - (b^2*x^2 + 2)*cosh
(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - ((a^
2 + 2)*cosh(b*x + a)^4 + 4*(a^2 + 2)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 +
 2)*sinh(b*x + a)^4 - 2*(a^2 + 2)*cosh(b*x + a)^2 + 2*(3*(a^2 + 2)*cosh(b*x
 + a)^2 - a^2 - 2)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 2)*cosh(b*x + a)^3 - (
a^2 + 2)*cosh(b*x + a))*sinh(b*x + a) + 2)*log(cosh(b*x + a) + sinh(b*x + a
) - 1) - ((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 -
 a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^
2)*sinh(b*x + a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 -
a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x
 + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh
(b*x + a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*
x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 +
4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x
```

```
 + a) - sinh(b*x + a)) + 2*(b^2*x^2 + 3*(b^2*x^2 + 2*b*x)*cosh(b*x + a)^2 -
 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x
+ a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(
b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x +
 a))*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**2*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.60

$$\begin{aligned}\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx &= -\frac{\left(bx^2e^{(3\,a)}+2\,xe^{(3\,a)}\right)e^{(3\,bx)}+\left(bx^2e^{a}-2\,xe^{a}\right)e^{(bx)}}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2} \\ &\quad -\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3(-e^{(bx+a)})}{2\,b^3} \\ &\quad +\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3(e^{(bx+a)})}{2\,b^3} \\ &\quad -\frac{\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}-1\right)}{b^3}\end{aligned}$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -((b*x^2*e^(3*a) + 2*x*e^(3*a))*e^(3*b*x) + (b*x^2*e^a - 2*x*e^a)*e^(b*x))/
(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^2*x^2*log(e^(b
*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3
+ 1/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog
(3, e^(b*x + a)))/b^3 - log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Giac [F]

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int x^2 \cosh\left(bx+a\right)^2 \operatorname{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
[Out] int((x^2*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.455 $\int x \coth^2(a+bx)\operatorname{\mathbf{csch}}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 82

$$\int x \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{x\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} + \frac{\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}$$

```
[Out] -x*arctanh(exp(b*x+a))/b-1/2*csch(b*x+a)/b^2-1/2*x*coth(b*x+a)*csch(b*x+a)/
b-1/2*polylog(2,-exp(b*x+a))/b^2+1/2*polylog(2,exp(b*x+a))/b^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5565, 4267, 2317, 2438, 4270}

$$\int x \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{x\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} + \frac{\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{\operatorname{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[In] Int[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -((x*ArcTanh[E^(a + b*x)])/b) - Csch[a + b*x]/(2*b^2) - (x*Coth[a + b*x]*Cs
ch[a + b*x])/(2*b) - PolyLog[2, -E^(a + b*x)]/(2*b^2) + PolyLog[2, E^(a + b
*x)]/(2*b^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5565

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x\text{csch}(a+bx)\,dx + \int x\text{csch}^3(a+bx)\,dx\\ &= -\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b}\\ &\quad - \frac{1}{2}\int x\text{csch}(a+bx)\,dx - \frac{\int \log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{\int \log\left(1+e^{a+bx}\right)\,dx}{b}\end{aligned}$$

$$= -\frac{x\text{arctanh}(e^{a+bx})}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{\int \log\left(1-e^{a+bx}\right)\,dx}{2b} - \frac{\int \log\left(1+e^{a+bx}\right)\,dx}{2b}$$

$$= -\frac{x\text{arctanh}(e^{a+bx})}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{2b^2} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{2b^2}$$

$$= -\frac{x\text{arctanh}(e^{a+bx})}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{x\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} + \frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\coth\left(\frac{1}{2}(a+bx)\right) + bx\text{csch}^2\left(\frac{1}{2}(a+bx)\right) - 4bx\log\left(1-e^{a+bx}\right) + 4bx\log\left(1+e^{a+bx}\right) + 4\,\text{PolyLog}\left(2,-e\right.}{8b^2}$$

```
[In] Integrate[x*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/8*(2*Coth[(a + b*x)/2] + b*x*Csch[(a + b*x)/2]^2 - 4*b*x*Log[1 - E^(a +
b*x)] + 4*b*x*Log[1 + E^(a + b*x)] + 4*PolyLog[2, -E^(a + b*x)] - 4*PolyLog
[2, E^(a + b*x)] + b*x*Sech[(a + b*x)/2]^2 - 2*Tanh[(a + b*x)/2])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(71) = 142.

Time = 1.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}bx+bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2} + \frac{\ln\left(1-e^{bx+a}\right)x}{2b} + \frac{\ln\left(1-e^{bx+a}\right)a}{2b^2} + \frac{\text{polylog}\left(2,e^{bx+a}\right)}{2b^2} - \frac{\ln\left(e^{bx+a}+1\right)x}{2b} - \frac{\ln\left(e^{bx+a}+1\right.}{2b^2}$

```
[In] int(x*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(b*x+a)*(exp(2*b*x+2*a)*b*x+b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1
)^2+1/2/b*ln(1-exp(b*x+a))*x+1/2/b^2*ln(1-exp(b*x+a))*a+1/2*polylog(2,exp(b
```

```
*x+a))/b^2-1/2/b*ln(exp(b*x+a)+1)*x-1/2/b^2*ln(exp(b*x+a)+1)*a-1/2*polylog(
2,-exp(b*x+a))/b^2+1/b^2*a*arctanh(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 842, normalized size of antiderivative = 10.27

$$\int x \coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*x + 1)*cosh(b*x + a)^3 + 6*(b*x + 1)*cosh(b*x + a)*sinh(b*x + a)
^2 + 2*(b*x + 1)*sinh(b*x + a)^3 + 2*(b*x - 1)*cosh(b*x + a) - (cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) + (cosh(b*
x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - co
sh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(co
sh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*s
inh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x
+ a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(
b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x + a)*cosh(b*x
+ a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a
)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x -
a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*
x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(3*(
b*x + 1)*cosh(b*x + a)^2 + b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4
*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x +
 a)^2 + 2*(3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh
(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int x\cosh^2{(a+bx)}\,\text{csch}^3{(a+bx)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(x*cosh(a + b*x)**2*csch(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\left(bxe^{(3\,a)}+e^{(3\,a)}\right)e^{(3\,bx)}+\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)}{2\,b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)}{2\,b^2}$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*b*
x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b*x*log(e^(b*x + a) + 1) + d
ilog(-e^(b*x + a)))/b^2 + 1/2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a
)))/b^2
```

Giac [F]

$$\int x\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int x\cosh{(bx+a)}^2\,\text{csch}{(bx+a)}^3\,dx$$

```
[In] integrate(x*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)^2*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \frac{x\cosh(a+b\,x)^2}{\sinh{(a+b\,x)}^3}\,dx$$

```
[In] int((x*cosh(a + b*x)^2)/sinh(a + b*x)^3,x)
[Out] int((x*cosh(a + b*x)^2)/sinh(a + b*x)^3, x)
```

3.456 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[Out] -1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[In] Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/2*ArcTanh[Cosh[a + b*x]]/b - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{1}{2}\int \text{csch}(a+bx)\,dx \\ &= -\frac{\text{arctanh}(\cosh(a+bx))}{2b} - \frac{\coth(a+bx)\text{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

```
[In] Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] -1/8*Csch[(a + b*x)/2]^2/b - Log[Cosh[(a + b*x)/2]]/(2*b) + Log[Sinh[(a + b
*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\,\text{csch}(bx+a)}{2}-\text{arctanh}\left(e^{bx+a}\right)}{b}$	45
default	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\,\text{csch}(bx+a)}{2}-\text{arctanh}\left(e^{bx+a}\right)}{b}$	45
risch	$-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{bx+a}+1\right)}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	65

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-arctanh(exp(b*x
+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\cosh(bx+a)^3 + 6\cosh(bx+a)\sinh(bx+a)^2 + 2\sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\right.}{}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \cosh^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) +
 log(e^(b*x + a) + e^(-b*x - a) + 2) - log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(cosh(a + b*x)^2/sinh(a + b*x)^3,x)
```

```
[Out] - atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b
*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a +
2*b*x) - 1))
```

3.457 $\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \text{Int}\left(\frac{\text{csch}(a+bx)}{x},x\right) + \text{Int}\left(\frac{\text{csch}^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)/x,x)+Unintegrable(csch(b*x+a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$$

```
[In] Int[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
[Out] Defer[Int][Csch[a + b*x]/x, x] + Defer[Int][Csch[a + b*x]^3/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{csch}(a+bx)}{x}\,dx + \int \frac{\text{csch}^3(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 47.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx$$

```
[In] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x,x]
[Out] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{3}}{x}dx$$

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 51.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x}\,dx = \int \frac{\cosh^{2}\left(a+bx\right)\operatorname{csch}^{3}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 8.72

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - e^(3*a))*e^(3*b*x) + (b*x*e^a + e^a)*e^(b*x))/(b^2*x^2*e^(
4*b*x + 4*a) - 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 2*integrate(1/4*(b^2*
x^2 + 2)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) + 2*integrate(1/4*(b^2*x^2 + 2
)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^2\operatorname{csch}(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\operatorname{csch}(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^2}{x\sinh(a+b\,x)^3}\,dx$$

```
[In] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^2/(x*sinh(a + b*x)^3), x)
```

3.458 $\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2} dx = \text{Int}\left(\frac{\text{csch}(a+bx)}{x^2}, x\right) + \text{Int}\left(\frac{\text{csch}^3(a+bx)}{x^2}, x\right)$$

```
[Out] Unintegrable(csch(b*x+a)/x^2,x)+Unintegrable(csch(b*x+a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2} dx = \int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2} dx$$

```
[In] Int[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
[Out] Defer[Int][Csch[a + b*x]/x^2, x] + Defer[Int][Csch[a + b*x]^3/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{csch}(a+bx)}{x^2} dx + \int \frac{\text{csch}^3(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 47.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2,x]
[Out] Integrate[(Coth[a + b*x]^2*Csch[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x^2}dx$$

```
[In] int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
[Out] int(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh(bx+a)^2\,\text{csch}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 61.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx = \int \frac{\cosh^2(a+bx)\,\text{csch}^3(a+bx)}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)**2*csch(b*x+a)**3/x**2,x)
[Out] Integral(cosh(a + b*x)**2*csch(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 8.83

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx=\int \frac{\cosh\left(bx+a\right)^2\text{csch}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -((b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x) + (b*x*e^a + 2*e^a)*e^(b*x))/(b^2*x^3
*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 2*integrate(1/4*(
b^2*x^2 + 6)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) + 2*integrate(1/4*(b^2*x^2
 + 6)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx=\int \frac{\cosh\left(bx+a\right)^2\text{csch}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^2*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(a+bx)\text{csch}(a+bx)}{x^2}\,dx=\int \frac{\cosh(a+b\,x)^2}{x^2\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^2/(x^2*sinh(a + b*x)^3), x)
```

3.459 $\int x^m \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \coth^3(a+bx)\,dx = \operatorname{Int}\left(x^m \coth^3(a+bx),x\right)$$

```
[Out] Unintegrable(x^m*coth(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \coth^3(a+bx)\,dx$$

```
[In] Int[x^m*Coth[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Coth[a + b*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m \coth^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 89.98 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \coth^3(a+bx)\,dx=\int x^m \coth^3(a+bx)\,dx$$

```
[In] Integrate[x^m*Coth[a + b*x]^3,x]
[Out] Integrate[x^m*Coth[a + b*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^3 dx$$

```
[In] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
[Out] int(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^3(a+bx)\,dx=\int x^m \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \coth^3(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(x**m*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 173, normalized size of antiderivative = 14.42

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) - 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) - m - 1) + integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m +
1)*e^(8*b*x + 8*a) - 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
- 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^3(a+bx)\,dx = \int x^m \cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^3\,dx$$

```
[In] integrate(x^m*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int x^m \coth^3(a+bx)\,dx = \int \frac{x^m \cosh(a+b\,x)^3}{\sinh{(a+b\,x)}^3}\,dx$$

```
[In] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
```

```
[Out] int((x^m*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.460 $\int x^3 \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 179

$$\int x^3 \coth^3(a+bx)\,dx = -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b} +\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} +\frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} -\frac{3x\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b-1/4*x^4-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*coth(b*x+a)^
2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*polylog(2,e
xp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3/2*x*polylog(3,ex
p(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3801, 3797, 2221, 2317, 2438, 30, 2611, 6744, 2320, 6724}

$$\int x^3 \coth^3(a+bx)\,dx = \frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3\,\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4} -\frac{3x\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} +\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2} +\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^3\coth^2(a+bx)}{2b}-\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}$$

```
[In] Int[x^3*Coth[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - x^4/4 - (3*x^2*Coth[a + b*x])/(2*b^2) - (x^3
*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a + b*x))])/b^3 + (x^3*Log[1 -
 E^(2*(a + b*x))])/b + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*Pol
yLog[2, E^(2*(a + b*x))])/(2*b^2) - (3*x*PolyLog[3, E^(2*(a + b*x))])/(2*b^
3) + (3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\coth^2(a+bx)}{2b}+\frac{3\int x^2\coth^2(a+bx)\,dx}{2b}+\int x^3\coth(a+bx)\,dx\\
&= -\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}\\
&\quad -2\int\frac{e^{2(a+bx)}x^3}{1-e^{2(a+bx)}}\,dx+\frac{3\int x\coth(a+bx)\,dx}{b^2}+\frac{3\int x^2\,dx}{2b}\\
&= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}\\
&\quad +\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{6\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b^2}-\frac{3\int x^2\log\left(1-e^{2(a+bx)}\right)\,dx}{b}
\end{aligned}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3\int\log\left(1-e^{2(a+bx)}\right)\,dx}{b^3} - \frac{3\int x\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)\,dx}{b^2}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} - \frac{3\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^4} + \frac{3\int\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)\,dx}{2b^3}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{3\text{Subst}\left(\int\frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{2(a+bx)}\right)}{4b^4}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3} + \frac{3\operatorname{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 422 vs. 2(179) = 358.

Time = 2.14 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.36

$$\int x^3\coth^3(a+bx)\,dx = \frac{1}{4}\left(x^4\coth(a) - \frac{2x^3\operatorname{csch}^2(a+bx)}{b} - \frac{2e^{2a}\left(6b^2e^{-2a}x^2 + b^4e^{-2a}x^4 - 6b\left(1-e^{-2a}\right)x\log\left(1-e^{-a-bx}\right) - 2b^3e^{-2a}\left(-1+e^{2a}\right)x^3\log\left(1-e^{-a-bx}\right) - \right.}{} + \frac{6x^2\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

```
[In] Integrate[x^3*Coth[a + b*x]^3,x]
```

[Out] (x^4*Coth[a] - (2*x^3*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b^2*x^2)/E^(2*a) + (b^4*x^4)/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log [1 + E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2* a) + 6*(1 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))*x^2* PolyLog[2, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)] + 6*b ^2*(1 - E^(-2*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*Poly Log[3, -E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, E^(-a - b*x)] + 12 *(1 - E^(-2*a))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-2*a))*PolyLog[4, E^ (-a - b*x)]))/(b^4*(-1 + E^(2*a))) + (6*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2)/4

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(161) = 322$.

Time = 0.53 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.09

method	result
risch	$-\frac{3a^2}{b^4}+\frac{\ln(e^{bx+a}+1)x^3}{b}-\frac{x^4}{4}-\frac{3x^2}{b^2}-\frac{3a\ln(e^{bx+a}-1)}{b^4}-\frac{a^3\ln(e^{bx+a}-1)}{b^4}+\frac{3\ln(e^{bx+a}+1)x}{b^3}+\frac{3\ln(1-e^{bx+a})x}{b^3}+\frac{3\ln}{}$

[In] int(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -3/b^4*a^2+1/b*ln(exp(b*x+a)+1)*x^3-1/4*x^4-3/b^2*x^2-3/b^4*a*ln(exp(b*x+a) -1)-1/b^4*a^3*ln(exp(b*x+a)-1)+3/b^3*ln(exp(b*x+a)+1)*x+3/b^3*ln(1-exp(b*x+ a))*x+3/b^4*ln(1-exp(b*x+a))*a-x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2+6/b^4*a*ln(exp(b*x+a))-6/b^3*a*x+2/b^4*a^3*ln(ex p(b*x+a))-2/b^3*a^3*x+3*x^2*polylog(2,-exp(b*x+a))/b^2+3*x^2*polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,-exp(b*x+a))/b^3-6*x*polylog(3,exp(b*x+a))/b^3+6* polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+1/b*ln(1-exp(b*x+a)) *x^3+1/b^4*ln(1-exp(b*x+a))*a^3-3/2/b^4*a^4+3*polylog(2,-exp(b*x+a))/b^4+3* polylog(2,exp(b*x+a))/b^4

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1985 vs. $2(159) = 318$.

Time = 0.29 (sec) , antiderivative size = 1985, normalized size of antiderivative = 11.09

$$\int x^3\coth^3(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")

[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4 *(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b

^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 - 2*(b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 2*(b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 4*(b^3*x^3 + (b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + 3*b*x)*cosh(b*x + a)^3 - (b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 3*a)*cosh(b*x + a)^2 - 2*(a^3 - 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 - (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 4*(b^3*x^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*((b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 - (b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^

```
4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)
^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*
x + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \coth^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.69

$$\begin{aligned}
&\int x^3 \coth^3(a+bx)\,dx \\
&= \frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 - 2\,\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\,\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4} \\
&+ \frac{b^3x^3\log\left(e^{(bx+a)} + 1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4} \\
&+ \frac{b^3x^3\log\left(-e^{(bx+a)} + 1\right) + 3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3(e^{(bx+a)}) + 6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4} \\
&+ \frac{3\,\left(bx\log\left(e^{(bx+a)} + 1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\,\left(bx\log\left(-e^{(bx+a)} + 1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}
\end{aligned}$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 - 2*(b^2*x^4*e^(2*a) + 4*b*
x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b
*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + (b^3*x^3*log(e^(b*x + a)
 + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*
polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*
dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x +
a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*
log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \coth^3(a+bx)\,dx = \int x^3 \cosh\left(bx+a\right)^3 \operatorname{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^3*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^3(a+bx)\,dx = \int \frac{x^3 \cosh(a+b\,x)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
[Out] int((x^3*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.461 $\int x^2 \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^2 \coth^3(a+bx)\,dx = \frac{x^2}{2b}-\frac{x^3}{3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}$$

```
[Out] 1/2*x^2/b-1/3*x^3-x*coth(b*x+a)/b^2-1/2*x^2*coth(b*x+a)^2/b+x^2*ln(1-exp(2*
b*x+2*a))/b+ln(sinh(b*x+a))/b^3+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*polylog
(3,exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3801, 3556, 30, 3797, 2221, 2611, 2320, 6724}

$$\int x^2 \coth^3(a+bx)\,dx = -\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{x\coth(a+bx)}{b^2}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2}{2b}-\frac{x^3}{3}$$

```
[In] Int[x^2*Coth[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - x^3/3 - (x*Coth[a + b*x])/b^2 - (x^2*Coth[a + b*x]^2)/(2*b) + (
x^2*Log[1 - E^(2*(a + b*x))])/b + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, E^
(2*(a + b*x))])/b^2 - PolyLog[3, E^(2*(a + b*x))]/(2*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
```

```
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\coth^2(a+bx)}{2b}+\frac{\int x\coth^2(a+bx)\,dx}{b}+\int x^2\coth(a+bx)\,dx\\
&= -\frac{x^3}{3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}-2\int\frac{e^{2(a+bx)}x^2}{1-e^{2(a+bx)}}\,dx+\frac{\int\coth(a+bx)\,dx}{b^2}\\
&\quad+\frac{\int x\,dx}{b}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}\\
&\quad+\frac{\log(\sinh(a+bx))}{b^3}-\frac{2\int x\log\left(1-e^{2(a+bx)}\right)\,dx}{b}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}\\
&\quad+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\int\text{PolyLog}\left(2,e^{2(a+bx)}\right)\,dx}{b^2}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}\\
&\quad+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^3}\\
&= \frac{x^2}{2b}-\frac{x^3}{3}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}\\
&\quad+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(114) = 228.

Time = 2.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.75

$$\int x^2 \coth^3(a+bx)\,dx = \frac{1}{3}x^3\coth(a) - \frac{x^2\mathrm{csch}^2(a+bx)}{2b} - \frac{e^{2a}\left(6be^{-2a}x + 6b(1-e^{-2a})\,x + 2b^3e^{-2a}x^3 - 3b^2e^{-2a}(-1+e^{2a})\,x^2\log\left(1-e^{-a-bx}\right) - 3b^2e^{-2a}(-1+e^{2a})\,x\right.}{} + \frac{x\mathrm{csch}(a)\mathrm{csch}(a+bx)\sinh(bx)}{b^2}$$

```
[In] Integrate[x^2*Coth[a + b*x]^3,x]
```

```
[Out] (x^3*Coth[a])/3 - (x^2*Csch[a + b*x]^2)/(2*b) - (E^(2*a)*((6*b*x)/E^(2*a) +
 6*b*(1 - E^(-2*a))*x + (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log
[1 - E^(-a - b*x)])/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x
)])/E^(2*a) - 3*(1 - E^(-2*a))*Log[1 - E^(a + b*x)] - 3*(1 - E^(-2*a))*Log[
1 + E^(a + b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1
- E^(-2*a))*x*PolyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a
 - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, E^(-a - b*x)]))/(3*b^3*(-1 + E^(2*a)
)) + (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(106) = 212.

Time = 0.52 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{x^3}{3} - \frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2} + \frac{\ln\left(e^{bx+a}+1\right)x^2}{b} + \frac{2x\,\mathrm{polylog}\left(2,-e^{bx+a}\right)}{b^2} + \frac{\ln\left(1-e^{bx+a}\right)x^2}{b} + \frac{2x\,\mathrm{polylog}\left(2,e^{bx+a}\right)}{b^2} +$

```
[In] int(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3-2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2
+1/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a
))*x^2+2*x*polylog(2,exp(b*x+a))/b^2+4/3/b^3*a^3+2/b^2*a^2*x+1/b^3*ln(exp(b
*x+a)-1)+1/b^3*ln(exp(b*x+a)+1)-2/b^3*ln(exp(b*x+a))-2*polylog(3,-exp(b*x+a
))/b^3-2*polylog(3,exp(b*x+a))/b^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(
exp(b*x+a))-1/b^3*ln(1-exp(b*x+a))*a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. 2(105) = 210.

Time = 0.28 (sec) , antiderivative size = 1467, normalized size of antiderivative = 12.87

$$\int x^2 \coth^3(a + bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 +
 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*
b*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 - 3*(b^3*x^3 +
2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 - 6*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(
(b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4
*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a
) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*((a^2 + 1)*cosh(b*x + a)^
4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 -
 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 - a^2 - 1)*si
nh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 - (a^2 + 1)*cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2
- a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)
^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
- a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))
*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^
4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 - 3*b^2
*x^2 + 2*a^3 + 3*b*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b
*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b
```

```
^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2
+ 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \coth^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x**2*cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(105) = 210$.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\begin{aligned}\int x^2 \coth^3(a+bx)\,dx &= -\frac{2}{3}\,x^3 + \frac{b^2x^3e^{(4\,bx+4\,a)} + b^2x^3 - 2\left(b^2x^3e^{(2\,a)} + 3\,bx^2e^{(2\,a)} + 3\,xe^{(2\,a)}\right)e^{(2\,bx)} + 6\,x}{3\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} \\ &\quad - \frac{2\,x}{b^2} + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3} \\ &\quad + \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3} \\ &\quad + \frac{\log\left(e^{(bx+a)}+1\right)}{b^3} + \frac{\log\left(e^{(bx+a)}-1\right)}{b^3}\end{aligned}$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 - 2*(b^2*x^3*e^(2*a) + 3*
b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) - 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*di
log(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x
+ a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 + log
(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Giac [F]

$$\int x^2 \coth^3(a+bx)\,dx = \int x^2 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3\,dx$$

```
[In] integrate(x^2*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^3(a+bx)\,dx = \int \frac{x^2 \cosh(a+b\,x)^3}{\sinh(a+b\,x)^3}\,dx$$

```
[In] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
[Out] int((x^2*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.462 $\int x\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x\coth^3(a+bx)\,dx=\frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

```
[Out] 1/2*x/b-1/2*x^2-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a)^2/b+x*ln(1-exp(2*b*x+
2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3801, 3554, 8, 3797, 2221, 2317, 2438}

$$\int x\coth^3(a+bx)\,dx=\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}-\frac{\coth(a+bx)}{2b^2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}-\frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}-\frac{x^2}{2}$$

```
[In] Int[x*Coth[a + b*x]^3,x]
```

```
[Out] x/(2*b) - x^2/2 - Coth[a + b*x]/(2*b^2) - (x*Coth[a + b*x]^2)/(2*b) + (x*Lo
g[1 - E^(2*(a + b*x))])/b + PolyLog[2, E^(2*(a + b*x))]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x\coth^2(a+bx)}{2b} + \frac{\int \coth^2(a+bx)\,dx}{2b} + \int x\coth(a+bx)\,dx$$

$$= -\frac{x^2}{2} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} - 2\int \frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx + \frac{\int 1\,dx}{2b}$$

$$= \frac{x}{2b} - \frac{x^2}{2} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\int \log\left(1-e^{2(a+bx)}\right)\,dx}{b}$$

$$= \frac{x}{2b} - \frac{x^2}{2} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^2}$$

$$= \frac{x}{2b} - \frac{x^2}{2} - \frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b} + \frac{x\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.60

$$\int x\coth^3(a+bx)\,dx = \frac{1}{2}\left(-\frac{2x^2}{-1+e^{2a}} + x^2\coth(a) - \frac{x\text{csch}^2(a+bx)}{b} + \frac{2x\log\left(1-e^{-a-bx}\right)}{b} + \frac{2x\log\left(1+e^{-a-bx}\right)}{b} - \frac{2\,\text{PolyLog}\left(2,-e^{-a-bx}\right)}{b^2} - \frac{2\,\text{PolyLog}\left(2,e^{-a-bx}\right)}{b^2} + \frac{\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

```
[In] Integrate[x*Coth[a + b*x]^3,x]
```

```
[Out] ((-2*x^2)/(-1 + E^(2*a)) + x^2*Coth[a] - (x*Csch[a + b*x]^2)/b + (2*x*Log[1
 - E^(-a - b*x)])/b + (2*x*Log[1 + E^(-a - b*x)])/b - (2*PolyLog[2, -E^(-a
- b*x)])/b^2 - (2*PolyLog[2, E^(-a - b*x)])/b^2 + (Csch[a]*Csch[a + b*x]*Si
nh[b*x])/b^2)/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(72) = 144.

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

method	result
risch	$-\frac{x^2}{2} - \frac{2\,e^{2bx+2a}bx+e^{2bx+2a}-1}{b^2\left(e^{2bx+2a}-1\right)^2} - \frac{2ax}{b} - \frac{a^2}{b^2} + \frac{\ln(e^{bx+a}+1)x}{b} + \frac{\text{polylog}(2,-e^{bx+a})}{b^2} + \frac{\ln(1-e^{bx+a})x}{b} + \frac{\ln(1-e^{bx+a})a}{b^2} +$

```
[In] int(x*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2-(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-2
/b*a*x-a^2/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2+1/b*ln(1-e
xp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2-1/b^2*a*ln(
exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 975, normalized size of antiderivative = 11.89

$$\int x \coth^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2
 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 2
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
 a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x
+ a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cos
h(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))
*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b*x + a)*c
osh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh
(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2
- b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)
*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 - (b^2*x^2 - 2*a^2 - 2*b*x - 1)*cosh
(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x + a)*si
nh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2*cosh
(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^2*cos
h(b*x + a))*sinh(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int x \coth^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.82

$$\int x \coth^3(a+bx)\,dx = -x^2 + \frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 - 2\left(b^2x^2e^{(2\,a)} + 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{2\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} + \frac{bx\log\left(e^{(bx+a)} + 1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)} + 1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2
*a) + b^2) + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*lo
g(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x \coth^3(a+bx)\,dx = \int x\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)^3*csch(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^3(a+bx)\,dx=\int \frac{x\cosh(a+b\,x)^3}{\sinh{(a+b\,x)}^3}\,dx$$

```
[In] int((x*cosh(a + b*x)^3)/sinh(a + b*x)^3,x)
[Out] int((x*cosh(a + b*x)^3)/sinh(a + b*x)^3, x)
```

3.463 $\int \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \coth^3(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \coth^3(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}-\frac{\coth^2(a+bx)}{2b}$$

```
[In] Int[Coth[a + b*x]^3,x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b + Log[Sinh[a + b*x]]/b
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth^2(a+bx)}{2b} + \int \coth(a+bx)\,dx \\ &= -\frac{\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \coth^3(a+bx)\,dx = -\frac{\coth^2(a+bx) - 2\log(\cosh(a+bx)) - 2\log(\tanh(a+bx))}{2b}$$

```
[In] Integrate[Coth[a + b*x]^3,x]
[Out] -1/2*(Coth[a + b*x]^2 - 2*Log[Cosh[a + b*x]] - 2*Log[Tanh[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	23
default	$\frac{\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	23
parallelrisch	$\frac{-2bx+2\ln(\tanh(bx+a))-2\ln(1-\tanh(bx+a))-\coth(bx+a)^2}{2b}$	43
risch	$-x-\frac{2a}{b}-\frac{2\,\mathrm{e}^{2bx+2a}}{b(\mathrm{e}^{2bx+2a}-1)^2}+\frac{\ln(\mathrm{e}^{2bx+2a}-1)}{b}$	54

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(sinh(b*x+a))-1/2*coth(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 12.81

$$\int \coth^3(a+bx)\,dx = -\frac{bx\cosh(bx+a)^4 + 4\,bx\cosh(bx+a)\sinh(bx+a)^3 + bx\sinh(bx+a)^4 - 2\,(bx-1)\cosh(bx+a)^2 + 2\,(\cdots}{\cdots}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 - (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [F]

$$\int \coth^3(a+bx)\,dx = \int \cosh^3(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.93

$$\int \coth^3(a+bx)\,dx = x + \frac{a}{b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-2*b*x -
 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \coth^3(a+bx)\,dx = -\frac{2\,bx + 2\,a + \frac{3\,e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 3}{\left(e^{(2\,bx+2\,a)} - 1\right)^2} - 2\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{2\,b}$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*b*x + 2*a + (3*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 3)/(e^(2*b*x +
 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \coth^3(a+bx)\,dx = \frac{\ln(\sinh(a+b\,x))}{b} - \frac{1}{2\,b\sinh{(a+b\,x)}^2}$$

```
[In] int(cosh(a + b*x)^3/sinh(a + b*x)^3,x)
[Out] log(sinh(a + b*x))/b - 1/(2*b*sinh(a + b*x)^2)
```

3.464 $\int \frac{\coth^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \text{Int}\left(\frac{\coth^3(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\coth^3(a+bx)}{x}\,dx$$

```
[In] Int[Coth[a + b*x]^3/x,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^3/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth^3(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\coth^3(a+bx)}{x}\,dx$$

```
[In] Integrate[Coth[a + b*x]^3/x,x]
[Out] Integrate[Coth[a + b*x]^3/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{3}}{x}dx$$

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{3}}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 118.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh^{3}\left(a+bx\right)\operatorname{csch}^{3}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x +
 a) + b^2*x^3), x) + integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3
), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh(bx+a)^3\operatorname{csch}(bx+a)^3}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\cosh(a+b\,x)^3}{x\sinh(a+b\,x)^3}\,dx$$

```
[In] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^3/(x*sinh(a + b*x)^3), x)
```

3.465 $\int \frac{\coth^3(a+bx)}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^3(a+bx)}{x^2} \, dx = \text{Int}\left(\frac{\coth^3(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(coth(b*x+a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x^2} \, dx = \int \frac{\coth^3(a+bx)}{x^2} \, dx$$

```
[In] Int[Coth[a + b*x]^3/x^2,x]
```

```
[Out] Defer[Int][Coth[a + b*x]^3/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\coth^3(a+bx)}{x^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth^3(a+bx)}{x^2}\,dx$$

```
[In] Integrate[Coth[a + b*x]^3/x^2,x]
[Out] Integrate[Coth[a + b*x]^3/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}dx$$

```
[In] int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
[Out] int(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh\left(bx+a\right)^{3}\operatorname{csch}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 159.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh^{3}\left(a+bx\right)\operatorname{csch}^{3}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(cosh(b*x+a)**3*csch(b*x+a)**3/x**2,x)
[Out] Integral(cosh(a + b*x)**3*csch(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 14.58

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e^
(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a)
+ b^2*x^3) - integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) +
integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\cosh{(bx+a)}^3 \operatorname{csch}{(bx+a)}^3}{x^2}\,dx$$

```
[In] integrate(cosh(b*x+a)^3*csch(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{{\cosh(a+b\,x)}^3}{x^2\,{\sinh\left(a+b\,x\right)}^3}\,dx$$

```
[In] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^3),x)
```

```
[Out] int(cosh(a + b*x)^3/(x^2*sinh(a + b*x)^3), x)
```

3.466 $\int x^m \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^m \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = \mathrm{Int}(x^m \mathrm{csch}(a+bx)\mathrm{sech}(a+bx),x)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = \int x^m \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x],x]
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^m \,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)\,dx$$

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a),x)
[Out] int(x^m*csch(b*x+a)*sech(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)\,\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^m \text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^m \,\text{csch}\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a),x)
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a + bx)\text{sech}(a + bx)\, dx = \int x^m \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)\, dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^m \text{csch}(a + bx)\text{sech}(a + bx)\, dx = \int x^m \operatorname{csch}(bx + a) \operatorname{sech}(bx + a)\, dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int x^m \text{csch}(a + bx)\text{sech}(a + bx)\, dx = \int \frac{x^m}{\cosh(a + b\,x)\,\sinh(a + b\,x)}\, dx$$

```
[In] int(x^m/(cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(x^m/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.467 $\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}$$

```
[Out] -2*x^3*arctanh(exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2
*x^2*polylog(2,exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3-3/2
*x*polylog(3,exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/b^4+3/4*pol
ylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5569, 4267, 2611, 6744, 2320, 6724}

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}$$

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/
(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2) + (3*x*PolyLog[3, -E^
(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) - (3*Po
lyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^
4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int x^3 \operatorname{csch}(2a+2bx)\,dx \\
&= -\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3\int x^2\log\left(1-e^{2a+2bx}\right)\,dx}{b}+\frac{3\int x^2\log\left(1+e^{2a+2bx}\right)\,dx}{b} \\
&= -\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} \\
&\quad+\frac{3\int x\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)\,dx}{b^2}-\frac{3\int x\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)\,dx}{b^2} \\
&= -\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} \\
&\quad+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} \\
&\quad-\frac{3\int\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)\,dx}{2b^3}+\frac{3\int\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)\,dx}{2b^3} \\
&= -\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} \\
&\quad+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} \\
&\quad-\frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-x)}{x}\,dx,x,e^{2a+2bx}\right)}{4b^4}+\frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{2a+2bx}\right)}{4b^4} \\
&= -\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} \\
&\quad+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} \\
&\quad-\frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int x^3 \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{4b^3x^3\log\left(1-e^{2(a+bx)}\right)-4b^3x^3\log\left(1+e^{2(a+bx)}\right)-6b^2x^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)+6b^2x^2\,\text{PolyLog}\left(2,e^{2(a+b}\right.}{}$$

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (4*b^3*x^3*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Log[1 + E^(2*(a + b*x))] -
6*b^2*x^2*PolyLog[2, -E^(2*(a + b*x))] + 6*b^2*x^2*PolyLog[2, E^(2*(a + b*x
))] + 6*b*x*PolyLog[3, -E^(2*(a + b*x))] - 6*b*x*PolyLog[3, E^(2*(a + b*x))
] - 3*PolyLog[4, -E^(2*(a + b*x))] + 3*PolyLog[4, E^(2*(a + b*x))])/(4*b^4)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.63

method	result
risch	$\frac{\ln(1-e^{bx+a})a^3}{b^4}+\frac{3x^2\,\text{polylog}(2,-e^{bx+a})}{b^2}-\frac{6x\,\text{polylog}(3,-e^{bx+a})}{b^3}-\frac{x^3\ln(1+e^{2bx+2a})}{b}-\frac{3x^2\,\text{polylog}(2,-e^{2bx+2a})}{2b^2}+\frac{3x\,\text{pol}}{}$

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*ln(1-exp(b*x+a))*a^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylog(3,-
exp(b*x+a))/b^3-x^3*ln(1+exp(2*b*x+2*a))/b-3/2*x^2*polylog(2,-exp(2*b*x+2*a
))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+1/b*ln(1-exp(b*x+a))*x^3+3*x^2*
polylog(2,exp(b*x+a))/b^2-6*x*polylog(3,exp(b*x+a))/b^3+1/b*ln(exp(b*x+a)+1
)*x^3-1/b^4*a^3*ln(exp(b*x+a)-1)-3/4*polylog(4,-exp(2*b*x+2*a))/b^4+6*polyl
og(4,exp(b*x+a))/b^4+6*polylog(4,-exp(b*x+a))/b^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.03

$$\int x^3 \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+3\,b^2x^2\text{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-3\,b^2x^2\text{Li}_2(i\,\cosh\left(\right.}{}$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^2*dilog(cosh(b*x
+ a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
- 3*b^2*x^2*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*b^2*x^2*dilog(-co
sh(b*x + a) - sinh(b*x + a)) + a^3*log(cosh(b*x + a) + sinh(b*x + a) + I) +
 a^3*log(cosh(b*x + a) + sinh(b*x + a) - I) - a^3*log(cosh(b*x + a) + sinh(
b*x + a) - 1) - 6*b*x*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*pol
ylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*b*x*polylog(3, -I*cosh(b*x +
 a) - I*sinh(b*x + a)) - 6*b*x*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) -
 (b^3*x^3 + a^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3*x^3 + a^
3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3)*log(-cosh(
b*x + a) - sinh(b*x + a) + 1) + 6*polylog(4, cosh(b*x + a) + sinh(b*x + a))
 - 6*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*polylog(4, -I*cosh(b
*x + a) - I*sinh(b*x + a)) + 6*polylog(4, -cosh(b*x + a) - sinh(b*x + a)))/
b^4
```

Sympy [F]

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx=\int x^3\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a),x)
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int x^3\text{csch}(a+bx)\text{sech}(a+bx)\,dx = -\frac{4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\text{Li}_3(-e^{(2\,bx+2\,a)})+3\,\text{Li}_4(-e^{(2\,bx+2\,a)})}{3\,b^4} + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\text{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\text{Li}_3(-e^{(bx+a)})+6\,\text{Li}_4(-e^{(bx+a)})}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\text{Li}_2\left(e^{(bx+a)}\right)-6\,bx\text{Li}_3(e^{(bx+a)})+6\,\text{Li}_4(e^{(bx+a)})}{b^4}$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] -1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)\,dx$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*csch(b*x + a)*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^3}{\cosh(a+b\,x)\,\sinh(a+b\,x)}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(x^3/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.468 $\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx=-\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}$$

```
[Out] -2*x^2*arctanh(exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+x*polylog
(2,exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(2*b*x+2*a))/b^3-1/2*polylog(3,exp
(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5569, 4267, 2611, 2320, 6724}

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx=-\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}+\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}$$

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2
+ (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 + PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3
) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int x^2\operatorname{csch}(2a+2bx)\,dx\\ &= -\frac{2x^2\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{2\int x\log\left(1-e^{2a+2bx}\right)\,dx}{b}+\frac{2\int x\log\left(1+e^{2a+2bx}\right)\,dx}{b}\end{aligned}$$

$$= -\frac{2x^2\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{x\,\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{x\,\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} + \frac{\int \mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)\,dx}{b^2} - \frac{\int \mathrm{PolyLog}\left(2,e^{2a+2bx}\right)\,dx}{b^2}$$

$$= -\frac{2x^2\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{x\,\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{x\,\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} + \frac{\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,-x)}{x}\,dx, x, e^{2a+2bx}\right)}{2b^3} - \frac{\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(2,x)}{x}\,dx, x, e^{2a+2bx}\right)}{2b^3}$$

$$= -\frac{2x^2\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{x\,\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{x\,\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} + \frac{\mathrm{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} - \frac{\mathrm{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int x^2\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2b^2x^2\log\left(1-e^{2(a+bx)}\right) - 2b^2x^2\log\left(1+e^{2(a+bx)}\right) - 2bx\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right) + 2bx\,\mathrm{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^3}$$

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x],x]
[Out] (2*b^2*x^2*Log[1 - E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + E^(2*(a + b*x))] -
2*b*x*PolyLog[2, -E^(2*(a + b*x))] + 2*b*x*PolyLog[2, E^(2*(a + b*x))] + Po
lyLog[3, -E^(2*(a + b*x))] - PolyLog[3, E^(2*(a + b*x))])/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(88) = 176.

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.92

method	result
risch	$\frac{\ln(e^{bx+a}+1)x^2}{b} + \frac{2x\,\mathrm{polylog}(2,-e^{bx+a})}{b^2} - \frac{x^2\ln(1+e^{2bx+2a})}{b} - \frac{x\,\mathrm{polylog}(2,-e^{2bx+2a})}{b^2} + \frac{\ln(1-e^{bx+a})x^2}{b} + \frac{2x\,\mathrm{polylog}(2,e}{b^2}$

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*ln(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2-x^2*ln(1+exp(2*b*x+
2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*ln(1-exp(b*x+a))*x^2+2*x*polyl
og(2,exp(b*x+a))/b^2+1/b^3*a^2*ln(exp(b*x+a)-1)-1/b^3*ln(1-exp(b*x+a))*a^2+
1/2*polylog(3,-exp(2*b*x+2*a))/b^3-2*polylog(3,exp(b*x+a))/b^3-2*polylog(3,
-exp(b*x+a))/b^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.62

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\text{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-2\,bx\text{Li}_2(i\,\cosh\left(bx+\right.}{}$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
[Out] (b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*dilog(cosh(b*x + a)
 + sinh(b*x + a)) - 2*b*x*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*b*x*
dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*b*x*dilog(-cosh(b*x + a) - si
nh(b*x + a)) - a^2*log(cosh(b*x + a) + sinh(b*x + a) + I) - a^2*log(cosh(b*
x + a) + sinh(b*x + a) - I) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) -
(b^2*x^2 - a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^2*x^2 - a^2
)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^2*x^2 - a^2)*log(-cosh(b
*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x + a))
+ 2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*polylog(3, -I*cosh(b*
x + a) - I*sinh(b*x + a)) - 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b
^3
```

Sympy [F]

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^2\,\text{csch}\left(a+bx\right)\text{sech}\left(a+bx\right)dx$$

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a),x)
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\begin{aligned}\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx\\ &= -\frac{2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\text{Li}_3\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^3}\\ &\quad+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\,\text{Li}_3\left(-e^{(bx+a)}\right)}{b^3}\\ &\quad+\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(e^{(bx+a)}\right)-2\,\text{Li}_3\left(e^{(bx+a)}\right)}{b^3}\end{aligned}$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) -
polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*d
ilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(b*x
 + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int x^2\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)\,\,dx$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)*sinh(a + b*x)),x)
```

```
[Out] int(x^2/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.469 $\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 58

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx=-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}$$

```
[Out] -2*x*arctanh(exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/2*polyl
og(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5569, 4267, 2317, 2438}

$$\int x\text{csch}(a+bx)\text{sech}(a+bx)\,dx=-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}$$

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] (-2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) +
PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int x\,\text{csch}(2a+2bx)\,dx\\
&= -\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\int \log\left(1-e^{2a+2bx}\right)\,dx}{b}+\frac{\int \log\left(1+e^{2a+2bx}\right)\,dx}{b}\\
&= -\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^2}+\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^2}\\
&= -\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int x\,\text{csch}(a+bx)\text{sech}(a+bx)\,dx = \frac{2bx\left(\log\left(1-e^{2(a+bx)}\right)-\log\left(1+e^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-e^{2(a+bx)}\right)+\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x],x]
[Out] (2*b*x*(Log[1 - E^(2*(a + b*x))] - Log[1 + E^(2*(a + b*x))]) - PolyLog[2, -
E^(2*(a + b*x))] + PolyLog[2, E^(2*(a + b*x))])/(2*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(51) = 102.

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.16

method	result
risch	$\frac{\ln(1-e^{bx+a})x}{b}+\frac{\ln(1-e^{bx+a})a}{b^2}+\frac{\mathrm{polylog}(2,e^{bx+a})}{b^2}+\frac{\ln(e^{bx+a}+1)x}{b}+\frac{\mathrm{polylog}(2,-e^{bx+a})}{b^2}-\frac{x\ln(1+e^{2bx+2a})}{b}-\frac{\mathrm{polylog}(2}{}$

[In] int(x*csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)

[Out] 1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+1/b*ln(exp(b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.86

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = \frac{bx\log(\cosh(bx+a)+\sinh(bx+a)+1)+a\log(\cosh(bx+a)+\sinh(bx+a)+i)+a\log(\cosh(bx+a)}{}$$

[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")

[Out] (b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + a*log(cosh(b*x + a) + sinh(b*x + a) + I) + a*log(cosh(b*x + a) + sinh(b*x + a) - I) - a*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (b*x + a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b*x + a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + dilog(-cosh(b*x + a) - sinh(b*x + a)))/b^2

Sympy [F]

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = \int x\,\mathrm{csch}\,(a+bx)\,\mathrm{sech}\,(a+bx)\,dx$$

[In] integrate(x*csch(b*x+a)*sech(b*x+a),x)

[Out] Integral(x*csch(a + b*x)*sech(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = -\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2} + \frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] -1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 + (b*x*
log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1
) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = \int x\,\mathrm{csch}\,(bx+a)\,\mathrm{sech}\,(bx+a)\,\,dx$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x*csch(b*x + a)*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}(a+bx)\,dx = \int \frac{x}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x/(cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(x/(cosh(a + b*x)*sinh(a + b*x)), x)
```

3.470 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}$$

```
[Out] ln(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2700, 29}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x],x]
```

```
[Out] Log[Tanh[a + b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = 2\left(-\frac{\log(\cosh(a+bx))}{2b} + \frac{\log(\sinh(a+bx))}{2b}\right)$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x],x]
[Out] 2*(-1/2*Log[Cosh[a + b*x]]/b + Log[Sinh[a + b*x]]/(2*b))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tanh(bx+a))}{b}$	12
default	$\frac{\ln(\tanh(bx+a))}{b}$	12
risch	$\frac{\ln(e^{2bx+2a}-1)}{b} - \frac{\ln(1+e^{2bx+2a})}{b}$	35

```
[In] int(csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right) - \log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
[Out] -(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x + a
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x)
[Out] Integral(csch(a + b*x)*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(2\,bx+2\,a)}+1\right) - \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] -(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1
)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
[Out] -(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.471 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = 2\text{Int}\left(\frac{\operatorname{csch}(2a+2bx)}{x},x\right)$$

```
[Out] 2*Unintegrable(csch(2*b*x+2*a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x])/x,x]
[Out] 2*Defer[Int][Csch[2*a + 2*b*x]/x, x]
```

Rubi steps

$$\text{integral} = 2\int \frac{\operatorname{csch}(2a+2bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}dx$$

```
[In] int(csch(b*x+a)*sech(b*x+a)/x,x)
[Out] int(csch(b*x+a)*sech(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x,x, algorithm="giac")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(1/(x*cosh(a + b*x)*sinh(a + b*x)), x)
```

3.472 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = 2\mathrm{Int}\left(\frac{\operatorname{csch}(2a+2bx)}{x^2},x\right)$$

```
[Out] 2*Unintegrable(csch(2*b*x+2*a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
[Out] 2*Defer[Int][Csch[2*a + 2*b*x]/x^2, x]
```

Rubi steps

$$\text{integral} = 2\int \frac{\operatorname{csch}(2a+2bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}dx$$

```
[In] int(csch(b*x+a)*sech(b*x+a)/x^2,x)
[Out] int(csch(b*x+a)*sech(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x**2,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(csch(b*x + a)*sech(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)),x)
[Out] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)), x)
```

3.473 $\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 102.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2,x]
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
[Out] int(x^m*csch(b*x+a)*sech(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2\,\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2\,\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)),x)
[Out] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.474 $\int x^3 \mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 226

$$\begin{aligned}\int x^3\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx =& -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\mathrm{arctanh}\left(e^{a+bx}\right)}{b}\\&-\frac{3x^2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\&+\frac{6ix\,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}-\frac{6ix\,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}\\&+\frac{3x^2\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6x\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}\\&-\frac{6i\,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{6i\,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}\\&-\frac{6x\,\mathrm{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{6\,\mathrm{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}\\&+\frac{6\,\mathrm{PolyLog}\left(4,e^{a+bx}\right)}{b^4}+\frac{x^3\mathrm{sech}(a+bx)}{b}\end{aligned}$$

```
[Out] -6*x^2*arctan(exp(b*x+a))/b^2-2*x^3*arctanh(exp(b*x+a))/b-3*x^2*polylog(2,-
exp(b*x+a))/b^2+6*I*x*polylog(2,-I*exp(b*x+a))/b^3-6*I*x*polylog(2,I*exp(b*
x+a))/b^3+3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp(b*x+a))/b^3-6*
I*polylog(3,-I*exp(b*x+a))/b^4+6*I*polylog(3,I*exp(b*x+a))/b^4-6*x*polylog(
3,exp(b*x+a))/b^3-6*polylog(4,-exp(b*x+a))/b^4+6*polylog(4,exp(b*x+a))/b^4+
x^3*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2702, 327, 213, 5570, 14, 6408, 12, 4267, 2611, 6744, 2320, 6724, 4265}

$$\begin{aligned}\int x^3\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=&-\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b}\\&-\frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}+\frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}\\&-\frac{6\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}+\frac{6\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}\\&+\frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}\\&-\frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}+\frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}\\&-\frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}\\&+\frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{x^3\text{sech}(a+bx)}{b}\end{aligned}$$

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-6*x^2*ArcTan[E^(a + b*x)])/b^2 - (2*x^3*ArcTanh[E^(a + b*x)])/b - (3*x^2*
PolyLog[2, -E^(a + b*x)])/b^2 + ((6*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^3
- ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*x^2*PolyLog[2, E^(a + b*x)])
/b^2 + (6*x*PolyLog[3, -E^(a + b*x)])/b^3 - ((6*I)*PolyLog[3, (-I)*E^(a + b
*x)])/b^4 + ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 - (6*x*PolyLog[3, E^(a +
b*x)])/b^3 - (6*PolyLog[4, -E^(a + b*x)])/b^4 + (6*PolyLog[4, E^(a + b*x)])
/b^4 + (x^3*Sech[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\text{arctanh}(\cosh(a+bx))}{b} + \frac{x^3\text{sech}(a+bx)}{b} \\
&\quad - 3\int x^2\left(-\frac{\text{arctanh}(\cosh(a+bx))}{b} + \frac{\text{sech}(a+bx)}{b}\right)dx \\
&= -\frac{x^3\text{arctanh}(\cosh(a+bx))}{b} + \frac{x^3\text{sech}(a+bx)}{b} \\
&\quad - 3\int\left(-\frac{x^2\text{arctanh}(\cosh(a+bx))}{b} + \frac{x^2\text{sech}(a+bx)}{b}\right)dx
\end{aligned}$$

$$= -\frac{x^3\text{arctanh}(\cosh(a+bx))}{b} + \frac{x^3\text{sech}(a+bx)}{b} + \frac{3\int x^2\text{arctanh}(\cosh(a+bx))\,dx}{b} - \frac{3\int x^2\text{sech}(a+bx)\,dx}{b}$$

$$= -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2} + \frac{x^3\text{sech}(a+bx)}{b} + \frac{(6i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b^2} - \frac{(6i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b^2} + \frac{\int bx^3\text{csch}(a+bx)\,dx}{b}$$

$$= -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2} + \frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{x^3\text{sech}(a+bx)}{b} - \frac{(6i)\int\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^3} + \frac{(6i)\int\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^3} + \int x^3\text{csch}(a+bx)\,dx$$

$$= -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{x^3\text{sech}(a+bx)}{b} - \frac{(6i)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{(6i)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{3\int x^2\log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{3\int x^2\log\left(1+e^{a+bx}\right)\,dx}{b}$$

$$= -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} + \frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} + \frac{x^3\text{sech}(a+bx)}{b} + \frac{6\int x\,\text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} - \frac{6\int x\,\text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2}$$

$$= -\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{3x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} + \frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} - \frac{6x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} + \frac{x^3\text{sech}(a+bx)}{b} - \frac{6\int\text{PolyLog}\left(3,-e^{a+bx}\right)\,dx}{b^3} + \frac{6\int\text{PolyLog}\left(3,e^{a+bx}\right)\,dx}{b^3}$$

$$= -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^3 \text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2 \text{PolyLog}\left(2, -e^{a+bx}\right)}{b^2} + \frac{6ix \text{PolyLog}\left(2, -ie^{a+bx}\right)}{b^3} - \frac{6ix \text{PolyLog}\left(2, ie^{a+bx}\right)}{b^3} + \frac{3x^2 \text{PolyLog}\left(2, e^{a+bx}\right)}{b^2} + \frac{6x \text{PolyLog}\left(3, -e^{a+bx}\right)}{b^3} - \frac{6i \text{PolyLog}\left(3, -ie^{a+bx}\right)}{b^4} + \frac{6i \text{PolyLog}\left(3, ie^{a+bx}\right)}{b^4} - \frac{6x \text{PolyLog}\left(3, e^{a+bx}\right)}{b^3} + \frac{x^3 \text{sech}(a+bx)}{b} - \frac{6\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x} dx, x, e^{a+bx}\right)}{b^4} + \frac{6\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{a+bx}\right)}{b^4}$$

$$= -\frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^3 \text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2 \text{PolyLog}\left(2, -e^{a+bx}\right)}{b^2} + \frac{6ix \text{PolyLog}\left(2, -ie^{a+bx}\right)}{b^3} - \frac{6ix \text{PolyLog}\left(2, ie^{a+bx}\right)}{b^3} + \frac{3x^2 \text{PolyLog}\left(2, e^{a+bx}\right)}{b^2} + \frac{6x \text{PolyLog}\left(3, -e^{a+bx}\right)}{b^3} - \frac{6i \text{PolyLog}\left(3, -ie^{a+bx}\right)}{b^4} + \frac{6i \text{PolyLog}\left(3, ie^{a+bx}\right)}{b^4} - \frac{6x \text{PolyLog}\left(3, e^{a+bx}\right)}{b^3} - \frac{6 \text{PolyLog}\left(4, -e^{a+bx}\right)}{b^4} + \frac{6 \text{PolyLog}\left(4, e^{a+bx}\right)}{b^4} + \frac{x^3 \text{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.16

$$\int x^3 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \frac{b^3x^3 \log\left(1-e^{a+bx}\right) - 3ib^2x^2 \log\left(1-ie^{a+bx}\right) + 3ib^2x^2 \log\left(1+ie^{a+bx}\right) - b^3x^3 \log\left(1+e^{a+bx}\right) - 3b^2x^2 \text{Pol}}{}$$

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (b^3*x^3*Log[1 - E^(a + b*x)] - (3*I)*b^2*x^2*Log[1 - I*E^(a + b*x)] + (3*I
)*b^2*x^2*Log[1 + I*E^(a + b*x)] - b^3*x^3*Log[1 + E^(a + b*x)] - 3*b^2*x^2
*PolyLog[2, -E^(a + b*x)] + (6*I)*b*x*PolyLog[2, (-I)*E^(a + b*x)] - (6*I)*
b*x*PolyLog[2, I*E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, E^(a + b*x)] + 6*b*x*P
olyLog[3, -E^(a + b*x)] - (6*I)*PolyLog[3, (-I)*E^(a + b*x)] + (6*I)*PolyLo
g[3, I*E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a + b*x)] - 6*PolyLog[4, -E^(a +
b*x)] + 6*PolyLog[4, E^(a + b*x)] + b^3*x^3*Sech[a + b*x])/b^4
```

Maple [F]

$$\int x^3 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^2 \, dx$$

[In] int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)

[Out] int(x^3*csch(b*x+a)*sech(b*x+a)^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(194) = 388.

Time = 0.30 (sec) , antiderivative size = 1280, normalized size of antiderivative = 5.66

$$\int x^3 \text{csch}(a+bx)\text{sech}^2(a+bx) \, dx = \text{Too large to display}$$

[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")

[Out] (2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(I*b*x*cosh(b*x + a)^2 + 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) + I*b*x*sinh(b*x + a)^2 + I*b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(-I*b*x*cosh(b*x + a)^2 - 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) - I*b*x*sinh(b*x + a)^2 - I*b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*x^3*cosh(b*x + a)^2 + 2*b^3*x^3*cosh(b*x + a)*sinh(b*x + a) + b^3*x^3*sinh(b*x + a)^2 + b^3*x^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*(I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*sinh(b*x + a) + I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) - 3*(-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(b*x + a)*sinh(b*x + a) - I*a^2*sinh(b*x + a)^2 - I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a^3*cosh(b*x + a)^2 + 2*a^3*cosh(b*x + a)*sinh(b*x + a) + a^3*sinh(b*x + a)^2 + a^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(-I*b^2*x^2 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + 2*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*(I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 + 2*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b^2*x^2 - I*a^2)*sinh(b*x + a)^2 - I*a^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + a^3 + (b^3*x^3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a) + (b^3*x^3 + a^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*polylog(4, -cosh(b*x + a

```
) - sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x
+ a) + b*x*sinh(b*x + a)^2 + b*x)*polylog(3, cosh(b*x + a) + sinh(b*x + a))
 - 6*(-I*cosh(b*x + a)^2 - 2*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a
)^2 - I)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(I*cosh(b*x + a)
^2 + 2*I*cosh(b*x + a)*sinh(b*x + a) + I*sinh(b*x + a)^2 + I)*polylog(3, -I
*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x
 + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*polylog(3, -cosh(b*x + a)
- sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a)
+ b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [F]

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^3\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^3\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2\,dx$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] 2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^3*x^3*log(e^(b*x + a) + 1) +
 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog
(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e
^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^
4 - 24*integrate(1/4*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Giac [F(-1)]

Timed out.

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \frac{x^3}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)),x)
[Out] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.475 $\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 146

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} + \frac{x^2\text{sech}(a+bx)}{b}$$

```
[Out] -4*x*arctan(exp(b*x+a))/b^2-2*x^2*arctanh(exp(b*x+a))/b-2*x*polylog(2,-exp(
b*x+a))/b^2+2*I*polylog(2,-I*exp(b*x+a))/b^3-2*I*polylog(2,I*exp(b*x+a))/b^
3+2*x*polylog(2,exp(b*x+a))/b^2+2*polylog(3,-exp(b*x+a))/b^3-2*polylog(3,ex
p(b*x+a))/b^3+x^2*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules

used = {2702, 327, 213, 5570, 14, 6408, 12, 4267, 2611, 2320, 6724, 4265, 2317, 2438}

$$\int x^2\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{x^2\text{sech}(a+bx)}{b}$$

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-4*x*ArcTan[E^(a + b*x)])/b^2 - (2*x^2*ArcTanh[E^(a + b*x)])/b - (2*x*Poly
Log[2, -E^(a + b*x)])/b^2 + ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 - ((2*
I)*PolyLog[2, I*E^(a + b*x)])/b^3 + (2*x*PolyLog[2, E^(a + b*x)])/b^2 + (2*
PolyLog[3, -E^(a + b*x)])/b^3 - (2*PolyLog[3, E^(a + b*x)])/b^3 + (x^2*Sech
[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\text{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b} \\
&\quad -2\int x\left(-\frac{\text{arctanh}(\cosh(a+bx))}{b}+\frac{\text{sech}(a+bx)}{b}\right)\,dx \\
&= -\frac{x^2\text{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b} \\
&\quad -2\int\left(-\frac{x\text{arctanh}(\cosh(a+bx))}{b}+\frac{x\text{sech}(a+bx)}{b}\right)\,dx \\
&= -\frac{x^2\text{arctanh}(\cosh(a+bx))}{b}+\frac{x^2\text{sech}(a+bx)}{b} \\
&\quad +\frac{2\int x\text{arctanh}(\cosh(a+bx))\,dx}{b}-\frac{2\int x\text{sech}(a+bx)\,dx}{b} \\
&= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{x^2\text{sech}(a+bx)}{b}+\frac{(2i)\int\log\left(1-ie^{a+bx}\right)\,dx}{b^2} \\
&\quad -\frac{(2i)\int\log\left(1+ie^{a+bx}\right)\,dx}{b^2}+\frac{\int bx^2\text{csch}(a+bx)\,dx}{b}
\end{aligned}$$

$$= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} + \frac{x^2\text{sech}(a+bx)}{b} + \frac{(2i)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{(2i)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \int x^2\text{csch}(a+bx)\,dx$$

$$= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{x^2\text{sech}(a+bx)}{b} - \frac{2\int x\log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{2\int x\log\left(1+e^{a+bx}\right)\,dx}{b}$$

$$= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{x^2\text{sech}(a+bx)}{b} + \frac{2\int \text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} - \frac{2\int \text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2}$$

$$= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{x^2\text{sech}(a+bx)}{b} + \frac{2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{a+bx}\right)}{b^3}$$

$$= -\frac{4x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{2x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{2x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} + \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} - \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{2x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} + \frac{2\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} - \frac{2\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} + \frac{x^2\text{sech}(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$
$$= \frac{b^2x^2\log\left(1-e^{a+bx}\right)-2ibx\log\left(1-ie^{a+bx}\right)+2ibx\log\left(1+ie^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\,\text{PolyLog}\left(2\right.}{}$$

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (b^2*x^2*Log[1 - E^(a + b*x)] - (2*I)*b*x*Log[1 - I*E^(a + b*x)] + (2*I)*b*
x*Log[1 + I*E^(a + b*x)] - b^2*x^2*Log[1 + E^(a + b*x)] - 2*b*x*PolyLog[2,
-E^(a + b*x)] + (2*I)*PolyLog[2, (-I)*E^(a + b*x)] - (2*I)*PolyLog[2, I*E^(
a + b*x)] + 2*b*x*PolyLog[2, E^(a + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*
PolyLog[3, E^(a + b*x)] + b^2*x^2*Sech[a + b*x])/b^3
```

Maple [F]

$$\int x^2\,\text{csch}\left(bx+a\right)\text{sech}\left(bx+a\right)^2dx$$

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] int(x^2*csch(b*x+a)*sech(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 937 vs. 2(126) = 252.

Time = 0.28 (sec) , antiderivative size = 937, normalized size of antiderivative = 6.42

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) + 2*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(cos
h(b*x + a) + sinh(b*x + a)) - 2*(I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh
(b*x + a) + I*sinh(b*x + a)^2 + I)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a))
 - 2*(-I*cosh(b*x + a)^2 - 2*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a
)^2 - I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^2
 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 + b*x)*dilog(-co
sh(b*x + a) - sinh(b*x + a)) - (b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*
x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 + b^2*x^2)*log(cosh(b*x + a)
```

```
 + sinh(b*x + a) + 1) - 2*(-I*a*cosh(b*x + a)^2 - 2*I*a*cosh(b*x + a)*sinh(
b*x + a) - I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x + a) + sinh(b*x + a) + I
) - 2*(I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh(b*x + a) + I*a*sinh(b
*x + a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (a^2*cosh(b*x + a
)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 + a^2)*log(co
sh(b*x + a) + sinh(b*x + a) - 1) - 2*((-I*b*x - I*a)*cosh(b*x + a)^2 + 2*(-
I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*sinh(b*x + a)^2 -
 I*b*x - I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((I*b*x + I*a)
*cosh(b*x + a)^2 + 2*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a) + (I*b*x + I
*a)*sinh(b*x + a)^2 + I*b*x + I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) +
 1) + (b^2*x^2 + (b^2*x^2 - a^2)*cosh(b*x + a)^2 + 2*(b^2*x^2 - a^2)*cosh(b
*x + a)*sinh(b*x + a) + (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*
x + a) - sinh(b*x + a) + 1) - 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 2*
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*pol
ylog(3, -cosh(b*x + a) - sinh(b*x + a)))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(
b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b^3)
```

Sympy [F]

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \,\text{csch}\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2 \,dx$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] 2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b^2*x^2*log(e^(b*x + a) + 1) +
 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log
(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/
b^3 - 8*integrate(1/2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Giac [F]

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2 \,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)),x)
[Out] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.476 $\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{x\text{sech}(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b^2-2*x*arctanh(exp(b*x+a))/b-polylog(2,-exp(b*x+a))/b
^2+polylog(2,exp(b*x+a))/b^2+x*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2702, 327, 213, 5570, 6406, 12, 4267, 2317, 2438, 3855}

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{x\text{sech}(a+bx)}{b}$$

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b^2) - (2*x*ArcTanh[E^(a + b*x)])/b - PolyLog[2, -E
^(a + b*x)]/b^2 + PolyLog[2, E^(a + b*x)]/b^2 + (x*Sech[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
```

```
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x\text{arctanh}(\cosh(a+bx))}{b}+\frac{x\text{sech}(a+bx)}{b} \\
&\quad -\int\left(-\frac{\text{arctanh}(\cosh(a+bx))}{b}+\frac{\text{sech}(a+bx)}{b}\right)dx \\
&= -\frac{x\text{arctanh}(\cosh(a+bx))}{b}+\frac{x\text{sech}(a+bx)}{b} \\
&\quad +\frac{\int\text{arctanh}(\cosh(a+bx))\,dx}{b}-\frac{\int\text{sech}(a+bx)\,dx}{b} \\
&= -\frac{\arctan(\sinh(a+bx))}{b^2}+\frac{x\text{sech}(a+bx)}{b}+\frac{\int bx\text{csch}(a+bx)\,dx}{b} \\
&= -\frac{\arctan(\sinh(a+bx))}{b^2}+\frac{x\text{sech}(a+bx)}{b}+\int x\text{csch}(a+bx)\,dx \\
&= -\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b} \\
&\quad -\frac{\int\log\left(1-e^{a+bx}\right)\,dx}{b}+\frac{\int\log\left(1+e^{a+bx}\right)\,dx}{b} \\
&= -\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{x\text{sech}(a+bx)}{b} \\
&\quad -\frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{a+bx}\right)}{b^2}+\frac{\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{a+bx}\right)}{b^2} \\
&= -\frac{\arctan(\sinh(a+bx))}{b^2}-\frac{2x\text{arctanh}\left(e^{a+bx}\right)}{b} \\
&\quad -\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{x\text{sech}(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \frac{-2\arctan\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)+bx\log\left(1-e^{a+bx}\right)-bx\log\left(1+e^{a+bx}\right)-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(\right.}{b^2}$$

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] (-2*ArcTan[Tanh[(a + b*x)/2]] + b*x*Log[1 - E^(a + b*x)] - b*x*Log[1 + E^(a
 + b*x)] - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)] + b*x*Sech[a
+ b*x])/b^2
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{2x\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{2\arctan\left(\mathrm{e}^{bx+a}\right)}{b^2}-\frac{\text{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{b^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}-\frac{\text{dilog}\left(\mathrm{e}^{bx+a}\right)}{b^2}-\frac{a\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^2}$	95

```
[In] int(x*csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2/b^2*arctan(exp(b*x+a))-1/b^2*dilog(ex
p(b*x+a)+1)-1/b*ln(exp(b*x+a)+1)*x-1/b^2*dilog(exp(b*x+a))-1/b^2*a*ln(exp(b
*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 401, normalized size of antiderivative = 5.99

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \frac{2\,bx\cosh\left(bx+a\right)+2\,bx\sinh\left(bx+a\right)-2\,\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2\right.}{}$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - 2*(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arctan(cosh(b*x + a) + sinh(b*x
 + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(b*x
```

```
+ a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) - (b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(
b*x + a)^2 + b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a*cosh(b*x + a)
^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 + a)*log(cosh(b*x
+ a) + sinh(b*x + a) - 1) + ((b*x + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b
*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2 + b*x + a)*log(-cosh(b*x
+ a) - sinh(b*x + a) + 1))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(
b*x + a) + b^2*sinh(b*x + a)^2 + b^2)
```

Sympy [F]

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx=\int x\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)**2,x)
[Out] Integral(x*csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)\,dx=\frac{2\,xe^{(bx+a)}}{be^{(2\,bx+2\,a)}+b}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}-\frac{2\arctan\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] 2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) + b) - (b*x*log(e^(b*x + a) + 1) + dilog
(-e^(b*x + a)))/b^2 + (b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
- 2*arctan(e^(b*x + a))/b^2
```

Giac [F]

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int x\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^2\,\,dx$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x*csch(b*x + a)*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x/(cosh(a + b*x)^2*sinh(a + b*x)),x)
[Out] int(x/(cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.477 $\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{b}+\frac{\text{sech}(a+bx)}{b}$$

```
[Out] -arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2702, 327, 213}

$$\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx=\frac{\text{sech}(a+bx)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*x]]/b) + Sech[a + b*x]/b
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= \frac{\text{sech}(a+bx)}{b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{b}\\ &= -\frac{\text{arctanh}(\cosh(a+bx))}{b}+\frac{\text{sech}(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\text{sech}(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
[Out] -(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{1}{\cosh(bx+a)}-2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	23
default	$\frac{\frac{1}{\cosh(bx+a)}-2\,\text{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	23
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b\left(1+\mathrm{e}^{2bx+2a}\right)}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	53

```
[In] int(csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\left(\cosh\left(bx+a\right)^2 + 2\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \sinh\left(bx+a\right)^2 + 1\right)\log\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right.}{b\cosh\left(bx\right.}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}+1)}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^(
-2*b*x - 2*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(23) = 46.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
[Out] 1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2) +
 log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)
[Out] (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b^
2)^(1/2))/b))/(-b^2)^(1/2)
```

3.478 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 35.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}dx$$

```
[In] int(csch(b*x+a)*sech(b*x+a)^2/x,x)
[Out] int(csch(b*x+a)*sech(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2/x,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 5.50

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) + b*x) + 8*integrate(1/4*e^(b*x + a)/(b*
x^2*e^(2*b*x + 2*a) + b*x^2), x) + 8*integrate(1/8/(x*e^(b*x + a) + x), x)
+ 8*integrate(1/8/(x*e^(b*x + a) - x), x)
```

Giac [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{1}{x\cosh(a+b\,x)^2\sinh(a+b\,x)}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.479 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}dx$$

```
[In] int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
[Out] int(csch(b*x+a)*sech(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**2/x**2,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.17

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) + b*x^2) + 8*integrate(1/2*e^(b*x + a)
/(b*x^3*e^(2*b*x + 2*a) + b*x^3), x) + 8*integrate(1/8/(x^2*e^(b*x + a) + x
^2), x) + 8*integrate(1/8/(x^2*e^(b*x + a) - x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)), x)
```

3.480 $\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx=\mathrm{Int}\big(x^m \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx),x\big)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx=\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
[Out] Defer[Int][x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

Rubi steps

$$\text{integral}=\int x^m \mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 80.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3,x]
[Out] Integrate[x^m*Csch[a + b*x]*Sech[a + b*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

```
[In] int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
[Out] int(x^m*csch(b*x+a)*sech(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^m \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**m*csch(b*x+a)*sech(b*x+a)**3,x)
[Out] Integral(x**m*csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)\,\text{sech}\,(bx+a)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)*sech(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)),x)
[Out] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.481 $\int x^3\mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 240

$$\int x^3\mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx=-\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{2x^3\mathrm{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3}+\frac{3\,\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}-\frac{3x^2\,\mathrm{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\,\mathrm{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}+\frac{3x\,\mathrm{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\,\mathrm{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}-\frac{3\,\mathrm{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\,\mathrm{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b-2*x^3*arctanh(exp(2*b*x+2*a))/b+3*x*ln(1+exp(2*b*x+2
*a))/b^3+3/2*polylog(2,-exp(2*b*x+2*a))/b^4-3/2*x^2*polylog(2,-exp(2*b*x+2*
a))/b^2+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*
a))/b^3-3/2*x*polylog(3,exp(2*b*x+2*a))/b^3-3/4*polylog(4,-exp(2*b*x+2*a))/
b^4+3/4*polylog(4,exp(2*b*x+2*a))/b^4-3/2*x^2*tanh(b*x+a)/b^2-1/2*x^3*tanh(
b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2700, 14, 5570, 2631, 12, 4267, 2611, 6744, 2320, 6724, 3801, 3799, 2221, 2317, 2438, 30}

$$\int x^3\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{3\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b^4}-\frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}+\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}+\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}-\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3x\log\left(e^{2(a+bx)}+1\right)}{b^3}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\tanh(a+bx)}{2b^2}-\frac{x^3\tanh^2(a+bx)}{2b}-\frac{3x^2}{2b^2}+\frac{x^3}{2b}$$

```
[In] Int[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) - (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b + (3*x*Lo
g[1 + E^(2*(a + b*x))])/b^3 + (3*PolyLog[2, -E^(2*(a + b*x))])/(2*b^4) - (3
*x^2*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^2) + (3*x^2*PolyLog[2, E^(2*a + 2*b
*x)])/(2*b^2) + (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) - (3*x*PolyLog[3
, E^(2*a + 2*b*x)])/(2*b^3) - (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) + (3
*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^4) - (3*x^2*Tanh[a + b*x])/(2*b^2) - (x^
3*Tanh[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} - 3\int x^2\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}\right)\,dx$$

$$= \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} - 3\int\left(\frac{x^2\log(\tanh(a+bx))}{b} - \frac{x^2\tanh^2(a+bx)}{2b}\right)\,dx$$

$$= \frac{x^3\log(\tanh(a+bx))}{b} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\int x^2\tanh^2(a+bx)\,dx}{2b} - \frac{3\int x^2\log(\tanh(a+bx))\,dx}{b}$$

$$= -\frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + \frac{3\int x\tanh(a+bx)\,dx}{b^2} + \frac{\int 2bx^3\operatorname{csch}(2a+2bx)\,dx}{b} + \frac{3\int x^2\,dx}{2b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} + 2\int x^3\operatorname{csch}(2a+2bx)\,dx + \frac{6\int\frac{e^{2(a+bx)}x}{1+e^{2(a+bx)}}\,dx}{b^2}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} - \frac{3\int\log\left(1+e^{2(a+bx)}\right)\,dx}{b^3} - \frac{3\int x^2\log\left(1-e^{2a+2bx}\right)\,dx}{b} + \frac{3\int x^2\log\left(1+e^{2a+2bx}\right)\,dx}{b}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} + \frac{3x\log\left(1+e^{2(a+bx)}\right)}{b^3} - \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2} + \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{3x^2\tanh(a+bx)}{2b^2} - \frac{x^3\tanh^2(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{2(a+bx)}\right)}{2b^4} + \frac{3\int x\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)\,dx}{b^2} - \frac{3\int x\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)\,dx}{b^2}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{3x \log(1+e^{2(a+bx)})}{b^3} + \frac{3 \operatorname{PolyLog}(2, -e^{2(a+bx)})}{2b^4} - \frac{3x^2 \operatorname{PolyLog}(2, -e^{2a+2bx})}{2b^2} + \frac{3x^2 \operatorname{PolyLog}(2, e^{2a+2bx})}{2b^2} + \frac{3x \operatorname{PolyLog}(3, -e^{2a+2bx})}{2b^3} - \frac{3x \operatorname{PolyLog}(3, e^{2a+2bx})}{2b^3} - \frac{3x^2 \tanh(a+bx)}{2b^2} - \frac{x^3 \tanh^2(a+bx)}{2b} - \frac{3 \int \operatorname{PolyLog}(3, -e^{2a+2bx})\, dx}{2b^3} + \frac{3 \int \operatorname{PolyLog}(3, e^{2a+2bx})\, dx}{2b^3}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{3x \log(1+e^{2(a+bx)})}{b^3} + \frac{3 \operatorname{PolyLog}(2, -e^{2(a+bx)})}{2b^4} - \frac{3x^2 \operatorname{PolyLog}(2, -e^{2a+2bx})}{2b^2} + \frac{3x^2 \operatorname{PolyLog}(2, e^{2a+2bx})}{2b^2} + \frac{3x \operatorname{PolyLog}(3, -e^{2a+2bx})}{2b^3} - \frac{3x \operatorname{PolyLog}(3, e^{2a+2bx})}{2b^3} - \frac{3x^2 \tanh(a+bx)}{2b^2} - \frac{x^3 \tanh^2(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-x)}{x}\, dx, x, e^{2a+2bx}\right)}{4b^4} + \frac{3\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\, dx, x, e^{2a+2bx}\right)}{4b^4}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{2x^3 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{3x \log(1+e^{2(a+bx)})}{b^3} + \frac{3 \operatorname{PolyLog}(2, -e^{2(a+bx)})}{2b^4} - \frac{3x^2 \operatorname{PolyLog}(2, -e^{2a+2bx})}{2b^2} + \frac{3x^2 \operatorname{PolyLog}(2, e^{2a+2bx})}{2b^2} + \frac{3x \operatorname{PolyLog}(3, -e^{2a+2bx})}{2b^3} - \frac{3x \operatorname{PolyLog}(3, e^{2a+2bx})}{2b^3} - \frac{3 \operatorname{PolyLog}(4, -e^{2a+2bx})}{4b^4} + \frac{3 \operatorname{PolyLog}(4, e^{2a+2bx})}{4b^4} - \frac{3x^2 \tanh(a+bx)}{2b^2} - \frac{x^3 \tanh^2(a+bx)}{2b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 524 vs. $2(240) = 480$.

Time = 6.40 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.18

$$\int x^3 \operatorname{csch}(a+bx) \operatorname{sech}^3(a+bx)\, dx =$$

$$-\frac{2e^{2a}\left(-\frac{3}{2}e^{-2a}x^2 + \frac{1}{4}b^2e^{-2a}x^4 - \frac{3(1+e^{-2a})x\log(1+e^{-2a-2bx})}{2b} + \frac{1}{2}b(1+e^{-2a})\,x^3\log\left(1+e^{-2a-2bx}\right) + \frac{3(1+e^{-2a})\,\mathrm{I}}{}\right.}{}$$

$$-\frac{e^{2a}\left(b^4e^{-2a}x^4 - 2b^3(1-e^{-2a})\,x^3\log\left(1-e^{-a-bx}\right) - 2b^3(1-e^{-2a})\,x^3\log\left(1+e^{-a-bx}\right) + 6b^2(1-e^{-2a})\,x^2\right.}{}$$

$$+ \frac{1}{4}x^4 \operatorname{csch}(a)\operatorname{sech}(a) + \frac{x^3 \operatorname{sech}^2(a+bx)}{2b} - \frac{3x^2 \operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(bx)}{2b^2}$$

```
[In] Integrate[x^3*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] (-2*E^(2*a)*((-3*x^2)/(2*E^(2*a)) + (b^2*x^4)/(4*E^(2*a)) - (3*(1 + E^(-2*a
))*x*Log[1 + E^(-2*a - 2*b*x)])/(2*b) + (b*(1 + E^(-2*a))*x^3*Log[1 + E^(-2
*a - 2*b*x)])/2 + (3*(1 + E^(-2*a))*PolyLog[2, -E^(-2*a - 2*b*x)])/(4*b^2)
- (3*(1 + E^(-2*a))*x^2*PolyLog[2, -E^(-2*a - 2*b*x)])/4 - (3*(1 + E^(-2*a)
)*x*PolyLog[3, -E^(-2*a - 2*b*x)])/(4*b) - (3*(1 + E^(-2*a))*PolyLog[4, -E^
(-2*a - 2*b*x)])/(8*b^2)))/(b^2*(1 + E^(2*a))) - (E^(2*a)*((b^4*x^4)/E^(2*a
) - 2*b^3*(1 - E^(-2*a))*x^3*Log[1 - E^(-a - b*x)] - 2*b^3*(1 - E^(-2*a))*x
^3*Log[1 + E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, -E^(-a - b*x
)] + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 - E^(-2*a)
)*x*PolyLog[3, -E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, E^(-a - b*
x)] + 12*(1 - E^(-2*a))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-2*a))*PolyL
og[4, E^(-a - b*x)]))/(2*b^4*(-1 + E^(2*a))) + (x^4*Csch[a]*Sech[a])/4 + (x
^3*Sech[a + b*x]^2)/(2*b) - (3*x^2*Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b^2)
```

Maple [A] (verified)

Time = 7.58 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.50

method	result
risch	$\frac{x^2(2\,\mathrm{e}^{2bx+2a}bx+3\,\mathrm{e}^{2bx+2a}+3)}{b^2(1+\mathrm{e}^{2bx+2a})^2}-\frac{3a^2}{b^4}-\frac{3x^2}{b^2}-\frac{6ax}{b^3}+\frac{6\,\mathrm{polylog}(4,\mathrm{e}^{bx+a})}{b^4}-\frac{3\,\mathrm{polylog}(4,-\mathrm{e}^{2bx+2a})}{4b^4}+\frac{6\,\mathrm{polylog}(4,-\mathrm{e}^{bx+a})}{b^4}$

```
[In] int(x^3*csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)+3)/b^2/(1+exp(2*b*x+2*a))^2-3/b^
4*a^2-3/b^2*x^2-6/b^3*a*x+6*polylog(4,exp(b*x+a))/b^4-3/4*polylog(4,-exp(2*
b*x+2*a))/b^4+6*polylog(4,-exp(b*x+a))/b^4+6/b^4*a*ln(exp(b*x+a))-1/b^4*a^3
*ln(exp(b*x+a)-1)-6*x*polylog(3,exp(b*x+a))/b^3-x^3*ln(1+exp(2*b*x+2*a))/b-
3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3
+1/b*ln(exp(b*x+a)+1)*x^3+3*x^2*polylog(2,-exp(b*x+a))/b^2-6*x*polylog(3,-e
xp(b*x+a))/b^3+3*x^2*polylog(2,exp(b*x+a))/b^2+1/b*ln(1-exp(b*x+a))*x^3+3*x
*ln(1+exp(2*b*x+2*a))/b^3+3/2*polylog(2,-exp(2*b*x+2*a))/b^4+1/b^4*ln(1-exp
(b*x+a))*a^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 3409, normalized size of antiderivative = 14.20

$$\int x^3\mathrm{csch}(a+bx)\mathrm{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 - (2*b^3*x^3 - 3*b^2*x^2 +
```

6*a^2)*cosh(b*x + a)^2 - (2*b^3*x^3 - 3*b^2*x^2 - 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 - 3*(b^2*x^2*cosh(b*x + a)^4 + 4
*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^
2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(
b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^2*x^2*cosh(b*x + a))*sinh(b*x +
a))*dilog(cosh(b*x + a) + sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^
4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
+ a)^4 + b^2*x^2 + 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^
2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a
)^3 + (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(I*cosh(b*x + a)
+ I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*co
sh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 + 2*(
b^2*x^2 - 1)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - 1)*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 + (b^2*x^2 - 1)*co
sh(b*x + a))*sinh(b*x + a) - 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) -
3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2
*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x^2*c
osh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3 + b^
2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) -
(b^3*x^3*cosh(b*x + a)^4 + 4*b^3*x^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*x
^3*sinh(b*x + a)^4 + 2*b^3*x^3*cosh(b*x + a)^2 + b^3*x^3 + 2*(3*b^3*x^3*cos
h(b*x + a)^2 + b^3*x^3)*sinh(b*x + a)^2 + 4*(b^3*x^3*cosh(b*x + a)^3 + b^3*
x^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3
+ (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3 - 3*a)*cosh(b*x + a)^2 + 2*(a^
3 + 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sinh(b*x + a)^2 + 4*((a^3 - 3*a)*c
osh(b*x + a)^3 + (a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*log(cosh(b
*x + a) + sinh(b*x + a) + I) - ((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)
*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(a^3
- 3*a)*cosh(b*x + a)^2 + 2*(a^3 + 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sin
h(b*x + a)^2 + 4*((a^3 - 3*a)*cosh(b*x + a)^3 + (a^3 - 3*a)*cosh(b*x + a))*
sinh(b*x + a) - 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (a^3*cosh(b*x
+ a)^4 + 4*a^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3*sinh(b*x + a)^4 + 2*a^3
*cosh(b*x + a)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 + a^3)*sinh(b*x + a)^2 +
4*(a^3*cosh(b*x + a)^3 + a^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + (b^3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x
+ a)^4 + 4*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^3*x^3 + a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 - 3*b*
x - 3*a)*cosh(b*x + a)^2 + 2*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 - 3*b*x - 3*
a)*cosh(b*x + a)^2 - 3*b*x - 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 + a
^3 - 3*b*x - 3*a)*cosh(b*x + a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x
+ a))*sinh(b*x + a) - 3*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (b^
3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 -
3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*
sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 + 2

```
*(b^3*x^3 + a^3 + 3*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 - 3*b*x -
 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x +
 a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*l
og(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cos
h(b*x + a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 +
 a^3)*sinh(b*x + a)^4 + a^3 + 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 + 2*(b^3*x^
3 + a^3 + 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3
+ a^3)*cosh(b*x + a)^3 + (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(
-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)
^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
+ 1)*polylog(4, cosh(b*x + a) + sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(cosh(b*x
 + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cos
h(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a
)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(
b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, -cosh(b*x + a) -
sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)
^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2
 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))
*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x
 + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x
*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x +
4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cos
h(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a
)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*
cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x
*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a
)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh
(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x +
a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(6*(b^2*x^2 - a^2)*cosh(
b*x + a)^3 - (2*b^3*x^3 - 3*b^2*x^2 + 6*a^2)*cosh(b*x + a))*sinh(b*x + a))/
(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x +
 a)^4 + 2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(
b*x + a)^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^3\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int x^3\,\text{csch}\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)*sech(b*x+a)**3,x)
[Out] Integral(x**3*csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.37

$$\begin{aligned}&\int x^3\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx\\&=-\frac{1}{2}x^4+\frac{3x^2+\left(2bx^3e^{(2a)}+3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}+\frac{b^4x^4-6b^2x^2}{2b^4}\\&\quad-\frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right)+6b^2x^2\text{Li}_2\left(-e^{(2bx+2a)}\right)-6bx\text{Li}_3(-e^{(2bx+2a)})+3\text{Li}_4(-e^{(2bx+2a)})}{3b^4}\\&\quad+\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3b^2x^2\text{Li}_2\left(-e^{(bx+a)}\right)-6bx\text{Li}_3(-e^{(bx+a)})+6\text{Li}_4(-e^{(bx+a)})}{b^4}\\&\quad+\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3b^2x^2\text{Li}_2\left(e^{(bx+a)}\right)-6bx\text{Li}_3(e^{(bx+a)})+6\text{Li}_4(e^{(bx+a)})}{b^4}\\&\quad+\frac{3\left(2bx\log\left(e^{(2bx+2a)}+1\right)+\text{Li}_2\left(-e^{(2bx+2a)}\right)\right)}{2b^4}\end{aligned}$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*x^4 + (3*x^2 + (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*
b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4 -
 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3/2*(2*b*x*log(e^(2*b*x + 2*a) + 1)
+ dilog(-e^(2*b*x + 2*a)))/b^4
```

Giac [F]

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

```
[In] integrate(x^3*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^3*csch(b*x + a)*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \frac{x^3}{\cosh(a+b\,x)^3\sinh(a+b\,x)}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)),x)
[Out] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.482 $\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{x^2}{2b} - \frac{2x^2\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{x\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} + \frac{\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} - \frac{\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b}$$

```
[Out] 1/2*x^2/b-2*x^2*arctanh(exp(2*b*x+2*a))/b+ln(cosh(b*x+a))/b^3-x*polylog(2,-
exp(2*b*x+2*a))/b^2+x*polylog(2,exp(2*b*x+2*a))/b^2+1/2*polylog(3,-exp(2*b*
x+2*a))/b^3-1/2*polylog(3,exp(2*b*x+2*a))/b^3-x*tanh(b*x+a)/b^2-1/2*x^2*tan
h(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2700, 14, 5570, 2631, 12, 4267, 2611, 2320, 6724, 3801, 3556, 30}

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{2x^2\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} + \frac{\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} - \frac{\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{x\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{x\tanh(a+bx)}{b^2} - \frac{x^2\tanh^2(a+bx)}{2b} + \frac{x^2}{2b}$$

```
[In] Int[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] x^2/(2*b) - (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b + Log[Cosh[a + b*x]]/b^3 - (
x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 + (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 +
 PolyLog[3, -E^(2*a + 2*b*x)]/(2*b^3) - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
 - (x*Tanh[a + b*x])/b^2 - (x^2*Tanh[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{x^2 \log(\tanh(a+bx))}{b} - \frac{x^2 \tanh^2(a+bx)}{2b} - 2\int x\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}\right)\,dx$$

$$= \frac{x^2 \log(\tanh(a+bx))}{b} - \frac{x^2 \tanh^2(a+bx)}{2b} - 2\int \left(\frac{x \log(\tanh(a+bx))}{b} - \frac{x \tanh^2(a+bx)}{2b}\right) dx$$

$$= \frac{x^2 \log(\tanh(a+bx))}{b} - \frac{x^2 \tanh^2(a+bx)}{2b} + \frac{\int x \tanh^2(a+bx)\, dx}{b} - \frac{2\int x \log(\tanh(a+bx))\, dx}{b}$$

$$= -\frac{x \tanh(a+bx)}{b^2} - \frac{x^2 \tanh^2(a+bx)}{2b} + \frac{\int \tanh(a+bx)\, dx}{b^2} + \frac{\int x\, dx}{b} + \frac{\int 2bx^2 \operatorname{csch}(2a+2bx)\, dx}{b}$$

$$= \frac{x^2}{2b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x \tanh(a+bx)}{b^2} - \frac{x^2 \tanh^2(a+bx)}{2b} + 2\int x^2 \operatorname{csch}(2a+2bx)\, dx$$

$$= \frac{x^2}{2b} - \frac{2x^2 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x \tanh(a+bx)}{b^2} - \frac{x^2 \tanh^2(a+bx)}{2b} - \frac{2\int x \log(1-e^{2a+2bx})\, dx}{b} + \frac{2\int x \log(1+e^{2a+2bx})\, dx}{b}$$

$$= \frac{x^2}{2b} - \frac{2x^2 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\, \text{PolyLog}(2, -e^{2a+2bx})}{b^2} + \frac{x\, \text{PolyLog}(2, e^{2a+2bx})}{b^2} - \frac{x \tanh(a+bx)}{b^2} - \frac{x^2 \tanh^2(a+bx)}{2b} + \frac{\int \text{PolyLog}(2, -e^{2a+2bx})\, dx}{b^2} - \frac{\int \text{PolyLog}(2, e^{2a+2bx})\, dx}{b^2}$$

$$= \frac{x^2}{2b} - \frac{2x^2 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\, \text{PolyLog}(2, -e^{2a+2bx})}{b^2} + \frac{x\, \text{PolyLog}(2, e^{2a+2bx})}{b^2} - \frac{x \tanh(a+bx)}{b^2} - \frac{x^2 \tanh^2(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\, dx, x, e^{2a+2bx}\right)}{2b^3} - \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\, dx, x, e^{2a+2bx}\right)}{2b^3}$$

$$= \frac{x^2}{2b} - \frac{2x^2 \operatorname{arctanh}(e^{2a+2bx})}{b} + \frac{\log(\cosh(a+bx))}{b^3} - \frac{x\, \text{PolyLog}(2, -e^{2a+2bx})}{b^2} + \frac{x\, \text{PolyLog}(2, e^{2a+2bx})}{b^2} + \frac{\text{PolyLog}(3, -e^{2a+2bx})}{2b^3} - \frac{\text{PolyLog}(3, e^{2a+2bx})}{2b^3} - \frac{x \tanh(a+bx)}{b^2} - \frac{x^2 \tanh^2(a+bx)}{2b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(148) = 296.

Time = 2.61 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.57

$$\int x^2 \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{1}{6}\Bigg(-\frac{2e^{2a}\left(2b^3e^{-2a}x^3 - 3b^2(1-e^{-2a})\,x^2\log\left(1-e^{-a-bx}\right) - 3b^2(1-e^{-2a})\,x^2\log\left(1+e^{-a-bx}\right) + 6b(1-e^{-2}\right.}{}$$
$$+\frac{-12be^{2a}x - 4b^3x^3 - 6b^2(1+e^{2a})\,x^2\log\left(1+e^{-2(a+bx)}\right) + 6\log\left(1+e^{2(a+bx)}\right) + 6e^{2a}\log\left(1+e^{2(a+bx)}\right) +}{b^3\left(1+e^{2a}\right)}$$
$$+ 2x^3\operatorname{csch}(a)\operatorname{sech}(a) + \frac{3x^2\operatorname{sech}^2(a+bx)}{b} - \frac{6x\operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(bx)}{b^2}\Bigg)$$

```
[In] Integrate[x^2*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] ((-2*E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E^(-a
- b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*b*(1 - E^(-2*a
))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, E^(-a - b*
x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog
[3, E^(-a - b*x)]))/(b^3*(-1 + E^(2*a))) + (-12*b*E^(2*a)*x - 4*b^3*x^3 - 6
*b^2*(1 + E^(2*a))*x^2*Log[1 + E^(-2*(a + b*x))] + 6*Log[1 + E^(2*(a + b*x)
)] + 6*E^(2*a)*Log[1 + E^(2*(a + b*x))] + 6*b*(1 + E^(2*a))*x*PolyLog[2, -E
^(-2*(a + b*x))] + 3*(1 + E^(2*a))*PolyLog[3, -E^(-2*(a + b*x))])/(b^3*(1 +
 E^(2*a))) + 2*x^3*Csch[a]*Sech[a] + (3*x^2*Sech[a + b*x]^2)/b - (6*x*Sech[
a]*Sech[a + b*x]*Sinh[b*x])/b^2)/6
```

Maple [A] (verified)

Time = 5.47 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73

method	result
risch	$\frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}+1\right)}{b^2\left(1+e^{2bx+2a}\right)^2} - \frac{2\ln\left(e^{bx+a}\right)}{b^3} - \frac{2\,\mathrm{polylog}\left(3,e^{bx+a}\right)}{b^3} + \frac{\mathrm{polylog}\left(3,-e^{2bx+2a}\right)}{2b^3} - \frac{2\,\mathrm{polylog}\left(3,-e^{bx+a}\right)}{b^3} + \frac{\ln(1-e^{b}}{b}$

```
[In] int(x^2*csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2-2/b^3*ln
(exp(b*x+a))-2*polylog(3,exp(b*x+a))/b^3+1/2*polylog(3,-exp(2*b*x+2*a))/b^3
-2*polylog(3,-exp(b*x+a))/b^3+1/b*ln(1-exp(b*x+a))*x^2+2*x*polylog(2,exp(b*
x+a))/b^2-x^2*ln(1+exp(2*b*x+2*a))/b-x*polylog(2,-exp(2*b*x+2*a))/b^2+1/b*l
n(exp(b*x+a)+1)*x^2+2*x*polylog(2,-exp(b*x+a))/b^2+1/b^3*a^2*ln(exp(b*x+a)-
1)+1/b^3*ln(1+exp(2*b*x+2*a))-1/b^3*ln(1-exp(b*x+a))*a^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 2523, normalized size of antiderivative = 17.05

$$\int x^2 \text{csch}(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
[Out] -(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 +
 2*(b*x + a)*sinh(b*x + a)^4 - 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 - 2*
(b^2*x^2 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 - 2*(b*
x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)
^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(
cosh(b*x + a) + sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*
x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) +
2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
 + a)^4 + 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x
+ a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*d
ilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
 a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) - (b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sinh(b*x + a)^3 +
 b^2*x^2*sinh(b*x + a)^4 + 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^2 + 2*(3*b^2*x
^2*cosh(b*x + a)^2 + b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*cosh(b*x + a)^3
+ b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^
3 + (a^2 - 1)*sinh(b*x + a)^4 + 2*(a^2 - 1)*cosh(b*x + a)^2 + 2*(3*(a^2 - 1
)*cosh(b*x + a)^2 + a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 - 1)*cosh(b*x
+ a)^3 + (a^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + si
nh(b*x + a) + I) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*s
inh(b*x + a)^3 + (a^2 - 1)*sinh(b*x + a)^4 + 2*(a^2 - 1)*cosh(b*x + a)^2 +
2*(3*(a^2 - 1)*cosh(b*x + a)^2 + a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 -
 1)*cosh(b*x + a)^3 + (a^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(
b*x + a) + sinh(b*x + a) - I) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*
sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 + 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*co
sh(b*x + a)^2 + a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 + a^2*c
osh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((b^2
*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x
```

```
+ a)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x +
a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x
+ a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + ((b^2*x^2
 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a
)^2 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a)
)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - ((b^2*x^2 -
a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 + 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2
 + 2*(b^2*x^2 + 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 -
a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 + (b^2*x^2 - a^2)*cosh(b*x + a))*s
inh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x +
a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -I*cosh(b*x + a
) - I*sinh(b*x + a)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
 + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3,
 -cosh(b*x + a) - sinh(b*x + a)) + 4*(2*(b*x + a)*cosh(b*x + a)^3 - (b^2*x^
2 - b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) + 2*a)/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int x^2\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**2*csch(b*x+a)*sech(b*x+a)**3,x)
[Out] Integral(x**2*csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$
$$=\frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}+x\right)}{b^2e^{(4bx+4a)}+2b^2e^{(2bx+2a)}+b^2}-\frac{2x}{b^2}$$
$$-\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\text{Li}_2\left(-e^{(2bx+2a)}\right)-\text{Li}_3\left(-e^{(2bx+2a)}\right)}{2b^3}$$
$$+\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\text{Li}_3\left(-e^{(bx+a)}\right)}{b^3}$$
$$+\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\text{Li}_2\left(e^{(bx+a)}\right)-2\text{Li}_3\left(e^{(bx+a)}\right)}{b^3}+\frac{\log\left(e^{(2bx+2a)}+1\right)}{b^3}$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
[Out] 2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) + x)/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2) - 2*x/b^2 - 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
+ 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 + (b^2*
x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x
 + a)))/b^3 + (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2
*polylog(3, e^(b*x + a)))/b^3 + log(e^(2*b*x + 2*a) + 1)/b^3
```

Giac [F]

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int x^2\,\text{csch}\left(bx+a\right)\text{sech}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^2*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*csch(b*x + a)*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int\frac{x^2}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)),x)
[Out] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.483 $\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\frac{x}{2b}-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}$$

```
[Out] 1/2*x/b-2*x*arctanh(exp(2*b*x+2*a))/b-1/2*polylog(2,-exp(2*b*x+2*a))/b^2+1/
2*polylog(2,exp(2*b*x+2*a))/b^2-1/2*tanh(b*x+a)/b^2-1/2*x*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2700, 14, 5570, 2628, 12, 4267, 2317, 2438, 3554, 8}

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=-\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{\tanh(a+bx)}{2b^2}-\frac{x\tanh^2(a+bx)}{2b}+\frac{x}{2b}$$

```
[In] Int[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] x/(2*b) - (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - PolyLog[2, -E^(2*a + 2*b*x)]/(
2*b^2) + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2) - Tanh[a + b*x]/(2*b^2) - (x*T
anh[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a+bx)}{2b} - \int\left(\frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}\right)dx \\
&= \frac{x\log(\tanh(a+bx))}{b} - \frac{x\tanh^2(a+bx)}{2b} + \frac{\int\tanh^2(a+bx)\,dx}{2b} - \frac{\int\log(\tanh(a+bx))\,dx}{b} \\
&= -\frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + \frac{\int 1\,dx}{2b} + \frac{\int 2bx\text{csch}(2a+2bx)\,dx}{b} \\
&= \frac{x}{2b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} + 2\int x\text{csch}(2a+2bx)\,dx \\
&= \frac{x}{2b} - \frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\int\log\left(1-e^{2a+2bx}\right)\,dx}{b} + \frac{\int\log\left(1+e^{2a+2bx}\right)\,dx}{b} \\
&= \frac{x}{2b} - \frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b} - \frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2a+2bx}\right)}{2b^2} + \frac{\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{2a+2bx}\right)}{2b^2} \\
&= \frac{x}{2b} - \frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2} + \frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{\tanh(a+bx)}{2b^2} - \frac{x\tanh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx$$
$$=\frac{2bx\log\left(1-e^{-2(a+bx)}\right)-2bx\log\left(1+e^{-2(a+bx)}\right)+\text{PolyLog}\left(2,-e^{-2(a+bx)}\right)-\text{PolyLog}\left(2,e^{-2(a+bx)}\right)+bx\text{se}}{2b^2}$$

```
[In] Integrate[x*Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] (2*b*x*Log[1 - E^(-2*(a + b*x))] - 2*b*x*Log[1 + E^(-2*(a + b*x))] + PolyLo
g[2, -E^(-2*(a + b*x))] - PolyLog[2, E^(-2*(a + b*x))] + b*x*Sech[a + b*x]^
2 - Tanh[a + b*x])/(2*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(82) = 164.

Time = 3.82 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.75

method	result
risch	$\frac{2\,\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}+1}{b^2\left(1+\mathrm{e}^{2bx+2a}\right)^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}+\frac{\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}-\frac{x}{}$

```
[In] int(x*csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)+1)/b^2/(1+exp(2*b*x+2*a))^2+1/b*ln(1-e
xp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+polylog(2,exp(b*x+a))/b^2+1/b*ln(exp(
b*x+a)+1)*x+polylog(2,-exp(b*x+a))/b^2-x*ln(1+exp(2*b*x+2*a))/b-1/2*polylog
(2,-exp(2*b*x+2*a))/b^2-1/b^2*a*ln(exp(b*x+a)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1543, normalized size of antiderivative = 16.24

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\text{Too large to display}$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
(2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
 a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cos
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*dilog
```

```
(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b
*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2
*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*d
ilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a
) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^4 + 4*cos
h(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^
4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 + 2*b*x*cosh(
b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x
*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 +
 a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh
(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a) + a)*lo
g(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(
b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 + a*cosh(b*x + a))*s
inh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a*cosh(b*x + a)
^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 + 2*a*cosh(b*x +
 a)^2 + 2*(3*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3
+ a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
+ (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*
cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x +
a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I*
sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a)*cosh(b*x + a)^2
+ 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^2 + b*x + 4*((b*x
 + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x
+ a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 + 2*(b*x + a
)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)
^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x
 + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 1)/(b^2*cosh(b*x + a)^
4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(
b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2
*cosh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int x\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)**3,x)
[Out] Integral(x*csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.49

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\frac{(2\,bxe^{(2\,a)}+e^{(2\,a)})e^{(2\,bx)}+1}{b^2e^{(4\,bx+4\,a)}+2\,b^2e^{(2\,bx+2\,a)}+b^2}-\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}+\frac{bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)}{b^2}+\frac{bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
[Out] ((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) + 1)/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2
*b*x + 2*a) + b^2) - 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
+ 2*a)))/b^2 + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*
log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int x\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3\,dx$$

```
[In] integrate(x*csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x*csch(b*x + a)*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x}{\cosh{(a+b\,x)}^3\,\sinh{(a+b\,x)}}\,dx$$

```
[In] int(x/(cosh(a + b*x)^3*sinh(a + b*x)),x)
[Out] int(x/(cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.484 $\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2700, 14}

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

```
[Out] Log[Tanh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, i\tanh(a+bx)\right)}{b}\\ &= \frac{\log(\tanh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{2\log(\cosh(a+bx)) - 2\log(\sinh(a+bx)) - \operatorname{sech}^2(a+bx)}{2b}$$

```
[In] Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
[Out] -1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
default	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
risch	$\frac{2\,e^{2bx+2a}}{b\left(1+e^{2bx+2a}\right)^2} - \frac{\ln(1+e^{2bx+2a})}{b} + \frac{\ln(e^{2bx+2a}-1)}{b}$	62

```
[In] int(csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 13.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 -\right.\right.}{}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + s
inh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a
)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x
 + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)
*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)
 + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x + a
)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)
*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] 2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b
))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

3.485 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 47.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^{3}}{x}dx$$

```
[In] int(csch(b*x+a)*sech(b*x+a)^3/x,x)
[Out] int(csch(b*x+a)*sech(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)\operatorname{sech}\left(bx+a\right)^{3}}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3/x,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 8.17

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] ((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^2*e^(4*b*x + 4*a) + 2*b^2*
x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/8*(b^2*x^2 - 1)/(b^2*x^3*e^
(2*b*x + 2*a) + b^2*x^3), x) - 16*integrate(1/16/(x*e^(b*x + a) + x), x) +
16*integrate(1/16/(x*e^(b*x + a) - x), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{1}{x\cosh(a+b\,x)^3\sinh(a+b\,x)}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.486 $\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 26.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2,x]
[Out] Integrate[(Csch[a + b*x]*Sech[a + b*x]^3)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}dx$$

```
[In] int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
[Out] int(csch(b*x+a)*sech(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)**3/x**2,x)
[Out] Integral(csch(a + b*x)*sech(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 8.61

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) - 1)/(b^2*x^3*e^(4*b*x + 4*a) + 2*b^2*
x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/8*(b^2*x^2 - 3)/(b^2*x^4*e^
(2*b*x + 2*a) + b^2*x^4), x) - 16*integrate(1/16/(x^2*e^(b*x + a) + x^2), x
) + 16*integrate(1/16/(x^2*e^(b*x + a) - x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)*sech(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{1}{x^2\cosh(a+b\,x)^3\sinh(a+b\,x)}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)), x)
```

3.487 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 45.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x],x]
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a),x)
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^2 \operatorname{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^2 \operatorname{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.488 $\int x^3 \mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 237

$$\int x^3\mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx=-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{6x^2\mathrm{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^3\mathrm{csch}(a+bx)}{b}-\frac{6x\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{3ix^2\,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix^2\,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{6x\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^3}+\frac{6\,\mathrm{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6ix\,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{6ix\,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}-\frac{6\,\mathrm{PolyLog}\left(3,e^{a+bx}\right)}{b^4}+\frac{6i\,\mathrm{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4}-\frac{6i\,\mathrm{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}$$

```
[Out] -2*x^3*arctan(exp(b*x+a))/b-6*x^2*arctanh(exp(b*x+a))/b^2-x^3*csch(b*x+a)/b
-6*x*polylog(2,-exp(b*x+a))/b^3+3*I*x^2*polylog(2,-I*exp(b*x+a))/b^2-3*I*x^
2*polylog(2,I*exp(b*x+a))/b^2+6*x*polylog(2,exp(b*x+a))/b^3+6*polylog(3,-ex
p(b*x+a))/b^4-6*I*x*polylog(3,-I*exp(b*x+a))/b^3+6*I*x*polylog(3,I*exp(b*x+
a))/b^3-6*polylog(3,exp(b*x+a))/b^4+6*I*polylog(4,-I*exp(b*x+a))/b^4-6*I*po
lylog(4,I*exp(b*x+a))/b^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {2701, 327, 213, 5570, 14, 5313, 12, 4265, 2611, 6744, 2320, 6724, 4267}

$$\begin{aligned}\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = &-\frac{2x^3\arctan\left(e^{a+bx}\right)}{b}-\frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2}\\&+\frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4}-\frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4}\\&+\frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4}-\frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}\\&-\frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}\\&-\frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}\\&+\frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}\\&-\frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{x^3\text{csch}(a+bx)}{b}\end{aligned}$$

```
[In] Int[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x^3*ArcTan[E^(a + b*x)])/b - (6*x^2*ArcTanh[E^(a + b*x)])/b^2 - (x^3*Cs
ch[a + b*x])/b - (6*x*PolyLog[2, -E^(a + b*x)])/b^3 + ((3*I)*x^2*PolyLog[2,
 (-I)*E^(a + b*x)])/b^2 - ((3*I)*x^2*PolyLog[2, I*E^(a + b*x)])/b^2 + (6*x*
PolyLog[2, E^(a + b*x)])/b^3 + (6*PolyLog[3, -E^(a + b*x)])/b^4 - ((6*I)*x*
PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[3, I*E^(a + b*x)])/b^3
 - (6*PolyLog[3, E^(a + b*x)])/b^4 + ((6*I)*PolyLog[4, (-I)*E^(a + b*x)])/b
^4 - ((6*I)*PolyLog[4, I*E^(a + b*x)])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3 \arctan(\sinh(a+bx))}{b} - \frac{x^3 \operatorname{csch}(a+bx)}{b} \\
&\quad - 3\int x^2 \left(-\frac{\arctan(\sinh(a+bx))}{b} - \frac{\operatorname{csch}(a+bx)}{b}\right) dx \\
&= -\frac{x^3 \arctan(\sinh(a+bx))}{b} - \frac{x^3 \operatorname{csch}(a+bx)}{b} \\
&\quad - 3\int \left(-\frac{x^2 \arctan(\sinh(a+bx))}{b} - \frac{x^2 \operatorname{csch}(a+bx)}{b}\right) dx
\end{aligned}$$

$$= -\frac{x^3\arctan(\sinh(a+bx))}{b} - \frac{x^3\text{csch}(a+bx)}{b} + \frac{3\int x^2\arctan(\sinh(a+bx))\,dx}{b} + \frac{3\int x^2\text{csch}(a+bx)\,dx}{b}$$

$$= -\frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6\int x\log\left(1-e^{a+bx}\right)\,dx}{b^2} + \frac{6\int x\log\left(1+e^{a+bx}\right)\,dx}{b^2} - \frac{\int bx^3\text{sech}(a+bx)\,dx}{b}$$

$$= -\frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\int\text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^3} - \frac{6\int\text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^3} - \int x^3\text{sech}(a+bx)\,dx$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{6\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{(3i)\int x^2\log\left(1-ie^{a+bx}\right)\,dx}{b} - \frac{(3i)\int x^2\log\left(1+ie^{a+bx}\right)\,dx}{b}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4} - \frac{(6i)\int x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} + \frac{(6i)\int x\,\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4} + \frac{(6i)\int\text{PolyLog}\left(3,-ie^{a+bx}\right)\,dx}{b^3} - \frac{(6i)\int\text{PolyLog}\left(3,ie^{a+bx}\right)\,dx}{b^3}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4} + \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{(6i)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4}$$

$$= -\frac{2x^3\arctan\left(e^{a+bx}\right)}{b} - \frac{6x^2\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{x^3\text{csch}(a+bx)}{b} - \frac{6x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{6x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} + \frac{6\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^4} - \frac{6ix\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{6ix\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{6\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^4} + \frac{6i\,\text{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} - \frac{6i\,\text{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.41

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \frac{-2b^3x^3\text{csch}(a) + 6b^2x^2\log\left(1-e^{a+bx}\right) - 2ib^3x^3\log\left(1-ie^{a+bx}\right) + 2ib^3x^3\log\left(1+ie^{a+bx}\right) - 6b^2x^2\log\left(1+\right.}{}$$

```
[In] Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*b^3*x^3*Csch[a] + 6*b^2*x^2*Log[1 - E^(a + b*x)] - (2*I)*b^3*x^3*Log[1
- I*E^(a + b*x)] + (2*I)*b^3*x^3*Log[1 + I*E^(a + b*x)] - 6*b^2*x^2*Log[1 +
 E^(a + b*x)] - 12*b*x*PolyLog[2, -E^(a + b*x)] + (6*I)*b^2*x^2*PolyLog[2,
(-I)*E^(a + b*x)] - (6*I)*b^2*x^2*PolyLog[2, I*E^(a + b*x)] + 12*b*x*PolyLo
g[2, E^(a + b*x)] + 12*PolyLog[3, -E^(a + b*x)] - (12*I)*b*x*PolyLog[3, (-I
)*E^(a + b*x)] + (12*I)*b*x*PolyLog[3, I*E^(a + b*x)] - 12*PolyLog[3, E^(a
+ b*x)] + (12*I)*PolyLog[4, (-I)*E^(a + b*x)] - (12*I)*PolyLog[4, I*E^(a +
b*x)] + b^3*x^3*Csch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] + b^3*x^3*Sech[a/
2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2])/(2*b^4)
```

Maple [F]

$$\int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

```
[In] int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
[Out] int(x^3*csch(b*x+a)^2*sech(b*x+a),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1309 vs. $2(197) = 394$.

Time = 0.31 (sec) , antiderivative size = 1309, normalized size of antiderivative = 5.52

$$\int x^3 \text{csch}^2(a+bx)\text{sech}(a+bx)\, dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
[Out] -(2*b^3*x^3*cosh(b*x + a) + 2*b^3*x^3*sinh(b*x + a) - 6*(b*x*cosh(b*x + a)^
2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(co
sh(b*x + a) + sinh(b*x + a)) + 3*(I*b^2*x^2*cosh(b*x + a)^2 + 2*I*b^2*x^2*c
osh(b*x + a)*sinh(b*x + a) + I*b^2*x^2*sinh(b*x + a)^2 - I*b^2*x^2)*dilog(I
*cosh(b*x + a) + I*sinh(b*x + a)) + 3*(-I*b^2*x^2*cosh(b*x + a)^2 - 2*I*b^2
*x^2*cosh(b*x + a)*sinh(b*x + a) - I*b^2*x^2*sinh(b*x + a)^2 + I*b^2*x^2)*d
ilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^2 + 2*b*x*c
osh(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a
) - sinh(b*x + a)) + 3*(b^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*s
inh(b*x + a) + b^2*x^2*sinh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(
b*x + a) + 1) - (I*a^3*cosh(b*x + a)^2 + 2*I*a^3*cosh(b*x + a)*sinh(b*x + a
) + I*a^3*sinh(b*x + a)^2 - I*a^3)*log(cosh(b*x + a) + sinh(b*x + a) + I) -
 (-I*a^3*cosh(b*x + a)^2 - 2*I*a^3*cosh(b*x + a)*sinh(b*x + a) - I*a^3*sinh
(b*x + a)^2 + I*a^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) - 3*(a^2*cosh(b
*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 - a^2)*
log(cosh(b*x + a) + sinh(b*x + a) - 1) - (-I*b^3*x^3 - I*a^3 + (I*b^3*x^3 +
 I*a^3)*cosh(b*x + a)^2 - 2*(-I*b^3*x^3 - I*a^3)*cosh(b*x + a)*sinh(b*x + a
) + (I*b^3*x^3 + I*a^3)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x +
 a) + 1) - (I*b^3*x^3 + I*a^3 + (-I*b^3*x^3 - I*a^3)*cosh(b*x + a)^2 - 2*(I
*b^3*x^3 + I*a^3)*cosh(b*x + a)*sinh(b*x + a) + (-I*b^3*x^3 - I*a^3)*sinh(b
*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 3*(b^2*x^2 - (b^2*
x^2 - a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
- (b^2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a)
 + 1) + 6*(I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x + a) + I*sinh(b*x
 + a)^2 - I)*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(-I*cosh(b*x
 + a)^2 - 2*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 + I)*polylog(
```

```
4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x + a)^2 + 2*cosh(b*x +
a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, cosh(b*x + a) + sinh(b*x
 + a)) + 6*(-I*b*x*cosh(b*x + a)^2 - 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) -
I*b*x*sinh(b*x + a)^2 + I*b*x)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)
) + 6*(I*b*x*cosh(b*x + a)^2 + 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) + I*b*x*
sinh(b*x + a)^2 - I*b*x)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 6
*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*po
lylog(3, -cosh(b*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh
(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 - b^4)
```

Sympy [F]

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^3\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)**2*sech(b*x+a),x)
[Out] Integral(x**3*csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^3\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
[Out] -2*x^3*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 3*(b^2*x^2*log(e^(b*x + a) + 1
) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^
2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))/b^4 - 8*integrate(1/4*x^3*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x^3\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^3\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*csch(b*x + a)^2*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx = \int \frac{x^3}{\cosh(a+b\,x)\,\sinh(a+b\,x)^2}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.489 $\int x^2\mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int x^2\mathrm{csch}^2(a+bx)\mathrm{sech}(a+bx)\,dx=-\frac{2x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{4x\mathrm{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\mathrm{csch}(a+bx)}{b}-\frac{2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2ix\,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{2ix\,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{2i\,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{2i\,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}$$

```
[Out] -2*x^2*arctan(exp(b*x+a))/b-4*x*arctanh(exp(b*x+a))/b^2-x^2*csch(b*x+a)/b-2
*polylog(2,-exp(b*x+a))/b^3+2*I*x*polylog(2,-I*exp(b*x+a))/b^2-2*I*x*polylo
g(2,I*exp(b*x+a))/b^2+2*polylog(2,exp(b*x+a))/b^3-2*I*polylog(3,-I*exp(b*x+
a))/b^3+2*I*polylog(3,I*exp(b*x+a))/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules

used = {2701, 327, 213, 5570, 14, 5313, 12, 4265, 2611, 2320, 6724, 4267, 2317, 2438}

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2x^2\arctan\left(e^{a+bx}\right)}{b} - \frac{4x\operatorname{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{2\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{2\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^3} - \frac{2i\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{2i\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{2ix\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{2ix\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} - \frac{x^2\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x^2*ArcTan[E^(a + b*x)])/b - (4*x*ArcTanh[E^(a + b*x)])/b^2 - (x^2*Csch
[a + b*x])/b - (2*PolyLog[2, -E^(a + b*x)])/b^3 + ((2*I)*x*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 - ((2*I)*x*PolyLog[2, I*E^(a + b*x)])/b^2 + (2*PolyLog[2,
 E^(a + b*x)])/b^3 - ((2*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 + ((2*I)*Poly
Log[3, I*E^(a + b*x)])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b} \\
&\quad -2\int x\left(-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\text{csch}(a+bx)}{b}\right)\,dx \\
&= -\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b} \\
&\quad -2\int\left(-\frac{x\arctan(\sinh(a+bx))}{b}-\frac{x\text{csch}(a+bx)}{b}\right)\,dx \\
&= -\frac{x^2\arctan(\sinh(a+bx))}{b}-\frac{x^2\text{csch}(a+bx)}{b} \\
&\quad +\frac{2\int x\arctan(\sinh(a+bx))\,dx}{b}+\frac{2\int x\text{csch}(a+bx)\,dx}{b} \\
&= -\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{csch}(a+bx)}{b}-\frac{2\int\log\left(1-e^{a+bx}\right)\,dx}{b^2} \\
&\quad +\frac{2\int\log\left(1+e^{a+bx}\right)\,dx}{b^2}-\frac{\int bx^2\text{sech}(a+bx)\,dx}{b}
\end{aligned}$$

$$= -\frac{4x\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \frac{2\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \int x^2\text{sech}(a+bx)\,dx$$

$$= -\frac{2x^2\arctan(e^{a+bx})}{b} - \frac{4x\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{2\,\text{PolyLog}(2,e^{a+bx})}{b^3} + \frac{(2i)\int x\log(1-ie^{a+bx})\,dx}{b} - \frac{(2i)\int x\log(1+ie^{a+bx})\,dx}{b}$$

$$= -\frac{2x^2\arctan(e^{a+bx})}{b} - \frac{4x\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{2ix\,\text{PolyLog}(2,-ie^{a+bx})}{b^2} - \frac{2ix\,\text{PolyLog}(2,ie^{a+bx})}{b^2} + \frac{2\,\text{PolyLog}(2,e^{a+bx})}{b^3} - \frac{(2i)\int \text{PolyLog}(2,-ie^{a+bx})\,dx}{b^2} + \frac{(2i)\int \text{PolyLog}(2,ie^{a+bx})\,dx}{b^2}$$

$$= -\frac{2x^2\arctan(e^{a+bx})}{b} - \frac{4x\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{2ix\,\text{PolyLog}(2,-ie^{a+bx})}{b^2} - \frac{2ix\,\text{PolyLog}(2,ie^{a+bx})}{b^2} + \frac{2\,\text{PolyLog}(2,e^{a+bx})}{b^3} - \frac{(2i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \frac{(2i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3}$$

$$= -\frac{2x^2\arctan(e^{a+bx})}{b} - \frac{4x\text{arctanh}(e^{a+bx})}{b^2} - \frac{x^2\text{csch}(a+bx)}{b} - \frac{2\,\text{PolyLog}(2,-e^{a+bx})}{b^3} + \frac{2ix\,\text{PolyLog}(2,-ie^{a+bx})}{b^2} - \frac{2ix\,\text{PolyLog}(2,ie^{a+bx})}{b^2} + \frac{2\,\text{PolyLog}(2,e^{a+bx})}{b^3} - \frac{2i\,\text{PolyLog}(3,-ie^{a+bx})}{b^3} + \frac{2i\,\text{PolyLog}(3,ie^{a+bx})}{b^3}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.61

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \frac{-2b^2x^2\text{csch}(a) + 4bx\log(1-e^{a+bx}) - 2ib^2x^2\log(1-ie^{a+bx}) + 2ib^2x^2\log(1+ie^{a+bx}) - 4bx\log(1+e^{a+b}}{}$$

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*b^2*x^2*Csch[a] + 4*b*x*Log[1 - E^(a + b*x)] - (2*I)*b^2*x^2*Log[1 - I*
E^(a + b*x)] + (2*I)*b^2*x^2*Log[1 + I*E^(a + b*x)] - 4*b*x*Log[1 + E^(a +
b*x)] - 4*PolyLog[2, -E^(a + b*x)] + (4*I)*b*x*PolyLog[2, (-I)*E^(a + b*x)]
```

```
 - (4*I)*b*x*PolyLog[2, I*E^(a + b*x)] + 4*PolyLog[2, E^(a + b*x)] - (4*I)*
PolyLog[3, (-I)*E^(a + b*x)] + (4*I)*PolyLog[3, I*E^(a + b*x)] + b^2*x^2*Cs
ch[a/2]*Csch[(a + b*x)/2]*Sinh[(b*x)/2] + b^2*x^2*Sech[a/2]*Sech[(a + b*x)/
2]*Sinh[(b*x)/2])/(2*b^3)
```

Maple [F]

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)\, dx$$

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

```
[Out] int(x^2*csch(b*x+a)^2*sech(b*x+a),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(129) = 258$.

Time = 0.28 (sec) , antiderivative size = 966, normalized size of antiderivative = 6.15

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b^2*x^2*cosh(b*x + a) + 2*b^2*x^2*sinh(b*x + a) - 2*(cosh(b*x + a)^2 +
2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) +
sinh(b*x + a)) + 2*(I*b*x*cosh(b*x + a)^2 + 2*I*b*x*cosh(b*x + a)*sinh(b*x
+ a) + I*b*x*sinh(b*x + a)^2 - I*b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x +
a)) + 2*(-I*b*x*cosh(b*x + a)^2 - 2*I*b*x*cosh(b*x + a)*sinh(b*x + a) - I*b
*x*sinh(b*x + a)^2 + I*b*x)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*dilo
g(-cosh(b*x + a) - sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x
 + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) - (-I*a^2*cosh(b*x + a)^2 - 2*I*a^2*cosh(b*x + a)*sinh(b*x + a
) - I*a^2*sinh(b*x + a)^2 + I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) -
 (I*a^2*cosh(b*x + a)^2 + 2*I*a^2*cosh(b*x + a)*sinh(b*x + a) + I*a^2*sinh(
b*x + a)^2 - I*a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 2*(a*cosh(b*x
+ a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a) + a*sinh(b*x + a)^2 - a)*log(cosh(
b*x + a) + sinh(b*x + a) - 1) - (-I*b^2*x^2 + (I*b^2*x^2 - I*a^2)*cosh(b*x
+ a)^2 - 2*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a) + (I*b^2*x^2 -
I*a^2)*sinh(b*x + a)^2 + I*a^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1)
- (I*b^2*x^2 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 - 2*(I*b^2*x^2 - I*a^2)
*cosh(b*x + a)*sinh(b*x + a) + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a)^2 - I*a^2
)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*((b*x + a)*cosh(b*x + a)^
```

```
2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a)*sinh(b*x + a)^2 - b
*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(-I*cosh(b*x + a)^2 - 2
*I*cosh(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 + I)*polylog(3, I*cosh(b
*x + a) + I*sinh(b*x + a)) + 2*(I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(
b*x + a) + I*sinh(b*x + a)^2 - I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x
+ a)))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(
b*x + a)^2 - b^3)
```

Sympy [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}^2\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a),x)
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\,\,dx$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
[Out] -2*x^2*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - 2*(b*x*log(e^(b*x + a) + 1) +
dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)
))/b^3 - 8*integrate(1/4*x^2*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)\,\,dx$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.490 $\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}$$

```
[Out] -2*x*arctan(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^2-x*csch(b*x+a)/b+I*polylo
g(2,-I*exp(b*x+a))/b^2-I*polylog(2,I*exp(b*x+a))/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2701, 327, 213, 5570, 5311, 12, 4265, 2317, 2438, 3855}

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}+\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{x\text{csch}(a+bx)}{b}$$

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (-2*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (x*Csch[a + b*x
])/b + (I*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (I*PolyLog[2, I*E^(a + b*x)])
/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\arctan(\sinh(a+bx))}{b}-\frac{x\text{csch}(a+bx)}{b} \\
&\quad -\int\left(-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\text{csch}(a+bx)}{b}\right)dx \\
&= -\frac{x\arctan(\sinh(a+bx))}{b}-\frac{x\text{csch}(a+bx)}{b}+\frac{\int\arctan(\sinh(a+bx))\,dx}{b}+\frac{\int\text{csch}(a+bx)\,dx}{b} \\
&= -\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}-\frac{\int bx\text{sech}(a+bx)\,dx}{b} \\
&= -\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b}-\int x\text{sech}(a+bx)\,dx \\
&= -\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b} \\
&\quad +\frac{i\int\log\left(1-ie^{a+bx}\right)\,dx}{b}-\frac{i\int\log\left(1+ie^{a+bx}\right)\,dx}{b} \\
&= -\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b} \\
&\quad +\frac{i\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^2}-\frac{i\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{a+bx}\right)}{b^2} \\
&= -\frac{2x\arctan\left(e^{a+bx}\right)}{b}-\frac{\text{arctanh}(\cosh(a+bx))}{b^2}-\frac{x\text{csch}(a+bx)}{b} \\
&\quad +\frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(79) = 158.

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.33

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$
$$= \frac{4a\arctan\left(e^{a+bx}\right) - bx\coth\left(\frac{1}{2}(a+bx)\right) - 2ia\log\left(1-ie^{a+bx}\right) - 2ibx\log\left(1-ie^{a+bx}\right) + 2ia\log\left(1+ie^{a+b}\right.}{}$$

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] (4*a*ArcTan[E^(a + b*x)] - b*x*Coth[(a + b*x)/2] - (2*I)*a*Log[1 - I*E^(a +
 b*x)] - (2*I)*b*x*Log[1 - I*E^(a + b*x)] + (2*I)*a*Log[1 + I*E^(a + b*x)]
+ (2*I)*b*x*Log[1 + I*E^(a + b*x)] - 2*Log[Cosh[(a + b*x)/2]] + 2*Log[Sinh[
(a + b*x)/2]] + (2*I)*PolyLog[2, (-I)*E^(a + b*x)] - (2*I)*PolyLog[2, I*E^(
a + b*x)] + b*x*Tanh[(a + b*x)/2])/(2*b^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(72) = 144.

Time = 0.99 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{2x\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{2a\arctan\left(e^{bx+a}\right)}{b^2}+\frac{i\,\text{dilog}\left(1+ie^{bx+a}\right)}{b^2}-\frac{i\ln\left(1-ie^{bx+a}\right)x}{b}-\frac{i\ln\left(1-ie^{bx+a}\right)a}{b^2}+\frac{i\ln\left(1+ie^{bx+a}\right)x}{b}+\frac{i\ln}{}$

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*x*exp(b*x+a)/(exp(2*b*x+2*a)-1)+2/b^2*a*arctan(exp(b*x+a))+I/b^2*dilog
(1+I*exp(b*x+a))-I/b*ln(1-I*exp(b*x+a))*x-I/b^2*ln(1-I*exp(b*x+a))*a+I/b*ln
(1+I*exp(b*x+a))*x+I/b^2*ln(1+I*exp(b*x+a))*a-I/b^2*dilog(1-I*exp(b*x+a))+1
/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b*x+a)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(66) = 132.

Time = 0.28 (sec) , antiderivative size = 567, normalized size of antiderivative = 7.18

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx =$$

$$-\frac{2\,bx\cosh{(bx+a)}+2\,bx\sinh{(bx+a)}-\left(-i\,\cosh{(bx+a)}^2-2i\,\cosh{(bx+a)}\sinh{(bx+a)}-i\,\sinh{(bx}}{}$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(b*x + a) + 2*b*x*sinh(b*x + a) - (-I*cosh(b*x + a)^2 - 2*I*cos
h(b*x + a)*sinh(b*x + a) - I*sinh(b*x + a)^2 + I)*dilog(I*cosh(b*x + a) + I
*sinh(b*x + a)) - (I*cosh(b*x + a)^2 + 2*I*cosh(b*x + a)*sinh(b*x + a) + I*
sinh(b*x + a)^2 - I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x
+ a) + sinh(b*x + a) + 1) - (I*a*cosh(b*x + a)^2 + 2*I*a*cosh(b*x + a)*sinh
(b*x + a) + I*a*sinh(b*x + a)^2 - I*a)*log(cosh(b*x + a) + sinh(b*x + a) +
I) - (-I*a*cosh(b*x + a)^2 - 2*I*a*cosh(b*x + a)*sinh(b*x + a) - I*a*sinh(b
*x + a)^2 + I*a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (cosh(b*x + a)^2
+ 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) - ((I*b*x + I*a)*cosh(b*x + a)^2 - 2*(-I*b*x - I*a)*cosh
(b*x + a)*sinh(b*x + a) + (I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x - I*a)*log(
I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((-I*b*x - I*a)*cosh(b*x + a)^2 -
2*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a) + (-I*b*x - I*a)*sinh(b*x + a)^
2 + I*b*x + I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^2*cosh(b*x
 + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x\operatorname{csch}^2{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a),x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\left(bx+a\right)^2\text{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*x*e^(b*x + a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^
2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 - 8*integrate(1/4*x*e^(b*x + a)/(e^(2
*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\left(bx+a\right)^2\text{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^2*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x}{\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}\,dx$$

```
[In] int(x/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] int(x/(cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.491 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[Out] -arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2701, 327, 213}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\operatorname{csch}(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

```
[Out] -(ArcTan[Sinh[a + b*x]]/b) - Csch[a + b*x]/b
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= -\frac{\text{csch}(a+bx)}{b} - \frac{i\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}\\ &= -\frac{\arctan(\sinh(a+bx))}{b} - \frac{\text{csch}(a+bx)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{\text{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	58

```
[In] int(csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx =$$
$$-\frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\arctan\left(\cosh(bx+a)+\sinh(bx\right.\right.}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
[Out] -2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(b*
cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a),x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
[Out] 2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\pi + \frac{4}{e^{(bx+a)}-e^{(-bx-a)}} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{2\,b}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
[Out] -1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a) -
 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(2*a + 2*b*x) - 1))
```

3.492 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 31.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)}{x}dx$$

```
[In] int(csch(b*x+a)^2*sech(b*x+a)/x,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)/x,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.89

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -2*e^(b*x + a)/(b*x*e^(2*b*x + 2*a) - b*x) - 8*integrate(1/4*e^(b*x + a)/(x
*e^(2*b*x + 2*a) + x), x) - 8*integrate(1/8/(b*x^2*e^(b*x + a) + b*x^2), x)
 - 8*integrate(1/8/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.493 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 24.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)}{x^{2}}dx$$

```
[In] int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)/x**2,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 6.33

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="maxima")
[Out] -2*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) - 8*integrate(1/4*e^(b*x + a
)/(x^2*e^(2*b*x + 2*a) + x^2), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) + b*
x^3), x) - 8*integrate(1/4/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)/x^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^2}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^2),x)
[Out] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^2), x)
```

3.494 $\int x^m \mathbf{csch}^2(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}\left(bx+a\right)^2 \operatorname{sech}\left(bx+a\right)^2 dx$$

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}\left(bx+a\right)^2 \operatorname{sech}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**2,x)
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2 \, dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \frac{x^m}{\cosh(a+b\,x)^2 \sinh(a+b\,x)^2}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.495 $\int x^3 \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int x^3 \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = -\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}+\frac{3x\,\mathrm{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\,\mathrm{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}$$

```
[Out] -2*x^3/b-2*x^3*coth(2*b*x+2*a)/b+3*x^2*ln(1-exp(4*b*x+4*a))/b^2+3/2*x*polyl
og(2,exp(4*b*x+4*a))/b^3-3/8*polylog(3,exp(4*b*x+4*a))/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5569, 4269, 3797, 2221, 2611, 2320, 6724}

$$\int x^3 \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = -\frac{3\,\mathrm{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}+\frac{3x\,\mathrm{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2x^3\coth(2a+2bx)}{b}-\frac{2x^3}{b}$$

```
[In] Int[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x^3)/b - (2*x^3*Coth[2*a + 2*b*x])/b + (3*x^2*Log[1 - E^(4*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(4*(a + b*x))])/(2*b^3) - (3*PolyLog[3, E^(4*(a +
b*x))])/(8*b^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 4\int x^3\text{csch}^2(2a+2bx)\,dx \\
&= -\frac{2x^3\coth(2a+2bx)}{b}+\frac{6\int x^2\coth(2a+2bx)\,dx}{b} \\
&= -\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}-\frac{12\int\frac{e^{2(2a+2bx)}x^2}{1-e^{2(2a+2bx)}}\,dx}{b} \\
&= -\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{6\int x\log\left(1-e^{2(2a+2bx)}\right)\,dx}{b^2} \\
&= -\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} \\
&\quad+\frac{3x\,\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\int\text{PolyLog}\left(2,e^{2(2a+2bx)}\right)\,dx}{2b^3} \\
&= -\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} \\
&\quad+\frac{3x\,\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2(2a+2bx)}\right)}{8b^4} \\
&= -\frac{2x^3}{b}-\frac{2x^3\coth(2a+2bx)}{b}+\frac{3x^2\log\left(1-e^{4(a+bx)}\right)}{b^2} \\
&\quad+\frac{3x\,\text{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}-\frac{3\,\text{PolyLog}\left(3,e^{4(a+bx)}\right)}{8b^4}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 307 vs. $2(85) = 170$.

Time = 1.48 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.61

$$
\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx
$$

$$
=4\left(-\frac{e^{4a}\left(8b^3e^{-4a}x^3-6b^2(1-e^{-4a})\,x^2\log\left(1-e^{-a-bx}\right)-6b^2(1-e^{-4a})\,x^2\log\left(1+e^{-a-bx}\right)-6b^2(1-e^{-4a}\right.}{}\right.
$$

$$
\left.+\frac{x^3\text{csch}(2a)\text{csch}(2a+2bx)\sinh(2bx)}{2b}\right)
$$

```
[In] Integrate[x^3*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] 4*(-1/8*(E^(4*a)*((8*b^3*x^3)/E^(4*a) - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 - E^
(-a - b*x)] - 6*b^2*(1 - E^(-4*a))*x^2*Log[1 + E^(-a - b*x)] - 6*b^2*(1 - E
```

```
^(-4*a))*x^2*Log[1 + E^(-2*(a + b*x))] + 12*b*(1 - E^(-4*a))*x*PolyLog[2, -
E^(-a - b*x)] + 12*b*(1 - E^(-4*a))*x*PolyLog[2, E^(-a - b*x)] + 6*b*(1 - E
^(-4*a))*x*PolyLog[2, -E^(-2*(a + b*x))] + 12*(1 - E^(-4*a))*PolyLog[3, -E^
(-a - b*x)] + 12*(1 - E^(-4*a))*PolyLog[3, E^(-a - b*x)] + 3*(1 - E^(-4*a))
*PolyLog[3, -E^(-2*(a + b*x))]))/(b^4*(-1 + E^(4*a))) + (x^3*Csch[2*a]*Csch
[2*a + 2*b*x]*Sinh[2*b*x])/(2*b))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(81) = 162.

Time = 6.98 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.09

method	result
risch	$-\frac{4x^3}{b\left(1+\mathrm{e}^{2bx+2a}\right)\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{8a^3}{b^4}+\frac{12x\,a^2}{b^3}-\frac{12a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^4}+\frac{3a^2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^4}-\frac{3\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^4}-\frac{4x^3}{b}-\frac{6\,\mathrm{polylo}}{}$

```
[In] int(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*x^3/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)+8/b^4*a^3+12*x/b^3*a^2-12/b^
4*a^2*ln(exp(b*x+a))+3/b^4*a^2*ln(exp(b*x+a)-1)-3/b^4*ln(1-exp(b*x+a))*a^2-
4*x^3/b-6*polylog(3,-exp(b*x+a))/b^4-6*polylog(3,exp(b*x+a))/b^4-3/2*polylo
g(3,-exp(2*b*x+2*a))/b^4+3*x^2*ln(1+exp(2*b*x+2*a))/b^2+3*x*polylog(2,-exp(
2*b*x+2*a))/b^3+3/b^2*ln(exp(b*x+a)+1)*x^2+6*x*polylog(2,-exp(b*x+a))/b^3+3
/b^2*ln(1-exp(b*x+a))*x^2+6*x*polylog(2,exp(b*x+a))/b^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1924, normalized size of antiderivative = 22.64

$$\int x^3 \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*(b^3*x^3 + a^3)*cosh(b*x + a)^4 + 16*(b^3*x^3 + a^3)*cosh(b*x + a)^3*si
nh(b*x + a) + 24*(b^3*x^3 + a^3)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 16*(b^3*
x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + 4*(b^3*x^3 + a^3)*sinh(b*x + a)^
4 - 4*a^3 - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) +
6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3
 + b*x*sinh(b*x + a)^4 - b*x)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x
*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a
)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x +
a)^4 - b*x)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)
^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x +
```

```
a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*di
log(-I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*co
sh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x
*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - b*x)*dilog(-cosh(b*x
 + a) - sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x +
a)^3*sinh(b*x + a) + 6*b^2*x^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^2*x^2*
cosh(b*x + a)*sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - b^2*x^2)*log(cosh
(b*x + a) + sinh(b*x + a) + 1) - 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x +
a)^3*sinh(b*x + a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x
 + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh
(b*x + a) + I) - 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)^3*sinh(b*x +
a) + 6*a^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + a^2*sinh(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) -
 3*(a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)^3*sinh(b*x + a) + 6*a^2*cosh(
b*x + a)^2*sinh(b*x + a)^2 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh
(b*x + a)^4 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2 - a
^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(
b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x
 + a)*sinh(b*x + a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*lo
g(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)
^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b^2*x^2 - a^2)*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x +
a)^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*log(-I*cosh(b*x + a
) - I*sinh(b*x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2
- a^2)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b^2*x^2 - a^2)*cosh(b*x + a)^2*si
nh(b*x + a)^2 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2
- a^2)*sinh(b*x + a)^4 - b^2*x^2 + a^2)*log(-cosh(b*x + a) - sinh(b*x + a)
+ 1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 -
1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x +
 a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*polylog(3, I*cosh(b*x + a) + I*s
inh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*co
sh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 - 1)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*polylog(3, -cosh(b
*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh(b*x + a)^3*sinh
(b*x + a) + 6*b^4*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^4*cosh(b*x + a)*sin
h(b*x + a)^3 + b^4*sinh(b*x + a)^4 - b^4)
```

Sympy [F]

$$\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int x^3 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)**2*sech(b*x+a)**2,x)
[Out] Integral(x**3*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(80) = 160$.

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\begin{aligned}\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx &= -\frac{4\,x^3}{be^{(4\,bx+4\,a)}-b}-\frac{4\,x^3}{b} \\ &\quad+\frac{3\,\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\text{Li}_3(-e^{(2\,bx+2\,a)})\right)}{2\,b^4} \\ &\quad+\frac{3\,\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\,\text{Li}_3(-e^{(bx+a)})\right)}{b^4} \\ &\quad+\frac{3\,\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\text{Li}_2\left(e^{(bx+a)}\right)-2\,\text{Li}_3(e^{(bx+a)})\right)}{b^4}\end{aligned}$$

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
[Out] -4*x^3/(b*e^(4*b*x + 4*a) - b) - 4*x^3/b + 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*
a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^4
 + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(
3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b
*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^2\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.496 $\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{2x^2}{b} - \frac{2x^2\coth(2a+2bx)}{b} + \frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{\operatorname{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3}$$

```
[Out] -2*x^2/b-2*x^2*coth(2*b*x+2*a)/b+2*x*ln(1-exp(4*b*x+4*a))/b^2+1/2*polylog(2
,exp(4*b*x+4*a))/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5569, 4269, 3797, 2221, 2317, 2438}

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\operatorname{PolyLog}\left(2,e^{4(a+bx)}\right)}{2b^3} + \frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2} - \frac{2x^2\coth(2a+2bx)}{b} - \frac{2x^2}{b}$$

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x^2)/b - (2*x^2*Coth[2*a + 2*b*x])/b + (2*x*Log[1 - E^(4*(a + b*x))])/b
^2 + PolyLog[2, E^(4*(a + b*x))]/(2*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 4\int x^2\text{csch}^2(2a+2bx)\,dx\\
&= -\frac{2x^2\coth(2a+2bx)}{b}+\frac{4\int x\coth(2a+2bx)\,dx}{b}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}-\frac{8\int\frac{e^{2(2a+2bx)}x}{1-e^{2(2a+2bx)}}\,dx}{b}\\
&= -\frac{2x^2}{b}-\frac{2x^2\coth(2a+2bx)}{b}+\frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2}-\frac{2\int\log\left(1-e^{2(2a+2bx)}\right)\,dx}{b^2}
\end{aligned}$$

$$= -\frac{2x^2}{b} - \frac{2x^2\coth(2a+2bx)}{b} + \frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2(2a+2bx)}\right)}{2b^3}$$
$$= -\frac{2x^2}{b} - \frac{2x^2\coth(2a+2bx)}{b} + \frac{2x\log\left(1-e^{4(a+bx)}\right)}{b^2} + \frac{\text{PolyLog}\left(2, e^{4(a+bx)}\right)}{2b^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(64) = 128.

Time = 0.91 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.38

$$\int x^2\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$
$$= 4\left(-\frac{e^{4a}\left(4b^2e^{-4a}x^2 - 2b(1-e^{-4a})\,x\log\left(1-e^{-a-bx}\right) - 2b(1-e^{-4a})\,x\log\left(1+e^{-a-bx}\right) - 2b(1-e^{-4a})\,x\log\right.}{}\right.$$
$$\left. + \frac{x^2\text{csch}(2a)\text{csch}(2a+2bx)\sinh(2bx)}{2b}\right)$$

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] 4*(-1/4*(E^(4*a)*((4*b^2*x^2)/E^(4*a) - 2*b*(1 - E^(-4*a))*x*Log[1 - E^(-a
- b*x)] - 2*b*(1 - E^(-4*a))*x*Log[1 + E^(-a - b*x)] - 2*b*(1 - E^(-4*a))*x
*Log[1 + E^(-2*(a + b*x))] + 2*(1 - E^(-4*a))*PolyLog[2, -E^(-a - b*x)] + 2
*(1 - E^(-4*a))*PolyLog[2, E^(-a - b*x)] + (1 - E^(-4*a))*PolyLog[2, -E^(-2
*(a + b*x))]))/(b^3*(-1 + E^(4*a))) + (x^2*Csch[2*a]*Csch[2*a + 2*b*x]*Sinh
[2*b*x])/(2*b))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(62) = 124.

Time = 4.99 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.11

method	result
risch	$-\frac{4x^2}{b(1+e^{2bx+2a})(e^{2bx+2a}-1)} - \frac{4x^2}{b} - \frac{8ax}{b^2} - \frac{4a^2}{b^3} + \frac{2\ln(e^{bx+a}+1)x}{b^2} + \frac{2\,\text{polylog}(2,-e^{bx+a})}{b^3} + \frac{2\ln(1-e^{bx+a})x}{b^2} + \frac{2\ln(1-}{b}$

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*x^2/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)-4*x^2/b-8*a*x/b^2-4/b^3*a^2+
2/b^2*ln(exp(b*x+a)+1)*x+2*polylog(2,-exp(b*x+a))/b^3+2/b^2*ln(1-exp(b*x+a)
)*x+2/b^3*ln(1-exp(b*x+a))*a+2*polylog(2,exp(b*x+a))/b^3+2*x*ln(1+exp(2*b*x
+2*a))/b^2+polylog(2,-exp(2*b*x+2*a))/b^3+8/b^3*a*ln(exp(b*x+a))-2/b^3*a*ln
(exp(b*x+a)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1327, normalized size of antiderivative = 20.73

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(2*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)^3*s
inh(b*x + a) + 12*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 8*(b^2*
x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + 2*(b^2*x^2 - a^2)*sinh(b*x + a)^
4 + 2*a^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x
 + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(I*cosh(b*x + a) + I*sinh(b*x +
 a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)
^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)
*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*
sinh(b*x + a)^3 + sinh(b*x + a)^4 - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)
) - (b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*x*cosh
(b*x + a)^2*sinh(b*x + a)^2 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sin
h(b*x + a)^4 - b*x)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (a*cosh(b*x +
a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a
)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - a)*log(cosh(b
*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sin
h(b*x + a) + 6*a*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b
*x + a)^3 + a*sinh(b*x + a)^4 - a)*log(cosh(b*x + a) + sinh(b*x + a) - I) +
 (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)^3*sinh(b*x + a) + 6*a*cosh(b*x + a)
^2*sinh(b*x + a)^2 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4
- a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b*x + a)*cosh(b*x + a)^4 +
4*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b*x + a)*cosh(b*x + a)^2*sin
h(b*x + a)^2 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b
*x + a)^4 - b*x - a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b*x + a
)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a) + 6*(b*x + a)
*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b*x + a)*sinh(b*x + a)^4 - b*x - a)*log(-I*cosh(b*x + a) - I*sinh(b*x
+ a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)^3*sinh(b
*x + a) + 6*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*(b*x + a)*cosh(b*
x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - b*x - a)*log(-cosh(b*x
 + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)^3*si
nh(b*x + a) + 6*b^3*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^3*cosh(b*x + a)*s
inh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - b^3)
```

Sympy [F]

$$\int x^2\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^2\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**2,x)
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.84

$$\begin{aligned}\int x^2\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=&-\frac{4\,x^2}{be^{(4\,bx+4\,a)}-b}-\frac{4\,x^2}{b}\\&+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^3}\\&+\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3}\\&+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\operatorname{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}\end{aligned}$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
[Out] -4*x^2/(b*e^(4*b*x + 4*a) - b) - 4*x^2/b + (2*b*x*log(e^(2*b*x + 2*a) + 1)
+ dilog(-e^(2*b*x + 2*a)))/b^3 + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*
x + a)))/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int x^2\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \mathrm{csch}^2(a+bx)\mathrm{sech}^2(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.497 $\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=-\frac{2x\coth(2a+2bx)}{b}+\frac{\log(\sinh(2a+2bx))}{b^2}$$

```
[Out] -2*x*coth(2*b*x+2*a)/b+ln(sinh(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5569, 4269, 3556}

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=\frac{\log(\sinh(2a+2bx))}{b^2}-\frac{2x\coth(2a+2bx)}{b}$$

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*x*Coth[2*a + 2*b*x])/b + Log[Sinh[2*a + 2*b*x]]/b^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 4\int x\text{csch}^2(2a+2bx)\,dx\\ &= -\frac{2x\coth(2a+2bx)}{b}+\frac{2\int\coth(2a+2bx)\,dx}{b}\\ &= -\frac{2x\coth(2a+2bx)}{b}+\frac{\log(\sinh(2a+2bx))}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx=\frac{-2bx\coth(2(a+bx))+\log(\sinh(2(a+bx)))}{b^2}$$

```
[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^2,x]
[Out] (-2*b*x*Coth[2*(a + b*x)] + Log[Sinh[2*(a + b*x)]])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 3.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

method	result	size
risch	$-\frac{4x}{b}-\frac{4a}{b^2}-\frac{4x}{b\left(1+e^{2bx+2a}\right)\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{4bx+4a}-1\right)}{b^2}$	62

```
[In] int(x*csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -4*x/b-4*a/b^2-4*x/b/(1+exp(2*b*x+2*a))/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(4*b
*x+4*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 9.73

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{4\,bx\cosh\left(bx+a\right)^4+16\,bx\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+24\,bx\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2+16\,bx\cosh}{b^2\cosh\left(b\right.}$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(4*b*x*cosh(b*x + a)^4 + 16*b*x*cosh(b*x + a)^3*sinh(b*x + a) + 24*b*x*cos
h(b*x + a)^2*sinh(b*x + a)^2 + 16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 4*b*x
*sinh(b*x + a)^4 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*c
osh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
 + a)^4 - 1)*log(4*cosh(b*x + a)*sinh(b*x + a)/(cosh(b*x + a)^2 - 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2)))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh
(b*x + a)^3*sinh(b*x + a) + 6*b^2*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b^2*c
osh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - b^2)
```

Sympy [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \int x\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)\,dx$$

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(30) = 60$.

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.90

$$\begin{aligned}\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx &= -\frac{4\,xe^{(4\,bx+4\,a)}}{be^{(4\,bx+4\,a)}-b}+\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}\\&\quad+\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}+\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\end{aligned}$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*x*e^(4*b*x + 4*a)/(b*e^(4*b*x + 4*a) - b) + log((e^(b*x + a) + 1)*e^(-a)
)/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2 + log((e^(2*b*x + 2*a) + 1)*e^(-2
*a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.40

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx$$
$$= -\frac{4\,bxe^{(4\,bx+4\,a)} - e^{(4\,bx+4\,a)}\log\left(e^{(4\,bx+4\,a)}-1\right) + \log\left(e^{(4\,bx+4\,a)}-1\right)}{b^2e^{(4\,bx+4\,a)} - b^2}$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*b*x*e^(4*b*x + 4*a) - e^(4*b*x + 4*a)*log(e^(4*b*x + 4*a) - 1) + log(e^
(4*b*x + 4*a) - 1))/(b^2*e^(4*b*x + 4*a) - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int x\text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{4\,a}\,\mathrm{e}^{4\,b\,x}-1\right)}{b^2} - \frac{4\,x}{b} - \frac{4\,x}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

```
[In] int(x/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] log(exp(4*a)*exp(4*b*x) - 1)/b^2 - (4*x)/b - (4*x)/(b*(exp(4*a + 4*b*x) - 1
))
```

3.498 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{b}-\frac{\tanh(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b-tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 14}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx=-\frac{\tanh(a+bx)}{b}-\frac{\coth(a+bx)}{b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b) - Tanh[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= \frac{i\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \text{csch}^2(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{2\coth(2(a+bx))}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
[Out] (-2*Coth[2*(a + b*x)])/b
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
risch	$-\frac{4}{b\left(1+e^{2bx+2a}\right)\left(e^{2bx+2a}-1\right)}$	32

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{4}{b\cosh{(bx+a)}^4 + 4\,b\cosh{(bx+a)}^3\sinh{(bx+a)} + 6\,b\cosh{(bx+a)}^2\sinh{(bx+a)}^2 + 4\,b\cosh{(bx+a)}\,\mathrm{s}}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^2{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{4}{b(e^{(-4\,bx-4\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 4/(b*(e^(-4*b*x - 4*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b(e^{(4\,bx+4\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] -4/(b*(e^(4*b*x + 4*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
[Out] -4/(b*(exp(4*a + 4*b*x) - 1))
```

3.499 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = 4\mathrm{Int}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x},x\right)$$

```
[Out] 4*Unintegrable(csch(2*b*x+2*a)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]
```

```
[Out] 4*Defer[Int][Csch[2*a + 2*b*x]^2/x, x]
```

Rubi steps

$$\text{integral} = 4\int \frac{\operatorname{csch}^2(2a+2bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 24.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{2}}{x}dx$$

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{2}}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2/x,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -4/(b*x*e^(4*b*x + 4*a) - b*x) + 16*integrate(1/8/(b*x^2*e^(2*b*x + 2*a) +
b*x^2), x) + 16*integrate(1/16/(b*x^2*e^(b*x + a) + b*x^2), x) - 16*integra
te(1/16/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{1}{x\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.500 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = 4\text{Int}\left(\frac{\operatorname{csch}^2(2a+2bx)}{x^2},x\right)$$

```
[Out] 4*Unintegrable(csch(2*b*x+2*a)^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] 4*Defer[Int][Csch[2*a + 2*b*x]^2/x^2, x]
```

Rubi steps

$$\text{integral} = 4\int \frac{\operatorname{csch}^2(2a+2bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{2}}{x^{2}}dx$$

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**2/x**2,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4/(b*x^2*e^(4*b*x + 4*a) - b*x^2) + 16*integrate(1/4/(b*x^3*e^(2*b*x + 2*a
) + b*x^3), x) + 16*integrate(1/8/(b*x^3*e^(b*x + a) + b*x^3), x) - 16*inte
grate(1/8/(b*x^3*e^(b*x + a) - b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{1}{x^2\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^2), x)
```

3.501 $\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 60.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
[Out] Integrate[x^m*Csch[a + b*x]^2*Sech[a + b*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

```
[In] int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
[Out] int(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**m*csch(b*x+a)**2*sech(b*x+a)**3,x)
[Out] Integral(x**m*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^2 \operatorname{sech}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^2 \operatorname{sech}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
[Out] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.502 $\int x^2 \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 206

$$\begin{aligned}\int x^2\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx &= -\frac{3x^2\arctan\left(e^{a+bx}\right)}{b}+\frac{\arctan(\sinh(a+bx))}{b^3}\\ &\quad-\frac{4x\mathrm{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{3x^2\mathrm{csch}(a+bx)}{2b}\\ &\quad-\frac{2\,\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{3ix\,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}\\ &\quad-\frac{3ix\,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2\,\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^3}\\ &\quad-\frac{3i\,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{3i\,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}\\ &\quad-\frac{x\mathrm{sech}(a+bx)}{b^2}+\frac{x^2\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)}{2b}\end{aligned}$$

```
[Out] -3*x^2*arctan(exp(b*x+a))/b+arctan(sinh(b*x+a))/b^3-4*x*arctanh(exp(b*x+a))
/b^2-3/2*x^2*csch(b*x+a)/b-2*polylog(2,-exp(b*x+a))/b^3+3*I*x*polylog(2,-I*
exp(b*x+a))/b^2-3*I*x*polylog(2,I*exp(b*x+a))/b^2+2*polylog(2,exp(b*x+a))/b
^3-3*I*polylog(3,-I*exp(b*x+a))/b^3+3*I*polylog(3,I*exp(b*x+a))/b^3-x*sech(
b*x+a)/b^2+1/2*x^2*csch(b*x+a)*sech(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2701, 294, 327, 213, 5570, 14, 5313, 12, 4265, 2611, 2320, 6724, 4267, 2317, 2438, 2702, 6406, 3855}

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^3}-\frac{3x^2\arctan\left(e^{a+bx}\right)}{b}-\frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2}-\frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3}+\frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3}-\frac{3i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3}+\frac{3i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}+\frac{3ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}-\frac{3ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}-\frac{x\text{sech}(a+bx)}{b^2}-\frac{3x^2\text{csch}(a+bx)}{2b}+\frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

```
[In] Int[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x^2*ArcTan[E^(a + b*x)])/b + ArcTan[Sinh[a + b*x]]/b^3 - (4*x*ArcTanh[E
^(a + b*x)])/b^2 - (3*x^2*Csch[a + b*x])/(2*b) - (2*PolyLog[2, -E^(a + b*x)
])/b^3 + ((3*I)*x*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*x*PolyLog[2, I
*E^(a + b*x)])/b^2 + (2*PolyLog[2, E^(a + b*x)])/b^3 - ((3*I)*PolyLog[3, (-
I)*E^(a + b*x)])/b^3 + ((3*I)*PolyLog[3, I*E^(a + b*x)])/b^3 - (x*Sech[a +
b*x])/b^2 + (x^2*Csch[a + b*x]*Sech[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
```

```
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3x^2\arctan(\sinh(a+bx))}{2b}-\frac{3x^2\operatorname{csch}(a+bx)}{2b}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} \\
&\quad -2\int x\left(-\frac{3\arctan(\sinh(a+bx))}{2b}-\frac{3\operatorname{csch}(a+bx)}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}\right)dx \\
&= -\frac{3x^2\arctan(\sinh(a+bx))}{2b}-\frac{3x^2\operatorname{csch}(a+bx)}{2b}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} \\
&\quad -2\int\left(-\frac{3x(\arctan(\sinh(a+bx))+\operatorname{csch}(a+bx))}{2b}+\frac{x\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}\right)dx \\
&= -\frac{3x^2\arctan(\sinh(a+bx))}{2b}-\frac{3x^2\operatorname{csch}(a+bx)}{2b}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} \\
&\quad -\frac{\int x\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx}{b}+\frac{3\int x(\arctan(\sinh(a+bx))+\operatorname{csch}(a+bx))\,dx}{b} \\
&= -\frac{3x^2\arctan(\sinh(a+bx))}{2b}+\frac{x\operatorname{arctanh}(\cosh(a+bx))}{b^2} \\
&\quad -\frac{3x^2\operatorname{csch}(a+bx)}{2b}-\frac{x\operatorname{sech}(a+bx)}{b^2}+\frac{x^2\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b} \\
&\quad +\frac{\int\left(-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}\right)dx}{b} \\
&\quad +\frac{3\int(x\arctan(\sinh(a+bx))+x\operatorname{csch}(a+bx))\,dx}{b}
\end{aligned}$$

$$= -\frac{3x^2\arctan(\sinh(a+bx))}{2b} + \frac{x\text{arctanh}(\cosh(a+bx))}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{\int \text{arctanh}(\cosh(a+bx))\,dx}{b^2} + \frac{\int \text{sech}(a+bx)\,dx}{b^2} + \frac{3\int x\arctan(\sinh(a+bx))\,dx}{b} + \frac{3\int x\text{csch}(a+bx)\,dx}{b}$$

$$= \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{\int bx\text{csch}(a+bx)\,dx}{b^2} - \frac{3\int \log\left(1-e^{a+bx}\right)\,dx}{b^2} + \frac{3\int \log\left(1+e^{a+bx}\right)\,dx}{b^2} - \frac{3\int bx^2\text{sech}(a+bx)\,dx}{2b}$$

$$= \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{3}{2}\int x^2\text{sech}(a+bx)\,dx - \frac{3\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \frac{3\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{\int x\text{csch}(a+bx)\,dx}{b}$$

$$= -\frac{3x^2\arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} + \frac{\int \log\left(1-e^{a+bx}\right)\,dx}{b^2} - \frac{\int \log\left(1+e^{a+bx}\right)\,dx}{b^2} + \frac{(3i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b} - \frac{(3i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b}$$

$$= -\frac{3x^2\arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{b^3} - \frac{(3i)\int \text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^2} + \frac{(3i)\int \text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^2}$$

$$= -\frac{3x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3} + \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{a+bx}\right)}{b^3}$$

$$= -\frac{3x^2 \arctan\left(e^{a+bx}\right)}{b} + \frac{\arctan(\sinh(a+bx))}{b^3} - \frac{4x\text{arctanh}\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\text{csch}(a+bx)}{2b} - \frac{2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^3} + \frac{3ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} - \frac{3ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^3} - \frac{3i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} + \frac{3i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{x\text{sech}(a+bx)}{b^2} + \frac{x^2\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 4.86 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.50

$$\int x^2\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \frac{4\arctan\left(e^{a+bx}\right) - 2b^2x^2\text{csch}(a) + 4bx\log\left(1-e^{a+bx}\right) - 3ib^2x^2\log\left(1-ie^{a+bx}\right) + 3ib^2x^2\log\left(1+ie^{a+bx}\right)}{}$$

```
[In] Integrate[x^2*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (4*ArcTan[E^(a + b*x)] - 2*b^2*x^2*Csch[a] + 4*b*x*Log[1 - E^(a + b*x)] - (
3*I)*b^2*x^2*Log[1 - I*E^(a + b*x)] + (3*I)*b^2*x^2*Log[1 + I*E^(a + b*x)]
- 4*b*x*Log[1 + E^(a + b*x)] - 4*PolyLog[2, -E^(a + b*x)] + (6*I)*b*x*PolyL
og[2, (-I)*E^(a + b*x)] - (6*I)*b*x*PolyLog[2, I*E^(a + b*x)] + 4*PolyLog[2
, E^(a + b*x)] - (6*I)*PolyLog[3, (-I)*E^(a + b*x)] + (6*I)*PolyLog[3, I*E^
(a + b*x)] - 2*b*x*Sech[a + b*x] + b^2*x^2*Csch[a/2]*Csch[(a + b*x)/2]*Sinh
[(b*x)/2] + b^2*x^2*Sech[a/2]*Sech[(a + b*x)/2]*Sinh[(b*x)/2] - b^2*x^2*Sec
h[a]*Sech[a + b*x]^2*Sinh[b*x] - b^2*x^2*Sech[a + b*x]*Tanh[a])/(2*b^3)
```

Maple [F]

$$\int x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)^3 \, dx$$

```
[In] int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

```
[Out] int(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3825 vs. $2(174) = 348$.

Time = 0.33 (sec) , antiderivative size = 3825, normalized size of antiderivative = 18.57

$$\int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^2*x^2*cosh(b*x + a)^3 + 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 + 1
0*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(3*b^2*x^2 + 2*b*x)
*sinh(b*x + a)^5 + 4*(b^2*x^2 + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*sinh
(b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x +
 a)^3)*sinh(b*x + a)^2 + 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 4*(cosh(b*x
+ a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x +
 a)^2 + 1)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(
b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) +
6*(I*b*x*cosh(b*x + a)^6 + 6*I*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + I*b*x*si
nh(b*x + a)^6 + I*b*x*cosh(b*x + a)^4 + (15*I*b*x*cosh(b*x + a)^2 + I*b*x)*
sinh(b*x + a)^4 - I*b*x*cosh(b*x + a)^2 + 4*(5*I*b*x*cosh(b*x + a)^3 + I*b*
x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*I*b*x*cosh(b*x + a)^4 + 6*I*b*x*cosh
(b*x + a)^2 - I*b*x)*sinh(b*x + a)^2 - I*b*x + 2*(3*I*b*x*cosh(b*x + a)^5 +
 2*I*b*x*cosh(b*x + a)^3 - I*b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh
(b*x + a) + I*sinh(b*x + a)) + 6*(-I*b*x*cosh(b*x + a)^6 - 6*I*b*x*cosh(b*x
 + a)*sinh(b*x + a)^5 - I*b*x*sinh(b*x + a)^6 - I*b*x*cosh(b*x + a)^4 + (-1
5*I*b*x*cosh(b*x + a)^2 - I*b*x)*sinh(b*x + a)^4 + I*b*x*cosh(b*x + a)^2 +
4*(-5*I*b*x*cosh(b*x + a)^3 - I*b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (-15*I
*b*x*cosh(b*x + a)^4 - 6*I*b*x*cosh(b*x + a)^2 + I*b*x)*sinh(b*x + a)^2 + I
*b*x + 2*(-3*I*b*x*cosh(b*x + a)^5 - 2*I*b*x*cosh(b*x + a)^3 + I*b*x*cosh(b
*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*(cosh
(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(
b*x + a)^2 + 1)*sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 -
```

1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + 4*(b*x*cosh(b*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 + b*x*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 + b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 - b*x + 2*(3*b*x*cosh(b*x + a)^5 + 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) - ((-3*I*a^2 + 2*I)*cosh(b*x + a)^6 - 6*(3*I*a^2 - 2*I)*cosh(b*x + a)*sinh(b*x + a)^5 + (-3*I*a^2 + 2*I)*sinh(b*x + a)^6 + (-3*I*a^2 + 2*I)*cosh(b*x + a)^4 - (15*(3*I*a^2 - 2*I)*cosh(b*x + a)^2 + 3*I*a^2 - 2*I)*sinh(b*x + a)^4 - 4*(5*(3*I*a^2 - 2*I)*cosh(b*x + a)^3 + (3*I*a^2 - 2*I)*cosh(b*x + a))*sinh(b*x + a)^3 + (3*I*a^2 - 2*I)*cosh(b*x + a)^2 - (15*(3*I*a^2 - 2*I)*cosh(b*x + a)^4 + 6*(3*I*a^2 - 2*I)*cosh(b*x + a)^2 - 3*I*a^2 + 2*I)*sinh(b*x + a)^2 + 3*I*a^2 - 2*(3*(3*I*a^2 - 2*I)*cosh(b*x + a)^5 + 2*(3*I*a^2 - 2*I)*cosh(b*x + a)^3 + (-3*I*a^2 + 2*I)*cosh(b*x + a))*sinh(b*x + a) - 2*I)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((3*I*a^2 - 2*I)*cosh(b*x + a)^6 - 6*(-3*I*a^2 + 2*I)*cosh(b*x + a)*sinh(b*x + a)^5 + (3*I*a^2 - 2*I)*sinh(b*x + a)^6 + (3*I*a^2 - 2*I)*cosh(b*x + a)^4 - (15*(-3*I*a^2 + 2*I)*cosh(b*x + a)^2 - 3*I*a^2 + 2*I)*sinh(b*x + a)^4 - 4*(5*(-3*I*a^2 + 2*I)*cosh(b*x + a)^3 + (-3*I*a^2 + 2*I)*cosh(b*x + a))*sinh(b*x + a)^3 + (-3*I*a^2 + 2*I)*cosh(b*x + a)^2 - (15*(-3*I*a^2 + 2*I)*cosh(b*x + a)^4 + 6*(-3*I*a^2 + 2*I)*cosh(b*x + a)^2 + 3*I*a^2 - 2*I)*sinh(b*x + a)^2 - 3*I*a^2 - 2*(3*(-3*I*a^2 + 2*I)*cosh(b*x + a)^5 + 2*(-3*I*a^2 + 2*I)*cosh(b*x + a)^3 + (3*I*a^2 - 2*I)*cosh(b*x + a))*sinh(b*x + a) + 2*I)*log(cosh(b*x + a) + sinh(b*x + a) - I) + 4*(a*cosh(b*x + a)^6 + 6*a*cosh(b*x + a)*sinh(b*x + a)^5 + a*sinh(b*x + a)^6 + a*cosh(b*x + a)^4 + (15*a*cosh(b*x + a)^2 + a)*sinh(b*x + a)^4 + 4*(5*a*cosh(b*x + a)^3 + a*cosh(b*x + a))*sinh(b*x + a)^3 - a*cosh(b*x + a)^2 + (15*a*cosh(b*x + a)^4 + 6*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^5 + 2*a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*((-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^6 + 6*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (-I*b^2*x^2 + I*a^2)*sinh(b*x + a)^6 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^4 + (-I*b^2*x^2 + 15*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 + I*a^2)*sinh(b*x + a)^4 + I*b^2*x^2 + 4*(5*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^3 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 + (15*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^4 + I*b^2*x^2 + 6*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^2 - I*a^2)*sinh(b*x + a)^2 - I*a^2 + 2*(3*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^5 + 2*(-I*b^2*x^2 + I*a^2)*cosh(b*x + a)^3 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 3*((I*b^2*x^2 - I*a^2)*cosh(b*x + a)^6 + 6*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)*sinh(b*x + a)^5 + (I*b^2*x^2 - I*a^2)*sinh(b*x + a)^6 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a)^4 + (I*b^2*x^2 + 15*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 - I*a^2)*sinh(b*x + a)^4 - I*b^2*x^2 + 4*(5*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^3 + (I*b^2*x^2 - I*a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + (-I*b^2*x^2

```
 + I*a^2)*cosh(b*x + a)^2 + (15*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^4 - I*b^2
*x^2 + 6*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^2 + I*a^2)*sinh(b*x + a)^2 + I*a
^2 + 2*(3*(I*b^2*x^2 - I*a^2)*cosh(b*x + a)^5 + 2*(I*b^2*x^2 - I*a^2)*cosh(
b*x + a)^3 + (-I*b^2*x^2 + I*a^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh
(b*x + a) - I*sinh(b*x + a) + 1) - 4*((b*x + a)*cosh(b*x + a)^6 + 6*(b*x +
a)*cosh(b*x + a)*sinh(b*x + a)^5 + (b*x + a)*sinh(b*x + a)^6 + (b*x + a)*co
sh(b*x + a)^4 + (15*(b*x + a)*cosh(b*x + a)^2 + b*x + a)*sinh(b*x + a)^4 +
4*(5*(b*x + a)*cosh(b*x + a)^3 + (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 -
 (b*x + a)*cosh(b*x + a)^2 + (15*(b*x + a)*cosh(b*x + a)^4 + 6*(b*x + a)*co
sh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 - b*x + 2*(3*(b*x + a)*cosh(b*x +
a)^5 + 2*(b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a)
 - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(-I*cosh(b*x + a)^6 - 6*I
*cosh(b*x + a)*sinh(b*x + a)^5 - I*sinh(b*x + a)^6 + (-15*I*cosh(b*x + a)^2
 - I)*sinh(b*x + a)^4 - I*cosh(b*x + a)^4 + 4*(-5*I*cosh(b*x + a)^3 - I*cos
h(b*x + a))*sinh(b*x + a)^3 + (-15*I*cosh(b*x + a)^4 - 6*I*cosh(b*x + a)^2
+ I)*sinh(b*x + a)^2 + I*cosh(b*x + a)^2 + 2*(-3*I*cosh(b*x + a)^5 - 2*I*co
sh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) + I)*polylog(3, I*cosh(b*x +
 a) + I*sinh(b*x + a)) + 6*(I*cosh(b*x + a)^6 + 6*I*cosh(b*x + a)*sinh(b*x
+ a)^5 + I*sinh(b*x + a)^6 + (15*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^4 + I
*cosh(b*x + a)^4 + 4*(5*I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a)^
3 + (15*I*cosh(b*x + a)^4 + 6*I*cosh(b*x + a)^2 - I)*sinh(b*x + a)^2 - I*co
sh(b*x + a)^2 + 2*(3*I*cosh(b*x + a)^5 + 2*I*cosh(b*x + a)^3 - I*cosh(b*x +
 a))*sinh(b*x + a) - I)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*
(6*b^2*x^2*cosh(b*x + a)^2 + 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^4 + 3*b^2*
x^2 - 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x + a)^6 + 6*b^3*cosh(b*x + a)*sinh
(b*x + a)^5 + b^3*sinh(b*x + a)^6 + b^3*cosh(b*x + a)^4 - b^3*cosh(b*x + a)
^2 + (15*b^3*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^4 + 4*(5*b^3*cosh(b*x + a
)^3 + b^3*cosh(b*x + a))*sinh(b*x + a)^3 - b^3 + (15*b^3*cosh(b*x + a)^4 +
6*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 2*(3*b^3*cosh(b*x + a)^5 + 2
*b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2 \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx = \int x^2 \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**2*csch(b*x+a)**2*sech(b*x+a)**3,x)
[Out] Integral(x**2*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int x^2 \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3\;dx$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -96*b^2*integrate(1/32*x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - (2
*b*x^2*e^(3*b*x + 3*a) + (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) + (3*b*x
^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2
*e^(2*b*x + 2*a) - b^2) - 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a))
)/b^3 + 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3 + 2*arctan(e
^(b*x + a))/b^3
```

Giac [F]

$$\int x^2 \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int x^2\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3\;dx$$

```
[In] integrate(x^2*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \int \frac{x^2}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.503 $\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx = -\frac{3x\arctan(e^{a+bx})}{b}-\frac{\mathrm{arctanh}(\cosh(a+bx))}{b^2}-\frac{3x\mathrm{csch}(a+bx)}{2b}+\frac{3i\,\mathrm{PolyLog}(2,-ie^{a+bx})}{2b^2}-\frac{3i\,\mathrm{PolyLog}(2,ie^{a+bx})}{2b^2}-\frac{\mathrm{sech}(a+bx)}{2b^2}+\frac{x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)}{2b}$$

```
[Out] -3*x*arctan(exp(b*x+a))/b-arctanh(cosh(b*x+a))/b^2-3/2*x*csch(b*x+a)/b+3/2*
I*polylog(2,-I*exp(b*x+a))/b^2-3/2*I*polylog(2,I*exp(b*x+a))/b^2-1/2*sech(b
*x+a)/b^2+1/2*x*csch(b*x+a)*sech(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2701, 294, 327, 213, 5570, 5311, 12, 4265, 2317, 2438, 3855, 2702}

$$\int x\mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx = -\frac{3x\arctan(e^{a+bx})}{b}-\frac{\mathrm{arctanh}(\cosh(a+bx))}{b^2}+\frac{3i\,\mathrm{PolyLog}(2,-ie^{a+bx})}{2b^2}-\frac{3i\,\mathrm{PolyLog}(2,ie^{a+bx})}{2b^2}-\frac{\mathrm{sech}(a+bx)}{2b^2}-\frac{3x\mathrm{csch}(a+bx)}{2b}+\frac{x\mathrm{csch}(a+bx)\mathrm{sech}^2(a+bx)}{2b}$$

```
[In] Int[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*x*ArcTan[E^(a + b*x)])/b - ArcTanh[Cosh[a + b*x]]/b^2 - (3*x*Csch[a + b
*x])/(2*b) + (((3*I)/2)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - (((3*I)/2)*Poly
Log[2, I*E^(a + b*x)])/b^2 - Sech[a + b*x]/(2*b^2) + (x*Csch[a + b*x]*Sech[
a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3x\arctan(\sinh(a+bx))}{2b} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} \\ &\quad - \int \left(-\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\text{csch}(a+bx)}{2b} + \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}\right) dx\end{aligned}$$

$$= -\frac{3x\arctan(\sinh(a+bx))}{2b} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx}{2b} + \frac{3\int \arctan(\sinh(a+bx))\,dx}{2b} + \frac{3\int \text{csch}(a+bx)\,dx}{2b}$$

$$= -\frac{3\text{arctanh}(\cosh(a+bx))}{2b^2} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b^2} - \frac{3\int bx\text{sech}(a+bx)\,dx}{2b}$$

$$= -\frac{3\text{arctanh}(\cosh(a+bx))}{2b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{3}{2}\int x\text{sech}(a+bx)\,dx - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b^2}$$

$$= -\frac{3x\arctan\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} + \frac{(3i)\int \log\left(1-ie^{a+bx}\right)\,dx}{2b} - \frac{(3i)\int \log\left(1+ie^{a+bx}\right)\,dx}{2b}$$

$$= -\frac{3x\arctan\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} + \frac{(3i)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{2b^2} - \frac{(3i)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{2b^2}$$

$$= -\frac{3x\arctan\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^2} - \frac{3x\text{csch}(a+bx)}{2b} + \frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} - \frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} - \frac{\text{sech}(a+bx)}{2b^2} + \frac{x\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.71

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{-6a\arctan\left(e^{a+bx}\right)+bx\coth\left(\frac{1}{2}(a+bx)\right)+3ia\log\left(1-ie^{a+bx}\right)+3ibx\log\left(1-ie^{a+bx}\right)-3ia\log\left(1+ie\right.}{}$$

[In] Integrate[x*Csch[a + b*x]^2*Sech[a + b*x]^3,x]

[Out] -1/2*(-6*a*ArcTan[E^(a + b*x)] + b*x*Coth[(a + b*x)/2] + (3*I)*a*Log[1 - I*E^(a + b*x)] + (3*I)*b*x*Log[1 - I*E^(a + b*x)] - (3*I)*a*Log[1 + I*E^(a + b*x)] - (3*I)*b*x*Log[1 + I*E^(a + b*x)] + 2*Log[Cosh[(a + b*x)/2]] - 2*Log[Sinh[(a + b*x)/2]] - (3*I)*PolyLog[2, (-I)*E^(a + b*x)] + (3*I)*PolyLog[2, I*E^(a + b*x)] + Sech[a + b*x] - b*x*Tanh[(a + b*x)/2] + b*x*Sech[a + b*x]*Tanh[a + b*x])/b^2

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(103) = 206.

Time = 8.93 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.93

method	result
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}bx+2\,e^{2bx+2a}bx+e^{4bx+4a}+3bx-1\right)}{b^2\left(1+e^{2bx+2a}\right)^2\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{bx+a}-1\right)}{b^2}-\frac{\ln\left(e^{bx+a}+1\right)}{b^2}+\frac{3a\arctan\left(e^{bx+a}\right)}{b^2}+\frac{3i\,\text{dilog}\left(1+ie^{bx+}\right.}{2b^2}$

[In] int(x*csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -exp(b*x+a)*(3*exp(4*b*x+4*a)*b*x+2*exp(2*b*x+2*a)*b*x+exp(4*b*x+4*a)+3*b*x-1)/b^2/(1+exp(2*b*x+2*a))^2/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(b*x+a)-1)-1/b^2*ln(exp(b*x+a)+1)+3/b^2*a*arctan(exp(b*x+a))+3/2*I/b^2*dilog(1+I*exp(b*x+a))-3/2*I/b*ln(1-I*exp(b*x+a))*x-3/2*I/b^2*ln(1-I*exp(b*x+a))*a+3/2*I/b*ln(1+I*exp(b*x+a))*x+3/2*I/b^2*ln(1+I*exp(b*x+a))*a-3/2*I/b^2*dilog(1-I*exp(b*x+a))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2227 vs. $2(97) = 194$.

Time = 0.29 (sec) , antiderivative size = 2227, normalized size of antiderivative = 18.56

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*x
 + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 + 4*b*x*cosh(b*x + a)^3 + 4*(5*(3*b
*x + 1)*cosh(b*x + a)^2 + b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x
+ a)^3 + 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(3*b*x - 1)*cosh(b*x + a)
 + 3*(I*cosh(b*x + a)^6 + 6*I*cosh(b*x + a)*sinh(b*x + a)^5 + I*sinh(b*x +
a)^6 + (15*I*cosh(b*x + a)^2 + I)*sinh(b*x + a)^4 + I*cosh(b*x + a)^4 + 4*(
5*I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a)^3 + (15*I*cosh(b*x + a
)^4 + 6*I*cosh(b*x + a)^2 - I)*sinh(b*x + a)^2 - I*cosh(b*x + a)^2 + 2*(3*I
*cosh(b*x + a)^5 + 2*I*cosh(b*x + a)^3 - I*cosh(b*x + a))*sinh(b*x + a) - I
)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 3*(-I*cosh(b*x + a)^6 - 6*I*co
sh(b*x + a)*sinh(b*x + a)^5 - I*sinh(b*x + a)^6 + (-15*I*cosh(b*x + a)^2 -
I)*sinh(b*x + a)^4 - I*cosh(b*x + a)^4 + 4*(-5*I*cosh(b*x + a)^3 - I*cosh(b
*x + a))*sinh(b*x + a)^3 + (-15*I*cosh(b*x + a)^4 - 6*I*cosh(b*x + a)^2 + I
)*sinh(b*x + a)^2 + I*cosh(b*x + a)^2 + 2*(-3*I*cosh(b*x + a)^5 - 2*I*cosh(
b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) + I)*dilog(-I*cosh(b*x + a) - I
*sinh(b*x + a)) + 2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + si
nh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + cosh(b*x + a)^4
+ 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)
^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b
*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(
b*x + a) + sinh(b*x + a) + 1) + 3*(-I*a*cosh(b*x + a)^6 - 6*I*a*cosh(b*x +
a)*sinh(b*x + a)^5 - I*a*sinh(b*x + a)^6 - I*a*cosh(b*x + a)^4 + (-15*I*a*c
osh(b*x + a)^2 - I*a)*sinh(b*x + a)^4 + 4*(-5*I*a*cosh(b*x + a)^3 - I*a*cos
h(b*x + a))*sinh(b*x + a)^3 + I*a*cosh(b*x + a)^2 + (-15*I*a*cosh(b*x + a)^
4 - 6*I*a*cosh(b*x + a)^2 + I*a)*sinh(b*x + a)^2 + 2*(-3*I*a*cosh(b*x + a)^
5 - 2*I*a*cosh(b*x + a)^3 + I*a*cosh(b*x + a))*sinh(b*x + a) + I*a)*log(cos
h(b*x + a) + sinh(b*x + a) + I) + 3*(I*a*cosh(b*x + a)^6 + 6*I*a*cosh(b*x +
 a)*sinh(b*x + a)^5 + I*a*sinh(b*x + a)^6 + I*a*cosh(b*x + a)^4 + (15*I*a*c
osh(b*x + a)^2 + I*a)*sinh(b*x + a)^4 + 4*(5*I*a*cosh(b*x + a)^3 + I*a*cosh
(b*x + a))*sinh(b*x + a)^3 - I*a*cosh(b*x + a)^2 + (15*I*a*cosh(b*x + a)^4
+ 6*I*a*cosh(b*x + a)^2 - I*a)*sinh(b*x + a)^2 + 2*(3*I*a*cosh(b*x + a)^5 +
 2*I*a*cosh(b*x + a)^3 - I*a*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(cosh(b
*x + a) + sinh(b*x + a) - I) - 2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*
x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + cos
```

```
h(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (15*
cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2
+ 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) -
 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*((-I*b*x - I*a)*cosh(b*x + a
)^6 + 6*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (-I*b*x - I*a)*sinh(
b*x + a)^6 + (-I*b*x - I*a)*cosh(b*x + a)^4 + (15*(-I*b*x - I*a)*cosh(b*x +
 a)^2 - I*b*x - I*a)*sinh(b*x + a)^4 + 4*(5*(-I*b*x - I*a)*cosh(b*x + a)^3
+ (-I*b*x - I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (I*b*x + I*a)*cosh(b*x +
a)^2 + (15*(-I*b*x - I*a)*cosh(b*x + a)^4 + 6*(-I*b*x - I*a)*cosh(b*x + a)^
2 + I*b*x + I*a)*sinh(b*x + a)^2 + I*b*x + 2*(3*(-I*b*x - I*a)*cosh(b*x + a
)^5 + 2*(-I*b*x - I*a)*cosh(b*x + a)^3 + (I*b*x + I*a)*cosh(b*x + a))*sinh(
b*x + a) + I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 3*((I*b*x + I*
a)*cosh(b*x + a)^6 + 6*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (I*b*x
 + I*a)*sinh(b*x + a)^6 + (I*b*x + I*a)*cosh(b*x + a)^4 + (15*(I*b*x + I*a)
*cosh(b*x + a)^2 + I*b*x + I*a)*sinh(b*x + a)^4 + 4*(5*(I*b*x + I*a)*cosh(b
*x + a)^3 + (I*b*x + I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (-I*b*x - I*a)*c
osh(b*x + a)^2 + (15*(I*b*x + I*a)*cosh(b*x + a)^4 + 6*(I*b*x + I*a)*cosh(b
*x + a)^2 - I*b*x - I*a)*sinh(b*x + a)^2 - I*b*x + 2*(3*(I*b*x + I*a)*cosh(
b*x + a)^5 + 2*(I*b*x + I*a)*cosh(b*x + a)^3 + (-I*b*x - I*a)*cosh(b*x + a)
)*sinh(b*x + a) - I*a)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*(5*(
3*b*x + 1)*cosh(b*x + a)^4 + 6*b*x*cosh(b*x + a)^2 + 3*b*x - 1)*sinh(b*x +
a))/(b^2*cosh(b*x + a)^6 + 6*b^2*cosh(b*x + a)*sinh(b*x + a)^5 + b^2*sinh(b
*x + a)^6 + b^2*cosh(b*x + a)^4 + (15*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x +
 a)^4 - b^2*cosh(b*x + a)^2 + 4*(5*b^2*cosh(b*x + a)^3 + b^2*cosh(b*x + a))
*sinh(b*x + a)^3 + (15*b^2*cosh(b*x + a)^4 + 6*b^2*cosh(b*x + a)^2 - b^2)*s
inh(b*x + a)^2 - b^2 + 2*(3*b^2*cosh(b*x + a)^5 + 2*b^2*cosh(b*x + a)^3 - b
^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x\,\text{csch}^2\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

```
[In] integrate(x*csch(b*x+a)**2*sech(b*x+a)**3,x)
```

```
[Out] Integral(x*csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3\,dx$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) + (3*b*x*e^a
- e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) + b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x +
 2*a) - b^2) - log((e^(b*x + a) + 1)*e^(-a))/b^2 + log((e^(b*x + a) - 1)*e^
(-a))/b^2 - 96*integrate(1/32*x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3\,dx$$

```
[In] integrate(x*csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^2*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^2(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.504 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b}-\frac{3\operatorname{csch}(a+bx)}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[Out] -3/2*arctan(sinh(b*x+a))/b-3/2*csch(b*x+a)/b+1/2*csch(b*x+a)*sech(b*x+a)^2/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2701, 294, 327, 213}

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b}-\frac{3\operatorname{csch}(a+bx)}{2b}+\frac{\operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sinh[a + b*x]])/(2*b) - (3*Csch[a + b*x])/(2*b) + (Csch[a + b*x]
*Sech[a + b*x]^2)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, -i\text{csch}(a+bx)\right)}{b}$$

$$= \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b}$$

$$= -\frac{3\text{csch}(a+bx)}{2b} + \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b}$$

$$= -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{3\text{csch}(a+bx)}{2b} + \frac{\text{csch}(a+bx)\text{sech}^2(a+bx)}{2b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
[Out] -((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)}{b}$	47
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)}{b}$	47
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}+2\,e^{2bx+2a}+3\right)}{b\left(1+e^{2bx+2a}\right)^2\left(e^{2bx+2a}-1\right)}+\frac{3i\ln\left(e^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(e^{bx+a}+i\right)}{2b}$	95

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp(
b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 511, normalized size of antiderivative = 10.43

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2+1\right)\sinh(bx}{}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
[Out] -(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
+ 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*cos
```

```
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*cosh(b*x
+ a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(b*cosh(b*
x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + b*cosh(b
*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*
x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{3\arctan\left(e^{(-bx-a)}\right)}{b}-\frac{3\,e^{(-bx-a)}+2\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}}{b(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}+1)}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
[Out] 3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*b
*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.08

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4e^{(bx+a)}-4e^{(-bx-a)}} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{4b}$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-b
*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2*a
) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan(
(exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(2*
a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

3.505 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 49.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{3}}{x}dx$$

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^{2}\operatorname{sech}\left(bx+a\right)^{3}}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^{2}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3/x,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 10.70

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx = \int \frac{\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) + (3*b*x*e^a
+ e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) + b^2*x^2*e^(4*b*x + 4*a) - b^2*x^
2*e^(2*b*x + 2*a) - b^2*x^2) - 32*integrate(1/32*(3*b^2*x^2*e^a - 2*e^a)*e^
(b*x)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) - 32*integrate(1/32/(b*x^2*e^
(b*x + a) + b*x^2), x) - 32*integrate(1/32/(b*x^2*e^(b*x + a) - b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx = \int \frac{\text{csch}\,(bx+a)^2\,\text{sech}\,(bx+a)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{csch}^2(a+bx)\text{sech}^3(a+bx)}{x}\,dx = \int \frac{1}{x\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.506 $\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}, x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$$

```
[In] Int[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 32.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2,x]
[Out] Integrate[(Csch[a + b*x]^2*Sech[a + b*x]^3)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x^2}dx$$

```
[In] int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
[Out] int(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^2\operatorname{sech}\left(bx+a\right)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)**2*sech(b*x+a)**3/x**2,x)
[Out] Integral(csch(a + b*x)**2*sech(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 10.75

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x*e^(3*b*x + 3*a) + (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) + (3*b*x*e^
a + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) + b^2*x^3*e^(4*b*x + 4*a) - b^
2*x^3*e^(2*b*x + 2*a) - b^2*x^3) - 32*integrate(3/32*(b^2*x^2*e^a - 2*e^a)*
e^(b*x)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) - 32*integrate(1/16/(b*x^3*
e^(b*x + a) + b*x^3), x) - 32*integrate(1/16/(b*x^3*e^(b*x + a) - b*x^3), x
)
```

Giac [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^2\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2*sech(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^2), x)
```

3.507 $\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 67.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x],x]
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)dx$$

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=\int x^m \operatorname{csch}^3\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a),x)
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^3),x)
[Out] int(x^m/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.508 $\int x^3 \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 240

$$\int x^3 \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{3x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{3x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} + \frac{3\,\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}-\frac{3\,\text{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}$$

```
[Out] -3/2*x^2/b^2+1/2*x^3/b+2*x^3*arctanh(exp(2*b*x+2*a))/b-3/2*x^2*coth(b*x+a)/
b^2-1/2*x^3*coth(b*x+a)^2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+3/2*polylog(2,exp(
2*b*x+2*a))/b^4+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/2*x^2*polylog(2,ex
p(2*b*x+2*a))/b^2-3/2*x*polylog(3,-exp(2*b*x+2*a))/b^3+3/2*x*polylog(3,exp(
2*b*x+2*a))/b^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4-3/4*polylog(4,exp(2*b*x+
2*a))/b^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2700, 14, 5570, 3801, 3797, 2221, 2317, 2438, 30, 2631, 12, 4267, 2611, 6744, 2320, 6724}

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{3\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4}-\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}-\frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}-\frac{3x^2}{2b^2}+\frac{x^3}{2b}$$

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] (-3*x^2)/(2*b^2) + x^3/(2*b) + (2*x^3*ArcTanh[E^(2*a + 2*b*x)])/b - (3*x^2*
Coth[a + b*x])/(2*b^2) - (x^3*Coth[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^(2*(a
 + b*x))])/b^3 + (3*PolyLog[2, E^(2*(a + b*x))])/(2*b^4) + (3*x^2*PolyLog[2
, -E^(2*a + 2*b*x)])/(2*b^2) - (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2)
- (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/(2*b^3) + (3*x*PolyLog[3, E^(2*a + 2*b
*x)])/(2*b^3) + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(4*b^4) - (3*PolyLog[4, E^
(2*a + 2*b*x)])/(4*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b} \\
&\quad -3\int x^2\left(-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}\right)dx \\
&= -\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b} \\
&\quad -3\int\left(-\frac{x^2\coth^2(a+bx)}{2b}-\frac{x^2\log(\tanh(a+bx))}{b}\right)dx \\
&= -\frac{x^3\coth^2(a+bx)}{2b}-\frac{x^3\log(\tanh(a+bx))}{b} \\
&\quad +\frac{3\int x^2\coth^2(a+bx)\,dx}{2b}+\frac{3\int x^2\log(\tanh(a+bx))\,dx}{b} \\
&= -\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b}+\frac{3\int x\coth(a+bx)\,dx}{b^2} \\
&\quad -\frac{\int 2bx^3\text{csch}(2a+2bx)\,dx}{b}+\frac{3\int x^2\,dx}{2b} \\
&= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b} \\
&\quad -2\int x^3\text{csch}(2a+2bx)\,dx-\frac{6\int\frac{e^{2(a+bx)}x}{1-e^{2(a+bx)}}\,dx}{b^2} \\
&= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2} \\
&\quad -\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}-\frac{3\int\log\left(1-e^{2(a+bx)}\right)\,dx}{b^3} \\
&\quad +\frac{3\int x^2\log\left(1-e^{2a+2bx}\right)\,dx}{b}-\frac{3\int x^2\log\left(1+e^{2a+2bx}\right)\,dx}{b} \\
&= -\frac{3x^2}{2b^2}+\frac{x^3}{2b}+\frac{2x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{3x^2\coth(a+bx)}{2b^2} \\
&\quad -\frac{x^3\coth^2(a+bx)}{2b}+\frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2} \\
&\quad -\frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}-\frac{3\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2(a+bx)}\right)}{2b^4} \\
&\quad -\frac{3\int x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)\,dx}{b^2}+\frac{3\int x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)\,dx}{b^2}
\end{aligned}
$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{3\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2} - \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} + \frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} + \frac{3\int\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)dx}{2b^3} - \frac{3\int\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)dx}{2b^3}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{3\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2} - \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} + \frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} + \frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-x)}{x}dx,x,e^{2a+2bx}\right)}{4b^4} - \frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,x)}{x}dx,x,e^{2a+2bx}\right)}{4b^4}$$

$$= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} + \frac{2x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\coth(a+bx)}{2b^2} - \frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3} + \frac{3\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2} - \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3} + \frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3} + \frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{4b^4} - \frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{4b^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 565 vs. $2(240) = 480$.

Time = 6.51 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.35

$$\int x^3\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{x^3\operatorname{csch}^2(a+bx)}{2b} + \frac{e^{2a}\left(-6b^2e^{-2a}x^2 + b^4e^{-2a}x^4 + 6b\left(1-e^{-2a}\right)x\log\left(1-e^{-a-bx}\right) - 2b^3e^{-2a}\left(-1+e^{2a}\right)x^3\log\left(1-e^{-a-bx}\right) + \cdots\right.}{\cdots} + \frac{e^{2a}\left(2b^4e^{-2a}x^4 + 4b^3\left(1+e^{-2a}\right)x^3\log\left(1+e^{-2(a+bx)}\right) - 6b^2\left(1+e^{-2a}\right)x^2\operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right) - 6b(1 \cdots\right.}{4b^4\left(1+e^{2a}\right)} - \frac{1}{4}x^4\operatorname{csch}(a)\operatorname{sech}(a) + \frac{3x^2\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{2b^2}$$

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*(x^3*Csch[a + b*x]^2)/b + (E^(2*a)*((-6*b^2*x^2)/E^(2*a) + (b^4*x^4)/E
^(2*a) + 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))
*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) + 6*b*(1 - E^(-2*a))*x*Log[1 + E^(-a -
b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*a) - 6*(1 - E
^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, -E
^(-a - b*x)] - 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)] + 6*b^2*(1 - E^(-2
*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, -E^(-a
 - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, E^(-a - b*x)] + 12*(1 - E^(-2*a
))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-2*a))*PolyLog[4, E^(-a - b*x)]))
/(2*b^4*(-1 + E^(2*a))) + (E^(2*a)*((2*b^4*x^4)/E^(2*a) + 4*b^3*(1 + E^(-2*
a))*x^3*Log[1 + E^(-2*(a + b*x))] - 6*b^2*(1 + E^(-2*a))*x^2*PolyLog[2, -E^
(-2*(a + b*x))] - 6*b*(1 + E^(-2*a))*x*PolyLog[3, -E^(-2*(a + b*x))] - 3*(1
 + E^(-2*a))*PolyLog[4, -E^(-2*(a + b*x))]))/(4*b^4*(1 + E^(2*a))) - (x^4*C
sch[a]*Sech[a])/4 + (3*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b^2)
```

Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{3a^2}{b^4}-\frac{\ln(\mathrm{e}^{bx+a}+1)x^3}{b}-\frac{3x^2}{b^2}-\frac{3a\ln(\mathrm{e}^{bx+a}-1)}{b^4}+\frac{a^3\ln(\mathrm{e}^{bx+a}-1)}{b^4}+\frac{3\ln(\mathrm{e}^{bx+a}+1)x}{b^3}+\frac{3\ln(1-\mathrm{e}^{bx+a})x}{b^3}+\frac{3\ln(1-\mathrm{e}^{bx+}}{b^4}$

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b^4*a^2-1/b*ln(exp(b*x+a)+1)*x^3-3/b^2*x^2-3/b^4*a*ln(exp(b*x+a)-1)+1/b^
4*a^3*ln(exp(b*x+a)-1)+3/b^3*ln(exp(b*x+a)+1)*x+3/b^3*ln(1-exp(b*x+a))*x+3/
b^4*ln(1-exp(b*x+a))*a-x^2*(2*exp(2*b*x+2*a)*b*x+3*exp(2*b*x+2*a)-3)/b^2/(e
xp(2*b*x+2*a)-1)^2+6/b^4*a*ln(exp(b*x+a))-6/b^3*a*x-3*x^2*polylog(2,-exp(b*
x+a))/b^2-3*x^2*polylog(2,exp(b*x+a))/b^2+6*x*polylog(3,-exp(b*x+a))/b^3+6*
x*polylog(3,exp(b*x+a))/b^3+3/2*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3/2*x*po
lylog(3,-exp(2*b*x+2*a))/b^3-6*polylog(4,-exp(b*x+a))/b^4-6*polylog(4,exp(b
*x+a))/b^4+x^3*ln(1+exp(2*b*x+2*a))/b-1/b*ln(1-exp(b*x+a))*x^3-1/b^4*ln(1-e
xp(b*x+a))*a^3+3/4*polylog(4,-exp(2*b*x+2*a))/b^4+3*polylog(2,-exp(b*x+a))/
b^4+3*polylog(2,exp(b*x+a))/b^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 3394, normalized size of antiderivative = 14.14

$$\int x^3 \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(3*(b^2*x^2 - a^2)*cosh(b*x + a)^4 + 12*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh
(b*x + a)^3 + 3*(b^2*x^2 - a^2)*sinh(b*x + a)^4 + (2*b^3*x^3 - 3*b^2*x^2 +
6*a^2)*cosh(b*x + a)^2 + (2*b^3*x^3 - 3*b^2*x^2 + 18*(b^2*x^2 - a^2)*cosh(b
*x + a)^2 + 6*a^2)*sinh(b*x + a)^2 - 3*a^2 + 3*((b^2*x^2 - 1)*cosh(b*x + a)
^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x
 + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x
^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x +
a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(cosh(b*x + a)
+ sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*sin
h(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^2 + b^2*x^
2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2*co
sh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) - 3*(b^2*x^2*cosh(b*x + a)^4 + 4*b^2*x^2*cosh(b*x + a)*
sinh(b*x + a)^3 + b^2*x^2*sinh(b*x + a)^4 - 2*b^2*x^2*cosh(b*x + a)^2 + b^2
*x^2 + 2*(3*b^2*x^2*cosh(b*x + a)^2 - b^2*x^2)*sinh(b*x + a)^2 + 4*(b^2*x^2
*cosh(b*x + a)^3 - b^2*x^2*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x
+ a) - I*sinh(b*x + a)) + 3*((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1
)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 -
 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 - (b^2*x^2 - 1
)*cosh(b*x + a))*sinh(b*x + a) - 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) +
 (b^3*x^3 + (b^3*x^3 - 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 - 3*b*x)*cosh(b*
x + a)*sinh(b*x + a)^3 + (b^3*x^3 - 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 - 3
*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 - 3*b*x)*cosh(b*x + a)^2 -
3*b*x)*sinh(b*x + a)^2 - 3*b*x + 4*((b^3*x^3 - 3*b*x)*cosh(b*x + a)^3 - (b^
3*x^3 - 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (a^3*cosh(b*x + a)^4 + 4*a^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3
*sinh(b*x + a)^4 - 2*a^3*cosh(b*x + a)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 -
 a^3)*sinh(b*x + a)^2 + 4*(a^3*cosh(b*x + a)^3 - a^3*cosh(b*x + a))*sinh(b*
x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (a^3*cosh(b*x + a)^4 + 4*a
^3*cosh(b*x + a)*sinh(b*x + a)^3 + a^3*sinh(b*x + a)^4 - 2*a^3*cosh(b*x + a
)^2 + a^3 + 2*(3*a^3*cosh(b*x + a)^2 - a^3)*sinh(b*x + a)^2 + 4*(a^3*cosh(b
*x + a)^3 - a^3*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
+ a) - I) - ((a^3 - 3*a)*cosh(b*x + a)^4 + 4*(a^3 - 3*a)*cosh(b*x + a)*sinh
(b*x + a)^3 + (a^3 - 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 - 3*a)*cosh(b*x +
```

a)^2 - 2*(a^3 - 3*(a^3 - 3*a)*cosh(b*x + a)^2 - 3*a)*sinh(b*x + a)^2 + 4*((
a^3 - 3*a)*cosh(b*x + a)^3 - (a^3 - 3*a)*cosh(b*x + a))*sinh(b*x + a) - 3*a
)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cosh(
b*x + a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a
^3)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 - 2*(b^3*x^3
+ a^3 - 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3 +
a^3)*cosh(b*x + a)^3 - (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(I*
cosh(b*x + a) + I*sinh(b*x + a) + 1) - (b^3*x^3 + (b^3*x^3 + a^3)*cosh(b*x
+ a)^4 + 4*(b^3*x^3 + a^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3)*
sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 + a^3)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^
3 - 3*(b^3*x^3 + a^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^3*x^3 + a^3)
*cosh(b*x + a)^3 - (b^3*x^3 + a^3)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cos
h(b*x + a) - I*sinh(b*x + a) + 1) + (b^3*x^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a
)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^3*x^3 + a^3 - 3*b*x - 3*a)*sinh(b*x + a)^4 + a^3 - 2*(b^3*x^3 +
a^3 - 3*b*x - 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3*x^3 + a^3 -
3*b*x - 3*a)*cosh(b*x + a)^2 - 3*b*x - 3*a)*sinh(b*x + a)^2 - 3*b*x + 4*((
b^3*x^3 + a^3 - 3*b*x - 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 - 3*b*x - 3*a
)*cosh(b*x + a))*sinh(b*x + a) - 3*a)*log(-cosh(b*x + a) - sinh(b*x + a) +
1) + 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh(b*x + a) + s
inh(b*x + a)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
+ 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, I*cosh
(b*x + a) + I*sinh(b*x + a)) - 6*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*
cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*po
lylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
b*x + a) + 1)*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x
+ a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*
cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4
*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b
*x + a) + sinh(b*x + a)) + 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sin
h(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(
b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh
(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 6
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*po
lylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b
*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a
)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b

```
*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -cosh(b*x + a) - s
inh(b*x + a)) + 2*(6*(b^2*x^2 - a^2)*cosh(b*x + a)^3 + (2*b^3*x^3 - 3*b^2*x
^2 + 6*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^4*cosh
(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)^2 + b
^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*x + a)
^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x^3\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a),x)
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.47

$$\begin{aligned}
&\int x^3\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx \\
&= -\frac{1}{2}x^4 + \frac{3x^2 - \left(2bx^3e^{(2a)} + 3x^2e^{(2a)}\right)e^{(2bx)}}{b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2} + \frac{b^4x^4 - 6b^2x^2}{2b^4} \\
&\quad + \frac{4b^3x^3\log\left(e^{(2bx+2a)}+1\right) + 6b^2x^2\text{Li}_2\left(-e^{(2bx+2a)}\right) - 6bx\text{Li}_3\left(-e^{(2bx+2a)}\right) + 3\,\text{Li}_4\left(-e^{(2bx+2a)}\right)}{3b^4} \\
&\quad - \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3b^2x^2\text{Li}_2\left(-e^{(bx+a)}\right) - 6bx\text{Li}_3\left(-e^{(bx+a)}\right) + 6\,\text{Li}_4\left(-e^{(bx+a)}\right)}{b^4} \\
&\quad - \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3b^2x^2\text{Li}_2\left(e^{(bx+a)}\right) - 6bx\text{Li}_3\left(e^{(bx+a)}\right) + 6\,\text{Li}_4\left(e^{(bx+a)}\right)}{b^4} \\
&\quad + \frac{3\left(bx\log\left(e^{(bx+a)}+1\right) + \text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\left(bx\log\left(-e^{(bx+a)}+1\right) + \text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}
\end{aligned}$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
[Out] -1/2*x^4 + (3*x^2 - (2*b*x^3*e^(2*a) + 3*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*
b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 1/2*(b^4*x^4 - 6*b^2*x^2)/b^4 +
 1/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a)
) - 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^
4 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - (b^3*x^3*log(-e
^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x +
a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(
-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x^3}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^3),x)
[Out] int(x^3/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.509 $\int x^2 \mathrm{csch}^3(a+bx)\mathrm{sech}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=\frac{x^2}{2b}+\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}$$

```
[Out] 1/2*x^2/b+2*x^2*arctanh(exp(2*b*x+2*a))/b-x*coth(b*x+a)/b^2-1/2*x^2*coth(b*
x+a)^2/b+ln(sinh(b*x+a))/b^3+x*polylog(2,-exp(2*b*x+2*a))/b^2-x*polylog(2,e
xp(2*b*x+2*a))/b^2-1/2*polylog(3,-exp(2*b*x+2*a))/b^3+1/2*polylog(3,exp(2*b
*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2700, 14, 5570, 3801, 3556, 30, 2631, 12, 4267, 2611, 2320, 6724}

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=\frac{2x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{2b^3}+\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}+\frac{\log(\sinh(a+bx))}{b^3}+\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{x\coth(a+bx)}{b^2}-\frac{x^2\coth^2(a+bx)}{2b}+\frac{x^2}{2b}$$

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x^2/(2*b) + (2*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - (x*Coth[a + b*x])/b^2 - (x
^2*Coth[a + b*x]^2)/(2*b) + Log[Sinh[a + b*x]]/b^3 + (x*PolyLog[2, -E^(2*a
+ 2*b*x)])/b^2 - (x*PolyLog[2, E^(2*a + 2*b*x)])/b^2 - PolyLog[3, -E^(2*a +
 2*b*x)]/(2*b^3) + PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2\coth^2(a+bx)}{2b}-\frac{x^2\log(\tanh(a+bx))}{b}\\
&\quad -2\int x\left(-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}\right)\,dx
\end{aligned}
$$

$$= -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} - 2\int\left(-\frac{x\coth^2(a+bx)}{2b} - \frac{x\log(\tanh(a+bx))}{b}\right)dx$$

$$= -\frac{x^2\coth^2(a+bx)}{2b} - \frac{x^2\log(\tanh(a+bx))}{b} + \frac{\int x\coth^2(a+bx)\,dx}{b} + \frac{2\int x\log(\tanh(a+bx))\,dx}{b}$$

$$= -\frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\int\coth(a+bx)\,dx}{b^2} + \frac{\int x\,dx}{b} - \frac{\int 2bx^2\text{csch}(2a+2bx)\,dx}{b}$$

$$= \frac{x^2}{2b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} - 2\int x^2\text{csch}(2a+2bx)\,dx$$

$$= \frac{x^2}{2b} + \frac{2x^2\text{arctanh}(e^{2a+2bx})}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{2\int x\log(1-e^{2a+2bx})\,dx}{b} - \frac{2\int x\log(1+e^{2a+2bx})\,dx}{b}$$

$$= \frac{x^2}{2b} + \frac{2x^2\text{arctanh}(e^{2a+2bx})}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\,\text{PolyLog}(2,-e^{2a+2bx})}{b^2} - \frac{x\,\text{PolyLog}(2,e^{2a+2bx})}{b^2} - \frac{\int\text{PolyLog}(2,-e^{2a+2bx})\,dx}{b^2} + \frac{\int\text{PolyLog}(2,e^{2a+2bx})\,dx}{b^2}$$

$$= \frac{x^2}{2b} + \frac{2x^2\text{arctanh}(e^{2a+2bx})}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\,\text{PolyLog}(2,-e^{2a+2bx})}{b^2} - \frac{x\,\text{PolyLog}(2,e^{2a+2bx})}{b^2} - \frac{\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^3} + \frac{\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^3}$$

$$= \frac{x^2}{2b} + \frac{2x^2\text{arctanh}(e^{2a+2bx})}{b} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\,\text{PolyLog}(2,-e^{2a+2bx})}{b^2} - \frac{x\,\text{PolyLog}(2,e^{2a+2bx})}{b^2} - \frac{\text{PolyLog}(3,-e^{2a+2bx})}{2b^3} + \frac{\text{PolyLog}(3,e^{2a+2bx})}{2b^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 388 vs. $2(148) = 296$.

Time = 2.95 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.62

$$\int x^2 \operatorname{csch}^3(a+bx) \operatorname{sech}(a+bx)\,dx = \frac{1}{6}\Bigg(-\frac{3x^2\operatorname{csch}^2(a+bx)}{b}$$
$$+\frac{2e^{2a}\left(-6be^{-2a}x-6b\left(1-e^{-2a}\right)x+2b^3e^{-2a}x^3-3b^2e^{-2a}\left(-1+e^{2a}\right)x^2\log\left(1-e^{-a-bx}\right)-3b^2e^{-2a}(-1+$$
$$+\frac{2b^2x^2\left(\frac{2bx}{1+e^{2a}}+3\log\left(1+e^{-2(a+bx)}\right)\right)-6bx\operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right)-3\operatorname{PolyLog}\left(3,-e^{-2(a+bx)}\right)}{b^3}$$
$$-2x^3\operatorname{csch}(a)\operatorname{sech}(a)+\frac{6x\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b^2}\Bigg)$$

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] ((-3*x^2*Csch[a + b*x]^2)/b + (2*E^(2*a)*((-6*b*x)/E^(2*a) - 6*b*(1 - E^(-2
*a))*x + (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a - b*
x)])/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a) + 3
*(1 - E^(-2*a))*Log[1 - E^(a + b*x)] + 3*(1 - E^(-2*a))*Log[1 + E^(a + b*x)
] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a))*x*P
olyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)] + 6*(1
 - E^(-2*a))*PolyLog[3, E^(-a - b*x)]))/(b^3*(-1 + E^(2*a))) + (2*b^2*x^2*(
(2*b*x)/(1 + E^(2*a)) + 3*Log[1 + E^(-2*(a + b*x))]) - 6*b*x*PolyLog[2, -E^
(-2*(a + b*x))] - 3*PolyLog[3, -E^(-2*(a + b*x))])/b^3 - 2*x^3*Csch[a]*Sech
[a] + (6*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2)/6
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{2x\left(e^{2bx+2a}bx+e^{2bx+2a}-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2}-\frac{a^2\ln\left(e^{bx+a}-1\right)}{b^3}-\frac{\ln\left(1-e^{bx+a}\right)x^2}{b}-\frac{2x\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2}+\frac{x^2\ln\left(1+e^{2bx+2a}\right)}{b}+\frac{x\operatorname{polylog}}{}$

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x*(exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-1/b^3*a
^2*ln(exp(b*x+a)-1)-1/b*ln(1-exp(b*x+a))*x^2-2*x*polylog(2,exp(b*x+a))/b^2+
x^2*ln(1+exp(2*b*x+2*a))/b+x*polylog(2,-exp(2*b*x+2*a))/b^2-1/b*ln(exp(b*x+
a)+1)*x^2-2*x*polylog(2,-exp(b*x+a))/b^2+1/b^3*ln(1-exp(b*x+a))*a^2+1/b^3*l
n(exp(b*x+a)-1)+1/b^3*ln(exp(b*x+a)+1)-2/b^3*ln(exp(b*x+a))+2*polylog(3,exp
```

(b*x+a))/b^3-1/2*polylog(3,-exp(2*b*x+2*a))/b^3+2*polylog(3,-exp(b*x+a))/b^
3

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 2562, normalized size of antiderivative = 17.31

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")

[Out] -(2*(b*x + a)*cosh(b*x + a)^4 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + 2*(b*x + a)*sinh(b*x + a)^4 + 2*(b^2*x^2 - b*x - 2*a)*cosh(b*x + a)^2 + 2*(b^2*x^2 + 6*(b*x + a)*cosh(b*x + a)^2 - b*x - 2*a)*sinh(b*x + a)^2 + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) + ((b^2*x^2 - 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 - 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 1)*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 - 1)*cosh(b*x + a)^3 - (b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 - a^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) - (a^2*cosh(b*x + a)^4 + 4*a^2*cosh(b*x + a)*sinh(b*x + a)^3 + a^2*sinh(b*x + a)^4 - 2*a^2*cosh(b*x + a)^2 + 2*(3*a^2*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 + a^2 + 4*(a^2*cosh(b*x + a)^3 - a^2*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((a^2 - 1)*cosh(b*x + a)^4 + 4*(a^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 - 1)*sinh(b*x + a)^4 - 2*(a^2 - 1)*cosh(b*x + a)^2 + 2*(3*(a^2 - 1)*cosh(b*x + a)^2 - a^2 + 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 - 1)*cosh(b*x + a)^3 - (a^2 - 1)*cosh(

```
b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - ((b^2
*x^2 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x
 + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x +
a)^2 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x
+ a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b^2*x^2
 - a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 (b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
 - a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a)
)*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b^2*x^2 -
a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b
^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^2
 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2 -
a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))*s
inh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
 a)) + 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cos
h(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -I*cosh(b*x + a
) - I*sinh(b*x + a)) - 2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
 + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3,
 -cosh(b*x + a) - sinh(b*x + a)) + 4*(2*(b*x + a)*cosh(b*x + a)^3 + (b^2*x^
2 - b*x - 2*a)*cosh(b*x + a))*sinh(b*x + a) + 2*a)/(b^3*cosh(b*x + a)^4 + 4
*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b^3*cosh(b*x +
 a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh
(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx=\int x^2\,\text{csch}^3\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a),x)
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.64

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = -\frac{2\left(\left(bx^2e^{(2a)}+xe^{(2a)}\right)e^{(2bx)}-x\right)}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}-\frac{2x}{b^2} + \frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\text{Li}_2\left(-e^{(2bx+2a)}\right)-\text{Li}_3\left(-e^{(2bx+2a)}\right)}{2b^3} - \frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\text{Li}_2\left(-e^{(bx+a)}\right)-2\text{Li}_3\left(-e^{(bx+a)}\right)}{b^3} - \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\text{Li}_2\left(e^{(bx+a)}\right)-2\text{Li}_3\left(e^{(bx+a)}\right)}{b^3} + \frac{\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}-1\right)}{b^3}$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*((b*x^2*e^(2*a) + x*e^(2*a))*e^(2*b*x) - x)/(b^2*e^(4*b*x + 4*a) - 2*b^2
*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + 1/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1)
 + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))/b^3 - (b^2
*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*
x + a)))/b^3 - (b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) -
2*polylog(3, e^(b*x + a)))/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a)
 - 1)/b^3
```

Giac [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x^2\,\text{csch}\left(bx+a\right)^3\text{sech}\left(bx+a\right)\,dx$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}^3(a+bx) \operatorname{sech}(a+bx) \, dx = \int \frac{x^2}{\cosh(a+bx) \sinh(a+bx)^3} \, dx$$

```
[In] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^3),x)
[Out] int(x^2/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.510 $\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \frac{x}{2b}+\frac{2x\text{arctanh}(e^{2a+2bx})}{b} - \frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b} + \frac{\text{PolyLog}(2,-e^{2a+2bx})}{2b^2}-\frac{\text{PolyLog}(2,e^{2a+2bx})}{2b^2}$$

```
[Out] 1/2*x/b+2*x*arctanh(exp(2*b*x+2*a))/b-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a)
^2/b+1/2*polylog(2,-exp(2*b*x+2*a))/b^2-1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2700, 14, 5570, 3554, 8, 2628, 12, 4267, 2317, 2438}

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \frac{2x\text{arctanh}(e^{2a+2bx})}{b}+\frac{\text{PolyLog}(2,-e^{2a+2bx})}{2b^2} - \frac{\text{PolyLog}(2,e^{2a+2bx})}{2b^2}-\frac{\coth(a+bx)}{2b^2} - \frac{x\coth^2(a+bx)}{2b}+\frac{x}{2b}$$

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] x/(2*b) + (2*x*ArcTanh[E^(2*a + 2*b*x)])/b - Coth[a + b*x]/(2*b^2) - (x*Cot
h[a + b*x]^2)/(2*b) + PolyLog[2, -E^(2*a + 2*b*x)]/(2*b^2) - PolyLog[2, E^(
2*a + 2*b*x)]/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x\coth^2(a+bx)}{2b}-\frac{x\log(\tanh(a+bx))}{b} \\
&\quad -\int\left(-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}\right)dx \\
&= -\frac{x\coth^2(a+bx)}{2b}-\frac{x\log(\tanh(a+bx))}{b}+\frac{\int\coth^2(a+bx)\,dx}{2b}+\frac{\int\log(\tanh(a+bx))\,dx}{b} \\
&= -\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{\int 1\,dx}{2b}-\frac{\int 2bx\text{csch}(2a+2bx)\,dx}{b} \\
&= \frac{x}{2b}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}-2\int x\text{csch}(2a+2bx)\,dx \\
&= \frac{x}{2b}+\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b} \\
&\quad +\frac{\int\log\left(1-e^{2a+2bx}\right)\,dx}{b}-\frac{\int\log\left(1+e^{2a+2bx}\right)\,dx}{b} \\
&= \frac{x}{2b}+\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b} \\
&\quad +\frac{\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^2}-\frac{\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^2} \\
&= \frac{x}{2b}+\frac{2x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b} \\
&\quad +\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int x\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\coth(a+bx)+bx\operatorname{csch}^2(a+bx)+2bx\log\left(1-e^{-2(a+bx)}\right)-2bx\log\left(1+e^{-2(a+bx)}\right)+\operatorname{PolyLog}\left(2,-e^{-2}\right.}{2b^2}$$

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 2*b*x*Log[1 - E^(-2*(a + b*x))]
 - 2*b*x*Log[1 + E^(-2*(a + b*x))] + PolyLog[2, -E^(-2*(a + b*x))] - PolyLo
g[2, E^(-2*(a + b*x))])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(82) = 164.

Time = 1.84 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}bx+\mathrm{e}^{2bx+2a}-1}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}-\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a}{b^2}-\frac{\operatorname{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)x}{b}-\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}$

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(2*exp(2*b*x+2*a)*b*x+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-1/b*ln(1-
exp(b*x+a))*x-1/b^2*ln(1-exp(b*x+a))*a-polylog(2,exp(b*x+a))/b^2-1/b*ln(exp
(b*x+a)+1)*x-polylog(2,-exp(b*x+a))/b^2+x*ln(1+exp(2*b*x+2*a))/b+1/2*polylo
g(2,-exp(2*b*x+2*a))/b^2+1/b^2*a*ln(exp(b*x+a)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1578, normalized size of antiderivative = 16.61

$$\int x\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -((2*b*x + 1)*cosh(b*x + a)^2 + 2*(2*b*x + 1)*cosh(b*x + a)*sinh(b*x + a) +
 (2*b*x + 1)*sinh(b*x + a)^2 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
+ a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*co
sh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilo
```

```
g(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*
dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - (cosh(b*x + a)^4 + 4*cosh(b*x +
a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
 a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x +
a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (cosh(b*x + a)^4 + 4*co
sh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*s
inh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*si
nh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)
^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh
(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*
x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3
+ a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sin
h(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*sinh(b*x + a) + a)*l
og(cosh(b*x + a) + sinh(b*x + a) + I) + (a*cosh(b*x + a)^4 + 4*a*cosh(b*x +
 a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cosh
(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a))*
sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I) - (a*cosh(b*x + a
)^4 + 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x
+ a)^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3
 - a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) -
1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3
 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)
*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x +
 a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(I*cosh(b*x + a) + I
*sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2
 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*
x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-I
*cosh(b*x + a) - I*sinh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^4 + 4*(b*x
 + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x +
a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a
)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)*cosh(b*x + a))*sinh(b*
x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 1)/(b^2*cosh(b*x + a)
^4 + 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh
(b*x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^
2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}^3\,(a+bx)\,\text{sech}\,(a+bx)\,dx$$

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a),x)
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\begin{aligned}\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = &-\frac{(2\,bxe^{(2\,a)}+e^{(2\,a)})e^{(2\,bx)}-1}{b^2e^{(4\,bx+4\,a)}-2\,b^2e^{(2\,bx+2\,a)}+b^2}\\ &+\frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{2\,b^2}\\ &-\frac{bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)}{b^2}\\ &-\frac{bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)}{b^2}\end{aligned}$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
[Out] -((2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x) - 1)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(
2*b*x + 2*a) + b^2) + 1/2*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
 + 2*a)))/b^2 - (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - (b*x
*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int x\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)\,dx$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^3(a+bx)\text{sech}(a+bx)\,dx = \int \frac{x}{\cosh(a+b\,x)\,\sinh(a+b\,x)^3}\,dx$$

```
[In] int(x/(cosh(a + b*x)*sinh(a + b*x)^3),x)
[Out] int(x/(cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.511 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2700, 14}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b - Log[Tanh[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, i\tanh(a+bx)\right)}{b}$$
$$= -\frac{\coth^2(a+bx)}{2b} - \frac{\log(\tanh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\operatorname{csch}^2(a+bx) - 2\log(\cosh(a+bx)) + 2\log(\sinh(a+bx))}{2b}$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
[Out] -1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
default	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
risch	$-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{2bx+2a}-1\right)}{b}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	62

```
[In] int(csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sinh(b*x+a)^2-ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 13.54

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx =$$
$$-\frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a\right.\right.}{}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a
)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a
) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(b*x +
a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a
)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b
*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*si
nh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\frac{e^{(2bx+2a)}+e^{(-2bx-2a)}-6}{e^{(2bx+2a)}+e^{(-2bx-2a)}-2}+\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)-\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)}{2b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
- 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x +
2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*a
 + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.512 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 55.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x}dx$$

```
[In] int(csch(b*x+a)^3*sech(b*x+a)/x,x)
[Out] int(csch(b*x+a)^3*sech(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)/x,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 9.28

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2
*x^2*e^(2*b*x + 2*a) + b^2*x^2) + 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*
e^(b*x + a) + b^2*x^3), x) - 16*integrate(1/16*(b^2*x^2 - 1)/(b^2*x^3*e^(b*
x + a) - b^2*x^3), x) - 16*integrate(1/8/(x*e^(2*b*x + 2*a) + x), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x}\,dx = \int \frac{1}{x\cosh\left(a+b\,x\right)\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.513 $\int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x^2}\,dx = \int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{csch}^3(a+bx)\text{sech}(a+bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 23.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)}{x^{2}}dx$$

```
[In] int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
[Out] int(csch(b*x+a)^3*sech(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^{3}\left(a+bx\right)\operatorname{sech}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)/x**2,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 9.44

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -2*((b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) + 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*
e^(b*x + a) + b^2*x^4), x) - 16*integrate(1/16*(b^2*x^2 - 3)/(b^2*x^4*e^(b*
x + a) - b^2*x^4), x) - 16*integrate(1/8/(x^2*e^(2*b*x + 2*a) + x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)}{x^2}\,dx=\int \frac{1}{x^2\cosh\left(a+b\,x\right)\,{\sinh\left(a+b\,x\right)}^3}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)*sinh(a + b*x)^3), x)
```

3.514 $\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 60.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^2 dx$$

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^2\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^m \operatorname{csch}^3\left(a+bx\right) \operatorname{sech}^2\left(a+bx\right) dx$$

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**2,x)
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^m\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
[Out] int(x^m/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.515 $\int x^3 \mathbf{csch}^3(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 317

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{6x^2\arctan(e^{a+bx})}{b^2} - \frac{6x\operatorname{arctanh}(e^{a+bx})}{b^3} + \frac{3x^3\operatorname{arctanh}(e^{a+bx})}{b} - \frac{3x^2\operatorname{csch}(a+bx)}{2b^2} - \frac{3\operatorname{PolyLog}(2,-e^{a+bx})}{b^4} + \frac{9x^2\operatorname{PolyLog}(2,-e^{a+bx})}{2b^2} - \frac{6ix\operatorname{PolyLog}(2,-ie^{a+bx})}{b^3} + \frac{6ix\operatorname{PolyLog}(2,ie^{a+bx})}{b^3} + \frac{3\operatorname{PolyLog}(2,e^{a+bx})}{b^4} - \frac{9x^2\operatorname{PolyLog}(2,e^{a+bx})}{2b^2} - \frac{9x\operatorname{PolyLog}(3,-e^{a+bx})}{b^3} + \frac{6i\operatorname{PolyLog}(3,-ie^{a+bx})}{b^4} - \frac{6i\operatorname{PolyLog}(3,ie^{a+bx})}{b^4} + \frac{9x\operatorname{PolyLog}(3,e^{a+bx})}{b^3} + \frac{9\operatorname{PolyLog}(4,-e^{a+bx})}{b^4} - \frac{9\operatorname{PolyLog}(4,e^{a+bx})}{b^4} - \frac{3x^3\operatorname{sech}(a+bx)}{2b} - \frac{x^3\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 6*x^2*arctan(exp(b*x+a))/b^2-6*x*arctanh(exp(b*x+a))/b^3+3*x^3*arctanh(exp(
b*x+a))/b-3/2*x^2*csch(b*x+a)/b^2-3*polylog(2,-exp(b*x+a))/b^4+9/2*x^2*poly
log(2,-exp(b*x+a))/b^2-6*I*x*polylog(2,-I*exp(b*x+a))/b^3+6*I*x*polylog(2,I
*exp(b*x+a))/b^3+3*polylog(2,exp(b*x+a))/b^4-9/2*x^2*polylog(2,exp(b*x+a))/
b^2-9*x*polylog(3,-exp(b*x+a))/b^3+6*I*polylog(3,-I*exp(b*x+a))/b^4-6*I*pol
ylog(3,I*exp(b*x+a))/b^4+9*x*polylog(3,exp(b*x+a))/b^3+9*polylog(4,-exp(b*x
+a))/b^4-9*polylog(4,exp(b*x+a))/b^4-3/2*x^3*sech(b*x+a)/b-1/2*x^3*csch(b*x
+a)^2*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {2702, 294, 327, 213, 5570, 14, 6408, 12, 4267, 2611, 6744, 2320, 6724, 6874, 4265, 2701, 5313, 2317, 2438}

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\frac{6x^2\arctan\left(e^{a+bx}\right)}{b^2}-\frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3}+\frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^4}+\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^4}+\frac{6i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4}-\frac{6i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4}+\frac{9\,\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^4}-\frac{9\,\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}-\frac{6ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{6ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{9x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{9x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{9x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{9x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{3x^2\text{csch}(a+bx)}{2b^2}-\frac{3x^3\text{sech}(a+bx)}{2b}-\frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (6*x^2*ArcTan[E^(a + b*x)])/b^2 - (6*x*ArcTanh[E^(a + b*x)])/b^3 + (3*x^3*A
rcTanh[E^(a + b*x)])/b - (3*x^2*Csch[a + b*x])/(2*b^2) - (3*PolyLog[2, -E^(
a + b*x)])/b^4 + (9*x^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) - ((6*I)*x*PolyLo
g[2, (-I)*E^(a + b*x)])/b^3 + ((6*I)*x*PolyLog[2, I*E^(a + b*x)])/b^3 + (3*
PolyLog[2, E^(a + b*x)])/b^4 - (9*x^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (9
*x*PolyLog[3, -E^(a + b*x)])/b^3 + ((6*I)*PolyLog[3, (-I)*E^(a + b*x)])/b^4
 - ((6*I)*PolyLog[3, I*E^(a + b*x)])/b^4 + (9*x*PolyLog[3, E^(a + b*x)])/b^
3 + (9*PolyLog[4, -E^(a + b*x)])/b^4 - (9*PolyLog[4, E^(a + b*x)])/b^4 - (3
*x^3*Sech[a + b*x])/(2*b) - (x^3*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{3x^3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{3x^3\operatorname{sech}(a+bx)}{2b} - \frac{x^3\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - 3\int x^2\left(\frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}\right)dx$$

$$= \frac{3x^3\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - 3\int \left(\frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b} - \frac{x^2(3+\text{csch}^2(a+bx))\,\text{sech}(a+bx)}{2b}\right)dx$$

$$= \frac{3x^3\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{3\int x^2(3+\text{csch}^2(a+bx))\,\text{sech}(a+bx)\,dx}{2b} - \frac{9\int x^2\text{arctanh}(\cosh(a+bx))\,dx}{2b}$$

$$= -\frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\int bx^3\text{csch}(a+bx)\,dx}{2b} + \frac{3\int\left(3x^2\text{sech}(a+bx)+x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\right)\,dx}{2b}$$

$$= -\frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3}{2}\int x^3\text{csch}(a+bx)\,dx + \frac{3\int x^2\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx}{2b} + \frac{9\int x^2\text{sech}(a+bx)\,dx}{2b}$$

$$= \frac{9x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\arctan(\sinh(a+bx))}{2b^2} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{(9i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b^2} + \frac{(9i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b^2} - \frac{3\int x\left(-\frac{\arctan(\sinh(a+bx))}{b} - \frac{\text{csch}(a+bx)}{b}\right)\,dx}{b} + \frac{9\int x^2\log\left(1-e^{a+bx}\right)\,dx}{2b} - \frac{9\int x^2\log\left(1+e^{a+bx}\right)\,dx}{2b}$$

$$= \frac{9x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\arctan(\sinh(a+bx))}{2b^2} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} + \frac{9x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{9ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{9ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{9x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{(9i)\int\text{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^3} - \frac{(9i)\int\text{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^3} - \frac{9\int x\,\text{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} + \frac{9\int x\,\text{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2} - \frac{3\int\left(-\frac{x\arctan(\sinh(a+bx))}{b} - \frac{x\text{csch}(a+bx)}{b}\right)\,dx}{b}$$

$$= \frac{9x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{3x^2\arctan(\sinh(a+bx))}{2b^2} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} + \frac{9x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{9ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{9ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{9x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{9x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{9x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{(9i)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{(9i)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{a+bx}\right)}{b^4} + \frac{9\int\text{PolyLog}\left(3,-e^{a+bx}\right)\,dx}{b^3} - \frac{9\int\text{PolyLog}\left(3,e^{a+bx}\right)\,dx}{b^3} + \frac{3\int x\arctan(\sinh(a+bx))\,dx}{b^2} + \frac{3\int x\text{csch}(a+bx)\,dx}{b^2}$$

$$= \frac{9x^2\arctan\left(e^{a+bx}\right)}{b^2} - \frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} + \frac{9x^2\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{9ix\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{9ix\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{9x^2\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{9x\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{9i\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} - \frac{9i\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} + \frac{9x\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{9\text{Subst}\left(\int\frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{9\text{Subst}\left(\int\frac{\text{PolyLog}(3,x)}{x}\,dx,x,e^{a+bx}\right)}{b^4} - \frac{3\int\log\left(1-e^{a+bx}\right)\,dx}{b^3} + \frac{3\int\log\left(1+e^{a+bx}\right)\,dx}{b^3} - \frac{3\int bx^2\text{sech}(a+bx)\,dx}{2b^2}$$

$$
\begin{aligned}
&= \frac{9x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} \\
&- \frac{3x^2\text{csch}(a+bx)}{2b^2} + \frac{9x^2 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} - \frac{9ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} \\
&+ \frac{9ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{9x^2 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{9x \operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} \\
&+ \frac{9i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} - \frac{9i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} + \frac{9x \operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\
&+ \frac{9 \operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} - \frac{9 \operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4} - \frac{3x^3\text{sech}(a+bx)}{2b} \\
&- \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^4} \\
&+ \frac{3\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{b^4} - \frac{3\int x^2\text{sech}(a+bx)\,dx}{2b} \\
&= \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} \\
&- \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{3 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^4} + \frac{9x^2 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} \\
&- \frac{9ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{9ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{3 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^4} \\
&- \frac{9x^2 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{9x \operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{9i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} \\
&- \frac{9i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} + \frac{9x \operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} + \frac{9 \operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} \\
&- \frac{9 \operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} \\
&+ \frac{(3i)\int x\log\left(1-ie^{a+bx}\right)\,dx}{b^2} - \frac{(3i)\int x\log\left(1+ie^{a+bx}\right)\,dx}{b^2} \\
&= \frac{6x^2 \arctan\left(e^{a+bx}\right)}{b^2} - \frac{6x\text{arctanh}\left(e^{a+bx}\right)}{b^3} + \frac{3x^3\text{arctanh}\left(e^{a+bx}\right)}{b} \\
&- \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{3 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^4} + \frac{9x^2 \operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2} \\
&- \frac{6ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{6ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} + \frac{3 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^4} \\
&- \frac{9x^2 \operatorname{PolyLog}\left(2,e^{a+bx}\right)}{2b^2} - \frac{9x \operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{9i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^4} \\
&- \frac{9i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^4} + \frac{9x \operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} + \frac{9 \operatorname{PolyLog}\left(4,-e^{a+bx}\right)}{b^4} \\
&- \frac{9 \operatorname{PolyLog}\left(4,e^{a+bx}\right)}{b^4} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} \\
&- \frac{(3i)\int \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)\,dx}{b^3} + \frac{(3i)\int \operatorname{PolyLog}\left(2,ie^{a+bx}\right)\,dx}{b^3}
\end{aligned}
$$

$$= \frac{6x^2 \arctan(e^{a+bx})}{b^2} - \frac{6x\text{arctanh}(e^{a+bx})}{b^3} + \frac{3x^3\text{arctanh}(e^{a+bx})}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{3\,\text{PolyLog}(2,-e^{a+bx})}{b^4} + \frac{9x^2\,\text{PolyLog}(2,-e^{a+bx})}{2b^2} - \frac{6ix\,\text{PolyLog}(2,-ie^{a+bx})}{b^3} + \frac{6ix\,\text{PolyLog}(2,ie^{a+bx})}{b^3} + \frac{3\,\text{PolyLog}(2,e^{a+bx})}{b^4} - \frac{9x^2\,\text{PolyLog}(2,e^{a+bx})}{2b^2} - \frac{9x\,\text{PolyLog}(3,-e^{a+bx})}{b^3} + \frac{9i\,\text{PolyLog}(3,-ie^{a+bx})}{b^4} - \frac{9i\,\text{PolyLog}(3,ie^{a+bx})}{b^4} + \frac{9x\,\text{PolyLog}(3,e^{a+bx})}{b^3} + \frac{9\,\text{PolyLog}(4,-e^{a+bx})}{b^4} - \frac{9\,\text{PolyLog}(4,e^{a+bx})}{b^4} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^4} + \frac{(3i)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{a+bx}\right)}{b^4}$$

$$= \frac{6x^2 \arctan(e^{a+bx})}{b^2} - \frac{6x\text{arctanh}(e^{a+bx})}{b^3} + \frac{3x^3\text{arctanh}(e^{a+bx})}{b} - \frac{3x^2\text{csch}(a+bx)}{2b^2} - \frac{3\,\text{PolyLog}(2,-e^{a+bx})}{b^4} + \frac{9x^2\,\text{PolyLog}(2,-e^{a+bx})}{2b^2} - \frac{6ix\,\text{PolyLog}(2,-ie^{a+bx})}{b^3} + \frac{6ix\,\text{PolyLog}(2,ie^{a+bx})}{b^3} + \frac{3\,\text{PolyLog}(2,e^{a+bx})}{b^4} - \frac{9x^2\,\text{PolyLog}(2,e^{a+bx})}{2b^2} - \frac{9x\,\text{PolyLog}(3,-e^{a+bx})}{b^3} + \frac{6i\,\text{PolyLog}(3,-ie^{a+bx})}{b^4} - \frac{6i\,\text{PolyLog}(3,ie^{a+bx})}{b^4} + \frac{9x\,\text{PolyLog}(3,e^{a+bx})}{b^3} + \frac{9\,\text{PolyLog}(4,-e^{a+bx})}{b^4} - \frac{9\,\text{PolyLog}(4,e^{a+bx})}{b^4} - \frac{3x^3\text{sech}(a+bx)}{2b} - \frac{x^3\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 6.44 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.47

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{3x^2\text{csch}(a)}{2b^2} - \frac{x^3\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{3i\left(b^2x^2\log\left(1-ie^{a+bx}\right) - b^2x^2\log\left(1+ie^{a+bx}\right) - 2bx\,\text{PolyLog}\left(2,-ie^{a+bx}\right) + 2bx\,\text{PolyLog}\left(2,ie^{a+bx}\right) + 2\right.}{b^4} - \frac{3\left(-\frac{x\log(1-e^{a+bx})}{b} + \frac{1}{2}bx^3\log\left(1-e^{a+bx}\right) + \frac{x\log(1+e^{a+bx})}{b} - \frac{1}{2}bx^3\log\left(1+e^{a+bx}\right) + \frac{\text{PolyLog}(2,-e^{a+bx})}{b^2} - \frac{3}{2}x^2\,\text{Po}\right.}{} - \frac{x^3\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{x^3\text{sech}(a+bx)}{b} + \frac{3x^2\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2} + \frac{3x^2\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2}$$

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (-3*x^2*Csch[a])/(2*b^2) - (x^3*Csch[a/2 + (b*x)/2]^2)/(8*b) + ((3*I)*(b^2*
x^2*Log[1 - I*E^(a + b*x)] - b^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*x*PolyLog
[2, (-I)*E^(a + b*x)] + 2*b*x*PolyLog[2, I*E^(a + b*x)] + 2*PolyLog[3, (-I)
*E^(a + b*x)] - 2*PolyLog[3, I*E^(a + b*x)]))/b^4 - (3*(-((x*Log[1 - E^(a +
 b*x)])/b) + (b*x^3*Log[1 - E^(a + b*x)])/2 + (x*Log[1 + E^(a + b*x)])/b -
(b*x^3*Log[1 + E^(a + b*x)])/2 + PolyLog[2, -E^(a + b*x)]/b^2 - (3*x^2*Poly
Log[2, -E^(a + b*x)])/2 - PolyLog[2, E^(a + b*x)]/b^2 + (3*x^2*PolyLog[2, E
^(a + b*x)])/2 + (3*x*PolyLog[3, -E^(a + b*x)])/b - (3*x*PolyLog[3, E^(a +
b*x)])/b - (3*PolyLog[4, -E^(a + b*x)])/b^2 + (3*PolyLog[4, E^(a + b*x)])/b
^2))/b^2 - (x^3*Sech[a/2 + (b*x)/2]^2)/(8*b) - (x^3*Sech[a + b*x])/b + (3*x
^2*Csch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (3*x^2*Sech[a/2]*
Sech[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```

Maple [F]

$$\int x^3 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

```
[In] int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] int(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5356 vs. $2(270) = 540$.

Time = 0.33 (sec) , antiderivative size = 5356, normalized size of antiderivative = 16.90

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^3\,\text{csch}^3\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

```
[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**2,x)
[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^3\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
[Out] (2*b*x^3*e^(3*b*x + 3*a) - 3*(b*x^3*e^(5*a) + x^2*e^(5*a))*e^(5*b*x) - 3*(b
*x^3*e^a - x^2*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b
^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*d
ilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x
+ a)))/b^4 - 3/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x +
a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3*(b
*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a
) + 1) + dilog(e^(b*x + a)))/b^4 + 96*integrate(1/16*x^2*e^(b*x + a)/(b*e^(
2*b*x + 2*a) + b), x)
```

Giac [F]

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x^3\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
[Out] int(x^3/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.516 $\int x^2 \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 197

$$\int x^2 \text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{4x\arctan\left(e^{a+bx}\right)}{b^2}+\frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3}-\frac{x\text{csch}(a+bx)}{b^2} + \frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

```
[Out] 4*x*arctan(exp(b*x+a))/b^2+3*x^2*arctanh(exp(b*x+a))/b-arctanh(cosh(b*x+a))
/b^3-x*csch(b*x+a)/b^2+3*x*polylog(2,-exp(b*x+a))/b^2-2*I*polylog(2,-I*exp(
b*x+a))/b^3+2*I*polylog(2,I*exp(b*x+a))/b^3-3*x*polylog(2,exp(b*x+a))/b^2-3
*polylog(3,-exp(b*x+a))/b^3+3*polylog(3,exp(b*x+a))/b^3-3/2*x^2*sech(b*x+a)
/b-1/2*x^2*csch(b*x+a)^2*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {2702, 294, 327, 213, 5570, 14, 6408, 12, 4267, 2611, 2320, 6724, 6874, 4265, 2317, 2438, 2701, 5311, 3855}

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \frac{4x\arctan\left(e^{a+bx}\right)}{b^2}-\frac{\text{arctanh}(\cosh(a+bx))}{b^3}+\frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3}+\frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{x\text{csch}(a+bx)}{b^2}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (4*x*ArcTan[E^(a + b*x)])/b^2 + (3*x^2*ArcTanh[E^(a + b*x)])/b - ArcTanh[Co
sh[a + b*x]]/b^3 - (x*Csch[a + b*x])/b^2 + (3*x*PolyLog[2, -E^(a + b*x)])/b
^2 - ((2*I)*PolyLog[2, (-I)*E^(a + b*x)])/b^3 + ((2*I)*PolyLog[2, I*E^(a +
b*x)])/b^3 - (3*x*PolyLog[2, E^(a + b*x)])/b^2 - (3*PolyLog[3, -E^(a + b*x)
])/b^3 + (3*PolyLog[3, E^(a + b*x)])/b^3 - (3*x^2*Sech[a + b*x])/(2*b) - (x
^2*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}\\
&\quad-2\int x\left(\frac{3\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3\text{sech}(a+bx)}{2b}-\frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}\right)dx\\
&=\frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}\\
&\quad-2\int\left(\frac{3x\text{arctanh}(\cosh(a+bx))}{2b}-\frac{x\left(3+\text{csch}^2(a+bx)\right)\text{sech}(a+bx)}{2b}\right)dx\\
&=\frac{3x^2\text{arctanh}(\cosh(a+bx))}{2b}-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}\\
&\quad+\frac{\int x\left(3+\text{csch}^2(a+bx)\right)\text{sech}(a+bx)\,dx}{b}-\frac{3\int x\text{arctanh}(\cosh(a+bx))\,dx}{b}\\
&=-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}\\
&\quad+\frac{\int\left(3x\text{sech}(a+bx)+x\text{csch}^2(a+bx)\text{sech}(a+bx)\right)\,dx}{b}-\frac{3\int bx^2\text{csch}(a+bx)\,dx}{2b}\\
&=-\frac{3x^2\text{sech}(a+bx)}{2b}-\frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}-\frac{3}{2}\int x^2\text{csch}(a+bx)\,dx\\
&\quad+\frac{\int x\text{csch}^2(a+bx)\text{sech}(a+bx)\,dx}{b}+\frac{3\int x\text{sech}(a+bx)\,dx}{b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{6x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{x\arctan(\sinh(a+bx))}{b^2} + \frac{3x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{x\operatorname{csch}(a+bx)}{b^2} \\
&\quad - \frac{3x^2\operatorname{sech}(a+bx)}{2b} - \frac{x^2\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{(3i)\int\log\left(1-ie^{a+bx}\right)\,dx}{b^2} \\
&\quad + \frac{(3i)\int\log\left(1+ie^{a+bx}\right)\,dx}{b^2} - \frac{\int\left(-\frac{\arctan(\sinh(a+bx))}{b} - \frac{\operatorname{csch}(a+bx)}{b}\right)dx}{b} \\
&\quad + \frac{3\int x\log\left(1-e^{a+bx}\right)\,dx}{b} - \frac{3\int x\log\left(1+e^{a+bx}\right)\,dx}{b} \\
&= \frac{6x\arctan\left(e^{a+bx}\right)}{b^2} - \frac{x\arctan(\sinh(a+bx))}{b^2} + \frac{3x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{x\operatorname{csch}(a+bx)}{b^2} \\
&\quad + \frac{3x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{3x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3x^2\operatorname{sech}(a+bx)}{2b} \\
&\quad - \frac{x^2\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{(3i)\operatorname{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3} \\
&\quad + \frac{(3i)\operatorname{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{a+bx}\right)}{b^3} + \frac{\int\arctan(\sinh(a+bx))\,dx}{b^2} \\
&\quad + \frac{\int\operatorname{csch}(a+bx)\,dx}{b^2} - \frac{3\int\operatorname{PolyLog}\left(2,-e^{a+bx}\right)\,dx}{b^2} + \frac{3\int\operatorname{PolyLog}\left(2,e^{a+bx}\right)\,dx}{b^2} \\
&= \frac{6x\arctan\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{arctanh}(\cosh(a+bx))}{b^3} \\
&\quad - \frac{x\operatorname{csch}(a+bx)}{b^2} + \frac{3x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{3i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} \\
&\quad + \frac{3i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{3x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3x^2\operatorname{sech}(a+bx)}{2b} \\
&\quad - \frac{x^2\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{a+bx}\right)}{b^3} \\
&\quad + \frac{3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{a+bx}\right)}{b^3} - \frac{\int bx\operatorname{sech}(a+bx)\,dx}{b^2} \\
&= \frac{6x\arctan\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\operatorname{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\operatorname{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\operatorname{csch}(a+bx)}{b^2} \\
&\quad + \frac{3x\operatorname{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{3i\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{3i\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} \\
&\quad - \frac{3x\operatorname{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3\operatorname{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{3\operatorname{PolyLog}\left(3,e^{a+bx}\right)}{b^3} \\
&\quad - \frac{3x^2\operatorname{sech}(a+bx)}{2b} - \frac{x^2\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b} - \frac{\int x\operatorname{sech}(a+bx)\,dx}{b}
\end{aligned}
$$

$$= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} + \frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{i\int\log\left(1-ie^{a+bx}\right)dx}{b^2} - \frac{i\int\log\left(1+ie^{a+bx}\right)dx}{b^2}$$

$$= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} + \frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{3i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{3i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{i\text{Subst}\left(\int\frac{\log(1-ix)}{x}dx,x,e^{a+bx}\right)}{b^3} - \frac{i\text{Subst}\left(\int\frac{\log(1+ix)}{x}dx,x,e^{a+bx}\right)}{b^3}$$

$$= \frac{4x\arctan\left(e^{a+bx}\right)}{b^2} + \frac{3x^2\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{arctanh}(\cosh(a+bx))}{b^3} - \frac{x\text{csch}(a+bx)}{b^2} + \frac{3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2} - \frac{2i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^3} - \frac{3x\,\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2} - \frac{3\,\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3} - \frac{3x^2\text{sech}(a+bx)}{2b} - \frac{x^2\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 6.34 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.73

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = -\frac{x\text{csch}(a)}{b^2} - \frac{x^2\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{2i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{b^3} + \frac{\frac{\log\left(1-e^{a+bx}\right)}{b}-\frac{3}{2}bx^2\log\left(1-e^{a+bx}\right)-\frac{\log\left(1+e^{a+bx}\right)}{b}+\frac{3}{2}bx^2\log\left(1+e^{a+bx}\right)+3x\,\text{PolyLog}\left(2,-e^{a+bx}\right)-3x\,\text{Po}}{b^2} - \frac{x^2\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{x^2\text{sech}(a+bx)}{b} + \frac{x\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b^2} + \frac{x\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{2b^2}$$

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] -((x*Csch[a])/b^2) - (x^2*Csch[a/2 + (b*x)/2]^2)/(8*b) + ((2*I)*(b*x*(Log[1
 - I*E^(a + b*x)] - Log[1 + I*E^(a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)]
+ PolyLog[2, I*E^(a + b*x)]))/b^3 + (Log[1 - E^(a + b*x)]/b - (3*b*x^2*Log[
1 - E^(a + b*x)])/2 - Log[1 + E^(a + b*x)]/b + (3*b*x^2*Log[1 + E^(a + b*x)
])/2 + 3*x*PolyLog[2, -E^(a + b*x)] - 3*x*PolyLog[2, E^(a + b*x)] - (3*Poly
Log[3, -E^(a + b*x)])/b + (3*PolyLog[3, E^(a + b*x)])/b)/b^2 - (x^2*Sech[a/
2 + (b*x)/2]^2)/(8*b) - (x^2*Sech[a + b*x])/b + (x*Csch[a/2]*Csch[a/2 + (b*
x)/2]*Sinh[(b*x)/2])/(2*b^2) + (x*Sech[a/2]*Sech[a/2 + (b*x)/2]*Sinh[(b*x)/
2])/(2*b^2)
```

Maple [F]

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 \, dx$$

```
[In] int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

```
[Out] int(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3804 vs. $2(173) = 346$.

Time = 0.32 (sec) , antiderivative size = 3804, normalized size of antiderivative = 19.31

$$\int x^2 \text{csch}^3(a+bx)\text{sech}^2(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b^2*x^2*cosh(b*x + a)^3 - 2*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^5 - 10
*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)*sinh(b*x + a)^4 - 2*(3*b^2*x^2 + 2*b*x)*
sinh(b*x + a)^5 + 4*(b^2*x^2 - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^2)*sinh(
b*x + a)^3 + 4*(3*b^2*x^2*cosh(b*x + a) - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x +
a)^3)*sinh(b*x + a)^2 - 2*(3*b^2*x^2 - 2*b*x)*cosh(b*x + a) - 6*(b*x*cosh(b
*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x
*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cos
h(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^
3 + (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2
+ b*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a
))*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(-I*cosh(b*x + a
)^6 - 6*I*cosh(b*x + a)*sinh(b*x + a)^5 - I*sinh(b*x + a)^6 + (-15*I*cosh(b
*x + a)^2 + I)*sinh(b*x + a)^4 + I*cosh(b*x + a)^4 + 4*(-5*I*cosh(b*x + a)^
3 + I*cosh(b*x + a))*sinh(b*x + a)^3 + (-15*I*cosh(b*x + a)^4 + 6*I*cosh(b*
```

x + a)^2 + I)*sinh(b*x + a)^2 + I*cosh(b*x + a)^2 + 2*(-3*I*cosh(b*x + a)^5
 + 2*I*cosh(b*x + a)^3 + I*cosh(b*x + a))*sinh(b*x + a) - I)*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) - 4*(I*cosh(b*x + a)^6 + 6*I*cosh(b*x + a)*sinh(
b*x + a)^5 + I*sinh(b*x + a)^6 + (15*I*cosh(b*x + a)^2 - I)*sinh(b*x + a)^4
 - I*cosh(b*x + a)^4 + 4*(5*I*cosh(b*x + a)^3 - I*cosh(b*x + a))*sinh(b*x +
 a)^3 + (15*I*cosh(b*x + a)^4 - 6*I*cosh(b*x + a)^2 - I)*sinh(b*x + a)^2 -
I*cosh(b*x + a)^2 + 2*(3*I*cosh(b*x + a)^5 - 2*I*cosh(b*x + a)^3 - I*cosh(b
*x + a))*sinh(b*x + a) + I)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 6*(
b*x*cosh(b*x + a)^6 + 6*b*x*cosh(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x +
a)^6 - b*x*cosh(b*x + a)^4 + (15*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4
 - b*x*cosh(b*x + a)^2 + 4*(5*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh
(b*x + a)^3 + (15*b*x*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b
*x + a)^2 + b*x + 2*(3*b*x*cosh(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*co
sh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) + ((3*b^2
*x^2 - 2)*cosh(b*x + a)^6 + 6*(3*b^2*x^2 - 2)*cosh(b*x + a)*sinh(b*x + a)^5
 + (3*b^2*x^2 - 2)*sinh(b*x + a)^6 - (3*b^2*x^2 - 2)*cosh(b*x + a)^4 - (3*b
^2*x^2 - 15*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 + 3*b^2*x^
2 + 4*(5*(3*b^2*x^2 - 2)*cosh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*s
inh(b*x + a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a)^2 + (15*(3*b^2*x^2 - 2)*cosh
(b*x + a)^4 - 3*b^2*x^2 - 6*(3*b^2*x^2 - 2)*cosh(b*x + a)^2 + 2)*sinh(b*x +
 a)^2 + 2*(3*(3*b^2*x^2 - 2)*cosh(b*x + a)^5 - 2*(3*b^2*x^2 - 2)*cosh(b*x +
 a)^3 - (3*b^2*x^2 - 2)*cosh(b*x + a))*sinh(b*x + a) - 2)*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) - 4*(I*a*cosh(b*x + a)^6 + 6*I*a*cosh(b*x + a)*sinh(b
*x + a)^5 + I*a*sinh(b*x + a)^6 - I*a*cosh(b*x + a)^4 + (15*I*a*cosh(b*x +
a)^2 - I*a)*sinh(b*x + a)^4 + 4*(5*I*a*cosh(b*x + a)^3 - I*a*cosh(b*x + a))
*sinh(b*x + a)^3 - I*a*cosh(b*x + a)^2 + (15*I*a*cosh(b*x + a)^4 - 6*I*a*co
sh(b*x + a)^2 - I*a)*sinh(b*x + a)^2 + 2*(3*I*a*cosh(b*x + a)^5 - 2*I*a*cos
h(b*x + a)^3 - I*a*cosh(b*x + a))*sinh(b*x + a) + I*a)*log(cosh(b*x + a) +
sinh(b*x + a) + I) - 4*(-I*a*cosh(b*x + a)^6 - 6*I*a*cosh(b*x + a)*sinh(b*x
 + a)^5 - I*a*sinh(b*x + a)^6 + I*a*cosh(b*x + a)^4 + (-15*I*a*cosh(b*x + a
)^2 + I*a)*sinh(b*x + a)^4 + 4*(-5*I*a*cosh(b*x + a)^3 + I*a*cosh(b*x + a))
*sinh(b*x + a)^3 + I*a*cosh(b*x + a)^2 + (-15*I*a*cosh(b*x + a)^4 + 6*I*a*c
osh(b*x + a)^2 + I*a)*sinh(b*x + a)^2 + 2*(-3*I*a*cosh(b*x + a)^5 + 2*I*a*c
osh(b*x + a)^3 + I*a*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(cosh(b*x + a)
+ sinh(b*x + a) - I) - ((3*a^2 - 2)*cosh(b*x + a)^6 + 6*(3*a^2 - 2)*cosh(b*
x + a)*sinh(b*x + a)^5 + (3*a^2 - 2)*sinh(b*x + a)^6 - (3*a^2 - 2)*cosh(b*x
 + a)^4 + (15*(3*a^2 - 2)*cosh(b*x + a)^2 - 3*a^2 + 2)*sinh(b*x + a)^4 + 4*
(5*(3*a^2 - 2)*cosh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x + a))*sinh(b*x + a)^3
 - (3*a^2 - 2)*cosh(b*x + a)^2 + (15*(3*a^2 - 2)*cosh(b*x + a)^4 - 6*(3*a^2
 - 2)*cosh(b*x + a)^2 - 3*a^2 + 2)*sinh(b*x + a)^2 + 3*a^2 + 2*(3*(3*a^2 -
2)*cosh(b*x + a)^5 - 2*(3*a^2 - 2)*cosh(b*x + a)^3 - (3*a^2 - 2)*cosh(b*x +
 a))*sinh(b*x + a) - 2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 4*((I*b*x
+ I*a)*cosh(b*x + a)^6 + 6*(I*b*x + I*a)*cosh(b*x + a)*sinh(b*x + a)^5 + (I
*b*x + I*a)*sinh(b*x + a)^6 + (-I*b*x - I*a)*cosh(b*x + a)^4 + (15*(I*b*x +
 I*a)*cosh(b*x + a)^2 - I*b*x - I*a)*sinh(b*x + a)^4 + 4*(5*(I*b*x + I*a)*c

```
osh(b*x + a)^3 + (-I*b*x - I*a)*cosh(b*x + a))*sinh(b*x + a)^3 + (-I*b*x -
I*a)*cosh(b*x + a)^2 + (15*(I*b*x + I*a)*cosh(b*x + a)^4 + 6*(-I*b*x - I*a)
*cosh(b*x + a)^2 - I*b*x - I*a)*sinh(b*x + a)^2 + I*b*x + 2*(3*(I*b*x + I*a
)*cosh(b*x + a)^5 + 2*(-I*b*x - I*a)*cosh(b*x + a)^3 + (-I*b*x - I*a)*cosh(
b*x + a))*sinh(b*x + a) + I*a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) -
 4*((-I*b*x - I*a)*cosh(b*x + a)^6 + 6*(-I*b*x - I*a)*cosh(b*x + a)*sinh(b*
x + a)^5 + (-I*b*x - I*a)*sinh(b*x + a)^6 + (I*b*x + I*a)*cosh(b*x + a)^4 +
 (15*(-I*b*x - I*a)*cosh(b*x + a)^2 + I*b*x + I*a)*sinh(b*x + a)^4 + 4*(5*(
-I*b*x - I*a)*cosh(b*x + a)^3 + (I*b*x + I*a)*cosh(b*x + a))*sinh(b*x + a)^
3 + (I*b*x + I*a)*cosh(b*x + a)^2 + (15*(-I*b*x - I*a)*cosh(b*x + a)^4 + 6*
(I*b*x + I*a)*cosh(b*x + a)^2 + I*b*x + I*a)*sinh(b*x + a)^2 - I*b*x + 2*(3
*(-I*b*x - I*a)*cosh(b*x + a)^5 + 2*(I*b*x + I*a)*cosh(b*x + a)^3 + (I*b*x
+ I*a)*cosh(b*x + a))*sinh(b*x + a) - I*a)*log(-I*cosh(b*x + a) - I*sinh(b*
x + a) + 1) - 3*((b^2*x^2 - a^2)*cosh(b*x + a)^6 + 6*(b^2*x^2 - a^2)*cosh(b
*x + a)*sinh(b*x + a)^5 + (b^2*x^2 - a^2)*sinh(b*x + a)^6 - (b^2*x^2 - a^2)
*cosh(b*x + a)^4 - (b^2*x^2 - 15*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sin
h(b*x + a)^4 + b^2*x^2 + 4*(5*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 -
a^2)*cosh(b*x + a))*sinh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a)^2 + (15
*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 - 6*(b^2*x^2 - a^2)*cosh(b*x + a
)^2 + a^2)*sinh(b*x + a)^2 - a^2 + 2*(3*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - 2
*(b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x
+ a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 6*(cosh(b*
x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*
sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^
3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x
+ a)) + 2*(6*b^2*x^2*cosh(b*x + a)^2 - 5*(3*b^2*x^2 + 2*b*x)*cosh(b*x + a)^
4 - 3*b^2*x^2 + 2*b*x)*sinh(b*x + a))/(b^3*cosh(b*x + a)^6 + 6*b^3*cosh(b*x
 + a)*sinh(b*x + a)^5 + b^3*sinh(b*x + a)^6 - b^3*cosh(b*x + a)^4 - b^3*cos
h(b*x + a)^2 + (15*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^4 + 4*(5*b^3*co
sh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a)^3 + b^3 + (15*b^3*cosh(b*x
 + a)^4 - 6*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2 + 2*(3*b^3*cosh(b*x
+ a)^5 - 2*b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**2,x)
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int x^2\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2\,dx$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
[Out] (2*b*x^2*e^(3*b*x + 3*a) - (3*b*x^2*e^(5*a) + 2*x*e^(5*a))*e^(5*b*x) - (3*b
*x^2*e^a - 2*x*e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b
^2*e^(2*b*x + 2*a) + b^2) + 3/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog
(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 3/2*(b^2*x^2*log(-e^(b*x
 + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - lo
g(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3 + 32*integrate(1/8*x*e^(b
*x + a)/(b*e^(2*b*x + 2*a) + b), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \int \frac{x^2}{\cosh(a+b\,x)^2\sinh(a+b\,x)^3}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
[Out] int(x^2/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.517 $\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^2}+\frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b}-\frac{\text{csch}(a+bx)}{2b^2}+\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{3x\text{sech}(a+bx)}{2b}-\frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

```
[Out] arctan(sinh(b*x+a))/b^2+3*x*arctanh(exp(b*x+a))/b-1/2*csch(b*x+a)/b^2+3/2*p
olylog(2,-exp(b*x+a))/b^2-3/2*polylog(2,exp(b*x+a))/b^2-3/2*x*sech(b*x+a)/b
-1/2*x*csch(b*x+a)^2*sech(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2702, 294, 327, 213, 5570, 6406, 12, 4267, 2317, 2438, 3855, 2701}

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b^2}+\frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{3\,\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3\,\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{\text{csch}(a+bx)}{2b^2}-\frac{3x\text{sech}(a+bx)}{2b}-\frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]

[Out] ArcTan[Sinh[a + b*x]]/b^2 + (3*x*ArcTanh[E^(a + b*x)])/b - Csch[a + b*x]/(2*b^2) + (3*PolyLog[2, -E^(a + b*x)])/(2*b^2) - (3*PolyLog[2, E^(a + b*x)])/(2*b^2) - (3*x*Sech[a + b*x])/(2*b) - (x*Csch[a + b*x]^2*Sech[a + b*x])/(2*b)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3x\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{3x\operatorname{sech}(a+bx)}{2b}-\frac{x\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}\\ &\quad-\int\left(\frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{3\operatorname{sech}(a+bx)}{2b}-\frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}\right)dx\end{aligned}$$

$$= \frac{3x\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx}{2b} - \frac{3\int \text{arctanh}(\cosh(a+bx))\,dx}{2b} + \frac{3\int \text{sech}(a+bx)\,dx}{2b}$$

$$= \frac{3\arctan(\sinh(a+bx))}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{i\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b^2} - \frac{3\int bx\text{csch}(a+bx)\,dx}{2b}$$

$$= \frac{3\arctan(\sinh(a+bx))}{2b^2} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3}{2}\int x\text{csch}(a+bx)\,dx - \frac{i\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(a+bx)\right)}{2b^2}$$

$$= \frac{\arctan(\sinh(a+bx))}{b^2} + \frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{3\int \log\left(1-e^{a+bx}\right)\,dx}{2b} - \frac{3\int \log\left(1+e^{a+bx}\right)\,dx}{2b}$$

$$= \frac{\arctan(\sinh(a+bx))}{b^2} + \frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{2b^2} - \frac{3\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{a+bx}\right)}{2b^2}$$

$$= \frac{\arctan(\sinh(a+bx))}{b^2} + \frac{3x\text{arctanh}\left(e^{a+bx}\right)}{b} - \frac{\text{csch}(a+bx)}{2b^2} + \frac{3\,\text{PolyLog}\left(2, -e^{a+bx}\right)}{2b^2} - \frac{3\,\text{PolyLog}\left(2, e^{a+bx}\right)}{2b^2} - \frac{3x\text{sech}(a+bx)}{2b} - \frac{x\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx =$$

$$-\frac{-16\arctan\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)+2\coth\left(\frac{1}{2}(a+bx)\right)+bx\text{csch}^2\left(\frac{1}{2}(a+bx)\right)+12bx\log\left(1-e^{a+bx}\right)-12$$

[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^2,x]

[Out] -1/8*(-16*ArcTan[Tanh[(a + b*x)/2]] + 2*Coth[(a + b*x)/2] + b*x*Csch[(a + b*x)/2]^2 + 12*b*x*Log[1 - E^(a + b*x)] - 12*b*x*Log[1 + E^(a + b*x)] - 12*PolyLog[2, -E^(a + b*x)] + 12*PolyLog[2, E^(a + b*x)] + b*x*Sech[(a + b*x)/2]^2 + 8*b*x*Sech[a + b*x] - 2*Tanh[(a + b*x)/2])/b^2

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}bx-2\,e^{2bx+2a}bx+e^{4bx+4a}+3bx-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)}+\frac{2\arctan\left(e^{bx+a}\right)}{b^2}+\frac{3a\ln\left(e^{bx+a}-1\right)}{2b^2}+\frac{3\,\text{dilog}\left(e^{bx+a}\right)}{2b^2}+\frac{3\,\text{dilog}\left(e^{bx+}\right.}{2b^2}$

[In] int(x*csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -exp(b*x+a)*(3*exp(4*b*x+4*a)*b*x-2*exp(2*b*x+2*a)*b*x+exp(4*b*x+4*a)+3*b*x-1)/b^2/(exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))+2/b^2*arctan(exp(b*x+a))+3/2/b^2*a*ln(exp(b*x+a)-1)+3/2/b^2*dilog(exp(b*x+a))+3/2/b^2*dilog(exp(b*x+a)+1)+3/2/b*ln(exp(b*x+a)+1)*x

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1694 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 1694, normalized size of antiderivative = 15.54

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(2*(3*b*x + 1)*cosh(b*x + a)^5 + 10*(3*b*x + 1)*cosh(b*x + a)*sinh(b*x + a)^4 + 2*(3*b*x + 1)*sinh(b*x + a)^5 - 4*b*x*cosh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^3 + 4*(5*(3*b*x + 1)*cosh(b*x + a)^3 - 3*b*x*cosh(b*x + a))*sinh(b*x + a)^2 - 4*(cosh(b*x + a)^6 + 6*cosh

```
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(
b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 2*(3*b*x - 1)*
cosh(b*x + a) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh
(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4
 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x
 + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(
b*x + a) + sinh(b*x + a)) - 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x +
 a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b
*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cos
h(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)
*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(b*x*cosh(b*x + a)^6 + 6*b*x*cos
h(b*x + a)*sinh(b*x + a)^5 + b*x*sinh(b*x + a)^6 - b*x*cosh(b*x + a)^4 + (1
5*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^4 - b*x*cosh(b*x + a)^2 + 4*(5*b
*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b*x*cosh(b*x
+ a)^4 - 6*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 2*(3*b*x*cosh
(b*x + a)^5 - 2*b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*log
(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*(a*cosh(b*x + a)^6 + 6*a*cosh(b*x +
 a)*sinh(b*x + a)^5 + a*sinh(b*x + a)^6 - a*cosh(b*x + a)^4 + (15*a*cosh(b*
x + a)^2 - a)*sinh(b*x + a)^4 + 4*(5*a*cosh(b*x + a)^3 - a*cosh(b*x + a))*s
inh(b*x + a)^3 - a*cosh(b*x + a)^2 + (15*a*cosh(b*x + a)^4 - 6*a*cosh(b*x +
 a)^2 - a)*sinh(b*x + a)^2 + 2*(3*a*cosh(b*x + a)^5 - 2*a*cosh(b*x + a)^3 -
 a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 3*((b*x + a)*cosh(b*x + a)^6 + 6*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^5
 + (b*x + a)*sinh(b*x + a)^6 - (b*x + a)*cosh(b*x + a)^4 + (15*(b*x + a)*co
sh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^4 + 4*(5*(b*x + a)*cosh(b*x + a)^3 -
 (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 - (b*x + a)*cosh(b*x + a)^2 + (15
*(b*x + a)*cosh(b*x + a)^4 - 6*(b*x + a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*
x + a)^2 + b*x + 2*(3*(b*x + a)*cosh(b*x + a)^5 - 2*(b*x + a)*cosh(b*x + a)
^3 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(
b*x + a) + 1) + 2*(5*(3*b*x + 1)*cosh(b*x + a)^4 - 6*b*x*cosh(b*x + a)^2 +
3*b*x - 1)*sinh(b*x + a))/(b^2*cosh(b*x + a)^6 + 6*b^2*cosh(b*x + a)*sinh(b
*x + a)^5 + b^2*sinh(b*x + a)^6 - b^2*cosh(b*x + a)^4 + (15*b^2*cosh(b*x +
a)^2 - b^2)*sinh(b*x + a)^4 - b^2*cosh(b*x + a)^2 + 4*(5*b^2*cosh(b*x + a)^
3 - b^2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*b^2*cosh(b*x + a)^4 - 6*b^2*co
sh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 2*(3*b^2*cosh(b*x + a)^5 - 2*b
^2*cosh(b*x + a)^3 - b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x\,\text{csch}^3\,(a+bx)\,\text{sech}^2\,(a+bx)\,dx$$

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.52

$$\begin{aligned}&\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx\\&=\frac{2\,bxe^{(3\,bx+3\,a)}-\left(3\,bxe^{(5\,a)}+e^{(5\,a)}\right)e^{(5\,bx)}-\left(3\,bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(6\,bx+6\,a)}-b^2e^{(4\,bx+4\,a)}-b^2e^{(2\,bx+2\,a)}+b^2}\\&\quad+\frac{3\,\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{2\,b^2}\\&\quad-\frac{3\,\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{2\,b^2}+\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b^2}\end{aligned}$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) + e^(5*a))*e^(5*b*x) - (3*b*x*e^a -
 e^a)*e^(b*x))/(b^2*e^(6*b*x + 6*a) - b^2*e^(4*b*x + 4*a) - b^2*e^(2*b*x +
2*a) + b^2) + 3/2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 - 3/
2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2 + 2*arctan(e^(b*x +
a))/b^2
```

Giac [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^3(a+bx)\text{sech}^2(a+bx)\,dx=\int \frac{x}{\cosh\left(a+b\,x\right)^2\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
[Out] int(x/(cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.518 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(cosh(b*x+a))/b-3/2*sech(b*x+a)/b-1/2*csch(b*x+a)^2*sech(b*x+a)/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2702, 294, 327, 213}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{3\operatorname{sech}(a+bx)}{2b} - \frac{\operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

```
[Out] (3*ArcTanh[Cosh[a + b*x]])/(2*b) - (3*Sech[a + b*x])/(2*b) - (Csch[a + b*x]
^2*Sech[a + b*x])/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \text{sech}(a+bx)\right)}{b}$$

$$= -\frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b}$$

$$= -\frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(a+bx)\right)}{2b}$$

$$= \frac{3\text{arctanh}(\cosh(a+bx))}{2b} - \frac{3\text{sech}(a+bx)}{2b} - \frac{\text{csch}^2(a+bx)\text{sech}(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}(a+bx)}{b}$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
[Out] -1/8*Csch[(a + b*x)/2]^2/b + (3*Log[Cosh[(a + b*x)/2]])/(2*b) - (3*Log[Sinh
[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	43
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}-2\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2\left(1+\mathrm{e}^{2bx+2a}\right)} - \frac{3\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b} + \frac{3\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}$	91

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 709, normalized size of antiderivative = 14.47

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 4\left(15\cosh(bx+a)^2-1\right)\sinh(}{}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12*(
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 - 1)
*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(
b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a
)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 6*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4
+ 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x +
a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2
*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a
) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\log\left(e^{(-bx-a)}+1\right)}{2\,b} - \frac{3\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{3\,e^{(-bx-a)} - 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b(e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} - 1)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a)
 - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*b*
x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(43) = 86$.

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4\,e^{(bx+a)}-4\,e^{(-bx-a)}} - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a)
)^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a) +
2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))/
(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.519 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x} dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}, x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x} dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x} dx$$

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 69.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)^{2}}{x}dx$$

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
[Out] int(csch(b*x+a)^3*sech(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)^{2}}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx = \int \frac{\operatorname{csch}^{3}\left(a+bx\right)\operatorname{sech}^{2}\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2/x,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - e^(5*a))*e^(5*b*x) - (3*b*x*e^a +
 e^a)*e^(b*x))/(b^2*x^2*e^(6*b*x + 6*a) - b^2*x^2*e^(4*b*x + 4*a) - b^2*x^2
*e^(2*b*x + 2*a) + b^2*x^2) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e^
(b*x + a) + b^2*x^3), x) - 32*integrate(1/64*(3*b^2*x^2 - 2)/(b^2*x^3*e^(b*
x + a) - b^2*x^3), x) - 32*integrate(1/16*e^(b*x + a)/(b*x^2*e^(2*b*x + 2*a
) + b*x^2), x)
```

Giac [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^2}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.520 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx = \operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}, x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$$

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 43.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2}{x^2}dx$$

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
[Out] int(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**2/x**2,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\,(bx+a)^3\operatorname{sech}\,(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] (2*b*x*e^(3*b*x + 3*a) - (3*b*x*e^(5*a) - 2*e^(5*a))*e^(5*b*x) - (3*b*x*e^a
 + 2*e^a)*e^(b*x))/(b^2*x^3*e^(6*b*x + 6*a) - b^2*x^3*e^(4*b*x + 4*a) - b^2
*x^3*e^(2*b*x + 2*a) + b^2*x^3) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*
e^(b*x + a) + b^2*x^4), x) - 32*integrate(3/64*(b^2*x^2 - 2)/(b^2*x^4*e^(b*
x + a) - b^2*x^4), x) - 32*integrate(1/8*e^(b*x + a)/(b*x^3*e^(2*b*x + 2*a)
 + b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 20.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}\,(bx+a)^3\operatorname{sech}\,(bx+a)^2}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh\left(a+b\,x\right)^2\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^2*sinh(a + b*x)^3), x)
```

3.521 $\int x^m \mathbf{csch}^3(a+bx)\mathbf{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \operatorname{Int}\left(x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] Int[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

Rubi steps

$$\text{integral} = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 167.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$

```
[In] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
[Out] Integrate[x^m*Csch[a + b*x]^3*Sech[a + b*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3\,dx$$

```
[In] int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
[Out] int(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(x**m*csch(b*x+a)**3*sech(b*x+a)**3,x)
[Out] Integral(x**m*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^m \operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^m*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^m*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \frac{x^m}{\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
[Out] int(x^m/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.522 $\int x^3 \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 240

$$\int x^3 \text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{6x\text{arctanh}(e^{2a+2bx})}{b^3} + \frac{4x^3\text{arctanh}(e^{2a+2bx})}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - \frac{3\,\text{PolyLog}(2,-e^{2a+2bx})}{2b^4} + \frac{3x^2\,\text{PolyLog}(2,-e^{2a+2bx})}{b^2} + \frac{3\,\text{PolyLog}(2,e^{2a+2bx})}{2b^4} - \frac{3x^2\,\text{PolyLog}(2,e^{2a+2bx})}{b^2} - \frac{3x\,\text{PolyLog}(3,-e^{2a+2bx})}{b^3} + \frac{3x\,\text{PolyLog}(3,e^{2a+2bx})}{b^3} + \frac{3\,\text{PolyLog}(4,-e^{2a+2bx})}{2b^4} - \frac{3\,\text{PolyLog}(4,e^{2a+2bx})}{2b^4}$$

```
[Out] -6*x*arctanh(exp(2*b*x+2*a))/b^3+4*x^3*arctanh(exp(2*b*x+2*a))/b-3*x^2*csch
(2*b*x+2*a)/b^2-2*x^3*coth(2*b*x+2*a)*csch(2*b*x+2*a)/b-3/2*polylog(2,-exp(
2*b*x+2*a))/b^4+3*x^2*polylog(2,-exp(2*b*x+2*a))/b^2+3/2*polylog(2,exp(2*b*
x+2*a))/b^4-3*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3*x*polylog(3,-exp(2*b*x+2*
a))/b^3+3*x*polylog(3,exp(2*b*x+2*a))/b^3+3/2*polylog(4,-exp(2*b*x+2*a))/b^
4-3/2*polylog(4,exp(2*b*x+2*a))/b^4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5569, 4271, 4267, 2317, 2438, 2611, 6744, 2320, 6724}

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{6x\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b^3}+\frac{4x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^4}+\frac{3\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^4} + \frac{3\operatorname{PolyLog}\left(4,-e^{2a+2bx}\right)}{2b^4}-\frac{3\operatorname{PolyLog}\left(4,e^{2a+2bx}\right)}{2b^4} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3}+\frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} - \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{3x^2\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b}$$

```
[In] Int[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (-6*x*ArcTanh[E^(2*a + 2*b*x)])/b^3 + (4*x^3*ArcTanh[E^(2*a + 2*b*x)])/b -
(3*x^2*Csch[2*a + 2*b*x])/b^2 - (2*x^3*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])
/b - (3*PolyLog[2, -E^(2*a + 2*b*x)])/(2*b^4) + (3*x^2*PolyLog[2, -E^(2*a +
 2*b*x)])/b^2 + (3*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^4) - (3*x^2*PolyLog[2,
 E^(2*a + 2*b*x)])/b^2 - (3*x*PolyLog[3, -E^(2*a + 2*b*x)])/b^3 + (3*x*Poly
Log[3, E^(2*a + 2*b*x)])/b^3 + (3*PolyLog[4, -E^(2*a + 2*b*x)])/(2*b^4) - (
3*PolyLog[4, E^(2*a + 2*b*x)])/(2*b^4)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 8\int x^3\operatorname{csch}^3(2a+2bx)\,dx$$

$$= -\frac{3x^2\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} - 4\int x^3\operatorname{csch}(2a+2bx)\,dx + \frac{6\int x\operatorname{csch}(2a+2bx)\,dx}{b^2}$$

$$= -\frac{6x\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} - \frac{3\int\log\left(1-e^{2a+2bx}\right)\,dx}{b^3} + \frac{3\int\log\left(1+e^{2a+2bx}\right)\,dx}{b^3} + \frac{6\int x^2\log\left(1-e^{2a+2bx}\right)\,dx}{b} - \frac{6\int x^2\log\left(1+e^{2a+2bx}\right)\,dx}{b}$$

$$= -\frac{6x\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} - \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{3\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^4} + \frac{3\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^4} - \frac{6\int x\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)\,dx}{b^2} + \frac{6\int x\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)\,dx}{b^2}$$

$$= -\frac{6x\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} - \frac{3\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^4} + \frac{3x^2\operatorname{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{3\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^4} - \frac{3x^2\operatorname{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{3x\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{3x\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3} + \frac{3\int\operatorname{PolyLog}\left(3,-e^{2a+2bx}\right)\,dx}{b^3} - \frac{3\int\operatorname{PolyLog}\left(3,e^{2a+2bx}\right)\,dx}{b^3}$$

$$= -\frac{6x\text{arctanh}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - \frac{3\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^4} + \frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{3\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^4} - \frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{3x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{3x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3} + \frac{3\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^4} - \frac{3\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx,x,e^{2a+2bx}\right)}{2b^4}$$

$$= -\frac{6x\text{arctanh}\left(e^{2a+2bx}\right)}{b^3} + \frac{4x^3\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{3x^2\text{csch}(2a+2bx)}{b^2} - \frac{2x^3\coth(2a+2bx)\text{csch}(2a+2bx)}{b} - \frac{3\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{2b^4} + \frac{3x^2\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} + \frac{3\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^4} - \frac{3x^2\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{3x\,\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{3x\,\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(4,-e^{2a+2bx}\right)}{2b^4} - \frac{3\,\text{PolyLog}\left(4,e^{2a+2bx}\right)}{2b^4}$$

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.14

$$\int x^3\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$$
$$= \frac{-b^3x^3\text{csch}^2(a+bx) + 6bx\log\left(1-e^{2(a+bx)}\right) - 4b^3x^3\log\left(1-e^{2(a+bx)}\right) - 6bx\log\left(1+e^{2(a+bx)}\right) + 4b^3x^3\log}{}$$

```
[In] Integrate[x^3*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (-(b^3*x^3*Csch[a + b*x]^2) + 6*b*x*Log[1 - E^(2*(a + b*x))] - 4*b^3*x^3*Lo
g[1 - E^(2*(a + b*x))] - 6*b*x*Log[1 + E^(2*(a + b*x))] + 4*b^3*x^3*Log[1 +
 E^(2*(a + b*x))] + (-3 + 6*b^2*x^2)*PolyLog[2, -E^(2*(a + b*x))] + (3 - 6*
b^2*x^2)*PolyLog[2, E^(2*(a + b*x))] - 6*b*x*PolyLog[3, -E^(2*(a + b*x))] +
 6*b*x*PolyLog[3, E^(2*(a + b*x))] + 3*PolyLog[4, -E^(2*(a + b*x))] - 3*Pol
yLog[4, E^(2*(a + b*x))] - 3*b^2*x^2*Csch[a]*Sech[a] - b^3*x^3*Sech[a + b*x
]^2 + 3*b^2*x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x] + 3*b^2*x^2*Sech[a]*Sech[a
+ b*x]*Sinh[b*x])/(2*b^4)
```

Maple [A] (verified)

Time = 29.30 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{2x^2 e^{2bx+2a}\left(2\,e^{4bx+4a} bx+3\,e^{4bx+4a}+2bx-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)^2}-\frac{12\,\mathrm{polylog}\left(4,e^{bx+a}\right)}{b^4}+\frac{3\,\mathrm{polylog}\left(4,-e^{2bx+2a}\right)}{2b^4}-\frac{12\,\mathrm{polylog}\left(4,-e^{bx+a}\right)}{b^4}-$ 2l

[In] int(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -2*x^2*exp(2*b*x+2*a)*(2*exp(4*b*x+4*a)*b*x+3*exp(4*b*x+4*a)+2*b*x-3)/b^2/(exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-12*polylog(4,exp(b*x+a))/b^4+3/2*polylog(4,-exp(2*b*x+2*a))/b^4-12*polylog(4,-exp(b*x+a))/b^4-2/b^4*ln(1-exp(b*x+a))*a^3+3/b^4*ln(1-exp(b*x+a))*a+3*polylog(2,exp(b*x+a))/b^4-3/2*polylog(2,-exp(2*b*x+2*a))/b^4+3*polylog(2,-exp(b*x+a))/b^4-3*x*ln(1+exp(2*b*x+2*a))/b^3+3/b^3*ln(1-exp(b*x+a))*x+3/b^3*ln(exp(b*x+a)+1)*x-2/b*ln(1-exp(b*x+a))*x^3-6*x^2*polylog(2,exp(b*x+a))/b^2+12*x*polylog(3,exp(b*x+a))/b^3+2*x^3*ln(1+exp(2*b*x+2*a))/b+3*x^2*polylog(2,-exp(2*b*x+2*a))/b^2-3*x*polylog(3,-exp(2*b*x+2*a))/b^3-2/b*ln(exp(b*x+a)+1)*x^3-6*x^2*polylog(2,-exp(b*x+a))/b^2+12*x*polylog(3,-exp(b*x+a))/b^3-3/b^4*a*ln(exp(b*x+a)-1)+2/b^4*a^3*ln(exp(b*x+a)-1)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 6764, normalized size of antiderivative = 28.18

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

[In] integrate(x**3*csch(b*x+a)**3*sech(b*x+a)**3,x)

[Out] Integral(x**3*csch(a + b*x)**3*sech(a + b*x)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.59

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= -\frac{2\left(\left(2\,bx^3e^{(6\,a)}+3\,x^2e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bx^3e^{(2\,a)}-3\,x^2e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2}$$
$$+\frac{2\left(4\,b^3x^3\log\left(e^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3(-e^{(2\,bx+2\,a)})+3\,\mathrm{Li}_4(-e^{(2\,bx+2\,a)})\right)}{3\,b^4}$$
$$-\frac{2\left(b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})\right)}{b^4}$$
$$-\frac{2\left(b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})\right)}{b^4}$$
$$-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)}{2\,b^4}$$
$$+\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x^3*e^(6*a) + 3*x^2*e^(6*a))*e^(6*b*x) + (2*b*x^3*e^(2*a) - 3*x^2*
e^(2*a))*e^(2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + 2
/3*(4*b^3*x^3*log(e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -e^(2*b*x + 2*a)) + 3*polylog(4, -e^(2*b*x + 2*a)))/b^4
- 2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*p
olylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - 2*(b^3*x^3*log(
-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x
+ a)) + 6*polylog(4, e^(b*x + a)))/b^4 - 3/2*(2*b*x*log(e^(2*b*x + 2*a) + 1
) + dilog(-e^(2*b*x + 2*a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(
b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int x^3\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3\,dx$$

```
[In] integrate(x^3*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \frac{x^3}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
[Out] int(x^3/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.523 $\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \frac{4x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\text{arctanh}(\cosh(2a+2bx))}{b^3} - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} + \frac{2x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} - \frac{2x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}$$

```
[Out] 4*x^2*arctanh(exp(2*b*x+2*a))/b-arctanh(cosh(2*b*x+2*a))/b^3-2*x*csch(2*b*x
+2*a)/b^2-2*x^2*coth(2*b*x+2*a)*csch(2*b*x+2*a)/b+2*x*polylog(2,-exp(2*b*x+
2*a))/b^2-2*x*polylog(2,exp(2*b*x+2*a))/b^2-polylog(3,-exp(2*b*x+2*a))/b^3+
polylog(3,exp(2*b*x+2*a))/b^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{5569, 4271, 3855, 4267, 2611, 2320, 6724\}$

$$\int x^2 \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(2a+2bx))}{b^3} + \frac{4x^2 \operatorname{arctanh}(e^{2a+2bx})}{b} - \frac{\operatorname{PolyLog}(3,-e^{2a+2bx})}{b^3} + \frac{\operatorname{PolyLog}(3,e^{2a+2bx})}{b^3} + \frac{2x\operatorname{PolyLog}(2,-e^{2a+2bx})}{b^2} - \frac{2x\operatorname{PolyLog}(2,e^{2a+2bx})}{b^2} - \frac{2x\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b}$$

```
[In] Int[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (4*x^2*ArcTanh[E^(2*a + 2*b*x)])/b - ArcTanh[Cosh[2*a + 2*b*x]]/b^3 - (2*x*
Csch[2*a + 2*b*x])/b^2 - (2*x^2*Coth[2*a + 2*b*x]*Csch[2*a + 2*b*x])/b + (2
*x*PolyLog[2, -E^(2*a + 2*b*x)])/b^2 - (2*x*PolyLog[2, E^(2*a + 2*b*x)])/b^
2 - PolyLog[3, -E^(2*a + 2*b*x)]/b^3 + PolyLog[3, E^(2*a + 2*b*x)]/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 8\int x^2\text{csch}^3(2a+2bx)\,dx \\
&= -\frac{2x\text{csch}(2a+2bx)}{b^2}-\frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} \\
&\quad -4\int x^2\text{csch}(2a+2bx)\,dx+\frac{2\int \text{csch}(2a+2bx)\,dx}{b^2} \\
&= \frac{4x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{arctanh}(\cosh(2a+2bx))}{b^3} \\
&\quad -\frac{2x\text{csch}(2a+2bx)}{b^2}-\frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} \\
&\quad +\frac{4\int x\log\left(1-e^{2a+2bx}\right)\,dx}{b}-\frac{4\int x\log\left(1+e^{2a+2bx}\right)\,dx}{b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\text{arctanh}(\cosh(2a+2bx))}{b^3} \\
&\quad - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} \\
&\quad + \frac{2x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} - \frac{2x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} \\
&\quad - \frac{2\int \text{PolyLog}\left(2,-e^{2a+2bx}\right)\,dx}{b^2} + \frac{2\int \text{PolyLog}\left(2,e^{2a+2bx}\right)\,dx}{b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\text{arctanh}(\cosh(2a+2bx))}{b^3} \\
&\quad - \frac{2x\text{csch}(2a+2bx)}{b^2} - \frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} \\
&\quad + \frac{2x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} - \frac{2x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2a+2bx}\right)}{b^3} + \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2a+2bx}\right)}{b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4x^2\text{arctanh}\left(e^{2a+2bx}\right)}{b} - \frac{\text{arctanh}(\cosh(2a+2bx))}{b^3} - \frac{2x\text{csch}(2a+2bx)}{b^2} \\
&\quad - \frac{2x^2\coth(2a+2bx)\text{csch}(2a+2bx)}{b} + \frac{2x\,\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2} \\
&\quad - \frac{2x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2} - \frac{\text{PolyLog}\left(3,-e^{2a+2bx}\right)}{b^3} + \frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.29

$$
\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{4\text{arctanh}\left(e^{2(a+bx)}\right) + b^2x^2\text{csch}^2(a+bx) + 4b^2x^2\log\left(1-e^{2(a+bx)}\right) - 4b^2x^2\log\left(1+e^{2(a+bx)}\right) - 4bx\,\text{PolyL}
$$

```
[In] Integrate[x^2*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(4*ArcTanh[E^(2*(a + b*x))] + b^2*x^2*Csch[a + b*x]^2 + 4*b^2*x^2*Log[
1 - E^(2*(a + b*x))] - 4*b^2*x^2*Log[1 + E^(2*(a + b*x))] - 4*b*x*PolyLog[2
, -E^(2*(a + b*x))] + 4*b*x*PolyLog[2, E^(2*(a + b*x))] + 2*PolyLog[3, -E^(
2*(a + b*x))] - 2*PolyLog[3, E^(2*(a + b*x))] + 2*b*x*Csch[a]*Sech[a] + b^2
*x^2*Sech[a + b*x]^2 - 2*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x] - 2*b*x*Sech[a
]*Sech[a + b*x]*Sinh[b*x])/b^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(144) = 288$.

Time = 21.85 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.01

method	result
risch	$-\frac{4x\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}bx+\mathrm{e}^{4bx+4a}+bx-1\right)}{b^{2}\left(\mathrm{e}^{2bx+2a}-1\right)^{2}\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}-\frac{2a^{2}\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^{3}}-\frac{2\ln\left(1-\mathrm{e}^{bx+a}\right)x^{2}}{b}-\frac{4x\,\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^{2}}+\frac{2x^{2}\ln\left(1+\mathrm{e}^{2bx+}\right.}{b}$

[In] int(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -4*x*exp(2*b*x+2*a)*(exp(4*b*x+4*a)*b*x+exp(4*b*x+4*a)+b*x-1)/b^2/(exp(2*b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-2/b^3*a^2*ln(exp(b*x+a)-1)-2/b*ln(1-exp(b*x+a))*x^2-4*x*polylog(2,exp(b*x+a))/b^2+2*x^2*ln(1+exp(2*b*x+2*a))/b+2*x*polylog(2,-exp(2*b*x+2*a))/b^2-2/b*ln(exp(b*x+a)+1)*x^2-4*x*polylog(2,-exp(b*x+a))/b^2+4*polylog(3,exp(b*x+a))/b^3-polylog(3,-exp(2*b*x+2*a))/b^3+4*polylog(3,-exp(b*x+a))/b^3+2/b^3*ln(1-exp(b*x+a))*a^2+1/b^3*ln(exp(b*x+a)-1)-1/b^3*ln(1+exp(2*b*x+2*a))+1/b^3*ln(exp(b*x+a)+1)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 4779, normalized size of antiderivative = 32.07

$$\int x^2\mathrm{csch}^3(a+bx)\mathrm{sech}^3(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")

[Out] -(4*(b^2*x^2 + b*x)*cosh(b*x + a)^6 + 80*(b^2*x^2 + b*x)*cosh(b*x + a)^3*sinh(b*x + a)^3 + 60*(b^2*x^2 + b*x)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 24*(b^2*x^2 + b*x)*cosh(b*x + a)*sinh(b*x + a)^5 + 4*(b^2*x^2 + b*x)*sinh(b*x + a)^6 + 4*(b^2*x^2 - b*x)*cosh(b*x + a)^2 + 4*(15*(b^2*x^2 + b*x)*cosh(b*x + a)^4 + b^2*x^2 - b*x)*sinh(b*x + a)^2 + 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x +

8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(I*cosh(b
*x + a) + I*sinh(b*x + a)) - 4*(b*x*cosh(b*x + a)^8 + 56*b*x*cosh(b*x + a)^
3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x
 + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35
*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*
x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*
x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7 - b*x*cosh(b*x + a
)^3)*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 4*(b*x*cosh
(b*x + a)^8 + 56*b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)
^2*sinh(b*x + a)^6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a
)^8 - 2*b*x*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a
)^4 + 8*(7*b*x*cosh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*
b*x*cosh(b*x + a)^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x
*cosh(b*x + a)^7 - b*x*cosh(b*x + a)^3)*sinh(b*x + a))*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) + ((2*b^2*x^2 - 1)*cosh(b*x + a)^8 + 56*(2*b^2*x^2 - 1)*c
osh(b*x + a)^3*sinh(b*x + a)^5 + 28*(2*b^2*x^2 - 1)*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*(2*b^2*x^2 - 1)*cosh(b*x + a)*sinh(b*x + a)^7 + (2*b^2*x^2 - 1
)*sinh(b*x + a)^8 - 2*(2*b^2*x^2 - 1)*cosh(b*x + a)^4 + 2*(35*(2*b^2*x^2 -
1)*cosh(b*x + a)^4 - 2*b^2*x^2 + 1)*sinh(b*x + a)^4 + 2*b^2*x^2 + 8*(7*(2*b
^2*x^2 - 1)*cosh(b*x + a)^5 - (2*b^2*x^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*(2*b^2*x^2 - 1)*cosh(b*x + a)^6 - 3*(2*b^2*x^2 - 1)*cosh(b*x + a)^
2)*sinh(b*x + a)^2 + 8*((2*b^2*x^2 - 1)*cosh(b*x + a)^7 - (2*b^2*x^2 - 1)*c
osh(b*x + a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 ((2*a^2 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a
)^5 + 28*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b
*x + a)*sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(
b*x + a)^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4
 + 8*(7*(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x +
 a)^3 + 4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*s
inh(b*x + a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(
b*x + a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + I) - ((2
*a^2 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5
+ 28*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b*x +
 a)*sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(b*x
+ a)^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4 + 8
(7(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(b*x
+ a)^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((2*a^2
 - 1)*cosh(b*x + a)^8 + 56*(2*a^2 - 1)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28
*(2*a^2 - 1)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(2*a^2 - 1)*cosh(b*x + a)*
sinh(b*x + a)^7 + (2*a^2 - 1)*sinh(b*x + a)^8 - 2*(2*a^2 - 1)*cosh(b*x + a)
^4 + 2*(35*(2*a^2 - 1)*cosh(b*x + a)^4 - 2*a^2 + 1)*sinh(b*x + a)^4 + 8*(7*
(2*a^2 - 1)*cosh(b*x + a)^5 - (2*a^2 - 1)*cosh(b*x + a))*sinh(b*x + a)^3 +
4*(7*(2*a^2 - 1)*cosh(b*x + a)^6 - 3*(2*a^2 - 1)*cosh(b*x + a)^2)*sinh(b*x

+ a)^2 + 2*a^2 + 8*((2*a^2 - 1)*cosh(b*x + a)^7 - (2*a^2 - 1)*cosh(b*x + a)
^3)*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b^2*x^2
 - a^2)*cosh(b*x + a)^8 + 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^
5 + 28*(b^2*x^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*
cosh(b*x + a)*sinh(b*x + a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^
2 - a^2)*cosh(b*x + a)^4 + 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2
+ a^2)*sinh(b*x + a)^4 + b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (
b^2*x^2 - a^2)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b
x + a)^6 - 3(b^2*x^2 - a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((
b^2*x^2 - a^2)*cosh(b*x + a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x
+ a))*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - 2*((b^2*x^2 - a^2)*cosh(
b*x + a)^8 + 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b^2*x
^2 - a^2)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)
*sinh(b*x + a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^2 - a^2)*cosh
(b*x + a)^4 + 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 + a^2)*sinh(b
*x + a)^4 + b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (b^2*x^2 - a^2
)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^6 - 3
*(b^2*x^2 - a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((b^2*x^2 - a^2
)*cosh(b*x + a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x + a))*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1) + 2*((b^2*x^2 - a^2)*cosh(b*x + a)^8 +
 56*(b^2*x^2 - a^2)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b^2*x^2 - a^2)*co
sh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x +
a)^7 + (b^2*x^2 - a^2)*sinh(b*x + a)^8 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)^4
+ 2*(35*(b^2*x^2 - a^2)*cosh(b*x + a)^4 - b^2*x^2 + a^2)*sinh(b*x + a)^4 +
b^2*x^2 + 8*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^5 - (b^2*x^2 - a^2)*cosh(b*x +
 a))*sinh(b*x + a)^3 + 4*(7*(b^2*x^2 - a^2)*cosh(b*x + a)^6 - 3*(b^2*x^2 -
a^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - a^2 + 8*((b^2*x^2 - a^2)*cosh(b*x +
 a)^7 - (b^2*x^2 - a^2)*cosh(b*x + a)^3)*sinh(b*x + a))*log(-cosh(b*x + a)
- sinh(b*x + a) + 1) - 4*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a
)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7
+ sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x
 + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh
(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cos
h(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a))
+ 4*(cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2
*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b
*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*
x + a) + 1)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 4*(cosh(b*x + a
)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)
^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a
)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(
b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polyl

```
og(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 4*(cosh(b*x + a)^8 + 56*cosh(b*
x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x
+ a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*
x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b
*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(
cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x
+ a) - sinh(b*x + a)) + 8*(3*(b^2*x^2 + b*x)*cosh(b*x + a)^5 + (b^2*x^2 - b
*x)*cosh(b*x + a))*sinh(b*x + a))/(b^3*cosh(b*x + a)^8 + 56*b^3*cosh(b*x +
a)^3*sinh(b*x + a)^5 + 28*b^3*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b^3*cosh(
b*x + a)*sinh(b*x + a)^7 + b^3*sinh(b*x + a)^8 - 2*b^3*cosh(b*x + a)^4 + 2*
(35*b^3*cosh(b*x + a)^4 - b^3)*sinh(b*x + a)^4 + 8*(7*b^3*cosh(b*x + a)^5 -
 b^3*cosh(b*x + a))*sinh(b*x + a)^3 + b^3 + 4*(7*b^3*cosh(b*x + a)^6 - 3*b^
3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b^3*cosh(b*x + a)^7 - b^3*cosh(b*x
+ a)^3)*sinh(b*x + a))
```

Sympy [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int x^2\operatorname{csch}^3{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

```
[In] integrate(x**2*csch(b*x+a)**3*sech(b*x+a)**3,x)
[Out] Integral(x**2*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.83

$$\begin{aligned}\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx &= -\frac{4\left(\left(bx^2e^{(6a)}+xe^{(6a)}\right)e^{(6bx)}+\left(bx^2e^{(2a)}-xe^{(2a)}\right)e^{(2bx)}\right)}{b^2e^{(8bx+8a)}-2b^2e^{(4bx+4a)}+b^2}\\&\quad+\frac{2b^2x^2\log\left(e^{(2bx+2a)}+1\right)+2bx\text{Li}_2(-e^{(2bx+2a)})-\text{Li}_3(-e^{(2bx+2a)})}{b^3}\\&\quad-\frac{2\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\text{Li}_2(-e^{(bx+a)})-2\text{Li}_3(-e^{(bx+a)})\right)}{b^3}\\&\quad-\frac{2\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\text{Li}_2(e^{(bx+a)})-2\text{Li}_3(e^{(bx+a)})\right)}{b^3}\\&\quad-\frac{\log\left(e^{(2bx+2a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{\log\left(e^{(bx+a)}-1\right)}{b^3}\end{aligned}$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*((b*x^2*e^(6*a) + x*e^(6*a))*e^(6*b*x) + (b*x^2*e^(2*a) - x*e^(2*a))*e^(
2*b*x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b^2*x^2*lo
g(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b
*x + 2*a)))/b^3 - 2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a
)) - 2*polylog(3, -e^(b*x + a)))/b^3 - 2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2
*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3 - log(e^(2*b*x + 2
*a) + 1)/b^3 + log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Giac [F]

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int x^2\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^3\,\,dx$$

```
[In] integrate(x^2*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x^2}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(x^2/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.524 $\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\frac{4x\text{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}+\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}$$

```
[Out] 4*x*arctanh(exp(2*b*x+2*a))/b-csch(2*b*x+2*a)/b^2-2*x*coth(2*b*x+2*a)*csch(
2*b*x+2*a)/b+polylog(2,-exp(2*b*x+2*a))/b^2-polylog(2,exp(2*b*x+2*a))/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5569, 4270, 4267, 2317, 2438}

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\frac{4x\text{arctanh}\left(e^{2a+2bx}\right)}{b}+\frac{\text{PolyLog}\left(2,-e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\text{csch}(2a+2bx)}{b}$$

```
[In] Int[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] (4*x*ArcTanh[E^(2*a + 2*b*x)])/b - Csch[2*a + 2*b*x]/b^2 - (2*x*Coth[2*a +
2*b*x]*Csch[2*a + 2*b*x])/b + PolyLog[2, -E^(2*a + 2*b*x)]/b^2 - PolyLog[2,
 E^(2*a + 2*b*x)]/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 8\int x\operatorname{csch}^3(2a+2bx)\,dx \\
&= -\frac{\operatorname{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b}-4\int x\operatorname{csch}(2a+2bx)\,dx \\
&= \frac{4x\operatorname{arctanh}\left(e^{2a+2bx}\right)}{b}-\frac{\operatorname{csch}(2a+2bx)}{b^2}-\frac{2x\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} \\
&\quad+\frac{2\int\log\left(1-e^{2a+2bx}\right)\,dx}{b}-\frac{2\int\log\left(1+e^{2a+2bx}\right)\,dx}{b}
\end{aligned}
$$

$$= \frac{4x\operatorname{arctanh}(e^{2a+2bx})}{b} - \frac{\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} + \frac{\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2a+2bx}\right)}{b^2} - \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2a+2bx}\right)}{b^2}$$

$$= \frac{4x\operatorname{arctanh}(e^{2a+2bx})}{b} - \frac{\operatorname{csch}(2a+2bx)}{b^2} - \frac{2x\coth(2a+2bx)\operatorname{csch}(2a+2bx)}{b} + \frac{\operatorname{PolyLog}(2, -e^{2a+2bx})}{b^2} - \frac{\operatorname{PolyLog}(2, e^{2a+2bx})}{b^2}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int x\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\coth(a+bx) + bx\operatorname{csch}^2(a+bx) + 4bx\log\left(1-e^{2(a+bx)}\right) - 4bx\log\left(1+e^{2(a+bx)}\right) - 2\operatorname{PolyLog}\left(2, -e^{2(a+b}\right.}{2b^2}$$

```
[In] Integrate[x*Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*(Coth[a + b*x] + b*x*Csch[a + b*x]^2 + 4*b*x*Log[1 - E^(2*(a + b*x))]
- 4*b*x*Log[1 + E^(2*(a + b*x))] - 2*PolyLog[2, -E^(2*(a + b*x))] + 2*PolyL
og[2, E^(2*(a + b*x))] + b*x*Sech[a + b*x]^2 - Tanh[a + b*x])/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(88) = 176.

Time = 15.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{2\,e^{2bx+2a}\left(2\,e^{4bx+4a}bx+e^{4bx+4a}+2bx-1\right)}{b^2\left(e^{2bx+2a}-1\right)^2\left(1+e^{2bx+2a}\right)^2} - \frac{2\ln\left(1-e^{bx+a}\right)x}{b} - \frac{2\ln\left(1-e^{bx+a}\right)a}{b^2} - \frac{2\operatorname{polylog}\left(2,e^{bx+a}\right)}{b^2} - \frac{2\ln\left(e^{bx+a}+1\right)x}{b} -$

```
[In] int(x*csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*exp(2*b*x+2*a)*(2*exp(4*b*x+4*a)*b*x+exp(4*b*x+4*a)+2*b*x-1)/b^2/(exp(2*
b*x+2*a)-1)^2/(1+exp(2*b*x+2*a))^2-2/b*ln(1-exp(b*x+a))*x-2/b^2*ln(1-exp(b*
x+a))*a-2*polylog(2,exp(b*x+a))/b^2-2/b*ln(exp(b*x+a)+1)*x-2*polylog(2,-exp
(b*x+a))/b^2+2*x*ln(1+exp(2*b*x+2*a))/b+polylog(2,-exp(2*b*x+2*a))/b^2+2/b^
2*a*ln(exp(b*x+a)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 3025, normalized size of antiderivative = 33.24

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*((2*b*x + 1)*cosh(b*x + a)^6 + 20*(2*b*x + 1)*cosh(b*x + a)^3*sinh(b*x +
 a)^3 + 15*(2*b*x + 1)*cosh(b*x + a)^2*sinh(b*x + a)^4 + 6*(2*b*x + 1)*cosh
(b*x + a)*sinh(b*x + a)^5 + (2*b*x + 1)*sinh(b*x + a)^6 + (2*b*x - 1)*cosh(
b*x + a)^2 + (15*(2*b*x + 1)*cosh(b*x + a)^4 + 2*b*x - 1)*sinh(b*x + a)^2 +
 (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(3
5*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x
+ a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x
 + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x +
 a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(
b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*
x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(
b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh
(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8
*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(I*cosh(b*x +
a) + I*sinh(b*x + a)) - (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)
^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 +
 sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x
+ a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(
b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh
(b*x + a)^3)*sinh(b*x + a) + 1)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) +
 (cosh(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(3
5*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x
+ a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x
 + a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x +
 a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*x*cosh(b*x + a)^8 + 56*
b*x*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b*x*cosh(b*x + a)^2*sinh(b*x + a)^
6 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x + a)^8 - 2*b*x*cosh(
b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 - b*x)*sinh(b*x + a)^4 + 8*(7*b*x*co
sh(b*x + a)^5 - b*x*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b*x*cosh(b*x + a)
^6 - 3*b*x*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*(b*x*cosh(b*x + a)^7
- b*x*cosh(b*x + a)^3)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1
) + (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b
*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x
```

```
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) + I
) + (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b
*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - I
) - (a*cosh(b*x + a)^8 + 56*a*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*a*cosh(b
*x + a)^2*sinh(b*x + a)^6 + 8*a*cosh(b*x + a)*sinh(b*x + a)^7 + a*sinh(b*x
+ a)^8 - 2*a*cosh(b*x + a)^4 + 2*(35*a*cosh(b*x + a)^4 - a)*sinh(b*x + a)^4
 + 8*(7*a*cosh(b*x + a)^5 - a*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*a*cosh(
b*x + a)^6 - 3*a*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(a*cosh(b*x + a)^7 -
a*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1
) - ((b*x + a)*cosh(b*x + a)^8 + 56*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a)
^5 + 28*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b*x + a)*cosh(b*x +
a)*sinh(b*x + a)^7 + (b*x + a)*sinh(b*x + a)^8 - 2*(b*x + a)*cosh(b*x + a)^
4 + 2*(35*(b*x + a)*cosh(b*x + a)^4 - b*x - a)*sinh(b*x + a)^4 + 8*(7*(b*x
+ a)*cosh(b*x + a)^5 - (b*x + a)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b*x
 + a)*cosh(b*x + a)^6 - 3*(b*x + a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x
+ 8*((b*x + a)*cosh(b*x + a)^7 - (b*x + a)*cosh(b*x + a)^3)*sinh(b*x + a) +
 a)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) - ((b*x + a)*cosh(b*x + a)^8
 + 56*(b*x + a)*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*(b*x + a)*cosh(b*x + a
)^2*sinh(b*x + a)^6 + 8*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^7 + (b*x + a)
*sinh(b*x + a)^8 - 2*(b*x + a)*cosh(b*x + a)^4 + 2*(35*(b*x + a)*cosh(b*x +
 a)^4 - b*x - a)*sinh(b*x + a)^4 + 8*(7*(b*x + a)*cosh(b*x + a)^5 - (b*x +
a)*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*(b*x + a)*cosh(b*x + a)^6 - 3*(b*x
 + a)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b*x + 8*((b*x + a)*cosh(b*x + a)^7
 - (b*x + a)*cosh(b*x + a)^3)*sinh(b*x + a) + a)*log(-I*cosh(b*x + a) - I*s
inh(b*x + a) + 1) + ((b*x + a)*cosh(b*x + a)^8 + 56*(b*x + a)*cosh(b*x + a)
^3*sinh(b*x + a)^5 + 28*(b*x + a)*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*(b*x
+ a)*cosh(b*x + a)*sinh(b*x + a)^7 + (b*x + a)*sinh(b*x + a)^8 - 2*(b*x + a
)*cosh(b*x + a)^4 + 2*(35*(b*x + a)*cosh(b*x + a)^4 - b*x - a)*sinh(b*x + a
)^4 + 8*(7*(b*x + a)*cosh(b*x + a)^5 - (b*x + a)*cosh(b*x + a))*sinh(b*x +
a)^3 + 4*(7*(b*x + a)*cosh(b*x + a)^6 - 3*(b*x + a)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + b*x + 8*((b*x + a)*cosh(b*x + a)^7 - (b*x + a)*cosh(b*x + a)^3)
*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(3*(2*b*x +
 1)*cosh(b*x + a)^5 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a))/(b^2*cosh(b
*x + a)^8 + 56*b^2*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*b^2*cosh(b*x + a)^2
*sinh(b*x + a)^6 + 8*b^2*cosh(b*x + a)*sinh(b*x + a)^7 + b^2*sinh(b*x + a)^
8 - 2*b^2*cosh(b*x + a)^4 + 2*(35*b^2*cosh(b*x + a)^4 - b^2)*sinh(b*x + a)^
4 + 8*(7*b^2*cosh(b*x + a)^5 - b^2*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*b^
2*cosh(b*x + a)^6 - 3*b^2*cosh(b*x + a)^2)*sinh(b*x + a)^2 + b^2 + 8*(b^2*c
```

```
osh(b*x + a)^7 - b^2*cosh(b*x + a)^3)*sinh(b*x + a))
```

Sympy [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = \int x\,\text{csch}^3\,(a+bx)\,\text{sech}^3\,(a+bx)\,dx$$

```
[In] integrate(x*csch(b*x+a)**3*sech(b*x+a)**3,x)
[Out] Integral(x*csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.80

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx = -\frac{2\left(\left(2\,bxe^{(6\,a)}+e^{(6\,a)}\right)e^{(6\,bx)}+\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}\right)}{b^2e^{(8\,bx+8\,a)}-2\,b^2e^{(4\,bx+4\,a)}+b^2} + \frac{2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\text{Li}_2\left(-e^{(2\,bx+2\,a)}\right)}{b^2} - \frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^2} - \frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2\left(e^{(bx+a)}\right)\right)}{b^2}$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
[Out] -2*((2*b*x*e^(6*a) + e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*
x))/(b^2*e^(8*b*x + 8*a) - 2*b^2*e^(4*b*x + 4*a) + b^2) + (2*b*x*log(e^(2*b
*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))/b^2 - 2*(b*x*log(e^(b*x + a) + 1)
 + dilog(-e^(b*x + a)))/b^2 - 2*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x +
 a)))/b^2
```

Giac [F]

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int x\,\text{csch}\,(bx+a)^3\,\text{sech}\,(bx+a)^3\,dx$$

```
[In] integrate(x*csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x*csch(b*x + a)^3*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^3(a+bx)\text{sech}^3(a+bx)\,dx=\int \frac{x}{\cosh\left(a+b\,x\right)^3\,\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
[Out] int(x/(cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.525 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}+\frac{\tanh^2(a+bx)}{2b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2700, 272, 45}

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\tanh^2(a+bx)}{2b}-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}$$

```
[In] Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b - (2*Log[Tanh[a + b*x]])/b + Tanh[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3}\,dx, x, i\tanh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)\,dx, x, -\tanh^2(a+bx)\right)}{2b}\\
&= -\frac{\coth^2(a+bx)}{2b} - \frac{2\log(\tanh(a+bx))}{b} + \frac{\tanh^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = 8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b} + \frac{\log(\cosh(a+bx))}{4b} - \frac{\log(\sinh(a+bx))}{4b} - \frac{\operatorname{sech}^2(a+bx)}{16b}\right)$$

```
[In] Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
[Out] 8*(-1/16*Csch[a + b*x]^2/b + Log[Cosh[a + b*x]]/(4*b) - Log[Sinh[a + b*x]]/
(4*b) - Sech[a + b*x]^2/(16*b))
```

Maple [A] (verified)

Time = 11.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^2}-\frac{1}{\cosh(bx+a)^2}-2\ln(\tanh(bx+a))}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^2}-\frac{1}{\cosh(bx+a)^2}-2\ln(\tanh(bx+a))}{b}$	43
risch	$-\frac{4\,\mathrm{e}^{2bx+2a}(\mathrm{e}^{4bx+4a}+1)}{b(\mathrm{e}^{2bx+2a}-1)^2(1+\mathrm{e}^{2bx+2a})^2}+\frac{2\ln(1+\mathrm{e}^{2bx+2a})}{b}-\frac{2\ln(\mathrm{e}^{2bx+2a}-1)}{b}$	87

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^2-1/cosh(b*x+a)^2-2*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 774, normalized size of antiderivative = 18.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
[Out] -2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b
*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*
x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b
*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)
*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)
*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^8 +
56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8
*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*x + a)^4 -
1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - cosh(b*x +
a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2)*sinh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x + a)^5 + cosh(b*x
 + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x + a)^3*sinh(b*x +
a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x + a)*sinh(b*x +
a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 -
b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a
)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 8*(b*
cosh(b*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{2\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{2\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
[Out] -2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4*a
) - e^(-8*b*x - 8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(39) = 78$.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
[Out] -(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*
a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x + 2
*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4\operatorname{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4\,e^{2a+2bx}}{b\,\left(e^{4a+4bx}-1\right)} - \frac{8\,e^{2a+2bx}}{b\,\left(e^{8a+8bx}-2\,e^{4a+4bx}+1\right)}$$

```
[In] int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] (4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8*b
*x) - 2*exp(4*a + 4*b*x) + 1))
```

3.526 $\int \frac{\mathbf{csch}^3(a+bx)\mathbf{sech}^3(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=8\text{Int}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x},x\right)$$

```
[Out] 8*Unintegrable(csch(2*b*x+2*a)^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
```

```
[Out] 8*Defer[Int][Csch[2*a + 2*b*x]^3/x, x]
```

Rubi steps

$$\text{integral}=8\int \frac{\operatorname{csch}^3(2a+2bx)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 51.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^3}{x}dx$$

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
[Out] int(csch(b*x+a)^3*sech(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3 \operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}^3\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3/x,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 10.40

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] -2*((2*b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (2*b*x*e^(2*a) + e^(2*a))*e^(2*b*
x))/(b^2*x^2*e^(8*b*x + 8*a) - 2*b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2) - 64*in
tegrate(1/32*(2*b^2*x^2 - 1)/(b^2*x^3*e^(2*b*x + 2*a) + b^2*x^3), x) + 64*i
ntegrate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) - 64*inte
grate(1/64*(2*b^2*x^2 - 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^3\operatorname{sech}\left(bx+a\right)^3}{x}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x}\,dx=\int \frac{1}{x\cosh\left(a+b\,x\right)^3\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.527 $\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = 8\mathrm{Int}\left(\frac{\operatorname{csch}^3(2a+2bx)}{x^2},x\right)$$

```
[Out] 8*Unintegrable(csch(2*b*x+2*a)^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] Int[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
```

```
[Out] 8*Defer[Int][Csch[2*a + 2*b*x]^3/x^2, x]
```

Rubi steps

$$\text{integral} = 8\int \frac{\operatorname{csch}^3(2a+2bx)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 38.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx$$

```
[In] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2,x]
[Out] Integrate[(Csch[a + b*x]^3*Sech[a + b*x]^3)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)^{3}}{x^{2}}dx$$

```
[In] int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
[Out] int(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}\left(bx+a\right)^{3}\operatorname{sech}\left(bx+a\right)^{3}}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx=\int \frac{\operatorname{csch}^{3}\left(a+bx\right)\operatorname{sech}^{3}\left(a+bx\right)}{x^{2}}\,dx$$

```
[In] integrate(csch(b*x+a)**3*sech(b*x+a)**3/x**2,x)
[Out] Integral(csch(a + b*x)**3*sech(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 10.30

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -4*((b*x*e^(6*a) - e^(6*a))*e^(6*b*x) + (b*x*e^(2*a) + e^(2*a))*e^(2*b*x))/
(b^2*x^3*e^(8*b*x + 8*a) - 2*b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3) - 64*integr
ate(1/32*(2*b^2*x^2 - 3)/(b^2*x^4*e^(2*b*x + 2*a) + b^2*x^4), x) + 64*integ
rate(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x) - 64*integrat
e(1/64*(2*b^2*x^2 - 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{\operatorname{csch}(bx+a)^3\operatorname{sech}(bx+a)^3}{x^2}\,dx$$

```
[In] integrate(csch(b*x+a)^3*sech(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3*sech(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cosh(a+b\,x)^3\sinh(a+b\,x)^3}\,dx$$

```
[In] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cosh(a + b*x)^3*sinh(a + b*x)^3), x)
```

3.528 $\int x \cosh^{\frac{5}{2}}(a+bx) \sinh(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int x \cosh^{\frac{5}{2}}(a+bx) \sinh(a+bx) \, dx = \frac{2x \cosh^{\frac{7}{2}}(a+bx)}{7b} + \frac{20i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2} - \frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2} - \frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2}$$

```
[Out] 2/7*x*cosh(b*x+a)^(7/2)/b+20/147*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a
+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/49*cosh(b*x+a)^(5/
2)*sinh(b*x+a)/b^2-20/147*sinh(b*x+a)*cosh(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5481, 2715, 2720}

$$\int x \cosh^{\frac{5}{2}}(a+bx) \sinh(a+bx) \, dx = \frac{20i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2} - \frac{4\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{49b^2} - \frac{20\sinh(a+bx)\sqrt{\cosh(a+bx)}}{147b^2} + \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[In] Int[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(7/2))/(7*b) + (((20*I)/147)*EllipticF[(I/2)*(a + b*x),
2])/b^2 - (20*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(147*b^2) - (4*Cosh[a + b*
x]^(5/2)*Sinh[a + b*x])/(49*b^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \cosh^{\frac{7}{2}}(a+bx)\,dx}{7b} \\
&= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2} - \frac{10\int \cosh^{\frac{3}{2}}(a+bx)\,dx}{49b} \\
&= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} - \frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2} \\
&\quad - \frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2} - \frac{10\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{147b} \\
&= \frac{2x\cosh^{\frac{7}{2}}(a+bx)}{7b} + \frac{20i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2} \\
&\quad - \frac{20\sqrt{\cosh(a+bx)}\sinh(a+bx)}{147b^2} - \frac{4\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{49b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$$
$$=\frac{40i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sqrt{\cosh(a+bx)}(63bx\cosh(a+bx)+21bx\cosh(3(a+bx))-46\sinh(a+bx)}{294b^2}$$

[In] Integrate[x*Cosh[a + b*x]^(5/2)*Sinh[a + b*x],x]

[Out] ((40*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(63*b*x*Cosh[a + b*x] + 21*b*x*Cosh[3*(a + b*x)] - 46*Sinh[a + b*x] - 6*Sinh[3*(a + b*x)]))/(294*b^2)

Maple [F]

$$\int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)dx$$

[In] int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)

[Out] int(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x)

Fricas [F(-2)]

Exception generated.

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\text{Exception raised: TypeError}$$

[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

Sympy [F(-1)]

Timed out.

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\text{Timed out}$$

[In] integrate(x*cosh(b*x+a)**(5/2)*sinh(b*x+a),x)

[Out] Timed out

Maxima [F]

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\cosh\left(bx+a\right)^{\frac{5}{2}}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*cosh(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)^(5/2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\cosh(a+b\,x)^{5/2}\sinh(a+b\,x)\,dx$$

```
[In] int(x*cosh(a + b*x)^(5/2)*sinh(a + b*x),x)
[Out] int(x*cosh(a + b*x)^(5/2)*sinh(a + b*x), x)
```

3.529 $\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx)\, dx = \frac{2x \cosh^{\frac{5}{2}}(a+bx)}{5b} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2} - \frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2}$$

```
[Out] 2/5*x*cosh(b*x+a)^(5/2)/b+12/25*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+
1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/25*cosh(b*x+a)^(3/2
)*sinh(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5481, 2715, 2719}

$$\int x \cosh^{\frac{3}{2}}(a+bx) \sinh(a+bx)\, dx = \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2} - \frac{4\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[In] Int[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(5/2))/(5*b) + (((12*I)/25)*EllipticE[(I/2)*(a + b*x), 2
])/b^2 - (4*Cosh[a + b*x]^(3/2)*Sinh[a + b*x])/(25*b^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \cosh^{\frac{5}{2}}(a+bx)\,dx}{5b} \\ &= \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2} - \frac{6\int \sqrt{\cosh(a+bx)}\,dx}{25b} \\ &= \frac{2x\cosh^{\frac{5}{2}}(a+bx)}{5b} + \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2} - \frac{4\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{25b^2}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.22

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(\left(1+e^{2(a+bx)}\right)\left(2+5bx+2e^{2(a+bx)}(-12+5bx)+e^{4(a+bx)}(-2+5bx)\right)+48e^{2(a+bx)}\sqrt{1+e^{2(a+bx)}}\right.}{50\sqrt{2}b^2\sqrt{e^{-a-bx}+e^{a+bx}}}$$

```
[In] Integrate[x*Cosh[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] ((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4*
(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a +
 b*x))*Sqrt[E^(-a - b*x) + E^(a + b*x)])
```

Maple [F]

$$\int x\cosh\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)dx$$

```
[In] int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
[Out] int(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)**(3/2)*sinh(b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\cosh\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh\left(bx+a\right)^{\frac{3}{2}}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*cosh(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)^(3/2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\cosh(a+b\,x)^{3/2}\sinh(a+b\,x)\,dx$$

```
[In] int(x*cosh(a + b*x)^(3/2)*sinh(a + b*x),x)
[Out] int(x*cosh(a + b*x)^(3/2)*sinh(a + b*x), x)
```

3.530 $\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}+\frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2}-\frac{4\sqrt{\cosh(a+bx)}\sinh(a+bx)}{9b^2}$$

```
[Out] 2/3*x*cosh(b*x+a)^(3/2)/b+4/9*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/
2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2-4/9*sinh(b*x+a)*cosh(b*
x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5481, 2715, 2720}

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2}-\frac{4\sinh(a+bx)\sqrt{\cosh(a+bx)}}{9b^2}+\frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[In] Int[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (2*x*Cosh[a + b*x]^(3/2))/(3*b) + (((4*I)/9)*EllipticF[(I/2)*(a + b*x), 2])
/b^2 - (4*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(9*b^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \cosh^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\cosh(a+bx)}\sinh(a+bx)}{9b^2} - \frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{9b}\\ &= \frac{2x\cosh^{\frac{3}{2}}(a+bx)}{3b} + \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2} - \frac{4\sqrt{\cosh(a+bx)}\sinh(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right) + 2\sqrt{\cosh(a+bx)}(3bx\cosh(a+bx) - 2\sinh(a+bx))}{9b^2}$$

```
[In] Integrate[x*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] ((4*I)*EllipticF[(I/2)*(a + b*x), 2] + 2*Sqrt[Cosh[a + b*x]]*(3*b*x*Cosh[a
+ b*x] - 2*Sinh[a + b*x]))/(9*b^2)
```

Maple [F]

$$\int x\sinh(bx+a)\sqrt{\cosh(bx+a)}dx$$

```
[In] int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
[Out] int(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\sinh(a+bx)\sqrt{\cosh(a+bx)}\,dx$$

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)**(1/2),x)
[Out] Integral(x*sinh(a + b*x)*sqrt(cosh(a + b*x)), x)
```

Maxima [F]

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\sqrt{\cosh(bx+a)}\sinh(bx+a)\,dx$$

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

Giac [F]

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\sqrt{\cosh\left(bx+a\right)}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*sinh(b*x+a)*cosh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(cosh(b*x + a))*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\cosh(a+bx)}\sinh(a+bx)\,dx = \int x\,\sqrt{\cosh\left(a+b\,x\right)}\,\sinh(a+b\,x)\,dx$$

```
[In] int(x*cosh(a + b*x)^(1/2)*sinh(a + b*x),x)
[Out] int(x*cosh(a + b*x)^(1/2)*sinh(a + b*x), x)
```

3.531 $\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2x\sqrt{\cosh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[Out] 4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*
a+1/2*b*x),2^(1/2))/b^2+2*x*cosh(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5481, 2719}

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2x\sqrt{\cosh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[In] Int[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
[Out] (2*x*Sqrt[Cosh[a + b*x]])/b + ((4*I)*EllipticE[(I/2)*(a + b*x), 2])/b^2
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
```

```
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x\sqrt{\cosh(a+bx)}}{b} - \frac{2\int\sqrt{\cosh(a+bx)}\,dx}{b}\\ &= \frac{2x\sqrt{\cosh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|\,2\right)}{b^2}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \frac{(\cosh(a+bx)-\sinh(a+bx))\left(4\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{\ldots}\right.}{b^2\sqrt{\cosh(a+b\ldots}}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((Cosh[a + b*x] - Sinh[a + b*x])*(4*Hypergeometric2F1[-1/4, 1/2, 3/4, -Cosh
[2*(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a +
 b*x)]] + (-2 + b*x)*(1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])))/(b^2*Sqr
t[Cosh[a + b*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(61) = 122.

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 6.76

method	result
risch	$\frac{(bx-2)\left(1+e^{2bx+2a}\right)\sqrt{2}\,e^{-bx-a}}{b^2\sqrt{\left(1+e^{2bx+2a}\right)e^{-bx-a}}} - \frac{2\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}} + \frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{bx+a}-i\right)}\,\sqrt{ie^{bx+a}}\left(-2i\,\text{EllipticE}\left(\sqrt{-i\left(e^{bx+a}+\ldots\right.}\right.\right.}{\sqrt{e^{3bx+3a}+e^{bx+a}}}\ldots}{b^2\sqrt{\left(1+e^{2bx+2a}\right)e^{-bx\ldots}}}$

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)/ex
p(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I*(
```

```
-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))^(1
/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))^(1/
2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2^(1/2
)/((exp(b*x+a)^2+1)/exp(b*x+a))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)/e
xp(b*x+a)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}} \, dx = \int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}} \, dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(1/2),x)
[Out] Integral(x*sinh(a + b*x)/sqrt(cosh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\sqrt{\cosh(a+bx)}} \, dx = \int \frac{x \sinh(bx+a)}{\sqrt{\cosh(bx+a)}} \, dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{x\sinh\left(bx+a\right)}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/sqrt(cosh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{x\sinh(a+b\,x)}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(1/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(1/2), x)
```

3.532 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

```
[Out] -4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b^2-2*x/b/cosh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5481, 2720}

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
```

```
1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x}{b\sqrt{\cosh(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{b}\\ &= -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\cosh(a+bx)}} - \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(3/2),x]
[Out] (-2*x)/(b*Sqrt[Cosh[a + b*x]]) - ((4*I)*EllipticF[(I/2)*(a + b*x), 2])/b^2
```

Maple [F]

$$\int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{3}{2}}}dx$$

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(3/2),x)

[Out] Integral(x*sinh(a + b*x)/cosh(a + b*x)**(3/2), x)

Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)

Giac [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(3/2),x, algorithm="giac")

[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sinh(a+b\,x)}{\cosh(a+b\,x)^{3/2}}\,dx$$

[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(3/2),x)

[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(3/2), x)

3.533 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)} dx = -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2} + \frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}$$

```
[Out] -2/3*x/b/cosh(b*x+a)^(3/2)+4/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1
/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/3*sinh(b*x+a)/b^2/co
sh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5481, 2716, 2719}

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)} dx = \frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2} + \frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}} - \frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*Cosh[a + b*x]^(3/2)) + (((4*I)/3)*EllipticE[(I/2)*(a + b*x), 2]
)/b^2 + (4*Sinh[a + b*x])/(3*b^2*Sqrt[Cosh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{2\int\frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\ &= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}-\frac{2\int\sqrt{\cosh(a+bx)}\,dx}{3b}\\ &= -\frac{2x}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{4iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}+\frac{4\sinh(a+bx)}{3b^2\sqrt{\cosh(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int\frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx=\frac{2\left(-bx+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(2(a+bx))\right)}{3b^2\cosh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(5/2),x]
[Out] (2*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + Sinh
[2*(a + b*x)]))/(3*b^2*Cosh[a + b*x]^(3/2))
```

Maple [F]

$$\int\frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}dx$$

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^{5/2}}\,dx$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(5/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(5/2), x)
```

3.534 $\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)} dx = -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{4i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2} + \frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/5*x/b/cosh(b*x+a)^(5/2)-4/15*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+
1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/15*sinh(b*x+a)/b^2/
cosh(b*x+a)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5481, 2716, 2720}

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)} dx = -\frac{4i \operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2} + \frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)} - \frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
```

```
[Out] (-2*x)/(5*b*Cosh[a + b*x]^(5/2)) - (((4*I)/15)*EllipticF[(I/2)*(a + b*x), 2
])/b^2 + (4*Sinh[a + b*x])/(15*b^2*Cosh[a + b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{15b}$$

$$= -\frac{2x}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{4i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2} + \frac{4\sinh(a+bx)}{15b^2\cosh^{\frac{3}{2}}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\left(-3bx - 2i\cosh^{\frac{5}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right) + \sinh(2(a+bx))\right)}{15b^2\cosh^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(7/2),x]
[Out] (2*(-3*b*x - (2*I)*Cosh[a + b*x]^(5/2)*EllipticF[(I/2)*(a + b*x), 2] + Sinh
[2*(a + b*x)]))/(15*b^2*Cosh[a + b*x]^(5/2))
```

Maple [F]

$$\int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}dx$$

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\, dx = \text{Timed out}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x \sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\, dx = \int \frac{x \sinh(bx+a)}{\cosh(bx+a)^{\frac{7}{2}}}\, dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{\cosh\left(a+b\,x\right)^{7/2}}\,dx$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(7/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(7/2), x)
```

3.535 $\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}$$

```
[Out] -2/7*x/b/cosh(b*x+a)^(7/2)+12/35*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a
+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b^2+4/35*sinh(b*x+a)/b^2
/cosh(b*x+a)^(5/2)+12/35*sinh(b*x+a)/b^2/cosh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5481, 2716, 2719}

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}-\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}$$

```
[In] Int[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
```

```
[Out] (-2*x)/(7*b*Cosh[a + b*x]^(7/2)) + (((12*I)/35)*EllipticE[(I/2)*(a + b*x),
2])/b^2 + (4*Sinh[a + b*x])/(35*b^2*Cosh[a + b*x]^(5/2)) + (12*Sinh[a + b*x
])/(35*b^2*Sqrt[Cosh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{2\int\frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx}{7b}\\
&= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\int\frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx}{35b}\\
&= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}-\frac{6\int\sqrt{\cosh(a+bx)}\,dx}{35b}\\
&= -\frac{2x}{7b\cosh^{\frac{7}{2}}(a+bx)}+\frac{12iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}+\frac{4\sinh(a+bx)}{35b^2\cosh^{\frac{5}{2}}(a+bx)}+\frac{12\sinh(a+bx)}{35b^2\sqrt{\cosh(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int\frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \frac{-20bx+24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+10\sinh(2(a+bx))+3\sinh(4(a+bx))}{70b^2\cosh^{\frac{7}{2}}(a+bx)}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Cosh[a + b*x]^(9/2),x]
[Out] (-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*(a + b*x), 2] + 10*Si
nh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(70*b^2*Cosh[a + b*x]^(7/2))
```

Maple [F]

$$\int \frac{x \sinh (bx + a)}{\cosh (bx + a)^{\frac{9}{2}}} dx$$

```
[In] int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
[Out] int(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a + bx)}{\cosh^{\frac{9}{2}}(a + bx)} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \sinh(a + bx)}{\cosh^{\frac{9}{2}}(a + bx)} dx = \text{Timed out}$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x \sinh(a + bx)}{\cosh^{\frac{9}{2}}(a + bx)} dx = \int \frac{x \sinh (bx + a)}{\cosh (bx + a)^{\frac{9}{2}}} dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sinh(bx+a)}{\cosh(bx+a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/cosh(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/cosh(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\cosh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{\cosh(a+b\,x)^{9/2}}\,dx$$

```
[In] int((x*sinh(a + b*x))/cosh(a + b*x)^(9/2),x)
[Out] int((x*sinh(a + b*x))/cosh(a + b*x)^(9/2), x)
```

3.536 $\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{35b^2}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\text{sech}(a+bx)}\sinh(a+bx)}{35b^2}+\frac{4\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}$$

```
[Out] -2/7*x*sech(b*x+a)^(7/2)/b+4/35*sech(b*x+a)^(5/2)*sinh(b*x+a)/b^2+12/35*sin
h(b*x+a)*sech(b*x+a)^(1/2)/b^2+12/35*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1
/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*se
ch(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5552, 3853, 3856, 2719}

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{4\sinh(a+bx)\text{sech}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{35b^2}+\frac{12i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{35b^2}-\frac{2x\text{sech}^{\frac{7}{2}}(a+bx)}{7b}$$

```
[In] Int[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
```

```
[Out] (((12*I)/35)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(7/2))/(7*b) + (12*Sqrt[Sech[a + b*x]]*Si
nh[a + b*x])/(35*b^2) + (4*Sech[a + b*x]^(5/2)*Sinh[a + b*x])/(35*b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int \operatorname{sech}^{\frac{7}{2}}(a+bx)\,dx}{7b}\\
&= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}+\frac{6\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx}{35b}\\
&= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{35b^2}\\
&\quad+\frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}-\frac{6\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{35b}\\
&= -\frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{35b^2}+\frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}\\
&\quad-\frac{\left(6\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{35b}
\end{aligned}
$$

$$= \frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{35b^2} - \frac{2x\operatorname{sech}^{\frac{7}{2}}(a+bx)}{7b} + \frac{12\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{35b^2} + \frac{4\operatorname{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{35b^2}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int x\operatorname{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{\operatorname{sech}^{\frac{7}{2}}(a+bx)\left(-20bx+24i\cosh^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+10\sinh(2(a+bx))+3\sinh(4(a+bx))\right)}{70b^2}$$

```
[In] Integrate[x*Sech[a + b*x]^(9/2)*Sinh[a + b*x],x]
[Out] (Sech[a + b*x]^(7/2)*(-20*b*x + (24*I)*Cosh[a + b*x]^(7/2)*EllipticE[(I/2)*
(a + b*x), 2] + 10*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)]))/(70*b^2)
```

Maple [F]

$$\int x\operatorname{sech}(bx+a)^{\frac{9}{2}}\sinh(bx+a)\,dx$$

```
[In] int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
[Out] int(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\operatorname{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(x*sech(b*x+a)**(9/2)*sinh(b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{9}{2}}\sinh\,(bx+a)\,\,dx$$

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\text{sech}\,(bx+a)^{\frac{9}{2}}\sinh\,(bx+a)\,\,dx$$

```
[In] integrate(x*sech(b*x+a)^(9/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^(9/2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{9}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,{\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)}^{9/2}\,dx$$

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(9/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(9/2), x)
```

3.537 $\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{15b^2} - \frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b} + \frac{4\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}$$

```
[Out] -2/5*x*sech(b*x+a)^(5/2)/b+4/15*sech(b*x+a)^(3/2)*sinh(b*x+a)/b^2-4/15*I*(c
osh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*
b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5552, 3853, 3856, 2720}

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{4\sinh(a+bx)\text{sech}^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{15b^2} - \frac{2x\text{sech}^{\frac{5}{2}}(a+bx)}{5b}$$

```
[In] Int[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

```
[Out] (((-4*I)/15)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b^2 - (2*x*Sech[a + b*x]^(5/2))/(5*b) + (4*Sech[a + b*x]^(3/2)*Sin
h[a + b*x])/(15*b^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x\operatorname{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx}{5b}\\
&= -\frac{2x\operatorname{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}+\frac{2\int \sqrt{\operatorname{sech}(a+bx)}\,dx}{15b}\\
&= -\frac{2x\operatorname{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}\\
&\quad+\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{15b}\\
&= -\frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{15b^2}\\
&\quad-\frac{2x\operatorname{sech}^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{15b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2\sqrt{\text{sech}(a+bx)}\left(2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+3bx\text{sech}^2(a+bx)-2\tanh(a+bx)\right)}{15b^2}$$

```
[In] Integrate[x*Sech[a + b*x]^(7/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*Sqrt[Sech[a + b*x]]*((2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x
), 2] + 3*b*x*Sech[a + b*x]^2 - 2*Tanh[a + b*x]))/(15*b^2)
```

Maple [F]

$$\int x\,\text{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\,dx$$

```
[In] int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*sech(b*x+a)**(7/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\mathrm{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\mathrm{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\;dx$$

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\mathrm{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\,\mathrm{sech}\,(bx+a)^{\frac{7}{2}}\sinh\,(bx+a)\;dx$$

```
[In] integrate(x*sech(b*x+a)^(7/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^(7/2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{sech}^{\frac{7}{2}}(a+bx)\sinh(a+bx)\,dx=\int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\,(a+b\,x)}\right)^{7/2}\,dx$$

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(7/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(7/2), x)
```

3.538 $\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\text{sech}(a+bx)}}{3b^2}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\text{sech}(a+bx)}\sinh(a+bx)}{3b^2}$$

```
[Out] -2/3*x*sech(b*x+a)^(3/2)/b+4/3*sinh(b*x+a)*sech(b*x+a)^(1/2)/b^2+4/3*I*(cos
h(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*
x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5552, 3853, 3856, 2719}

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx=\frac{4\sinh(a+bx)\sqrt{\text{sech}(a+bx)}}{3b^2}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b^2}-\frac{2x\text{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

```
[In] Int[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (((4*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
b*x]])/b^2 - (2*x*Sech[a + b*x]^(3/2))/(3*b) + (4*Sqrt[Sech[a + b*x]]*Sinh[
a + b*x])/(3*b^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx}{3b}\\
&= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}-\frac{2\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{3b}\\
&= -\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}\\
&\quad-\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{3b}\\
&= \frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b^2}\\
&\quad-\frac{2x\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{3b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \frac{2\text{sech}^{\frac{3}{2}}(a+bx)\left(-bx+2i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(2(a+bx))\right)}{3b^2}$$

```
[In] Integrate[x*Sech[a + b*x]^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (2*Sech[a + b*x]^(3/2)*(-(b*x) + (2*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*
(a + b*x), 2] + Sinh[2*(a + b*x)]))/(3*b^2)
```

Maple [F]

$$\int x\,\text{sech}\,(bx+a)^{\frac{5}{2}}\sinh\,(bx+a)\,dx$$

```
[In] int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

```
[Out] int(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*sech(b*x+a)**(5/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\,\text{sech}\,(bx+a)^{\frac{5}{2}}\sinh\,(bx+a)\;dx$$

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\,\text{sech}\,(bx+a)^{\frac{5}{2}}\sinh\,(bx+a)\;dx$$

```
[In] integrate(x*sech(b*x+a)^(5/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^(5/2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{5}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\,(a+b\,x)}\right)^{5/2}\,dx$$

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(5/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(5/2), x)
```

3.539 $\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2x\sqrt{\text{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{b^2}$$

```
[Out] -2*x*sech(b*x+a)^(1/2)/b-4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b
*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^
(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5552, 3856, 2720}

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2x\sqrt{\text{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b^2}$$

```
[In] Int[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (-2*x*Sqrt[Sech[a + b*x]])/b - ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(
a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2\int\sqrt{\operatorname{sech}(a+bx)}\,dx}{b}\\
&= -\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{b}\\
&= -\frac{2x\sqrt{\operatorname{sech}(a+bx)}}{b}-\frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int x\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = -\frac{2\left(bx+2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}$$

```
[In] Integrate[x*Sech[a + b*x]^(3/2)*Sinh[a + b*x],x]
[Out] (-2*(b*x + (2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2])*Sqrt[Se
ch[a + b*x]])/b^2
```

Maple [F]

$$\int x\,\mathrm{sech}\,(bx+a)^{\frac{3}{2}}\sinh\,(bx+a)\,dx$$

```
[In] int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
[Out] int(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*sech(b*x+a)**(3/2)*sinh(b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int x\mathrm{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\,\mathrm{sech}\,(bx+a)^{\frac{3}{2}}\sinh\,(bx+a)\,dx$$

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

Giac [F]

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\,\text{sech}\,(bx+a)^{\frac{3}{2}}\sinh\,(bx+a)\;dx$$

```
[In] integrate(x*sech(b*x+a)^(3/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sech(b*x + a)^(3/2)*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)\,dx = \int x\sinh(a+b\,x)\,\left(\frac{1}{\cosh\,(a+b\,x)}\right)^{3/2}\,dx$$

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(3/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(3/2), x)
```

3.540 $\int x\sqrt{\textbf{sech}(a+bx)}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx=\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}$$

```
[Out] 2*x/b/sech(b*x+a)^(1/2)+4*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*
x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(
1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5552, 3856, 2719}

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx=\frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}}+\frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b^2}$$

```
[In] Int[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (2*x)/(b*Sqrt[Sech[a + b*x]]) + ((4*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*
(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{b}\\ &= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{b}\\ &= \frac{2x}{b\sqrt{\operatorname{sech}(a+bx)}} + \frac{4i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b^2}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \frac{\sqrt{2}e^{-a-bx}\sqrt{\frac{e^{a+bx}}{1+e^{2(a+bx)}}}\left(\left(1+e^{2(a+bx)}\right)(-2+bx)+4\sqrt{1+e^{2(a+bx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2(a+bx)}\right)\right)}{b^2}$$

```
[In] Integrate[x*Sqrt[Sech[a + b*x]]*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[2]*E^(-a - b*x)*Sqrt[E^(a + b*x)/(1 + E^(2*(a + b*x)))]*((1 + E^(2*(a
 + b*x)))*(-2 + b*x) + 4*Sqrt[1 + E^(2*(a + b*x))]*Hypergeometric2F1[-1/4,
1/2, 3/4, -E^(2*(a + b*x))]))/b^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.39

method	result
risch	$\frac{(bx-2)\left(1+e^{2bx+2a}\right)\sqrt{2}\sqrt{\frac{e^{bx+a}}{1+e^{2bx+2a}}}\,e^{-bx-a}}{b^2} - \frac{2\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\sqrt{2}\sqrt{i\left(e^{bx+a}-i\right)}\sqrt{ie^{bx+a}}\left(-2i\,\mathrm{EllipticE} \cdots\right.}{\sqrt{e^{3bx+3 \cdots}}}\cdots\right.}{\cdots}$

```
[In] int(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2+1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)/ex
p(b*x+a)-2/b^2*(-2*(exp(b*x+a)^2+1)/((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)+I*(
-I*(exp(b*x+a)+I))^(1/2)*2^(1/2)*(I*(exp(b*x+a)-I))^(1/2)*(I*exp(b*x+a))^(1
/2)/(exp(b*x+a)^3+exp(b*x+a))^(1/2)*(-2*I*EllipticE((-I*(exp(b*x+a)+I))^(1/
2),1/2*2^(1/2))+I*EllipticF((-I*(exp(b*x+a)+I))^(1/2),1/2*2^(1/2))))*2^(1/2
)*(exp(b*x+a)/(exp(b*x+a)^2+1))^(1/2)*((exp(b*x+a)^2+1)*exp(b*x+a))^(1/2)/e
xp(b*x+a)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \int x\sinh{(a+bx)}\sqrt{\operatorname{sech}{(a+bx)}}\,dx$$

```
[In] integrate(x*sech(b*x+a)**(1/2)*sinh(b*x+a),x)
```

```
[Out] Integral(x*sinh(a + b*x)*sqrt(sech(a + b*x)), x)
```

Maxima [F]

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \int x\sqrt{\operatorname{sech}\left(bx+a\right)}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)
```

Giac [F]

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \int x\sqrt{\operatorname{sech}\left(bx+a\right)}\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(x*sech(b*x+a)^(1/2)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x*sqrt(sech(b*x + a))*sinh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)\,dx = \int x\sinh(a+b\,x)\,\sqrt{\frac{1}{\cosh\left(a+b\,x\right)}}\,dx$$

```
[In] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(1/2),x)
[Out] int(x*sinh(a + b*x)*(1/cosh(a + b*x))^(1/2), x)
```

3.541 $\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{9b^2} - \frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}$$

```
[Out] 2/3*x/b/sech(b*x+a)^(3/2)-4/9*sinh(b*x+a)/b^2/sech(b*x+a)^(1/2)+4/9*I*(cosh
(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x
),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5552, 3854, 3856, 2720}

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = -\frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}} + \frac{4i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{9b^2} + \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Sech[a + b*x]^(3/2)) + (((4*I)/9)*Sqrt[Cosh[a + b*x]]*EllipticF[
(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(9*b^2*Sqr
t[Sech[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx}{3b} \\
&= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{2\int \sqrt{\operatorname{sech}(a+bx)}\,dx}{9b} \\
&= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}} - \frac{\left(2\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{9b} \\
&= \frac{2x}{3b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{4i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{9b^2} \\
&\quad - \frac{4\sinh(a+bx)}{9b^2\sqrt{\operatorname{sech}(a+bx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{x\sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx = \frac{\sqrt{\text{sech}(a+bx)}\left(3bx+3bx\cosh(2(a+bx))+4i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)-2\sinh(2(a+bx))\right)}{9b^2}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Sqrt[Sech[a + b*x]],x]
[Out] (Sqrt[Sech[a + b*x]]*(3*b*x + 3*b*x*Cosh[2*(a + b*x)] + (4*I)*Sqrt[Cosh[a +
 b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 2*Sinh[2*(a + b*x)]))/(9*b^2)
```

Maple [F]

$$\int \frac{x\sinh(bx+a)}{\sqrt{\text{sech}(bx+a)}}dx$$

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx = \int \frac{x\sinh(a+bx)}{\sqrt{\text{sech}(a+bx)}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(1/2),x)
[Out] Integral(x*sinh(a + b*x)/sqrt(sech(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x\sinh\left(bx+a\right)}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x\sinh\left(bx+a\right)}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/sqrt(sech(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{x\sinh(a+b\,x)}{\sqrt{\frac{1}{\cosh(a+b\,x)}}}\,dx$$

```
[In] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(1/2),x)
[Out] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(1/2), x)
```

3.542 $\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} + \frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{25b^2} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/sech(b*x+a)^(5/2)-4/25*sinh(b*x+a)/b^2/sech(b*x+a)^(3/2)+12/25*I*(c
osh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*
b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5552, 3854, 3856, 2719}

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = -\frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{12i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{25b^2} + \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)}$$

```
[In] Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Sech[a + b*x]^(5/2)) + (((12*I)/25)*Sqrt[Cosh[a + b*x]]*Elliptic
E[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(25*b^2*
Sech[a + b*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx}{5b} \\
&= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)} - \frac{6\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx}{25b} \\
&= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)} \\
&\quad - \frac{\left(6\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx}{25b} \\
&= \frac{2x}{5b\operatorname{sech}^{\frac{5}{2}}(a+bx)} + \frac{12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{25b^2} - \frac{4\sinh(a+bx)}{25b^2\operatorname{sech}^{\frac{3}{2}}(a+bx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.49

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{e^{-3(a+bx)}\left(\left(1+e^{2(a+bx)}\right)\left(2+5bx+2e^{2(a+bx)}(-12+5bx)+e^{4(a+bx)}(-2+5bx)\right)+48e^{2(a+bx)}\sqrt{1+e^{2(a+bx)}}\right.}{100b^2}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(3/2),x]
```

```
[Out] (((1 + E^(2*(a + b*x)))*(2 + 5*b*x + 2*E^(2*(a + b*x))*(-12 + 5*b*x) + E^(4
*(a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 + E^(2*(a + b*x))]*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -E^(2*(a + b*x))])*Sqrt[Sech[a + b*x]])/(10
0*b^2*E^(3*(a + b*x)))
```

Maple [F]

$$\int \frac{x \sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}dx$$

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh{\left(a+bx\right)}}{\operatorname{sech}^{\frac{3}{2}}{\left(a+bx\right)}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(3/2),x)
[Out] Integral(x*sinh(a + b*x)/sech(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh\left(bx+a\right)}{\operatorname{sech}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh\left(bx+a\right)}{\operatorname{sech}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{{\left(\frac{1}{\cosh(a+b\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(3/2),x)
[Out] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(3/2), x)
```

3.543 $\int \frac{x \sinh(a+bx)}{\text{sech}^{\frac{5}{2}}(a+bx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\begin{aligned}\int \frac{x \sinh(a+bx)}{\text{sech}^{\frac{5}{2}}(a+bx)} dx = & \frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)} \\ & + \frac{20i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{147b^2} \\ & - \frac{4\sinh(a+bx)}{49b^2\text{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\text{sech}(a+bx)}}\end{aligned}$$

```
[Out] 2/7*x/b/sech(b*x+a)^(7/2)-4/49*sinh(b*x+a)/b^2/sech(b*x+a)^(5/2)-20/147*sin
h(b*x+a)/b^2/sech(b*x+a)^(1/2)+20/147*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(
1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*s
ech(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5552, 3854, 3856, 2720}

$$\begin{aligned}\int \frac{x \sinh(a+bx)}{\text{sech}^{\frac{5}{2}}(a+bx)} dx = & -\frac{4\sinh(a+bx)}{49b^2\text{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\text{sech}(a+bx)}} \\ & + \frac{20i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{147b^2} \\ & + \frac{2x}{7b\text{sech}^{\frac{7}{2}}(a+bx)}\end{aligned}$$

```
[In] Int[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Sech[a + b*x]^(7/2)) + (((20*I)/147)*Sqrt[Cosh[a + b*x]]*Ellipti
cF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]])/b^2 - (4*Sinh[a + b*x])/(49*b^2
*Sech[a + b*x]^(5/2)) - (20*Sinh[a + b*x])/(147*b^2*Sqrt[Sech[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5552

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sinh[(a_.) + (b_.)*(x_)^
(n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sech[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x}{7b\,\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\text{sech}^{\frac{7}{2}}(a+bx)}\,dx}{7b}\\
&= \frac{2x}{7b\,\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{49b^2\text{sech}^{\frac{5}{2}}(a+bx)}-\frac{10\int \frac{1}{\text{sech}^{\frac{3}{2}}(a+bx)}\,dx}{49b}\\
&= \frac{2x}{7b\,\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{49b^2\text{sech}^{\frac{5}{2}}(a+bx)}-\frac{20\sinh(a+bx)}{147b^2\sqrt{\text{sech}(a+bx)}}-\frac{10\int \sqrt{\text{sech}(a+bx)}\,dx}{147b}\\
&= \frac{2x}{7b\,\text{sech}^{\frac{7}{2}}(a+bx)}-\frac{4\sinh(a+bx)}{49b^2\text{sech}^{\frac{5}{2}}(a+bx)}-\frac{20\sinh(a+bx)}{147b^2\sqrt{\text{sech}(a+bx)}}\\
&\quad-\frac{\left(10\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx}{147b}
\end{aligned}
$$

$$= \frac{2x}{7b\operatorname{sech}^{\frac{7}{2}}(a+bx)} + \frac{20i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{147b^2} - \frac{4\sinh(a+bx)}{49b^2\operatorname{sech}^{\frac{5}{2}}(a+bx)} - \frac{20\sinh(a+bx)}{147b^2\sqrt{\operatorname{sech}(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{sech}(a+bx)}\left(63bx+84bx\cosh(2(a+bx))+21bx\cosh(4(a+bx))+80i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a\right.\right.}{588b^2}$$

```
[In] Integrate[(x*Sinh[a + b*x])/Sech[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*(63*b*x + 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
+ b*x)] + (80*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2] - 52*Sin
h[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

Maple [F]

$$\int \frac{x\sinh(bx+a)}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}dx$$

```
[In] int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

```
[Out] int(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh{(a+bx)}}{\operatorname{sech}^{\frac{5}{2}}{(a+bx)}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)**(5/2),x)
[Out] Integral(x*sinh(a + b*x)/sech(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh{(bx+a)}}{\operatorname{sech}{(bx+a)}^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh{(bx+a)}}{\operatorname{sech}{(bx+a)}^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*sinh(b*x+a)/sech(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*sinh(b*x + a)/sech(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(a+bx)}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sinh(a+b\,x)}{{\left(\frac{1}{\cosh(a+b\,x)}\right)}^{5/2}}\,dx$$

```
[In] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(5/2),x)
[Out] int((x*sinh(a + b*x))/(1/cosh(a + b*x))^(5/2), x)
```

3.544 $\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx=\frac{20i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{147b^2\sqrt{\sinh(a+bx)}}+\frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2}-\frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] -4/49*cosh(b*x+a)*sinh(b*x+a)^(5/2)/b^2+2/7*x*sinh(b*x+a)^(7/2)/b-20/147*I*
(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellip
ticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b^2/sinh(
b*x+a)^(1/2)+20/147*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5480, 2715, 2721, 2720}

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx=-\frac{4\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{49b^2}+\frac{20\sqrt{\sinh(a+bx)}\cosh(a+bx)}{147b^2}+\frac{20i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{147b^2\sqrt{\sinh(a+bx)}}+\frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}$$

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((20*I)/147)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b^2*Sqrt[Sinh[a + b*x]]) + (20*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(147*b^2)
 - (4*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(49*b^2) + (2*x*Sinh[a + b*x]^(7/2
))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sinh^{\frac{7}{2}}(a+bx)\,dx}{7b}\\
&= -\frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b} + \frac{10\int \sinh^{\frac{3}{2}}(a+bx)\,dx}{49b}\\
&= \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2} - \frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2}\\
&\quad + \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b} - \frac{10\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{147b}\\
&= \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2} - \frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2}\\
&\quad + \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b} - \frac{\left(10\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{147b\sqrt{\sinh(a+bx)}}
\end{aligned}
$$

$$= \frac{20i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{147b^2\sqrt{\sinh(a+bx)}} + \frac{20\cosh(a+bx)\sqrt{\sinh(a+bx)}}{147b^2} - \frac{4\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sinh^{\frac{7}{2}}(a+bx)}{7b}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{63bx-84bx\cosh(2(a+bx))+21bx\cosh(4(a+bx))-80i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+}}{588b^2\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(5/2),x]
[Out] (63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a + b*x)] - (80*I)*Elli
pticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + 52*Sinh[2*(
a + b*x)] - 6*Sinh[4*(a + b*x)])/(588*b^2*Sqrt[Sinh[a + b*x]])
```

Maple [F]

$$\int x\cosh(bx+a)\sinh(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \int x\cosh(bx+a)\sinh(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(5/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,1
,1,0]%%%} / %%%{1,[0,0,0,2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)\,dx = \int x\cosh(a+b\,x)\,\sinh(a+b\,x)^{5/2}\,dx$$

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^(5/2),x)
[Out] int(x*cosh(a + b*x)*sinh(a + b*x)^(5/2), x)
```

3.545 $\int x \cosh(a+bx) \sinh^{\frac{3}{2}}(a+bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x \cosh(a+bx) \sinh^{\frac{3}{2}}(a+bx)\, dx = -\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{25b^2\sqrt{i\sinh(a+bx)}} - \frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -4/25*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b^2+2/5*x*sinh(b*x+a)^(5/2)/b+12/25*I*(
sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipt
icE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*
x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5480, 2715, 2721, 2719}

$$\int x \cosh(a+bx) \sinh^{\frac{3}{2}}(a+bx)\, dx = -\frac{4\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{25b^2} - \frac{12i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}$$

```
[In] Int[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((-12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^
2*Sqrt[I*Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(25*b^2) +
 (2*x*Sinh[a + b*x]^(5/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int\sinh^{\frac{5}{2}}(a+bx)\,dx}{5b} \\
&= -\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} + \frac{6\int\sqrt{\sinh(a+bx)}\,dx}{25b} \\
&= -\frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b} \\
&\quad + \frac{\left(6\sqrt{\sinh(a+bx)}\right)\int\sqrt{i\sinh(a+bx)}\,dx}{25b\sqrt{i\sinh(a+bx)}} \\
&= -\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{25b^2\sqrt{i\sinh(a+bx)}} \\
&\quad - \frac{4\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sinh^{\frac{5}{2}}(a+bx)}{5b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.95 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.46

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(\left(-1+e^{2(a+bx)}\right)\left(2+5bx+e^{2(a+bx)}(24-10bx)+e^{4(a+bx)}(-2+5bx)\right)+48e^{2(a+bx)}\sqrt{1-e^{2(a+bx)}}\right.}{50\sqrt{2}b^2\sqrt{-e^{-a-bx}+e^{a+bx}}}$$

```
[In] Integrate[x*Cosh[a + b*x]*Sinh[a + b*x]^(3/2),x]
[Out] ((-1 + E^(2*(a + b*x)))*(2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(
a + b*x))*(-2 + 5*b*x)) + 48*E^(2*(a + b*x))*Sqrt[1 - E^(2*(a + b*x))]*Hype
rgeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/(50*Sqrt[2]*b^2*E^(3*(a + b
*x))*Sqrt[-E^(-a - b*x) + E^(a + b*x)])
```

Maple [F]

$$\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}dx$$

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=\int x\sinh^{\frac{3}{2}}\left(a+bx\right)\cosh\left(a+bx\right)dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(3/2),x)
[Out] Integral(x*sinh(a + b*x)**(3/2)*cosh(a + b*x), x)
```

Maxima [F]

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=\int x\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*sinh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,\sinh(a+b\,x)^{3/2}\,dx$$

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^(3/2),x)
[Out] int(x*cosh(a + b*x)*sinh(a + b*x)^(3/2), x)
```

3.546 $\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=-\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{9b^2\sqrt{\sinh(a+bx)}}-\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 2/3*x*sinh(b*x+a)^(3/2)/b+4/9*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))
*(I*sinh(b*x+a))^(1/2)/b^2/sinh(b*x+a)^(1/2)-4/9*cosh(b*x+a)*sinh(b*x+a)^(1
/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5480, 2715, 2721, 2720}

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=-\frac{4\sqrt{\sinh(a+bx)}\cosh(a+bx)}{9b^2}-\frac{4i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{9b^2\sqrt{\sinh(a+bx)}}+\frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}$$

```
[In] Int[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (((-4*I)/9)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b^
2*Sqrt[Sinh[a + b*x]]) - (4*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(9*b^2) + (2
*x*Sinh[a + b*x]^(3/2))/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \sinh^{\frac{3}{2}}(a+bx)\,dx}{3b} \\
&= -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2} + \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{9b} \\
&= -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2} + \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b} + \frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{9b\sqrt{\sinh(a+bx)}} \\
&= -\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{9b^2\sqrt{\sinh(a+bx)}} \\
&\quad -\frac{4\cosh(a+bx)\sqrt{\sinh(a+bx)}}{9b^2} + \frac{2x\sinh^{\frac{3}{2}}(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \frac{2\left(2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}+3bx\sinh^2(a+bx)-\sinh(2(a+bx))\right)}{9b^2\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[x*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]],x]
[Out] (2*((2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]
 + 3*b*x*Sinh[a + b*x]^2 - Sinh[2*(a + b*x)]))/(9*b^2*Sqrt[Sinh[a + b*x]])
```

Maple [F]

$$\int x\cosh(bx+a)\sqrt{\sinh(bx+a)}dx$$

```
[In] int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
[Out] int(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx = \int x\sqrt{\sinh(a+bx)}\cosh(a+bx)\,dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)**(1/2),x)
[Out] Integral(x*sqrt(sinh(a + b*x))*cosh(a + b*x), x)
```

Maxima [F]

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=\int x\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=\int x\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sqrt{\sinh(a+bx)}\,dx=\int x\cosh(a+b\,x)\,\sqrt{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(x*cosh(a + b*x)*sinh(a + b*x)^(1/2),x)
[Out] int(x*cosh(a + b*x)*sinh(a + b*x)^(1/2), x)
```

3.547 $\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b^2\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*x*sinh(b*x+a)^(1/2)/b-4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2
*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sin
h(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5480, 2721, 2719}

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2x\sqrt{\sinh(a+bx)}}{b}+\frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}}$$

```
[In] Int[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*x*Sqrt[Sinh[a + b*x]])/b + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*S
qrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x\sqrt{\sinh(a+bx)}}{b} - \frac{2\int\sqrt{\sinh(a+bx)}\,dx}{b}\\ &= \frac{2x\sqrt{\sinh(a+bx)}}{b} - \frac{\left(2\sqrt{\sinh(a+bx)}\right)\int\sqrt{i\sinh(a+bx)}\,dx}{b\sqrt{i\sinh(a+bx)}}\\ &= \frac{2x\sqrt{\sinh(a+bx)}}{b} + \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b^2\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int\frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \frac{(-\cosh(a+bx)+\sinh(a+bx))\left(-2(-2+bx)\sinh(a+bx)(\cosh(a+bx)+\sinh(a+bx))+4\sqrt{2}\,\text{Hyperge}\right.}{b^2\sqrt{\text{sinh}}}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-Cosh[a + b*x] + Sinh[a + b*x])*(-2*(-2 + b*x)*Sinh[a + b*x]*(Cosh[a + b*
x] + Sinh[a + b*x]) + 4*Sqrt[2]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cosh[2*(a
 + b*x)] + Sinh[2*(a + b*x)]]*Sqrt[-(Sinh[a + b*x]*(Cosh[a + b*x] + Sinh[a
+ b*x]))]))/(b^2*Sqrt[Sinh[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(92) = 184$.

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.23

method	result
risch	$\frac{(bx-2)\left(\mathrm{e}^{2bx+2a}-1\right)\sqrt{2}\,\mathrm{e}^{-bx-a}}{b^{2}\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{-bx-a}}}+\frac{2\left(\frac{2\,\mathrm{e}^{2bx+2a}-2}{\sqrt{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}}-\frac{\sqrt{\mathrm{e}^{bx+a}+1}\,\sqrt{-2\,\mathrm{e}^{bx+a}+2}\,\sqrt{-\mathrm{e}^{bx+a}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{bx+a}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{Ellipt}\right.}{\sqrt{\mathrm{e}^{3bx+3a}-\mathrm{e}^{bx+a}}}\right.}{b^{2}\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{-bx-a}}}$

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^(1/2)/ex
p(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)-(exp
(b*x+a)+1)^(1/2)*(-2*exp(b*x+a)+2)^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)^3-
exp(b*x+a))^(1/2)*(-2*EllipticE((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))+EllipticF
((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(b*x+a)^2-1)/exp(b*x+a))^
(1/2)*(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)/exp(b*x+a)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh{(a+bx)}}{\sqrt{\sinh{(a+bx)}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sqrt(sinh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh{(bx+a)}}{\sqrt{\sinh{(bx+a)}}}\,dx$$

[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh{(bx+a)}}{\sqrt{\sinh{(bx+a)}}}\,dx$$

[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(x*cosh(b*x + a)/sqrt(sinh(b*x + a)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{x\cosh(a+b\,x)}{\sqrt{\sinh{(a+b\,x)}}}\,dx$$

[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(1/2),x)

[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(1/2), x)

3.548 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}$$

```
[Out] -2*x/b/sinh(b*x+a)^(1/2)+4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/
2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I
*sinh(b*x+a))^(1/2)/b^2/sinh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5480, 2721, 2720}

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b^2\sqrt{\sinh(a+bx)}}$$

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[Sinh[a + b*x]]) - ((4*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2
]*Sqrt[I*Sinh[a + b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{2\int\frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{b}\\ &= -\frac{2x}{b\sqrt{\sinh(a+bx)}}+\frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int\frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{b\sqrt{\sinh(a+bx)}}\\ &= -\frac{2x}{b\sqrt{\sinh(a+bx)}}-\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int\frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=\frac{-2bx+4i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}}{b^2\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(3/2),x]
[Out] (-2*b*x + (4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a +
 b*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Maple [F]

$$\int\frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{3}{2}}}dx$$

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh{(a+bx)}}{\sinh^{\frac{3}{2}}{(a+bx)}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cosh(a + b*x)/sinh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh{(bx+a)}}{\sinh{(bx+a)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh{(bx+a)}}{\sinh{(bx+a)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh{(a+b\,x)}^{3/2}}\,dx$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(3/2),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(3/2), x)
```

3.549 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}} - \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{3b^2\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2/3*x/b/sinh(b*x+a)^(3/2)-4/3*cosh(b*x+a)/b^2/sinh(b*x+a)^(1/2)+4/3*I*(sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE
(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a
))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5480, 2716, 2721, 2719}

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}} - \frac{4i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}} - \frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*Sinh[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(3*b^2*Sqrt[Sinh[a + b
*x]]) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]]
)/(b^2*Sqrt[I*Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}+\frac{2\int \sqrt{\sinh(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}+\frac{\left(2\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{3b\sqrt{i\sinh(a+bx)}}\\
&= -\frac{2x}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{3b^2\sqrt{\sinh(a+bx)}}-\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{3b^2\sqrt{i\sinh(a+bx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(bx+2iE\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)(i\sinh(a+bx))^{3/2}+\sinh(2(a+bx))\right)}{3b^2\sinh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(5/2),x]
[Out] (-2*(b*x + (2*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a + b*
x])^(3/2) + Sinh[2*(a + b*x)]))/(3*b^2*Sinh[a + b*x]^(3/2))
```

Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}dx$$

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(5/2),x)
[Out] Integral(x*cosh(a + b*x)/sinh(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x \cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\, dx = \int \frac{x \cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x \cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\, dx = \int \frac{x \cosh(bx+a)}{\sinh(bx+a)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \cosh(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\, dx = \int \frac{x \cosh(a+b\,x)}{\sinh(a+b\,x)^{5/2}}\, dx$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(5/2),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(5/2), x)
```

3.550 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)} + \frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{15b^2\sqrt{\sinh(a+bx)}}$$

```
[Out] -2/5*x/b/sinh(b*x+a)^(5/2)-4/15*cosh(b*x+a)/b^2/sinh(b*x+a)^(3/2)-4/15*I*(s
in(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b^2/sinh(b*
x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5480, 2716, 2721, 2720}

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)} + \frac{4i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{15b^2\sqrt{\sinh(a+bx)}} - \frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
```

```
[Out] (-2*x)/(5*b*Sinh[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(15*b^2*Sinh[a + b*x]^
(3/2)) + (((4*I)/15)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b
*x]])/(b^2*Sqrt[Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\
&= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx}{15b}\\
&= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}-\frac{\left(2\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{15b\sqrt{\sinh(a+bx)}}\\
&= -\frac{2x}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{4\cosh(a+bx)}{15b^2\sinh^{\frac{3}{2}}(a+bx)}+\frac{4i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{15b^2\sqrt{\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\left(3bx-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)(i\sinh(a+bx))^{5/2}+\sinh(2(a+bx))\right)}{15b^2\sinh^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(7/2),x]
[Out] (-2*(3*b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[a +
b*x])^(5/2) + Sinh[2*(a + b*x)]))/(15*b^2*Sinh[a + b*x]^(5/2))
```

Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}dx$$

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh(a+b\,x)^{7/2}}\,dx$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(7/2),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(7/2), x)
```

3.551 $\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} + \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{35b^2\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2/7*x/b/sinh(b*x+a)^(7/2)-4/35*cosh(b*x+a)/b^2/sinh(b*x+a)^(5/2)+12/35*cos
h(b*x+a)/b^2/sinh(b*x+a)^(1/2)-12/35*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))*sinh(b*x+a)^(1/2)/b^2/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5480, 2716, 2721, 2719}

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} + \frac{12i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}} - \frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)}$$

```
[In] Int[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
```

```
[Out] (-2*x)/(7*b*Sinh[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(35*b^2*Sinh[a + b*x]^
(5/2)) + (12*Cosh[a + b*x])/(35*b^2*Sqrt[Sinh[a + b*x]]) + (((12*I)/35)*Ell
```

```
ipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b^2*Sqrt[I*Sinh[a +
 b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx}{7b} \\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} - \frac{6\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx}{35b} \\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} - \frac{6\int \sqrt{\sinh(a+bx)}\,dx}{35b} \\
&= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} \\
&\quad - \frac{\left(6\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{35b\sqrt{i\sinh(a+bx)}}
\end{aligned}
$$

$$= -\frac{2x}{7b\sinh^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{35b^2\sinh^{\frac{5}{2}}(a+bx)} + \frac{12\cosh(a+bx)}{35b^2\sqrt{\sinh(a+bx)}} + \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{35b^2\sqrt{i\sinh(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2\left(5bx-6\cosh(a+bx)\sinh^3(a+bx)+6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\sinh^3(a+bx)+\text{s}\right.}{35b^2\sinh^{\frac{7}{2}}(a+bx)}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Sinh[a + b*x]^(9/2),x]
[Out] (-2*(5*b*x - 6*Cosh[a + b*x]*Sinh[a + b*x]^3 + 6*EllipticE[((-2*I)*a + Pi -
 (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]*Sinh[a + b*x]^3 + Sinh[2*(a + b*x)]
))/(35*b^2*Sinh[a + b*x]^(7/2))
```

Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{9}{2}}}dx$$

```
[In] int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
[Out] int(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\sinh(bx+a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/sinh(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)/sinh(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sinh^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{\sinh(a+b\,x)^{9/2}}\,dx$$

```
[In] int((x*cosh(a + b*x))/sinh(a + b*x)^(9/2),x)
[Out] int((x*cosh(a + b*x))/sinh(a + b*x)^(9/2), x)
```

3.552 $\int x \cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx=\frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{35b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] -4/35*cosh(b*x+a)*csch(b*x+a)^(5/2)/b^2-2/7*x*csch(b*x+a)^(7/2)/b+12/35*cos
h(b*x+a)*csch(b*x+a)^(1/2)/b^2-12/35*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5553, 3853, 3856, 2719}

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx=-\frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2}+\frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2}+\frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}-\frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b}$$

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

```
[Out] (12*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(35*b^2) - (4*Cosh[a + b*x]*Csch[a +
 b*x]^(5/2))/(35*b^2) - (2*x*Csch[a + b*x]^(7/2))/(7*b) + (((12*I)/35)*Elli
pticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a +
b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int \text{csch}^{\frac{7}{2}}(a+bx)\,dx}{7b}\\
&= -\frac{4\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}-\frac{6\int \text{csch}^{\frac{3}{2}}(a+bx)\,dx}{35b}\\
&= \frac{12\cosh(a+bx)\sqrt{\text{csch}(a+bx)}}{35b^2}-\frac{4\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)}{35b^2}\\
&\quad -\frac{2x\text{csch}^{\frac{7}{2}}(a+bx)}{7b}-\frac{6\int \frac{1}{\sqrt{\text{csch}(a+bx)}}\,dx}{35b}
\end{aligned}$$

$$= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2} - \frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b} - \frac{6\int\sqrt{i\sinh(a+bx)}\,dx}{35b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

$$= \frac{12\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{35b^2} - \frac{4\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\operatorname{csch}^{\frac{7}{2}}(a+bx)}{7b} + \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{35b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx = -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(-6\cosh(a+bx)+6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}+\operatorname{csch}^3(a+bx)(5bx\right.}{35b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(9/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(-6*Cosh[a + b*x] + 6*EllipticE[((-2*I)*a + Pi - (2
*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Csch[a + b*x]^3*(5*b*x + Sinh[2*(a +
 b*x)])))/(35*b^2)
```

Maple [F]

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{9}{2}}\,dx$$

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{9}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx=\int x\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{\frac{9}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

Giac [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx=\int x\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{\frac{9}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{9}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,{\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)}^{9/2}\,dx$$

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(9/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(9/2), x)
```

3.553 $\int x\cosh(a+bx)\mathbf{csch}^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx = -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b} + \frac{4i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{15b^2}$$

```
[Out] -4/15*cosh(b*x+a)*csch(b*x+a)^(3/2)/b^2-2/5*x*csch(b*x+a)^(5/2)/b-4/15*I*(s
in(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))
^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5553, 3853, 3856, 2720}

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx = -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2} + \frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{15b^2} - \frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b}$$

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(15*b^2) - (2*x*Csch[a + b*x]^(5/2))
/(5*b) + (((4*I)/15)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2,
2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b} + \frac{2\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx}{5b} \\
&= -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sqrt{\text{csch}(a+bx)}\,dx}{15b} \\
&= -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b} \\
&\quad - \frac{\left(2\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{15b} \\
&= -\frac{4\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{2x\text{csch}^{\frac{5}{2}}(a+bx)}{5b} \\
&\quad + \frac{4i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{15b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{7}{2}}(a+bx)\,dx = -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(2\coth(a+bx)+3bx\operatorname{csch}^2(a+bx)+2i\,\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+\dots}\right.}{15b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(7/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(2*Coth[a + b*x] + 3*b*x*Csch[a + b*x]^2 + (2*I)*El
lipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(15*b^2)
```

Maple [F]

$$\int x\cosh(bx+a)\operatorname{csch}(bx+a)^{\frac{7}{2}}\,dx$$

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

```
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{7}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

Giac [F]

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx=\int x\cosh(bx+a)\,\text{csch}\,(bx+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{7}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,\left(\frac{1}{\sinh(a+b\,x)}\right)^{7/2}\,dx$$

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(7/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(7/2), x)
```

3.554 $\int x \cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2} - \frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} - \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{3b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2/3*x*csch(b*x+a)^(3/2)/b-4/3*cosh(b*x+a)*csch(b*x+a)^(1/2)/b^2+4/3*I*(sin
(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE
(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x+a
))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5553, 3853, 3856, 2719}

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2} - \frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}} - \frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/(3*b^2) - (2*x*Csch[a + b*x]^(3/2))/
(3*b) - (((4*I)/3)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a +
 b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx}{3b}\\
&= -\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{3b}\\
&= -\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int \sqrt{i\sinh(a+bx)}\,dx}{3b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\
&= -\frac{4\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{3b^2}-\frac{2x\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)}{3b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\sqrt{\text{csch}(a+bx)}\left(2\cosh(a+bx)+bx\text{csch}(a+bx)-2E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{3b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(5/2),x]
[Out] (-2*Sqrt[Csch[a + b*x]]*(2*Cosh[a + b*x] + b*x*Csch[a + b*x] - 2*EllipticE[
((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b^2)
```

Maple [F]

$$\int x\cosh(bx+a)\,\text{csch}(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\text{csch}^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x\cosh(a+bx)\mathrm{csch}^{\frac{5}{2}}(a+bx)\,dx=\int x\cosh\left(bx+a\right)\mathrm{csch}\left(bx+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

Giac [F]

$$\int x\cosh(a+bx)\mathrm{csch}^{\frac{5}{2}}(a+bx)\,dx=\int x\cosh\left(bx+a\right)\mathrm{csch}\left(bx+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\mathrm{csch}^{\frac{5}{2}}(a+bx)\,dx=\int x\cosh(a+b\,x)\,\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)^{5/2}\,dx$$

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(5/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(5/2), x)
```

3.555 $\int x \cosh(a+bx)\mathbf{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\text{csch}(a+bx)}}{b}-\frac{4i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b^2}$$

```
[Out] -2*x*csch(b*x+a)^(1/2)/b+4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/
2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*cs
ch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5553, 3856, 2720}

$$\int x\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\text{csch}(a+bx)}}{b}-\frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b^2}$$

```
[In] Int[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csch[a + b*x]])/b - ((4*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{2\int\sqrt{\operatorname{csch}(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{\left(2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int\frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{b}\\ &= -\frac{2x\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{4i\sqrt{\operatorname{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int x\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(bx-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}\right)}{b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Csch[a + b*x]^(3/2),x]
[Out] (-2*Sqrt[Csch[a + b*x]]*(b*x - (2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/
4, 2]*Sqrt[I*Sinh[a + b*x]]))/b^2
```

Maple [F]

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{3}{2}} dx$$

[In] int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)

[Out] int(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F(-1)]

Timed out.

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\, dx = \text{Timed out}$$

[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int x \cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)\, dx = \int x \cosh(bx+a) \operatorname{csch}(bx+a)^{\frac{3}{2}}\, dx$$

[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)

Giac [F]

$$\int x \cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = \int x \cosh(bx+a)\,\text{csch}(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*csch(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)\,dx = \int x \cosh(a+b\,x)\,\left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2}\,dx$$

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(3/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(3/2), x)
```

3.556 $\int x \cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx=\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*x/b/csch(b*x+a)^(1/2)-4*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2
*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2
/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5553, 3856, 2719}

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx=\frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}}+\frac{4iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

```
[In] Int[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csch[a + b*x]]) + ((4*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]
)/(b^2*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{b} \\
&= \frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}} - \frac{2\int \sqrt{i\sinh(a+bx)}\,dx}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}} \\
&= \frac{2x}{b\sqrt{\operatorname{csch}(a+bx)}} + \frac{4iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.90 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \frac{\sqrt{2}e^{-a-bx}\sqrt{\frac{e^{a+bx}}{-1+e^{2(a+bx)}}}\left(\left(-1+e^{2(a+bx)}\right)(-2+bx)-4\sqrt{1-e^{2(a+bx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},e^{2(a+bx)}\right)\right)}{b^2}$$

```
[In] Integrate[x*Cosh[a + b*x]*Sqrt[Csch[a + b*x]],x]
[Out] (Sqrt[2]*E^(-a - b*x)*Sqrt[E^(a + b*x)/(-1 + E^(2*(a + b*x)))]*((-1 + E^(2*
(a + b*x)))*(-2 + b*x) - 4*Sqrt[1 - E^(2*(a + b*x))]*Hypergeometric2F1[-1/4
, 1/2, 3/4, E^(2*(a + b*x))]))/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(92) = 184$.

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.23

method	result
risch	$\frac{(bx-2)\left(e^{2bx+2a}-1\right)\sqrt{2}\,\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}\,e^{-bx-a}}{b^{2}}+\frac{2\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{e^{bx+a}\left(e^{2bx+2a}-1\right)}}-\frac{\sqrt{e^{bx+a}+1}\,\sqrt{-2\,e^{bx+a}+2}\,\sqrt{-e^{bx+a}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{bx+a}}\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{}$

[In] int(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (b*x-2)*(exp(b*x+a)^2-1)/b^2*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(1/2)/exp(b*x+a)+2/b^2*(2*(exp(b*x+a)^2-1)/(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)-(exp(b*x+a)+1)^(1/2)*(-2*exp(b*x+a)+2)^(1/2)*(-exp(b*x+a))^(1/2)/(exp(b*x+a)^3-exp(b*x+a))^(1/2)*(-2*EllipticE((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(b*x+a)+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(b*x+a)/(exp(b*x+a)^2-1))^(1/2)*(exp(b*x+a)*(exp(b*x+a)^2-1))^(1/2)/exp(b*x+a)

Fricas [F(-2)]

Exception generated.

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

Sympy [F]

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x\cosh\left(a+bx\right)\sqrt{\operatorname{csch}\left(a+bx\right)}\,dx$$

[In] integrate(x*cosh(b*x+a)*csch(b*x+a)**(1/2),x)

[Out] Integral(x*cosh(a + b*x)*sqrt(csch(a + b*x)), x)

Maxima [F]

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x\cosh(bx+a)\sqrt{\operatorname{csch}(bx+a)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

Giac [F]

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x\cosh(bx+a)\sqrt{\operatorname{csch}(bx+a)}\,dx$$

```
[In] integrate(x*cosh(b*x+a)*csch(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)*sqrt(csch(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}\,dx = \int x\cosh(a+b\,x)\,\sqrt{\frac{1}{\sinh(a+b\,x)}}\,dx$$

```
[In] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(1/2),x)
[Out] int(x*cosh(a + b*x)*(1/sinh(a + b*x))^(1/2), x)
```

3.557 $\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{4i\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{9b^2}$$

```
[Out] 2/3*x/b/csch(b*x+a)^(3/2)-4/9*cosh(b*x+a)/b^2/csch(b*x+a)^(1/2)+4/9*I*(sin(
1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(
cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1
/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5553, 3854, 3856, 2720}

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = -\frac{4\cosh(a+bx)}{9b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{4i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{9b^2} + \frac{2x}{3b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
```

[Out] (2*x)/(3*b*Csch[a + b*x]^(3/2)) - (4*Cosh[a + b*x])/(9*b^2*Sqrt[Csch[a + b*x]]) - (((4*I)/9)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b^2

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\text{integral} = \frac{2x}{3b\text{csch}^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\text{csch}^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$

$$= \frac{2x}{3b\text{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\text{csch}(a+bx)}} + \frac{2\int \sqrt{\text{csch}(a+bx)}\,dx}{9b}$$

$$= \frac{2x}{3b\text{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\text{csch}(a+bx)}} + \frac{\left(2\sqrt{\text{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{9b}$$

$$= \frac{2x}{3b\text{csch}^{\frac{3}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{9b^2\sqrt{\text{csch}(a+bx)}} - \frac{4i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{9b^2}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{6bx-4\coth(a+bx)-\frac{4i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)}{(i\sinh(a+bx))^{3/2}}}{9b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Sqrt[Csch[a + b*x]],x]
[Out] (6*b*x - 4*Coth[a + b*x] - ((4*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4,
2])/(I*Sinh[a + b*x])^(3/2))/(9*b^2*Csch[a + b*x]^(3/2))
```

Maple [F]

$$\int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}dx$$

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(1/2),x)
[Out] Integral(x*cosh(a + b*x)/sqrt(csch(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)/sqrt(csch(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{x\cosh(a+b\,x)}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

```
[In] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(1/2),x)
[Out] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(1/2), x)
```

3.558 $\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{25b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2/5*x/b/csch(b*x+a)^(5/2)-4/25*cosh(b*x+a)/b^2/csch(b*x+a)^(3/2)+12/25*I*(s
in(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b^2/csch(b*x+a)^(1/2)/(I*sinh(b*x
+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5553, 3854, 3856, 2719}

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = -\frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{12iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}} + \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)}$$

```
[In] Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csch[a + b*x]^(5/2)) - (4*Cosh[a + b*x])/(25*b^2*Csch[a + b*x]^(
3/2)) - (((12*I)/25)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b^2*Sqrt[Csch[a
 + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx}{5b} \\
&= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx}{25b} \\
&= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6\int \sqrt{i\sinh(a+bx)}\,dx}{25b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}} \\
&= \frac{2x}{5b\operatorname{csch}^{\frac{5}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{25b^2\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{12iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{25b^2\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{e^{-2(a+bx)}\left(2+5bx+e^{2(a+bx)}(24-10bx)+e^{4(a+bx)}(-2+5bx)-\frac{48e^{2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},e^{2(a+bx)}\right)}{\sqrt{1-e^{2(a+bx)}}}\right)}{50b^2\sqrt{\operatorname{csch}(a+bx)}}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(3/2),x]
```

```
[Out] (2 + 5*b*x + E^(2*(a + b*x))*(24 - 10*b*x) + E^(4*(a + b*x))*(-2 + 5*b*x) -
 (48*E^(2*(a + b*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, E^(2*(a + b*x))])/Sq
rt[1 - E^(2*(a + b*x))])/(50*b^2*E^(2*(a + b*x))*Sqrt[Csch[a + b*x]])
```

Maple [F]

$$\int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{3}{2}}}dx$$

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh{(a+bx)}}{\operatorname{csch}^{\frac{3}{2}}{(a+bx)}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(3/2),x)
[Out] Integral(x*cosh(a + b*x)/csch(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh{(bx+a)}}{\operatorname{csch}{(bx+a)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh{(bx+a)}}{\operatorname{csch}{(bx+a)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{{\left(\frac{1}{\sinh(a+b\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(3/2),x)
[Out] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(3/2), x)
```

3.559 $\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{20i\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{147b^2}$$

```
[Out] 2/7*x/b/csch(b*x+a)^(7/2)-4/49*cosh(b*x+a)/b^2/csch(b*x+a)^(5/2)+20/147*cos
h(b*x+a)/b^2/csch(b*x+a)^(1/2)-20/147*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(
1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),
2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5553, 3854, 3856, 2720}

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = -\frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} + \frac{20i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{147b^2} + \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)}$$

```
[In] Int[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csch[a + b*x]^(7/2)) - (4*Cosh[a + b*x])/(49*b^2*Csch[a + b*x]^(
5/2)) + (20*Cosh[a + b*x])/(147*b^2*Sqrt[Csch[a + b*x]]) + (((20*I)/147)*Sq
rt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]
])/b^2
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\operatorname{csch}^{\frac{7}{2}}(a+bx)}\,dx}{7b} \\
&= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{10\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx}{49b} \\
&= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} - \frac{10\int \sqrt{\operatorname{csch}(a+bx)}\,dx}{147b} \\
&= \frac{2x}{7b\operatorname{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\operatorname{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\operatorname{csch}(a+bx)}} \\
&\quad - \frac{\left(10\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{147b}
\end{aligned}$$

$$= \frac{2x}{7b\text{csch}^{\frac{7}{2}}(a+bx)} - \frac{4\cosh(a+bx)}{49b^2\text{csch}^{\frac{5}{2}}(a+bx)} + \frac{20\cosh(a+bx)}{147b^2\sqrt{\text{csch}(a+bx)}} + \frac{20i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{147b^2}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{x\cosh(a+bx)}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\text{csch}(a+bx)}\big(63bx - 84bx\cosh(2(a+bx)) + 21bx\cosh(4(a+bx)) - 80i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx\right.}{588b^2}$$

```
[In] Integrate[(x*Cosh[a + b*x])/Csch[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csch[a + b*x]]*(63*b*x - 84*b*x*Cosh[2*(a + b*x)] + 21*b*x*Cosh[4*(a
+ b*x)] - (80*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a
+ b*x]] + 52*Sinh[2*(a + b*x)] - 6*Sinh[4*(a + b*x)]))/(588*b^2)
```

Maple [F]

$$\int \frac{x\cosh(bx+a)}{\text{csch}(bx+a)^{\frac{5}{2}}}dx$$

```
[In] int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

```
[Out] int(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cosh(a+bx)}{\text{csch}^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(bx+a)}{\operatorname{csch}(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*cosh(b*x+a)/csch(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cosh(b*x + a)/csch(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(a+bx)}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cosh(a+b\,x)}{{\left(\frac{1}{\sinh(a+b\,x)}\right)}^{5/2}}\,dx$$

```
[In] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(5/2),x)
[Out] int((x*cosh(a + b*x))/(1/sinh(a + b*x))^(5/2), x)
```

3.560 $\int \sqrt{\sinh(x)\tanh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] 2*coth(x)*(sinh(x)*tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4483, 4485, 2669}

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[In] Int[Sqrt[Sinh[x]*Tanh[x]],x]
```

```
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
```

```
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{\sinh(x)\tanh(x)}\int\sqrt{-\sinh(x)\tanh(x)}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\ &= \frac{\sqrt{\sinh(x)\tanh(x)}\int\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= 2\coth(x)\sqrt{\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int\sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[In] Integrate[Sqrt[Sinh[x]*Tanh[x]],x]
[Out] 2*Coth[x]*Sqrt[Sinh[x]*Tanh[x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

method	result	size
risch	$\frac{\sqrt{2}\sqrt{\frac{\left(e^{2x}-1\right)^{2}e^{-x}}{1+e^{2x}}}\,\left(1+e^{2x}\right)}{e^{2x}-1}$	42

```
[In] int((sinh(x)*tanh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*((exp(2*x)-1)^2*exp(-x)/(1+exp(2*x)))^(1/2)/(exp(2*x)-1)*(1+exp(2*x
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2} + 2\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 1\right)}{\sqrt{\cosh\left(x\right)^{3} + 3\cosh\left(x\right)\sinh\left(x\right)^{2} + \sinh\left(x\right)^{3} + \left(3\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right) + \cosh\left(x\right)}}$$

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))
```

Sympy [F]

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = \int \sqrt{\sinh\left(x\right)\tanh\left(x\right)}\,dx$$

```
[In] integrate((sinh(x)*tanh(x))**(1/2),x)
[Out] Integral(sqrt(sinh(x)*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(11) = 22.

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = -\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{e^{\left(-2x\right)} + 1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{e^{\left(-2x\right)} + 1}}$$

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="maxima")
[Out] -sqrt(2)*e^(1/2*x)/sqrt(e^(-2*x) + 1) - sqrt(2)*e^(-3/2*x)/sqrt(e^(-2*x) +
1)
```

Giac [F]

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = \int \sqrt{\sinh(x)\tanh(x)}\,dx$$

```
[In] integrate((sinh(x)*tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(x)*tanh(x)), x)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\sinh(x)\tanh(x)}\,dx = 2\coth(x)\sqrt{-\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)(e^{2x}-1)}\sqrt{\frac{1}{e^{2x}+1}}$$

```
[In] int((sinh(x)*tanh(x))^(1/2),x)
```

```
[Out] 2*coth(x)*(-(exp(-x)/2 - exp(x)/2)*(exp(2*x) - 1))^(1/2)*(1/(exp(2*x) + 1))
^(1/2)
```

3.561 $\int (\sinh(x)\tanh(x))^{3/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \frac{8}{3}\operatorname{csch}(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] 8/3*csch(x)*(sinh(x)*tanh(x))^(1/2)+2/3*sinh(x)*(sinh(x)*tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4483, 4485, 2678, 2669}

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{8}{3}\operatorname{csch}(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[In] Int[(Sinh[x]*Tanh[x])^(3/2),x]
[Out] (8*Csch[x]*Sqrt[Sinh[x]*Tanh[x]])/3 + (2*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
```

```
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{3/2}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\
&= -\frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{3/2}(i\tanh(x))^{3/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}-\frac{\left(4\sqrt{\sinh(x)\tanh(x)}\right)\int\frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{8}{3}\text{csch}(x)\sqrt{\sinh(x)\tanh(x)}+\frac{2}{3}\sinh(x)\sqrt{\sinh(x)\tanh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int(\sinh(x)\tanh(x))^{3/2}\,dx=\frac{2}{3}\left(1+4\text{csch}^2(x)\right)\sinh(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[In] Integrate[(Sinh[x]*Tanh[x])^(3/2),x]
[Out] (2*(1 + 4*Csch[x]^2)*Sinh[x]*Sqrt[Sinh[x]*Tanh[x]])/3
```

Maple [F]

$$\int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

```
[In] int((sinh(x)*tanh(x))^(3/2),x)
[Out] int((sinh(x)*tanh(x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 7\right)\sinh(x)^2 \right.}{3\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 + \right.}}$$

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x)
+ cosh(x))*(cosh(x) + sinh(x)))
```

Sympy [F]

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

```
[In] integrate((sinh(x)*tanh(x))**(3/2),x)
[Out] Integral((sinh(x)*tanh(x))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = -\frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{3}{2}}} - \frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-2x)}+1\right)^{\frac{3}{2}}} - \frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-2x)}+1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}$$

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*e^(3/2*x)/(e^(-2*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(e^(-2
*x) + 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(e^(-2*x) + 1)^(3/2) - 1/6*sqrt(2)*
e^(-9/2*x)/(e^(-2*x) + 1)^(3/2)
```

Giac [F]

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \int (\sinh(x)\tanh(x))^{\frac{3}{2}}\,dx$$

```
[In] integrate((sinh(x)*tanh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)*tanh(x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\sinh(x)\tanh(x))^{3/2}\,dx = \int (\sinh(x)\,\tanh(x))^{3/2}\,dx$$

```
[In] int((sinh(x)*tanh(x))^(3/2),x)
```

```
[Out] int((sinh(x)*tanh(x))^(3/2), x)
```

3.562 $\int (\sinh(x)\tanh(x))^{5/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = -\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[Out] -64/15*coth(x)*(sinh(x)*tanh(x))^(1/2)+16/15*(sinh(x)*tanh(x))^(1/2)*tanh(x
)+2/5*sinh(x)^2*(sinh(x)*tanh(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4483, 4485, 2678, 2674, 2669}

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} - \frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[In] Int[(Sinh[x]*Tanh[x])^(5/2),x]
```

```
[Out] (-64*Coth[x]*Sqrt[Sinh[x]*Tanh[x]])/15 + (16*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])
/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/5
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\sinh(x)\tanh(x)}\int(-\sinh(x)\tanh(x))^{5/2}\,dx}{\sqrt{-\sinh(x)\tanh(x)}}\\
&= \frac{\sqrt{\sinh(x)\tanh(x)}\int(i\sinh(x))^{5/2}(i\tanh(x))^{5/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{\left(8\sqrt{\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}(i\tanh(x))^{5/2}\,dx}{5\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)}+\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}\\
&\quad-\frac{\left(32\sqrt{\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{15\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}
\end{aligned}$$

$$= -\frac{64}{15}\coth(x)\sqrt{\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{\sinh(x)\tanh(x)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = -\frac{2}{15}\left(-5-3\cosh^2(x)+32\coth^2(x)\right)\tanh(x)\sqrt{\sinh(x)\tanh(x)}$$

```
[In] Integrate[(Sinh[x]*Tanh[x])^(5/2),x]
[Out] (-2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Tanh[x]*Sqrt[Sinh[x]*Tanh[x]])/15
```

Maple [F]

$$\int (\sinh(x)\tanh(x))^{\frac{5}{2}}\,dx$$

```
[In] int((sinh(x)*tanh(x))^(5/2),x)
[Out] int((sinh(x)*tanh(x))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.06

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 - 9\right)\sin\right.}{}$$

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="fricas")
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3*cosh
(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^
2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)))
```

Sympy [F(-1)]

Timed out.

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((sinh(x)*tanh(x))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(38) = 76.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.06

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = -\frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,\left(e^{(-2\,x)}+1\right)^{\frac{5}{2}}}$$

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="maxima")
[Out] -1/20*sqrt(2)*e^(5/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(e^(-2
*x) + 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(e^(-2*x) + 1)^(5/2) + 41/6*sqrt(2
)*e^(-7/2*x)/(e^(-2*x) + 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(e^(-2*x) + 1)^
(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(e^(-2*x) + 1)^(5/2)
```

Giac [F]

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \int (\sinh(x)\tanh(x))^{\frac{5}{2}}\,dx$$

```
[In] integrate((sinh(x)*tanh(x))^(5/2),x, algorithm="giac")
[Out] integrate((sinh(x)*tanh(x))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\sinh(x)\tanh(x))^{5/2}\,dx = \int (\sinh(x)\ \tanh(x))^{5/2}\,dx$$

```
[In] int((sinh(x)*tanh(x))^(5/2),x)
[Out] int((sinh(x)*tanh(x))^(5/2), x)
```

3.563 $\int \sqrt{\cosh(x)\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = 2\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

```
[Out] 2*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4483, 4485, 2669}

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = 2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[In] Int[Sqrt[Cosh[x]*Coth[x]],x]
```

```
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
```

```
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{\cosh(x)\coth(x)}\int\sqrt{-i\cosh(x)\coth(x)}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\ &= \frac{\sqrt{\cosh(x)\coth(x)}\int\sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\ &= 2\sqrt{\cosh(x)\coth(x)}\tanh(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int\sqrt{\cosh(x)\coth(x)}\,dx = \frac{2\sqrt{\cosh(x)\coth(x)}\left(-1+\sqrt[4]{-\sinh^2(x)}\right)\tanh(x)}{\sqrt[4]{-\sinh^2(x)}}$$

```
[In] Integrate[Sqrt[Cosh[x]*Coth[x]],x]
[Out] (2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(1
/4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

method	result	size
risch	$\frac{\sqrt{2}\sqrt{\frac{\left(1+e^{2x}\right)^{2}e^{-x}}{e^{2x}-1}}\left(e^{2x}-1\right)}{1+e^{2x}}$	42

```
[In] int((coth(x)*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*((1+exp(2*x))^2*exp(-x)/(exp(2*x)-1))^(1/2)/(1+exp(2*x))*(exp(2*x)-
1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.23

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)}{\sqrt{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)-\cosh\left(x\right)}}$$

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

Sympy [F]

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \int \sqrt{\cosh\left(x\right)\coth\left(x\right)}\,dx$$

```
[In] integrate((cosh(x)*coth(x))**(1/2),x)
[Out] Integral(sqrt(cosh(x)*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.15

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \frac{\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{\sqrt{e^{(-x)}+1}\sqrt{-e^{(-x)}+1}}$$

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="maxima")
[Out] sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x)
/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

Giac [F]

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = \int \sqrt{\cosh\left(x\right)\coth\left(x\right)}\,dx$$

```
[In] integrate((cosh(x)*coth(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cosh(x)*coth(x)), x)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \sqrt{\cosh(x)\coth(x)}\,dx = 4\,e^x\sinh(x)\,\sqrt{\frac{e^{-x}}{2\,\left(e^{2\,x}-1\right)}}$$

```
[In] int((cosh(x)*coth(x))^(1/2),x)
[Out] 4*exp(x)*sinh(x)*(exp(-x)/(2*(exp(2*x) - 1)))^(1/2)
```

3.564 $\int (\cosh(x)\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

```
[Out] 2/3*cosh(x)*(cosh(x)*coth(x))^(1/2)-8/3*sech(x)*(cosh(x)*coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4483, 4485, 2678, 2669}

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\operatorname{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

```
[In] Int[(Cosh[x]*Coth[x])^(3/2),x]
```

```
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
```

```
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
&& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int(-i\cosh(x)\coth(x))^{3/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}} \\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int\cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(4i\sqrt{\cosh(x)\coth(x)}\right)\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int(\cosh(x)\coth(x))^{3/2}\,dx = \frac{2}{3}\left(-4+\cosh^2(x)\right)\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

```
[In] Integrate[(Cosh[x]*Coth[x])^(3/2),x]
[Out] (2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Maple [F]

$$\int (\coth (x) \cosh (x))^{\frac{3}{2}}\, dx$$

```
[In] int((coth(x)*cosh(x))^(3/2),x)
```

```
[Out] int((coth(x)*cosh(x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int (\cosh(x)\coth(x))^{3/2}\, dx = \frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 7\right)\sinh(x)^2 -\right.}{3\sqrt{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right.}}$$

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*(cosh(x) + sinh(x)))
```

Sympy [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{3/2}\, dx = \text{Timed out}$$

```
[In] integrate((cosh(x)*coth(x))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int (\cosh(x)\coth(x))^{3/2}\, dx = \frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}} - \frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}} + \frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}$$

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(2
)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5/2
*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((e^(
-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

Giac [F]

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \int (\cosh(x)\coth(x))^{\frac{3}{2}}\,dx$$

```
[In] integrate((cosh(x)*coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cosh(x)*coth(x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{3/2}\,dx = \int (\cosh(x)\,\coth(x))^{3/2}\,dx$$

```
[In] int((cosh(x)*coth(x))^(3/2),x)
```

```
[Out] int((cosh(x)*coth(x))^(3/2), x)
```

3.565 $\int (\cosh(x)\coth(x))^{5/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

```
[Out] -16/15*coth(x)*(cosh(x)*coth(x))^(1/2)+2/5*cosh(x)^2*coth(x)*(cosh(x)*coth(
x))^(1/2)+64/15*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4483, 4485, 2678, 2674, 2669}

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[In] Int[(Cosh[x]*Coth[x])^(5/2),x]
```

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cosh(x)\coth(x)}\int(-i\cosh(x)\coth(x))^{5/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}\\
&= -\frac{\sqrt{\cosh(x)\coth(x)}\int\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{\left(8\sqrt{\cosh(x)\coth(x)}\right)\int\sqrt{\cosh(x)}(-i\coth(x))^{5/2}\,dx}{5\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}\\
&= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}\\
&\quad+\frac{\left(32\sqrt{\cosh(x)\coth(x)}\right)\int\sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{15\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}
\end{aligned}$$

$$= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \frac{1}{15}\sqrt{\cosh(x)\coth(x)}\Big(-10\coth(x) + 6\cosh(x)\sinh(x) + 57\text{csch}(x)\text{sech}(x)\left(-\sinh^2(x)\right)^{3/4} + 64\tanh(x)\Big)$$

```
[In] Integrate[(Cosh[x]*Coth[x])^(5/2),x]
[Out] (Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[x
]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

Maple [F]

$$\int \left(\coth(x)\cosh(x)\right)^{\frac{5}{2}}dx$$

```
[In] int((coth(x)*cosh(x))^(5/2),x)
[Out] int((coth(x)*cosh(x))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 5.18

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 + 9\right)\sinh\right.}{}$$

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="fricas")
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3*cosh
(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^
2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

Sympy [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((cosh(x)*coth(x))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.26

$$\begin{aligned}\int (\cosh(x)\coth(x))^{5/2}\,dx &= \frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} \\ &+ \frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} \\ &+ \frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} \\ &- \frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}}\end{aligned}$$

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="maxima")
[Out] 1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt(
2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-3/
2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/((e
^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x) +
1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)^(5/2
)*(-e^(-x) + 1)^(5/2))
```

Giac [F]

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \int (\cosh(x)\coth(x))^{\frac{5}{2}}\,dx$$

```
[In] integrate((cosh(x)*coth(x))^(5/2),x, algorithm="giac")
[Out] integrate((cosh(x)*coth(x))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\cosh(x)\coth(x))^{5/2}\,dx = \int (\cosh(x)\ \coth(x))^{5/2}\,dx$$

```
[In] int((cosh(x)*coth(x))^(5/2),x)
[Out] int((cosh(x)*coth(x))^(5/2), x)
```

3.566 $\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = -\frac{2(b+c)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{\log(a+b\sinh(x))}{b}$$

```
[Out] ln(a+b*sinh(x))/b-2*(b+c)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4486, 2739, 632, 212, 2747, 31}

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\log(a+b\sinh(x))}{b} - \frac{2(b+c)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] Int[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + L
og[a + b*Sinh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\left(1+\frac{b}{c}\right)c}{a+b\sinh(x)}+\frac{\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= (b+c)\int \frac{1}{a+b\sinh(x)}\,dx+\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}+(2(b+c))\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{\log(a+b\sinh(x))}{b}-(4(b+c))\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2(b+c)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(x))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{2(b+c)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\log(a+b\sinh(x))}{b}$$

```
[In] Integrate[(b + c + Cosh[x])/(a + b*Sinh[x]),x]
[Out] (2*(b + c)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + L
og[a + b*Sinh[x]]/b
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result
parts	$\frac{\ln(a+b\sinh(x))}{b}+\frac{2(b+c)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-b^2-cb\right)\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}$
risch	$\frac{x}{b}-\frac{2x\,a^2b}{a^2b^2+b^4}-\frac{2x\,b^3}{a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2+b^6+2b^5c+b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2+b^2)b}+\frac{b\ln\left(\mathrm{e}^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3}}{b^2(b}\right.}{a^2+b^2}$

```
[In] int((b+c+cosh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*sinh(x))/b+2*(b+c)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(48) = 96$.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.21

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^2b+b^3}$$

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*co
sh(x) + a)*sinh(x) - b)) - (a^2 + b^2)*x + (a^2 + b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.48 (sec) , antiderivative size = 585, normalized size of antiderivative = 11.25

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2)
)), Eq(a, 0) & Eq(b, 0)), ((b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*log
(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), ((c*x + sinh(x))/a, Eq(b, 0
)), (2*I*b/(b*tanh(x/2) - I*b) + 2*I*c/(b*tanh(x/2) - I*b) + x*tanh(x/2)/(b
*tanh(x/2) - I*b) - I*x/(b*tanh(x/2) - I*b) - 2*log(tanh(x/2) + 1)*tanh(x/2
)/(b*tanh(x/2) - I*b) + 2*I*log(tanh(x/2) + 1)/(b*tanh(x/2) - I*b) + 2*log(
tanh(x/2) - I)*tanh(x/2)/(b*tanh(x/2) - I*b) - 2*I*log(tanh(x/2) - I)/(b*ta
nh(x/2) - I*b), Eq(a, -I*b)), (-2*I*b/(b*tanh(x/2) + I*b) - 2*I*c/(b*tanh(x
/2) + I*b) + x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*x/(b*tanh(x/2) + I*b) - 2*
log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) + I*b) - 2*I*log(tanh(x/2) + 1)/(
b*tanh(x/2) + I*b) + 2*log(tanh(x/2) + I)*tanh(x/2)/(b*tanh(x/2) + I*b) + 2
*I*log(tanh(x/2) + I)/(b*tanh(x/2) + I*b), Eq(a, I*b)), (-b*log(tanh(x/2) -
 b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + b*log(tanh(x/2) - b/a + sqr
t(a**2 + b**2)/a)/sqrt(a**2 + b**2) - c*log(tanh(x/2) - b/a - sqrt(a**2 + b
**2)/a)/sqrt(a**2 + b**2) + c*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sq
rt(a**2 + b**2) + x/b - 2*log(tanh(x/2) + 1)/b + log(tanh(x/2) - b/a - sqrt
(a**2 + b**2)/a)/b + log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/b, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(48) = 96.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.35

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{c\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{\log\left(b\sinh\left(x\right)+a\right)}{b}$$

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sq
rt(a^2 + b^2) + c*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/sqrt(a^2 + b^2) + log(b*sinh(x) + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx = \frac{(b+c)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} - \frac{x}{b} + \frac{\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{b}$$

```
[In] integrate((b+c+cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (b + c)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/sqrt(a^2 + b^2) - x/b + log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.42

$$\begin{aligned}\int \frac{b+c+\cosh(x)}{a+b\sinh(x)}\,dx &= \frac{\ln\left(a^2\,e^x-b\,\sqrt{a^2+b^2}+b^2\,e^x+a\,e^x\,\sqrt{a^2+b^2}\right)\,\left(b^2\,\sqrt{a^2+b^2}+a^2+b^2+b\,c\,\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}\\&\quad-\frac{\ln\left(b\,\sqrt{a^2+b^2}+a^2\,e^x+b^2\,e^x-a\,e^x\,\sqrt{a^2+b^2}\right)\,\left(b^2\,\sqrt{a^2+b^2}-a^2-b^2+b\,c\,\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}\\&\quad-\frac{x}{b}\end{aligned}$$

```
[In] int((b + c + cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (log(a^2*exp(x) - b*(a^2 + b^2)^(1/2) + b^2*exp(x) + a*exp(x)*(a^2 + b^2)^(
1/2))*(b^2*(a^2 + b^2)^(1/2) + a^2 + b^2 + b*c*(a^2 + b^2)^(1/2)))/(a^2*b +
 b^3) - (log(b*(a^2 + b^2)^(1/2) + a^2*exp(x) + b^2*exp(x) - a*exp(x)*(a^2
+ b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) - a^2 - b^2 + b*c*(a^2 + b^2)^(1/2)))/
(a^2*b + b^3) - x/b
```

3.567 $\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = \frac{2(b+c)\text{arctanh}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{\log(a-b\sinh(x))}{b}$$

```
[Out] -ln(a-b*sinh(x))/b+2*(b+c)*arctanh((b+a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+
b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4486, 2739, 632, 212, 2747, 31}

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = \frac{2(b+c)\text{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{\log(a-b\sinh(x))}{b}$$

```
[In] Int[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(b + a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Lo
g[a - b*Sinh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\left(1+\frac{b}{c}\right) c}{a-b\sinh(x)}+\frac{\cosh(x)}{a-b\sinh(x)}\right) dx \\
&= (b+c)\int \frac{1}{a-b\sinh(x)}\, dx+\int \frac{\cosh(x)}{a-b\sinh(x)}\, dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\, dx, x, -b\sinh(x)\right)}{b}+(2(b+c))\text{Subst}\left(\int \frac{1}{a-2bx-ax^2}\, dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= -\frac{\log(a-b\sinh(x))}{b}-(4(b+c))\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\, dx, x, -2b-2a\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{2(b+c)\text{arctanh}\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\log(a-b\sinh(x))}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx=-\frac{2(b+c)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{\log(-a+b\sinh(x))}{b}$$

```
[In] Integrate[(b + c + Cosh[x])/(a - b*Sinh[x]),x]
```

```
[Out] (-2*(b + c)*ArcTan[(b + a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] -
Log[-a + b*Sinh[x]]/b
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result
parts	$-\frac{\ln(a-b\sinh(x))}{b}+\frac{2(b+c)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-b^2-cb\right)\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$-\frac{x}{b}+\frac{2x\,a^2b}{a^2b^2+b^4}+\frac{2x\,b^3}{a^2b^2+b^4}-\frac{\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2+b^6+2b^5c+b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2+b^2)b}-\frac{b\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3\cdots}}{b^2(\cdots}\right.}{a^2+b^2\cdots}$

```
[In] int((b+c+cosh(x))/(a-b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a-b*sinh(x))/b+2*(b+c)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)+2*b
)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.28

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)-ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)-a)}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)-b}\right)\cdots}{a^2b+b^3}$$

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*c
osh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b*co
sh(x) - a)*sinh(x) - b)) + (a^2 + b^2)*x - (a^2 + b^2)*log(2*(b*sinh(x) - a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.30 (sec) , antiderivative size = 586, normalized size of antiderivative = 11.06

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(c*log(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x/2)
)), Eq(a, 0) & Eq(b, 0)), (-(b*log(tanh(x/2)) + c*log(tanh(x/2)) + x - 2*lo
g(tanh(x/2) + 1) + log(tanh(x/2)))/b, Eq(a, 0)), ((c*x + sinh(x))/a, Eq(b,
0)), (2*I*b/(b*tanh(x/2) + I*b) + 2*I*c/(b*tanh(x/2) + I*b) - x*tanh(x/2)/(
b*tanh(x/2) + I*b) - I*x/(b*tanh(x/2) + I*b) + 2*log(tanh(x/2) + 1)*tanh(x/
2)/(b*tanh(x/2) + I*b) + 2*I*log(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) - 2*log
(tanh(x/2) + I)*tanh(x/2)/(b*tanh(x/2) + I*b) - 2*I*log(tanh(x/2) + I)/(b*t
anh(x/2) + I*b), Eq(a, -I*b)), (-2*I*b/(b*tanh(x/2) - I*b) - 2*I*c/(b*tanh(
x/2) - I*b) - x*tanh(x/2)/(b*tanh(x/2) - I*b) + I*x/(b*tanh(x/2) - I*b) + 2
*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) - I*b) - 2*I*log(tanh(x/2) + 1)/
(b*tanh(x/2) - I*b) - 2*log(tanh(x/2) - I)*tanh(x/2)/(b*tanh(x/2) - I*b) +
2*I*log(tanh(x/2) - I)/(b*tanh(x/2) - I*b), Eq(a, I*b)), (-b*log(tanh(x/2)
+ b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + b*log(tanh(x/2) + b/a + sq
rt(a**2 + b**2)/a)/sqrt(a**2 + b**2) - c*log(tanh(x/2) + b/a - sqrt(a**2 +
b**2)/a)/sqrt(a**2 + b**2) + c*log(tanh(x/2) + b/a + sqrt(a**2 + b**2)/a)/s
qrt(a**2 + b**2) - x/b + 2*log(tanh(x/2) + 1)/b - log(tanh(x/2) + b/a - sqr
t(a**2 + b**2)/a)/b - log(tanh(x/2) + b/a + sqrt(a**2 + b**2)/a)/b, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(47) = 94$.

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx = -\frac{b\log\left(\frac{be^{(-x)}+a-\sqrt{a^2+b^2}}{be^{(-x)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{c\log\left(\frac{be^{(-x)}+a-\sqrt{a^2+b^2}}{be^{(-x)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{\log\left(b\sinh\left(x\right)-a\right)}{b}$$

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*log((b*e^(-x) + a - sqrt(a^2 + b^2))/(b*e^(-x) + a + sqrt(a^2 + b^2)))/s
qrt(a^2 + b^2) - c*log((b*e^(-x) + a - sqrt(a^2 + b^2))/(b*e^(-x) + a + sqr
t(a^2 + b^2)))/sqrt(a^2 + b^2) - log(b*sinh(x) - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx=-\frac{(b+c)\log\left(\frac{\left|2\,be^x-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}+\frac{x}{b}-\frac{\log\left(\left|be^{(2\,x)}-2\,ae^x-b\right|\right)}{b}$$

```
[In] integrate((b+c+cosh(x))/(a-b*sinh(x)),x, algorithm="giac")
```

```
[Out] -(b + c)*log(abs(2*b*e^x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x - 2*a + 2*s
qrt(a^2 + b^2)))/sqrt(a^2 + b^2) + x/b - log(abs(b*e^(2*x) - 2*a*e^x - b))/
b
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.34

$$\begin{aligned}\int \frac{b+c+\cosh(x)}{a-b\sinh(x)}\,dx\\ &=\frac{x}{b}\\ &+\frac{\ln\left(b\sqrt{a^2+b^2}+a^2\,\mathrm{e}^x+b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2+b^2}\right)\left(b^2\sqrt{a^2+b^2}-a^2-b^2+b\,c\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}\\ &-\frac{\ln\left(b\sqrt{a^2+b^2}-a^2\,\mathrm{e}^x-b^2\,\mathrm{e}^x+a\,\mathrm{e}^x\sqrt{a^2+b^2}\right)\left(b^2\sqrt{a^2+b^2}+a^2+b^2+b\,c\sqrt{a^2+b^2}\right)}{a^2\,b+b^3}\end{aligned}$$

```
[In] int((b + c + cosh(x))/(a - b*sinh(x)),x)
```

```
[Out] x/b + (log(b*(a^2 + b^2)^(1/2) + a^2*exp(x) + b^2*exp(x) + a*exp(x)*(a^2 +
b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) - a^2 - b^2 + b*c*(a^2 + b^2)^(1/2)))/(a
^2*b + b^3) - (log(b*(a^2 + b^2)^(1/2) - a^2*exp(x) - b^2*exp(x) + a*exp(x)
*(a^2 + b^2)^(1/2))*(b^2*(a^2 + b^2)^(1/2) + a^2 + b^2 + b*c*(a^2 + b^2)^(1
/2)))/(a^2*b + b^3)
```

3.568 $\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2(b+c)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{\log(a+b\cosh(x))}{b}$$

```
[Out] ln(a+b*cosh(x))/b+2*(b+c)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b
)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4486, 2738, 214, 2747, 31}

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2(b+c)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{\log(a+b\cosh(x))}{b}$$

```
[In] Int[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]) + Log[a + b*Cosh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b+c}{a+b\cosh(x)}+\frac{\sinh(x)}{a+b\cosh(x)}\right)dx\\
&= (b+c)\int \frac{1}{a+b\cosh(x)}\,dx+\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{b}+(2(b+c))\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{\log(a+b\cosh(x))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx=-\frac{2(b+c)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\log(a+b\cosh(x))}{b}$$

```
[In] Integrate[(b + c + Sinh[x])/(a + b*Cosh[x]),x]
[Out] (-2*(b + c)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
+ Log[a + b*Cosh[x]]/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(47) = 94$.

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.82

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{b}-\frac{\ln(\tanh(\frac{x}{2})+1)}{b}+\frac{\ln\left(\tanh(\frac{x}{2})^2a-\tanh(\frac{x}{2})^2b-a-b\right)-\frac{2\left(-b^2-cb\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh(\frac{x}{2})}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}$
risch	$\frac{x}{b}+\frac{2x\,a^2b}{-a^2b^2+b^4}-\frac{2x\,b^3}{-a^2b^2+b^4}+\frac{\ln\left(e^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2-b^6-2b^5c-b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2-b^2)b}-\frac{b\ln\left(e^x-\frac{-a\,b^2-cab+\sqrt{a^2b^4+2\cdots}}{\cdots}\right)}{a^2-\cdots}$

[In] int((b+c+sinh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)

[Out] -1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)+2/b*(1/2*ln(tanh(1/2*x)^2*a-ta
nh(1/2*x)^2*b-a-b)-(-b^2-b*c)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)
/((a+b)*(a-b))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 5.07

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^2-b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a\cdots}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.\cdots}{a^2b-b^3}\right.$$
$$-\frac{2\,\sqrt{-a^2+b^2}(b^2+bc)\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(a^2-b^2)x-(a^2-b^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)\cdots}{a^2b-b^3}$$

[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")

[Out] [(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) + b)) - (a^2 - b^2)*x + (a^2 - b^2)*log(2*(b*cosh(x) +
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*(b^2 + b*c)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 - b^
2)*x - (a^2 - b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3
)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(49) = 98$.

Time = 16.66 (sec) , antiderivative size = 840, normalized size of antiderivative = 14.74

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*c*atan(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x
/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2) + c*tanh(x/2)/b + x/b - 2*lo
g(tanh(x/2) + 1)/b, Eq(a, b)), (-1/tanh(x/2) - c/(b*tanh(x/2)) + x/b - 2*lo
g(tanh(x/2) + 1)/b + 2*log(tanh(x/2))/b, Eq(a, -b)), ((c*x + cosh(x))/a, Eq
(b, 0)), (a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b))
 - b**2*sqrt(a/(a - b) + b/(a - b))) + a*sqrt(a/(a - b) + b/(a - b))*log(-sq
rt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b
**2*sqrt(a/(a - b) + b/(a - b))) + a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a
/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*
sqrt(a/(a - b) + b/(a - b))) - 2*a*sqrt(a/(a - b) + b/(a - b))*log(tanh(x/2
) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b)))
 - b**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
 b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + b**2*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
 - b) + b/(a - b))) - b*c*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b
*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + b*c*log(
sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) -
 b**2*sqrt(a/(a - b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*s
qrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - b*sqrt(a/(
a - b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt
(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - b*sqrt(a/(a -
 b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(
a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*b*sqrt(a/(a - b
) + b/(a - b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx = \frac{2\,(b+c)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}} - \frac{x}{b} + \frac{\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

```
[In] integrate((b+c+sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*(b + c)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - x/b + log
(b*e^(2*x) + 2*a*e^x + b)/b
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.47

$$\int \frac{b+c+\sinh(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{\ln\left(b\sqrt{(a+b)\,(a-b)}+a^{2}\,\mathrm{e}^{x}-b^{2}\,\mathrm{e}^{x}+a\,\mathrm{e}^{x}\sqrt{(a+b)\,(a-b)}\right)\,\left(b^{2}\sqrt{(a+b)\,(a-b)}+a^{2}-b^{2}+b\,c\,\sqrt{(a\ldots}{a^{2}\,b-b^{3}}$$
$$-\frac{x}{b}$$
$$-\frac{\ln\left(b\sqrt{(a+b)\,(a-b)}-a^{2}\,\mathrm{e}^{x}+b^{2}\,\mathrm{e}^{x}+a\,\mathrm{e}^{x}\sqrt{(a+b)\,(a-b)}\right)\,\left(b^{2}\sqrt{(a+b)\,(a-b)}-a^{2}+b^{2}+b\,c\,\sqrt{\ldots}{a^{2}\,b-b^{3}}$$

```
[In] int((b + c + sinh(x))/(a + b*cosh(x)),x)
```

```
[Out] (log(b*((a + b)*(a - b))^(1/2) + a^2*exp(x) - b^2*exp(x) + a*exp(x)*((a + b
)*(a - b))^(1/2))*(b^2*((a + b)*(a - b))^(1/2) + a^2 - b^2 + b*c*((a + b)*(
a - b))^(1/2)))/(a^2*b - b^3) - x/b - (log(b*((a + b)*(a - b))^(1/2) - a^2*
exp(x) + b^2*exp(x) + a*exp(x)*((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a -
b))^(1/2) - a^2 + b^2 + b*c*((a + b)*(a - b))^(1/2)))/(a^2*b - b^3)
```

3.569 $\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx=\frac{2(b+c)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

```
[Out] -ln(a-b*cosh(x))/b+2*(b+c)*arctanh((a+b)^(1/2)*tanh(1/2*x)/(a-b)^(1/2))/(a-
b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4486, 2738, 214, 2747, 31}

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx=\frac{2(b+c)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}$$

```
[In] Int[(b + c + Sinh[x])/(a - b*Cosh[x]),x]
```

```
[Out] (2*(b + c)*ArcTanh[(Sqrt[a + b]*Tanh[x/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[
a + b]) - Log[a - b*Cosh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{-b-c}{-a+b\cosh(x)}+\frac{\sinh(x)}{a-b\cosh(x)}\right) dx \\
&= (-b-c)\int \frac{1}{-a+b\cosh(x)}\,dx+\int \frac{\sinh(x)}{a-b\cosh(x)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,-b\cosh(x)\right)}{b} \\
&\quad -(2(b+c))\text{Subst}\left(\int \frac{1}{-a+b-(-a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{\log(a-b\cosh(x))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = -\frac{2(b+c)\arctan\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{\log(a-b\cosh(x))}{b}$$

[In] Integrate[(b + c + Sinh[x])/(a - b*Cosh[x]),x]

[Out] (-2*(b + c)*ArcTan[((a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - Log[a - b*Cosh[x]]/b

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(49) = 98.

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

method	result
default	$\frac{\frac{2(-a-b)\ln\left(\tanh\left(\frac{x}{2}\right)^2 a+\tanh\left(\frac{x}{2}\right)^2 b-a+b\right)}{2a+2b}-\frac{2\left(-b^2-cb\right)\operatorname{arctanh}\left(\frac{(a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$-\frac{x}{b}-\frac{2x\,a^2 b}{-a^2b^2+b^4}+\frac{2x\,b^3}{-a^2b^2+b^4}-\frac{\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2a^2b^3c+a^2b^2c^2-b^6-2b^5c-b^4c^2}}{b^2(b+c)}\right)a^2}{(a^2-b^2)b}+\frac{b\ln\left(\mathrm{e}^x+\frac{-a\,b^2-cab+\sqrt{a^2b^4+2}}{}\right.}{a^2}$

[In] int((b+c+sinh(x))/(a-b*cosh(x)),x,method=_RETURNVERBOSE)

[Out] 2/b*(1/2*(-a-b)/(a+b)*ln(tanh(1/2*x)^2*a+tanh(1/2*x)^2*b-a+b)-(-b^2-b*c)/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))+1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.07

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \left[\frac{\sqrt{a^2-b^2}(b^2+bc)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2-2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)-ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)-a)}{b\cosh(x)^2+b\sinh(x)^2-2\,a\cosh(x)+2\,(b\cosh(x)-a)\sinh(x)+b}\right)}{a^2b-b^3}\right.$$

[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="fricas")

```
[Out] [(sqrt(a^2 - b^2)*(b^2 + b*c)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 - 2*a*b*co
sh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) - a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) - a))/(b*cosh(x)^2 + b*sinh(x)^2 - 2*a*cosh(x) + 2*(b*c
osh(x) - a)*sinh(x) + b)) + (a^2 - b^2)*x - (a^2 - b^2)*log(2*(b*cosh(x) -
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*(b^2 + b*c)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) - a)/(a^2 - b^2)) + (a^2 - b^2
)*x - (a^2 - b^2)*log(2*(b*cosh(x) - a)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(49) = 98$.

Time = 15.82 (sec) , antiderivative size = 840, normalized size of antiderivative = 14.24

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*c*atan(tanh(x/2)) + x - 2*log(tanh(x/2) + 1) + log(tanh(x
/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-tanh(x/2) - c*tanh(x/2)/b - x/b + 2*l
og(tanh(x/2) + 1)/b, Eq(a, -b)), (1/tanh(x/2) + c/(b*tanh(x/2)) - x/b + 2*l
og(tanh(x/2) + 1)/b - 2*log(tanh(x/2))/b, Eq(a, b)), ((c*x + cosh(x))/a, Eq
(b, 0)), (-a*x*sqrt(a/(a + b) - b/(a + b))/(a*b*sqrt(a/(a + b) - b/(a + b))
 + b**2*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(-s
qrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) +
b**2*sqrt(a/(a + b) - b/(a + b))) - a*sqrt(a/(a + b) - b/(a + b))*log(sqrt(
a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2
*sqrt(a/(a + b) - b/(a + b))) + 2*a*sqrt(a/(a + b) - b/(a + b))*log(tanh(x/
2) + 1)/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))
) - b**2*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b)
- b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + b**2*log(sqrt(a/(a + b)
- b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a
 + b) - b/(a + b))) - b*c*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*
b*sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + b*c*log
(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/(a + b) - b/(a + b))
+ b**2*sqrt(a/(a + b) - b/(a + b))) - b*x*sqrt(a/(a + b) - b/(a + b))/(a*b*
sqrt(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/
(a + b) - b/(a + b))*log(-sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqr
t(a/(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) - b*sqrt(a/(a
+ b) - b/(a + b))*log(sqrt(a/(a + b) - b/(a + b)) + tanh(x/2))/(a*b*sqrt(a/
(a + b) - b/(a + b)) + b**2*sqrt(a/(a + b) - b/(a + b))) + 2*b*sqrt(a/(a +
b) - b/(a + b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a + b) - b/(a + b)) + b**2*
sqrt(a/(a + b) - b/(a + b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = -\frac{2\,(b+c)\arctan\left(\frac{be^{x}-a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}} + \frac{x}{b} - \frac{\log\left(be^{(2\,x)}-2\,ae^{x}+b\right)}{b}$$

```
[In] integrate((b+c+sinh(x))/(a-b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*(b + c)*arctan((b*e^x - a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) + x/b - lo
g(b*e^(2*x) - 2*a*e^x + b)/b
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.37

$$\int \frac{b+c+\sinh(x)}{a-b\cosh(x)}\,dx = \frac{x}{b} - \frac{\ln\left(b\sqrt{(a+b)\,(a-b)}+a^{2}\,e^{x}-b^{2}\,e^{x}-a\,e^{x}\sqrt{(a+b)\,(a-b)}\right)\left(b^{2}\sqrt{(a+b)\,(a-b)}+a^{2}-b^{2}+b\,c\,\sqrt{(}\right.}{a^{2}\,b-b^{3}} + \frac{\ln\left(b\sqrt{(a+b)\,(a-b)}-a^{2}\,e^{x}+b^{2}\,e^{x}-a\,e^{x}\sqrt{(a+b)\,(a-b)}\right)\left(b^{2}\sqrt{(a+b)\,(a-b)}-a^{2}+b^{2}+b\,c\,\sqrt{(}\right.}{a^{2}\,b-b^{3}}$$

```
[In] int((b + c + sinh(x))/(a - b*cosh(x)),x)
```

```
[Out] x/b - (log(b*((a + b)*(a - b))^(1/2) + a^2*exp(x) - b^2*exp(x) - a*exp(x)*(
(a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a - b))^(1/2) + a^2 - b^2 + b*c*((a
+ b)*(a - b))^(1/2)))/(a^2*b - b^3) + (log(b*((a + b)*(a - b))^(1/2) - a^2*
exp(x) + b^2*exp(x) - a*exp(x)*((a + b)*(a - b))^(1/2))*(b^2*((a + b)*(a -
b))^(1/2) - a^2 + b^2 + b*c*((a + b)*(a - b))^(1/2)))/(a^2*b - b^3)
```

3.570 $\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = \frac{\log(a+b\sinh(x))}{b} - \frac{x\cosh(x)}{a+b\sinh(x)}$$

```
[Out] ln(a+b*sinh(x))/b-x*cosh(x)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5755, 2747, 31}

$$\int \frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx = \frac{\log(a+b\sinh(x))}{b} - \frac{x\cosh(x)}{a+b\sinh(x)}$$

```
[In] Int[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 5755

```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sinh[(c_.) + (d_.)*(x_)]))/((a_) +
(b_.)*Sinh[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[B*(e + f*x)*(Cosh[c +
d*x]/(a*d*(a + b*Sinh[c + d*x]))), x] - Dist[B*(f/(a*d)), Int[Cosh[c + d*x]
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ
[a*A + b*B, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\cosh(x)}{a+b\sinh(x)}+\int\frac{\cosh(x)}{a+b\sinh(x)}\,dx\\ &= -\frac{x\cosh(x)}{a+b\sinh(x)}+\frac{\text{Subst}\left(\int\frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int\frac{x(b-a\sinh(x))}{(a+b\sinh(x))^2}\,dx=\frac{\log(a+b\sinh(x))}{b}-\frac{x\cosh(x)}{a+b\sinh(x)}$$

```
[In] Integrate[(x*(b - a*Sinh[x]))/(a + b*Sinh[x])^2,x]
[Out] Log[a + b*Sinh[x]]/b - (x*Cosh[x])/(a + b*Sinh[x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

method	result	size
risch	$-\frac{2x}{b}+\frac{2x(a\,e^{x}-b)}{b(b\,e^{2x}+2a\,e^{x}-b)}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$	58
parallelrisch	$\frac{(a+b\sinh(x))\ln\left(\frac{-2b\sinh(x)-2a}{\cosh(x)+1}\right)+(-2b\sinh(x)-2a)\ln(1-\coth(x)+\operatorname{csch}(x))-x(a+b\cosh(x)+b\sinh(x))}{b(a+b\sinh(x))}$	70

```
[In] int(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*x/b+2*x*(a*exp(x)-b)/b/(b*exp(2*x)+2*a*exp(x)-b)+1/b*ln(exp(2*x)+2*a/b*e
xp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.36

$$\int \frac{x(b - a\sinh(x))}{(a + b\sinh(x))^2}\,dx = -\frac{2\,bx\cosh\left(x\right)^2 + 2\,bx\sinh\left(x\right)^2 + 2\,ax\cosh\left(x\right) - \left(b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 2\,a\cosh\left(x\right) + 2\,(b\cosh\left(x\right)\cdots}{b^2\cosh\left(x\right)^2 + b^2\sinh\left(x\right)^2 + 2\,ab\cosh\left(x\right) - b^2 + 2\,(b^2 c\cdots}$$

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*(b*sinh(x) + a)/
(cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 + b^
2*sinh(x)^2 + 2*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(b - a\sinh(x))}{(a + b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{x(b - a\sinh(x))}{(a + b\sinh(x))^2}\,dx = -\frac{2\,\left(bxe^{(2\,x)} + axe^{x}\right)}{b^2e^{(2\,x)} + 2\,abe^{x} - b^2} + \frac{\log\left(\frac{be^{(2\,x)} + 2\,ae^{x} - b}{b}\right)}{b}$$

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(b*x*e^(2*x) + a*x*e^x)/(b^2*e^(2*x) + 2*a*b*e^x - b^2) + log((b*e^(2*x)
 + 2*a*e^x - b)/b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.84

$$\int \frac{x(b - a\sinh(x))}{(a + b\sinh(x))^2}\,dx = -\frac{2\,bxe^{(2\,x)} - be^{(2\,x)}\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right) - 2\,ae^{x}\log\left(-be^{(2\,x)} - 2\,ae^{x} + b\right) + 2\,bx + b\log\left(-be^{(2\,x)} - 2\,a\right.}{b^2e^{(2\,x)} + 2\,abe^{x} - b^2}$$

```
[In] integrate(x*(b-a*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x + b) - 2*a*e^x*log(-b*
e^(2*x) - 2*a*e^x + b) + 2*b*x + b*log(-b*e^(2*x) - 2*a*e^x + b))/(b^2*e^(2
*x) + 2*a*b*e^x - b^2)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{x(b - a\sinh(x))}{(a + b\sinh(x))^2}\,dx = \frac{\ln\left(2\,a\,\mathrm{e}^{x} - b + b\,\mathrm{e}^{2\,x}\right)}{b} - \frac{\frac{2\,\left(x\,a^2\,b + x\,b^3\right)}{a^2\,b + b^3} - \frac{2\,\mathrm{e}^{x}\,\left(x\,a^3\,b + x\,a\,b^3\right)}{b\,\left(a^2\,b + b^3\right)}}{2\,a\,\mathrm{e}^{x} - b + b\,\mathrm{e}^{2\,x}} - \frac{2\,x}{b}$$

```
[In] int((x*(b - a*sinh(x)))/(a + b*sinh(x))^2,x)
```

```
[Out] log(2*a*exp(x) - b + b*exp(2*x))/b - ((2*(b^3*x + a^2*b*x))/(a^2*b + b^3) -
 (2*exp(x)*(a*b^3*x + a^3*b*x))/(b*(a^2*b + b^3)))/(2*a*exp(x) - b + b*exp(
2*x)) - (2*x)/b
```

3.571 $\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = -\frac{\log(a+b\cosh(x))}{b}+\frac{x\sinh(x)}{a+b\cosh(x)}$$

```
[Out] -ln(a+b*cosh(x))/b+x*sinh(x)/(a+b*cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5756, 2747, 31}

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \frac{x\sinh(x)}{a+b\cosh(x)}-\frac{\log(a+b\cosh(x))}{b}$$

```
[In] Int[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 5756

```
Int[((Cosh[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cosh[(c
_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[B*(e + f*x)*(Sinh[c +
d*x]/(a*d*(a + b*Cosh[c + d*x]))), x] - Dist[B*(f/(a*d)), Int[Sinh[c + d*x]
/(a + b*Cosh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ
[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sinh(x)}{a+b\cosh(x)} - \int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\ &= \frac{x\sinh(x)}{a+b\cosh(x)} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}\\ &= -\frac{\log(a+b\cosh(x))}{b} + \frac{x\sinh(x)}{a+b\cosh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = -\frac{\log(a+b\cosh(x))}{b} + \frac{x\sinh(x)}{a+b\cosh(x)}$$

```
[In] Integrate[(x*(b + a*Cosh[x]))/(a + b*Cosh[x])^2,x]
[Out] -(Log[a + b*Cosh[x]]/b) + (x*Sinh[x])/(a + b*Cosh[x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

method	result	size
risch	$\frac{2x}{b} - \frac{2x(a\,e^{x}+b)}{b(b\,e^{2x}+2a\,e^{x}+b)} - \frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$	55
parallelrisch	$\frac{(-b\cosh(x)-a)\ln\left(\frac{-2b\cosh(x)-2a}{\cosh(x)+1}\right)+(2b\cosh(x)+2a)\ln(1-\coth(x)+\operatorname{csch}(x))+x(a+b\cosh(x)+b\sinh(x))}{b(a+b\cosh(x))}$	72

```
[In] int(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*x/b-2*x*(a*exp(x)+b)/b/(b*exp(2*x)+2*a*exp(x)+b)-1/b*ln(exp(2*x)+2*a/b*ex
p(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.16

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \frac{2\,bx\cosh\left(x\right)^2+2\,bx\sinh\left(x\right)^2+2\,ax\cosh\left(x\right)-\left(b\cosh\left(x\right)^2+b\sinh\left(x\right)^2+2\,a\cosh\left(x\right)+2\left(b\cosh\left(x\right)+a\right.\right.}{b^2\cosh\left(x\right)^2+b^2\sinh\left(x\right)^2+2\,ab\cosh\left(x\right)+b^2+2\left(b^2\cosh\right.}$$

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh(x)^2 + 2*b*x*sinh(x)^2 + 2*a*x*cosh(x) - (b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*log(2*(b*cosh(x) + a)/(
cosh(x) - sinh(x))) + 2*(2*b*x*cosh(x) + a*x)*sinh(x))/(b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(b+a\cosh(x))}{(a+b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int \frac{x(b + a\cosh(x))}{(a + b\cosh(x))^2}\,dx = \frac{2\,bxe^{(2\,x)} - be^{(2\,x)}\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right) - 2\,ae^{x}\log\left(-be^{(2\,x)} - 2\,ae^{x} - b\right) - 2\,bx - b\log\left(-be^{(2\,x)} - 2\,ae^{x}\right.}{b^2e^{(2\,x)} + 2\,abe^{x} + b^2}$$

```
[In] integrate(x*(b+a*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] (2*b*x*e^(2*x) - b*e^(2*x)*log(-b*e^(2*x) - 2*a*e^x - b) - 2*a*e^x*log(-b*e
^(2*x) - 2*a*e^x - b) - 2*b*x - b*log(-b*e^(2*x) - 2*a*e^x - b))/(b^2*e^(2*
x) + 2*a*b*e^x + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.20

$$\int \frac{x(b + a\cosh(x))}{(a + b\cosh(x))^2}\,dx = \frac{2\,x}{b} + \frac{\frac{2\,\left(b^3\,x - a^2\,b\,x\right)}{a^2\,b - b^3} + \frac{2\,\mathrm{e}^{x}\,\left(a\,b^3\,x - a^3\,b\,x\right)}{b\,\left(a^2\,b - b^3\right)}}{b + 2\,a\,\mathrm{e}^{x} + b\,\mathrm{e}^{2\,x}} - \frac{\ln\left(b + 2\,a\,\mathrm{e}^{x} + b\,\mathrm{e}^{2\,x}\right)}{b}$$

```
[In] int((x*(b + a*cosh(x)))/(a + b*cosh(x))^2,x)
```

```
[Out] (2*x)/b + ((2*(b^3*x - a^2*b*x))/(a^2*b - b^3) + (2*exp(x)*(a*b^3*x - a^3*b
*x))/(b*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) - log(b + 2*a*exp(x)
+ b*exp(2*x))/b
```

3.572 $\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)}\,dx=\frac{b\arctan(\sinh(x))}{c}+\frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}$$

```
[Out] b*arctan(sinh(x))/c+2*(a*c-b*d)*arctanh((c-d)^(1/2)*tanh(1/2*x)/(c+d)^(1/2)
)/c/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2907, 3080, 3855, 2738, 214}

$$\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)}\,dx=\frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}+\frac{b\arctan(\sinh(x))}{c}$$

```
[In] Int[(a + b*Sech[x])/(c + d*Cosh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/c + (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tanh[x/2])/Sqrt
[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(b+a\cosh(x))\text{sech}(x)}{c+d\cosh(x)}\,dx \\
&= \frac{b\int \text{sech}(x)\,dx}{c} + \frac{(ac-bd)\int \frac{1}{c+d\cosh(x)}\,dx}{c} \\
&= \frac{b\arctan(\sinh(x))}{c} + \frac{(2(ac-bd))\text{Subst}\left(\int \frac{1}{c+d-(c-d)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c} \\
&= \frac{b\arctan(\sinh(x))}{c} + \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx=\frac{2\left(b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{(-ac+bd)\arctan\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}\right)}{c}$$

```
[In] Integrate[(a + b*Sech[x])/(c + d*Cosh[x]),x]
[Out] (2*(b*ArcTan[Tanh[x/2]] + ((-(a*c) + b*d)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-
c^2 + d^2]])/Sqrt[-c^2 + d^2]))/c
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result
default	$\frac{2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{2(-ac+bd)\operatorname{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}$
risch	$\frac{ib\ln(\mathrm{e}^{x}+i)}{c}-\frac{ib\ln(\mathrm{e}^{x}-i)}{c}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^2-d^2}\,c-c^2+d^2}{\sqrt{c^2-d^2}\,d}\right)a}{\sqrt{c^2-d^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^2-d^2}\,c-c^2+d^2}{\sqrt{c^2-d^2}\,d}\right)bd}{\sqrt{c^2-d^2}\,c}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{c^2-d^2}\,c+c^2-d^2}{\sqrt{c^2-d^2}\,d}\right)a}{\sqrt{c^2-d^2}}+$

```
[In] int((a+b*sech(x))/(c+d*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 2*b/c*arctan(tanh(1/2*x))-2*(-a*c+b*d)/c/((c+d)*(c-d))^(1/2)*arctanh((c-d)*
tanh(1/2*x)/((c+d)*(c-d))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.02

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx$$

$$=\left[-\frac{(ac-bd)\sqrt{c^2-d^2}\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2-d^2+2\,\left(d^2\cosh(x)+cd\right)\sinh(x)+2\,\sqrt{c^2-d^2}(d\cosh(x)+d\sinh(x}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\,(d\cosh(x)+c)\sinh(x)+d}\right.}{c^3-cd^2}\right.$$

$$-\frac{2\left((ac-bd)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cosh(x)+d\sinh(x)+c)}{c^2-d^2}\right)-(bc^2-bd^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right.\right.}{c^3-cd^2}$$

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((a*c - b*d)*sqrt(c^2 - d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*c
osh(x) + 2*c^2 - d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 - d^2)*(d
*cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*
cosh(x) + c)*sinh(x) + d)) - 2*(b*c^2 - b*d^2)*arctan(cosh(x) + sinh(x)))/(
c^3 - c*d^2), -2*((a*c - b*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*
cosh(x) + d*sinh(x) + c)/(c^2 - d^2)) - (b*c^2 - b*d^2)*arctan(cosh(x) + si
nh(x)))/(c^3 - c*d^2)]
```

Sympy [F]

$$\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)}\,dx=\int \frac{a+b\,\text{sech}\,(x)}{c+d\cosh\,(x)}\,dx$$

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x)
```

```
[Out] Integral((a + b*sech(x))/(c + d*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{a+b\text{sech}(x)}{c+d\cosh(x)}\,dx=\frac{2\,b\arctan\left(e^{x}\right)}{c}+\frac{2\,(ac-bd)\arctan\left(\frac{de^{x}+c}{\sqrt{-c^{2}+d^{2}}}\right)}{\sqrt{-c^{2}+d^{2}}c}$$

```
[In] integrate((a+b*sech(x))/(c+d*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^x)/c + 2*(a*c - b*d)*arctan((d*e^x + c)/sqrt(-c^2 + d^2))/(sqr
t(-c^2 + d^2)*c)
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 636, normalized size of antiderivative = 10.26

$$\int \frac{a+b\operatorname{sech}(x)}{c+d\cosh(x)}\,dx = \frac{\ln\left(\frac{\sqrt{(c+d)\,(c-d)}\,(a\,c-b\,d)\left(\frac{32\,\left(a^2\,c^2\,d-2\,a\,b\,c\,d^2-4\,e^x\,b^2\,c^3-2\,b^2\,c^2\,d+3\,e^x\,b^2\,c\,d^2+2\,b^2\,d^3\right)}{d^5}+\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\,c^2\,\left(2\,b\,d^2-4\,a\,c^2\,e^x+a\,d^2\,e^x-2\,a\,c\,d+\dots\right)}{d^5}\dots\right.}{\dots}\right.}{c\,d^2-c^3}\dots\right)}{\dots} - \frac{\ln\left(-\frac{32\,b\,(a\,c-b\,d)\,(2\,b\,d-a\,d\,e^x+4\,b\,c\,e^x)}{d^5}-\frac{\sqrt{(c+d)\,(c-d)}\,(a\,c-b\,d)\left(\frac{32\,\left(a^2\,c^2\,d-2\,a\,b\,c\,d^2-4\,e^x\,b^2\,c^3-2\,b^2\,c^2\,d+3\,e^x\,b^2\,c\,d^2+2\,b^2\,d^3\right)}{d^5}-\frac{\sqrt{(c+\dots}}{\dots}\right.}{\dots}\right)}{\dots} - \frac{b\,\ln\left(e^x-i\right)\,1i}{c}+\frac{b\,\ln\left(e^x+1i\right)\,1i}{c}$$

```
[In] int((a + b/cosh(x))/(c + d*cosh(x)),x)
```

```
[Out] (b*log(exp(x) + 1i)*1i)/c - (b*log(exp(x) - 1i)*1i)/c + (log((((c + d)*(c -
 d))^(1/2)*(a*c - b*d)*((32*(2*b^2*d^3 + a^2*c^2*d - 2*b^2*c^2*d - 4*b^2*c^
3*exp(x) + 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2))/d^5 + (((c + d)*(c - d))^(1/2
)*((32*c^2*(2*b*d^2 - 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*a*c*d + 3*b*c*d*exp
(x)))/d^5 - (32*c^2*((c + d)*(c - d))^(1/2)*(a*c - b*d)*(3*c^2*d - 2*d^3 +
4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c*d^2 - c^3)))*(a*c - b*d))/(c*d^2 -
c^3)))/(c*d^2 - c^3) - (32*b*(a*c - b*d)*(2*b*d - a*d*exp(x) + 4*b*c*exp(x)
))/d^5)*((c + d)*(c - d))^(1/2)*(a*c - b*d))/(c*d^2 - c^3) - (log(- (32*b*(
a*c - b*d)*(2*b*d - a*d*exp(x) + 4*b*c*exp(x)))/d^5 - (((c + d)*(c - d))^(1
/2)*(a*c - b*d)*((32*(2*b^2*d^3 + a^2*c^2*d - 2*b^2*c^2*d - 4*b^2*c^3*exp(x
) + 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2))/d^5 - (((c + d)*(c - d))^(1/2)*((32*
c^2*(2*b*d^2 - 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*a*c*d + 3*b*c*d*exp(x)))/d
^5 + (32*c^2*((c + d)*(c - d))^(1/2)*(a*c - b*d)*(3*c^2*d - 2*d^3 + 4*c^3*e
xp(x) - 3*c*d^2*exp(x)))/(d^5*(c*d^2 - c^3)))*(a*c - b*d))/(c*d^2 - c^3)))/
(c*d^2 - c^3))*((c + d)*(c - d))^(1/2)*(a*c - b*d))/(c*d^2 - c^3)
```

3.573 $\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=-\frac{b\text{arctanh}(\cosh(x))}{c}-\frac{2(ac-bd)\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}$$

```
[Out] -b*arctanh(cosh(x))/c-2*(a*c-b*d)*arctanh((d-c*tanh(1/2*x))/(c^2+d^2)^(1/2)
)/c/(c^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2907, 3080, 3855, 2739, 632, 212}

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=-\frac{2(ac-bd)\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}-\frac{b\text{arctanh}(\cosh(x))}{c}$$

```
[In] Int[(a + b*Csch[x])/(c + d*Sinh[x]),x]
```

```
[Out] -((b*ArcTanh[Cosh[x]])/c) - (2*(a*c - b*d)*ArcTanh[(d - c*Tanh[x/2])/Sqrt[c
^2 + d^2]])/(c*Sqrt[c^2 + d^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \frac{\operatorname{csch}(x)(ib+ia\sinh(x))}{c+d\sinh(x)}\,dx\right)\\
&= \frac{b\int \operatorname{csch}(x)\,dx}{c}+\frac{(ac-bd)\int \frac{1}{c+d\sinh(x)}\,dx}{c}\\
&= -\frac{b\operatorname{arctanh}(\cosh(x))}{c}+\frac{(2(ac-bd))\operatorname{Subst}\left(\int \frac{1}{c+2dx-cx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= -\frac{b\operatorname{arctanh}(\cosh(x))}{c}-\frac{(4(ac-bd))\operatorname{Subst}\left(\int \frac{1}{4(c^2+d^2)-x^2}\,dx,x,2d-2c\tanh\left(\frac{x}{2}\right)\right)}{c}\\
&= -\frac{b\operatorname{arctanh}(\cosh(x))}{c}-\frac{2(ac-bd)\operatorname{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx = \frac{\frac{2(ac-bd)\arctan\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)}{\sqrt{-c^2-d^2}}+b\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{c}$$

```
[In] Integrate[(a + b*Csch[x])/(c + d*Sinh[x]),x]
[Out] ((2*(a*c - b*d)*ArcTan[(d - c*Tanh[x/2])/Sqrt[-c^2 - d^2]])/Sqrt[-c^2 - d^2
] + b*(-Log[Cosh[x/2]] + Log[Sinh[x/2]]))/c
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

method	result
default	$\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{(-2ac+2bd)\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}$
parts	$\frac{2a\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c}-\frac{2bd\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^2+d^2}}\right)}{c\sqrt{c^2+d^2}}$
risch	$\frac{b\ln(e^x-1)}{c}-\frac{b\ln(e^x+1)}{c}+\frac{\ln\left(e^x+\frac{\sqrt{c^2+d^2}\,c-c^2-d^2}{\sqrt{c^2+d^2}\,d}\right)a}{\sqrt{c^2+d^2}}-\frac{\ln\left(e^x+\frac{\sqrt{c^2+d^2}\,c-c^2-d^2}{\sqrt{c^2+d^2}\,d}\right)bd}{\sqrt{c^2+d^2}\,c}-\frac{\ln\left(e^x+\frac{\sqrt{c^2+d^2}\,c+c^2+d^2}{\sqrt{c^2+d^2}\,d}\right)a}{\sqrt{c^2+d^2}}+\ln\left(e\right.$

```
[In] int((a+b*csch(x))/(c+d*sinh(x)),x,method=_RETURNVERBOSE)
[Out] b/c*ln(tanh(1/2*x))-(-2*a*c+2*b*d)/c/(c^2+d^2)^(1/2)*arctanh(1/2*(2*c*tanh(
1/2*x)-2*d)/(c^2+d^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(54) = 108.

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{a+b\operatorname{csch}(x)}{c+d\sinh(x)}\,dx = -\frac{(ac-bd)\sqrt{c^2+d^2}\log\left(\frac{d^2\cosh(x)^2+d^2\sinh(x)^2+2\,cd\cosh(x)+2\,c^2+d^2+2\left(d^2\cosh(x)+cd\right)\sinh(x)+2\sqrt{c^2+d^2}\left(d\cosh(x)+d\sinh(x)+\right.}{d\cosh(x)^2+d\sinh(x)^2+2\,c\cosh(x)+2\left(d\cosh(x)+c\right)\sinh(x)-d}\right.}{c^3+cd^2}$$

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a*c - b*d)*sqrt(c^2 + d^2)*log((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*co
sh(x) + 2*c^2 + d^2 + 2*(d^2*cosh(x) + c*d)*sinh(x) + 2*sqrt(c^2 + d^2)*(d*
cosh(x) + d*sinh(x) + c))/(d*cosh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*c
osh(x) + c)*sinh(x) - d)) + (b*c^2 + b*d^2)*log(cosh(x) + sinh(x) + 1) - (b
*c^2 + b*d^2)*log(cosh(x) + sinh(x) - 1))/(c^3 + c*d^2)
```

Sympy [F]

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=\int \frac{a+b\,\text{csch}\,(x)}{c+d\sinh\,(x)}\,dx$$

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x)
```

```
[Out] Integral((a + b*csch(x))/(c + d*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=-b\left(\frac{d\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}c}+\frac{\log\left(e^{(-x)}+1\right)}{c}-\frac{\log\left(e^{(-x)}-1\right)}{c}\right)+\frac{a\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}$$

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*(d*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2))
)/(sqrt(c^2 + d^2)*c) + log(e^(-x) + 1)/c - log(e^(-x) - 1)/c) + a*log((d*e
^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2)))/sqrt(c^2 + d
^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{a+b\text{csch}(x)}{c+d\sinh(x)}\,dx=-\frac{b\log\left(e^x+1\right)}{c}+\frac{b\log\left(\left|e^x-1\right|\right)}{c}+\frac{(ac-bd)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}c}$$

```
[In] integrate((a+b*csch(x))/(c+d*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(e^x + 1)/c + b*log(abs(e^x - 1))/c + (a*c - b*d)*log(abs(2*d*e^x + 2
*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/(sqrt(c^2 +
 d^2)*c)
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 539, normalized size of antiderivative = 9.29

$$\int \frac{a+b\mathrm{csch}(x)}{c+d\sinh(x)}\,dx = \frac{b\,\ln\left(\mathrm{e}^x-1\right)}{c} - \frac{b\,\ln\left(\mathrm{e}^x+1\right)}{c}$$

$$-\frac{\ln\left(\frac{\left(\frac{32\left(a^2c^2d-2abcd^2-4\,\mathrm{e}^x b^2c^3+2b^2c^2d-3\,\mathrm{e}^x b^2cd^2+2b^2d^3\right)}{d^5}-\frac{(ac-bd)\left(\frac{32c^2\left(2bd^2+4ac^2\mathrm{e}^x+ad^2\mathrm{e}^x-2acd-3bcd\mathrm{e}^x\right)}{d^5}+\frac{32c(ac-bd)\left(-4\mathrm{e}\right.}{d}\right.}{c\sqrt{c^2+d^2}}}{c\sqrt{c^2+d^2}}\right.}{c^3+c\,d^2}$$

$$+\frac{\ln\left(\frac{32b(ac-bd)(ad\mathrm{e}^x-2bd+4bc\mathrm{e}^x)}{d^5}-\frac{\left(\frac{32\left(a^2c^2d-2abcd^2-4\,\mathrm{e}^x b^2c^3+2b^2c^2d-3\,\mathrm{e}^x b^2cd^2+2b^2d^3\right)}{d^5}+\frac{(ac-bd)\left(\frac{32c^2\left(2bd^2+4ac^2\mathrm{e}^x+ad\right.}{d^5}\right.}{}\right.}{c\sqrt{c^2+d^2}}\right.}{c^3+c\,d^2}$$

```
[In] int((a + b/sinh(x))/(c + d*sinh(x)),x)
```

```
[Out] (b*log(exp(x) - 1))/c - (b*log(exp(x) + 1))/c - (log((((32*(2*b^2*d^3 + a^2
*c^2*d + 2*b^2*c^2*d - 4*b^2*c^3*exp(x) - 3*b^2*c*d^2*exp(x) - 2*a*b*c*d^2)
)/d^5 - ((a*c - b*d)*((32*c^2*(2*b*d^2 + 4*a*c^2*exp(x) + a*d^2*exp(x) - 2*
a*c*d - 3*b*c*d*exp(x)))/d^5 + (32*c*(a*c - b*d)*(3*c^2*d + 2*d^3 - 4*c^3*e
xp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c*(c^2 + d^2)^(1/2)))*(
a*c - b*d))/(c*(c^2 + d^2)^(1/2)) + (32*b*(a*c - b*d)*(a*d*exp(x) - 2*b*d +
 4*b*c*exp(x)))/d^5)*(a*c - b*d)*(c^2 + d^2)^(1/2))/(c*d^2 + c^3) + (log((3
2*b*(a*c - b*d)*(a*d*exp(x) - 2*b*d + 4*b*c*exp(x)))/d^5 - (((32*(2*b^2*d^3
 + a^2*c^2*d + 2*b^2*c^2*d - 4*b^2*c^3*exp(x) - 3*b^2*c*d^2*exp(x) - 2*a*b*
c*d^2))/d^5 + ((a*c - b*d)*((32*c^2*(2*b*d^2 + 4*a*c^2*exp(x) + a*d^2*exp(x
) - 2*a*c*d - 3*b*c*d*exp(x)))/d^5 - (32*c*(a*c - b*d)*(3*c^2*d + 2*d^3 - 4
*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c*(c^2 + d^2)^(1/
2)))*(a*c - b*d))/(c*(c^2 + d^2)^(1/2)))*(a*c - b*d)*(c^2 + d^2)^(1/2))/(c*
d^2 + c^3)
```

3.574 $\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)} \, dx = -x+\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)$$

```
[Out] -x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3250, 3260, 212}

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)} \, dx = \sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)-x$$

```
[In] Int[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
```

```
b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x + 2\int \frac{1}{1-\sinh^2(x)}\,dx \\ &= -x + 2\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) \\ &= -x + \sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -2\left(\frac{x}{2} - \frac{\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\right)$$

```
[In] Integrate[(1 + Sinh[x]^2)/(1 - Sinh[x]^2),x]
[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(15) = 30.

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3+2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{2}$
default	$\sqrt{2}\,\text{arctanh}\left(\frac{(2\tanh(\frac{x}{2})-2)\sqrt{2}}{4}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\,\text{arctanh}\left(\frac{(2\tanh(\frac{x}{2})+2)\sqrt{2}}{4}\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right.$

```
[In] int((1+sinh(x)^2)/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] -x+1/2*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)-3-2*2^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^2-4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^2-2\sqrt{2}+3}{\cosh\left(x\right)^2+\sinh\left(x\right)^2-3}\right) - x$$

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(15) = 30$.

Time = 3.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 12.53

$$\begin{aligned}\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = &-\frac{1331714x}{941664\sqrt{2}+1331714}-\frac{941664\sqrt{2}x}{941664\sqrt{2}+1331714}\\&+\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}\\&+\frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}\\&+\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}\\&+\frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}\\&-\frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714}\\&-\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714}\\&-\frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714}\\&-\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714}\end{aligned}$$

```
[In] integrate((1+sinh(x)**2)/(1-sinh(x)**2),x)
```

```
[Out] -1331714*x/(941664*sqrt(2) + 1331714) - 941664*sqrt(2)*x/(941664*sqrt(2) +
1331714) + 941664*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) +
 665857*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 9
41664*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 665857*sqrt
(2)*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) - 665857*sqrt(2
)*log(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh
(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 665857*sqrt(2)*log(tanh(x
/2) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh(x/2) - sqrt
(2) + 1)/(941664*sqrt(2) + 1331714)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = \frac{1}{2}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = -\frac{1}{2}\,\sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right)-x$$

```
[In] integrate((1+sinh(x)^2)/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1+\sinh^2(x)}{1-\sinh^2(x)}\,dx = \frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}+\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{2}\right)}{2} - \frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}-\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{2}\right)}{2} - x$$

```
[In] int(-(sinh(x)^2 + 1)/(sinh(x)^2 - 1),x)
[Out] (2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log(
8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

3.575 $\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)} dx$

Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)} dx = -x + 2\tanh(x)$$

```
[Out] -x+2*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3250, 3254, 3852, 8}

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)} dx = 2\tanh(x) - x$$

```
[In] Int[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
```

```
[Out] -x + 2*Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x + 2\int \frac{1}{1+\sinh^2(x)}\,dx\\ &= -x + 2\int \operatorname{sech}^2(x)\,dx\\ &= -x + 2i\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)\\ &= -x + 2\tanh(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -\frac{x}{2} - \frac{1}{2}\text{arctanh}(\tanh(x)) + 2\tanh(x)$$

```
[In] Integrate[(1 - Sinh[x]^2)/(1 + Sinh[x]^2),x]
[Out] -1/2*x - ArcTanh[Tanh[x]]/2 + 2*Tanh[x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x+2\tanh\left(x\right)$	9
risch	$-x-\frac{4}{1+{\mathrm e}^{2 x}}$	15
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	34

```
[In] int((1-sinh(x)^2)/(1+sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] -x+2*tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -\frac{(x+2)\cosh(x) - 2\sinh(x)}{\cosh(x)}$$

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="fricas")
[Out] -((x + 2)*cosh(x) - 2*sinh(x))/cosh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(5) = 10$.

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 5.12

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -\frac{x\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} - \frac{x}{\tanh^2\left(\frac{x}{2}\right)+1} + \frac{4\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

```
[In] integrate((1-sinh(x)**2)/(1+sinh(x)**2),x)
[Out] -x*tanh(x/2)**2/(tanh(x/2)**2 + 1) - x/(tanh(x/2)**2 + 1) + 4*tanh(x/2)/(ta
nh(x/2)**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x + \frac{4}{e^{(-2x)}+1}$$

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="maxima")
[Out] -x + 4/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x - \frac{4}{e^{(2\,x)}+1}$$

```
[In] integrate((1-sinh(x)^2)/(1+sinh(x)^2),x, algorithm="giac")
[Out] -x - 4/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1-\sinh^2(x)}{1+\sinh^2(x)}\,dx = -x - \frac{4}{e^{2\,x}+1}$$

```
[In] int(-(sinh(x)^2 - 1)/(sinh(x)^2 + 1),x)
[Out] - x - 4/(exp(2*x) + 1)
```

3.576 $\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -x+2\coth(x)$$

```
[Out] -x+2*coth(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3250, 3254, 3852, 8}

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = 2\coth(x)-x$$

```
[In] Int[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
```

```
[Out] -x + 2*Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x + 2\int \frac{1}{1-\cosh^2(x)}\,dx\\ &= -x - 2\int \operatorname{csch}^2(x)\,dx\\ &= -x + 2i\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\\ &= -x + 2\coth(x)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = \coth(x) + \coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \tanh^2(x)\right)$$

```
[In] Integrate[(1 + Cosh[x]^2)/(1 - Cosh[x]^2),x]
[Out] Coth[x] + Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x+2\coth\left(x\right)$	9
risch	$-x+\frac{4}{{\mathrm e}^{2x}-1}$	15
default	$\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)}$	28

[In] int((1+cosh(x)^2)/(1-cosh(x)^2),x,method=_RETURNVERBOSE)

[Out] -x+2*coth(x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -\frac{(x+2)\sinh(x) - 2\cosh(x)}{\sinh(x)}$$

[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="fricas")

[Out] -((x + 2)*sinh(x) - 2*cosh(x))/sinh(x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -x + \tanh\left(\frac{x}{2}\right) + \frac{1}{\tanh\left(\frac{x}{2}\right)}$$

[In] integrate((1+cosh(x)**2)/(1-cosh(x)**2),x)

[Out] -x + tanh(x/2) + 1/tanh(x/2)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -x - \frac{4}{e^{(-2x)} - 1}$$

[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="maxima")

[Out] -x - 4/(e^(-2*x) - 1)

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = -x + \frac{4}{e^{(2\,x)} - 1}$$

```
[In] integrate((1+cosh(x)^2)/(1-cosh(x)^2),x, algorithm="giac")
[Out] -x + 4/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cosh^2(x)}{1-\cosh^2(x)}\,dx = \frac{4}{e^{2\,x} - 1} - x$$

```
[In] int(-(cosh(x)^2 + 1)/(cosh(x)^2 - 1),x)
[Out] 4/(exp(2*x) - 1) - x
```

3.577 $\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -x+\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

```
[Out] -x+arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3250, 3260, 212}

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)-x$$

```
[In] Int[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
```

```
b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -x+2\int \frac{1}{1+\cosh^2(x)}\,dx\\ &= -x+2\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \coth(x)\right)\\ &= -x+\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -2\left(\frac{x}{2}-\frac{\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

```
[In] Integrate[(1 - Cosh[x]^2)/(1 + Cosh[x]^2),x]
[Out] -2*(x/2 - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3-2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{2}$
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\mathrm{t}\right.\right.}{4}$

```
[In] int((1-cosh(x)^2)/(1+cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/2*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx =\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^2-4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^2+2\sqrt{2}-3}{\cosh\left(x\right)^2+\sinh\left(x\right)^2+3}\right) - x$$

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^
2 + 3)) - x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(17) = 34.

Time = 1.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = -x - \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{2} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{2}$$

```
[In] integrate((1-cosh(x)**2)/(1+cosh(x)**2),x)
```

```
[Out] -x - sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/2 + sqrt(2)*log(
4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.37

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{3}{16}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right) - \frac{5}{16}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(-2\,x)}-3}{2\,\sqrt{2}+e^{(-2\,x)}+3}\right) - 2\,x + \frac{1}{4}\,\log\left(e^{(4\,x)}+6\,e^{(2\,x)}+1\right) - \frac{1}{4}\,\log\left(6\,e^{(-2\,x)}+e^{(-4\,x)}+1\right)$$

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - 5/
16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) - 2*
x + 1/4*log(e^(4*x) + 6*e^(2*x) + 1) - 1/4*log(6*e^(-2*x) + e^(-4*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{1}{2}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right) - x$$

```
[In] integrate((1-cosh(x)^2)/(1+cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) - x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1-\cosh^2(x)}{1+\cosh^2(x)}\,dx = \frac{\sqrt{2}\,\ln\left(-8\,\mathrm{e}^{2\,x}-\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}+4\right)}{2}\right)}{2} - x - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}+4\right)}{2}-8\,\mathrm{e}^{2\,x}\right)}{2}$$

```
[In] int(-(cosh(x)^2 - 1)/(cosh(x)^2 + 1),x)
```

```
[Out] (2^(1/2)*log(- 8*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/2))/2 - x - (2^(1/2
)*log((2^(1/2)*(12*exp(2*x) + 4))/2 - 8*exp(2*x)))/2
```

3.578 $\int \frac{a+b\text{sech}^2(x)}{c+d\cosh(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{a+b\text{sech}^2(x)}{c+d\cosh(x)}\,dx=-\frac{bd\arctan(\sinh(x))}{c^2}+\frac{2(ac^2+bd^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}+\frac{b\tanh(x)}{c}$$

```
[Out] -b*d*arctan(sinh(x))/c^2+2*(a*c^2+b*d^2)*arctanh((c-d)^(1/2)*tanh(1/2*x)/(c
+d)^(1/2))/c^2/(c-d)^(1/2)/(c+d)^(1/2)+b*tanh(x)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4319, 3135, 3080, 3855, 2738, 214}

$$\int \frac{a+b\text{sech}^2(x)}{c+d\cosh(x)}\,dx=\frac{2(ac^2+bd^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}}-\frac{bd\arctan(\sinh(x))}{c^2}+\frac{b\tanh(x)}{c}$$

```
[In] Int[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

```
[Out] -((b*d*ArcTan[Sinh[x]])/c^2) + (2*(a*c^2 + b*d^2)*ArcTanh[(Sqrt[c - d]*Tanh
[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c - d]*Sqrt[c + d]) + (b*Tanh[x])/c
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4319

```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Cos[a + b*x]^2)/Cos[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \int \frac{(b + a\cosh^2(x))\,\text{sech}^2(x)}{c + d\cosh(x)}\,dx$$

$$= \frac{b\tanh(x)}{c} + \frac{\int \frac{(-bd+ac\cosh(x))\text{sech}(x)}{c+d\cosh(x)}\,dx}{c}$$

$$= \frac{b\tanh(x)}{c} - \frac{(bd)\int \operatorname{sech}(x)\,dx}{c^2} + \left(a + \frac{bd^2}{c^2}\right)\int \frac{1}{c + d\cosh(x)}\,dx$$

$$= -\frac{bd\arctan(\sinh(x))}{c^2} + \frac{b\tanh(x)}{c} + \left(2\left(a + \frac{bd^2}{c^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{c + d - (c-d)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= -\frac{bd\arctan(\sinh(x))}{c^2} + \frac{2\left(a + \frac{bd^2}{c^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tanh\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}} + \frac{b\tanh(x)}{c}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{a + b\operatorname{sech}^2(x)}{c + d\cosh(x)}\,dx = -\frac{2\left(b + a\cosh^2(x)\right)\operatorname{sech}(x)\left(2\left(bd\sqrt{-c^2+d^2}\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \left(ac^2 + bd^2\right)\arctan\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)\right)\cosh(\right.}{c^2\sqrt{-c^2+d^2}(a + 2b + a\cosh(2x))}$$

```
[In] Integrate[(a + b*Sech[x]^2)/(c + d*Cosh[x]),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*Sech[x]*(2*(b*d*Sqrt[-c^2 + d^2]*ArcTan[Tanh[x/2]] +
(a*c^2 + b*d^2)*ArcTan[((c - d)*Tanh[x/2])/Sqrt[-c^2 + d^2]])*Cosh[x] - b*c
*Sqrt[-c^2 + d^2]*Sinh[x]))/(c^2*Sqrt[-c^2 + d^2]*(a + 2*b + a*Cosh[2*x]))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2\left(-a\,c^2-b\,d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tanh\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c^2\sqrt{(c+d)(c-d)}} - \frac{2b\left(-\frac{c\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2} + d\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{c^2}$
risch	$-\frac{2b}{c\left(1+e^{2x}\right)} + \frac{ibd\ln\left(e^x-i\right)}{c^2} - \frac{ibd\ln\left(e^x+i\right)}{c^2} + \frac{\ln\left(e^x+\frac{\sqrt{c^2-d^2}\,c-c^2+d^2}{\sqrt{c^2-d^2}\,d}\right)a}{\sqrt{c^2-d^2}} + \frac{\ln\left(e^x+\frac{\sqrt{c^2-d^2}\,c-c^2+d^2}{\sqrt{c^2-d^2}\,d}\right)b\,d^2}{\sqrt{c^2-d^2}\,c^2} - \frac{\ln\left(e^x+\frac{\sqrt{c^2-d^2}}{\sqrt{c^2}}\right.}{\sqrt{c^2-}}$

```
[In] int((a+sech(x)^2*b)/(c+d*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-a*c^2-b*d^2)/c^2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tanh(1/2*x)/((c+d)*
(c-d))^(1/2))-2*b/c^2*(-c*tanh(1/2*x)/(1+tanh(1/2*x)^2)+d*arctan(tanh(1/2*x
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(64) = 128.

Time = 0.48 (sec) , antiderivative size = 598, normalized size of antiderivative = 8.08

$$\int \frac{a+b\text{sech}^2(x)}{c+d\cosh(x)}\,dx$$
$$= \left[-\frac{2\,bc^3-2\,bcd^2-\left(ac^2+bd^2+\left(ac^2+bd^2\right)\cosh\left(x\right)^2+2\left(ac^2+bd^2\right)\cosh\left(x\right)\sinh\left(x\right)+\left(ac^2+bd^2\right)\sinh\right.}{}\right.$$
$$-\frac{2\left(bc^3-bcd^2+\left(ac^2+bd^2+\left(ac^2+bd^2\right)\cosh\left(x\right)^2+2\left(ac^2+bd^2\right)\cosh\left(x\right)\sinh\left(x\right)+\left(ac^2+bd^2\right)\sinh\left(\right.\right.\right.}{c^4-}$$

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(2*b*c^3 - 2*b*c*d^2 - (a*c^2 + b*d^2 + (a*c^2 + b*d^2)*cosh(x)^2 + 2*(a*
c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 - d^2)*l
og((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 - d^2 + 2*(d^2*co
sh(x) + c*d)*sinh(x) - 2*sqrt(c^2 - d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*co
sh(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) + d)) + 2*(
b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d - b*d^3)*cosh(x)
*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))/(c^4 - c
^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cosh(x)*sinh(x) + (c
^4 - c^2*d^2)*sinh(x)^2), -2*(b*c^3 - b*c*d^2 + (a*c^2 + b*d^2 + (a*c^2 + b
*d^2)*cosh(x)^2 + 2*(a*c^2 + b*d^2)*cosh(x)*sinh(x) + (a*c^2 + b*d^2)*sinh(
x)^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c)
/(c^2 - d^2)) + (b*c^2*d - b*d^3 + (b*c^2*d - b*d^3)*cosh(x)^2 + 2*(b*c^2*d
 - b*d^3)*cosh(x)*sinh(x) + (b*c^2*d - b*d^3)*sinh(x)^2)*arctan(cosh(x) + s
inh(x)))/(c^4 - c^2*d^2 + (c^4 - c^2*d^2)*cosh(x)^2 + 2*(c^4 - c^2*d^2)*cos
h(x)*sinh(x) + (c^4 - c^2*d^2)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{a+b\text{sech}^2(x)}{c+d\cosh(x)}\,dx=\int \frac{a+b\,\text{sech}^2\left(x\right)}{c+d\cosh\left(x\right)}\,dx$$

```
[In] integrate((a+b*sech(x)**2)/(c+d*cosh(x)),x)
```

```
[Out] Integral((a + b*sech(x)**2)/(c + d*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{sech}^2(x)}{c + d\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{a + b\operatorname{sech}^2(x)}{c + d\cosh(x)}\,dx = -\frac{2\,bd\arctan\left(e^x\right)}{c^2} + \frac{2\left(ac^2 + bd^2\right)\arctan\left(\frac{de^x + c}{\sqrt{-c^2 + d^2}}\right)}{\sqrt{-c^2 + d^2}\,c^2} - \frac{2\,b}{c\left(e^{(2\,x)} + 1\right)}$$

```
[In] integrate((a+b*sech(x)^2)/(c+d*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b*d*arctan(e^x)/c^2 + 2*(a*c^2 + b*d^2)*arctan((d*e^x + c)/sqrt(-c^2 + d
^2))/(sqrt(-c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 704, normalized size of antiderivative = 9.51

$$\int \frac{a+b\,\text{sech}^2(x)}{c+d\cosh(x)}\,dx$$

$$= \frac{\ln\left(\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d-2\,b^2\,c^2\,d^2+3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}-\frac{\sqrt{(c+d)\,(c-d)}\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\left(2\,b\,d^3+4\,a\,c^3\,e^x+2\,a\,c^2\,d-a\,c\,d\cdots\right.}{d^5}\cdots\right.}{\cdots}\right.}{c^2\,(c^2-d^2)}\cdots\right)}{\cdots}$$

$$-\frac{2\,b}{c\,\left(e^{2\,x}+1\right)}$$

$$-\frac{\ln\left(-\frac{32\,b\left(a\,c^2+b\,d^2\right)\left(2\,b\,d+a\,c\,e^x+4\,b\,c\,e^x\right)}{c^3\,d^3}-\frac{\sqrt{(c+d)\,(c-d)}\left(\frac{32\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,e^x\,b^2\,c^3\,d-2\,b^2\,c^2\,d^2+3\,e^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}+\frac{\sqrt{(c+d)\,(c-\cdots}}{\cdots}\right.}{\cdots}\cdots\right)}{\cdots}$$

$$+\frac{b\,d\,\ln\left(e^x-i\right)\,1i}{c^2}-\frac{b\,d\,\ln\left(e^x+1i\right)\,1i}{c^2}$$

```
[In] int((a + b/cosh(x)^2)/(c + d*cosh(x)),x)
```

```
[Out] (log((((c + d)*(c - d))^(1/2)*((32*(a^2*c^4 + 2*b^2*d^4 - 2*b^2*c^2*d^2 + 3
*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) - (((c +
 d)*(c - d))^(1/2)*(a*c^2 + b*d^2)*((32*c*(2*b*d^3 + 4*a*c^3*exp(x) + 2*a*c
^2*d - a*c*d^2*exp(x) + 3*b*c*d^2*exp(x)))/d^5 + (32*((c + d)*(c - d))^(1/2
)*(a*c^2 + b*d^2)*(3*c^2*d - 2*d^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(
c^2 - d^2))))/(c^2*(c^2 - d^2)))*(a*c^2 + b*d^2))/(c^2*(c^2 - d^2)) - (32*b
*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) + 4*b*c*exp(x)))/(c^3*d^3))*((c + d)*(
c - d))^(1/2)*(a*c^2 + b*d^2))/(c^4 - c^2*d^2) - (2*b)/(c*(exp(2*x) + 1)) -
 (log(- (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) + 4*b*c*exp(x)))/(c^3*d^3
) - (((c + d)*(c - d))^(1/2)*((32*(a^2*c^4 + 2*b^2*d^4 - 2*b^2*c^2*d^2 + 3*
b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) + (((c +
d)*(c - d))^(1/2)*(a*c^2 + b*d^2)*((32*c*(2*b*d^3 + 4*a*c^3*exp(x) + 2*a*c^
2*d - a*c*d^2*exp(x) + 3*b*c*d^2*exp(x)))/d^5 - (32*((c + d)*(c - d))^(1/2)
*(a*c^2 + b*d^2)*(3*c^2*d - 2*d^3 + 4*c^3*exp(x) - 3*c*d^2*exp(x)))/(d^5*(c
^2 - d^2))))/(c^2*(c^2 - d^2)))*(a*c^2 + b*d^2))/(c^2*(c^2 - d^2)))*((c + d
)*(c - d))^(1/2)*(a*c^2 + b*d^2))/(c^4 - c^2*d^2) + (b*d*log(exp(x) - 1i)*1
i)/c^2 - (b*d*log(exp(x) + 1i)*1i)/c^2
```

3.579 $\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=\frac{bd\text{arctanh}(\cosh(x))}{c^2}-\frac{2(ac^2+bd^2)\,\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}-\frac{b\coth(x)}{c}$$

```
[Out] b*d*arctanh(cosh(x))/c^2-b*coth(x)/c-2*(a*c^2+b*d^2)*arctanh((d-c*tanh(1/2*
x))/(c^2+d^2)^(1/2))/c^2/(c^2+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4318, 3135, 3080, 3855, 2739, 632, 212}

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=-\frac{2(ac^2+bd^2)\,\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{c^2\sqrt{c^2+d^2}}+\frac{bd\text{arctanh}(\cosh(x))}{c^2}-\frac{b\coth(x)}{c}$$

```
[In] Int[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
```

```
[Out] (b*d*ArcTanh[Cosh[x]])/c^2 - (2*(a*c^2 + b*d^2)*ArcTanh[(d - c*Tanh[x/2])/S
qrt[c^2 + d^2]])/(c^2*Sqrt[c^2 + d^2]) - (b*Coth[x])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4318

```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Sin[a + b*x]^2)/Sin[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = -\int \frac{\text{csch}^2(x)\left(-b-a\sinh^2(x)\right)}{c+d\sinh(x)}\,dx$$

$$= -\frac{b\coth(x)}{c} - \frac{i\int \frac{\text{csch}(x)(-ibd+iac\sinh(x))}{c+d\sinh(x)}\,dx}{c}$$

$$= -\frac{b\coth(x)}{c} - \frac{(bd)\int \text{csch}(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\sinh(x)}\,dx$$

$$= \frac{bd\text{arctanh}(\cosh(x))}{c^2} - \frac{b\coth(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx-cx^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{bd\text{arctanh}(\cosh(x))}{c^2} - \frac{b\coth(x)}{c} - \left(4\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{4(c^2+d^2)-x^2}\,dx, x, 2d-2c\tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{bd\text{arctanh}(\cosh(x))}{c^2} - \frac{2\left(a+\frac{bd^2}{c^2}\right)\text{arctanh}\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}} - \frac{b\coth(x)}{c}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx = \frac{\text{csch}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\left(-bc\cosh(x)+\left(\frac{2(ac^2+bd^2)\arctan\left(\frac{d-c\tanh\left(\frac{x}{2}\right)}{\sqrt{-c^2-d^2}}\right)}{\sqrt{-c^2-d^2}}+bd\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\sinh}{2c^2}$$

```
[In] Integrate[(a + b*Csch[x]^2)/(c + d*Sinh[x]),x]
[Out] (Csch[x/2]*Sech[x/2]*(-(b*c*Cosh[x]) + ((2*(a*c^2 + b*d^2)*ArcTan[(d - c*Ta
nh[x/2])/Sqrt[-c^2 - d^2]])/Sqrt[-c^2 - d^2] + b*d*(Log[Cosh[x/2]] - Log[Si
nh[x/2]]))*Sinh[x]))/(2*c^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

method	result
default	$-\frac{b\tanh\left(\frac{x}{2}\right)}{2c}-\frac{\left(-4a\,c^{2}-4b\,d^{2}\right)\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)}{2c^{2}\sqrt{c^{2}+d^{2}}}-\frac{b}{2c\tanh\left(\frac{x}{2}\right)}-\frac{bd\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c^{2}}$
parts	$\frac{2a\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)}{\sqrt{c^{2}+d^{2}}}+b\left(-\frac{\tanh\left(\frac{x}{2}\right)}{2c}-\frac{1}{2c\tanh\left(\frac{x}{2}\right)}-\frac{d\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{c^{2}}+\frac{2d^{2}\operatorname{arctanh}\left(\frac{2c\tanh\left(\frac{x}{2}\right)-2d}{2\sqrt{c^{2}+d^{2}}}\right)}{c^{2}\sqrt{c^{2}+d^{2}}}\right)$
risch	$-\frac{2b}{c\left(e^{2x}-1\right)}+\frac{bd\ln\left(e^{x}+1\right)}{c^{2}}-\frac{bd\ln\left(e^{x}-1\right)}{c^{2}}+\frac{\ln\left(e^{x}+\frac{\sqrt{c^{2}+d^{2}}\,c-c^{2}-d^{2}}{\sqrt{c^{2}+d^{2}}\,d}\right)a}{\sqrt{c^{2}+d^{2}}}+\frac{\ln\left(e^{x}+\frac{\sqrt{c^{2}+d^{2}}\,c-c^{2}-d^{2}}{\sqrt{c^{2}+d^{2}}\,d}\right)b\,d^{2}}{\sqrt{c^{2}+d^{2}}\,c^{2}}-\frac{\ln\left(e^{x}+\frac{\sqrt{c^{2}+d^{2}}}{\sqrt{c}}\right.}{\sqrt{c^{2}-}}$

```
[In] int((a+b*csch(x)^2)/(c+d*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/c*tanh(1/2*x)-1/2/c^2*(-4*a*c^2-4*b*d^2)/(c^2+d^2)^(1/2)*arctanh(1/2
*(2*c*tanh(1/2*x)-2*d)/(c^2+d^2)^(1/2))-1/2*b/c/tanh(1/2*x)-1/c^2*b*d*ln(ta
nh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(65) = 130$.

Time = 0.48 (sec) , antiderivative size = 401, normalized size of antiderivative = 5.81

$$\int \frac{a+b\operatorname{csch}^2(x)}{c+d\sinh(x)}\,dx = \frac{2\,bc^3+2\,bcd^2+\left(ac^2+bd^2-\left(ac^2+bd^2\right)\cosh\left(x\right)^2-2\left(ac^2+bd^2\right)\cosh\left(x\right)\sinh\left(x\right)-\left(ac^2+bd^2\right)\sinh\left(x\right)^2\right.}{}$$

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*c^3 + 2*b*c*d^2 + (a*c^2 + b*d^2 - (a*c^2 + b*d^2)*cosh(x)^2 - 2*(a*c^
2 + b*d^2)*cosh(x)*sinh(x) - (a*c^2 + b*d^2)*sinh(x)^2)*sqrt(c^2 + d^2)*log
((d^2*cosh(x)^2 + d^2*sinh(x)^2 + 2*c*d*cosh(x) + 2*c^2 + d^2 + 2*(d^2*cosh
(x) + c*d)*sinh(x) - 2*sqrt(c^2 + d^2)*(d*cosh(x) + d*sinh(x) + c))/(d*cosh
(x)^2 + d*sinh(x)^2 + 2*c*cosh(x) + 2*(d*cosh(x) + c)*sinh(x) - d)) + (b*c^
2*d + b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*sin
h(x) - (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (b*c^2*d +
 b*d^3 - (b*c^2*d + b*d^3)*cosh(x)^2 - 2*(b*c^2*d + b*d^3)*cosh(x)*sinh(x)
- (b*c^2*d + b*d^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(c^4 + c^2*d^2 -
 (c^4 + c^2*d^2)*cosh(x)^2 - 2*(c^4 + c^2*d^2)*cosh(x)*sinh(x) - (c^4 + c^2
*d^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=\int \frac{a+b\,\text{csch}^2\left(x\right)}{c+d\sinh\left(x\right)}\,dx$$

```
[In] integrate((a+b*csch(x)**2)/(c+d*sinh(x)),x)
[Out] Integral((a + b*csch(x)**2)/(c + d*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.29

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx = b\left(\frac{d^2\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}c^2}+\frac{d\log\left(e^{(-x)}+1\right)}{c^2}-\frac{d\log\left(e^{(-x)}-1\right)}{c^2}+\frac{2}{ce^{(-2\,x)}-c}\right)+\frac{a\log\left(\frac{de^{(-x)}-c-\sqrt{c^2+d^2}}{de^{(-x)}-c+\sqrt{c^2+d^2}}\right)}{\sqrt{c^2+d^2}}$$

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="maxima")
[Out] b*(d^2*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c + sqrt(c^2 + d^2)
))/(sqrt(c^2 + d^2)*c^2) + d*log(e^(-x) + 1)/c^2 - d*log(e^(-x) - 1)/c^2 +
2/(c*e^(-2*x) - c)) + a*log((d*e^(-x) - c - sqrt(c^2 + d^2))/(d*e^(-x) - c
+ sqrt(c^2 + d^2)))/sqrt(c^2 + d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.58

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=\frac{bd\log\left(e^x+1\right)}{c^2}-\frac{bd\log\left(|e^x-1|\right)}{c^2}+\frac{\left(ac^2+bd^2\right)\log\left(\frac{\left|2\,de^x+2\,c-2\,\sqrt{c^2+d^2}\right|}{\left|2\,de^x+2\,c+2\,\sqrt{c^2+d^2}\right|}\right)}{\sqrt{c^2+d^2}c^2}-\frac{2\,b}{c(e^{(2\,x)}-1)}$$

```
[In] integrate((a+b*csch(x)^2)/(c+d*sinh(x)),x, algorithm="giac")
[Out] b*d*log(e^x + 1)/c^2 - b*d*log(abs(e^x - 1))/c^2 + (a*c^2 + b*d^2)*log(abs(
2*d*e^x + 2*c - 2*sqrt(c^2 + d^2))/abs(2*d*e^x + 2*c + 2*sqrt(c^2 + d^2)))/
(sqrt(c^2 + d^2)*c^2) - 2*b/(c*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \frac{a+b\text{csch}^2(x)}{c+d\sinh(x)}\,dx=\frac{b\,d\,\ln\left(\mathrm{e}^x+1\right)}{c^2}-\frac{b\,d\,\ln\left(\mathrm{e}^x-1\right)}{c^2}-\frac{2\,b}{c\,\left(\mathrm{e}^{2\,x}-1\right)}$$

$$-\frac{\ln\left(\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,\mathrm{e}^x\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2-3\,\mathrm{e}^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}-\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\,\left(4\,a\,c^3\,\mathrm{e}^x-2\,b\,d^3-2\,a\,c^2\,d+a\,c\,d^2\,\mathrm{e}^x+3\,b\,c\,d^2\,\mathrm{e}\right.}{d^5}\right.}{c^2\,\sqrt{c^2+c}}\right.}{c^2\,\sqrt{c^2+d^2}}\right.}{c^4+c^2\,d^2}$$

$$+\frac{\ln\left(-\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,\left(a^2\,c^4+2\,a\,b\,c^2\,d^2-4\,\mathrm{e}^x\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2-3\,\mathrm{e}^x\,b^2\,c\,d^3+2\,b^2\,d^4\right)}{c^2\,d^4}+\frac{\left(a\,c^2+b\,d^2\right)\left(\frac{32\,c\,\left(4\,a\,c^3\,\mathrm{e}^x-2\,b\,d^3-2\,a\,c^2\,d+a\,c\,d^2\,\mathrm{e}^x+3\,b\,c\,d\right.}{d^5}\right.}{c^2\,\sqrt{c^2}}\right.}{c^2\,\sqrt{c^2+d^2}}\right.}{c^4+c^2\,c}$$

```
[In] int((a + b/sinh(x)^2)/(c + d*sinh(x)),x)
```

```
[Out] (b*d*log(exp(x) + 1))/c^2 - (b*d*log(exp(x) - 1))/c^2 - (2*b)/(c*(exp(2*x)
- 1)) - (log(((a*c^2 + b*d^2)*((32*(a^2*c^4 + 2*b^2*d^4 + 2*b^2*c^2*d^2 - 3
*b^2*c*d^3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) - ((a*c^
2 + b*d^2)*((32*c*(4*a*c^3*exp(x) - 2*b*d^3 - 2*a*c^2*d + a*c*d^2*exp(x) +
3*b*c*d^2*exp(x)))/d^5 + (32*(a*c^2 + b*d^2)*(3*c^2*d + 2*d^3 - 4*c^3*exp(x
) - 3*c*d^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c^2*(c^2 + d^2)^(1/2))))/(c
^2*(c^2 + d^2)^(1/2)) - (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) - 4*b*c*e
xp(x)))/(c^3*d^3))*(a*c^2 + b*d^2)*(c^2 + d^2)^(1/2))/(c^4 + c^2*d^2) + (lo
g(- ((a*c^2 + b*d^2)*((32*(a^2*c^4 + 2*b^2*d^4 + 2*b^2*c^2*d^2 - 3*b^2*c*d^
3*exp(x) - 4*b^2*c^3*d*exp(x) + 2*a*b*c^2*d^2))/(c^2*d^4) + ((a*c^2 + b*d^2
)*((32*c*(4*a*c^3*exp(x) - 2*b*d^3 - 2*a*c^2*d + a*c*d^2*exp(x) + 3*b*c*d^2
*exp(x)))/d^5 - (32*(a*c^2 + b*d^2)*(3*c^2*d + 2*d^3 - 4*c^3*exp(x) - 3*c*d
^2*exp(x)))/(d^5*(c^2 + d^2)^(1/2))))/(c^2*(c^2 + d^2)^(1/2))))/(c^2*(c^2 +
 d^2)^(1/2)) - (32*b*(a*c^2 + b*d^2)*(2*b*d + a*c*exp(x) - 4*b*c*exp(x)))/(
c^3*d^3))*(a*c^2 + b*d^2)*(c^2 + d^2)^(1/2))/(c^4 + c^2*d^2)
```

3.580 $\int (a\cosh(x)+b\sinh(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int (a\cosh(x)+b\sinh(x))\,dx=b\cosh(x)+a\sinh(x)$$

```
[Out] b*cosh(x)+a*sinh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2717, 2718}

$$\int (a\cosh(x)+b\sinh(x))\,dx=a\sinh(x)+b\cosh(x)$$

```
[In] Int[a*Cosh[x] + b*Sinh[x],x]
```

```
[Out] b*Cosh[x] + a*Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = a\int \cosh(x)\,dx + b\int \sinh(x)\,dx$$
$$= b\cosh(x) + a\sinh(x)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh(x) + a\sinh(x)$$

```
[In] Integrate[a*Cosh[x] + b*Sinh[x],x]
[Out] b*Cosh[x] + a*Sinh[x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$b\cosh\left(x\right)+a\sinh\left(x\right)$	10
parts	$b\cosh\left(x\right)+a\sinh\left(x\right)$	10
meijerg	$a\sinh\left(x\right)-b\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x)}{\sqrt{\pi}}\right)$	23
risch	$\frac{\left(a\,{\mathrm e}^{2x}+b\,{\mathrm e}^{2x}-a+b\right){\mathrm e}^{-x}}{2}$	24

```
[In] int(a*cosh(x)+b*sinh(x),x,method=_RETURNVERBOSE)
[Out] b*cosh(x)+a*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh\left(x\right) + a\sinh\left(x\right)$$

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="fricas")
[Out] b*cosh(x) + a*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int (a\cosh(x) + b\sinh(x))\,dx = a\sinh{(x)} + b\cosh{(x)}$$

```
[In] integrate(a*cosh(x)+b*sinh(x),x)
[Out] a*sinh(x) + b*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh{(x)} + a\sinh{(x)}$$

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="maxima")
[Out] b*cosh(x) + a*sinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int (a\cosh(x) + b\sinh(x))\,dx = \frac{1}{2}\,b\big(e^{(-x)} + e^{x}\big) - \frac{1}{2}\,a\big(e^{(-x)} - e^{x}\big)$$

```
[In] integrate(a*cosh(x)+b*sinh(x),x, algorithm="giac")
[Out] 1/2*b*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (a\cosh(x) + b\sinh(x))\,dx = b\cosh(x) + a\sinh(x)$$

```
[In] int(a*cosh(x) + b*sinh(x),x)
[Out] b*cosh(x) + a*sinh(x)
```

3.581 $\int (a\cosh(x) + b\sinh(x))^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int (a\cosh(x)+b\sinh(x))^2 \, dx = \frac{1}{2}(a^2-b^2)\,x+\frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))$$

```
[Out] 1/2*(a^2-b^2)*x+1/2*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3152, 8}

$$\int (a\cosh(x)+b\sinh(x))^2 \, dx = \frac{1}{2}x(a^2-b^2)+\frac{1}{2}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^2,x]
[Out] ((a^2 - b^2)*x)/2 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x]))/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{2}\left(a^2-b^2\right)\int 1\,dx\\ &= \frac{1}{2}\left(a^2-b^2\right)x+\frac{1}{2}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int (a\cosh(x)+b\sinh(x))^2\,dx=\frac{1}{4}\left(2(a-b)(a+b)x+2ab\cosh(2x)+\left(a^2+b^2\right)\sinh(2x)\right)$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^2,x]
[Out] (2*(a - b)*(a + b)*x + 2*a*b*Cosh[2*x] + (a^2 + b^2)*Sinh[2*x])/4
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
default	$a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ab\cosh\left(x\right)^2+b^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$	37
parts	$a^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ab\cosh\left(x\right)^2+b^2\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$	37
risch	$\frac{a^2x}{2}-\frac{b^2x}{2}+\frac{a^2\mathrm{e}^{2x}}{8}+\frac{b\,\mathrm{e}^{2x}a}{4}+\frac{b^2\mathrm{e}^{2x}}{8}-\frac{\mathrm{e}^{-2x}a^2}{8}+\frac{\mathrm{e}^{-2x}ab}{4}-\frac{\mathrm{e}^{-2x}b^2}{8}$	66

```
[In] int((a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] a^2*(1/2*cosh(x)*sinh(x)+1/2*x)+a*b*cosh(x)^2+b^2*(1/2*cosh(x)*sinh(x)-1/2*
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int (a\cosh(x)+b\sinh(x))^2\,dx = \frac{1}{2}\,ab\cosh\left(x\right)^2 + \frac{1}{2}\,ab\sinh\left(x\right)^2 + \frac{1}{2}\,\left(a^2+b^2\right)\cosh\left(x\right)\sinh\left(x\right) + \frac{1}{2}\,\left(a^2-b^2\right)x$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
[Out] 1/2*a*b*cosh(x)^2 + 1/2*a*b*sinh(x)^2 + 1/2*(a^2 + b^2)*cosh(x)*sinh(x) + 1
/2*(a^2 - b^2)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int (a\cosh(x)+b\sinh(x))^2\,dx = -\frac{a^2x\sinh^2\left(x\right)}{2} + \frac{a^2x\cosh^2\left(x\right)}{2} + \frac{a^2\sinh\left(x\right)\cosh\left(x\right)}{2} + ab\cosh^2\left(x\right) + \frac{b^2x\sinh^2\left(x\right)}{2} - \frac{b^2x\cosh^2\left(x\right)}{2} + \frac{b^2\sinh\left(x\right)\cosh\left(x\right)}{2}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**2,x)
[Out] -a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*sinh(x)*cosh(x)/2 + a*b*c
osh(x)**2 + b**2*x*sinh(x)**2/2 - b**2*x*cosh(x)**2/2 + b**2*sinh(x)*cosh(x
)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int (a\cosh(x)+b\sinh(x))^2\,dx = ab\cosh\left(x\right)^2 + \frac{1}{8}\,a^2\left(4\,x+e^{(2\,x)}-e^{(-2\,x)}\right) - \frac{1}{8}\,b^2\left(4\,x-e^{(2\,x)}+e^{(-2\,x)}\right)$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
[Out] a*b*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*b^2*(4*x - e^(2*x)
 + e^(-2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.00

$$\int (a\cosh(x)+b\sinh(x))^2\,dx = \frac{1}{8}\,a^2e^{(2\,x)}+\frac{1}{4}\,abe^{(2\,x)}+\frac{1}{8}\,b^2e^{(2\,x)}+\frac{1}{2}\,(a^2-b^2)x - \frac{1}{8}\,(2\,a^2e^{(2\,x)}-2\,b^2e^{(2\,x)}+a^2-2\,ab+b^2)e^{(-2\,x)}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*x) + 1/4*a*b*e^(2*x) + 1/8*b^2*e^(2*x) + 1/2*(a^2 - b^2)*x - 1
/8*(2*a^2*e^(2*x) - 2*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int (a\cosh(x)+b\sinh(x))^2\,dx = \frac{a^2\sinh(2\,x)}{4}+\frac{b^2\sinh(2\,x)}{4}+\frac{a^2\,x}{2}-\frac{b^2\,x}{2}+\frac{a\,b\cosh(2\,x)}{2}$$

```
[In] int((a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] (a^2*sinh(2*x))/4 + (b^2*sinh(2*x))/4 + (a^2*x)/2 - (b^2*x)/2 + (a*b*cosh(2
*x))/2
```

3.582 $\int (a\cosh(x)+b\sinh(x))^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int (a\cosh(x)+b\sinh(x))^3\,dx=\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))+\frac{1}{3}(b\cosh(x)+a\sinh(x))^3$$

```
[Out] (a^2-b^2)*(b*cosh(x)+a*sinh(x))+1/3*(b*cosh(x)+a*sinh(x))^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3151}

$$\int (a\cosh(x)+b\sinh(x))^3\,dx=\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))+\frac{1}{3}(a\sinh(x)+b\cosh(x))^3$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a^2 - b^2)*(b*Cosh[x] + a*Sinh[x]) + (b*Cosh[x] + a*Sinh[x])^3/3
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=i\text{Subst}\left(\int \left(a^2-b^2-x^2\right)\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)\\&=\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))+\frac{1}{3}(b\cosh(x)+a\sinh(x))^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int (a\cosh(x)+b\sinh(x))^3\,dx=\frac{1}{12}\left(9b(a^2-b^2)\cosh(x)+b(3a^2+b^2)\cosh(3x)+9a(a^2-b^2)\sinh(x)+a(a^2+3b^2)\sinh(3x)\right)$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^3,x]
[Out] (9*b*(a^2 - b^2)*Cosh[x] + b*(3*a^2 + b^2)*Cosh[3*x] + 9*a*(a^2 - b^2)*Sinh
[x] + a*(a^2 + 3*b^2)*Sinh[3*x])/12
```

Maple [A] (verified)

Time = 5.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result
default	$a^3\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+a^2b\cosh(x)^3+a\,b^2\sinh(x)^3+b^3\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x)$
parts	$a^3\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+a^2b\cosh(x)^3+a\,b^2\sinh(x)^3+b^3\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x)$
risch	$\frac{e^{3x}a^3}{24}+\frac{e^{3x}a^2b}{8}+\frac{e^{3x}a\,b^2}{8}+\frac{e^{3x}b^3}{24}+\frac{3a^3e^x}{8}+\frac{3a^2b\,e^x}{8}-\frac{3\,e^xb^2a}{8}-\frac{3b^3e^x}{8}-\frac{3\,e^{-x}a^3}{8}+\frac{3\,e^{-x}a^2b}{8}+\frac{3\,e^{-x}a\,b^2}{8}-\frac{3\,e^{-}}{8}$

```
[In] int((a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] a^3*(2/3+1/3*cosh(x)^2)*sinh(x)+a^2*b*cosh(x)^3+a*b^2*sinh(x)^3+b^3*(-2/3+1
/3*sinh(x)^2)*cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

$$\int (a\cosh(x)+b\sinh(x))^3\,dx=\frac{1}{12}\left(3\,a^2b+b^3\right)\cosh(x)^3+\frac{1}{4}\left(3\,a^2b+b^3\right)\cosh(x)\sinh(x)^2+\frac{1}{12}\left(a^3+3\,ab^2\right)\sinh(x)^3+\frac{3}{4}\left(a^2b-b^3\right)\cosh(x)+\frac{1}{4}\left(3\,a^3-3\,ab^2+\left(a^3+3\,ab^2\right)\cosh(x)^2\right)\sinh(x)$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
[Out] 1/12*(3*a^2*b + b^3)*cosh(x)^3 + 1/4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^2 + 1/
12*(a^3 + 3*a*b^2)*sinh(x)^3 + 3/4*(a^2*b - b^3)*cosh(x) + 1/4*(3*a^3 - 3*a
*b^2 + (a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = -\frac{2a^3\sinh^3(x)}{3} + a^3\sinh(x)\cosh^2(x) + a^2b\cosh^3(x) + ab^2\sinh^3(x) + b^3\sinh^2(x)\cosh(x) - \frac{2b^3\cosh^3(x)}{3}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] -2*a**3*sinh(x)**3/3 + a**3*sinh(x)*cosh(x)**2 + a**2*b*cosh(x)**3 + a*b**2
*sinh(x)**3 + b**3*sinh(x)**2*cosh(x) - 2*b**3*cosh(x)**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = a^2b\cosh(x)^3 + ab^2\sinh(x)^3 + \frac{1}{24}\,b^3\left(e^{(3\,x)} - 9\,e^{(-x)} + e^{(-3\,x)} - 9\,e^x\right) + \frac{1}{24}\,a^3\left(e^{(3\,x)} - 9\,e^{(-x)} - e^{(-3\,x)} + 9\,e^x\right)$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] a^2*b*cosh(x)^3 + a*b^2*sinh(x)^3 + 1/24*b^3*(e^(3*x) - 9*e^(-x) + e^(-3*x)
 - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = \frac{1}{24}\,a^3e^{(3\,x)} + \frac{1}{8}\,a^2be^{(3\,x)} + \frac{1}{8}\,ab^2e^{(3\,x)} + \frac{1}{24}\,b^3e^{(3\,x)} + \frac{3}{8}\,a^3e^x + \frac{3}{8}\,a^2be^x - \frac{3}{8}\,ab^2e^x - \frac{3}{8}\,b^3e^x - \frac{1}{24}\left(9\,a^3e^{(2\,x)} - 9\,a^2be^{(2\,x)} - 9\,ab^2e^{(2\,x)} + 9\,b^3e^{(2\,x)} + a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)e^{(-3\,x)}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*a^3*e^(3*x) + 1/8*a^2*b*e^(3*x) + 1/8*a*b^2*e^(3*x) + 1/24*b^3*e^(3*x)
 + 3/8*a^3*e^x + 3/8*a^2*b*e^x - 3/8*a*b^2*e^x - 3/8*b^3*e^x - 1/24*(9*a^3*
e^(2*x) - 9*a^2*b*e^(2*x) - 9*a*b^2*e^(2*x) + 9*b^3*e^(2*x) + a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int (a\cosh(x)+b\sinh(x))^3\,dx = \cosh(x)^3\left(a^2\,b-\frac{2\,b^3}{3}\right)+\sinh(x)^3\left(a\,b^2-\frac{2\,a^3}{3}\right) + a^3\cosh(x)^2\sinh(x)+b^3\cosh(x)\,\sinh(x)^2$$

```
[In] int((a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] cosh(x)^3*(a^2*b - (2*b^3)/3) + sinh(x)^3*(a*b^2 - (2*a^3)/3) + a^3*cosh(x)
^2*sinh(x) + b^3*cosh(x)*sinh(x)^2
```

3.583 $\int (a\cosh(x)+b\sinh(x))^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int (a\cosh(x)+b\sinh(x))^4\,dx=\frac{3}{8}\left(a^2-b^2\right)^2x+\frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3$$

```
[Out] 3/8*(a^2-b^2)^2*x+3/8*(a^2-b^2)*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))
+1/4*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3152, 8}

$$\int (a\cosh(x)+b\sinh(x))^4\,dx=\frac{3}{8}x\left(a^2-b^2\right)^2+\frac{3}{8}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))+\frac{1}{4}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^3$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^4,x]
```

```
[Out] (3*(a^2 - b^2)^2*x)/8 + (3*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] +
 b*Sinh[x]))/8 + ((b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3 \\ &\quad+\frac{1}{4}\left(3\left(a^2-b^2\right)\right)\int(a\cosh(x)+b\sinh(x))^2\,dx \\ &= \frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x)) \\ &\quad+\frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3+\frac{1}{8}\left(3\left(a^2-b^2\right)^2\right)\int 1\,dx \\ &= \frac{3}{8}\left(a^2-b^2\right)^2x+\frac{3}{8}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x)) \\ &\quad+\frac{1}{4}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\begin{aligned}\int(a\cosh(x)+b\sinh(x))^4\,dx &= \frac{1}{32}\big(12(a-b)^2(a+b)^2x+16ab\left(a^2-b^2\right)\cosh(2x) \\ &\quad+4ab\left(a^2+b^2\right)\cosh(4x)+8\left(a^4-b^4\right)\sinh(2x) \\ &\quad+\left(a^4+6a^2b^2+b^4\right)\sinh(4x)\big)\end{aligned}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^4,x]
```

```
[Out] (12*(a - b)^2*(a + b)^2*x + 16*a*b*(a^2 - b^2)*Cosh[2*x] + 4*a*b*(a^2 + b^2
)*Cosh[4*x] + 8*(a^4 - b^4)*Sinh[2*x] + (a^4 + 6*a^2*b^2 + b^4)*Sinh[4*x])/
32
```

Maple [A] (verified)

Time = 24.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result
default	$a^{4}\left(\left(\frac{\cosh(x)^{3}}{4}+\frac{3\cosh(x)}{8}\right)\sinh\left(x\right)+\frac{3x}{8}\right)+a^{3}b\cosh\left(x\right)^{4}+6a^{2}b^{2}\left(\frac{\cosh(x)^{3}\sinh(x)}{4}-\frac{\cosh(x)\sinh(x)}{8}-\frac{x}{8}\right)$
parts	$a^{4}\left(\left(\frac{\cosh(x)^{3}}{4}+\frac{3\cosh(x)}{8}\right)\sinh\left(x\right)+\frac{3x}{8}\right)+a^{3}b\cosh\left(x\right)^{4}+6a^{2}b^{2}\left(\frac{\cosh(x)^{3}\sinh(x)}{4}-\frac{\cosh(x)\sinh(x)}{8}-\frac{x}{8}\right)$
risch	$\frac{3x\,a^{4}}{8}-\frac{3a^{2}b^{2}x}{4}+\frac{3b^{4}x}{8}+\frac{e^{4x}a^{4}}{64}+\frac{e^{4x}a^{3}b}{16}+\frac{3\,e^{4x}a^{2}b^{2}}{32}+\frac{e^{4x}a\,b^{3}}{16}+\frac{e^{4x}b^{4}}{64}+\frac{e^{2x}a^{4}}{8}+\frac{e^{2x}a^{3}b}{4}-\frac{e^{2x}a\,b^{3}}{4}-\frac{e^{2x}b^{4}}{8}-$

```
[In] int((a*cosh(x)+b*sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*((1/4*cosh(x)^3+3/8*cosh(x))*sinh(x)+3/8*x)+a^3*b*cosh(x)^4+6*a^2*b^2*(
1/4*cosh(x)^3*sinh(x)-1/8*cosh(x)*sinh(x)-1/8*x)+a*b^3*sinh(x)^4+b^4*((1/4*
sinh(x)^3-3/8*sinh(x))*cosh(x)+3/8*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(66) = 132.

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.33

$$\int (a\cosh(x)+b\sinh(x))^4\,dx$$
$$=\frac{1}{8}\left(a^{3}b+ab^{3}\right)\cosh\left(x\right)^{4}+\frac{1}{8}\left(a^{4}+6\,a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)\sinh\left(x\right)^{3}+\frac{1}{8}\left(a^{3}b+ab^{3}\right)\sinh\left(x\right)^{4}$$
$$+\frac{1}{2}\left(a^{3}b-ab^{3}\right)\cosh\left(x\right)^{2}+\frac{1}{4}\left(2\,a^{3}b-2\,ab^{3}+3\left(a^{3}b+ab^{3}\right)\cosh\left(x\right)^{2}\right)\sinh\left(x\right)^{2}$$
$$+\frac{3}{8}\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)x+\frac{1}{8}\left(\left(a^{4}+6\,a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)^{3}+4\left(a^{4}-b^{4}\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] 1/8*(a^3*b + a*b^3)*cosh(x)^4 + 1/8*(a^4 + 6*a^2*b^2 + b^4)*cosh(x)*sinh(x)
^3 + 1/8*(a^3*b + a*b^3)*sinh(x)^4 + 1/2*(a^3*b - a*b^3)*cosh(x)^2 + 1/4*(2
*a^3*b - 2*a*b^3 + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x)^2 + 3/8*(a^4 - 2*a^
2*b^2 + b^4)*x + 1/8*((a^4 + 6*a^2*b^2 + b^4)*cosh(x)^3 + 4*(a^4 - b^4)*cos
h(x))*sinh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(71) = 142$.

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.68

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = \frac{3a^4x\sinh^4(x)}{8} - \frac{3a^4x\sinh^2(x)\cosh^2(x)}{4} + \frac{3a^4x\cosh^4(x)}{8} - \frac{3a^4\sinh^3(x)\cosh(x)}{8} + \frac{5a^4\sinh(x)\cosh^3(x)}{8} + a^3b\cosh^4(x) - \frac{3a^2b^2x\sinh^4(x)}{4} + \frac{3a^2b^2x\sinh^2(x)\cosh^2(x)}{2} - \frac{3a^2b^2x\cosh^4(x)}{4} + \frac{3a^2b^2\sinh^3(x)\cosh(x)}{4} + \frac{3a^2b^2\sinh(x)\cosh^3(x)}{4} + ab^3\sinh^4(x) + \frac{3b^4x\sinh^4(x)}{8} - \frac{3b^4x\sinh^2(x)\cosh^2(x)}{4} + \frac{3b^4x\cosh^4(x)}{8} + \frac{5b^4\sinh^3(x)\cosh(x)}{8} - \frac{3b^4\sinh(x)\cosh^3(x)}{8}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**4,x)
```

```
[Out] 3*a**4*x*sinh(x)**4/8 - 3*a**4*x*sinh(x)**2*cosh(x)**2/4 + 3*a**4*x*cosh(x)
**4/8 - 3*a**4*sinh(x)**3*cosh(x)/8 + 5*a**4*sinh(x)*cosh(x)**3/8 + a**3*b*
cosh(x)**4 - 3*a**2*b**2*x*sinh(x)**4/4 + 3*a**2*b**2*x*sinh(x)**2*cosh(x)*
*2/2 - 3*a**2*b**2*x*cosh(x)**4/4 + 3*a**2*b**2*sinh(x)**3*cosh(x)/4 + 3*a*
*2*b**2*sinh(x)*cosh(x)**3/4 + a*b**3*sinh(x)**4 + 3*b**4*x*sinh(x)**4/8 -
3*b**4*x*sinh(x)**2*cosh(x)**2/4 + 3*b**4*x*cosh(x)**4/8 + 5*b**4*sinh(x)**
3*cosh(x)/8 - 3*b**4*sinh(x)*cosh(x)**3/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = a^3b\cosh(x)^4 + ab^3\sinh(x)^4 + \frac{1}{64}a^4\left(24x + e^{(4x)} + 8e^{(2x)} - 8e^{(-2x)} - e^{(-4x)}\right) + \frac{1}{64}b^4\left(24x + e^{(4x)} - 8e^{(2x)} + 8e^{(-2x)} - e^{(-4x)}\right) - \frac{3}{32}a^2b^2\left(8x - e^{(4x)} + e^{(-4x)}\right)$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] a^3*b*cosh(x)^4 + a*b^3*sinh(x)^4 + 1/64*a^4*(24*x + e^(4*x) + 8*e^(2*x) -
8*e^(-2*x) - e^(-4*x)) + 1/64*b^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x)
- e^(-4*x)) - 3/32*a^2*b^2*(8*x - e^(4*x) + e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(66) = 132$.

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.89

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = \frac{1}{64}\,a^4e^{(4\,x)}+\frac{1}{16}\,a^3be^{(4\,x)}+\frac{3}{32}\,a^2b^2e^{(4\,x)}+\frac{1}{16}\,ab^3e^{(4\,x)} + \frac{1}{64}\,b^4e^{(4\,x)}+\frac{1}{8}\,a^4e^{(2\,x)}+\frac{1}{4}\,a^3be^{(2\,x)}-\frac{1}{4}\,ab^3e^{(2\,x)}-\frac{1}{8}\,b^4e^{(2\,x)}+\frac{3}{8}\,\left(a^4-2\,a^2b^2+b^4\right)x - \frac{1}{64}\,\left(18\,a^4e^{(4\,x)}-36\,a^2b^2e^{(4\,x)}+18\,b^4e^{(4\,x)}+8\,a^4e^{(2\,x)}-16\,a^3be^{(2\,x)}+16\,ab^3e^{(2\,x)}-8\,b^4e^{(2\,x)}+a^4-4\,a^3b$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/64*a^4*e^(4*x) + 1/16*a^3*b*e^(4*x) + 3/32*a^2*b^2*e^(4*x) + 1/16*a*b^3*e
^(4*x) + 1/64*b^4*e^(4*x) + 1/8*a^4*e^(2*x) + 1/4*a^3*b*e^(2*x) - 1/4*a*b^3
*e^(2*x) - 1/8*b^4*e^(2*x) + 3/8*(a^4 - 2*a^2*b^2 + b^4)*x - 1/64*(18*a^4*e
^(4*x) - 36*a^2*b^2*e^(4*x) + 18*b^4*e^(4*x) + 8*a^4*e^(2*x) - 16*a^3*b*e^(
2*x) + 16*a*b^3*e^(2*x) - 8*b^4*e^(2*x) + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b
^3 + b^4)*e^(-4*x)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int (a\cosh(x)+b\sinh(x))^4\,dx = \cosh(x)\,\sinh(x)^3\,\left(-\frac{3\,a^4}{8}+\frac{3\,a^2\,b^2}{4}+\frac{5\,b^4}{8}\right) - \cosh(x)^4\,\left(a\,b^3-a^3\,b\right) + \cosh(x)^3\sinh(x)\,\left(\frac{5\,a^4}{8}+\frac{3\,a^2\,b^2}{4}-\frac{3\,b^4}{8}\right) + \frac{3\,x\cosh(x)^4\,\left(a^2-b^2\right)^2}{8}+\frac{3\,x\sinh(x)^4\,\left(a^2-b^2\right)^2}{8} + 2\,a\,b^3\cosh(x)^2\sinh(x)^2-\frac{3\,x\cosh(x)^2\sinh(x)^2\,\left(a^2-b^2\right)^2}{4}$$

```
[In] int((a*cosh(x) + b*sinh(x))^4,x)
```

```
[Out] cosh(x)*sinh(x)^3*((5*b^4)/8 - (3*a^4)/8 + (3*a^2*b^2)/4) - cosh(x)^4*(a*b^
3 - a^3*b) + cosh(x)^3*sinh(x)*((5*a^4)/8 - (3*b^4)/8 + (3*a^2*b^2)/4) + (3
*x*cosh(x)^4*(a^2 - b^2)^2)/8 + (3*x*sinh(x)^4*(a^2 - b^2)^2)/8 + 2*a*b^3*c
osh(x)^2*sinh(x)^2 - (3*x*cosh(x)^2*sinh(x)^2*(a^2 - b^2)^2)/4
```

3.584 $\int (a\cosh(x)+b\sinh(x))^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 61

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = \left(a^2-b^2\right)^2 (b\cosh(x)+a\sinh(x)) + \frac{2}{3}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))^3 + \frac{1}{5}(b\cosh(x)+a\sinh(x))^5$$

```
[Out] (a^2-b^2)^2*(b*cosh(x)+a*sinh(x))+2/3*(a^2-b^2)*(b*cosh(x)+a*sinh(x))^3+1/5
*(b*cosh(x)+a*sinh(x))^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3151, 200}

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = \frac{2}{3}\left(a^2-b^2\right)(a\sinh(x)+b\cosh(x))^3 + \left(a^2-b^2\right)^2 (a\sinh(x)+b\cosh(x)) + \frac{1}{5}(a\sinh(x)+b\cosh(x))^5$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^5,x]
```

```
[Out] (a^2 - b^2)^2*(b*Cosh[x] + a*Sinh[x]) + (2*(a^2 - b^2)*(b*Cosh[x] + a*Sinh[
x])^3)/3 + (b*Cosh[x] + a*Sinh[x])^5/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\text{Subst}\left(\int \left(a^2-b^2-x^2\right)^2\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)\\
&= i\text{Subst}\left(\int \left(a^4\left(1+\frac{-2a^2b^2+b^4}{a^4}\right)-2a^2\left(1-\frac{b^2}{a^2}\right)x^2+x^4\right)\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)\\
&= \left(a^2-b^2\right)^2(b\cosh(x)+a\sinh(x))\\
&\quad+\frac{2}{3}\left(a^2-b^2\right)(b\cosh(x)+a\sinh(x))^3+\frac{1}{5}(b\cosh(x)+a\sinh(x))^5
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 133 vs. $2(61) = 122$.

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.18

$$\begin{aligned}
\int (a\cosh(x)+b\sinh(x))^5\,dx &= \frac{1}{240}\Big(150b\left(a^2-b^2\right)^2\cosh(x)-25b\left(-3a^4+2a^2b^2+b^4\right)\cosh(3x)\\
&\quad+3b\left(5a^4+10a^2b^2+b^4\right)\cosh(5x)+150a\left(a^2-b^2\right)^2\sinh(x)\\
&\quad+25a\left(a^4+2a^2b^2-3b^4\right)\sinh(3x)\\
&\quad+3a\left(a^4+10a^2b^2+5b^4\right)\sinh(5x)\Big)
\end{aligned}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^5,x]
```

```
[Out] (150*b*(a^2 - b^2)^2*Cosh[x] - 25*b*(-3*a^4 + 2*a^2*b^2 + b^4)*Cosh[3*x] +
3*b*(5*a^4 + 10*a^2*b^2 + b^4)*Cosh[5*x] + 150*a*(a^2 - b^2)^2*Sinh[x] + 25
*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sinh[3*x] + 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Sin
h[5*x])/240
```

Maple [A] (verified)

Time = 180.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

method	result
parts	$a^{5}\left(\frac{8}{15}+\frac{\cosh(x)^{4}}{5}+\frac{4\cosh(x)^{2}}{15}\right)\sinh\left(x\right)+b^{5}\left(\frac{8}{15}+\frac{\sinh(x)^{4}}{5}-\frac{4\sinh(x)^{2}}{15}\right)\cosh\left(x\right)+a^{4}b\cosh\left(x\right)^{5}+10a^{3}$
default	$a^{5}\left(\frac{8}{15}+\frac{\cosh(x)^{4}}{5}+\frac{4\cosh(x)^{2}}{15}\right)\sinh\left(x\right)+a^{4}b\cosh\left(x\right)^{5}+10a^{3}b^{2}\left(\frac{\cosh(x)^{4}\sinh(x)}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(x)^{2}}{3}\right)\sinh(x)}{5}\right)$
risch	$\frac{5b^{5}\mathrm{e}^{x}}{16}+\frac{5\,\mathrm{e}^{x}a\,b^{4}}{16}+\frac{5\,\mathrm{e}^{x}a^{4}b}{16}-\frac{5\,\mathrm{e}^{x}a^{3}b^{2}}{8}-\frac{5\,\mathrm{e}^{x}a^{2}b^{3}}{8}+\frac{5\,\mathrm{e}^{x}a^{5}}{16}+\frac{\mathrm{e}^{-5x}b^{5}}{160}+\frac{\mathrm{e}^{5x}a^{5}}{160}+\frac{5\,\mathrm{e}^{3x}a^{5}}{96}+\frac{5\,\mathrm{e}^{-x}a^{4}b}{16}+\frac{5\,\mathrm{e}^{-x}a^{3}b^{2}}{8}-$

```
[In] int((a*cosh(x)+b*sinh(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(8/15+1/5*cosh(x)^4+4/15*cosh(x)^2)*sinh(x)+b^5*(8/15+1/5*sinh(x)^4-4/1
5*sinh(x)^2)*cosh(x)+a^4*b*cosh(x)^5+10*a^3*b^2*(1/5*sinh(x)^5+1/3*sinh(x)^
3)+10*a^2*b^3*(1/5*cosh(x)^5-1/3*cosh(x)^3)+a*b^4*sinh(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(57) = 114.

Time = 0.25 (sec) , antiderivative size = 298, normalized size of antiderivative = 4.89

$$\begin{aligned}&\int (a\cosh(x)+b\sinh(x))^5\,dx\\&=\frac{1}{80}\left(5\,a^{4}b+10\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)^{5}+\frac{1}{16}\left(5\,a^{4}b+10\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)\sinh\left(x\right)^{4}\\&\quad+\frac{1}{80}\left(a^{5}+10\,a^{3}b^{2}+5\,ab^{4}\right)\sinh\left(x\right)^{5}+\frac{5}{48}\left(3\,a^{4}b-2\,a^{2}b^{3}-b^{5}\right)\cosh\left(x\right)^{3}\\&\quad+\frac{1}{48}\left(5\,a^{5}+10\,a^{3}b^{2}-15\,ab^{4}+6\left(a^{5}+10\,a^{3}b^{2}+5\,ab^{4}\right)\cosh\left(x\right)^{2}\right)\sinh\left(x\right)^{3}\\&\quad+\frac{1}{16}\left(2\left(5\,a^{4}b+10\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)^{3}+5\left(3\,a^{4}b-2\,a^{2}b^{3}-b^{5}\right)\cosh\left(x\right)\right)\sinh\left(x\right)^{2}\\&\quad+\frac{5}{8}\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)\\&\quad+\frac{1}{16}\left(10\,a^{5}-20\,a^{3}b^{2}+10\,ab^{4}+\left(a^{5}+10\,a^{3}b^{2}+5\,ab^{4}\right)\cosh\left(x\right)^{4}+5\left(a^{5}+2\,a^{3}b^{2}-3\,ab^{4}\right)\cosh\left(x\right)^{2}\right)\sinh\left(x\right)\end{aligned}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
```

```
[Out] 1/80*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(x)^5 + 1/16*(5*a^4*b + 10*a^2*b^3 +
b^5)*cosh(x)*sinh(x)^4 + 1/80*(a^5 + 10*a^3*b^2 + 5*a*b^4)*sinh(x)^5 + 5/48
*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)^3 + 1/48*(5*a^5 + 10*a^3*b^2 - 15*a*b^
4 + 6*(a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^2)*sinh(x)^3 + 1/16*(2*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(x)^3 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))*sinh
(x)^2 + 5/8*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x) + 1/16*(10*a^5 - 20*a^3*b^2 +
```

```
 10*a*b^4 + (a^5 + 10*a^3*b^2 + 5*a*b^4)*cosh(x)^4 + 5*(a^5 + 2*a^3*b^2 - 3
*a*b^4)*cosh(x)^2)*sinh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.82

$$\int (a\cosh(x) + b\sinh(x))^5\,dx = \frac{8a^5\sinh^5(x)}{15} - \frac{4a^5\sinh^3(x)\cosh^2(x)}{3} + a^5\sinh(x)\cosh^4(x) + a^4b\cosh^5(x) - \frac{4a^3b^2\sinh^5(x)}{3} + \frac{10a^3b^2\sinh^3(x)\cosh^2(x)}{3} + \frac{10a^2b^3\sinh^2(x)\cosh^3(x)}{3} - \frac{4a^2b^3\cosh^5(x)}{3} + ab^4\sinh^5(x) + b^5\sinh^4(x)\cosh(x) - \frac{4b^5\sinh^2(x)\cosh^3(x)}{3} + \frac{8b^5\cosh^5(x)}{15}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**5,x)
```

```
[Out] 8*a**5*sinh(x)**5/15 - 4*a**5*sinh(x)**3*cosh(x)**2/3 + a**5*sinh(x)*cosh(x
)**4 + a**4*b*cosh(x)**5 - 4*a**3*b**2*sinh(x)**5/3 + 10*a**3*b**2*sinh(x)*
*3*cosh(x)**2/3 + 10*a**2*b**3*sinh(x)**2*cosh(x)**3/3 - 4*a**2*b**3*cosh(x
)**5/3 + a*b**4*sinh(x)**5 + b**5*sinh(x)**4*cosh(x) - 4*b**5*sinh(x)**2*co
sh(x)**3/3 + 8*b**5*cosh(x)**5/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.13

$$\begin{aligned}\int (a\cosh(x) + b\sinh(x))^5\,dx &= a^4b\cosh(x)^5 + ab^4\sinh(x)^5 \\ &\quad + \frac{1}{48}\left(\left(5e^{(-2x)} - 30e^{(-4x)} + 3\right)e^{(5x)} + 30e^{(-x)} - 5e^{(-3x)} - 3e^{(-5x)}\right)a^3b^2 \\ &\quad - \frac{1}{48}\left(\left(5e^{(-2x)} + 30e^{(-4x)} - 3\right)e^{(5x)} + 30e^{(-x)} + 5e^{(-3x)} - 3e^{(-5x)}\right)a^2b^3 \\ &\quad + \frac{1}{480}a^5\left(3e^{(5x)} + 25e^{(3x)} - 150e^{(-x)} - 25e^{(-3x)} - 3e^{(-5x)} + 150e^{x}\right) \\ &\quad + \frac{1}{480}b^5\left(3e^{(5x)} - 25e^{(3x)} + 150e^{(-x)} - 25e^{(-3x)} + 3e^{(-5x)} + 150e^{x}\right)\end{aligned}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
```

```
[Out] a^4*b*cosh(x)^5 + a*b^4*sinh(x)^5 + 1/48*((5*e^(-2*x) - 30*e^(-4*x) + 3)*e^
(5*x) + 30*e^(-x) - 5*e^(-3*x) - 3*e^(-5*x))*a^3*b^2 - 1/48*((5*e^(-2*x) +
30*e^(-4*x) - 3)*e^(5*x) + 30*e^(-x) + 5*e^(-3*x) - 3*e^(-5*x))*a^2*b^3 + 1
/480*a^5*(3*e^(5*x) + 25*e^(3*x) - 150*e^(-x) - 25*e^(-3*x) - 3*e^(-5*x) +
150*e^x) + 1/480*b^5*(3*e^(5*x) - 25*e^(3*x) + 150*e^(-x) - 25*e^(-3*x) + 3
*e^(-5*x) + 150*e^x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(57) = 114$.

Time = 0.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 5.64

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = \frac{1}{160}\,a^5e^{(5\,x)} + \frac{1}{32}\,a^4be^{(5\,x)} + \frac{1}{16}\,a^3b^2e^{(5\,x)} + \frac{1}{16}\,a^2b^3e^{(5\,x)} + \frac{1}{32}\,ab^4e^{(5\,x)} + \frac{1}{160}\,b^5e^{(5\,x)} + \frac{5}{96}\,a^5e^{(3\,x)} + \frac{5}{32}\,a^4be^{(3\,x)} + \frac{5}{48}\,a^3b^2e^{(3\,x)} - \frac{5}{48}\,a^2b^3e^{(3\,x)} - \frac{5}{32}\,ab^4e^{(3\,x)} - \frac{5}{96}\,b^5e^{(3\,x)} + \frac{5}{16}\,a^5e^{x} + \frac{5}{16}\,a^4be^{x} - \frac{5}{8}\,a^3b^2e^{x} - \frac{5}{8}\,a^2b^3e^{x} + \frac{5}{16}\,ab^4e^{x} + \frac{5}{16}\,b^5e^{x} - \frac{1}{480}\left(150\,a^5e^{(4\,x)} - 150\,a^4be^{(4\,x)} - 300\,a^3b^2e^{(4\,x)} + 300\,a^2b^3e^{(4\,x)} + 150\,ab^4e^{(4\,x)} - 150\,b^5e^{(4\,x)} + 25\,a^5e^{(2\,x)} -\right.$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

```
[Out] 1/160*a^5*e^(5*x) + 1/32*a^4*b*e^(5*x) + 1/16*a^3*b^2*e^(5*x) + 1/16*a^2*b^
3*e^(5*x) + 1/32*a*b^4*e^(5*x) + 1/160*b^5*e^(5*x) + 5/96*a^5*e^(3*x) + 5/3
2*a^4*b*e^(3*x) + 5/48*a^3*b^2*e^(3*x) - 5/48*a^2*b^3*e^(3*x) - 5/32*a*b^4*
e^(3*x) - 5/96*b^5*e^(3*x) + 5/16*a^5*e^x + 5/16*a^4*b*e^x - 5/8*a^3*b^2*e^
x - 5/8*a^2*b^3*e^x + 5/16*a*b^4*e^x + 5/16*b^5*e^x - 1/480*(150*a^5*e^(4*x
) - 150*a^4*b*e^(4*x) - 300*a^3*b^2*e^(4*x) + 300*a^2*b^3*e^(4*x) + 150*a*b
^4*e^(4*x) - 150*b^5*e^(4*x) + 25*a^5*e^(2*x) - 75*a^4*b*e^(2*x) + 50*a^3*b
^2*e^(2*x) + 50*a^2*b^3*e^(2*x) - 75*a*b^4*e^(2*x) + 25*b^5*e^(2*x) + 3*a^5
 - 15*a^4*b + 30*a^3*b^2 - 30*a^2*b^3 + 15*a*b^4 - 3*b^5)*e^(-5*x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.92

$$\int (a\cosh(x)+b\sinh(x))^5\,dx = \cosh(x)^5\left(a^4\,b-\frac{4\,a^2\,b^3}{3}+\frac{8\,b^5}{15}\right) + \sinh(x)^5\left(\frac{8\,a^5}{15}-\frac{4\,a^3\,b^2}{3}+a\,b^4\right) - \cosh(x)^2\sinh(x)^3\left(\frac{4\,a^5}{3}-\frac{10\,a^3\,b^2}{3}\right) + a^5\cosh(x)^4\sinh(x) - \cosh(x)^3\sinh(x)^2\left(\frac{4\,b^5}{3}-\frac{10\,a^2\,b^3}{3}\right) + b^5\cosh(x)\,\sinh(x)^4$$

```
[In] int((a*cosh(x) + b*sinh(x))^5,x)
```

```
[Out] cosh(x)^5*(a^4*b + (8*b^5)/15 - (4*a^2*b^3)/3) + sinh(x)^5*(a*b^4 + (8*a^5)
/15 - (4*a^3*b^2)/3) - cosh(x)^2*sinh(x)^3*((4*a^5)/3 - (10*a^3*b^2)/3) + a
^5*cosh(x)^4*sinh(x) - cosh(x)^3*sinh(x)^2*((4*b^5)/3 - (10*a^2*b^3)/3) + b
^5*cosh(x)*sinh(x)^4
```

3.585 $\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx=\frac{\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3153, 212}

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx=\frac{\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-1),x]
```

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)\\ &= \frac{\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-1),x]
[Out] (2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sqrt[a
 + b])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	39
risch	$-\frac{\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	70

```
[In] int(1/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.89

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{-a^2+b^2}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b}\right)}{a^2-b^2}, -\frac{2\,\arctan\left(\frac{\sqrt{a^2-b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))/(a^2 - b
^2), -2*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/sqrt(a^
2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(31) = 62.

Time = 2.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.76

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\log\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for}\ a=0 \wedge b=0 \\ \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} & \text{for}\ a=0 \\ -\frac{1}{-b\sinh(x)+b\cosh(x)} & \text{for}\ a=-b \\ -\frac{1}{b\sinh(x)+b\cosh(x)} & \text{for}\ a=b \\ \frac{\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*log(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tanh(x/2))/b, Eq(
a, 0)), (-1/(-b*sinh(x) + b*cosh(x)), Eq(a, -b)), (-1/(b*sinh(x) + b*cosh(x
)), Eq(a, b)), (log(tanh(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b*
*2) - log(tanh(x/2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True)
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{a\cosh(x) + b\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^x\sqrt{a^2-b^2}}{a-b}\right)}{\sqrt{a^2-b^2}}$$

```
[In] int(1/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] (2*atan((exp(x)*(a^2 - b^2)^(1/2))/(a - b)))/(a^2 - b^2)^(1/2)
```

3.586 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

```
[Out] sinh(x)/a/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3154}

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-2),x]
```

```
[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx=\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-2),x]
[Out] Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{2}{(a\,e^{2x}+b\,e^{2x}+a-b)(a+b)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a\right)}$	29

```
[In] int(1/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/(a*exp(2*x)+b*exp(2*x)+a-b)/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2}{(a^2+2\,ab+b^2)\cosh{(x)}^2+2\,(a^2+2\,ab+b^2)\cosh{(x)}\sinh{(x)}+(a^2+2\,ab+b^2)\sinh{(x)}^2+a^2-b^2}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
[Out] -2/((a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x) +
 (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(14) = 28$.

Time = 136.94 (sec) , antiderivative size = 604, normalized size of antiderivative = 35.53

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)}\right) \\ \frac{-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)}}{b^2} \\ \frac{x\tanh^4\left(\frac{x}{2}\right)}{2b^2\sinh^2(x)\tanh^4\left(\frac{x}{2}\right) + 4b^2\sinh^2(x)\tanh^2\left(\frac{x}{2}\right) + 2b^2\sinh^2(x) - 8b^2\sinh(x)\cosh(x)\tanh^3\left(\frac{x}{2}\right) - 8b^2\sinh(x)\cosh(x)\tanh\left(\frac{x}{2}\right) + 8b^2\cosh^2(x)\,\mathrm{ta}} \\ \frac{2\tanh\left(\frac{x}{2}\right)}{a^2\tanh^2\left(\frac{x}{2}\right) + a^2 + 2ab\tanh\left(\frac{x}{2}\right)} \end{cases}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*(-tanh(x/2)/2 - 1/(2*tanh(x/2))), Eq(a, 0) & Eq(b, 0)), ((-t
anh(x/2)/2 - 1/(2*tanh(x/2)))/b**2, Eq(a, 0)), (x*tanh(x/2)**4/(2*b**2*sinh
(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b**2*sinh(x)**2 -
8*b**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2) + 8*
b**2*cosh(x)**2*tanh(x/2)**2) - 2*x*tanh(x/2)**2/(2*b**2*sinh(x)**2*tanh(x/
2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b**2*sinh(x)**2 - 8*b**2*sinh(x)
*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2) + 8*b**2*cosh(x)**
2*tanh(x/2)**2) + x/(2*b**2*sinh(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tan
h(x/2)**2 + 2*b**2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**
2*sinh(x)*cosh(x)*tanh(x/2) + 8*b**2*cosh(x)**2*tanh(x/2)**2) + 2*tanh(x/2)
**3/(2*b**2*sinh(x)**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b*
*2*sinh(x)**2 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x
)*tanh(x/2) + 8*b**2*cosh(x)**2*tanh(x/2)**2) + 2*tanh(x/2)/(2*b**2*sinh(x)
**2*tanh(x/2)**4 + 4*b**2*sinh(x)**2*tanh(x/2)**2 + 2*b**2*sinh(x)**2 - 8*b
**2*sinh(x)*cosh(x)*tanh(x/2)**3 - 8*b**2*sinh(x)*cosh(x)*tanh(x/2) + 8*b**
2*cosh(x)**2*tanh(x/2)**2), Eq(a, -2*b*tanh(x/2)/(tanh(x/2)**2 + 1))), (2*t
anh(x/2)/(a**2*tanh(x/2)**2 + a**2 + 2*a*b*tanh(x/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2}{a^2-b^2+(a^2-2\,ab+b^2)e^{(-2\,x)}}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
[Out] 2/(a^2 - b^2 + (a^2 - 2*a*b + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2}{(ae^{(2\,x)}+be^{(2\,x)}+a-b)(a+b)}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
[Out] -2/((a*e^(2*x) + b*e^(2*x) + a - b)*(a + b))
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2}{(a+b)\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^2,x)
[Out] -2/((a + b)*(a - b + exp(2*x)*(a + b)))
```

3.587 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}+\frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}$$

```
[Out] 1/2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+1/2*(b*co
sh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3155, 3153, 212}

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}+\frac{a\sinh(x)+b\cosh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

```
[Out] ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]]/(2*(a^2 - b^2)^(3/2)) + (b*
Cosh[x] + a*Sinh[x])/(2*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{2\left(a^2-b^2\right)}\\
&= \frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}+\frac{i\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{2\left(a^2-b^2\right)}\\
&= \frac{\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}+\frac{b\cosh(x)+a\sinh(x)}{2\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int\frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx=\frac{1}{2}\left(\frac{2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{\sinh(x)}{a(a\cosh(x)+b\sinh(x))^2}+\frac{b}{a(a-b)(a+b)(a\cosh(x)+b\sinh(x))}\right)$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-3),x]
```

```
[Out] ((2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a
+ b)^(3/2)) + Sinh[x]/(a*(a*Cosh[x] + b*Sinh[x])^2) + b/(a*(a - b)*(a + b)*
(a*Cosh[x] + b*Sinh[x])))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(69) = 138$.

Time = 32.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.90

method	result	size
risch	$\frac{e^{x}\left(a\,e^{2x}+b\,e^{2x}-a+b\right)}{(a-b)(a+b)\left(a\,e^{2x}+b\,e^{2x}+a-b\right)^{2}}-\frac{\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{2\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{2\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	146
default	$\frac{-\frac{\left(a^{2}-2b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}}{\left(a^{2}-b^{2}\right)a}+\frac{b\left(a^{2}+2b^{2}\right)\tanh\left(\frac{x}{2}\right)^{2}}{\left(a^{2}-b^{2}\right)a^{2}}+\frac{\left(a^{2}+2b^{2}\right)\tanh\left(\frac{x}{2}\right)}{\left(a^{2}-b^{2}\right)a}+\frac{2b}{2a^{2}-2b^{2}}}{\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)^{2}}+\frac{\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{2}-b^{2}\right)^{\frac{3}{2}}}$	167

```
[In] int(1/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)*(a*exp(x)^2+b*exp(x)^2-a+b)/(a-b)/(a+b)/(a*exp(x)^2+b*exp(x)^2+a-b)^
2-1/2/(-a^2+b^2)^(1/2)/(a+b)/(a-b)*ln(exp(x)-(a-b)/(-a^2+b^2)^(1/2))+1/2/(-
a^2+b^2)^(1/2)/(a+b)/(a-b)*ln(exp(x)+(a-b)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 1495, normalized size of antiderivative = 19.42

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 6*(a^3 + a^2*b - a*b^2 - b^
3)*cosh(x)*sinh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3 + ((a^2 + 2*
a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a
*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*co
sh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)) - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x) -
2*(a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2)*sin
h(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 + (a^6
 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^4 + 4*(
a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)*sinh
(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*sin
h(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 2*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2
```

```
*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^
3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x))*sinh(x)), ((a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^3 + 3*(a^3 + a^2*
b - a*b^2 - b^3)*cosh(x)*sinh(x)^2 + (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^3
- ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3
+ (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a
*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 2*a*b + b^2 + 4*((a^2 +
2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arct
an(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x) - (a^3 - a^2*b - a*b^2 + b^3 - 3*(a^3 + a^2*b - a*b^2 -
b^3)*cosh(x)^2)*sinh(x))/(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2
*a*b^5 + b^6 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b
^6)*cosh(x)^4 + 4*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5
+ b^6)*cosh(x)*sinh(x)^3 + (a^6 + 2*a^5*b - a^4*b^2 - 4*a^3*b^3 - a^2*b^4 +
 2*a*b^5 + b^6)*sinh(x)^4 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2
 + 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*(a^6 + 2*a^5*b - a^4*b^2 - 4*a^
3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^2)*sinh(x)^2 + 4*((a^6 + 2*a^5*b -
 a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5 + b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{ae^{(3x)}+be^{(3x)}-ae^x+be^x}{(a^2-b^2)(ae^{(2x)}+be^{(2x)}+a-b)^2}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + (a*e^(3*x) + b*
e^(3*x) - a*e^x + b*e^x)/((a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.04

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{e^x}{(a+b)\ (a-b)\ (a-b+e^{2x}\ (a+b))} - \frac{2\,e^x}{(a+b)\ \left(e^{4x}\,(a+b)^2+(a-b)^2+2\,e^{2x}\ (a+b)\ (a-b)\right)} - \frac{\mathrm{atan}\left(\frac{e^x\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{-a^3+a^2\,b+a\,b^2-b^3}\right)}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] exp(x)/((a + b)*(a - b)*(a - b + exp(2*x)*(a + b))) - (2*exp(x))/((a + b)*(
exp(4*x)*(a + b)^2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b))) - atan((exp(x
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a*b^2 + a^2*b - a^3 - b^3))/(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.588 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = \frac{b\cosh(x)+a\sinh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3} + \frac{2\sinh(x)}{3a(a^2-b^2)(a\cosh(x)+b\sinh(x))}$$

```
[Out] 1/3*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^3+2/3*sinh(x)/a/(
a^2-b^2)/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3155, 3154}

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = \frac{2\sinh(x)}{3a(a^2-b^2)(a\cosh(x)+b\sinh(x))} + \frac{a\sinh(x)+b\cosh(x)}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^3}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-4),x]
```

```
[Out] (b*Cosh[x] + a*Sinh[x])/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^3) + (2*Sinh
[x])/(3*a*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\cosh(x)+a\sinh(x)}{3\,(a^2-b^2)\,(a\cosh(x)+b\sinh(x))^3}+\frac{2\int\frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{3\,(a^2-b^2)}\\ &= \frac{b\cosh(x)+a\sinh(x)}{3\,(a^2-b^2)\,(a\cosh(x)+b\sinh(x))^3}+\frac{2\sinh(x)}{3a\,(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int\frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx=\frac{ab\cosh(3x)+(2a^2-b^2+(a^2+b^2)\cosh(2x))\sinh(x)}{3a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^3}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-4),x]
[Out] (a*b*Cosh[3*x] + (2*a^2 - b^2 + (a^2 + b^2)*Cosh[2*x])*Sinh[x])/(3*a*(a - b
)*(a + b)*(a*Cosh[x] + b*Sinh[x])^3)
```

Maple [A] (verified)

Time = 137.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{4(3a\,e^{2x}+3b\,e^{2x}+a-b)}{3(a\,e^{2x}+b\,e^{2x}+a-b)^3(a+b)^2}$	46
default	$-\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^5}{a}-\frac{2b\tanh\left(\frac{x}{2}\right)^4}{a^2}-\frac{2\left(a^2+2b^2\right)\tanh\left(\frac{x}{2}\right)^3}{3a^3}-\frac{2b\tanh\left(\frac{x}{2}\right)^2}{a^2}-\frac{\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)^3}$	87

```
[In] int(1/(a*cosh(x)+b*sinh(x))^4,x,method=_RETURNVERBOSE)
[Out] -4/3*(3*a*exp(2*x)+3*b*exp(2*x)+a-b)/(a*exp(2*x)+b*exp(2*x)+a-b)^3/(a+b)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 527, normalized size of antiderivative = 7.87

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx =$$

$$-\frac{}{3\left((a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\cosh{(x)}^5+5\,(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b\right.}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -8/3*((2*a + b)*cosh(x) + (a + 2*b)*sinh(x))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^4 + (a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^5 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (3*a^5 + 9*a^4*b + 6*a^3*b^2 - 6*a^2*b^
3 - 9*a*b^4 - 3*b^5 + 10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*cosh(x)^2)*sinh(x)^3 + (10*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*cosh(x)^3 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*cosh(x))*sinh(x)^2 + 2*(2*a^5 + a^4*b - 4*a^3*b^2 - 2*a^2*b^3
+ 2*a*b^4 + b^5)*cosh(x) + (2*a^5 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 2*a*b
^4 + 4*b^5 + 5*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*co
sh(x)^4 + 9*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)
^2)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(63) = 126.

Time = 0.22 (sec) , antiderivative size = 498, normalized size of antiderivative = 7.43

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx$$
$$= \frac{4\,(a-b)e^{(-2\,x)}}{a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)e^{(-2\,x)}+3\,(a^5-3\,a^4b+2\,a}$$
$$+\frac{4\,a}{3\,(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)e^{(-2\,x)}+3\,(a^5-3\,a^4b}$$
$$+\frac{4\,b}{3\,(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5+3\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)e^{(-2\,x)}+3\,(a^5-3\,a^4b}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4*(a - b)*e^(-2*x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10
*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x)) + 4/3*a/(a^5 + a^4*b - 2*a
^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
 a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4
 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*e^(-6*x)) + 4/3*b/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*
(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*e^(-2*x) + 3*(a^5 - 3*a
^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*e^(-4*x) + (a^5 - 5*a^4*b + 1
0*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*e^(-6*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = -\frac{4\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}+a-b\right)}{3\,(a^2+2\,ab+b^2)\left(ae^{(2\,x)}+be^{(2\,x)}+a-b\right)^3}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*a*e^(2*x) + 3*b*e^(2*x) + a - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) +
b*e^(2*x) + a - b)^3)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^4}\,dx = -\frac{a\left(4\,\mathrm{e}^{2\,x}+\frac{4}{3}\right)+b\left(4\,\mathrm{e}^{2\,x}-\frac{4}{3}\right)}{(a+b)^2\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)^3}$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^4,x)
```

```
[Out] -(a*(4*exp(2*x) + 4/3) + b*(4*exp(2*x) - 4/3))/((a + b)^2*(a - b + a*exp(2*
x) + b*exp(2*x))^3)
```

3.589 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 112

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx=\frac{3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}+\frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}$$

```
[Out] 3/8*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/4*(b*co
sh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^4+3/8*(b*cosh(x)+a*sinh(x)
)/(a^2-b^2)^2/(a*cosh(x)+b*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3155, 3153, 212}

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx=\frac{3\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}+\frac{3(a\sinh(x)+b\cosh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+\frac{a\sinh(x)+b\cosh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

```
[Out] (3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(5/2)) +
 (b*Cosh[x] + a*Sinh[x])/(4*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^4) + (3*(b*
Cosh[x] + a*Sinh[x]))/(8*(a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3\int\frac{1}{(a\cosh(x)+b\sinh(x))^3}\,dx}{4\left(a^2-b^2\right)}\\
&= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}\\
&\quad+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}+\frac{3\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{8\left(a^2-b^2\right)^2}\\
&= \frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}\\
&\quad+\frac{(3i)\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{8\left(a^2-b^2\right)^2}\\
&= \frac{3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{5/2}}+\frac{b\cosh(x)+a\sinh(x)}{4\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^4}\\
&\quad+\frac{3(b\cosh(x)+a\sinh(x))}{8\left(a^2-b^2\right)^2(a\cosh(x)+b\sinh(x))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{1}{8}\left(\frac{6\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{b(2(a-b)(a+b)+3(a\cosh(x)+b\sinh(x))^2)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^3} + \frac{\sinh(x)\left(2+\frac{3(a\cosh(x)+b\sinh(x))^2}{(a-b)(a+b)}\right)}{a(a\cosh(x)+b\sinh(x))^4}\right)$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-5),x]
```

```
[Out] ((6*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(5/2)*(a
+ b)^(5/2)) + (b*(2*(a - b)*(a + b) + 3*(a*Cosh[x] + b*Sinh[x])^2))/(a*(a -
 b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])^3) + (Sinh[x]*(2 + (3*(a*Cosh[x] +
b*Sinh[x])^2)/((a - b)*(a + b))))/(a*(a*Cosh[x] + b*Sinh[x])^4))/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(102) = 204.

Time = 0.29 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.12

$$\frac{-\frac{(5a^4-16a^2b^2+8b^4)\tanh\left(\frac{x}{2}\right)^7}{4a(a^4-2a^2b^2+b^4)} - \frac{3b(a^4-16a^2b^2+8b^4)\tanh\left(\frac{x}{2}\right)^6}{4(a^4-2a^2b^2+b^4)a^2} + \frac{(3a^6+36a^4b^2+56a^2b^4-32b^6)\tanh\left(\frac{x}{2}\right)^5}{4a^3(a^4-2a^2b^2+b^4)} + \frac{b(15a^6+114a^4b^2-8a^2b^4-16b^6)}{4a^4(a^4-2a^2b^2+b^4)}}{\left(\tanh\left(\frac{x}{2}\right)^2 a + 2}$$

```
[In] int(1/(a*cosh(x)+b*sinh(x))^5,x)
```

```
[Out] 2*(-1/8*(5*a^4-16*a^2*b^2+8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^7-3/8*b*
(a^4-16*a^2*b^2+8*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^6+1/8/a^3*(3*a^6
+36*a^4*b^2+56*a^2*b^4-32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^5+1/8/a^4*b*
(15*a^6+114*a^4*b^2-8*a^2*b^4-16*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^4-1/8
/a^3*(3*a^6-84*a^4*b^2-56*a^2*b^4+32*b^6)/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)^3
+1/8*b*(23*a^4+64*a^2*b^2-24*b^4)/(a^4-2*a^2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/8
*(5*a^4+24*a^2*b^2-8*b^4)/a/(a^4-2*a^2*b^2+b^4)*tanh(1/2*x)+1/8*(5*a^2-2*b^
2)*b/(a^4-2*a^2*b^2+b^4))/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)^4+3/4/(a^4-2*
a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3408 vs. 2(102) = 204.

Time = 0.35 (sec) , antiderivative size = 6874, normalized size of antiderivative = 61.38

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**5,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(102) = 204.

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.11

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{4\,(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{3\,a^3e^{(7\,x)}+9\,a^2be^{(7\,x)}+9\,ab^2e^{(7\,x)}+3\,b^3e^{(7\,x)}+11\,a^3e^{(5\,x)}+11\,a^2be^{(5\,x)}-11\,ab^2e^{(5\,x)}-11\,b^3e^{(5\,x)}-11\,a}{4\,(a^4-2\,a^2b^2+b^4)(ae^{(2\,x)}+be^{(2}}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^5,x, algorithm="giac")
```

```
[Out] 3/4*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a
^2 - b^2)) + 1/4*(3*a^3*e^(7*x) + 9*a^2*b*e^(7*x) + 9*a*b^2*e^(7*x) + 3*b^3
*e^(7*x) + 11*a^3*e^(5*x) + 11*a^2*b*e^(5*x) - 11*a*b^2*e^(5*x) - 11*b^3*e^
(5*x) - 11*a^3*e^(3*x) + 11*a^2*b*e^(3*x) + 11*a*b^2*e^(3*x) - 11*b^3*e^(3*
x) - 3*a^3*e^x + 9*a^2*b*e^x - 9*a*b^2*e^x + 3*b^3*e^x)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*e^(2*x) + b*e^(2*x) + a - b)^4)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.16

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^5}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5-a^4\,b-2\,a^3\,b^2+2\,a^2\,b^3+a\,b^4-b^5}\right)}{4\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}$$
$$-\frac{4\,\mathrm{e}^{3\,x}}{(a+b)\,\left(\mathrm{e}^{8\,x}\,(a+b)^4+(a-b)^4+4\,\mathrm{e}^{2\,x}\,(a+b)\,(a-b)^3+4\,\mathrm{e}^{6\,x}\,(a+b)^3\,(a-b)+6\,\mathrm{e}^{4\,x}\,(a+b)^2\,(a-b)^2\right)}$$
$$-\frac{2\,\mathrm{e}^x}{(a+b)^2\,\left(\mathrm{e}^{6\,x}\,(a+b)^3+(a-b)^3+3\,\mathrm{e}^{2\,x}\,(a+b)\,(a-b)^2+3\,\mathrm{e}^{4\,x}\,(a+b)^2\,(a-b)\right)}$$
$$+\frac{3\,\mathrm{e}^x}{4\,(a+b)^2\,(a-b)^2\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$
$$+\frac{\mathrm{e}^x}{2\,(a+b)^2\,(a-b)\,\left(\mathrm{e}^{4\,x}\,(a+b)^2+(a-b)^2+2\,\mathrm{e}^{2\,x}\,(a+b)\,(a-b)\right)}$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^5,x)
```

```
[Out] (3*atan((exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*
b^2)^(1/2))/(a*b^4 - a^4*b + a^5 - b^5 + 2*a^2*b^3 - 2*a^3*b^2)))/(4*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)) - (4*exp(3
*x))/((a + b)*(exp(8*x)*(a + b)^4 + (a - b)^4 + 4*exp(2*x)*(a + b)*(a - b)^
3 + 4*exp(6*x)*(a + b)^3*(a - b) + 6*exp(4*x)*(a + b)^2*(a - b)^2)) - (2*ex
p(x))/((a + b)^2*(exp(6*x)*(a + b)^3 + (a - b)^3 + 3*exp(2*x)*(a + b)*(a -
b)^2 + 3*exp(4*x)*(a + b)^2*(a - b))) + (3*exp(x))/(4*(a + b)^2*(a - b)^2*(
a - b + exp(2*x)*(a + b))) + exp(x)/(2*(a + b)^2*(a - b)*(exp(4*x)*(a + b)^
2 + (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

3.590 $\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx=-\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*
b))*EllipticE(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x)
)^(1/2)/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3157, 2719}

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx=-\frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[In] Int[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x
]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a\cosh(x)+b\sinh(x)}\int\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}\,dx}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

$$= -\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.17

$$\int\sqrt{a\cosh(x)+b\sinh(x)}\,dx$$

$$= \frac{b(-a^2+b^2)\ {}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cosh^2\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)\right)\sinh\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)+\sqrt{-\sinh^2\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right.}}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{}}$$

```
[In] Integrate[Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] (b*(-a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a
]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a^3*Sqrt[
1 - b^2/a^2]*Cosh[x] - 2*a*(a^2 - b^2)*Cosh[x + ArcTanh[b/a]] + 2*a^2*b*Sqr
t[1 - b^2/a^2]*Sinh[x] + a^2*b*Sinh[x + ArcTanh[b/a]] - b^3*Sinh[x + ArcTan
h[b/a]]))/(a*b*Sqrt[1 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x +
 ArcTanh[b/a]]^2])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

method	result
default	$-\frac{\sqrt{a^2-b^2}\,\cosh(x)}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$
risch	$\sqrt{2}\,\sqrt{\left(a\,e^{2x}+b\,e^{2x}+a-b\right)e^{-x}}+\frac{(2a-2b)\left(-\frac{2\left(a\,e^{2x}+b\,e^{2x}+a-b\right)}{(a-b)\sqrt{\left(a\,e^{2x}+b\,e^{2x}+a-b\right)e^{x}}}+\frac{\left(-\frac{a+b}{a-b}+\frac{2a+2b}{a-b}\right)\sqrt{-(a+b)(a-b)}\sqrt{\frac{\left(e^{x}+\sqrt{-}\right.}{\sqrt{-}}}}{}\right.}{}$

[In] int((a*cosh(x)+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)

[Out] -(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx = -2\sqrt{2}\sqrt{a+b}\,\text{weierstrassZeta}\left(-\frac{4(a-b)}{a+b},0,\text{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b},0,\cosh(x)+\sinh(x)\right)\right)-2\sqrt{a\cosh(x)+b\sinh(x)}$$

[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(2)*sqrt(a + b)*weierstrassZeta(-4*(a - b)/(a + b), 0, weierstrassPI
nverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x))) - 2*sqrt(a*cosh(x) + b*si
nh(x))

Sympy [F]

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\,dx = \int \sqrt{a\cosh(x) + b\sinh(x)}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**(1/2),x)
[Out] Integral(sqrt(a*cosh(x) + b*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\,dx = \int \sqrt{a\cosh(x) + b\sinh(x)}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

Giac [F]

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\,dx = \int \sqrt{a\cosh(x) + b\sinh(x)}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*cosh(x) + b*sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh(x) + b\sinh(x)}\,dx = \int \sqrt{a\cosh(x) + b\sinh(x)}\,dx$$

```
[In] int((a*cosh(x) + b*sinh(x))^(1/2),x)
[Out] int((a*cosh(x) + b*sinh(x))^(1/2), x)
```

3.591 $\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 103

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i(a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] 2/3*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^(1/2)-2/3*I*(a^2-b^2)*(cos(
1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Ellipt
icF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x))/(a^2-b^
2)^(1/2))^(1/2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3152, 3157, 2720}

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\frac{2}{3}(a\sinh(x)+b\cosh(x))\sqrt{a\cosh(x)+b\sinh(x)}-\frac{2i(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)}{3\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3 - (((2*I)/3)*(a^2
 - b^2)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)} \\
&\quad+\frac{1}{3}(a^2-b^2)\int\frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx \\
&= \frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)} \\
&\quad+\frac{\left((a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\right)\int\frac{1}{\sqrt{\cosh(x+i\tan^{-1}(a,-ib))}}\,dx}{3\sqrt{a\cosh(x)+b\sinh(x)}} \\
&= \frac{2}{3}(b\cosh(x)+a\sinh(x))\sqrt{a\cosh(x)+b\sinh(x)} \\
&\quad-\frac{2i(a^2-b^2)\operatorname{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{3\sqrt{a\cosh(x)+b\sinh(x)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\frac{2}{3}\Bigg(b\cosh(x)$$
$$-\sqrt{1-\frac{a^2}{b^2}}\,b\sqrt{\cosh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)\right)\operatorname{sech}\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)$$
$$+a\sinh(x)\Bigg)\sqrt{a\cosh(x)+b\sinh(x)}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*(b*Cosh[x] - Sqrt[1 - a^2/b^2]*b*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/
b]] + a*Sinh[x])*Sqrt[a*Cosh[x] + b*Sinh[x]])/3
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{-\sqrt{a^2-b^2}\,\sinh(x)^3}\left(\cosh(x)\sqrt{-\sqrt{a^2-b^2}\,\sinh(x)^3}\,\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,(a^2-b^2)+\sinh(x)(a^2-b^2)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\cos}{\sqrt{-\sqrt{a^2-b^2}\,\sinh(x}}\right.\right.}{2\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\sinh(x)^2\sqrt{a^2-b^2}\,\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$

```
[In] int((a*cosh(x)+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)*(cosh(x)*(-(a^2-b^2)^(1/2)*sinh(x)^
3)^(1/2)*(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*(a^2-b^2)+sinh(x)*(a^2-b^2)^(3/2)*
arctan((sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)
^(1/2)))/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)/sinh(x)^2/(a^2-b^2)^(1/2)/(-sinh(x
)*(a^2-b^2)^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}(a-b)\cosh\left(x\right) + \sqrt{2}(a-b)\sinh\left(x\right)\right)\sqrt{a+b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(a-b)}{a+b}, 0, \cosh\left(\right.\right.}{}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(sqrt(2)*(a - b)*cosh(x) + sqrt(2)*(a - b)*sinh(x))*sqrt(a + b)*weie
rstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x)) + ((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)*sqrt(a*cosh(x)
 + b*sinh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \int \left(a\cosh\left(x\right) + b\sinh\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x) + b*sinh(x))**(3/2), x)
```

Maxima [F]

$$\int (a\cosh(x) + b\sinh(x))^{3/2}\,dx = \int \left(a\cosh\left(x\right) + b\sinh\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

Giac [F]

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\int (a\cosh{(x)}+b\sinh{(x)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((a*cosh(x) + b*sinh(x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx=\int (a\cosh(x)+b\sinh(x))^{3/2}\,dx$$

```
[In] int((a*cosh(x) + b*sinh(x))^(3/2),x)
[Out] int((a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.592 $\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 103

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2} - \frac{6i(a^2-b^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] 2/5*(b*cosh(x)+a*sinh(x))*(a*cosh(x)+b*sinh(x))^(3/2)-6/5*I*(a^2-b^2)*(cos(
1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Ellipt
icE(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x))^(1/2)/((
a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3152, 3157, 2719}

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{2}{5}(a\sinh(x)+b\cosh(x))(a\cosh(x)+b\sinh(x))^{3/2} - \frac{6i(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(5/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x])*(a*Cosh[x] + b*Sinh[x])^(3/2))/5 - (((6*I)/5)*(a
^2 - b^2)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh
[x]])/Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2} \\
&\quad +\frac{1}{5}\left(3\left(a^2-b^2\right)\right)\int \sqrt{a\cosh(x)+b\sinh(x)}\,dx \\
&= \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2} \\
&\quad +\frac{\left(3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}\right)\int \sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}\,dx}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}} \\
&= \frac{2}{5}(b\cosh(x)+a\sinh(x))(a\cosh(x)+b\sinh(x))^{3/2} \\
&\quad -\frac{6i(a^2-b^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|\,2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{5\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.87

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \frac{(a\cosh(x)+b\sinh(x))\left(6a(a^2-b^2)+2ab^2\cosh(2x)+b(a^2+b^2)\sinh(2x)\right)-\frac{3(a-b)^2(a+b)^2}{\cdots}}{\cdots}$$

[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(5/2),x]

[Out] ((a*Cosh[x] + b*Sinh[x])*(6*a*(a^2 - b^2) + 2*a*b^2*Cosh[2*x] + b*(a^2 + b^2)*Sinh[2*x]) - (3*(a - b)^2*(a + b)^2*(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x + ArcTanh[b/a]] + Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a*Cosh[x + ArcTanh[b/a]] - b*Sinh[x + ArcTanh[b/a]])))/(a*Sqrt[1 - b^2/a^2]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2]))/(5*b*Sqrt[a*Cosh[x] + b*Sinh[x]])

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{((a+b)(a-b))^{\frac{3}{2}}\left(\frac{\cosh(x)^3}{3}-\cosh(x)\right)}{\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	42

[In] int((a*cosh(x)+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*((a+b)*(a-b))^(3/2)*(1/3*cosh(x)^3-cosh(x))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.66

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = -\frac{12\left(\sqrt{2}(a^2-b^2)\cosh{(x)}^2+2\sqrt{2}(a^2-b^2)\cosh{(x)}\sinh{(x)}+\sqrt{2}(a^2-b^2)\sinh{(x)}^2\right)\sqrt{a+b}\text{weierstrassZet}}{}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/10*(12*(sqrt(2)*(a^2 - b^2)*cosh(x)^2 + 2*sqrt(2)*(a^2 - b^2)*cosh(x)*si
nh(x) + sqrt(2)*(a^2 - b^2)*sinh(x)^2)*sqrt(a + b)*weierstrassZeta(-4*(a -
b)/(a + b), 0, weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x)
)) - ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)
^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 12*(a^2 - b^2)*cosh(x)^2 + 6*((a^2 + 2
*a*b + b^2)*cosh(x)^2 - 2*a^2 + 2*b^2)*sinh(x)^2 - a^2 + 2*a*b - b^2 + 4*((
a^2 + 2*a*b + b^2)*cosh(x)^3 - 6*(a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a*cosh(
x) + b*sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*cosh(x)+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx = \int (a\cosh{(x)}+b\sinh{(x)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + b*sinh(x))^(5/2), x)
```

Giac [F]

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx=\int (a\cosh{(x)}+b\sinh{(x)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((a*cosh(x) + b*sinh(x))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx=\int (a\cosh(x)+b\sinh(x))^{5/2}\,dx$$

```
[In] int((a*cosh(x) + b*sinh(x))^(5/2),x)
[Out] int((a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.593 $\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*
b))*EllipticF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x
))/(a^2-b^2)^(1/2))^(1/2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3157, 2720}

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = -\frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[In] Int[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*Sinh[
x])/Sqrt[a^2 - b^2]])/Sqrt[a*Cosh[x] + b*Sinh[x]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int\frac{1}{\sqrt{\cosh(x+i\tan^{-1}(a,-ib))}}\,dx}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

$$= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{\sqrt{a\cosh(x)+b\sinh(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int\frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$
$$= \frac{2\sqrt{\cosh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)\right)\operatorname{sech}\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)\sqrt{a\cosh(x)+b\sinh(x)}}{\sqrt{1-\frac{a^2}{b^2}}b}$$

```
[In] Integrate[1/Sqrt[a*Cosh[x] + b*Sinh[x]],x]
[Out] (2*Sqrt[Cosh[x + ArcTanh[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sin
h[x + ArcTanh[a/b]]^2]*Sech[x + ArcTanh[a/b]]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
(Sqrt[1 - a^2/b^2]*b)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\sqrt{-\sqrt{a^2-b^2}\,\sinh(x)^3}\,\arctan\left(\frac{\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\cosh(x)}{\sqrt{-\sqrt{a^2-b^2}\,\sinh(x)^3}}\right)}{\sqrt{\sinh(x)\sqrt{a^2-b^2}}\,\sinh(x)\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	97

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2)/(sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctan((
sinh(x)*(a^2-b^2)^(1/2))^(1/2)*cosh(x)/(-(a^2-b^2)^(1/2)*sinh(x)^3)^(1/2))/
sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}\left(-\frac{4(a-b)}{a+b},0,\cosh(x)+\sinh(x)\right)}{\sqrt{a+b}}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x))/sqr
t(a + b)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh\left(x\right) + b\sinh\left(x\right)}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*cosh(x) + b*sinh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh(x) + b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh\left(x\right) + b\sinh\left(x\right)}}\,dx$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^(1/2),x)
[Out] int(1/(a*cosh(x) + b*sinh(x))^(1/2), x)
```

3.594 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}} + \frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[Out] 2*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(1/2)+2*I*(cos(1/2*
I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*EllipticE(
sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*(a*cosh(x)+b*sinh(x))^(1/2)/(a^2-b
^2)/((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3155, 3157, 2719}

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx = \frac{2(a\sinh(x)+b\cosh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}} + \frac{2i\sqrt{a\cosh(x)+b\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]]) + ((2
*I)*EllipticE[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[a*Cosh[x] + b*Sinh[x]])/
((a^2 - b^2)*Sqrt[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(b\cosh(x)+a\sinh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{\int\sqrt{a\cosh(x)+b\sinh(x)}\,dx}{-a^2+b^2}\\
&= \frac{2(b\cosh(x)+a\sinh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{\sqrt{a\cosh(x)+b\sinh(x)}\int\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}\,dx}{(-a^2+b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}\\
&= \frac{2(b\cosh(x)+a\sinh(x))}{(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}+\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib))\middle|2\right)\sqrt{a\cosh(x)+b\sinh(x)}}{(a^2-b^2)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int\frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx=\frac{b\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cosh^2\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)\right)\sinh\left(x+\operatorname{arctanh}\left(\frac{b}{a}\right)\right)-\sqrt{-\sin}}{ab\sqrt{1-\frac{b^2}{a^2}}\sqrt{a\cos}}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-3/2),x]
```

```
[Out] (b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cosh[x + ArcTanh[b/a]]^2]*Sinh[x
+ ArcTanh[b/a]] - Sqrt[-Sinh[x + ArcTanh[b/a]]^2]*(2*a*Sqrt[1 - b^2/a^2]*Co
sh[x] - 2*a*Cosh[x + ArcTanh[b/a]] + b*Sinh[x + ArcTanh[b/a]]))/(a*b*Sqrt[1
 - b^2/a^2]*Sqrt[a*Cosh[x] + b*Sinh[x]]*Sqrt[-Sinh[x + ArcTanh[b/a]]^2])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\operatorname{arctanh}(\cosh(x))}{\sqrt{a^2-b^2}\,\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	33

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2-b^2)^(1/2)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)*arctanh(cosh(x))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{3/2}}\,dx = \frac{2\left(\left(\sqrt{2}(a+b)\cosh\left(x\right)^2+2\sqrt{2}(a+b)\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}(a+b)\sinh\right.\right.}{}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((sqrt(2)*(a + b)*cosh(x)^2 + 2*sqrt(2)*(a + b)*cosh(x)*sinh(x) + sqrt(2)
*(a + b)*sinh(x)^2 + sqrt(2)*(a - b))*sqrt(a + b)*weierstrassZeta(-4*(a - b
)/(a + b), 0, weierstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x))
) + 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2)*s
qrt(a*cosh(x) + b*sinh(x)))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3 + a^2*b - a*b
^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)*sinh(x) + (a^3
+ a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh{(x)} + b\sinh{(x)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(3/2),x)
[Out] Integral((a*cosh(x) + b*sinh(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh{(x)} + b\sinh{(x)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh{(x)} + b\sinh{(x)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((a*cosh(x) + b*sinh(x))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh{(x)} + b\sinh{(x)})^{3/2}}\,dx$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^(3/2),x)
[Out] int(1/(a*cosh(x) + b*sinh(x))^(3/2), x)
```

3.595 $\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 116

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2(b\cosh(x)+a\sinh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}} - \frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[Out] 2/3*(b*cosh(x)+a*sinh(x))/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(3/2)-2/3*I*(cos(
1/2*I*x-1/2*arctan(a,-I*b))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(a,-I*b))*Ellipt
icF(sin(1/2*I*x-1/2*arctan(a,-I*b)),2^(1/2))*((a*cosh(x)+b*sinh(x))/(a^2-b^
2)^(1/2))^(1/2)/(a^2-b^2)/(a*cosh(x)+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3155, 3157, 2720}

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2(a\sinh(x)+b\cosh(x))}{3(a^2-b^2)(a\cosh(x)+b\sinh(x))^{3/2}} - \frac{2i\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(a,-ib)),2\right)}{3(a^2-b^2)\sqrt{a\cosh(x)+b\sinh(x)}}$$

```
[In] Int[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

```
[Out] (2*(b*Cosh[x] + a*Sinh[x]))/(3*(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x])^(3/2)) -
 (((2*I)/3)*EllipticF[(I*x - ArcTan[a, (-I)*b])/2, 2]*Sqrt[(a*Cosh[x] + b*S
inh[x])/Sqrt[a^2 - b^2]])/((a^2 - b^2)*Sqrt[a*Cosh[x] + b*Sinh[x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\int\frac{1}{\sqrt{a\cosh(x)+b\sinh(x)}}\,dx}{3\left(a^2-b^2\right)}\\&=\frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}+\frac{\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}\int\frac{1}{\sqrt{\cosh\left(x+i\tan^{-1}(a,-ib)\right)}}\,dx}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}\\&=\frac{2(b\cosh(x)+a\sinh(x))}{3\left(a^2-b^2\right)(a\cosh(x)+b\sinh(x))^{3/2}}\\&\quad-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(a,-ib)\right),2\right)\sqrt{\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}}}{3\left(a^2-b^2\right)\sqrt{a\cosh(x)+b\sinh(x)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int\frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx=$$
$$-\frac{2\left(\sqrt{1-\frac{a^2}{b^2}}b(b\cosh(x)+a\sinh(x))+\sqrt{\cosh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\sinh^2\left(x+\operatorname{arctanh}\left(\frac{a}{b}\right)\right)\right)\operatorname{sec}\right.}{3\sqrt{1-\frac{a^2}{b^2}}b(-a+b)(a+b)(a\cosh(x)+b\sinh(x))^{3/2}}$$

```
[In] Integrate[(a*Cosh[x] + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*(Sqrt[1 - a^2/b^2]*b*(b*Cosh[x] + a*Sinh[x]) + Sqrt[Cosh[x + ArcTanh[a/
b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, -Sinh[x + ArcTanh[a/b]]^2]*Sech
[x + ArcTanh[a/b]]*(a*Cosh[x] + b*Sinh[x])^2))/(3*Sqrt[1 - a^2/b^2]*b*(-a +
 b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^(3/2))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.32

method	result	size
default	$-\frac{\cosh(x)}{(a^2-b^2)\sinh(x)\sqrt{-\sinh(x)\sqrt{a^2-b^2}}}$	37

```
[In] int(1/(a*cosh(x)+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cosh(x)/(a^2-b^2)/sinh(x)/(-sinh(x)*(a^2-b^2)^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 679, normalized size of antiderivative = 5.85

$$\int \frac{1}{(a\cosh(x)+b\sinh(x))^{5/2}}\,dx = \frac{2\left(\left(\sqrt{2}(a^2+2\,ab+b^2)\cosh(x)^4+4\sqrt{2}(a^2+2\,ab+b^2)\cosh(x)\sinh(x)^3\right.\right.}{3\left(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+(a^5+3\,a\right.}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((sqrt(2)*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*sqrt(2)*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^3 + sqrt(2)*(a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*sqrt(2)*(a^2
 - b^2)*cosh(x)^2 + 2*(3*sqrt(2)*(a^2 + 2*a*b + b^2)*cosh(x)^2 + sqrt(2)*(a
^2 - b^2))*sinh(x)^2 + 4*(sqrt(2)*(a^2 + 2*a*b + b^2)*cosh(x)^3 + sqrt(2)*(
a^2 - b^2)*cosh(x))*sinh(x) + sqrt(2)*(a^2 - 2*a*b + b^2))*sqrt(a + b)*weie
rstrassPInverse(-4*(a - b)/(a + b), 0, cosh(x) + sinh(x)) + 2*((a^2 + 2*a*b
 + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^2 + (a^2 + 2*a*b
+ b^2)*sinh(x)^3 - (a^2 - b^2)*cosh(x) + (3*(a^2 + 2*a*b + b^2)*cosh(x)^2 -
 a^2 + b^2)*sinh(x))*sqrt(a*cosh(x) + b*sinh(x)))/(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^
4 - b^5)*cosh(x)^4 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
^5)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 -
b^5)*sinh(x)^4 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh
(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 + 3*a
^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^
5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3 + (a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x) + b\sinh(x))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*cosh(x)+b*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((a*cosh(x) + b*sinh(x))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x) + b\sinh(x))^{5/2}}\,dx$$

```
[In] int(1/(a*cosh(x) + b*sinh(x))^(5/2),x)
[Out] int(1/(a*cosh(x) + b*sinh(x))^(5/2), x)
```

3.596 $\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}$$

```
[Out] a*cosh(d*x+c)/d+a*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2717, 2718}

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\sinh(c+dx)}{d}+\frac{a\cosh(c+dx)}{d}$$

```
[In] Int[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
[Out] (a*Cosh[c + d*x])/d + (a*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = a\int \cosh(c+dx)\,dx + a\int \sinh(c+dx)\,dx$$
$$= \frac{a\cosh(c+dx)}{d} + \frac{a\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx = \frac{a\cosh(c)\cosh(dx)}{d} + \frac{a\cosh(dx)\sinh(c)}{d} + \frac{a\cosh(c)\sinh(dx)}{d} + \frac{a\sinh(c)\sinh(dx)}{d}$$

```
[In] Integrate[a*Cosh[c + d*x] + a*Sinh[c + d*x],x]
[Out] (a*Cosh[c]*Cosh[d*x])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d
 + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

method	result	size
risch	$\frac{e^{dx+c}a}{d}$	12
gosper	$\frac{a(\cosh(dx+c)+\sinh(dx+c))}{d}$	19
derivativedivides	$\frac{a\cosh(dx+c)+a\sinh(dx+c)}{d}$	22
default	$\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	24
parts	$\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	24
meijerg	$\frac{(\cosh(c)\sqrt{\pi}\,a+\sqrt{\pi}\,\sinh(c)a)\sinh(dx)}{d\sqrt{\pi}}-\frac{i(-i\cosh(c)\sqrt{\pi}\,a-i\sqrt{\pi}\,\sinh(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}$	66

```
[In] int(a*cosh(d*x+c)+a*sinh(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*exp(d*x+c)*a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh(dx+c)+a\sinh(dx+c)}{d}$$

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="fricas")
[Out] (a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=a\left(\begin{cases}\frac{\sinh(c+dx)}{d} & \text{for } d\neq 0\\ x\cosh(c) & \text{otherwise}\end{cases}\right)+a\left(\begin{cases}\frac{\cosh(c+dx)}{d} & \text{for } d\neq 0\\ x\sinh(c) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x)
[Out] a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) + a*Piecewise((
cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx=\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$$

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="maxima")
[Out] a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx = \frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

```
[In] integrate(a*cosh(d*x+c)+a*sinh(d*x+c),x, algorithm="giac")
[Out] 1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x - c
)/d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int (a\cosh(c+dx)+a\sinh(c+dx))\,dx = \frac{a\,\mathrm{e}^{c+d\,x}}{d}$$

```
[In] int(a*cosh(c + d*x) + a*sinh(c + d*x),x)
[Out] (a*exp(c + d*x))/d
```

3.597 $\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^2}{2d}$$

```
[Out] 1/2*(a*cosh(d*x+c)+a*sinh(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3150}

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{(a\sinh(c+dx)+a\cosh(c+dx))^2}{2d}$$

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^2/(2*d)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^2}{2d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2(\cosh(c+dx)+\sinh(c+dx))^2}{2d}$$

[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2,x]

[Out] (a^2*(Cosh[c + d*x] + Sinh[c + d*x])^2)/(2*d)

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{a^2 e^{2dx+2c}}{2d}$	18
gosper	$\frac{a^2(\cosh(dx+c)+\sinh(dx+c))^2}{2d}$	24
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\cosh(dx+c)^2+a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	70
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\cosh(dx+c)^2+a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	70
parts	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{a^2\cosh(dx+c)^2}{d}$	75

[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/2*a^2/d*exp(2*d*x+2*c)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2\cosh(dx+c)+a^2\sinh(dx+c)}{2\,(d\cosh(dx+c)-d\sinh(dx+c))}$$

[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*(a^2*cosh(d*x + c) + a^2*sinh(d*x + c))/(d*cosh(d*x + c) - d*sinh(d*x + c))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx = \begin{cases} \frac{a^2\sinh^2(c+dx)}{d}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{d} & \text{for } d\neq 0\\ x(a\sinh(c)+a\cosh(c))^2 & \text{otherwise}\end{cases}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
[Out] Piecewise((a**2*sinh(c + d*x)**2/d + a**2*sinh(c + d*x)*cosh(c + d*x)/d, Ne
(d, 0)), (x*(a*sinh(c) + a*cosh(c))**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx = \frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) + \frac{a^2\cosh(dx+c)^2}{d}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*cosh(d*x + c)^2/d
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx = \frac{a^2e^{(2dx+2c)}}{2d}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
[Out] 1/2*a^2*e^(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^2\,dx=\frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{2\,d}$$

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^2,x)
[Out] (a^2*exp(2*c + 2*d*x))/(2*d)
```

3.598 $\int (a\cosh (c+dx)+a\sinh (c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int (a\cosh (c+dx)+a\sinh (c+dx))^3\,dx=\frac{(a\cosh (c+dx)+a\sinh (c+dx))^3}{3d}$$

```
[Out] 1/3*(a*cosh(d*x+c)+a*sinh(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3150}

$$\int (a\cosh (c+dx)+a\sinh (c+dx))^3\,dx=\frac{(a\sinh (c+dx)+a\cosh (c+dx))^3}{3d}$$

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^3/(3*d)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{(a\cosh (c+dx)+a\sinh (c+dx))^3}{3d}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{a^3(\cosh(c+dx)+\sinh(c+dx))^3}{3d}$$

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3,x]
[Out] (a^3*(Cosh[c + d*x] + Sinh[c + d*x])^3)/(3*d)
```

Maple [A] (verified)

Time = 6.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{a^3 e^{3dx+3c}}{3d}$	18
gosper	$\frac{a^3(\cosh(dx+c)+\sinh(dx+c))^3}{3d}$	24
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+a^3\cosh(dx+c)^3+a^3\sinh(dx+c)^3+a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$	74
default	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+a^3\cosh(dx+c)^3+a^3\sinh(dx+c)^3+a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$	74
parts	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)}{d}+\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{a^3\cosh(dx+c)^3}{d}+\frac{a^3\sinh(dx+c)^3}{d}$	82

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/3/d*a^3*exp(3*d*x+3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx =\frac{a^3\cosh(dx+c)^2+2\,a^3\cosh(dx+c)\sinh(dx+c)+a^3\sinh(dx+c)^2}{3\,(d\cosh(dx+c)-d\sinh(dx+c))}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x
 + c)^2)/(d*cosh(d*x + c) - d*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx = \begin{cases} \frac{a^3\sinh^3(c+dx)}{3d} + \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x(a\sinh(c)+a\cosh(c))^3 & \text{otherwise} \end{cases}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)**2*cosh(c + d*x
)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d + a**3*cosh(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(a*sinh(c) + a*cosh(c))**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(24) = 48$.

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.62

$$\begin{aligned}\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx &= \frac{a^3\cosh(dx+c)^3}{d} + \frac{a^3\sinh(dx+c)^3}{d} \\ &+ \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) \\ &+ \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)\end{aligned}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/d
 + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 1/24*a^3*(e^(
3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{a^3e^{(3\,dx+3\,c)}}{3\,d}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
[Out] 1/3*a^3*e^(3*d*x + 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^3\,dx=\frac{a^3\,\mathrm{e}^{3\,c+3\,d\,x}}{3\,d}$$

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^3,x)
[Out] (a^3*exp(3*c + 3*d*x))/(3*d)
```

3.599 $\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^n}{dn}$$

```
[Out] (a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3150}

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a\sinh(c+dx)+a\cosh(c+dx))^n}{dn}$$

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
```

```
[Out] (a*Cosh[c + d*x] + a*Sinh[c + d*x])^n/(d*n)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{(a\cosh(c+dx)+a\sinh(c+dx))^n}{dn}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx=\frac{(a(\cosh(c+dx)+\sinh(c+dx)))^n}{dn}$$

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^n,x]
[Out] (a*(Cosh[c + d*x] + Sinh[c + d*x]))^n/(d*n)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$	27
derivativedivides	$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$	27
default	$\frac{(a\cosh(dx+c)+a\sinh(dx+c))^n}{dn}$	27

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] (a*cosh(d*x+c)+a*sinh(d*x+c))^n/d/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx =\frac{\cosh(dnx+cn+n\log(a))+\sinh(dnx+cn+n\log(a))}{dn}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] (cosh(d*n*x + c*n + n*log(a)) + sinh(d*n*x + c*n + n*log(a)))/(d*n)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0 \wedge n=0 \\ x(a\sinh{(c)}+a\cosh{(c)})^n & \text{for } d=0 \\ x & \text{for } n=0 \\ \frac{(a\sinh{(c+dx)}+a\cosh{(c+dx)})^n}{dn} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**n,x)
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(a*sinh(c) + a*cosh(c))**n, Eq(d, 0)
), (x, Eq(n, 0)), ((a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx = \frac{a^n e^{((dx+c)n)}}{dn}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="maxima")
[Out] a^n*e^((d*x + c)*n)/(d*n)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a\cosh(c+dx)+a\sinh(c+dx))^n\,dx = \frac{\left(ae^{(dx+c)}\right)^n}{dn}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^n,x, algorithm="giac")
[Out] (a*e^(d*x + c))^n/(d*n)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int (a \cosh(c+dx) + a \sinh(c+dx))^n \, dx = \frac{\left(a\,\mathrm{e}^{c+d\,x}\right)^n}{d\,n}$$

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^n,x)
```

```
[Out] (a*exp(c + d*x))^n/(d*n)
```

3.600 $\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

```
[Out] -1/d/(a*cosh(d*x+c)+a*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3150}

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{1}{d(a\sinh(c+dx)+a\cosh(c+dx))}$$

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]
```

```
[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{1}{d(a\cosh(c+dx)+a\sinh(c+dx))}$$

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-1),x]
[Out] -(1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{e^{-dx-c}}{ad}$	18
gosper	$-\frac{1}{da(\cosh(dx+c)+\sinh(dx+c))}$	24
derivativedivides	$-\frac{1}{d(a\cosh(dx+c)+a\sinh(dx+c))}$	25
default	$-\frac{1}{d(a\cosh(dx+c)+a\sinh(dx+c))}$	25

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/a/d*exp(-d*x-c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{1}{ad\cosh(dx+c)+ad\sinh(dx+c)}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="fricas")
[Out] -1/(a*d*cosh(d*x + c) + a*d*sinh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \begin{cases} -\frac{1}{ad\sinh{(c+dx)}+ad\cosh{(c+dx)}} & \text{for } d\neq 0 \\ \frac{x}{a\sinh{(c)}+a\cosh{(c)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((-1/(a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(a*sin
h(c) + a*cosh(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{e^{(-dx-c)}}{ad}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^(-d*x - c)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = -\frac{e^{(-dx-c)}}{ad}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -e^(-d*x - c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=-\frac{e^{-c-dx}}{a\,d}$$

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x)),x)
[Out] -exp(- c - d*x)/(a*d)
```

3.601 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

```
[Out] -1/2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3150}

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2d(a\sinh(c+dx)+a\cosh(c+dx))^2}$$

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/2*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2d(a\cosh(c+dx)+a\sinh(c+dx))^2}$$

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-2),x]
[Out] -1/2*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^2)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-2dx-2c}}{2a^2d}$	18
gosper	$-\frac{1}{2a^2(\cosh(dx+c)+\sinh(dx+c))^2 d}$	24
derivativedivides	$-\frac{1}{2d(a\cosh(dx+c)+a\sinh(dx+c))^2}$	25
default	$-\frac{1}{2d(a\cosh(dx+c)+a\sinh(dx+c))^2}$	25

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/2/a^2/d*exp(-2*d*x-2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{1}{2\left(a^2d\cosh\left(dx+c\right)^2+2\,a^2d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2d\sinh\left(dx+c\right)^2\right)}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*s
inh(d*x + c)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = \begin{cases} -\frac{1}{2a^2d\sinh^2(c+dx)+4a^2d\sinh(c+dx)\cosh(c+dx)+2a^2d\cosh^2(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sinh(c + d*x)**2 + 4*a**2*d*sinh(c + d*x)*cosh(c +
d*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**2
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{e^{(-2\,dx-2\,c)}}{2\,a^2d}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*e^(-2*d*x - 2*c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^2}\,dx = -\frac{e^{-2\,c-2\,d\,x}}{2\,a^2\,d}$$

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^2,x)
[Out] -exp(- 2*c - 2*d*x)/(2*a^2*d)
```

3.602 $\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx=-\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

```
[Out] -1/3/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3150}

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx=-\frac{1}{3d(a\sinh(c+dx)+a\cosh(c+dx))^3}$$

```
[In] Int[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
```

```
[Out] -1/3*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=-\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{1}{3d(a\cosh(c+dx)+a\sinh(c+dx))^3}$$

```
[In] Integrate[(a*Cosh[c + d*x] + a*Sinh[c + d*x])^(-3),x]
[Out] -1/3*1/(d*(a*Cosh[c + d*x] + a*Sinh[c + d*x])^3)
```

Maple [A] (verified)

Time = 19.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{e^{-3dx-3c}}{3a^{3}d}$	18
gosper	$-\frac{1}{3a^{3}\left(\cosh(dx+c)+\sinh(dx+c)\right)^{3}d}$	24
derivativedivides	$-\frac{1}{3d\left(a\cosh(dx+c)+a\sinh(dx+c)\right)^{3}}$	25
default	$-\frac{1}{3d\left(a\cosh(dx+c)+a\sinh(dx+c)\right)^{3}}$	25

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/3/a^3/d*exp(-3*d*x-3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{1}{3\left(a^{3}d\cosh\left(dx+c\right)^{3}+3\,a^{3}d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,a^{3}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+a^{3}d\sinh\right.}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="fricas")
[Out] -1/3/(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3
*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*d*sinh(d*x + c)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(24) = 48$.

Time = 0.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = \begin{cases} -\frac{1}{3a^3d\sinh^3(c+dx)+9a^3d\sinh^2(c+dx)\cosh(c+dx)+9a^3d\sinh(c+dx)\cosh^2(c+dx)+3a^3d\cosh^3(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(a\sinh(c)+a\cosh(c))^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**3,x)
[Out] Piecewise((-1/(3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(c
 + d*x) + 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x)*
*3), Ne(d, 0)), (x/(a*sinh(c) + a*cosh(c))**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="maxima")
[Out] -1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx = -\frac{e^{(-3\,dx-3\,c)}}{3\,a^3d}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^3,x, algorithm="giac")
[Out] -1/3*e^(-3*d*x - 3*c)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a\cosh(c+dx)+a\sinh(c+dx))^3}\,dx=-\frac{e^{-3\,c-3\,d\,x}}{3\,a^3\,d}$$

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^3,x)
[Out] -exp(- 3*c - 3*d*x)/(3*a^3*d)
```

3.603 $\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}{d}$$

```
[Out] 2*(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3150}

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}{d}$$

```
[In] Int[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a(\cosh(c+dx)+\sinh(c+dx))}}{d}$$

```
[In] Integrate[Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
[Out] (2*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])/d
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2a\,\mathrm{e}^{dx+c}}{d\sqrt{a\,\mathrm{e}^{dx+c}}}$	23
gosper	$\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$	25
derivativedivides	$\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$	25
default	$\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{d}$	25

```
[In] int((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a/d*exp(d*x+c)/(a*exp(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx=\frac{2\sqrt{a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)}}{d}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/d
```

Sympy [F]

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \int \sqrt{a\sinh(c+dx)+a\cosh(c+dx)}\,dx$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \frac{2\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \frac{2\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate((a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(a)*e^(1/2*d*x + 1/2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \sqrt{a\cosh(c+dx)+a\sinh(c+dx)}\,dx = \frac{2\sqrt{a\,\mathrm{e}^{c+d\,x}}}{d}$$

```
[In] int((a*cosh(c + d*x) + a*sinh(c + d*x))^(1/2),x)
[Out] (2*(a*exp(c + d*x))^(1/2))/d
```

3.604 $\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}$$

```
[Out] -2/d/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{3150\}$

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\sqrt{a\sinh(c+dx)+a\cosh(c+dx)}}$$

```
[In] Int[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
```

```
[Out] -2/(d*Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]])
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2}{d\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx=-\frac{2}{d\sqrt{a(\cosh(c+dx)+\sinh(c+dx))}}$$

```
[In] Integrate[1/Sqrt[a*Cosh[c + d*x] + a*Sinh[c + d*x]],x]
[Out] -2/(d*Sqrt[a*(Cosh[c + d*x] + Sinh[c + d*x])])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{2}{d\sqrt{a\,\mathrm{e}^{dx+c}}}$	16
gosper	$-\frac{2}{d\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}$	25
derivativedivides	$-\frac{2}{d\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}$	25
default	$-\frac{2}{d\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}$	25

```
[In] int(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d/(a*exp(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx=-\frac{2\sqrt{a\cosh(dx+c)+a\sinh(dx+c)}}{ad\cosh(dx+c)+ad\sinh(dx+c)}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a*cosh(d*x + c) + a*sinh(d*x + c))/(a*d*cosh(d*x + c) + a*d*sinh(d*
x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

```
[In] integrate(1/(a*cosh(d*x+c)+a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] -2*e^(-1/2*d*x - 1/2*c)/(sqrt(a)*d)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a\sinh(c+dx)}}\,dx = -\frac{2}{d\,\sqrt{a\,\mathrm{e}^{c+d\,x}}}$$

```
[In] int(1/(a*cosh(c + d*x) + a*sinh(c + d*x))^(1/2),x)
[Out] -2/(d*(a*exp(c + d*x))^(1/2))
```

3.605 $\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 24

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=-\frac{a\cosh(c+dx)}{d}+\frac{a\sinh(c+dx)}{d}$$

```
[Out] -a*cosh(d*x+c)/d+a*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2717, 2718}

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx=\frac{a\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d}$$

```
[In] Int[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d) + (a*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = a\int \cosh(c+dx)\,dx - a\int \sinh(c+dx)\,dx$$
$$= -\frac{a\cosh(c+dx)}{d} + \frac{a\sinh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int (a\cosh(c+dx) - a\sinh(c+dx))\,dx = -\frac{a\cosh(c)\cosh(dx)}{d} + \frac{a\cosh(dx)\sinh(c)}{d} + \frac{a\cosh(c)\sinh(dx)}{d} - \frac{a\sinh(c)\sinh(dx)}{d}$$

```
[In] Integrate[a*Cosh[c + d*x] - a*Sinh[c + d*x],x]
[Out] -((a*Cosh[c]*Cosh[d*x])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x]
)/d - (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{a\,\mathrm{e}^{-dx-c}}{d}$	16
gosper	$-\frac{a(\cosh(dx+c)-\sinh(dx+c))}{d}$	22
derivativedivides	$\frac{a\sinh(dx+c)-a\cosh(dx+c)}{d}$	23
default	$-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	25
parts	$-\frac{a\cosh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}$	25
meijerg	$\frac{(\cosh(c)\sqrt{\pi}\,a-\sqrt{\pi}\,\sinh(c)a)\sinh(dx)}{d\sqrt{\pi}}-\frac{i(i\cosh(c)\sqrt{\pi}\,a-i\sqrt{\pi}\,\sinh(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}$	67

```
[In] int(a*cosh(d*x+c)-a*sinh(d*x+c),x,method=_RETURNVERBOSE)
[Out] -a/d*exp(-d*x-c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx = -\frac{a}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)}$$

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="fricas")
[Out] -a/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx = a\left(\begin{cases}\frac{\sinh\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\cosh\left(c\right) & \text{otherwise}\end{cases}\right) - a\left(\begin{cases}\frac{\cosh\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\sinh\left(c\right) & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x)
[Out] a*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) - a*Piecewise((
cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx = -\frac{a\cosh\left(dx+c\right)}{d}+\frac{a\sinh\left(dx+c\right)}{d}$$

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="maxima")
[Out] -a*cosh(d*x + c)/d + a*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx = -\frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

```
[In] integrate(a*cosh(d*x+c)-a*sinh(d*x+c),x, algorithm="giac")
[Out] -1/2*a*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(e^(d*x + c)/d - e^(-d*x -
c)/d)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (a\cosh(c+dx)-a\sinh(c+dx))\,dx = -\frac{a\,\mathrm{e}^{-c-d\,x}}{d}$$

```
[In] int(a*cosh(c + d*x) - a*sinh(c + d*x),x)
[Out] -(a*exp(- c - d*x))/d
```

3.606 $\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

```
[Out] -1/2*(a*cosh(d*x+c)-a*sinh(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3150}

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]
```

```
[Out] -1/2*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^2}{2d}$$

[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2,x]

[Out] -1/2*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2/d

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{a^{2} {\mathrm e}^{-2 d x -2 c}}{2 d}$	18
gosper	$-\frac{a^{2} \left(\cosh \left(d x +c \right)-\sinh \left(d x +c \right)\right)^{2}}{2 d}$	26
derivativedivides	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-a^{2} \cosh \left(d x +c \right)^{2}+a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}$	71
default	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-a^{2} \cosh \left(d x +c \right)^{2}+a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}$	71
parts	$\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{a^{2} \left(\frac{\cosh \left(d x +c \right) \sinh \left(d x +c \right)}{2}-\frac{d x}{2}-\frac{c}{2}\right)}{d}-\frac{a^{2} \cosh \left(d x +c \right)^{2}}{d}$	76

[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/d*exp(-2*d*x-2*c)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = -\frac{a^2}{2\left(d\cosh\left(dx+c\right)^2+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^2\right)}$$

[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*a^2/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = \begin{cases} -\frac{a^2\sinh^2(c+dx)}{d}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{d} & \text{for } d\neq 0\\ x(-a\sinh(c)+a\cosh(c))^2 & \text{otherwise}\end{cases}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
[Out] Piecewise((-a**2*sinh(c + d*x)**2/d + a**2*sinh(c + d*x)*cosh(c + d*x)/d, N
e(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.30

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = \frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) - \frac{a^2\cosh(dx+c)^2}{d}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*a^2*(4*x - e^(
2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - a^2*cosh(d*x + c)^2/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx = -\frac{a^2e^{(-2dx-2c)}}{2d}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*a^2*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^2\,dx=-\frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{2\,d}$$

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^2,x)
[Out] -(a^2*exp(- 2*c - 2*d*x))/(2*d)
```

3.607 $\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

```
[Out] -1/3*(a*cosh(d*x+c)-a*sinh(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3150}

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]
[Out] -1/3*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{(a\cosh(c+dx)-a\sinh(c+dx))^3}{3d}$$

[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3,x]

[Out] -1/3*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3/d

Maple [A] (verified)

Time = 6.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	siz
risch	$-\frac{a^3 e^{-3dx-3c}}{3d}$	18
gosper	$-\frac{a^3(\cosh(dx+c)-\sinh(dx+c))^3}{3d}$	26
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)-a^3\cosh(dx+c)^3+a^3\sinh(dx+c)^3-a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$	76
default	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)-a^3\cosh(dx+c)^3+a^3\sinh(dx+c)^3-a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$	76
parts	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)}{d}-\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}-\frac{a^3\cosh(dx+c)^3}{d}+\frac{a^3\sinh(dx+c)^3}{d}$	84

[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -1/3/d*a^3*exp(-3*d*x-3*c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^3\,dx=-\frac{a^3}{3\left(d\cosh(dx+c)^3+3\,d\cosh(dx+c)^2\sinh(dx+c)+3\,d\cosh(dx+c)\sinh(dx+c)^2+d\sinh(dx+c)^3\right.}$$

[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")

[Out] -1/3*a^3/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(22) = 44$.

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx = \begin{cases} \frac{a^3\sinh^3(c+dx)}{3d} - \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{a^3\cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(-a\sinh(c) + a\cosh(c))^3 & \text{otherwise} \end{cases}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**3/(3*d) - a**3*sinh(c + d*x)**2*cosh(c + d*x
)/d + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d - a**3*cosh(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(-a*sinh(c) + a*cosh(c))**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 5.44

$$\begin{aligned}\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx &= -\frac{a^3\cosh(dx+c)^3}{d} + \frac{a^3\sinh(dx+c)^3}{d} \\ &\quad + \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) \\ &\quad - \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)\end{aligned}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^3*cosh(d*x + c)^3/d + a^3*sinh(d*x + c)^3/d + 1/24*a^3*(e^(3*d*x + 3*c)/
d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) - 1/24*a^3*(e^
(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx = -\frac{a^3e^{(-3\,dx-3\,c)}}{3\,d}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
[Out] -1/3*a^3*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^3\,dx = -\frac{a^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{3\,d}$$

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^3,x)
[Out] -(a^3*exp(- 3*c - 3*d*x))/(3*d)
```

3.608 $\int (a\cosh(c+dx) - a\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 28

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^n\,dx = -\frac{(a\cosh(c+dx) - a\sinh(c+dx))^n}{dn}$$

```
[Out] -(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3150}

$$\int (a\cosh(c+dx) - a\sinh(c+dx))^n\,dx = -\frac{(a\cosh(c+dx) - a\sinh(c+dx))^n}{dn}$$

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
```

```
[Out] -((a*Cosh[c + d*x] - a*Sinh[c + d*x])^n/(d*n))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a\cosh(c+dx) - a\sinh(c+dx))^n}{dn}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{(a(\cosh(c+dx)-\sinh(c+dx)))^n}{dn}$$

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^n,x]
[Out] -((a*(Cosh[c + d*x] - Sinh[c + d*x]))^n/(d*n))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$	29
derivativedivides	$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$	29
default	$-\frac{(a\cosh(dx+c)-a\sinh(dx+c))^n}{dn}$	29

```
[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] -(a*cosh(d*x+c)-a*sinh(d*x+c))^n/d/n
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx = -\frac{\cosh(-dnx-cn+n\log(a))+\sinh(-dnx-cn+n\log(a))}{dn}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] -(cosh(-d*n*x - c*n + n*log(a)) + sinh(-d*n*x - c*n + n*log(a)))/(d*n)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0 \wedge n=0 \\ x(-a\sinh(c)+a\cosh(c))^n & \text{for } d=0 \\ x & \text{for } n=0 \\ -\frac{(-a\sinh(c+dx)+a\cosh(c+dx))^n}{dn} & \text{otherwise} \end{cases}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(-a*sinh(c) + a*cosh(c))**n, Eq(d, 0
)), (x, Eq(n, 0)), (-(-a*sinh(c + d*x) + a*cosh(c + d*x))**n/(d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx = -\frac{a^n e^{(-(dx+c)n)}}{dn}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -a^n*e^(-(d*x + c)*n)/(d*n)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx = -\frac{e^{(-dnx-cn+n\log(a))}}{dn}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -e^(-d*n*x - c*n + n*log(a))/(d*n)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (a\cosh(c+dx)-a\sinh(c+dx))^n\,dx=-\frac{\left(a\,\mathrm{e}^{-c-d\,x}\right)^n}{d\,n}$$

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^n,x)
[Out] -(a*exp(- c - d*x))^n/(d*n)
```

3.609 $\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

```
[Out] 1/d/(a*cosh(d*x+c)-a*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3150}

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]
```

```
[Out] 1/(d*(a*Cosh[c + d*x] - a*Sinh[c + d*x]))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{d(a\cosh(c+dx)-a\sinh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{1}{ad\cosh(c+dx)-ad\sinh(c+dx)}$$

[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-1),x]

[Out] (a*d*Cosh[c + d*x] - a*d*Sinh[c + d*x])^(-1)

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{e^{dx+c}}{ad}$	14
derivativedivides	$-\frac{2}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	22
default	$-\frac{2}{da\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	22
gosper	$\frac{1}{da(\cosh(dx+c)-\sinh(dx+c))}$	25

[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/a/d*exp(d*x+c)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{\cosh(dx+c)+\sinh(dx+c)}{ad}$$

[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="fricas")

[Out] (cosh(d*x + c) + sinh(d*x + c))/(a*d)

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \begin{cases} \frac{1}{-ad\sinh{(c+dx)}+ad\cosh{(c+dx)}} & \text{for } d\neq 0 \\ \frac{x}{-a\sinh{(c)}+a\cosh{(c)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((1/(-a*d*sinh(c + d*x) + a*d*cosh(c + d*x)), Ne(d, 0)), (x/(-a*si
nh(c) + a*cosh(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \frac{e^{(dx+c)}}{ad}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(d*x + c)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \frac{e^{(dx+c)}}{ad}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] e^(d*x + c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=\frac{\mathrm{e}^{c+d\,x}}{a\,d}$$

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x)),x)
[Out] exp(c + d*x)/(a*d)
```

3.610 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx=\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

```
[Out] 1/2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3150}

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx=\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
```

```
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{1}{2d(a\cosh(c+dx)-a\sinh(c+dx))^2}$$

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-2),x]
[Out] 1/(2*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^2)
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{e^{2dx+2c}}{2a^2d}$	18
gosper	$\frac{1}{2a^2(\cosh(dx+c)-\sinh(dx+c))^2 d}$	26
derivativedivides	$\frac{\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{d\,a^2}$	36
default	$\frac{\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{d\,a^2}$	36

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2/a^2/d*exp(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{\cosh(dx+c)+\sinh(dx+c)}{2\left(a^2d\cosh(dx+c)-a^2d\sinh(dx+c)\right)}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(cosh(d*x + c) + sinh(d*x + c))/(a^2*d*cosh(d*x + c) - a^2*d*sinh(d*x +
 c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \begin{cases} \frac{1}{2a^2d\sinh^2(c+dx)-4a^2d\sinh(c+dx)\cosh(c+dx)+2a^2d\cosh^2(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{(-a\sinh(c)+a\cosh(c))^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**2,x)
[Out] Piecewise((1/(2*a**2*d*sinh(c + d*x)**2 - 4*a**2*d*sinh(c + d*x)*cosh(c + d
*x) + 2*a**2*d*cosh(c + d*x)**2), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**2
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*e^(2*d*x + 2*c)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx = \frac{e^{(2\,dx+2\,c)}}{2\,a^2d}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^2,x, algorithm="giac")
[Out] 1/2*e^(2*d*x + 2*c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^2}\,dx=\frac{e^{2\,c+2\,d\,x}}{2\,a^2\,d}$$

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^2,x)
[Out] exp(2*c + 2*d*x)/(2*a^2*d)
```

3.611 $\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

```
[Out] 1/3/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3150}

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

```
[In] Int[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
```

```
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{1}{3d(a\cosh(c+dx)-a\sinh(c+dx))^3}$$

```
[In] Integrate[(a*Cosh[c + d*x] - a*Sinh[c + d*x])^(-3),x]
[Out] 1/(3*d*(a*Cosh[c + d*x] - a*Sinh[c + d*x])^3)
```

Maple [A] (verified)

Time = 19.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{e^{3dx+3c}}{3a^3d}$	18
gosper	$\frac{1}{3a^3(\cosh(dx+c)-\sinh(dx+c))^3d}$	26
derivativedivides	$\frac{-\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{8}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{a^3d}$	55
default	$\frac{-\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{8}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}}{a^3d}$	55

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/3/a^3/d*exp(3*d*x+3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}{3\left(a^3d\cosh(dx+c)-a^3d\sinh(dx+c)\right)}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)/(a^
3*d*cosh(d*x + c) - a^3*d*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(22) = 44$.

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \begin{cases} \frac{1}{-3a^3d\sinh^3{(c+dx)}+9a^3d\sinh^2{(c+dx)}\cosh{(c+dx)}-9a^3d\sinh{(c+dx)}\cosh^2{(c+dx)}+3a^3d\cosh^3{(c+dx)}} & \text{for } d\neq 0 \\ \frac{x}{(-a\sinh{(c)}+a\cosh{(c)})^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**3,x)
[Out] Piecewise((1/(-3*a**3*d*sinh(c + d*x)**3 + 9*a**3*d*sinh(c + d*x)**2*cosh(c
 + d*x) - 9*a**3*d*sinh(c + d*x)*cosh(c + d*x)**2 + 3*a**3*d*cosh(c + d*x)*
*3), Ne(d, 0)), (x/(-a*sinh(c) + a*cosh(c))**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{e^{(3\,dx+3\,c)}}{3\,a^3d}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="maxima")
[Out] 1/3*e^(3*d*x + 3*c)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx = \frac{e^{(3\,dx+3\,c)}}{3\,a^3d}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^3,x, algorithm="giac")
[Out] 1/3*e^(3*d*x + 3*c)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\cosh(c+dx)-a\sinh(c+dx))^3}\,dx=\frac{\mathrm{e}^{3\,c+3\,d\,x}}{3\,a^3\,d}$$

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^3,x)
[Out] exp(3*c + 3*d*x)/(3*a^3*d)
```

3.612 $\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

```
[Out] -2*(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3150}

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

```
[In] Int[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
[Out] (-2*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=-\frac{2\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}{d}$$

[In] Integrate[Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]

[Out] (-2*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])/d

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{2\sqrt{{\mathrm e}^{-dx-c}a}}{d}$	19
gosper	$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$	26
derivativedivides	$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$	26
default	$-\frac{2\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}{d}$	26

[In] int((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(exp(-d*x-c)*a)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx=-\frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}}{d}$$

[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/d

Sympy [F]

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = \int \sqrt{-a\sinh{(c+dx)}+a\cosh{(c+dx)}}\,dx$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate((a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(a)*e^(-1/2*d*x - 1/2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sqrt{a\cosh(c+dx)-a\sinh(c+dx)}\,dx = -\frac{2\sqrt{a\,\mathrm{e}^{-c-d\,x}}}{d}$$

```
[In] int((a*cosh(c + d*x) - a*sinh(c + d*x))^(1/2),x)
[Out] -(2*(a*exp(- c - d*x))^(1/2))/d
```

3.613 $\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx = \frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

```
[Out] 2/d/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3150}

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx = \frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

```
[In] Int[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
[Out] 2/(d*Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]])
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{d\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2}{d\sqrt{a(\cosh(c+dx)-\sinh(c+dx))}}$$

```
[In] Integrate[1/Sqrt[a*Cosh[c + d*x] - a*Sinh[c + d*x]],x]
[Out] 2/(d*Sqrt[a*(Cosh[c + d*x] - Sinh[c + d*x])])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{2}{\sqrt{{\rm e}^{-dx-c}a}\,d}$	19
gosper	$\frac{2}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$	26
derivativedivides	$\frac{2}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$	26
default	$\frac{2}{d\sqrt{a\cosh(dx+c)-a\sinh(dx+c)}}$	26

```
[In] int(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(exp(-d*x-c)*a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}{ad}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c))/(
a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{-a\sinh\left(c+dx\right)+a\cosh\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(-a*sinh(c + d*x) + a*cosh(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}d}$$

```
[In] integrate(1/(a*cosh(d*x+c)-a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\cosh(c+dx)-a\sinh(c+dx)}}\,dx=\frac{2\,\mathrm{e}^{c+d\,x}\,\sqrt{a\,\mathrm{e}^{-c-d\,x}}}{a\,d}$$

```
[In] int(1/(a*cosh(c + d*x) - a*sinh(c + d*x))^(1/2),x)
[Out] (2*exp(c + d*x)*(a*exp(- c - d*x))^(1/2))/(a*d)
```

3.614 $\int (a\mathbf{sech}(x) + b\tanh(x))^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 124

$$\int (a\text{sech}(x)+b\tanh(x))^5\,dx=\frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)\arctan(\sinh(x))+b^5\log(\cosh(x))-\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)-\frac{1}{4}\text{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4-\frac{1}{8}\text{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\sinh(x)\right)$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))+b^5*ln(cosh(x))-1/8*a*b^2*(
3*a^2+7*b^2)*sinh(x)-1/4*sech(x)^4*(b-a*sinh(x))*(a+b*sinh(x))^4-1/8*sech(x
)^2*(a+b*sinh(x))^2*(2*b*(a^2+2*b^2)-a*(3*a^2+5*b^2)*sinh(x))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4476, 2747, 753, 833, 788, 649, 210, 266}

$$\int (a\text{sech}(x)+b\tanh(x))^5\,dx=-\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)-\frac{1}{8}\text{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\sinh(x)\right)+\frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)\arctan(\sinh(x))-\frac{1}{4}\text{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4+b^5\log(\cosh(x))$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^5,x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/8 + b^5*Log[Cosh[x]] - (a
*b^2*(3*a^2 + 7*b^2)*Sinh[x])/8 - (Sech[x]^4*(b - a*Sinh[x])*(a + b*Sinh[x]
)^4)/4 - (Sech[x]^2*(a + b*Sinh[x])^2*(2*b*(a^2 + 2*b^2) - a*(3*a^2 + 5*b^2
)*Sinh[x]))/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^5(x)(a+b\sinh(x))^5\,dx \\
&= -\left(b^5\,\text{Subst}\left(\int \frac{(a+x)^5}{(-b^2-x^2)^3}\,dx, x, b\sinh(x)\right)\right) \\
&= -\frac{1}{4}\operatorname{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 \\
&\quad -\frac{1}{4}b^3\,\text{Subst}\left(\int \frac{(a+x)^3\left(-3a^2-4b^2+ax\right)}{(-b^2-x^2)^2}\,dx, x, b\sinh(x)\right) \\
&= -\frac{1}{4}\operatorname{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 \\
&\quad -\frac{1}{8}\operatorname{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\sinh(x)\right) \\
&\quad -\frac{1}{8}b\,\text{Subst}\left(\int \frac{(a+x)\left(3a^4+7a^2b^2+8b^4-a(3a^2+7b^2)\,x\right)}{-b^2-x^2}\,dx, x, b\sinh(x)\right) \\
&= -\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)-\frac{1}{4}\operatorname{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 \\
&\quad -\frac{1}{8}\operatorname{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\sinh(x)\right) \\
&\quad +\frac{1}{8}b\,\text{Subst}\left(\int \frac{-ab^2(3a^2+7b^2)-a(3a^4+7a^2b^2+8b^4)-(3a^4+7a^2b^2+8b^4-a^2(3a^2+7b^2))\,x}{-b^2-x^2}\,d \right. \\
&= -\frac{1}{8}ab^2\left(3a^2+7b^2\right)\sinh(x)-\frac{1}{4}\operatorname{sech}^4(x)(b-a\sinh(x))(a+b\sinh(x))^4 \\
&\quad -\frac{1}{8}\operatorname{sech}^2(x)(a+b\sinh(x))^2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\sinh(x)\right) \\
&\quad -b^5\,\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x)\right) \\
&\quad -\frac{1}{8}\left(ab\left(3a^4+10a^2b^2+15b^4\right)\right)\text{Subst}\left(\int \frac{1}{-b^2-x^2}\,dx, x, b\sinh(x)\right)
\end{aligned}$$

$$= \frac{1}{8}a(3a^4 + 10a^2b^2 + 15b^4)\arctan(\sinh(x)) + b^5\log(\cosh(x)) - \frac{1}{8}ab^2(3a^2 + 7b^2)\sinh(x) - \frac{1}{4}\text{sech}^4(x)(b - a\sinh(x))(a + b\sinh(x))^4 - \frac{1}{8}\text{sech}^2(x)(a + b\sinh(x))^2\left(2b(a^2 + 2b^2) - a(3a^2 + 5b^2)\sinh(x)\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 355 vs. 2(124) = 248.

Time = 1.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.86

$$\int (a\text{sech}(x) + b\tanh(x))^5\,dx$$

$$= \frac{4\text{sech}^4(x)(b + a\sinh(x))(a + b\sinh(x))^6 + \frac{2\text{sech}^2(x)(a+b\sinh(x))^6(6a^2b-2b^3+a(3a^2-5b^2)\sinh(x))}{a^2+b^2} + \frac{b\left(\frac{(a^2+b^2)^2((3a^5+10\ldots}{\ldots}\right.}{\ldots}}{\ldots}$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^5,x]
```

```
[Out] (4*Sech[x]^4*(b + a*Sinh[x])*(a + b*Sinh[x])^6 + (2*Sech[x]^2*(a + b*Sinh[x
])^6*(6*a^2*b - 2*b^3 + a*(3*a^2 - 5*b^2)*Sinh[x]))/(a^2 + b^2) + (b*(((a^2
 + b^2)^2*((3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 8*b^4*Sqrt[-b^2])*Log[Sqrt[-b^2
] - b*Sinh[x]] + (-3*a^5 - 10*a^3*b^2 - 15*a*b^4 + 8*(-b^2)^(5/2))*Log[Sqrt
[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] - 10*a*b*(9*a^6 + 6*a^4*b^2 + 8*a^2*b^4 +
3*b^6)*Sinh[x] - 8*b^2*(15*a^6 - 4*a^4*b^2 + 2*a^2*b^4 + b^6)*Sinh[x]^2 + 1
0*a*b^3*(-9*a^4 + 8*a^2*b^2 + b^4)*Sinh[x]^3 + 4*b^4*(-9*a^4 + 12*a^2*b^2 +
 b^4)*Sinh[x]^4 + 2*a*b^5*(-3*a^2 + 5*b^2)*Sinh[x]^5))/(a^2 + b^2))/(16*(a^
2 + b^2))
```

Maple [A] (verified)

Time = 198.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

method	result
parts	$a^5\left(\left(\frac{\text{sech}(x)^3}{4} + \frac{3\,\text{sech}(x)}{8}\right)\tanh(x) + \frac{3\arctan(e^x)}{4}\right) + b^5\left(-\frac{\tanh(x)^4}{4} - \frac{\tanh(x)^2}{2} - \frac{\ln(\tanh(x)-1)}{2} - \frac{\ln(1+\tanh(x}{2}\right.$
default	$a^5\left(\left(\frac{\text{sech}(x)^3}{4} + \frac{3\,\text{sech}(x)}{8}\right)\tanh(x) + \frac{3\arctan(e^x)}{4}\right) - \frac{5a^4b}{4\cosh(x)^4} + 10a^3b^2\left(-\frac{\sinh(x)}{3\cosh(x)^4} + \frac{\left(\frac{\text{sech}(x)^3}{4} + \frac{3\,\text{sech}(x)}{8}\right)}{3}\right.$
risch	$-b^5x + \frac{e^x(3a^5e^{6x}+10a^3b^2e^{6x}-25a\,b^4e^{6x}-80a^2b^3e^{5x}+16b^5e^{5x}+11a^5e^{4x}-70a^3b^2e^{4x}+15a\,b^4e^{4x}-80\,e^{3x}a^4b+16b^5e^{3x}-11a^5e^{2x}+7}{4(1+e^{2x})^4}$

```
[In] int((a*sech(x)+b*tanh(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*((1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x)))+b^5*(-1/4*tanh
(x)^4-1/2*tanh(x)^2-1/2*ln(tanh(x)-1)-1/2*ln(1+tanh(x)))-5/4*a^4*b*sech(x)^
4+10*a^3*b^2*(-1/3*sinh(x)/cosh(x)^4+1/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x
)+1/4*arctan(exp(x)))+5/2*a^2*b^3*tanh(x)^4+5*a*b^4*(-sinh(x)^3/cosh(x)^4-s
inh(x)/cosh(x)^4+(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2040 vs. $2(116) = 232$.

Time = 0.27 (sec) , antiderivative size = 2040, normalized size of antiderivative = 16.45

$$\int (a\text{sech}(x) + b\tanh(x))^5\,dx = \text{Too large to display}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^5*x*cosh(x)^8 + 4*b^5*x*sinh(x)^8 - (3*a^5 + 10*a^3*b^2 - 25*a*b^
4)*cosh(x)^7 + (32*b^5*x*cosh(x) - 3*a^5 - 10*a^3*b^2 + 25*a*b^4)*sinh(x)^7
 + 16*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^6 + (112*b^5*x*cosh(x)^2 + 16*b^5*x
 + 80*a^2*b^3 - 16*b^5 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x))*sinh(x)
^6 + 4*b^5*x - (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + (224*b^5*x*cosh
(x)^3 - 11*a^5 + 70*a^3*b^2 - 15*a*b^4 - 21*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)
*cosh(x)^2 + 96*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 8*(3*b^5*x +
 10*a^4*b - 2*b^5)*cosh(x)^4 + (280*b^5*x*cosh(x)^4 + 24*b^5*x + 80*a^4*b -
 16*b^5 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^3 + 240*(b^5*x + 5*a^2
*b^3 - b^5)*cosh(x)^2 - 5*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)
^4 + (11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (224*b^5*x*cosh(x)^5 + 11
*a^5 - 70*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^4
 + 320*(b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^3 - 10*(11*a^5 - 70*a^3*b^2 + 15*a
*b^4)*cosh(x)^2 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x))*sinh(x)^3 + 16*(
b^5*x + 5*a^2*b^3 - b^5)*cosh(x)^2 + (112*b^5*x*cosh(x)^6 + 16*b^5*x - 21*(
3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^5 + 80*a^2*b^3 - 16*b^5 + 240*(b^5*x
 + 5*a^2*b^3 - b^5)*cosh(x)^4 - 10*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)
^3 + 48*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^2 + 3*(11*a^5 - 70*a^3*b^2 + 1
5*a*b^4)*cosh(x))*sinh(x)^2 - ((3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^8 +
8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 + 10*a^3*b^2 +
 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 4*(3*a
^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*s
inh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a
^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 6*(
3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^5 + 30*a^3*b^2 + 45*a*b^4
 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(3*a^5 + 10*a^3*b^2 +
15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(
```

```
x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)
^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 3*a^5 + 10*a^3*b^2 +
15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(3*a^5 + 10*a^3
*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 + 10*a^3*b^2 + 15*a*b^4)*
cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(3*a^5 + 10*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(
x))*arctan(cosh(x) + sinh(x)) + (3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x) - 4
*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sinh(x)^8 + 4*b^5*cosh(x)^6
 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cosh(x)^2 + b^5)*sinh(x)^6
+ 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4
 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*cosh(x)^5 + 10*b^5*cosh(x
)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6 + 15*b^5*cosh(x)^4 + 9*
b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 + 3*b^5*cosh(x)^5 + 3*b^5
*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (32
*b^5*x*cosh(x)^7 - 7*(3*a^5 + 10*a^3*b^2 - 25*a*b^4)*cosh(x)^6 + 96*(b^5*x
+ 5*a^2*b^3 - b^5)*cosh(x)^5 + 3*a^5 + 10*a^3*b^2 - 25*a*b^4 - 5*(11*a^5 -
70*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 32*(3*b^5*x + 10*a^4*b - 2*b^5)*cosh(x)^
3 + 3*(11*a^5 - 70*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 32*(b^5*x + 5*a^2*b^3 -
b^5)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*
cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x
)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)
^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
+ 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^5\,dx = \int (a\,\text{sech}\,(x) + b\tanh\,(x))^5\;dx$$

```
[In] integrate((a*sech(x)+b*tanh(x))**5,x)
[Out] Integral((a*sech(x) + b*tanh(x))**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.25

$$\int (a\text{sech}(x) + b\tanh(x))^5\,dx$$
$$= \frac{5}{2}\,a^2b^3\tanh\left(x\right)^4 + b^5\left(x + \frac{4\left(e^{(-2\,x)} + e^{(-4\,x)} + e^{(-6\,x)}\right)}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} + \log\left(e^{(-2\,x)} + 1\right)\right)$$
$$- \frac{5}{4}\,ab^4\left(\frac{5\,e^{(-x)} - 3\,e^{(-3\,x)} + 3\,e^{(-5\,x)} - 5\,e^{(-7\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} + 3\arctan\left(e^{(-x)}\right)\right)$$
$$+ \frac{1}{4}\,a^5\left(\frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} - 3\arctan\left(e^{(-x)}\right)\right)$$
$$+ \frac{5}{2}\,a^3b^2\left(\frac{e^{(-x)} - 7\,e^{(-3\,x)} + 7\,e^{(-5\,x)} - e^{(-7\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} - \arctan\left(e^{(-x)}\right)\right) - \frac{20\,a^4b}{\left(e^{(-x)} + e^{x}\right)^4}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*a^2*b^3*tanh(x)^4 + b^5*(x + 4*(e^(-2*x) + e^(-4*x) + e^(-6*x))/(4*e^(-
2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + log(e^(-2*x) + 1)) - 5/4*a
*b^4*((5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x))/(4*e^(-2*x) + 6*e^(
-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 3*arctan(e^(-x))) + 1/4*a^5*((3*e^(-x)
 + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(
-6*x) + e^(-8*x) + 1) - 3*arctan(e^(-x))) + 5/2*a^3*b^2*((e^(-x) - 7*e^(-3*
x) + 7*e^(-5*x) - e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x
) + 1) - arctan(e^(-x))) - 20*a^4*b/(e^(-x) + e^x)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.94

$$\int (a\text{sech}(x) + b\tanh(x))^5\,dx = \frac{1}{2}\,b^5\log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)$$
$$+ \frac{1}{16}\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^5 + 10\,a^3b^2 + 15\,ab^4\right)$$
$$- \frac{3\,b^5\left(e^{(-x)} - e^{x}\right)^4 + 3\,a^5\left(e^{(-x)} - e^{x}\right)^3 + 10\,a^3b^2\left(e^{(-x)} - e^{x}\right)^3 - 25\,ab^4\left(e^{(-x)} - e^{x}\right)^3 + 80\,a^2b^3\left(e^{(-x)} - e^{x}\right.}{4\left(\left(e^{(-x)} -\right.\right.}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log((e^(-x) - e^x)^2 + 4) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e
^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b^4) - 1/4*(3*b^5*(e^(-x) - e^x)^4 + 3*a
^5*(e^(-x) - e^x)^3 + 10*a^3*b^2*(e^(-x) - e^x)^3 - 25*a*b^4*(e^(-x) - e^x)
^3 + 80*a^2*b^3*(e^(-x) - e^x)^2 + 8*b^5*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
- e^x) - 40*a^3*b^2*(e^(-x) - e^x) - 60*a*b^4*(e^(-x) - e^x) + 80*a^4*b + 1
60*a^2*b^3)/((e^(-x) - e^x)^2 + 4)^2
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.99

$$\int (a\mathrm{sech}(x) + b\tanh(x))^5\,dx = \frac{e^x\,(4\,a^5 - 40\,a^3\,b^2 + 20\,a\,b^4) - 20\,a^4\,b - 4\,b^5 + 40\,a^2\,b^3}{4\,e^{2\,x} + 6\,e^{4\,x} + 4\,e^{6\,x} + e^{8\,x} + 1} + b^5\,\ln\left(\left(\frac{3\,a^5\,e^x}{4} - 2\,\sqrt{-\frac{9\,a^{10}}{64} - \frac{15\,a^8\,b^2}{16} - \frac{95\,a^6\,b^4}{32} - \frac{75\,a^4\,b^6}{16} - \frac{225\,a^2\,b^8}{64}} + \frac{15\,a\,b^4\,e^x}{4} + \frac{5\,a^3\,b^2\,e^x}{2}\right)\left(2\,\sqrt{-\frac{9\,a^{10}}{64} - \frac{15\,a^8\,b^2}{16} - \frac{95\,a^6\,b^4}{32} - \frac{75\,a^4\,b^6}{16} - \frac{225\,a^2\,b^8}{64}} + \frac{3\,a^5\,e^x}{4} + \frac{15\,a\,b^4\,e^x}{4} + \frac{5\,a^3\,b^2\,e^x}{2}\right)\right) - \frac{e^x\,(6\,a^5 - 60\,a^3\,b^2 + 30\,a\,b^4) - 40\,a^4\,b - 8\,b^5 + 80\,a^2\,b^3}{3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1} - b^5\,x + \frac{\mathrm{atan}\left(\frac{4\,e^x\,\left(\frac{3\,a^5}{4} + \frac{5\,a^3\,b^2}{2} + \frac{15\,a\,b^4}{4}\right)}{\sqrt{9\,a^{10} + 60\,a^8\,b^2 + 190\,a^6\,b^4 + 300\,a^4\,b^6 + 225\,a^2\,b^8}}\right)\sqrt{9\,a^{10} + 60\,a^8\,b^2 + 190\,a^6\,b^4 + 300\,a^4\,b^6 + 225\,a^2\,b^8}}{4} + \frac{e^x\,\left(\frac{3\,a^5}{4} + \frac{5\,a^3\,b^2}{2} - \frac{25\,a\,b^4}{4}\right) + 4\,b^5 - 20\,a^2\,b^3}{e^{2\,x} + 1} + \frac{e^x\,\left(\frac{a^5}{2} - 25\,a^3\,b^2 + \frac{45\,a\,b^4}{2}\right) - 20\,a^4\,b - 8\,b^5 + 60\,a^2\,b^3}{2\,e^{2\,x} + e^{4\,x} + 1}$$

```
[In] int((b*tanh(x) + a/cosh(x))^5,x)
```

```
[Out] (exp(x)*(20*a*b^4 + 4*a^5 - 40*a^3*b^2) - 20*a^4*b - 4*b^5 + 40*a^2*b^3)/(4
*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + b^5*log(((3*a^5*exp(x
))/4 - 2*(- (9*a^10)/64 - (225*a^2*b^8)/64 - (75*a^4*b^6)/16 - (95*a^6*b^4)
/32 - (15*a^8*b^2)/16)^(1/2) + (15*a*b^4*exp(x))/4 + (5*a^3*b^2*exp(x))/2)*
(2*(- (9*a^10)/64 - (225*a^2*b^8)/64 - (75*a^4*b^6)/16 - (95*a^6*b^4)/32 -
(15*a^8*b^2)/16)^(1/2) + (3*a^5*exp(x))/4 + (15*a*b^4*exp(x))/4 + (5*a^3*b^
2*exp(x))/2)) - (exp(x)*(30*a*b^4 + 6*a^5 - 60*a^3*b^2) - 40*a^4*b - 8*b^5
+ 80*a^2*b^3)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - b^5*x + (atan((4*e
xp(x)*((15*a*b^4)/4 + (3*a^5)/4 + (5*a^3*b^2)/2))/(9*a^10 + 225*a^2*b^8 + 3
00*a^4*b^6 + 190*a^6*b^4 + 60*a^8*b^2)^(1/2))*(9*a^10 + 225*a^2*b^8 + 300*a
^4*b^6 + 190*a^6*b^4 + 60*a^8*b^2)^(1/2))/4 + (exp(x)*((3*a^5)/4 - (25*a*b^
4)/4 + (5*a^3*b^2)/2) + 4*b^5 - 20*a^2*b^3)/(exp(2*x) + 1) + (exp(x)*((45*a
*b^4)/2 + a^5/2 - 25*a^3*b^2) - 20*a^4*b - 8*b^5 + 60*a^2*b^3)/(2*exp(2*x)
+ exp(4*x) + 1)
```

3.615 $\int (a\text{sech}(x) + b\tanh(x))^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 100

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx = b^4x - \frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x) - \frac{1}{3}b^2\left(2a^2+3b^2\right)\cosh(x)\sinh(x) - \frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 + \frac{1}{3}\text{sech}(x)(a+b\sinh(x))^2\left(ab+\left(2a^2+3b^2\right)\sinh(x)\right)$$

```
[Out] b^4*x-4/3*a*b*(a^2+2*b^2)*cosh(x)-1/3*b^2*(2*a^2+3*b^2)*cosh(x)*sinh(x)-1/3
*sech(x)^3*(b-a*sinh(x))*(a+b*sinh(x))^3+1/3*sech(x)*(a+b*sinh(x))^2*(a*b+(
2*a^2+3*b^2)*sinh(x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4476, 2770, 2940, 2813}

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx = -\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x) - \frac{1}{3}b^2\left(2a^2+3b^2\right)\sinh(x)\cosh(x) + \frac{1}{3}\text{sech}(x)(a+b\sinh(x))^2\left(\left(2a^2+3b^2\right)\sinh(x)+ab\right) - \frac{1}{3}\text{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 + b^4x$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^4,x]
```

```
[Out] b^4*x - (4*a*b*(a^2 + 2*b^2)*Cosh[x])/3 - (b^2*(2*a^2 + 3*b^2)*Cosh[x]*Sinh
[x])/3 - (Sech[x]^3*(b - a*Sinh[x])*(a + b*Sinh[x])^3)/3 + (Sech[x]*(a + b*
Sinh[x])^2*(a*b + (2*a^2 + 3*b^2)*Sinh[x]))/3
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \operatorname{sech}^4(x)(a+b\sinh(x))^4\,dx\\ &= -\frac{1}{3}\operatorname{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3\\ &\quad-\frac{1}{3}\int \operatorname{sech}^2(x)(a+b\sinh(x))^2\left(-2a^2-3b^2+ab\sinh(x)\right)\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{3}\operatorname{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 \\
&\quad +\frac{1}{3}\operatorname{sech}(x)(a+b\sinh(x))^2\left(ab+\left(2a^2+3b^2\right)\sinh(x)\right) \\
&\quad +\frac{1}{3}\int(a+b\sinh(x))\left(-2ab^2-2b\left(2a^2+3b^2\right)\sinh(x)\right)\,dx \\
&= b^4x-\frac{4}{3}ab\left(a^2+2b^2\right)\cosh(x)-\frac{1}{3}b^2\left(2a^2+3b^2\right)\cosh(x)\sinh(x) \\
&\quad -\frac{1}{3}\operatorname{sech}^3(x)(b-a\sinh(x))(a+b\sinh(x))^3 \\
&\quad +\frac{1}{3}\operatorname{sech}(x)(a+b\sinh(x))^2\left(ab+\left(2a^2+3b^2\right)\sinh(x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$
\begin{aligned}
\int(a\operatorname{sech}(x)+b\tanh(x))^4\,dx &= \frac{1}{3}\left(3b^4x-12ab^3\operatorname{sech}(x)-4ab\left(a^2-b^2\right)\operatorname{sech}^3(x)\right. \\
&\quad +2\left(a^4+3a^2b^2-2b^4\right)\tanh(x) \\
&\quad \left.+\left(a^4-6a^2b^2+b^4\right)\operatorname{sech}^2(x)\tanh(x)\right)
\end{aligned}
$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^4,x]
```

```
[Out] (3*b^4*x - 12*a*b^3*Sech[x] - 4*a*b*(a^2 - b^2)*Sech[x]^3 + 2*(a^4 + 3*a^2*
b^2 - 2*b^4)*Tanh[x] + (a^4 - 6*a^2*b^2 + b^4)*Sech[x]^2*Tanh[x])/3
```

Maple [A] (verified)

Time = 52.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result
parts	$a^4\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)+b^4\left(-\frac{\tanh(x)^3}{3}-\tanh(x)-\frac{\ln(\tanh(x)-1)}{2}+\frac{\ln(1+\tanh(x))}{2}\right)-\frac{4a^3b\operatorname{sech}(x)^3}{3}+$
default	$a^4\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)-\frac{4a^3b}{3\cosh(x)^3}+6a^2b^2\left(-\frac{\sinh(x)}{2\cosh(x)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^2}{3}\right)\tanh(x)}{2}\right)+4a\,b^3\left(-\frac{\sinh(x)^2}{\cosh(x)^3}\right.$
risch	$b^4x-\frac{4\left(6a\,b^3\mathrm{e}^{5x}+9\,\mathrm{e}^{4x}a^2b^2-3\,\mathrm{e}^{4x}b^4+8a^3b\,\mathrm{e}^{3x}+4a\,b^3\mathrm{e}^{3x}+3\,\mathrm{e}^{2x}a^4-3\,\mathrm{e}^{2x}b^4+6a\,b^3\mathrm{e}^{x}+a^4+3a^2b^2-2b^4\right)}{3\left(1+\mathrm{e}^{2x}\right)^3}$

```
[In] int((a*sech(x)+b*tanh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(2/3+1/3*sech(x)^2)*tanh(x)+b^4*(-1/3*tanh(x)^3-tanh(x)-1/2*ln(tanh(x)-
1)+1/2*ln(1+tanh(x)))-4/3*a^3*b*sech(x)^3+2*a^2*b^2*tanh(x)^3+4*a*b^3*(1/3*
sech(x)^3-sech(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(93) = 186$.

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.07

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx = -\frac{24\,ab^3\cosh(x)^2 + 16\,a^3b + 8\,ab^3 - (3\,b^4x - 2\,a^4 - 6\,a^2b^2 + 4\,b^4)\cosh(x)^3 - 2\,(a^4 + 3\,a^2b^2 - 2\,b^4)\sinh(x$$

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
[Out] -1/3*(24*a*b^3*cosh(x)^2 + 16*a^3*b + 8*a*b^3 - (3*b^4*x - 2*a^4 - 6*a^2*b^
2 + 4*b^4)*cosh(x)^3 - 2*(a^4 + 3*a^2*b^2 - 2*b^4)*sinh(x)^3 + 3*(8*a*b^3 -
 (3*b^4*x - 2*a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^2 - 3*(3*b^4*x - 2*
a^4 - 6*a^2*b^2 + 4*b^4)*cosh(x) - 6*(a^4 - 3*a^2*b^2 + (a^4 + 3*a^2*b^2 -
2*b^4)*cosh(x)^2)*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*cosh(x))
```

Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^4\,dx = \int (a\,\text{sech}\,(x) + b\tanh\,(x))^4\,dx$$

```
[In] integrate((a*sech(x)+b*tanh(x))**4,x)
[Out] Integral((a*sech(x) + b*tanh(x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(93) = 186$.

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

$$\begin{aligned}\int (a\text{sech}(x) + b\tanh(x))^4\,dx &= 2\,a^2b^2\tanh(x)^3 + \frac{1}{3}\,b^4\left(3\,x - \frac{4\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2)}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1}\right)\\ &\quad - \frac{8}{3}\,ab^3\left(\frac{3\,e^{(-x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{2\,e^{(-3\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{3\,e^{(-5\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} +}\right.\\ &\quad + \frac{4}{3}\,a^4\left(\frac{3\,e^{(-2\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{1}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1}\right)\\ &\quad - \frac{32\,a^3b}{3\,(e^{(-x)} + e^x)^3}\end{aligned}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] 2*a^2*b^2*tanh(x)^3 + 1/3*b^4*(3*x - 4*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(
-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) - 8/3*a*b^3*(3*e^(-x)/(3*e^(-2*x) + 3*e
^(-4*x) + e^(-6*x) + 1) + 2*e^(-3*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
1) + 3*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1)) + 4/3*a^4*(3*e^(-
2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 1/(3*e^(-2*x) + 3*e^(-4*x)
+ e^(-6*x) + 1)) - 32/3*a^3*b/(e^(-x) + e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int (a\mathrm{sech}(x) + b\tanh(x))^4\,dx = b^4x - \frac{4\left(6\,ab^3e^{(5\,x)} + 9\,a^2b^2e^{(4\,x)} - 3\,b^4e^{(4\,x)} + 8\,a^3be^{(3\,x)} + 4\,ab^3e^{(3\,x)} + 3\,a^4e^{(2\,x)} - 3\,b^4e^{(2\,x)} + 6\,ab^3e^{x} + a^4 + 3}{3\left(e^{(2\,x)} + 1\right)^3}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

```
[Out] b^4*x - 4/3*(6*a*b^3*e^(5*x) + 9*a^2*b^2*e^(4*x) - 3*b^4*e^(4*x) + 8*a^3*b*
e^(3*x) + 4*a*b^3*e^(3*x) + 3*a^4*e^(2*x) - 3*b^4*e^(2*x) + 6*a*b^3*e^x + a
^4 + 3*a^2*b^2 - 2*b^4)/(e^(2*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.45

$$\int (a\mathrm{sech}(x) + b\tanh(x))^4\,dx = \frac{\mathrm{e}^{x}\left(\frac{32\,a\,b^3}{3} - \frac{32\,a^3\,b}{3}\right) - 4\,a^4 - 4\,b^4 + 24\,a^2\,b^2}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} - \frac{12\,a^2\,b^2 + 8\,\mathrm{e}^{x}\,a\,b^3 - 4\,b^4}{\mathrm{e}^{2\,x} + 1} + b^4\,x - \frac{\mathrm{e}^{x}\left(\frac{32\,a\,b^3}{3} - \frac{32\,a^3\,b}{3}\right) - \frac{8\,a^4}{3} - \frac{8\,b^4}{3} + 16\,a^2\,b^2}{3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1}$$

```
[In] int((b*tanh(x) + a/cosh(x))^4,x)
```

```
[Out] (exp(x)*((32*a*b^3)/3 - (32*a^3*b)/3) - 4*a^4 - 4*b^4 + 24*a^2*b^2)/(2*exp(
2*x) + exp(4*x) + 1) - (12*a^2*b^2 - 4*b^4 + 8*a*b^3*exp(x))/(exp(2*x) + 1)
 + b^4*x - (exp(x)*((32*a*b^3)/3 - (32*a^3*b)/3) - (8*a^4)/3 - (8*b^4)/3 +
16*a^2*b^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
```

3.616 $\int (a\text{sech}(x)+b\tanh(x))^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int (a\text{sech}(x)+b\tanh(x))^3\,dx=\frac{1}{2}a(a^2+3b^2)\arctan(\sinh(x))+b^3\log(\cosh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2$$

```
[Out] 1/2*a*(a^2+3*b^2)*arctan(sinh(x))+b^3*ln(cosh(x))-1/2*a*b^2*sinh(x)-1/2*sec
h(x)^2*(b-a*sinh(x))*(a+b*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4476, 2747, 753, 788, 649, 210, 266}

$$\int (a\text{sech}(x)+b\tanh(x))^3\,dx=\frac{1}{2}a(a^2+3b^2)\arctan(\sinh(x))-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2+b^3\log(\cosh(x))$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/2 + b^3*Log[Cosh[x]] - (a*b^2*Sinh[x])/2
- (Sech[x]^2*(b - a*Sinh[x])*(a + b*Sinh[x])^2)/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\text{integral} = \int \operatorname{sech}^3(x)(a+b\sinh(x))^3\,dx$$

$$= b^3\text{Subst}\left(\int \frac{(a+x)^3}{(-b^2-x^2)^2}\,dx, x, b\sinh(x)\right)$$

$$
\begin{aligned}
&= -\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2 \\
&\quad+\frac{1}{2}b\text{Subst}\left(\int\frac{(a+x)\left(-a^2-2b^2+ax\right)}{-b^2-x^2}\,dx,x,b\sinh(x)\right) \\
&= -\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2 \\
&\quad-\frac{1}{2}b\text{Subst}\left(\int\frac{ab^2-a(-a^2-2b^2)+2b^2x}{-b^2-x^2}\,dx,x,b\sinh(x)\right) \\
&= -\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2 \\
&\quad-b^3\text{Subst}\left(\int\frac{x}{-b^2-x^2}\,dx,x,b\sinh(x)\right) \\
&\quad-\frac{1}{2}\left(ab\left(a^2+3b^2\right)\right)\text{Subst}\left(\int\frac{1}{-b^2-x^2}\,dx,x,b\sinh(x)\right) \\
&= \frac{1}{2}a\left(a^2+3b^2\right)\arctan(\sinh(x))+b^3\log(\cosh(x)) \\
&\quad-\frac{1}{2}ab^2\sinh(x)-\frac{1}{2}\text{sech}^2(x)(b-a\sinh(x))(a+b\sinh(x))^2
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. $2(58) = 116$.

Time = 1.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.34

$$
\begin{aligned}
&\int(a\text{sech}(x)+b\tanh(x))^3\,dx \\
&= \frac{1}{4}\left(\frac{b\left(\left(a^3+3ab^2-2(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-\left(a^3+3ab^2+2(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}\right. \\
&\quad+\frac{2a^4b\text{sech}^2(x)}{a^2+b^2}+\frac{a(2a^4-4a^2b^2-7b^4+b^4\cosh(2x))\,\text{sech}(x)\tanh(x)}{a^2+b^2} \\
&\quad\left.-\frac{2b(-4a^4-2a^2b^2+b^4+ab^3\sinh(x))\tanh^2(x)}{a^2+b^2}\right)
\end{aligned}
$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^3,x]
```

```
[Out] ((b*((a^3 + 3*a*b^2 - 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[x]] - (a^3 +
3*a*b^2 + 2*(-b^2)^(3/2))*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (2*a^4
*b*Sech[x]^2)/(a^2 + b^2) + (a*(2*a^4 - 4*a^2*b^2 - 7*b^4 + b^4*Cosh[2*x])*
Sech[x]*Tanh[x])/(a^2 + b^2) - (2*b*(-4*a^4 - 2*a^2*b^2 + b^4 + a*b^3*Sinh[
x])*Tanh[x]^2)/(a^2 + b^2))/4
```

Maple [A] (verified)

Time = 13.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result
default	$a^{3}\left(\frac{\operatorname{sech}(x)\tanh(x)}{2}+\arctan\left(\mathrm{e}^{x}\right)\right)-\frac{3a^{2}b}{2\cosh(x)^{2}}+3a\,b^{2}\left(-\frac{\sinh(x)}{\cosh(x)^{2}}+\frac{\operatorname{sech}(x)\tanh(x)}{2}+\arctan\left(\mathrm{e}^{x}\right)\right)+b^{3}\Big($
parts	$a^{3}\left(\frac{\operatorname{sech}(x)\tanh(x)}{2}+\arctan\left(\mathrm{e}^{x}\right)\right)+b^{3}\left(-\frac{\tanh(x)^{2}}{2}-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(1+\tanh(x))}{2}\right)+3a\,b^{2}\Big(-\frac{\sinh(x)}{\cosh(x)^{2}}+$
risch	$-b^{3}x+\frac{\mathrm{e}^{x}\left(a^{3}\mathrm{e}^{2x}-3a\,b^{2}\mathrm{e}^{2x}-6a^{2}b\,\mathrm{e}^{x}+2b^{3}\mathrm{e}^{x}-a^{3}+3a\,b^{2}\right)}{\left(1+\mathrm{e}^{2x}\right)^{2}}+\frac{i\ln(\mathrm{e}^{x}+i)a^{3}}{2}+\frac{3i\ln(\mathrm{e}^{x}+i)a\,b^{2}}{2}+\ln\left(\mathrm{e}^{x}+i\right)b^{3}-\frac{i\ln(\mathrm{e}^{x}-i)}{2}$

```
[In] int((a*sech(x)+b*tanh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2*sech(x)*tanh(x)+arctan(exp(x)))-3/2*a^2*b/cosh(x)^2+3*a*b^2*(-1/co
sh(x)^2*sinh(x)+1/2*sech(x)*tanh(x)+arctan(exp(x)))+b^3*(ln(cosh(x))-1/2*ta
nh(x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 502, normalized size of antiderivative = 8.66

$$\int (a\mathrm{sech}(x)+b\tanh(x))^3\,dx = -\frac{b^{3}x\cosh\left(x\right)^{4}+b^{3}x\sinh\left(x\right)^{4}+b^{3}x-\left(a^{3}-3\,ab^{2}\right)\cosh\left(x\right)^{3}+\left(4\,b^{3}x\cosh\left(x\right)-a^{3}+3\,ab^{2}\right)\sinh\left(x\right)^{3}+2}{}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] -(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + b^3*x - (a^3 - 3*a*b^2)*cosh(x)^3 + (
4*b^3*x*cosh(x) - a^3 + 3*a*b^2)*sinh(x)^3 + 2*(b^3*x + 3*a^2*b - b^3)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 6*a^2*b - 2*b^3 - 3*(a^3 - 3*a*b^2)*
cosh(x))*sinh(x)^2 - ((a^3 + 3*a*b^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)
*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*
cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 + 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x
) + sinh(x)) + (a^3 - 3*a*b^2)*cosh(x) - (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sin
h(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*
sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
 - sinh(x))) + (4*b^3*x*cosh(x)^3 + a^3 - 3*a*b^2 - 3*(a^3 - 3*a*b^2)*cosh(
x)^2 + 4*(b^3*x + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cos
h(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (a\mathrm{sech}(x) + b\tanh(x))^3\,dx = \int (a\,\mathrm{sech}\,(x) + b\tanh\,(x))^3\,dx$$

```
[In] integrate((a*sech(x)+b*tanh(x))**3,x)
[Out] Integral((a*sech(x) + b*tanh(x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.07

$$\int (a\mathrm{sech}(x) + b\tanh(x))^3\,dx = \frac{3}{2}\,a^2b\tanh\,(x)^2 + b^3\left(x + \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \log\left(e^{(-2\,x)} + 1\right)\right) - 3\,ab^2\left(\frac{e^{(-x)} - e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \arctan\left(e^{(-x)}\right)\right) + a^3\left(\frac{e^{(-x)} - e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} - \arctan\left(e^{(-x)}\right)\right)$$

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
[Out] 3/2*a^2*b*tanh(x)^2 + b^3*(x + 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + log
(e^(-2*x) + 1)) - 3*a*b^2*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
+ arctan(e^(-x))) + a^3*((e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) -
arctan(e^(-x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.02

$$\int (a\mathrm{sech}(x) + b\tanh(x))^3\,dx = \frac{1}{2}\,b^3\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right) + \frac{1}{4}\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^3 + 3\,ab^2\right) - \frac{b^3\left(e^{(-x)} - e^x\right)^2 + 2\,a^3\left(e^{(-x)} - e^x\right) - 6\,ab^2\left(e^{(-x)} - e^x\right) + 12\,a^2b}{2\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log((e^(-x) - e^x)^2 + 4) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^
(-x)))*(a^3 + 3*a*b^2) - 1/2*(b^3*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) -
 6*a*b^2*(e^(-x) - e^x) + 12*a^2*b)/((e^(-x) - e^x)^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.02

$$\int (a\mathrm{sech}(x)+b\tanh(x))^3\,dx = \mathrm{atan}\left(\frac{\mathrm{e}^x\,(a^3+3\,a\,b^2)}{\sqrt{a^6+6\,a^4\,b^2+9\,a^2\,b^4}}\right)\sqrt{a^6+6\,a^4\,b^2+9\,a^2\,b^4} + \frac{\mathrm{e}^x\,(6\,a\,b^2-2\,a^3)+6\,a^2\,b-2\,b^3}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + b^3\,\ln\left(\left(a^3\,\mathrm{e}^x-2\,\sqrt{-\frac{a^6}{4}-\frac{3\,a^4\,b^2}{2}-\frac{9\,a^2\,b^4}{4}} + 3\,a\,b^2\,\mathrm{e}^x\right)\left(2\,\sqrt{-\frac{a^6}{4}-\frac{3\,a^4\,b^2}{2}-\frac{9\,a^2\,b^4}{4}}+a^3\,\mathrm{e}^x + 3\,a\,b^2\,\mathrm{e}^x\right)\right) - b^3\,x - \frac{\mathrm{e}^x\,(3\,a\,b^2-a^3)+6\,a^2\,b-2\,b^3}{\mathrm{e}^{2\,x}+1}$$

```
[In] int((b*tanh(x) + a/cosh(x))^3,x)
```

```
[Out] atan((exp(x)*(3*a*b^2 + a^3))/(a^6 + 9*a^2*b^4 + 6*a^4*b^2)^(1/2))*(a^6 + 9
*a^2*b^4 + 6*a^4*b^2)^(1/2) + (exp(x)*(6*a*b^2 - 2*a^3) + 6*a^2*b - 2*b^3)/
(2*exp(2*x) + exp(4*x) + 1) + b^3*log((a^3*exp(x) - 2*(- a^6/4 - (9*a^2*b^4
)/4 - (3*a^4*b^2)/2)^(1/2) + 3*a*b^2*exp(x))*(2*(- a^6/4 - (9*a^2*b^4)/4 -
(3*a^4*b^2)/2)^(1/2) + a^3*exp(x) + 3*a*b^2*exp(x))) - b^3*x - (exp(x)*(3*a
*b^2 - a^3) + 6*a^2*b - 2*b^3)/(exp(2*x) + 1)
```

3.617 $\int (a\text{sech}(x) + b\tanh(x))^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int (a\text{sech}(x) + b\tanh(x))^2 \, dx = b^2x - ab\cosh(x) - \text{sech}(x)(b - a\sinh(x))(a + b\sinh(x))$$

```
[Out] b^2*x-a*b*cosh(x)-sech(x)*(b-a*sinh(x))*(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4476, 2770, 2718}

$$\int (a\text{sech}(x) + b\tanh(x))^2 \, dx = -ab\cosh(x) - \text{sech}(x)(b - a\sinh(x))(a + b\sinh(x)) + b^2x$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^2,x]
```

```
[Out] b^2*x - a*b*Cosh[x] - Sech[x]*(b - a*Sinh[x])*(a + b*Sinh[x])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
```

```
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \text{sech}^2(x)(a+b\sinh(x))^2\,dx \\
&= -\text{sech}(x)(b-a\sinh(x))(a+b\sinh(x)) - \int \left(-b^2+ab\sinh(x)\right)\,dx \\
&= b^2x - \text{sech}(x)(b-a\sinh(x))(a+b\sinh(x)) - (ab)\int \sinh(x)\,dx \\
&= b^2x - ab\cosh(x) - \text{sech}(x)(b-a\sinh(x))(a+b\sinh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (a\text{sech}(x)+b\tanh(x))^2\,dx = b^2\text{arctanh}(\tanh(x)) - 2ab\text{sech}(x) + \left(a^2-b^2\right)\tanh(x)$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^2,x]
[Out] b^2*ArcTanh[Tanh[x]] - 2*a*b*Sech[x] + (a^2 - b^2)*Tanh[x]
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$a^2\tanh(x) - \frac{2ab}{\cosh(x)} + b^2(x-\tanh(x))$	26
risch	$b^2x - \frac{2(2b\,e^x a + a^2 - b^2)}{1+e^{2x}}$	32
parts	$a^2\tanh(x) + b^2\left(-\tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(1+\tanh(x))}{2}\right) - 2ab\,\text{sech}(x)$	37

```
[In] int((a*sech(x)+b*tanh(x))^2,x,method=_RETURNVERBOSE)
[Out] a^2*tanh(x)-2*a*b/cosh(x)+b^2*(x-tanh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int (a\text{sech}(x) + b\tanh(x))^2\,dx = -\frac{2\,ab - (b^2x - a^2 + b^2)\cosh(x) - (a^2 - b^2)\sinh(x)}{\cosh(x)}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
[Out] -(2*a*b - (b^2*x - a^2 + b^2)*cosh(x) - (a^2 - b^2)*sinh(x))/cosh(x)
```

Sympy [F]

$$\int (a\text{sech}(x) + b\tanh(x))^2\,dx = \int (a\,\text{sech}(x) + b\tanh(x))^2\,dx$$

```
[In] integrate((a*sech(x)+b*tanh(x))**2,x)
[Out] Integral((a*sech(x) + b*tanh(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int (a\text{sech}(x) + b\tanh(x))^2\,dx = b^2\left(x - \frac{2}{e^{(-2\,x)} + 1}\right) - \frac{4\,ab}{e^{(-x)} + e^x} + \frac{2\,a^2}{e^{(-2\,x)} + 1}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
[Out] b^2*(x - 2/(e^(-2*x) + 1)) - 4*a*b/(e^(-x) + e^x) + 2*a^2/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a\operatorname{sech}(x)+b\tanh(x))^2\,dx = b^2x - \frac{2\,(2\,abe^x + a^2 - b^2)}{e^{(2\,x)}+1}$$

```
[In] integrate((a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
[Out] b^2*x - 2*(2*a*b*e^x + a^2 - b^2)/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int (a\operatorname{sech}(x)+b\tanh(x))^2\,dx = b^2\,x - \frac{2\,a^2 + 4\,\mathrm{e}^x\,a\,b - 2\,b^2}{\mathrm{e}^{2\,x}+1}$$

```
[In] int((b*tanh(x) + a/cosh(x))^2,x)
[Out] b^2*x - (2*a^2 - 2*b^2 + 4*a*b*exp(x))/(exp(2*x) + 1)
```

3.618 $\int (a\mathbf{sech}(x) + b\tanh(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = a\arctan(\sinh(x)) + b\log(\cosh(x))$$

```
[Out] a*arctan(sinh(x))+b*ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3855, 3556}

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = a\arctan(\sinh(x)) + b\log(\cosh(x))$$

```
[In] Int[a*Sech[x] + b*Tanh[x],x]
```

```
[Out] a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\int \operatorname{sech}(x)\,dx + b\int \tanh(x)\,dx\\ &= a\arctan(\sinh(x)) + b\log(\cosh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (a\operatorname{sech}(x) + b\tanh(x))\,dx = a\arctan(\sinh(x)) + b\log(\cosh(x))$$

```
[In] Integrate[a*Sech[x] + b*Tanh[x],x]
[Out] a*ArcTan[Sinh[x]] + b*Log[Cosh[x]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result
default	$a \arctan\left(\sinh\left(x\right)\right)+b \ln\left(\cosh\left(x\right)\right)$
parts	$a \arctan\left(\sinh\left(x\right)\right)+b \ln\left(\cosh\left(x\right)\right)$
risch	$i a \ln\left({\mathrm e}^{x}+i\right)-i a \ln\left({\mathrm e}^{x}-i\right)-b x +b \ln\left(1+{\mathrm e}^{2 x}\right)$
parallelrisch	$-i a \ln\left(-i+\coth\left(x\right)-\operatorname{csch}\left(x\right)\right)+i a \ln\left(i+\coth\left(x\right)-\operatorname{csch}\left(x\right)\right)-b \left(x +\ln\left(1-\tanh\left(x\right)\right)\right)$

```
[In] int(a*sech(x)+b*tanh(x),x,method=_RETURNVERBOSE)
[Out] a*arctan(sinh(x))+b*ln(cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int (a\operatorname{sech}(x) + b\tanh(x))\,dx = -bx + 2\,a\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + b\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right)$$

```
[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="fricas")
[Out] -b*x + 2*a*arctan(cosh(x) + sinh(x)) + b*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = 2a\,\text{atan}\left(\tanh\left(\frac{x}{2}\right)\right) + b(x - \log\left(\tanh\left(x\right) + 1\right))$$

```
[In] integrate(a*sech(x)+b*tanh(x),x)
[Out] 2*a*atan(tanh(x/2)) + b*(x - log(tanh(x) + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = a\arctan\left(\sinh\left(x\right)\right) + b\log\left(\cosh\left(x\right)\right)$$

```
[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="maxima")
[Out] a*arctan(sinh(x)) + b*log(cosh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = -b\big(x - \log\left(e^{(2x)} + 1\right)\big) + 2\,a\arctan\left(e^{x}\right)$$

```
[In] integrate(a*sech(x)+b*tanh(x),x, algorithm="giac")
[Out] -b*(x - log(e^(2*x) + 1)) + 2*a*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int (a\text{sech}(x) + b\tanh(x))\,dx = b\,\ln\left(4\,a^2\,e^{2\,x} + 4\,a^2\right) - b\,x + 2\,\text{atan}\left(\frac{a\,e^{x}}{\sqrt{a^2}}\right)\,\sqrt{a^2}$$

```
[In] int(b*tanh(x) + a/cosh(x),x)
[Out] b*log(4*a^2*exp(2*x) + 4*a^2) - b*x + 2*atan((a*exp(x))/(a^2)^(1/2))*(a^2)^
(1/2)
```

3.619 $\int \frac{1}{a\mathbf{sech}(x)+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{a\text{sech}(x)+b\tanh(x)}\,dx=\frac{\log(a+b\sinh(x))}{b}$$

```
[Out] ln(a+b*sinh(x))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3238, 2747, 31}

$$\int \frac{1}{a\text{sech}(x)+b\tanh(x)}\,dx=\frac{\log(a+b\sinh(x))}{b}$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\text{sech}(x)+b\tanh(x)}\,dx = \frac{\log(a+b\sinh(x))}{b}$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-1),x]
[Out] Log[a + b*Sinh[x]]/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
risch	$-\frac{x}{b}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b}$	27
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}$	50

```
[In] int(1/(a*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -x/b+1/b*ln(exp(2*x)+2*a/b*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = -\frac{x - \log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="fricas")
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = \begin{cases} \frac{x}{b} + \frac{\log\left(\frac{a\,\text{sech}\,(x)}{b}+\tanh\,(x)\right)}{b} - \frac{\log\,(\tanh\,(x)+1)}{b} & \text{for } b \neq 0 \\ \frac{\tanh\,(x)}{a\,\text{sech}\,(x)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x)
[Out] Piecewise((x/b + log(a*sech(x)/b + tanh(x))/b - log(tanh(x) + 1)/b, Ne(b, 0
)), (tanh(x)/(a*sech(x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = \frac{x}{b} + \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="maxima")
[Out] x/b + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{b}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x)),x, algorithm="giac")
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{a\text{sech}(x) + b\tanh(x)}\,dx = -\frac{x - \ln\left(2\,a\,\mathrm{e}^{x} - b + b\,\mathrm{e}^{2\,x}\right)}{b}$$

```
[In] int(1/(b*tanh(x) + a/cosh(x)),x)
[Out] -(x - log(2*a*exp(x) - b + b*exp(2*x)))/b
```

3.620 $\int \frac{1}{(a\mathbf{sech}(x)+b\tanh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 62

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx=\frac{x}{b^2}+\frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}$$

```
[Out] x/b^2-cosh(x)/b/(a+b*sinh(x))+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {4476, 2772, 2814, 2739, 632, 212}

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx=\frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\text{integral} = \int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$$

$$= -\frac{\cosh(x)}{b(a+b\sinh(x))} + \frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{x}{b^2} - \frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}$$

$$= \frac{x}{b^2} - \frac{\cosh(x)}{b(a+b\sinh(x))} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

$$= \frac{x}{b^2} - \frac{\cosh(x)}{b(a+b\sinh(x))} + \frac{(4a)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b - 2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

$$= \frac{x}{b^2} + \frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\cosh(x)}{b(a+b\sinh(x))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 502, normalized size of antiderivative = 8.10

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx =$$

$$-\frac{\cosh(x)\left(-2a\sqrt{a-ib}\sqrt{a+ib}\text{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}(a+b\sinh(x))+2a(a-ib)\text{arcta}\right.}{}$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-2),x]
```

```
[Out] -((Cosh[x]*(-2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]
))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]*(
a + b*Sinh[x]) + 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x
]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqr
t[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/
(a + I*b))]*(-2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a -
 I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] - 2*(-1)^(1/4)*b^(3/2)
*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a -
I*b))])/Sqrt[b]]*Sinh[x] + Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]]*Sq
rt[-((b*(I + Sinh[x]))/(a - I*b))])))/((a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sq
rt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh
[x]))/(a - I*b))]*(a + b*Sinh[x])))
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result	size
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}+\frac{\frac{2\left(\frac{b^{2}\tanh\left(\frac{x}{2}\right)}{a}+b\right)}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}}{b^{2}}$	101
risch	$\frac{x}{b^{2}}+\frac{2a\,{\mathrm e}^{x}-2b}{b^{2}\left(b\,{\mathrm e}^{2x}+2a\,{\mathrm e}^{x}-b\right)}+\frac{a\ln\left({\mathrm e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{b\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}-\frac{a\ln\left({\mathrm e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{b\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}$	140

```
[In] int(1/(a*sech(x)+b*tanh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)+2/b^2*((b^2/a*tanh(1/2*x)+
b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
anh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(58) = 116$.

Time = 0.26 (sec) , antiderivative size = 362, normalized size of antiderivative = 5.84

$$\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^2}\,dx =$$
$$-\frac{(a^2b+b^3)x\cosh(x)^2+(a^2b+b^3)x\sinh(x)^2-2a^2b-2b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2a^2\cosh(x)\right.}{}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx=\int \frac{1}{(a\,\text{sech}\,(x)+b\tanh\,(x))^2}\,dx$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**2,x)
[Out] Integral((a*sech(x) + b*tanh(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx=-\frac{2\left(ae^{(-x)}+b\right)}{2\,ab^2e^{(-x)}-b^3e^{(-2\,x)}+b^3}-\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2}+\frac{x}{b^2}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="maxima")
[Out] -2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x) -
 a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^
2) + x/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx=-\frac{a\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{x}{b^2}+\frac{2\,(ae^x-b)}{(be^{(2\,x)}+2\,ae^x-b)b^2}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^2,x, algorithm="giac")
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*
e^x - b)*b^2)
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^2}\,dx = \frac{x}{b^2}-\frac{\frac{2}{b}-\frac{2\,a\,e^x}{b^2}}{2\,a\,e^x-b+b\,e^{2\,x}}-\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}-\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}+\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}+\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}$$

```
[In] int(1/(b*tanh(x) + a/cosh(x))^2,x)
```

```
[Out] x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2*
a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 +
b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^
2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
```

3.621 $\int \frac{1}{(a\mathbf{sech}(x)+b\tanh(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^3}\,dx=\frac{\log(a+b\sinh(x))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}$$

```
[Out] ln(a+b*sinh(x))/b^3+1/2*(-a^2-b^2)/b^3/(a+b*sinh(x))^2+2*a/b^3/(a+b*sinh(x)
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2747, 711}

$$\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^3}\,dx=-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}+\frac{\log(a+b\sinh(x))}{b^3}$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-3),x]
```

```
[Out] Log[a + b*Sinh[x]]/b^3 - (a^2 + b^2)/(2*b^3*(a + b*Sinh[x])^2) + (2*a)/(b^3
*(a + b*Sinh[x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh^3(x)}{(a+b\sinh(x))^3}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^3}\,dx, x, b\sinh(x)\right)}{b^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2-b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)\,dx, x, b\sinh(x)\right)}{b^3} \\
&= \frac{\log(a+b\sinh(x))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\sinh(x))^2}+\frac{2a}{b^3(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^3}\,dx = -\frac{-\log(a+b\sinh(x))+\frac{-3a^2+b^2-4ab\sinh(x)}{2(a+b\sinh(x))^2}}{b^3}$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-3),x]
[Out] -((-Log[a + b*Sinh[x]] + (-3*a^2 + b^2 - 4*a*b*Sinh[x])/(2*(a + b*Sinh[x])^
2))/b^3)
```

Maple [A] (verified)

Time = 11.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{x}{b^3}+\frac{2\,\mathrm{e}^{x}\left(2b\,\mathrm{e}^{2x}a+3a^2\mathrm{e}^{x}-b^2\mathrm{e}^{x}-2ab\right)}{b^3\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)^2}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b^3}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^3}+\frac{\frac{2\left(\frac{b\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)^3}{a}-\frac{b^2\left(3a^2-b^2\right)\tanh\left(\frac{x}{2}\right)^2}{a^2}-\frac{b\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)}{a}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)^2}+\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{b^3}-\frac{\ln(}{}$

```
[In] int(1/(a*sech(x)+b*tanh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x/b^3+2/b^3*exp(x)*(2*b*exp(2*x)*a+3*a^2*exp(x)-b^2*exp(x)-2*a*b)/(b*exp(2
*x)+2*a*exp(x)-b)^2+1/b^3*ln(exp(2*x)+2*a/b*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 11.31

$$\int \frac{1}{(a\mathrm{sech}(x)+b\tanh(x))^3}\,dx =$$
$$-\frac{b^2x\cosh(x)^4+b^2x\sinh(x)^4+4(abx-ab)\cosh(x)^3+4(b^2x\cosh(x)+abx-ab)\sinh(x)^3+b^2x-2(}{}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^4 + b^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(b^2*x*
cosh(x) + a*b*x - a*b)*sinh(x)^3 + b^2*x - 2*(3*a^2 - b^2 - (2*a^2 - b^2)*x
)*cosh(x)^2 + 2*(3*b^2*x*cosh(x)^2 - 3*a^2 + b^2 + (2*a^2 - b^2)*x + 6*(a*b
*x - a*b)*cosh(x))*sinh(x)^2 - 4*(a*b*x - a*b)*cosh(x) - (b^2*cosh(x)^4 + b
^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(b^2*cosh(x) + a*b)*sinh(x)^3 - 4*a*b*co
sh(x) + 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 6*a*b*cosh(x) + 2*
a^2 - b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + 3*a*b*cosh(x)^2 - a*b + (2*
a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 4
*(b^2*x*cosh(x)^3 - a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 + a*b - (3*a^2 - b^2
- (2*a^2 - b^2)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 + 4*a*b
^4*cosh(x)^3 - 4*a*b^4*cosh(x) + b^5 + 4*(b^5*cosh(x) + a*b^4)*sinh(x)^3 +
2*(2*a^2*b^3 - b^5)*cosh(x)^2 + 2*(3*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) + 2*a^
2*b^3 - b^5)*sinh(x)^2 + 4*(b^5*cosh(x)^3 + 3*a*b^4*cosh(x)^2 - a*b^4 + (2*
a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(48) = 96$.

Time = 1.30 (sec) , antiderivative size = 651, normalized size of antiderivative = 13.56

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**3,x)
```

```
[Out] Piecewise((2*a**2*x*sech(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*s
ech(x) + 2*b**5*tanh(x)**2) + 2*a**2*log(a*sech(x)/b + tanh(x))*sech(x)**2/
(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 2
*a**2*log(tanh(x) + 1)*sech(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x
)*sech(x) + 2*b**5*tanh(x)**2) + a**2*sech(x)**2/(2*a**2*b**3*sech(x)**2 +
4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) + 4*a*b*x*tanh(x)*sech(x)/(2*
a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) + 4*a*
b*log(a*sech(x)/b + tanh(x))*tanh(x)*sech(x)/(2*a**2*b**3*sech(x)**2 + 4*a*
b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 4*a*b*log(tanh(x) + 1)*tanh(x)*
sech(x)/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)
**2) + 2*b**2*x*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(
x) + 2*b**5*tanh(x)**2) + 2*b**2*log(a*sech(x)/b + tanh(x))*tanh(x)**2/(2*a
**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - 2*b**
2*log(tanh(x) + 1)*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*b**4*tanh(x)*se
ch(x) + 2*b**5*tanh(x)**2) - b**2*tanh(x)**2/(2*a**2*b**3*sech(x)**2 + 4*a*
b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2) - b**2/(2*a**2*b**3*sech(x)**2 +
4*a*b**4*tanh(x)*sech(x) + 2*b**5*tanh(x)**2), Ne(b, 0)), ((-2*tanh(x)**3/(
3*sech(x)**3) + tanh(x)/sech(x)**3)/a**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^3}\,dx = \frac{2\left(2\,abe^{(-x)} - 2\,abe^{(-3\,x)} + (3\,a^2 - b^2)e^{(-2\,x)}\right)}{4\,ab^4e^{(-x)} - 4\,ab^4e^{(-3\,x)} + b^5e^{(-4\,x)} + b^5 + 2\,(2\,a^2b^3 - b^5)e^{(-2\,x)}} + \frac{x}{b^3} + \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^3}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*e^(-x) - 2*a*b*e^(-3*x) + (3*a^2 - b^2)*e^(-2*x))/(4*a*b^4*e^(-x)
- 4*a*b^4*e^(-3*x) + b^5*e^(-4*x) + b^5 + 2*(2*a^2*b^3 - b^5)*e^(-2*x)) + x
/b^3 + log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^3}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2a\right|\right)}{b^3} - \frac{3b\left(e^{(-x)} - e^x\right)^2 - 4a\left(e^{(-x)} - e^x\right) + 4b}{2\left(b\left(e^{(-x)} - e^x\right) - 2a\right)^2 b^2}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^3,x, algorithm="giac")
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 - 1/2*(3*b*(e^(-x) - e^x)^2 - 4*a*(e^
(-x) - e^x) + 4*b)/((b*(e^(-x) - e^x) - 2*a)^2*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^3}\,dx = \int \frac{1}{\left(b\tanh(x) + \frac{a}{\cosh(x)}\right)^3}\,dx$$

```
[In] int(1/(b*tanh(x) + a/cosh(x))^3,x)
[Out] int(1/(b*tanh(x) + a/cosh(x))^3, x)
```

3.622 $\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 146

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx = \frac{x}{b^4}+\frac{a(2a^2+3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3}+\frac{a\cosh^3(x)}{2b\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\cosh(x)\left(2(a^2+b^2)+ab\sinh(x)\right)}{2b^3\left(a^2+b^2\right)(a+b\sinh(x))}$$

```
[Out] x/b^4+a*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^4/(a^2+b
^2)^(3/2)-1/3*cosh(x)^3/b/(a+b*sinh(x))^3+1/2*a*cosh(x)^3/b/(a^2+b^2)/(a+b*
sinh(x))^2-1/2*cosh(x)*(2*a^2+2*b^2+a*b*sinh(x))/b^3/(a^2+b^2)/(a+b*sinh(x)
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4476, 2772, 2943, 2942, 2814, 2739, 632, 212}

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^4}\,dx = \frac{a(2a^2+3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}} + \frac{a\cosh^3(x)}{2b\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{\cosh(x)\,(2(a^2+b^2)+ab\sinh(x))}{2b^3\,(a^2+b^2)\,(a+b\sinh(x))} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{x}{b^4}$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

```
[Out] x/b^4 + (a*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^4
*(a^2 + b^2)^(3/2)) - Cosh[x]^3/(3*b*(a + b*Sinh[x])^3) + (a*Cosh[x]^3)/(2*
b*(a^2 + b^2)*(a + b*Sinh[x])^2) - (Cosh[x]*(2*(a^2 + b^2) + a*b*Sinh[x]))/
(2*b^3*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\text{integral} = \int \frac{\cosh^4(x)}{(a+b\sinh(x))^4}\,dx$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{\int \frac{\cosh^2(x)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{b}$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b(a^2+b^2)(a+b\sinh(x))^2} + \frac{i\int \frac{\cosh^2(x)(-2ib+ia\sinh(x))}{(a+b\sinh(x))^2}dx}{2b(a^2+b^2)}$$

$$= -\frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b(a^2+b^2)(a+b\sinh(x))^2} - \frac{\cosh(x)(2(a^2+b^2)+ab\sinh(x))}{2b^3(a^2+b^2)(a+b\sinh(x))} + \frac{i\int \frac{iab-2i(a^2+b^2)\sinh(x)}{a+b\sinh(x)}dx}{2b^3(a^2+b^2)}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b(a^2+b^2)(a+b\sinh(x))^2} - \frac{\cosh(x)(2(a^2+b^2)+ab\sinh(x))}{2b^3(a^2+b^2)(a+b\sinh(x))} - \frac{(a(2a^2+3b^2))\int \frac{1}{a+b\sinh(x)}dx}{2b^4(a^2+b^2)}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b(a^2+b^2)(a+b\sinh(x))^2} - \frac{\cosh(x)(2(a^2+b^2)+ab\sinh(x))}{2b^3(a^2+b^2)(a+b\sinh(x))} - \frac{(a(2a^2+3b^2))\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^4(a^2+b^2)}$$

$$= \frac{x}{b^4} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b(a^2+b^2)(a+b\sinh(x))^2} - \frac{\cosh(x)(2(a^2+b^2)+ab\sinh(x))}{2b^3(a^2+b^2)(a+b\sinh(x))} + \frac{(2a(2a^2+3b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^4(a^2+b^2)}$$

$$= \frac{x}{b^4} + \frac{a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4(a^2+b^2)^{3/2}} - \frac{\cosh^3(x)}{3b(a+b\sinh(x))^3} + \frac{a\cosh^3(x)}{2b(a^2+b^2)(a+b\sinh(x))^2} - \frac{\cosh(x)(2(a^2+b^2)+ab\sinh(x))}{2b^3(a^2+b^2)(a+b\sinh(x))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.17 (sec) , antiderivative size = 3430, normalized size of antiderivative = 23.49

$$\int \frac{1}{(a\operatorname{sech}(x)+b\tanh(x))^4}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-4),x]
```

[Out] ((-I)*Sech[x]*(a + b*Sinh[x])^4*(((I/3)*b*(((-I)*b)/(a - I*b) - (b*Sinh[x])
/(a - I*b))^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*
a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a +
b*Sinh[x])^3) - (((I/2)*a*b^3*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))
^(5/2)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((a^2 + b^2)*(((-I)
*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a
+ b*Sinh[x])^2) - (-(((((3*I)*a^2*b^5)/(a^2 + b^2)^2 - ((2*I)*b^5*(3*a^2 +
2*b^2))/(a^2 + b^2)^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(5/2)*(
(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b))^(5/2))/((((-I)*a*b)/(a - I*b) - b^
2/(a - I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b))*(a + b*Sinh[x]))) - ((1
6*Sqrt[2]*(a - I*b)*b^6*(3*a^2 + 4*b^2)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(
a - I*b))^(5/2)*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(
a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((5*(1/(2*(
1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) +
(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1)
))/8 + (((15*I)/32)*b^3*(((-I)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/
(a - I*b)))/b + ((a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^2
)/(3*b^2) + ((-1)^(1/4)*Sqrt[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I
*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sq
rt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a
- I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b])))/((a - I*b)^3*((
(-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^3*(1 - ((I/2)*(a - I*b)*(((-I)*b
)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2)))/(5*(a + I*b)*(a^2 + b^2)^3*Sq
rt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]) + (I*(((4
*I)*a*b^7*(3*a^2 + 4*b^2))/(a^2 + b^2)^3 - (I*a*b^7*(6*a^2 + 7*b^2))/(a^2 +
b^2)^3)*((-4*Sqrt[2]*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b))^(3/2)*Sq
rt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)
/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)*((3/(4*(1 - ((I/2)*(a - I*b)*
(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^2) + (1 - ((I/2)*(a - I*b)
*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(-1))/2 - (3*b^2*(((-I)*(
a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b + ((-1)^(1/4)*Sqrt
[2]*Sqrt[a - I*b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b)
- (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[((-I)*b)/(a - I*b) - (b*S
inh[x])/(a - I*b)])/(Sqrt[b]*Sqrt[1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b)
- (b*Sinh[x])/(a - I*b)))/b])))/(8*(a - I*b)^2*(((-I)*b)/(a - I*b) - (b*Sin
h[x])/(a - I*b))^2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/
(a - I*b)))/b)^2)))/(3*(a + I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b
*Sinh[x])/(a + I*b)))/b]) - (I*((I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*(
(I*a*b)/(a - I*b) + b^2/(a - I*b))*(((-I)*((I*a*b)/(a + I*b) - b^2/(a + I*b
))*(((2*I)*Sqrt[a - I*b]*ArcTanh[(Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (
b*Sinh[x])/(a - I*b)])/(Sqrt[a + I*b]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a
+ I*b)])])/(Sqrt[a + I*b]*b) - ((2*I)*Sqrt[(I*a*b)/(a + I*b) - b^2/(a + I*
b)]*ArcTanh[(Sqrt[(I*a*b)/(a + I*b) - b^2/(a + I*b)]*Sqrt[((-I)*b)/(a - I*b
) - (b*Sinh[x])/(a - I*b)])/(Sqrt[(I*a*b)/(a - I*b) + b^2/(a - I*b)]*Sqrt[(
I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)])])/(b*Sqrt[(I*a*b)/(a - I*b) + b^2/

```
(a - I*b)]])))/b + ((2*I)*Sqrt[2]*(a - I*b)*Sqrt[((-I)*b)/(a - I*b) - (b*Sin
h[x])/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)]*(1 - ((I/2)*
(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(3/2)*(-(((-1)^(
3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*
Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - I*b]*Sqrt[((-I)*
b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a -
I*b) - (b*Sinh[x])/(a - I*b)))/b)^(3/2))) + 1/(2*(1 - ((I/2)*(a - I*b)*(((-
I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b))))/((a + I*b)*b*Sqrt[((-I)*(a
+ I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/(a + I*b)))/b]))))/b - (4*Sqrt[2]*Sqrt
[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*Sqrt[(I*b)/(a + I*b) - (b*Sinh
[x])/(a + I*b)]*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a
- I*b)))/b)^(5/2)*((-3*(-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*
Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)])/(Sqrt[2]*Sqrt[b])])/(4*Sq
rt[2]*Sqrt[a - I*b]*Sqrt[((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)]*(1 - (
(I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)^(5/2)) + (
3/(2*(1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b)
^2) + (1 - ((I/2)*(a - I*b)*(((-I)*b)/(a - I*b) - (b*Sinh[x])/(a - I*b)))/b
)^(-1))/4))/((a + I*b)*Sqrt[((-I)*(a + I*b)*((I*b)/(a + I*b) - (b*Sinh[x])/
(a + I*b)))/b]))))/b))/b)/((((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*(((-I)*a*b
)/(a + I*b) + b^2/(a + I*b))))/(2*(((-I)*a*b)/(a - I*b) - b^2/(a - I*b))*((
(-I)*a*b)/(a + I*b) + b^2/(a + I*b))))/(3*(((-I)*a*b)/(a - I*b) - b^2/(a -
I*b))*(((-I)*a*b)/(a + I*b) + b^2/(a + I*b)))))/((1 - (a + b*Sinh[x])/(a -
I*b))^(3/2)*(1 - (a + b*Sinh[x])/(a + I*b))^(3/2)*(a*Sech[x] + b*Tanh[x])^4
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(135) = 270$.

Time = 90.76 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.45

method	result
default	$\frac{\frac{2\left(\frac{b^2(a^4+2a^2b^2+2b^4)\tanh(\frac{x}{2})^5}{2a(a^2+b^2)}+\frac{b(2a^6-3a^4b^2-4a^2b^4-4b^6)\tanh(\frac{x}{2})^4}{2(a^2+b^2)a^2}-\frac{b^2(18a^6+3a^4b^2-4a^2b^4-4b^6)\tanh(\frac{x}{2})^3}{3a^3(a^2+b^2)}-\frac{b(2a^6-8a^4b^2-7a^2b^4-2\cdots}{a^2(a^2+b^2)\cdots}\cdots}{(\tanh(\frac{x}{2})^2a-2b\tanh(\frac{x}{2})-a)^3}\cdots}{b^4}$
risch	$\frac{x}{b^4}+\frac{18\,e^{5x}a^3b^2+15a\,b^4e^{5x}+54a^4b\,e^{4x}+27a^2b^3e^{4x}-12b^5e^{4x}+44\,e^{3x}a^5-34a^3b^2e^{3x}-48\,e^{3x}a\,b^4-78a^4b\,e^{2x}-36a^2b^3e^{2x}+12b^5e^{2x}+\cdots}{3b^4(a^2+b^2)(b\,e^{2x}+2a\,e^{x}-b)^3}$

```
[In] int(1/(a*sech(x)+b*tanh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*((1/2*b^2*(a^4+2*a^2*b^2+2*b^4)/a/(a^2+b^2)*tanh(1/2*x)^5+1/2*b*(2*a^
6-3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)/a^2*tanh(1/2*x)^4-1/3/a^3*b^2*(18*a^
6+3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)*tanh(1/2*x)^3-1/a^2*b*(2*a^6-8*a^4*b
^2-7*a^2*b^4-2*b^6)/(a^2+b^2)*tanh(1/2*x)^2+1/2/a*b^2*(11*a^4+8*a^2*b^2+2*b
```

```
^4)/(a^2+b^2)*tanh(1/2*x)+1/6*b*(6*a^4+5*a^2*b^2+2*b^4)/(a^2+b^2))/(tanh(1/
2*x)^2*a-2*b*tanh(1/2*x)-a)^3-1/2*a*(2*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1
/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/b^4*ln(tanh(1/2*x)+1)-1/b^4*ln
(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2978 vs. $2(137) = 274$.

Time = 0.29 (sec) , antiderivative size = 2978, normalized size of antiderivative = 20.40

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="fricas")
[Out] -1/6*(6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^6 + 6*(a^4*b^3 + 2*a^2*b^5 +
b^7)*x*sinh(x)^6 - 22*a^4*b^3 - 38*a^2*b^5 - 16*b^7 + 6*(6*a^5*b^2 + 11*a^3
*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^5 + 6*(6*a^5*b^
2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x) + 6*(a^5
*b^2 + 2*a^3*b^4 + a*b^6)*x)*sinh(x)^5 + 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b
^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^4 + 6*(18
*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^7 + 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*
cosh(x)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x + 5*(6*a^5*b^2 + 11
*a^3*b^4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x))*sinh(x)^4
+ 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^
3*b^4 - 3*a*b^6)*x)*cosh(x)^3 + 4*(22*a^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b
^6 + 30*(a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^3 + 15*(6*a^5*b^2 + 11*a^3*b^
4 + 5*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^2 + 6*(2*a^7 + a^5
*b^2 - 4*a^3*b^4 - 3*a*b^6)*x + 6*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5 - 4*b^
7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - 6*(26
*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^
5 - b^7)*x)*cosh(x)^2 - 6*(26*a^6*b + 38*a^4*b^3 + 8*a^2*b^5 - 4*b^7 - 15*(
a^4*b^3 + 2*a^2*b^5 + b^7)*x*cosh(x)^4 - 10*(6*a^5*b^2 + 11*a^3*b^4 + 5*a*b
^6 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^3 - 6*(18*a^6*b + 27*a^4*b^
3 + 5*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x
)^2 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x - 2*(22*a^7 + 5*a^5*b^2 -
 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3*a*b^6)*x)*cosh(
x))*sinh(x)^2 + 3*((2*a^3*b^3 + 3*a*b^5)*cosh(x)^6 + (2*a^3*b^3 + 3*a*b^5)*
sinh(x)^6 - 2*a^3*b^3 - 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 6*(
2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^5 + 3*(8*a^5
*b + 10*a^3*b^3 - 3*a*b^5)*cosh(x)^4 + 3*(8*a^5*b + 10*a^3*b^3 - 3*a*b^5 +
5*(2*a^3*b^3 + 3*a*b^5)*cosh(x)^2 + 10*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sin
h(x)^4 + 4*(4*a^6 - 9*a^2*b^4)*cosh(x)^3 + 4*(4*a^6 - 9*a^2*b^4 + 5*(2*a^3*
b^3 + 3*a*b^5)*cosh(x)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 3*(8*a^5*
b + 10*a^3*b^3 - 3*a*b^5)*cosh(x))*sinh(x)^3 - 3*(8*a^5*b + 10*a^3*b^3 - 3*
```

```
a*b^5)*cosh(x)^2 - 3*(8*a^5*b + 10*a^3*b^3 - 3*a*b^5 - 5*(2*a^3*b^3 + 3*a*b
^5)*cosh(x)^4 - 20*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 - 6*(8*a^5*b + 10*a^3*
b^3 - 3*a*b^5)*cosh(x)^2 - 4*(4*a^6 - 9*a^2*b^4)*cosh(x))*sinh(x)^2 + 6*(2*
a^4*b^2 + 3*a^2*b^4)*cosh(x) + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*
b^5)*cosh(x)^5 + 5*(2*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(8*a^5*b + 10*a^3*
b^3 - 3*a*b^5)*cosh(x)^3 + 2*(4*a^6 - 9*a^2*b^4)*cosh(x)^2 - (8*a^5*b + 10*
a^3*b^3 - 3*a*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
+ 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b^3 + 2*a^2*b^5 +
 b^7)*x + 6*(16*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6 + 6*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*x)*cosh(x) + 6*(16*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6 + 6*(a^4*b^3 + 2*
a^2*b^5 + b^7)*x*cosh(x)^5 + 5*(6*a^5*b^2 + 11*a^3*b^4 + 5*a*b^6 + 6*(a^5*b
^2 + 2*a^3*b^4 + a*b^6)*x)*cosh(x)^4 + 4*(18*a^6*b + 27*a^4*b^3 + 5*a^2*b^5
 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)*cosh(x)^3 + 2*(22*a
^7 + 5*a^5*b^2 - 41*a^3*b^4 - 24*a*b^6 + 6*(2*a^7 + a^5*b^2 - 4*a^3*b^4 - 3
*a*b^6)*x)*cosh(x)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*x - 2*(26*a^6*b + 38
*a^4*b^3 + 8*a^2*b^5 - 4*b^7 + 3*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*x)
*cosh(x))*sinh(x))/(a^4*b^7 + 2*a^2*b^9 + b^11 - (a^4*b^7 + 2*a^2*b^9 + b^1
1)*cosh(x)^6 - (a^4*b^7 + 2*a^2*b^9 + b^11)*sinh(x)^6 - 6*(a^5*b^6 + 2*a^3*
b^8 + a*b^10)*cosh(x)^5 - 6*(a^5*b^6 + 2*a^3*b^8 + a*b^10 + (a^4*b^7 + 2*a^
2*b^9 + b^11)*cosh(x))*sinh(x)^5 - 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b
^11)*cosh(x)^4 - 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11 + 5*(a^4*b^7 +
 2*a^2*b^9 + b^11)*cosh(x)^2 + 10*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x))*s
inh(x)^4 - 4*(2*a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x)^3 - 4*(2*
a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10 + 5*(a^4*b^7 + 2*a^2*b^9 + b^11)*c
osh(x)^3 + 15*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x)^2 + 3*(4*a^6*b^5 + 7*a
^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x))*sinh(x)^3 + 3*(4*a^6*b^5 + 7*a^4*b^7 +
2*a^2*b^9 - b^11)*cosh(x)^2 + 3*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11 -
 5*(a^4*b^7 + 2*a^2*b^9 + b^11)*cosh(x)^4 - 20*(a^5*b^6 + 2*a^3*b^8 + a*b^1
0)*cosh(x)^3 - 6*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x)^2 - 4*(
2*a^7*b^4 + a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x))*sinh(x)^2 - 6*(a^5*b^6
 + 2*a^3*b^8 + a*b^10)*cosh(x) - 6*(a^5*b^6 + 2*a^3*b^8 + a*b^10 + (a^4*b^7
 + 2*a^2*b^9 + b^11)*cosh(x)^5 + 5*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*cosh(x)^4
 + 2*(4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*b^9 - b^11)*cosh(x)^3 + 2*(2*a^7*b^4 +
a^5*b^6 - 4*a^3*b^8 - 3*a*b^10)*cosh(x)^2 - (4*a^6*b^5 + 7*a^4*b^7 + 2*a^2*
b^9 - b^11)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx=\int \frac{1}{(a\,\text{sech}\,(x)+b\tanh\,(x))^4}\,dx$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**4,x)
[Out] Integral((a*sech(x) + b*tanh(x))**(-4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(137)=274$.

Time = 0.32 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.57

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx=-\frac{(2\,a^2+3\,b^2)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{2\,(a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{11\,a^2b^3+8\,b^5+3\,(16\,a^3b^2+11\,ab^4)e^{(-x)}+6\,(13\,a^4b+6\,a^2b^3-2\,b^5)e^{(-2\,x)}+2\,(22\,a^5-17\,a^3b^2-24\,ab^4}{3\,(a^2b^7+b^9+6\,(a^3b^6+ab^8)e^{(-x)}+3\,(4\,a^4b^5+3\,a^2b^7-b^9)e^{(-2\,x)}+4\,(2\,a^5b^4-a^3b^6-3\,ab^8)e^{(-3\,x)}-3\,(4} + \frac{x}{b^4}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="maxima")
[Out] -1/2*(2*a^2 + 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a +
 sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 1/3*(11*a^2*b^3 + 8*
b^5 + 3*(16*a^3*b^2 + 11*a*b^4)*e^(-x) + 6*(13*a^4*b + 6*a^2*b^3 - 2*b^5)*e
^(-2*x) + 2*(22*a^5 - 17*a^3*b^2 - 24*a*b^4)*e^(-3*x) - 3*(18*a^4*b + 9*a^2
*b^3 - 4*b^5)*e^(-4*x) + 3*(6*a^3*b^2 + 5*a*b^4)*e^(-5*x))/(a^2*b^7 + b^9 +
 6*(a^3*b^6 + a*b^8)*e^(-x) + 3*(4*a^4*b^5 + 3*a^2*b^7 - b^9)*e^(-2*x) + 4*
(2*a^5*b^4 - a^3*b^6 - 3*a*b^8)*e^(-3*x) - 3*(4*a^4*b^5 + 3*a^2*b^7 - b^9)*
e^(-4*x) + 6*(a^3*b^6 + a*b^8)*e^(-5*x) - (a^2*b^7 + b^9)*e^(-6*x)) + x/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx=-\frac{(2\,a^3+3\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{18\,a^3b^2e^{(5\,x)}+15\,ab^4e^{(5\,x)}+54\,a^4be^{(4\,x)}+27\,a^2b^3e^{(4\,x)}-12\,b^5e^{(4\,x)}+44\,a^5e^{(3\,x)}-34\,a^3b^2e^{(3\,x)}-48\,ab^4e^{(3}}{3\,(a^2b^4+b^6)(be^{(2\,x)}+2\,ae^x} + \frac{x}{b^4}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3 + 3*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e
^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(18*
a^3*b^2*e^(5*x) + 15*a*b^4*e^(5*x) + 54*a^4*b*e^(4*x) + 27*a^2*b^3*e^(4*x)
- 12*b^5*e^(4*x) + 44*a^5*e^(3*x) - 34*a^3*b^2*e^(3*x) - 48*a*b^4*e^(3*x) -
 78*a^4*b*e^(2*x) - 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) + 48*a^3*b^2*e^x +
33*a*b^4*e^x - 11*a^2*b^3 - 8*b^5)/((a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x -
b)^3) + x/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^4}\,dx=\int \frac{1}{\left(b\tanh\left(x\right)+\frac{a}{\cosh(x)}\right)^4}\,dx$$

```
[In] int(1/(b*tanh(x) + a/cosh(x))^4,x)
```

```
[Out] int(1/(b*tanh(x) + a/cosh(x))^4, x)
```

3.623 $\int \frac{1}{(a\mathbf{sech}(x)+b\tanh(x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx = \frac{\log(a+b\sinh(x))}{b^5} - \frac{(a^2+b^2)^2}{4b^5(a+b\sinh(x))^4} + \frac{4a(a^2+b^2)}{3b^5(a+b\sinh(x))^3} - \frac{3a^2+b^2}{b^5(a+b\sinh(x))^2} + \frac{4a}{b^5(a+b\sinh(x))}$$

```
[Out] ln(a+b*sinh(x))/b^5-1/4*(a^2+b^2)^2/b^5/(a+b*sinh(x))^4+4/3*a*(a^2+b^2)/b^5
/(a+b*sinh(x))^3+(-3*a^2-b^2)/b^5/(a+b*sinh(x))^2+4*a/b^5/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2747, 711}

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx = -\frac{(a^2+b^2)^2}{4b^5(a+b\sinh(x))^4} + \frac{4a(a^2+b^2)}{3b^5(a+b\sinh(x))^3} - \frac{3a^2+b^2}{b^5(a+b\sinh(x))^2} + \frac{4a}{b^5(a+b\sinh(x))} + \frac{\log(a+b\sinh(x))}{b^5}$$

```
[In] Int[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

```
[Out] Log[a + b*Sinh[x]]/b^5 - (a^2 + b^2)^2/(4*b^5*(a + b*Sinh[x])^4) + (4*a*(a^
2 + b^2))/(3*b^5*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(b^5*(a + b*Sinh[x])^2)
 + (4*a)/(b^5*(a + b*Sinh[x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh^5(x)}{(a+b\sinh(x))^5}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-b^2-x^2)^2}{(a+x)^5}\,dx, x, b\sinh(x)\right)}{b^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)^2}{(a+x)^5} - \frac{4a(a^2+b^2)}{(a+x)^4} + \frac{2(3a^2+b^2)}{(a+x)^3} - \frac{4a}{(a+x)^2} + \frac{1}{a+x}\right)\,dx, x, b\sinh(x)\right)}{b^5} \\
&= \frac{\log(a+b\sinh(x))}{b^5} - \frac{(a^2+b^2)^2}{4b^5(a+b\sinh(x))^4} + \frac{4a(a^2+b^2)}{3b^5(a+b\sinh(x))^3} \\
&\quad - \frac{3a^2+b^2}{b^5(a+b\sinh(x))^2} + \frac{4a}{b^5(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx = \frac{\log(a+b\sinh(x))-\frac{(a^2+b^2)^2}{4(a+b\sinh(x))^4}+\frac{4a(a^2+b^2)}{3(a+b\sinh(x))^3}-\frac{3a^2+b^2}{(a+b\sinh(x))^2}+\frac{4a}{a+b\sinh(x)}}{b^5}$$

```
[In] Integrate[(a*Sech[x] + b*Tanh[x])^(-5),x]
```

```
[Out] (Log[a + b*Sinh[x]] - (a^2 + b^2)^2/(4*(a + b*Sinh[x])^4) + (4*a*(a^2 + b^2
))/(3*(a + b*Sinh[x])^3) - (3*a^2 + b^2)/(a + b*Sinh[x])^2 + (4*a)/(a + b*S
inh[x]))/b^5
```

Maple [A] (verified)

Time = 289.96 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{x}{b^5}+\frac{4\left(6a\,b^3\mathrm{e}^{6x}+27a^2b^2\mathrm{e}^{5x}-3\,\mathrm{e}^{5x}b^4+44a^3b\,\mathrm{e}^{4x}-22a\,b^3\mathrm{e}^{4x}+25a^4\mathrm{e}^{3x}-56a^2b^2\mathrm{e}^{3x}+3\,\mathrm{e}^{3x}b^4-44\,\mathrm{e}^{2x}a^3b+22\,\mathrm{e}^{2x}a\,b^3+27a^2b^2\mathrm{e}^{x}-3\,\mathrm{e}^{\cdots}\right.}{3b^5\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)^4}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^5}+\frac{2\left(\frac{\left(a^4-b^4\right)b\tanh\left(\frac{x}{2}\right)^7}{a}-\frac{b^2\left(7a^4-3b^4\right)\tanh\left(\frac{x}{2}\right)^6}{a^2}-\frac{b\left(9a^6-52a^4b^2-a^2b^4+12b^6\right)\tanh\left(\frac{x}{2}\right)^5}{3a^3}+\frac{2b^2\left(21a^6-25a^4b^2-7a^2b^4\cdots\right.}{3a^4}\right.}{\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\cdots\right.}$

```
[In] int(1/(a*sech(x)+b*tanh(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*x+4/3*(6*a*b^3*exp(6*x)+27*a^2*b^2*exp(5*x)-3*exp(5*x)*b^4+44*a^3*b*
exp(4*x)-22*a*b^3*exp(4*x)+25*a^4*exp(3*x)-56*a^2*b^2*exp(3*x)+3*exp(3*x)*b
^4-44*exp(2*x)*a^3*b+22*exp(2*x)*a*b^3+27*a^2*b^2*exp(x)-3*exp(x)*b^4-6*a*b
^3)/b^5*exp(x)/(b*exp(2*x)+2*a*exp(x)-b)^4+1/b^5*ln(exp(2*x)+2*a/b*exp(x)-1
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. 2(91) = 182.

Time = 0.28 (sec) , antiderivative size = 2640, normalized size of antiderivative = 27.79

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^5} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^8 + 3*b^4*x*sinh(x)^8 + 24*(a*b^3*x - a*b^3)*cosh(x)^
7 + 24*(b^4*x*cosh(x) + a*b^3*x - a*b^3)*sinh(x)^7 - 12*(9*a^2*b^2 - b^4 -
(6*a^2*b^2 - b^4)*x)*cosh(x)^6 + 12*(7*b^4*x*cosh(x)^2 - 9*a^2*b^2 + b^4 +
(6*a^2*b^2 - b^4)*x + 14*(a*b^3*x - a*b^3)*cosh(x))*sinh(x)^6 - 8*(22*a^3*b
 - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^5 + 8*(21*b^4*x*cosh(x)^3 -
22*a^3*b + 11*a*b^3 + 63*(a*b^3*x - a*b^3)*cosh(x)^2 + 3*(4*a^3*b - 3*a*b^3
)*x - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x)^5 + 3*b^4*
x - 2*(50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)*cos
h(x)^4 + 2*(105*b^4*x*cosh(x)^4 - 50*a^4 + 112*a^2*b^2 - 6*b^4 + 420*(a*b^3
*x - a*b^3)*cosh(x)^3 - 90*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^
2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x - 20*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b
 - 3*a*b^3)*x)*cosh(x))*sinh(x)^4 + 8*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3
*a*b^3)*x)*cosh(x)^3 + 8*(21*b^4*x*cosh(x)^5 + 105*(a*b^3*x - a*b^3)*cosh(x
)^4 + 22*a^3*b - 11*a*b^3 - 30*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh
(x)^3 - 10*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^2 - 3*(4
*a^3*b - 3*a*b^3)*x - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2
 + 3*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x
)*cosh(x)^2 + 4*(21*b^4*x*cosh(x)^6 + 126*(a*b^3*x - a*b^3)*cosh(x)^5 - 45*
(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^4 - 27*a^2*b^2 + 3*b^4 - 20
*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^3 - 3*(50*a^4 - 11
2*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)*cosh(x)^2 + 3*(6*a^2*
b^2 - b^4)*x + 6*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x))*s
inh(x)^2 - 24*(a*b^3*x - a*b^3)*cosh(x) - 3*(b^4*cosh(x)^8 + b^4*sinh(x)^8
+ 8*a*b^3*cosh(x)^7 + 8*(b^4*cosh(x) + a*b^3)*sinh(x)^7 + 4*(6*a^2*b^2 - b^
4)*cosh(x)^6 + 4*(7*b^4*cosh(x)^2 + 14*a*b^3*cosh(x) + 6*a^2*b^2 - b^4)*sin
h(x)^6 + 8*(4*a^3*b - 3*a*b^3)*cosh(x)^5 + 8*(7*b^4*cosh(x)^3 + 21*a*b^3*co
sh(x)^2 + 4*a^3*b - 3*a*b^3 + 3*(6*a^2*b^2 - b^4)*cosh(x))*sinh(x)^5 - 8*a*
b^3*cosh(x) + 2*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x)^4 + 2*(35*b^4*cosh(x)^
4 + 140*a*b^3*cosh(x)^3 + 8*a^4 - 24*a^2*b^2 + 3*b^4 + 30*(6*a^2*b^2 - b^4)
*cosh(x)^2 + 20*(4*a^3*b - 3*a*b^3)*cosh(x))*sinh(x)^4 + b^4 - 8*(4*a^3*b -
 3*a*b^3)*cosh(x)^3 + 8*(7*b^4*cosh(x)^5 + 35*a*b^3*cosh(x)^4 - 4*a^3*b + 3
*a*b^3 + 10*(6*a^2*b^2 - b^4)*cosh(x)^3 + 10*(4*a^3*b - 3*a*b^3)*cosh(x)^2
+ (8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x)^3 + 4*(6*a^2*b^2 - b^4)*cos
h(x)^2 + 4*(7*b^4*cosh(x)^6 + 42*a*b^3*cosh(x)^5 + 15*(6*a^2*b^2 - b^4)*cos
h(x)^4 + 6*a^2*b^2 - b^4 + 20*(4*a^3*b - 3*a*b^3)*cosh(x)^3 + 3*(8*a^4 - 24
```

```
*a^2*b^2 + 3*b^4)*cosh(x)^2 - 6*(4*a^3*b - 3*a*b^3)*cosh(x))*sinh(x)^2 + 8*
(b^4*cosh(x)^7 + 7*a*b^3*cosh(x)^6 + 3*(6*a^2*b^2 - b^4)*cosh(x)^5 + 5*(4*a
^3*b - 3*a*b^3)*cosh(x)^4 - a*b^3 + (8*a^4 - 24*a^2*b^2 + 3*b^4)*cosh(x)^3
- 3*(4*a^3*b - 3*a*b^3)*cosh(x)^2 + (6*a^2*b^2 - b^4)*cosh(x))*sinh(x))*log
(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*x*cosh(x)^7 + 21*(a*b^3*
x - a*b^3)*cosh(x)^6 - 9*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x)^5
- 3*a*b^3*x - 5*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^4 +
 3*a*b^3 - (50*a^4 - 112*a^2*b^2 + 6*b^4 - 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x
)*cosh(x)^3 + 3*(22*a^3*b - 11*a*b^3 - 3*(4*a^3*b - 3*a*b^3)*x)*cosh(x)^2 -
 3*(9*a^2*b^2 - b^4 - (6*a^2*b^2 - b^4)*x)*cosh(x))*sinh(x))/(b^9*cosh(x)^8
 + b^9*sinh(x)^8 + 8*a*b^8*cosh(x)^7 - 8*a*b^8*cosh(x) + b^9 + 8*(b^9*cosh(
x) + a*b^8)*sinh(x)^7 + 4*(6*a^2*b^7 - b^9)*cosh(x)^6 + 4*(7*b^9*cosh(x)^2
+ 14*a*b^8*cosh(x) + 6*a^2*b^7 - b^9)*sinh(x)^6 + 8*(4*a^3*b^6 - 3*a*b^8)*c
osh(x)^5 + 8*(7*b^9*cosh(x)^3 + 21*a*b^8*cosh(x)^2 + 4*a^3*b^6 - 3*a*b^8 +
3*(6*a^2*b^7 - b^9)*cosh(x))*sinh(x)^5 + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)
*cosh(x)^4 + 2*(35*b^9*cosh(x)^4 + 140*a*b^8*cosh(x)^3 + 8*a^4*b^5 - 24*a^2
*b^7 + 3*b^9 + 30*(6*a^2*b^7 - b^9)*cosh(x)^2 + 20*(4*a^3*b^6 - 3*a*b^8)*co
sh(x))*sinh(x)^4 - 8*(4*a^3*b^6 - 3*a*b^8)*cosh(x)^3 + 8*(7*b^9*cosh(x)^5 +
 35*a*b^8*cosh(x)^4 - 4*a^3*b^6 + 3*a*b^8 + 10*(6*a^2*b^7 - b^9)*cosh(x)^3
+ 10*(4*a^3*b^6 - 3*a*b^8)*cosh(x)^2 + (8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cos
h(x))*sinh(x)^3 + 4*(6*a^2*b^7 - b^9)*cosh(x)^2 + 4*(7*b^9*cosh(x)^6 + 42*a
*b^8*cosh(x)^5 + 6*a^2*b^7 - b^9 + 15*(6*a^2*b^7 - b^9)*cosh(x)^4 + 20*(4*a
^3*b^6 - 3*a*b^8)*cosh(x)^3 + 3*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cosh(x)^2
- 6*(4*a^3*b^6 - 3*a*b^8)*cosh(x))*sinh(x)^2 + 8*(b^9*cosh(x)^7 + 7*a*b^8*c
osh(x)^6 - a*b^8 + 3*(6*a^2*b^7 - b^9)*cosh(x)^5 + 5*(4*a^3*b^6 - 3*a*b^8)*
cosh(x)^4 + (8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*cosh(x)^3 - 3*(4*a^3*b^6 - 3*a
*b^8)*cosh(x)^2 + (6*a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2162 vs. $2(92) = 184$.

Time = 6.20 (sec) , antiderivative size = 2162, normalized size of antiderivative = 22.76

$$\int \frac{1}{(a\operatorname{sech}(x) + b\tanh(x))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))**5,x)
```

```
[Out] Piecewise((36*a**4*x*sech(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*ta
nh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)
**3*sech(x) + 36*b**9*tanh(x)**4) + 36*a**4*log(a*sech(x)/b + tanh(x))*sech
(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a*
*2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tan
h(x)**4) - 36*a**4*log(tanh(x) + 1)*sech(x)**4/(36*a**4*b**5*sech(x)**4 + 1
44*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144
```

*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 20*a**4*sech(x)**4/(36*a
4*b5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh
(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 1
44*a**3*b*x*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tan
h(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)*
*3*sech(x) + 36*b**9*tanh(x)**4) + 144*a**3*b*log(a*sech(x)/b + tanh(x))*ta
nh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**
3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 3
6*b**9*tanh(x)**4) - 144*a**3*b*log(tanh(x) + 1)*tanh(x)*sech(x)**3/(36*a**
4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x
)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 44*
a**3*b*tanh(x)*sech(x)**3/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*
sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*se
ch(x) + 36*b**9*tanh(x)**4) + 216*a**2*b**2*x*tanh(x)**2*sech(x)**2/(36*a**
4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x
)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 216
*a**2*b**2*log(a*sech(x)/b + tanh(x))*tanh(x)**2*sech(x)**2/(36*a**4*b**5*s
ech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sec
h(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 216*a**2*b*
*2*log(tanh(x) + 1)*tanh(x)**2*sech(x)**2/(36*a**4*b**5*sech(x)**4 + 144*a*
*3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b*
*8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 6*a**2*b**2*sech(x)**2/(36*a*
*4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(
x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 14
4*a*b**3*x*tanh(x)**3*sech(x)/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh
(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**
3*sech(x) + 36*b**9*tanh(x)**4) + 144*a*b**3*log(a*sech(x)/b + tanh(x))*tan
h(x)**3*sech(x)/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3
 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36
*b**9*tanh(x)**4) - 144*a*b**3*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36*a**4
*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)
2*sech(x)2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 52*a
*b**3*tanh(x)**3*sech(x)/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*s
ech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sec
h(x) + 36*b**9*tanh(x)**4) - 24*a*b**3*tanh(x)*sech(x)/(36*a**4*b**5*sech(x
)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)*
*2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) + 36*b**4*x*tanh(x
)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2
*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(
x)**4) + 36*b**4*log(a*sech(x)/b + tanh(x))*tanh(x)**4/(36*a**4*b**5*sech(x
)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)*
*2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 36*b**4*log(tanh
(x) + 1)*tanh(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x
)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x)
+ 36*b**9*tanh(x)**4) - 28*b**4*tanh(x)**4/(36*a**4*b**5*sech(x)**4 + 144*a

```
**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)**2*sech(x)**2 + 144*a*b
**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 18*b**4*tanh(x)**2/(36*a**4*
b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b**7*tanh(x)*
*2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)**4) - 9*b**
4/(36*a**4*b**5*sech(x)**4 + 144*a**3*b**6*tanh(x)*sech(x)**3 + 216*a**2*b*
*7*tanh(x)**2*sech(x)**2 + 144*a*b**8*tanh(x)**3*sech(x) + 36*b**9*tanh(x)*
*4), Ne(b, 0)), ((8*tanh(x)**5/(15*sech(x)**5) - 4*tanh(x)**3/(3*sech(x)**5
) + tanh(x)/sech(x)**5)/a**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(91) = 182.

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.13

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx = \frac{4\left(6\,ab^3e^{(-x)}-6\,ab^3e^{(-7\,x)}+3\,(9\,a^2b^2-b^4)e^{(-2\,x)}+22\,(2\,a^3b-ab^3)e^{(-3\,x)}+(25\,a^4-56\,a^2b^2+3\right.}{3\,(8\,ab^8e^{(-x)}-8\,ab^8e^{(-7\,x)}+b^9e^{(-8\,x)}+b^9+4\,(6\,a^2b^7-b^9)e^{(-2\,x)}+8\,(4\,a^3b^6-3\,ab^8)e^{(-3\,x)}+2\,(8\,a^4b^5-24}+\frac{x}{b^5}+\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^5}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 4/3*(6*a*b^3*e^(-x) - 6*a*b^3*e^(-7*x) + 3*(9*a^2*b^2 - b^4)*e^(-2*x) + 22*
(2*a^3*b - a*b^3)*e^(-3*x) + (25*a^4 - 56*a^2*b^2 + 3*b^4)*e^(-4*x) - 22*(2
*a^3*b - a*b^3)*e^(-5*x) + 3*(9*a^2*b^2 - b^4)*e^(-6*x))/(8*a*b^8*e^(-x) -
8*a*b^8*e^(-7*x) + b^9*e^(-8*x) + b^9 + 4*(6*a^2*b^7 - b^9)*e^(-2*x) + 8*(4
*a^3*b^6 - 3*a*b^8)*e^(-3*x) + 2*(8*a^4*b^5 - 24*a^2*b^7 + 3*b^9)*e^(-4*x)
- 8*(4*a^3*b^6 - 3*a*b^8)*e^(-5*x) + 4*(6*a^2*b^7 - b^9)*e^(-6*x)) + x/b^5
+ log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a\text{sech}(x)+b\tanh(x))^5}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^5} - \frac{25\,b^3\left(e^{(-x)}-e^x\right)^4-104\,ab^2\left(e^{(-x)}-e^x\right)^3+168\,a^2b\left(e^{(-x)}-e^x\right)^2+48\,b^3\left(e^{(-x)}-e^x\right)^2-96\,a^3\left(e^{(-x)}-e^x\right)}{12\,\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)^4b^4}$$

```
[In] integrate(1/(a*sech(x)+b*tanh(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b^5 - 1/12*(25*b^3*(e^(-x) - e^x)^4 - 104
*a*b^2*(e^(-x) - e^x)^3 + 168*a^2*b*(e^(-x) - e^x)^2 + 48*b^3*(e^(-x) - e^x
)^2 - 96*a^3*(e^(-x) - e^x) - 64*a*b^2*(e^(-x) - e^x) + 32*a^2*b + 48*b^3)/
((b*(e^(-x) - e^x) - 2*a)^4*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\text{sech}(x) + b\tanh(x))^5} \, dx = \int \frac{1}{\left(b\tanh\left(x\right) + \frac{a}{\cosh(x)}\right)^5} \, dx$$

```
[In] int(1/(b*tanh(x) + a/cosh(x))^5,x)
[Out] int(1/(b*tanh(x) + a/cosh(x))^5, x)
```

3.624 $\int (\operatorname{sech}(x) + i\tanh(x))^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int (\operatorname{sech}(x) + i\tanh(x))^5 \, dx = i\log(i + \sinh(x)) - \frac{2i}{(1 - i\sinh(x))^2} + \frac{4i}{1 - i\sinh(x)}$$

```
[Out] I*ln(I+sinh(x))-2*I/(1-I*sinh(x))^2+4*I/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2746, 45}

$$\int (\operatorname{sech}(x) + i\tanh(x))^5 \, dx = \frac{4i}{1 - i\sinh(x)} - \frac{2i}{(1 - i\sinh(x))^2} + i\log(\sinh(x) + i)$$

```
[In] Int[(Sech[x] + I*Tanh[x])^5,x]
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^5(x)(1+i\sinh(x))^5\,dx \\
&= -\left(i\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, i\sinh(x)\right)\right) \\
&= -\left(i\text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, i\sinh(x)\right)\right) \\
&= i\log(i+\sinh(x))-\frac{2i}{(1-i\sinh(x))^2}+\frac{4i}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\begin{aligned}
\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx &= \arctan(\sinh(x))+i\log(\cosh(x))-\frac{5}{4}i\operatorname{sech}^4(x) \\
&\quad+\operatorname{sech}(x)\tanh(x)-\operatorname{sech}^3(x)\tanh(x) \\
&\quad-\frac{1}{2}i\tanh^2(x)-5\operatorname{sech}(x)\tanh^3(x)-\frac{11}{4}i\tanh^4(x)
\end{aligned}$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^5,x]
```

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]] - ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
Sech[x]^3*Tanh[x] - (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 - ((11*I)/4)*Tanh
[x]^4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(34) = 68.

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

$$\frac{8\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)}{3}+2\arctan(\mathrm{e}^x)+\frac{5i}{4\cosh(x)^4}-\frac{5\sinh(x)}{3\cosh(x)^4}+\frac{5i\sinh(x)^2}{\cosh(x)^4}-\frac{5\sinh(x)^3}{\cosh(x)^4}+i\ln(\cosh($$

```
[In] int((sech(x)+I*tanh(x))^5,x)
```

```
[Out] 8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))+5/4*I/cosh(x)^4-5/
3*sinh(x)/cosh(x)^4+5*I*sinh(x)^2/cosh(x)^4-5*sinh(x)^3/cosh(x)^4+I*ln(cosh
(x))-1/2*I*tanh(x)^2-1/4*I*tanh(x)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx$$
$$=\frac{-ixe^{(4x)}+4(x-2)e^{(3x)}-2(-3ix+4i)e^{(2x)}-4(x-2)e^x-2\left(-ie^{(4x)}+4e^{(3x)}+6ie^{(2x)}-4e^x-i\right)\mathrm{lo}}{e^{(4x)}+4ie^{(3x)}-6e^{(2x)}-4ie^x+1}$$

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(-3*I*x + 4*I)*e^(2*x) - 4*(x - 2)*e^
x - 2*(-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) - I*x
)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int (\operatorname{sech}(x)+i\tanh(x))^5\,dx=\int (i\tanh(x)+\operatorname{sech}(x))^5\,dx$$

```
[In] integrate((sech(x)+I*tanh(x))**5,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 5.88

$$\int (\operatorname{sech}(x) + i\tanh(x))^5\,dx = -\frac{5}{2}i\,\tanh(x)^4 + i\,x - \frac{5\left(5\,e^{(-x)} - 3\,e^{(-3\,x)} + 3\,e^{(-5\,x)} - 5\,e^{(-7\,x)}\right)}{4\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)} + \frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)} - \frac{5\left(e^{(-x)} - 7\,e^{(-3\,x)} + 7\,e^{(-5\,x)} - e^{(-7\,x)}\right)}{2\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)} + \frac{4i\left(e^{(-2\,x)} + e^{(-4\,x)} + e^{(-6\,x)}\right)}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} - \frac{20i}{\left(e^{(-x)} + e^{x}\right)^4} - 2\,\arctan\left(e^{(-x)}\right) + i\,\log\left(e^{(-2\,x)} + 1\right)$$

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*I*tanh(x)^4 + I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*
x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x) +
 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6
*x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*
e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 4*I*(e^(-2*x) + e^(-4*
x) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 20*I
/(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) + I*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int (\operatorname{sech}(x) + i\tanh(x))^5\,dx = -i\,x - \frac{8\left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x}\right)}{\left(e^{x} + i\right)^4} + 2i\,\log\left(e^{x} + i\right)$$

```
[In] integrate((sech(x)+I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -I*x - 8*(e^(3*x) + I*e^(2*x) - e^x)/(e^x + I)^4 + 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int (\mathrm{sech}(x) + i\tanh(x))^5\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,2\mathrm{i} + \frac{16\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^x\,2\mathrm{i}} - \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 + \mathrm{e}^{3\,x}\,4\mathrm{i} - \mathrm{e}^x\,4\mathrm{i}} - \frac{8}{\mathrm{e}^x + 1\mathrm{i}} + \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^x - \mathrm{i}}$$

```
[In] int((tanh(x)*1i + 1/cosh(x))^5,x)
```

```
[Out] log(exp(x) + 1i)*2i - x*1i + 16i/(exp(2*x) + exp(x)*2i - 1) - 8i/(exp(3*x)*
4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 8/(exp(x) + 1i) + 16/(exp(2*x
)*3i + exp(3*x) - 3*exp(x) - 1i)
```

3.625 $\int (\text{sech}(x)+i\tanh(x))^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int (\text{sech}(x)+i\tanh(x))^4\,dx=x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[Out] x-2/3*I*cosh(x)^3/(1-I*sinh(x))^3+2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4476, 2749, 2759, 8}

$$\int (\text{sech}(x)+i\tanh(x))^4\,dx=x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[In] Int[(Sech[x] + I*Tanh[x])^4,x]
```

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
```

```
0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^4(x)(1+i\sinh(x))^4\,dx\\
&= \int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}\,dx\\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} - \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} + \frac{2i\cosh(x)}{1-i\sinh(x)} + \int 1\,dx\\
&= x - \frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} + \frac{2i\cosh(x)}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int (\operatorname{sech}(x)+i\tanh(x))^4\,dx = \frac{3(-8i+3x)\cosh\left(\frac{x}{2}\right)+(16i-3x)\cosh\left(\frac{3x}{2}\right)-6i(-4i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^4,x]
[Out] (3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$-2\left(\frac{2}{3}+\frac{\operatorname{sech}(x)^{2}}{3}\right)\tanh(x)-\frac{4i}{3\cosh(x)^{3}}+\frac{3\sinh(x)}{\cosh(x)^{3}}-4i\left(-\frac{\sinh(x)^{2}}{\cosh(x)^{3}}-\frac{2}{3\cosh(x)^{3}}\right)+x-\tanh(x)-\frac{\tanh}{3}$$

```
[In] int((sech(x)+I*tanh(x))^4,x)
```

```
[Out] -2*(2/3+1/3*sech(x)^2)*tanh(x)-4/3*I/cosh(x)^3+3*sinh(x)/cosh(x)^3-4*I*(-si
nh(x)^2/cosh(x)^3-2/3/cosh(x)^3)+x-tanh(x)-1/3*tanh(x)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int (\operatorname{sech}(x)+i\tanh(x))^{4}\,dx=\frac{3\,xe^{(3\,x)}-3\,(-3i\,x-8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}-3i\,x-16i}{3\,\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*e^(3*x) - 3*(-3*I*x - 8*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16*
I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [F]

$$\int (\operatorname{sech}(x)+i\tanh(x))^{4}\,dx=\int \left(i\tanh(x)+\operatorname{sech}(x)\right)^{4}\,dx$$

```
[In] integrate((sech(x)+I*tanh(x))**4,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(28) = 56.

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.76

$$\int (\operatorname{sech}(x) + i\tanh(x))^4\,dx = -2\tanh(x)^3 + x - \frac{4\left(3e^{(-2x)} + 3e^{(-4x)} + 2\right)}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} + \frac{8ie^{(-x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{4e^{(-2x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{16ie^{(-3x)}}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} + \frac{8ie^{(-5x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{4}{3\left(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1\right)} - \frac{32i}{3\left(e^{(-x)} + e^{x}\right)^3}$$

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*
x) + e^(-6*x) + 1) + 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) +
4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 16/3*I*e^(-3*x)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x)
 + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 32/3*I/(e
^(-x) + e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int (\operatorname{sech}(x) + i\tanh(x))^4\,dx = x - \frac{8\left(-3ie^{(2x)} + 3e^{x} + 2i\right)}{3\left(e^{x} + i\right)^3}$$

```
[In] integrate((sech(x)+I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(-3*I*e^(2*x) + 3*e^x + 2*I)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (\mathrm{sech}(x) + i\tanh(x))^4\,dx = x + \frac{\frac{e^{2x}\,8i}{3} - \frac{8}{3}i}{e^{2x}\,3i + e^{3x} - 3\,e^{x} - i} + \frac{e^{x}\,8i}{3\,\left(e^{2x} - 1 + e^{x}\,2i\right)} + \frac{8i}{3\,\left(e^{x} + 1i\right)}$$

```
[In] int((tanh(x)*1i + 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + (ex
p(x)*8i)/(3*(exp(2*x) + exp(x)*2i - 1)) + 8i/(3*(exp(x) + 1i))
```

3.626 $\int (\text{sech}(x) + i\tanh(x))^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int (\text{sech}(x) + i\tanh(x))^3 \, dx = -i\log(i + \sinh(x)) - \frac{2i}{1 - i\sinh(x)}$$

```
[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4476, 2746, 45}

$$\int (\text{sech}(x) + i\tanh(x))^3 \, dx = -\frac{2i}{1 - i\sinh(x)} - i\log(\sinh(x) + i)$$

```
[In] Int[(Sech[x] + I*Tanh[x])^3,x]
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^3(x)(1+i\sinh(x))^3\,dx \\
&= -\left(i\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, i\sinh(x)\right)\right) \\
&= -\left(i\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, i\sinh(x)\right)\right) \\
&= -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx = -\arctan(\sinh(x)) - i\log(\cosh(x)) - \frac{3}{2}i\operatorname{sech}^2(x) + 2\operatorname{sech}(x)\tanh(x) + \frac{1}{2}i\tanh^2(x)$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^3,x]
[Out] -ArcTan[Sinh[x]] - I*Log[Cosh[x]] - ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x]
 + (I/2)*Tanh[x]^2
```

Maple [A] (verified)

Time = 73.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result
risch	$ix+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}+i\right)^{2}}-2i\ln\left(\mathrm{e}^{x}+i\right)$
default	$-\operatorname{sech}\left(x\right)\tanh\left(x\right)-2\arctan\left(\mathrm{e}^{x}\right)-\frac{3i}{2\cosh\left(x\right)^{2}}+\frac{3\sinh\left(x\right)}{\cosh\left(x\right)^{2}}-i\left(\ln\left(\cosh\left(x\right)\right)-\frac{\tanh\left(x\right)^{2}}{2}\right)$
parts	$-\operatorname{sech}\left(x\right)\tanh\left(x\right)-2\arctan\left(\mathrm{e}^{x}\right)-i\left(-\frac{\tanh\left(x\right)^{2}}{2}-\frac{\ln\left(\tanh\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}\right)+\frac{3i\tanh\left(x\right)^{2}}{2}+\frac{3\sinh(}{\cosh(x}$

```
[In] int((sech(x)+I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I*x+4*exp(x)/(exp(x)+I)^2-2*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx=\frac{i\,xe^{(2\,x)}-2\,(x-2)e^{x}-2\,\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(2\,x)}+2i\,e^{x}-1}$$

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*x
)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int (\operatorname{sech}(x)+i\tanh(x))^3\,dx=\int\left(i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{3}\,dx$$

```
[In] integrate((sech(x)+I*tanh(x))**3,x)
```

```
[Out] Integral((I*tanh(x) + sech(x))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.81

$$\int (\operatorname{sech}(x) + i\tanh(x))^3\,dx = \frac{3}{2}i\,\tanh(x)^2 - i\,x + \frac{4\left(e^{(-x)} - e^{(-3\,x)}\right)}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} - \frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + 2\,\arctan\left(e^{(-x)}\right) - i\,\log\left(e^{(-2\,x)} + 1\right)$$

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*I*tanh(x)^2 - I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) -
 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) - I*log(e^(-2*
x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int (\operatorname{sech}(x) + i\tanh(x))^3\,dx = i\,x + \frac{4\,e^x}{(e^x + i)^2} - 2i\,\log\left(e^x + i\right)$$

```
[In] integrate((sech(x)+I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int (\operatorname{sech}(x) + i\tanh(x))^3\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x + 1\mathrm{i}}$$

```
[In] int((tanh(x)*1i + 1/cosh(x))^3,x)
```

```
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

3.627 $\int (\text{sech}(x) + i\tanh(x))^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int (\text{sech}(x) + i\tanh(x))^2 \, dx = -x - \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

```
[Out] -x-2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4476, 2749, 2759, 8}

$$\int (\text{sech}(x) + i\tanh(x))^2 \, dx = -x - \frac{2i\cosh(x)}{1 - i\sinh(x)}$$

```
[In] Int[(Sech[x] + I*Tanh[x])^2,x]
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \operatorname{sech}^2(x)(1+i\sinh(x))^2\,dx\\ &= \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\ &= -\frac{2i\cosh(x)}{1-i\sinh(x)}-\int 1\,dx\\ &= -x-\frac{2i\cosh(x)}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (\operatorname{sech}(x)+i\tanh(x))^2\,dx = -\operatorname{arctanh}(\tanh(x))-2i\operatorname{sech}(x)+2\tanh(x)$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^2,x]
[Out] -ArcTanh[Tanh[x]] - (2*I)*Sech[x] + 2*Tanh[x]
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x-\frac{4i}{e^{x}+i}$	15
default	$2\tanh\left(x\right)-\frac{2i}{\cosh\left(x\right)}-x$	16
parts	$2\tanh\left(x\right)-2i\,\mathrm{sech}\left(x\right)+\frac{\ln\left(\tanh\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	25

```
[In] int((sech(x)+I*tanh(x))^2,x,method=_RETURNVERBOSE)
[Out] -x-4*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (\mathrm{sech}(x)+i\tanh(x))^2\,dx=-\frac{xe^{x}+i\,x+4i}{e^{x}+i}$$

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [F]

$$\int (\mathrm{sech}(x)+i\tanh(x))^2\,dx=\int\left(i\tanh\left(x\right)+\mathrm{sech}\left(x\right)\right)^{2}\,dx$$

```
[In] integrate((sech(x)+I*tanh(x))**2,x)
[Out] Integral((I*tanh(x) + sech(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int (\operatorname{sech}(x) + i\tanh(x))^2 \, dx = -x - \frac{4i}{e^{(-x)} + e^x} + \frac{4}{e^{(-2x)} + 1}$$

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="maxima")
[Out] -x - 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int (\operatorname{sech}(x) + i\tanh(x))^2 \, dx = -x - \frac{4i}{e^x + i}$$

```
[In] integrate((sech(x)+I*tanh(x))^2,x, algorithm="giac")
[Out] -x - 4*I/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int (\operatorname{sech}(x) + i\tanh(x))^2 \, dx = -x - \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

```
[In] int((tanh(x)*1i + 1/cosh(x))^2,x)
[Out] - x - 4i/(exp(x) + 1i)
```

3.628 $\int (\mathbf{sech}(x) + i\tanh(x))\, dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (\mathrm{sech}(x) + i\tanh(x))\, dx = \arctan(\sinh(x)) + i\log(\cosh(x))$$

```
[Out] arctan(sinh(x))+I*ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3855, 3556}

$$\int (\mathrm{sech}(x) + i\tanh(x))\, dx = \arctan(\sinh(x)) + i\log(\cosh(x))$$

```
[In] Int[Sech[x] + I*Tanh[x],x]
```

```
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int \tanh(x)\,dx + \int \operatorname{sech}(x)\,dx\\ &= \arctan(\sinh(x)) + i\log(\cosh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = \arctan(\sinh(x)) + i\log(\cosh(x))$$

```
[In] Integrate[Sech[x] + I*Tanh[x],x]
[Out] ArcTan[Sinh[x]] + I*Log[Cosh[x]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan(\sinh(x)) + i\ln(\cosh(x))$	11
parts	$\arctan(\sinh(x)) + i\ln(\cosh(x))$	11
risch	$i\ln(\mathrm{e}^{x}+i) - i\ln(\mathrm{e}^{x}-i) - ix + i\ln(1+\mathrm{e}^{2x})$	34
parallelrisch	$-\frac{i(\ln(1-\tanh(x))+\ln(1+\tanh(x))+2\ln(-i+\coth(x)-\operatorname{csch}(x))-2\ln(i+\coth(x)-\operatorname{csch}(x)))}{2}$	41

```
[In] int(sech(x)+I*tanh(x),x,method=_RETURNVERBOSE)
[Out] arctan(sinh(x))+I*ln(cosh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = -i\,x + 2i\,\log\left(e^{x} + i\right)$$

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="fricas")
[Out] -I*x + 2*I*log(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = i(x - \log(\tanh(x) + 1)) + 2\,\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(sech(x)+I*tanh(x),x)
[Out] I*(x - log(tanh(x) + 1)) + 2*atan(tanh(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = \arctan(\sinh(x)) + i\,\log(\cosh(x))$$

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="maxima")
[Out] arctan(sinh(x)) + I*log(cosh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = -i\,x + 2\,\arctan(e^x) + i\,\log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(sech(x)+I*tanh(x),x, algorithm="giac")
[Out] -I*x + 2*arctan(e^x) + I*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (\operatorname{sech}(x) + i\tanh(x))\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,2\mathrm{i}$$

```
[In] int(tanh(x)*1i + 1/cosh(x),x)
[Out] log(exp(x) + 1i)*2i - x*1i
```

3.629 $\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx = -i\log(i-\sinh(x))$$

```
[Out] -I*ln(I-sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3238, 2746, 31}

$$\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx = -i\log(-\sinh(x)+i)$$

```
[In] Int[(Sech[x] + I*Tanh[x])^(-1),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh(x)}{1+i\sinh(x)}\,dx\\ &= -\left(i\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, i\sinh(x)\right)\right)\\ &= -i\log(i-\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\operatorname{sech}(x)+i\tanh(x)}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - i\log(\cosh(x))$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-1),x]
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
risch	$ix-2i\ln\left(e^{x}-i\right)$	15
default	$-2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	33

```
[In] int(1/(sech(x)+I*tanh(x)),x,method=_RETURNVERBOSE)
[Out] I*x-2*I*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\operatorname{sech}(x) + i \tanh(x)} \, dx = i\,x - 2i\,\log\left(e^{x} - i\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="fricas")
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\operatorname{sech}(x) + i \tanh(x)} \, dx = -ix + i\log\left(\tanh\left(x\right) + 1\right) - i\log\left(\tanh\left(x\right) - i\operatorname{sech}\left(x\right)\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x)),x)
[Out] -I*x + I*log(tanh(x) + 1) - I*log(tanh(x) - I*sech(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\operatorname{sech}(x) + i \tanh(x)} \, dx = -i\,x - 2i\,\log\left(i\,e^{(-x)} - 1\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="maxima")
[Out] -I*x - 2*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\operatorname{sech}(x) + i\tanh(x)}\, dx = i\, x - 2i\, \log\left(e^{x} - i\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x)),x, algorithm="giac")
[Out] I*x - 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\operatorname{sech}(x) + i\tanh(x)}\, dx = x\, 1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\, 2\mathrm{i}$$

```
[In] int(1/(tanh(x)*1i + 1/cosh(x)),x)
[Out] x*1i - log(exp(x) - 1i)*2i
```

3.630 $\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx=-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] -x+2*I*cosh(x)/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2759, 8}

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx=-x+\frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[In] Int[(Sech[x] + I*Tanh[x])^(-2),x]
```

```
[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\ &= \frac{2i\cosh(x)}{1+i\sinh(x)} - \int 1\,dx\\ &= -x + \frac{2i\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx = -x + \frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-2),x]
[Out] -x + (4*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A] (verified)

Time = 7.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x+\frac{4i}{e^{x}-i}$	15
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{4}{\tanh\left(\frac{x}{2}\right)-i}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	29

```
[In] int(1/(sech(x)+I*tanh(x))^2,x,method=_RETURNVERBOSE)
[Out] -x+4*I/(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx = -\frac{xe^x - i\,x - 4i}{e^x - i}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x - I*x - 4*I)/(e^x - I)
```

Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx = \int \frac{1}{(i\tanh(x)+\operatorname{sech}(x))^2}\,dx$$

```
[In] integrate(1/(sech(x)+I*tanh(x))**2,x)
[Out] Integral((I*tanh(x) + sech(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx = -x + \frac{4i}{e^{(-x)} + i}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="maxima")
[Out] -x + 4*I/(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^2}\,dx = -x + \frac{4i}{e^x - i}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^2,x, algorithm="giac")
[Out] -x + 4*I/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^2}\,dx = -x + \frac{4\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^2,x)
[Out] 4i/(exp(x) - 1i) - x
```

3.631 $\int \frac{1}{(\mathrm{sech}(x)+i\tanh(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{(\mathrm{sech}(x)+i\tanh(x))^3}\,dx = i\log(i-\sinh(x)) + \frac{2i}{1+i\sinh(x)}$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4476, 2746, 45}

$$\int \frac{1}{(\mathrm{sech}(x)+i\tanh(x))^3}\,dx = \frac{2i}{1+i\sinh(x)} + i\log(-\sinh(x)+i)$$

```
[In] Int[(Sech[x] + I*Tanh[x])^(-3),x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx \\
&= -\left(i\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, i\sinh(x)\right)\right) \\
&= -\left(i\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, i\sinh(x)\right)\right) \\
&= i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{1}{(\text{sech}(x)+i\tanh(x))^3}\,dx = -2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x)) + \frac{2i}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^2}$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-3),x]
[Out] -2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I)/(Cosh[x/2] + I*Sinh[x/2])^2
```

Maple [A] (verified)

Time = 166.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-i\right)^{2}}+2i\ln\left(\mathrm{e}^{x}-i\right)$	26
default	$2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{4}{\tanh\left(\frac{x}{2}\right)-i}-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	56

```
[In] int(1/(sech(x)+I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I*x+4/(exp(x)-I)^2*exp(x)+2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = \frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}-2\left(-i\,e^{(2\,x)}-2\,e^{x}+i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) + I
*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(17) = 34$.

Time = 0.98 (sec) , antiderivative size = 432, normalized size of antiderivative = 15.43

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^3}\,dx = -\frac{2ix\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{4x\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{2ix\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)+1)\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{4\log(\tanh(x)+1)\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)+1)\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)-i\operatorname{sech}(x))\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} - \frac{4\log(\tanh(x)-i\operatorname{sech}(x))\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)-i\operatorname{sech}(x))\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{i\tanh^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{i\operatorname{sech}^2(x)}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)} + \frac{i}{-2\tanh^2(x)+4i\tanh(x)\operatorname{sech}(x)+2\operatorname{sech}^2(x)}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))**3,x)
```

```
[Out] -2*I*x*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*
x*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*
I*x*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) + 2*I*l
og(tanh(x) + 1)*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)
**2) + 4*log(tanh(x) + 1)*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech
(x) + 2*sech(x)**2) - 2*I*log(tanh(x) + 1)*sech(x)**2/(-2*tanh(x)**2 + 4*I*
tanh(x)*sech(x) + 2*sech(x)**2) - 2*I*log(tanh(x) - I*sech(x))*tanh(x)**2/(
-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*log(tanh(x) - I*sec
h(x))*tanh(x)*sech(x)/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

```
+ 2*I*log(tanh(x) - I*sech(x))*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech
(x) + 2*sech(x)**2) + I*tanh(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2
*sech(x)**2) + I*sech(x)**2/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x
)**2) + I/(-2*tanh(x)**2 + 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^3}\,dx = i\,x - \frac{4\,e^{(-x)}}{2i\,e^{(-x)} + e^{(-2\,x)} - 1} + 2i\,\log\left(e^{(-x)} + i\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="maxima")
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^3}\,dx = -i\,x + \frac{4\,e^{x}}{(e^{x} - i)^2} + 2i\,\log\left(e^{x} - i\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^3,x, algorithm="giac")
[Out] -I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{(\operatorname{sech}(x) + i\tanh(x))^3}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} + \frac{4}{\mathrm{e}^{x} - \mathrm{i}}$$

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^3,x)
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

3.632 $\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4} dx = x + \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[Out] x+2/3*I*cosh(x)^3/(1+I*sinh(x))^3-2*I*cosh(x)/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2759, 8}

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4} dx = x + \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)}$$

```
[In] Int[(Sech[x] + I*Tanh[x])^(-4),x]
```

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
```

```
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}\,dx\\ &= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\ &= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}+\int 1\,dx\\ &= x+\frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3}-\frac{2i\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx = \frac{3(8i+3x)\cosh\left(\frac{x}{2}\right)-(16i+3x)\cosh\left(\frac{3x}{2}\right)+6i(4i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-4),x]
```

```
[Out] (3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

```
[In] int(1/(sech(x)+I*tanh(x))^4,x)
[Out] 8*I/(tanh(1/2*x)-I)^2-16/3/(tanh(1/2*x)-I)^3+ln(tanh(1/2*x)+1)-ln(tanh(1/2*
x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx=\frac{3\,xe^{(3\,x)}-3\,(3i\,x+8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}+3i\,x+16i}{3\,(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i)}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="fricas")
[Out] 1/3*(3*x*e^(3*x) - 3*(3*I*x + 8*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*I
)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)
```

Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx=\int \frac{1}{(i\tanh{(x)}+\operatorname{sech}{(x)})^4}\,dx$$

```
[In] integrate(1/(sech(x)+I*tanh(x))**4,x)
[Out] Integral((I*tanh(x) + sech(x))**(-4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx=x-\frac{8\left(3\,e^{(-x)}-3i\,e^{(-2\,x)}+2i\right)}{3\,(3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i)}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="maxima")
[Out] x - 8/3*(3*e^(-x) - 3*I*e^(-2*x) + 2*I)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x)
 + I)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx = x-\frac{8\left(3i\,e^{(2\,x)}+3\,e^{x}-2i\right)}{3\,(e^{x}-i)^3}$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^4,x, algorithm="giac")
[Out] x - 8/3*(3*I*e^(2*x) + 3*e^x - 2*I)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^4}\,dx = x+\frac{\frac{e^{2\,x}\,8i}{3}-\frac{8}{3}i}{e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i}-\frac{8i}{3\,\left(e^{x}-i\right)}+\frac{e^{x}\,8i}{3\,\left(1-e^{2\,x}+e^{x}\,2i\right)}$$

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^4,x)
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 8i/
(3*(exp(x) - 1i)) + (exp(x)*8i)/(3*(exp(x)*2i - exp(2*x) + 1))
```

3.633 $\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx=-i\log(i-\sinh(x))+\frac{2i}{(1+i\sinh(x))^2}-\frac{4i}{1+i\sinh(x)}$$

```
[Out] -I*ln(I-sinh(x))+2*I/(1+I*sinh(x))^2-4*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2746, 45}

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx=-\frac{4i}{1+i\sinh(x)}+\frac{2i}{(1+i\sinh(x))^2}-i\log(-\sinh(x)+i)$$

```
[In] Int[(Sech[x] + I*Tanh[x])^(-5),x]
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh^5(x)}{(1+i\sinh(x))^5}\,dx \\
&= -\left(i\text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, i\sinh(x)\right)\right) \\
&= -\left(i\text{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)\,dx, x, i\sinh(x)\right)\right) \\
&= -i\log(i-\sinh(x))+\frac{2i}{(1+i\sinh(x))^2}-\frac{4i}{1+i\sinh(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$
\begin{aligned}
&\int \frac{1}{(\text{sech}(x)+i\tanh(x))^5}\,dx \\
&= 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))+\frac{-2i+4\sinh(x)}{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^4}
\end{aligned}
$$

```
[In] Integrate[(Sech[x] + I*Tanh[x])^(-5),x]
[Out] 2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + (-2*I + 4*Sinh[x])/(Cosh[x/2] + I*Si
nh[x/2])^4
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^4}-2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^2}+\frac{16}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^3}+i\ln\left(\tanh\right.$$

[In] int(1/(sech(x)+I*tanh(x))^5,x)

[Out] I*ln(tanh(1/2*x)-1)+8*I/(tanh(1/2*x)-I)^4-2*I*ln(tanh(1/2*x)-I)-8*I/(tanh(1
/2*x)-I)^2+16/(tanh(1/2*x)-I)^3+I*ln(tanh(1/2*x)+1)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{1}{(\mathrm{sech}(x)+i\tanh(x))^5}\,dx = \frac{i\,xe^{(4\,x)}+4\,(x-2)e^{(3\,x)}-2\,(3i\,x-4i)e^{(2\,x)}-4\,(x-2)e^{x}-2\,\left(i\,e^{(4\,x)}+4\,e^{(3\,x)}-6i\,e^{(2\,x)}-4\,e^{x}+i\right)\log\left(e^{x}\right.}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="fricas")

[Out] (I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(3*I*x - 4*I)*e^(2*x) - 4*(x - 2)*e^x
- 2*(I*e^(4*x) + 4*e^(3*x) - 6*I*e^(2*x) - 4*e^x + I)*log(e^x - I) + I*x)/(
e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(29) = 58.

Time = 4.26 (sec) , antiderivative size = 1445, normalized size of antiderivative = 34.40

$$\int \frac{1}{(\mathrm{sech}(x)+i\tanh(x))^5}\,dx = \text{Too large to display}$$

[In] integrate(1/(sech(x)+I*tanh(x))**5,x)

[Out] -36*I*x*tanh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)*
*2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*x*tanh(x)**
3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x
)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 216*I*x*tanh(x)**2*sech(
x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2

+ 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*x*tanh(x)*sech(x)**3/(36
*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*
tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*x*sech(x)**4/(36*tanh(x)**4 - 14
4*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)*
*3 + 36*sech(x)**4) + 36*I*log(tanh(x) + 1)*tanh(x)**4/(36*tanh(x)**4 - 144
*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**
3 + 36*sech(x)**4) + 144*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36*tanh(x)**4
 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sec
h(x)**3 + 36*sech(x)**4) - 216*I*log(tanh(x) + 1)*tanh(x)**2*sech(x)**2/(36
*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*
tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*log(tanh(x) + 1)*tanh(x)*sech(x)*
*3/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 +
144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*log(tanh(x) + 1)*sech(x)**
4/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 1
44*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*log(tanh(x) - I*sech(x))*ta
nh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)
2 + 144*I*tanh(x)*sech(x)3 + 36*sech(x)**4) - 144*log(tanh(x) - I*sech(
x))*tanh(x)**3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh
(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 216*I*log(t
anh(x) - I*sech(x))*tanh(x)**2*sech(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3
*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x
)**4) + 144*log(tanh(x) - I*sech(x))*tanh(x)*sech(x)**3/(36*tanh(x)**4 - 14
4*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)*
*3 + 36*sech(x)**4) - 36*I*log(tanh(x) - I*sech(x))*sech(x)**4/(36*tanh(x)*
*4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*s
ech(x)**3 + 36*sech(x)**4) + 28*I*tanh(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)
3*sech(x) - 216*tanh(x)2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sec
h(x)**4) + 52*tanh(x)**3*sech(x)/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x)
- 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 1
8*I*tanh(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*s
ech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 44*tanh(x)*sech(x)*
*3/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 +
144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 24*tanh(x)*sech(x)/(36*tanh(x)*
*4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*tanh(x)*s
ech(x)**3 + 36*sech(x)**4) - 20*I*sech(x)**4/(36*tanh(x)**4 - 144*I*tanh(x)
3*sech(x) - 216*tanh(x)2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sec
h(x)**4) - 6*I*sech(x)**2/(36*tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*t
anh(x)**2*sech(x)**2 + 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 9*I/(36*
tanh(x)**4 - 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 + 144*I*t
anh(x)*sech(x)**3 + 36*sech(x)**4)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = -i\,x - \frac{8\left(e^{(-x)} - i\,e^{(-2\,x)} - e^{(-3\,x)}\right)}{-4i\,e^{(-x)} - 6\,e^{(-2\,x)} + 4i\,e^{(-3\,x)} + e^{(-4\,x)} + 1} - 2i\,\log\left(e^{(-x)} + i\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="maxima")
[Out] -I*x - 8*(e^(-x) - I*e^(-2*x) - e^(-3*x))/(-4*I*e^(-x) - 6*e^(-2*x) + 4*I*e
^(-3*x) + e^(-4*x) + 1) - 2*I*log(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = i\,x - \frac{8\left(e^{(3\,x)} - i\,e^{(2\,x)} - e^{x}\right)}{(e^{x} - i)^4} - 2i\,\log\left(e^{x} - i\right)$$

```
[In] integrate(1/(sech(x)+I*tanh(x))^5,x, algorithm="giac")
[Out] I*x - 8*(e^(3*x) - I*e^(2*x) - e^x)/(e^x - I)^4 - 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{1}{(\operatorname{sech}(x)+i\tanh(x))^5}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i} - \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i} - \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} - \mathrm{i}} + \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 - \mathrm{e}^{3\,x}\,4\mathrm{i} + \mathrm{e}^{x}\,4\mathrm{i}} + \frac{16\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{8}{\mathrm{e}^{x} - \mathrm{i}}$$

```
[In] int(1/(tanh(x)*1i + 1/cosh(x))^5,x)
[Out] x*1i - log(exp(x) - 1i)*2i - 16/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) +
8i/(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) + 16i/(exp(x)*2i -
 exp(2*x) + 1) - 8/(exp(x) - 1i)
```

3.634 $\int (\operatorname{sech}(x) - i \tanh(x))^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int (\operatorname{sech}(x) - i \tanh(x))^5 \, dx = -i \log(i - \sinh(x)) + \frac{2i}{(1 + i \sinh(x))^2} - \frac{4i}{1 + i \sinh(x)}$$

```
[Out] -I*ln(I-sinh(x))+2*I/(1+I*sinh(x))^2-4*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4476, 2746, 45}

$$\int (\operatorname{sech}(x) - i \tanh(x))^5 \, dx = -\frac{4i}{1 + i \sinh(x)} + \frac{2i}{(1 + i \sinh(x))^2} - i \log(-\sinh(x) + i)$$

```
[In] Int[(Sech[x] - I*Tanh[x])^5,x]
[Out] (-I)*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])^2 - (4*I)/(1 + I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^5(x)(1-i\sinh(x))^5\,dx \\
&= i\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, -i\sinh(x)\right) \\
&= i\text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, -i\sinh(x)\right) \\
&= -i\log(i-\sinh(x))+\frac{2i}{(1+i\sinh(x))^2}-\frac{4i}{1+i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\begin{aligned}
\int (\operatorname{sech}(x)-i\tanh(x))^5\,dx &= \arctan(\sinh(x))-i\log(\cosh(x))+\frac{5}{4}i\operatorname{sech}^4(x) \\
&\quad+\operatorname{sech}(x)\tanh(x)-\operatorname{sech}^3(x)\tanh(x) \\
&\quad+\frac{1}{2}i\tanh^2(x)-5\operatorname{sech}(x)\tanh^3(x)+\frac{11}{4}i\tanh^4(x)
\end{aligned}$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^5,x]
```

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]] + ((5*I)/4)*Sech[x]^4 + Sech[x]*Tanh[x] -
Sech[x]^3*Tanh[x] + (I/2)*Tanh[x]^2 - 5*Sech[x]*Tanh[x]^3 + ((11*I)/4)*Tanh
[x]^4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(36) = 72.

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\frac{8\left(\frac{\operatorname{sech}(x)^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)}{3}+2\arctan(\mathrm{e}^x)-\frac{5i}{4\cosh(x)^4}-\frac{5\sinh(x)}{3\cosh(x)^4}-\frac{5i\sinh(x)^2}{\cosh(x)^4}-\frac{5\sinh(x)^3}{\cosh(x)^4}-i\ln(\cosh$$

```
[In] int((sech(x)-I*tanh(x))^5,x)
```

```
[Out] 8/3*(1/4*sech(x)^3+3/8*sech(x))*tanh(x)+2*arctan(exp(x))-5/4*I/cosh(x)^4-5/
3*sinh(x)/cosh(x)^4-5*I*sinh(x)^2/cosh(x)^4-5*sinh(x)^3/cosh(x)^4-I*ln(cosh
(x))+1/2*I*tanh(x)^2+1/4*I*tanh(x)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int(\operatorname{sech}(x)-i\tanh(x))^5\,dx$$
$$=\frac{ixe^{(4x)}+4(x-2)e^{(3x)}-2(3ix-4i)e^{(2x)}-4(x-2)e^x-2\left(ie^{(4x)}+4e^{(3x)}-6ie^{(2x)}-4e^x+i\right)\log(e}{e^{(4x)}-4ie^{(3x)}-6e^{(2x)}+4ie^x+1}$$

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(3*I*x - 4*I)*e^(2*x) - 4*(x - 2)*e^x
- 2*(I*e^(4*x) + 4*e^(3*x) - 6*I*e^(2*x) - 4*e^x + I)*log(e^x - I) + I*x)/(
e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [F]

$$\int(\operatorname{sech}(x)-i\tanh(x))^5\,dx=\int(-i\tanh(x)+\operatorname{sech}(x))^5\,dx$$

```
[In] integrate((sech(x)-I*tanh(x))**5,x)
```

```
[Out] Integral((-I*tanh(x) + sech(x))**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 5.60

$$\int (\operatorname{sech}(x) - i\tanh(x))^5\,dx = \frac{5}{2}i\tanh(x)^4 - ix - \frac{5\left(5e^{(-x)} - 3e^{(-3x)} + 3e^{(-5x)} - 5e^{(-7x)}\right)}{4\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} + \frac{3e^{(-x)} + 11e^{(-3x)} - 11e^{(-5x)} - 3e^{(-7x)}}{4\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} - \frac{5\left(e^{(-x)} - 7e^{(-3x)} + 7e^{(-5x)} - e^{(-7x)}\right)}{2\left(4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1\right)} - \frac{4i\left(e^{(-2x)} + e^{(-4x)} + e^{(-6x)}\right)}{4e^{(-2x)} + 6e^{(-4x)} + 4e^{(-6x)} + e^{(-8x)} + 1} + \frac{20i}{\left(e^{(-x)} + e^{x}\right)^4} - 2\arctan\left(e^{(-x)}\right) - i\log\left(e^{(-2x)} + 1\right)$$

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*I*tanh(x)^4 - I*x - 5/4*(5*e^(-x) - 3*e^(-3*x) + 3*e^(-5*x) - 5*e^(-7*x
))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 1/4*(3*e^(-x) +
11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*
x) + e^(-8*x) + 1) - 5/2*(e^(-x) - 7*e^(-3*x) + 7*e^(-5*x) - e^(-7*x))/(4*e
^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 4*I*(e^(-2*x) + e^(-4*x
) + e^(-6*x))/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 20*I/
(e^(-x) + e^x)^4 - 2*arctan(e^(-x)) - I*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (\operatorname{sech}(x) - i\tanh(x))^5\,dx = ix - \frac{8\left(e^{(3x)} - ie^{(2x)} - e^{x}\right)}{\left(e^{x} - i\right)^4} - 2i\log\left(e^{x} - i\right)$$

```
[In] integrate((sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] I*x - 8*(e^(3*x) - I*e^(2*x) - e^x)/(e^x - I)^4 - 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int (\mathrm{sech}(x) - i\tanh(x))^5\, dx = x\, 1\mathrm{i} - \ln\left(\mathrm{e}^x - \mathrm{i}\right)\, 2\mathrm{i} - \frac{16}{\mathrm{e}^{2\,x}\, 3\mathrm{i} - \mathrm{e}^{3\,x} + 3\,\mathrm{e}^x - \mathrm{i}} + \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 - \mathrm{e}^{3\,x}\, 4\mathrm{i} + \mathrm{e}^x\, 4\mathrm{i}} + \frac{16\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^x\, 2\mathrm{i}} - \frac{8}{\mathrm{e}^x - \mathrm{i}}$$

```
[In] int(-(tanh(x)*1i - 1/cosh(x))^5,x)
```

```
[Out] x*1i - log(exp(x) - 1i)*2i - 16/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) +
8i/(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) + 16i/(exp(x)*2i -
 exp(2*x) + 1) - 8/(exp(x) - 1i)
```

3.635 $\int (\text{sech}(x) - i \tanh(x))^4 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int (\text{sech}(x) - i \tanh(x))^4 \, dx = x + \frac{2i \cosh^3(x)}{3(1 + i \sinh(x))^3} - \frac{2i \cosh(x)}{1 + i \sinh(x)}$$

```
[Out] x+2/3*I*cosh(x)^3/(1+I*sinh(x))^3-2*I*cosh(x)/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4476, 2749, 2759, 8}

$$\int (\text{sech}(x) - i \tanh(x))^4 \, dx = x + \frac{2i \cosh^3(x)}{3(1 + i \sinh(x))^3} - \frac{2i \cosh(x)}{1 + i \sinh(x)}$$

```
[In] Int[(Sech[x] - I*Tanh[x])^4,x]
```

```
[Out] x + (((2*I)/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3 - ((2*I)*Cosh[x])/(1 + I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
```

```
0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^4(x)(1-i\sinh(x))^4\,dx\\
&= \int \frac{\cosh^4(x)}{(1+i\sinh(x))^4}\,dx\\
&= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \int \frac{\cosh^2(x)}{(1+i\sinh(x))^2}\,dx\\
&= \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)} + \int 1\,dx\\
&= x + \frac{2i\cosh^3(x)}{3(1+i\sinh(x))^3} - \frac{2i\cosh(x)}{1+i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int (\operatorname{sech}(x)-i\tanh(x))^4\,dx = \frac{3(8i+3x)\cosh\left(\frac{x}{2}\right)-(16i+3x)\cosh\left(\frac{3x}{2}\right)+6i(4i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^4,x]
[Out] (3*(8*I + 3*x)*Cosh[x/2] - (16*I + 3*x)*Cosh[(3*x)/2] + (6*I)*(4*I + 2*x +
x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 278.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result
risch	$x-\frac{8i\left(-3i\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}-2\right)}{3\left(\mathrm{e}^{x}-i\right)^{3}}$
parts	$\left(\frac{2}{3}+\frac{\mathrm{sech}(x)^{2}}{3}\right)\tanh\left(x\right)-\frac{7\tanh(x)^{3}}{3}-\tanh\left(x\right)-\frac{\ln(\tanh(x)-1)}{2}+\frac{\ln(1+\tanh(x))}{2}+\frac{4i\,\mathrm{sech}(x)^{3}}{3}+4i\left(\frac{\mathrm{sech}(x)^{3}}{3}\right.$
default	$-2\left(\frac{2}{3}+\frac{\mathrm{sech}(x)^{2}}{3}\right)\tanh\left(x\right)+\frac{4i}{3\cosh(x)^{3}}+\frac{3\sinh(x)}{\cosh(x)^{3}}+4i\left(-\frac{\sinh(x)^{2}}{\cosh(x)^{3}}-\frac{2}{3\cosh(x)^{3}}\right)+x-\tanh\left(x\right)-\frac{\tanh(x}{3}$

[In] int((sech(x)-I*tanh(x))^4,x,method=_RETURNVERBOSE)

[Out] x-8/3*I*(-3*I*exp(x)+3*exp(2*x)-2)/(exp(x)-I)^3

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int(\mathrm{sech}(x)-i\tanh(x))^{4}\,dx=\frac{3\,xe^{(3\,x)}-3\,(3i\,x+8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}+3i\,x+16i}{3\,\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i\right)}$$

[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="fricas")

[Out] 1/3*(3*x*e^(3*x) - 3*(3*I*x + 8*I)*e^(2*x) - 3*(3*x + 8)*e^x + 3*I*x + 16*I)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)

Sympy [F]

$$\int(\mathrm{sech}(x)-i\tanh(x))^{4}\,dx=\int\left(-i\tanh\left(x\right)+\mathrm{sech}\left(x\right)\right)^{4}\,dx$$

[In] integrate((sech(x)-I*tanh(x))**4,x)

[Out] Integral((-I*tanh(x) + sech(x))**4, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(28) = 56.

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.76

$$\int (\mathrm{sech}(x) - i\tanh(x))^4\,dx = -2\,\tanh(x)^3 + x - \frac{4\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + 2)}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} - \frac{8i\,e^{(-x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} - \frac{16i\,e^{(-3\,x)}}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} - \frac{8i\,e^{(-5\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} + \frac{4}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)} + \frac{32i}{3\,(e^{(-x)} + e^{x})^3}$$

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="maxima")
```

```
[Out] -2*tanh(x)^3 + x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*
x) + e^(-6*x) + 1) - 8*I*e^(-x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) +
4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 16/3*I*e^(-3*x)/(3*e^
(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 8*I*e^(-5*x)/(3*e^(-2*x) + 3*e^(-4*x)
 + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 32/3*I/(e
^(-x) + e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int (\mathrm{sech}(x) - i\tanh(x))^4\,dx = x - \frac{8\,\left(3i\,e^{(2\,x)} + 3\,e^{x} - 2i\right)}{3\,(e^{x} - i)^3}$$

```
[In] integrate((sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*I*e^(2*x) + 3*e^x - 2*I)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int (\mathrm{sech}(x) - i\tanh(x))^4\,dx = x + \frac{\frac{e^{2x}\,8i}{3} - \frac{8}{3}i}{e^{2x}\,3i - e^{3x} + 3\,e^{x} - i} - \frac{8i}{3\,(e^{x} - i)} + \frac{e^{x}\,8i}{3\,(1 - e^{2x} + e^{x}\,2i)}$$

```
[In] int((tanh(x)*1i - 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 8i/
(3*(exp(x) - 1i)) + (exp(x)*8i)/(3*(exp(x)*2i - exp(2*x) + 1))
```

3.636 $\int (\text{sech}(x) - i \tanh(x))^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int (\text{sech}(x) - i \tanh(x))^3 \, dx = i \log(i - \sinh(x)) + \frac{2i}{1 + i \sinh(x)}$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2746, 45}

$$\int (\text{sech}(x) - i \tanh(x))^3 \, dx = \frac{2i}{1 + i \sinh(x)} + i \log(-\sinh(x) + i)$$

```
[In] Int[(Sech[x] - I*Tanh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \text{sech}^3(x)(1 - i\sinh(x))^3\,dx\\ &= i\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, -i\sinh(x)\right)\\ &= i\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2} + \frac{1}{-1+x}\right)\,dx, x, -i\sinh(x)\right)\\ &= i\log(i - \sinh(x)) + \frac{2i}{1 + i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int (\text{sech}(x) - i\tanh(x))^3\,dx = -\arctan(\sinh(x)) + i\log(\cosh(x)) + \frac{3}{2}i\text{sech}^2(x) + 2\text{sech}(x)\tanh(x) - \frac{1}{2}i\tanh^2(x)$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^3,x]
```

```
[Out] -ArcTan[Sinh[x]] + I*Log[Cosh[x]] + ((3*I)/2)*Sech[x]^2 + 2*Sech[x]*Tanh[x]
 - (I/2)*Tanh[x]^2
```

Maple [A] (verified)

Time = 15.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-i)^{2}}+2i\ln\left(\mathrm{e}^{x}-i\right)$
default	$-\operatorname{sech}\left(x\right)\tanh\left(x\right)-2\arctan\left(\mathrm{e}^{x}\right)+\frac{3i}{2\cosh(x)^{2}}+\frac{3\sinh(x)}{\cosh(x)^{2}}+i\left(\ln\left(\cosh\left(x\right)\right)-\frac{\tanh(x)^{2}}{2}\right)$
parts	$-\operatorname{sech}\left(x\right)\tanh\left(x\right)-2\arctan\left(\mathrm{e}^{x}\right)+i\left(-\frac{\tanh(x)^{2}}{2}-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(1+\tanh(x))}{2}\right)-\frac{3i\tanh(x)^{2}}{2}+\frac{3\sin}{\cosh}$

```
[In] int((sech(x)-I*tanh(x))^3,x,method=_RETURNVERBOSE)
[Out] -I*x+4/(exp(x)-I)^2*exp(x)+2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int (\operatorname{sech}(x)-i\tanh(x))^3\,dx = \frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}-2\left(-i\,e^{(2\,x)}-2\,e^{x}+i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}$$

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="fricas")
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) + I
*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int (\operatorname{sech}(x)-i\tanh(x))^3\,dx = \int \left(-i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^3\,dx$$

```
[In] integrate((sech(x)-I*tanh(x))**3,x)
[Out] Integral((-I*tanh(x) + sech(x))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx = -\frac{3}{2}i\tanh(x)^2 + i\,x + \frac{4\left(e^{(-x)} - e^{(-3\,x)}\right)}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + 2\arctan\left(e^{(-x)}\right) + i\log\left(e^{(-2\,x)} + 1\right)$$

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*I*tanh(x)^2 + I*x + 4*(e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1)
+ 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2*arctan(e^(-x)) + I*log(e^(-2
*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx = -i\,x + \frac{4\,e^x}{(e^x - i)^2} + 2i\log\left(e^x - i\right)$$

```
[In] integrate((sech(x)-I*tanh(x))^3,x, algorithm="giac")
```

```
[Out] -I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int (\operatorname{sech}(x) - i\tanh(x))^3\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^x - \mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x - \mathrm{i}}$$

```
[In] int(-(tanh(x)*1i - 1/cosh(x))^3,x)
```

```
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

3.637 $\int (\operatorname{sech}(x) - i \tanh(x))^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int (\operatorname{sech}(x) - i \tanh(x))^2 \, dx = -x + \frac{2i \cosh(x)}{1 + i \sinh(x)}$$

```
[Out] -x+2*I*cosh(x)/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4476, 2749, 2759, 8}

$$\int (\operatorname{sech}(x) - i \tanh(x))^2 \, dx = -x + \frac{2i \cosh(x)}{1 + i \sinh(x)}$$

```
[In] Int[(Sech[x] - I*Tanh[x])^2,x]
```

```
[Out] -x + ((2*I)*Cosh[x])/(1 + I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \operatorname{sech}^2(x)(1 - i\sinh(x))^2\,dx \\
&= \int \frac{\cosh^2(x)}{(1 + i\sinh(x))^2}\,dx \\
&= \frac{2i\cosh(x)}{1 + i\sinh(x)} - \int 1\,dx \\
&= -x + \frac{2i\cosh(x)}{1 + i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = -\operatorname{arctanh}(\tanh(x)) + 2i\operatorname{sech}(x) + 2\tanh(x)$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^2,x]
[Out] -ArcTanh[Tanh[x]] + (2*I)*Sech[x] + 2*Tanh[x]
```

Maple [A] (verified)

Time = 6.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x+\frac{4i}{\mathrm{e}^{x}-i}$	15
default	$2\tanh\left(x\right)+\frac{2i}{\cosh\left(x\right)}-x$	16
parts	$2\tanh\left(x\right)+2i\,\mathrm{sech}\left(x\right)+\frac{\ln\left(\tanh\left(x\right)-1\right)}{2}-\frac{\ln\left(1+\tanh\left(x\right)\right)}{2}$	25

```
[In] int((sech(x)-I*tanh(x))^2,x,method=_RETURNVERBOSE)
[Out] -x+4*I/(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (\mathrm{sech}(x)-i\tanh(x))^2\,dx=-\frac{xe^{x}-i\,x-4i}{e^{x}-i}$$

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x - I*x - 4*I)/(e^x - I)
```

Sympy [F]

$$\int (\mathrm{sech}(x)-i\tanh(x))^2\,dx=\int \left(-i\tanh\left(x\right)+\mathrm{sech}\left(x\right)\right)^{2}\,dx$$

```
[In] integrate((sech(x)-I*tanh(x))**2,x)
[Out] Integral((-I*tanh(x) + sech(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = -x + \frac{4i}{e^{(-x)} + e^x} + \frac{4}{e^{(-2\,x)} + 1}$$

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="maxima")
[Out] -x + 4*I/(e^(-x) + e^x) + 4/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = -x + \frac{4i}{e^x - i}$$

```
[In] integrate((sech(x)-I*tanh(x))^2,x, algorithm="giac")
[Out] -x + 4*I/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int (\operatorname{sech}(x) - i\tanh(x))^2\,dx = -x + \frac{4\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

```
[In] int((tanh(x)*1i - 1/cosh(x))^2,x)
[Out] 4i/(exp(x) - 1i) - x
```

3.638 $\int (\operatorname{sech}(x) - i\tanh(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = \arctan(\sinh(x)) - i\log(\cosh(x))$$

```
[Out] arctan(sinh(x))-I*ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3855, 3556}

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = \arctan(\sinh(x)) - i\log(\cosh(x))$$

```
[In] Int[Sech[x] - I*Tanh[x],x]
```

```
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\left(i\int \tanh(x)\,dx\right) + \int \text{sech}(x)\,dx$$
$$= \arctan(\sinh(x)) - i\log(\cosh(x))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\text{sech}(x) - i\tanh(x))\,dx = \arctan(\sinh(x)) - i\log(\cosh(x))$$

```
[In] Integrate[Sech[x] - I*Tanh[x],x]
[Out] ArcTan[Sinh[x]] - I*Log[Cosh[x]]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan(\sinh(x)) - i\ln(\cosh(x))$	11
parts	$\arctan(\sinh(x)) - i\ln(\cosh(x))$	11
risch	$i\ln(e^x + i) - i\ln(e^x - i) + ix - i\ln(1 + e^{2x})$	34
parallelrisch	$\frac{i(\ln(1-\tanh(x))+\ln(1+\tanh(x))-2\ln(-i+\coth(x)-\text{csch}(x))+2\ln(i+\coth(x)-\text{csch}(x)))}{2}$	41

```
[In] int(sech(x)-I*tanh(x),x,method=_RETURNVERBOSE)
[Out] arctan(sinh(x))-I*ln(cosh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (\text{sech}(x) - i\tanh(x))\,dx = i\,x - 2i\,\log\left(e^x - i\right)$$

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="fricas")
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = -i(x - \log(\tanh(x) + 1)) + 2\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(sech(x)-I*tanh(x),x)
[Out] -I*(x - log(tanh(x) + 1)) + 2*atan(tanh(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = \arctan(\sinh(x)) - i\log(\cosh(x))$$

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="maxima")
[Out] arctan(sinh(x)) - I*log(cosh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = i\,x + 2\arctan(e^x) - i\log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(sech(x)-I*tanh(x),x, algorithm="giac")
[Out] I*x + 2*arctan(e^x) - I*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int (\operatorname{sech}(x) - i\tanh(x))\,dx = x\,1\mathrm{i} - \ln(\mathrm{e}^x - \mathrm{i})\ 2\mathrm{i}$$

```
[In] int(1/cosh(x) - tanh(x)*1i,x)
[Out] x*1i - log(exp(x) - 1i)*2i
```

3.639 $\int \frac{1}{\operatorname{sech}(x)-i\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{\operatorname{sech}(x)-i\tanh(x)}\,dx = i\log(i+\sinh(x))$$

```
[Out] I*ln(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3238, 2746, 31}

$$\int \frac{1}{\operatorname{sech}(x)-i\tanh(x)}\,dx = i\log(\sinh(x)+i)$$

```
[In] Int[(Sech[x] - I*Tanh[x])^(-1),x]
```

```
[Out] I*Log[I + Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\cosh(x)}{1 - i\sinh(x)}\,dx \\ &= i\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -i\sinh(x)\right) \\ &= i\log(i + \sinh(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x))$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-1),x]
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
risch	$-ix + 2i\ln(e^x + i)$	15
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - i\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + 2i\ln\left(\tanh\left(\frac{x}{2}\right) + i\right)$	33

```
[In] int(1/(sech(x)-I*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -I*x+2*I*ln(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = -i\,x + 2i\,\log\left(e^{x} + i\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="fricas")
[Out] -I*x + 2*I*log(e^x + I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(7) = 14.

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = ix - i\log\left(\tanh\left(x\right) + 1\right) + i\log\left(\tanh\left(x\right) + i\operatorname{sech}\left(x\right)\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x)),x)
[Out] I*x - I*log(tanh(x) + 1) + I*log(tanh(x) + I*sech(x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = i\,x + 2i\,\log\left(i\,e^{(-x)} + 1\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="maxima")
[Out] I*x + 2*I*log(I*e^(-x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = -i\,x + 2i\,\log\left(e^x + i\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x)),x, algorithm="giac")
[Out] -I*x + 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{\operatorname{sech}(x) - i\tanh(x)}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,2\mathrm{i}$$

```
[In] int(-1/(tanh(x)*1i - 1/cosh(x)),x)
[Out] log(exp(x) + 1i)*2i - x*1i
```

3.640 $\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^2}\,dx=-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[Out] -x-2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2759, 8}

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^2}\,dx=-x-\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[In] Int[(Sech[x] - I*Tanh[x])^(-2),x]
```

```
[Out] -x - ((2*I)*Cosh[x])/(1 - I*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx\\ &= -\frac{2i\cosh(x)}{1-i\sinh(x)} - \int 1\,dx\\ &= -x - \frac{2i\cosh(x)}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^2}\,dx = -x + \frac{4\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-2),x]
[Out] -x + (4*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
```

Maple [A] (verified)

Time = 9.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-x-\frac{4i}{{\mathrm e}^{x}+i}$	15
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	29

```
[In] int(1/(sech(x)-I*tanh(x))^2,x,method=_RETURNVERBOSE)
[Out] -x-4*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -\frac{xe^x + i\,x + 4i}{e^x + i}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="fricas")
[Out] -(x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = \int \frac{1}{(-i\tanh{(x)} + \operatorname{sech}{(x)})^2}\,dx$$

```
[In] integrate(1/(sech(x)-I*tanh(x))**2,x)
[Out] Integral((-I*tanh(x) + sech(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -x - \frac{4i}{e^{(-x)} - i}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="maxima")
[Out] -x - 4*I/(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^2}\,dx = -x - \frac{4i}{e^x + i}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^2,x, algorithm="giac")
[Out] -x - 4*I/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\operatorname{sech}(x) - i \tanh(x))^2} \, dx = -x - \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

```
[In] int(1/(tanh(x)*1i - 1/cosh(x))^2,x)
[Out] - x - 4i/(exp(x) + 1i)
```

3.641 $\int \frac{1}{(\mathbf{sech}(x)-i\tanh(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^3}\,dx=-i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}$$

```
[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2746, 45}

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^3}\,dx=-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

```
[In] Int[(Sech[x] - I*Tanh[x])^(-3),x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx\\ &= i\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -i\sinh(x)\right)\\ &= i\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, -i\sinh(x)\right)\\ &= -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^3}\,dx = -2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - i\log(\cosh(x)) + \frac{2}{i+\sinh(x)}$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-3),x]
[Out] -2*ArcTan[Tanh[x/2]] - I*Log[Cosh[x]] + 2/(I + Sinh[x])
```

Maple [A] (verified)

Time = 44.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
risch	$ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+i)^{2}}-2i\ln\left(\mathrm{e}^{x}+i\right)$	26
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	56

```
[In] int(1/(sech(x)-I*tanh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I*x+4*exp(x)/(exp(x)+I)^2-2*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = \frac{i\,xe^{(2\,x)} - 2\,(x-2)e^x - 2\left(i\,e^{(2\,x)} - 2\,e^x - i\right)\log\left(e^x + i\right) - i\,x}{e^{(2\,x)} + 2i\,e^x - 1}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*x
)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(19) = 38$.

Time = 0.97 (sec) , antiderivative size = 432, normalized size of antiderivative = 16.62

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = \frac{2ix\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{4x\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{2ix\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)+1)\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} + \frac{4\log(\tanh(x)+1)\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)+1)\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} + \frac{2i\log(\tanh(x)+i\operatorname{sech}(x))\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{4\log(\tanh(x)+i\operatorname{sech}(x))\tanh(x)\operatorname{sech}(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{2i\log(\tanh(x)+i\operatorname{sech}(x))\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{i\tanh^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{i\operatorname{sech}^2(x)}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)} - \frac{i}{-2\tanh^2(x) - 4i\tanh(x)\operatorname{sech}(x) + 2\operatorname{sech}^2(x)}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))**3,x)
```

```
[Out] 2*I*x*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*x
*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I
*x*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 2*I*lo
g(tanh(x) + 1)*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)*
*2) + 4*log(tanh(x) + 1)*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(
x) + 2*sech(x)**2) + 2*I*log(tanh(x) + 1)*sech(x)**2/(-2*tanh(x)**2 - 4*I*t
anh(x)*sech(x) + 2*sech(x)**2) + 2*I*log(tanh(x) + I*sech(x))*tanh(x)**2/(-
2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) - 4*log(tanh(x) + I*sech
(x))*tanh(x)*sech(x)/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2) -
 2*I*log(tanh(x) + I*sech(x))*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(
x) + 2*sech(x)**2) - I*tanh(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*
sech(x)**2) - I*sech(x)**2/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)
**2) - I/(-2*tanh(x)**2 - 4*I*tanh(x)*sech(x) + 2*sech(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = -i\,x - \frac{4\,e^{(-x)}}{-2i\,e^{(-x)} + e^{(-2\,x)} - 1} - 2i\,\log\left(e^{(-x)} - i\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="maxima")
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = i\,x + \frac{4\,e^{x}}{(e^{x} + i)^2} - 2i\,\log\left(e^{x} + i\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^3,x, algorithm="giac")
[Out] I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{1}{(\operatorname{sech}(x) - i\tanh(x))^3}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} + \frac{4}{\mathrm{e}^{x} + 1\mathrm{i}}$$

```
[In] int(-1/(tanh(x)*1i - 1/cosh(x))^3,x)
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

3.642 $\int \frac{1}{(\text{sech}(x)-i\tanh(x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^4}\,dx = x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[Out] x-2/3*I*cosh(x)^3/(1-I*sinh(x))^3+2*I*cosh(x)/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2759, 8}

$$\int \frac{1}{(\text{sech}(x)-i\tanh(x))^4}\,dx = x-\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3}+\frac{2i\cosh(x)}{1-i\sinh(x)}$$

```
[In] Int[(Sech[x] - I*Tanh[x])^(-4),x]
```

```
[Out] x - (((2*I)/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3 + ((2*I)*Cosh[x])/(1 - I*Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
```

```
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh^4(x)}{(1-i\sinh(x))^4}\,dx \\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} - \int \frac{\cosh^2(x)}{(1-i\sinh(x))^2}\,dx \\
&= -\frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} + \frac{2i\cosh(x)}{1-i\sinh(x)} + \int 1\,dx \\
&= x - \frac{2i\cosh^3(x)}{3(1-i\sinh(x))^3} + \frac{2i\cosh(x)}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^4}\,dx = \frac{3(-8i+3x)\cosh\left(\frac{x}{2}\right)+(16i-3x)\cosh\left(\frac{3x}{2}\right)-6i(-4i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-4),x]
[Out] (3*(-8*I + 3*x)*Cosh[x/2] + (16*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-4*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

```
[In] int(1/(sech(x)-I*tanh(x))^4,x)
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-8*I/(tanh(1/2*x)+I)^2-16/3/(tanh(1/2*x
)+I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^4}\,dx=\frac{3\,xe^{(3\,x)}-3\,(-3i\,x-8i)e^{(2\,x)}-3\,(3\,x+8)e^{x}-3i\,x-16i}{3\,(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i)}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="fricas")
[Out] 1/3*(3*x*e^(3*x) - 3*(-3*I*x - 8*I)*e^(2*x) - 3*(3*x + 8)*e^x - 3*I*x - 16*
I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [F]

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^4}\,dx=\int \frac{1}{\left(-i\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^{4}}\,dx$$

```
[In] integrate(1/(sech(x)-I*tanh(x))**4,x)
[Out] Integral((-I*tanh(x) + sech(x))**(-4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^4}\,dx=x-\frac{8\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-2i\right)}{3\,(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i)}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="maxima")
[Out] x - 8/3*(3*e^(-x) + 3*I*e^(-2*x) - 2*I)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x)
 - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{(\operatorname{sech}(x) - i \tanh(x))^4} \, dx = x - \frac{8 \left(-3i \, e^{(2 \, x)} + 3 \, e^{x} + 2i\right)}{3 \, (e^{x} + i)^{3}}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(-3*I*e^(2*x) + 3*e^x + 2*I)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{1}{(\operatorname{sech}(x) - i \tanh(x))^4} \, dx = x + \frac{\frac{e^{2\,x}\,8\mathrm{i}}{3} - \frac{8}{3}\mathrm{i}}{e^{2\,x}\,3\mathrm{i} + e^{3\,x} - 3\,e^{x} - \mathrm{i}} + \frac{e^{x}\,8\mathrm{i}}{3\,\left(e^{2\,x} - 1 + e^{x}\,2\mathrm{i}\right)} + \frac{8\mathrm{i}}{3\,\left(e^{x} + 1\mathrm{i}\right)}$$

```
[In] int(1/(tanh(x)*1i - 1/cosh(x))^4,x)
```

```
[Out] x + ((exp(2*x)*8i)/3 - 8i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + (ex
p(x)*8i)/(3*(exp(2*x) + exp(x)*2i - 1)) + 8i/(3*(exp(x) + 1i))
```

3.643 $\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^5}\,dx = i\log(i+\sinh(x)) - \frac{2i}{(1-i\sinh(x))^2} + \frac{4i}{1-i\sinh(x)}$$

```
[Out] I*ln(I+sinh(x))-2*I/(1-I*sinh(x))^2+4*I/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2746, 45}

$$\int \frac{1}{(\mathrm{sech}(x)-i\tanh(x))^5}\,dx = \frac{4i}{1-i\sinh(x)} - \frac{2i}{(1-i\sinh(x))^2} + i\log(\sinh(x)+i)$$

```
[In] Int[(Sech[x] - I*Tanh[x])^(-5),x]
```

```
[Out] I*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])^2 + (4*I)/(1 - I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\cosh^5(x)}{(1-i\sinh(x))^5}\,dx\\
&= i\text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, -i\sinh(x)\right)\\
&= i\text{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)\,dx, x, -i\sinh(x)\right)\\
&= i\log(i+\sinh(x)) - \frac{2i}{(1-i\sinh(x))^2} + \frac{4i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\begin{aligned}&\int \frac{1}{(\text{sech}(x)-i\tanh(x))^5}\,dx\\
&= 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + i\log(\cosh(x)) + \frac{2i+4\sinh(x)}{\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^4}\end{aligned}$$

```
[In] Integrate[(Sech[x] - I*Tanh[x])^(-5),x]
[Out] 2*ArcTan[Tanh[x/2]] + I*Log[Cosh[x]] + (2*I + 4*Sinh[x])/(Cosh[x/2] - I*Sin
h[x/2])^4
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}+2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{8i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^4}+\frac{}{(\mathrm{t}}$$

```
[In] int(1/(sech(x)-I*tanh(x))^5,x)
```

```
[Out] -I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+8*I/(tanh(1/2*x)+I)^2+2*I*ln(tanh(
1/2*x)+I)-8*I/(tanh(1/2*x)+I)^4+16/(tanh(1/2*x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^5}\,dx = \frac{-ixe^{(4x)}+4(x-2)e^{(3x)}-2(-3ix+4i)e^{(2x)}-4(x-2)e^{x}-2\left(-ie^{(4x)}+4e^{(3x)}+6ie^{(2x)}-4e^{x}-i\right)}{e^{(4x)}+4ie^{(3x)}-6e^{(2x)}-4ie^{x}+1}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(4*x) + 4*(x - 2)*e^(3*x) - 2*(-3*I*x + 4*I)*e^(2*x) - 4*(x - 2)*e^
x - 2*(-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) - I*x
)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(29) = 58.

Time = 4.42 (sec) , antiderivative size = 1445, normalized size of antiderivative = 36.12

$$\int \frac{1}{(\operatorname{sech}(x)-i\tanh(x))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(sech(x)-I*tanh(x))**5,x)
```

```
[Out] 36*I*x*tanh(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**
2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*x*tanh(x)**3
*sech(x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)
**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 216*I*x*tanh(x)**2*sech(x
)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2
```

```
- 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 144*x*tanh(x)*sech(x)**3/(36*
tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*t
anh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*x*sech(x)**4/(36*tanh(x)**4 + 144
*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**
3 + 36*sech(x)**4) - 36*I*log(tanh(x) + 1)*tanh(x)**4/(36*tanh(x)**4 + 144*
I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3
 + 36*sech(x)**4) + 144*log(tanh(x) + 1)*tanh(x)**3*sech(x)/(36*tanh(x)**4
+ 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech
(x)**3 + 36*sech(x)**4) + 216*I*log(tanh(x) + 1)*tanh(x)**2*sech(x)**2/(36*
tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*t
anh(x)*sech(x)**3 + 36*sech(x)**4) - 144*log(tanh(x) + 1)*tanh(x)*sech(x)**
3/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 1
44*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 36*I*log(tanh(x) + 1)*sech(x)**4
/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 14
4*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 36*I*log(tanh(x) + I*sech(x))*tan
h(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)*
*2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 144*log(tanh(x) + I*sech(x
))*tanh(x)**3*sech(x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(
x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 216*I*log(ta
nh(x) + I*sech(x))*tanh(x)**2*sech(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*
sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)
**4) + 144*log(tanh(x) + I*sech(x))*tanh(x)*sech(x)**3/(36*tanh(x)**4 + 144
*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**
3 + 36*sech(x)**4) + 36*I*log(tanh(x) + I*sech(x))*sech(x)**4/(36*tanh(x)**
4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*se
ch(x)**3 + 36*sech(x)**4) - 28*I*tanh(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)*
*3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech
(x)**4) + 52*tanh(x)**3*sech(x)/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) -
 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 18
*I*tanh(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*se
ch(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 44*tanh(x)*sech(x)**
3/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 1
44*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) + 24*tanh(x)*sech(x)/(36*tanh(x)**
4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*se
ch(x)**3 + 36*sech(x)**4) + 20*I*sech(x)**4/(36*tanh(x)**4 + 144*I*tanh(x)*
*3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech
(x)**4) + 6*I*sech(x)**2/(36*tanh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*ta
nh(x)**2*sech(x)**2 - 144*I*tanh(x)*sech(x)**3 + 36*sech(x)**4) - 9*I/(36*t
anh(x)**4 + 144*I*tanh(x)**3*sech(x) - 216*tanh(x)**2*sech(x)**2 - 144*I*ta
nh(x)*sech(x)**3 + 36*sech(x)**4)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{1}{(\mathrm{sech}(x) - i\tanh(x))^5}\,dx = i\,x - \frac{8\left(e^{(-x)} + i\,e^{(-2\,x)} - e^{(-3\,x)}\right)}{4i\,e^{(-x)} - 6\,e^{(-2\,x)} - 4i\,e^{(-3\,x)} + e^{(-4\,x)} + 1} + 2i\,\log\left(e^{(-x)} - i\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="maxima")
```

```
[Out] I*x - 8*(e^(-x) + I*e^(-2*x) - e^(-3*x))/(4*I*e^(-x) - 6*e^(-2*x) - 4*I*e^(
-3*x) + e^(-4*x) + 1) + 2*I*log(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\mathrm{sech}(x) - i\tanh(x))^5}\,dx = -i\,x - \frac{8\left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x}\right)}{(e^{x} + i)^4} + 2i\,\log\left(e^{x} + i\right)$$

```
[In] integrate(1/(sech(x)-I*tanh(x))^5,x, algorithm="giac")
```

```
[Out] -I*x - 8*(e^(3*x) + I*e^(2*x) - e^x)/(e^x + I)^4 + 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{1}{(\mathrm{sech}(x) - i\tanh(x))^5}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\,2\mathrm{i} + \frac{16\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{8\mathrm{i}}{\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 + \mathrm{e}^{3\,x}\,4\mathrm{i} - \mathrm{e}^{x}\,4\mathrm{i}} - \frac{8}{\mathrm{e}^{x} + 1\mathrm{i}} + \frac{16}{\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^{x} - \mathrm{i}}$$

```
[In] int(-1/(tanh(x)*1i - 1/cosh(x))^5,x)
```

```
[Out] log(exp(x) + 1i)*2i - x*1i + 16i/(exp(2*x) + exp(x)*2i - 1) - 8i/(exp(3*x)*
4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 8/(exp(x) + 1i) + 16/(exp(2*x
)*3i + exp(3*x) - 3*exp(x) - 1i)
```

3.644 $\int (a\coth(x)+b\text{csch}(x))^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 124

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx=-\frac{1}{8}b(15a^4-10a^2b^2+3b^4)\,\text{arctanh}(\cosh(x))+\frac{1}{8}a^2b(7a^2-3b^2)\cosh(x)-\frac{1}{8}(b+a\cosh(x))^2\left(2a(2a^2-b^2)+b(5a^2-3b^2)\cosh(x)\right)\text{csch}^2(x)-\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\text{csch}^4(x)+a^5\log(\sinh(x))$$

```
[Out] -1/8*b*(15*a^4-10*a^2*b^2+3*b^4)*arctanh(cosh(x))+1/8*a^2*b*(7*a^2-3*b^2)*c
osh(x)-1/8*(b+a*cosh(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^2-3*b^2)*cosh(x))*csch(x
)^2-1/4*(b+a*cosh(x))^4*(a+b*cosh(x))*csch(x)^4+a^5*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4477, 2747, 753, 833, 788, 649, 210, 266}

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx=a^5\log(\sinh(x))+\frac{1}{8}a^2b(7a^2-3b^2)\cosh(x)-\frac{1}{8}\text{csch}^2(x)(a\cosh(x)+b)^2\left(b(5a^2-3b^2)\cosh(x)+2a(2a^2-b^2)\right)-\frac{1}{8}b(15a^4-10a^2b^2+3b^4)\,\text{arctanh}(\cosh(x))-\frac{1}{4}\text{csch}^4(x)(a\cosh(x)+b)^4(a+b\cosh(x))$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^5,x]
```

```
[Out] -1/8*(b*(15*a^4 - 10*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]]) + (a^2*b*(7*a^2 - 3
*b^2)*Cosh[x])/8 - ((b + a*Cosh[x])^2*(2*a*(2*a^2 - b^2) + b*(5*a^2 - 3*b^2
)*Cosh[x])*Csch[x]^2)/8 - ((b + a*Cosh[x])^4*(a + b*Cosh[x])*Csch[x]^4)/4 +
 a^5*Log[Sinh[x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\text{integral} = -\left(i\int (ib+ia\cosh(x))^5\text{csch}^5(x)\,dx\right)$$

$$= -\left(a^5\text{Subst}\left(\int \frac{(ib+x)^5}{(-a^2-x^2)^3}\,dx, x, ia\cosh(x)\right)\right)$$

$$= -\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\text{csch}^4(x) - \frac{1}{4}a^3\text{Subst}\left(\int \frac{(ib+x)^3\,(-4a^2+3b^2+ibx)}{(-a^2-x^2)^2}\,dx, x, ia\cosh(x)\right)$$

$$= -\frac{1}{8}(b+a\cosh(x))^2\,\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\text{csch}^2(x) - \frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\text{csch}^4(x) - \frac{1}{8}a\text{Subst}\left(\int \frac{(ib+x)\,(8a^4-7a^2b^2+3b^4-ib(7a^2-3b^2)\,x)}{-a^2-x^2}\,dx, x, ia\cosh(x)\right)$$

$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cosh(x) - \frac{1}{8}(b+a\cosh(x))^2\,\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cosh(x)\right)\text{csch}^2(x) - \frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\text{csch}^4(x) + \frac{1}{8}a\text{Subst}\left(\int \frac{-ia^2b(7a^2-3b^2)-ib(8a^4-7a^2b^2+3b^4)-(8a^4-7a^2b^2+3b^4+b^2(7a^2-3b^2))\,x}{-a^2-x^2}\,d\right.$$

$$
\begin{aligned}
&= \frac{1}{8}a^2b(7a^2-3b^2)\cosh(x)-\frac{1}{8}(b+a\cosh(x))^2\,(2a(2a^2-b^2)\\
&\qquad +b(5a^2-3b^2)\cosh(x))\operatorname{csch}^2(x)-\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\operatorname{csch}^4(x)\\
&\quad -a^5\text{Subst}\left(\int\frac{x}{-a^2-x^2}\,dx,x,ia\cosh(x)\right)\\
&\quad -\frac{1}{8}(iab(15a^4-10a^2b^2+3b^4))\,\text{Subst}\left(\int\frac{1}{-a^2-x^2}\,dx,x,ia\cosh(x)\right)\\
&= -\frac{1}{8}b(15a^4-10a^2b^2+3b^4)\,\text{arctanh}(\cosh(x))+\frac{1}{8}a^2b(7a^2-3b^2)\cosh(x)\\
&\quad -\frac{1}{8}(b+a\cosh(x))^2\,(2a(2a^2-b^2)+b(5a^2-3b^2)\cosh(x))\operatorname{csch}^2(x)\\
&\quad -\frac{1}{4}(b+a\cosh(x))^4(a+b\cosh(x))\operatorname{csch}^4(x)+a^5\log(\sinh(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$
\begin{aligned}
\int(a\coth(x)+b\text{csch}(x))^5\,dx &= \frac{1}{64}\left(-2(7a-3b)(a+b)^4\text{csch}^2\left(\frac{x}{2}\right)-(a+b)^5\text{csch}^4\left(\frac{x}{2}\right)\right.\\
&\quad +8(a-b)^3\,(8a^2+9ab+3b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)\\
&\quad +8(a+b)^3\,(8a^2-9ab+3b^2)\log\left(\sinh\left(\frac{x}{2}\right)\right)\\
&\quad \left.+2(a-b)^4(7a+3b)\text{sech}^2\left(\frac{x}{2}\right)-(a-b)^5\text{sech}^4\left(\frac{x}{2}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^5,x]
```

```
[Out] (-2*(7*a - 3*b)*(a + b)^4*Csch[x/2]^2 - (a + b)^5*Csch[x/2]^4 + 8*(a - b)^3
*(8*a^2 + 9*a*b + 3*b^2)*Log[Cosh[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*b^
2)*Log[Sinh[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sech[x/2]^2 - (a - b)^5*Sech[x/
2]^4)/64
```

Maple [A] (verified)

Time = 55.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.28

method	result
parts	$a^5\left(-\frac{\coth(x)^4}{4}-\frac{\coth(x)^2}{2}-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(1+\coth(x))}{2}\right)+b^5\left(\left(-\frac{\operatorname{csch}(x)^3}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth(x)-\frac{3\operatorname{arctanh}}{4}\right.$
default	$a^5\left(\ln(\sinh(x))-\frac{\coth(x)^2}{2}-\frac{\coth(x)^4}{4}\right)+5a^4b\left(-\frac{\cosh(x)^3}{\sinh(x)^4}+\frac{\cosh(x)}{\sinh(x)^4}+\left(-\frac{\operatorname{csch}(x)^3}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth(x)\right.$
risch	$-a^5x-\frac{e^x(25a^4b\,e^{6x}+10a^2b^3e^{6x}-3b^5e^{6x}+16\,e^{5x}a^5+80\,e^{5x}a^3b^2+15a^4b\,e^{4x}+70a^2b^3e^{4x}+11b^5e^{4x}-16\,e^{3x}a^5+80\,e^{3x}a\,b^4+15a^4b\,e^{2x}}{4(e^{2x}-1)^4}$

```
[In] int((a*coth(x)+b*csch(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-1/4*coth(x)^4-1/2*coth(x)^2-1/2*ln(coth(x)-1)-1/2*ln(1+coth(x)))+b^5*
((-1/4*csch(x)^3+3/8*csch(x))*coth(x)-3/4*arctanh(exp(x)))+5*a^4*b*(-1/sinh
(x)^4*cosh(x)^3+1/sinh(x)^4*cosh(x)+(-1/4*csch(x)^3+3/8*csch(x))*coth(x)-3/
4*arctanh(exp(x)))-5/2*a^3*b^2*coth(x)^4+10*a^2*b^3*(-1/3/sinh(x)^4*cosh(x)
-1/3*(-1/4*csch(x)^3+3/8*csch(x))*coth(x)+1/4*arctanh(exp(x)))-5/4*b^4*csch
(x)^4*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2716 vs. $2(116) = 232$.

Time = 0.28 (sec) , antiderivative size = 2716, normalized size of antiderivative = 21.90

$$\int (a\coth(x)+b\operatorname{csch}(x))^5\,dx = \text{Too large to display}$$

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^5*x*cosh(x)^8 + 8*a^5*x*sinh(x)^8 + 2*(25*a^4*b + 10*a^2*b^3 - 3*
b^5)*cosh(x)^7 + 2*(32*a^5*x*cosh(x) + 25*a^4*b + 10*a^2*b^3 - 3*b^5)*sinh(
x)^7 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^6 + 2*(112*a^5*x*cosh(x)^2 - 16
*a^5*x + 16*a^5 + 80*a^3*b^2 + 7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*s
inh(x)^6 + 8*a^5*x + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^5 + 2*(224*
a^5*x*cosh(x)^3 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 21*(25*a^4*b + 10*a^2*b^
3 - 3*b^5)*cosh(x)^2 - 96*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(x)^5 + 16
*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^4 + 2*(280*a^5*x*cosh(x)^4 + 24*a^5*x
 - 16*a^5 + 80*a*b^4 + 35*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - 240*(
a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 5*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cos
h(x))*sinh(x)^4 + 2*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 + 2*(224*a^5
*x*cosh(x)^5 + 15*a^4*b + 70*a^2*b^3 + 11*b^5 + 35*(25*a^4*b + 10*a^2*b^3 -
 3*b^5)*cosh(x)^4 - 320*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^3 + 10*(15*a^4*b
+ 70*a^2*b^3 + 11*b^5)*cosh(x)^2 + 32*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x))
*sinh(x)^3 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^2 + 2*(112*a^5*x*cosh(x)^
6 - 16*a^5*x + 21*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 + 16*a^5 + 80*a
```

^3*b^2 - 240*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^4 + 10*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^3 + 48*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^2 + 3*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^2 + 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x) - ((8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^8 + 8*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)*sinh(x)^7 + (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*sinh(x)^8 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 - 7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^5 + 8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + 6*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 + 2*(24*a^5 - 45*a^4*b + 30*a^2*b^3 - 9*b^5 + 35*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 - 30*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 - 10*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 + 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 4*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2 + 4*(7*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 - 15*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^4 + 9*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^7 - 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^5 + 3*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^3 - (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^8 + 8*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)*sinh(x)^7 + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*sinh(x)^8 - 4*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^6 - 4*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 - 7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 - 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x)^5 + 8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + 6*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 + 2*(24*a^5 + 45*a^4*b - 30*a^2*b^3 + 9*b^5 + 35*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 - 30*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^5 - 10*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 + 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x)^3 - 4*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^2 + 4*(7*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^6 - 8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 - 15*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^4 + 9*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^7 - 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^5 + 3*(8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x)^3 - (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(32*a^5*x*cosh(x)^7 + 7*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*cosh(x)^6 - 96*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x)^5 + 25*a^4*b + 10*a^2*b^3 - 3*b^5 + 5*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^4 + 32*(3*a^5*x - 2*a^5 + 10*a*b^4)*cosh(x)^3 + 3*(15*a^4*b + 70*a^2*b^3 + 11*b^5)*cosh(x)^2 - 32*(a^5*x - a^5 - 5*a^3*b^2)*cosh(x))*sinh(

```
x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh
(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)
^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x
)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 -
1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cos
h(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx=\int (a\coth{(x)}+b\,\text{csch}{(x)})^5\,dx$$

```
[In] integrate((a*coth(x)+b*csch(x))**5,x)
[Out] Integral((a*coth(x) + b*csch(x))**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(116) = 232.

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.66

$$\begin{aligned}\int (a\coth(x)+b\text{csch}(x))^5\,dx &= -\frac{5}{2}\,a^3b^2\coth{(x)}^4\\ &+a^5\left(x+\frac{4\left(e^{(-2x)}-e^{(-4x)}+e^{(-6x)}\right)}{4\,e^{(-2x)}-6\,e^{(-4x)}+4\,e^{(-6x)}-e^{(-8x)}-1}+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)\right)\\ &+\frac{5}{8}\,a^4b\left(\frac{2\left(5\,e^{(-x)}+3\,e^{(-3x)}+3\,e^{(-5x)}+5\,e^{(-7x)}\right)}{4\,e^{(-2x)}-6\,e^{(-4x)}+4\,e^{(-6x)}-e^{(-8x)}-1}-3\,\log\left(e^{(-x)}+1\right)+3\,\log\left(e^{(-x)}-1\right)\right)\\ &-\frac{1}{8}\,b^5\left(\frac{2\left(3\,e^{(-x)}-11\,e^{(-3x)}-11\,e^{(-5x)}+3\,e^{(-7x)}\right)}{4\,e^{(-2x)}-6\,e^{(-4x)}+4\,e^{(-6x)}-e^{(-8x)}-1}+3\,\log\left(e^{(-x)}+1\right)-3\,\log\left(e^{(-x)}-1\right)\right)\\ &+\frac{5}{4}\,a^2b^3\left(\frac{2\left(e^{(-x)}+7\,e^{(-3x)}+7\,e^{(-5x)}+e^{(-7x)}\right)}{4\,e^{(-2x)}-6\,e^{(-4x)}+4\,e^{(-6x)}-e^{(-8x)}-1}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)\right)\\ &-\frac{20\,ab^4}{\left(e^{(-x)}-e^{x}\right)^4}\end{aligned}$$

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
[Out] -5/2*a^3*b^2*coth(x)^4 + a^5*(x + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(
-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + log(e^(-x) + 1) + log(e^(
-x) - 1)) + 5/8*a^4*b*(2*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 3*log(e^(-x) + 1) +
```

```
 3*log(e^(-x) - 1)) - 1/8*b^5*(2*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*
e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3*log(e^(
-x) + 1) - 3*log(e^(-x) - 1)) + 5/4*a^2*b^3*(2*(e^(-x) + 7*e^(-3*x) + 7*e^(
-5*x) + e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + l
og(e^(-x) + 1) - log(e^(-x) - 1)) - 20*a*b^4/(e^(-x) - e^x)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(116) = 232$.

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.89

$$\int (a\coth(x) + b\text{csch}(x))^5\,dx = \frac{1}{16}\left(8\,a^5 - 15\,a^4b + 10\,a^2b^3 - 3\,b^5\right)\log\left(e^{(-x)} + e^x + 2\right) + \frac{1}{16}\left(8\,a^5 + 15\,a^4b - 10\,a^2b^3 + 3\,b^5\right)\log\left(e^{(-x)} + e^x - 2\right) - \frac{3\,a^5\left(e^{(-x)} + e^x\right)^4 + 25\,a^4b\left(e^{(-x)} + e^x\right)^3 + 10\,a^2b^3\left(e^{(-x)} + e^x\right)^3 - 3\,b^5\left(e^{(-x)} + e^x\right)^3 - 8\,a^5\left(e^{(-x)} + e^x\right)^2 +}{4\left(\left(e^{(-x)} +\right.\right.}$$

```
[In] integrate((a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(e^(-x) + e^x + 2) + 1/16*(
8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(e^(-x) + e^x - 2) - 1/4*(3*a^5*(
e^(-x) + e^x)^4 + 25*a^4*b*(e^(-x) + e^x)^3 + 10*a^2*b^3*(e^(-x) + e^x)^3 -
 3*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 80*a^3*b^2*(e^(-x) + e^x
)^2 - 60*a^4*b*(e^(-x) + e^x) + 40*a^2*b^3*(e^(-x) + e^x) + 20*b^5*(e^(-x)
+ e^x) - 160*a^3*b^2 + 80*a*b^4)/((e^(-x) + e^x)^2 - 4)^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.16

$$\int (a\coth(x)+b\text{csch}(x))^5\,dx = \ln\left(\frac{15\,a^4\,b}{4}+\frac{3\,b^5}{4}-\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5\,\mathrm{e}^x}{4}-\frac{15\,a^4\,b\,\mathrm{e}^x}{4}+\frac{5\,a^2\,b^3\,\mathrm{e}^x}{2}\right)\left(a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{\mathrm{e}^x\,(20\,a^4\,b+40\,a^2\,b^3+4\,b^5)+20\,a\,b^4+4\,a^5+40\,a^3\,b^2}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}-\frac{\mathrm{e}^x\,(30\,a^4\,b+60\,a^2\,b^3+6\,b^5)+40\,a\,b^4+8\,a^5+80\,a^3\,b^2}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}-a^5\,x-\ln\left(\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5}{4}-\frac{15\,a^4\,b}{4}-\frac{3\,b^5\,\mathrm{e}^x}{4}-\frac{15\,a^4\,b\,\mathrm{e}^x}{4}+\frac{5\,a^2\,b^3\,\mathrm{e}^x}{2}\right)\left(-a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{\mathrm{e}^x\,\left(\frac{25\,a^4\,b}{4}+\frac{5\,a^2\,b^3}{2}-\frac{3\,b^5}{4}\right)+4\,a^5+20\,a^3\,b^2}{\mathrm{e}^{2\,x}-1}-\frac{\mathrm{e}^x\,\left(\frac{45\,a^4\,b}{2}+25\,a^2\,b^3+\frac{b^5}{2}\right)+20\,a\,b^4+8\,a^5+60\,a^3\,b^2}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}$$

```
[In] int((b/sinh(x) + a*coth(x))^5,x)
```

```
[Out] log((15*a^4*b)/4 + (3*b^5)/4 - (5*a^2*b^3)/2 - (3*b^5*exp(x))/4 - (15*a^4*b
*exp(x))/4 + (5*a^2*b^3*exp(x))/2)*((15*a^4*b)/8 + a^5 + (3*b^5)/8 - (5*a^2
*b^3)/4) - (exp(x)*(20*a^4*b + 4*b^5 + 40*a^2*b^3) + 20*a*b^4 + 4*a^5 + 40*
a^3*b^2)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - (exp(x)*(3
0*a^4*b + 6*b^5 + 60*a^2*b^3) + 40*a*b^4 + 8*a^5 + 80*a^3*b^2)/(3*exp(2*x)
- 3*exp(4*x) + exp(6*x) - 1) - a^5*x - log((5*a^2*b^3)/2 - (3*b^5)/4 - (15*
a^4*b)/4 - (3*b^5*exp(x))/4 - (15*a^4*b*exp(x))/4 + (5*a^2*b^3*exp(x))/2)*(
(15*a^4*b)/8 - a^5 + (3*b^5)/8 - (5*a^2*b^3)/4) - (exp(x)*((25*a^4*b)/4 - (
3*b^5)/4 + (5*a^2*b^3)/2) + 4*a^5 + 20*a^3*b^2)/(exp(2*x) - 1) - (exp(x)*((
45*a^4*b)/2 + b^5/2 + 25*a^2*b^3) + 20*a*b^4 + 8*a^5 + 60*a^3*b^2)/(exp(4*x
) - 2*exp(2*x) + 1)
```

3.645 $\int (a\coth(x)+b\text{csch}(x))^4\,dx$

Optimal result

Integrand size = 11, antiderivative size = 101

$$\int (a\coth(x)+b\text{csch}(x))^4\,dx = a^4x-\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\text{csch}(x) -\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\text{csch}^3(x) +\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\cosh(x)\sinh(x)$$

```
[Out] a^4*x-1/3*(b+a*cosh(x))^2*(a*b+(3*a^2-2*b^2)*cosh(x))*csch(x)-1/3*(b+a*cosh
(x))^3*(a+b*cosh(x))*csch(x)^3+4/3*a*b*(2*a^2-b^2)*sinh(x)+1/3*a^2*(3*a^2-2
*b^2)*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4477, 2770, 2940, 2813}

$$\int (a\coth(x)+b\text{csch}(x))^4\,dx = a^4x+\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sinh(x)\cosh(x) -\frac{1}{3}\text{csch}(x)(a\cosh(x)+b)^2\left(\left(3a^2-2b^2\right)\cosh(x)+ab\right) -\frac{1}{3}\text{csch}^3(x)(a\cosh(x)+b)^3(a+b\cosh(x))$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^4,x]
```

```
[Out] a^4*x - ((b + a*Cosh[x])^2*(a*b + (3*a^2 - 2*b^2)*Cosh[x])*Csch[x])/3 - ((b
 + a*Cosh[x])^3*(a + b*Cosh[x])*Csch[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sinh[x]
)/3 + (a^2*(3*a^2 - 2*b^2)*Cosh[x]*Sinh[x])/3
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (ib+ia\cosh(x))^4\text{csch}^4(x)\,dx\\
&= -\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\text{csch}^3(x)\\
&\quad+\frac{1}{3}\int (ib+ia\cosh(x))^2\left(-3a^2+2b^2-ab\cosh(x)\right)\text{csch}^2(x)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\operatorname{csch}(x) \\
&\quad -\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\operatorname{csch}^3(x) \\
&\quad +\frac{1}{3}\int(ib+ia\cosh(x))\left(-2ia^2b-2ia\left(3a^2-2b^2\right)\cosh(x)\right)\,dx \\
&= a^4x-\frac{1}{3}(b+a\cosh(x))^2\left(ab+\left(3a^2-2b^2\right)\cosh(x)\right)\operatorname{csch}(x) \\
&\quad -\frac{1}{3}(b+a\cosh(x))^3(a+b\cosh(x))\operatorname{csch}^3(x) \\
&\quad +\frac{4}{3}ab\left(2a^2-b^2\right)\sinh(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int(a\coth(x)+b\operatorname{csch}(x))^4\,dx = -\frac{1}{12}\operatorname{csch}^3(x)&\left(-8a^3b+16ab^3+6b^2\left(3a^2+b^2\right)\cosh(x)\right. \\
&+24a^3b\cosh(2x)+4a^4\cosh(3x)+6a^2b^2\cosh(3x) \\
&\left.-2b^4\cosh(3x)+9a^4x\sinh(x)-3a^4x\sinh(3x)\right)
\end{aligned}
$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^4,x]
```

```
[Out] -1/12*(Csch[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cosh[x] + 24*a^
3*b*Cosh[2*x] + 4*a^4*Cosh[3*x] + 6*a^2*b^2*Cosh[3*x] - 2*b^4*Cosh[3*x] + 9
*a^4*x*Sinh[x] - 3*a^4*x*Sinh[3*x]))
```

Maple [A] (verified)

Time = 20.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

method	result
parts	$a^4\left(-\frac{\coth(x)^3}{3}-\coth(x)-\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}\right)+b^4\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^2}{3}\right)\coth(x)-2a^2b^2\coth(x$
default	$a^4\left(x-\coth(x)-\frac{\coth(x)^3}{3}\right)+4a^3b\left(-\frac{\cosh(x)^2}{\sinh(x)^3}+\frac{2}{3\sinh(x)^3}\right)+6a^2b^2\left(-\frac{\cosh(x)}{2\sinh(x)^3}-\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(x)^2}{3}\right)\coth(x}{2}\right.$
risch	$x\,a^4-\frac{4\left(6a^3b\,e^{5x}+3\,e^{4x}a^4+9\,e^{4x}a^2b^2-4a^3b\,e^{3x}+8a\,b^3e^{3x}-3\,e^{2x}a^4+3\,e^{2x}b^4+6a^3b\,e^{x}+2a^4+3a^2b^2-b^4\right)}{3\left(e^{2x}-1\right)^3}$

```
[In] int((a*coth(x)+b*csch(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/3*coth(x)^3-coth(x)-1/2*ln(coth(x)-1)+1/2*ln(1+coth(x)))+b^4*(2/3-1
/3*csch(x)^2)*coth(x)-2*a^2*b^2*coth(x)^3-4/3*b^3*csch(x)^3*a+4*a^3*b*(-1/3
*csch(x)^3-csch(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(93) = 186$.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.07

$$\int (a\coth(x) + b\text{csch}(x))^4\,dx = -\frac{24\,a^3b\cosh{(x)}^2 - 8\,a^3b + 16\,ab^3 + 2\,(2\,a^4 + 3\,a^2b^2 - b^4)\cosh{(x)}^3 - (3\,a^4x + 4\,a^4 + 6\,a^2b^2 - 2\,b^4)\sinh{(x}}{}$$

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
[Out] -1/3*(24*a^3*b*cosh(x)^2 - 8*a^3*b + 16*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)
*cosh(x)^3 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4)*sinh(x)^3 + 6*(4*a^3*b +
 (2*a^4 + 3*a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 6*(3*a^2*b^2 + b^4)*cosh(x)
 + 3*(3*a^4*x + 4*a^4 + 6*a^2*b^2 - 2*b^4 - (3*a^4*x + 4*a^4 + 6*a^2*b^2 -
2*b^4)*cosh(x)^2)*sinh(x))/(sinh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x))
```

Sympy [F]

$$\int (a\coth(x) + b\text{csch}(x))^4\,dx = \int (a\coth{(x)} + b\operatorname{csch}{(x)})^4\,dx$$

```
[In] integrate((a*coth(x)+b*csch(x))**4,x)
[Out] Integral((a*coth(x) + b*csch(x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(93) = 186$.

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.12

$$\begin{aligned}\int (a\coth(x) + b\text{csch}(x))^4\,dx &= -2\,a^2b^2\coth{(x)}^3 + \frac{1}{3}\,a^4\left(3\,x - \frac{4\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2)}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1}\right)\\ &+ \frac{8}{3}\,a^3b\left(\frac{3\,e^{(-x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} - \frac{2\,e^{(-3\,x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} + \frac{3\,e^{(-5\,x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} -}\right.\\ &+ \frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} - \frac{1}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1}\right)\\ &+ \frac{32\,ab^3}{3\,(e^{(-x)} - e^x)^3}\end{aligned}$$

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*a^2*b^2*coth(x)^3 + 1/3*a^4*(3*x - 4*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^
(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 8/3*a^3*b*(3*e^(-x)/(3*e^(-2*x) - 3*
e^(-4*x) + e^(-6*x) - 1) - 2*e^(-3*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) -
 1) + 3*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 4/3*b^4*(3*e^(
-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 1/(3*e^(-2*x) - 3*e^(-4*x)
 + e^(-6*x) - 1)) + 32/3*a*b^3/(e^(-x) - e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int (a\coth(x) + b\text{csch}(x))^4\,dx = a^4x - \frac{4\left(6\,a^3be^{(5\,x)} + 3\,a^4e^{(4\,x)} + 9\,a^2b^2e^{(4\,x)} - 4\,a^3be^{(3\,x)} + 8\,ab^3e^{(3\,x)} - 3\,a^4e^{(2\,x)} + 3\,b^4e^{(2\,x)} + 6\,a^3be^x + 2\,a^4 + \right.}{3\left(e^{(2\,x)} - 1\right)^3}$$

```
[In] integrate((a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

```
[Out] a^4*x - 4/3*(6*a^3*b*e^(5*x) + 3*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x) - 4*a^3*b*
e^(3*x) + 8*a*b^3*e^(3*x) - 3*a^4*e^(2*x) + 3*b^4*e^(2*x) + 6*a^3*b*e^x + 2
*a^4 + 3*a^2*b^2 - b^4)/(e^(2*x) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\int (a\coth(x) + b\text{csch}(x))^4\,dx = a^4\,x - \frac{4\,a^4 + 8\,\mathrm{e}^x\,a^3\,b + 12\,a^2\,b^2}{\mathrm{e}^{2\,x} - 1} - \frac{\mathrm{e}^x\,\left(\frac{32\,a^3\,b}{3} + \frac{32\,a\,b^3}{3}\right) + 4\,a^4 + 4\,b^4 + 24\,a^2\,b^2}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{\mathrm{e}^x\,\left(\frac{32\,a^3\,b}{3} + \frac{32\,a\,b^3}{3}\right) + \frac{8\,a^4}{3} + \frac{8\,b^4}{3} + 16\,a^2\,b^2}{3\,\mathrm{e}^{2\,x} - 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} - 1}$$

```
[In] int((b/sinh(x) + a*coth(x))^4,x)
```

```
[Out] a^4*x - (4*a^4 + 12*a^2*b^2 + 8*a^3*b*exp(x))/(exp(2*x) - 1) - (exp(x)*((32
*a*b^3)/3 + (32*a^3*b)/3) + 4*a^4 + 4*b^4 + 24*a^2*b^2)/(exp(4*x) - 2*exp(2
*x) + 1) - (exp(x)*((32*a*b^3)/3 + (32*a^3*b)/3) + (8*a^4)/3 + (8*b^4)/3 +
16*a^2*b^2)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)
```

3.646 $\int (a\coth(x)+b\text{csch}(x))^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx=-\frac{1}{2}b(3a^2-b^2)\,\text{arctanh}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)-\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x)+a^3\log(\sinh(x))$$

```
[Out] -1/2*b*(3*a^2-b^2)*arctanh(cosh(x))+1/2*a^2*b*cosh(x)-1/2*(b+a*cosh(x))^2*(
a+b*cosh(x))*csch(x)^2+a^3*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4477, 2747, 753, 788, 649, 210, 266}

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx=a^3\log(\sinh(x))-\frac{1}{2}b(3a^2-b^2)\,\text{arctanh}(\cosh(x))+\frac{1}{2}a^2b\cosh(x)-\frac{1}{2}\text{csch}^2(x)(a\cosh(x)+b)^2(a+b\cosh(x))$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^3,x]
```

```
[Out] -1/2*(b*(3*a^2 - b^2)*ArcTanh[Cosh[x]]) + (a^2*b*Cosh[x])/2 - ((b + a*Cosh[
x])^2*(a + b*Cosh[x])*Csch[x]^2)/2 + a^3*Log[Sinh[x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\text{integral} = i\int (ib + ia\cosh(x))^3\text{csch}^3(x)\,dx$$

$$= a^3\text{Subst}\left(\int \frac{(ib+x)^3}{(-a^2-x^2)^2}\,dx, x, ia\cosh(x)\right)$$

$$
\begin{aligned}
&= -\frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x) \\
&\quad + \frac{1}{2}a\text{Subst}\left(\int \frac{(ib+x)\left(-2a^2+b^2+ibx\right)}{-a^2-x^2}\,dx,x,ia\cosh(x)\right) \\
&= \frac{1}{2}a^2b\cosh(x) - \frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x) \\
&\quad - \frac{1}{2}a\text{Subst}\left(\int \frac{ia^2b-ib(-2a^2+b^2)+2a^2x}{-a^2-x^2}\,dx,x,ia\cosh(x)\right) \\
&= \frac{1}{2}a^2b\cosh(x) - \frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x) \\
&\quad - a^3\text{Subst}\left(\int \frac{x}{-a^2-x^2}\,dx,x,ia\cosh(x)\right) \\
&\quad - \frac{1}{2}\left(iab\left(3a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-a^2-x^2}\,dx,x,ia\cosh(x)\right) \\
&= -\frac{1}{2}b\left(3a^2-b^2\right)\text{arctanh}(\cosh(x)) + \frac{1}{2}a^2b\cosh(x) \\
&\quad - \frac{1}{2}(b+a\cosh(x))^2(a+b\cosh(x))\text{csch}^2(x) + a^3\log(\sinh(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$
\int (a\coth(x)+b\text{csch}(x))^3\,dx = \frac{1}{8}\left(-(a+b)^3\text{csch}^2\left(\frac{x}{2}\right)+4(a-b)^2(2a+b)\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4(2a-b)(a+b)^2\log\left(\sinh\left(\frac{x}{2}\right)\right)+(a-b)^3\text{sech}^2\left(\frac{x}{2}\right)\right)
$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^3,x]
[Out] (-((a + b)^3*Csch[x/2]^2) + 4*(a - b)^2*(2*a + b)*Log[Cosh[x/2]] + 4*(2*a -
 b)*(a + b)^2*Log[Sinh[x/2]] + (a - b)^3*Sech[x/2]^2)/8
```

Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result
default	$a^3\left(\ln(\sinh(x))-\frac{\coth(x)^2}{2}\right)+3a^2b\left(-\frac{\cosh(x)}{\sinh(x)^2}+\frac{\text{csch}(x)\coth(x)}{2}-\text{arctanh}(e^x)\right)-\frac{3ab^2}{2\sinh(x)^2}+b^3\left(-\frac{\text{csch}}{}\right.$
parts	$a^3\left(-\frac{\coth(x)^2}{2}-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(1+\coth(x))}{2}\right)+b^3\left(-\frac{\text{csch}(x)\coth(x)}{2}+\text{arctanh}(e^x)\right)+3a^2b\left(-\frac{\cosh(x)}{\sinh(x)^2}+\right.$
risch	$-a^3x-\frac{e^x\left(3a^2b\,e^{2x}+b^3e^{2x}+2a^3e^x+6\,e^xb^2a+3a^2b+b^3\right)}{\left(e^{2x}-1\right)^2}+\ln(e^x-1)\,a^3+\frac{3\ln(e^x-1)a^2b}{2}-\frac{\ln(e^x-1)b^3}{2}+\ln(e^x+1)$

```
[In] int((a*coth(x)+b*csch(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(ln(sinh(x))-1/2*coth(x)^2)+3*a^2*b*(-1/sinh(x)^2*cosh(x)+1/2*csch(x)*c
oth(x)-arctanh(exp(x)))-3/2*a*b^2/sinh(x)^2+b^3*(-1/2*csch(x)*coth(x)+arcta
nh(exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 674, normalized size of antiderivative = 11.42

$$\int (a\coth(x) + b\text{csch}(x))^3 \, dx =$$
$$-\frac{2\,a^3x\cosh\left(x\right)^4 + 2\,a^3x\sinh\left(x\right)^4 + 2\,a^3x + 2\,(3\,a^2b + b^3)\cosh\left(x\right)^3 + 2\,(4\,a^3x\cosh\left(x\right) + 3\,a^2b + b^3)\sinh}{}$$

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*x*cosh(x)^4 + 2*a^3*x*sinh(x)^4 + 2*a^3*x + 2*(3*a^2*b + b^3)*c
osh(x)^3 + 2*(4*a^3*x*cosh(x) + 3*a^2*b + b^3)*sinh(x)^3 - 4*(a^3*x - a^3 -
 3*a*b^2)*cosh(x)^2 + 2*(6*a^3*x*cosh(x)^2 - 2*a^3*x + 2*a^3 + 6*a*b^2 + 3*
(3*a^2*b + b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^2*b + b^3)*cosh(x) - ((2*a^3 -
3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (2
*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3 - 3*a^2*
b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b + b^3)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^3 - 3*a^2
*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - ((2*a^3 + 3*a^2*b
- b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 3
*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 - 2*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3 - (2*a^3 + 3*a^2*b - b^3
)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*a^3*x*cosh(x)^3 + 3*a
^2*b + b^3 + 3*(3*a^2*b + b^3)*cosh(x)^2 - 4*(a^3*x - a^3 - 3*a*b^2)*cosh(x
))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (a\coth(x)+b\text{csch}(x))^3\,dx=\int (a\coth(x)+b\,\text{csch}(x))^3\,\,dx$$

```
[In] integrate((a*coth(x)+b*csch(x))**3,x)
[Out] Integral((a*coth(x) + b*csch(x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(53) = 106$.

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.58

$$\begin{aligned}
&\int (a\coth(x)+b\text{csch}(x))^3\,dx\\
&=-\frac{3}{2}\,ab^2\coth(x)^2+a^3\left(x+\frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)\right)\\
&\quad+\frac{1}{2}\,b^3\left(\frac{2\left(e^{(-x)}+e^{(-3\,x)}\right)}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)\right)\\
&\quad+\frac{3}{2}\,a^2b\left(\frac{2\left(e^{(-x)}+e^{(-3\,x)}\right)}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)\right)
\end{aligned}$$

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
[Out] -3/2*a*b^2*coth(x)^2 + a^3*(x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + lo
g(e^(-x) + 1) + log(e^(-x) - 1)) + 1/2*b^3*(2*(e^(-x) + e^(-3*x))/(2*e^(-2*
x) - e^(-4*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)) + 3/2*a^2*b*(2*(e^(
-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - log(e^(-x) + 1) + log(e^(-x)
- 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.95

$$\begin{aligned}
&\int (a\coth(x)+b\text{csch}(x))^3\,dx\\
&=\frac{1}{4}\left(2\,a^3-3\,a^2b+b^3\right)\log\left(e^{(-x)}+e^x+2\right)+\frac{1}{4}\left(2\,a^3+3\,a^2b-b^3\right)\log\left(e^{(-x)}+e^x-2\right)\\
&\quad-\frac{a^3\left(e^{(-x)}+e^x\right)^2+6\,a^2b\left(e^{(-x)}+e^x\right)+2\,b^3\left(e^{(-x)}+e^x\right)+12\,ab^2}{2\left(\left(e^{(-x)}+e^x\right)^2-4\right)}
\end{aligned}$$

```
[In] integrate((a*coth(x)+b*csch(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3 - 3*a^2*b + b^3)*log(e^(-x) + e^x + 2) + 1/4*(2*a^3 + 3*a^2*b -
b^3)*log(e^(-x) + e^x - 2) - 1/2*(a^3*(e^(-x) + e^x)^2 + 6*a^2*b*(e^(-x) +
e^x) + 2*b^3*(e^(-x) + e^x) + 12*a*b^2)/((e^(-x) + e^x)^2 - 4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

$$\int (a\coth(x)+b\mathrm{csch}(x))^3\,dx = \ln\left(b^3-3\,a^2\,b+b^3\,\mathrm{e}^x-3\,a^2\,b\,\mathrm{e}^x\right)\,\left(a^3-\frac{3\,a^2\,b}{2}+\frac{b^3}{2}\right) - \frac{6\,a\,b^2+2\,a^3+\mathrm{e}^x\,\left(3\,a^2\,b+b^3\right)}{\mathrm{e}^{2\,x}-1} - \frac{\mathrm{e}^x\,\left(6\,a^2\,b+2\,b^3\right)+6\,a\,b^2+2\,a^3}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-a^3\,x + \ln\left(3\,a^2\,b-b^3+b^3\,\mathrm{e}^x-3\,a^2\,b\,\mathrm{e}^x\right)\,\left(a^3+\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)$$

```
[In] int((b/sinh(x) + a*coth(x))^3,x)
```

```
[Out] log(b^3 - 3*a^2*b + b^3*exp(x) - 3*a^2*b*exp(x))*(a^3 - (3*a^2*b)/2 + b^3/2
) - (6*a*b^2 + 2*a^3 + exp(x)*(3*a^2*b + b^3))/(exp(2*x) - 1) - (exp(x)*(6*
a^2*b + 2*b^3) + 6*a*b^2 + 2*a^3)/(exp(4*x) - 2*exp(2*x) + 1) - a^3*x + log
(3*a^2*b - b^3 + b^3*exp(x) - 3*a^2*b*exp(x))*((3*a^2*b)/2 + a^3 - b^3/2)
```

3.647 $\int (a\coth(x) + b\text{csch}(x))^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int (a\coth(x) + b\text{csch}(x))^2 \, dx = a^2x - (b + a\cosh(x))(a + b\cosh(x))\text{csch}(x) + ab\sinh(x)$$

```
[Out] a^2*x-(b+a*cosh(x))*(a+b*cosh(x))*csch(x)+a*b*sinh(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4477, 2770, 2717}

$$\int (a\coth(x) + b\text{csch}(x))^2 \, dx = a^2x + ab\sinh(x) - \text{csch}(x)(a\cosh(x) + b)(a + b\cosh(x))$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^2,x]
[Out] a^2*x - (b + a*Cosh[x])*(a + b*Cosh[x])*Csch[x] + a*b*Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
```

```
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (ib + ia\cosh(x))^2 \text{csch}^2(x)\,dx \\
&= -((b + a\cosh(x))(a + b\cosh(x))\text{csch}(x)) - \int \left(-a^2 - ab\cosh(x)\right)\,dx \\
&= a^2x - (b + a\cosh(x))(a + b\cosh(x))\text{csch}(x) + (ab)\int \cosh(x)\,dx \\
&= a^2x - (b + a\cosh(x))(a + b\cosh(x))\text{csch}(x) + ab\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int (a\coth(x) + b\text{csch}(x))^2\,dx = -\left(\left(a^2 + b^2\right)\coth(x)\right) + a(ax - 2b\text{csch}(x))$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^2,x]
[Out] -((a^2 + b^2)*Coth[x]) + a*(a*x - 2*b*Csch[x])
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
default	$a^2(x - \coth(x)) - \frac{2ab}{\sinh(x)} - b^2\coth(x)$	27
risch	$a^2x - \frac{2(2b\,\mathrm{e}^x a + a^2 + b^2)}{\mathrm{e}^{2x} - 1}$	30
parts	$a^2\left(-\coth(x) - \frac{\ln(\coth(x)-1)}{2} + \frac{\ln(1+\coth(x))}{2}\right) - b^2\coth(x) - 2b\,\text{csch}(x)\,a$	38

```
[In] int((a*coth(x)+b*csch(x))^2,x,method=_RETURNVERBOSE)
[Out] a^2*(x-coth(x))-2*a*b/sinh(x)-b^2*coth(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx=-\frac{2\,ab+(a^2+b^2)\cosh{(x)}-(a^2x+a^2+b^2)\sinh{(x)}}{\sinh{(x)}}$$

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
[Out] -(2*a*b + (a^2 + b^2)*cosh(x) - (a^2*x + a^2 + b^2)*sinh(x))/sinh(x)
```

Sympy [F]

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx=\int (a\coth{(x)}+b\,\text{csch}{(x)})^2\;dx$$

```
[In] integrate((a*coth(x)+b*csch(x))**2,x)
[Out] Integral((a*coth(x) + b*csch(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int (a\coth(x)+b\text{csch}(x))^2\,dx=a^2\left(x+\frac{2}{e^{(-2\,x)}-1}\right)+\frac{4\,ab}{e^{(-x)}-e^x}+\frac{2\,b^2}{e^{(-2\,x)}-1}$$

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
[Out] a^2*(x + 2/(e^(-2*x) - 1)) + 4*a*b/(e^(-x) - e^x) + 2*b^2/(e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a\coth(x) + b\text{csch}(x))^2 \, dx = a^2x - \frac{2\,(2\,abe^x + a^2 + b^2)}{e^{(2\,x)} - 1}$$

```
[In] integrate((a*coth(x)+b*csch(x))^2,x, algorithm="giac")
[Out] a^2*x - 2*(2*a*b*e^x + a^2 + b^2)/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int (a\coth(x) + b\text{csch}(x))^2 \, dx = a^2\,x - \frac{2\,a^2 + 4\,\mathrm{e}^x\,a\,b + 2\,b^2}{\mathrm{e}^{2\,x} - 1}$$

```
[In] int((b/sinh(x) + a*coth(x))^2,x)
[Out] a^2*x - (2*a^2 + 2*b^2 + 4*a*b*exp(x))/(exp(2*x) - 1)
```

3.648 $\int (a\coth(x) + b\mathbf{csch}(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (a\coth(x) + b\text{csch}(x))\,dx = -b\text{arctanh}(\cosh(x)) + a\log(\sinh(x))$$

```
[Out] -b*arctanh(cosh(x))+a*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3556, 3855}

$$\int (a\coth(x) + b\text{csch}(x))\,dx = a\log(\sinh(x)) - b\text{arctanh}(\cosh(x))$$

```
[In] Int[a*Coth[x] + b*Csch[x],x]
```

```
[Out] -(b*ArcTanh[Cosh[x]]) + a*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \coth(x)\,dx + b\int \operatorname{csch}(x)\,dx \\
&= -b\operatorname{arctanh}(\cosh(x)) + a\log(\sinh(x))
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$
\begin{aligned}
\int (a\coth(x) + b\operatorname{csch}(x))\,dx = &-b\log\left(\cosh\left(\frac{x}{2}\right)\right) + a\log(\cosh(x)) \\
&+ b\log\left(\sinh\left(\frac{x}{2}\right)\right) + a\log(\tanh(x))
\end{aligned}
$$

```
[In] Integrate[a*Coth[x] + b*Csch[x],x]
[Out] -(b*Log[Cosh[x/2]]) + a*Log[Cosh[x]] + b*Log[Sinh[x/2]] + a*Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$a\ln(\sinh(x)) + b\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	14
parts	$a\ln(\sinh(x)) + b\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	14
parallelrisch	$b\ln(\coth(x) - \operatorname{csch}(x)) - a(x - \ln(\tanh(x)) + \ln(1 - \tanh(x)))$	29
risch	$-ax + a\ln(e^{2x} - 1) + b\ln(e^{x} - 1) - b\ln(e^{x} + 1)$	30

```
[In] int(a*coth(x)+b*csch(x),x,method=_RETURNVERBOSE)
[Out] a*ln(sinh(x))+b*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int (a\coth(x) + b\text{csch}(x))\,dx = -ax + (a-b)\log(\cosh(x) + \sinh(x) + 1) + (a+b)\log(\cosh(x) + \sinh(x) - 1)$$

```
[In] integrate(a*coth(x)+b*csch(x),x, algorithm="fricas")
[Out] -a*x + (a - b)*log(cosh(x) + sinh(x) + 1) + (a + b)*log(cosh(x) + sinh(x) -
 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int (a\coth(x) + b\text{csch}(x))\,dx = a(x - \log(\tanh(x) + 1) + \log(\tanh(x))) + b\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(a*coth(x)+b*csch(x),x)
[Out] a*(x - log(tanh(x) + 1) + log(tanh(x))) + b*log(tanh(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int (a\coth(x) + b\text{csch}(x))\,dx = a\log(\sinh(x)) + b\log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

```
[In] integrate(a*coth(x)+b*csch(x),x, algorithm="maxima")
[Out] a*log(sinh(x)) + b*log(tanh(1/2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int (a\coth(x) + b\text{csch}(x))\,dx = -a\big(x - \log\left(\left|e^{(2\,x)} - 1\right|\right)\big) - b(\log\left(e^{x} + 1\right) - \log\left(\left|e^{x} - 1\right|\right))$$

```
[In] integrate(a*coth(x)+b*csch(x),x, algorithm="giac")
[Out] -a*(x - log(abs(e^(2*x) - 1))) - b*(log(e^x + 1) - log(abs(e^x - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int (a\coth(x) + b\text{csch}(x))\,dx = \ln\left(-2\,b - 2\,b\,\mathrm{e}^{x}\right)\,(a - b) - a\,x + \ln\left(2\,b - 2\,b\,\mathrm{e}^{x}\right)\,(a + b)$$

```
[In] int(b/sinh(x) + a*coth(x),x)
[Out] log(- 2*b - 2*b*exp(x))*(a - b) - a*x + log(2*b - 2*b*exp(x))*(a + b)
```

3.649 $\int \frac{1}{a\coth(x)+b\mathbf{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{a\coth(x)+b\mathrm{csch}(x)}\,dx=\frac{\log(b+a\cosh(x))}{a}$$

```
[Out] ln(b+a*cosh(x))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3239, 2747, 31}

$$\int \frac{1}{a\coth(x)+b\mathrm{csch}(x)}\,dx=\frac{\log(a\cosh(x)+b)}{a}$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-1),x]
```

```
[Out] Log[b + a*Cosh[x]]/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int \frac{\sinh(x)}{ib + ia\cosh(x)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{ib+x}\,dx, x, ia\cosh(x)\right)}{a}\\ &= \frac{\log(b + a\cosh(x))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = \frac{\log(b + a\cosh(x))}{a}$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-1),x]
[Out] Log[b + a*Cosh[x]]/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
risch	$-\frac{x}{a} + \frac{\ln\left(e^{2x} + \frac{2b\,e^{x}}{a} + 1\right)}{a}$	27
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2} a - \tanh\left(\frac{x}{2}\right)^{2} b + a + b\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a}$	51

```
[In] int(1/(a*coth(x)+b*csch(x)),x,method=_RETURNVERBOSE)
[Out] -x/a+1/a*ln(exp(2*x)+2*b/a*exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = -\frac{x - \log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [F]

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = \int \frac{1}{a\coth(x) + b\,\text{csch}(x)}\,dx$$

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x)
[Out] Integral(1/(a*coth(x) + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = \frac{x}{a} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="maxima")
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{1}{a\coth(x) + b\text{csch}(x)}\,dx = \frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x)),x, algorithm="giac")
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{a\coth(x)+b\text{csch}(x)}\,dx=-\frac{x-\ln\left(a+2\,b\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}\right)}{a}$$

```
[In] int(1/(b/sinh(x) + a*coth(x)),x)
[Out] -(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

3.650 $\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx=\frac{x}{a^2}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{a(b+a\cosh(x))}$$

```
[Out] x/a^2-sinh(x)/a/(b+a*cosh(x))-2*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2
))/a^2/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {4477, 2772, 2814, 2738, 211}

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^2}\,dx=-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{x}{a^2}-\frac{\sinh(x)}{a(a\cosh(x)+b)}$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*
Sqrt[a + b]) - Sinh[x]/(a*(b + a*Cosh[x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\sinh^2(x)}{(ib+ia\cosh(x))^2}\,dx\\
&= -\frac{\sinh(x)}{a(b+a\cosh(x))}+\frac{i\int \frac{\cosh(x)}{ib+ia\cosh(x)}\,dx}{a}\\
&= \frac{x}{a^2}-\frac{\sinh(x)}{a(b+a\cosh(x))}-\frac{(ib)\int \frac{1}{ib+ia\cosh(x)}\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{\sinh(x)}{a(b+a\cosh(x))}-\frac{(2ib)\text{Subst}\left(\int \frac{1}{ia+ib-(-ia+ib)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{x}{a^2}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{a(b+a\cosh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}\,dx=\frac{x+\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{a\sinh(x)}{b+a\cosh(x)}}{a^2}$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-2),x]
```

```
[Out] (x + (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (
a*Sinh[x])/(b + a*Cosh[x]))/a^2
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{-\frac{2\tanh\left(\frac{x}{2}\right)a}{\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^2}$	96
risch	$\frac{x}{a^2}+\frac{2\,e^{x}b+2a}{a^2(a\,e^{2x}+2\,e^{x}b+a)}-\frac{b\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^2}+\frac{b\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^2}$	148

```
[In] int(1/(a*coth(x)+b*csch(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^2*(-tanh(1/2*x)*a/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)-b/((a+b)*(a-b))
^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))-1/a^2*ln(tanh(1/2*x)-
1)+1/a^2*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(57) = 114.

Time = 0.27 (sec) , antiderivative size = 682, normalized size of antiderivative = 10.18

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}\,dx$$
$$=\Bigg[\frac{(a^3-ab^2)x\cosh(x)^2+(a^3-ab^2)x\sinh(x)^2+2\,a^3-2\,ab^2-\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,b^2\cosh(x)\right.}{}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="fricas")
```

```
[Out] [((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 -
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b + 2*(a*b*cosh(x) + b^
2)*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cos
h(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*
cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*c
osh(x) + b)*sinh(x) + a)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b - b^3
)*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b^3)*x)*
sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^2)*sinh(
x)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*c
osh(x))*sinh(x)), ((a^3 - a*b^2)*x*cosh(x)^2 + (a^3 - a*b^2)*x*sinh(x)^2 +
2*a^3 - 2*a*b^2 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*b^2*cosh(x) + a*b +
2*(a*b*cosh(x) + b^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(
x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2)*x + 2*(a^2*b - b^3 + (a^2*b - b^3)
*x)*cosh(x) + 2*(a^2*b - b^3 + (a^3 - a*b^2)*x*cosh(x) + (a^2*b - b^3)*x)*s
inh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^2 + (a^5 - a^3*b^2)*sinh(x
)^2 + 2*(a^4*b - a^2*b^3)*cosh(x) + 2*(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*co
sh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^2}\,dx = \int \frac{1}{\left(a\coth\left(x\right) + b\operatorname{csch}\left(x\right)\right)^2}\,dx$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))**2,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}\,dx = -\frac{2\,b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^2}+\frac{x}{a^2}+\frac{2\,(be^x+a)}{(ae^{(2\,x)}+2\,be^x+a)a^2}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) + x/a^2 + 2*
(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^2}\,dx = \frac{x}{a^2}+\frac{\frac{2}{a}+\frac{2\,b\,e^x}{a^2}}{a+2\,b\,e^x+a\,e^{2\,x}}+\frac{b\,\ln\left(\frac{2\,b\,e^x}{a^3}-\frac{2\,b\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b\,\ln\left(\frac{2\,b\,e^x}{a^3}+\frac{2\,b\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(1/(b/sinh(x) + a*coth(x))^2,x)
```

```
[Out] x/a^2 + (2/a + (2*b*exp(x))/a^2)/(a + 2*b*exp(x) + a*exp(2*x)) + (b*log((2*
b*exp(x))/a^3 - (2*b*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a
^2*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b*exp(x))/a^3 + (2*b*(a + b*exp
(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(1/2)*(b - a)^(1/2))
```

3.651 $\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^3}\,dx=\frac{a^2-b^2}{2a^3(b+a\cosh(x))^2}+\frac{2b}{a^3(b+a\cosh(x))}+\frac{\log(b+a\cosh(x))}{a^3}$$

```
[Out] 1/2*(a^2-b^2)/a^3/(b+a*cosh(x))^2+2*b/a^3/(b+a*cosh(x))+ln(b+a*cosh(x))/a^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4477, 2747, 711}

$$\int \frac{1}{(a\coth(x)+b\operatorname{csch}(x))^3}\,dx=\frac{2b}{a^3(a\cosh(x)+b)}+\frac{\log(a\cosh(x)+b)}{a^3}+\frac{a^2-b^2}{2a^3(a\cosh(x)+b)^2}$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-3),x]
```

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cosh[x])^2) + (2*b)/(a^3*(b + a*Cosh[x])) + Log[b
 + a*Cosh[x]]/a^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\sinh^3(x)}{(ib+ia\cosh(x))^3}\,dx\right)\\
&= -\frac{\text{Subst}\left(\int \frac{-a^2-x^2}{(ib+x)^3}\,dx, x, ia\cosh(x)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(ib+x)^3}+\frac{2ib}{(ib+x)^2}-\frac{1}{ib+x}\right)dx, x, ia\cosh(x)\right)}{a^3}\\
&= \frac{a^2-b^2}{2a^3(b+a\cosh(x))^2}+\frac{2b}{a^3(b+a\cosh(x))}+\frac{\log(b+a\cosh(x))}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = \frac{a^2+3b^2+a^2\log(b+a\cosh(x))+2b^2\log(b+a\cosh(x))+a^2\cosh(2x)\log(b+a\cosh(x))+4ab\cosh(x)(1+}{2a^3(b+a\cosh(x))^2}$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-3),x]
```

```
[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cosh[x]] + 2*b^2*Log[b + a*Cosh[x]] + a^2*Cosh
[2*x]*Log[b + a*Cosh[x]] + 4*a*b*Cosh[x]*(1 + Log[b + a*Cosh[x]]))/(2*a^3*(
b + a*Cosh[x])^2)
```

Maple [A] (verified)

Time = 10.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{x}{a^{3}}+\frac{2\,{\mathrm e}^{x} \left(2 b \,{\mathrm e}^{2 x} a +a^{2} {\mathrm e}^{x}+3 b^{2} {\mathrm e}^{x}+2 a b \right)}{a^{3} \left(a \,{\mathrm e}^{2 x}+2 \,{\mathrm e}^{x} b +a \right)^{2}}+\frac{\ln \left({\mathrm e}^{2 x}+\frac{2 b \,{\mathrm e}^{x}}{a}+1\right)}{a^{3}}$
default	$\frac{\frac{2 a^{2} \left(a +b \right)}{\left(a -b \right) \left(\tanh \left(\frac{x}{2}\right)^{2} a -\tanh \left(\frac{x}{2}\right)^{2} b +a +b \right)^{2}}-\frac{2 a}{\tanh \left(\frac{x}{2}\right)^{2} a -\tanh \left(\frac{x}{2}\right)^{2} b +a +b}+\ln \left(\tanh \left(\frac{x}{2}\right)^{2} a -\tanh \left(\frac{x}{2}\right)^{2} b +a +b \right)}{a^{3}}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{a^{3}}-\frac{\ln(}{}$

[In] int(1/(a*coth(x)+b*csch(x))^3,x,method=_RETURNVERBOSE)

[Out] -x/a^3+2/a^3*exp(x)*(2*b*exp(2*x)*a+a^2*exp(x)+3*b^2*exp(x)+2*a*b)/(a*exp(2*x)+2*exp(x)*b+a)^2+1/a^3*ln(exp(2*x)+2*b/a*exp(x)+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(48) = 96.

Time = 0.25 (sec) , antiderivative size = 521, normalized size of antiderivative = 10.42

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = -\frac{a^2x\cosh(x)^4+a^2x\sinh(x)^4+4\,(abx-ab)\cosh(x)^3+4\,(a^2x\cosh(x)+abx-ab)\sinh(x)^3+a^2x-2}{}$$

[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="fricas")

[Out] -(a^2*x*cosh(x)^4 + a^2*x*sinh(x)^4 + 4*(a*b*x - a*b)*cosh(x)^3 + 4*(a^2*x*cosh(x) + a*b*x - a*b)*sinh(x)^3 + a^2*x - 2*(a^2 + 3*b^2 - (a^2 + 2*b^2)*x)*cosh(x)^2 + 2*(3*a^2*x*cosh(x)^2 - a^2 - 3*b^2 + (a^2 + 2*b^2)*x + 6*(a*b*x - a*b)*cosh(x))*sinh(x)^2 + 4*(a*b*x - a*b)*cosh(x) - (a^2*cosh(x)^4 + a^2*sinh(x)^4 + 4*a*b*cosh(x)^3 + 4*(a^2*cosh(x) + a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(a^2 + 2*b^2)*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + 6*a*b*cosh(x) + a^2 + 2*b^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + 3*a*b*cosh(x)^2 + a*b + (a^2 + 2*b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 4*(a^2*x*cosh(x)^3 + a*b*x + 3*(a*b*x - a*b)*cosh(x)^2 - a*b - (a^2 + 3*b^2 - (a^2 + 2*b^2)*x)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + a^5*sinh(x)^4 + 4*a^4*b*cosh(x)^3 + 4*a^4*b*cosh(x) + a^5 + 4*(a^5*cosh(x) + a^4*b)*sinh(x)^3 + 2*(a^5 + 2*a^3*b^2)*cosh(x)^2 + 2*(3*a^5*cosh(x)^2 + 6*a^4*b*cosh(x) + a^5 + 2*a^3*b^2)*sinh(x)^2 + 4*(a^5*cosh(x)^3 + 3*a^4*b*cosh(x)^2 + a^4*b + (a^5 + 2*a^3*b^2)*cosh(x))*sinh(x))

Sympy [F]

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx=\int \frac{1}{(a\coth(x)+b\,\text{csch}\,(x))^3}\,dx$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))**3,x)
[Out] Integral((a*coth(x) + b*csch(x))**(-3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.22

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = \frac{2\left(2\,abe^{(-x)}+2\,abe^{(-3\,x)}+(a^2+3\,b^2)e^{(-2\,x)}\right)}{4\,a^4be^{(-x)}+4\,a^4be^{(-3\,x)}+a^5e^{(-4\,x)}+a^5+2\,(a^5+2\,a^3b^2)e^{(-2\,x)}}+\frac{x}{a^3}+\frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="maxima")
[Out] 2*(2*a*b*e^(-x) + 2*a*b*e^(-3*x) + (a^2 + 3*b^2)*e^(-2*x))/(4*a^4*b*e^(-x)
+ 4*a^4*b*e^(-3*x) + a^5*e^(-4*x) + a^5 + 2*(a^5 + 2*a^3*b^2)*e^(-2*x)) + x
/a^3 + log(2*b*e^(-x) + a*e^(-2*x) + a)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx = \frac{\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^3}-\frac{3\,a\left(e^{(-x)}+e^x\right)^2+4\,b\left(e^{(-x)}+e^x\right)-4\,a}{2\,\left(a\left(e^{(-x)}+e^x\right)+2\,b\right)^2a^2}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^3,x, algorithm="giac")
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a^3 - 1/2*(3*a*(e^(-x) + e^x)^2 + 4*b*(e^(
-x) + e^x) - 4*a)/((a*(e^(-x) + e^x) + 2*b)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^3}\,dx=\int \frac{1}{\left(\frac{b}{\sinh(x)}+a\coth\left(x\right)\right)^3}\,dx$$

```
[In] int(1/(b/sinh(x) + a*coth(x))^3,x)
[Out] int(1/(b/sinh(x) + a*coth(x))^3, x)
```

3.652 $\int \frac{1}{(a\coth(x)+b\mathrm{csch}(x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 159

$$\int \frac{1}{(a\coth(x)+b\mathrm{csch}(x))^4}\,dx = \frac{x}{a^4}-\frac{b(3a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} -\frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2a^3(a^2-b^2)(b+a\cosh(x))} -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3}-\frac{b\sinh^3(x)}{2a(a^2-b^2)(b+a\cosh(x))^2}$$

```
[Out] x/a^4-b*(3*a^2-2*b^2)*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4/(a-b)
^(3/2)/(a+b)^(3/2)-1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/a^3/(a^2-b^2)/(b+a
*cosh(x))-1/3*sinh(x)^3/a/(b+a*cosh(x))^3-1/2*b*sinh(x)^3/a/(a^2-b^2)/(b+a*
cosh(x))^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {4477, 2772, 2943, 2942, 2814, 2738, 211}

$$\int \frac{1}{(a\coth(x)+b\mathrm{csch}(x))^4}\,dx = \frac{x}{a^4}-\frac{b\sinh^3(x)}{2a(a^2-b^2)(a\cosh(x)+b)^2} -\frac{b(3a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} -\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2a^3(a^2-b^2)(a\cosh(x)+b)}-\frac{\sinh^3(x)}{3a(a\cosh(x)+b)^3}$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-4),x]
```

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(2
*a^3*(a^2 - b^2)*(b + a*Cosh[x])) - Sinh[x]^3/(3*a*(b + a*Cosh[x])^3) - (b*
Sinh[x]^3)/(2*a*(a^2 - b^2)*(b + a*Cosh[x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sinh^4(x)}{(ib+ia\cosh(x))^4}\,dx\\
&= -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{i\int \frac{\cosh(x)\sinh^2(x)}{(ib+ia\cosh(x))^3}\,dx}{a}\\
&= -\frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2} + \frac{i\int \frac{(2ia+ib\cosh(x))\sinh^2(x)}{(ib+ia\cosh(x))^2}\,dx}{2a\left(a^2-b^2\right)}\\
&= -\frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3}\\
&\quad - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2} - \frac{i\int \frac{ab-2(a^2-b^2)\cosh(x)}{ib+ia\cosh(x)}\,dx}{2a^3\left(a^2-b^2\right)}\\
&= \frac{x}{a^4} - \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3}\\
&\quad - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2} - \frac{(ib(3a^2-2b^2))\int \frac{1}{ib+ia\cosh(x)}\,dx}{2a^4\left(a^2-b^2\right)}\\
&= \frac{x}{a^4} - \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2a^3\left(a^2-b^2\right)(b+a\cosh(x))}\\
&\quad - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)(b+a\cosh(x))^2}\\
&\quad - \frac{(ib(3a^2-2b^2))\,\text{Subst}\left(\int \frac{1}{ia+ib-(-ia+ib)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^4\left(a^2-b^2\right)}
\end{aligned}$$

$$= \frac{x}{a^4} - \frac{b(3a^2-2b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2a^3\left(a^2-b^2\right)\left(b+a\cosh(x)\right)} - \frac{\sinh^3(x)}{3a(b+a\cosh(x))^3} - \frac{b\sinh^3(x)}{2a\left(a^2-b^2\right)\left(b+a\cosh(x)\right)^2}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx = \frac{\left(2a(a^2-b^2)+7ab(b+a\cosh(x))-\frac{a(8a^2-11b^2)(b+a\cosh(x))^2}{(a-b)(a+b)}+6x(b+a\cosh(x))^3\text{csch}(x)-\frac{6b(-3a^2+2b^2)\,\text{arct}}{}\right.}{6a^4(b+a\cosh(x))^3}$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-4),x]
```

```
[Out] ((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cosh[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cosh
[x])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cosh[x])^3*Csch[x] - (6*b*(-3*a^2 +
2*b^2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cosh[x])^3*Csch[
x])/(a^2 - b^2)^(3/2))*Sinh[x])/(6*a^4*(b + a*Cosh[x])^3)
```

Maple [A] (verified)

Time = 40.02 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

method	result
default	$\frac{\frac{2\left(-\frac{\left(2a^3-a^2b-3ab^2+2b^3\right)a\tanh\left(\frac{x}{2}\right)^5}{2(a+b)}-\frac{2a\left(5a^2-3b^2\right)\tanh\left(\frac{x}{2}\right)^3}{3}-\frac{\left(2a^3+a^2b-3ab^2-2b^3\right)a\tanh\left(\frac{x}{2}\right)}{2(a-b)}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^3}-\frac{b\left(3a^2-2b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^2-b^2\right)\sqrt{(a+b)(a-b)}}}{a^4}$
risch	$\frac{x}{a^4}+\frac{15a^4b\,e^{5x}-18a^2b^3e^{5x}+12a^5e^{4x}+27a^3b^2e^{4x}-54a\,b^4e^{4x}+48\,e^{3x}a^4b-34a^2b^3e^{3x}-44b^5e^{3x}+12a^5e^{2x}+36a^3b^2e^{2x}-78a\,b^4e^{2x}+3}{3a^4\left(a\,e^{2x}+2\,e^xb+a\right)^3\left(a^2-b^2\right)}$

```
[In] int(1/(a*coth(x)+b*csch(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^4*((-1/2*(2*a^3-a^2*b-3*a*b^2+2*b^3)*a/(a+b)*tanh(1/2*x)^5-2/3*a*(5*a^2
-3*b^2)*tanh(1/2*x)^3-1/2*(2*a^3+a^2*b-3*a*b^2-2*b^3)*a/(a-b)*tanh(1/2*x))/
(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)^3-1/2*b*(3*a^2-2*b^2)/(a^2-b^2)/((a+b
)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))-1/a^4*ln(tanh
(1/2*x)-1)+1/a^4*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. 2(141) = 282.

Time = 0.35 (sec) , antiderivative size = 5830, normalized size of antiderivative = 36.67

$$\int \frac{1}{(a\coth(x) + b\mathrm{csch}(x))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(a\coth(x) + b\mathrm{csch}(x))^4}\,dx = \int \frac{1}{(a\coth(x) + b\operatorname{csch}(x))^4}\,dx$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))**4,x)
[Out] Integral((a*coth(x) + b*csch(x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\coth(x) + b\mathrm{csch}(x))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.52

$$\int \frac{1}{(a\coth(x) + b\mathrm{csch}(x))^4}\,dx = -\frac{(3\,a^2b - 2\,b^3)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^6 - a^4b^2)\sqrt{a^2-b^2}} + \frac{15\,a^4be^{(5\,x)} - 18\,a^2b^3e^{(5\,x)} + 12\,a^5e^{(4\,x)} + 27\,a^3b^2e^{(4\,x)} - 54\,ab^4e^{(4\,x)} + 48\,a^4be^{(3\,x)} - 34\,a^2b^3e^{(3\,x)} - 44\,b^5e^{(3}}{3\,(a^6 - a^4b^2)(ae^{(2\,x)} + 2\,be^x} + \frac{x}{a^4}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^4,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - 2*b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^6 - a^4*b^2)*sqr
t(a^2 - b^2)) + 1/3*(15*a^4*b*e^(5*x) - 18*a^2*b^3*e^(5*x) + 12*a^5*e^(4*x)
 + 27*a^3*b^2*e^(4*x) - 54*a*b^4*e^(4*x) + 48*a^4*b*e^(3*x) - 34*a^2*b^3*e^
(3*x) - 44*b^5*e^(3*x) + 12*a^5*e^(2*x) + 36*a^3*b^2*e^(2*x) - 78*a*b^4*e^(
2*x) + 33*a^4*b*e^x - 48*a^2*b^3*e^x + 8*a^5 - 11*a^3*b^2)/((a^6 - a^4*b^2)
*(a*e^(2*x) + 2*b*e^x + a)^3) + x/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^4}\,dx=\int \frac{1}{\left(\frac{b}{\sinh(x)}+a\coth\left(x\right)\right)^4}\,dx$$

```
[In] int(1/(b/sinh(x) + a*coth(x))^4,x)
```

```
[Out] int(1/(b/sinh(x) + a*coth(x))^4, x)
```

3.653 $\int \frac{1}{(a\coth(x)+b\,\mathrm{csch}(x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 98

$$\int \frac{1}{(a\coth(x)+b\,\mathrm{csch}(x))^5}\,dx = -\frac{(a^2-b^2)^2}{4a^5(b+a\cosh(x))^4} - \frac{4b(a^2-b^2)}{3a^5(b+a\cosh(x))^3} + \frac{a^2-3b^2}{a^5(b+a\cosh(x))^2} + \frac{4b}{a^5(b+a\cosh(x))} + \frac{\log(b+a\cosh(x))}{a^5}$$

```
[Out] -1/4*(a^2-b^2)^2/a^5/(b+a*cosh(x))^4-4/3*b*(a^2-b^2)/a^5/(b+a*cosh(x))^3+(a
^2-3*b^2)/a^5/(b+a*cosh(x))^2+4*b/a^5/(b+a*cosh(x))+ln(b+a*cosh(x))/a^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4477, 2747, 711}

$$\int \frac{1}{(a\coth(x)+b\,\mathrm{csch}(x))^5}\,dx = \frac{4b}{a^5(a\cosh(x)+b)} + \frac{\log(a\cosh(x)+b)}{a^5} - \frac{(a^2-b^2)^2}{4a^5(a\cosh(x)+b)^4} - \frac{4b(a^2-b^2)}{3a^5(a\cosh(x)+b)^3} + \frac{a^2-3b^2}{a^5(a\cosh(x)+b)^2}$$

```
[In] Int[(a*Coth[x] + b*Csch[x])^(-5),x]
```

```
[Out] -1/4*(a^2 - b^2)^2/(a^5*(b + a*Cosh[x])^4) - (4*b*(a^2 - b^2))/(3*a^5*(b +
a*Cosh[x])^3) + (a^2 - 3*b^2)/(a^5*(b + a*Cosh[x])^2) + (4*b)/(a^5*(b + a*C
osh[x])) + Log[b + a*Cosh[x]]/a^5
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \frac{\sinh^5(x)}{(ib+ia\cosh(x))^5}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a^2-x^2)^2}{(ib+x)^5}\,dx, x, ia\cosh(x)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(ib+x)^5}+\frac{4ib(-a^2+b^2)}{(ib+x)^4}+\frac{2(a^2-3b^2)}{(ib+x)^3}-\frac{4ib}{(ib+x)^2}+\frac{1}{ib+x}\right)dx, x, ia\cosh(x)\right)}{a^5} \\
&= -\frac{(a^2-b^2)^2}{4a^5(b+a\cosh(x))^4}-\frac{4b(a^2-b^2)}{3a^5(b+a\cosh(x))^3} \\
&\quad +\frac{a^2-3b^2}{a^5(b+a\cosh(x))^2}+\frac{4b}{a^5(b+a\cosh(x))}+\frac{\log(b+a\cosh(x))}{a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = \frac{-3a^4+2a^2b^2+25b^4+12b^4\log(b+a\cosh(x))+12a^4\cosh^4(x)\log(b+a\cosh(x))+48a^3b\cosh^3(x)(1+\text{lo}}{12a}$$

```
[In] Integrate[(a*Coth[x] + b*Csch[x])^(-5),x]
```

```
[Out] (-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cosh[x]] + 12*a^4*Cosh[x]^4
*Log[b + a*Cosh[x]] + 48*a^3*b*Cosh[x]^3*(1 + Log[b + a*Cosh[x]]) + 12*a^2*
Cosh[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cosh[x]]) + 8*a*b*Cosh[x]*(a^2 + 1
1*b^2 + 6*b^2*Log[b + a*Cosh[x]]))/(12*a^5*(b + a*Cosh[x])^4)
```

Maple [A] (verified)

Time = 126.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{x}{a^5}+\frac{4(6a^3b\,\mathrm{e}^{6x}+3a^4\mathrm{e}^{5x}+27a^2b^2\mathrm{e}^{5x}+22a^3b\,\mathrm{e}^{4x}+44a\,b^3\mathrm{e}^{4x}+3a^4\mathrm{e}^{3x}+56a^2b^2\mathrm{e}^{3x}+25\,\mathrm{e}^{3x}b^4+22\,\mathrm{e}^{2x}a^3b+44\,\mathrm{e}^{2x}a\,b^3+3a^4\mathrm{e}^{x}+27a^2b^2}{3a^5(a\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}b+a)^4}$
default	$\frac{-\frac{2a^2}{\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^2}-\frac{2a}{\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b}+\frac{8a^3\left(3a^2+2ab-b^2\right)}{3(a-b)^2\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^3}+\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b\right.}{a^5}$

```
[In] int(1/(a*coth(x)+b*csch(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -x/a^5+4/3*(6*a^3*b*exp(6*x)+3*a^4*exp(5*x)+27*a^2*b^2*exp(5*x)+22*a^3*b*ex
p(4*x)+44*a*b^3*exp(4*x)+3*a^4*exp(3*x)+56*a^2*b^2*exp(3*x)+25*exp(3*x)*b^4
+22*exp(2*x)*a^3*b+44*exp(2*x)*a*b^3+3*a^4*exp(x)+27*a^2*b^2*exp(x)+6*a^3*b
)/a^5*exp(x)/(a*exp(2*x)+2*exp(x)*b+a)^4+1/a^5*ln(exp(2*x)+2*b/a*exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2564 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 2564, normalized size of antiderivative = 26.16

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^4*x*cosh(x)^8 + 3*a^4*x*sinh(x)^8 + 24*(a^3*b*x - a^3*b)*cosh(x)^
7 + 24*(a^4*x*cosh(x) + a^3*b*x - a^3*b)*sinh(x)^7 - 12*(a^4 + 9*a^2*b^2 -
(a^4 + 6*a^2*b^2)*x)*cosh(x)^6 + 12*(7*a^4*x*cosh(x)^2 - a^4 - 9*a^2*b^2 +
(a^4 + 6*a^2*b^2)*x + 14*(a^3*b*x - a^3*b)*cosh(x))*sinh(x)^6 - 8*(11*a^3*b
 + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^5 + 8*(21*a^4*x*cosh(x)^3 -
11*a^3*b - 22*a*b^3 + 63*(a^3*b*x - a^3*b)*cosh(x)^2 + 3*(3*a^3*b + 4*a*b^3
)*x - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x)^5 + 3*a^4*
x - 2*(6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cos
h(x)^4 + 2*(105*a^4*x*cosh(x)^4 - 6*a^4 - 112*a^2*b^2 - 50*b^4 + 420*(a^3*b
*x - a^3*b)*cosh(x)^3 - 90*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^
2 + 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x - 20*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b
 + 4*a*b^3)*x)*cosh(x))*sinh(x)^4 - 8*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4
```

*a*b^3)*x)*cosh(x)^3 + 8*(21*a^4*x*cosh(x)^5 + 105*(a^3*b*x - a^3*b)*cosh(x
)^4 - 11*a^3*b - 22*a*b^3 - 30*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh
(x)^3 - 10*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^2 + 3*(3
*a^3*b + 4*a*b^3)*x - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2
 + 8*b^4)*x)*cosh(x))*sinh(x)^3 - 12*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x
)*cosh(x)^2 + 4*(21*a^4*x*cosh(x)^6 + 126*(a^3*b*x - a^3*b)*cosh(x)^5 - 45*
(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^4 - 3*a^4 - 27*a^2*b^2 - 20
*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^3 - 3*(6*a^4 + 112
*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)*cosh(x)^2 + 3*(a^4 +
6*a^2*b^2)*x - 6*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x))*s
inh(x)^2 + 24*(a^3*b*x - a^3*b)*cosh(x) - 3*(a^4*cosh(x)^8 + a^4*sinh(x)^8
+ 8*a^3*b*cosh(x)^7 + 8*(a^4*cosh(x) + a^3*b)*sinh(x)^7 + 4*(a^4 + 6*a^2*b^
2)*cosh(x)^6 + 4*(7*a^4*cosh(x)^2 + 14*a^3*b*cosh(x) + a^4 + 6*a^2*b^2)*sin
h(x)^6 + 8*(3*a^3*b + 4*a*b^3)*cosh(x)^5 + 8*(7*a^4*cosh(x)^3 + 21*a^3*b*co
sh(x)^2 + 3*a^3*b + 4*a*b^3 + 3*(a^4 + 6*a^2*b^2)*cosh(x))*sinh(x)^5 + 8*a^
3*b*cosh(x) + 2*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^4 + 2*(35*a^4*cosh(x)^
4 + 140*a^3*b*cosh(x)^3 + 3*a^4 + 24*a^2*b^2 + 8*b^4 + 30*(a^4 + 6*a^2*b^2)
cosh(x)^2 + 20(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + a^4 + 8*(3*a^3*b +
 4*a*b^3)*cosh(x)^3 + 8*(7*a^4*cosh(x)^5 + 35*a^3*b*cosh(x)^4 + 3*a^3*b + 4
*a*b^3 + 10*(a^4 + 6*a^2*b^2)*cosh(x)^3 + 10*(3*a^3*b + 4*a*b^3)*cosh(x)^2
+ (3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 6*a^2*b^2)*cos
h(x)^2 + 4*(7*a^4*cosh(x)^6 + 42*a^3*b*cosh(x)^5 + 15*(a^4 + 6*a^2*b^2)*cos
h(x)^4 + a^4 + 6*a^2*b^2 + 20*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 3*(3*a^4 + 24
*a^2*b^2 + 8*b^4)*cosh(x)^2 + 6*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 8*
(a^4*cosh(x)^7 + 7*a^3*b*cosh(x)^6 + 3*(a^4 + 6*a^2*b^2)*cosh(x)^5 + 5*(3*a
^3*b + 4*a*b^3)*cosh(x)^4 + a^3*b + (3*a^4 + 24*a^2*b^2 + 8*b^4)*cosh(x)^3
+ 3*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (a^4 + 6*a^2*b^2)*cosh(x))*sinh(x))*log
(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 8*(3*a^4*x*cosh(x)^7 + 21*(a^3*b*
x - a^3*b)*cosh(x)^6 - 9*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x)^5
+ 3*a^3*b*x - 5*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^4 -
 3*a^3*b - (6*a^4 + 112*a^2*b^2 + 50*b^4 - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x
)*cosh(x)^3 - 3*(11*a^3*b + 22*a*b^3 - 3*(3*a^3*b + 4*a*b^3)*x)*cosh(x)^2 -
 3*(a^4 + 9*a^2*b^2 - (a^4 + 6*a^2*b^2)*x)*cosh(x))*sinh(x))/(a^9*cosh(x)^8
 + a^9*sinh(x)^8 + 8*a^8*b*cosh(x)^7 + 8*a^8*b*cosh(x) + a^9 + 8*(a^9*cosh(
x) + a^8*b)*sinh(x)^7 + 4*(a^9 + 6*a^7*b^2)*cosh(x)^6 + 4*(7*a^9*cosh(x)^2
+ 14*a^8*b*cosh(x) + a^9 + 6*a^7*b^2)*sinh(x)^6 + 8*(3*a^8*b + 4*a^6*b^3)*c
osh(x)^5 + 8*(7*a^9*cosh(x)^3 + 21*a^8*b*cosh(x)^2 + 3*a^8*b + 4*a^6*b^3 +
3*(a^9 + 6*a^7*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)
cosh(x)^4 + 2(35*a^9*cosh(x)^4 + 140*a^8*b*cosh(x)^3 + 3*a^9 + 24*a^7*b^2
 + 8*a^5*b^4 + 30*(a^9 + 6*a^7*b^2)*cosh(x)^2 + 20*(3*a^8*b + 4*a^6*b^3)*co
sh(x))*sinh(x)^4 + 8*(3*a^8*b + 4*a^6*b^3)*cosh(x)^3 + 8*(7*a^9*cosh(x)^5 +
 35*a^8*b*cosh(x)^4 + 3*a^8*b + 4*a^6*b^3 + 10*(a^9 + 6*a^7*b^2)*cosh(x)^3
+ 10*(3*a^8*b + 4*a^6*b^3)*cosh(x)^2 + (3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cos
h(x))*sinh(x)^3 + 4*(a^9 + 6*a^7*b^2)*cosh(x)^2 + 4*(7*a^9*cosh(x)^6 + 42*a
^8*b*cosh(x)^5 + a^9 + 6*a^7*b^2 + 15*(a^9 + 6*a^7*b^2)*cosh(x)^4 + 20*(3*a

```
^8*b + 4*a^6*b^3)*cosh(x)^3 + 3*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cosh(x)^2
+ 6*(3*a^8*b + 4*a^6*b^3)*cosh(x))*sinh(x)^2 + 8*(a^9*cosh(x)^7 + 7*a^8*b*c
osh(x)^6 + a^8*b + 3*(a^9 + 6*a^7*b^2)*cosh(x)^5 + 5*(3*a^8*b + 4*a^6*b^3)*
cosh(x)^4 + (3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*cosh(x)^3 + 3*(3*a^8*b + 4*a^6
*b^3)*cosh(x)^2 + (a^9 + 6*a^7*b^2)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^5}\,dx = \int \frac{1}{(a\coth(x) + b\,\text{csch}(x))^5}\,dx$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))**5,x)
```

```
[Out] Integral((a*coth(x) + b*csch(x))**(-5), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(94) = 188.

Time = 0.22 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.91

$$\int \frac{1}{(a\coth(x) + b\text{csch}(x))^5}\,dx = \frac{4\left(6\,a^3be^{(-x)} + 6\,a^3be^{(-7\,x)} + 3\,(a^4 + 9\,a^2b^2)e^{(-2\,x)} + 22\,(a^3b + 2\,ab^3)e^{(-3\,x)} + (3\,a^4 + 56\,a^2b^2 + 25 \cdots\right.}{3\,(8\,a^8be^{(-x)} + 8\,a^8be^{(-7\,x)} + a^9e^{(-8\,x)} + a^9 + 4\,(a^9 + 6\,a^7b^2)e^{(-2\,x)} + 8\,(3\,a^8b + 4\,a^6b^3)e^{(-3\,x)} + 2\,(3\,a^9 + 24\,a \cdots} + \frac{x}{a^5} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^5}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="maxima")
```

```
[Out] 4/3*(6*a^3*b*e^(-x) + 6*a^3*b*e^(-7*x) + 3*(a^4 + 9*a^2*b^2)*e^(-2*x) + 22*
(a^3*b + 2*a*b^3)*e^(-3*x) + (3*a^4 + 56*a^2*b^2 + 25*b^4)*e^(-4*x) + 22*(a
^3*b + 2*a*b^3)*e^(-5*x) + 3*(a^4 + 9*a^2*b^2)*e^(-6*x))/(8*a^8*b*e^(-x) +
8*a^8*b*e^(-7*x) + a^9*e^(-8*x) + a^9 + 4*(a^9 + 6*a^7*b^2)*e^(-2*x) + 8*(3
*a^8*b + 4*a^6*b^3)*e^(-3*x) + 2*(3*a^9 + 24*a^7*b^2 + 8*a^5*b^4)*e^(-4*x)
+ 8*(3*a^8*b + 4*a^6*b^3)*e^(-5*x) + 4*(a^9 + 6*a^7*b^2)*e^(-6*x)) + x/a^5
+ log(2*b*e^(-x) + a*e^(-2*x) + a)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = \frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a^5} - \frac{25\,a^3\left(e^{(-x)}+e^{x}\right)^4+104\,a^2b\left(e^{(-x)}+e^{x}\right)^3-48\,a^3\left(e^{(-x)}+e^{x}\right)^2+168\,ab^2\left(e^{(-x)}+e^{x}\right)^2-64\,a^2b\left(e^{(-x)}+\cdots\right.}{12\left(a\left(e^{(-x)}+e^{x}\right)+2\,b\right)^4a^4}$$

```
[In] integrate(1/(a*coth(x)+b*csch(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a^5 - 1/12*(25*a^3*(e^(-x) + e^x)^4 + 104*
a^2*b*(e^(-x) + e^x)^3 - 48*a^3*(e^(-x) + e^x)^2 + 168*a*b^2*(e^(-x) + e^x)
^2 - 64*a^2*b*(e^(-x) + e^x) + 96*b^3*(e^(-x) + e^x) + 48*a^3 - 32*a*b^2)/(
(a*(e^(-x) + e^x) + 2*b)^4*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\coth(x)+b\text{csch}(x))^5}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(x)}+a\coth\left(x\right)\right)^5}\,dx$$

```
[In] int(1/(b/sinh(x) + a*coth(x))^5,x)
```

```
[Out] int(1/(b/sinh(x) + a*coth(x))^5, x)
```

3.654 $\int (\coth(x) + \mathbf{csch}(x))^5 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx = -\frac{2}{(1-\cosh(x))^2}+\frac{4}{1-\cosh(x)}+\log(1-\cosh(x))$$

```
[Out] -2/(1-cosh(x))^2+4/(1-cosh(x))+ln(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4477, 2746, 45}

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx = \frac{4}{1-\cosh(x)}-\frac{2}{(1-\cosh(x))^2}+\log(1-\cosh(x))$$

```
[In] Int[(Coth[x] + Csch[x])^5,x]
```

```
[Out] -2/(1 - Cosh[x])^2 + 4/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int (i+i\cosh(x))^5\text{csch}^5(x)\,dx\right)\\
&= -\text{Subst}\left(\int \frac{(i+x)^2}{(i-x)^3}\,dx, x, i\cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{i-x}+\frac{4}{(-i+x)^3}-\frac{4i}{(-i+x)^2}\right)\,dx, x, i\cosh(x)\right)\\
&= \frac{2}{(i-i\cosh(x))^2}+\frac{4i}{i-i\cosh(x)}+\log(1-\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int (\coth(x)+\text{csch}(x))^5\,dx = -2\text{csch}^2\left(\frac{x}{2}\right)-\frac{1}{2}\text{csch}^4\left(\frac{x}{2}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^5,x]
[Out] -2*Csch[x/2]^2 - Csch[x/2]^4/2 + 2*Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

method	result
risch	$-x-\frac{8\,\mathrm{e}^{x}\left(\mathrm{e}^{2x}-\mathrm{e}^{x}+1\right)}{\left(\mathrm{e}^{x}-1\right)^{4}}+2\ln\left(\mathrm{e}^{x}-1\right)$
default	$\ln\left(\sinh\left(x\right)\right)-\frac{\coth(x)^{2}}{2}-\frac{\coth(x)^{4}}{4}-\frac{5\cosh(x)^{3}}{\sinh(x)^{4}}+\frac{5\cosh(x)}{3\sinh(x)^{4}}+\frac{8\left(-\frac{\operatorname{csch}(x)^{3}}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth(x)}{3}-2\,\operatorname{arctanh}\left(\mathrm{e}^{x}\right.$
parts	$-\frac{11\coth(x)^{4}}{4}-\frac{\coth(x)^{2}}{2}-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(1+\coth(x))}{2}+\frac{8\left(-\frac{\operatorname{csch}(x)^{3}}{4}+\frac{3\operatorname{csch}(x)}{8}\right)\coth(x)}{3}-2\,\operatorname{arctanh}\left(\mathrm{e}^{x}\right)+\frac{5}{3}$

[In] int((coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)

[Out] -x-8*exp(x)*(exp(2*x)-exp(x)+1)/(exp(x)-1)^4+2*ln(exp(x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 270, normalized size of antiderivative = 9.64

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx =$$
$$-\frac{x\cosh\left(x\right)^{4}+x\sinh\left(x\right)^{4}-4\left(x-2\right)\cosh\left(x\right)^{3}+4\left(x\cosh\left(x\right)-x+2\right)\sinh\left(x\right)^{3}+2\left(3\,x-4\right)\cosh\left(x\right)^{2}+}{}$$

[In] integrate((coth(x)+csch(x))^5,x, algorithm="fricas")

[Out] -(x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) + 3
*x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*sinh
(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh
(x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(x)^2
+ (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1)*sinh
(x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh
(x) + 1)

Sympy [F]

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx=\int (\coth(x)+\operatorname{csch}(x))^5\,dx$$

```
[In] integrate((coth(x)+csch(x))**5,x)
[Out] Integral((coth(x) + csch(x))**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(24)=48$.

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.43

$$\begin{aligned}\int (\coth(x)+\operatorname{csch}(x))^5\,dx=&-\frac{5}{2}\coth(x)^4+x\\&+\frac{5\left(5e^{(-x)}+3e^{(-3x)}+3e^{(-5x)}+5e^{(-7x)}\right)}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)}\\&-\frac{3e^{(-x)}-11e^{(-3x)}-11e^{(-5x)}+3e^{(-7x)}}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)}\\&+\frac{5\left(e^{(-x)}+7e^{(-3x)}+7e^{(-5x)}+e^{(-7x)}\right)}{2\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)}\\&+\frac{4\left(e^{(-2x)}-e^{(-4x)}+e^{(-6x)}\right)}{4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1}\\&-\frac{20}{\left(e^{(-x)}-e^{x}\right)^4}+2\log\left(e^{(-x)}-1\right)\end{aligned}$$

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="maxima")
[Out] -5/2*coth(x)^4 + x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11*
e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x)
- e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e
^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 20/(e^(-x)
 - e^x)^4 + 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx = -x-\frac{8\left(e^{(3x)}-e^{(2x)}+e^{x}\right)}{\left(e^{x}-1\right)^4}+2\log\left(\left|e^{x}-1\right|\right)$$

```
[In] integrate((coth(x)+csch(x))^5,x, algorithm="giac")
[Out] -x - 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 + 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.89

$$\int (\coth(x)+\operatorname{csch}(x))^5\,dx = 2\ln\left(\mathrm{e}^{x}-1\right)-x+\frac{16}{3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1}-\frac{16}{\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1}-\frac{8}{6\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}-4\,\mathrm{e}^{x}+1}-\frac{8}{\mathrm{e}^{x}-1}$$

```
[In] int((coth(x) + 1/sinh(x))^5,x)
[Out] 2*log(exp(x) - 1) - x + 16/(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1) - 16/(exp
(2*x) - 2*exp(x) + 1) - 8/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) +
1) - 8/(exp(x) - 1)
```

3.655 $\int (\coth(x) + \mathbf{csch}(x))^4 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 30

$$\int (\coth(x) + \operatorname{csch}(x))^4 \, dx = x + \frac{2\sinh(x)}{1 - \cosh(x)} + \frac{2\sinh^3(x)}{3(1 - \cosh(x))^3}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))+2/3*sinh(x)^3/(1-cosh(x))^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4477, 2749, 2759, 8}

$$\int (\coth(x) + \operatorname{csch}(x))^4 \, dx = x + \frac{2\sinh^3(x)}{3(1 - \cosh(x))^3} + \frac{2\sinh(x)}{1 - \cosh(x)}$$

```
[In] Int[(Coth[x] + Csch[x])^4,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (i+i\cosh(x))^4\operatorname{csch}^4(x)\,dx \\
&= \int \frac{\sinh^4(x)}{(i-i\cosh(x))^4}\,dx \\
&= \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx \\
&= \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} + \int 1\,dx \\
&= x + \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx = x - \frac{8}{3}\coth\left(\frac{x}{2}\right) - \frac{2}{3}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^4,x]
[Out] x - (8*Coth[x/2])/3 - (2*Coth[x/2]*Csch[x/2]^2)/3
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
risch	$x-\frac{8(3\,\mathrm{e}^{2x}-3\,\mathrm{e}^{x}+2)}{3(\mathrm{e}^{x}-1)^{3}}$
parts	$-\frac{7\coth(x)^{3}}{3}-\coth(x)-\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}+\left(\frac{2}{3}-\frac{\mathrm{csch}(x)^{2}}{3}\right)\coth(x)-\frac{8\,\mathrm{csch}(x)^{3}}{3}-4\,\mathrm{csch}(x$
default	$x-\coth(x)-\frac{\coth(x)^{3}}{3}-\frac{4\cosh(x)^{2}}{\sinh(x)^{3}}+\frac{4}{3\sinh(x)^{3}}-\frac{3\cosh(x)}{\sinh(x)^{3}}-2\left(\frac{2}{3}-\frac{\mathrm{csch}(x)^{2}}{3}\right)\coth(x)$

```
[In] int((coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
[Out] x-8/3*(3*exp(2*x)-3*exp(x)+2)/(exp(x)-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int(\coth(x)+\mathrm{csch}(x))^4\,dx = \frac{3\,x\cosh(x)^2+3\,x\sinh(x)^2-4\,(3\,x+10)\cosh(x)+2\,(3\,x\cosh(x)-3\,x-4)\sinh(x)+9\,x+24}{3\,\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\,\cosh(x)+3\right)}$$

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="fricas")
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
- 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 4*cosh(x) + 3)
```

Sympy [F]

$$\int(\coth(x)+\mathrm{csch}(x))^4\,dx=\int(\coth(x)+\mathrm{csch}(x))^4\,dx$$

```
[In] integrate((coth(x)+csch(x))**4,x)
[Out] Integral((coth(x) + csch(x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.10

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx = -2\coth(x)^3 + x - \frac{4\left(3e^{(-2x)} - 3e^{(-4x)} - 2\right)}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} + \frac{8e^{(-x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} + \frac{4e^{(-2x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} - \frac{16e^{(-3x)}}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} + \frac{8e^{(-5x)}}{3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1} - \frac{4}{3\left(3e^{(-2x)} - 3e^{(-4x)} + e^{(-6x)} - 1\right)} + \frac{32}{3\left(e^{(-x)} - e^{x}\right)^3}$$

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*
x) + e^(-6*x) - 1) + 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*
e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 16/3*e^(-3*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) + 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(
-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 32/3/(e^(-x) -
e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx = x - \frac{8\left(3e^{(2x)} - 3e^{x} + 2\right)}{3\left(e^{x} - 1\right)^3}$$

```
[In] integrate((coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int (\coth(x)+\operatorname{csch}(x))^4\,dx = x-\frac{8\,e^x}{3\,\left(e^{2\,x}-2\,e^x+1\right)}+\frac{\frac{8\,e^{2\,x}}{3}+\frac{8}{3}}{3\,e^{2\,x}-e^{3\,x}-3\,e^x+1}-\frac{8}{3\,\left(e^x-1\right)}$$

```
[In] int((coth(x) + 1/sinh(x))^4,x)
```

```
[Out] x - (8*exp(x))/(3*(exp(2*x) - 2*exp(x) + 1)) + ((8*exp(2*x))/3 + 8/3)/(3*ex
p(2*x) - exp(3*x) - 3*exp(x) + 1) - 8/(3*(exp(x) - 1))
```

3.656 $\int (\coth(x) + \operatorname{csch}(x))^3 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = \frac{2}{1 - \cosh(x)} + \log(1 - \cosh(x))$$

```
[Out] 2/(1-cosh(x))+ln(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4477, 2746, 45}

$$\int (\coth(x) + \operatorname{csch}(x))^3 \, dx = \frac{2}{1 - \cosh(x)} + \log(1 - \cosh(x))$$

```
[In] Int[(Coth[x] + Csch[x])^3,x]
[Out] 2/(1 - Cosh[x]) + Log[1 - Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int (i+i\cosh(x))^3\text{csch}^3(x)\,dx\\ &= \text{Subst}\left(\int \frac{i+x}{(i-x)^2}\,dx, x, i\cosh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{2i}{(-i+x)^2}+\frac{1}{-i+x}\right)dx, x, i\cosh(x)\right)\\ &= \frac{2i}{i-i\cosh(x)}+\log(1-\cosh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (\coth(x)+\text{csch}(x))^3\,dx = -\text{csch}^2\left(\frac{x}{2}\right)+2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^3,x]
[Out] -Csch[x/2]^2 + 2*Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

method	result	size
risch	$-x-\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-1)^{2}}+2\ln\left(\mathrm{e}^{x}-1\right)$	22
default	$\ln\left(\sinh\left(x\right)\right)-\frac{\coth(x)^{2}}{2}-\frac{3\cosh(x)}{\sinh(x)^{2}}+\text{csch}\left(x\right)\coth\left(x\right)-2\,\text{arctanh}\left(\mathrm{e}^{x}\right)-\frac{3}{2\sinh(x)^{2}}$	35
parts	$-2\coth\left(x\right)^{2}-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(1+\coth(x))}{2}+\text{csch}\left(x\right)\coth\left(x\right)-2\,\text{arctanh}\left(\mathrm{e}^{x}\right)-\frac{3\cosh(x)}{\sinh(x)^{2}}$	40

```
[In] int((coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x-4*exp(x)/(exp(x)-1)^2+2*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(16) = 32$.

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 5.06

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx = -\frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\,\text{co}\right.}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x}$$

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(
x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh
(x)^2 - 2*cosh(x) + 1)
```

Sympy [F]

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx = \int (\coth(x) + \operatorname{csch}(x))^3\,dx$$

```
[In] integrate((coth(x)+csch(x))**3,x)
```

```
[Out] Integral((coth(x) + csch(x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int (\coth(x)+\operatorname{csch}(x))^3\,dx = -\frac{3}{2}\coth(x)^2 + x + \frac{4\left(e^{(-x)} + e^{(-3x)}\right)}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + 2\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*coth(x)^2 + x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*
e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (\coth(x) + \text{csch}(x))^3 \, dx = -x - \frac{4 e^x}{(e^x - 1)^2} + 2 \log(|e^x - 1|)$$

```
[In] integrate((coth(x)+csch(x))^3,x, algorithm="giac")
[Out] -x - 4*e^x/(e^x - 1)^2 + 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int (\coth(x) + \text{csch}(x))^3 \, dx = 2 \ln(e^x - 1) - x - \frac{4}{e^{2x} - 2 e^x + 1} - \frac{4}{e^x - 1}$$

```
[In] int((coth(x) + 1/sinh(x))^3,x)
[Out] 2*log(exp(x) - 1) - x - 4/(exp(2*x) - 2*exp(x) + 1) - 4/(exp(x) - 1)
```

3.657 $\int (\coth(x) + \mathbf{csch}(x))^2 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4477, 2749, 2759, 8}

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{2\sinh(x)}{1 - \cosh(x)}$$

```
[In] Int[(Coth[x] + Csch[x])^2,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int (i + i\cosh(x))^2 \operatorname{csch}^2(x)\,dx\\ &= -\int \frac{\sinh^2(x)}{(i - i\cosh(x))^2}\,dx\\ &= \frac{2\sinh(x)}{1-\cosh(x)} + \int 1\,dx\\ &= x + \frac{2\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (\coth(x) + \operatorname{csch}(x))^2\,dx = x - 2\coth\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^2,x]
[Out] x - 2*Coth[x/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$x-\frac{4}{{\mathrm e}^{x}-1}$	11
default	$x-2\coth\left(x\right)-\frac{2}{\sinh\left(x\right)}$	13
parts	$-2\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}+\frac{\ln\left(1+\coth\left(x\right)\right)}{2}-2\,\operatorname{csch}\left(x\right)$	24

```
[In] int((coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
[Out] x-4/(exp(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (\coth(x)+\operatorname{csch}(x))^2\,dx=\frac{x\cosh\left(x\right)+x\sinh\left(x\right)-x-4}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int (\coth(x)+\operatorname{csch}(x))^2\,dx=\int\left(\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{2}\,dx$$

```
[In] integrate((coth(x)+csch(x))**2,x)
[Out] Integral((coth(x) + csch(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{4}{e^{(-x)} - e^x} + \frac{4}{e^{(-2x)} - 1}$$

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="maxima")
[Out] x + 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{4}{e^x - 1}$$

```
[In] integrate((coth(x)+csch(x))^2,x, algorithm="giac")
[Out] x - 4/(e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{4}{e^x - 1}$$

```
[In] int((coth(x) + 1/sinh(x))^2,x)
[Out] x - 4/(exp(x) - 1)
```

3.658 $\int (\coth(x) + \mathrm{csch}(x))\, dx$

Optimal result

Integrand size = 5, antiderivative size = 9

$$\int (\coth(x) + \mathrm{csch}(x))\, dx = -\mathrm{arctanh}(\cosh(x)) + \log(\sinh(x))$$

```
[Out] -arctanh(cosh(x))+ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3556, 3855}

$$\int (\coth(x) + \mathrm{csch}(x))\, dx = \log(\sinh(x)) - \mathrm{arctanh}(\cosh(x))$$

```
[In] Int[Coth[x] + Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] + Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \coth(x)\,dx + \int \operatorname{csch}(x)\,dx \\ &= -\operatorname{arctanh}(\cosh(x)) + \log(\sinh(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int (\coth(x)+\operatorname{csch}(x))\,dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))+\log\left(\sinh\left(\frac{x}{2}\right)\right)+\log(\tanh(x))$$

```
[In] Integrate[Coth[x] + Csch[x],x]
[Out] -Log[Cosh[x/2]] + Log[Cosh[x]] + Log[Sinh[x/2]] + Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\ln(\sinh(x)) + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$	10
parts	$\ln(\sinh(x)) + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$	10
risch	$-x + \ln(e^{2x} - 1) + \ln(e^{x} - 1) - \ln(e^{x} + 1)$	24
parallelrisch	$-x + \ln(\tanh(x)) + \ln(\coth(x) - \operatorname{csch}(x)) - \ln(1 - \tanh(x))$	25

```
[In] int(coth(x)+csch(x),x,method=_RETURNVERBOSE)
[Out] ln(sinh(x))+ln(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int (\coth(x) + \operatorname{csch}(x))\,dx = -x + 2\log(\cosh(x) + \sinh(x) - 1)$$

```
[In] integrate(coth(x)+csch(x),x, algorithm="fricas")
[Out] -x + 2*log(cosh(x) + sinh(x) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int (\coth(x) + \operatorname{csch}(x))\, dx = x - \log(\tanh(x) + 1) + \log\left(\tanh\left(\frac{x}{2}\right)\right) + \log(\tanh(x))$$

```
[In] integrate(coth(x)+csch(x),x)
[Out] x - log(tanh(x) + 1) + log(tanh(x/2)) + log(tanh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (\coth(x) + \operatorname{csch}(x))\, dx = \log(\sinh(x)) + \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

```
[In] integrate(coth(x)+csch(x),x, algorithm="maxima")
[Out] log(sinh(x)) + log(tanh(1/2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.78

$$\int (\coth(x) + \operatorname{csch}(x))\, dx = -x - \log(e^x + 1) + \log\left(\left|e^{(2\,x)} - 1\right|\right) + \log(|e^x - 1|)$$

```
[In] integrate(coth(x)+csch(x),x, algorithm="giac")
[Out] -x - log(e^x + 1) + log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int (\coth(x) + \operatorname{csch}(x))\, dx = 2 \ln(e^x - 1) - x$$

```
[In] int(coth(x) + 1/sinh(x),x)
[Out] 2*log(exp(x) - 1) - x
```

3.659 $\int \frac{1}{\coth(x)+\operatorname{csch}(x)} dx$

Optimal result

Integrand size = 7, antiderivative size = 5

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)} dx = \log(1+\cosh(x))$$

```
[Out] ln(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3239, 2746, 31}

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)} dx = \log(\cosh(x)+1)$$

```
[In] Int[(Coth[x] + Csch[x])^(-1),x]
[Out] Log[1 + Cosh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= i\int \frac{\sinh(x)}{i+i\cosh(x)}\,dx \\ &= \text{Subst}\left(\int \frac{1}{i+x}\,dx, x, i\cosh(x)\right) \\ &= \log(1+\cosh(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^(-1),x]
[Out] 2*Log[Cosh[x/2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.40

method	result	size
risch	$-x+2\ln\left(e^{x}+1\right)$	12
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	20

```
[In] int(1/(coth(x)+csch(x)),x,method=_RETURNVERBOSE)
[Out] -x+2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(5) = 10$.

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.60

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = -x + 2\,\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)$$

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="fricas")
[Out] -x + 2*log(cosh(x) + sinh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = \int \frac{1}{\coth\left(x\right) + \operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(1/(coth(x)+csch(x)),x)
[Out] Integral(1/(coth(x) + csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = x + 2\,\log\left(e^{(-x)} + 1\right)$$

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="maxima")
[Out] x + 2*log(e^(-x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{1}{\coth(x)+\operatorname{csch}(x)}\,dx = -x + 2\,\log\left(e^{x} + 1\right)$$

```
[In] integrate(1/(coth(x)+csch(x)),x, algorithm="giac")
[Out] -x + 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{1}{\coth(x) + \operatorname{csch}(x)}\,dx = 2\,\ln\left(e^x + 1\right) - x$$

```
[In] int(1/(coth(x) + 1/sinh(x)),x)
[Out] 2*log(exp(x) + 1) - x
```

3.660 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^2} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2} \, dx = x-\frac{2\sinh(x)}{1+\cosh(x)}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4477, 2759, 8}

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2} \, dx = x-\frac{2\sinh(x)}{\cosh(x)+1}$$

```
[In] Int[(Coth[x] + Csch[x])^(-2),x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{\sinh^2(x)}{(i + i\cosh(x))^2}\,dx \\ &= -\frac{2\sinh(x)}{1+\cosh(x)} + \int 1\,dx \\ &= x - \frac{2\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = x - 2\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^(-2),x]
[Out] x - 2*Tanh[x/2]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
risch	$x + \frac{4}{e^x+1}$	11
default	$-2\tanh\left(\frac{x}{2}\right) - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	24

```
[In] int(1/(coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
[Out] x+4/(exp(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = \frac{x\cosh{(x)} + x\sinh{(x)} + x + 4}{\cosh{(x)} + \sinh{(x)} + 1}$$

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = \int \frac{1}{\left(\coth{(x)} + \operatorname{csch}{(x)}\right)^2}\,dx$$

```
[In] integrate(1/(coth(x)+csch(x))**2,x)
[Out] Integral((coth(x) + csch(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = x - \frac{4}{e^{(-x)} + 1}$$

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="maxima")
[Out] x - 4/(e^(-x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^2}\,dx = x + \frac{4}{e^x + 1}$$

```
[In] integrate(1/(coth(x)+csch(x))^2,x, algorithm="giac")
[Out] x + 4/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(\coth(x) + \operatorname{csch}(x))^2} \, dx = x + \frac{4}{e^x + 1}$$

```
[In] int(1/(coth(x) + 1/sinh(x))^2,x)
[Out] x + 4/(exp(x) + 1)
```

3.661 $\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx=\frac{2}{1+\cosh(x)}+\log(1+\cosh(x))$$

```
[Out] 2/(1+cosh(x))+ln(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4477, 2746, 45}

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx=\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

```
[In] Int[(Coth[x] + Csch[x])^(-3),x]
```

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\sinh^3(x)}{(i+i\cosh(x))^3}\,dx\right)\\
&= -\text{Subst}\left(\int \frac{i-x}{(i+x)^2}\,dx, x, i\cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{-i-x}+\frac{2i}{(i+x)^2}\right)\,dx, x, i\cosh(x)\right)\\
&= \frac{2i}{i+i\cosh(x)}+\log(1+\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^(-3),x]
[Out] 2*Log[Cosh[x/2]] + Sech[x/2]^2
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

method	result	size
risch	$-x+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+1)^{2}}+2\ln\left(\mathrm{e}^{x}+1\right)$	22
default	$-\tanh\left(\frac{x}{2}\right)^{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	28

```
[In] int(1/(coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

[Out] -x+4*exp(x)/(exp(x)+1)^2+2*ln(exp(x)+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 6.36

$$\int \frac{1}{(\coth(x) + \operatorname{csch}(x))^3} \, dx = -\frac{x \cosh(x)^2 + x \sinh(x)^2 + 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2 \operatorname{co}\right.}{\cosh(x)^2 + 2(\cosh(x)+1)\sinh(x) + \sinh(x}$$

[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="fricas")

[Out] -(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)

Sympy [F]

$$\int \frac{1}{(\coth(x) + \operatorname{csch}(x))^3} \, dx = \int \frac{1}{(\coth(x) + \operatorname{csch}(x))^3} \, dx$$

[In] integrate(1/(coth(x)+csch(x))**3,x)

[Out] Integral((coth(x) + csch(x))**(-3), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{(\coth(x) + \operatorname{csch}(x))^3} \, dx = x + \frac{4 e^{(-x)}}{2 e^{(-x)} + e^{(-2x)} + 1} + 2 \log\left(e^{(-x)} + 1\right)$$

[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="maxima")

[Out] x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = -x + \frac{4\,e^x}{(e^x+1)^2} + 2\,\log\left(e^x+1\right)$$

```
[In] integrate(1/(coth(x)+csch(x))^3,x, algorithm="giac")
[Out] -x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^3}\,dx = 2\,\ln\left(e^x+1\right) - x - \frac{4}{e^{2\,x}+2\,e^x+1} + \frac{4}{e^x+1}$$

```
[In] int(1/(coth(x) + 1/sinh(x))^3,x)
[Out] 2*log(exp(x) + 1) - x - 4/(exp(2*x) + 2*exp(x) + 1) + 4/(exp(x) + 1)
```

3.662 $\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4} dx$

Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4} dx = x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))-2/3*sinh(x)^3/(1+cosh(x))^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4477, 2759, 8}

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4} dx = x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[In] Int[(Coth[x] + Csch[x])^(-4),x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\sinh^4(x)}{(i+i\cosh(x))^4}\,dx\\ &= -\frac{2\sinh^3(x)}{3(1+\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i+i\cosh(x))^2}\,dx\\ &= -\frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3} + \int 1\,dx\\ &= x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8}{3}\tanh\left(\frac{x}{2}\right) + \frac{2}{3}\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^(-4),x]
[Out] x - (8*Tanh[x/2])/3 + (2*Sech[x/2]^2*Tanh[x/2])/3
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.08

method	result	size
parallelrisch	0	2
risch	$x+\frac{8\,\mathrm{e}^{2x}+8\,\mathrm{e}^{x}+\frac{16}{3}}{(\mathrm{e}^{x}+1)^{3}}$	23
default	$-\frac{2\tanh\left(\frac{x}{2}\right)^{3}}{3}-2\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	32

```
[In] int(1/(coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
[Out] 0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = \frac{3\,x\cosh(x)^2+3\,x\sinh(x)^2+4\,(3\,x+10)\cosh(x)+2\,(3\,x\cosh(x)+3\,x+4)\sinh(x)+9\,x+24}{3\,(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\,\cosh(x)+3)}$$

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="fricas")
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
+ 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 4*cosh(x) + 3)
```

Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = \int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx$$

```
[In] integrate(1/(coth(x)+csch(x))**4,x)
[Out] Integral((coth(x) + csch(x))**(-4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\,(3\,e^{(-x)}+3\,e^{(-2\,x)}+2)}{3\,(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1)}$$

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="maxima")
[Out] x - 8/3*(3*e^(-x) + 3*e^(-2*x) + 2)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x+\frac{8\left(3\,e^{(2\,x)}+3\,e^{x}+2\right)}{3\,(e^{x}+1)^{3}}$$

```
[In] integrate(1/(coth(x)+csch(x))^4,x, algorithm="giac")
[Out] x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^4}\,dx = x+\frac{\frac{8\,\mathrm{e}^{2\,x}}{3}+\frac{8}{3}}{3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1}+\frac{8\,\mathrm{e}^{x}}{3\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}+\frac{8}{3\,\left(\mathrm{e}^{x}+1\right)}$$

```
[In] int(1/(coth(x) + 1/sinh(x))^4,x)
[Out] x + ((8*exp(2*x))/3 + 8/3)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) + (8*exp(
x))/(3*(exp(2*x) + 2*exp(x) + 1)) + 8/(3*(exp(x) + 1))
```

3.663 $\int \frac{1}{(\coth(x)+\mathbf{csch}(x))^5} dx$

Optimal result

Integrand size = 7, antiderivative size = 22

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5} dx = -\frac{2}{(1+\cosh(x))^2}+\frac{4}{1+\cosh(x)}+\log(1+\cosh(x))$$

```
[Out] -2/(1+cosh(x))^2+4/(1+cosh(x))+ln(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4477, 2746, 45}

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5} dx = \frac{4}{\cosh(x)+1}-\frac{2}{(\cosh(x)+1)^2}+\log(\cosh(x)+1)$$

```
[In] Int[(Coth[x] + Csch[x])^(-5),x]
```

```
[Out] -2/(1 + Cosh[x])^2 + 4/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \frac{\sinh^5(x)}{(i+i\cosh(x))^5}\,dx \\
&= \text{Subst}\left(\int \frac{(i-x)^2}{(i+x)^3}\,dx, x, i\cosh(x)\right) \\
&= \text{Subst}\left(\int \left(-\frac{4}{(i+x)^3}-\frac{4i}{(i+x)^2}+\frac{1}{i+x}\right)\,dx, x, i\cosh(x)\right) \\
&= \frac{2}{(i+i\cosh(x))^2}+\frac{4i}{i+i\cosh(x)}+\log(1+\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right)+2\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{1}{2}\operatorname{sech}^4\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Coth[x] + Csch[x])^(-5),x]
[Out] 2*Log[Cosh[x/2]] + 2*Sech[x/2]^2 - Sech[x/2]^4/2
```

Maple [A] (verified)

Time = 10.66 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.09

method	result	size
parallelrisch	0	2
risch	$-x+\frac{8\,{\rm e}^{x}\left(1+{\rm e}^{x}+{\rm e}^{2x}\right)}{\left({\rm e}^{x}+1\right)^{4}}+2\ln\left({\rm e}^{x}+1\right)$	30
default	$-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{2}-\tanh\left(\frac{x}{2}\right)^{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	36

```
[In] int(1/(coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
[Out] 0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 12.09

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = -\frac{x\cosh(x)^4 + x\sinh(x)^4 + 4(x-2)\cosh(x)^3 + 4(x\cosh(x)+x-2)\sinh(x)^3 + 2(3x-4)\cosh(x)^2 + \cdots}{\cdots}$$

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="fricas")
[Out] -(x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*s
inh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) + 3
*x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*sinh
(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh
(x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(x)^2
+ (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1)*sinh
(x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh
(x) + 1)
```

Sympy [F]

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = \int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx$$

```
[In] integrate(1/(coth(x)+csch(x))**5,x)
[Out] Integral((coth(x) + csch(x))**(-5), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(22) = 44$.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = x + \frac{8\left(e^{(-x)} + e^{(-2x)} + e^{(-3x)}\right)}{4\,e^{(-x)} + 6\,e^{(-2x)} + 4\,e^{(-3x)} + e^{(-4x)} + 1} + 2\,\log\left(e^{(-x)} + 1\right)$$

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="maxima")
[Out] x + 8*(e^(-x) + e^(-2*x) + e^(-3*x))/(4*e^(-x) + 6*e^(-2*x) + 4*e^(-3*x) +
e^(-4*x) + 1) + 2*log(e^(-x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = -x + \frac{8\left(e^{(3x)} + e^{(2x)} + e^{x}\right)}{(e^{x}+1)^4} + 2\,\log\left(e^{x}+1\right)$$

```
[In] integrate(1/(coth(x)+csch(x))^5,x, algorithm="giac")
[Out] -x + 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 + 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.59

$$\int \frac{1}{(\coth(x)+\operatorname{csch}(x))^5}\,dx = 2\,\ln\left(\mathrm{e}^{x}+1\right) - x - \frac{16}{\mathrm{e}^{2x}+2\,\mathrm{e}^{x}+1} - \frac{8}{6\,\mathrm{e}^{2x}+4\,\mathrm{e}^{3x}+\mathrm{e}^{4x}+4\,\mathrm{e}^{x}+1} + \frac{8}{\mathrm{e}^{x}+1} + \frac{16}{3\,\mathrm{e}^{2x}+\mathrm{e}^{3x}+3\,\mathrm{e}^{x}+1}$$

```
[In] int(1/(coth(x) + 1/sinh(x))^5,x)
[Out] 2*log(exp(x) + 1) - x - 16/(exp(2*x) + 2*exp(x) + 1) - 8/(6*exp(2*x) + 4*ex
p(3*x) + exp(4*x) + 4*exp(x) + 1) + 8/(exp(x) + 1) + 16/(3*exp(2*x) + exp(3
*x) + 3*exp(x) + 1)
```

3.664 $\int (-\coth(x)+\mathbf{csch}(x))^5\,dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx=\frac{2}{(1+\cosh(x))^2}-\frac{4}{1+\cosh(x)}-\log(1+\cosh(x))$$

```
[Out] 2/(1+cosh(x))^2-4/(1+cosh(x))-ln(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4477, 2746, 45}

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx=-\frac{4}{\cosh(x)+1}+\frac{2}{(\cosh(x)+1)^2}-\log(\cosh(x)+1)$$

```
[In] Int[(-Coth[x] + Csch[x])^5,x]
[Out] 2/(1 + Cosh[x])^2 - 4/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int (i-i\cosh(x))^5\operatorname{csch}^5(x)\,dx\right)\\
&= \operatorname{Subst}\left(\int \frac{(i+x)^2}{(i-x)^3}\,dx, x, -i\cosh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{1}{i-x}+\frac{4}{(-i+x)^3}-\frac{4i}{(-i+x)^2}\right)\,dx, x, -i\cosh(x)\right)\\
&= -\frac{2}{(i+i\cosh(x))^2}-\frac{4i}{i+i\cosh(x)}-\log(1+\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = -2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\tanh^2\left(\frac{x}{2}\right)+\frac{1}{2}\tanh^4\left(\frac{x}{2}\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^5,x]
[Out] -2*Log[Cosh[x/2]] + Tanh[x/2]^2 + Tanh[x/2]^4/2
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result
risch	$x-\frac{8\,\mathrm{e}^{x}\left(1+\mathrm{e}^{x}+\mathrm{e}^{2x}\right)}{\left(\mathrm{e}^{x}+1\right)^{4}}-2\ln\left(\mathrm{e}^{x}+1\right)$
parts	$\frac{8\left(-\frac{\mathrm{csch}(x)^{3}}{4}+\frac{3\,\mathrm{csch}(x)}{8}\right)\coth(x)}{3}-2\,\mathrm{arctanh}\left(\mathrm{e}^{x}\right)+\frac{11\coth(x)^{4}}{4}+\frac{\coth(x)^{2}}{2}+\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}+\frac{5\,\mathrm{co}}{3\sin}$
default	$-\ln\left(\sinh\left(x\right)\right)+\frac{\coth(x)^{2}}{2}+\frac{\coth(x)^{4}}{4}-\frac{5\cosh(x)^{3}}{\sinh(x)^{4}}+\frac{5\cosh(x)}{3\sinh(x)^{4}}+\frac{8\left(-\frac{\mathrm{csch}(x)^{3}}{4}+\frac{3\,\mathrm{csch}(x)}{8}\right)\coth(x)}{3}-2\,\mathrm{arctanh}\,($

```
[In] int((-coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
[Out] x-8*exp(x)*(1+exp(x)+exp(2*x))/(exp(x)+1)^4-2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 11.04

$$\int(-\coth(x)+\operatorname{csch}(x))^5\,dx$$
$$=\frac{x\cosh\left(x\right)^{4}+x\sinh\left(x\right)^{4}+4\left(x-2\right)\cosh\left(x\right)^{3}+4\left(x\cosh\left(x\right)+x-2\right)\sinh\left(x\right)^{3}+2\left(3\,x-4\right)\cosh\left(x\right)^{2}+2}{}$$

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="fricas")
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) + x - 2)*si
nh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 + 6*(x - 2)*cosh(x) + 3*
x - 4)*sinh(x)^2 + 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) + 1)*sinh(
x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh(
x) + 1)*log(cosh(x) + sinh(x) + 1) + 4*(x*cosh(x)^3 + 3*(x - 2)*cosh(x)^2 +
 (3*x - 4)*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) + 1)*sinh(
x)^3 + sinh(x)^4 + 4*cosh(x)^3 + 6*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x) + 4*cosh(
x) + 1)
```

Sympy [F]

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = -\int 5\coth(x)\operatorname{csch}^4(x)\,dx - \int \left(-10\coth^2(x)\operatorname{csch}^3(x)\right)\,dx - \int 10\coth^3(x)\operatorname{csch}^2(x)\,dx - \int \left(-5\coth^4(x)\operatorname{csch}(x)\right)\,dx - \int \coth^5(x)\,dx - \int \left(-\operatorname{csch}^5(x)\right)\,dx$$

```
[In] integrate((-coth(x)+csch(x))**5,x)
```

```
[Out] -Integral(5*coth(x)*csch(x)**4, x) - Integral(-10*coth(x)**2*csch(x)**3, x)
 - Integral(10*coth(x)**3*csch(x)**2, x) - Integral(-5*coth(x)**4*csch(x),
x) - Integral(coth(x)**5, x) - Integral(-csch(x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(24) = 48$.

Time = 0.21 (sec) , antiderivative size = 238, normalized size of antiderivative = 9.92

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = \frac{5}{2}\coth(x)^4 - x + \frac{5\left(5e^{(-x)}+3e^{(-3x)}+3e^{(-5x)}+5e^{(-7x)}\right)}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)} - \frac{3e^{(-x)}-11e^{(-3x)}-11e^{(-5x)}+3e^{(-7x)}}{4\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)} + \frac{5\left(e^{(-x)}+7e^{(-3x)}+7e^{(-5x)}+e^{(-7x)}\right)}{2\left(4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1\right)} - \frac{4\left(e^{(-2x)}-e^{(-4x)}+e^{(-6x)}\right)}{4e^{(-2x)}-6e^{(-4x)}+4e^{(-6x)}-e^{(-8x)}-1} + \frac{20}{\left(e^{(-x)}-e^{x}\right)^4} - 2\log\left(e^{(-x)}+1\right)$$

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*coth(x)^4 - x + 5/4*(5*e^(-x) + 3*e^(-3*x) + 3*e^(-5*x) + 5*e^(-7*x))/(
4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 1/4*(3*e^(-x) - 11*e
^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) -
 e^(-8*x) - 1) + 5/2*(e^(-x) + 7*e^(-3*x) + 7*e^(-5*x) + e^(-7*x))/(4*e^(-2
*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 4*(e^(-2*x) - e^(-4*x) + e^
(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 20/(e^(-x)
- e^x)^4 - 2*log(e^(-x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = x - \frac{8\left(e^{(3x)}+e^{(2x)}+e^{x}\right)}{\left(e^{x}+1\right)^4} - 2\log\left(e^{x}+1\right)$$

```
[In] integrate((-coth(x)+csch(x))^5,x, algorithm="giac")
[Out] x - 8*(e^(3*x) + e^(2*x) + e^x)/(e^x + 1)^4 - 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.21

$$\int (-\coth(x)+\operatorname{csch}(x))^5\,dx = x - 2\ln\left(e^{x}+1\right) + \frac{16}{e^{2x}+2e^{x}+1} + \frac{8}{6e^{2x}+4e^{3x}+e^{4x}+4e^{x}+1} - \frac{8}{e^{x}+1} - \frac{16}{3e^{2x}+e^{3x}+3e^{x}+1}$$

```
[In] int(-(coth(x) - 1/sinh(x))^5,x)
[Out] x - 2*log(exp(x) + 1) + 16/(exp(2*x) + 2*exp(x) + 1) + 8/(6*exp(2*x) + 4*ex
p(3*x) + exp(4*x) + 4*exp(x) + 1) - 8/(exp(x) + 1) - 16/(3*exp(2*x) + exp(3
*x) + 3*exp(x) + 1)
```

3.665 $\int (-\coth(x) + \operatorname{csch}(x))^4 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int (-\coth(x) + \operatorname{csch}(x))^4 \, dx = x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))-2/3*sinh(x)^3/(1+cosh(x))^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4477, 2749, 2759, 8}

$$\int (-\coth(x) + \operatorname{csch}(x))^4 \, dx = x - \frac{2\sinh^3(x)}{3(\cosh(x)+1)^3} - \frac{2\sinh(x)}{\cosh(x)+1}$$

```
[In] Int[(-Coth[x] + Csch[x])^4,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x]) - (2*Sinh[x]^3)/(3*(1 + Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (i - i\cosh(x))^4 \operatorname{csch}^4(x)\,dx \\
&= \int \frac{\sinh^4(x)}{(i + i\cosh(x))^4}\,dx \\
&= -\frac{2\sinh^3(x)}{3(1+\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i + i\cosh(x))^2}\,dx \\
&= -\frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3} + \int 1\,dx \\
&= x - \frac{2\sinh(x)}{1+\cosh(x)} - \frac{2\sinh^3(x)}{3(1+\cosh(x))^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int (-\coth(x) + \operatorname{csch}(x))^4\,dx = 2\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right) - \frac{2}{3}\tanh^3\left(\frac{x}{2}\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^4,x]
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2] - (2*Tanh[x/2]^3)/3
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result
risch	$x+\frac{8\,{\mathrm e}^{2 x}+8\,{\mathrm e}^{x}+\frac{16}{3}}{\left({\mathrm e}^{x}+1\right)^{3}}$
parts	$-\frac{7 \coth \left(x \right)^{3}}{3}-\coth \left(x \right)-\frac{\ln \left(\coth \left(x \right)-1\right)}{2}+\frac{\ln \left(1+\coth \left(x \right)\right)}{2}+\left(\frac{2}{3}-\frac{\operatorname{csch}\left(x \right)^{2}}{3}\right) \coth \left(x \right)+\frac{8 \operatorname{csch}\left(x \right)^{3}}{3}+4\, \operatorname{csch}(x$
default	$x-\coth \left(x \right)-\frac{\coth \left(x \right)^{3}}{3}+\frac{4 \cosh \left(x \right)^{2}}{\sinh \left(x \right)^{3}}-\frac{4}{3 \sinh \left(x \right)^{3}}-\frac{3 \cosh \left(x \right)}{\sinh \left(x \right)^{3}}-2 \left(\frac{2}{3}-\frac{\operatorname{csch}\left(x \right)^{2}}{3}\right) \coth \left(x \right)$

```
[In] int((-coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
[Out] x+8/3*(3*exp(2*x)+3*exp(x)+2)/(exp(x)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int (-\coth(x)+\operatorname{csch}(x))^4\,dx = \frac{3\,x\cosh\left(x\right)^{2}+3\,x\sinh\left(x\right)^{2}+4\,{\left(3\,x+10\right)}\cosh\left(x\right)+2\,{\left(3\,x\cosh\left(x\right)+3\,x+4\right)}\sinh\left(x\right)+9\,x+24}{3\,{\left(\cosh\left(x\right)^{2}+2\,{\left(\cosh\left(x\right)+1\right)}\sinh\left(x\right)+\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)+3\right)}}$$

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="fricas")
[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 + 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x)
+ 3*x + 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 4*cosh(x) + 3)
```

Sympy [F]

$$\int (-\coth(x)+\operatorname{csch}(x))^4\,dx = \int \left(-\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{4}\,dx$$

```
[In] integrate((-coth(x)+csch(x))**4,x)
[Out] Integral((-coth(x) + csch(x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(24) = 48.

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 7.04

$$\int (-\coth(x)+\operatorname{csch}(x))^4\,dx = -2\,\coth{(x)}^3 + x - \frac{4\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2)}{3\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1)} - \frac{8\,e^{(-x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} + \frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} + \frac{16\,e^{(-3\,x)}}{3\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1)} - \frac{8\,e^{(-5\,x)}}{3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1} - \frac{4}{3\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1)} - \frac{32}{3\,(e^{(-x)} - e^{x})^3}$$

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="maxima")
```

```
[Out] -2*coth(x)^3 + x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*
x) + e^(-6*x) - 1) - 8*e^(-x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*
e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 16/3*e^(-3*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) - 8*e^(-5*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(
-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 32/3/(e^(-x) -
e^x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (-\coth(x)+\operatorname{csch}(x))^4\,dx = x + \frac{8\,(3\,e^{(2\,x)} + 3\,e^{x} + 2)}{3\,(e^{x} + 1)^3}$$

```
[In] integrate((-coth(x)+csch(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*e^(2*x) + 3*e^x + 2)/(e^x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int (-\coth(x)+\text{csch}(x))^4\,dx = x+\frac{\frac{8\,{\mathrm{e}}^{2\,x}}{3}+\frac{8}{3}}{3\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^x+1}+\frac{8\,{\mathrm{e}}^x}{3\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^x+1\right)}+\frac{8}{3\,\left({\mathrm{e}}^x+1\right)}$$

```
[In] int((coth(x) - 1/sinh(x))^4,x)
```

```
[Out] x + ((8*exp(2*x))/3 + 8/3)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) + (8*exp(
x))/(3*(exp(2*x) + 2*exp(x) + 1)) + 8/(3*(exp(x) + 1))
```

3.666 $\int (-\coth(x) + \text{csch}(x))^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx=-\frac{2}{1+\cosh(x)}-\log(1+\cosh(x))$$

```
[Out] -2/(1+cosh(x))-ln(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4477, 2746, 45}

$$\int (-\coth(x)+\operatorname{csch}(x))^3\,dx=-\frac{2}{\cosh(x)+1}-\log(\cosh(x)+1)$$

```
[In] Int[(-Coth[x] + Csch[x])^3,x]
[Out] -2/(1 + Cosh[x]) - Log[1 + Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int (i-i\cosh(x))^3\text{csch}^3(x)\,dx\\ &= -\text{Subst}\left(\int \frac{i+x}{(i-x)^2}\,dx, x, -i\cosh(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{2i}{(-i+x)^2}+\frac{1}{-i+x}\right)\,dx, x, -i\cosh(x)\right)\\ &= -\frac{2i}{i+i\cosh(x)}-\log(1+\cosh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (-\coth(x)+\text{csch}(x))^3\,dx = -2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\tanh^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^3,x]
[Out] -2*Log[Cosh[x/2]] + Tanh[x/2]^2
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

method	result	size
risch	$x-\frac{4\,{\rm e}^{x}}{({\rm e}^{x}+1)^{2}}-2\ln\left({\rm e}^{x}+1\right)$	20
default	$-\ln\left(\sinh\left(x\right)\right)+\frac{\coth(x)^{2}}{2}-\frac{3\cosh(x)}{\sinh(x)^{2}}+\text{csch}\left(x\right)\coth\left(x\right)-2\,\text{arctanh}\left({\rm e}^{x}\right)+\frac{3}{2\sinh(x)^{2}}$	37
parts	$\text{csch}\left(x\right)\coth\left(x\right)-2\,\text{arctanh}\left({\rm e}^{x}\right)+2\coth\left(x\right)^{2}+\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}-\frac{3\cosh(x)}{\sinh(x)^{2}}$	40

```
[In] int((-coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
[Out] x-4*exp(x)/(exp(x)+1)^2-2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int (-\coth(x) + \operatorname{csch}(x))^3 \, dx$$
$$= \frac{x \cosh{(x)}^2 + x \sinh{(x)}^2 + 2\,(x-2)\cosh{(x)} - 2\left(\cosh{(x)}^2 + 2\,(\cosh{(x)}+1)\sinh{(x)} + \sinh{(x)}^2 + 2\,\cosh\right.}{\cosh{(x)}^2 + 2\,(\cosh{(x)}+1)\sinh{(x)} + \sinh{(x)}^2 +}$$

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="fricas")
[Out] (x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
+ 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(x
*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(
x)^2 + 2*cosh(x) + 1)
```

Sympy [F]

$$\int (-\coth(x) + \operatorname{csch}(x))^3 \, dx = -\int 3\coth{(x)}\operatorname{csch}^2{(x)}\, dx - \int \left(-3\coth^2{(x)}\operatorname{csch}{(x)}\right) dx - \int \coth^3{(x)}\, dx - \int \left(-\operatorname{csch}^3{(x)}\right) dx$$

```
[In] integrate((-coth(x)+csch(x))**3,x)
[Out] -Integral(3*coth(x)*csch(x)**2, x) - Integral(-3*coth(x)**2*csch(x), x) - I
ntegral(coth(x)**3, x) - Integral(-csch(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int (-\coth(x) + \operatorname{csch}(x))^3 \, dx = \frac{3}{2}\coth{(x)}^2 - x + \frac{4\left(e^{(-x)} + e^{(-3\,x)}\right)}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - 2\,\log\left(e^{(-x)} + 1\right)$$

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="maxima")
[Out] 3/2*coth(x)^2 - x + 4*(e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - 2*e
^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*log(e^(-x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (-\coth(x) + \operatorname{csch}(x))^3\, dx = x - \frac{4\, e^x}{(e^x + 1)^2} - 2\, \log\left(e^x + 1\right)$$

```
[In] integrate((-coth(x)+csch(x))^3,x, algorithm="giac")
[Out] x - 4*e^x/(e^x + 1)^2 - 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int (-\coth(x) + \operatorname{csch}(x))^3\, dx = x - 2\, \ln\left(e^x + 1\right) + \frac{4}{e^{2\,x} + 2\, e^x + 1} - \frac{4}{e^x + 1}$$

```
[In] int(-(coth(x) - 1/sinh(x))^3,x)
[Out] x - 2*log(exp(x) + 1) + 4/(exp(2*x) + 2*exp(x) + 1) - 4/(exp(x) + 1)
```

3.667 $\int (-\coth(x) + \mathbf{csch}(x))^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{2\sinh(x)}{1 + \cosh(x)}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4477, 2749, 2759, 8}

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{2\sinh(x)}{\cosh(x) + 1}$$

```
[In] Int[(-Coth[x] + Csch[x])^2,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int (i - i\cosh(x))^2 \operatorname{csch}^2(x)\,dx \\
&= -\int \frac{\sinh^2(x)}{(i + i\cosh(x))^2}\,dx \\
&= -\frac{2\sinh(x)}{1+\cosh(x)} + \int 1\,dx \\
&= x - \frac{2\sinh(x)}{1+\cosh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int (-\coth(x) + \operatorname{csch}(x))^2\,dx = 2\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^2,x]
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
risch	$x+\frac{4}{{\mathrm e}^{x}+1}$	11
default	$x-2\coth\left(x\right)+\frac{2}{\sinh\left(x\right)}$	13
parts	$-2\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{2}+\frac{\ln\left(1+\coth\left(x\right)\right)}{2}+2\,\operatorname{csch}\left(x\right)$	24

```
[In] int((-coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
[Out] x+4/(exp(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int(-\coth(x)+\operatorname{csch}(x))^2\,dx=\frac{x\cosh\left(x\right)+x\sinh\left(x\right)+x+4}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [F]

$$\int(-\coth(x)+\operatorname{csch}(x))^2\,dx=\int\left(-\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^{2}\,dx$$

```
[In] integrate((-coth(x)+csch(x))**2,x)
[Out] Integral((-coth(x) + csch(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x - \frac{4}{e^{(-x)} - e^x} + \frac{4}{e^{(-2\,x)} - 1}$$

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="maxima")
[Out] x - 4/(e^(-x) - e^x) + 4/(e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{4}{e^x + 1}$$

```
[In] integrate((-coth(x)+csch(x))^2,x, algorithm="giac")
[Out] x + 4/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (-\coth(x) + \operatorname{csch}(x))^2 \, dx = x + \frac{4}{\mathrm{e}^x + 1}$$

```
[In] int((coth(x) - 1/sinh(x))^2,x)
[Out] x + 4/(exp(x) + 1)
```

3.668 $\int (-\coth(x)+\operatorname{csch}(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (-\coth(x)+\operatorname{csch}(x))\,dx = -\operatorname{arctanh}(\cosh(x))-\log(\sinh(x))$$

```
[Out] -arctanh(cosh(x))-ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3556, 3855}

$$\int (-\coth(x)+\operatorname{csch}(x))\,dx = -\operatorname{arctanh}(\cosh(x))-\log(\sinh(x))$$

```
[In] Int[-Coth[x] + Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] - Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \coth(x)\,dx + \int \text{csch}(x)\,dx\\ &= -\text{arctanh}(\cosh(x)) - \log(\sinh(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\begin{aligned}\int (-\coth(x) + \text{csch}(x))\,dx = &-\log\left(\cosh\left(\frac{x}{2}\right)\right) - \log(\cosh(x))\\ &+ \log\left(\sinh\left(\frac{x}{2}\right)\right) - \log(\tanh(x))\end{aligned}$$

```
[In] Integrate[-Coth[x] + Csch[x],x]
[Out] -Log[Cosh[x/2]] - Log[Cosh[x]] + Log[Sinh[x/2]] - Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	12
parts	$-\ln\left(\sinh\left(x\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	12
parallelrisch	$x-\ln\left(\tanh\left(x\right)\right)+\ln\left(\coth\left(x\right)-\text{csch}\left(x\right)\right)+\ln\left(1-\tanh\left(x\right)\right)$	23
risch	$x-\ln\left(\mathrm{e}^{2x}-1\right)+\ln\left(\mathrm{e}^{x}-1\right)-\ln\left(\mathrm{e}^{x}+1\right)$	24

```
[In] int(-coth(x)+csch(x),x,method=_RETURNVERBOSE)
[Out] -ln(sinh(x))+ln(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-\coth(x) + \operatorname{csch}(x))\, dx = x - 2 \log(\cosh(x) + \sinh(x) + 1)$$

```
[In] integrate(-coth(x)+csch(x),x, algorithm="fricas")
[Out] x - 2*log(cosh(x) + sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-\coth(x) + \operatorname{csch}(x))\, dx = -x + \log(\tanh(x) + 1) + \log\left(\tanh\left(\frac{x}{2}\right)\right) - \log(\tanh(x))$$

```
[In] integrate(-coth(x)+csch(x),x)
[Out] -x + log(tanh(x) + 1) + log(tanh(x/2)) - log(tanh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-\coth(x) + \operatorname{csch}(x))\, dx = -\log(\sinh(x)) + \log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

```
[In] integrate(-coth(x)+csch(x),x, algorithm="maxima")
[Out] -log(sinh(x)) + log(tanh(1/2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int (-\coth(x) + \operatorname{csch}(x))\, dx = x - \log(e^x + 1) - \log\left(\left|e^{(2x)} - 1\right|\right) + \log(|e^x - 1|)$$

```
[In] integrate(-coth(x)+csch(x),x, algorithm="giac")
[Out] x - log(e^x + 1) - log(abs(e^(2*x) - 1)) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-\coth(x) + \operatorname{csch}(x))\, dx = x - 2\, \ln(e^x + 1)$$

```
[In] int(1/sinh(x) - coth(x),x)
[Out] x - 2*log(exp(x) + 1)
```

3.669 $\int \frac{1}{-\coth(x)+\text{csch}(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{1}{-\coth(x)+\text{csch}(x)}\,dx=-\log(1-\cosh(x))$$

```
[Out] -ln(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3239, 2746, 31}

$$\int \frac{1}{-\coth(x)+\text{csch}(x)}\,dx=-\log(1-\cosh(x))$$

```
[In] Int[(-Coth[x] + Csch[x])^(-1),x]
```

```
[Out] -Log[1 - Cosh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int \frac{\sinh(x)}{i - i\cosh(x)}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{i+x}\,dx, x, -i\cosh(x)\right)\\ &= -\log(1-\cosh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^(-1),x]
[Out] -2*Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
risch	$x - 2\ln(e^{x} - 1)$	10
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - 2\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	23

```
[In] int(1/(-coth(x)+csch(x)),x,method=_RETURNVERBOSE)
[Out] x-2*ln(exp(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = x-2\,\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="fricas")
[Out] x - 2*log(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -\int \frac{1}{\coth\left(x\right)-\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(1/(-coth(x)+csch(x)),x)
[Out] -Integral(1/(coth(x) - csch(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = -x-2\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="maxima")
[Out] -x - 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = x-2\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(1/(-coth(x)+csch(x)),x, algorithm="giac")
[Out] x - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\coth(x)+\operatorname{csch}(x)}\,dx = x-2\,\ln\left(\mathrm{e}^x-1\right)$$

```
[In] int(-1/(coth(x) - 1/sinh(x)),x)
[Out] x - 2*log(exp(x) - 1)
```

3.670 $\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4477, 2759, 8}

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[In] Int[(-Coth[x] + Csch[x])^(-2),x]
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{\sinh^2(x)}{(i - i\cosh(x))^2}\,dx \\ &= \frac{2\sinh(x)}{1-\cosh(x)} + \int 1\,dx \\ &= x + \frac{2\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = -2\coth\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^(-2),x]
[Out] -2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$x-\frac{4}{{\mathrm e}^{x}-1}$	11
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)}$	26

```
[In] int(1/(-coth(x)+csch(x))^2,x,method=_RETURNVERBOSE)
[Out] x-4/(exp(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = \frac{x\cosh(x)+x\sinh(x)-x-4}{\cosh(x)+\sinh(x)-1}$$

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = \int \frac{1}{\left(-\coth(x)+\operatorname{csch}(x)\right)^2}\,dx$$

```
[In] integrate(1/(-coth(x)+csch(x))**2,x)
[Out] Integral((-coth(x) + csch(x))**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x+\frac{4}{e^{(-x)}-1}$$

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="maxima")
[Out] x + 4/(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x-\frac{4}{e^x-1}$$

```
[In] integrate(1/(-coth(x)+csch(x))^2,x, algorithm="giac")
[Out] x - 4/(e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^2}\,dx = x - \frac{4}{e^x - 1}$$

```
[In] int(1/(coth(x) - 1/sinh(x))^2,x)
[Out] x - 4/(exp(x) - 1)
```

3.671 $\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3} dx = -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

```
[Out] -2/(1-cosh(x))-ln(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4477, 2746, 45}

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3} dx = -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

```
[In] Int[(-Coth[x] + Csch[x])^(-3),x]
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\sinh^3(x)}{(i-i\cosh(x))^3}\,dx\right)\\
&= \text{Subst}\left(\int \frac{i-x}{(i+x)^2}\,dx, x, -i\cosh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-i-x}+\frac{2i}{(i+x)^2}\right)\,dx, x, -i\cosh(x)\right)\\
&= -\frac{2i}{i-i\cosh(x)}-\log(1-\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = \coth^2\left(\frac{x}{2}\right)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)-2\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^(-3),x]
[Out] Coth[x/2]^2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$x+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-1)^{2}}-2\ln\left(\mathrm{e}^{x}-1\right)$	20
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)^{2}}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	29

```
[In] int(1/(-coth(x)+csch(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x+4*exp(x)/(exp(x)-1)^2-2*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = \frac{x\cosh{(x)}^2 + x\sinh{(x)}^2 - 2\,(x-2)\cosh{(x)} - 2\left(\cosh{(x)}^2 + 2\,(\cosh{(x)}-1)\sinh{(x)} + \sinh{(x)}^2 - 2\cosh\right.}{\cosh{(x)}^2 + 2\,(\cosh{(x)}-1)\sinh{(x)} + \sinh{(x)}^2}$$

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
- 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(x
*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 2*cosh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = -\int \frac{1}{\coth^3{(x)} - 3\coth^2{(x)}\operatorname{csch}{(x)} + 3\coth{(x)}\operatorname{csch}^2{(x)} - \operatorname{csch}^3{(x)}}\,dx$$

```
[In] integrate(1/(-coth(x)+csch(x))**3,x)
```

```
[Out] -Integral(1/(coth(x)**3 - 3*coth(x)**2*csch(x) + 3*coth(x)*csch(x)**2 - csc
h(x)**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^3}\,dx = -x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="maxima")
```

```
[Out] -x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-\coth(x) + \operatorname{csch}(x))^3} \, dx = x + \frac{4\, e^x}{(e^x - 1)^2} - 2\, \log\left(|e^x - 1|\right)$$

```
[In] integrate(1/(-coth(x)+csch(x))^3,x, algorithm="giac")
[Out] x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-\coth(x) + \operatorname{csch}(x))^3} \, dx = x - 2\, \ln\left(e^x - 1\right) + \frac{4}{e^{2\,x} - 2\,e^x + 1} + \frac{4}{e^x - 1}$$

```
[In] int(-1/(coth(x) - 1/sinh(x))^3,x)
[Out] x - 2*log(exp(x) - 1) + 4/(exp(2*x) - 2*exp(x) + 1) + 4/(exp(x) - 1)
```

3.672 $\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4} dx = x+\frac{2\sinh(x)}{1-\cosh(x)}+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))+2/3*sinh(x)^3/(1-cosh(x))^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4477, 2759, 8}

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4} dx = x+\frac{2\sinh^3(x)}{3(1-\cosh(x))^3}+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[In] Int[(-Coth[x] + Csch[x])^(-4),x]
[Out] x + (2*Sinh[x])/(1 - Cosh[x]) + (2*Sinh[x]^3)/(3*(1 - Cosh[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sinh^4(x)}{(i - i\cosh(x))^4}\,dx \\
&= \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} - \int \frac{\sinh^2(x)}{(i-i\cosh(x))^2}\,dx \\
&= \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3} + \int 1\,dx \\
&= x + \frac{2\sinh(x)}{1-\cosh(x)} + \frac{2\sinh^3(x)}{3(1-\cosh(x))^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = -\frac{2}{3}\coth^3\left(\frac{x}{2}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[(-Coth[x] + Csch[x])^(-4),x]
[Out] (-2*Coth[x/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x/2]^2])/3
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
risch	$x-\frac{8\left(3\,{\mathrm e}^{2 x}-3\,{\mathrm e}^{x}+2\right)}{3\left({\mathrm e}^{x}-1\right)^{3}}$	23
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2}{3\tanh\left(\frac{x}{2}\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	34

```
[In] int(1/(-coth(x)+csch(x))^4,x,method=_RETURNVERBOSE)
```

[Out] x-8/3*(3*exp(2*x)-3*exp(x)+2)/(exp(x)-1)^3

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = \frac{3\,x\cosh\left(x\right)^2+3\,x\sinh\left(x\right)^2-4\,(3\,x+10)\cosh\left(x\right)+2\,(3\,x\cosh\left(x\right)-3\,x-4)\sinh\left(x\right)+9\,x+24}{3\,\left(\cosh\left(x\right)^2+2\,(\cosh\left(x\right)-1)\sinh\left(x\right)+\sinh\left(x\right)^2-4\,\cosh\left(x\right)+3\right)}$$

[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="fricas")

[Out] 1/3*(3*x*cosh(x)^2 + 3*x*sinh(x)^2 - 4*(3*x + 10)*cosh(x) + 2*(3*x*cosh(x) - 3*x - 4)*sinh(x) + 9*x + 24)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 4*cosh(x) + 3)

Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = \int \frac{1}{\left(-\coth\left(x\right)+\operatorname{csch}\left(x\right)\right)^4}\,dx$$

[In] integrate(1/(-coth(x)+csch(x))**4,x)

[Out] Integral((-coth(x) + csch(x))**(-4), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = x-\frac{8\,\left(3\,e^{(-x)}-3\,e^{(-2\,x)}-2\right)}{3\,(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1)}$$

[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="maxima")

[Out] x - 8/3*(3*e^(-x) - 3*e^(-2*x) - 2)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\left(3\,e^{(2\,x)} - 3\,e^x + 2\right)}{3\,(e^x-1)^3}$$

```
[In] integrate(1/(-coth(x)+csch(x))^4,x, algorithm="giac")
[Out] x - 8/3*(3*e^(2*x) - 3*e^x + 2)/(e^x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^4}\,dx = x - \frac{8\,\mathrm{e}^x}{3\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^x+1\right)} + \frac{\frac{8\,\mathrm{e}^{2\,x}}{3}+\frac{8}{3}}{3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^x+1} - \frac{8}{3\,\left(\mathrm{e}^x-1\right)}$$

```
[In] int(1/(coth(x) - 1/sinh(x))^4,x)
[Out] x - (8*exp(x))/(3*(exp(2*x) - 2*exp(x) + 1)) + ((8*exp(2*x))/3 + 8/3)/(3*ex
p(2*x) - exp(3*x) - 3*exp(x) + 1) - 8/(3*(exp(x) - 1))
```

3.673 $\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5} dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5} dx = \frac{2}{(1-\cosh(x))^2} - \frac{4}{1-\cosh(x)} - \log(1-\cosh(x))$$

```
[Out] 2/(1-cosh(x))^2-4/(1-cosh(x))-ln(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4477, 2746, 45}

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5} dx = -\frac{4}{1-\cosh(x)} + \frac{2}{(1-\cosh(x))^2} - \log(1-\cosh(x))$$

```
[In] Int[(-Coth[x] + Csch[x])^(-5),x]
```

```
[Out] 2/(1 - Cosh[x])^2 - 4/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i\int \frac{\sinh^5(x)}{(i-i\cosh(x))^5}\,dx \\
&= -\text{Subst}\left(\int \frac{(i-x)^2}{(i+x)^3}\,dx, x, -i\cosh(x)\right) \\
&= -\text{Subst}\left(\int \left(-\frac{4}{(i+x)^3}-\frac{4i}{(i+x)^2}+\frac{1}{i+x}\right)\,dx, x, -i\cosh(x)\right) \\
&= -\frac{2}{(i-i\cosh(x))^2}-\frac{4i}{i-i\cosh(x)}-\log(1-\cosh(x))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$
\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = \coth^2\left(\frac{x}{2}\right)+\frac{1}{2}\coth^4\left(\frac{x}{2}\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right)-2\log\left(\tanh\left(\frac{x}{2}\right)\right)
$$

```
[In] Integrate[(-Coth[x] + Csch[x])^(-5),x]
[Out] Coth[x/2]^2 + Coth[x/2]^4/2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

Maple [A] (verified)

Time = 10.36 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.07

method	result	size
parallelrisch	0	2
risch	$x+\frac{8\,\mathrm{e}^{x}\left(\mathrm{e}^{2x}-\mathrm{e}^{x}+1\right)}{\left(\mathrm{e}^{x}-1\right)^{4}}-2\ln\left(\mathrm{e}^{x}-1\right)$	30
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)^{4}}+\frac{1}{\tanh\left(\frac{x}{2}\right)^{2}}-2\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	37

```
[In] int(1/(-coth(x)+csch(x))^5,x,method=_RETURNVERBOSE)
[Out] 0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 8.97

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = \frac{x\cosh(x)^4 + x\sinh(x)^4 - 4(x-2)\cosh(x)^3 + 4(x\cosh(x) - x + 2)\sinh(x)^3 + 2(3x-4)\cosh(x)^2 + 2}{}$$

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="fricas")
[Out] (x*cosh(x)^4 + x*sinh(x)^4 - 4*(x - 2)*cosh(x)^3 + 4*(x*cosh(x) - x + 2)*si
nh(x)^3 + 2*(3*x - 4)*cosh(x)^2 + 2*(3*x*cosh(x)^2 - 6*(x - 2)*cosh(x) + 3*
x - 4)*sinh(x)^2 - 4*(x - 2)*cosh(x) - 2*(cosh(x)^4 + 4*(cosh(x) - 1)*sinh(
x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh(
x) + 1)*log(cosh(x) + sinh(x) - 1) + 4*(x*cosh(x)^3 - 3*(x - 2)*cosh(x)^2 +
 (3*x - 4)*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^4 + 4*(cosh(x) - 1)*sinh(
x)^3 + sinh(x)^4 - 4*cosh(x)^3 + 6*(cosh(x)^2 - 2*cosh(x) + 1)*sinh(x)^2 +
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x)^2 + 3*cosh(x) - 1)*sinh(x) - 4*cosh(
x) + 1)
```

Sympy [F]

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = -\int \frac{1}{\coth^5(x) - 5\coth^4(x)\operatorname{csch}(x) + 10\coth^3(x)\operatorname{csch}^2(x) - 10\coth^2(x)\operatorname{csch}^3(x) + 5\coth(x)\operatorname{csch}^4(x)}$$

```
[In] integrate(1/(-coth(x)+csch(x))**5,x)
[Out] -Integral(1/(coth(x)**5 - 5*coth(x)**4*csch(x) + 10*coth(x)**3*csch(x)**2 -
 10*coth(x)**2*csch(x)**3 + 5*coth(x)*csch(x)**4 - csch(x)**5), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = -x - \frac{8\left(e^{(-x)} - e^{(-2x)} + e^{(-3x)}\right)}{4\,e^{(-x)} - 6\,e^{(-2x)} + 4\,e^{(-3x)} - e^{(-4x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="maxima")
[Out] -x - 8*(e^(-x) - e^(-2*x) + e^(-3*x))/(4*e^(-x) - 6*e^(-2*x) + 4*e^(-3*x) -
 e^(-4*x) - 1) - 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = x + \frac{8\left(e^{(3x)} - e^{(2x)} + e^{x}\right)}{(e^{x} - 1)^4} - 2\,\log\left(|e^{x} - 1|\right)$$

```
[In] integrate(1/(-coth(x)+csch(x))^5,x, algorithm="giac")
[Out] x + 8*(e^(3*x) - e^(2*x) + e^x)/(e^x - 1)^4 - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.63

$$\int \frac{1}{(-\coth(x)+\operatorname{csch}(x))^5}\,dx = x - 2\,\ln\left(\mathrm{e}^{x} - 1\right) - \frac{16}{3\,\mathrm{e}^{2x} - \mathrm{e}^{3x} - 3\,\mathrm{e}^{x} + 1} + \frac{16}{\mathrm{e}^{2x} - 2\,\mathrm{e}^{x} + 1} + \frac{8}{6\,\mathrm{e}^{2x} - 4\,\mathrm{e}^{3x} + \mathrm{e}^{4x} - 4\,\mathrm{e}^{x} + 1} + \frac{8}{\mathrm{e}^{x} - 1}$$

```
[In] int(-1/(coth(x) - 1/sinh(x))^5,x)
[Out] x - 2*log(exp(x) - 1) - 16/(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1) + 16/(exp
(2*x) - 2*exp(x) + 1) + 8/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) +
1) + 8/(exp(x) - 1)
```

3.674 $\int (\mathbf{csch}(x) + \sinh(x))\, dx$

Optimal result

Integrand size = 5, antiderivative size = 8

$$\int (\operatorname{csch}(x) + \sinh(x))\, dx = -\operatorname{arctanh}(\cosh(x)) + \cosh(x)$$

```
[Out] -arctanh(cosh(x))+cosh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3855, 2718}

$$\int (\operatorname{csch}(x) + \sinh(x))\, dx = \cosh(x) - \operatorname{arctanh}(\cosh(x))$$

```
[In] Int[Csch[x] + Sinh[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \text{csch}(x)\, dx + \int \sinh(x)\, dx \\ &= -\text{arctanh}(\cosh(x)) + \cosh(x) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int (\text{csch}(x) + \sinh(x))\, dx = \cosh(x) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x] + Sinh[x],x]
[Out] Cosh[x] - Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \cosh(x)$	9
parts	$\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \cosh(x)$	9
parallelrisch	$\cosh(x) + \ln(\coth(x) - \text{csch}(x)) + 1$	13
risch	$\ln(e^x - 1) - \ln(e^x + 1) + \frac{e^x}{2} + \frac{e^{-x}}{2}$	24

```
[In] int(csch(x)+sinh(x),x,method=_RETURNVERBOSE)
[Out] ln(tanh(1/2*x))+cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 6.62

$$\int (\text{csch}(x) + \sinh(x))\, dx = \frac{\cosh(x)^2 - 2(\cosh(x) + \sinh(x))\log(\cosh(x) + \sinh(x) + 1) + 2(\cosh(x) + \sinh(x))\log(\cosh(x) + \text{si}}{2(\cosh(x) + \sinh(x))}$$

```
[In] integrate(csch(x)+sinh(x),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(cosh
(x) + sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx=\log\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(x)$$

```
[In] integrate(csch(x)+sinh(x),x)
[Out] log(tanh(x/2)) + cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx=\cosh(x)+\log\left(\tanh\left(\frac{1}{2}\,x\right)\right)$$

```
[In] integrate(csch(x)+sinh(x),x, algorithm="maxima")
[Out] cosh(x) + log(tanh(1/2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int (\operatorname{csch}(x)+\sinh(x))\,dx=\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-\log\left(e^{x}+1\right)+\log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)+sinh(x),x, algorithm="giac")
[Out] 1/2*e^(-x) + 1/2*e^x - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int (\operatorname{csch}(x) + \sinh(x))\,dx = \ln(2 - 2\,e^x) - \ln(-2\,e^x - 2) + \frac{e^{-x}}{2} + \frac{e^x}{2}$$

```
[In] int(sinh(x) + 1/sinh(x),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + exp(-x)/2 + exp(x)/2
```

3.675 $\int (\operatorname{csch}(x) + \sinh(x))^2 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 22

$$\int (\operatorname{csch}(x) + \sinh(x))^2 \, dx = \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{1}{2}\cosh^2(x)\coth(x)$$

```
[Out] 3/2*x-3/2*coth(x)+1/2*cosh(x)^2*coth(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {296, 331, 212}

$$\int (\operatorname{csch}(x) + \sinh(x))^2 \, dx = \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{1}{2}\cosh^2(x)\coth(x)$$

```
[In] Int[(Csch[x] + Sinh[x])^2,x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 + (Cosh[x]^2*Coth[x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2 (1-x^2)^2} \, dx, x, \tanh(x)\right) \\
&= \frac{1}{2}\cosh^2(x)\coth(x) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{x^2 (1-x^2)} \, dx, x, \tanh(x)\right) \\
&= -\frac{3\coth(x)}{2} + \frac{1}{2}\cosh^2(x)\coth(x) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \tanh(x)\right) \\
&= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{1}{2}\cosh^2(x)\coth(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (\operatorname{csch}(x) + \sinh(x))^2 \, dx = \frac{3x}{2} - \coth(x) + \frac{1}{4}\sinh(2x)$$

```
[In] Integrate[(Csch[x] + Sinh[x])^2,x]
[Out] (3*x)/2 - Coth[x] + Sinh[2*x]/4
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

method	result	size
default	$-\coth(x)+\frac{3x}{2}+\frac{\cosh(x)\sinh(x)}{2}$	15
parts	$-\coth(x)+\frac{3x}{2}+\frac{\cosh(x)\sinh(x)}{2}$	15
parallelrisch	$\frac{\coth(x)\cosh(2x)}{4}-\frac{5\coth(x)}{4}+\frac{3x}{2}$	17
risch	$\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}-\frac{2}{e^{2x}-1}$	27

```
[In] int((csch(x)+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -coth(x)+3/2*x+1/2*cosh(x)*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx=\frac{\cosh(x)^3+3\cosh(x)\sinh(x)^2+4(3x+2)\sinh(x)-9\cosh(x)}{8\sinh(x)}$$

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="fricas")
[Out] 1/8*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 4*(3*x + 2)*sinh(x) - 9*cosh(x))/sin
h(x)
```

Sympy [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx=\int (\sinh(x)+\operatorname{csch}(x))^2\,dx$$

```
[In] integrate((csch(x)+sinh(x))**2,x)
[Out] Integral((sinh(x) + csch(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (\operatorname{csch}(x)+\sinh(x))^2\,dx=\frac{3}{2}x+\frac{2}{e^{(-2x)}-1}+\frac{1}{8}e^{(2x)}-\frac{1}{8}e^{(-2x)}$$

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="maxima")
[Out] 3/2*x + 2/(e^(-2*x) - 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int (\operatorname{csch}(x) + \sinh(x))^2\, dx = \frac{3}{2}\, x - \frac{3\, e^{(4\, x)} + 14\, e^{(2\, x)} - 1}{8\, (e^{(4\, x)} - e^{(2\, x)})} + \frac{1}{8}\, e^{(2\, x)}$$

```
[In] integrate((csch(x)+sinh(x))^2,x, algorithm="giac")
[Out] 3/2*x - 1/8*(3*e^(4*x) + 14*e^(2*x) - 1)/(e^(4*x) - e^(2*x)) + 1/8*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (\operatorname{csch}(x) + \sinh(x))^2\, dx = \frac{3\, x}{2} - \frac{\mathrm{e}^{-2\, x}}{8} + \frac{\mathrm{e}^{2\, x}}{8} - \frac{2}{\mathrm{e}^{2\, x} - 1}$$

```
[In] int((sinh(x) + 1/sinh(x))^2,x)
[Out] (3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - 2/(exp(2*x) - 1)
```

3.676 $\int (\mathbf{csch}(x) + \sinh(x))^3 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 34

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = -\frac{5}{2}\operatorname{arctanh}(\cosh(x))+\frac{5\cosh(x)}{2}+\frac{5\cosh^3(x)}{6}-\frac{1}{2}\cosh^3(x)\coth^2(x)$$

```
[Out] -5/2*arctanh(cosh(x))+5/2*cosh(x)+5/6*cosh(x)^3-1/2*cosh(x)^3*coth(x)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4482, 2672, 294, 308, 212}

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = -\frac{5}{2}\operatorname{arctanh}(\cosh(x))+\frac{5\cosh^3(x)}{6}+\frac{5\cosh(x)}{2}-\frac{1}{2}\cosh^3(x)\coth^2(x)$$

```
[In] Int[(Csch[x] + Sinh[x])^3,x]
```

```
[Out] (-5*ArcTanh[Cosh[x]])/2 + (5*Cosh[x])/2 + (5*Cosh[x]^3)/6 - (Cosh[x]^3*Coth
[x]^2)/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \cosh^3(x)\coth^3(x)\,dx \\
&= \text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cosh(x)\right) \\
&= -\frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cosh(x)\right) \\
&= \frac{5\cosh(x)}{2} + \frac{5\cosh^3(x)}{6} - \frac{1}{2}\cosh^3(x)\coth^2(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(x)\right) \\
&= -\frac{5}{2}\text{arctanh}(\cosh(x)) + \frac{5\cosh(x)}{2} + \frac{5\cosh^3(x)}{6} - \frac{1}{2}\cosh^3(x)\coth^2(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = \frac{1}{48}\operatorname{csch}^2(x)\left(-50\cosh(x)+25\cosh(3x)+\cosh(5x)+60\log\left(\cosh\left(\frac{x}{2}\right)\right)-60\cosh(2x)\log\left(\cosh\left(\frac{x}{2}\right)\right)-60\log\left(\sinh\left(\frac{x}{2}\right)\right)+60\cosh(2x)\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

[In] Integrate[(Csch[x] + Sinh[x])^3,x]

[Out] (Csch[x]^2*(-50*Cosh[x] + 25*Cosh[3*x] + Cosh[5*x] + 60*Log[Cosh[x/2]] - 60*Cosh[2*x]*Log[Cosh[x/2]] - 60*Log[Sinh[x/2]] + 60*Cosh[2*x]*Log[Sinh[x/2]]))/48

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\operatorname{csch}(x)\coth(x)}{2}-5\,\operatorname{arctanh}(e^x)+3\cosh(x)+\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x)$	28
parts	$-\frac{\operatorname{csch}(x)\coth(x)}{2}-5\,\operatorname{arctanh}(e^x)+3\cosh(x)+\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x)$	28
parallelrisch	$\frac{5}{12}+\frac{5\ln(\coth(x)-\operatorname{csch}(x))}{2}+\frac{\cosh(3x)}{12}+\frac{9\cosh(x)}{4}-\frac{\operatorname{csch}(x)\coth(x)}{2}$	29
risch	$\frac{e^{3x}}{24}+\frac{9\,e^{x}}{8}+\frac{9\,e^{-x}}{8}+\frac{e^{-3x}}{24}-\frac{e^{x}(1+e^{2x})}{(e^{2x}-1)^2}-\frac{5\ln(e^x+1)}{2}+\frac{5\ln(e^x-1)}{2}$	56

[In] int((csch(x)+sinh(x))^3,x,method=_RETURNVERBOSE)

[Out] -1/2*csch(x)*coth(x)-5*arctanh(exp(x))+3*cosh(x)+(-2/3+1/3*sinh(x)^2)*cosh(x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 616, normalized size of antiderivative = 18.12

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx = \text{Too large to display}$$

[In] integrate((csch(x)+sinh(x))^3,x, algorithm="fricas")

[Out] 1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 5)*
sinh(x)^8 + 25*cosh(x)^8 + 40*(3*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 10*(21*
cosh(x)^4 + 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5 +
350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 175*cosh(x)^4 -
75*cosh(x)^2 - 5)*sinh(x)^4 - 50*cosh(x)^4 + 40*(3*cosh(x)^7 + 35*cosh(x)^5
 - 25*cosh(x)^3 - 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 140*cosh(x)^6 - 1
50*cosh(x)^4 - 60*cosh(x)^2 + 5)*sinh(x)^2 + 25*cosh(x)^2 - 60*(cosh(x)^7 +
 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)
^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 +
 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(
x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*log(cosh(x) + si
nh(x) + 1) + 60*(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(x)^
2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*cosh(x)^3 - 2*cosh(x))*sinh(x)^4 + (3
5*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)^5 - 20*
cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^
2)*sinh(x))*log(cosh(x) + sinh(x) - 1) + 10*(cosh(x)^9 + 20*cosh(x)^7 - 30*
cosh(x)^5 - 20*cosh(x)^3 + 5*cosh(x))*sinh(x) + 1)/(cosh(x)^7 + 7*cosh(x)*s
inh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 - 2)*sinh(x)^5 - 2*cosh(x)^5 + 5*(7*co
sh(x)^3 - 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^
3 + cosh(x)^3 + (21*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*co
sh(x)^6 - 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))

Sympy [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx=\int (\sinh(x)+\operatorname{csch}(x))^3\,dx$$

[In] integrate((csch(x)+sinh(x))**3,x)

[Out] Integral((sinh(x) + csch(x))**3, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(26) = 52.

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (\operatorname{csch}(x)+\sinh(x))^3\,dx=\frac{e^{(-x)}+e^{(-3x)}}{2e^{(-2x)}-e^{(-4x)}-1}+\frac{1}{24}e^{(3x)}+\frac{9}{8}e^{(-x)}+\frac{1}{24}e^{(-3x)}+\frac{9}{8}e^{x}-\frac{5}{2}\log\left(e^{(-x)}+1\right)+\frac{5}{2}\log\left(e^{(-x)}-1\right)$$

[In] integrate((csch(x)+sinh(x))^3,x, algorithm="maxima")

[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/24*e^(3*x) + 9/8*e^(-x)
 + 1/24*e^(-3*x) + 9/8*e^x - 5/2*log(e^(-x) + 1) + 5/2*log(e^(-x) - 1)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

$$\int (\operatorname{csch}(x) + \sinh(x))^3 \, dx = \frac{1}{24} \left(e^{(-x)} + e^x\right)^3 - \frac{e^{(-x)} + e^x}{\left(e^{(-x)} + e^x\right)^2 - 4} + e^{(-x)} + e^x - \frac{5}{4} \log\left(e^{(-x)} + e^x + 2\right) + \frac{5}{4} \log\left(e^{(-x)} + e^x - 2\right)$$

```
[In] integrate((csch(x)+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*(e^(-x) + e^x)^3 - (e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + e^(-x) + e^
x - 5/4*log(e^(-x) + e^x + 2) + 5/4*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int (\operatorname{csch}(x) + \sinh(x))^3 \, dx = \frac{5 \ln(5 - 5 e^x)}{2} - \frac{5 \ln(-5 e^x - 5)}{2} + \frac{9 e^{-x}}{8} + \frac{e^{-3x}}{24} + \frac{e^{3x}}{24} + \frac{9 e^x}{8} - \frac{e^x}{e^{2x} - 1} - \frac{2 e^x}{e^{4x} - 2 e^{2x} + 1}$$

```
[In] int((sinh(x) + 1/sinh(x))^3,x)
```

```
[Out] (5*log(5 - 5*exp(x)))/2 - (5*log(- 5*exp(x) - 5))/2 + (9*exp(-x))/8 + exp(-
3*x)/24 + exp(3*x)/24 + (9*exp(x))/8 - exp(x)/(exp(2*x) - 1) - (2*exp(x))/(
exp(4*x) - 2*exp(2*x) + 1)
```

3.677 $\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = 2\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

```
[Out] 2*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4482, 4483, 4485, 2669}

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = 2\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[In] Int[Sqrt[Csch[x] + Sinh[x]],x]
[Out] 2*Sqrt[Cosh[x]*Coth[x]]*Tanh[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{\cosh(x)\coth(x)}\,dx \\
&= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{-i\cosh(x)\coth(x)}\,dx}{\sqrt{-i\cosh(x)\coth(x)}} \\
&= \frac{\sqrt{\cosh(x)\coth(x)}\int \sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= 2\sqrt{\cosh(x)\coth(x)}\tanh(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = \frac{2\sqrt{\cosh(x)\coth(x)}\left(-1+\sqrt[4]{-\sinh^2(x)}\right)\tanh(x)}{\sqrt[4]{-\sinh^2(x)}}$$

```
[In] Integrate[Sqrt[Csch[x] + Sinh[x]],x]
[Out] (2*Sqrt[Cosh[x]*Coth[x]]*(-1 + (-Sinh[x]^2)^(1/4))*Tanh[x])/(-Sinh[x]^2)^(1
/4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{\frac{\left(1+{\mathrm e}^{2 x}\right)^{2} {\mathrm e}^{-x}}{{\mathrm e}^{2 x}-1}}\,\left({\mathrm e}^{2 x}-1\right)}{1+{\mathrm e}^{2 x}}$	42

```
[In] int((csch(x)+sinh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*((1+exp(2*x))^2*exp(-x)/(exp(2*x)-1))^(1/2)/(1+exp(2*x))*(exp(2*x)-
1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.23

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)}{\sqrt{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)-\cosh\left(x\right)}}$$

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/sqrt(cosh(x)^3
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))
```

Sympy [F]

$$\int \sqrt{\operatorname{csch}(x)+\sinh(x)}\,dx = \int \sqrt{\sinh\left(x\right)+\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate((csch(x)+sinh(x))**(1/2),x)
[Out] Integral(sqrt(sinh(x) + csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.15

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx = \frac{\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{e^{(-x)} + 1}\sqrt{-e^{(-x)} + 1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{\sqrt{e^{(-x)} + 1}\sqrt{-e^{(-x)} + 1}}$$

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="maxima")
[Out] sqrt(2)*e^(1/2*x)/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1)) - sqrt(2)*e^(-3/2*x)
/(sqrt(e^(-x) + 1)*sqrt(-e^(-x) + 1))
```

Giac [F]

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx = \int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx$$

```
[In] integrate((csch(x)+sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csch(x) + sinh(x)), x)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{csch}(x) + \sinh(x)}\, dx = 2\tanh(x)\,\sqrt{\sinh(x) + \frac{1}{\sinh(x)}}$$

```
[In] int((sinh(x) + 1/sinh(x))^(1/2),x)
[Out] 2*tanh(x)*(sinh(x) + 1/sinh(x))^(1/2)
```

3.678 $\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

```
[Out] 2/3*cosh(x)*(cosh(x)*coth(x))^(1/2)-8/3*sech(x)*(cosh(x)*coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4482, 4483, 4485, 2678, 2669}

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)}-\frac{8}{3}\operatorname{sech}(x)\sqrt{\cosh(x)\coth(x)}$$

```
[In] Int[(Csch[x] + Sinh[x])^(3/2),x]
[Out] (2*Cosh[x]*Sqrt[Cosh[x]*Coth[x]])/3 - (8*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
```

```
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (\cosh(x)\coth(x))^{3/2}\,dx \\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int(-i\cosh(x)\coth(x))^{3/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}} \\
&= \frac{\left(i\sqrt{\cosh(x)\coth(x)}\right)\int\cosh^{\frac{3}{2}}(x)(-i\coth(x))^{3/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(4i\sqrt{\cosh(x)\coth(x)}\right)\int\frac{(-i\coth(x))^{3/2}}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{\cosh(x)}\sqrt{-i\coth(x)}} \\
&= \frac{2}{3}\cosh(x)\sqrt{\cosh(x)\coth(x)} - \frac{8}{3}\sqrt{\cosh(x)\coth(x)}\text{sech}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\frac{2}{3}\left(-4+\cosh^2(x)\right)\sqrt{\cosh(x)\coth(x)}\operatorname{sech}(x)$$

```
[In] Integrate[(Csch[x] + Sinh[x])^(3/2),x]
[Out] (2*(-4 + Cosh[x]^2)*Sqrt[Cosh[x]*Coth[x]]*Sech[x])/3
```

Maple [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^{\frac{3}{2}}\,dx$$

```
[In] int((csch(x)+sinh(x))^(3/2),x)
[Out] int((csch(x)+sinh(x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2}\,dx=\frac{\sqrt{\frac{1}{2}}\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-7\right)\sinh(x)^2-14\cosh\right.}{3\sqrt{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(3\cosh(x)^2-1\right)\sinh(x)}}$$

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)/(sq
rt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x)
- cosh(x))*(cosh(x) + sinh(x)))
```

Sympy [F(-1)]

Timed out.

$$\int (\operatorname{csch}(x) + \sinh(x))^{3/2} \, dx = \text{Timed out}$$

```
[In] integrate((csch(x)+sinh(x))**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int (\operatorname{csch}(x)+\sinh(x))^{3/2} \, dx = \frac{\sqrt{2}e^{\left(\frac{3}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}} - \frac{5\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}} + \frac{5\sqrt{2}e^{\left(-\frac{5}{2}x\right)}}{2\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}x\right)}}{6\left(e^{(-x)}+1\right)^{\frac{3}{2}}\left(-e^{(-x)}+1\right)^{\frac{3}{2}}}$$

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*e^(3/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 5/2*sqrt(2
)*e^(-1/2*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) + 5/2*sqrt(2)*e^(-5/2
*x)/((e^(-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2)) - 1/6*sqrt(2)*e^(-9/2*x)/((e^(
-x) + 1)^(3/2)*(-e^(-x) + 1)^(3/2))
```

Giac [F]

$$\int (\operatorname{csch}(x) + \sinh(x))^{3/2} \, dx = \int (\operatorname{csch}(x) + \sinh(x))^{\frac{3}{2}} \, dx$$

```
[In] integrate((csch(x)+sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((csch(x) + sinh(x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\mathrm{csch}(x) + \sinh(x))^{3/2}\,dx = \int \left(\sinh(x) + \frac{1}{\sinh(x)}\right)^{3/2}dx$$

```
[In] int((sinh(x) + 1/sinh(x))^(3/2),x)
[Out] int((sinh(x) + 1/sinh(x))^(3/2), x)
```

3.679 $\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx=-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

```
[Out] -16/15*coth(x)*(cosh(x)*coth(x))^(1/2)+2/5*cosh(x)^2*coth(x)*(cosh(x)*coth(
x))^(1/2)+64/15*(cosh(x)*coth(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4482, 4483, 4485, 2678, 2674, 2669}

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx=\frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)}-\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)}+\frac{64}{15}\tanh(x)\sqrt{\cosh(x)\coth(x)}$$

```
[In] Int[(Csch[x] + Sinh[x])^(5/2),x]
```

```
[Out] (-16*Coth[x]*Sqrt[Cosh[x]*Coth[x]])/15 + (2*Cosh[x]^2*Coth[x]*Sqrt[Cosh[x]*
Coth[x]])/5 + (64*Sqrt[Cosh[x]*Coth[x]]*Tanh[x])/15
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule steps

$$\text{integral} = \int (\cosh(x)\coth(x))^{5/2}\,dx$$

$$= -\frac{\sqrt{\cosh(x)\coth(x)}\int(-i\cosh(x)\coth(x))^{5/2}\,dx}{\sqrt{-i\cosh(x)\coth(x)}}$$

$$= -\frac{\sqrt{\cosh(x)\coth(x)}\int\cosh^{\frac{5}{2}}(x)(-i\coth(x))^{5/2}\,dx}{\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$= \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} - \frac{\left(8\sqrt{\cosh(x)\coth(x)}\right)\int\sqrt{\cosh(x)}(-i\coth(x))^{5/2}\,dx}{5\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{\left(32\sqrt{\cosh(x)\coth(x)}\right)\int\sqrt{\cosh(x)}\sqrt{-i\coth(x)}\,dx}{15\sqrt{\cosh(x)}\sqrt{-i\coth(x)}}$$

$$= -\frac{16}{15}\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{2}{5}\cosh^2(x)\coth(x)\sqrt{\cosh(x)\coth(x)} + \frac{64}{15}\sqrt{\cosh(x)\coth(x)}\tanh(x)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (\operatorname{csch}(x)+\sinh(x))^{5/2}\,dx = \frac{1}{15}\sqrt{\cosh(x)\coth(x)}\Big(-10\coth(x) + 6\cosh(x)\sinh(x) + 57\operatorname{csch}(x)\operatorname{sech}(x)\left(-\sinh^2(x)\right)^{3/4} + 64\tanh(x)\Big)$$

```
[In] Integrate[(Csch[x] + Sinh[x])^(5/2),x]
```

```
[Out] (Sqrt[Cosh[x]*Coth[x]]*(-10*Coth[x] + 6*Cosh[x]*Sinh[x] + 57*Csch[x]*Sech[x
]*(-Sinh[x]^2)^(3/4) + 64*Tanh[x]))/15
```

Maple [F]

$$\int (\operatorname{csch}(x)+\sinh(x))^{\frac{5}{2}}\,dx$$

```
[In] int((csch(x)+sinh(x))^(5/2),x)
```

```
[Out] int((csch(x)+sinh(x))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(38) = 76.

Time = 0.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 5.18

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2+9\right)\sinh(x)^6 + 108\right.}{}$$

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 + 9)*sinh(x)^6 + 108*cosh(x)^6 + 24*(7*cosh(x)^3 + 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 + 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 + 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 + 135*cosh(x)^4 - 151*cosh(x)^2 + 9)*sinh(x)^2 + 108*cosh(x)^2 + 8*(3*cosh
(x)^7 + 81*cosh(x)^5 - 151*cosh(x)^3 + 27*cosh(x))*sinh(x) + 3)/((cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^
2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*sqrt(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x)))
```

Sympy [F(-1)]

Timed out.

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2}\, dx = \text{Timed out}$$

```
[In] integrate((csch(x)+sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.26

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2}\, dx = \frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,(e^{(-x)}+1)^{\frac{5}{2}}(-e^{(-x)}+1)^{\frac{5}{2}}}$$

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*e^(5/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 7/4*sqrt(
2)*e^(1/2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 41/6*sqrt(2)*e^(-3/
2*x)/((e^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) + 41/6*sqrt(2)*e^(-7/2*x)/((e
^(-x) + 1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 7/4*sqrt(2)*e^(-11/2*x)/((e^(-x) +
1)^(5/2)*(-e^(-x) + 1)^(5/2)) - 1/20*sqrt(2)*e^(-15/2*x)/((e^(-x) + 1)^(5/2
)*(-e^(-x) + 1)^(5/2))
```

Giac [F]

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2}\,dx = \int (\operatorname{csch}(x) + \sinh(x))^{\frac{5}{2}}\,dx$$

```
[In] integrate((csch(x)+sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((csch(x) + sinh(x))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (\operatorname{csch}(x) + \sinh(x))^{5/2}\,dx = \int \left(\sinh(x) + \frac{1}{\sinh(x)}\right)^{5/2}\,dx$$

```
[In] int((sinh(x) + 1/sinh(x))^(5/2),x)
[Out] int((sinh(x) + 1/sinh(x))^(5/2), x)
```

3.680 $\int (-\cosh(x) + \text{sech}(x)) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int (-\cosh(x) + \text{sech}(x)) \, dx = \arctan(\sinh(x)) - \sinh(x)$$

```
[Out] arctan(sinh(x))-sinh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2717, 3855}

$$\int (-\cosh(x) + \text{sech}(x)) \, dx = \arctan(\sinh(x)) - \sinh(x)$$

```
[In] Int[-Cosh[x] + Sech[x],x]
```

```
[Out] ArcTan[Sinh[x]] - Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\int \cosh(x)\,dx + \int \text{sech}(x)\,dx$$
$$= \arctan(\sinh(x)) - \sinh(x)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (-\cosh(x) + \text{sech}(x))\,dx = \arctan(\sinh(x)) - \sinh(x)$$

```
[In] Integrate[-Cosh[x] + Sech[x],x]
[Out] ArcTan[Sinh[x]] - Sinh[x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\arctan(\sinh(x)) - \sinh(x)$	9
parts	$\arctan(\sinh(x)) - \sinh(x)$	9
risch	$-\frac{e^{x}}{2} + \frac{e^{-x}}{2} + i\ln(e^{x} + i) - i\ln(e^{x} - i)$	30
parallelrisch	$-i\ln(-i + \coth(x) - \text{csch}(x)) + i\ln(i + \coth(x) - \text{csch}(x)) - \sinh(x)$	32

```
[In] int(-cosh(x)+sech(x),x,method=_RETURNVERBOSE)
[Out] arctan(sinh(x))-sinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(8) = 16.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 5.25

$$\int (-\cosh(x) + \text{sech}(x))\,dx = \frac{4(\cosh(x) + \sinh(x))\arctan(\cosh(x) + \sinh(x)) - \cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + 1}{2(\cosh(x) + \sinh(x))}$$

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="fricas")
[Out] 1/2*(4*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) - cosh(x)^2 - 2*cosh(x
)*sinh(x) - sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int (-\cosh(x) + \mathrm{sech}(x))\, dx = -\sinh{(x)} + 2\,\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] integrate(-cosh(x)+sech(x),x)
[Out] -sinh(x) + 2*atan(tanh(x/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (-\cosh(x) + \mathrm{sech}(x))\, dx = \arctan\left(\sinh\left(x\right)\right) - \sinh\left(x\right)$$

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="maxima")
[Out] arctan(sinh(x)) - sinh(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int (-\cosh(x) + \mathrm{sech}(x))\, dx = 2\, \arctan\left(e^{x}\right) + \frac{1}{2}\, e^{(-x)} - \frac{1}{2}\, e^{x}$$

```
[In] integrate(-cosh(x)+sech(x),x, algorithm="giac")
[Out] 2*arctan(e^x) + 1/2*e^(-x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int (-\cosh(x) + \mathrm{sech}(x))\, dx = \frac{\mathrm{e}^{-x}}{2} + 2\,\mathrm{atan}(\mathrm{e}^{x}) - \frac{\mathrm{e}^{x}}{2}$$

```
[In] int(1/cosh(x) - cosh(x),x)
[Out] exp(-x)/2 + 2*atan(exp(x)) - exp(x)/2
```

3.681 $\int (-\cosh(x) + \mathbf{sech}(x))^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx=-\frac{3x}{2}+\frac{3\tanh(x)}{2}+\frac{1}{2}\sinh^2(x)\tanh(x)$$

```
[Out] -3/2*x+3/2*tanh(x)+1/2*sinh(x)^2*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {294, 327, 212}

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx=-\frac{3x}{2}+\frac{3\tanh(x)}{2}+\frac{1}{2}\sinh^2(x)\tanh(x)$$

```
[In] Int[(-Cosh[x] + Sech[x])^2,x]
```

```
[Out] (-3*x)/2 + (3*Tanh[x])/2 + (Sinh[x]^2*Tanh[x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\sinh^2(x)\tanh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{3x}{2} + \frac{3\tanh(x)}{2} + \frac{1}{2}\sinh^2(x)\tanh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int (-\cosh(x)+\operatorname{sech}(x))^2\,dx = -\frac{3x}{2} + \frac{1}{4}\sinh(2x) + \tanh(x)$$

```
[In] Integrate[(-Cosh[x] + Sech[x])^2,x]
[Out] (-3*x)/2 + Sinh[2*x]/4 + Tanh[x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\cosh(x)\sinh(x)}{2}-\frac{3x}{2}+\tanh(x)$	13
parallelrisch	$-\frac{3x}{2}+\frac{\sinh(2x)}{4}+\tanh(x)$	13
parts	$\frac{\cosh(x)\sinh(x)}{2}-\frac{3x}{2}+\tanh(x)$	13
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{e^{-2x}}{8}-\frac{2}{1+e^{2x}}$	27

```
[In] int((-cosh(x)+sech(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*cosh(x)*sinh(x)-3/2*x+tanh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int(-\cosh(x)+\operatorname{sech}(x))^2\,dx=\frac{\sinh(x)^3-4(3x+2)\cosh(x)+3\left(\cosh(x)^2+3\right)\sinh(x)}{8\cosh(x)}$$

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="fricas")
[Out] 1/8*(sinh(x)^3 - 4*(3*x + 2)*cosh(x) + 3*(cosh(x)^2 + 3)*sinh(x))/cosh(x)
```

Sympy [F]

$$\int(-\cosh(x)+\operatorname{sech}(x))^2\,dx=\int(-\cosh(x)+\operatorname{sech}(x))^2\,dx$$

```
[In] integrate((-cosh(x)+sech(x))**2,x)
[Out] Integral((-cosh(x) + sech(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int(-\cosh(x)+\operatorname{sech}(x))^2\,dx=-\frac{3}{2}\,x+\frac{2}{e^{(-2\,x)}+1}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="maxima")
[Out] -3/2*x + 2/(e^(-2*x) + 1) + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (-\cosh(x) + \operatorname{sech}(x))^2\, dx = -\frac{3}{2}\, x + \frac{3\, e^{(4\,x)} - 14\, e^{(2\,x)} - 1}{8\,(e^{(4\,x)} + e^{(2\,x)})} + \frac{1}{8}\, e^{(2\,x)}$$

```
[In] integrate((-cosh(x)+sech(x))^2,x, algorithm="giac")
[Out] -3/2*x + 1/8*(3*e^(4*x) - 14*e^(2*x) - 1)/(e^(4*x) + e^(2*x)) + 1/8*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (-\cosh(x) + \operatorname{sech}(x))^2\, dx = \frac{e^{2\,x}}{8} - \frac{e^{-2\,x}}{8} - \frac{3\,x}{2} - \frac{2}{e^{2\,x} + 1}$$

```
[In] int((cosh(x) - 1/cosh(x))^2,x)
[Out] exp(2*x)/8 - exp(-2*x)/8 - (3*x)/2 - 2/(exp(2*x) + 1)
```

3.682 $\int (-\cosh(x) + \textbf{sech}(x))^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx = -\frac{5}{2} \arctan(\sinh(x)) + \frac{5\sinh(x)}{2} - \frac{5\sinh^3(x)}{6} + \frac{1}{2}\sinh^3(x)\tanh^2(x)$$

```
[Out] -5/2*arctan(sinh(x))+5/2*sinh(x)-5/6*sinh(x)^3+1/2*sinh(x)^3*tanh(x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4482, 2672, 294, 308, 209}

$$\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx = -\frac{5}{2} \arctan(\sinh(x)) - \frac{5\sinh^3(x)}{6} + \frac{5\sinh(x)}{2} + \frac{1}{2}\sinh^3(x)\tanh^2(x)$$

```
[In] Int[(-Cosh[x] + Sech[x])^3,x]
```

```
[Out] (-5*ArcTan[Sinh[x]])/2 + (5*Sinh[x])/2 - (5*Sinh[x]^3)/6 + (Sinh[x]^3*Tanh[
x]^2)/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \sinh^3(x)\tanh^3(x)\,dx\\
&= -\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \sinh(x)\right)\\
&= \frac{5\sinh(x)}{2} - \frac{5\sinh^3(x)}{6} + \frac{1}{2}\sinh^3(x)\tanh^2(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{5}{2}\arctan(\sinh(x)) + \frac{5\sinh(x)}{2} - \frac{5\sinh^3(x)}{6} + \frac{1}{2}\sinh^3(x)\tanh^2(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx = -\frac{1}{48}\operatorname{sech}^2(x)(60\arctan(\sinh(x)) + 60\arctan(\sinh(x))\cosh(2x) - 50\sinh(x) - 25\sinh(3x) + \sinh(5x))$$

[In] Integrate[(-Cosh[x] + Sech[x])^3,x]

[Out] -1/48*(Sech[x]^2*(60*ArcTan[Sinh[x]] + 60*ArcTan[Sinh[x]]*Cosh[2*x] - 50*Sinh[x] - 25*Sinh[3*x] + Sinh[5*x]))

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
default	$-\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+3\sinh(x)-5\arctan(e^x)+\frac{\operatorname{sech}(x)\tanh(x)}{2}$	29
parts	$-\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)+3\sinh(x)-5\arctan(e^x)+\frac{\operatorname{sech}(x)\tanh(x)}{2}$	29
parallelrisch	$\frac{\operatorname{sech}(x)\tanh(x)}{2}+\frac{5i\ln(-i+\coth(x)-\operatorname{csch}(x))}{2}-\frac{5i\ln(i+\coth(x)-\operatorname{csch}(x))}{2}+\frac{9\sinh(x)}{4}-\frac{\sinh(3x)}{12}$	44
risch	$-\frac{e^{3x}}{24}+\frac{9e^x}{8}-\frac{9e^{-x}}{8}+\frac{e^{-3x}}{24}+\frac{(e^{2x}-1)e^x}{(1+e^{2x})^2}+\frac{5i\ln(e^x-i)}{2}-\frac{5i\ln(e^x+i)}{2}$	59

[In] int((-cosh(x)+sech(x))^3,x,method=_RETURNVERBOSE)

[Out] -(2/3+1/3*cosh(x)^2)*sinh(x)+3*sinh(x)-5*arctan(exp(x))+1/2*sech(x)*tanh(x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 486, normalized size of antiderivative = 14.29

$$\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx = -\frac{\cosh(x)^{10}+10\cosh(x)\sinh(x)^9+\sinh(x)^{10}+5\left(9\cosh(x)^2-5\right)\sinh(x)^8-25\cosh(x)^8+40\left(3\cosh\right.}{}$$

[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="fricas")

[Out] -1/24*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 - 5)*sinh(x)^8 - 25*cosh(x)^8 + 40*(3*cosh(x)^3 - 5*cosh(x))*sinh(x)^7 + 10*(21

```
*cosh(x)^4 - 70*cosh(x)^2 - 5)*sinh(x)^6 - 50*cosh(x)^6 + 4*(63*cosh(x)^5 -
 350*cosh(x)^3 - 75*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 - 175*cosh(x)^4 -
 75*cosh(x)^2 + 5)*sinh(x)^4 + 50*cosh(x)^4 + 40*(3*cosh(x)^7 - 35*cosh(x)^
5 - 25*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 - 140*cosh(x)^6 -
150*cosh(x)^4 + 60*cosh(x)^2 + 5)*sinh(x)^2 + 120*(cosh(x)^7 + 7*cosh(x)*si
nh(x)^6 + sinh(x)^7 + (21*cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cos
h(x)^3 + 2*cosh(x))*sinh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3
 + cosh(x)^3 + (21*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cos
h(x)^6 + 10*cosh(x)^4 + 3*cosh(x)^2)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2
5*cosh(x)^2 + 10*(cosh(x)^9 - 20*cosh(x)^7 - 30*cosh(x)^5 + 20*cosh(x)^3 +
5*cosh(x))*sinh(x) - 1)/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*
cosh(x)^2 + 2)*sinh(x)^5 + 2*cosh(x)^5 + 5*(7*cosh(x)^3 + 2*cosh(x))*sinh(x
)^4 + (35*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^3 + cosh(x)^3 + (21*cosh(x)
^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 + 10*cosh(x)^4 + 3*
cosh(x)^2)*sinh(x))
```

Sympy [F]

$$\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx = -\int 3\cosh(x)\operatorname{sech}^2(x)\,dx - \int \left(-3\cosh^2(x)\operatorname{sech}(x)\right)\,dx - \int \cosh^3(x)\,dx - \int \left(-\operatorname{sech}^3(x)\right)\,dx$$

```
[In] integrate((-cosh(x)+sech(x))**3,x)
```

```
[Out] -Integral(3*cosh(x)*sech(x)**2, x) - Integral(-3*cosh(x)**2*sech(x), x) - I
ntegral(cosh(x)**3, x) - Integral(-sech(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int (-\cosh(x)+\operatorname{sech}(x))^3\,dx = \frac{e^{(-x)} - e^{(-3x)}}{2e^{(-2x)} + e^{(-4x)} + 1} + 5\arctan\left(e^{(-x)}\right) - \frac{1}{24}e^{(3x)} - \frac{9}{8}e^{(-x)} + \frac{1}{24}e^{(-3x)} + \frac{9}{8}e^{x}$$

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) - e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + 5*arctan(e^(-x)) - 1/24*e
^(3*x) - 9/8*e^(-x) + 1/24*e^(-3*x) + 9/8*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx = -\frac{5}{4}\pi + \frac{1}{24}\left(e^{(-x)} - e^x\right)^3 - \frac{e^{(-x)} - e^x}{\left(e^{(-x)} - e^x\right)^2 + 4} - \frac{5}{2}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right) - e^{(-x)} + e^x$$

```
[In] integrate((-cosh(x)+sech(x))^3,x, algorithm="giac")
```

```
[Out] -5/4*pi + 1/24*(e^(-x) - e^x)^3 - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) - 5
/2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - e^(-x) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int (-\cosh(x) + \operatorname{sech}(x))^3 \, dx = \frac{e^{-3x}}{24} - \frac{9\,e^{-x}}{8} - \frac{e^{3x}}{24} - 5\,\operatorname{atan}(e^x) + \frac{9\,e^x}{8} + \frac{e^x}{e^{2x} + 1} - \frac{2\,e^x}{2\,e^{2x} + e^{4x} + 1}$$

```
[In] int(-(cosh(x) - 1/cosh(x))^3,x)
```

```
[Out] exp(-3*x)/24 - (9*exp(-x))/8 - exp(3*x)/24 - 5*atan(exp(x)) + (9*exp(x))/8
+ exp(x)/(exp(2*x) + 1) - (2*exp(x))/(2*exp(2*x) + exp(4*x) + 1)
```

3.683 $\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx=2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] 2*coth(x)*(-sinh(x)*tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4482, 4485, 2669}

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx=2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[In] Int[Sqrt[-Cosh[x] + Sech[x]],x]
[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{-\sinh(x)\tanh(x)}\,dx\\ &= \frac{\sqrt{-\sinh(x)\tanh(x)}\int \sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\ &= 2\coth(x)\sqrt{-\sinh(x)\tanh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx = 2\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[In] Integrate[Sqrt[-Cosh[x] + Sech[x]],x]
[Out] 2*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.60 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

method	result	size
risch	$\frac{\sqrt{2}\sqrt{-\frac{\left(e^{2x}-1\right)^{2} e^{-x}}{1+e^{2x}}}\,\left(1+e^{2x}\right)}{e^{2x}-1}$	43

```
[In] int((-cosh(x)+sech(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*(-(exp(2*x)-1)^2*exp(-x)/(1+exp(2*x)))^(1/2)/(exp(2*x)-1)*(1+exp(2*
x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \sqrt{-\cosh(x) + \operatorname{sech}(x)}\, dx$$
$$= 2\sqrt{\frac{1}{2}}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\sqrt{-\frac{1}{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + (}}$$

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-1/(cosh(x
)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x)
))
```

Sympy [F]

$$\int \sqrt{-\cosh(x) + \operatorname{sech}(x)}\, dx = \int \sqrt{-\cosh(x) + \operatorname{sech}(x)}\, dx$$

```
[In] integrate((-cosh(x)+sech(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-cosh(x) + sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \sqrt{-\cosh(x) + \operatorname{sech}(x)}\, dx = -\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{-e^{(-2x)} - 1}} - \frac{\sqrt{2}e^{\left(-\frac{3}{2}x\right)}}{\sqrt{-e^{(-2x)} - 1}}$$

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*e^(1/2*x)/sqrt(-e^(-2*x) - 1) - sqrt(2)*e^(-3/2*x)/sqrt(-e^(-2*x)
- 1)
```

Giac [F]

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx=\int \sqrt{-\cosh\left(x\right)+\operatorname{sech}\left(x\right)}\,dx$$

```
[In] integrate((-cosh(x)+sech(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-cosh(x) + sech(x)), x)
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \sqrt{-\cosh(x)+\operatorname{sech}(x)}\,dx=2\coth(x)\,\sqrt{\frac{1}{\cosh\left(x\right)}-\cosh\left(x\right)}$$

```
[In] int((1/cosh(x) - cosh(x))^(1/2),x)
[Out] 2*coth(x)*(1/cosh(x) - cosh(x))^(1/2)
```

3.684 $\int (-\cosh(x) + \text{sech}(x))^{3/2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (-\cosh(x) + \text{sech}(x))^{3/2} \, dx = -\frac{8}{3}\text{csch}(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] -8/3*csch(x)*(-sinh(x)*tanh(x))^(1/2)-2/3*sinh(x)*(-sinh(x)*tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4482, 4485, 2678, 2669}

$$\int (-\cosh(x) + \text{sech}(x))^{3/2} \, dx = -\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{8}{3}\text{csch}(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[In] Int[(-Cosh[x] + Sech[x])^(3/2),x]
```

```
[Out] (-8*Csch[x]*Sqrt[-(Sinh[x]*Tanh[x])])/3 - (2*Sinh[x]*Sqrt[-(Sinh[x]*Tanh[x]
)])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-\sinh(x)\tanh(x))^{3/2}\,dx \\
&= \frac{\sqrt{-\sinh(x)\tanh(x)}\int (i\sinh(x))^{3/2}(i\tanh(x))^{3/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}} \\
&= -\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{\left(4\sqrt{-\sinh(x)\tanh(x)}\right)\int \frac{(i\tanh(x))^{3/2}}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}} \\
&= -\frac{8}{3}\text{csch}(x)\sqrt{-\sinh(x)\tanh(x)}-\frac{2}{3}\sinh(x)\sqrt{-\sinh(x)\tanh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int (-\cosh(x)+\text{sech}(x))^{3/2}\,dx=\frac{2}{3}\coth(x)\left(1+4\text{csch}^2(x)\right)(-\sinh(x)\tanh(x))^{3/2}$$

```
[In] Integrate[(-Cosh[x] + Sech[x])^(3/2),x]
[Out] (2*Coth[x]*(1 + 4*Csch[x]^2)*(-(Sinh[x]*Tanh[x]))^(3/2))/3
```

Maple [F]

$$\int (-\cosh{(x)} + \operatorname{sech}{(x)})^{\frac{3}{2}}\, dx$$

```
[In] int((-cosh(x)+sech(x))^(3/2),x)
```

```
[Out] int((-cosh(x)+sech(x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int (-\cosh(x) + \operatorname{sech}(x))^{3/2}\, dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(\cosh{(x)}^4 + 4\cosh{(x)}\sinh{(x)}^3 + \sinh{(x)}^4 + 2\left(3\cosh{(x)}^2 + 7\right)\sinh{(x)}^2 + 14\cosh{(x)}^2 + 4\left(\cosh{(}\right.}{3\left(\cosh{(x)} + \sinh{(}\right.}$$

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 7)*sinh(x)^2 + 14*cosh(x)^2 + 4*(cosh(x)^3 + 7*cosh(x))*sinh(x) + 1)*sq
rt(-1/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh
(x) + cosh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int (-\cosh(x) + \operatorname{sech}(x))^{3/2}\, dx = \int \left(-\cosh{(x)} + \operatorname{sech}{(x)}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((-cosh(x)+sech(x))**(3/2),x)
```

```
[Out] Integral((-cosh(x) + sech(x))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int (-\cosh(x) + \operatorname{sech}(x))^{3/2}\, dx = -\frac{\sqrt{2}e^{\left(\frac{3}{2}\,x\right)}}{6\left(-e^{(-2\,x)} - 1\right)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{1}{2}\,x\right)}}{2\left(-e^{(-2\,x)} - 1\right)^{\frac{3}{2}}} - \frac{5\,\sqrt{2}e^{\left(-\frac{5}{2}\,x\right)}}{2\left(-e^{(-2\,x)} - 1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{9}{2}\,x\right)}}{6\left(-e^{(-2\,x)} - 1\right)^{\frac{3}{2}}}$$

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="maxima")
[Out] -1/6*sqrt(2)*e^(3/2*x)/(-e^(-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-1/2*x)/(-e^(
-2*x) - 1)^(3/2) - 5/2*sqrt(2)*e^(-5/2*x)/(-e^(-2*x) - 1)^(3/2) - 1/6*sqrt(
2)*e^(-9/2*x)/(-e^(-2*x) - 1)^(3/2)
```

Giac [F]

$$\int (-\cosh(x) + \operatorname{sech}(x))^{3/2}\, dx = \int (-\cosh(x) + \operatorname{sech}(x))^{\frac{3}{2}}\, dx$$

```
[In] integrate((-cosh(x)+sech(x))^(3/2),x, algorithm="giac")
[Out] integrate((-cosh(x) + sech(x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\cosh(x) + \operatorname{sech}(x))^{3/2}\, dx = \int \left(\frac{1}{\cosh(x)} - \cosh(x)\right)^{3/2} dx$$

```
[In] int((1/cosh(x) - cosh(x))^(3/2),x)
[Out] int((1/cosh(x) - cosh(x))^(3/2), x)
```

3.685 $\int (-\cosh(x)+\mathbf{sech}(x))^{5/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int (-\cosh(x)+\operatorname{sech}(x))^{5/2}\,dx = -\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[Out] -64/15*coth(x)*(-sinh(x)*tanh(x))^(1/2)+16/15*(-sinh(x)*tanh(x))^(1/2)*tanh
(x)+2/5*sinh(x)^2*(-sinh(x)*tanh(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {4482, 4485, 2678, 2674, 2669}

$$\int (-\cosh(x)+\operatorname{sech}(x))^{5/2}\,dx = \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)} - \frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)}$$

```
[In] Int[(-Cosh[x] + Sech[x])^(5/2),x]
```

```
[Out] (-64*Coth[x]*Sqrt[-(Sinh[x]*Tanh[x])])/15 + (16*Tanh[x]*Sqrt[-(Sinh[x]*Tanh
[x])])/15 + (2*Sinh[x]^2*Tanh[x]*Sqrt[-(Sinh[x]*Tanh[x])])/5
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (-\sinh(x)\tanh(x))^{5/2}\,dx\\
&= \frac{\sqrt{-\sinh(x)\tanh(x)}\int (i\sinh(x))^{5/2}(i\tanh(x))^{5/2}\,dx}{\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}\\
&\quad+\frac{\left(8\sqrt{-\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}(i\tanh(x))^{5/2}\,dx}{5\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}\\
&= \frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)}+\frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}\\
&\quad-\frac{\left(32\sqrt{-\sinh(x)\tanh(x)}\right)\int\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}\,dx}{15\sqrt{i\sinh(x)}\sqrt{i\tanh(x)}}
\end{aligned}$$

$$= -\frac{64}{15}\coth(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{16}{15}\tanh(x)\sqrt{-\sinh(x)\tanh(x)} + \frac{2}{5}\sinh^2(x)\tanh(x)\sqrt{-\sinh(x)\tanh(x)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\,dx = \frac{2}{15}\left(-5-3\cosh^2(x)+32\coth^2(x)\right)\operatorname{csch}(x)(-\sinh(x)\tanh(x))^{3/2}$$

```
[In] Integrate[(-Cosh[x] + Sech[x])^(5/2),x]
[Out] (2*(-5 - 3*Cosh[x]^2 + 32*Coth[x]^2)*Csch[x]*(-(Sinh[x]*Tanh[x]))^(3/2))/15
```

Maple [F]

$$\int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}}\,dx$$

```
[In] int((-cosh(x)+sech(x))^(5/2),x)
[Out] int((-cosh(x)+sech(x))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.85

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\cosh(x)^8 + 24\cosh(x)\sinh(x)^7 + 3\sinh(x)^8 + 12\left(7\cosh(x)^2 - 9\right)\sinh(x)^6 - 1\right.}{}$$

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="fricas")
[Out] 1/30*sqrt(1/2)*(3*cosh(x)^8 + 24*cosh(x)*sinh(x)^7 + 3*sinh(x)^8 + 12*(7*co
sh(x)^2 - 9)*sinh(x)^6 - 108*cosh(x)^6 + 24*(7*cosh(x)^3 - 27*cosh(x))*sinh
(x)^5 + 2*(105*cosh(x)^4 - 810*cosh(x)^2 - 151)*sinh(x)^4 - 302*cosh(x)^4 +
 8*(21*cosh(x)^5 - 270*cosh(x)^3 - 151*cosh(x))*sinh(x)^3 + 12*(7*cosh(x)^6
 - 135*cosh(x)^4 - 151*cosh(x)^2 - 9)*sinh(x)^2 - 108*cosh(x)^2 + 8*(3*cosh
(x)^7 - 81*cosh(x)^5 - 151*cosh(x)^3 - 27*cosh(x))*sinh(x) + 3)*sqrt(-1/(co
sh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cos
h(x)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sin
h(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((-cosh(x)+sech(x))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\,dx = -\frac{\sqrt{2}e^{\left(\frac{5}{2}\,x\right)}}{20\,\left(-e^{(-2\,x)} - 1\right)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(\frac{1}{2}\,x\right)}}{4\,\left(-e^{(-2\,x)} - 1\right)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{3}{2}\,x\right)}}{6\,\left(-e^{(-2\,x)} - 1\right)^{\frac{5}{2}}} + \frac{41\,\sqrt{2}e^{\left(-\frac{7}{2}\,x\right)}}{6\,\left(-e^{(-2\,x)} - 1\right)^{\frac{5}{2}}} + \frac{7\,\sqrt{2}e^{\left(-\frac{11}{2}\,x\right)}}{4\,\left(-e^{(-2\,x)} - 1\right)^{\frac{5}{2}}} - \frac{\sqrt{2}e^{\left(-\frac{15}{2}\,x\right)}}{20\,\left(-e^{(-2\,x)} - 1\right)^{\frac{5}{2}}}$$

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="maxima")
[Out] -1/20*sqrt(2)*e^(5/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(1/2*x)/(-e^(
-2*x) - 1)^(5/2) + 41/6*sqrt(2)*e^(-3/2*x)/(-e^(-2*x) - 1)^(5/2) + 41/6*sqr
t(2)*e^(-7/2*x)/(-e^(-2*x) - 1)^(5/2) + 7/4*sqrt(2)*e^(-11/2*x)/(-e^(-2*x)
- 1)^(5/2) - 1/20*sqrt(2)*e^(-15/2*x)/(-e^(-2*x) - 1)^(5/2)
```

Giac [F]

$$\int (-\cosh(x) + \operatorname{sech}(x))^{5/2}\,dx = \int (-\cosh(x) + \operatorname{sech}(x))^{\frac{5}{2}}\,dx$$

```
[In] integrate((-cosh(x)+sech(x))^(5/2),x, algorithm="giac")
[Out] integrate((-cosh(x) + sech(x))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (-\cosh(x)+\operatorname{sech}(x))^{5/2}\,dx=\int \left(\frac{1}{\cosh(x)}-\cosh(x)\right)^{5/2}\,dx$$

```
[In] int((1/cosh(x) - cosh(x))^(5/2),x)
[Out] int((1/cosh(x) - cosh(x))^(5/2), x)
```

3.686 $\int \frac{1}{\sinh(x)+\tanh(x)} dx$

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \frac{1}{\sinh(x)+\tanh(x)} dx = -\frac{1}{2}\operatorname{arctanh}(\cosh(x)) - \frac{1}{2(1+\cosh(x))}$$

```
[Out] -1/2*arctanh(cosh(x))-1/2/(1+cosh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4482, 2785, 2686, 30, 2691, 3855}

$$\int \frac{1}{\sinh(x)+\tanh(x)} dx = -\frac{1}{2}\operatorname{arctanh}(\cosh(x)) + \frac{\operatorname{csch}^2(x)}{2} - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[(Sinh[x] + Tanh[x])^(-1),x]
[Out] -1/2*ArcTanh[Cosh[x]] - (Coth[x]*Csch[x])/2 + Csch[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\coth(x)}{-i-i\cosh(x)}\,dx\right)\\
&= \int \coth^2(x)\text{csch}(x)\,dx - \int \coth(x)\text{csch}^2(x)\,dx\\
&= -\frac{1}{2}\coth(x)\text{csch}(x) + \frac{1}{2}\int \text{csch}(x)\,dx - \text{Subst}(\int x\,dx, x, -i\text{csch}(x))\\
&= -\frac{1}{2}\text{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\text{csch}(x) + \frac{\text{csch}^2(x)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx=-\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{4}\text{sech}^2\left(\frac{x}{2}\right)$$

```
[In] Integrate[(Sinh[x] + Tanh[x])^(-1),x]
[Out] -1/2*Log[Cosh[x/2]] + Log[Sinh[x/2]]/2 - Sech[x/2]^2/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)^{2}}{4}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$	17
risch	$-\frac{\mathrm{e}^{x}}{\left(\mathrm{e}^{x}+1\right)^{2}}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2}$	26

```
[In] int(1/(sinh(x)+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.33

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = -\frac{\left(\cosh\left(x\right)^2+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^2+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\right.}{2\left(\cosh\left(x\right)^2+2\left(\cosh\left(x\right)+1\right.\right.}$$

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="fricas")
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(cosh(
x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = \int \frac{1}{\sinh{(x)}+\tanh{(x)}}\,dx$$

```
[In] integrate(1/(sinh(x)+tanh(x)),x)
[Out] Integral(1/(sinh(x) + tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = -\frac{e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1} - \frac{1}{2}\,\log\left(e^{(-x)}+1\right) + \frac{1}{2}\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="maxima")
[Out] -e^(-x)/(2*e^(-x) + e^(-2*x) + 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) -
1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx = \frac{e^{(-x)}+e^{x}-2}{4\left(e^{(-x)}+e^{x}+2\right)} - \frac{1}{4}\,\log\left(e^{(-x)}+e^{x}+2\right) + \frac{1}{4}\,\log\left(e^{(-x)}+e^{x}-2\right)$$

```
[In] integrate(1/(sinh(x)+tanh(x)),x, algorithm="giac")
[Out] 1/4*(e^(-x) + e^x - 2)/(e^(-x) + e^x + 2) - 1/4*log(e^(-x) + e^x + 2) + 1/4
*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sinh(x)+\tanh(x)}\,dx=\frac{\ln\left(1-\mathrm{e}^x\right)}{2}-\frac{\ln\left(-\mathrm{e}^x-1\right)}{2}+\frac{1}{\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1}-\frac{1}{\mathrm{e}^x+1}$$

```
[In] int(1/(sinh(x) + tanh(x)),x)
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + 1/(exp(2*x) + 2*exp(x) + 1) - 1/(
exp(x) + 1)
```

3.687 $\int \frac{1}{\sinh(x)-\tanh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{1}{\sinh(x)-\tanh(x)}\,dx=-\frac{1}{2}\text{arctanh}(\cosh(x))+\frac{1}{2(1-\cosh(x))}$$

```
[Out] -1/2*arctanh(cosh(x))+1/2/(1-cosh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4482, 2785, 2686, 30, 2691, 3855}

$$\int \frac{1}{\sinh(x)-\tanh(x)}\,dx=-\frac{1}{2}\text{arctanh}(\cosh(x))-\frac{1}{2}\text{csch}^2(x)-\frac{1}{2}\coth(x)\text{csch}(x)$$

```
[In] Int[(Sinh[x] - Tanh[x])^(-1),x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]] - (Coth[x]*Csch[x])/2 - Csch[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\coth(x)}{i-i\cosh(x)}\,dx\right)\\
&= \int \coth^2(x)\text{csch}(x)\,dx + \int \coth(x)\text{csch}^2(x)\,dx\\
&= -\frac{1}{2}\coth(x)\text{csch}(x) + \frac{1}{2}\int \text{csch}(x)\,dx + \text{Subst}(\int x\,dx, x, -i\text{csch}(x))\\
&= -\frac{1}{2}\text{arctanh}(\cosh(x)) - \frac{1}{2}\coth(x)\text{csch}(x) - \frac{\text{csch}^2(x)}{2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = -\frac{1}{4}\operatorname{csch}^2\left(\frac{x}{2}\right)\left(1 - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \cosh(x)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right) - \log\left(\sinh\left(\frac{x}{2}\right)\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

```
[In] Integrate[(Sinh[x] - Tanh[x])^(-1),x]
[Out] -1/4*(Csch[x/2]^2*(1 - Log[Cosh[x/2]] + Cosh[x]*(Log[Cosh[x/2]] - Log[Sinh[
x/2]]) + Log[Sinh[x/2]]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{4\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$	17
risch	$-\frac{e^{x}}{\left(e^{x}-1\right)^{2}}+\frac{\ln\left(e^{x}-1\right)}{2}-\frac{\ln\left(e^{x}+1\right)}{2}$	26

```
[In] int(1/(sinh(x)-tanh(x)),x,method=_RETURNVERBOSE)
[Out] -1/4/tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = -\frac{\left(\cosh(x)^2 + 2\left(\cosh(x) - 1\right)\sinh(x) + \sinh(x)^2 - 2\cosh(x) + 1\right)\log\left(\cosh(x) + \sinh(x) + 1\right) - (\cos}{2\left(\cosh(x)^2 + 2\left(\cosh(x) - \right.\right.}$$

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="fricas")
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
- 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(cosh(
x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = \int \frac{1}{\sinh(x) - \tanh(x)}\,dx$$

```
[In] integrate(1/(sinh(x)-tanh(x)),x)
[Out] Integral(1/(sinh(x) - tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = \frac{e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="maxima")
[Out] e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = -\frac{e^{(-x)} + e^{x} + 2}{4\left(e^{(-x)} + e^{x} - 2\right)} - \frac{1}{4}\,\log\left(e^{(-x)} + e^{x} + 2\right) + \frac{1}{4}\,\log\left(e^{(-x)} + e^{x} - 2\right)$$

```
[In] integrate(1/(sinh(x)-tanh(x)),x, algorithm="giac")
[Out] -1/4*(e^(-x) + e^x + 2)/(e^(-x) + e^x - 2) - 1/4*log(e^(-x) + e^x + 2) + 1/
4*log(e^(-x) + e^x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sinh(x) - \tanh(x)}\,dx = \frac{\ln(1 - e^x)}{2} - \frac{\ln(-e^x - 1)}{2} - \frac{1}{e^{2x} - 2e^x + 1} - \frac{1}{e^x - 1}$$

```
[In] int(1/(sinh(x) - tanh(x)),x)
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - 1/(exp(2*x) - 2*exp(x) + 1) - 1/(
exp(x) - 1)
```

3.688 $\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3176, 3212}

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{-bx+a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x]),x]
[Out] (-(b*x) + a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a\ln(a+b\tanh(x))-a\ln(1-\tanh(x))-(a+b)x}{a^2-b^2}$	39
risch	$\frac{x}{a+b}-\frac{2ax}{a^2-b^2}+\frac{a\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	54
default	$-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4a+4b}-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4a-4b}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{(a+b)(a-b)}$	70

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] (a*ln(a+b*tanh(x))-a*ln(1-tanh(x))-(a+b)*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{(a+b)x - a\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.67

$$\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b \\ \frac{a\log\left(\cosh(x)+\frac{b\sinh(x)}{a}\right)}{a^2-b^2} - \frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-x*sinh(x)/(-2*b*
sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - cosh(x)/(
-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2*b*sinh(x) + 2*b*cosh
(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + cosh(x)/(2*b*sinh(x) + 2*b*c
osh(x)), Eq(a, b)), (a*log(cosh(x) + b*sinh(x)/a)/(a**2 - b**2) - b*x/(a**2
 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{a\log\left(-(a-b)e^{(-2\,x)} - a - b\right)}{a^2 - b^2} + \frac{x}{a+b}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
[Out] a*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{a\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^2 - b^2} - \frac{x}{a-b}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) - x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = -\frac{b\,x - a\,\ln\left(a\cosh(x) + b\sinh(x)\right)}{a^2 - b^2}$$

```
[In] int(sinh(x)/(a*cosh(x) + b*sinh(x)),x)
[Out] -(b*x - a*log(a*cosh(x) + b*sinh(x)))/(a^2 - b^2)
```

3.689 $\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

```
[Out] -a^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3178, 3153, 212, 2718}

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

```
[In] Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sinh(x)}{a^2-b^2} - \frac{a^2\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int\sinh(x)\,dx}{a^2-b^2}\\ &= -\frac{b\cosh(x)}{a^2-b^2} + \frac{a\sinh(x)}{a^2-b^2} - \frac{(ia^2)\,\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\ &= -\frac{a^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{b\cosh(x)}{a^2-b^2} + \frac{a\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int\frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{-\sqrt{a-b}b(a+b)\cosh(x)+a\left(-2a\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)+\sqrt{a-b}(a+b)\sinh(x)\right)}{(a-b)^{3/2}(a+b)^2}$$

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-(Sqrt[a - b]*b*(a + b)*Cosh[x]) + a*(-2*a*Sqrt[a + b]*ArcTan[(b + a*Tanh[
x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*(a + b)*Sinh[x]))/((a - b)^(
3/2)*(a + b)^2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2a^{2}\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a+b)(a-b)\sqrt{a^{2}-b^{2}}}$	93
risch	$\frac{e^{x}}{2a+2b}-\frac{e^{-x}}{2(a-b)}-\frac{a^{2}\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{a^{2}\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	122

```
[In] int(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -8/(8*a-8*b)/(tanh(1/2*x)+1)-8/(8*a+8*b)/(tanh(1/2*x)-1)-2*a^2/(a+b)/(a-b)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

$$\int \frac{\sinh^{2}(x)}{a\cosh(x)+b\sinh(x)}\,dx = \left[-\frac{a^{3}+a^{2}b-ab^{2}-b^{3}-(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)^{2}-2(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)\sinh(x)-(a^{3}-}{2((a^{4}-2}\right.$$

$$-\frac{a^{3}+a^{2}b-ab^{2}-b^{3}-(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)^{2}-2(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)\sinh(x)-(a^{3}-}{2((a^{4}-2a^{2}b^{2}+b^{4})\cosh(x)+(a}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a^2*cosh(x) + a^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(58) = 116.

Time = 127.11 (sec) , antiderivative size = 685, normalized size of antiderivative = 9.26

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\cosh(x) \\ \frac{\cosh(x)}{b} \\ -\frac{\sinh^2(x)}{-3b\sinh(x)+3b\cosh(x)} - \frac{2\sinh(x)\cosh(x)}{-3b\sinh(x)+3b\cosh(x)} + \frac{2\cosh^2(x)}{-3b\sinh(x)+3b\cosh(x)} \\ -\frac{\sinh^2(x)}{3b\sinh(x)+3b\cosh(x)} + \frac{2\sinh(x)\cosh(x)}{3b\sinh(x)+3b\cosh(x)} + \frac{2\cosh^2(x)}{3b\sinh(x)+3b\cosh(x)} \\ -\frac{a^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)+b^2\sqrt{-a^2+b^2}} + \frac{a^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2} \end{cases}$$

```
[In] integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*cosh(x), Eq(a, 0) & Eq(b, 0)), (cosh(x)/b, Eq(a, 0)), (-sinh
(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)) - 2*sinh(x)*cosh(x)/(-3*b*sinh(x) + 3*b
*cosh(x)) + 2*cosh(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)), Eq(a, -b)), (-sinh(x
)**2/(3*b*sinh(x) + 3*b*cosh(x)) + 2*sinh(x)*cosh(x)/(3*b*sinh(x) + 3*b*cos
h(x)) + 2*cosh(x)**2/(3*b*sinh(x) + 3*b*cosh(x)), Eq(a, b)), (-a**2*log(tan
h(x/2) + b/a - sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*
tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)*
*2 + b**2*sqrt(-a**2 + b**2)) + a**2*log(tanh(x/2) + b/a - sqrt(-a**2 + b**
2)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**
2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + a**2*log(tan
h(x/2) + b/a + sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*
tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)*
*2 + b**2*sqrt(-a**2 + b**2)) - a**2*log(tanh(x/2) + b/a + sqrt(-a**2 + b**
2)/a)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**
2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - 2*a*sqrt(-a*
*2 + b**2)*tanh(x/2)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**
2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2))
 + 2*b*sqrt(-a**2 + b**2)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt
(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b
**2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,a^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^x}{2\,a+2\,b}-\frac{e^{-x}}{2\,a-2\,b}-\frac{a^2\,\ln\left(-\frac{2\,a^2}{(a+b)^{5/2}\,\sqrt{b-a}}-\frac{2\,a^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}} + \frac{a^2\,\ln\left(\frac{2\,a^2}{(a+b)^{5/2}\,\sqrt{b-a}}-\frac{2\,a^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

```
[In] int(sinh(x)^2/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (a^2*log(- (2*a^2)/((a + b)^(5/2
)*(b - a)^(1/2)) - (2*a^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3
/2)*(b - a)^(3/2)) + (a^2*log((2*a^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2*a^
2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))
```

3.690 $\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}-\frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}$$

```
[Out] a^2*b*x/(a^2-b^2)^2+1/2*b*x/(a^2-b^2)-a^3*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
^2-1/2*b*cosh(x)*sinh(x)/(a^2-b^2)+1/2*a*sinh(x)^2/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3178, 3176, 3212, 2715, 8}

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2bx}{(a^2-b^2)^2}+\frac{bx}{2(a^2-b^2)}+\frac{a\sinh^2(x)}{2(a^2-b^2)}-\frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)}-\frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 + (b*x)/(2*(a^2 - b^2)) - (a^3*Log[a*Cosh[x] + b*Si
nh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^2)
/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sinh^2(x)}{2(a^2-b^2)} - \frac{a^2\int\frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int\sinh^2(x)\,dx}{a^2-b^2} \\
&= \frac{a^2bx}{(a^2-b^2)^2} - \frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)} + \frac{a\sinh^2(x)}{2(a^2-b^2)} - \frac{(ia^3)\int\frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{b\int 1\,dx}{2(a^2-b^2)} \\
&= \frac{a^2bx}{(a^2-b^2)^2} + \frac{bx}{2(a^2-b^2)} - \frac{a^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} - \frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)} + \frac{a\sinh^2(x)}{2(a^2-b^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{6a^2bx-2b^3x+a(a^2-b^2)\cosh(2x)-4a^3\log(a\cosh(x)+b\sinh(x))+(-a^2b+b^3)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (6*a^2*b*x - 2*b^3*x + a*(a^2 - b^2)*Cosh[2*x] - 4*a^3*Log[a*Cosh[x] + b*Si
nh[x]] + (-(a^2*b) + b^3)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{ax}{(a+b)^2}-\frac{xb}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}+\frac{e^{-2x}}{8a-8b}+\frac{2a^3x}{a^4-2a^2b^2+b^4}-\frac{a^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{8}{(16a+16b)(\tanh(\frac{x}{2})-1)^2}+\frac{16}{(32a+32b)(\tanh(\frac{x}{2})-1)}+\frac{(2a+b)\ln(\tanh(\frac{x}{2})-1)}{2(a+b)^2}-\frac{16}{(32a-32b)(\tanh(\frac{x}{2})+1)}+\frac{8}{(16a-16b)(\tan$

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a*x/(a+b)^2-1/2*x/(a+b)^2*b+1/8/(a+b)*exp(2*x)+1/8/(a-b)*exp(-2*x)+2*a^3/(
a^4-2*a^2*b^2+b^4)*x-a^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(95) = 190.

Time = 0.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.34

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(2*a^3 + 3*a^
```

```
2*b - b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(2*a^3 + 3*a^2*b - b^3)*x)*sinh(x)^2 - 8*(a^3*cosh
(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(2*a^
3 + 3*a^2*b - b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} - \frac{(2a+b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} + \frac{e^{(-2x)}}{8(a-b)}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(2*a + b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} + \frac{(2a-b)x}{2(a^2-2ab+b^2)} - \frac{\left(4ae^{(2x)}-2be^{(2x)}-a+b\right)e^{(-2x)}}{8(a^2-2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*
(2*a - b)*x/(a^2 - 2*a*b + b^2) - 1/8*(4*a*e^(2*x) - 2*b*e^(2*x) - a + b)*e
^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{e^{-2x}}{8a-8b}+\frac{e^{2x}}{8a+8b}-\frac{a^3\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4}+\frac{x\,(2a-b)}{2\,(a-b)^2}$$

```
[In] int(sinh(x)^3/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*a - 8*b) + exp(2*x)/(8*a + 8*b) - (a^3*log(a - b + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(2*a - b))/(2*(a - b)^2)
```

3.691 $\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3177, 3212}

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x]),x]
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{-b\ln(a+b\tanh(x))+\ln(1-\tanh(x))b+(a+b)x}{a^2-b^2}$	38
risch	$\frac{x}{a+b}+\frac{2xb}{a^2-b^2}-\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55
default	$-\frac{2\ln(\tanh(\frac{x}{2})-1)}{2a+2b}+\frac{2\ln(\tanh(\frac{x}{2})+1)}{2a-2b}-\frac{b\ln\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)}{(a-b)(a+b)}$	71

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] (-b*ln(a+b*tanh(x))+ln(1-tanh(x))*b+(a+b)*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(29) = 58.

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.85

$$\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\log(\sinh(x)) & \text{for } a=0\wedge b=0 \\ \frac{\log(\sinh(x))}{b} & \text{for } a=0 \\ \frac{x\sinh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{x\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{-2b\sinh(x)+2b\cosh(x)} & \text{for } a=-b \\ \frac{x\sinh(x)}{2b\sinh(x)+2b\cosh(x)} + \frac{x\cosh(x)}{2b\sinh(x)+2b\cosh(x)} - \frac{\cosh(x)}{2b\sinh(x)+2b\cosh(x)} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{b\log\left(\cosh(x)+\frac{b\sinh(x)}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (log(sinh(x))/b, Eq(a, 0
)), (x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - x*cosh(x)/(-2*b*sinh(x) + 2*b
*cosh(x)) - cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, -b)), (x*sinh(x)/(2
*b*sinh(x) + 2*b*cosh(x)) + x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - cosh(x)
/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, b)), (a*x/(a**2 - b**2) - b*log(cosh(x)
 + b*sinh(x)/a)/(a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\cosh(x)}{a\cosh(x) + b\sinh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)} - a - b\right)}{a^2 - b^2} + \frac{x}{a+b}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)}{a\cosh(x) + b\sinh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} + a - b\right|\right)}{a^2 - b^2} + \frac{x}{a-b}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\cosh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{a\,x - b\,\ln\left(a\cosh(x) + b\sinh(x)\right)}{a^2 - b^2}$$

```
[In] int(cosh(x)/(a*cosh(x) + b*sinh(x)),x)
[Out] (a*x - b*log(a*cosh(x) + b*sinh(x)))/(a^2 - b^2)
```

3.692 $\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

```
[Out] -b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3179, 2717, 3153, 212}

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{b^2\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

```
[In] Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2))
- (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\int\cosh(x)\,dx}{a^2-b^2}-\frac{b^2\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{(ib^2)\,\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= -\frac{b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int\frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{2b^2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{b\cosh(x)}{-a^2+b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x]),x]
[Out] (-2*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)
*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+1\right)(2a-2b)}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)(2a+2b)}-\frac{2b^{2}\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)(a+b)\sqrt{a^{2}-b^{2}}}$	93
risch	$\frac{\mathrm{e}^{x}}{2a+2b}-\frac{\mathrm{e}^{-x}}{2(a-b)}-\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{b^{2}\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	122

```
[In] int(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(tanh(1/2*x)+1)/(2*a-2*b)-2/(tanh(1/2*x)-1)/(2*a+2*b)-2*b^2/(a-b)/(a+b)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

$$\int \frac{\cosh^{2}(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\left[-\frac{a^{3}+a^{2}b-ab^{2}-b^{3}-(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)^{2}-2(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)\sinh(x)-(a^{3}-}{2((a^{4}-2}\right.$$
$$-\frac{a^{3}+a^{2}b-ab^{2}-b^{3}-(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)^{2}-2(a^{3}-a^{2}b-ab^{2}+b^{3})\cosh(x)\sinh(x)-(a^{3}-}{2((a^{4}-2a^{2}b^{2}+b^{4})\cosh(x)+(a}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(58) = 116$.

Time = 123.84 (sec) , antiderivative size = 774, normalized size of antiderivative = 10.46

$$\int \frac{\cosh^2(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x
/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((lo
g(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)**2
 - 1) - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), (2*sinh(x)**2/(-3*b*sinh(x) + 3
*b*cosh(x)) - 2*sinh(x)*cosh(x)/(-3*b*sinh(x) + 3*b*cosh(x)) - cosh(x)**2/(
-3*b*sinh(x) + 3*b*cosh(x)), Eq(a, -b)), (2*sinh(x)**2/(3*b*sinh(x) + 3*b*c
osh(x)) + 2*sinh(x)*cosh(x)/(3*b*sinh(x) + 3*b*cosh(x)) - cosh(x)**2/(3*b*s
inh(x) + 3*b*cosh(x)), Eq(a, b)), (-2*a*sqrt(-a**2 + b**2)*tanh(x/2)/(a**2*
sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2
 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - b**2*log(tanh(x/2) + b/a
 - sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2
 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sq
rt(-a**2 + b**2)) + b**2*log(tanh(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2*
sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2
 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + b**2*log(tanh(x/2) + b/a
 + sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2
 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sq
rt(-a**2 + b**2)) - b**2*log(tanh(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(a**2*
sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2
 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + 2*b*sqrt(-a**2 + b**2)/(
a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(
-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}} - \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")

[Out] -2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^x}{2\,a+2\,b} - \frac{e^{-x}}{2\,a-2\,b} - \frac{b^2\,\ln\left(-\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{b-a}} - \frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}} + \frac{b^2\,\ln\left(\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{b-a}} - \frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

[In] int(cosh(x)^2/(a*cosh(x) + b*sinh(x)),x)

[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (b^2*log(- (2*b^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2*b^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2*b^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))

3.693 $\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{ab^2x}{(a^2-b^2)^2}+\frac{ax}{2(a^2-b^2)}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)}$$

```
[Out] -a*b^2*x/(a^2-b^2)^2+1/2*a*x/(a^2-b^2)-1/2*b*cosh(x)^2/(a^2-b^2)+b^3*ln(a*c
osh(x)+b*sinh(x))/(a^2-b^2)^2+1/2*a*cosh(x)*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3179, 2715, 8, 3177, 3212}

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{ab^2x}{(a^2-b^2)^2}+\frac{ax}{2(a^2-b^2)}-\frac{b\cosh^2(x)}{2(a^2-b^2)}+\frac{a\sinh(x)\cosh(x)}{2(a^2-b^2)}+\frac{b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (a*x)/(2*(a^2 - b^2)) - (b*Cosh[x]^2)/(2*(a^2
- b^2)) + (b^3*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[
x])/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral} = -\frac{b\cosh^2(x)}{2(a^2-b^2)} + \frac{a\int \cosh^2(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} - \frac{b\cosh^2(x)}{2(a^2-b^2)} + \frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)} + \frac{(ib^3)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{a\int 1\,dx}{2(a^2-b^2)}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{ax}{2(a^2-b^2)} - \frac{b\cosh^2(x)}{2(a^2-b^2)} + \frac{b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2a^3x-6ab^2x+(-a^2b+b^3)\cosh(2x)+4b^3\log(a\cosh(x)+b\sinh(x))+a(a^2-b^2)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (2*a^3*x - 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] + 4*b^3*Log[a*Cosh[x] + b
*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{xb}{(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}-\frac{2b^3x}{a^4-2a^2b^2+b^4}+\frac{b^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{1}{(2a+2b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-a-2b)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{1}{(2a-2b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{2}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+\right.}$

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/(a+b)^2+x/(a+b)^2*b+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)-2*b^3/(a
^4-2*a^2*b^2+b^4)*x+b^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.28

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 - 3*a*b^
```

```
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 + 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} + \frac{(a+2b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a + 2*b)*
x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} + \frac{(a-2b)x}{2(a^2-2ab+b^2)} - \frac{\left(2ae^{(2x)}-4be^{(2x)}+a-b\right)e^{(-2x)}}{8(a^2-2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(
a - 2*b)*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) + a - b)*e^
(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}+\frac{b^3\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{x\,(a-2\,b)}{2\,(a-b)^2}$$

```
[In] int(cosh(x)^3/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (b^3*log(a - b + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(a - 2*b))/(2*(a - b)^2)
```

3.694 $\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}+\frac{b\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[Out] arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3189, 3855, 3153, 210}

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{b\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\arctan(\sinh(x))}{a}$$

```
[In] Int[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int\left(\frac{i\text{sech}(x)}{a}-\frac{ib}{a(b\cosh(x)+a\sinh(x))}\right)dx\right)\\
&= \frac{\int\text{sech}(x)\,dx}{a}-\frac{b\int\frac{1}{b\cosh(x)+a\sinh(x)}\,dx}{a}\\
&= \frac{\arctan(\sinh(x))}{a}-\frac{(ib)\text{Subst}\left(\int\frac{1}{-a^2+b^2-x^2}\,dx,x,-ia\cosh(x)-ib\sinh(x)\right)}{a}\\
&= \frac{\arctan(\sinh(x))}{a}+\frac{b\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int\frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{a}$$

```
[In] Integrate[Tanh[x]/(b*Cosh[x] + a*Sinh[x]),x]
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2b\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{a\sqrt{-a^{2}+b^{2}}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	54
risch	$\frac{b\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}-\frac{b\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}+\frac{i\ln(\mathrm{e}^{x}+i)}{a}-\frac{i\ln(\mathrm{e}^{x}-i)}{a}$	102

```
[In] int(tanh(x)/(b*cosh(x)+a*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))+
2/a*arctan(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.00

$$\int\frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^{2}-b^{2}}b\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+2\,\sqrt{a^{2}-b^{2}}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-a+b}\right)+2\,(a^{2}-b^{2})\,\mathrm{arctan}}{a^{3}-ab^{2}}\right.$$
$$\left.-\frac{2\left(\sqrt{-a^{2}+b^{2}}b\arctan\left(\frac{\sqrt{-a^{2}+b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)-(a^{2}-b^{2})\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^{3}-ab^{2}}\right]$$

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]

$$\int \frac{\tanh(x)}{b\cosh(x) + a\sinh(x)}\,dx = \int \frac{\tanh{(x)}}{a\sinh{(x)} + b\cosh{(x)}}\,dx$$

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a*sinh(x) + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh(x)}{b\cosh(x) + a\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\tanh(x)}{b\cosh(x) + a\sinh(x)}\,dx = -\frac{2\,b\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}a} + \frac{2\,\arctan\left(e^{x}\right)}{a}$$

```
[In] integrate(tanh(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.28

$$\int \frac{\tanh(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x+32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}-\frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}+\frac{\ln\left(32\,a\,b\,\mathrm{e}^x-32\,a^2\,\mathrm{e}^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a}-\frac{\ln\left(32\,a^2\,\mathrm{e}^x-32\,a\,b\,\mathrm{e}^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a}$$

```
[In] int(tanh(x)/(b*cosh(x) + a*sinh(x)),x)
```

```
[Out] (log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*32
i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(x)
 + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) + (b*
log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2
- b^2)^(1/2))
```

3.695 $\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{b}+\frac{a\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

```
[Out] -arctanh(cosh(x))/b+a*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/b/(a^2
-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3189, 3855, 3153, 212}

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{a\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

```
[In] Int[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (a*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]]
)/(b*Sqrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int\left(-\frac{i\text{csch}(x)}{b}-\frac{a}{b(ib\cosh(x)+ia\sinh(x))}\right)dx\\
&=\frac{\int\text{csch}(x)\,dx}{b}-\frac{(ia)\int\frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{b}\\
&=-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{a\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{b}\\
&=-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{a\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int\frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx=\frac{-\frac{2a\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)}{b}$$

```
[In] Integrate[Coth[x]/(b*Cosh[x] + a*Sinh[x]),x]
[Out] ((-2*a*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*
Sqrt[a + b]) - Log[Cosh[x/2]] + Log[Sinh[x/2]])/b
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{2a\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{b\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$	53
risch	$\frac{\ln(\mathrm{e}^{x}-1)}{b}-\frac{\ln(\mathrm{e}^{x}+1)}{b}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}-\frac{a\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}$	97

```
[In] int(coth(x)/(b*cosh(x)+a*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))+
1/b*ln(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.69

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^{2}-b^{2}}a\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+2\,\sqrt{a^{2}-b^{2}}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-a+b}\right)-(a^{2}-b^{2})\log\left(\mathrm{co}\right.}{a^{2}b-b^{3}}\right.$$
$$-\frac{2\sqrt{-a^{2}+b^{2}}a\arctan\left(\frac{\sqrt{-a^{2}+b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)+(a^{2}-b^{2})\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a^{2}-b^{2})\log\left(\right.}{a^{2}b-b^{3}}$$

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*a*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) - (a^2 -
 b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 - b^2)*log(cosh(x) + sinh(x) - 1))/
(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*a*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh
(x) + (a + b)*sinh(x))) + (a^2 - b^2)*log(cosh(x) + sinh(x) + 1) - (a^2 - b
^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b - b^3)]
```

Sympy [F]

$$\int \frac{\coth(x)}{b\cosh(x) + a\sinh(x)}\,dx = \int \frac{\coth(x)}{a\sinh(x) + b\cosh(x)}\,dx$$

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x)
```

```
[Out] Integral(coth(x)/(a*sinh(x) + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth(x)}{b\cosh(x) + a\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{\coth(x)}{b\cosh(x) + a\sinh(x)}\,dx = -\frac{2\,a\arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b} - \frac{\log(e^x+1)}{b} + \frac{\log(|e^x-1|)}{b}$$

```
[In] integrate(coth(x)/(b*cosh(x)+a*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) - log(e^
x + 1)/b + log(abs(e^x - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.47

$$\int \frac{\coth(x)}{b\cosh(x)+a\sinh(x)}\,dx = \frac{\ln\left(32\,a\,b-32\,b^2+32\,b^2\,\mathrm{e}^x-32\,a\,b\,\mathrm{e}^x\right)}{b} - \frac{\ln\left(32\,a\,b-32\,b^2-32\,b^2\,\mathrm{e}^x+32\,a\,b\,\mathrm{e}^x\right)}{b} - \frac{a\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-32\,a\,b\,\sqrt{a^2-b^2}\right)\,\sqrt{a^2-b^2}}{a^2\,b-b^3} + \frac{a\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x+32\,a\,b\,\sqrt{a^2-b^2}\right)\,\sqrt{a^2-b^2}}{a^2\,b-b^3}$$

```
[In] int(coth(x)/(b*cosh(x) + a*sinh(x)),x)
```

```
[Out] log(32*a*b - 32*b^2 + 32*b^2*exp(x) - 32*a*b*exp(x))/b - log(32*a*b - 32*b^
2 - 32*b^2*exp(x) + 32*a*b*exp(x))/b - (a*log(32*a*b^2*exp(x) + 32*a^2*b*ex
p(x) - 32*a*b*(a^2 - b^2)^(1/2))*(a^2 - b^2)^(1/2))/(a^2*b - b^3) + (a*log(
32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2))*(a^2 - b^2)^(
1/2))/(a^2*b - b^3)
```

3.696 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}$$

```
[Out] -b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-a/(a^2-b^2
)/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3233, 3153, 212}

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{b\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}$$

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)) -
a/((a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}-\frac{b\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{a}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}-\frac{(ib)\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= -\frac{b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{a}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.89

$$\int\frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a\sqrt{a-b}(a+b)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2b^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}$$

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
[Out] -((a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt
[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/
(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] + b
*Sinh[x])))
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{-8b\tanh\left(\frac{x}{2}\right)-8a}{\left(4a^{2}-4b^{2}\right)\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)}-\frac{8b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\left(4a^{2}-4b^{2}\right)\sqrt{a^{2}-b^{2}}}$	99
risch	$-\frac{2a\,\mathrm{e}^{x}}{(a-b)(a+b)(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b)}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}+\frac{b\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)}$	132

[In] int(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] 4*(-2*b*tanh(1/2*x)-2*a)/(4*a^2-4*b^2)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-8*b/(4*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 594, normalized size of antiderivative = 9.00

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \left[\frac{\left((ab+b^2)\cosh(x)^2+2(ab+b^2)\cosh(x)\sinh(x)+(ab+b^2)\sinh(x)^2+ab-b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{(a+b)\cos\cdots}{\cdots}\right)\cdots}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)\cosh(x)^2+2(a^5\cdots}\right.$$

[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")

[Out] [(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b - b^2)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2), 2*(((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a*b - b^2)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^3 - a*b^2)*cosh(x) - (a^3 - a*b^2)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} - \frac{2\,ae^x}{(a^2-b^2)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 2*a*e^x/((
a^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(b^2\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+a\,b\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}\right)}{a^4\,\sqrt{b^2}-2\,a^2\,{\left(b^2\right)}^{3/2}+b^4\,\sqrt{b^2}+a\,b\,{\left(b^2\right)}^{3/2}-a\,b^3\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}} - \frac{2\,a\,\mathrm{e}^x}{(a+b)\,\left(a-b\right)\,\left(a-b+\mathrm{e}^{2\,x}\,\left(a+b\right)\right)}$$

```
[In] int(sinh(x)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] - (2*atan((exp(x)*(b^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a*b*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)))/(a^4*(b^2)^(1/2) - 2*a^2*(b^2)^(3/2
) + b^4*(b^2)^(1/2) + a*b*(b^2)^(3/2) - a*b^3*(b^2)^(1/2)))*(b^2)^(1/2))/(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - (2*a*exp(x))/((a + b)*(a - b)*(a
- b + exp(2*x)*(a + b)))
```

3.697 $\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{a}{(a^2-b^2)\,(b+a\coth(x))} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2-a/(a^2-b^2)/(b+a*coth(x))-2*a*b*ln(a*cosh(x)+b*sinh
(x))/(a^2-b^2)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3164, 3564, 3612, 3611}

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{x(a^2+b^2)}{(a^2-b^2)^2} - \frac{a}{(a^2-b^2)\,(a\coth(x)+b)} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - a/((a^2 - b^2)*(b + a*Coth[x])) - (2*a*b*Lo
g[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2
```

<u>Rule 3164</u>

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
```

```
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{1}{(-ib-ia\coth(x))^2}\,dx \\
&= -\frac{a}{(a^2-b^2)(b+a\coth(x))} - \frac{\int \frac{-ib+ia\coth(x)}{-ib-ia\coth(x)}\,dx}{a^2-b^2} \\
&= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{a}{(a^2-b^2)(b+a\coth(x))} - \frac{(2iab)\int \frac{-a-b\coth(x)}{-ib-ia\coth(x)}\,dx}{(a^2-b^2)^2} \\
&= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{a}{(a^2-b^2)(b+a\coth(x))} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x-2ab\log(a\cosh(x)+b\sinh(x))-\frac{a(a-b)(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}}{(a-b)^2(a+b)^2}$$

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] - (a*(a - b)*(a + b)*Sinh
[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{(a+b)^2}+\frac{\ln(\tanh(\frac{x}{2})+1)}{(a-b)^2}+\frac{2a\left(\frac{(-a^2+b^2)\tanh(\frac{x}{2})}{\tanh(\frac{x}{2})^2 a+2b\tanh(\frac{x}{2})+a}-b\ln\left(\tanh(\frac{x}{2})^2 a+2b\tanh(\frac{x}{2})+a\right)\right)}{(a-b)^2(a+b)^2}$
parallelrisch	$\frac{(-2\tanh(x)a\,b^3-2a^2b^2)\ln(a+b\tanh(x))+(2\tanh(x)a\,b^3+2a^2b^2)\ln(1-\tanh(x))+(x\,b^2(a+b)\tanh(x)+(b^2x+a(-1+x)b+a^2)}{(a-b)^2(a+b)^2(a+b\tanh(x))b}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{2a^2}{(a-b)(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{2ab\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$

```
[In] int(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^2*ln(tanh(1/2*x)-1)+1/(a-b)^2*ln(tanh(1/2*x)+1)+2*a/(a-b)^2/(a+b)^
2*((-a^2+b^2)*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-b*ln(tanh(1/2
*x)^2*a+2*b*tanh(1/2*x)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.12

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh(x)^2+2\,(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh(x)\sinh(x)+(a^3+3\,a^2b+3\,ab^2+}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+(a^5+a^4b-2\,a^3b^2-2\,a^2b}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*a
^3 - 2*a^2*b + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b +
a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh
(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
 a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*
sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(56) = 112$.

Time = 0.82 (sec) , antiderivative size = 983, normalized size of antiderivative = 14.46

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b**2, Eq(a, 0)), (2*x*sinh(x)**2
/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 4*x*si
nh(x)*cosh(x)/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)
**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2
*cosh(x)**2) + 3*sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) +
8*b**2*cosh(x)**2) - cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x
) + 8*b**2*cosh(x)**2), Eq(a, -b)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 + 16
*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*si
nh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8
*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - 3*sinh(x)
**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + cos
h(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2),
Eq(a, b)), ((x - sinh(x)/cosh(x))/a**2, Eq(b, 0)), (a**4*cosh(x)/(a**5*b*co
sh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b
**5*cosh(x) + b**6*sinh(x)) + a**3*b*x*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*
sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6
*sinh(x)) + a**2*b**2*x*sinh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**
3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) - 2*a
**2*b**2*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sin
h(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*si
nh(x)) - a**2*b**2*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**
3*cosh(x) - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + a*b**3*x
*cosh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2
*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) - 2*a*b**3*log(cosh(x) + b*s
```

```
inh(x)/a)*sinh(x)/(a**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x)
 - 2*a**2*b**4*sinh(x) + a*b**5*cosh(x) + b**6*sinh(x)) + b**4*x*sinh(x)/(a
**5*b*cosh(x) + a**4*b**2*sinh(x) - 2*a**3*b**3*cosh(x) - 2*a**2*b**4*sinh(
x) + a*b**5*cosh(x) + b**6*sinh(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{2\,a^2}{a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}} + \frac{x}{a^2+2\,ab+b^2}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*a^2/(a^4
- 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 + 2*
a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.66

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4} + \frac{x}{a^2-2\,ab+b^2} + \frac{2\left(abe^{(2\,x)}+a^2-ab\right)}{(a^3-a^2b-ab^2+b^3)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x/
(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a^2 - a*b)/((a^3 - a^2*b - a*b^2 + b
^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.59

$$\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\frac{a^2\cosh(x)}{b\,(a^2-b^2)}+\frac{a\,x\cosh(x)\,(a^2+b^2)}{(a^2-b^2)^2}+\frac{b\,x\sinh(x)\,(a^2+b^2)}{(a^2-b^2)^2}}{a\cosh\left(x\right)+b\sinh\left(x\right)} + \ln\left(a\cosh(x)+b\sinh(x)\right)\,\left(\frac{1}{2\,{(a+b)}^2}-\frac{1}{2\,{(a-b)}^2}\right)$$

```
[In] int(sinh(x)^2/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] ((a^2*cosh(x))/(b*(a^2 - b^2)) + (a*x*cosh(x)*(a^2 + b^2))/(a^2 - b^2)^2 +
(b*x*sinh(x)*(a^2 + b^2))/(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x)) + log(a*co
sh(x) + b*sinh(x))*(1/(2*(a + b)^2) - 1/(2*(a - b)^2))
```

3.698 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 195

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{3a^2b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{(2a^2+b^2)\cosh(x)}{-a^2b^2+b^4} + \frac{a(a^2+2b^2)\sinh(x)}{b^3(a^2-b^2)} - \frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} + \frac{a^3}{(a-b)^2b^3\left(1+\tanh\left(\frac{x}{2}\right)\right)} + \frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}$$

```
[Out] 3*a^2*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+(2*a^
2+b^2)*cosh(x)/(-a^2*b^2+b^4)+a*(a^2+2*b^2)*sinh(x)/b^3/(a^2-b^2)-a^3/b^3/(
a+b)^2/(1-tanh(1/2*x))+a^3/(a-b)^2/b^3/(1+tanh(1/2*x))+2*a^2*(a+b*tanh(1/2*
x))/(a^2-b^2)^2/(a+2*b*tanh(1/2*x)+a*tanh(1/2*x)^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.54, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules

used = {4486, 2717, 2718, 6874, 652, 632, 210, 3179, 3153, 212}

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{b^3(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{2a^2(3a^2-b^2)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}} + \frac{2a^2b\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{3a^2\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} + \frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)} + \frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{\cosh(x)}{b^2}$$

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-3*a^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/
2)) + (2*a^2*b*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (2*a^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2
- b^2)^(5/2)) + Cosh[x]/b^2 - (3*a^2*Cosh[x])/(b^2*(a^2 - b^2)) - (2*a*Sinh
[x])/b^3 + (3*a^3*Sinh[x])/(b^3*(a^2 - b^2)) - a^3/(b^3*(a + b)^2*(1 - Tanh
[x/2])) + a^3/((a - b)^2*b^3*(1 + Tanh[x/2])) + (2*a^2*(a + b*Tanh[x/2]))/(
(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/2]^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
```

```
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = i\int\left(\frac{2ia\cosh(x)}{b^3}-\frac{i\sinh(x)}{b^2}-\frac{ia^3\cosh^3(x)}{b^3(ia\cosh(x)+ib\sinh(x))^2}-\frac{3ia^2\cosh^2(x)}{b^3(a\cosh(x)+b\sinh(x))}\right)dx$$

$$=-\frac{(2a)\int\cosh(x)\,dx}{b^3}+\frac{(3a^2)\int\frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{b^3}+\frac{a^3\int\frac{\cosh^3(x)}{(ia\cosh(x)+ib\sinh(x))^2}\,dx}{b^3}+\frac{\int\sinh(x)\,dx}{b^2}$$

$$=\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2(a^2-b^2)}-\frac{2a\sinh(x)}{b^3}+\frac{(2a^3)\,\text{Subst}\left(\int\frac{(-1-x^2)^3}{(1-x^2)^2(a+2bx+ax^2)^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}+\frac{(3a^3)\int\cosh(x)\,dx}{b^3(a^2-b^2)}-\frac{(3a^2)\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{b(a^2-b^2)}$$

$$=\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2(a^2-b^2)}-\frac{2a\sinh(x)}{b^3}+\frac{3a^3\sinh(x)}{b^3(a^2-b^2)}+\frac{(2a^3)\,\text{Subst}\left(\int\left(-\frac{1}{2(a+b)^2(-1+x)^2}-\frac{1}{2(a-b)^2(1+x)^2}+\frac{2b^3x}{a(-a^2+b^2)(a+2bx+ax^2)^2}+\frac{3a^2b^2-b^4}{a(a^2-b^2)^2(a+2bx+ax^2)}\right)\,dx,x\right.}{b^3}-\frac{(3ia^2)\,\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{b(a^2-b^2)}$$

$$=-\frac{3a^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}}+\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2(a^2-b^2)}-\frac{2a\sinh(x)}{b^3}+\frac{3a^3\sinh(x)}{b^3(a^2-b^2)}-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{(a-b)^2b^3\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{(4a^2)\,\text{Subst}\left(\int\frac{x}{(a+2bx+ax^2)^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}+\frac{(2a^2(3a^2-b^2))\,\text{Subst}\left(\int\frac{1}{a+2bx+ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b(a^2-b^2)^2}$$

$$=-\frac{3a^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}}+\frac{\cosh(x)}{b^2}-\frac{3a^2\cosh(x)}{b^2(a^2-b^2)}-\frac{2a\sinh(x)}{b^3}+\frac{3a^3\sinh(x)}{b^3(a^2-b^2)}-\frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{a^3}{(a-b)^2b^3\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}+\frac{(2a^2b)\,\text{Subst}\left(\int\frac{1}{a+2bx+ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2)^2}-\frac{(4a^2(3a^2-b^2))\,\text{Subst}\left(\int\frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tanh\left(\frac{x}{2}\right)\right)}{b(a^2-b^2)^2}$$

$$= -\frac{3a^2 \arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{2a^2(3a^2-b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}} + \frac{\cosh(x)}{b^2} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)} - \frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} + \frac{a^3}{(a-b)^2b^3\left(1+\tanh\left(\frac{x}{2}\right)\right)} + \frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)} - \frac{(4a^2b)\,\text{Subst}\left(\int\frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2}$$

$$= -\frac{3a^2 \arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}} + \frac{2a^2b\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{2a^2(3a^2-b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}} + \frac{\cosh(x)}{b^2} - \frac{3a^2\cosh(x)}{b^2\left(a^2-b^2\right)} - \frac{2a\sinh(x)}{b^3} + \frac{3a^3\sinh(x)}{b^3\left(a^2-b^2\right)} - \frac{a^3}{b^3(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} + \frac{a^3}{(a-b)^2b^3\left(1+\tanh\left(\frac{x}{2}\right)\right)} + \frac{2a^2\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$

$$= \frac{a\sqrt{a-b}(a^3+a^2b+ab^2+b^3)\cosh^2(x) - b\cosh(x)\left(-6a^3\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right) + (a-b)^{3/2}(a+b)^2 \cdots\right.}{(a-b)^{5/2}(a+b)^3(a \cdots}$$

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*Sqrt[a - b]*(a^3 + a^2*b + a*b^2 + b^3)*Cosh[x]^2 - b*Cosh[x]*(-6*a^3*Sq
rt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^(3/
2)*(a + b)^2*Sinh[x]) + a*(a^2*Sqrt[a - b]*(a + b) + 6*a*b^2*Sqrt[a + b]*Ar
cTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] - 2*Sqrt[a - b]*b
^2*(a + b)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.62

method	result
default	$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{4a^2\left(\frac{-\frac{b\tanh\left(\frac{x}{2}\right)}{2}-\frac{a}{2}}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a}-\frac{3b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2}+\frac{e^{-x}}{2a^2-4ab+2b^2}+\frac{2a^3e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{3b\,a^2\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+\frac{3b\,a^2\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-}$

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)+1/(a-b)^2/(tanh(1/2*x)+1)-4*a^2/(a+b)^2/(a-b)^2*
((-1/2*b*tanh(1/2*x)-1/2*a)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-3/2*b/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(180) = 360.

Time = 0.29 (sec) , antiderivative size = 1633, normalized size of antiderivative = 8.37

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 - a*b
^4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)
^2 - 6*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sinh(x)^2
 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b - a^2*b
^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b
^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
+ 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
```

```
+ 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2 + 6*(a^5 - a*b^
4 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^
2 - 12*((a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^3*b + a^2*b^2)*cosh(x)*sinh(x)^2
 + (a^3*b + a^2*b^2)*sinh(x)^3 + (a^3*b - a^2*b^2)*cosh(x) + (a^3*b - a^2*b
^2 + 3*(a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^
2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 4*((a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x))*sinh(x))/(
(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)
*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5
 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b -
3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)
*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{6\,a^2b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{e^x}{2\,(a^2+2\,ab+b^2)} + \frac{5\,a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 6*a^2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sq
rt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(5*a^3*e^(2*x) + 3*a^2*b
*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((a^4
 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\mathrm{e}^{-x}}{2\,{(a-b)}^2}+\frac{\mathrm{e}^x}{2\,{(a+b)}^2} + \frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,\mathrm{e}^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^4\,b^2}-b^5\,\sqrt{a^4\,b^2}+2\,a^2\,b^3\,\sqrt{a^4\,b^2}-2\,a^3\,b^2\,\sqrt{a^4\,b^2}+a\,b^4\,\sqrt{a^4\,b^2}-a^4\,b\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} + \frac{2\,a^3\,\mathrm{e}^x}{{(a+b)}^2\,{(a-b)}^2\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

```
[In] int(sinh(x)^3/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*(a - b)^2) + exp(x)/(2*(a + b)^2) + (6*atan((a^2*b*exp(x)*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^4*
b^2)^(1/2) - b^5*(a^4*b^2)^(1/2) + 2*a^2*b^3*(a^4*b^2)^(1/2) - 2*a^3*b^2*(a
^4*b^2)^(1/2) + a*b^4*(a^4*b^2)^(1/2) - a^4*b*(a^4*b^2)^(1/2)))*(a^4*b^2)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
 + (2*a^3*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.699 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2} dx = \frac{a\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{b}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}$$

```
[Out] a*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+b/(a^2-b^2)
/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3234, 3153, 212}

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2} dx = \frac{a\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{b}{(a^2-b^2)(a\cosh(x)+b\sinh(x))}$$

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + b/(
(a^2 - b^2)*(a*Cosh[x] + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}+\frac{a\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}+\frac{(ia)\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= \frac{a\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{b}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.94

$$
\int\frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\sqrt{a-b}b(a+b)+2a^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))}
$$

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a
 - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(S
qrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x] + b*S
inh[x]))
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\frac{2b^2\tanh\left(\frac{x}{2}\right)}{a\left(a^2-b^2\right)}+\frac{2b}{a^2-b^2}}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}}$	98
risch	$\frac{2b\,e^{x}}{(a-b)(a+b)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{a\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{a\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	132

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b^2/a/(a^2-b^2)*tanh(1/2*x)+b/(a^2-b^2))/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x
)+a)+2/(a^2-b^2)^(3/2)*a*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(60) = 120.

Time = 0.26 (sec) , antiderivative size = 596, normalized size of antiderivative = 9.31

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \left[\frac{\left((a^2+ab)\cosh(x)^2+2(a^2+ab)\cosh(x)\sinh(x)+(a^2+ab)\sinh(x)^2+a^2-ab\right)\sqrt{-a^2+b^2}\log\left(\frac{(a+b)}{\cdots}\right.}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)\cosh(x)^2+2(a^5\cdots}\right.$$
$$-\frac{2\left(\left((a^2+ab)\cosh(x)^2+2(a^2+ab)\cosh(x)\sinh(x)+(a^2+ab)\sinh(x)^2+a^2-ab\right)\sqrt{a}\cdots\right.}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)\cosh(x)^2+2(a^5+a^4b\cdots}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh
(x)^2 + a^2 - a*b)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a - b)) + 2*(a^2*b - b^3)*cosh(x) + 2*(a^2*b - b^3)*sinh(x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4
 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^
5)*sinh(x)^2), -2*(((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) +
 (a^2 + a*b)*sinh(x)^2 + a^2 - a*b)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/
((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3
)*sinh(x))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*
```

```
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3
*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2
- 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,a\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{2\,be^x}{(a^2-b^2)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
[Out] 2*a*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*b*e^x/((a
^2 - b^2)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.86

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(a^2\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+a\,b\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}\right)}{a^4\,\sqrt{a^2}-2\,b^2\,{\left(a^2\right)}^{3/2}+b^4\,\sqrt{a^2}+a\,b\,{\left(a^2\right)}^{3/2}-a^3\,b\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}} + \frac{2\,b\,\mathrm{e}^x}{(a+b)\,(a-b)\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

```
[In] int(cosh(x)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] (2*atan((exp(x)*(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a*b*(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)))/(a^4*(a^2)^(1/2) - 2*b^2*(a^2)^(3/2)
+ b^4*(a^2)^(1/2) + a*b*(a^2)^(3/2) - a^3*b*(a^2)^(1/2)))*(a^2)^(1/2))/(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + (2*b*exp(x))/((a + b)*(a - b)*(a -
b + exp(2*x)*(a + b)))
```

3.700 $\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,(a+b\tanh(x))}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2-2*a*b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^2+b/(a^2-b^
2)/(a+b*tanh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3165, 3564, 3612, 3611}

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{x(a^2+b^2)}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,(a+b\tanh(x))} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b
^2)^2 + b/((a^2 - b^2)*(a + b*Tanh[x]))
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
```

```
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+b\tanh(x))^2}\,dx \\
&= \frac{b}{(a^2-b^2)\,(a+b\tanh(x))} + \frac{\int \frac{a-b\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2} \\
&= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,(a+b\tanh(x))} - \frac{(2iab)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{(a^2-b^2)^2} \\
&= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{2ab\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,(a+b\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\,x-2ab\log(a\cosh(x)+b\sinh(x))+\frac{b^2(-a^2+b^2)\sinh(x)}{a(a\cosh(x)+b\sinh(x))}}{(a-b)^2(a+b)^2}$$

[In] Integrate[Cosh[x]^2/(a*Cosh[x] + b*Sinh[x])^2,x]

[Out] ((a^2 + b^2)*x - 2*a*b*Log[a*Cosh[x] + b*Sinh[x]] + (b^2*(-a^2 + b^2)*Sinh[x])/(a*(a*Cosh[x] + b*Sinh[x])))/((a - b)^2*(a + b)^2)

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{(a+b)^2}+\frac{\ln(\tanh(\frac{x}{2})+1)}{(a-b)^2}-\frac{2b\left(\frac{b(a^2-b^2)\tanh(\frac{x}{2})}{a\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)}+a\ln\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)\right)}{(a+b)^2(a-b)^2}$
parallelrisch	$\frac{(-2b^2\tanh(x)a^2-2a^3b)\ln(a+b\tanh(x))+(2b^2\tanh(x)a^2+2a^3b)\ln(1-\tanh(x))+(b(a^2x+a(-1+x)b+b^2)\tanh(x)+a^2x(a+b))(}{(a-b)^2(a+b)^2(a+b\tanh(x))a}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{2b^2}{(a-b)(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{2ab\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$

[In] int(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] -1/(a+b)^2*ln(tanh(1/2*x)-1)+1/(a-b)^2*ln(tanh(1/2*x)+1)-2*b/(a+b)^2/(a-b)^2*(b*(a^2-b^2)/a*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+a*ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.19

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^3+3a^2b+3ab^2+b^3)x\cosh(x)^2+2(a^3+3a^2b+3ab^2+b^3)x\cosh(x)\sinh(x)+(a^3+3a^2b+3ab^2+}{a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5+(a^5+a^4b-2a^3b^2-2a^2b^3}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*a
*b^2 - 2*b^3 + (a^3 + a^2*b - a*b^2 - b^3)*x - 2*(a^2*b - a*b^2 + (a^2*b +
a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^2)*sinh
(x)^2)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
 a*b^4 + b^5)*cosh(x)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
b^5)*cosh(x)*sinh(x) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*
sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(56) = 112$.

Time = 0.76 (sec) , antiderivative size = 952, normalized size of antiderivative = 14.21

$$\int \frac{\cosh^2(x)}{(a\cosh(x) + b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((x - cosh(x)/s
inh(x))/b**2, Eq(a, 0)), (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(
x)*cosh(x) + 8*b**2*cosh(x)**2) - 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 -
16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh(
x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) - sinh(x)**2/(8*b**2*s
inh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 3*cosh(x)**2/(8*
b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, -b)),
 (2*x*sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh
(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x)
+ 8*b**2*cosh(x)**2) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*
cosh(x) + 8*b**2*cosh(x)**2) + sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh
(x)*cosh(x) + 8*b**2*cosh(x)**2) - 3*cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**
2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, b)), (a**3*x*cosh(x)/(a**5*co
sh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4
*cosh(x) + b**5*sinh(x)) + a**2*b*x*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
- 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)
) - 2*a**2*b*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**5*cosh(x) + a**4*b*sinh
(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sin
h(x)) + a**2*b*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x)
 - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a*b**2*x*cosh(x)/
(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x)
+ a*b**4*cosh(x) + b**5*sinh(x)) - 2*a*b**2*log(cosh(x) + b*sinh(x)/a)*sinh
(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh
```

```
(x) + a*b**4*cosh(x) + b**5*sinh(x)) + b**3*x*sinh(x)/(a**5*cosh(x) + a**4*
b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b*
*5*sinh(x)) - b**3*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cos
h(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{2\,b^2}{a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}} + \frac{x}{a^2+2\,ab+b^2}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 2*b^2/(a^4
- 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) + x/(a^2 + 2*
a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,ab\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4} + \frac{x}{a^2-2\,ab+b^2} + \frac{2\,\left(abe^{(2\,x)}+ab-b^2\right)}{(a^3-a^2b-ab^2+b^3)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + x/
(a^2 - 2*a*b + b^2) + 2*(a*b*e^(2*x) + a*b - b^2)/((a^3 - a^2*b - a*b^2 + b
^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\frac{b\cosh(x)}{a^2-b^2}+\frac{x\sinh(x)\,(a^2\,b+b^3)}{(a^2-b^2)^2}+\frac{a\,x\cosh(x)\,(a^2+b^2)}{(a^2-b^2)^2}}{a\cosh\,(x)+b\sinh\,(x)} + \ln\left(a\cosh(x)+b\sinh(x)\right)\,\left(\frac{1}{2\,(a+b)^2}-\frac{1}{2\,(a-b)^2}\right)$$

```
[In] int(cosh(x)^2/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] ((b*cosh(x))/(a^2 - b^2) + (x*sinh(x)*(a^2*b + b^3))/(a^2 - b^2)^2 + (a*x*c
osh(x)*(a^2 + b^2))/(a^2 - b^2)^2)/(a*cosh(x) + b*sinh(x)) + log(a*cosh(x)
+ b*sinh(x))*(1/(2*(a + b)^2) - 1/(2*(a - b)^2))
```

3.701 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3ab^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)} - \frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}$$

```
[Out] -3*a*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+1/(a
+b)^2/(1-tanh(1/2*x))-1/(a-b)^2/(1+tanh(1/2*x))-2*b^3*(a+b*tanh(1/2*x))/a/(
a^2-b^2)^2/(a+2*b*tanh(1/2*x)+a*tanh(1/2*x)^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6874, 652, 632, 210}

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2b^2(3a^2-b^2)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}} - \frac{2b^4\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}} - \frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)\right)} + \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}$$

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) -
(2*b^2*(3*a^2 - b^2)*ArcTan[(b + a*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b
^2)^(5/2)) + 1/((a + b)^2*(1 - Tanh[x/2])) - 1/((a - b)^2*(1 + Tanh[x/2]))
- (2*b^3*(a + b*Tanh[x/2]))/(a*(a^2 - b^2)^2*(a + 2*b*Tanh[x/2] + a*Tanh[x/
2]^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{(1+x^2)^3}{(1-x^2)^2 (a+2bx+ax^2)^2} \, dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{1}{2(a+b)^2(-1+x)^2} + \frac{1}{2(a-b)^2(1+x)^2} - \frac{2b^3x}{a(-a^2+b^2)(a+2bx+ax^2)^2} + \frac{-3a^2b^2+b^4}{a(a^2-b^2)^2(a+2bx+ax^2)}\right) dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)} + \frac{(4b^3)\,\text{Subst}\left(\int \frac{x}{(a+2bx+ax^2)^2} \, dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)} - \frac{(2b^2(3a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2} \, dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)^2}$$

$$= \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)} - \frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)} - \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2} \, dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)^2} + \frac{(4b^2(3a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2} \, dx, x, 2b+2a\tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)^2}$$

$$= -\frac{2b^2(3a^2-b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{5/2}} + \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)} - \frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)} + \frac{(4b^4)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2} \, dx, x, 2b+2a\tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)^2}$$

$$= -\frac{2b^4 \arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}} - \frac{2b^2(3a^2-b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}} + \frac{1}{(a+b)^2\left(1-\tanh\left(\frac{x}{2}\right)\right)} - \frac{1}{(a-b)^2\left(1+\tanh\left(\frac{x}{2}\right)\right)} - \frac{2b^3\left(a+b\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2\left(a+2b\tanh\left(\frac{x}{2}\right)+a\tanh^2\left(\frac{x}{2}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{-\sqrt{a-b}b^3(a+b) - 2a^2\sqrt{a-b}b(a+b)\cosh^2(x) - 6ab^3\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x) + \sqrt{a-b}b(a^3}{(a-b)^{5/2}(a+b)^3(a}$$

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-(Sqrt[a - b]*b^3*(a + b)) - 2*a^2*Sqrt[a - b]*b*(a + b)*Cosh[x]^2 - 6*a*b
^3*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]
+ Sqrt[a - b]*b*(a^3 + a^2*b + a*b^2 + b^3)*Sinh[x]^2 + a*Cosh[x]*(-6*a*b^2
*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + (a - b)^
(3/2)*(a + b)^2*Sinh[x]))/((a - b)^(5/2)*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
default	$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{2b^2\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}+b}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a} + \frac{3a\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2} - \frac{e^{-x}}{2(a^2-2ab+b^2)} - \frac{2b^3 e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)} - \frac{3b^2 a\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2} + \frac{3b^2 a\ln\left(e^x-\frac{a-b}{\sqrt{-a^2-}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a}$

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)-1/(a-b)^2/(tanh(1/2*x)+1)-2*b^2/(a+b)^2/(a-b)^2*
((b^2/a*tanh(1/2*x)+b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+3*a/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(118) = 236.

Time = 0.29 (sec) , antiderivative size = 1645, normalized size of antiderivative = 12.37

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*b -
b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x
)^2 + 6*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^
2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b^2 - a*
b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 +
b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))/((a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*
b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
 -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2 + 6*(a^4*b -
b^5 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x
)^2 - 12*((a^2*b^2 + a*b^3)*cosh(x)^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)*sinh(x)
^2 + (a^2*b^2 + a*b^3)*sinh(x)^3 + (a^2*b^2 - a*b^3)*cosh(x) + (a^2*b^2 - a
*b^3 + 3*(a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(
a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 4*((a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))*sinh(x))
/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^
7)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3
+ 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2
 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b
- 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^
```

```
6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^
2)*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{6\,ab^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{e^x}{2\,(a^2+2\,ab+b^2)} - \frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+5\,b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
[Out] -6*a*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x) + 3*a^2*b*
e^(2*x) + 3*a*b^2*e^(2*x) + 5*b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^3)/((a^
4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.92

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$=\frac{e^x}{2\,(a+b)^2}-\frac{e^{-x}}{2\,(a-b)^2}$$
$$-\frac{6\,\mathrm{atan}\left(\frac{a\,b^2\,e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^2\,b^4}-b^5\,\sqrt{a^2\,b^4}+2\,a^2\,b^3\,\sqrt{a^2\,b^4}-2\,a^3\,b^2\,\sqrt{a^2\,b^4}+a\,b^4\,\sqrt{a^2\,b^4}-a^4\,b\,\sqrt{a^2\,b^4}}\right)\,\sqrt{a^2\,b^4}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}$$
$$-\frac{2\,b^3\,e^x}{(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

```
[In] int(cosh(x)^3/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(x)/(2*(a + b)^2) - exp(-x)/(2*(a - b)^2) - (6*atan((a*b^2*exp(x)*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^2*
b^4)^(1/2) - b^5*(a^2*b^4)^(1/2) + 2*a^2*b^3*(a^2*b^4)^(1/2) - 2*a^3*b^2*(a
^2*b^4)^(1/2) + a*b^4*(a^2*b^4)^(1/2) - a^4*b*(a^2*b^4)^(1/2)))*(a^2*b^4)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
 - (2*b^3*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.702 $\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx=\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

```
[Out] 1/2*tanh(x)^2/a/(a+b*tanh(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3166, 37}

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx=\frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}$$

```
[In] Int[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] Tanh[x]^2/(2*a*(a + b*Tanh[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3166

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[x^m*((a + b*x
```

```
)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{x}{(a-ibx)^3}\,dx, x, i\tanh(x)\right)\\ &= \frac{\tanh^2(x)}{2a(a+b\tanh(x))^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{a^2-b^2+b^2\cosh(2x)+ab\sinh(2x)}{2a(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

```
[In] Integrate[Sinh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -1/2*(a^2 - b^2 + b^2*Cosh[2*x] + a*b*Sinh[2*x])/(a*(a - b)*(a + b)*(a*Cosh
[x] + b*Sinh[x])^2)
```

Maple [A] (verified)

Time = 12.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{2\tanh\left(\frac{x}{2}\right)^2}{a\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a\right)^2}$	31
risch	$-\frac{2\left(a\,e^{2x}+b\,e^{2x}-b\right)}{\left(a\,e^{2x}+b\,e^{2x}+a-b\right)^2(a+b)^2}$	43

```
[In] int(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*tanh(1/2*x)^2/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 216, normalized size of antiderivative = 11.37

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx =$$

$$-\frac{}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\cosh(x)^3+3(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\cosh(x)\sinh(x)^2+(a}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(a*cosh(x) + (a + 2*b)*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh
(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 + 4*
a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2 - 4
*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*s
inh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(17) = 34.

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.79

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx =$$

$$-\frac{2(a-b)e^{(-2x)}}{a^4-2a^2b^2+b^4+2(a^4-2a^3b+2ab^3-b^4)e^{(-2x)}+(a^4-4a^3b+6a^2b^2-4ab^3+b^4)e^{(-4x)}}$$
$$-\frac{2b}{a^4-2a^2b^2+b^4+2(a^4-2a^3b+2ab^3-b^4)e^{(-2x)}+(a^4-4a^3b+6a^2b^2-4ab^3+b^4)e^{(-4x)}}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b
^4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) - 2*b/
(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.63

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{2\left(ae^{(2x)}+be^{(2x)}-b\right)}{(a^2+2ab+b^2)(ae^{(2x)}+be^{(2x)}+a-b)^2}$$

```
[In] integrate(sinh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*x) + b*e^(2*x) - b)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
+ a - b)^2)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{2b-\mathrm{e}^{2x}\,(2a+2b)}{(a+b)^2\,(a-b+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x})^2}$$

```
[In] int(sinh(x)/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] (2*b - exp(2*x)*(2*a + 2*b))/((a + b)^2*(a - b + a*exp(2*x) + b*exp(2*x))^2
)
```

3.703 $\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{b(3a^2+b^2)\,x}{(a^2-b^2)^3} - \frac{a}{2\,(a^2-b^2)\,(b+a\coth(x))^2} + \frac{2ab}{(a^2-b^2)^2\,(b+a\coth(x))} + \frac{a(a^2+3b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}$$

```
[Out] -b*(3*a^2+b^2)*x/(a^2-b^2)^3-1/2*a/(a^2-b^2)/(b+a*coth(x))^2+2*a*b/(a^2-b^2
)^2/(b+a*coth(x))+a*(a^2+3*b^2)*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3164, 3564, 3610, 3612, 3611}

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{bx(3a^2+b^2)}{(a^2-b^2)^3} + \frac{2ab}{(a^2-b^2)^2\,(a\coth(x)+b)} - \frac{a}{2\,(a^2-b^2)\,(a\coth(x)+b)^2} + \frac{a(a^2+3b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}$$

```
[In] Int[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -((b*(3*a^2 + b^2)*x)/(a^2 - b^2)^3) - a/(2*(a^2 - b^2)*(b + a*Coth[x])^2)
+ (2*a*b)/((a^2 - b^2)^2*(b + a*Coth[x])) + (a*(a^2 + 3*b^2)*Log[a*Cosh[x]
+ b*Sinh[x]])/(a^2 - b^2)^3
```

Rule 3164

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = i\int \frac{1}{(-ib-ia\coth(x))^3}\,dx$$

$$= -\frac{a}{2\left(a^2-b^2\right)(b+a\coth(x))^2}+\frac{i\int \frac{-ib+ia\coth(x)}{(-ib-ia\coth(x))^2}\,dx}{a^2-b^2}$$

$$= -\frac{a}{2\left(a^2-b^2\right)\left(b+a\coth(x)\right)^2}+\frac{2ab}{\left(a^2-b^2\right)^2\left(b+a\coth(x)\right)}+\frac{i\int\frac{-a^2-b^2+2ab\coth(x)}{-ib-ia\coth(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= -\frac{b(3a^2+b^2)\,x}{\left(a^2-b^2\right)^3}-\frac{a}{2\left(a^2-b^2\right)\left(b+a\coth(x)\right)^2}+\frac{2ab}{\left(a^2-b^2\right)^2\left(b+a\coth(x)\right)}+\frac{\left(ia(a^2+3b^2)\right)\int\frac{-a-b\coth(x)}{-ib-ia\coth(x)}\,dx}{\left(a^2-b^2\right)^3}$$

$$= -\frac{b(3a^2+b^2)\,x}{\left(a^2-b^2\right)^3}-\frac{a}{2\left(a^2-b^2\right)\left(b+a\coth(x)\right)^2}+\frac{2ab}{\left(a^2-b^2\right)^2\left(b+a\coth(x)\right)}+\frac{a(a^2+3b^2)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int\frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{b(3a^2+b^2)\,x}{(a-b)^3(a+b)^3}+\frac{(a^3+3ab^2)\log(a\cosh(x)+b\sinh(x))}{\left(a^2-b^2\right)^3}+\frac{a^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2}+\frac{3ab\sinh(x)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}$$

```
[In] Integrate[Sinh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
[Out] -((b*(3*a^2 + b^2)*x)/((a - b)^3*(a + b)^3)) + ((a^3 + 3*a*b^2)*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 + a^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*
Sinh[x])^2) + (3*a*b*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^3}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{(a-b)^3}+\frac{2a\left(\frac{2ba\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^4+6a^2b^2-5b^4\right)\tanh\left(\frac{x}{2}\right)^2+2ba\left(a^2-b^2\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)^2}+\frac{\left(a^2 \cdots\right.}{\cdots}\right.}{(a+b)^3(a-b)^3}$
parallelrisch	$\frac{2b^2a\left(a^2+3b^2\right)(a+b\tanh(x))^2\ln(a+b\tanh(x))-2b^2a\left(a^2+3b^2\right)(a+b\tanh(x))^2\ln(1-\tanh(x))+\left(-2x\,b^4(a+b)^2\tanh(x)^2+2(-\cdots\right.}{2(a-b)^3(a+b)^3b^2(a+b\cdots}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2a^3x}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{6ax\,b^2}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2a^2\left(a^2\mathrm{e}^{2x}-2b\,\mathrm{e}^{2x}a-3b^2\mathrm{e}^{2x}-3ab+3b^2\right)}{(a-b)^2\left(a^3+3a^2b+3a\,b^2+b^3\right)\left(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b\right)^2}$

```
[In] int(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^3*ln(tanh(1/2*x)-1)-1/(a-b)^3*ln(tanh(1/2*x)+1)+2*a/(a+b)^3/(a-b)^
3*((2*b*a*(a^2-b^2)*tanh(1/2*x)^3+(-a^4+6*a^2*b^2-5*b^4)*tanh(1/2*x)^2+2*b*
a*(a^2-b^2)*tanh(1/2*x))/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)^2+1/2*(a^2+3*b
^2)*ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 1268, normalized size of antiderivative = 12.19

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4
*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(x
)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)^4
 + 6*a^4*b - 12*a^3*b^2 + 6*a^2*b^3 - 2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^
3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 -
2*(a^5 - 3*a^4*b - a^3*b^2 + 3*a^2*b^3 - 3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*
b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +
a*b^4 + b^5)*x - (a^5 - 2*a^4*b + 4*a^3*b^2 - 6*a^2*b^3 + 3*a*b^4 + (a^5 +
2*a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + 4
*a^3*b^2 + 6*a^2*b^3 + 3*a*b^4)*cosh(x)*sinh(x)^3 + (a^5 + 2*a^4*b + 4*a^3*
b^2 + 6*a^2*b^3 + 3*a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(x
)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 + 3*(a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*b
^3 + 3*a*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + 4*a^3*b^2 + 6*a^2*
b^3 + 3*a*b^4)*cosh(x)^3 + (a^5 + 2*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x))*lo
g(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 - (a^5 - 3*a^4*b - a^3*b^2
 + 3*a^2*b^3 - (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*c
osh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2
*b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 +
 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^4 + 4*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^
5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)*sinh(x)^3 + (a^8 + 2
*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*sin
h(x)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 2*(a
^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 3*(a^8 + 2*a^7*b - 2*a^6*b^2
 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^2)*sinh(x)^2
+ 4*((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b
^7 - b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(
x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3813 vs. $2(88) = 176$.

Time = 1.86 (sec) , antiderivative size = 3813, normalized size of antiderivative = 36.66

$$\int \frac{\sinh^3(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b**3, Eq(a, 0)), (-3*x*sinh(x)**
3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(
x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)**2*cosh(x)/(-24*b**3*sinh(x)**3 +
 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)*
*3) - 9*x*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh
(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 3*x*cosh(x)**3/(-2
4*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2
 + 24*b**3*cosh(x)**3) - 7*sinh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x
)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 6*sinh(x)
*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sin
h(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 3*cosh(x)**3/(-24*b**3*sinh(x)**3 +
 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)*
*3), Eq(a, -b)), (3*x*sinh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*c
osh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)**2*
cosh(x)/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*
cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(24*b**3*sinh(x)*
*3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh
(x)**3) + 3*x*cosh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) +
 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 7*sinh(x)**3/(24*b**3*s
inh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b*
*3*cosh(x)**3) + 6*sinh(x)*cosh(x)**2/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)
**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 3*cosh(x)*
*3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(
x)**2 + 24*b**3*cosh(x)**3), Eq(a, b)), (2*a**5*log(cosh(x) + b*sinh(x)/a)*
cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh
(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**
4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a*
*2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*
b**8*sinh(x)**2) - a**5*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*co
sh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh
(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**
5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b
**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 2*a**5*cosh(x)**2/(2*a**8*cosh(x
)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh
(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**
```

4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a*
*2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 6*a**4
*b*x*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2
*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**
4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) +
 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x)
 - 2*b**8*sinh(x)**2) + 4*a**4*b*log(cosh(x) + b*sinh(x)/a)*sinh(x)*cosh(x)
/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6
*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)*
*2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*si
nh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(
x)**2) - 12*a**3*b**2*x*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(
x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3
*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**
3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 -
4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 2*a**3*b**2*log(cosh(x) + b
*sinh(x)/a)*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a*
*6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x)
 - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*co
sh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*
cosh(x) - 2*b**8*sinh(x)**2) + 6*a**3*b**2*log(cosh(x) + b*sinh(x)/a)*cosh(
x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**
2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sin
h(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b*
*6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*
sinh(x)**2) + 4*a**3*b**2*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*
cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*si
nh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b
5*sinh(x)*cosh(x) + 6*a2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a
*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 2*a**3*b**2*cosh(x)**2/(2*a**8
*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b*
*2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a
4*b4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2
 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) -
 6*a**2*b**3*x*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2
*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(
x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)
*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(
x)*cosh(x) - 2*b**8*sinh(x)**2) - 2*a**2*b**3*x*cosh(x)**2/(2*a**8*cosh(x)*
*2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x
)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*
cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2
*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 12*a**2*
b**3*log(cosh(x) + b*sinh(x)/a)*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7
*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a

```
**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2
+ 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(
x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 4*a*b**4*x*sinh(x)*
cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)
**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*s
inh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*
b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**
8*sinh(x)**2) + 6*a*b**4*log(cosh(x) + b*sinh(x)/a)*sinh(x)**2/(2*a**8*cosh
(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*co
sh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b
**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*
a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 3*a*
b**4*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2
*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**
4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) +
 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x)
 - 2*b**8*sinh(x)**2) - 2*b**5*x*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*s
inh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*
b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12
*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**
2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(102) = 204$.

Time = 0.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{(a^3+3\,ab^2)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{2\left(3\,a^3b+3\,a^2b^2+\left(a^4+2\,a^3b-3\right.\right.}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\left(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^5\right.} + \frac{x}{a^3+3\,a^2b+3\,ab^2+b^3}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] (a^3 + 3*a*b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + 2*(3*a^3*b + 3*a^2*b^2 + (a^4 + 2*a^3*b - 3*a^2*b^2)*e^(-2*x))/(a
^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 +
2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^
7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 +
3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.41

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{(a^3+3\,ab^2)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{x}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{3\,a^4e^{(4\,x)}+3\,a^3be^{(4\,x)}+9\,a^2b^2e^{(4\,x)}+9\,ab^3e^{(4\,x)}+2\,a^4e^{(2\,x)}+10\,a^3be^{(2\,x)}+6\,a^2b^2e^{(2\,x)}-18\,ab^3e^{(2\,x)}+3\,a^4}{2\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)(ae^{(2\,x)}+be^{(2\,x)}+a-b)^2}$$

```
[In] integrate(sinh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (a^3 + 3*a*b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6) - x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/2*(3*a^4*e^(4*x) +
 3*a^3*b*e^(4*x) + 9*a^2*b^2*e^(4*x) + 9*a*b^3*e^(4*x) + 2*a^4*e^(2*x) + 10
*a^3*b*e^(2*x) + 6*a^2*b^2*e^(2*x) - 18*a*b^3*e^(2*x) + 3*a^4 + 3*a^3*b - 1
5*a^2*b^2 + 9*a*b^3)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*(
a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(a^3+3\,a\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6} - \frac{x}{(a-b)^3} - \frac{2\,(3\,a^2\,b-a^3)}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)} - \frac{2\,a^3}{(a+b)^3\,(a-b)\,\left(e^{4\,x}\,(a+b)^2+(a-b)^2+2\,e^{2\,x}\,(a+b)\,(a-b)\right)}$$

```
[In] int(sinh(x)^3/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] (log(a - b + a*exp(2*x) + b*exp(2*x))*(3*a*b^2 + a^3))/(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2) - x/(a - b)^3 - (2*(3*a^2*b - a^3))/((a + b)^3*(a - b)^2*(a
 - b + exp(2*x)*(a + b))) - (2*a^3)/((a + b)^3*(a - b)*(exp(4*x)*(a + b)^2
+ (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

3.704 $\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3} \, dx = -\frac{\coth^2(x)}{2b(b+a\coth(x))^2}$$

```
[Out] -1/2*coth(x)^2/b/(b+a*coth(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3167, 37}

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3} \, dx = -\frac{\coth^2(x)}{2b(a\coth(x)+b)^2}$$

```
[In] Int[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] -1/2*Coth[x]^2/(b*(b + a*Coth[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
```

```
a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\text{integral} = i\text{Subst}\left(\int \frac{x}{(-ib+ax)^3}\,dx, x, -i\coth(x)\right)$$
$$= -\frac{\coth^2(x)}{2b(b+a\coth(x))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{b\cosh(2x)+a\sinh(2x)}{2(a-b)(a+b)(a\cosh(x)+b\sinh(x))^2}$$

```
[In] Integrate[Cosh[x]/(a*Cosh[x] + b*Sinh[x])^3,x]
[Out] (b*Cosh[2*x] + a*Sinh[2*x])/(2*(a - b)*(a + b)*(a*Cosh[x] + b*Sinh[x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.

Time = 5.86 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

method	result	size
risch	$-\frac{2(a\,e^{2x}+b\,e^{2x}+a)}{(a\,e^{2x}+b\,e^{2x}+a-b)^2(a+b)^2}$	41
default	$-\frac{2\left(-\frac{\tanh(\frac{x}{2})^3}{a}-\frac{b\tanh(\frac{x}{2})^2}{a^2}-\frac{\tanh(\frac{x}{2})}{a}\right)}{\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)^2}$	55

```
[In] int(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] -2*(a*exp(2*x)+b*exp(2*x)+a)/(a*exp(2*x)+b*exp(2*x)+a-b)^2/(a+b)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 11.37

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx =$$

$$-\frac{}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\cosh(x)^3+3(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\cosh(x)\sinh(x)^2+(a}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*((2*a + b)*cosh(x) + b*sinh(x))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)*sinh
(x)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(x)^3 + (3*a^4 + 4*
a^3*b - 2*a^2*b^2 - 4*a*b^3 - b^4)*cosh(x) + (a^4 + 4*a^3*b + 2*a^2*b^2 - 4
*a*b^3 - 3*b^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(x)^2)*s
inh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.79

$$\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$$

$$= \frac{2(a-b)e^{(-2x)}}{a^4-2a^2b^2+b^4+2(a^4-2a^3b+2ab^3-b^4)e^{(-2x)}+(a^4-4a^3b+6a^2b^2-4ab^3+b^4)e^{(-4x)}} + \frac{2a}{a^4-2a^2b^2+b^4+2(a^4-2a^3b+2ab^3-b^4)e^{(-2x)}+(a^4-4a^3b+6a^2b^2-4ab^3+b^4)e^{(-4x)}}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(a - b)*e^(-2*x)/(a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^
4)*e^(-2*x) + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x)) + 2*a/(
a^4 - 2*a^2*b^2 + b^4 + 2*(a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x) + (a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = -\frac{2\left(ae^{(2x)} + be^{(2x)} + a\right)}{(a^2 + 2ab + b^2)(ae^{(2x)} + be^{(2x)} + a - b)^2}$$

```
[In] integrate(cosh(x)/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
[Out] -2*(a*e^(2*x) + b*e^(2*x) + a)/((a^2 + 2*a*b + b^2)*(a*e^(2*x) + b*e^(2*x)
+ a - b)^2)
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{\cosh(x)}{(a\cosh(x) + b\sinh(x))^3}\,dx = -\frac{2a + e^{2x}(2a + 2b)}{(a+b)^2 (a - b + a e^{2x} + b e^{2x})^2}$$

```
[In] int(cosh(x)/(a*cosh(x) + b*sinh(x))^3,x)
[Out] -(2*a + exp(2*x)*(2*a + 2*b))/((a + b)^2*(a - b + a*exp(2*x) + b*exp(2*x))^
2)
```

3.705 $\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx=\frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3}-\frac{b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{b}{2\,(a^2-b^2)\,(a+b\tanh(x))^2}+\frac{2ab}{(a^2-b^2)^2\,(a+b\tanh(x))}$$

```
[Out] a*(a^2+3*b^2)*x/(a^2-b^2)^3-b*(3*a^2+b^2)*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
^3+1/2*b/(a^2-b^2)/(a+b*tanh(x))^2+2*a*b/(a^2-b^2)^2/(a+b*tanh(x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3165, 3564, 3610, 3612, 3611}

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx=\frac{ax(a^2+3b^2)}{(a^2-b^2)^3}+\frac{2ab}{(a^2-b^2)^2\,(a+b\tanh(x))}+\frac{b}{2\,(a^2-b^2)\,(a+b\tanh(x))^2}-\frac{b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}$$

```
[In] Int[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[x] + b*Sinh
[x]])/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*(a + b*Tanh[x])^2) + (2*a*b)/((a^2 -
 b^2)^2*(a + b*Tanh[x]))
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(a+b\tanh(x))^3}\,dx \\
&= \frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{\int \frac{a-b\tanh(x)}{(a+b\tanh(x))^2}\,dx}{a^2-b^2} \\
&= \frac{b}{2\left(a^2-b^2\right)(a+b\tanh(x))^2}+\frac{2ab}{\left(a^2-b^2\right)^2(a+b\tanh(x))}+\frac{\int \frac{a^2+b^2-2ab\tanh(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2}
\end{aligned}$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2\,(a^2-b^2)\,(a+b\tanh(x))^2} + \frac{2ab}{(a^2-b^2)^2\,(a+b\tanh(x))} - \frac{(ib(3a^2+b^2))\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{(a^2-b^2)^3}$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} - \frac{b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{b}{2\,(a^2-b^2)\,(a+b\tanh(x))^2} + \frac{2ab}{(a^2-b^2)^2\,(a+b\tanh(x))}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \frac{a(a^2+3b^2)\,x}{(a-b)^3(a+b)^3} + \frac{(-3a^2b-b^3)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{b^3}{2(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))^2} - \frac{3b^2\sinh(x)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}$$

```
[In] Integrate[Cosh[x]^3/(a*Cosh[x] + b*Sinh[x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/((a - b)^3*(a + b)^3) + ((-3*a^2*b - b^3)*Log[a*Cosh[x]
 + b*Sinh[x]])/(a^2 - b^2)^3 - b^3/(2*(a - b)^2*(a + b)^2*(a*Cosh[x] + b*Si
nh[x])^2) - (3*b^2*Sinh[x])/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.52

method	result
parallelrisch	$\frac{-3(a+b\tanh(x))^2 b\left(a^2+\frac{b^2}{3}\right)\ln(a+b\tanh(x))+3(a+b\tanh(x))^2 b\left(a^2+\frac{b^2}{3}\right)\ln(1-\tanh(x))+(a+b)\left(x\,b^2(a+b)^2\tanh(x)^2+2(b^2\right.}{(a-b)^3(a+b)^3(a+b\tanh(x))^2}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^3}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{(a-b)^3}-\frac{2b\left(\frac{\frac{b\left(3a^4-4a^2b^2+b^4\right)\tanh\left(\frac{x}{2}\right)^3}{a}+\frac{b^2\left(5a^4-6a^2b^2+b^4\right)\tanh\left(\frac{x}{2}\right)^2}{a^2}+\frac{b\left(3a^4-4a^2b^2+b^4\right)\mathrm{t}}{a}}{\left(\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a\right)^2}\right.}{(a+b)^3(a-b)^3}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6b\,a^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^2\left(3a^2\mathrm{e}^{2x}+2b\,\mathrm{e}^{2x}a-b^2\mathrm{e}^{2x}+3a^2-3ab\right)}{(a-b)^2\left(a^3+3a^2b+3a\,b^2+b^3\right)\left(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b\right)^2}$

```
[In] int(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3*(a+b*tanh(x))^2*b*(a^2+1/3*b^2)*ln(a+b*tanh(x))+3*(a+b*tanh(x))^2*b*(a^
2+1/3*b^2)*ln(1-tanh(x))+(a+b)*(x*b^2*(a+b)^2*tanh(x)^2+2*(b^2*(-1+x)+a*(1+
2*x)*b+a^2*x)*b*a*tanh(x)+1/2*b^4-1/2*a*b^3+a^2*(x-5/2)*b^2+2*a^3*(x+5/4)*b
+x*a^4))/(a-b)^3/(a+b)^3/(a+b*tanh(x))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1269 vs. $2(102) = 204$.

Time = 0.26 (sec) , antiderivative size = 1269, normalized size of antiderivative = 12.20

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^4 + 4*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)*sinh(x)
^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*sinh(x)^4
+ 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5
 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x)^2 + 2
*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 + 3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a
*b^4 + b^5)*x - (3*a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b
 + 6*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^4*b + 6*a^3*b^
2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (3*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + 2*a*b^4 + b^5)*sinh(x)^4 + 2*(3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x)
^2 + 2*(3*a^4*b - 2*a^2*b^3 - b^5 + 3*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2*
a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2
*a*b^4 + b^5)*cosh(x)^3 + (3*a^4*b - 2*a^2*b^3 - b^5)*cosh(x))*sinh(x))*log
(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^5 + 5*a^4*b + 10*a^
3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x*cosh(x)^3 + (3*a^3*b^2 - a^2*b^3 - 3*
a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*co
sh(x))*sinh(x))/(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*
b^6 + 2*a*b^7 - b^8 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 +
2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^4 + 4*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5
*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)*sinh(x)^3 + (a^8 + 2*
a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*sinh
(x)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 2*(a^
8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 3*(a^8 + 2*a^7*b - 2*a^6*b^2
- 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*cosh(x)^2)*sinh(x)^2 +
 4*((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^
7 - b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x
))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3840 vs. 2(88) = 176.

Time = 1.77 (sec) , antiderivative size = 3840, normalized size of antiderivative = 36.92

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)**3/(a*cosh(x)+b*sinh(x))**3,x)
[Out] Piecewise((zoo*(log(sinh(x)) - cosh(x)**2/(2*sinh(x)**2)), Eq(a, 0) & Eq(b,
 0)), ((log(sinh(x)) - cosh(x)**2/(2*sinh(x)**2))/b**3, Eq(a, 0)), (3*x*sin
h(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)
*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*x*sinh(x)**2*cosh(x)/(-24*b**3*sinh(x
)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*co
sh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**
2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 3*x*cosh(x)*
*3/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh
(x)**2 + 24*b**3*cosh(x)**3) - sinh(x)**3/(-24*b**3*sinh(x)**3 + 72*b**3*si
nh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 6*sin
h(x)*cosh(x)**2/(-24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3
*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*cosh(x)**3/(-24*b**3*sinh(x)*
*3 + 72*b**3*sinh(x)**2*cosh(x) - 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh
(x)**3), Eq(a, -b)), (3*x*sinh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)*
*2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)
**2*cosh(x)/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh
(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + 9*x*sinh(x)*cosh(x)**2/(24*b**3*sinh
(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*
cosh(x)**3) + 3*x*cosh(x)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(
x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) + sinh(x)**3/(24*b**3
*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*
b**3*cosh(x)**3) - 6*sinh(x)*cosh(x)**2/(24*b**3*sinh(x)**3 + 72*b**3*sinh(
x)**2*cosh(x) + 72*b**3*sinh(x)*cosh(x)**2 + 24*b**3*cosh(x)**3) - 9*cosh(x
)**3/(24*b**3*sinh(x)**3 + 72*b**3*sinh(x)**2*cosh(x) + 72*b**3*sinh(x)*cos
h(x)**2 + 24*b**3*cosh(x)**3), Eq(a, b)), (2*a**5*x*cosh(x)**2/(2*a**8*cosh
(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*co
sh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b
**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*
a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 4*a*
*4*b*x*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a*
*6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x)
- 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*co
sh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*
cosh(x) - 2*b**8*sinh(x)**2) - 6*a**4*b*log(cosh(x) + b*sinh(x)/a)*cosh(x)*
*2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 -
```

6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x
)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*
sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sin
h(x)**2) + 3*a**4*b*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x
) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*
cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*si
nh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*
sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 2*a**3*b**2*x*sinh(x)**2/(2*a**8*cos
h(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*c
osh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*
b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2
*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) + 6*a
3*b2*x*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**
6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) -
6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cos
h(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*c
osh(x) - 2*b**8*sinh(x)**2) - 12*a**3*b**2*log(cosh(x) + b*sinh(x)/a)*sinh(
x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh
(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**
4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a*
*2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*
b**8*sinh(x)**2) + 12*a**2*b**3*x*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a*
*7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12
*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**
2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cos
h(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 6*a**2*b**3*log(c
osh(x) + b*sinh(x)/a)*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh
(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x
)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*
sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**
7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 2*a**2*b**3*log(cosh(x) + b*sinh(x
)/a)*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2
*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**
4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) +
6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x)
- 2*b**8*sinh(x)**2) - 2*a**2*b**3*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*
b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a*
*5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 +
12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x
)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 4*a**2*b**3*cosh(x)*
*2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 -
6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x
)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*
sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sin
h(x)**2) + 6*a*b**4*x*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh

```
(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x
)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*
sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**
7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2) - 4*a*b**4*log(cosh(x) + b*sinh(x)/a
)*sinh(x)*cosh(x)/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b*
*2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a
**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x)
 + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(
x) - 2*b**8*sinh(x)**2) - 2*b**5*log(cosh(x) + b*sinh(x)/a)*sinh(x)**2/(2*a
**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6
*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 +
6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)
**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2
) + 2*b**5*sinh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b*sinh(x)*cosh(x) + 2*a**
6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**5*b**3*sinh(x)*cosh(x) -
 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 + 12*a**3*b**5*sinh(x)*cos
h(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)**2 - 4*a*b**7*sinh(x)*c
osh(x) - 2*b**8*sinh(x)**2) + b**5*cosh(x)**2/(2*a**8*cosh(x)**2 + 4*a**7*b
*sinh(x)*cosh(x) + 2*a**6*b**2*sinh(x)**2 - 6*a**6*b**2*cosh(x)**2 - 12*a**
5*b**3*sinh(x)*cosh(x) - 6*a**4*b**4*sinh(x)**2 + 6*a**4*b**4*cosh(x)**2 +
12*a**3*b**5*sinh(x)*cosh(x) + 6*a**2*b**6*sinh(x)**2 - 2*a**2*b**6*cosh(x)
**2 - 4*a*b**7*sinh(x)*cosh(x) - 2*b**8*sinh(x)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(102) = 204$.

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.81

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx = -\frac{(3\,a^2b+b^3)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{2\,(3\,a^2b^2+3\,ab^3+(3\,a^2b^2-2\,ab^3}{a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\,(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^5} + \frac{x}{a^3+3\,a^2b+3\,ab^2+b^3}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - 2*(3*a^2*b^2 + 3*a*b^3 + (3*a^2*b^2 - 2*a*b^3 - b^4)*e^(-2*x))/(
a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 +
 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b
^7)*e^(-2*x) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 +
 3*a*b^6 - b^7)*e^(-4*x)) + x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(102) = 204$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.41

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$$
$$= -\frac{(3\,a^2b+b^3)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{x}{a^3-3\,a^2b+3\,ab^2-b^3}$$
$$+\frac{9\,a^3be^{(4\,x)}+9\,a^2b^2e^{(4\,x)}+3\,ab^3e^{(4\,x)}+3\,b^4e^{(4\,x)}+18\,a^3be^{(2\,x)}-6\,a^2b^2e^{(2\,x)}-10\,ab^3e^{(2\,x)}-2\,b^4e^{(2\,x)}+9\,a^3}{2\,(a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5)(ae^{(2\,x)}+be^{(2\,x)}+a-b)^2}$$

```
[In] integrate(cosh(x)^3/(a*cosh(x)+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b + b^3)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6) + x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/2*(9*a^3*b*e^(4*x
) + 9*a^2*b^2*e^(4*x) + 3*a*b^3*e^(4*x) + 3*b^4*e^(4*x) + 18*a^3*b*e^(2*x)
- 6*a^2*b^2*e^(2*x) - 10*a*b^3*e^(2*x) - 2*b^4*e^(2*x) + 9*a^3*b - 15*a^2*b
^2 + 3*a*b^3 + 3*b^4)/((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
(a*e^(2*x) + b*e^(2*x) + a - b)^2)
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)}{(a\cosh(x)+b\sinh(x))^3}\,dx$$
$$= \frac{x}{(a-b)^3}-\frac{\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,(3\,a^2\,b+b^3)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}$$
$$+\frac{2\,(3\,a\,b^2-b^3)}{(a+b)^3\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$
$$+\frac{2\,b^3}{(a+b)^3\,(a-b)\,\left(e^{4\,x}\,(a+b)^2+(a-b)^2+2\,e^{2\,x}\,(a+b)\,(a-b)\right)}$$

```
[In] int(cosh(x)^3/(a*cosh(x) + b*sinh(x))^3,x)
```

```
[Out] x/(a - b)^3 - (log(a - b + a*exp(2*x) + b*exp(2*x))*(3*a^2*b + b^3))/(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*(3*a*b^2 - b^3))/((a + b)^3*(a - b)^2*(a
 - b + exp(2*x)*(a + b))) + (2*b^3)/((a + b)^3*(a - b)*(exp(4*x)*(a + b)^2
+ (a - b)^2 + 2*exp(2*x)*(a + b)*(a - b)))
```

3.706 $\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{ab\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} - \frac{b\sinh(x)}{a^2-b^2}$$

```
[Out] a*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(x)
/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3188, 2717, 2718, 3153, 212}

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{ab\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \frac{b\sinh(x)}{a^2-b^2} + \frac{a\cosh(x)}{a^2-b^2}$$

```
[In] Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \sinh(x)\,dx}{a^2-b^2}-\frac{b\int \cosh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}+\frac{(iab)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= \frac{ab\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2ab\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} + \frac{b\sinh(x)}{-a^2+b^2}$$

```
[In] Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*
(a + b)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) + (b*Sinh[x])/(-a^2 + b^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{2ab\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a+b)(a-b)\sqrt{a^2-b^2}}+\frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	92
risch	$\frac{e^{x}}{2a+2b}+\frac{e^{-x}}{2a-2b}-\frac{ba\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{ba\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	120

```
[In] int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2
)^(1/2))+4/(4*a-4*b)/(tanh(1/2*x)+1)-4/(4*a+4*b)/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.93

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \left[\frac{a^3+a^2b-ab^2-b^3+(a^3-a^2b-ab^2+b^3)\cosh(x)^2+2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)+(a^3-}{2((a^4-2}\right.$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cos
```

h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)
*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x
) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^
2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(58) = 116$.

Time = 124.53 (sec) , antiderivative size = 678, normalized size of antiderivative = 9.42

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\sinh(x) \\ \frac{\sinh(x)}{b} \\ -\frac{\sinh^2(x)}{-3b\sinh(x)+3b\cosh(x)}+\frac{\sinh(x)\cosh(x)}{-3b\sinh(x)+3b\cosh(x)}-\frac{\cosh^2(x)}{-3b\sinh(x)+3b\cosh(x)} \\ \frac{\sinh^2(x)}{3b\sinh(x)+3b\cosh(x)}+\frac{\sinh(x)\cosh(x)}{3b\sinh(x)+3b\cosh(x)}+\frac{\cosh^2(x)}{3b\sinh(x)+3b\cosh(x)} \\ \frac{ab\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)+b^2\sqrt{-a^2+b^2}} - \frac{ab\log\left(\tanh\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{a^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)-a^2\sqrt{-a^2+b^2}-b^2\sqrt{-a^2+b^2}\tanh^2\left(\frac{x}{2}\right)} \end{cases}$$

[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x)

[Out] Piecewise((zoo*sinh(x), Eq(a, 0) & Eq(b, 0)), (sinh(x)/b, Eq(a, 0)), (-sinh
(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)) + sinh(x)*cosh(x)/(-3*b*sinh(x) + 3*b*c
osh(x)) - cosh(x)**2/(-3*b*sinh(x) + 3*b*cosh(x)), Eq(a, -b)), (sinh(x)**2/
(3*b*sinh(x) + 3*b*cosh(x)) + sinh(x)*cosh(x)/(3*b*sinh(x) + 3*b*cosh(x)) +
 cosh(x)**2/(3*b*sinh(x) + 3*b*cosh(x)), Eq(a, b)), (a*b*log(tanh(x/2) + b/
a - sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**
2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*s
qrt(-a**2 + b**2)) - a*b*log(tanh(x/2) + b/a - sqrt(-a**2 + b**2)/a)/(a**2*
sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2
 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - a*b*log(tanh(x/2) + b/a
+ sqrt(-a**2 + b**2)/a)*tanh(x/2)**2/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2
- a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqr
t(-a**2 + b**2)) + a*b*log(tanh(x/2) + b/a + sqrt(-a**2 + b**2)/a)/(a**2*sq
rt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a**2 +
 b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) - 2*a*sqrt(-a**2 + b**2)/(a*
*2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2) - b**2*sqrt(-a
2 + b2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)) + 2*b*sqrt(-a**2 + b**2

```
)*tanh(x/2)/(a**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 - a**2*sqrt(-a**2 + b**2)
 - b**2*sqrt(-a**2 + b**2)*tanh(x/2)**2 + b**2*sqrt(-a**2 + b**2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{2\,ab\arctan\left(\frac{ae^x + be^x}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{\frac{3}{2}}} + \frac{e^{(-x)}}{2\,(a - b)} + \frac{e^x}{2\,(a + b)}$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 1/2*e^(-x
)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.18

$$\int \frac{\cosh(x)\sinh(x)}{a\cosh(x) + b\sinh(x)}\,dx = \frac{e^x}{2\,a + 2\,b} + \frac{e^{-x}}{2\,a - 2\,b} + \frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{a^6 - 3\,a^4\,b^2 + 3\,a^2\,b^4 - b^6}}{a^3\,\sqrt{a^2\,b^2} + b^3\,\sqrt{a^2\,b^2} - a\,b^2\,\sqrt{a^2\,b^2} - a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{a^6 - 3\,a^4\,b^2 + 3\,a^2\,b^4 - b^6}}$$

```
[In] int((cosh(x)*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2) -
 a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.707 $\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2x}{(a^2-b^2)^2}-\frac{ax}{2(a^2-b^2)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}+\frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)}-\frac{b\sinh^2(x)}{2(a^2-b^2)}$$

```
[Out] -a*b^2*x/(a^2-b^2)^2-1/2*a*x/(a^2-b^2)+a^2*b*ln(a*cosh(x)+b*sinh(x))/(a^2-b
^2)^2+1/2*a*cosh(x)*sinh(x)/(a^2-b^2)-1/2*b*sinh(x)^2/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3188, 2644, 30, 2715, 8, 3176, 3212}

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ax}{2(a^2-b^2)}-\frac{ab^2x}{(a^2-b^2)^2}-\frac{b\sinh^2(x)}{2(a^2-b^2)}+\frac{a\sinh(x)\cosh(x)}{2(a^2-b^2)}+\frac{a^2b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) - (a*x)/(2*(a^2 - b^2)) + (a^2*b*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2 - b^2)^2 + (a*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) - (b*Sinh[
x]^2)/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral} = \frac{a\int \sinh^2(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)} + \frac{(ia^2b)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} - \frac{a\int 1\,dx}{2(a^2-b^2)} + \frac{b\text{Subst}(\int x\,dx, x, i\sinh(x))}{a^2-b^2}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} - \frac{ax}{2(a^2-b^2)} + \frac{a^2b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{a\cosh(x)\sinh(x)}{2(a^2-b^2)} - \frac{b\sinh^2(x)}{2(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(-a^2b+b^3)\cosh(2x)+a(-2(a^2+b^2)x+4ab\log(a\cosh(x)+b\sinh(x))+(a^2-b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(-2*(a^2 + b^2)*x + 4*a*b*Log[a*Cosh[x] + b
*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{ax}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}-\frac{2a^2bx}{a^4-2a^2b^2+b^4}+\frac{a^2b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{8}{(16a+16b)(\tanh(\frac{x}{2})-1)}+\frac{4}{(\tanh(\frac{x}{2})-1)^2(8a+8b)}+\frac{a\ln(\tanh(\frac{x}{2})-1)}{2(a+b)^2}-\frac{4}{(\tanh(\frac{x}{2})+1)^2(8a-8b)}+\frac{8}{(16a-16b)(\tanh(\frac{x}{2})+1)}-$

```
[In] int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*a*x/(a+b)^2+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)-2*a^2*b/(a^4-2*a^2*
b^2+b^4)*x+a^2*b/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(96) = 192$.

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.27

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$= \frac{(a^3-a^2b-ab^2+b^3)\cosh{(x)}^4+4\,(a^3-a^2b-ab^2+b^3)\cosh{(x)}\sinh{(x)}^3+(a^3-a^2b-ab^2+b^3)\sinh{(x}}{}$$

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 + 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\,(a^2+2\,ab+b^2)} + \frac{e^{(2\,x)}}{8\,(a+b)} - \frac{e^{(-2\,x)}}{8\,(a-b)}$$

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*a*x/(a^2
 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\,(a^2-2\,ab+b^2)} + \frac{\left(2\,ae^{(2\,x)}-a+b\right)e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)} + \frac{e^{(2\,x)}}{8\,(a+b)}$$

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*a*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
+ b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{2\,x}}{8\,a+8\,b} - \frac{e^{-2\,x}}{8\,a-8\,b} - \frac{a\,x}{2\,{(a-b)}^2} + \frac{a^2\,b\,\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^4-2\,a^2\,b^2+b^4}$$

```
[In] int((cosh(x)*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (a*x)/(2*(a - b)^2) + (a^2*b
*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.708 $\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^3b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] -a^3*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2*
cosh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*sin
h(x)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{b\sinh^3(x)}{3(a^2-b^2)} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} - \frac{a\cosh(x)}{a^2-b^2} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a^3b\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

```
[In] Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
```

```
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \sinh^3(x)\,dx}{a^2-b^2}-\frac{b\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{a^2b\sinh(x)}{(a^2-b^2)^2}-\frac{(a^3b)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}-\frac{(ab^2)\int \sinh(x)\,dx}{(a^2-b^2)^2}\\
&\quad -\frac{a\text{Subst}\left(\int (1-x^2)\,dx,x,\cosh(x)\right)}{a^2-b^2}-\frac{(ib)\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{a^2-b^2}\\
&= -\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3(a^2-b^2)}\\
&\quad -\frac{(ia^3b)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2}\\
&= -\frac{a^3b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{ab^2\cosh(x)}{(a^2-b^2)^2}\\
&\quad -\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh(x)}{(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3(a^2-b^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$$

$$= \frac{-3a\sqrt{a-b}\sqrt{a+b}(3a^2+b^2)\cosh(x)+a\sqrt{a-b}\sqrt{a+b}(a^2-b^2)\cosh(3x)+b\left(-24a^3\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] + a*Sqrt[a - b]*Sqrt[a
+ b]*(a^2 - b^2)*Cosh[3*x] + b*(-24*a^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] + 3*Sqrt[a - b]*Sqrt[a + b]*(5*a^2 - b^2)*Sinh[x] - Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result
default	$-\frac{2a^3 b \arctan\left(\frac{2a \tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2 (a+b)^2 \sqrt{a^2-b^2}} - \frac{8}{(16a-16b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{16}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3 (16a-16b)} - \frac{a}{2(a-b)^2 \left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{}{3(\tanh(}$
risch	$\frac{e^{3x}}{24a+24b} - \frac{3\,e^{x} a}{8(a+b)^2} - \frac{e^{x} b}{8(a+b)^2} - \frac{3\,e^{-x} a}{8(a-b)^2} + \frac{e^{-x} b}{8(a-b)^2} + \frac{e^{-3x}}{24a-24b} - \frac{b\,a^3 \ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2 (a-b)^2} + \frac{b\,a^3 \ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2 (a-b)^2}$

```
[In] int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(
a^2-b^2)^(1/2))-8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/3/(tanh(1/2*x)+1)^3/(16*
a-16*b)-1/2*a/(a-b)^2/(tanh(1/2*x)+1)-16/3/(tanh(1/2*x)-1)^3/(16*a+16*b)-8/
(16*a+16*b)/(tanh(1/2*x)-1)^2+1/2*a/(a+b)^2/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 1861, normalized size of antiderivative = 13.58

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*
```

```
a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5
*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(
x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 - 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(3*a^5 - 5*
a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5
+ 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cos
h(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,a^3b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} - \frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-12\,abe^x-3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^x
- 12*a*b*e^x - 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.91

$$\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-3\,x}}{24\,a-24\,b} + \frac{e^{3\,x}}{24\,a+24\,b} - \frac{e^x\,(3\,a+b)}{8\,{(a+b)}^2} - \frac{e^{-x}\,(3\,a-b)}{8\,{(a-b)}^2} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^6\,b^2}-b^5\,\sqrt{a^6\,b^2}+2\,a^2\,b^3\,\sqrt{a^6\,b^2}-2\,a^3\,b^2\,\sqrt{a^6\,b^2}+a\,b^4\,\sqrt{a^6\,b^2}-a^4\,b\,\sqrt{a^6\,b^2}}\right)\,\sqrt{a^6\,b^2}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}$$

```
[In] int((cosh(x)*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(x)*(3*a + b))/(8*(a
 + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) - (2*atan((a^3*b*exp(x)*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^6*
b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^2*(a
^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b^2)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

3.709 $\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2bx}{(a^2-b^2)^2} - \frac{bx}{2(a^2-b^2)} - \frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} - \frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)} + \frac{a\sinh^2(x)}{2(a^2-b^2)}$$

```
[Out] a^2*b*x/(a^2-b^2)^2-1/2*b*x/(a^2-b^2)-a*b^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^
2)^2-1/2*b*cosh(x)*sinh(x)/(a^2-b^2)+1/2*a*sinh(x)^2/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3188, 2715, 8, 2644, 30, 3177, 3212}

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2bx}{(a^2-b^2)^2} - \frac{bx}{2(a^2-b^2)} + \frac{a\sinh^2(x)}{2(a^2-b^2)} - \frac{b\sinh(x)\cosh(x)}{2(a^2-b^2)} - \frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 - b^2)^2 - (b*x)/(2*(a^2 - b^2)) - (a*b^2*Log[a*Cosh[x] + b*
Sinh[x]])/(a^2 - b^2)^2 - (b*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)) + (a*Sinh[x]^
2)/(2*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral} = \frac{a \int \cosh(x)\sinh(x)\,dx}{a^2-b^2} - \frac{b \int \cosh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}$$

$$= \frac{a^2bx}{(a^2-b^2)^2} - \frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)} - \frac{(iab^2)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} - \frac{a\text{Subst}(\int x\,dx, x, i\sinh(x))}{a^2-b^2} - \frac{b\int 1\,dx}{2(a^2-b^2)}$$

$$= \frac{a^2bx}{(a^2-b^2)^2} - \frac{bx}{2(a^2-b^2)} - \frac{ab^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} - \frac{b\cosh(x)\sinh(x)}{2(a^2-b^2)} + \frac{a\sinh^2(x)}{2(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a(a^2-b^2)\cosh(2x)+b(2(a^2+b^2)x-4ab\log(a\cosh(x)+b\sinh(x))+(-a^2+b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 - b^2)*Cosh[2*x] + b*(2*(a^2 + b^2)*x - 4*a*b*Log[a*Cosh[x] + b*Sin
h[x]] + (-a^2 + b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

method	result
risch	$\frac{xb}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}+\frac{e^{-2x}}{8a-8b}+\frac{2a\,b^2x}{a^4-2a^2b^2+b^4}-\frac{a\,b^2\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+$

```
[In] int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(a+b)^2*b+1/8/(a+b)*exp(2*x)+1/8/(a-b)*exp(-2*x)+2*a*b^2/(a^4-2*a^2*b
^2+b^4)*x-a*b^2/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.27

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^2*b + 2*a*
b^2 + b^3)*x*cosh(x)^2 + a^3 + a^2*b - a*b^2 - b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^2*b + 2*a*b^2 + b^3)*x)*sinh(x)^2 - 8*(a*b^2*co
sh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^2*b + 2*a*b^2 + b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} + \frac{bx}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} + \frac{e^{(-2x)}}{8(a-b)}$$

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*b*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) + 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^2\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}+\frac{bx}{2(a^2-2ab+b^2)} - \frac{\left(2be^{(2x)}-a+b\right)e^{(-2x)}}{8(a^2-2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}$$

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/
2*b*x/(a^2 - 2*a*b + b^2) - 1/8*(2*b*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
 + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-2x}}{8a-8b}+\frac{e^{2x}}{8a+8b}+\frac{bx}{2(a-b)^2}-\frac{ab^2\ln\left(a-b+ae^{2x}+be^{2x}\right)}{a^4-2a^2b^2+b^4}$$

```
[In] int((cosh(x)^2*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*a - 8*b) + exp(2*x)/(8*a + 8*b) + (b*x)/(2*(a - b)^2) - (a*b^2
*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.710 $\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a^2b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} - \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] a^2*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a^2*b
*cosh(x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)-a*b^2*sinh(x)/(a^2-b^2)^2+1/
3*a*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3188, 2645, 30, 2644, 2717, 2718, 3153, 212}

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^2b^2\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a\sinh^3(x)}{3(a^2-b^2)} - \frac{ab^2\sinh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2b\cosh(x)}{(a^2-b^2)^2}$$

```
[In] Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^2*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*S
inh[x])/(a^2 - b^2)^2 + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
```

```
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2}-\frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{(a^2b)\int \sinh(x)\,dx}{(a^2-b^2)^2}-\frac{(ab^2)\int \cosh(x)\,dx}{(a^2-b^2)^2}+\frac{(a^2b^2)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&\quad+\frac{(ia)\mathrm{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{a^2-b^2}-\frac{b\mathrm{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)}{a^2-b^2}\\
&= \frac{a^2b\cosh(x)}{(a^2-b^2)^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}\\
&\quad+\frac{(ia^2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2}\\
&= \frac{a^2b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2b\cosh(x)}{(a^2-b^2)^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$$
$$=\frac{3\sqrt{a-b}b\sqrt{a+b}(3a^2+b^2)\cosh(x)-\sqrt{a-b}b\sqrt{a+b}(a^2-b^2)\cosh(3x)+a\left(24ab^2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)-\right.}{12(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (3*Sqrt[a - b]*b*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] - Sqrt[a - b]*b*Sqrt[a +
 b]*(a^2 - b^2)*Cosh[3*x] + a*(24*a*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a -
b]*Sqrt[a + b])] - 3*Sqrt[a - b]*Sqrt[a + b]*(a^2 + 3*b^2)*Sinh[x] + Sqrt[a
 - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.38

method	result
default	$-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(8a+8b)}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}(8a+8b)}-\frac{b}{2(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}(8a-8b)}+\frac{4}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}(8a}$
risch	$\frac{e^{3x}}{24a+24b}-\frac{e^{x}a}{8(a+b)^{2}}+\frac{e^{x}b}{8(a+b)^{2}}+\frac{a\,e^{-x}}{8a^{2}-16ab+8b^{2}}+\frac{b\,e^{-x}}{8a^{2}-16ab+8b^{2}}-\frac{e^{-3x}}{24(a-b)}-\frac{b^{2}a^{2}\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)^{2}}+\frac{b^{2}a^{2}\ln\left(e^{x}+\right.}{\sqrt{-a^{2}+b^{2}}\,(a}$

[In] int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] -8/3/(tanh(1/2*x)-1)^3/(8*a+8*b)-4/(tanh(1/2*x)-1)^2/(8*a+8*b)-1/2*b/(a+b)^2/(tanh(1/2*x)-1)-8/3/(tanh(1/2*x)+1)^3/(8*a-8*b)+4/(tanh(1/2*x)+1)^2/(8*a-8*b)+1/2/(a-b)^2*b/(tanh(1/2*x)+1)+2*a^2*b^2/(a+b)^2/(a-b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(114) = 228.

Time = 0.28 (sec) , antiderivative size = 1847, normalized size of antiderivative = 15.14

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")

[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x))*sinh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^2*b^2*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)*sinh(x)^2 + a^2*b^2*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5

```
 - 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^3 + (a
^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*
b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*
b^3 - a*b^4 - b^5 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^
5)*cosh(x)^4 - 3*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 +
 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^
5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x))*sinh(x)^3 + 3
*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2 + 3*(a^5
 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3
*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 +
2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^2*b^2*cosh(x)^3 + 3
*a^2*b^2*cosh(x)^2*sinh(x) + 3*a^2*b^2*cosh(x)*sinh(x)^2 + a^2*b^2*sinh(x)^
3)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x
))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*
(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*cosh(x)^3 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x))/((a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2\,a^2b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{\left(3\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-3\,a^2e^x+3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 3*a^2*e^x
 + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{\mathrm{e}^{3\,x}}{24\,a+24\,b} - \frac{\mathrm{e}^{-3\,x}}{24\,a-24\,b} - \frac{\mathrm{e}^x\,(a-b)}{8\,{(a+b)}^2} + \frac{2\,\mathrm{atan}\left(\frac{a^2\,b^2\,\mathrm{e}^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^4\,b^4}-b^5\,\sqrt{a^4\,b^4}+2\,a^2\,b^3\,\sqrt{a^4\,b^4}-2\,a^3\,b^2\,\sqrt{a^4\,b^4}+a\,b^4\,\sqrt{a^4\,b^4}-a^4\,b\,\sqrt{a^4\,b^4}}\right)\,\sqrt{a^4\,b^4}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} + \frac{\mathrm{e}^{-x}\,(a+b)}{8\,{(a-b)}^2}$$

```
[In] int((cosh(x)^2*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) - (exp(x)*(a - b))/(8*(a +
 b)^2) + (2*atan((a^2*b^2*exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10
*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^4*b^4)^(1/2) - b^5*(a^4*b^4)^(1/2) + 2
*a^2*b^3*(a^4*b^4)^(1/2) - 2*a^3*b^2*(a^4*b^4)^(1/2) + a*b^4*(a^4*b^4)^(1/2
) - a^4*b*(a^4*b^4)^(1/2)))*(a^4*b^4)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*
a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (exp(-x)*(a + b))/(8*(a - b)^2)
```

3.711 $\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2b^3x}{(a^2-b^2)^3}-\frac{a^2bx}{2(a^2-b^2)^2}+\frac{bx}{8(a^2-b^2)}+\frac{a^3b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{a^2b\cosh(x)\sinh(x)}{2(a^2-b^2)^2}+\frac{b\cosh(x)\sinh(x)}{8(a^2-b^2)}-\frac{b\cosh^3(x)\sinh(x)}{4(a^2-b^2)}-\frac{ab^2\sinh^2(x)}{2(a^2-b^2)^2}+\frac{a\sinh^4(x)}{4(a^2-b^2)}$$

```
[Out] -a^2*b^3*x/(a^2-b^2)^3-1/2*a^2*b*x/(a^2-b^2)^2+1/8*b*x/(a^2-b^2)+a^3*b^2*ln
(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3+1/2*a^2*b*cosh(x)*sinh(x)/(a^2-b^2)^2+1/8
*b*cosh(x)*sinh(x)/(a^2-b^2)-1/4*b*cosh(x)^3*sinh(x)/(a^2-b^2)-1/2*a*b^2*si
nh(x)^2/(a^2-b^2)^2+1/4*a*sinh(x)^4/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3188, 2648, 2715, 8, 2644, 30, 3176, 3212}

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{bx}{8(a^2-b^2)}-\frac{a^2bx}{2(a^2-b^2)^2}+\frac{a\sinh^4(x)}{4(a^2-b^2)}-\frac{ab^2\sinh^2(x)}{2(a^2-b^2)^2}-\frac{b\sinh(x)\cosh^3(x)}{4(a^2-b^2)}+\frac{b\sinh(x)\cosh(x)}{8(a^2-b^2)}+\frac{a^2b\sinh(x)\cosh(x)}{2(a^2-b^2)^2}-\frac{a^2b^3x}{(a^2-b^2)^3}+\frac{a^3b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a^2*b^3*x)/(a^2 - b^2)^3) - (a^2*b*x)/(2*(a^2 - b^2)^2) + (b*x)/(8*(a^2
- b^2)) + (a^3*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*b*Cosh[
x]*Sinh[x])/(2*(a^2 - b^2)^2) + (b*Cosh[x]*Sinh[x])/(8*(a^2 - b^2)) - (b*Co
sh[x]^3*Sinh[x])/(4*(a^2 - b^2)) - (a*b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (a
*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\int \cosh(x)\sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} \\
&= -\frac{b\cosh^3(x)\sinh(x)}{4\,(a^2-b^2)} + \frac{(a^2b)\int \sinh^2(x)\,dx}{(a^2-b^2)^2} - \frac{(ab^2)\int \cosh(x)\sinh(x)\,dx}{(a^2-b^2)^2} \\
&\quad + \frac{(a^2b^2)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{a\text{Subst}\left(\int x^3\,dx, x, i\sinh(x)\right)}{a^2-b^2} + \frac{b\int \cosh^2(x)\,dx}{4\,(a^2-b^2)} \\
&= -\frac{a^2b^3x}{(a^2-b^2)^3} + \frac{a^2b\cosh(x)\sinh(x)}{2\,(a^2-b^2)^2} + \frac{b\cosh(x)\sinh(x)}{8\,(a^2-b^2)} \\
&\quad - \frac{b\cosh^3(x)\sinh(x)}{4\,(a^2-b^2)} + \frac{a\sinh^4(x)}{4\,(a^2-b^2)} + \frac{(ia^3b^2)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3} \\
&\quad - \frac{(a^2b)\int 1\,dx}{2\,(a^2-b^2)^2} + \frac{(ab^2)\,\text{Subst}\left(\int x\,dx, x, i\sinh(x)\right)}{(a^2-b^2)^2} + \frac{b\int 1\,dx}{8\,(a^2-b^2)} \\
&= -\frac{a^2b^3x}{(a^2-b^2)^3} - \frac{a^2bx}{2\,(a^2-b^2)^2} + \frac{bx}{8\,(a^2-b^2)} + \frac{a^3b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\
&\quad + \frac{a^2b\cosh(x)\sinh(x)}{2\,(a^2-b^2)^2} + \frac{b\cosh(x)\sinh(x)}{8\,(a^2-b^2)} - \frac{b\cosh^3(x)\sinh(x)}{4\,(a^2-b^2)} - \frac{ab^2\sinh^2(x)}{2\,(a^2-b^2)^2} \\
&\quad + \frac{a\sinh^4(x)}{4\,(a^2-b^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{-4a(a^4-b^4)\cosh(2x)+a(a^2-b^2)^2\cosh(4x)-b\Big(4(3a^4x+6a^2b^2x-b^4x-8a^3b\log(a\cosh(x)+b\sinh(x}{32(a-b)^3(a+b)^3}$$

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (-4*a*(a^4 - b^4)*Cosh[2*x] + a*(a^2 - b^2)^2*Cosh[4*x] - b*(4*(3*a^4*x + 6
*a^2*b^2*x - b^4*x - 8*a^3*b*Log[a*Cosh[x] + b*Sinh[x]]) - 8*a^2*(a^2 - b^2
)*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)
```

Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{3axb}{8(a+b)^3}-\frac{b^2x}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}-\frac{e^{2x}a}{16(a+b)^2}-\frac{e^{-2x}a}{16(a-b)^2}+\frac{e^{-4x}}{64a-64b}-\frac{2a^3b^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a^3b^2\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$
default	$\frac{4}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^4(16a+16b)}+\frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{-a-3b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{a-b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b(3a+b)\ln(\tanh}{8(a+b)^3}$

```
[In] int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*a*x/(a+b)^3*b-1/8*b^2*x/(a+b)^3+1/64/(a+b)*exp(4*x)-1/16/(a+b)^2*exp(2
*x)*a-1/16/(a-b)^2*exp(-2*x)*a+1/64/(a-b)*exp(-4*x)-2*a^3*b^2/(a^6-3*a^4*b^
2+3*a^2*b^4-b^6)*x+a^3*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)+(a-b)/
(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. 2(180) = 360.

Time = 0.27 (sec) , antiderivative size = 1158, normalized size of antiderivative = 5.97

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
```

```
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(a^5 - 2*a^4*b
+ 2*a^2*b^3 - a*b^4)*cosh(x)^6 - 4*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4 - 7*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 - 8
*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(a^5 - 2*a^4*b + 2*a^2*b
^3 - a*b^4)*cosh(x))*sinh(x)^5 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^
4 + b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^4 - 30*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^2 - 4*(3*a^4*b + 8*a^3*
b^2 + 6*a^2*b^3 - b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh
(x)^3 - 4*(3*a^4*b + 8*a^3*b^2 + 6*a^2*b^3 - b^5)*x*cosh(x))*sinh(x)^3 - 4*
(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 - a^5 - 2*a^4*b + 2*a^2*b^3 + a*b^4
 - 15*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^4 - 12*(3*a^4*b + 8*a^3*b
^2 + 6*a^2*b^3 - b^5)*x*cosh(x)^2)*sinh(x)^2 + 64*(a^3*b^2*cosh(x)^4 + 4*a^
3*b^2*cosh(x)^3*sinh(x) + 6*a^3*b^2*cosh(x)^2*sinh(x)^2 + 4*a^3*b^2*cosh(x)
*sinh(x)^3 + a^3*b^2*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - si
nh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^7
- 3*(a^5 - 2*a^4*b + 2*a^2*b^3 - a*b^4)*cosh(x)^5 - 4*(3*a^4*b + 8*a^3*b^2
+ 6*a^2*b^3 - b^5)*x*cosh(x)^3 - (a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*cosh(x
))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(3ab+b^2)x}{8(a^3+3a^2b+3ab^2+b^3)} - \frac{\left(4ae^{(-2x)}-a-b\right)e^{(4x)}}{64(a^2+2ab+b^2)} - \frac{4ae^{(-2x)}-(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^3*b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- 1/8*(3*a*b + b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*a*e^(-2*x)
- a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*a*e^(-2*x) - (a - b)*e^(-4*x
))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^2\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(3ab-b^2)x}{8(a^3-3a^2b+3ab^2-b^3)} + \frac{\left(18abe^{(4x)}-6b^2e^{(4x)}-4a^2e^{(2x)}+4abe^{(2x)}+a^2-2ab+b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}-4ae^{(2x)}}{64(a^2+2ab+b^2)}$$

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^3*b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - 1/8*(3*a*b - b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(18*a
*b*e^(4*x) - 6*b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*a*b*e^(2*x) + a^2 - 2*a*b +
b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x)
 - 4*a*e^(2*x))/(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-4x}}{64a-64b} + \frac{e^{4x}}{64a+64b} - \frac{x(3ab-b^2)}{8(a-b)^3} - \frac{a\,e^{2x}}{16(a+b)^2} - \frac{a\,e^{-2x}}{16(a-b)^2} + \frac{a^3b^2\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$$

```
[In] int((cosh(x)^2*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-4*x)/(64*a - 64*b) + exp(4*x)/(64*a + 64*b) - (x*(3*a*b - b^2))/(8*(a
- b)^3) - (a*exp(2*x))/(16*(a + b)^2) - (a*exp(-2*x))/(16*(a - b)^2) + (a^3
*b^2*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)
```

3.712 $\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] -a*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2*
cosh(x)/(a^2-b^2)^2+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*sinh(x)/(a^2-b^2)^2-b*s
inh(x)/(a^2-b^2)-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3188, 2713, 2645, 30, 3179, 2717, 3153, 212}

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ab^3\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{b\sinh^3(x)}{3(a^2-b^2)} - \frac{b\sinh(x)}{a^2-b^2} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2}$$

```
[In] Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] -((a*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 + (a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a^2*b*
```

```
Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x])/(a^2 - b^2) - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^3(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} \\
&= -\frac{ab^2\cosh(x)}{(a^2-b^2)^2} + \frac{(a^2b)\int \cosh(x)\,dx}{(a^2-b^2)^2} - \frac{(ab^3)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} \\
&\quad + \frac{a\text{Subst}\left(\int x^2\,dx, x, \cosh(x)\right)}{a^2-b^2} - \frac{(ib)\text{Subst}\left(\int (1-x^2)\,dx, x, -i\sinh(x)\right)}{a^2-b^2} \\
&= -\frac{ab^2\cosh(x)}{(a^2-b^2)^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)} \\
&\quad - \frac{(iab^3)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2} \\
&= -\frac{ab^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2} \\
&\quad + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh(x)}{a^2-b^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{1}{12}\left(-\frac{24ab^3\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{3a(a^2-5b^2)\cosh(x)}{(a-b)^2(a+b)^2} + \frac{a\cosh(3x)}{(a-b)(a+b)} + \frac{3b(a^2+3b^2)\sinh(x)}{(a-b)^2(a+b)^2} - \frac{a^2b\sinh(3x)}{(a-b)^2(a+b)^2} + \frac{b^3\sinh(3x)}{(a-b)^2(a+b)^2}\right)$$

[In] Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x]),x]

[Out] ((-24*a*b^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) + (3*a*(a^2 - 5*b^2)*Cosh[x])/((a - b)^2*(a + b)^2) + (a*Cosh[3*x])/((a - b)*(a + b)) + (3*b*(a^2 + 3*b^2)*Sinh[x])/((a - b)^2*(a + b)^2) - (a^2*b*Sinh[3*x])/((a - b)^2*(a + b)^2) + (b^3*Sinh[3*x])/((a - b)^2*(a + b)^2))/12

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27

method	result
risch	$\frac{e^{3x}}{24a+24b}+\frac{e^{x}a}{8(a+b)^{2}}+\frac{3\,e^{x}b}{8(a+b)^{2}}+\frac{e^{-x}a}{8(a-b)^{2}}-\frac{3\,e^{-x}b}{8(a-b)^{2}}+\frac{e^{-3x}}{24a-24b}-\frac{b^{3}a\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)^{2}}+\frac{b^{3}a\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)}$
default	$-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}(4a+4b)}-\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{a+2b}{2(a+b)^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}(}$

[In] int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] 1/24/(a+b)*exp(x)^3+1/8/(a+b)^2*exp(x)*a+3/8/(a+b)^2*exp(x)*b+1/8/(a-b)^2/exp(x)*a-3/8/(a-b)^2/exp(x)*b+1/24/(a-b)/exp(x)^3-1/(-a^2+b^2)^(1/2)*b^3*a/(a+b)^2/(a-b)^2*ln(exp(x)+(a-b)/(-a^2+b^2)^(1/2))+1/(-a^2+b^2)^(1/2)*b^3*a/(a+b)^2/(a-b)^2*ln(exp(x)-(a-b)/(-a^2+b^2)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(129) = 258$.

Time = 0.30 (sec) , antiderivative size = 1829, normalized size of antiderivative = 13.35

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")

[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*co

```
sh(x)^2 + 3*(a^5 - a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5 + 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 + a^4*b
- 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a*b^3
*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(a^
5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^3 + (a^5 - a^4
*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(
x)^4 + 3*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5 + 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^5 + a^4*
b - 6*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(a^5 -
a^4*b - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x)^2 + 3*(a^5 - a^4*b
 - 6*a^3*b^2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 + a^4*b - 6*a^3*b^2 + 2*a^2*b^3
+ 5*a*b^4 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a*b^3*cosh(x)^3 + 3*a*b^3*cos
h(x)^2*sinh(x) + 3*a*b^3*cosh(x)*sinh(x)^2 + a*b^3*sinh(x)^3)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(a^5 + a^4*b - 6
*a^3*b^2 + 2*a^2*b^3 + 5*a*b^4 - 3*b^5)*cosh(x)^3 + (a^5 - a^4*b - 6*a^3*b^
2 - 2*a^2*b^3 + 5*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{2\,ab^3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{\left(3\,ae^{(2\,x)}-9\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+3\,a^2e^x+12\,abe^x+9\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) + 1/24*(3*a*e^(2*x) - 9*b*e^(2*x) + a - b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 3*a^2*e^x
+ 12*a*b*e^x + 9*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-3\,x}}{24\,a-24\,b} + \frac{e^{3\,x}}{24\,a+24\,b} + \frac{e^x\,(a+3\,b)}{8\,{(a+b)}^2} - \frac{2\,\mathrm{atan}\left(\frac{a\,b^3\,e^x\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}}{a^5\,\sqrt{a^2\,b^6}-b^5\,\sqrt{a^2\,b^6}+2\,a^2\,b^3\,\sqrt{a^2\,b^6}-2\,a^3\,b^2\,\sqrt{a^2\,b^6}+a\,b^4\,\sqrt{a^2\,b^6}-a^4\,b\,\sqrt{a^2\,b^6}}\right)\,\sqrt{a^2\,b^6}}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} + \frac{e^{-x}\,(a-3\,b)}{8\,{(a-b)}^2}$$

```
[In] int((cosh(x)^3*sinh(x))/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) + (exp(x)*(a + 3*b))/(8*(a
 + b)^2) - (2*atan((a*b^3*exp(x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10
*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^2*b^6)^(1/2) - b^5*(a^2*b^6)^(1/2) + 2
*a^2*b^3*(a^2*b^6)^(1/2) - 2*a^3*b^2*(a^2*b^6)^(1/2) + a*b^4*(a^2*b^6)^(1/2
) - a^4*b*(a^2*b^6)^(1/2)))*(a^2*b^6)^(1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*
a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (exp(-x)*(a - 3*b))/(8*(a - b)^2)
```

3.713 $\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 194

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{a^3b^2x}{(a^2-b^2)^3} - \frac{ab^2x}{2(a^2-b^2)^2} - \frac{ax}{8(a^2-b^2)} - \frac{b\cosh^4(x)}{4(a^2-b^2)} - \frac{a^2b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{ab^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2} - \frac{a\cosh(x)\sinh(x)}{8(a^2-b^2)} + \frac{a\cosh^3(x)\sinh(x)}{4(a^2-b^2)} + \frac{a^2b\sinh^2(x)}{2(a^2-b^2)^2}$$

```
[Out] a^3*b^2*x/(a^2-b^2)^3-1/2*a*b^2*x/(a^2-b^2)^2-1/8*a*x/(a^2-b^2)-1/4*b*cosh(
x)^4/(a^2-b^2)-a^2*b^3*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-1/2*a*b^2*cosh(x
)*sinh(x)/(a^2-b^2)^2-1/8*a*cosh(x)*sinh(x)/(a^2-b^2)+1/4*a*cosh(x)^3*sinh(
x)/(a^2-b^2)+1/2*a^2*b*sinh(x)^2/(a^2-b^2)^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3188, 2645, 30, 2648, 2715, 8, 2644, 3177, 3212}

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{ax}{8(a^2-b^2)} - \frac{ab^2x}{2(a^2-b^2)^2} + \frac{a^2b\sinh^2(x)}{2(a^2-b^2)^2} - \frac{b\cosh^4(x)}{4(a^2-b^2)} + \frac{a\sinh(x)\cosh^3(x)}{4(a^2-b^2)} - \frac{a\sinh(x)\cosh(x)}{8(a^2-b^2)} - \frac{ab^2\sinh(x)\cosh(x)}{2(a^2-b^2)^2} - \frac{a^2b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{a^3b^2x}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^3*b^2*x)/(a^2 - b^2)^3 - (a*b^2*x)/(2*(a^2 - b^2)^2) - (a*x)/(8*(a^2 - b
^2)) - (b*Cosh[x]^4)/(4*(a^2 - b^2)) - (a^2*b^3*Log[a*Cosh[x] + b*Sinh[x]])
/(a^2 - b^2)^3 - (a*b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*Cosh[x]*Sin
h[x])/(8*(a^2 - b^2)) + (a*Cosh[x]^3*Sinh[x])/(4*(a^2 - b^2)) + (a^2*b*Sinh
[x]^2)/(2*(a^2 - b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\int \cosh^2(x)\sinh^2(x)\,dx}{a^2-b^2}-\frac{b\int \cosh^3(x)\sinh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{a\cosh^3(x)\sinh(x)}{4(a^2-b^2)}+\frac{(a^2b)\int \cosh(x)\sinh(x)\,dx}{(a^2-b^2)^2}-\frac{(ab^2)\int \cosh^2(x)\,dx}{(a^2-b^2)^2}\\
&\quad+\frac{(a^2b^2)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}-\frac{a\int \cosh^2(x)\,dx}{4(a^2-b^2)}-\frac{b\text{Subst}\left(\int x^3\,dx,x,\cosh(x)\right)}{a^2-b^2}\\
&= \frac{a^3b^2x}{(a^2-b^2)^3}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{ab^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}-\frac{a\cosh(x)\sinh(x)}{8(a^2-b^2)}\\
&\quad+\frac{a\cosh^3(x)\sinh(x)}{4(a^2-b^2)}-\frac{(ia^2b^3)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\\
&\quad-\frac{(a^2b)\,\text{Subst}(\int x\,dx,x,i\sinh(x))}{(a^2-b^2)^2}-\frac{(ab^2)\int 1\,dx}{2(a^2-b^2)^2}-\frac{a\int 1\,dx}{8(a^2-b^2)}\\
&= \frac{a^3b^2x}{(a^2-b^2)^3}-\frac{ab^2x}{2(a^2-b^2)^2}-\frac{ax}{8(a^2-b^2)}-\frac{b\cosh^4(x)}{4(a^2-b^2)}-\frac{a^2b^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}\\
&\quad-\frac{ab^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}-\frac{a\cosh(x)\sinh(x)}{8(a^2-b^2)}+\frac{a\cosh^3(x)\sinh(x)}{4(a^2-b^2)}+\frac{a^2b\sinh^2(x)}{2(a^2-b^2)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.65

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{4b(a^4-b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)+a\Big(-4((a^4-6a^2b^2-3b^4)\,x+8ab^3\log(a\cosh(x)+b\sinh(x))}{32(a-b)^3(a+b)^3}$$

[In] Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x]),x]

[Out] (4*b*(a^4 - b^4)*Cosh[2*x] - b*(a^2 - b^2)^2*Cosh[4*x] + a*(-4*((a^4 - 6*a^2*b^2 - 3*b^4)*x + 8*a*b^3*Log[a*Cosh[x] + b*Sinh[x]]) + 8*b^2*(-a^2 + b^2)*Sinh[2*x] + (a^2 - b^2)^2*Sinh[4*x]))/(32*(a - b)^3*(a + b)^3)

Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{a^2 x}{8(a+b)^3}-\frac{3axb}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}+\frac{e^{2x}b}{16(a+b)^2}+\frac{e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2a^2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{a^2b^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}$
default	$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^4(8a+8b)}+\frac{8}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^3(16a+16b)}-\frac{-3a-5b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-a-3b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a(a+3b)\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{8(a+b)^3}$

[In] int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] -1/8*a^2*x/(a+b)^3-3/8*a*x/(a+b)^3*b+1/64/(a+b)*exp(4*x)+1/16/(a+b)^2*exp(2*x)*b+1/16/(a-b)^2*exp(-2*x)*b-1/64/(a-b)*exp(-4*x)+2*a^2*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*x-a^2*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)+(a-b)/(a+b))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(180) = 360.

Time = 0.28 (sec) , antiderivative size = 1162, normalized size of antiderivative = 5.99

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")

[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -

```
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(a^4*b - 2*a^3*
b^2 + 2*a*b^4 - b^5)*cosh(x)^6 + 4*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5 + 7*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 - 8
*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(a^4*b - 2*a^3*b^2 + 2*a
*b^4 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^4 + 30*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 - 6*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^5 + 10*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh
(x)^3 - 4*(a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*x*cosh(x))*sinh(x)^3 + 4*
(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b
^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5
 + 15*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^4 - 12*(a^5 - 6*a^3*b^2 -
 8*a^2*b^3 - 3*a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^2*b^3*cosh(x)^4 + 4*a^
2*b^3*cosh(x)^3*sinh(x) + 6*a^2*b^3*cosh(x)^2*sinh(x)^2 + 4*a^2*b^3*cosh(x)
*sinh(x)^3 + a^2*b^3*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - si
nh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^7
+ 3*(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*cosh(x)^5 - 4*(a^5 - 6*a^3*b^2 - 8*
a^2*b^3 - 3*a*b^4)*x*cosh(x)^3 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*cosh(x
))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(a^2+3ab)x}{8(a^3+3a^2b+3ab^2+b^3)} + \frac{\left(4be^{(-2x)}+a+b\right)e^{(4x)}}{64(a^2+2ab+b^2)} + \frac{4be^{(-2x)}-(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
[Out] -a^2*b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - 1/8*(a^2 + 3*a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*b*e^(-2*x)
 + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*b*e^(-2*x) - (a - b)*e^(-4*
x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = -\frac{a^2b^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{(a^2-3ab)x}{8(a^3-3a^2b+3ab^2-b^3)} + \frac{\left(6a^2e^{(4x)}-18abe^{(4x)}+4abe^{(2x)}-4b^2e^{(2x)}-a^2+2ab-b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}+4be^{(2x)}}{64(a^2+2ab+b^2)}$$

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
[Out] -a^2*b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - 1/8*(a^2 - 3*a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(6*a
^2*e^(4*x) - 18*a*b*e^(4*x) + 4*a*b*e^(2*x) - 4*b^2*e^(2*x) - a^2 + 2*a*b -
 b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x
) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{4x}}{64a+64b} - \frac{e^{-4x}}{64a-64b} + \frac{x\,(3ab-a^2)}{8\,(a-b)^3} + \frac{b\,e^{2x}}{16\,(a+b)^2} + \frac{b\,e^{-2x}}{16\,(a-b)^2} - \frac{a^2\,b^3\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}$$

```
[In] int((cosh(x)^3*sinh(x)^2)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (x*(3*a*b - a^2))/(8*(a
- b)^3) + (b*exp(2*x))/(16*(a + b)^2) + (b*exp(-2*x))/(16*(a - b)^2) - (a^2
*b^3*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)
```

3.714 $\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 212

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=\frac{a^3b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{a^3b^2\cosh(x)}{(a^2-b^2)^3}-\frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2}-\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{a\cosh^5(x)}{5(a^2-b^2)}-\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a^2b\sinh^3(x)}{3(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3(a^2-b^2)}-\frac{b\sinh^5(x)}{5(a^2-b^2)}$$

```
[Out] a^3*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+a^3*b
^2*cosh(x)/(a^2-b^2)^3-1/3*a*b^2*cosh(x)^3/(a^2-b^2)^2-1/3*a*cosh(x)^3/(a^2
-b^2)+1/5*a*cosh(x)^5/(a^2-b^2)-a^2*b^3*sinh(x)/(a^2-b^2)^3+1/3*a^2*b*sinh(
x)^3/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)-1/5*b*sinh(x)^5/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3188, 2644, 14, 2645, 30, 2717, 2718, 3153, 212}

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx=-\frac{b\sinh^5(x)}{5(a^2-b^2)}-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a^2b\sinh^3(x)}{3(a^2-b^2)^2}+\frac{a\cosh^5(x)}{5(a^2-b^2)}-\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\cosh^3(x)}{3(a^2-b^2)^2}-\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a^3b^3\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{a^3b^2\cosh(x)}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] (a^3*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2)
 + (a^3*b^2*Cosh[x])/(a^2 - b^2)^3 - (a*b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) -
(a*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*Cosh[x]^5)/(5*(a^2 - b^2)) - (a^2*b^3*Si
nh[x])/(a^2 - b^2)^3 + (a^2*b*Sinh[x]^3)/(3*(a^2 - b^2)^2) - (b*Sinh[x]^3)/
(3*(a^2 - b^2)) - (b*Sinh[x]^5)/(5*(a^2 - b^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \cosh^2(x)\sinh^3(x)\,dx}{a^2-b^2}-\frac{b\int \cosh^3(x)\sinh^2(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{(a^2b)\int \cosh(x)\sinh^2(x)\,dx}{(a^2-b^2)^2}-\frac{(ab^2)\int \cosh^2(x)\sinh(x)\,dx}{(a^2-b^2)^2}+\frac{(a^2b^2)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&\quad-\frac{a\text{Subst}\left(\int x^2(1-x^2)\ dx,x,\cosh(x)\right)}{a^2-b^2}-\frac{(ib)\text{Subst}\left(\int x^2(1-x^2)\ dx,x,i\sinh(x)\right)}{a^2-b^2}\\
&= \frac{(a^3b^2)\int \sinh(x)\,dx}{(a^2-b^2)^3}-\frac{(a^2b^3)\int \cosh(x)\,dx}{(a^2-b^2)^3}+\frac{(a^3b^3)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\\
&\quad+\frac{(ia^2b)\,\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{(a^2-b^2)^2}-\frac{(ab^2)\,\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)}{(a^2-b^2)^2}\\
&\quad-\frac{a\text{Subst}\left(\int (x^2-x^4)\ dx,x,\cosh(x)\right)}{a^2-b^2}-\frac{(ib)\text{Subst}\left(\int (x^2-x^4)\ dx,x,i\sinh(x)\right)}{a^2-b^2}\\
&= \frac{a^3b^2\cosh(x)}{(a^2-b^2)^3}-\frac{ab^2\cosh^3(x)}{3\,(a^2-b^2)^2}-\frac{a\cosh^3(x)}{3\,(a^2-b^2)}+\frac{a\cosh^5(x)}{5\,(a^2-b^2)}-\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a^2b\sinh^3(x)}{3\,(a^2-b^2)^2}\\
&\quad-\frac{b\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\sinh^5(x)}{5\,(a^2-b^2)}+\frac{(ia^3b^3)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3}\\
&= \frac{a^3b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{a^3b^2\cosh(x)}{(a^2-b^2)^3}-\frac{ab^2\cosh^3(x)}{3\,(a^2-b^2)^2}-\frac{a\cosh^3(x)}{3\,(a^2-b^2)}\\
&\quad+\frac{a\cosh^5(x)}{5\,(a^2-b^2)}-\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3}+\frac{a^2b\sinh^3(x)}{3\,(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3\,(a^2-b^2)}-\frac{b\sinh^5(x)}{5\,(a^2-b^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{1}{32}\left(\frac{4ab(3a^4+10a^2b^2+3b^4)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}+\frac{2a(a^4+10a^2b^2+5b^4)\cosh(x)}{(a-b)^3(a+b)^3} - \frac{2a(a^2+3b^2)\cosh(3x)}{3(a-b)^2(a+b)^2}+\frac{2a\cosh(5x)}{5(a-b)(a+b)}+\frac{2b(5a^4+10a^2b^2+b^4)\sinh(x)}{(-a+b)^3(a+b)^3} -3\left(\frac{4ab\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{2a\cosh(x)}{a^2-b^2}+\frac{2b\sinh(x)}{-a^2+b^2}\right)+\frac{2b(3a^2+b^2)\sinh(3x)}{3(a-b)^2(a+b)^2}-\frac{2b\sinh(5x)}{5(a-b)(a+b)}\right)$$

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((4*a*b*(3*a^4 + 10*a^2*b^2 + 3*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*
Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) + (2*a*(a^4 + 10*a^2*b^2 + 5*b
^4)*Cosh[x])/((a - b)^3*(a + b)^3) - (2*a*(a^2 + 3*b^2)*Cosh[3*x])/(3*(a -
b)^2*(a + b)^2) + (2*a*Cosh[5*x])/(5*(a - b)*(a + b)) + (2*b*(5*a^4 + 10*a^
2*b^2 + b^4)*Sinh[x])/((-a + b)^3*(a + b)^3) - 3*((4*a*b*ArcTan[(b + a*Tanh
[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*(a + b)^(3/2)) + (2*a*Cos
h[x])/(a^2 - b^2) + (2*b*Sinh[x])/(-a^2 + b^2)) + (2*b*(3*a^2 + b^2)*Sinh[3
*x])/(3*(a - b)^2*(a + b)^2) - (2*b*Sinh[5*x])/(5*(a - b)*(a + b)))/32
```

Maple [A] (verified)

Time = 31.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

method	result
default	$-\frac{16}{5\left(\tanh\left(\frac{x}{2}\right)-1\right)^5(16a+16b)}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^4(8a+8b)}-\frac{a+3b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{5a+7b}{12(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{a(a+3}{8(a+b)^3(\tan}$
risch	$\frac{e^{5x}}{160a+160b}-\frac{e^{3x}a}{96(a+b)^2}+\frac{e^{3x}b}{96(a+b)^2}-\frac{e^{x}a^2}{16(a+b)^3}-\frac{e^{x}ab}{4(a+b)^3}-\frac{e^{x}b^2}{16(a+b)^3}-\frac{e^{-x}a^2}{16(a^3-3a^2b+3ab^2-b^3)}+\frac{e^{-x}ab}{4a^3-12a^2b+12ab}$

```
[In] int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -16/5/(tanh(1/2*x)-1)^5/(16*a+16*b)-4/(tanh(1/2*x)-1)^4/(8*a+8*b)-1/8*(a+3*
b)/(a+b)^2/(tanh(1/2*x)-1)^2-1/12*(5*a+7*b)/(a+b)^2/(tanh(1/2*x)-1)^3+1/8*a
*(a+3*b)/(a+b)^3/(tanh(1/2*x)-1)-4/(tanh(1/2*x)+1)^4/(8*a-8*b)+16/5/(tanh(1
/2*x)+1)^5/(16*a-16*b)-1/12*(-5*a+7*b)/(a-b)^2/(tanh(1/2*x)+1)^3-1/8*(a-3*b
)/(a-b)^2/(tanh(1/2*x)+1)^2-1/8*a*(a-3*b)/(a-b)^3/(tanh(1/2*x)+1)+2*a^3*b^3
```

```
/(a+b)^3/(a-b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)
^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2440 vs. $2(196) = 392$.

Time = 0.32 (sec) , antiderivative size = 4935, normalized size of antiderivative = 23.28

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] [1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*
b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5*(a^7 - 3*
a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^
8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6
- b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a
*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b
^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*
b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 +
 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 - 3*a*b^6
- 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 +
 a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 21*a^4*b^3 + 2
1*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 14*(a^7 - 3*a^6*
b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2)*s
inh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*
b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*
a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 + a^6*b - 9*a^5*b^2
+ 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x))*sinh(x)^5 - 30*
(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)
*cosh(x)^4 - 10*(3*a^7 - 3*a^6*b - 27*a^5*b^2 - 21*a^4*b^3 + 21*a^3*b^4 + 2
7*a^2*b^5 + 3*a*b^6 - 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a
^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 35*(a^7 - 3*a^6*b + a^5*b^2 +
 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 45*(a^7 + a^6
*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^2
)*sinh(x)^4 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^
2*b^5 - a*b^6 + b^7)*cosh(x)^7 - 7*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5
*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 15*(a^7 + a^6*b - 9*a^5*b^2
 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^3 - 3*(a^7 - a^
6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x))
```

sinh(x)^3 - 5(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x)^2 + 5*(27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^5*b^2 + 5*a^4*b^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 90*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^4 - 36*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 480*(a^3*b^3*cosh(x)^5 + 5*a^3*b^3*cosh(x)^4*sinh(x) + 10*a^3*b^3*cosh(x)^3*sinh(x)^2 + 10*a^3*b^3*cosh(x)^2*sinh(x)^3 + 5*a^3*b^3*cosh(x)*sinh(x)^4 + a^3*b^3*sinh(x)^5)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 10*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^9 - 4*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^7 - 18*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^5 - 12*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^3 - (a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4*sinh(x) + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3*sinh(x)^2 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^4 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^5), 1/480*(3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^10 + 30*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + 3*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 5*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7 - 27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^2)*sinh(x)^8 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + 3*a^7 + 3*a^6*b - 9*a^5*b^2 - 9*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 - 3*a*b^6 - 3*b^7 - 30*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^6 - 10*(3*a^7 + 3*a^6*b - 27*a^5*b^2 + 21*a^4*b^3 + 21*a^3*b^4 - 27*a^2*b^5 + 3*a*b^6 + 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^5 - 70*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 45*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x))*sinh(x)^5 - 30*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*

```
b^6 - b^7)*cosh(x)^4 - 10*(3*a^7 - 3*a^6*b - 27*a^5*b^2 - 21*a^4*b^3 + 21*a
^3*b^4 + 27*a^2*b^5 + 3*a*b^6 - 3*b^7 - 63*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4
*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 35*(a^7 - 3*a^6*b +
 a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 45*
(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)
*cosh(x)^2)*sinh(x)^4 + 40*(9*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*
b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^7 - 7*(a^7 - 3*a^6*b + a^5*b^2 + 5*a
^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 15*(a^7 + a^6*b -
 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^3 - 3
*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7
)*cosh(x))*sinh(x)^3 - 5*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 +
 a^2*b^5 + 3*a*b^6 + b^7)*cosh(x)^2 + 5*(27*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^
5*b^2 + 5*a^4*b^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b
 + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 9
0*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*cosh(x)^4 - 36*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4*b^3 + 7*a^3*b^4 + 9*a^2*
b^5 + a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 - 960*(a^3*b^3*cosh(x)^5 + 5*a^3*b^
3*cosh(x)^4*sinh(x) + 10*a^3*b^3*cosh(x)^3*sinh(x)^2 + 10*a^3*b^3*cosh(x)^2
*sinh(x)^3 + 5*a^3*b^3*cosh(x)*sinh(x)^4 + a^3*b^3*sinh(x)^5)*sqrt(a^2 - b^
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 10*(3*(a^7
 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cos
h(x)^9 - 4*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a
*b^6 - b^7)*cosh(x)^7 - 18*(a^7 + a^6*b - 9*a^5*b^2 + 7*a^4*b^3 + 7*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cosh(x)^5 - 12*(a^7 - a^6*b - 9*a^5*b^2 - 7*a^4
*b^3 + 7*a^3*b^4 + 9*a^2*b^5 + a*b^6 - b^7)*cosh(x)^3 - (a^7 + 3*a^6*b + a^
5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6
*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4*sinh(x) + 10*(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3*sinh(x)^2 + 10*(a^8 - 4*a^6*b^2 +
 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^4 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*sinh(x)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{2\,a^3b^3\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} - \frac{\left(30\,a^2e^{(4\,x)}-120\,abe^{(4\,x)}+30\,b^2e^{(4\,x)}+5\,a^2e^{(2\,x)}-5\,b^2e^{(2\,x)}-3\,a^2+6\,ab-3\,b^2\right)e^{(-5\,x)}}{480\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{3\,a^4e^{(5\,x)}+12\,a^3be^{(5\,x)}+18\,a^2b^2e^{(5\,x)}+12\,ab^3e^{(5\,x)}+3\,b^4e^{(5\,x)}-5\,a^4e^{(3\,x)}-10\,a^3be^{(3\,x)}+10\,ab^3e^{(3\,x)}+5}{480\,(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4}$$

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b^3*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*sqrt(a^2 - b^2)) - 1/480*(30*a^2*e^(4*x) - 120*a*b*e^(4*x) + 30
*b^2*e^(4*x) + 5*a^2*e^(2*x) - 5*b^2*e^(2*x) - 3*a^2 + 6*a*b - 3*b^2)*e^(-5
*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/480*(3*a^4*e^(5*x) + 12*a^3*b*e^(5*
x) + 18*a^2*b^2*e^(5*x) + 12*a*b^3*e^(5*x) + 3*b^4*e^(5*x) - 5*a^4*e^(3*x)
- 10*a^3*b*e^(3*x) + 10*a*b^3*e^(3*x) + 5*b^4*e^(3*x) - 30*a^4*e^x - 180*a^
3*b*e^x - 300*a^2*b^2*e^x - 180*a*b^3*e^x - 30*b^4*e^x)/(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^3(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx = \frac{e^{-5x}}{160a-160b}+\frac{e^{5x}}{160a+160b} + \frac{2\,\mathrm{atan}\left(\frac{a^3 b^3 e^x \sqrt{a^{14}-7a^{12}b^2+21a^{10}b^4-35a^8b^6+35a^6b^8-21a^4b^{10}+7a^2b^{12}-b^{14}}}{a^7\sqrt{a^6b^6}+b^7\sqrt{a^6b^6}-3a^2b^5\sqrt{a^6b^6}+3a^3b^4\sqrt{a^6b^6}+3a^4b^3\sqrt{a^6b^6}-3a^5b^2\sqrt{a^6b^6}-ab^6\sqrt{a^6b^6}-a^6b\sqrt{a^6b^6}}\right)\sqrt{a^6b^6}}{\sqrt{a^{14}-7a^{12}b^2+21a^{10}b^4-35a^8b^6+35a^6b^8-21a^4b^{10}+7a^2b^{12}-b^{14}}} - \frac{e^{-x}(a^2-4ab+b^2)}{16(a-b)^3}-\frac{e^{-3x}(a+b)}{96(a-b)^2}-\frac{e^{3x}(a-b)}{96(a+b)^2}-\frac{e^x(a^2+4ab+b^2)}{16(a+b)^3}$$

```
[In] int((cosh(x)^3*sinh(x)^3)/(a*cosh(x) + b*sinh(x)),x)
```

```
[Out] exp(-5*x)/(160*a - 160*b) + exp(5*x)/(160*a + 160*b) + (2*atan((a^3*b^3*exp
(x)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*
a^10*b^4 - 7*a^12*b^2)^(1/2))/(a^7*(a^6*b^6)^(1/2) + b^7*(a^6*b^6)^(1/2) -
3*a^2*b^5*(a^6*b^6)^(1/2) + 3*a^3*b^4*(a^6*b^6)^(1/2) + 3*a^4*b^3*(a^6*b^6)
^(1/2) - 3*a^5*b^2*(a^6*b^6)^(1/2) - a*b^6*(a^6*b^6)^(1/2) - a^6*b*(a^6*b^6
)^(1/2)))*(a^6*b^6)^(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6
*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) - (exp(-x)*(a^2 - 4*a*b
 + b^2))/(16*(a - b)^3) - (exp(-3*x)*(a + b))/(96*(a - b)^2) - (exp(3*x)*(a
 - b))/(96*(a + b)^2) - (exp(x)*(4*a*b + a^2 + b^2))/(16*(a + b)^3)
```

3.715 $\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2abx}{(a^2-b^2)^2} + \frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}$$

```
[Out] -2*a*b*x/(a^2-b^2)^2+a^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^2+b^2*ln(a*cosh(
x)+b*sinh(x))/(a^2-b^2)^2+b*sinh(x)/(a^2-b^2)/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3190, 3177, 3212, 3176, 3154}

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2abx}{(a^2-b^2)^2} + \frac{b\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))} + \frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2}$$

```
[In] Int[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*x)/(a^2 - b^2)^2 + (a^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 +
 (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^2 + (b*Sinh[x])/((a^2 - b^2)*
(a*Cosh[x] + b*Sinh[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2} \\
&= -\frac{2abx}{(a^2-b^2)^2} + \frac{b\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))} \\
&\quad + \frac{(ia^2)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{(ib^2)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} \\
&= -\frac{2abx}{(a^2-b^2)^2} + \frac{a^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} \\
&\quad + \frac{b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^2} + \frac{b\sinh(x)}{(a^2-b^2)\,(a\cosh(x)+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$
\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{-2abx+(a^2+b^2)\log(a\cosh(x)+b\sinh(x))+\frac{(a-b)b(a+b)\sinh(x)}{a\cosh(x)+b\sinh(x)}}{(a-b)^2(a+b)^2}
$$

```
[In] Integrate[(Cosh[x]*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
[Out] (-2*a*b*x + (a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]] + ((a - b)*b*(a + b)*Sin
h[x])/(a*Cosh[x] + b*Sinh[x]))/((a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{(a^2+b^2)(a+b\tanh(x))\ln(a+b\tanh(x))-(a^2+b^2)(a+b\tanh(x))\ln(1-\tanh(x))-(((x+1)b+a(-1+x))b\tanh(x)+ax(a+b))(a+}{(a-b)^2(a+b)^2(a+b\tanh(x))}$
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{(a+b)^2}-\frac{\ln(\tanh(\frac{x}{2})+1)}{(a-b)^2}+\frac{\frac{2b(a^2-b^2)\tanh(\frac{x}{2})}{\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a}+(a^2+b^2)\ln\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x\,a^2}{a^4-2a^2b^2+b^4}-\frac{2b^2x}{a^4-2a^2b^2+b^4}-\frac{2ab}{(a-b)(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}+\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)a^2}{a^4-2a^2b^2+b^4}+\frac{b^2\ln(e}{a^4-2}$

```
[In] int(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2+b^2)*(a+b*tanh(x))*ln(a+b*tanh(x))-(a^2+b^2)*(a+b*tanh(x))*ln(1-tanh(
x))-(((x+1)*b+a*(-1+x))*b*tanh(x)+a*x*(a+b))*(a+b))/(a-b)^2/(a+b)^2/(a+b*ta
nh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(93) = 186$.

Time = 0.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.04

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh{(x)}^2+2\,(a^3+3\,a^2b+3\,ab^2+b^3)x\cosh{(x)}\sinh{(x)}+(a^3+3\,a^2b+3\,ab^2}{a^5-a^4b-2\,a^3b^2+2\,a^2b^3+ab^4-b^5+(a^5}$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*cosh(x)^2 + 2*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*x*cosh(x)*sinh(x) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x*sinh(x)^2 + 2*
a^2*b - 2*a*b^2 + (a^3 + a^2*b - a*b^2 - b^3)*x - (a^3 - a^2*b + a*b^2 - b^
3 + (a^3 + a^2*b + a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 + a^2*b + a*b^2 + b^3)*c
osh(x)*sinh(x) + (a^3 + a^2*b + a*b^2 + b^3)*sinh(x)^2)*log(2*(a*cosh(x) +
b*sinh(x))/(cosh(x) - sinh(x))))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5 + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2 +
2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*cosh(x)*sinh(x) + (a^
5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(83) = 166$.

Time = 0.84 (sec) , antiderivative size = 962, normalized size of antiderivative = 10.34

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Piecewise((zoo*log(sinh(x)), Eq(a, 0) & Eq(b, 0)), (log(sinh(x))/b**2, Eq(a
, 0)), (-2*x*sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*cosh(x) + 8*b*
*2*cosh(x)**2) + 4*x*sinh(x)*cosh(x)/(8*b**2*sinh(x)**2 - 16*b**2*sinh(x)*c
osh(x) + 8*b**2*cosh(x)**2) - 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 - 16*b**2*s
inh(x)*cosh(x) + 8*b**2*cosh(x)**2) + sinh(x)**2/(8*b**2*sinh(x)**2 - 16*b*
*2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + cosh(x)**2/(8*b**2*sinh(x)**2 - 1
6*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2), Eq(a, -b)), (2*x*sinh(x)**2/(8
```

```
*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2) + 4*x*sinh(
x)*cosh(x)/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*cosh(x)**2
) + 2*x*cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**2*co
sh(x)**2) + sinh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8*b**
2*cosh(x)**2) + cosh(x)**2/(8*b**2*sinh(x)**2 + 16*b**2*sinh(x)*cosh(x) + 8
*b**2*cosh(x)**2), Eq(a, b)), (a**3*log(cosh(x) + b*sinh(x)/a)*cosh(x)/(a**
5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*
b**4*cosh(x) + b**5*sinh(x)) - a**3*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x)
- 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)
) - 2*a**2*b*x*cosh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x)
 - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a**2*b*log(cosh(x
) + b*sinh(x)/a)*sinh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(
x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) + b**5*sinh(x)) - 2*a*b**2*x*sinh
(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh
(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a*b**2*log(cosh(x) + b*sinh(x)/a)*co
sh(x)/(a**5*cosh(x) + a**4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*si
nh(x) + a*b**4*cosh(x) + b**5*sinh(x)) + a*b**2*cosh(x)/(a**5*cosh(x) + a**
4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) +
b**5*sinh(x)) + b**3*log(cosh(x) + b*sinh(x)/a)*sinh(x)/(a**5*cosh(x) + a**
4*b*sinh(x) - 2*a**3*b**2*cosh(x) - 2*a**2*b**3*sinh(x) + a*b**4*cosh(x) +
b**5*sinh(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,ab}{a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,x)}} + \frac{(a^2+b^2)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} + \frac{x}{a^2+2\,ab+b^2}$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b/(a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*x)) +
(a^2 + b^2)*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + x/(a^2
 + 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{(a^2+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} - \frac{x}{a^2-2ab+b^2} - \frac{a^2e^{(2x)}+b^2e^{(2x)}+a^2-b^2}{(a^3-a^2b-ab^2+b^3)(ae^{(2x)}+be^{(2x)}+a-b)}$$

```
[In] integrate(cosh(x)*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4)
 - x/(a^2 - 2*a*b + b^2) - (a^2*e^(2*x) + b^2*e^(2*x) + a^2 - b^2)/((a^3 -
a^2*b - a*b^2 + b^3)*(a*e^(2*x) + b*e^(2*x) + a - b))
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \ln\left(a\cosh(x)+b\sinh(x)\right)\left(\frac{1}{2\,(a+b)^2}+\frac{1}{2\,(a-b)^2}\right) - \frac{\frac{a\cosh(x)}{a^2-b^2}+\frac{2\,a^2\,b\,x\cosh(x)}{(a^2-b^2)^2}+\frac{2\,a\,b^2\,x\sinh(x)}{(a^2-b^2)^2}}{a\cosh\left(x\right)+b\sinh\left(x\right)}$$

```
[In] int((cosh(x)*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] log(a*cosh(x) + b*sinh(x))*(1/(2*(a + b)^2) + 1/(2*(a - b)^2)) - ((a*cosh(x
))/(a^2 - b^2) + (2*a^2*b*x*cosh(x))/(a^2 - b^2)^2 + (2*a*b^2*x*sinh(x))/(a
^2 - b^2)^2)/(a*cosh(x) + b*sinh(x))
```

3.716 $\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 165

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{2ab^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{2ab\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\sinh(x)}{(a^2-b^2)^2} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} - \frac{a^2b}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}$$

```
[Out] -a^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-2*a*b^2*
arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-2*a*b*cosh(x)
/(a^2-b^2)^2+a^2*sinh(x)/(a^2-b^2)^2+b^2*sinh(x)/(a^2-b^2)^2-a^2*b/(a^2-b^2
)^2/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3190, 3188, 2717, 2718, 3153, 212, 3178, 3233}

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2ab^2\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a^2\sinh(x)}{(a^2-b^2)^2} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} - \frac{2ab\cosh(x)}{(a^2-b^2)^2} - \frac{a^2b}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))} - \frac{a^3\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

```
[In] Int[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -((a^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2))
- (2*a*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) - (2*a*b*Cosh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x])/(a^2 - b^2)^2 + (b^2*Sin
h[x])/(a^2 - b^2)^2 - (a^2*b)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^2\sinh(x)}{(a^2-b^2)^2}-\frac{a^2b}{(a^2-b^2)^2\,(a\cosh(x)+b\sinh(x))}-\frac{a^3\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&\quad -2\frac{(ab)\int \sinh(x)\,dx}{(a^2-b^2)^2}+\frac{b^2\int \cosh(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab^2)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&= -\frac{2ab\cosh(x)}{(a^2-b^2)^2}+\frac{a^2\sinh(x)}{(a^2-b^2)^2}+\frac{b^2\sinh(x)}{(a^2-b^2)^2}-\frac{a^2b}{(a^2-b^2)^2\,(a\cosh(x)+b\sinh(x))}\\
&\quad -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2}\\
&\quad -2\frac{(iab^2)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2}\\
&= -\frac{a^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{2ab^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{2ab\cosh(x)}{(a^2-b^2)^2}\\
&\quad +\frac{a^2\sinh(x)}{(a^2-b^2)^2}+\frac{b^2\sinh(x)}{(a^2-b^2)^2}-\frac{a^2b}{(a^2-b^2)^2\,(a\cosh(x)+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= \frac{-2a^2\sqrt{a-b}b\sqrt{a+b}\cosh^2(x)+a\cosh(x)\left(-2a(a^2+2b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)+\sqrt{a-b}\sqrt{a+b}(a^2-b^2)\,\mathrm{si}\right.}{(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Integrate[(Cosh[x]*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^2*Sqrt[a - b]*b*Sqrt[a + b]*Cosh[x]^2 + a*Cosh[x]*(-2*a*(a^2 + 2*b^2)
*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)*Sinh[x]) + b*(-(a^2*Sqrt[a - b]*Sqrt[a + b]) - 2*a*(a^2 + 2
*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x] + Sqrt[a
- b]*Sqrt[a + b]*(a^2 + b^2)*Sinh[x]^2))/((a - b)^(5/2)*(a + b)^(5/2)*(a*Co
sh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

method	result
default	$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2a\left(\frac{b^2\tanh\left(\frac{x}{2}\right)+ab}{\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a}+\frac{(a^2+2b^2)\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2}-\frac{e^{-x}}{2(a^2-2ab+b^2)}-\frac{2a^2b\,e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{a^3\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}-\frac{2b^2a\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b}}\right.}{\sqrt{-a^2+b^2}\,(a+b)^2(a-}$

```
[In] int(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)-1/(a-b)^2/(tanh(1/2*x)+1)-2*a/(a-b)^2/(a+b)^2*((
b^2*tanh(1/2*x)+a*b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+(a^2+2*b^2)/(a^2-b
^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(157) = 314$.

Time = 0.29 (sec) , antiderivative size = 1819, normalized size of antiderivative = 11.02

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
[Out] [-1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x)^2
 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*
cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2 + (a^4
+ a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b
^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b + 2*a^2*b
^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a - b)) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x))/((a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
 b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*
b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
 -1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - (a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 4*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 - (a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x)^2
 + 2*(5*a^4*b - 4*a^2*b^3 - b^5 - 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*
cosh(x)^3 + 3*(a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cosh(x)*sinh(x)^2 + (a^4
+ a^3*b + 2*a^2*b^2 + 2*a*b^3)*sinh(x)^3 + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b
^3)*cosh(x) + (a^4 - a^3*b + 2*a^2*b^2 - 2*a*b^3 + 3*(a^4 + a^3*b + 2*a^2*b
^2 + 2*a*b^3)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((
a + b)*cosh(x) + (a + b)*sinh(x))) - 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^
3 + a*b^4 - b^5)*cosh(x)^3 - (5*a^4*b - 4*a^2*b^3 - b^5)*cosh(x))*sinh(x))/
((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7
)*cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^
```

```
5 - a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b -
 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6
*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2
)*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,(a^3+2\,ab^2)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{e^x}{2\,(a^2+2\,ab+b^2)} - \frac{a^3e^{(2\,x)}+7\,a^2be^{(2\,x)}+3\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

```
[In] integrate(cosh(x)*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(a^3 + 2*a*b^2)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) - 1/2*(a^3*e^(2*x)
+ 7*a^2*b*e^(2*x) + 3*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b
^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.41

$$\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\mathrm{e}^{x}}{2\,(a+b)^2} - \frac{\mathrm{e}^{-x}}{2\,(a-b)^2} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\left(a^3\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}+2\,a\,b^2\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-\cdots}\right.}{a^5\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}-b^5\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}+2\,a^2\,b^3\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}-2\,a^3\,b^2\,\sqrt{a^6+4\,a^4\,b^2+4\,a^2\,b^4}+a\,b^4\,\sqrt{a^6+4\,a^4\,b^2+4\cdots}}\cdots\right)}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} - \frac{2\,a^2\,b\,\mathrm{e}^{x}}{(a+b)^2\,(a-b)^2\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

```
[In] int((cosh(x)*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(x)/(2*(a + b)^2) - exp(-x)/(2*(a - b)^2) - (2*atan((exp(x)*(a^3*(a^10 -
 b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*a*b^2*(a
^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))/(a^5*
(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - b^5*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/
2) + 2*a^2*b^3*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - 2*a^3*b^2*(a^6 + 4*a^2
*b^4 + 4*a^4*b^2)^(1/2) + a*b^4*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - a^4*b
*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2)))*(a^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2))
/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - (2
*a^2*b*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.717 $\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 215

$$
\begin{aligned}
\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \frac{a^3bx}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3}+\frac{abx}{(a^2-b^2)^2}+\frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3} \\
&\quad -\frac{a^2b}{(a^2-b^2)^2\,(b+a\coth(x))}-\frac{a^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\
&\quad -\frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\
&\quad -\frac{ab\cosh(x)\sinh(x)}{(a^2-b^2)^2}+\frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2}+\frac{b^2\sinh^2(x)}{2\,(a^2-b^2)^2}
\end{aligned}
$$

```
[Out] a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3+a*b*x/(a^2-b^2)^2+a*b*(a^2+b^2)*x/(
a^2-b^2)^3-a^2*b/(a^2-b^2)^2/(b+a*coth(x))-a^4*ln(a*cosh(x)+b*sinh(x))/(a^2
-b^2)^3-3*a^2*b^2*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-a*b*cosh(x)*sinh(x)/(
a^2-b^2)^2+1/2*a^2*sinh(x)^2/(a^2-b^2)^2+1/2*b^2*sinh(x)^2/(a^2-b^2)^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules

used = {3190, 3188, 2644, 30, 2715, 8, 3176, 3212, 3178, 3164, 3564, 3612, 3611}

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{abx(a^2+b^2)}{(a^2-b^2)^3}+\frac{abx}{(a^2-b^2)^2}+\frac{a^2\sinh^2(x)}{2(a^2-b^2)^2}+\frac{b^2\sinh^2(x)}{2(a^2-b^2)^2} - \frac{a^2b}{(a^2-b^2)^2(a\coth(x)+b)}-\frac{ab\sinh(x)\cosh(x)}{(a^2-b^2)^2} - \frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3} - \frac{a^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{a^3bx}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 + (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 - (a^2*b)/((a^2 - b^2)^2*(b + a*Coth[x])
) - (a^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (3*a^2*b^2*Log[a*Cosh[
x] + b*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2
*Sinh[x]^2)/(2*(a^2 - b^2)^2) + (b^2*Sinh[x]^2)/(2*(a^2 - b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3164

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
```

```
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
```

```
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{a^3\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}-2\frac{(ab)\int \sinh^2(x)\,dx}{\left(a^2-b^2\right)^2}\\
&\quad+\frac{b^2\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2}-\frac{\left(ab^2\right)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}-\frac{(ab)\int \frac{1}{(-ib-ia\coth(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{ab^3x}{\left(a^2-b^2\right)^3}-\frac{a^2b}{\left(a^2-b^2\right)^2(b+a\coth(x))}\\
&\quad+\frac{a^2\sinh^2(x)}{2\left(a^2-b^2\right)^2}-\frac{\left(ia^4\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3}\\
&\quad-\frac{\left(ia^2b^2\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3}-2\left(\frac{ab\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2}-\frac{(ab)\int 1\,dx}{2\left(a^2-b^2\right)^2}\right)\\
&\quad-\frac{(ab)\int \frac{-ib+ia\coth(x)}{-ib-ia\coth(x)}\,dx}{\left(a^2-b^2\right)^2}-\frac{b^2\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)}{\left(a^2-b^2\right)^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^3bx}{(a^2-b^2)^3} + \frac{ab^3x}{(a^2-b^2)^3} + \frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3} \\
&\quad - \frac{a^2b}{(a^2-b^2)^2\,(b+a\coth(x))} - \frac{a^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\
&\quad - \frac{a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2} + \frac{b^2\sinh^2(x)}{2\,(a^2-b^2)^2} \\
&\quad - 2\left(-\frac{abx}{2\,(a^2-b^2)^2} + \frac{ab\cosh(x)\sinh(x)}{2\,(a^2-b^2)^2}\right) - \frac{(2ia^2b^2)\int \frac{-a-b\coth(x)}{-ib-ia\coth(x)}\,dx}{(a^2-b^2)^3} \\
&= \frac{a^3bx}{(a^2-b^2)^3} + \frac{ab^3x}{(a^2-b^2)^3} + \frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3} - \frac{a^2b}{(a^2-b^2)^2\,(b+a\coth(x))} \\
&\quad - \frac{a^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} \\
&\quad + \frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2} + \frac{b^2\sinh^2(x)}{2\,(a^2-b^2)^2} - 2\left(-\frac{abx}{2\,(a^2-b^2)^2} + \frac{ab\cosh(x)\sinh(x)}{2\,(a^2-b^2)^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$
\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a(a^2-b^2)^2\cosh(3x) + a\cosh(x)\left(a^4+2a^2b^2-3b^4+24a^3bx+8ab^3x-8a^2(a^2+3b^2)\log(a\cosh(x)+b\sinh\right.}{8(a-b)^3(a+b}
$$

```
[In] Integrate[(Cosh[x]*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*(a^2 - b^2)^2*Cosh[3*x] + a*Cosh[x]*(a^4 + 2*a^2*b^2 - 3*b^4 + 24*a^3*b*
x + 8*a*b^3*x - 8*a^2*(a^2 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*b*((a^2
 - b^2)^2*Cosh[2*x] + 2*a*(3*a^3 - 3*a*b^2 - 6*a^2*b*x - 2*b^3*x + 2*a*(a^2
 + 3*b^2)*Log[a*Cosh[x] + b*Sinh[x]]))*Sinh[x])/(8*(a - b)^3*(a + b)^3*(a*C
osh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

method	result
default	$\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}+\frac{a\ln(\tanh(\frac{x}{2})-1)}{(a+b)^3}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)}+$
risch	$-\frac{ax}{(a+b)(a^2+2ab+b^2)}+\frac{e^{2x}}{8a^2+16ab+8b^2}+\frac{e^{-2x}}{8a^2-16ab+8b^2}+\frac{2a^4x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{6a^2x\,b^2}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{}{(a-b)^2(a^3+3}$

```
[In] int(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2+a/(a+b)^3*ln(tanh
(1/2*x)-1)+1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+a/(a-b
)^3*ln(tanh(1/2*x)+1)-2*a^2/(a+b)^3/(a-b)^3*(b*(a^2-b^2)*tanh(1/2*x)/(tanh(
1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(a^2+3*b^2)*ln(tanh(1/2*x)^2*a+2*b*tanh(1
/2*x)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. $2(211) = 422$.

Time = 0.28 (sec) , antiderivative size = 1655, normalized size of antiderivative = 7.70

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(x
)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b
^3 - 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*c
osh(x))*sinh(x)^3 + (a^5 + 19*a^4*b - 14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*cosh(x)^2 + (a^5 + 19*a^4*b -
14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
 3*a*b^4 + b^5 + 8*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(
x)^2 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*x)*sinh(x)^2 - 8*((a^5 + a^4*b
 + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^4 + 4*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^
3)*cosh(x)*sinh(x)^3 + (a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*sinh(x)^4 + (a
^5 - a^4*b + 3*a^3*b^2 - 3*a^2*b^3)*cosh(x)^2 + (a^5 - a^4*b + 3*a^3*b^2 -
```

```
3*a^2*b^3 + 6*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^2)*sinh(x)^2 +
2*(2*(a^5 + a^4*b + 3*a^3*b^2 + 3*a^2*b^3)*cosh(x)^3 + (a^5 - a^4*b + 3*a^3
*b^2 - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x)
- sinh(x))) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^5 +
 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*x)*cosh(x)^3 + (a^5 + 19*a^4*b -
14*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 8*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b
^4)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^3 + (a^7 + a^6*b
- 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^4 +
(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)
*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
 a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3 + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x))*sinh(x
))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{ax}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{(a^4+3\,a^2b^2)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{a^4-2\,a^3b+2\,ab^3-b^4+(a^4-20\,a^3b+6\,a^2b^2-4\,ab^3+b^4)e^{(-2\,x)}}{8\left((a^6-3\,a^4b^2+3\,a^2b^4-b^6)e^{(-2\,x)}+(a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6)e^{(-4\,x)}\right)} + \frac{e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)}$$

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -a*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a^4 + 3*a^2*b^2)*log(-(a - b)*e^(-2
*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*a
*b^3 - b^4 + (a^4 - 20*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b
^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{ax}{a^3-3a^2b+3ab^2-b^3} - \frac{(a^4+3a^2b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{e^{(2x)}}{8(a^2+2ab+b^2)} + \frac{2a^3e^{(4x)}-4a^2be^{(4x)}+2ab^2e^{(4x)}+3a^3e^{(2x)}+11a^2be^{(2x)}+ab^2e^{(2x)}+b^3e^{(2x)}+a^3+a^2b-ab^2-b^3}{8(a^4-2a^2b^2+b^4)(ae^{(4x)}+be^{(4x)}+ae^{(2x)}-be^{(2x)})}$$

```
[In] integrate(cosh(x)*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] a*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a^4 + 3*a^2*b^2)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a^2
 + 2*a*b + b^2) + 1/8*(2*a^3*e^(4*x) - 4*a^2*b*e^(4*x) + 2*a*b^2*e^(4*x) +
3*a^3*e^(2*x) + 11*a^2*b*e^(2*x) + a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*
b - a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x
) - b*e^(2*x)))
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int \frac{\cosh(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{2x}}{8(a+b)^2} + \frac{e^{-2x}}{8(a-b)^2} + \frac{a\,x}{(a-b)^3} - \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\,(a^4+3a^2b^2)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{2a^3b}{(a+b)^3(a-b)^2\,(a-b+e^{2x}\,(a+b))}$$

```
[In] int((cosh(x)*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*(a + b)^2) + exp(-2*x)/(8*(a - b)^2) + (a*x)/(a - b)^3 - (log(a
 - b + a*exp(2*x) + b*exp(2*x))*(a^4 + 3*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2) + (2*a^3*b)/((a + b)^3*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.718 $\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 163

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2a^2b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{b^2\cosh(x)}{(a^2-b^2)^2} - \frac{2ab\sinh(x)}{(a^2-b^2)^2} + \frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}$$

```
[Out] 2*a^2*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+b^3*a
rctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a^2*cosh(x)/(a
^2-b^2)^2+b^2*cosh(x)/(a^2-b^2)^2-2*a*b*sinh(x)/(a^2-b^2)^2+a*b^2/(a^2-b^2)
^2/(a*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3190, 3179, 2717, 3153, 212, 3188, 2718, 3234}

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2a^2b\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{b^3\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{2ab\sinh(x)}{(a^2-b^2)^2} + \frac{b^2\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}$$

```
[In] Int[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (2*a^2*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
+ (a^2*Cosh[x])/(a^2 - b^2)^2 + (b^2*Cosh[x])/(a^2 - b^2)^2 - (2*a*b*Sinh[x
])/(a^2 - b^2)^2 + (a*b^2)/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{b^2\cosh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}+\frac{a^2\int \sinh(x)\,dx}{(a^2-b^2)^2}\\
&\quad -2\frac{(ab)\int \cosh(x)\,dx}{(a^2-b^2)^2}+2\frac{(a^2b)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}+\frac{b^3\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&= \frac{a^2\cosh(x)}{(a^2-b^2)^2}+\frac{b^2\cosh(x)}{(a^2-b^2)^2}-\frac{2ab\sinh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}\\
&\quad +2\frac{(ia^2b)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2}\\
&\quad +\frac{(ib^3)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2}\\
&= \frac{2a^2b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a^2\cosh(x)}{(a^2-b^2)^2}\\
&\quad +\frac{b^2\cosh(x)}{(a^2-b^2)^2}-\frac{2ab\sinh(x)}{(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a\sqrt{a-b}(a+b)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2b^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{4(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))} + \frac{1}{4}\left(\frac{6b(3a^2+b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{4(a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2}-\frac{8ab\sinh(x)}{(a-b)^2(a+b)^2}+\frac{a(a^2+3b^2)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}\right)$$

```
[In] Integrate[(Cosh[x]^2*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -1/4*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(S
qrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2
])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x]
+ b*Sinh[x])) + ((6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*S
qrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) + (4*(a^2 + b^2)*Cosh[x])/((a -
 b)^2*(a + b)^2) - (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (a*(a^2 + 3*b^2)
)/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/4
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80

method	result
default	$-\frac{1}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{4b\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{2}+\frac{ab}{2}}{\tanh\left(\frac{x}{2}\right)^2 a+2b\tanh\left(\frac{x}{2}\right)+a}+\frac{\left(2a^2+b^2\right)\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{e^x}{2a^2+4ab+2b^2}+\frac{e^{-x}}{2a^2-4ab+2b^2}+\frac{2a\,b^2 e^x}{(a-b)^2(a^2+2ab+b^2)(a\,e^{2x}+b\,e^{2x}+a-b)}-\frac{2b\,a^2\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}-\frac{b^3\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a}$

```
[In] int(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)^2/(tanh(1/2*x)-1)+1/(a-b)^2/(tanh(1/2*x)+1)+4*b/(a+b)^2/(a-b)^2*((
1/2*b^2*tanh(1/2*x)+1/2*a*b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(2*a^2
+b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(155) = 310$.

Time = 0.29 (sec) , antiderivative size = 1805, normalized size of antiderivative = 11.07

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
[Out] [1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x)^2
+ 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 2*((2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*c
osh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (2*a^3
*b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^
4)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a^2*b^2 +
a*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 +
2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x)
 + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
b)*sinh(x)^2 + a - b)) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x))/((a^7 + a^6*
b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3
+ 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 -
b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3
+ 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3*a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b - 3*a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)),
1/2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - a^4*b - 2*a
^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 4*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^3 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*sinh(x)^4 + 2*(a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x)^2 +
 2*(a^5 + 4*a^3*b^2 - 5*a*b^4 + 3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*
b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 4*((2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*co
sh(x)^3 + 3*(2*a^3*b + 2*a^2*b^2 + a*b^3 + b^4)*cosh(x)*sinh(x)^2 + (2*a^3*
b + 2*a^2*b^2 + a*b^3 + b^4)*sinh(x)^3 + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4
)*cosh(x) + (2*a^3*b - 2*a^2*b^2 + a*b^3 - b^4 + 3*(2*a^3*b + 2*a^2*b^2 + a
*b^3 + b^4)*cosh(x)^2)*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a
+ b)*cosh(x) + (a + b)*sinh(x))) + 4*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3
+ a*b^4 - b^5)*cosh(x)^3 + (a^5 + 4*a^3*b^2 - 5*a*b^4)*cosh(x))*sinh(x))/((
a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*
cosh(x)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5
```

```
- a*b^6 - b^7)*cosh(x)*sinh(x)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3
*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^3 + (a^7 - a^6*b - 3*a^5*b^2 +
3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x) + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 3*(a^7 + a^6*b
 - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*
sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2\,(2\,a^2b+b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{e^x}{2\,(a^2+2\,ab+b^2)} + \frac{a^3e^{(2\,x)}+3\,a^2be^{(2\,x)}+7\,ab^2e^{(2\,x)}+b^3e^{(2\,x)}+a^3+a^2b-ab^2-b^3}{2\,(a^4-2\,a^2b^2+b^4)(ae^{(3\,x)}+be^{(3\,x)}+ae^x-be^x)}$$

```
[In] integrate(cosh(x)^2*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(2*a^2*b + b^3)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2
 + b^4)*sqrt(a^2 - b^2)) + 1/2*e^x/(a^2 + 2*a*b + b^2) + 1/2*(a^3*e^(2*x) +
 3*a^2*b*e^(2*x) + 7*a*b^2*e^(2*x) + b^3*e^(2*x) + a^3 + a^2*b - a*b^2 - b^
3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(3*x) + b*e^(3*x) + a*e^x - b*e^x))
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.44

$$\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{-x}}{2\,(a-b)^2}+\frac{e^{x}}{2\,(a+b)^2} + \frac{2\,\mathrm{atan}\left(\frac{e^{x}\,\left(b^3\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}+2\,a^2\,b\,\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{1}}\right.}{a^5\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}-b^5\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}+2\,a^2\,b^3\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}-2\,a^3\,b^2\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+b^6}+a\,b^4\,\sqrt{4\,a^4\,b^2+4\,a^2\,b^4+}}\right.}{\sqrt{a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}}} + \frac{2\,a\,b^2\,e^{x}}{(a+b)^2\,(a-b)^2\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

```
[In] int((cosh(x)^2*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*(a - b)^2) + exp(x)/(2*(a + b)^2) + (2*atan((exp(x)*(b^3*(a^10 -
 b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*a^2*b*(a
^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))/(a^5*
(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - b^5*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/
2) + 2*a^2*b^3*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - 2*a^3*b^2*(b^6 + 4*a^2
*b^4 + 4*a^4*b^2)^(1/2) + a*b^4*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2) - a^4*b
*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2)))*(b^6 + 4*a^2*b^4 + 4*a^4*b^2)^(1/2))
/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + (2
*a*b^2*exp(x))/((a + b)^2*(a - b)^2*(a - b + exp(2*x)*(a + b)))
```

3.719 $\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 205

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{4a^2b^2x}{(a^2-b^2)^3} - \frac{a^2x}{2(a^2-b^2)^2} + \frac{b^2x}{2(a^2-b^2)^2} + \frac{2a^3b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{2ab^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{a^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2} + \frac{b^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2} - \frac{ab\sinh^2(x)}{(a^2-b^2)^2} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}$$

```
[Out] -4*a^2*b^2*x/(a^2-b^2)^3-1/2*a^2*x/(a^2-b^2)^2+1/2*b^2*x/(a^2-b^2)^2+2*a^3*
b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3+2*a*b^3*ln(a*cosh(x)+b*sinh(x))/(a^2-
b^2)^3+1/2*a^2*cosh(x)*sinh(x)/(a^2-b^2)^2+1/2*b^2*cosh(x)*sinh(x)/(a^2-b^2
)^2-a*b*sinh(x)^2/(a^2-b^2)^2+a*b^2*sinh(x)/(a^2-b^2)^2/(a*cosh(x)+b*sinh(x
))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3190, 3188, 2715, 8, 2644, 30, 3177, 3212, 3176, 3154}

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{a^2x}{2(a^2-b^2)^2}-\frac{4a^2b^2x}{(a^2-b^2)^3}+\frac{b^2x}{2(a^2-b^2)^2}-\frac{ab\sinh^2(x)}{(a^2-b^2)^2}+\frac{a^2\sinh(x)\cosh(x)}{2(a^2-b^2)^2}+\frac{ab^2\sinh(x)}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}+\frac{b^2\sinh(x)\cosh(x)}{2(a^2-b^2)^2}+\frac{2ab^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{2a^3b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-4*a^2*b^2*x)/(a^2 - b^2)^3 - (a^2*x)/(2*(a^2 - b^2)^2) + (b^2*x)/(2*(a^2
 - b^2)^2) + (2*a^3*b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (2*a*b^3*L
og[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 + (a^2*Cosh[x]*Sinh[x])/(2*(a^2 -
b^2)^2) + (b^2*Cosh[x]*Sinh[x])/(2*(a^2 - b^2)^2) - (a*b*Sinh[x]^2)/(a^2 -
b^2)^2 + (a*b^2*Sinh[x])/((a^2 - b^2)^2*(a*Cosh[x] + b*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
```

```
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^2\int \sinh^2(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab)\int \cosh(x)\sinh(x)\,dx}{(a^2-b^2)^2}+2\frac{(a^2b)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&\quad+\frac{b^2\int \cosh^2(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab^2)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}+\frac{(a^2b^2)\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{(a^2-b^2)^2}\\
&= \frac{a^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}+\frac{b^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}+\frac{ab^2\sinh(x)}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}\\
&\quad+2\left(-\frac{a^2b^2x}{(a^2-b^2)^3}+\frac{(ia^3b)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right)\\
&\quad-2\left(\frac{a^2b^2x}{(a^2-b^2)^3}-\frac{(iab^3)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right)\\
&\quad-\frac{a^2\int 1\,dx}{2(a^2-b^2)^2}+2\frac{(ab)\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)}{(a^2-b^2)^2}+\frac{b^2\int 1\,dx}{2(a^2-b^2)^2}\\
&= -\frac{a^2x}{2(a^2-b^2)^2}+\frac{b^2x}{2(a^2-b^2)^2}+2\left(-\frac{a^2b^2x}{(a^2-b^2)^3}+\frac{a^3b\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}\right)\\
&\quad-2\left(\frac{a^2b^2x}{(a^2-b^2)^3}-\frac{ab^3\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}\right)+\frac{a^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}\\
&\quad+\frac{b^2\cosh(x)\sinh(x)}{2(a^2-b^2)^2}-\frac{ab\sinh^2(x)}{(a^2-b^2)^2}+\frac{ab^2\sinh(x)}{(a^2-b^2)^2(a\cosh(x)+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx=\frac{1}{8}\left(-\frac{4(a^4+6a^2b^2+b^4)\,x}{(a-b)^3(a+b)^3}-\frac{4ab\cosh(2x)}{(a-b)^2(a+b)^2}+\frac{16ab(a^2+b^2)\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}+\frac{(a^4+6a^2b^2+b^4)\sinh(x)}{a(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}-\frac{\sinh(x)}{a^2\cosh(x)+ab\sinh(x)}+\frac{2(a^2+b^2)\sinh(2x)}{(a-b)^2(a+b)^2}\right)$$

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((-4*(a^4 + 6*a^2*b^2 + b^4)*x)/((a - b)^3*(a + b)^3) - (4*a*b*Cosh[2*x])/(
(a - b)^2*(a + b)^2) + (16*a*b*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]])/(a^2
 - b^2)^3 + ((a^4 + 6*a^2*b^2 + b^4)*Sinh[x])/(a*(a - b)^2*(a + b)^2*(a*Cos
h[x] + b*Sinh[x])) - Sinh[x]/(a^2*Cosh[x] + a*b*Sinh[x]) + (2*(a^2 + b^2)*S
inh[2*x])/((a - b)^2*(a + b)^2))/8
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

method	result
default	$\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}+\frac{(a-b)\ln(\tanh(\frac{x}{2})-1)}{2(a+b)^3}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)}$
risch	$-\frac{ax}{2(a+b)(a^2+2ab+b^2)}+\frac{xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2x}}{8a^2+16ab+8b^2}-\frac{e^{-2x}}{8(a^2-2ab+b^2)}-\frac{4a^3bx}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{4ab^3}{a^6-3a^4b^2+3}$

```
[In] int(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2+1/2*(a-b)/(a+b)^3
*ln(tanh(1/2*x)-1)-1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/2/(a-b)^2/(tanh(1/2*x)+1
)+1/2/(a-b)^3*(-a-b)*ln(tanh(1/2*x)+1)+2*a*b/(a+b)^3/(a-b)^3*(b*(a^2-b^2)*t
anh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(2*a^2+2*b^2)*ln(tanh(1/
2*x)^2*a+2*b*tanh(1/2*x)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1726 vs. 2(197) = 394.

Time = 0.29 (sec) , antiderivative size = 1726, normalized size of antiderivative = 8.42

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^
3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*x)*cosh(x)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 - 4*(a^5 + 5
```

```
*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b +
 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x))*sinh(x)^3 - (a^5 + 3*a^4*b + 18*a^3*b
^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3
- 3*a*b^4 - b^5)*x)*cosh(x)^2 - (a^5 + 3*a^4*b + 18*a^3*b^2 - 18*a^2*b^3 -
3*a*b^4 - b^5 - 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^4 - 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 +
 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x)^2 + 4*(a^5 +
 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*x)*sinh(x)^2 + 16*((a^4*b
 + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^4 + 4*(a^4*b + a^3*b^2 + a^2*b^3 + a*
b^4)*cosh(x)*sinh(x)^3 + (a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*sinh(x)^4 + (a
^4*b - a^3*b^2 + a^2*b^3 - a*b^4)*cosh(x)^2 + (a^4*b - a^3*b^2 + a^2*b^3 -
a*b^4 + 6*(a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^2 + 2*(2*(
a^4*b + a^3*b^2 + a^2*b^3 + a*b^4)*cosh(x)^3 + (a^4*b - a^3*b^2 + a^2*b^3 -
 a*b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))
) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*
(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 - 4*(a^5 + 5*a^4*b +
 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*x)*cosh(x)^3 - (a^5 + 3*a^4*b + 1
8*a^3*b^2 - 18*a^2*b^3 - 3*a*b^4 - b^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b -
 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(
x)^3 + (a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6
 - b^7)*sinh(x)^4 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^
2*b^5 - a*b^6 + b^7)*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a
^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 +
 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^
6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^
3 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
b^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$
$$= -\frac{(a-b)x}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{2\,(a^3b+ab^3)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}$$
$$+ \frac{a^4-2\,a^3b+2\,ab^3-b^4+(a^4-4\,a^3b+22\,a^2b^2-4\,ab^3+b^4)e^{(-2\,x)}}{8\left((a^6-3\,a^4b^2+3\,a^2b^4-b^6)e^{(-2\,x)}+(a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6)e^{(-4\,x)}\right)}$$
$$- \frac{e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)}$$

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*(a - b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*b + a*b^3)*log(-(a
- b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a
^3*b + 2*a*b^3 - b^4 + (a^4 - 4*a^3*b + 22*a^2*b^2 - 4*a*b^3 + b^4)*e^(-2*x
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2
 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) - 1/8*e^(-2*x)/(a^2 - 2*a
*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{(a+b)x}{2\,(a^3-3\,a^2b+3\,ab^2-b^3)}$$
$$+ \frac{2\,(a^3b+ab^3)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{e^{(2\,x)}}{8\,(a^2+2\,ab+b^2)}$$
$$+ \frac{a^3e^{(4\,x)}-3\,a^2be^{(4\,x)}+3\,ab^2e^{(4\,x)}-b^3e^{(4\,x)}-8\,a^2be^{(2\,x)}-8\,ab^2e^{(2\,x)}-a^3-a^2b+ab^2+b^3}{8\,(a^4-2\,a^2b^2+b^4)(ae^{(4\,x)}+be^{(4\,x)}+ae^{(2\,x)}-be^{(2\,x)})}$$

```
[In] integrate(cosh(x)^2*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(a^3*b + a*b^3)*log(abs(
a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e
^(2*x)/(a^2 + 2*a*b + b^2) + 1/8*(a^3*e^(4*x) - 3*a^2*b*e^(4*x) + 3*a*b^2*e
^(4*x) - b^3*e^(4*x) - 8*a^2*b*e^(2*x) - 8*a*b^2*e^(2*x) - a^3 - a^2*b + a*
b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x) - b*
e^(2*x)))
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

$$\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{2x}}{8(a+b)^2} - \frac{e^{-2x}}{8(a-b)^2} + \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\,\left(2a^3b+2ab^3\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{x(a+b)}{2(a-b)^3} - \frac{2a^2b^2}{(a+b)^3(a-b)^2\,\left(a-b+e^{2x}\,(a+b)\right)}$$

```
[In] int((cosh(x)^2*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*(a + b)^2) - exp(-2*x)/(8*(a - b)^2) + (log(a - b + a*exp(2*x)
+ b*exp(2*x))*(2*a*b^3 + 2*a^3*b))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (x
*(a + b))/(2*(a - b)^3) - (2*a^2*b^2)/((a + b)^3*(a - b)^2*(a - b + exp(2*x
)*(a + b)))
```

3.720 $\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 261

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2a^4b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{3a^2b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{4a^2b^2\cosh(x)}{(a^2-b^2)^3} - \frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{2a^3b\sinh(x)}{(a^2-b^2)^3} + \frac{2ab^3\sinh(x)}{(a^2-b^2)^3} - \frac{2ab\sinh^3(x)}{3(a^2-b^2)^2} - \frac{a^3b^2}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}$$

```
[Out] -2*a^4*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)-3*a^
2*b^3*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)-4*a^2*b
^2*cosh(x)/(a^2-b^2)^3-a^2*cosh(x)/(a^2-b^2)^2+1/3*a^2*cosh(x)^3/(a^2-b^2)^
2+1/3*b^2*cosh(x)^3/(a^2-b^2)^2+2*a^3*b*sinh(x)/(a^2-b^2)^3+2*a*b^3*sinh(x)
/(a^2-b^2)^3-2/3*a*b*sinh(x)^3/(a^2-b^2)^2-a^3*b^2/(a^2-b^2)^3/(a*cosh(x)+b
*sinh(x))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3190, 3188, 2645, 30, 2644, 2717, 2718, 3153, 212, 2713, 3178, 3233}

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3a^2b^3\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{2ab\sinh^3(x)}{3(a^2-b^2)^2} + \frac{a^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\cosh^3(x)}{3(a^2-b^2)^2} - \frac{a^2\cosh(x)}{(a^2-b^2)^2} - \frac{4a^2b^2\cosh(x)}{(a^2-b^2)^3} + \frac{2ab^3\sinh(x)}{(a^2-b^2)^3} - \frac{2a^4b\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2a^3b\sinh(x)}{(a^2-b^2)^3} - \frac{a^3b^2}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}$$

```
[In] Int[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^4*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2
) - (3*a^2*b^3*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(7/2) - (4*a^2*b^2*Cosh[x])/(a^2 - b^2)^3 - (a^2*Cosh[x])/(a^2 - b^2)^2 +
(a^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Cosh[x]^3)/(3*(a^2 - b^2)^2) + (2*
a^3*b*Sinh[x])/(a^2 - b^2)^3 + (2*a*b^3*Sinh[x])/(a^2 - b^2)^3 - (2*a*b*Sin
h[x]^3)/(3*(a^2 - b^2)^2) - (a^3*b^2)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x]
))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^2\int \sinh^3(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab)\int \cosh(x)\sinh^2(x)\,dx}{(a^2-b^2)^2}+2\frac{(a^2b)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&\quad+\frac{b^2\int \cosh^2(x)\sinh(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab^2)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}+\frac{(a^2b^2)\int \frac{\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{(a^2-b^2)^2}\\
&= -\frac{a^3b^2}{(a^2-b^2)^3\,(a\cosh(x)+b\sinh(x))}\\
&\quad+2\left(\frac{a^3b\sinh(x)}{(a^2-b^2)^3}-\frac{(a^4b)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}-\frac{(a^2b^2)\int \sinh(x)\,dx}{(a^2-b^2)^3}\right)\\
&\quad-\frac{(a^2b^3)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\\
&\quad-2\left(\frac{(a^2b^2)\int \sinh(x)\,dx}{(a^2-b^2)^3}-\frac{(ab^3)\int \cosh(x)\,dx}{(a^2-b^2)^3}+\frac{(a^2b^3)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right)\\
&\quad-\frac{a^2\text{Subst}\left(\int (1-x^2)\,dx,x,\cosh(x)\right)}{(a^2-b^2)^2}\\
&\quad-2\frac{(iab)\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{(a^2-b^2)^2}+\frac{b^2\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)}{(a^2-b^2)^2}
\end{aligned}$$

$$= -\frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\cosh^3(x)}{3(a^2-b^2)^2} - \frac{2ab\sinh^3(x)}{3(a^2-b^2)^2} - \frac{a^3b^2}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} + 2\left(-\frac{a^2b^2\cosh(x)}{(a^2-b^2)^3} + \frac{a^3b\sinh(x)}{(a^2-b^2)^3} - \frac{(ia^4b)\,\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3}\right) - \frac{(ia^2b^3)\,\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3} - 2\left(\frac{a^2b^2\cosh(x)}{(a^2-b^2)^3} - \frac{ab^3\sinh(x)}{(a^2-b^2)^3} + \frac{(ia^2b^3)\,\text{Subst}\left(\int\frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3}\right)$$

$$= -\frac{a^2b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{a^2\cosh(x)}{(a^2-b^2)^2} + \frac{a^2\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\cosh^3(x)}{3(a^2-b^2)^2} - \frac{2ab\sinh^3(x)}{3(a^2-b^2)^2} - \frac{a^3b^2}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} + 2\left(-\frac{a^4b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{a^2b^2\cosh(x)}{(a^2-b^2)^3} + \frac{a^3b\sinh(x)}{(a^2-b^2)^3}\right) - 2\left(\frac{a^2b^3\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{a^2b^2\cosh(x)}{(a^2-b^2)^3} - \frac{ab^3\sinh(x)}{(a^2-b^2)^3}\right)$$

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$$

$$= \frac{1}{16}\left(-\frac{6b(3a^2+b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{10b(5a^4+10a^2b^2+b^4)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} - \frac{4(a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2} - \frac{8(a^4+6a^2b^2+b^4)\cosh(x)}{(a-b)^3(a+b)^3} + \frac{4(a^2+b^2)\cosh(3x)}{3(a-b)^2(a+b)^2} + \frac{8ab\sinh(x)}{(a-b)^2(a+b)^2} + \frac{32ab(a^2+b^2)\sinh(x)}{(a-b)^3(a+b)^3} - \frac{a(a^2+3b^2)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{a(a^4+10a^2b^2+5b^4)}{(a-b)^3(a+b)^3(a\cosh(x)+b\sinh(x))} + \frac{2\left(a\sqrt{a-b}(a+b)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2b^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)\right)}{(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))} - \frac{8ab\sinh(3x)}{3(a-b)^2(a+b)^2}\right)$$

```
[In] Integrate[(Cosh[x]^2*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] ((-6*b*(3*a^2 + b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(
(a - b)^(5/2)*(a + b)^(5/2)) - (10*b*(5*a^4 + 10*a^2*b^2 + b^4)*ArcTan[(b +
 a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(7/2)*(a + b)^(7/2)) - (
4*(a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^4)*C
osh[x])/((a - b)^3*(a + b)^3) + (4*(a^2 + b^2)*Cosh[3*x])/(3*(a - b)^2*(a +
 b)^2) + (8*a*b*Sinh[x])/((a - b)^2*(a + b)^2) + (32*a*b*(a^2 + b^2)*Sinh[x
])/((a - b)^3*(a + b)^3) - (a*(a^2 + 3*b^2))/((a - b)^2*(a + b)^2*(a*Cosh[x
] + b*Sinh[x])) - (a*(a^4 + 10*a^2*b^2 + 5*b^4))/((a - b)^3*(a + b)^3*(a*Co
sh[x] + b*Sinh[x])) + (2*(a*Sqrt[a - b]*(a + b) + 2*a*b*Sqrt[a + b]*ArcTan[
(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*b^2*Sqrt[a + b]*Ar
cTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x]))/((a - b)^(3/2)*
(a + b)^2*(a*Cosh[x] + b*Sinh[x])) - (8*a*b*Sinh[3*x])/(3*(a - b)^2*(a + b)
^2))/16
```

Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.80

method	result
default	$-\frac{1}{3(a+b)^2(\tanh(\frac{x}{2})-1)^3}-\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}-\frac{-a+b}{2(a+b)^3(\tanh(\frac{x}{2})-1)}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}+\frac{1}{3(a-b)^2(\tanh(\frac{x}{2})+1}$
risch	$\frac{e^{3x}}{24a^2+48ab+24b^2}-\frac{3\,e^{x}a}{8(a+b)(a^2+2ab+b^2)}+\frac{e^{x}b}{8(a+b)(a^2+2ab+b^2)}-\frac{3\,e^{-x}a}{8(a^3-3a^2b+3a\,b^2-b^3)}-\frac{e^{-x}b}{8(a^3-3a^2b+3a\,b^2-b^3)}+\frac{}{24a^2-}$

```
[In] int(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2-1/2/(a+b)^3*(-
a+b)/(tanh(1/2*x)-1)-1/2/(a-b)^2/(tanh(1/2*x)+1)^2+1/3/(a-b)^2/(tanh(1/2*x)
+1)^3-1/2*(a+b)/(a-b)^3/(tanh(1/2*x)+1)-4*a^2*b/(a+b)^3/(a-b)^3*((1/2*b^2*t
anh(1/2*x)+1/2*a*b)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(2*a^2+3*b^2)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2503 vs. $2(247) = 494$.

Time = 0.33 (sec) , antiderivative size = 5061, normalized size of antiderivative = 19.39

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{2\,a^3b^2e^x}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)(ae^{(2\,x)}+be^{(2\,x)}+a-b)} - \frac{(9\,ae^{(2\,x)}+3\,be^{(2\,x)}-a+b)e^{(-3\,x)}}{24\,(a^3-3\,a^2b+3\,ab^2-b^3)} - \frac{2\,(2\,a^4b+3\,a^2b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} + \frac{a^4e^{(3\,x)}+4\,a^3be^{(3\,x)}+6\,a^2b^2e^{(3\,x)}+4\,ab^3e^{(3\,x)}+b^4e^{(3\,x)}-9\,a^4e^x-24\,a^3be^x-18\,a^2b^2e^x+3\,b^4e^x}{24\,(a^6+6\,a^5b+15\,a^4b^2+20\,a^3b^3+15\,a^2b^4+6\,ab^5+b^6)}$$

```
[In] integrate(cosh(x)^2*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a^3*b^2*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x)
+ a - b)) - 1/24*(9*a*e^(2*x) + 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - 2*(2*a^4*b + 3*a^2*b^3)*arctan((a*e^x + b*e^x)/sqrt(a^
2 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(a^4
*e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*e^(3
*x) - 9*a^4*e^x - 24*a^3*b*e^x - 18*a^2*b^2*e^x + 3*b^4*e^x)/(a^6 + 6*a^5*b
 + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.27

$$\int \frac{\cosh^2(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{3\,x}}{24\,{(a+b)}^2} + \frac{e^{-3\,x}}{24\,{(a-b)}^2} - \frac{2\,\mathrm{atan}\left(\frac{e^x\,\left(2\,a^4\,b\,\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}}\right.}{a^7\,\sqrt{4\,a^8\,b^2+12\,a^6\,b^4+9\,a^4\,b^6}+b^7\,\sqrt{4\,a^8\,b^2+12\,a^6\,b^4+9\,a^4\,b^6}-3\,a^2\,b^5\,\sqrt{4\,a^8\,b^2+12\,a^6\,b^4+9\,a^4\,b^6}+3\,a^3\,b^4\,\sqrt{4\,a^8\,b^2+12\,a^6\,b^4+9\,a^4\,b^6}+}\right.}{\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b}} - \frac{e^x\,(3\,a-b)}{8\,{(a+b)}^3} - \frac{e^{-x}\,(3\,a+b)}{8\,{(a-b)}^3} - \frac{2\,a^3\,b^2\,e^x}{{(a+b)}^3\,{(a-b)}^3\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

```
[In] int((cosh(x)^2*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(3*x)/(24*(a + b)^2) + exp(-3*x)/(24*(a - b)^2) - (2*atan((exp(x)*(2*a^4
*b*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a
^10*b^4 - 7*a^12*b^2)^(1/2) + 3*a^2*b^3*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2)))/(a^7*(9*
a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + b^7*(9*a^4*b^6 + 12*a^6*b^4 + 4*a
^8*b^2)^(1/2) - 3*a^2*b^5*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + 3*a^
3*b^4*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) + 3*a^4*b^3*(9*a^4*b^6 + 1
```

```
2*a^6*b^4 + 4*a^8*b^2)^(1/2) - 3*a^5*b^2*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^
2)^(1/2) - a*b^6*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2) - a^6*b*(9*a^4*
b^6 + 12*a^6*b^4 + 4*a^8*b^2)^(1/2)))*(9*a^4*b^6 + 12*a^6*b^4 + 4*a^8*b^2)^
(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 +
21*a^10*b^4 - 7*a^12*b^2)^(1/2) - (exp(x)*(3*a - b))/(8*(a + b)^3) - (exp(-
x)*(3*a + b))/(8*(a - b)^3) - (2*a^3*b^2*exp(x))/((a + b)^3*(a - b)^3*(a -
b + exp(2*x)*(a + b)))
```

3.721 $\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 215

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a^3bx}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3}-\frac{abx}{(a^2-b^2)^2}+\frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3}+\frac{b^2\cosh^2(x)}{2\,(a^2-b^2)^2}-\frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}-\frac{b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3}-\frac{ab\cosh(x)\sinh(x)}{(a^2-b^2)^2}+\frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2}+\frac{ab^2}{(a^2-b^2)^2\,(a+b\tanh(x))}$$

```
[Out] a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3-a*b*x/(a^2-b^2)^2+a*b*(a^2+b^2)*x/(
a^2-b^2)^3+1/2*b^2*cosh(x)^2/(a^2-b^2)^2-3*a^2*b^2*ln(a*cosh(x)+b*sinh(x))/
(a^2-b^2)^3-b^4*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^3-a*b*cosh(x)*sinh(x)/(a^
2-b^2)^2+1/2*a^2*sinh(x)^2/(a^2-b^2)^2+a*b^2/(a^2-b^2)^2/(a+b*tanh(x))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules

used = {3190, 3179, 2715, 8, 3177, 3212, 3188, 2644, 30, 3165, 3564, 3612, 3611}

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{abx(a^2+b^2)}{(a^2-b^2)^3} - \frac{abx}{(a^2-b^2)^2} + \frac{a^2\sinh^2(x)}{2(a^2-b^2)^2} + \frac{b^2\cosh^2(x)}{2(a^2-b^2)^2} + \frac{ab^2}{(a^2-b^2)^2(a+b\tanh(x))} - \frac{ab\sinh(x)\cosh(x)}{(a^2-b^2)^2} - \frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{ab^3x}{(a^2-b^2)^3} + \frac{a^3bx}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b^2)^3 - (a*b*x)/(a^2 - b^2)^2 +
 (a*b*(a^2 + b^2)*x)/(a^2 - b^2)^3 + (b^2*Cosh[x]^2)/(2*(a^2 - b^2)^2) - (3
*a^2*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^3 - (b^4*Log[a*Cosh[x] + b
*Sinh[x]])/(a^2 - b^2)^3 - (a*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[
x]^2)/(2*(a^2 - b^2)^2) + (a*b^2)/((a^2 - b^2)^2*(a + b*Tanh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
```

```
FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
```

```
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2}+\frac{a^2\int \cosh(x)\sinh(x)\,dx}{\left(a^2-b^2\right)^2}-2\frac{(ab)\int \cosh^2(x)\,dx}{\left(a^2-b^2\right)^2}\\
&\quad+\frac{\left(a^2b\right)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{b^3\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{(ab)\int \frac{1}{(a+b\tanh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^3bx}{\left(a^2-b^2\right)^3}+\frac{ab^3x}{\left(a^2-b^2\right)^3}+\frac{b^2\cosh^2(x)}{2\left(a^2-b^2\right)^2}\\
&\quad+\frac{ab^2}{\left(a^2-b^2\right)^2(a+b\tanh(x))}-\frac{\left(ia^2b^2\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3}\\
&\quad-\frac{\left(ib^4\right)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{\left(a^2-b^2\right)^3}-\frac{a^2\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)}{\left(a^2-b^2\right)^2}\\
&\quad-2\left(\frac{ab\cosh(x)\sinh(x)}{2\left(a^2-b^2\right)^2}+\frac{(ab)\int 1\,dx}{2\left(a^2-b^2\right)^2}\right)+\frac{(ab)\int \frac{a-b\tanh(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2}
\end{aligned}$$

$$= \frac{a^3bx}{(a^2-b^2)^3} + \frac{ab^3x}{(a^2-b^2)^3} + \frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3} + \frac{b^2\cosh^2(x)}{2\,(a^2-b^2)^2} - \frac{a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2} - 2\left(\frac{abx}{2\,(a^2-b^2)^2} + \frac{ab\cosh(x)\sinh(x)}{2\,(a^2-b^2)^2}\right) + \frac{ab^2}{(a^2-b^2)^2\,(a+b\tanh(x))} - \frac{(2ia^2b^2)\int\frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{(a^2-b^2)^3}$$

$$= \frac{a^3bx}{(a^2-b^2)^3} + \frac{ab^3x}{(a^2-b^2)^3} + \frac{ab(a^2+b^2)\,x}{(a^2-b^2)^3} + \frac{b^2\cosh^2(x)}{2\,(a^2-b^2)^2} - \frac{3a^2b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} - \frac{b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^3} + \frac{a^2\sinh^2(x)}{2\,(a^2-b^2)^2} - 2\left(\frac{abx}{2\,(a^2-b^2)^2} + \frac{ab\cosh(x)\sinh(x)}{2\,(a^2-b^2)^2}\right) + \frac{ab^2}{(a^2-b^2)^2\,(a+b\tanh(x))}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{a\cosh(x)\left((a^4-b^4)\cosh(2x)-4b(-a(a^2+3b^2)\,x+b(3a^2+b^2)\log(a\cosh(x)+b\sinh(x))+a(a^2-b^2)\cos\right.}{4(}$$

```
[In] Integrate[(Cosh[x]^3*Sinh[x])/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (a*Cosh[x]*((a^4 - b^4)*Cosh[2*x] - 4*b*(-(a*(a^2 + 3*b^2)*x) + b*(3*a^2 +
b^2)*Log[a*Cosh[x] + b*Sinh[x]] + a*(a^2 - b^2)*Cosh[x]*Sinh[x])) + b*Sinh[
x]*((a^4 - b^4)*Cosh[2*x] + 4*b*(-(a^2*b) + b^3 + a^3*x + 3*a*b^2*x - b*(3*
a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) - 2*a*b*(a^2 - b^2)*Sinh[2*x]))/(4*(
a - b)^3*(a + b)^3*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.83

method	result
default	$\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}-\frac{b\ln(\tanh(\frac{x}{2})-1)}{(a+b)^3}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}-\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)}+$
risch	$\frac{xb}{(a+b)(a^2+2ab+b^2)}+\frac{e^{2x}}{8a^2+16ab+8b^2}+\frac{e^{-2x}}{8a^2-16ab+8b^2}+\frac{6a^2xb^2}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^4x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{}{(a-b)^2(a^3+3a^2}$

```
[In] int(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^2/(tanh(1/2*x)-1)+1/2/(a+b)^2/(tanh(1/2*x)-1)^2-b/(a+b)^3*ln(tanh
(1/2*x)-1)+1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/2/(a-b)^2/(tanh(1/2*x)+1)+1/(a-b
)^3*b*ln(tanh(1/2*x)+1)-2*b^2/(a+b)^3/(a-b)^3*(b*(a^2-b^2)*tanh(1/2*x)/(tan
h(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(3*a^2+b^2)*ln(tanh(1/2*x)^2*a+2*b*tanh
(1/2*x)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1661 vs. $2(211) = 422$.

Time = 0.28 (sec) , antiderivative size = 1661, normalized size of antiderivative = 7.73

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(x
)^4 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 15*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + 8*(a^4*b + 4*a^3*b^2
 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b
^3 - 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*c
osh(x))*sinh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^
5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*cosh(x)^2 + (a^5 + 3*a^4*b + 2
*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 15*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 -
 3*a*b^4 + b^5 + 8*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(
x)^2 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*x)*sinh(x)^2 - 8*((3*a^3*b^2 +
 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^4 + 4*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^
5)*cosh(x)*sinh(x)^3 + (3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*sinh(x)^4 + (3
*a^3*b^2 - 3*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + (3*a^3*b^2 - 3*a^2*b^3 + a*
```

```
b^4 - b^5 + 6*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 +
2*(2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 - 3*a^2*b
^3 + a*b^4 - b^5)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x)
- sinh(x))) + 2*(3*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5 + 8*(a^4*b
 + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*x)*cosh(x)^3 + (a^5 + 3*a^4*b + 2
*a^3*b^2 + 14*a^2*b^3 - 19*a*b^4 - b^5 + 8*(a^4*b + 2*a^3*b^2 - 2*a*b^4 - b
^5)*x)*cosh(x))*sinh(x))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^4 + 4*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)*sinh(x)^3 + (a^7 + a^6*b
- 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*sinh(x)^4 +
(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)
*cosh(x)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 -
 a*b^6 + b^7 + 6*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b
^5 - a*b^6 - b^7)*cosh(x)^2)*sinh(x)^2 + 2*(2*(a^7 + a^6*b - 3*a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*cosh(x)^3 + (a^7 - a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x))*sinh(x
))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3*sinh(x)/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx \\
&= \frac{bx}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{(3\,a^2b^2+b^4)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} \\
&\quad + \frac{a^4-2\,a^3b+2\,ab^3-b^4+(a^4-4\,a^3b+6\,a^2b^2-20\,ab^3+b^4)e^{(-2\,x)}}{8\,((a^6-3\,a^4b^2+3\,a^2b^4-b^6)e^{(-2\,x)}+(a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6)e^{(-4\,x)})} \\
&\quad + \frac{e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)}
\end{aligned}$$

```
[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima")
```

[Out] b*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^2*b^2 + b^4)*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*(a^4 - 2*a^3*b + 2*a*b^3 - b^4 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 20*a*b^3 + b^4)*e^(-2*x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*e^(-2*x) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6)*e^(-4*x)) + 1/8*e^(-2*x)/(a^2 - 2*a*b + b^2)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{bx}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{(3\,a^2b^2+b^4)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{e^{(2\,x)}}{8\,(a^2+2\,ab+b^2)} - \frac{2\,a^2be^{(4\,x)}-4\,ab^2e^{(4\,x)}+2\,b^3e^{(4\,x)}-a^3e^{(2\,x)}-a^2be^{(2\,x)}-11\,ab^2e^{(2\,x)}-3\,b^3e^{(2\,x)}-a^3-a^2b+ab^2+b^3}{8\,(a^4-2\,a^2b^2+b^4)(ae^{(4\,x)}+be^{(4\,x)}+ae^{(2\,x)}-be^{(2\,x)})}$$

[In] integrate(cosh(x)^3*sinh(x)/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")

[Out] b*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^2*b^2 + b^4)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1/8*e^(2*x)/(a^2 + 2*a*b + b^2) - 1/8*(2*a^2*b*e^(4*x) - 4*a*b^2*e^(4*x) + 2*b^3*e^(4*x) - a^3*e^(2*x) - a^2*b*e^(2*x) - 11*a*b^2*e^(2*x) - 3*b^3*e^(2*x) - a^3 - a^2*b + a*b^2 + b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*e^(4*x) + b*e^(4*x) + a*e^(2*x) - b*e^(2*x)))

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int \frac{\cosh^3(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{\mathrm{e}^{2\,x}}{8\,{(a+b)}^2} + \frac{\mathrm{e}^{-2\,x}}{8\,{(a-b)}^2} + \frac{b\,x}{{(a-b)}^3} - \frac{\ln\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,\left(3\,a^2\,b^2+b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6} + \frac{2\,a\,b^3}{{(a+b)}^3\,{(a-b)}^2\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

[In] int((cosh(x)^3*sinh(x))/(a*cosh(x) + b*sinh(x))^2,x)

[Out] exp(2*x)/(8*(a + b)^2) + exp(-2*x)/(8*(a - b)^2) + (b*x)/(a - b)^3 - (log(a - b + a*exp(2*x) + b*exp(2*x))*(b^4 + 3*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*a*b^3)/((a + b)^3*(a - b)^2*(a - b + exp(2*x)*(a + b)))

3.722 $\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 259

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{3a^3b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2ab^4\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2a^3b\cosh(x)}{(a^2-b^2)^3} + \frac{2ab^3\cosh(x)}{(a^2-b^2)^3} - \frac{2ab\cosh^3(x)}{3(a^2-b^2)^2} - \frac{4a^2b^2\sinh(x)}{(a^2-b^2)^3} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} + \frac{a^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{a^2b^3}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}$$

```
[Out] 3*a^3*b^2*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+2*a
*b^4*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)+2*a^3*b*
cosh(x)/(a^2-b^2)^3+2*a*b^3*cosh(x)/(a^2-b^2)^3-2/3*a*b*cosh(x)^3/(a^2-b^2)
^2-4*a^2*b^2*sinh(x)/(a^2-b^2)^3+b^2*sinh(x)/(a^2-b^2)^2+1/3*a^2*sinh(x)^3/
(a^2-b^2)^2+1/3*b^2*sinh(x)^3/(a^2-b^2)^2+a^2*b^3/(a^2-b^2)^3/(a*cosh(x)+b*
sinh(x))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3190, 3188, 2713, 2645, 30, 3179, 2717, 3153, 212, 2644, 2718, 3234}

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{2ab^4\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{b^2\sinh^3(x)}{3(a^2-b^2)^2}+\frac{a^2\sinh^3(x)}{3(a^2-b^2)^2}+\frac{b^2\sinh(x)}{(a^2-b^2)^2}-\frac{4a^2b^2\sinh(x)}{(a^2-b^2)^3}-\frac{2ab\cosh^3(x)}{3(a^2-b^2)^2}+\frac{2ab^3\cosh(x)}{(a^2-b^2)^3}+\frac{a^2b^3}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}+\frac{3a^3b^2\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\frac{2a^3b\cosh(x)}{(a^2-b^2)^3}$$

```
[In] Int[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (3*a^3*b^2*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/
2) + (2*a*b^4*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(7/2) + (2*a^3*b*Cosh[x])/(a^2 - b^2)^3 + (2*a*b^3*Cosh[x])/(a^2 - b^2)^3 -
 (2*a*b*Cosh[x]^3)/(3*(a^2 - b^2)^2) - (4*a^2*b^2*Sinh[x])/(a^2 - b^2)^3 +
(b^2*Sinh[x])/(a^2 - b^2)^2 + (a^2*Sinh[x]^3)/(3*(a^2 - b^2)^2) + (b^2*Sinh
[x]^3)/(3*(a^2 - b^2)^2) + (a^2*b^3)/((a^2 - b^2)^3*(a*Cosh[x] + b*Sinh[x])
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\cosh^2(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}-\frac{b\int \frac{\cosh^3(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}+\frac{(ab)\int \frac{\cosh^2(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2}\\
&= \frac{a^2\int \cosh(x)\sinh^2(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab)\int \cosh^2(x)\sinh(x)\,dx}{(a^2-b^2)^2}+2\frac{(a^2b)\int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}\\
&\quad+\frac{b^2\int \cosh^3(x)\,dx}{(a^2-b^2)^2}-2\frac{(ab^2)\int \frac{\cosh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2}+\frac{(a^2b^2)\int \frac{\cosh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{(a^2-b^2)^2}\\
&= \frac{a^2b^3}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}+\frac{(a^3b^2)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\\
&\quad+2\left(\frac{(a^3b)\int \sinh(x)\,dx}{(a^2-b^2)^3}-\frac{(a^2b^2)\int \cosh(x)\,dx}{(a^2-b^2)^3}+\frac{(a^3b^2)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right)\\
&\quad-2\left(-\frac{ab^3\cosh(x)}{(a^2-b^2)^3}+\frac{(a^2b^2)\int \cosh(x)\,dx}{(a^2-b^2)^3}-\frac{(ab^4)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right)\\
&\quad+\frac{(ia^2)\,\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{(a^2-b^2)^2}-2\frac{(ab)\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)}{(a^2-b^2)^2}\\
&\quad+\frac{(ib^2)\,\text{Subst}\left(\int (1-x^2)\,dx,x,-i\sinh(x)\right)}{(a^2-b^2)^2}
\end{aligned}$$

$$= -\frac{2ab\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} + \frac{a^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{a^2b^3}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} + \frac{(ia^3b^2)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3} + 2\left(\frac{a^3b\cosh(x)}{(a^2-b^2)^3} - \frac{a^2b^2\sinh(x)}{(a^2-b^2)^3} + \frac{(ia^3b^2)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3}\right) - 2\left(-\frac{ab^3\cosh(x)}{(a^2-b^2)^3} + \frac{a^2b^2\sinh(x)}{(a^2-b^2)^3} - \frac{(iab^4)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^3}\right)$$

$$= \frac{a^3b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{2ab\cosh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\sinh(x)}{(a^2-b^2)^2} + \frac{a^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{b^2\sinh^3(x)}{3(a^2-b^2)^2} + \frac{a^2b^3}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} + 2\left(\frac{a^3b^2\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{a^3b\cosh(x)}{(a^2-b^2)^3} - \frac{a^2b^2\sinh(x)}{(a^2-b^2)^3}\right) - 2\left(-\frac{ab^4\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - \frac{ab^3\cosh(x)}{(a^2-b^2)^3} + \frac{a^2b^2\sinh(x)}{(a^2-b^2)^3}\right)$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{\sqrt{a-b}b(a+b)+2a^2\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\cosh(x)+2ab\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\sinh(x)}{8(a-b)^{3/2}(a+b)^2(a\cosh(x)+b\sinh(x))} + \frac{1}{16}\left(-\frac{6a(a^2+3b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{8ab\cosh(x)}{(a-b)^2(a+b)^2} + \frac{4(a^2+b^2)\sinh(x)}{(a-b)^2(a+b)^2} - \frac{b(3a^2+b^2)}{(a-b)^2(a+b)^2(a\cosh(x)+b\sinh(x))}\right) + \frac{1}{16}\left(\frac{10a(a^4+10a^2b^2+5b^4)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} + \frac{32ab(a^2+}{(a-b}\right.$$

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^2)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] -1/8*(Sqrt[a - b]*b*(a + b) + 2*a^2*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2]])/(S
qrt[a - b]*Sqrt[a + b])]*Cosh[x] + 2*a*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2
```

```
])/(Sqrt[a - b]*Sqrt[a + b])]*Sinh[x])/((a - b)^(3/2)*(a + b)^2*(a*Cosh[x]
+ b*Sinh[x])) + ((-6*a*(a^2 + 3*b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*
Sqrt[a + b])])/((a - b)^(5/2)*(a + b)^(5/2)) - (8*a*b*Cosh[x])/((a - b)^2*(
a + b)^2) + (4*(a^2 + b^2)*Sinh[x])/((a - b)^2*(a + b)^2) - (b*(3*a^2 + b^2
))/((a - b)^2*(a + b)^2*(a*Cosh[x] + b*Sinh[x])))/16 + ((10*a*(a^4 + 10*a^2
*b^2 + 5*b^4)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)
^(7/2)*(a + b)^(7/2)) + (32*a*b*(a^2 + b^2)*Cosh[x])/((a - b)^3*(a + b)^3)
- (8*a*b*Cosh[3*x])/(3*(a - b)^2*(a + b)^2) - (8*(a^4 + 6*a^2*b^2 + b^4)*Si
nh[x])/((a - b)^3*(a + b)^3) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/((a - b)^3*(a
 + b)^3*(a*Cosh[x] + b*Sinh[x])) + (4*(a^2 + b^2)*Sinh[3*x])/(3*(a - b)^2*(
a + b)^2))/16
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76

method	result
default	$-\frac{1}{3(a+b)^2(\tanh(\frac{x}{2})-1)^3}-\frac{1}{2(a+b)^2(\tanh(\frac{x}{2})-1)^2}-\frac{b}{(a+b)^3(\tanh(\frac{x}{2})-1)}+\frac{1}{2(a-b)^2(\tanh(\frac{x}{2})+1)^2}-\frac{1}{3(a-b)^2(\tanh(\frac{x}{2})+$
risch	$\frac{e^{3x}}{24a^2+48ab+24b^2}-\frac{e^{x}a}{8(a+b)(a^2+2ab+b^2)}+\frac{3\,e^{x}b}{8(a+b)(a^2+2ab+b^2)}+\frac{a\,e^{-x}}{8a^3-24a^2b+24a\,b^2-8b^3}+\frac{3\,e^{-x}b}{8(a^3-3a^2b+3a\,b^2-b^3)}-$ 24

```
[In] int(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+b)^2/(tanh(1/2*x)-1)^3-1/2/(a+b)^2/(tanh(1/2*x)-1)^2-b/(a+b)^3/(tan
h(1/2*x)-1)+1/2/(a-b)^2/(tanh(1/2*x)+1)^2-1/3/(a-b)^2/(tanh(1/2*x)+1)^3+1/(
a-b)^3*b/(tanh(1/2*x)+1)+2*a*b^2/(a+b)^3/(a-b)^3*((b^2*tanh(1/2*x)+a*b)/(ta
nh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+(3*a^2+2*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2488 vs. $2(245) = 490$.

Time = 0.36 (sec) , antiderivative size = 5031, normalized size of antiderivative = 19.42

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3*sinh(x)**2/(a*cosh(x)+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= \frac{2\,a^2b^3e^x}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)(ae^{(2\,x)}+be^{(2\,x)}+a-b)} \\ &\quad+\frac{(3\,ae^{(2\,x)}+9\,be^{(2\,x)}-a+b)e^{(-3\,x)}}{24\,(a^3-3\,a^2b+3\,ab^2-b^3)}+\frac{2\,(3\,a^3b^2+2\,ab^4)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}} \\ &\quad+\frac{a^4e^{(3\,x)}+4\,a^3be^{(3\,x)}+6\,a^2b^2e^{(3\,x)}+4\,ab^3e^{(3\,x)}+b^4e^{(3\,x)}-3\,a^4e^x+18\,a^2b^2e^x+24\,ab^3e^x+9\,b^4e^x}{24\,(a^6+6\,a^5b+15\,a^4b^2+20\,a^3b^3+15\,a^2b^4+6\,ab^5+b^6)}\end{aligned}$$

```
[In] integrate(cosh(x)^3*sinh(x)^2/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
[Out] 2*a^2*b^3*e^x/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*e^(2*x) + b*e^(2*x) +
 a - b)) + 1/24*(3*a*e^(2*x) + 9*b*e^(2*x) - a + b)*e^(-3*x)/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) + 2*(3*a^3*b^2 + 2*a*b^4)*arctan((a*e^x + b*e^x)/sqrt(a^2
 - b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/24*(a^4*
e^(3*x) + 4*a^3*b*e^(3*x) + 6*a^2*b^2*e^(3*x) + 4*a*b^3*e^(3*x) + b^4*e^(3*
x) - 3*a^4*e^x + 18*a^2*b^2*e^x + 24*a*b^3*e^x + 9*b^4*e^x)/(a^6 + 6*a^5*b
+ 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.28

$$\int \frac{\cosh^3(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{3x}}{24\,(a+b)^2} - \frac{e^{-3x}}{24\,(a-b)^2} - \frac{e^{x}\,(a-3\,b)}{8\,(a+b)^3}$$

$$+ \frac{2\,\mathrm{atan}\left(\frac{e^{x}\,\left(2\,a\,b^4\,\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b}\right.}{a^7\,\sqrt{9\,a^6\,b^4+12\,a^4\,b^6+4\,a^2\,b^8}+b^7\,\sqrt{9\,a^6\,b^4+12\,a^4\,b^6+4\,a^2\,b^8}-3\,a^2\,b^5\,\sqrt{9\,a^6\,b^4+12\,a^4\,b^6+4\,a^2\,b^8}+3\,a^3\,b^4\,\sqrt{9\,a^6\,b^4+12\,a^4\,b^6+4\,a^2\,b^8}}\right.}{\sqrt{a^{14}-7\,a^{12}\,b^2+21\,a^{10}}}$$

$$+ \frac{e^{-x}\,(a+3\,b)}{8\,(a-b)^3} + \frac{2\,a^2\,b^3\,e^{x}}{(a+b)^3\,(a-b)^3\,\left(a-b+e^{2\,x}\,(a+b)\right)}$$

```
[In] int((cosh(x)^3*sinh(x)^2)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(3*x)/(24*(a + b)^2) - exp(-3*x)/(24*(a - b)^2) - (exp(x)*(a - 3*b))/(8*
(a + b)^3) + (2*atan((exp(x)*(2*a*b^4*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^
10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) + 3*a^3*b^2*
(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10
*b^4 - 7*a^12*b^2)^(1/2)))/(a^7*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2)
+ b^7*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - 3*a^2*b^5*(4*a^2*b^8 + 1
2*a^4*b^6 + 9*a^6*b^4)^(1/2) + 3*a^3*b^4*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^
4)^(1/2) + 3*a^4*b^3*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - 3*a^5*b^2
*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2) - a*b^6*(4*a^2*b^8 + 12*a^4*b^6
 + 9*a^6*b^4)^(1/2) - a^6*b*(4*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2)))*(4
*a^2*b^8 + 12*a^4*b^6 + 9*a^6*b^4)^(1/2))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^
4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)^(1/2) + (exp(-
x)*(a + 3*b))/(8*(a - b)^3) + (2*a^2*b^3*exp(x))/((a + b)^3*(a - b)^3*(a -
b + exp(2*x)*(a + b)))
```

3.723 $\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 314

$$\begin{aligned}\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx &= -\frac{6a^3b^3x}{(a^2-b^2)^4}-\frac{a^3bx}{(a^2-b^2)^3}+\frac{ab^3x}{(a^2-b^2)^3}+\frac{abx}{4(a^2-b^2)^2}\\&+\frac{b^2\cosh^4(x)}{4(a^2-b^2)^2}+\frac{3a^4b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4}\\&+\frac{3a^2b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4}+\frac{a^3b\cosh(x)\sinh(x)}{(a^2-b^2)^3}\\&+\frac{ab^3\cosh(x)\sinh(x)}{(a^2-b^2)^3}+\frac{ab\cosh(x)\sinh(x)}{4(a^2-b^2)^2}\\&-\frac{ab\cosh^3(x)\sinh(x)}{2(a^2-b^2)^2}-\frac{2a^2b^2\sinh^2(x)}{(a^2-b^2)^3}\\&+\frac{a^2\sinh^4(x)}{4(a^2-b^2)^2}+\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))}\end{aligned}$$

```
[Out] -6*a^3*b^3*x/(a^2-b^2)^4-a^3*b*x/(a^2-b^2)^3+a*b^3*x/(a^2-b^2)^3+1/4*a*b*x/
(a^2-b^2)^2+1/4*b^2*cosh(x)^4/(a^2-b^2)^2+3*a^4*b^2*ln(a*cosh(x)+b*sinh(x))
/(a^2-b^2)^4+3*a^2*b^4*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)^4+a^3*b*cosh(x)*si
nh(x)/(a^2-b^2)^3+a*b^3*cosh(x)*sinh(x)/(a^2-b^2)^3+1/4*a*b*cosh(x)*sinh(x)
/(a^2-b^2)^2-1/2*a*b*cosh(x)^3*sinh(x)/(a^2-b^2)^2-2*a^2*b^2*sinh(x)^2/(a^2
-b^2)^3+1/4*a^2*sinh(x)^4/(a^2-b^2)^2+a^2*b^3*sinh(x)/(a^2-b^2)^3/(a*cosh(x
)+b*sinh(x))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3190, 3188, 2645, 30, 2648, 2715, 8, 2644, 3177, 3212, 3176, 3154}

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{abx}{4(a^2-b^2)^2}+\frac{a^2\sinh^4(x)}{4(a^2-b^2)^2}-\frac{2a^2b^2\sinh^2(x)}{(a^2-b^2)^3} +\frac{b^2\cosh^4(x)}{4(a^2-b^2)^2}-\frac{ab\sinh(x)\cosh^3(x)}{2(a^2-b^2)^2} +\frac{ab\sinh(x)\cosh(x)}{4(a^2-b^2)^2}+\frac{3a^2b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4} +\frac{ab^3x}{(a^2-b^2)^3}+\frac{a^2b^3\sinh(x)}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} +\frac{ab^3\sinh(x)\cosh(x)}{(a^2-b^2)^3}+\frac{3a^4b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4} -\frac{a^3bx}{(a^2-b^2)^3}+\frac{a^3b\sinh(x)\cosh(x)}{(a^2-b^2)^3}-\frac{6a^3b^3x}{(a^2-b^2)^4}$$

```
[In] Int[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-6*a^3*b^3*x)/(a^2 - b^2)^4 - (a^3*b*x)/(a^2 - b^2)^3 + (a*b^3*x)/(a^2 - b
^2)^3 + (a*b*x)/(4*(a^2 - b^2)^2) + (b^2*Cosh[x]^4)/(4*(a^2 - b^2)^2) + (3*
a^4*b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)^4 + (3*a^2*b^4*Log[a*Cosh[x
] + b*Sinh[x]])/(a^2 - b^2)^4 + (a^3*b*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*
b^3*Cosh[x]*Sinh[x])/(a^2 - b^2)^3 + (a*b*Cosh[x]*Sinh[x])/(4*(a^2 - b^2)^2
) - (a*b*Cosh[x]^3*Sinh[x])/(2*(a^2 - b^2)^2) - (2*a^2*b^2*Sinh[x]^2)/(a^2
- b^2)^3 + (a^2*Sinh[x]^4)/(4*(a^2 - b^2)^2) + (a^2*b^3*Sinh[x])/((a^2 - b^
2)^3*(a*Cosh[x] + b*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
```

```
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \frac{\cosh^2(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} - \frac{b\int \frac{\cosh^3(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\cosh^2(x)\sinh^2(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{a^2-b^2} \\
&= \frac{a^2\int \cosh(x)\sinh^3(x)\,dx}{(a^2-b^2)^2} - 2\frac{(ab)\int \cosh^2(x)\sinh^2(x)\,dx}{(a^2-b^2)^2} + 2\frac{(a^2b)\int \frac{\cosh(x)\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} \\
&\quad + \frac{b^2\int \cosh^3(x)\sinh(x)\,dx}{(a^2-b^2)^2} - 2\frac{(ab^2)\int \frac{\cosh^2(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{(a^2b^2)\int \frac{\cosh(x)\sinh(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx}{(a^2-b^2)^2}
\end{aligned}$$

$$= \frac{(a^3b^2)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3} + 2\left(\frac{(a^3b)\int \sinh^2(x)\,dx}{(a^2-b^2)^3} - \frac{(a^2b^2)\int \cosh(x)\sinh(x)\,dx}{(a^2-b^2)^3} + \frac{(a^3b^2)\int \frac{\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right) - \frac{(a^2b^3)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3} - 2\left(\frac{(a^2b^2)\int \cosh(x)\sinh(x)\,dx}{(a^2-b^2)^3} - \frac{(ab^3)\int \cosh^2(x)\,dx}{(a^2-b^2)^3} + \frac{(a^2b^3)\int \frac{\cosh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^3}\right) + \frac{(a^3b^3)\int \frac{1}{(a\cosh(x)+b\sinh(x))^2}\,dx}{(a^2-b^2)^3} + \frac{a^2\text{Subst}\left(\int x^3\,dx, x, i\sinh(x)\right)}{(a^2-b^2)^2} - 2\left(\frac{ab\cosh^3(x)\sinh(x)}{4(a^2-b^2)^2} - \frac{(ab)\int \cosh^2(x)\,dx}{4(a^2-b^2)^2}\right) + \frac{b^2\text{Subst}\left(\int x^3\,dx, x, \cosh(x)\right)}{(a^2-b^2)^2}$$

$$= -\frac{2a^3b^3x}{(a^2-b^2)^4} + \frac{b^2\cosh^4(x)}{4(a^2-b^2)^2} + \frac{a^2\sinh^4(x)}{4(a^2-b^2)^2} + \frac{a^2b^3\sinh(x)}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} + \frac{(ia^4b^2)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^4} + \frac{(ia^2b^4)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^4} + 2\left(-\frac{a^3b^3x}{(a^2-b^2)^4} + \frac{a^3b\cosh(x)\sinh(x)}{2(a^2-b^2)^3} + \frac{(ia^4b^2)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^4} - \frac{(a^3b)\int 1\,dx}{2(a^2-b^2)^3} + \frac{(a^2b^2)\,\text{Subst}(\int x\,dx, x, i\sinh(x))}{(a^2-b^2)^3}\right) - 2\left(\frac{a^3b^3x}{(a^2-b^2)^4} - \frac{ab^3\cosh(x)\sinh(x)}{2(a^2-b^2)^3} - \frac{(ia^2b^4)\int \frac{-ib\cosh(x)-ia\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^4} - \frac{(a^2b^2)\,\text{Subst}(\int x\,dx, x, i\sinh(x))}{(a^2-b^2)^3} - \frac{(ab^3)\int 1\,dx}{2(a^2-b^2)^3}\right) - 2\left(-\frac{ab\cosh(x)\sinh(x)}{8(a^2-b^2)^2} + \frac{ab\cosh^3(x)\sinh(x)}{4(a^2-b^2)^2} - \frac{(ab)\int 1\,dx}{8(a^2-b^2)^2}\right)$$

$$= -\frac{2a^3b^3x}{(a^2-b^2)^4} + \frac{b^2\cosh^4(x)}{4(a^2-b^2)^2} + \frac{a^4b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4} + \frac{a^2b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4} + \frac{a^2\sinh^4(x)}{4(a^2-b^2)^2} + \frac{a^2b^3\sinh(x)}{(a^2-b^2)^3(a\cosh(x)+b\sinh(x))} - 2\left(-\frac{abx}{8(a^2-b^2)^2} - \frac{ab\cosh(x)\sinh(x)}{8(a^2-b^2)^2} + \frac{ab\cosh^3(x)\sinh(x)}{4(a^2-b^2)^2}\right) + 2\left(-\frac{a^3b^3x}{(a^2-b^2)^4} - \frac{a^3bx}{2(a^2-b^2)^3} + \frac{a^4b^2\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4} + \frac{a^3b\cosh(x)\sinh(x)}{2(a^2-b^2)^3} - \frac{a^2b^2\sinh^2(x)}{2(a^2-b^2)^3}\right) - 2\left(\frac{a^3b^3x}{(a^2-b^2)^4} - \frac{ab^3x}{2(a^2-b^2)^3} - \frac{a^2b^4\log(a\cosh(x)+b\sinh(x))}{(a^2-b^2)^4} - \frac{ab^3\cosh(x)\sinh(x)}{2(a^2-b^2)^3} + \frac{a^2b^2\sinh^2(x)}{2(a^2-b^2)^3}\right)$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.17

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{-3a(a^2-b^2)^2(a^2+3b^2)\cosh(3x) + a^7\cosh(5x) - 3a^5b^2\cosh(5x) + 3a^3b^4\cosh(5x) - ab^6\cosh(5x) - 4a\cosh}{}$$

```
[In] Integrate[(Cosh[x]^3*Sinh[x]^3)/(a*Cosh[x] + b*Sinh[x])^2,x]
```

```
[Out] (-3*a*(a^2 - b^2)^2*(a^2 + 3*b^2)*Cosh[3*x] + a^7*Cosh[5*x] - 3*a^5*b^2*Cos
h[5*x] + 3*a^3*b^4*Cosh[5*x] - a*b^6*Cosh[5*x] - 4*a*Cosh[x]*(a^6 + 9*a^4*b
^2 - 5*a^2*b^4 - 5*b^6 + 12*a^5*b*x + 72*a^3*b^3*x + 12*a*b^5*x - 48*a^2*b^
2*(a^2 + b^2)*Log[a*Cosh[x] + b*Sinh[x]]) + 20*a^6*b*Sinh[x] + 84*a^4*b^3*S
inh[x] - 100*a^2*b^5*Sinh[x] - 4*b^7*Sinh[x] - 48*a^5*b^2*x*Sinh[x] - 288*a
^3*b^4*x*Sinh[x] - 48*a*b^6*x*Sinh[x] + 192*a^4*b^3*Log[a*Cosh[x] + b*Sinh[
x]]*Sinh[x] + 192*a^2*b^5*Log[a*Cosh[x] + b*Sinh[x]]*Sinh[x] + 9*a^6*b*Sinh
[3*x] - 15*a^4*b^3*Sinh[3*x] + 3*a^2*b^5*Sinh[3*x] + 3*b^7*Sinh[3*x] - a^6*
b*Sinh[5*x] + 3*a^4*b^3*Sinh[5*x] - 3*a^2*b^5*Sinh[5*x] + b^7*Sinh[5*x])/(6
4*(a - b)^4*(a + b)^4*(a*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 17.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.88

method	result
default	$\frac{1}{4(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{-a-5b}{8(a+b)^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{a-3b}{8(a+b)^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3ab\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4(a+b)^4}+$
risch	$-\frac{3abx}{4(a^2+2ab+b^2)(a+b)^2}+\frac{e^{4x}}{64(a+b)^2}-\frac{e^{2x}a}{16(a+b)^3}+\frac{e^{2x}b}{16(a+b)^3}-\frac{e^{-2x}a}{16(a^3-3a^2b+3ab^2-b^3)}-\frac{e^{-2x}b}{16(a^3-3a^2b+3ab^2-b^3)}+\frac{}{64a^2}$

```
[In] int(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a+b)^2/(tanh(1/2*x)-1)^4+1/2/(a+b)^2/(tanh(1/2*x)-1)^3-1/8*(-a-5*b)/(a
+b)^3/(tanh(1/2*x)-1)^2-1/8*(a-3*b)/(a+b)^3/(tanh(1/2*x)-1)+3/4*a*b/(a+b)^4
*ln(tanh(1/2*x)-1)+1/4/(a-b)^2/(tanh(1/2*x)+1)^4-1/2/(a-b)^2/(tanh(1/2*x)+1
)^3-1/8*(-a-3*b)/(a-b)^3/(tanh(1/2*x)+1)-1/8*(-a+5*b)/(a-b)^3/(tanh(1/2*x)+
1)^2-3/4*a*b/(a-b)^4*ln(tanh(1/2*x)+1)+2*a^2*b^2/(a+b)^4/(a-b)^4*(b*(a^2-b^
2)*tanh(1/2*x)/(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)+1/2*(3*a^2+3*b^2)*ln(tan
h(1/2*x)^2*a+2*b*tanh(1/2*x)+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. 2(304) = 608.

Time = 0.31 (sec) , antiderivative size = 4001, normalized size of antiderivative = 12.74

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/64*((a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6
+ b^7)*cosh(x)^10 + 10*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3
*a^2*b^5 - a*b^6 + b^7)*cosh(x)*sinh(x)^9 + (a^7 - a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*sinh(x)^10 - 3*(a^7 - 3*a^6*b
+ a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^8 - 3*
(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7
- 15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^2)*sinh(x)^8 + 24*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^3 - (a^7 - 3*a^6*b + a^5*b^2 +
5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x))*sinh(x)^7 + a^7 +
 a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7 - 4*(a
^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^
7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)
```

cosh(x)^6 - 2(2*a^7 - 10*a^6*b + 18*a^5*b^2 - 10*a^4*b^3 - 10*a^3*b^4 + 1
8*a^2*b^5 - 10*a*b^6 + 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3
*a^3*b^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^4 + 42*(a^7 - 3*a^6*b + a^5*b^2
 + 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^2 + 24*(a^6*b +
 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*sinh(x)^6 + 12
(21(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*b^6 +
 b^7)*cosh(x)^5 - 14*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - a^2
*b^5 + 3*a*b^6 - b^7)*cosh(x)^3 - 2*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^4*b^3
- 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*a^4*b^
3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x))*sinh(x)^5 - 4*(a^7 + 5*a^6*
b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a
^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*cosh(x)^4
- 2*(2*a^7 + 10*a^6*b + 18*a^5*b^2 + 74*a^4*b^3 - 74*a^3*b^4 - 18*a^2*b^5 -
 10*a*b^6 - 2*b^7 - 105*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^6 + 105*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b
^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^4 + 30*(a^7 - 5*a^6*b + 9
*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b +
5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^2 + 24*
(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*sinh(x)^
4 + 8*(15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 - a*
b^6 + b^7)*cosh(x)^7 - 21*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4
- a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^5 - 10*(a^7 - 5*a^6*b + 9*a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b + 5*a^5*b^2 + 10*
a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^3 - 2*(a^7 + 5*a^6*b +
 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7 + 12*(a^6*
b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)*x)*cosh(x))*sinh
(x)^3 - 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*
b^6 + b^7)*cosh(x)^2 + 3*(15*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b
^4 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^8 - a^7 - 3*a^6*b - a^5*b^2 + 5*a^4*b
^3 + 5*a^3*b^4 - a^2*b^5 - 3*a*b^6 - b^7 - 28*(a^7 - 3*a^6*b + a^5*b^2 + 5*
a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^6 - 20*(a^7 - 5*a^6*
b + 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6
*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^4
- 8*(a^7 + 5*a^6*b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*
b^6 - b^7 + 12*(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b
^6)*x)*cosh(x)^2)*sinh(x)^2 + 192*((a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*
cosh(x)^6 + 6*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)*sinh(x)^5 + (
a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*sinh(x)^6 + (a^5*b^2 - a^4*b^3 + a^3
*b^4 - a^2*b^5)*cosh(x)^4 + (a^5*b^2 - a^4*b^3 + a^3*b^4 - a^2*b^5 + 15*(a^
5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5*b^2 +
 a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^3 + (a^5*b^2 - a^4*b^3 + a^3*b^4 - a^
2*b^5)*cosh(x))*sinh(x)^3 + 3*(5*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*co
sh(x)^4 + 2*(a^5*b^2 - a^4*b^3 + a^3*b^4 - a^2*b^5)*cosh(x)^2)*sinh(x)^2 +
2*(3*(a^5*b^2 + a^4*b^3 + a^3*b^4 + a^2*b^5)*cosh(x)^5 + 2*(a^5*b^2 - a^4*b
^3 + a^3*b^4 - a^2*b^5)*cosh(x)^3)*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(

```
cosh(x) - sinh(x))) + 2*(5*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 - 3*a^2*b^5 - a*b^6 + b^7)*cosh(x)^9 - 12*(a^7 - 3*a^6*b + a^5*b^2 + 5*a^4
*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*cosh(x)^7 - 12*(a^7 - 5*a^6*b +
 9*a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7 + 12*(a^6*b
+ 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^5 + a*b^6)*x)*cosh(x)^5 - 8
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 37*a^4*b^3 - 37*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6
 - b^7 + 12*(a^6*b + 3*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - 3*a^2*b^5 - a*b^6)
*x)*cosh(x)^3 - 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^
5 + 3*a*b^6 + b^7)*cosh(x))*sinh(x))/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3
+ 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^6 +
6*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6
- 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)*sinh(x)^5 + (a^9 + a^8*b - 4*a^7*b^2 - 4
*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sin
h(x)^6 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a
^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^4 + (a^9 - a^8*b - 4*a^7*b^2 + 4*
a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9 + 15*
(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^9 + a^8*b - 4*a^7*b
^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^
9)*cosh(x)^3 + (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5
 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x))*sinh(x)^3 + 3*(5*(a^9 + a^
8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7
 + a*b^8 + b^9)*cosh(x)^4 + 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cosh(x)^2)*sinh(x)^2
 + 2*(3*(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^
3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*cosh(x)^5 + 2*(a^9 - a^8*b - 4*a^7*b^2 + 4
*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*cos
h(x)^3)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3*sinh(x)**3/(a*cosh(x)+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3\,abx}{4\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)} + \frac{3\,(a^4b^2+a^2b^4)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{4\,(a+b)e^{(-2\,x)}-(a-b)e^{(-4\,x)}}{64\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-3\,(a^6-4\,a^5b+5\,a^4b^2-5\,a^2b^4+4\,ab^5-b^6)e^{(-2\,x)}-4\,(}{64\,((a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)e^{(-4\,x)}+(a^8-2\,a^7b-2\,a^6b^2+6\,a^5b^3-}$$

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="maxima"
)
```

```
[Out] -3/4*a*b*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b
^4)*log(-(a - b)*e^(-2*x) - a - b)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
 + b^8) - 1/64*(4*(a + b)*e^(-2*x) - (a - b)*e^(-4*x))/(a^3 - 3*a^2*b + 3*a
*b^2 - b^3) + 1/64*(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5
 + b^6 - 3*(a^6 - 4*a^5*b + 5*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*e^(-2*x)
 - 4*(a^6 - 6*a^5*b + 15*a^4*b^2 - 52*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)
*e^(-4*x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*e^(-4*x) + (a^8
 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)
*e^(-6*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = -\frac{3\,abx}{4\,(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)} + \frac{\left(36\,abe^{(4\,x)}-4\,a^2e^{(2\,x)}+4\,b^2e^{(2\,x)}+a^2-2\,ab+b^2\right)e^{(-4\,x)}}{64\,(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)} + \frac{3\,(a^4b^2+a^2b^4)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} + \frac{a^2e^{(4\,x)}+2\,abe^{(4\,x)}+b^2e^{(4\,x)}-4\,a^2e^{(2\,x)}+4\,b^2e^{(2\,x)}}{64\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)} - \frac{3\,a^5b^2e^{(2\,x)}+3\,a^4b^3e^{(2\,x)}+3\,a^3b^4e^{(2\,x)}+3\,a^2b^5e^{(2\,x)}+3\,a^5b^2-a^4b^3+a^3b^4-3\,a^2b^5}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)(ae^{(2\,x)}+be^{(2\,x)}+a-b)}$$

```
[In] integrate(cosh(x)^3*sinh(x)^3/(a*cosh(x)+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -3/4*a*b*x/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 1/64*(36*a*b*e^(4*
x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^4 - 4*a
^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^4*b^2 + a^2*b^4)*log(abs(a*e^(2*x)
 + b*e^(2*x) + a - b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 1/
64*(a^2*e^(4*x) + 2*a*b*e^(4*x) + b^2*e^(4*x) - 4*a^2*e^(2*x) + 4*b^2*e^(2*
x))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (3*a^5*b^2*e^(2*x) + 3*a^
4*b^3*e^(2*x) + 3*a^3*b^4*e^(2*x) + 3*a^2*b^5*e^(2*x) + 3*a^5*b^2 - a^4*b^3
 + a^3*b^4 - 3*a^2*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a
*e^(2*x) + b*e^(2*x) + a - b))
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \frac{\cosh^3(x)\sinh^3(x)}{(a\cosh(x)+b\sinh(x))^2}\,dx = \frac{e^{4x}}{64\,(a+b)^2}+\frac{e^{-4x}}{64\,(a-b)^2} + \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\,\left(3\,a^4\,b^2+3\,a^2\,b^4\right)}{a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8} - \frac{e^{-2x}\,(a+b)}{16\,(a-b)^3}-\frac{e^{2x}\,(a-b)}{16\,(a+b)^3}-\frac{3\,a\,b\,x}{4\,(a-b)^4} - \frac{2\,a^3\,b^3}{(a+b)^4\,(a-b)^3\,\left(a-b+e^{2x}\,(a+b)\right)}$$

```
[In] int((cosh(x)^3*sinh(x)^3)/(a*cosh(x) + b*sinh(x))^2,x)
```

```
[Out] exp(4*x)/(64*(a + b)^2) + exp(-4*x)/(64*(a - b)^2) + (log(a - b + a*exp(2*x
) + b*exp(2*x))*(3*a^2*b^4 + 3*a^4*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
 - 4*a^6*b^2) - (exp(-2*x)*(a + b))/(16*(a - b)^3) - (exp(2*x)*(a - b))/(16
*(a + b)^3) - (3*a*b*x)/(4*(a - b)^4) - (2*a^3*b^3)/((a + b)^4*(a - b)^3*(a
 - b + exp(2*x)*(a + b)))
```

3.724 $\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = -\frac{cCx}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] -c*C*x/(b^2-c^2)+b*C*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan((c*cosh(x)+
b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3216, 3153, 212}

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{A\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*C*x)/(b^2 - c^2)) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]]
)/Sqrt[b^2 - c^2] + (b*C*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}+A\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx\\
&= -\frac{cCx}{b^2-c^2}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}\\
&\quad+(iA)\mathrm{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cosh(x)-ib\sinh(x)\right)\\
&= -\frac{cCx}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bC\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$
\int\frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{\sqrt{b-c}\sqrt{b+c}}+\frac{C(-cx+b\log(b\cosh(x)+c\sinh(x)))}{b^2-c^2}
$$

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
[Out] (2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/(Sqrt[b - c]*Sqrt
[b + c]) + (C*(-(c*x) + b*Log[b*Cosh[x] + c*Sinh[x]]))/(b^2 - c^2)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

method	result
default	$-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}-\frac{2C\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{Cb\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)+\frac{2\left(b^{2}A-A\,c^{2}\right)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\sqrt{b^{2}-c^{2}}}}{(b-c)(b+c)}$
risch	$\frac{Cx}{b+c}-\frac{2xC\,b^{3}}{b^{4}-2b^{2}c^{2}+c^{4}}+\frac{2xCb\,c^{2}}{b^{4}-2b^{2}c^{2}+c^{4}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)bC}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^{x}-\right.}{(}$

[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)

[Out] -2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)-2*C/(2*b-2*c)*ln(tanh(1/2*x)+1)+2/(b-c)/(b+c)*(1/2*C*b*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+(A*b^2-A*c^2)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.91

$$\int \frac{A+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$$
$$=\left[\frac{Cb\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\sqrt{-b^{2}+c^{2}}A\log\left(\frac{(b+c)\cosh(x)^{2}+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^{2}-2\,\sqrt{-b^{2}+c^{2}}(\cosh(x}{(b+c)\cosh(x)^{2}+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^{2}+b-}\right.}{b^{2}-c^{2}}\right.$$

[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")

[Out] [(C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - sqrt(-b^2 + c^2)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (C*b + C*c)*x)/(b^2 - c^2), (C*b*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) - 2*sqrt(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - (C*b + C*c)*x)/(b^2 - c^2)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(66) = 132$.

Time = 26.64 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.59

$$\int \frac{A + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Cx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)} - \frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} + \frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{C\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{C\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2} + \frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2} + \frac{Cbx}{b^2-c^2} - \frac{2Cb\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2} + \frac{Cb\log\left(\right.}{} \end{cases}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh
(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh(x
)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) -
 C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*sinh(x) + 2
*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2*c*si
nh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c)), (-
A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c*
*2) + A*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**
2 - c**2) + C*b*x/(b**2 - c**2) - 2*C*b*log(tanh(x/2) + 1)/(b**2 - c**2) +
C*b*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*log(tan
h(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - C*c*x/(b**2 - c**2), T
rue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{A + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{Cb\log\left(be^{(2\,x)} + ce^{(2\,x)} + b - c\right)}{b^2 - c^2} + \frac{2\,A\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2 - c^2}} - \frac{Cx}{b - c}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] C*b*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c*
e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) - C*x/(b - c)
```

Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int \frac{A + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}} - \frac{C\,x}{b-c} + \frac{C\,b^3\,\ln\left(4\,A^2\,b - 4\,A^2\,c + 4\,A^2\,b\,\mathrm{e}^{2\,x} + 4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4 - 2\,b^2\,c^2 + c^4} - \frac{C\,b\,c^2\,\ln\left(4\,A^2\,b - 4\,A^2\,c + 4\,A^2\,b\,\mathrm{e}^{2\,x} + 4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4 - 2\,b^2\,c^2 + c^4}$$

```
[In] int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2)
^(1/2))/(b^2 - c^2)^(1/2) - (C*x)/(b - c) + (C*b^3*log(4*A^2*b - 4*A^2*c +
4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (C*b*c^2*lo
g(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*
b^2*c^2)
```

3.725 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=-\frac{cC\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}-\frac{bC-Ac\cosh(x)-Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

```
[Out] -c*C*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(-b*C+A*
c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3233, 3153, 212}

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=-\frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}-\frac{cC\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] -((c*C*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2))
- (b*C - A*c*Cosh[x] - A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{bC - Ac\cosh(x) - Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} - \frac{(cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2} \\
&= -\frac{bC - Ac\cosh(x) - Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} - \frac{(icC)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) - ib\sinh(x)\right)}{b^2-c^2} \\
&= -\frac{cC\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} - \frac{bC - Ac\cosh(x) - Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.89

$$\int \frac{A + C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{b^2\sqrt{b-c}(b+c)C + 2b^2c\sqrt{b+c}C\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)\cosh(x) + \left(-A(b-c)^{3/2}(b+c)^2 + 2bc^2\sqrt{b+c}C\right.}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] -((b^2*Sqrt[b - c]*(b + c)*C + 2*b^2*c*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2
])/(Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (-(A*(b - c)^(3/2)*(b + c)^2) + 2*b
*c^2*Sqrt[b + c]*C*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sin
h[x])/(b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x])))
```

Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

method	result	size
default	$-\frac{2\left(-\frac{\left(b^{2} A-A\,c^{2}-C c b\right) \tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}+\frac{b C}{b^{2}-c^{2}}\right)}{\tanh\left(\frac{x}{2}\right)^{2} b+2 c \tanh\left(\frac{x}{2}\right)+b}-\frac{2 C c \arctan\left(\frac{2 b \tanh\left(\frac{x}{2}\right)+2 c}{2 \sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$	115
risch	$-\frac{2\left(C b\,{\mathrm e}^{x}+b A-A c\right)}{\left(b-c\right)\left(b+c\right)\left(b\,{\mathrm e}^{2 x}+{\mathrm e}^{2 x} c+b-c\right)}-\frac{c C \ln\left({\mathrm e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,\left(b+c\right)\left(b-c\right)}+\frac{c C \ln\left({\mathrm e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,\left(b+c\right)\left(b-c\right)}$	144

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(A*b^2-A*c^2-C*b*c)/b/(b^2-c^2)*tanh(1/2*x)+b*C/(b^2-c^2))/(tanh(1/2*x
)^2*b+2*c*tanh(1/2*x)+b)-2*C*c/(b^2-c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x)+
2*c)/(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(76) = 152.

Time = 0.27 (sec) , antiderivative size = 679, normalized size of antiderivative = 8.28

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$$
$$=\left[-\frac{2\,Ab^{3}-2\,Ab^{2}c-2\,Abc^{2}+2\,Ac^{3}-\left(Cbc-Cc^{2}+\left(Cbc+Cc^{2}\right)\cosh\left(x\right)^{2}+2\,\left(Cbc+Cc^{2}\right)\cosh\left(x\right)\sinh\left(\right.\right.}{b^{5}-b^{4}c-2\,b^{3}c^{2}+2\,b^{2}c^{3}+bc^{4}-c^{5}+\left(b^{5}+b^{4}c-2\,b^{3}c^{2}-2\,b^{2}c^{3}\right.}\right.$$
$$-\frac{2\,\left(Ab^{3}-Ab^{2}c-Abc^{2}+Ac^{3}-\left(Cbc-Cc^{2}+\left(Cbc+Cc^{2}\right)\cosh\left(x\right)^{2}+2\,\left(Cbc+Cc^{2}\right)\cosh\left(x\right)\sinh\left(x\right)+\right.\right.}{b^{5}-b^{4}c-2\,b^{3}c^{2}+2\,b^{2}c^{3}+bc^{4}-c^{5}+\left(b^{5}+b^{4}c-2\,b^{3}c^{2}-2\,b^{2}c^{3}+bc^{4}+c^{5}\right)\cosh\left(x\right)^{2}+2\,(}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (C*b*c - C*c^2 + (C*b*c + C*
c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh(x
)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(
C*b^3 - C*b*c^2)*cosh(x) + 2*(C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4*c - 2*b^
3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sin
h(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - (C*b*c - C*c^2 + (C*b*c +
C*c^2)*cosh(x)^2 + 2*(C*b*c + C*c^2)*cosh(x)*sinh(x) + (C*b*c + C*c^2)*sinh
```

```
(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*si
nh(x))) + (C*b^3 - C*b*c^2)*cosh(x) + (C*b^3 - C*b*c^2)*sinh(x))/(b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2
*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = -\frac{2\,Cc\arctan\left(\frac{be^{x}+ce^{x}}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} - \frac{2\,(Cbe^{x} + Ab - Ac)}{(b^2-c^2)(be^{(2\,x)} + ce^{(2\,x)} + b - c)}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
[Out] -2*C*c*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2*(C*b*e
^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{C\,c\,\ln\left(\frac{2\,C\,c}{(b+c)^{5/2}\,\sqrt{c-b}}-\frac{2\,C\,c\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{C\,c\,\ln\left(-\frac{2\,C\,c}{(b+c)^{5/2}\,\sqrt{c-b}}-\frac{2\,C\,c\,\mathrm{e}^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\frac{2\,A}{b+c}+\frac{2\,C\,b\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)}$$

```
[In] int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (C*c*log((2*C*c)/((b + c)^(5/2)*(c - b)^(1/2)) - (2*C*c*exp(x))/(b*c^2 - b^
2*c - b^3 + c^3)))/((b + c)^(3/2)*(c - b)^(3/2)) - (C*c*log(- (2*C*c)/((b +
 c)^(5/2)*(c - b)^(1/2)) - (2*C*c*exp(x))/(b*c^2 - b^2*c - b^3 + c^3)))/((b
 + c)^(3/2)*(c - b)^(3/2)) - ((2*A)/(b + c) + (2*C*b*exp(x))/((b + c)*(b -
c)))/(b - c + exp(2*x)*(b + c))
```

3.726 $\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} - \frac{bC-Ac\cosh(x)-Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} - \frac{c^2C\cosh(x)+bcC\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(-b
*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(-c^2*C*cosh(
x)-b*c*C*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3236, 3232, 3153, 212}

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} - \frac{-Ab\sinh(x)-Ac\cosh(x)+bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} - \frac{bcC\sinh(x)+c^2C\cosh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) -
 (b*C - A*c*Cosh[x] - A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) - (c^2*C*Cosh[x] + b*c*C*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3236

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{bC - Ac\cosh(x) - Ab\sinh(x)}{2\,(b^2-c^2)\,(b\cosh(x)+c\sinh(x))^2} + \frac{\int \frac{-2cC+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\,(b^2-c^2)}\\
&= -\frac{bC - Ac\cosh(x) - Ab\sinh(x)}{2\,(b^2-c^2)\,(b\cosh(x)+c\sinh(x))^2}\\
&\quad - \frac{c^2C\cosh(x)+bcC\sinh(x)}{(b^2-c^2)^2\,(b\cosh(x)+c\sinh(x))} + \frac{A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{2\,(b^2-c^2)}
\end{aligned}$$

$$= -\frac{bC - Ac\cosh(x) - Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} - \frac{c^2C\cosh(x) + bcC\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))} + \frac{(iA)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) - ib\sinh(x)\right)}{2(b^2-c^2)}$$

$$= \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} - \frac{bC - Ac\cosh(x) - Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} - \frac{c^2C\cosh(x) + bcC\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{A + C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}} + \frac{-b^2C + A(b^2-c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2} + \frac{c(A-2C\sinh(x))}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}\right)$$

```
[In] Integrate[(A + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (-(b^2*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Co
sh[x] + c*Sinh[x])^2) + (c*(A - 2*C*Sinh[x]))/(b*(b - c)*(b + c)*(b*Cosh[x]
 + c*Sinh[x])))/2
```

Maple [A] (verified)

Time = 31.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

method	result
default	$\frac{-\frac{A\left(b^2-2c^2\right)\tanh\left(\frac{x}{2}\right)^3}{\left(b^2-c^2\right)b}+\frac{\left(A\,b^2c+2A\,c^3+2C\,b^3-2Cb\,c^2\right)\tanh\left(\frac{x}{2}\right)^2}{b^2\left(b^2-c^2\right)}+\frac{A\left(b^2+2c^2\right)\tanh\left(\frac{x}{2}\right)}{b\left(b^2-c^2\right)}+\frac{2Ac}{2b^2-2c^2}}{\left(\tanh\left(\frac{x}{2}\right)^2b+2c\tanh\left(\frac{x}{2}\right)+b\right)^2}+\frac{A\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{\frac{3}{2}}}$
risch	$\frac{A\,b^2e^{3x}+2Abc\,e^{3x}+A\,c^2e^{3x}-2C\,b^2e^{2x}+2C\,c^2e^{2x}-A\,e^xb^2+A\,e^xc^2+2Ccb-2C\,c^2}{(b-c)(b\,e^{2x}+e^{2x}c+b-c)^2(b^2+2cb+c^2)}-\frac{A\ln\left(e^x-\frac{b-c}{\sqrt{-b^2+c^2}}\right)}{2\sqrt{-b^2+c^2}\,(b+c)(b-c)}+\frac{A\ln\left(e^x+\frac{b-c}{\sqrt{-b^2+c}}\right.}{2\sqrt{-b^2+c^2}\,(b+c)(b}$

```
[In] int((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/2*A*(b^2-2*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^2*c+2*A*c^3+2*C*b^
3-2*C*b*c^2)/b^2/(b^2-c^2)*tanh(1/2*x)^2+1/2*A*(b^2+2*c^2)/b/(b^2-c^2)*tanh
(1/2*x)+1/2*A*c/(b^2-c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2
)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 1855, normalized size of antiderivative = 15.08

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(4*C*b^2*c - 8*C*b*c^2 + 4*C*c^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^
3)*cosh(x)^3 + 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 4*(C*b^3 -
 C*b^2*c - C*b*c^2 + C*c^3)*cosh(x)^2 - 2*(2*C*b^3 - 2*C*b^2*c - 2*C*b*c^2
+ 2*C*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 + ((A*
b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh
(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(
A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*c
osh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c
^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*c
osh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
 - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) - 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - 2*(A*b^3 - A*
b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 +
 4*(C*b^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b
^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4
*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2
- 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c -
 b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*
c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3
*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)
*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x)), (2*
C*b^2*c - 4*C*b*c^2 + 2*C*c^3 + (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)
^3 + (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 - 2*(C*b^3 - C*b^2*c - C
*b*c^2 + C*c^3)*cosh(x)^2 - (2*C*b^3 - 2*C*b^2*c - 2*C*b*c^2 + 2*C*c^3 - 3*
(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A*b^2 + 2*A*b*c
+ A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh(x)^3 + (A*b^2
 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(A*b^2 - A*c^2)
*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^2)*sinh
(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c^2)*cosh(x))*s
```

```
inh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*s
inh(x))) - (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) - (A*b^3 - A*b^2*c -
 A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 + 4*(C*b
^3 - C*b^2*c - C*b*c^2 + C*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2
+ 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^
3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3
*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^
2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3
*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6 +
 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh(x
)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*co
sh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} + \frac{Ab^2e^{(3\,x)}+2\,Abce^{(3\,x)}+Ac^2e^{(3\,x)}-2\,Cb^2e^{(2\,x)}+2\,Cc^2e^{(2\,x)}-Ab^2e^x+Ac^2e^x+2\,Cbc-2\,Cc^2}{(b^3+b^2c-bc^2-c^3)(be^{(2\,x)}+ce^{(2\,x)}+b-c)^2}$$

```
[In] integrate((A+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*C*b^2*e^(2*x) + 2*C*c^2*e^(2*x) - A
*b^2*e^x + A*c^2*e^x + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e
^(2*x) + c*e^(2*x) + b - c)^2)
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.76

$$\int \frac{A+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}} - \frac{\frac{C}{(b+c)^2}-\frac{A\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)} - \frac{\frac{2\,A\,\mathrm{e}^x}{b+c}-\frac{C}{b+c}+\frac{C\,\mathrm{e}^{2\,x}}{b+c}}{\mathrm{e}^{4\,x}\,(b+c)^2+(b-c)^2+2\,\mathrm{e}^{2\,x}\,(b+c)\,(b-c)}$$

```
[In] int((A + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] (atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2)
 + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))/(
b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - (C/(b + c)^2 - (A*exp(x))/((b +
c)*(b - c)))/(b - c + exp(2*x)*(b + c)) - ((2*A*exp(x))/(b + c) - C/(b + c)
 + (C*exp(2*x))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*exp(2*x)*(b +
c)*(b - c))
```

3.727 $\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{bBx}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] b*B*x/(b^2-c^2)-B*c*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan((c*cosh(x)+b
*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 3153, 212}

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{A\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{bBx}{b^2-c^2}-\frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/S
qrt[b^2 - c^2] - (B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3217

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[c*B*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bBx}{b^2-c^2} - \frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2} + A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx \\
&= \frac{bBx}{b^2-c^2} - \frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad + (iA)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x)-ib\sinh(x)\right) \\
&= \frac{bBx}{b^2-c^2} + \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}} - \frac{Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$
\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{bBx+2A\sqrt{b-c}\sqrt{b+c}\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)-Bc\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}
$$

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*
Sqrt[b + c])] - B*c*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

method	result
default	$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{-Bc\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)+\frac{2\left(b^{2}A-A\,c^{2}\right)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\sqrt{b^{2}-c^{2}}}}{(b-c)(b+c)}$
risch	$\frac{Bx}{b+c}+\frac{2xB\,b^{2}c}{b^{4}-2b^{2}c^{2}+c^{4}}-\frac{2xB\,c^{3}}{b^{4}-2b^{2}c^{2}+c^{4}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)Bc}{(b+c)(b-c)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{A(b+c)}\right)\sqrt{-A^{2}b^{2}+A^{2}c^{2}}}{(b+c)(b-c)}-\frac{\ln\left(\mathrm{e}^{x}-\ldots\right.}{(\ldots}$

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*B/(2*b-2*c)*ln(tanh(1/2*x)+1)+2/(b-c)/(b
+c)*(-1/2*B*c*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+(A*b^2-A*c^2)/(b^2-c^2)
^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.92

$$\int\frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx$$

$$=\left[-\frac{Bc\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)+\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\,\sqrt{-b^2+c^2}(\cosh\ldots}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+\ldots}\right.}{b^2-c^2}\right.$$

$$\left.-\frac{Bc\log\left(\frac{2\,(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)+2\,\sqrt{b^2-c^2}A\arctan\left(\frac{\sqrt{b^2-c^2}}{(b+c)\cosh(x)+(b+c)\sinh(x)}\right)-(Bb+Bc)x}{b^2-c^2}\right]$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + sqrt(-b^2 + c^2
)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2
- 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
 + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - (B*b + B*c)*x)/(b^2 -
 c^2), -(B*c*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + 2*sqrt(b^
2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - (B
*b + B*c)*x)/(b^2 - c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(66) = 132.

Time = 30.01 (sec) , antiderivative size = 697, normalized size of antiderivative = 8.71

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)}+\frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)}-\frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} \\ \frac{Ab^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}}-\frac{Ab^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}}-\frac{Ac^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-b^2+c^2}}+\frac{Ac^2\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\right.}{b^2\sqrt{-b^2+c^2}-c^2\sqrt{-}} \end{cases}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2
) + 1) + B*log(tanh(x/2)))/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) + 2*c*cosh(x))
 + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c*sinh(x) + 2
*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*
c*sinh(x) + 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*co
sh(x)/(2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)),
Eq(b, c)), (A*b**2*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-
b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - A*b**2*log(tanh(x/2) + c/b + sqrt
(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - A*c
**2*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2) -
c**2*sqrt(-b**2 + c**2)) + A*c**2*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/
b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) + B*b*x*sqrt(-b**2 +
 c**2)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - B*c*x*sqrt(-b*
*2 + c**2)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) + 2*B*c*sqrt
(-b**2 + c**2)*log(tanh(x/2) + 1)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**
2 + c**2)) - B*c*sqrt(-b**2 + c**2)*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2
)/b)/(b**2*sqrt(-b**2 + c**2) - c**2*sqrt(-b**2 + c**2)) - B*c*sqrt(-b**2 +
 c**2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2*sqrt(-b**2 + c**2)
 - c**2*sqrt(-b**2 + c**2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx = -\frac{Bc\log\left(be^{(2x)}+ce^{(2x)}+b-c\right)}{b^2-c^2}+\frac{2A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{Bx}{b-c}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*c*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2) + 2*A*arctan((b*e^x + c
*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + B*x/(b - c)
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \frac{A+B\cosh(x)}{b\cosh(x)+c\sinh(x)}\,dx &= \frac{2\,\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-c^2}}+\frac{B\,x}{b-c}\\&\quad+\frac{B\,c^3\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}\\&\quad-\frac{B\,b^2\,c\,\ln\left(4\,A^2\,b-4\,A^2\,c+4\,A^2\,b\,\mathrm{e}^{2\,x}+4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4-2\,b^2\,c^2+c^4}\end{aligned}$$

```
[In] int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2)
^(1/2))/(b^2 - c^2)^(1/2) + (B*x)/(b - c) + (B*c^3*log(4*A^2*b - 4*A^2*c +
4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (B*b^2*c*lo
g(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*
b^2*c^2)
```

3.728 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{bB\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

```
[Out] b*B*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*c+A*c*
cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3234, 3153, 212}

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{(b^2-c^2)(b\cosh(x)+c\sinh(x))}+\frac{bB\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (b*B*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
B*c + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} + \frac{(bB)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}\\ &= \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} + \frac{(ibB)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) - ib\sinh(x)\right)}{b^2-c^2}\\ &= \frac{bB\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} + \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

$$\int \frac{A + B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{bB\sqrt{b-c}c(b+c) + 2b^3B\sqrt{b+c}\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)\cosh(x) + \left(A(b-c)^{3/2}(b+c)^2 + 2b^2Bc\sqrt{b+c}\,\text{arcta}\right.}{b(b-c)^{3/2}(b+c)^2(b\cosh(x)+c\sinh(x))}$$

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (b*B*Sqrt[b - c]*c*(b + c) + 2*b^3*B*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(
Sqrt[b - c]*Sqrt[b + c])]*Cosh[x] + (A*(b - c)^(3/2)*(b + c)^2 + 2*b^2*B*c*
Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])*Sinh[x])/(
b*(b - c)^(3/2)*(b + c)^2*(b*Cosh[x] + c*Sinh[x]))
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result	size
default	$-\frac{2\left(-\frac{\left(b^{2} A-A c^{2}+B c^{2}\right) \tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}-\frac{B c}{b^{2}-c^{2}}\right)}{\tanh\left(\frac{x}{2}\right)^{2} b+2 c \tanh\left(\frac{x}{2}\right)+b}+\frac{2 b B \arctan\left(\frac{2 b \tanh\left(\frac{x}{2}\right)+2 c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$	116
risch	$-\frac{2(-B c\,e^{x}+b A-A c)}{(b-c)(b+c)(b\,e^{2x}+e^{2x} c+b-c)}-\frac{b B \ln\left(e^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{b B \ln\left(e^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}$	145

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(A*b^2-A*c^2+B*c^2)/b/(b^2-c^2)*tanh(1/2*x)-B*c/(b^2-c^2))/(tanh(1/2*x
)^2*b+2*c*tanh(1/2*x)+b)+2*b*B/(b^2-c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x)+
2*c)/(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 8.72

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$$

$$= \left[-\frac{2\,Ab^3-2\,Ab^2c-2\,Abc^2+2\,Ac^3-\left(Bb^2-Bbc+(Bb^2+Bbc)\cosh(x)^2+2\,(Bb^2+Bbc)\cosh(x)\sinh(\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+(b^5+b^4c-2\,b^3c^2-2\,b^2c^3}\right.$$

$$-\frac{2\left(Ab^3-Ab^2c-Abc^2+Ac^3+\left(Bb^2-Bbc+(Bb^2+Bbc)\cosh(x)^2+2\,(Bb^2+Bbc)\cosh(x)\sinh(x)+\right.\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+(b^5+b^4c-2\,b^3c^2-2\,b^2c^3+bc^4+c^5)\cosh(x)^2+2\,(}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 - (B*b^2 - B*b*c + (B*b^2 + B*
b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh(x
)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) +
(b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c
)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - 2*(
B*b^2*c - B*c^3)*cosh(x) - 2*(B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^
3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5
)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sin
h(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 - B*b*c + (B*b^2 +
B*b*c)*cosh(x)^2 + 2*(B*b^2 + B*b*c)*cosh(x)*sinh(x) + (B*b^2 + B*b*c)*sinh
```

```
(x)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*si
nh(x))) - (B*b^2*c - B*c^3)*cosh(x) - (B*b^2*c - B*c^3)*sinh(x))/(b^5 - b^4
*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2
*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*
c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 +
 c^5)*sinh(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\, dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^2}\, dx = \frac{2\, Bb \arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{(b^2 - c^2)^{\frac{3}{2}}} + \frac{2\,(Bce^x - Ab + Ac)}{(b^2 - c^2)(be^{(2\,x)} + ce^{(2\,x)} + b - c)}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*B*b*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + 2*(B*c*e^
x - A*b + A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.15

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{B\,b\,\ln\left(\frac{2\,B\,b}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,B\,b\,e^x}{-b^3-b^2\,c+b\,c^2+c^3}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{B\,b\,\ln\left(\frac{2\,B\,b\,e^x}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,B\,b}{(b+c)^{5/2}\,\sqrt{c-b}}\right)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\frac{2\,A}{b+c}-\frac{2\,B\,c\,e^x}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}$$

```
[In] int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (B*b*log((2*B*b)/((b + c)^(5/2)*(c - b)^(1/2)) + (2*B*b*exp(x))/(b*c^2 - b^
2*c - b^3 + c^3)))/((b + c)^(3/2)*(c - b)^(3/2)) - (B*b*log((2*B*b*exp(x))/
(b*c^2 - b^2*c - b^3 + c^3) - (2*B*b)/((b + c)^(5/2)*(c - b)^(1/2))))/((b +
 c)^(3/2)*(c - b)^(3/2)) - ((2*A)/(b + c) - (2*B*c*exp(x))/((b + c)*(b - c)
))/(b - c + exp(2*x)*(b + c))
```

3.729 $\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} + \frac{Bc+Ac\cosh(x)+Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{bBc\cosh(x)+b^2B\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(B*
c+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(b*B*c*cosh(x)
+b^2*B*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3237, 3232, 3153, 212}

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} + \frac{Ab\sinh(x)+Ac\cosh(x)+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{b^2B\sinh(x)+bBc\cosh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2
) + (b*B*c*Cosh[x] + b^2*B*Sinh[x])/((b^2 - c^2)^2*(b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3237

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2
- b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x
], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 -
 b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{\int \frac{2bB+Ab\cosh(x)+Ac\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2(b^2-c^2)} \\ &= \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} \\ &\quad + \frac{bBc\cosh(x) + b^2B\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))} + \frac{A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{2(b^2-c^2)}\end{aligned}$$

$$= \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{2\left(b^2 - c^2\right)\left(b\cosh(x) + c\sinh(x)\right)^2} + \frac{bBc\cosh(x) + b^2B\sinh(x)}{\left(b^2 - c^2\right)^2\left(b\cosh(x) + c\sinh(x)\right)} + \frac{(iA)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) - ib\sinh(x)\right)}{2\left(b^2 - c^2\right)}$$

$$= \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2 - c^2\right)^{3/2}} + \frac{Bc + Ac\cosh(x) + Ab\sinh(x)}{2\left(b^2 - c^2\right)\left(b\cosh(x) + c\sinh(x)\right)^2} + \frac{bBc\cosh(x) + b^2B\sinh(x)}{\left(b^2 - c^2\right)^2\left(b\cosh(x) + c\sinh(x)\right)}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}} + \frac{Ac + 2bB\sinh(x)}{b(b-c)(b+c)(b\cosh(x) + c\sinh(x))} + \frac{bBc + A(b^2 - c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x) + c\sinh(x))^2}\right)$$

```
[In] Integrate[(A + B*Cosh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (A*c + 2*b*B*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*Sin
h[x])) + (b*B*c + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*(b*Cosh[x] + c*
Sinh[x])^2))/2
```

Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.76

method	result
risch	$\frac{A\,b^2 e^{3x}+2Abc\,e^{3x}+A\,c^2 e^{3x}-2B\,b^2 e^{2x}+2B\,c^2 e^{2x}-A\,e^{x} b^2+A\,e^{x} c^2-2B\,b^2+2bBc}{(b-c)(b\,e^{2x}+e^{2x}c+b-c)^2(b^2+2cb+c^2)} - \frac{A\ln\left(e^{x}-\frac{b-c}{\sqrt{-b^2+c^2}}\right)}{2\sqrt{-b^2+c^2}\,(b+c)(b-c)} + \frac{A\ln\left(e^{x}+\frac{b-c}{\sqrt{-b^2+c}}\right.}{2\sqrt{-b^2+c^2}\,(b+c)(b}$
default	$\frac{-\frac{\left(b^2A-2A\,c^2-2B\,b^2+2B\,c^2\right)\tanh\left(\frac{x}{2}\right)^3}{\left(b^2-c^2\right)b}+\frac{c\left(b^2A+2A\,c^2+2B\,b^2-2B\,c^2\right)\tanh\left(\frac{x}{2}\right)^2}{b^2\left(b^2-c^2\right)}+\frac{\left(b^2A+2A\,c^2+2B\,b^2-2B\,c^2\right)\tanh\left(\frac{x}{2}\right)}{b\left(b^2-c^2\right)}+\frac{2Ac}{2b^2-2c^2}}{\left(\tanh\left(\frac{x}{2}\right)^2 b+2c\tanh\left(\frac{x}{2}\right)+b\right)^2} + \frac{A\,a}{}$

```
[In] int((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (A*b^2*exp(x)^3+2*A*b*c*exp(x)^3+A*c^2*exp(x)^3-2*B*b^2*exp(x)^2+2*B*c^2*ex
p(x)^2-A*exp(x)*b^2+A*exp(x)*c^2-2*B*b^2+2*b*B*c)/(b-c)/(b*exp(x)^2+exp(x)^
2*c+b-c)^2/(b^2+2*b*c+c^2)-1/2/(-b^2+c^2)^(1/2)*A/(b+c)/(b-c)*ln(exp(x)-(b-
c)/(-b^2+c^2)^(1/2))+1/2/(-b^2+c^2)^(1/2)*A/(b+c)/(b-c)*ln(exp(x)+(b-c)/(-b
^2+c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 1855, normalized size of antiderivative = 15.46

$$\int \frac{A + B\cosh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*B*b^3 - 8*B*b^2*c + 4*B*b*c^2 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c
^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 4*(B*b^3
- B*b^2*c - B*b*c^2 + B*c^3)*cosh(x)^2 + 2*(2*B*b^3 - 2*B*b^2*c - 2*B*b*c^2
 + 2*B*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A
*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sin
h(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*
(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*
cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*
c^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*
cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)
) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)
^2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + 2*(A*b^3 - A
*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2
+ 4*(B*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c -
b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 -
4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2
 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c
- b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4
*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 +
3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2
)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
 c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x)), -(
2*B*b^3 - 4*B*b^2*c + 2*B*b*c^2 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(
x)^3 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*sinh(x)^3 + 2*(B*b^3 - B*b^2*c -
 B*b*c^2 + B*c^3)*cosh(x)^2 + (2*B*b^3 - 2*B*b^2*c - 2*B*b*c^2 + 2*B*c^3 -
3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 + ((A*b^2 + 2*A*b*
c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh(x)^3 + (A*b
^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(A*b^2 - A*c^
2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^2)*si
```

```
nh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c^2)*cosh(x))
*sinh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)
*sinh(x))) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + (A*b^3 - A*b^2*c
 - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 + 4*(B
*b^3 - B*b^2*c - B*b*c^2 + B*c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^
2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*
c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b
^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*
c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 +
 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6
 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh
(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*
cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} + \frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Bb^2e^{(2x)}+2\,Bc^2e^{(2x)}-Ab^2e^x+Ac^2e^x-2\,Bb^2+2\,Bbc}{(b^3+b^2c-bc^2-c^3)\left(be^{(2x)}+ce^{(2x)}+b-c\right)^2}$$

```
[In] integrate((A+B*cosh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) + 2*B*c^2*e^(2*x) - A
*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*c)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e
^(2*x) + c*e^(2*x) + b - c)^2)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.80

$$\int \frac{A+B\cosh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}} - \frac{\frac{B}{(b+c)^2}-\frac{A\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)} - \frac{\frac{B}{b+c}+\frac{2\,A\,\mathrm{e}^x}{b+c}+\frac{B\,\mathrm{e}^{2\,x}}{b+c}}{\mathrm{e}^{4\,x}\,(b+c)^2+(b-c)^2+2\,\mathrm{e}^{2\,x}\,(b+c)\,(b-c)}$$

```
[In] int((A + B*cosh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] (atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2)
 + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))/(
b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - (B/(b + c)^2 - (A*exp(x))/((b +
c)*(b - c)))/(b - c + exp(2*x)*(b + c)) - (B/(b + c) + (2*A*exp(x))/(b + c)
 + (B*exp(2*x))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*exp(2*x)*(b +
c)*(b - c))
```

3.730 $\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}(\cosh(x)+\sinh(x))^2$$

```
[Out] 1/2*(cosh(x)+sinh(x))^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4470}

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}(\sinh(x)+\cosh(x))^2$$

```
[In] Int[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] (Cosh[x] + Sinh[x])^2/2
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\text{integral} = \frac{1}{2}(\cosh(x)+\sinh(x))^2$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}\cosh(2x)+\frac{1}{2}\sinh(2x)$$

```
[In] Integrate[(Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
[Out] Cosh[2*x]/2 + Sinh[2*x]/2
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{e^{2x}}{2}$	7
parallelrisch	$-\frac{1}{\tanh(x)-1}$	9
gosper	$-\frac{\sinh(x)+\cosh(x)}{2(\sinh(x)-\cosh(x))}$	17
default	$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{2}{\tanh\left(\frac{x}{2}\right)-1}$	22

```
[In] int((sinh(x)+cosh(x))/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{\cosh\left(x\right)+\sinh\left(x\right)}{2\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)}\,dx = \frac{\cosh{(x)}}{-\sinh{(x)} + \cosh{(x)}}$$

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
[Out] cosh(x)/(-sinh(x) + cosh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)}\,dx = \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
[Out] 1/2*e^(2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)}\,dx = \frac{1}{2}\,e^{(2\,x)}$$

```
[In] integrate((cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
[Out] 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)}\,dx = \frac{e^{2\,x}}{2}$$

```
[In] int((cosh(x) + sinh(x))/(cosh(x) - sinh(x)),x)
[Out] exp(2*x)/2
```

3.731 $\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)} dx = -\frac{1}{2(\cosh(x)+\sinh(x))^2}$$

```
[Out] -1/2/(cosh(x)+sinh(x))^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{4470\}$

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)} dx = -\frac{1}{2(\sinh(x)+\cosh(x))^2}$$

```
[In] Int[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -1/2*1/(Cosh[x] + Sinh[x])^2
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\text{integral} = -\frac{1}{2(\cosh(x)+\sinh(x))^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx = -\frac{1}{2}\cosh(2x)+\frac{1}{2}\sinh(2x)$$

```
[In] Integrate[(Cosh[x] - Sinh[x])/(Cosh[x] + Sinh[x]),x]
[Out] -1/2*Cosh[2*x] + Sinh[2*x]/2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{e^{-2x}}{2}$	7
parallelrisch	$-\frac{1}{1+\tanh(x)}$	9
gosper	$\frac{\sinh(x)-\cosh(x)}{2\sinh(x)+2\cosh(x)}$	17
default	$\frac{2}{\tanh(\frac{x}{2})+1}-\frac{2}{(\tanh(\frac{x}{2})+1)^2}$	22

```
[In] int((cosh(x)-sinh(x))/(sinh(x)+cosh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\cosh(x)-\sinh(x)}{\cosh(x)+\sinh(x)}\,dx = -\frac{1}{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] -1/2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{\cosh(x)}{\sinh(x) + \cosh(x)}$$

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x)
[Out] -cosh(x)/(sinh(x) + cosh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{1}{2} \, e^{(-2 \, x)}$$

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] -1/2*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{1}{2} \, e^{(-2 \, x)}$$

```
[In] integrate((cosh(x)-sinh(x))/(cosh(x)+sinh(x)),x, algorithm="giac")
[Out] -1/2*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} \, dx = -\frac{e^{-2\,x}}{2}$$

```
[In] int((cosh(x) - sinh(x))/(cosh(x) + sinh(x)),x)
[Out] -exp(-2*x)/2
```

3.732 $\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx=-i\log(\cosh(x)+i\sinh(x))$$

```
[Out] -I*ln(cosh(x)+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3212}

$$\int \frac{\cosh(x)-i\sinh(x)}{\cosh(x)+i\sinh(x)}\,dx=-i\log(\cosh(x)+i\sinh(x))$$

```
[In] Int[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
```

```
[Out] (-I)*Log[Cosh[x] + I*Sinh[x]]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral}=-i\log(\cosh(x)+i\sinh(x))$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = \arctan(\tanh(x)) - \frac{1}{2}i\log(\cosh(2x))$$

```
[In] Integrate[(Cosh[x] - I*Sinh[x])/(Cosh[x] + I*Sinh[x]),x]
[Out] ArcTan[Tanh[x]] - (I/2)*Log[Cosh[2*x]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
risch	$ix - i\ln\left(\mathrm{e}^{2x} - i\right)$	17
parallelrisch	$\frac{i(\ln(1-\tanh(x))+\ln(1+\tanh(x))-2\ln(\tanh(x)-i))}{2}$	25
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-i\ln\left(2i\tanh\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)^{2}+1\right)$	41

```
[In] int((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x,method=_RETURNVERBOSE)
[Out] I*x-I*ln(exp(2*x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = i\,x - i\,\log\left(e^{(2\,x)} - i\right)$$

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="fricas")
[Out] I*x - I*log(e^(2*x) - I)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = ix - i\log\left(e^{2x} - i\right)$$

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x)
[Out] I*x - I*log(exp(2*x) - I)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = -i\,\log\left(\cosh\left(x\right) + i\,\sinh\left(x\right)\right)$$

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="maxima")
[Out] -I*log(cosh(x) + I*sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = i\,x - i\,\log\left(e^{(2\,x)} - i\right)$$

```
[In] integrate((cosh(x)-I*sinh(x))/(cosh(x)+I*sinh(x)),x, algorithm="giac")
[Out] I*x - I*log(e^(2*x) - I)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(x) - i\sinh(x)}{\cosh(x) + i\sinh(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - \mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int((cosh(x) - sinh(x)*1i)/(cosh(x) + sinh(x)*1i),x)
[Out] x*1i - log(exp(2*x) - 1i)*1i
```

3.733 $\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3212}

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{x(bB-cC)}{b^2-c^2} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2
 - c^2)
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{integral} = \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{(bB-cC)x+(-Bc+bC)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]

[Out] ((b*B - c*C)*x + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c
))

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{(-Bc+bC)\ln(c\tanh(x)+b)+(Bc-bC)\ln(1-\tanh(x))+x(b+c)(B-C)}{b^2-c^2}$	56
default	$\frac{2(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2(B-C)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{2\left(-\frac{Bc}{2}+\frac{bC}{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2b+2c\tanh\left(\frac{x}{2}\right)+b\right)}{(b-c)(b+c)}$	91
risch	$\frac{Bx}{b+c}+\frac{Cx}{b+c}+\frac{2xBc}{b^2-c^2}-\frac{2xbC}{b^2-c^2}-\frac{\ln\left(e^{2x}+\frac{b-c}{b+c}\right)Bc}{b^2-c^2}+\frac{\ln\left(e^{2x}+\frac{b-c}{b+c}\right)bC}{b^2-c^2}$	113

[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)

[Out] ((-B*c+C*b)*ln(c*tanh(x)+b)+(B*c-C*b)*ln(1-tanh(x))+x*(b+c)*(B-C))/(b^2-c^2
)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx=\frac{((B-C)b+(B-C)c)x+(Cb-Bc)\log\left(\frac{2(b\cosh(x)+c\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{b^2-c^2}$$

[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas"
)

[Out] (((B - C)*b + (B - C)*c)*x + (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cos
h(x) - sinh(x))))/(b^2 - c^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(39) = 78$.

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 6.15

$$\int \frac{B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \begin{cases} \tilde{\infty}(B\log(\sinh(x)) + Cx) \\ \frac{B\log(\sinh(x))+Cx}{c} \\ \frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} + \frac{Cx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \\ \frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{\ }{2c\sinh} \\ \frac{Bbx}{b^2-c^2} - \frac{Bc\log\left(\cosh(x)+\frac{c\sinh(x)}{b}\right)}{b^2-c^2} + \frac{Cb\log\left(\cosh(x)+\frac{c\sinh(x)}{b}\right)}{b^2-c^2} - \frac{Ccx}{b^2-c^2} \end{cases}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(B*log(sinh(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*log(sinh(x
)) + C*x)/c, Eq(b, 0)), (B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cos
h(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x))
- C*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*
c*cosh(x)) - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (B*x*sinh(
x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) -
B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh
(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*cosh(x)/(2*c*sinh(x) + 2
*c*cosh(x)), Eq(b, c)), (B*b*x/(b**2 - c**2) - B*c*log(cosh(x) + c*sinh(x)/
b)/(b**2 - c**2) + C*b*log(cosh(x) + c*sinh(x)/b)/(b**2 - c**2) - C*c*x/(b*
*2 - c**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.64

$$\int \frac{B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = C\left(\frac{b\log\left(-(b-c)e^{(-2x)} - b - c\right)}{b^2 - c^2} + \frac{x}{b+c}\right) - B\left(\frac{c\log\left(-(b-c)e^{(-2x)} - b - c\right)}{b^2 - c^2} - \frac{x}{b+c}\right)$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxima"
)
```

```
[Out] C*(b*log(-(b - c)*e^(-2*x) - b - c)/(b^2 - c^2) + x/(b + c)) - B*(c*log(-(b
 - c)*e^(-2*x) - b - c)/(b^2 - c^2) - x/(b + c))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{(B-C)x}{b-c} + \frac{(Cb-Bc)\log\left(\left|be^{(2x)} + ce^{(2x)} + b - c\right|\right)}{b^2-c^2}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(abs(b*e^(2*x) + c*e^(2*x) + b - c))/(b^
2 - c^2)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{x\,(B\,b - C\,c)}{b^2-c^2} - \frac{\ln\left(b\cosh(x) + c\sinh(x)\right)\,(B\,c - C\,b)}{b^2-c^2}$$

```
[In] int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (x*(B*b - C*c))/(b^2 - c^2) - (log(b*cosh(x) + c*sinh(x))*(B*c - C*b))/(b^2
 - c^2)
```

3.734 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=\frac{(bB-cC)\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{Bc-bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

```
[Out] (B*b-C*c)*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*
c-C*b)/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3232, 3153, 212}

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx=\frac{(bB-cC)\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{Bc-bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C)/((b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bc-bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}+\frac{(bB-cC)\int\frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}\\
&= \frac{Bc-bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}\\
&\quad+\frac{(i(bB-cC))\text{Subst}\left(\int\frac{1}{b^2-c^2-x^2}\,dx,x,-ic\cosh(x)-ib\sinh(x)\right)}{b^2-c^2}\\
&= \frac{(bB-cC)\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}+\frac{Bc-bC}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\begin{aligned}
\int\frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx &= \frac{2(bB-cC)\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}}\\
&\quad+\frac{Bc-bC}{(b-c)(b+c)(b\cosh(x)+c\sinh(x))}
\end{aligned}$$

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
[Out] (2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c
)^(3/2)*(b + c)^(3/2)) + (B*c - b*C)/((b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x
]))
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

method	result
default	$\frac{\frac{2c(Bc-bC)\tanh\left(\frac{x}{2}\right)}{\left(b^{2}-c^{2}\right)b}+\frac{2(Bc-bC)}{b^{2}-c^{2}}}{\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b}+\frac{2(Bb-Cc)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2c}{2\sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2\,\mathrm{e}^{x}(Bc-bC)}{(b-c)(b+c)(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+b-c)}-\frac{bB\ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{cC\ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{bB\ln\left(\mathrm{e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}-\frac{cC\ln\left(\mathrm{e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}$

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*(B*c-C*b)/(b^2-c^2)/b*tanh(1/2*x)+(B*c-C*b)/(b^2-c^2))/(tanh(1/2*x)^2*
b+2*c*tanh(1/2*x)+b)+2*(B*b-C*c)/(b^2-c^2)^(3/2)*arctan(1/2*(2*b*tanh(1/2*x
)+2*c)/(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(75) = 150.

Time = 0.29 (sec) , antiderivative size = 749, normalized size of antiderivative = 9.60

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$$
$$=\left[-\frac{(Bb^2-(B+C)bc+Cc^2+(Bb^2+(B-C)bc-Cc^2)\cosh(x)^2+2\,(Bb^2+(B-C)bc-Cc^2)\cosh(x)\cdots}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+(b^5\cdots}\right.$$
$$-\frac{2\left((Bb^2-(B+C)bc+Cc^2+(Bb^2+(B-C)bc-Cc^2)\cosh(x)^2+2\,(Bb^2+(B-C)bc-Cc^2)\cosh(x\cdots\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+(b^5+b^4c-2\,b^3c^2-2\,b^2c^3\cdots}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="frica
s")
```

```
[Out] [-((B*b^2 - (B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 +
 2*(B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C
*c^2)*sinh(x)^2)*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x
)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b
+ c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b
 - c)) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c
 - C*b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 -
 c^5 + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b
^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 +
b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2), -2*((B*b^2 - (B +
```

```
C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B -
C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*
sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x)))
 + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b^2*c - C*b*c^2
 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^
5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c
 - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*
b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \frac{2\,(Bb - Cc)\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{(b^2 - c^2)^{\frac{3}{2}}} - \frac{2\,(Cbe^x - Bce^x)}{(b^2 - c^2)(be^{(2\,x)} + ce^{(2\,x)} + b - c)}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="giac"
)
```

```
[Out] 2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2
*(C*b*e^x - B*c*e^x)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b - c))
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.55

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{\ln\left(\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,\mathrm{e}^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\ln\left(\frac{2\,\mathrm{e}^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} + \frac{2\,\mathrm{e}^x\,(B\,c-C\,b)}{(b+c)\,\left(b-c\right)\,\left(b-c+\mathrm{e}^{2\,x}\,\left(b+c\right)\right)}$$

```
[In] int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (log((2*(B*b - C*c))/((b + c)^(5/2)*(c - b)^(1/2)) + (2*exp(x)*(B*b - C*c))
/(b*c^2 - b^2*c - b^3 + c^3))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) -
(log((2*exp(x)*(B*b - C*c))/(b*c^2 - b^2*c - b^3 + c^3) - (2*(B*b - C*c))/(
(b + c)^(5/2)*(c - b)^(1/2)))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) +
(2*exp(x)*(B*c - C*b))/((b + c)*(b - c)*(b - c + exp(2*x)*(b + c)))
```

3.735 $\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{(bB-cC)\sinh(x)}{b(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*(B*c-C*b)/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(B*b-C*c)*sinh(x)/b/(b^2-c^
2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3235, 12, 3154}

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{Bc-bC}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{\sinh(x)(bB-cC)}{b(b^2-c^2)(b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (B*c - b*C)/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x])^2) + ((b*B - c*C)*Sinh[x
])/(b*(b^2 - c^2)*(b*Cosh[x] + c*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{\int \frac{2(bB-cC)}{(b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(b^2-c^2\right)}\\
&= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\int \frac{1}{(b\cosh(x)+c\sinh(x))^2}\,dx}{b^2-c^2}\\
&= \frac{Bc-bC}{2\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))^2}+\frac{(bB-cC)\sinh(x)}{b\left(b^2-c^2\right)(b\cosh(x)+c\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\left(-b^2+c^2\right)C+c(bB-cC)\cosh(2x)+b(bB-cC)\sinh(2x)}{2b(b-c)(b+c)(b\cosh(x)+c\sinh(x))^2}$$

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
[Out] ((-b^2 + c^2)*C + c*(b*B - c*C)*Cosh[2*x] + b*(b*B - c*C)*Sinh[2*x])/(2*b*(
b - c)*(b + c)*(b*Cosh[x] + c*Sinh[x])^2)
```

Maple [A] (verified)

Time = 14.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\left(-\frac{B\tanh\left(\frac{x}{2}\right)^{3}}{b}-\frac{(Bc+bC)\tanh\left(\frac{x}{2}\right)^{2}}{b^{2}}-\frac{B\tanh\left(\frac{x}{2}\right)}{b}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}b+2c\tanh\left(\frac{x}{2}\right)+b\right)^{2}}$	63
risch	$-\frac{2\left(Bb\,\mathrm{e}^{2x}+\mathrm{e}^{2x}Bc+\mathrm{e}^{2x}bC+Cc\,\mathrm{e}^{2x}+Bb-Cc\right)}{\left(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+b-c\right)^{2}(b+c)^{2}}$	63

```
[In] int((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-B/b*tanh(1/2*x)^3-(B*c+C*b)/b^2*tanh(1/2*x)^2-B/b*tanh(1/2*x))/(tanh(1
/2*x)^2*b+2*c*tanh(1/2*x)+b)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(69) = 138.

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.27

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx =$$

$$-\frac{}{(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4)\cosh(x)^3+3\,(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4)\cosh(x)\sinh(x)^2+(b^4}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="frica
s")
```

```
[Out] -2*(((2*B + C)*b + B*c)*cosh(x) + (C*b + (B + 2*C)*c)*sinh(x))/((b^4 + 4*b^
3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)*sinh(x)^2 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 +
c^4)*sinh(x)^3 + (3*b^4 + 4*b^3*c - 2*b^2*c^2 - 4*b*c^3 - c^4)*cosh(x) + (b
^4 + 4*b^3*c + 2*b^2*c^2 - 4*b*c^3 - 3*c^4 + 3*(b^4 + 4*b^3*c + 6*b^2*c^2 +
 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(69) = 138$.

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.75

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx$$
$$= 2\,B\left(\frac{(b-c)e^{(-2\,x)}}{b^4 - 2\,b^2c^2 + c^4 + 2\,(b^4 - 2\,b^3c + 2\,bc^3 - c^4)e^{(-2\,x)} + (b^4 - 4\,b^3c + 6\,b^2c^2 - 4\,bc^3 + c^4)e^{(-4\,x)}} + \frac{}{b^4 - 2\,b\cdots}\right.$$
$$-2\,C\left(\frac{(b-c)e^{(-2\,x)}}{b^4 - 2\,b^2c^2 + c^4 + 2\,(b^4 - 2\,b^3c + 2\,bc^3 - c^4)e^{(-2\,x)} + (b^4 - 4\,b^3c + 6\,b^2c^2 - 4\,bc^3 + c^4)e^{(-4\,x)}} + \frac{}{b^4 - \cdots}\right.$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="maxim
a")
[Out] 2*B*((b - c)*e^(-2*x)/(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 -
 c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + b/
(b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4
- 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x))) - 2*C*((b - c)*e^(-2*x)/(
b^4 - 2*b^2*c^2 + c^4 + 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 -
 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)*e^(-4*x)) + c/(b^4 - 2*b^2*c^2 + c^4
+ 2*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(-2*x) + (b^4 - 4*b^3*c + 6*b^2*c^2 -
 4*b*c^3 + c^4)*e^(-4*x)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = -\frac{2\left(Bbe^{(2\,x)} + Cbe^{(2\,x)} + Bce^{(2\,x)} + Cce^{(2\,x)} + Bb - Cc\right)}{(b^2 + 2\,bc + c^2)\left(be^{(2\,x)} + ce^{(2\,x)} + b - c\right)^2}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="giac"
)
[Out] -2*(B*b*e^(2*x) + C*b*e^(2*x) + B*c*e^(2*x) + C*c*e^(2*x) + B*b - C*c)/((b^
2 + 2*b*c + c^2)*(b*e^(2*x) + c*e^(2*x) + b - c)^2)
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx=-\frac{b\,(2\,B+2\,B\,\mathrm{e}^{2\,x}+2\,C\,\mathrm{e}^{2\,x})+c\,(2\,B\,\mathrm{e}^{2\,x}-2\,C+2\,C\,\mathrm{e}^{2\,x})}{(b+c)^2\,(b-c+b\,\mathrm{e}^{2\,x}+c\,\mathrm{e}^{2\,x})^2}$$

```
[In] int((B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] -(b*(2*B + 2*B*exp(2*x) + 2*C*exp(2*x)) + c*(2*B*exp(2*x) - 2*C + 2*C*exp(2
*x)))/((b + c)^2*(b - c + b*exp(2*x) + c*exp(2*x))^2)
```

3.736 $\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2}+\frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(b*cosh(x)+c*sinh(x))/(b^2-c^2)+A*arctan(
(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3215, 3153, 212}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx = \frac{A\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}}+\frac{x(bB-cC)}{b^2-c^2} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 -
c^2]])/Sqrt[b^2 - c^2] - ((B*c - b*C)*Log[b*Cosh[x] + c*Sinh[x]])/(b^2 - c^
2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3215

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[(c*B - b*C)*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 +
c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2} + A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx \\
&= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad + (iA)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x)-ib\sinh(x)\right) \\
&= \frac{(bB-cC)x}{b^2-c^2} + \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{\sqrt{b^2-c^2}} - \frac{(Bc-bC)\log(b\cosh(x)+c\sinh(x))}{b^2-c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$
\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx
= \frac{(bB-cC)x+2A\sqrt{b-c}\sqrt{b+c}\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)+(-Bc+bC)\log(b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}
$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x]),x]
[Out] ((b*B - c*C)*x + 2*A*Sqrt[b - c]*Sqrt[b + c]*ArcTan[(c + b*Tanh[x/2])/(Sqrt
[b - c]*Sqrt[b + c])] + (-(B*c) + b*C)*Log[b*Cosh[x] + c*Sinh[x]])/((b - c)
*(b + c))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.90

method	result
default	$\frac{2(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{2(B-C)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b-2c}+\frac{\frac{\left(-bBc+C\,b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2b+2c\tanh\left(\frac{x}{2}\right)+b\right)}{b}+\frac{2\left(b^2A-A\,c^2-B\,c^2+Ccb-\frac{(-b\ldots}{}\right.}{}}{(b-c)(b+c)}$
risch	$\frac{Bx}{b+c}+\frac{Cx}{b+c}+\frac{2xB\,b^2c}{b^4-2b^2c^2+c^4}-\frac{2xB\,c^3}{b^4-2b^2c^2+c^4}-\frac{2xC\,b^3}{b^4-2b^2c^2+c^4}+\frac{2xCb\,c^2}{b^4-2b^2c^2+c^4}-\frac{\ln\left(e^x+\frac{\sqrt{-A^2b^2+A^2c^2}}{A(b+c)}\right)Bc}{(b+c)(b-c)}+\frac{\ln\left(e^x+\frac{\sqrt{-\ldots}}{}\right)}{(b+\ldots}$

[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)

[Out] 2*(-B-C)/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*(B-C)/(2*b-2*c)*ln(tanh(1/2*x)+1)+2/(b-c)/(b+c)*(1/2*(-B*b*c+C*b^2)/b*ln(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+(b^2*A-A*c^2-B*c^2+C*c*b-(-B*b*c+C*b^2)*c/b)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.87

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{b\cosh(x)+c\sinh(x)}\,dx$$

$$= \left[-\frac{\sqrt{-b^2+c^2}A\log\left(\frac{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{-b^2+c^2}(\cosh(x)+\sinh(x))-b+c}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+b-c}\right)-((B-C)b+\ldots}{b^2-c^2}\right.$$

$$-\frac{2\sqrt{b^2-c^2}A\arctan\left(\frac{\sqrt{b^2-c^2}}{(b+c)\cosh(x)+(b+c)\sinh(x)}\right)-((B-C)b+(B-C)c)x-(Cb-Bc)\log\left(\frac{2\,(b\cosh(x)+c\sin\ldots}{\cosh(x)-\sinh(\ldots}\right.}{b^2-c^2}$$

[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="fricas")

[Out] [-(sqrt(-b^2 + c^2)*A*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) - ((B - C)*b + (B - C)*c)*x - (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))))/(b^2 - c^2), -(2*sqrt(b^2 - c^2)*A*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh(x))) - ((B - C)*b + (B - C)*c)*x - (C*b - B*c)*log(2*(b*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))))/(b^2 - c^2)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(73) = 146$.

Time = 30.67 (sec) , antiderivative size = 643, normalized size of antiderivative = 6.99

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right) + B\log\left(\tanh\left(\frac{x}{2}\right)\right) + Cx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{2A}{-2c\sinh(x)+2c\cosh(x)} + \frac{Bx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Bx\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{-2c\sinh(x)+2c\cosh(x)} - \frac{Cx\sinh(x)}{-2c\sinh(x)+2c\cosh(x)} \\ -\frac{2A}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Bx\cosh(x)}{2c\sinh(x)+2c\cosh(x)} - \frac{B\cosh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{Cx\sinh(x)}{2c\sinh(x)+2c\cosh(x)} + \frac{}{2c} \\ -\frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}-\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2} + \frac{A\sqrt{-b^2+c^2}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{c}{b}+\frac{\sqrt{-b^2+c^2}}{b}\right)}{b^2-c^2} + \frac{Bbx}{b^2-c^2} - \frac{Bcx}{b^2-c^2} + \frac{2Bc\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2-c^2} \end{cases}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(ta
nh(x/2) + 1) + B*log(tanh(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(-2*c*sinh(x) +
2*c*cosh(x)) + B*x*sinh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - B*x*cosh(x)/(-2*c
*sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)) - C*x*sinh
(x)/(-2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x))
 - C*cosh(x)/(-2*c*sinh(x) + 2*c*cosh(x)), Eq(b, -c)), (-2*A/(2*c*sinh(x) +
 2*c*cosh(x)) + B*x*sinh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + B*x*cosh(x)/(2*c*
sinh(x) + 2*c*cosh(x)) - B*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*sinh(x
)/(2*c*sinh(x) + 2*c*cosh(x)) + C*x*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)) + C
*cosh(x)/(2*c*sinh(x) + 2*c*cosh(x)), Eq(b, c)), (-A*sqrt(-b**2 + c**2)*log
(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + A*sqrt(-b**2 + c**
2)*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + B*b*x/(b**2
- c**2) - B*c*x/(b**2 - c**2) + 2*B*c*log(tanh(x/2) + 1)/(b**2 - c**2) - B*
c*log(tanh(x/2) + c/b - sqrt(-b**2 + c**2)/b)/(b**2 - c**2) - B*c*log(tanh(
x/2) + c/b + sqrt(-b**2 + c**2)/b)/(b**2 - c**2) + C*b*x/(b**2 - c**2) - 2*
C*b*log(tanh(x/2) + 1)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b - sqrt(-b**2
 + c**2)/b)/(b**2 - c**2) + C*b*log(tanh(x/2) + c/b + sqrt(-b**2 + c**2)/b)
/(b**2 - c**2) - C*c*x/(b**2 - c**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{2\,A\arctan\left(\frac{be^x + ce^x}{\sqrt{b^2 - c^2}}\right)}{\sqrt{b^2 - c^2}} + \frac{(B - C)x}{b - c} + \frac{(Cb - Bc)\log\left(be^{(2\,x)} + ce^{(2\,x)} + b - c\right)}{b^2 - c^2}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x)),x, algorithm="giac"
)
```

```
[Out] 2*A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/sqrt(b^2 - c^2) + (B - C)*x/(b
- c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + b - c)/(b^2 - c^2)
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.28

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{b\cosh(x) + c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^2-c^2}}{b\,\sqrt{A^2}-c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2 - c^2}} + \frac{B\,x}{b-c} - \frac{C\,x}{b-c} + \frac{B\,c^3\,\ln\left(4\,A^2\,b - 4\,A^2\,c + 4\,A^2\,b\,\mathrm{e}^{2\,x} + 4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4 - 2\,b^2\,c^2 + c^4} + \frac{C\,b^3\,\ln\left(4\,A^2\,b - 4\,A^2\,c + 4\,A^2\,b\,\mathrm{e}^{2\,x} + 4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4 - 2\,b^2\,c^2 + c^4} - \frac{B\,b^2\,c\,\ln\left(4\,A^2\,b - 4\,A^2\,c + 4\,A^2\,b\,\mathrm{e}^{2\,x} + 4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4 - 2\,b^2\,c^2 + c^4} - \frac{C\,b\,c^2\,\ln\left(4\,A^2\,b - 4\,A^2\,c + 4\,A^2\,b\,\mathrm{e}^{2\,x} + 4\,A^2\,c\,\mathrm{e}^{2\,x}\right)}{b^4 - 2\,b^2\,c^2 + c^4}$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan((A*exp(x)*(b^2 - c^2)^(1/2))/(b*(A^2)^(1/2) - c*(A^2)^(1/2)))*(A^2)
^(1/2))/(b^2 - c^2)^(1/2) + (B*x)/(b - c) - (C*x)/(b - c) + (B*c^3*log(4*A^
2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^
2) + (C*b^3*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(
b^4 + c^4 - 2*b^2*c^2) - (B*b^2*c*log(4*A^2*b - 4*A^2*c + 4*A^2*b*exp(2*x)
+ 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2) - (C*b*c^2*log(4*A^2*b - 4*A^2
*c + 4*A^2*b*exp(2*x) + 4*A^2*c*exp(2*x)))/(b^4 + c^4 - 2*b^2*c^2)
```

3.737 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{(bB-cC)\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} + \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}$$

```
[Out] (B*b-C*c)*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+(B*
c-b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3232, 3153, 212}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} + \frac{(bB-cC)\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((b*B - c*C)*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(
3/2) + (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/((b^2 - c^2)*(b*Cosh[x] + c*
Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} + \frac{(bB-cC)\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2} \\
&= \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))} \\
&\quad + \frac{(i(bB-cC))\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) - ib\sinh(x)\right)}{b^2-c^2} \\
&= \frac{(bB-cC)\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} + \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{(b^2-c^2)\,(b\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$
\begin{aligned}
\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx &= \frac{2(bB-cC)\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}} \\
&\quad + \frac{b(Bc-bC) + A(b^2-c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x)+c\sinh(x))}
\end{aligned}
$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c
)^(3/2)*(b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)
*(b + c)*(b*Cosh[x] + c*Sinh[x]))
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

method	result
default	$-\frac{2\left(-\frac{\left(b^{2} A-A c^{2}+B c^{2}-C c b\right) \tanh\left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}-\frac{B c-b C}{b^{2}-c^{2}}\right)}{\tanh\left(\frac{x}{2}\right)^{2} b+2 c \tanh\left(\frac{x}{2}\right)+b}+\frac{2(B b-C c) \arctan\left(\frac{2 b \tanh\left(\frac{x}{2}\right)+2 c}{2 \sqrt{b^{2}-c^{2}}}\right)}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{2\left(-B c\, \mathrm{e}^{x}+C b\, \mathrm{e}^{x}+b A-A c\right)}{(b-c)(b+c)\left(b\, \mathrm{e}^{2 x}+\mathrm{e}^{2 x} c+b-c\right)}-\frac{b B \ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{c C \ln\left(\mathrm{e}^{x}-\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}+\frac{b B \ln\left(\mathrm{e}^{x}+\frac{b-c}{\sqrt{-b^{2}+c^{2}}}\right)}{\sqrt{-b^{2}+c^{2}}\,(b+c)(b-c)}-\frac{c C \ln\left(\mathrm{e}^{x}+\frac{}{\sqrt{-}}\right.}{\sqrt{-b^{2}+c^{2}}\,(b+}$

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -2*(-(A*b^2-A*c^2+B*c^2-C*b*c)/b/(b^2-c^2)*tanh(1/2*x)-(B*c-C*b)/(b^2-c^2))
/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)+2*(B*b-C*c)/(b^2-c^2)^(3/2)*arctan(1/2
*(2*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(84) = 168.

Time = 0.29 (sec) , antiderivative size = 799, normalized size of antiderivative = 9.08

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx$$
$$=\left[-\frac{2\,Ab^3-2\,Ab^2c-2\,Abc^2+2\,Ac^3+\left(Bb^2-(B+C)bc+Cc^2+\left(Bb^2+(B-C)bc-Cc^2\right)\cosh\left(x\right)^2+2\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c}\right.$$
$$-\frac{2\left(Ab^3-Ab^2c-Abc^2+Ac^3+\left(Bb^2-(B+C)bc+Cc^2+\left(Bb^2+(B-C)bc-Cc^2\right)\cosh\left(x\right)^2+2\left(Bb\right.\right.\right.}{b^5-b^4c-2\,b^3c^2+2\,b^2c^3+bc^4-c^5+\left(b^5+b^4c-\right.}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="fri
cas")
```

```
[Out] [-(2*A*b^3 - 2*A*b^2*c - 2*A*b*c^2 + 2*A*c^3 + (B*b^2 - (B + C)*b*c + C*c^2
 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 + (B - C)*b*c - C*c^2
)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x)^2)*sqrt(-b^2 + c^
2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x)) - b + c)/((b + c)*cosh(x)^2 + 2*(b
```

```
+ c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + b - c)) + 2*(C*b^3 - B*b^2*c - C
*b*c^2 + B*c^3)*cosh(x) + 2*(C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*sinh(x))/(b
^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + (b^5 + b^4*c - 2*b^3*c^2
 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*
c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 +
 b*c^4 + c^5)*sinh(x)^2), -2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 + (B*b^2 -
(B + C)*b*c + C*c^2 + (B*b^2 + (B - C)*b*c - C*c^2)*cosh(x)^2 + 2*(B*b^2 +
(B - C)*b*c - C*c^2)*cosh(x)*sinh(x) + (B*b^2 + (B - C)*b*c - C*c^2)*sinh(x
)^2)*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b + c)*cosh(x) + (b + c)*sinh
(x))) + (C*b^3 - B*b^2*c - C*b*c^2 + B*c^3)*cosh(x) + (C*b^3 - B*b^2*c - C*
b*c^2 + B*c^3)*sinh(x))/(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5
+ (b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2 + 2*(b^5 +
b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)*sinh(x) + (b^5 + b^4*c
 - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*sinh(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,(Bb-Cc)\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} - \frac{2\,(Cbe^x-Bce^x+Ab-Ac)}{(b^2-c^2)(be^{(2\,x)}+ce^{(2\,x)}+b-c)}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^2,x, algorithm="gia
c")
```

```
[Out] 2*(B*b - C*c)*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) - 2
*(C*b*e^x - B*c*e^x + A*b - A*c)/((b^2 - c^2)*(b*e^(2*x) + c*e^(2*x) + b -
c))
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.39

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^2}\,dx = \frac{\ln\left(\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}+\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\ln\left(\frac{2\,e^x\,(B\,b-C\,c)}{-b^3-b^2\,c+b\,c^2+c^3}-\frac{2\,(B\,b-C\,c)}{(b+c)^{5/2}\,\sqrt{c-b}}\right)\,(B\,b-C\,c)}{(b+c)^{3/2}\,(c-b)^{3/2}} - \frac{\frac{2\,A}{b+c}-\frac{2\,e^x\,(B\,c-C\,b)}{(b+c)\,(b-c)}}{b-c+e^{2\,x}\,(b+c)}$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] (log((2*(B*b - C*c))/((b + c)^(5/2)*(c - b)^(1/2)) + (2*exp(x)*(B*b - C*c))
/(b*c^2 - b^2*c - b^3 + c^3))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) -
(log((2*exp(x)*(B*b - C*c))/(b*c^2 - b^2*c - b^3 + c^3) - (2*(B*b - C*c))/(
(b + c)^(5/2)*(c - b)^(1/2)))*(B*b - C*c))/((b + c)^(3/2)*(c - b)^(3/2)) -
((2*A)/(b + c) - (2*exp(x)*(B*c - C*b))/((b + c)*(b - c)))/(b - c + exp(2*x
)*(b + c))
```

3.738 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} + \frac{Bc-bC+Ac\cosh(x)+Ab\sinh(x)}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{c(bB-cC)\cosh(x)+b(bB-cC)\sinh(x)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*A*arctan((c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)+1/2*(B*
c-b*C+A*c*cosh(x)+A*b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x))^2+(c*(B*b-C*
c)*cosh(x)+b*(B*b-C*c)*sinh(x))/(b^2-c^2)^2/(b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3235, 3232, 3153, 212}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{b\sinh(x)+c\cosh(x)}{\sqrt{b^2-c^2}}\right)}{2(b^2-c^2)^{3/2}} + \frac{Ab\sinh(x)+Ac\cosh(x)-bC+Bc}{2(b^2-c^2)(b\cosh(x)+c\sinh(x))^2} + \frac{b\sinh(x)(bB-cC)+c\cosh(x)(bB-cC)}{(b^2-c^2)^2(b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (A*ArcTan[(c*Cosh[x] + b*Sinh[x])/Sqrt[b^2 - c^2]])/(2*(b^2 - c^2)^(3/2)) +
 (B*c - b*C + A*c*Cosh[x] + A*b*Sinh[x])/(2*(b^2 - c^2)*(b*Cosh[x] + c*Sinh
[x])^2) + (c*(b*B - c*C)*Cosh[x] + b*(b*B - c*C)*Sinh[x])/((b^2 - c^2)^2*(b
*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\text{integral} = \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{2\left(b^2 - c^2\right)\left(b\cosh(x) + c\sinh(x)\right)^2} + \frac{\int \frac{2(bB - cC) + Ab\cosh(x) + Ac\sinh(x)}{(b\cosh(x) + c\sinh(x))^2}\,dx}{2\left(b^2 - c^2\right)}$$

$$= \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{2\left(b^2 - c^2\right)(b\cosh(x) + c\sinh(x))^2} + \frac{c(bB - cC)\cosh(x) + b(bB - cC)\sinh(x)}{\left(b^2 - c^2\right)^2 (b\cosh(x) + c\sinh(x))} + \frac{A\int \frac{1}{b\cosh(x)+c\sinh(x)}\,dx}{2\left(b^2 - c^2\right)}$$

$$= \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{2\left(b^2 - c^2\right)(b\cosh(x) + c\sinh(x))^2} + \frac{c(bB - cC)\cosh(x) + b(bB - cC)\sinh(x)}{\left(b^2 - c^2\right)^2 (b\cosh(x) + c\sinh(x))} + \frac{(iA)\text{Subst}\left(\int \frac{1}{b^2-c^2-x^2}\,dx, x, -ic\cosh(x) - ib\sinh(x)\right)}{2\left(b^2 - c^2\right)}$$

$$= \frac{A\arctan\left(\frac{c\cosh(x)+b\sinh(x)}{\sqrt{b^2-c^2}}\right)}{2\left(b^2 - c^2\right)^{3/2}} + \frac{Bc - bC + Ac\cosh(x) + Ab\sinh(x)}{2\left(b^2 - c^2\right)(b\cosh(x) + c\sinh(x))^2} + \frac{c(bB - cC)\cosh(x) + b(bB - cC)\sinh(x)}{\left(b^2 - c^2\right)^2 (b\cosh(x) + c\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2A\arctan\left(\frac{c+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-c}\sqrt{b+c}}\right)}{(b-c)^{3/2}(b+c)^{3/2}} + \frac{b(Bc - bC) + A(b^2 - c^2)\sinh(x)}{b(b-c)(b+c)(b\cosh(x) + c\sinh(x))^2} + \frac{Ac + 2(bB - cC)\sinh(x)}{b(b-c)(b+c)(b\cosh(x) + c\sinh(x))}\right)$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*A*ArcTan[(c + b*Tanh[x/2])/(Sqrt[b - c]*Sqrt[b + c])])/((b - c)^(3/2)*(
b + c)^(3/2)) + (b*(B*c - b*C) + A*(b^2 - c^2)*Sinh[x])/(b*(b - c)*(b + c)*
(b*Cosh[x] + c*Sinh[x])^2) + (A*c + 2*(b*B - c*C)*Sinh[x])/(b*(b - c)*(b +
c)*(b*Cosh[x] + c*Sinh[x])))/2
```

Maple [A] (verified)

Time = 12.66 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.69

method	result
default	$\frac{-\frac{\left(b^{2} A-2 A c^{2}-2 B b^{2}+2 B c^{2}\right) \tanh \left(\frac{x}{2}\right)^{3}}{\left(b^{2}-c^{2}\right) b}+\frac{\left(A b^{2} c+2 A c^{3}+2 B b^{2} c-2 B c^{3}+2 C b^{3}-2 C b c^{2}\right) \tanh \left(\frac{x}{2}\right)^{2}}{b^{2}\left(b^{2}-c^{2}\right)}+\frac{\left(b^{2} A+2 A c^{2}+2 B b^{2}-2 B c^{2}\right) \tanh \left(\frac{x}{2}\right)}{b\left(b^{2}-c^{2}\right)}+\frac{2}{2b^{2}}}{\left(\tanh \left(\frac{x}{2}\right)^{2} b+2 c \tanh \left(\frac{x}{2}\right)+b\right)^{2}}$
risch	$\frac{A b^{2} e^{3 x}+2 A b c e^{3 x}+A c^{2} e^{3 x}-2 B b^{2} e^{2 x}+2 B c^{2} e^{2 x}-2 C b^{2} e^{2 x}+2 C c^{2} e^{2 x}-A e^{x} b^{2}+A e^{x} c^{2}-2 B b^{2}+2 b B c+2 C c b-2 C c^{2}}{(b-c)\left(b e^{2 x}+e^{2 x} c+b-c\right)^{2}\left(b^{2}+2 c b+c^{2}\right)}-\frac{A \ln \left(e^{x}-\frac{}{\sqrt{}}\right.}{2 \sqrt{-b^{2}+c^{2}}\,(}$

```
[In] int((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 2*(-1/2*(A*b^2-2*A*c^2-2*B*b^2+2*B*c^2)/(b^2-c^2)/b*tanh(1/2*x)^3+1/2*(A*b^
2*c+2*A*c^3+2*B*b^2*c-2*B*c^3+2*C*b^3-2*C*b*c^2)/b^2/(b^2-c^2)*tanh(1/2*x)^
2+1/2*(A*b^2+2*A*c^2+2*B*b^2-2*B*c^2)/b/(b^2-c^2)*tanh(1/2*x)+1/2*A*c/(b^2-
c^2))/(tanh(1/2*x)^2*b+2*c*tanh(1/2*x)+b)^2+A/(b^2-c^2)^(3/2)*arctan(1/2*(2
*b*tanh(1/2*x)+2*c)/(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(127) = 254.

Time = 0.30 (sec) , antiderivative size = 1931, normalized size of antiderivative = 14.30

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="fri
cas")
```

```
[Out] [-1/2*(4*B*b^3 - 4*(2*B + C)*b^2*c + 4*(B + 2*C)*b*c^2 - 4*C*c^3 - 2*(A*b^3
 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - 2*(A*b^3 + A*b^2*c - A*b*c^2 - A*
c^3)*sinh(x)^3 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c
^3)*cosh(x)^2 + 2*(2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*c^2 + 2*(B
 + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x)^2 - ((A*
b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*cosh(x)*sinh
(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A*c^2 + 2*(
A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c + A*c^2)*c
osh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A*b^2 - A*c
^2)*cosh(x))*sinh(x))*sqrt(-b^2 + c^2)*log(((b + c)*cosh(x)^2 + 2*(b + c)*c
osh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(-b^2 + c^2)*(cosh(x) + sinh(x))
 - b + c)/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^
2 + b - c)) + 2*(A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh(x) + 2*(A*b^3 - A*
b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^2 +
```

```
4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*c^3)*cosh(x))*sin
h(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b*c^5 + c^6 + (b^6
 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^4 + 4*(
b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)*sinh
(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*sin
h(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 + 2*(b^6 - 3*b^4*c
^2 + 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2
*b*c^5 + c^6)*cosh(x)^2)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^
3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6
)*cosh(x))*sinh(x)), -(2*B*b^3 - 2*(2*B + C)*b^2*c + 2*(B + 2*C)*b*c^2 - 2*
C*c^3 - (A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x)^3 - (A*b^3 + A*b^2*c -
A*b*c^2 - A*c^3)*sinh(x)^3 + 2*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2
 + (B + C)*c^3)*cosh(x)^2 + (2*(B + C)*b^3 - 2*(B + C)*b^2*c - 2*(B + C)*b*
c^2 + 2*(B + C)*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A*c^3)*cosh(x))*sinh(x
)^2 + ((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^4 + 4*(A*b^2 + 2*A*b*c + A*c^2)*co
sh(x)*sinh(x)^3 + (A*b^2 + 2*A*b*c + A*c^2)*sinh(x)^4 + A*b^2 - 2*A*b*c + A
*c^2 + 2*(A*b^2 - A*c^2)*cosh(x)^2 + 2*(A*b^2 - A*c^2 + 3*(A*b^2 + 2*A*b*c
+ A*c^2)*cosh(x)^2)*sinh(x)^2 + 4*((A*b^2 + 2*A*b*c + A*c^2)*cosh(x)^3 + (A
*b^2 - A*c^2)*cosh(x))*sinh(x))*sqrt(b^2 - c^2)*arctan(sqrt(b^2 - c^2)/((b
+ c)*cosh(x) + (b + c)*sinh(x))) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3)*cosh
(x) + (A*b^3 - A*b^2*c - A*b*c^2 + A*c^3 - 3*(A*b^3 + A*b^2*c - A*b*c^2 - A
*c^3)*cosh(x)^2 + 4*((B + C)*b^3 - (B + C)*b^2*c - (B + C)*b*c^2 + (B + C)*
c^3)*cosh(x))*sinh(x))/(b^6 - 2*b^5*c - b^4*c^2 + 4*b^3*c^3 - b^2*c^4 - 2*b
*c^5 + c^6 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6
)*cosh(x)^4 + 4*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 +
c^6)*cosh(x)*sinh(x)^3 + (b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2
*b*c^5 + c^6)*sinh(x)^4 + 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6)*cosh(x)^2 +
 2*(b^6 - 3*b^4*c^2 + 3*b^2*c^4 - c^6 + 3*(b^6 + 2*b^5*c - b^4*c^2 - 4*b^3*
c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^2)*sinh(x)^2 + 4*((b^6 + 2*b^5*c - b
^4*c^2 - 4*b^3*c^3 - b^2*c^4 + 2*b*c^5 + c^6)*cosh(x)^3 + (b^6 - 3*b^4*c^2
+ 3*b^2*c^4 - c^6)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.36

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{A\arctan\left(\frac{be^x+ce^x}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{\frac{3}{2}}} + \frac{Ab^2e^{(3x)}+2\,Abce^{(3x)}+Ac^2e^{(3x)}-2\,Bb^2e^{(2x)}-2\,Cb^2e^{(2x)}+2\,Bc^2e^{(2x)}+2\,Cc^2e^{(2x)}-Ab^2e^x+Ac^2e^x-}{(b^3+b^2c-bc^2-c^3)(be^{(2x)}+ce^{(2x)}+b-c)^2}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(b*cosh(x)+c*sinh(x))^3,x, algorithm="gia
c")
```

```
[Out] A*arctan((b*e^x + c*e^x)/sqrt(b^2 - c^2))/(b^2 - c^2)^(3/2) + (A*b^2*e^(3*x
) + 2*A*b*c*e^(3*x) + A*c^2*e^(3*x) - 2*B*b^2*e^(2*x) - 2*C*b^2*e^(2*x) + 2
*B*c^2*e^(2*x) + 2*C*c^2*e^(2*x) - A*b^2*e^x + A*c^2*e^x - 2*B*b^2 + 2*B*b*
c + 2*C*b*c - 2*C*c^2)/((b^3 + b^2*c - b*c^2 - c^3)*(b*e^(2*x) + c*e^(2*x)
+ b - c)^2)
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.66

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(b\cosh(x)+c\sinh(x))^3}\,dx = \frac{\mathrm{atan}\left(\frac{A\,\mathrm{e}^x\,\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}}{b^3\,\sqrt{A^2}+c^3\,\sqrt{A^2}-b\,c^2\,\sqrt{A^2}-b^2\,c\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^6-3\,b^4\,c^2+3\,b^2\,c^4-c^6}} - \frac{\frac{B-C}{b+c}+\frac{2\,A\,\mathrm{e}^x}{b+c}+\frac{\mathrm{e}^{2\,x}\,(B+C)}{b+c}}{\mathrm{e}^{4\,x}\,{(b+c)}^2+{(b-c)}^2+2\,\mathrm{e}^{2\,x}\,(b+c)\,(b-c)} - \frac{\frac{B+C}{{(b+c)}^2}-\frac{A\,\mathrm{e}^x}{(b+c)\,(b-c)}}{b-c+\mathrm{e}^{2\,x}\,(b+c)}$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] (atan((A*exp(x)*(b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2))/(b^3*(A^2)^(1/2)
 + c^3*(A^2)^(1/2) - b*c^2*(A^2)^(1/2) - b^2*c*(A^2)^(1/2)))*(A^2)^(1/2))/(
b^6 - c^6 + 3*b^2*c^4 - 3*b^4*c^2)^(1/2) - ((B - C)/(b + c) + (2*A*exp(x))/
(b + c) + (exp(2*x)*(B + C))/(b + c))/(exp(4*x)*(b + c)^2 + (b - c)^2 + 2*e
xp(2*x)*(b + c)*(b - c)) - ((B + C)/(b + c)^2 - (A*exp(x))/((b + c)*(b - c)
))/(b - c + exp(2*x)*(b + c))
```

3.739 $\int (a+b\cosh(x)+c\sinh(x))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 119

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{2}a\left(2a^2+3b^2-3c^2\right)x+\frac{1}{6}c\left(11a^2+4b^2-4c^2\right)\cosh(x) + \frac{1}{6}b\left(11a^2+4b^2-4c^2\right)\sinh(x) + \frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh(x)) + \frac{1}{3}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^2$$

```
[Out] 1/2*a*(2*a^2+3*b^2-3*c^2)*x+1/6*c*(11*a^2+4*b^2-4*c^2)*cosh(x)+1/6*b*(11*a^
2+4*b^2-4*c^2)*sinh(x)+5/6*(a*c*cosh(x)+a*b*sinh(x))*(a+b*cosh(x)+c*sinh(x)
)+1/3*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3199, 3225, 2717, 2718}

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{2}ax\left(2a^2+3b^2-3c^2\right)+\frac{1}{6}b\sinh(x)\left(11a^2+4b^2-4c^2\right) + \frac{1}{6}c\cosh(x)\left(11a^2+4b^2-4c^2\right) + \frac{1}{3}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^2 + \frac{5}{6}(ab\sinh(x)+ac\cosh(x))(a+b\cosh(x)+c\sinh(x))$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2 - 3*c^2)*x)/2 + (c*(11*a^2 + 4*b^2 - 4*c^2)*Cosh[x])/6 +
(b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a*b*Sinh[x])*(a
+ b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*S
inh[x])^2)/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^2 \\
&\quad+\frac{1}{3}\int (a+b\cosh(x)+c\sinh(x))\left(3a^2+2b^2-2c^2+5ab\cosh(x)+5ac\sinh(x)\right)\,dx \\
&= \frac{5}{6}(ac\cosh(x)+ab\sinh(x))(a+b\cosh(x)+c\sinh(x)) \\
&\quad+\frac{1}{3}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^2 \\
&\quad+\frac{\int \left(3a^2(2a^2+3b^2-3c^2)+ab(11a^2+4b^2-4c^2)\cosh(x)+ac(11a^2+4b^2-4c^2)\sinh(x)\right)\,dx}{6a}
\end{aligned}$$

$$= \frac{1}{2}a(2a^2 + 3b^2 - 3c^2)x + \frac{5}{6}(ac\cosh(x) + ab\sinh(x))(a + b\cosh(x) + c\sinh(x)) + \frac{1}{3}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x))^2 + \frac{1}{6}(b(11a^2 + 4b^2 - 4c^2))\int\cosh(x)\,dx + \frac{1}{6}(c(11a^2 + 4b^2 - 4c^2))\int\sinh(x)\,dx$$

$$= \frac{1}{2}a(2a^2 + 3b^2 - 3c^2)x + \frac{1}{6}c(11a^2 + 4b^2 - 4c^2)\cosh(x) + \frac{1}{6}b(11a^2 + 4b^2 - 4c^2)\sinh(x) + \frac{5}{6}(ac\cosh(x) + ab\sinh(x))(a + b\cosh(x) + c\sinh(x)) + \frac{1}{3}(c\cosh(x) + b\sinh(x))(a + b\cosh(x) + c\sinh(x))^2$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int (a + b\cosh(x) + c\sinh(x))^3\,dx = \frac{1}{12}\left(6a(2a^2 + 3b^2 - 3c^2)x + 9c(4a^2 + b^2 - c^2)\cosh(x) + 18abc\cosh(2x) + c(3b^2 + c^2)\cosh(3x) + 9b(4a^2 + b^2 - c^2)\sinh(x) + 9a(b^2 + c^2)\sinh(2x) + b(b^2 + 3c^2)\sinh(3x)\right)$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*b^2 - 3*c^2)*x + 9*c*(4*a^2 + b^2 - c^2)*Cosh[x] + 18*a*b*c
*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 9*b*(4*a^2 + b^2 - c^2)*Sinh[x] +
9*a*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])/12
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
parts	$a^3x + c^3\left(-\frac{2}{3} + \frac{\sinh(x)^2}{3}\right)\cosh(x) + c\,b^2\cosh(x)^3 + 3a\,b^2\left(\frac{\cosh(x)\sinh(x)}{2} + \frac{x}{2}\right) + \frac{b(c\sinh(x)+a)^3}{c} + b^3\left(\frac{2}{3}\right.$
default	$a^3x + 3\sinh(x)\,a^2b + 3c\,a^2\cosh(x) + 3a\,b^2\left(\frac{\cosh(x)\sinh(x)}{2} + \frac{x}{2}\right) + 3cab\cosh(x)^2 + 3a\,c^2\left(\frac{\cosh(x)\sinh}{2}\right.$
risch	$\frac{3a\,b^2e^{2x}}{8} + a^3x - \frac{3a\,c^2x}{2} + \frac{3a\,b^2x}{2} + \frac{e^{3x}b^3}{24} + \frac{3a^2b\,e^x}{2} + \frac{3b^3e^x}{8} - \frac{3\,e^{-2x}a\,b^2}{8} + \frac{e^{3x}c^2b}{8} + \frac{3\,e^{2x}a\,c^2}{8} + \frac{3\,e^xc\,a^2}{2} + \frac{3\,e^xc}{8}$

```
[In] int((a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+c*b^2*cosh(x)^3+3*a*b^2*(1/2*cosh(x)
*sinh(x)+1/2*x)+b*(c*sinh(x)+a)^3/c+b^3*(2/3+1/3*cosh(x)^2)*sinh(x)+3*c*a^2
*cosh(x)+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.34

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = \frac{3}{2}\,abc\cosh\left(x\right)^2+\frac{1}{12}\left(3\,b^2c+c^3\right)\cosh\left(x\right)^3+\frac{1}{12}\left(b^3+3\,bc^2\right)\sinh\left(x\right)^3 + \frac{1}{4}\left(6\,abc+\left(3\,b^2c+c^3\right)\cosh\left(x\right)\right)\sinh\left(x\right)^2 + \frac{1}{2}\left(2\,a^3+3\,ab^2-3\,ac^2\right)x-\frac{3}{4}\left(c^3-\left(4\,a^2+b^2\right)c\right)\cosh\left(x\right) + \frac{1}{4}\left(12\,a^2b+3\,b^3-3\,bc^2+\left(b^3+3\,bc^2\right)\cosh\left(x\right)^2+6\left(ab^2+ac^2\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
[Out] 3/2*a*b*c*cosh(x)^2 + 1/12*(3*b^2*c + c^3)*cosh(x)^3 + 1/12*(b^3 + 3*b*c^2)
*sinh(x)^3 + 1/4*(6*a*b*c + (3*b^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(2*a^3
 + 3*a*b^2 - 3*a*c^2)*x - 3/4*(c^3 - (4*a^2 + b^2)*c)*cosh(x) + 1/4*(12*a^2
*b + 3*b^3 - 3*b*c^2 + (b^3 + 3*b*c^2)*cosh(x)^2 + 6*(a*b^2 + a*c^2)*cosh(x
))*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.65

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = a^3x+3a^2b\sinh\left(x\right)+3a^2c\cosh\left(x\right)-\frac{3ab^2x\sinh^2\left(x\right)}{2}+\frac{3ab^2x\cosh^2\left(x\right)}{2}+\frac{3ab^2\sinh\left(x\right)\cosh\left(x\right)}{2}+3abc\cosh^2\left(x\right)+\frac{3ac^2x\sinh^2\left(x\right)}{2}-\frac{3ac^2x\cosh^2\left(x\right)}{2}+\frac{3ac^2\sinh\left(x\right)\cosh\left(x\right)}{2}-\frac{2b^3\sinh^3\left(x\right)}{3}+b^3\sinh\left(x\right)\cosh^2\left(x\right)+b^2c\cosh^3\left(x\right)+bc^2\sinh^3\left(x\right)+c^3\sinh^2\left(x\right)\cosh\left(x\right)-\frac{2c^3\cosh^3\left(x\right)}{3}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**3,x)
[Out] a**3*x + 3*a**2*b*sinh(x) + 3*a**2*c*cosh(x) - 3*a*b**2*x*sinh(x)**2/2 + 3*
a*b**2*x*cosh(x)**2/2 + 3*a*b**2*sinh(x)*cosh(x)/2 + 3*a*b*c*cosh(x)**2 + 3
*a*c**2*x*sinh(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + 3*a*c**2*sinh(x)*cosh(x)
/2 - 2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + b**2*c*cosh(x)**3 + b*
c**2*sinh(x)**3 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = b^2c\cosh(x)^3+bc^2\sinh(x)^3+a^3x+\frac{1}{24}c^3\left(e^{(3x)}-9e^{(-x)}+e^{(-3x)}-9e^{x}\right)+\frac{1}{24}b^3\left(e^{(3x)}-9e^{(-x)}-e^{(-3x)}+9e^{x}\right)+3(c\cosh(x)+b\sinh(x))a^2+\frac{3}{8}\left(8bc\cosh(x)^2+b^2\left(4x+e^{(2x)}-e^{(-2x)}\right)-c^2\left(4x-e^{(2x)}+e^{(-2x)}\right)\right)a$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
e^(-3*x) - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*(c
*cosh(x) + b*sinh(x))*a^2 + 3/8*(8*b*c*cosh(x)^2 + b^2*(4*x + e^(2*x) - e^(
-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(109) = 218.

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.84

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{24}b^3e^{(3x)}+\frac{1}{8}b^2ce^{(3x)}+\frac{1}{8}bc^2e^{(3x)}+\frac{1}{24}c^3e^{(3x)}+\frac{3}{8}ab^2e^{(2x)}+\frac{3}{4}abce^{(2x)}+\frac{3}{8}ac^2e^{(2x)}+\frac{3}{2}a^2be^{x}+\frac{3}{8}b^3e^{x}+\frac{3}{2}a^2ce^{x}+\frac{3}{8}b^2ce^{x}-\frac{3}{8}bc^2e^{x}-\frac{3}{8}c^3e^{x}+\frac{1}{2}\left(2a^3+3ab^2-3ac^2\right)x-\frac{1}{24}\left(b^3-3b^2c+3bc^2-c^3+9\left(4a^2b+b^3-4a^2c-b^2c-bc^2+c^3\right)e^{(2x)}+9\left(ab^2-2abc+ac^2\right)e^{x}\right)e^{(-3x)}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*x) + 1/8*b^2*c*e^(3*x) + 1/8*b*c^2*e^(3*x) + 1/24*c^3*e^(3*x)
 + 3/8*a*b^2*e^(2*x) + 3/4*a*b*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 3/2*a^2*b*e^
x + 3/8*b^3*e^x + 3/2*a^2*c*e^x + 3/8*b^2*c*e^x - 3/8*b*c^2*e^x - 3/8*c^3*e
^x + 1/2*(2*a^3 + 3*a*b^2 - 3*a*c^2)*x - 1/24*(b^3 - 3*b^2*c + 3*b*c^2 - c^
3 + 9*(4*a^2*b + b^3 - 4*a^2*c - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(a*b^2 -
2*a*b*c + a*c^2)*e^x)*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int (a+b\cosh(x)+c\sinh(x))^3\,dx = a^3\,x+\cosh(x)^3\left(b^2\,c-\frac{2\,c^3}{3}\right)+\sinh(x)^3\left(b\,c^2-\frac{2\,b^3}{3}\right)+b^3\cosh(x)^2\sinh(x)+c^3\cosh(x)\,\sinh(x)^2+3\,a^2\,c\cosh(x)+3\,a^2\,b\sinh(x)+\frac{3\,a\cosh(x)\,\sinh(x)\,\left(b^2+c^2\right)}{2}+\frac{3\,a\,x\cosh(x)^2\,\left(b^2-c^2\right)}{2}+3\,a\,b\,c\cosh(x)^2-\frac{3\,a\,x\sinh(x)^2\,\left(b^2-c^2\right)}{2}$$

```
[In] int((a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] a^3*x + cosh(x)^3*(b^2*c - (2*c^3)/3) + sinh(x)^3*(b*c^2 - (2*b^3)/3) + b^3
*cosh(x)^2*sinh(x) + c^3*cosh(x)*sinh(x)^2 + 3*a^2*c*cosh(x) + 3*a^2*b*sinh
(x) + (3*a*cosh(x)*sinh(x)*(b^2 + c^2))/2 + (3*a*x*cosh(x)^2*(b^2 - c^2))/2
 + 3*a*b*c*cosh(x)^2 - (3*a*x*sinh(x)^2*(b^2 - c^2))/2
```

3.740 $\int (a+b\cosh(x)+c\sinh(x))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{2}\left(2a^2+b^2-c^2\right)x+\frac{3}{2}ac\cosh(x)+\frac{3}{2}ab\sinh(x) + \frac{1}{2}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))$$

```
[Out] 1/2*(2*a^2+b^2-c^2)*x+3/2*a*c*cosh(x)+3/2*a*b*sinh(x)+1/2*(c*cosh(x)+b*sinh
(x))*(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3199, 2717, 2718}

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{2}x\left(2a^2+b^2-c^2\right) + \frac{1}{2}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x)) + \frac{3}{2}ab\sinh(x)+\frac{3}{2}ac\cosh(x)$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((2*a^2 + b^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a*b*Sinh[x])/2 + ((c*Cos
h[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x]))/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x)) \\
&\quad+\frac{1}{2}\int\left(2a^2+b^2-c^2+3ab\cosh(x)+3ac\sinh(x)\right)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2-c^2\right)x+\frac{1}{2}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x)) \\
&\quad+\frac{1}{2}(3ab)\int\cosh(x)\,dx+\frac{1}{2}(3ac)\int\sinh(x)\,dx \\
&= \frac{1}{2}\left(2a^2+b^2-c^2\right)x+\frac{3}{2}ac\cosh(x)+\frac{3}{2}ab\sinh(x) \\
&\quad+\frac{1}{2}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int(a+b\cosh(x)+c\sinh(x))^2\,dx=\frac{1}{4}\left(2\left(2a^2+b^2-c^2\right)x+8ac\cosh(x)+2bc\cosh(2x)+8ab\sinh(x)+\left(b^2+c^2\right)\sinh(2x)\right)$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2 - c^2)*x + 8*a*c*Cosh[x] + 2*b*c*Cosh[2*x] + 8*a*b*Sinh[x]
+ (b^2 + c^2)*Sinh[2*x])/4
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result
default	$a^{2}x+2ab\sinh\left(x\right)+2ac\cosh\left(x\right)+b^{2}\left(\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}+\frac{x}{2}\right)+cb\cosh\left(x\right)^{2}+c^{2}\left(\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}-\frac{x}{2}\right)$
parts	$a^{2}x+2b\left(\frac{c\sinh\left(x\right)^{2}}{2}+a\sinh\left(x\right)\right)+b^{2}\left(\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}+\frac{x}{2}\right)+c^{2}\left(\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}-\frac{x}{2}\right)+2ac\cosh\left(x\right)$
risch	$a^{2}x+\frac{b^{2}x}{2}-\frac{c^{2}x}{2}+\frac{b^{2}{\mathrm e}^{2x}}{8}+\frac{{\mathrm e}^{2x}cb}{4}+\frac{{\mathrm e}^{2x}c^{2}}{8}+b\,{\mathrm e}^{x}a+{\mathrm e}^{x}ac-{\mathrm e}^{-x}ab+{\mathrm e}^{-x}ac-\frac{{\mathrm e}^{-2x}b^{2}}{8}+\frac{{\mathrm e}^{-2x}cb}{4}-\frac{{\mathrm e}^{-2x}c^{2}}{8}$

```
[In] int((a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2*a*b*sinh(x)+2*a*c*cosh(x)+b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+c*b*cosh(
x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx=\frac{1}{2}\,bc\cosh\left(x\right)^{2}+\frac{1}{2}\,bc\sinh\left(x\right)^{2}+2\,ac\cosh\left(x\right)+\frac{1}{2}\,\left(2\,a^{2}+b^{2}-c^{2}\right)x+\frac{1}{2}\,\left(4\,ab+\left(b^{2}+c^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*c*cosh(x)^2 + 1/2*b*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(2*a^2 + b^2 -
c^2)*x + 1/2*(4*a*b + (b^2 + c^2)*cosh(x))*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx=a^{2}x+2ab\sinh\left(x\right)+2ac\cosh\left(x\right)-\frac{b^{2}x\sinh^{2}\left(x\right)}{2}+\frac{b^{2}x\cosh^{2}\left(x\right)}{2}+\frac{b^{2}\sinh\left(x\right)\cosh\left(x\right)}{2}+bc\cosh^{2}\left(x\right)+\frac{c^{2}x\sinh^{2}\left(x\right)}{2}-\frac{c^{2}x\cosh^{2}\left(x\right)}{2}+\frac{c^{2}\sinh\left(x\right)\cosh\left(x\right)}{2}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] a**2*x + 2*a*b*sinh(x) + 2*a*c*cosh(x) - b**2*x*sinh(x)**2/2 + b**2*x*cosh(
x)**2/2 + b**2*sinh(x)*cosh(x)/2 + b*c*cosh(x)**2 + c**2*x*sinh(x)**2/2 - c
**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = bc\cosh(x)^2 + \frac{1}{8}b^2\left(4x + e^{(2x)} - e^{(-2x)}\right) - \frac{1}{8}c^2\left(4x - e^{(2x)} + e^{(-2x)}\right) + a^2x + 2(c\cosh(x) + b\sinh(x))a$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + b*sinh(x))*a
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{8}b^2e^{(2x)} + \frac{1}{4}bce^{(2x)} + \frac{1}{8}c^2e^{(2x)} + abe^x + ace^x + \frac{1}{2}\left(2a^2 + b^2 - c^2\right)x - \frac{1}{8}\left(b^2 - 2bc + c^2 + 8(ab - ac)e^x\right)e^{(-2x)}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*e^(2*x) + 1/4*b*c*e^(2*x) + 1/8*c^2*e^(2*x) + a*b*e^x + a*c*e^x + 1
/2*(2*a^2 + b^2 - c^2)*x - 1/8*(b^2 - 2*b*c + c^2 + 8*(a*b - a*c)*e^x)*e^(-
2*x)
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a+b\cosh(x)+c\sinh(x))^2\,dx = x\,a^2+2\sinh(x)\;a\,b+2\,a\,c\cosh(x)+\frac{\sinh(x)\;b^2\cosh(x)}{2}+\frac{x\,b^2}{2}+b\,c\cosh(x)^2+\frac{\sinh(x)\;c^2\cosh(x)}{2}-\frac{x\,c^2}{2}$$

```
[In] int((a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] a^2*x + (b^2*x)/2 - (c^2*x)/2 + 2*a*b*sinh(x) + b*c*cosh(x)^2 + (b^2*cosh(x
)*sinh(x))/2 + (c^2*cosh(x)*sinh(x))/2 + 2*a*c*cosh(x)
```

3.741 $\int (a+b\cosh(x)+c\sinh(x))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 12

$$\int (a+b\cosh(x)+c\sinh(x))\,dx = ax+c\cosh(x)+b\sinh(x)$$

```
[Out] a*x+c*cosh(x)+b*sinh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2717, 2718}

$$\int (a+b\cosh(x)+c\sinh(x))\,dx = ax+b\sinh(x)+c\cosh(x)$$

```
[In] Int[a + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] a*x + c*Cosh[x] + b*Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= ax + b\int \cosh(x)\,dx + c\int \sinh(x)\,dx \\ &= ax + c\cosh(x) + b\sinh(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = ax + c\cosh(x) + b\sinh(x)$$

```
[In] Integrate[a + b*Cosh[x] + c*Sinh[x],x]
[Out] a*x + c*Cosh[x] + b*Sinh[x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$ax + c\cosh(x) + b\sinh(x)$	13
parts	$ax + c\cosh(x) + b\sinh(x)$	13
risch	$\frac{(b\,e^{2x} + e^{2x}c + 2ax\,e^{x} - b + c)\,e^{-x}}{2}$	30

```
[In] int(a+b*cosh(x)+c*sinh(x),x,method=_RETURNVERBOSE)
[Out] a*x+c*cosh(x)+b*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = ax + c\cosh(x) + b\sinh(x)$$

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="fricas")
[Out] a*x + c*cosh(x) + b*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = ax + b\sinh(x) + c\cosh(x)$$

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x)
[Out] a*x + b*sinh(x) + c*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = ax + c\cosh(x) + b\sinh(x)$$

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="maxima")
[Out] a*x + c*cosh(x) + b*sinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = ax + \frac{1}{2}\,c\left(e^{(-x)} + e^{x}\right) - \frac{1}{2}\,b\left(e^{(-x)} - e^{x}\right)$$

```
[In] integrate(a+b*cosh(x)+c*sinh(x),x, algorithm="giac")
[Out] a*x + 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + b\cosh(x) + c\sinh(x))\,dx = a\,x + c\cosh(x) + b\sinh(x)$$

```
[In] int(a + b*cosh(x) + c*sinh(x),x)
[Out] a*x + c*cosh(x) + b*sinh(x)
```

3.742 $\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[Out] -2*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3203, 632, 212}

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= -\left(4\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-1),x]
[Out] (2*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
 - c^2]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{2\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$	53
risch	$\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2+c^2}-a^2+b^2-c^2}{(b+c)\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2+c^2}+a^2-b^2+c^2}{(b+c)\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$	139

```
[In] int(1/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)
^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.86

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \left[\frac{\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+ac+(b^2+2\,bc+c^2)\cosh(x))\sinh(x)-2\,\sqrt{a^2-b^2+}}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)+a)\sinh(x)+b-c}\right.}{\sqrt{a^2-b^2+c^2}} \right.$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2
 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*c
osh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x
) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*c
osh(x) + a)*sinh(x) + b - c))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 - c^
2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a
^2 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{1}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{2\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{1}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a}{\sqrt{-a^2+b^2-c^2}} + \frac{b\,e^x}{\sqrt{-a^2+b^2-c^2}} + \frac{c\,e^x}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}}$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (2*atan(a/(b^2 - a^2 - c^2)^(1/2) + (b*exp(x))/(b^2 - a^2 - c^2)^(1/2) + (c
*exp(x))/(b^2 - a^2 - c^2)^(1/2)))/(b^2 - a^2 - c^2)^(1/2)
```

3.743 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2a\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*a*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(3/2)
+(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3208, 12, 3203, 632, 212}

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2a\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{b\sinh(x)+c\cosh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 +
c^2)^(3/2) - (c*Cosh[x] + b*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*
Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}+\frac{\int \frac{a}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}+\frac{a\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}+\frac{(2a)\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}
\end{aligned}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))} - \frac{(4a)\text{Subst}\left(\int \frac{1}{4(a^2-b^2+c^2)-x^2}\,dx, x, 2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}$$

$$= -\frac{2a\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{c\cosh(x)+b\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2a\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}} + \frac{-ac+(b^2-c^2)\sinh(x)}{b\,(-a^2+b^2-c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (-2*a*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2
- c^2)^(3/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*
Cosh[x] + c*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.

Time = 3.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

method	result
default	$-\frac{2\left(-\frac{\left(ab-b^2+c^2\right)\tanh\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-c^2b}-\frac{ac}{a^3-a^2b-a\,b^2+a\,c^2+b^3-c^2b}\right)}{\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2a\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}}$
risch	$\frac{2a\,\mathrm{e}^{x}+2b-2c}{(a^2-b^2+c^2)(b\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+b-c)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\left(a^2-b^2+c^2\right)^{\frac{3}{2}}-a^4+2a^2b^2-2c^2a^2-b^4+2b^2c^2-c^4}{(b+c)\left(a^2-b^2+c^2\right)^{\frac{3}{2}}}\right)}{(a^2-b^2+c^2)^{\frac{3}{2}}}-$ [illegible]

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(a*b-b^2+c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(1/2*x)-a*c/(a^3-a
^2*b-a*b^2+a*c^2+b^3-b*c^2))/(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b-2*c*tanh(1/2*
x)-a-b)-2*a/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2
*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 1268, normalized size of antiderivative = 14.09

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b - 2*b^3 + 2*b*c^2 - 2*c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)^
2 + (a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sinh(
x))*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c
 + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*
c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)
*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2
*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(a^2 - b^2)*c +
2*(a^3 - a*b^2 + a*c^2)*cosh(x) + 2*(a^3 - a*b^2 + a*c^2)*sinh(x))/(a^4*b -
 2*a^2*b^3 + b^5 + b*c^4 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 +
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3
)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^3 + b^5 + b
*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b
^4)*c)*sinh(x)^2 - (a^4 - 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*b^2 + a*b^4 +
 a*c^4 + 2*(a^3 - a*b^2)*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4
+ 2*(a^3 - a*b^2)*c^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b
^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x))*sinh(x)
), 2*(a^2*b - b^3 + b*c^2 - c^3 + (2*a^2*cosh(x) + (a*b + a*c)*cosh(x)^2 +
(a*b + a*c)*sinh(x)^2 + a*b - a*c + 2*(a^2 + (a*b + a*c)*cosh(x))*sinh(x))*
sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
+ c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2 - b^2)*c + (a^3 - a*b^2 + a*c^2
)*cosh(x) + (a^3 - a*b^2 + a*c^2)*sinh(x))/(a^4*b - 2*a^2*b^3 + b^5 + b*c^4
 - c^5 - 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4*b - 2*a^2*b^3 + b^5
 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2
 + b^4)*c)*cosh(x)^2 + (a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^
2)*c^3 + 2*(a^2*b - b^3)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*sinh(x)^2 - (a^4
- 2*a^2*b^2 + b^4)*c + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)
*c^2)*cosh(x) + 2*(a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 + 2*(a^3 - a*b^2)*c^2 +
(a^4*b - 2*a^2*b^3 + b^5 + b*c^4 + c^5 + 2*(a^2 - b^2)*c^3 + 2*(a^2*b - b^3
)*c^2 + (a^4 - 2*a^2*b^2 + b^4)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,a\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}} + \frac{2\,(ae^x+b-c)}{(a^2-b^2+c^2)(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c)}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2 + c^2)*s
qrt(-a^2 + b^2 - c^2)) + 2*(a*e^x + b - c)/((a^2 - b^2 + c^2)*(b*e^(2*x) +
c*e^(2*x) + 2*a*e^x + b - c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\int \frac{1}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^2,x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.744 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 146

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx=-\frac{(2a^2+b^2-c^2)\,\mathrm{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}-\frac{c\cosh(x)+b\sinh(x)}{2\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^2}-\frac{3(ac\cosh(x)+ab\sinh(x))}{2\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -(2*a^2+b^2-c^2)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^
2+c^2)^(5/2)+1/2*(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x
))^2-3/2*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3208, 3232, 3203, 632, 212}

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx=-\frac{(2a^2+b^2-c^2)\,\mathrm{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}-\frac{3(ab\sinh(x)+ac\cosh(x))}{2\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))}-\frac{b\sinh(x)+c\cosh(x)}{2\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^2}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] -((((2*a^2 + b^2 - c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2
]])/(a^2 - b^2 + c^2)^(5/2)) - (c*Cosh[x] + b*Sinh[x])/(2*(a^2 - b^2 + c^2)
*(a + b*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a*b*Sinh[x]))/(2*(a^2 -
 b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\text{integral} = -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{\int \frac{-2a+b\cosh(x)+c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(a^2-b^2+c^2\right)}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x))}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))} + \frac{\left(2a^2+b^2-c^2\right)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{2\left(a^2-b^2+c^2\right)^2}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x))}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))} + \frac{\left(2a^2+b^2-c^2\right)\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^2}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x))}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))} - \frac{\left(2\left(2a^2+b^2-c^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^2}$$

$$= -\frac{\left(2a^2+b^2-c^2\right)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{5/2}} - \frac{c\cosh(x)+b\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+ab\sinh(x))}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{1}{2}\left(\frac{2(2a^2+b^2-c^2)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}} + \frac{-ac+(b^2-c^2)\sinh(x)}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))^2} + \frac{c(2a^2+b^2-c^2)-3a(b^2-c^2)\sinh(x)}{b\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}\right)$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] ((2*(2*a^2 + b^2 - c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c
^2]])/(-a^2 + b^2 - c^2)^(5/2) + (-(a*c) + (b^2 - c^2)*Sinh[x])/(b*(-a^2 +
b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (c*(2*a^2 + b^2 - c^2) - 3*a*(b
^2 - c^2)*Sinh[x])/(b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(138) = 276.

Time = 21.91 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.95

method	result
default	$-\frac{2\left(-\frac{\left(4a^3b-7a^2b^2+5c^2a^2+2a\,b^3-2c^2ab+b^4-3b^2c^2+2c^4\right)\tanh\left(\frac{x}{2}\right)^3}{2(a-b)\left(a^4-2a^2b^2+2c^2a^2+b^4-2b^2c^2+c^4\right)}-\frac{c\left(4a^4-12a^3b+13a^2b^2-7c^2a^2-6a\,b^3+6c^2ab+b^4+b^2c^2-2c^4\right)\tanh\left(\frac{x}{2}\right)}{2\left(a^4-2a^2b^2+2c^2a^2+b^4-2b^2c^2+c^4\right)\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh(}$
risch	$\frac{2\,e^{3x}a^2b+2a^2c\,e^{3x}+e^{3x}b^3+e^{3x}c\,b^2-e^{3x}c^2b-e^{3x}c^3+6a^3e^{2x}+3a\,b^2e^{2x}-3\,e^{2x}a\,c^2+10a^2b\,e^{x}-10\,e^{x}c\,a^2-b^3e^{x}+e^{x}c\,b^2+e^{x}c^2b-e^{x}c^3+3}{(a^2-b^2+c^2)^2(b\,e^{2x}+e^{2x}c+2a\,e^{x}+b-c)^2}$

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(4*a^3*b-7*a^2*b^2+5*a^2*c^2+2*a*b^3-2*a*b*c^2+b^4-3*b^2*c^2+2*c^4
)/(a-b)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*c*(4*
a^4-12*a^3*b+13*a^2*b^2-7*a^2*c^2-6*a*b^3+6*a*b*c^2+b^4+b^2*c^2-2*c^4)/(a^4
-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(
4*a^4*b-5*a^3*b^2+11*a^3*c^2-3*a^2*b^3-3*a^2*b*c^2+5*a*b^4-7*a*b^2*c^2+2*a*
c^4-b^5-b^3*c^2+2*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tanh(1/2*x)+1/2*c*(4*a^4-3*a^2*b^2+a^2*c^2-b^4+b^2*c^2)/(a^4-2*a^
2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(tanh(1/2*x)^2*a-tanh(1
/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*a^2+b^2-c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b
```

```
^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)
/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3633 vs. $2(138) = 276$.

Time = 0.33 (sec) , antiderivative size = 7379, normalized size of antiderivative = 50.54

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^3} \, dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(138) = 276.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(2\,a^2+b^2-c^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)\sqrt{-a^2+b^2-c^2}} + \frac{2\,a^2be^{(3\,x)}+b^3e^{(3\,x)}+2\,a^2ce^{(3\,x)}+b^2ce^{(3\,x)}-bc^2e^{(3\,x)}-c^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,ab^2e^{(2\,x)}-3\,ac^2e^{(2\,x)}+10}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)(be^{(2\,x)}+ce^{(2}}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*a^2 + b^2 - c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2))
+ (2*a^2*b*e^(3*x) + b^3*e^(3*x) + 2*a^2*c*e^(3*x) + b^2*c*e^(3*x) - b*c^2*
e^(3*x) - c^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a*b^2*e^(2*x) - 3*a*c^2*e^(2*x) +
 10*a^2*b*e^x - b^3*e^x - 10*a^2*c*e^x + b^2*c*e^x + b*c^2*e^x - c^3*e^x +
3*a*b^2 - 6*a*b*c + 3*a*c^2)/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^
2 + c^4)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \int \frac{1}{{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)}^3}\,dx$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.745 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 220

$$\begin{aligned}\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx &= -\frac{a(2a^2+3b^2-3c^2)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{7/2}} \\ &\quad -\frac{c\cosh(x)+b\sinh(x)}{3\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^3} \\ &\quad -\frac{5(ac\cosh(x)+ab\sinh(x))}{6\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))^2} \\ &\quad -\frac{c(11a^2+4b^2-4c^2)\cosh(x)+b(11a^2+4b^2-4c^2)\sinh(x)}{6\,(a^2-b^2+c^2)^3\,(a+b\cosh(x)+c\sinh(x))}\end{aligned}$$

```
[Out] -a*(2*a^2+3*b^2-3*c^2)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(
a^2-b^2+c^2)^(7/2)+1/3*(-c*cosh(x)-b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*
sinh(x))^3-5/6*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sin
h(x))^2+1/6*(-c*(11*a^2+4*b^2-4*c^2)*cosh(x)-b*(11*a^2+4*b^2-4*c^2)*sinh(x)
)/(a^2-b^2+c^2)^3/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3208, 3235, 3232, 3203, 632, 212}

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx$$
$$=-\frac{a(2a^2+3b^2-3c^2)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{7/2}}$$
$$-\frac{b\sinh(x)\,(11a^2+4b^2-4c^2)+c\cosh(x)\,(11a^2+4b^2-4c^2)}{6\,(a^2-b^2+c^2)^3\,(a+b\cosh(x)+c\sinh(x))}$$
$$-\frac{5(ab\sinh(x)+ac\cosh(x))}{6\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))^2}$$
$$-\frac{b\sinh(x)+c\cosh(x)}{3\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^3}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2
 + c^2]])/(a^2 - b^2 + c^2)^(7/2)) - (c*Cosh[x] + b*Sinh[x])/(3*(a^2 - b^2
+ c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a*b*Sinh[x]))/(6*
(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2) - (c*(11*a^2 + 4*b^2 - 4
*c^2)*Cosh[x] + b*(11*a^2 + 4*b^2 - 4*c^2)*Sinh[x])/(6*(a^2 - b^2 + c^2)^3*
(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^3}-\frac{\int\frac{-3a+2b\cosh(x)+2c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx}{3\left(a^2-b^2+c^2\right)}\\ &= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^3}\\ &\quad-\frac{5(ac\cosh(x)+ab\sinh(x))}{6\left(a^2-b^2+c^2\right)^2\left(a+b\cosh(x)+c\sinh(x)\right)^2}\\ &\quad+\frac{\int\frac{2\left(3a^2+2b^2-2c^2\right)-5ab\cosh(x)-5ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{6\left(a^2-b^2+c^2\right)^2}\end{aligned}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^3} - \frac{5(ac\cosh(x)+ab\sinh(x))}{6\left(a^2-b^2+c^2\right)^2\left(a+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c(11a^2+4b^2-4c^2)\cosh(x)+b(11a^2+4b^2-4c^2)\sinh(x)}{6\left(a^2-b^2+c^2\right)^3\left(a+b\cosh(x)+c\sinh(x)\right)} + \frac{\left(a(2a^2+3b^2-3c^2)\right)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{2\left(a^2-b^2+c^2\right)^3}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^3} - \frac{5(ac\cosh(x)+ab\sinh(x))}{6\left(a^2-b^2+c^2\right)^2\left(a+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c(11a^2+4b^2-4c^2)\cosh(x)+b(11a^2+4b^2-4c^2)\sinh(x)}{6\left(a^2-b^2+c^2\right)^3\left(a+b\cosh(x)+c\sinh(x)\right)} + \frac{\left(a(2a^2+3b^2-3c^2)\right)\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^3}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^3} - \frac{5(ac\cosh(x)+ab\sinh(x))}{6\left(a^2-b^2+c^2\right)^2\left(a+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c(11a^2+4b^2-4c^2)\cosh(x)+b(11a^2+4b^2-4c^2)\sinh(x)}{6\left(a^2-b^2+c^2\right)^3\left(a+b\cosh(x)+c\sinh(x)\right)} - \frac{\left(2a(2a^2+3b^2-3c^2)\right)\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^3}$$

$$= -\frac{a(2a^2+3b^2-3c^2)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{7/2}} - \frac{c\cosh(x)+b\sinh(x)}{3\left(a^2-b^2+c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)^3} - \frac{5(ac\cosh(x)+ab\sinh(x))}{6\left(a^2-b^2+c^2\right)^2\left(a+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c(11a^2+4b^2-4c^2)\cosh(x)+b(11a^2+4b^2-4c^2)\sinh(x)}{6\left(a^2-b^2+c^2\right)^3\left(a+b\cosh(x)+c\sinh(x)\right)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. $2(220) = 440$.

Time = 0.63 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.22

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = -\frac{a(2a^2+3b^2-3c^2)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{7/2}} - \frac{-44a^5c-82a^3b^2c-24ab^4c+82a^3c^3+48ab^2c^3-24ac^5-30a^2bc(2a^2+3b^2-3c^2)\cosh(x)-6ac(a^2(-7}{}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -((a*(2*a^2 + 3*b^2 - 3*c^2)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^
2 - c^2]])/(-a^2 + b^2 - c^2)^(7/2)) - (-44*a^5*c - 82*a^3*b^2*c - 24*a*b^4
*c + 82*a^3*c^3 + 48*a*b^2*c^3 - 24*a*c^5 - 30*a^2*b*c*(2*a^2 + 3*b^2 - 3*c
^2)*Cosh[x] - 6*a*c*(a^2*(-7*b^2 + 11*c^2) + 2*(b^4 + b^2*c^2 - 2*c^4))*Cos
h[2*x] + 22*a^2*b^3*c*Cosh[3*x] + 8*b^5*c*Cosh[3*x] - 22*a^2*b*c^3*Cosh[3*x
] - 16*b^3*c^3*Cosh[3*x] + 8*b*c^5*Cosh[3*x] + 72*a^4*b^2*Sinh[x] - 9*a^2*b
^4*Sinh[x] + 12*b^6*Sinh[x] - 132*a^4*c^2*Sinh[x] - 72*a^2*b^2*c^2*Sinh[x]
- 36*b^4*c^2*Sinh[x] + 81*a^2*c^4*Sinh[x] + 36*b^2*c^4*Sinh[x] - 12*c^6*Sin
h[x] + 54*a^3*b^3*Sinh[2*x] + 6*a*b^5*Sinh[2*x] - 78*a^3*b*c^2*Sinh[2*x] -
48*a*b^3*c^2*Sinh[2*x] + 42*a*b*c^4*Sinh[2*x] + 11*a^2*b^4*Sinh[3*x] + 4*b^
6*Sinh[3*x] - 4*b^4*c^2*Sinh[3*x] - 11*a^2*c^4*Sinh[3*x] - 4*b^2*c^4*Sinh[3
*x] + 4*c^6*Sinh[3*x])/(24*b*(a^2 - b^2 + c^2)^3*(a + b*Cosh[x] + c*Sinh[x]
)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1587 vs. $2(212) = 424$.

Time = 118.96 (sec) , antiderivative size = 1588, normalized size of antiderivative = 7.22

method	result	size
risch	Expression too large to display	1588
default	Expression too large to display	1598

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(33*a^2*b*c^2+102*a^4*b*exp(x)^2+36*a^2*b^3*exp(x)^2+30*a^4*b*exp(x)^4+
45*a^2*b^3*exp(x)^4+8*b^2*c^3+15*exp(x)*a*b^4-12*b*c^4+8*b^3*c^2-11*c^3*a^2
+4*b^5+6*a^3*b^2*exp(x)^5+9*a*b^4*exp(x)^5+82*a^3*b^2*exp(x)^3+24*a*b^4*exp
(x)^3+60*a^3*b^2*exp(x)+11*a^2*b^3+12*b^5*exp(x)^2+4*c^5+44*a^5*exp(x)^3-12
*b^4*c-33*a^2*b^2*c-12*c^5*exp(x)^2-12*b^4*c*exp(x)^2-24*b^3*c^2*exp(x)^2-1
02*a^4*c*exp(x)^2-9*a*c^4*exp(x)^5+24*a*c^4*exp(x)^3+6*a^3*c^2*exp(x)^5+24*
```

```
b^2*c^3*exp(x)^2+36*a^2*c^3*exp(x)^2+12*b*c^4*exp(x)^2-82*a^3*c^2*exp(x)^3-
45*a^2*c^3*exp(x)^4+30*a^4*c*exp(x)^4+60*a^3*c^2*exp(x)-15*a*c^4*exp(x)+12*
a^3*b*c*exp(x)^5+18*a*b^3*c*exp(x)^5-18*a*b*c^3*exp(x)^5+45*a^2*b^2*c*exp(x
)^4-45*a^2*b*c^2*exp(x)^4-48*a*b^2*c^2*exp(x)^3-36*a^2*b^2*c*exp(x)^2-36*a^
2*b*c^2*exp(x)^2-120*a^3*b*c*exp(x)-30*a*b^3*c*exp(x)+30*a*b*c^3*exp(x))/(a
^2-b^2+c^2)^3/(b*exp(x)^2+exp(x)^2*c+2*a*exp(x)+b-c)^3+1/(a^2-b^2+c^2)^(7/2
)*a^3*ln(exp(x)+((a^2-b^2+c^2)^(7/2)*a-a^8+4*a^6*b^2-4*a^6*c^2-6*a^4*b^4+12
*a^4*b^2*c^2-6*a^4*c^4+4*a^2*b^6-12*a^2*b^4*c^2+12*a^2*b^2*c^4-4*a^2*c^6-b^
8+4*c^2*b^6-6*b^4*c^4+4*c^6*b^2-c^8)/(a^2-b^2+c^2)^(7/2)/(b+c))+3/2/(a^2-b^
2+c^2)^(7/2)*a*ln(exp(x)+((a^2-b^2+c^2)^(7/2)*a-a^8+4*a^6*b^2-4*a^6*c^2-6*a
^4*b^4+12*a^4*b^2*c^2-6*a^4*c^4+4*a^2*b^6-12*a^2*b^4*c^2+12*a^2*b^2*c^4-4*a
^2*c^6-b^8+4*c^2*b^6-6*b^4*c^4+4*c^6*b^2-c^8)/(a^2-b^2+c^2)^(7/2)/(b+c))*b^
2-3/2/(a^2-b^2+c^2)^(7/2)*a*ln(exp(x)+((a^2-b^2+c^2)^(7/2)*a-a^8+4*a^6*b^2-
4*a^6*c^2-6*a^4*b^4+12*a^4*b^2*c^2-6*a^4*c^4+4*a^2*b^6-12*a^2*b^4*c^2+12*a^
2*b^2*c^4-4*a^2*c^6-b^8+4*c^2*b^6-6*b^4*c^4+4*c^6*b^2-c^8)/(a^2-b^2+c^2)^(7
/2)/(b+c))*c^2-1/(a^2-b^2+c^2)^(7/2)*a^3*ln(exp(x)+((a^2-b^2+c^2)^(7/2)*a+a
^8-4*a^6*b^2+4*a^6*c^2+6*a^4*b^4-12*a^4*b^2*c^2+6*a^4*c^4-4*a^2*b^6+12*a^2*
b^4*c^2-12*a^2*b^2*c^4+4*a^2*c^6+b^8-4*c^2*b^6+6*b^4*c^4-4*c^6*b^2+c^8)/(a^
2-b^2+c^2)^(7/2)/(b+c))-3/2/(a^2-b^2+c^2)^(7/2)*a*ln(exp(x)+((a^2-b^2+c^2)^
(7/2)*a+a^8-4*a^6*b^2+4*a^6*c^2+6*a^4*b^4-12*a^4*b^2*c^2+6*a^4*c^4-4*a^2*b^
6+12*a^2*b^4*c^2-12*a^2*b^2*c^4+4*a^2*c^6+b^8-4*c^2*b^6+6*b^4*c^4-4*c^6*b^2
+c^8)/(a^2-b^2+c^2)^(7/2)/(b+c))*b^2+3/2/(a^2-b^2+c^2)^(7/2)*a*ln(exp(x)+((
a^2-b^2+c^2)^(7/2)*a+a^8-4*a^6*b^2+4*a^6*c^2+6*a^4*b^4-12*a^4*b^2*c^2+6*a^4
*c^4-4*a^2*b^6+12*a^2*b^4*c^2-12*a^2*b^2*c^4+4*a^2*c^6+b^8-4*c^2*b^6+6*b^4*
c^4-4*c^6*b^2+c^8)/(a^2-b^2+c^2)^(7/2)/(b+c))*c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11492 vs. $2(210) = 420$.

Time = 0.49 (sec) , antiderivative size = 23093, normalized size of antiderivative = 104.97

$$\int \frac{1}{(a + b\cosh(x) + c\sinh(x))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(210) = 420.

Time = 0.30 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.26

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx = \frac{(2a^3+3ab^2-3ac^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^6-3a^4b^2+3a^2b^4-b^6+3a^4c^2-6a^2b^2c^2+3b^4c^2+3a^2c^4-3b^2c^4+c^6)\sqrt{-a^2+b^2-c^2}} + \frac{6a^3b^2e^{(5x)}+9ab^4e^{(5x)}+12a^3bce^{(5x)}+18ab^3ce^{(5x)}+6a^3c^2e^{(5x)}-18abc^3e^{(5x)}-9ac^4e^{(5x)}+30a^4be^{(4x}}{}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
[Out] (2*a^3 + 3*a*b^2 - 3*a*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^
2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2 - 6*a^2*b^2*c^2 + 3*b^4
*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*sqrt(-a^2 + b^2 - c^2)) + 1/3*(6*a^3*b^
2*e^(5*x) + 9*a*b^4*e^(5*x) + 12*a^3*b*c*e^(5*x) + 18*a*b^3*c*e^(5*x) + 6*a
^3*c^2*e^(5*x) - 18*a*b*c^3*e^(5*x) - 9*a*c^4*e^(5*x) + 30*a^4*b*e^(4*x) +
45*a^2*b^3*e^(4*x) + 30*a^4*c*e^(4*x) + 45*a^2*b^2*c*e^(4*x) - 45*a^2*b*c^2
```

```
*e^(4*x) - 45*a^2*c^3*e^(4*x) + 44*a^5*e^(3*x) + 82*a^3*b^2*e^(3*x) + 24*a*
b^4*e^(3*x) - 82*a^3*c^2*e^(3*x) - 48*a*b^2*c^2*e^(3*x) + 24*a*c^4*e^(3*x)
+ 102*a^4*b*e^(2*x) + 36*a^2*b^3*e^(2*x) + 12*b^5*e^(2*x) - 102*a^4*c*e^(2*
x) - 36*a^2*b^2*c*e^(2*x) - 12*b^4*c*e^(2*x) - 36*a^2*b*c^2*e^(2*x) - 24*b^
3*c^2*e^(2*x) + 36*a^2*c^3*e^(2*x) + 24*b^2*c^3*e^(2*x) + 12*b*c^4*e^(2*x)
- 12*c^5*e^(2*x) + 60*a^3*b^2*e^x + 15*a*b^4*e^x - 120*a^3*b*c*e^x - 30*a*b
^3*c*e^x + 60*a^3*c^2*e^x + 30*a*b*c^3*e^x - 15*a*c^4*e^x + 11*a^2*b^3 + 4*
b^5 - 33*a^2*b^2*c - 12*b^4*c + 33*a^2*b*c^2 + 8*b^3*c^2 - 11*a^2*c^3 + 8*b
^2*c^3 - 12*b*c^4 + 4*c^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 3*a^4*c^2
- 6*a^2*b^2*c^2 + 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 + c^6)*(b*e^(2*x) + c*e
^(2*x) + 2*a*e^x + b - c)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^4}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{cosh}\left(x\right)+c\,\mathrm{sinh}\left(x\right)\right)}^4} \,d x$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^4,x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^4, x)
```

3.746 $\int (a+a\cosh(x)+c\sinh(x))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{2}a\left(5a^2-3c^2\right)x+\frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x) + \frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x) + \frac{5}{6}\left(ac\cosh(x)+a^2\sinh(x)\right)(a+a\cosh(x)+c\sinh(x)) + \frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2$$

```
[Out] 1/2*a*(5*a^2-3*c^2)*x+1/6*c*(15*a^2-4*c^2)*cosh(x)+1/6*a*(15*a^2-4*c^2)*sin
h(x)+5/6*(a*c*cosh(x)+a^2*sinh(x))*(a+a*cosh(x)+c*sinh(x))+1/3*(c*cosh(x)+a
*sinh(x))*(a+a*cosh(x)+c*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3199, 3225, 2717, 2718}

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{2}ax\left(5a^2-3c^2\right)+\frac{1}{6}a\left(15a^2-4c^2\right)\sinh(x) + \frac{1}{6}c\left(15a^2-4c^2\right)\cosh(x) + \frac{5}{6}\left(a^2\sinh(x)+ac\cosh(x)\right)(a\cosh(x)+a+c\sinh(x)) + \frac{1}{3}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))^2$$

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (a*(5*a^2 - 3*c^2)*x)/2 + (c*(15*a^2 - 4*c^2)*Cosh[x])/6 + (a*(15*a^2 - 4*c
^2)*Sinh[x])/6 + (5*(a*c*Cosh[x] + a^2*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])
)/6 + ((c*Cosh[x] + a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x])^2)/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2 \\
&\quad+\frac{1}{3}\int(a+a\cosh(x)+c\sinh(x))\left(5a^2-2c^2+5a^2\cosh(x)+5ac\sinh(x)\right)\,dx \\
&= \frac{5}{6}\left(ac\cosh(x)+a^2\sinh(x)\right)(a+a\cosh(x)+c\sinh(x)) \\
&\quad+\frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2 \\
&\quad+\frac{\int\left(3a^2(5a^2-3c^2)+a^2(15a^2-4c^2)\cosh(x)+ac(15a^2-4c^2)\sinh(x)\right)\,dx}{6a}
\end{aligned}
$$

$$= \frac{1}{2}a(5a^2-3c^2)x + \frac{5}{6}(ac\cosh(x)+a^2\sinh(x))(a+a\cosh(x)+c\sinh(x)) + \frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2 + \frac{1}{6}(a(15a^2-4c^2))\int\cosh(x)\,dx + \frac{1}{6}(c(15a^2-4c^2))\int\sinh(x)\,dx$$

$$= \frac{1}{2}a(5a^2-3c^2)x + \frac{1}{6}c(15a^2-4c^2)\cosh(x) + \frac{1}{6}a(15a^2-4c^2)\sinh(x) + \frac{5}{6}(ac\cosh(x)+a^2\sinh(x))(a+a\cosh(x)+c\sinh(x)) + \frac{1}{3}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))^2$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{12}(30a^3x-18ac^2x-9c(-5a^2+c^2)\cosh(x) + 18a^2c\cosh(2x)+3a^2c\cosh(3x)+c^3\cosh(3x) + 45a^3\sinh(x)-9ac^2\sinh(x)+9a^3\sinh(2x) + 9ac^2\sinh(2x)+a^3\sinh(3x)+3ac^2\sinh(3x))$$

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (30*a^3*x - 18*a*c^2*x - 9*c*(-5*a^2 + c^2)*Cosh[x] + 18*a^2*c*Cosh[2*x] +
3*a^2*c*Cosh[3*x] + c^3*Cosh[3*x] + 45*a^3*Sinh[x] - 9*a*c^2*Sinh[x] + 9*a^
3*Sinh[2*x] + 9*a*c^2*Sinh[2*x] + a^3*Sinh[3*x] + 3*a*c^2*Sinh[3*x])/12
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result
parts	$a^3x + c^3\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh(x) + c\,a^2\cosh(x)^3 + 3a^3\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right) + \frac{a(c\sinh(x)+a)^3}{c} + a^3\left(\frac{2}{3}\right.$
default	$a^3x + 3\sinh(x)\,a^3 + 3c\,a^2\cosh(x) + 3a^3\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right) + 3c\,a^2\cosh(x)^2 + 3a\,c^2\left(\frac{\cosh(x)\sinh(x)}{2}\right.$
risch	$\frac{5a^3x}{2} - \frac{3a\,c^2x}{2} + \frac{e^{3x}a^3}{24} + \frac{a^2c\,e^{3x}}{8} + \frac{e^{3x}a\,c^2}{8} + \frac{e^{3x}c^3}{24} + \frac{3a^3e^{2x}}{8} + \frac{3\,e^{2x}c\,a^2}{4} + \frac{3\,e^{2x}a\,c^2}{8} + \frac{15a^3e^x}{8} + \frac{15\,e^xc\,a^2}{8} - \frac{3a\,e^x}{8}$

```
[In] int((a+a*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+c*a^2*cosh(x)^3+3*a^3*(1/2*cosh(x)*s
inh(x)+1/2*x)+a*(c*sinh(x)+a)^3/c+a^3*(2/3+1/3*cosh(x)^2)*sinh(x)+3*c*a^2*c
osh(x)+3*a*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.37

$$\int (a + a\cosh(x) + c\sinh(x))^3\,dx = \frac{3}{2}\,a^2c\cosh{(x)}^2 + \frac{1}{12}\,\left(3\,a^2c + c^3\right)\cosh{(x)}^3 + \frac{1}{12}\,\left(a^3 + 3\,ac^2\right)\sinh{(x)}^3 + \frac{1}{4}\,\left(6\,a^2c + \left(3\,a^2c + c^3\right)\cosh{(x)}\right)\sinh{(x)}^2 + \frac{1}{2}\,\left(5\,a^3 - 3\,ac^2\right)x + \frac{3}{4}\,\left(5\,a^2c - c^3\right)\cosh{(x)} + \frac{1}{4}\,\left(15\,a^3 - 3\,ac^2 + \left(a^3 + 3\,ac^2\right)\cosh{(x)}^2 + 6\,\left(a^3 + ac^2\right)\cosh{(x)}\right)\sinh{(x)}$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 3/2*a^2*c*cosh(x)^2 + 1/12*(3*a^2*c + c^3)*cosh(x)^3 + 1/12*(a^3 + 3*a*c^2)
*sinh(x)^3 + 1/4*(6*a^2*c + (3*a^2*c + c^3)*cosh(x))*sinh(x)^2 + 1/2*(5*a^3
 - 3*a*c^2)*x + 3/4*(5*a^2*c - c^3)*cosh(x) + 1/4*(15*a^3 - 3*a*c^2 + (a^3
+ 3*a*c^2)*cosh(x)^2 + 6*(a^3 + a*c^2)*cosh(x))*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.80

$$\int (a + a\cosh(x) + c\sinh(x))^3\,dx = -\frac{3a^3x\sinh^2{(x)}}{2} + \frac{3a^3x\cosh^2{(x)}}{2} + a^3x - \frac{2a^3\sinh^3{(x)}}{3} + a^3\sinh{(x)}\cosh^2{(x)} + \frac{3a^3\sinh{(x)}\cosh{(x)}}{2} + 3a^3\sinh{(x)} + a^2c\cosh^3{(x)} + 3a^2c\cosh^2{(x)} + 3a^2c\cosh{(x)} + \frac{3ac^2x\sinh^2{(x)}}{2} - \frac{3ac^2x\cosh^2{(x)}}{2} + ac^2\sinh^3{(x)} + \frac{3ac^2\sinh{(x)}\cosh{(x)}}{2} + c^3\sinh^2{(x)}\cosh{(x)} - \frac{2c^3\cosh^3{(x)}}{3}$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] -3*a**3*x*sinh(x)**2/2 + 3*a**3*x*cosh(x)**2/2 + a**3*x - 2*a**3*sinh(x)**3
/3 + a**3*sinh(x)*cosh(x)**2 + 3*a**3*sinh(x)*cosh(x)/2 + 3*a**3*sinh(x) +
a**2*c*cosh(x)**3 + 3*a**2*c*cosh(x)**2 + 3*a**2*c*cosh(x) + 3*a*c**2*x*sin
h(x)**2/2 - 3*a*c**2*x*cosh(x)**2/2 + a*c**2*sinh(x)**3 + 3*a*c**2*sinh(x)*
cosh(x)/2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(x)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int (a + a\cosh(x) + c\sinh(x))^3\,dx = a^2c\cosh{(x)}^3 + ac^2\sinh{(x)}^3 + a^3x + \frac{1}{24}\,c^3\left(e^{(3\,x)} - 9\,e^{(-x)} + e^{(-3\,x)} - 9\,e^x\right) + \frac{1}{24}\,a^3\left(e^{(3\,x)} - 9\,e^{(-x)} - e^{(-3\,x)} + 9\,e^x\right) + 3\,(c\cosh{(x)} + a\sinh{(x)})a^2 + \frac{3}{8}\left(8\,ac\cosh{(x)}^2 + a^2\left(4\,x + e^{(2\,x)} - e^{(-2\,x)}\right) - c^2\left(4\,x - e^{(2\,x)} + e^{(-2\,x)}\right)\right)a$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] a^2*c*cosh(x)^3 + a*c^2*sinh(x)^3 + a^3*x + 1/24*c^3*(e^(3*x) - 9*e^(-x) +
e^(-3*x) - 9*e^x) + 1/24*a^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + 3*(c
*cosh(x) + a*sinh(x))*a^2 + 3/8*(8*a*c*cosh(x)^2 + a^2*(4*x + e^(2*x) - e^(
-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.77

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = \frac{1}{24}\,a^3e^{(3\,x)} + \frac{1}{8}\,a^2ce^{(3\,x)} + \frac{1}{8}\,ac^2e^{(3\,x)} + \frac{1}{24}\,c^3e^{(3\,x)} + \frac{3}{8}\,a^3e^{(2\,x)} + \frac{3}{4}\,a^2ce^{(2\,x)} + \frac{3}{8}\,ac^2e^{(2\,x)} + \frac{15}{8}\,a^3e^x + \frac{15}{8}\,a^2ce^x - \frac{3}{8}\,ac^2e^x - \frac{3}{8}\,c^3e^x + \frac{1}{2}\left(5\,a^3 - 3\,ac^2\right)x - \frac{1}{24}\left(a^3 - 3\,a^2c + 3\,ac^2 - c^3 + 9\left(5\,a^3 - 5\,a^2c - ac^2 + c^3\right)e^{(2\,x)} + 9\left(a^3 - 2\,a^2c + ac^2\right)e^x\right)e^{(-3\,x)}$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/24*a^3*e^(3*x) + 1/8*a^2*c*e^(3*x) + 1/8*a*c^2*e^(3*x) + 1/24*c^3*e^(3*x)
 + 3/8*a^3*e^(2*x) + 3/4*a^2*c*e^(2*x) + 3/8*a*c^2*e^(2*x) + 15/8*a^3*e^x +
 15/8*a^2*c*e^x - 3/8*a*c^2*e^x - 3/8*c^3*e^x + 1/2*(5*a^3 - 3*a*c^2)*x - 1
/24*(a^3 - 3*a^2*c + 3*a*c^2 - c^3 + 9*(5*a^3 - 5*a^2*c - a*c^2 + c^3)*e^(2
*x) + 9*(a^3 - 2*a^2*c + a*c^2)*e^x)*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int (a+a\cosh(x)+c\sinh(x))^3\,dx = 3\,a^3\sinh(x)+a^3\,x+\cosh(x)^3\left(a^2\,c-\frac{2\,c^3}{3}\right)+\sinh(x)^3\left(a\,c^2-\frac{2\,a^3}{3}\right)+a^3\cosh(x)^2\sinh(x)+c^3\cosh(x)\,\sinh(x)^2+3\,a^2\,c\cosh(x)+3\,a^2\,c\cosh(x)^2+\frac{3\,a\cosh(x)\,\sinh(x)\,\left(a^2+c^2\right)}{2}+\frac{3\,a\,x\cosh(x)^2\left(a^2-c^2\right)}{2}-\frac{3\,a\,x\sinh(x)^2\left(a^2-c^2\right)}{2}$$

```
[In] int((a + a*cosh(x) + c*sinh(x))^3,x)
```

```
[Out] 3*a^3*sinh(x) + a^3*x + cosh(x)^3*(a^2*c - (2*c^3)/3) + sinh(x)^3*(a*c^2 -
(2*a^3)/3) + a^3*cosh(x)^2*sinh(x) + c^3*cosh(x)*sinh(x)^2 + 3*a^2*c*cosh(x
) + 3*a^2*c*cosh(x)^2 + (3*a*cosh(x)*sinh(x)*(a^2 + c^2))/2 + (3*a*x*cosh(x
)^2*(a^2 - c^2))/2 - (3*a*x*sinh(x)^2*(a^2 - c^2))/2
```

3.747 $\int (a+a\cosh(x)+c\sinh(x))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (a+a\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{2}\left(3a^2-c^2\right)x+\frac{3}{2}ac\cosh(x)+\frac{3}{2}a^2\sinh(x) + \frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))$$

```
[Out] 1/2*(3*a^2-c^2)*x+3/2*a*c*cosh(x)+3/2*a^2*sinh(x)+1/2*(c*cosh(x)+a*sinh(x))
*(a+a*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3199, 2717, 2718}

$$\int (a+a\cosh(x)+c\sinh(x))^2\,dx = \frac{1}{2}x\left(3a^2-c^2\right)+\frac{3}{2}a^2\sinh(x)+\frac{3}{2}ac\cosh(x) + \frac{1}{2}(a\sinh(x)+c\cosh(x))(a\cosh(x)+a+c\sinh(x))$$

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] ((3*a^2 - c^2)*x)/2 + (3*a*c*Cosh[x])/2 + (3*a^2*Sinh[x])/2 + ((c*Cosh[x] +
 a*Sinh[x])*(a + a*Cosh[x] + c*Sinh[x]))/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x)) \\
&\quad+\frac{1}{2}\int \left(3a^2-c^2+3a^2\cosh(x)+3ac\sinh(x)\right)\,dx \\
&= \frac{1}{2}\left(3a^2-c^2\right)x+\frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x)) \\
&\quad+\frac{1}{2}\left(3a^2\right)\int \cosh(x)\,dx+\frac{1}{2}(3ac)\int \sinh(x)\,dx \\
&= \frac{1}{2}\left(3a^2-c^2\right)x+\frac{3}{2}ac\cosh(x)+\frac{3}{2}a^2\sinh(x) \\
&\quad+\frac{1}{2}(c\cosh(x)+a\sinh(x))(a+a\cosh(x)+c\sinh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\begin{aligned}
\int (a+a\cosh(x)+c\sinh(x))^2\,dx &= \frac{1}{2}\left(3a^2-c^2\right)x+2ac\cosh(x)+\frac{1}{2}ac\cosh(2x) \\
&\quad+2a^2\sinh(x)+\frac{1}{4}\left(a^2+c^2\right)\sinh(2x)
\end{aligned}$$

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^2,x]
[Out] ((3*a^2 - c^2)*x)/2 + 2*a*c*Cosh[x] + (a*c*Cosh[2*x])/2 + 2*a^2*Sinh[x] + (
(a^2 + c^2)*Sinh[2*x])/4
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result
default	$a^{2}x+2a^{2}\sinh\left(x\right)+2ac\cosh\left(x\right)+a^{2}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+ac\cosh\left(x\right)^{2}+c^{2}\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)$
parts	$a^{2}x+a^{2}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+2a\left(\frac{c\sinh(x)^{2}}{2}+a\sinh\left(x\right)\right)+c^{2}\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+2ac\cosh\left(x\right)$
risch	$\frac{3a^{2}x}{2}-\frac{c^{2}x}{2}+\frac{a^{2}{\rm e}^{2x}}{8}+\frac{{\rm e}^{2x}ac}{4}+\frac{{\rm e}^{2x}c^{2}}{8}+a^{2}{\rm e}^{x}+{\rm e}^{x}ac-{\rm e}^{-x}a^{2}+{\rm e}^{-x}ac-\frac{{\rm e}^{-2x}a^{2}}{8}+\frac{{\rm e}^{-2x}ac}{4}-\frac{{\rm e}^{-2x}c^{2}}{8}$

```
[In] int((a+a*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2*a^2*sinh(x)+2*a*c*cosh(x)+a^2*(1/2*cosh(x)*sinh(x)+1/2*x)+a*c*cosh(
x)^2+c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int(a+a\cosh(x)+c\sinh(x))^{2}\,dx=\frac{1}{2}\,ac\cosh\left(x\right)^{2}+\frac{1}{2}\,ac\sinh\left(x\right)^{2}+2\,ac\cosh\left(x\right)+\frac{1}{2}\left(3\,a^{2}-c^{2}\right)x+\frac{1}{2}\left(4\,a^{2}+\left(a^{2}+c^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*c*cosh(x)^2 + 1/2*a*c*sinh(x)^2 + 2*a*c*cosh(x) + 1/2*(3*a^2 - c^2)*x
 + 1/2*(4*a^2 + (a^2 + c^2)*cosh(x))*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int(a+a\cosh(x)+c\sinh(x))^{2}\,dx=-\frac{a^{2}x\sinh^{2}\left(x\right)}{2}+\frac{a^{2}x\cosh^{2}\left(x\right)}{2}+a^{2}x+\frac{a^{2}\sinh\left(x\right)\cosh\left(x\right)}{2}+2a^{2}\sinh\left(x\right)+ac\cosh^{2}\left(x\right)+2ac\cosh\left(x\right)+\frac{c^{2}x\sinh^{2}\left(x\right)}{2}-\frac{c^{2}x\cosh^{2}\left(x\right)}{2}+\frac{c^{2}\sinh\left(x\right)\cosh\left(x\right)}{2}$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] -a**2*x*sinh(x)**2/2 + a**2*x*cosh(x)**2/2 + a**2*x + a**2*sinh(x)*cosh(x)/
2 + 2*a**2*sinh(x) + a*c*cosh(x)**2 + 2*a*c*cosh(x) + c**2*x*sinh(x)**2/2 -
 c**2*x*cosh(x)**2/2 + c**2*sinh(x)*cosh(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int (a + a\cosh(x) + c\sinh(x))^2\,dx = ac\cosh{(x)}^2 + \frac{1}{8}\,a^2\big(4\,x + e^{(2\,x)} - e^{(-2\,x)}\big) - \frac{1}{8}\,c^2\big(4\,x - e^{(2\,x)} + e^{(-2\,x)}\big) + a^2x + 2\,(c\cosh{(x)} + a\sinh{(x)})a$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] a*c*cosh(x)^2 + 1/8*a^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + a^2*x + 2*(c*cosh(x) + a*sinh(x))*a
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int (a + a\cosh(x) + c\sinh(x))^2\,dx = \frac{1}{8}\,a^2e^{(2\,x)} + \frac{1}{4}\,ace^{(2\,x)} + \frac{1}{8}\,c^2e^{(2\,x)} + a^2e^x + ace^x + \frac{1}{2}\,\big(3\,a^2 - c^2\big)x - \frac{1}{8}\,\big(a^2 - 2\,ac + c^2 + 8\,\big(a^2 - ac\big)e^x\big)e^{(-2\,x)}$$

```
[In] integrate((a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*e^(2*x) + 1/4*a*c*e^(2*x) + 1/8*c^2*e^(2*x) + a^2*e^x + a*c*e^x + 1
/2*(3*a^2 - c^2)*x - 1/8*(a^2 - 2*a*c + c^2 + 8*(a^2 - a*c)*e^x)*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int (a + a\cosh(x) + c\sinh(x))^2\,dx = 2\,a^2\sinh(x) + \frac{3\,a^2\,x}{2} - \frac{c^2\,x}{2} + a\,c\cosh(x)^2 + \frac{a^2\cosh(x)\,\sinh(x)}{2} + \frac{c^2\cosh(x)\,\sinh(x)}{2} + 2\,a\,c\cosh(x)$$

```
[In] int((a + a*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] 2*a^2*sinh(x) + (3*a^2*x)/2 - (c^2*x)/2 + a*c*cosh(x)^2 + (a^2*cosh(x)*sinh
(x))/2 + (c^2*cosh(x)*sinh(x))/2 + 2*a*c*cosh(x)
```

3.748 $\int (a+a\cosh(x)+c\sinh(x))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 12

$$\int (a+a\cosh(x)+c\sinh(x))\,dx = ax+c\cosh(x)+a\sinh(x)$$

```
[Out] a*x+c*cosh(x)+a*sinh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2717, 2718}

$$\int (a+a\cosh(x)+c\sinh(x))\,dx = ax+a\sinh(x)+c\cosh(x)$$

```
[In] Int[a + a*Cosh[x] + c*Sinh[x],x]
```

```
[Out] a*x + c*Cosh[x] + a*Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + a\int \cosh(x)\,dx + c\int \sinh(x)\,dx\\ &= ax + c\cosh(x) + a\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + c\cosh(x) + a\sinh(x)$$

```
[In] Integrate[a + a*Cosh[x] + c*Sinh[x],x]
[Out] a*x + c*Cosh[x] + a*Sinh[x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$ax+c\cosh\left(x\right)+a\sinh\left(x\right)$	13
parts	$ax+c\cosh\left(x\right)+a\sinh\left(x\right)$	13
risch	$\frac{\left(a\,{\mathrm e}^{2x}+{\mathrm e}^{2x}c+2ax\,{\mathrm e}^{x}-a+c\right){\mathrm e}^{-x}}{2}$	30

```
[In] int(a+a*cosh(x)+c*sinh(x),x,method=_RETURNVERBOSE)
[Out] a*x+c*cosh(x)+a*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + c\cosh\left(x\right) + a\sinh\left(x\right)$$

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="fricas")
[Out] a*x + c*cosh(x) + a*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + a\sinh{(x)} + c\cosh{(x)}$$

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x)
[Out] a*x + a*sinh(x) + c*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + c\cosh{(x)} + a\sinh{(x)}$$

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="maxima")
[Out] a*x + c*cosh(x) + a*sinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = ax + \frac{1}{2}\,c\big(e^{(-x)} + e^{x}\big) - \frac{1}{2}\,a\big(e^{(-x)} - e^{x}\big)$$

```
[In] integrate(a+a*cosh(x)+c*sinh(x),x, algorithm="giac")
[Out] a*x + 1/2*c*(e^(-x) + e^x) - 1/2*a*(e^(-x) - e^x)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + a\cosh(x) + c\sinh(x))\,dx = a\,x + c\cosh(x) + a\sinh(x)$$

```
[In] int(a + a*cosh(x) + c*sinh(x),x)
[Out] a*x + c*cosh(x) + a*sinh(x)
```

3.749 $\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

```
[Out] ln(a+c*tanh(1/2*x))/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3203, 31}

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$$

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
```

```
[Out] Log[a + c*Tanh[x/2]]/c
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = -\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{c} + \frac{\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}{c}$$

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-1),x]
[Out] -(Log[Cosh[x/2]]/c) + Log[a*Cosh[x/2] + c*Sinh[x/2]]/c
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c}$	14
risch	$\frac{\ln\left(e^{x}+\frac{a-c}{a+c}\right)}{c}-\frac{\ln\left(e^{x}+1\right)}{c}$	31

```
[In] int(1/(a+a*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+c*tanh(1/2*x))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx = \frac{\log\left((a+c)\cosh\left(x\right)+(a+c)\sinh\left(x\right)+a-c\right)-\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{c}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] (log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - log(cosh(x) + sinh(x) + 1
))/c
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{a + a\cosh(x) + c\sinh(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{c} + \tanh\left(\frac{x}{2}\right)\right)}{c} & \text{for } c \neq 0 \\ \frac{\tanh\left(\frac{x}{2}\right)}{a} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x)
[Out] Piecewise((log(a/c + tanh(x/2))/c, Ne(c, 0)), (tanh(x/2)/a, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \frac{1}{a + a\cosh(x) + c\sinh(x)}\,dx = \frac{\log\left(-(a-c)e^{(-x)} - a - c\right)}{c} - \frac{\log\left(e^{(-x)} + 1\right)}{c}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="maxima")
[Out] log(-(a - c)*e^(-x) - a - c)/c - log(e^(-x) + 1)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{a + a\cosh(x) + c\sinh(x)}\,dx = \frac{(a+c)\log\left(|ae^x + ce^x + a - c|\right)}{ac + c^2} - \frac{\log\left(e^x + 1\right)}{c}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x)),x, algorithm="giac")
[Out] (a + c)*log(abs(a*e^x + c*e^x + a - c))/(a*c + c^2) - log(e^x + 1)/c
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int \frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx=-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-c^2}+a\,\mathrm{e}^x\,\sqrt{-c^2}+c\,\mathrm{e}^x\,\sqrt{-c^2}}{c^2}\right)}{\sqrt{-c^2}}$$

```
[In] int(1/(a + a*cosh(x) + c*sinh(x)),x)
[Out] -(2*atan((a*(-c^2)^(1/2) + a*exp(x)*(-c^2)^(1/2) + c*exp(x)*(-c^2)^(1/2))/c
^2))/(-c^2)^(1/2)
```

3.750 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx=\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}$$

```
[Out] a*ln(a+c*tanh(1/2*x))/c^3+(-c*cosh(x)-a*sinh(x))/c^2/(a+a*cosh(x)+c*sinh(x)
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3208, 12, 3203, 31}

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx=\frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{a\sinh(x)+c\cosh(x)}{c^2(a\cosh(x)+a+c\sinh(x))}$$

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (a*Log[a + c*Tanh[x/2]])/c^3 - (c*Cosh[x] + a*Sinh[x])/(c^2*(a + a*Cosh[x]
+ c*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{\int\frac{a}{a+a\cosh(x)+c\sinh(x)}\,dx}{c^2}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{a\int\frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx}{c^2}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}+\frac{(2a)\text{Subst}\left(\int\frac{1}{2a+2cx}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c^2}\\
&= \frac{a\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^3}-\frac{c\cosh(x)+a\sinh(x)}{c^2(a+a\cosh(x)+c\sinh(x))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int\frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx$$
$$=\frac{2a\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)\right)+\frac{c(-a^2+c^2)\sinh\left(\frac{x}{2}\right)}{a\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)}-c\tanh\left(\frac{x}{2}\right)}{2c^3}$$

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-2),x]
[Out] (2*a*(-Log[Cosh[x/2]] + Log[a*Cosh[x/2] + c*Sinh[x/2]]) + (c*(-a^2 + c^2)*S
inh[x/2])/(a*(a*Cosh[x/2] + c*Sinh[x/2])) - c*Tanh[x/2])/(2*c^3)
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2c^{2}}+\frac{a\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{c^{3}}-\frac{-a^{2}+c^{2}}{2c^{3}\left(a+c\tanh\left(\frac{x}{2}\right)\right)}$	49
risch	$\frac{2a\,\mathrm{e}^{x}+2a-2c}{c^{2}\left(a\,\mathrm{e}^{2x}+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+a-c\right)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a-c}{a+c}\right)}{c^{3}}-\frac{a\ln\left(\mathrm{e}^{x}+1\right)}{c^{3}}$	71

[In] int(1/(a+a*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] -1/2/c^2*tanh(1/2*x)+a*ln(a+c*tanh(1/2*x))/c^3-1/2/c^3*(-a^2+c^2)/(a+c*tanh(1/2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.49

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,ac\cosh\left(x\right)+2\,ac\sinh\left(x\right)+2\,ac-2\,c^2+\left(2\,a^2\cosh\left(x\right)+\left(a^2+ac\right)\cosh\left(x\right)^2+\left(a^2+ac\right)\sinh\left(x\right)^2+a^2-\right.}{}$$

[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")

[Out] (2*a*c*cosh(x) + 2*a*c*sinh(x) + 2*a*c - 2*c^2 + (2*a^2*cosh(x) + (a^2 + a*c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 + (a^2 + a*c)*cosh(x))*sinh(x))*log((a + c)*cosh(x) + (a + c)*sinh(x) + a - c) - (2*a^2*cosh(x) + (a^2 + a*c)*cosh(x)^2 + (a^2 + a*c)*sinh(x)^2 + a^2 - a*c + 2*(a^2 + (a^2 + a*c)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1))/(2*a*c^3*cosh(x) + a*c^3 - c^4 + (a*c^3 + c^4)*cosh(x)^2 + (a*c^3 + c^4)*sinh(x)^2 + 2*(a*c^3 + (a*c^3 + c^4)*cosh(x))*sinh(x))

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^2}\,dx = \text{Timed out}$$

[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**2,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^2}\,dx = -\frac{2\left(ae^{(-x)} + a + c\right)}{2\,ac^2e^{(-x)} + ac^2 + c^3 + (ac^2 - c^3)e^{(-2\,x)}} + \frac{a\log\left(-(a-c)e^{(-x)} - a - c\right)}{c^3} - \frac{a\log\left(e^{(-x)} + 1\right)}{c^3}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + a + c)/(2*a*c^2*e^(-x) + a*c^2 + c^3 + (a*c^2 - c^3)*e^(-2*x
)) + a*log(-(a - c)*e^(-x) - a - c)/c^3 - a*log(e^(-x) + 1)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^2}\,dx = \frac{(a^2 + ac)\log\left(|ae^x + ce^x + a - c|\right)}{ac^3 + c^4} - \frac{a\log\left(e^x + 1\right)}{c^3} + \frac{2\left(ae^x + a - c\right)}{(ae^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + a - c)c^2}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + a*c)*log(abs(a*e^x + c*e^x + a - c))/(a*c^3 + c^4) - a*log(e^x + 1)/
c^3 + 2*(a*e^x + a - c)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^2}\,dx = \int \frac{1}{{\left(a + a\cosh\left(x\right) + c\,\sinh\left(x\right)\right)}^2}\,dx$$

```
[In] int(1/(a + a*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int(1/(a + a*cosh(x) + c*sinh(x))^2, x)
```

3.751 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = \frac{(3a^2-c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a^2\sinh(x))}{2c^4(a+a\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*(3*a^2-c^2)*ln(a+c*tanh(1/2*x))/c^5+1/2*(-c*cosh(x)-a*sinh(x))/c^2/(a+a
*cosh(x)+c*sinh(x))^2-3/2*(a*c*cosh(x)+a^2*sinh(x))/c^4/(a+a*cosh(x)+c*sinh
(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3208, 3232, 3203, 31}

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = -\frac{3(a^2\sinh(x)+ac\cosh(x))}{2c^4(a\cosh(x)+a+c\sinh(x))} + \frac{(3a^2-c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{a\sinh(x)+c\cosh(x)}{2c^2(a\cosh(x)+a+c\sinh(x))^2}$$

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] ((3*a^2 - c^2)*Log[a + c*Tanh[x/2]])/(2*c^5) - (c*Cosh[x] + a*Sinh[x])/(2*c
^2*(a + a*Cosh[x] + c*Sinh[x])^2) - (3*(a*c*Cosh[x] + a^2*Sinh[x]))/(2*c^4*
(a + a*Cosh[x] + c*Sinh[x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2}-\frac{\int\frac{-2a+a\cosh(x)+c\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^2}\,dx}{2c^2}\\ &= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2}-\frac{3(ac\cosh(x)+a^2\sinh(x))}{2c^4(a+a\cosh(x)+c\sinh(x))}\\ &\quad+\frac{(3a^2-c^2)\int\frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx}{2c^4}\end{aligned}$$

$$= -\frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a^2\sinh(x))}{2c^4(a+a\cosh(x)+c\sinh(x))} + \frac{(3a^2-c^2)\,\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c^4}$$

$$= \frac{(3a^2-c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5} - \frac{c\cosh(x)+a\sinh(x)}{2c^2(a+a\cosh(x)+c\sinh(x))^2} - \frac{3(ac\cosh(x)+a^2\sinh(x))}{2c^4(a+a\cosh(x)+c\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = \frac{4(-3a^2+c^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)+4(3a^2-c^2)\log\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)-c^2\text{sech}^2\left(\frac{x}{2}\right)+\frac{(a-c)c^2(a+c)}{\left(a\cosh\left(\frac{x}{2}\right)+c\sinh\left(\frac{x}{2}\right)\right)^2}}{8c^5}$$

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (4*(-3*a^2 + c^2)*Log[Cosh[x/2]] + 4*(3*a^2 - c^2)*Log[a*Cosh[x/2] + c*Sinh
[x/2]] - c^2*Sech[x/2]^2 + ((a - c)*c^2*(a + c))/(a*Cosh[x/2] + c*Sinh[x/2]
)^2 + (6*c*(-a^2 + c^2)*Sinh[x/2])/(a*Cosh[x/2] + c*Sinh[x/2]) - 6*a*c*Tanh
[x/2])/(8*c^5)
```

Maple [A] (verified)

Time = 11.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
default	$-\frac{-\frac{\tanh\left(\frac{x}{2}\right)^2 c}{2}+3a\tanh\left(\frac{x}{2}\right)}{4c^4}+\frac{(6a^2-2c^2)\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{4c^5}-\frac{a^4-2c^2a^2+c^4}{8c^5\left(a+c\tanh\left(\frac{x}{2}\right)\right)^2}+\frac{a(a^2-c^2)}{c^5\left(a+c\tanh\left(\frac{x}{2}\right)\right)}$
risch	$\frac{3\,e^{3x}a^3+3a^2c\,e^{3x}-e^{3x}a\,c^2-e^{3x}c^3+9a^3e^{2x}-3\,e^{2x}a\,c^2+9a^3e^{x}-9\,e^{x}c\,a^2+a\,e^{x}c^2-e^{x}c^3+3a^3-6c\,a^2+3a\,c^2}{(a\,e^{2x}+e^{2x}c+2a\,e^{x}+a-c)^2c^4}-\frac{3\ln(e^{x}+1)a^2}{2c^5}+\frac{\ln(e^{x}+}{2c^3}$

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c^4*(-1/2*tanh(1/2*x)^2*c+3*a*tanh(1/2*x))+1/4*(6*a^2-2*c^2)/c^5*ln(a+
c*tanh(1/2*x))-1/8/c^5*(a^4-2*a^2*c^2+c^4)/(a+c*tanh(1/2*x))^2+a/c^5*(a^2-c
^2)/(a+c*tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1504 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 1504, normalized size of antiderivative = 16.90

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*c - 12*a^2*c^2 + 6*a*c^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)
*cosh(x)^3 + 2*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*sinh(x)^3 + 6*(3*a^3*c -
 a*c^3)*cosh(x)^2 + 6*(3*a^3*c - a*c^3 + (3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4
)*cosh(x))*sinh(x)^2 + 2*(9*a^3*c - 9*a^2*c^2 + a*c^3 - c^4)*cosh(x) + ((3*
a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2
*a^2*c^2 - 2*a*c^3 - c^4)*sinh(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3
 - c^4 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3
*c - a^2*c^2 - a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x
))*sinh(x)^3 + 2*(9*a^4 - 6*a^2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2
 + c^4 + 3*(3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^2 + 6*(3*a
^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x))*sinh(x)^2 + 4*(3*a^4 - 3*a^3*c - a
^2*c^2 + a*c^3)*cosh(x) + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3 + (3*a^4 + 6
*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a^3*c - a^2*c^
2 - a*c^3)*cosh(x)^2 + (9*a^4 - 6*a^2*c^2 + c^4)*cosh(x))*sinh(x))*log((a +
 c)*cosh(x) + (a + c)*sinh(x) + a - c) - ((3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*
a*c^3 - c^4)*cosh(x)^4 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*sinh
(x)^4 + 3*a^4 - 6*a^3*c + 2*a^2*c^2 + 2*a*c^3 - c^4 + 4*(3*a^4 + 3*a^3*c -
a^2*c^2 - a*c^3)*cosh(x)^3 + 4*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3 + (3*a^4
+ 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x))*sinh(x)^3 + 2*(9*a^4 - 6*a^
2*c^2 + c^4)*cosh(x)^2 + 2*(9*a^4 - 6*a^2*c^2 + c^4 + 3*(3*a^4 + 6*a^3*c +
2*a^2*c^2 - 2*a*c^3 - c^4)*cosh(x)^2 + 6*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3
)*cosh(x))*sinh(x)^2 + 4*(3*a^4 - 3*a^3*c - a^2*c^2 + a*c^3)*cosh(x) + 4*(3
*a^4 - 3*a^3*c - a^2*c^2 + a*c^3 + (3*a^4 + 6*a^3*c + 2*a^2*c^2 - 2*a*c^3 -
 c^4)*cosh(x)^3 + 3*(3*a^4 + 3*a^3*c - a^2*c^2 - a*c^3)*cosh(x)^2 + (9*a^4
- 6*a^2*c^2 + c^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + 2*(9*a^3*
c - 9*a^2*c^2 + a*c^3 - c^4 + 3*(3*a^3*c + 3*a^2*c^2 - a*c^3 - c^4)*cosh(x)
^2 + 6*(3*a^3*c - a*c^3)*cosh(x))*sinh(x))/(a^2*c^5 - 2*a*c^6 + c^7 + (a^2*
c^5 + 2*a*c^6 + c^7)*cosh(x)^4 + (a^2*c^5 + 2*a*c^6 + c^7)*sinh(x)^4 + 4*(a
^2*c^5 + a*c^6)*cosh(x)^3 + 4*(a^2*c^5 + a*c^6 + (a^2*c^5 + 2*a*c^6 + c^7)*
cosh(x))*sinh(x)^3 + 2*(3*a^2*c^5 - c^7)*cosh(x)^2 + 2*(3*a^2*c^5 - c^7 + 3
*(a^2*c^5 + 2*a*c^6 + c^7)*cosh(x)^2 + 6*(a^2*c^5 + a*c^6)*cosh(x))*sinh(x)
^2 + 4*(a^2*c^5 - a*c^6)*cosh(x) + 4*(a^2*c^5 - a*c^6 + (a^2*c^5 + 2*a*c^6
+ c^7)*cosh(x)^3 + 3*(a^2*c^5 + a*c^6)*cosh(x)^2 + (3*a^2*c^5 - c^7)*cosh(x
))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.79

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = -\frac{3\,a^3+6\,a^2c+3\,ac^2+(9\,a^3+9\,a^2c+ac^2+c^3)e^{(-x)}+3\,(3\,a^3-ac^2)e^{(-2\,x)}+(3\,a^3-3\,a^2c-ac^2+c^3)e}{a^2c^4+2\,ac^5+c^6+4\,(a^2c^4+ac^5)e^{(-x)}+2\,(3\,a^2c^4-c^6)e^{(-2\,x)}+4\,(a^2c^4-ac^5)e^{(-3\,x)}+(a^2c^4-2\,ac^5+c^6}+\frac{(3\,a^2-c^2)\log\left(-(a-c)e^{(-x)}-a-c\right)}{2\,c^5}-\frac{(3\,a^2-c^2)\log\left(e^{(-x)}+1\right)}{2\,c^5}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^3 + 6*a^2*c + 3*a*c^2 + (9*a^3 + 9*a^2*c + a*c^2 + c^3)*e^(-x) + 3*(3
*a^3 - a*c^2)*e^(-2*x) + (3*a^3 - 3*a^2*c - a*c^2 + c^3)*e^(-3*x))/(a^2*c^4
 + 2*a*c^5 + c^6 + 4*(a^2*c^4 + a*c^5)*e^(-x) + 2*(3*a^2*c^4 - c^6)*e^(-2*x
) + 4*(a^2*c^4 - a*c^5)*e^(-3*x) + (a^2*c^4 - 2*a*c^5 + c^6)*e^(-4*x)) + 1/
2*(3*a^2 - c^2)*log(-(a - c)*e^(-x) - a - c)/c^5 - 1/2*(3*a^2 - c^2)*log(e^
(-x) + 1)/c^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(81) = 162$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx = \frac{(3\,a^3+3\,a^2c-ac^2-c^3)\log\left(|ae^x+ce^x+a-c|\right)}{2\,(ac^5+c^6)}-\frac{(3\,a^2-c^2)\log\left(e^x+1\right)}{2\,c^5}+\frac{3\,a^3e^{(3\,x)}+3\,a^2ce^{(3\,x)}-ac^2e^{(3\,x)}-c^3e^{(3\,x)}+9\,a^3e^{(2\,x)}-3\,ac^2e^{(2\,x)}+9\,a^3e^x-9\,a^2ce^x+ac^2e^x-c^3e^x+3\,a}{\left(ae^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+a-c\right)^2c^4}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^3 + 3*a^2*c - a*c^2 - c^3)*log(abs(a*e^x + c*e^x + a - c))/(a*c^5
+ c^6) - 1/2*(3*a^2 - c^2)*log(e^x + 1)/c^5 + (3*a^3*e^(3*x) + 3*a^2*c*e^(3
*x) - a*c^2*e^(3*x) - c^3*e^(3*x) + 9*a^3*e^(2*x) - 3*a*c^2*e^(2*x) + 9*a^3
*e^x - 9*a^2*c*e^x + a*c^2*e^x - c^3*e^x + 3*a^3 - 6*a^2*c + 3*a*c^2)/((a*e
^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^2*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^3}\,dx=\int \frac{1}{\left(a+a\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

```
[In] int(1/(a + a*cosh(x) + c*sinh(x))^3,x)
[Out] int(1/(a + a*cosh(x) + c*sinh(x))^3, x)
```

3.752 $\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 140

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = \frac{a(5a^2-3c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7} - \frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3} - \frac{5(ac\cosh(x)+a^2\sinh(x))}{6c^4(a+a\cosh(x)+c\sinh(x))^2} - \frac{c(15a^2-4c^2)\cosh(x)+a(15a^2-4c^2)\sinh(x)}{6c^6(a+a\cosh(x)+c\sinh(x))}$$

```
[Out] 1/2*a*(5*a^2-3*c^2)*ln(a+c*tanh(1/2*x))/c^7+1/3*(-c*cosh(x)-a*sinh(x))/c^2/
(a+a*cosh(x)+c*sinh(x))^3-5/6*(a*c*cosh(x)+a^2*sinh(x))/c^4/(a+a*cosh(x)+c*
sinh(x))^2+1/6*(-c*(15*a^2-4*c^2)*cosh(x)-a*(15*a^2-4*c^2)*sinh(x))/c^6/(a+
a*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used

= {3208, 3235, 3232, 3203, 31}

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = -\frac{5(a^2\sinh(x)+ac\cosh(x))}{6c^4(a\cosh(x)+a+c\sinh(x))^2} + \frac{a(5a^2-3c^2)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7} - \frac{a(15a^2-4c^2)\sinh(x)+c(15a^2-4c^2)\cosh(x)}{6c^6(a\cosh(x)+a+c\sinh(x))} - \frac{a\sinh(x)+c\cosh(x)}{3c^2(a\cosh(x)+a+c\sinh(x))^3}$$

```
[In] Int[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (a*(5*a^2 - 3*c^2)*Log[a + c*Tanh[x/2]])/(2*c^7) - (c*Cosh[x] + a*Sinh[x])/
(3*c^2*(a + a*Cosh[x] + c*Sinh[x])^3) - (5*(a*c*Cosh[x] + a^2*Sinh[x]))/(6*
c^4*(a + a*Cosh[x] + c*Sinh[x])^2) - (c*(15*a^2 - 4*c^2)*Cosh[x] + a*(15*a^
2 - 4*c^2)*Sinh[x])/(6*c^6*(a + a*Cosh[x] + c*Sinh[x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
```

```
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3}-\frac{\int\frac{-3a+2a\cosh(x)+2c\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^3}\,dx}{3c^2}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3}-\frac{5(ac\cosh(x)+a^2\sinh(x))}{6c^4(a+a\cosh(x)+c\sinh(x))^2}\\
&\quad+\frac{\int\frac{2(5a^2-2c^2)-5a^2\cosh(x)-5ac\sinh(x)}{(a+a\cosh(x)+c\sinh(x))^2}\,dx}{6c^4}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3}-\frac{5(ac\cosh(x)+a^2\sinh(x))}{6c^4(a+a\cosh(x)+c\sinh(x))^2}\\
&\quad-\frac{c(15a^2-4c^2)\cosh(x)+a(15a^2-4c^2)\sinh(x)}{6c^6(a+a\cosh(x)+c\sinh(x))}\\
&\quad-\frac{\left(a\left(3-\frac{5a^2}{c^2}\right)\right)\int\frac{1}{a+a\cosh(x)+c\sinh(x)}\,dx}{2c^4}\\
&= -\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3}-\frac{5(ac\cosh(x)+a^2\sinh(x))}{6c^4(a+a\cosh(x)+c\sinh(x))^2}\\
&\quad-\frac{c(15a^2-4c^2)\cosh(x)+a(15a^2-4c^2)\sinh(x)}{6c^6(a+a\cosh(x)+c\sinh(x))}\\
&\quad-\frac{\left(a\left(3-\frac{5a^2}{c^2}\right)\right)\operatorname{Subst}\left(\int\frac{1}{2a+2cx}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c^4}\\
&= -\frac{a\left(3-\frac{5a^2}{c^2}\right)\log\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^5}-\frac{c\cosh(x)+a\sinh(x)}{3c^2(a+a\cosh(x)+c\sinh(x))^3}\\
&\quad-\frac{5(ac\cosh(x)+a^2\sinh(x))}{6c^4(a+a\cosh(x)+c\sinh(x))^2}-\frac{c(15a^2-4c^2)\cosh(x)+a(15a^2-4c^2)\sinh(x)}{6c^6(a+a\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(140) = 280$.

Time = 0.41 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.14

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^4}\,dx$$
$$= \frac{192(-5a^3 + 3ac^2)\log\left(\cosh\left(\frac{x}{2}\right)\right) + 192a(5a^2 - 3c^2)\log\left(a\cosh\left(\frac{x}{2}\right) + c\sinh\left(\frac{x}{2}\right)\right) - \frac{c\operatorname{sech}^6\left(\frac{x}{2}\right)(-150a^5c + 130a^3}{}}{}$$

```
[In] Integrate[(a + a*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (192*(-5*a^3 + 3*a*c^2)*Log[Cosh[x/2]] + 192*a*(5*a^2 - 3*c^2)*Log[a*Cosh[x
/2] + c*Sinh[x/2]] - (c*Sech[x/2]^6*(-150*a^5*c + 130*a^3*c^3 - 24*a*c^5 +
(-75*a^5*c + 75*a^3*c^3 + 12*a*c^5)*Cosh[x] + 6*a*c*(25*a^4 - 15*a^2*c^2 +
4*c^4)*Cosh[2*x] + 75*a^5*c*Cosh[3*x] - 35*a^3*c^3*Cosh[3*x] + 4*a*c^5*Cosh
[3*x] + 150*a^6*Sinh[x] - 255*a^4*c^2*Sinh[x] + 129*a^2*c^4*Sinh[x] - 12*c^
6*Sinh[x] + 120*a^6*Sinh[2*x] - 72*a^4*c^2*Sinh[2*x] + 36*a^2*c^4*Sinh[2*x]
 + 30*a^6*Sinh[3*x] + 37*a^4*c^2*Sinh[3*x] - 27*a^2*c^4*Sinh[3*x] + 4*c^6*S
inh[3*x]))/(a*(a + c*Tanh[x/2])^3))/(384*c^7)
```

Maple [A] (verified)

Time = 44.91 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\frac{c^2\tanh\left(\frac{x}{2}\right)^3}{3}-2ac\tanh\left(\frac{x}{2}\right)^2+10a^2\tanh\left(\frac{x}{2}\right)-3c^2\tanh\left(\frac{x}{2}\right)}{8c^6}+\frac{a(5a^2-3c^2)\ln\left(a+c\tanh\left(\frac{x}{2}\right)\right)}{2c^7}-\frac{3a(a^4-2c^2a^2+c^4)}{8c^7\left(a+c\tanh\left(\frac{x}{2}\right)\right)^2}-\frac{-a^6+3}{24c^7(}$
risch	$\frac{41a^3c^2+15a^5-3c^3a^2+4c^5+75\,e^{x}a^5+15\,e^{5x}a^5+150\,e^{3x}a^5+75a^5e^{4x}+150a^5e^{2x}-12a\,c^4-45a^4c+60a^2c^3e^{2x}-130a^3c^2e^{3x}-45a^2c^3e^{4x}}{}$

```
[In] int(1/(a+a*cosh(x)+c*sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c^6*(1/3*c^2*tanh(1/2*x)^3-2*a*c*tanh(1/2*x)^2+10*a^2*tanh(1/2*x)-3*c^
2*tanh(1/2*x))+1/2*a*(5*a^2-3*c^2)*ln(a+c*tanh(1/2*x))/c^7-3/8*a/c^7*(a^4-2
*a^2*c^2+c^4)/(a+c*tanh(1/2*x))^2-1/24/c^7*(-a^6+3*a^4*c^2-3*a^2*c^4+c^6)/(
a+c*tanh(1/2*x))^3-1/8*(-15*a^4+18*a^2*c^2-3*c^4)/c^7/(a+c*tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4015 vs. $2(130) = 260$.

Time = 0.30 (sec) , antiderivative size = 4015, normalized size of antiderivative = 28.68

$$\int \frac{1}{(a + a\cosh(x) + c\sinh(x))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(30*a^5*c - 90*a^4*c^2 + 82*a^3*c^3 - 6*a^2*c^4 - 24*a*c^5 + 8*c^6 + 6*
(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 6*(5*a
^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3*a*c^5)*sinh(x)^5 + 30*(5*a^5*
c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 30*(5*a^5*c + 5*a^4*c^2
- 3*a^3*c^3 - 3*a^2*c^4 + (5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3
*a*c^5)*cosh(x))*sinh(x)^4 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)^3
 + 4*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5 + 15*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c
^3 - 6*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 30*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 -
 3*a^2*c^4)*cosh(x))*sinh(x)^3 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^3*c^3 + 1
0*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x)^2 + 12*(25*a^5*c - 25*a^4*c^2 - 10*a^3
*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6 + 5*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 -
 6*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a
^2*c^4)*cosh(x)^2 + (75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x))*sinh(x)^2 +
 30*(5*a^5*c - 10*a^4*c^2 + 4*a^3*c^3 + 2*a^2*c^4 - a*c^5)*cosh(x) + 3*((5*
a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^6 +
(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*sinh(x)^6
 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3 - 9*a^2*c^4 + 3*a*c^5 + 6*(5*a
^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^5 + 6*(5*a^6 + 1
0*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^
2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x)^5 + 3*(25*a^6 + 25*a^
5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^4 + 3*(25*a^6
+ 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5 + 5*(5*a^6 + 15*
a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 10*(5*a^6
 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x))*sinh(x)^4 + 4*(25
*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^3 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c
^4 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*co
sh(x)^3 + 15*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)
^2 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*
cosh(x))*sinh(x)^3 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2
*c^4 - 3*a*c^5)*cosh(x)^2 + 3*(25*a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3
+ 3*a^2*c^4 - 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^
2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 -
 3*a^2*c^4)*cosh(x)^3 + 6*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*
a^2*c^4 + 3*a*c^5)*cosh(x)^2 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x))
*sinh(x)^2 + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4)*cosh(
```

x) + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3*c^3 - 3*a^2*c^4 + (5*a^6 + 15*
a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^5 + 5*(5*a^6
+ 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^4 + 2*(25*a^6 + 25*
a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^3 + 2*(25*a^
6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^2 + (25*a^6 - 25*a^5*c - 20*a^4*c^2 + 2
0*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x))*sinh(x))*log((a + c)*cosh(x) + (a
 + c)*sinh(x) + a - c) - 3*((5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*
a^2*c^4 - 3*a*c^5)*cosh(x)^6 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 -
 9*a^2*c^4 - 3*a*c^5)*sinh(x)^6 + 5*a^6 - 15*a^5*c + 12*a^4*c^2 + 4*a^3*c^3
 - 9*a^2*c^4 + 3*a*c^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^
2*c^4)*cosh(x)^5 + 6*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4
+ (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)
)*sinh(x)^5 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 +
3*a*c^5)*cosh(x)^4 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2
*c^4 + 3*a*c^5 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 -
 3*a*c^5)*cosh(x)^2 + 10*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*
c^4)*cosh(x))*sinh(x)^4 + 4*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^3 + 4
*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4 + 5*(5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^
3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^3 + 15*(5*a^6 + 10*a^5*c + 2*a^4*c^2 -
 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^2 + 3*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*
a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x))*sinh(x)^3 + 3*(25*a^6 - 25*a^5*c -
20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x)^2 + 3*(25*a^6 - 25*a
^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5 + 5*(5*a^6 + 15*a^5*c
+ 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 - 3*a*c^5)*cosh(x)^4 + 20*(5*a^6 + 10*
a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c^4)*cosh(x)^3 + 6*(25*a^6 + 25*a^5*c
 - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4 + 3*a*c^5)*cosh(x)^2 + 4*(25*a^6 - 3
0*a^4*c^2 + 9*a^2*c^4)*cosh(x))*sinh(x)^2 + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2
 + 6*a^3*c^3 - 3*a^2*c^4)*cosh(x) + 6*(5*a^6 - 10*a^5*c + 2*a^4*c^2 + 6*a^3
*c^3 - 3*a^2*c^4 + (5*a^6 + 15*a^5*c + 12*a^4*c^2 - 4*a^3*c^3 - 9*a^2*c^4 -
 3*a*c^5)*cosh(x)^5 + 5*(5*a^6 + 10*a^5*c + 2*a^4*c^2 - 6*a^3*c^3 - 3*a^2*c
^4)*cosh(x)^4 + 2*(25*a^6 + 25*a^5*c - 20*a^4*c^2 - 20*a^3*c^3 + 3*a^2*c^4
+ 3*a*c^5)*cosh(x)^3 + 2*(25*a^6 - 30*a^4*c^2 + 9*a^2*c^4)*cosh(x)^2 + (25*
a^6 - 25*a^5*c - 20*a^4*c^2 + 20*a^3*c^3 + 3*a^2*c^4 - 3*a*c^5)*cosh(x))*si
nh(x))*log(cosh(x) + sinh(x) + 1) + 6*(25*a^5*c - 50*a^4*c^2 + 20*a^3*c^3 +
 10*a^2*c^4 - 5*a*c^5 + 5*(5*a^5*c + 10*a^4*c^2 + 2*a^3*c^3 - 6*a^2*c^4 - 3
*a*c^5)*cosh(x)^4 + 20*(5*a^5*c + 5*a^4*c^2 - 3*a^3*c^3 - 3*a^2*c^4)*cosh(x
)^3 + 2*(75*a^5*c - 65*a^3*c^3 + 12*a*c^5)*cosh(x)^2 + 4*(25*a^5*c - 25*a^4
*c^2 - 10*a^3*c^3 + 10*a^2*c^4 + 2*a*c^5 - 2*c^6)*cosh(x))*sinh(x))/(a^3*c^
7 - 3*a^2*c^8 + 3*a*c^9 - c^10 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cos
h(x)^6 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*sinh(x)^6 + 6*(a^3*c^7 + 2*
a^2*c^8 + a*c^9)*cosh(x)^5 + 6*(a^3*c^7 + 2*a^2*c^8 + a*c^9 + (a^3*c^7 + 3*
a^2*c^8 + 3*a*c^9 + c^10)*cosh(x))*sinh(x)^5 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a
*c^9 - c^10)*cosh(x)^4 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10 + 5*(a^3*c
^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^2 + 10*(a^3*c^7 + 2*a^2*c^8 + a*c^
9)*cosh(x))*sinh(x)^4 + 4*(5*a^3*c^7 - 3*a*c^9)*cosh(x)^3 + 4*(5*a^3*c^7 -

```
3*a*c^9 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^3 + 15*(a^3*c^7
+ 2*a^2*c^8 + a*c^9)*cosh(x)^2 + 3*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*c
osh(x))*sinh(x)^3 + 3*(5*a^3*c^7 - 5*a^2*c^8 - a*c^9 + c^10)*cosh(x)^2 + 3*
(5*a^3*c^7 - 5*a^2*c^8 - a*c^9 + c^10 + 5*(a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 +
c^10)*cosh(x)^4 + 20*(a^3*c^7 + 2*a^2*c^8 + a*c^9)*cosh(x)^3 + 6*(5*a^3*c^7
 + 5*a^2*c^8 - a*c^9 - c^10)*cosh(x)^2 + 4*(5*a^3*c^7 - 3*a*c^9)*cosh(x))*s
inh(x)^2 + 6*(a^3*c^7 - 2*a^2*c^8 + a*c^9)*cosh(x) + 6*(a^3*c^7 - 2*a^2*c^8
 + a*c^9 + (a^3*c^7 + 3*a^2*c^8 + 3*a*c^9 + c^10)*cosh(x)^5 + 5*(a^3*c^7 +
2*a^2*c^8 + a*c^9)*cosh(x)^4 + 2*(5*a^3*c^7 + 5*a^2*c^8 - a*c^9 - c^10)*cos
h(x)^3 + 2*(5*a^3*c^7 - 3*a*c^9)*cosh(x)^2 + (5*a^3*c^7 - 5*a^2*c^8 - a*c^9
 + c^10)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(130) = 260.

Time = 0.23 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.48

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = -\frac{15\,a^5+45\,a^4c+41\,a^3c^2+3\,a^2c^3-12\,ac^4-4\,c^5+15\,(5\,a^5+10\,a^4c+4\,a^3c^2-2\,a^2c^3-ac^4)e^{(-x)}+6\,(25}{3\,(a^3c^6+3\,a^2c^7+3\,ac^8+c^9+6\,(a^3c^6+2\,a^2c^7+ac^8)e^{(-x)}+3\,(5\,a^3c^6+5\,a^2} + \frac{(5\,a^3-3\,ac^2)\log\left(-(a-c)e^{(-x)}-a-c\right)}{2\,c^7} - \frac{(5\,a^3-3\,ac^2)\log\left(e^{(-x)}+1\right)}{2\,c^7}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(15*a^5 + 45*a^4*c + 41*a^3*c^2 + 3*a^2*c^3 - 12*a*c^4 - 4*c^5 + 15*(5
*a^5 + 10*a^4*c + 4*a^3*c^2 - 2*a^2*c^3 - a*c^4)*e^(-x) + 6*(25*a^5 + 25*a^
4*c - 10*a^3*c^2 - 10*a^2*c^3 + 2*a*c^4 + 2*c^5)*e^(-2*x) + 2*(75*a^5 - 65*
a^3*c^2 + 12*a*c^4)*e^(-3*x) + 15*(5*a^5 - 5*a^4*c - 3*a^3*c^2 + 3*a^2*c^3)
*e^(-4*x) + 3*(5*a^5 - 10*a^4*c + 2*a^3*c^2 + 6*a^2*c^3 - 3*a*c^4)*e^(-5*x)
)/(a^3*c^6 + 3*a^2*c^7 + 3*a*c^8 + c^9 + 6*(a^3*c^6 + 2*a^2*c^7 + a*c^8)*e^
(-x) + 3*(5*a^3*c^6 + 5*a^2*c^7 - a*c^8 - c^9)*e^(-2*x) + 4*(5*a^3*c^6 - 3*
a*c^8)*e^(-3*x) + 3*(5*a^3*c^6 - 5*a^2*c^7 - a*c^8 + c^9)*e^(-4*x) + 6*(a^3
```

```
*c^6 - 2*a^2*c^7 + a*c^8)*e^(-5*x) + (a^3*c^6 - 3*a^2*c^7 + 3*a*c^8 - c^9)*
e^(-6*x)) + 1/2*(5*a^3 - 3*a*c^2)*log(-(a - c)*e^(-x) - a - c)/c^7 - 1/2*(5
*a^3 - 3*a*c^2)*log(e^(-x) + 1)/c^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(130) = 260.

Time = 0.27 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.69

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = \frac{(5a^4+5a^3c-3a^2c^2-3ac^3)\log(|ae^x+ce^x+a-c|)}{2(ac^7+c^8)} - \frac{(5a^3-3ac^2)\log(e^x+1)}{2c^7} + \frac{15a^5e^{(5x)}+30a^4ce^{(5x)}+6a^3c^2e^{(5x)}-18a^2c^3e^{(5x)}-9ac^4e^{(5x)}+75a^5e^{(4x)}+75a^4ce^{(4x)}-45a^3c^2e^{(4x)}}{}$$

```
[In] integrate(1/(a+a*cosh(x)+c*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^4 + 5*a^3*c - 3*a^2*c^2 - 3*a*c^3)*log(abs(a*e^x + c*e^x + a - c))
/(a*c^7 + c^8) - 1/2*(5*a^3 - 3*a*c^2)*log(e^x + 1)/c^7 + 1/3*(15*a^5*e^(5*
x) + 30*a^4*c*e^(5*x) + 6*a^3*c^2*e^(5*x) - 18*a^2*c^3*e^(5*x) - 9*a*c^4*e^
(5*x) + 75*a^5*e^(4*x) + 75*a^4*c*e^(4*x) - 45*a^3*c^2*e^(4*x) - 45*a^2*c^3
*e^(4*x) + 150*a^5*e^(3*x) - 130*a^3*c^2*e^(3*x) + 24*a*c^4*e^(3*x) + 150*a
^5*e^(2*x) - 150*a^4*c*e^(2*x) - 60*a^3*c^2*e^(2*x) + 60*a^2*c^3*e^(2*x) +
12*a*c^4*e^(2*x) - 12*c^5*e^(2*x) + 75*a^5*e^x - 150*a^4*c*e^x + 60*a^3*c^2
*e^x + 30*a^2*c^3*e^x - 15*a*c^4*e^x + 15*a^5 - 45*a^4*c + 41*a^3*c^2 - 3*a
^2*c^3 - 12*a*c^4 + 4*c^5)/((a*e^(2*x) + c*e^(2*x) + 2*a*e^x + a - c)^3*c^6
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx = \int \frac{1}{(a+a\cosh(x)+c\sinh(x))^4}\,dx$$

```
[In] int(1/(a + a*cosh(x) + c*sinh(x))^4,x)
```

```
[Out] int(1/(a + a*cosh(x) + c*sinh(x))^4, x)
```

3.753 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$
\begin{aligned}
&\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 \, dx \\
&= \frac{35}{8}\left(b^2-c^2\right)^2 x+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x) \\
&\quad+\frac{35}{24}\left(b^2-c^2\right)\left(c\cosh(x)+b\sinh(x)\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) \\
&\quad+\frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 \\
&\quad+\frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3
\end{aligned}
$$

```
[Out] 35/8*(b^2-c^2)^2*x+35/8*c*(b^2-c^2)^(3/2)*cosh(x)+35/8*b*(b^2-c^2)^(3/2)*si
nh(x)+35/24*(b^2-c^2)*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^
(1/2))+7/12*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2
-c^2)^(1/2))^2+1/4*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/
2))^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3192, 2717, 2718\}$

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx$$
$$= \frac{35}{8}x\left(b^2-c^2\right)^2+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x)$$
$$+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{1}{4}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3$$
$$+\frac{7}{12}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2$$
$$+\frac{35}{24}\left(b^2-c^2\right)(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

```
[Out] (35*(b^2 - c^2)^2*x)/8 + (35*c*(b^2 - c^2)^(3/2)*Cosh[x])/8 + (35*b*(b^2 -
c^2)^(3/2)*Sinh[x])/8 + (35*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 -
 c^2] + b*Cosh[x] + c*Sinh[x]))/24 + (7*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2)/12 + ((c*Cosh[x] + b*Sinh
[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3)/4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3$$
$$+\frac{1}{4}\left(7\sqrt{b^2-c^2}\right)\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx$$

$$
\begin{aligned}
&= \frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 \\
&\quad + \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 \\
&\quad + \frac{1}{12}\left(35\left(b^2-c^2\right)\right)\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx \\
&= \frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) \\
&\quad + \frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 \\
&\quad + \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 \\
&\quad + \frac{1}{8}\left(35\left(b^2-c^2\right)^{3/2}\right)\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx \\
&= \frac{35}{8}\left(b^2-c^2\right)^2x+\frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) \\
&\quad + \frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 \\
&\quad + \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 \\
&\quad + \frac{1}{8}\left(35b\left(b^2-c^2\right)^{3/2}\right)\int\cosh(x)\,dx+\frac{1}{8}\left(35c\left(b^2-c^2\right)^{3/2}\right)\int\sinh(x)\,dx \\
&= \frac{35}{8}\left(b^2-c^2\right)^2x+\frac{35}{8}c\left(b^2-c^2\right)^{3/2}\cosh(x)+\frac{35}{8}b\left(b^2-c^2\right)^{3/2}\sinh(x) \\
&\quad + \frac{35}{24}\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) \\
&\quad + \frac{7}{12}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 \\
&\quad + \frac{1}{4}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.11

$$
\begin{aligned}
&\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx \\
&= \frac{35}{8}(b-c)^2(b+c)^2x+7(b-c)c(b+c)\sqrt{b^2-c^2}\cosh(x)+\frac{7}{2}bc\left(b^2-c^2\right)\cosh(2x) \\
&\quad + \frac{1}{3}c\sqrt{b^2-c^2}\left(3b^2+c^2\right)\cosh(3x)+\frac{1}{8}bc\left(b^2+c^2\right)\cosh(4x)+7b(b-c)(b+c)\sqrt{b^2-c^2}\sinh(x) \\
&\quad + \frac{7}{4}\left(b^4-c^4\right)\sinh(2x)+\frac{1}{3}b\sqrt{b^2-c^2}\left(b^2+3c^2\right)\sinh(3x)+\frac{1}{32}\left(b^4+6b^2c^2+c^4\right)\sinh(4x)
\end{aligned}
$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^4,x]
```

```
[Out] (35*(b - c)^2*(b + c)^2*x)/8 + 7*(b - c)*c*(b + c)*Sqrt[b^2 - c^2]*Cosh[x]
+ (7*b*c*(b^2 - c^2)*Cosh[2*x])/2 + (c*Sqrt[b^2 - c^2]*(3*b^2 + c^2)*Cosh[3
*x])/3 + (b*c*(b^2 + c^2)*Cosh[4*x])/8 + 7*b*(b - c)*(b + c)*Sqrt[b^2 - c^2
]*Sinh[x] + (7*(b^4 - c^4)*Sinh[2*x])/4 + (b*Sqrt[b^2 - c^2]*(b^2 + 3*c^2)*
Sinh[3*x])/3 + ((b^4 + 6*b^2*c^2 + c^4)*Sinh[4*x])/32
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.71

method	result
parts	$(b^2-c^2)^2 x+c^4\left(\left(\frac{\sinh(x)^3}{4}-\frac{3\sinh(x)}{8}\right)\cosh(x)+\frac{3x}{8}\right)+4b^3\left(\frac{c\sinh(x)^4}{4}+\frac{\sqrt{b^2-c^2}\,\sinh(x)^3}{3}+\frac{c\sinh(x)^2}{2}+\right.$
default	$b^4x+c^4x-2b^2c^2x+4\sqrt{b^2-c^2}\,b^3\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh(x)-6b^2c^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+4\sqrt{b^2-c^2}$
risch	$-\frac{35b^2c^2x}{4}+\frac{e^{4x}b^4}{64}+\frac{7e^{2x}b^4}{8}+\frac{35b^4x}{8}+\frac{35c^4x}{8}+\frac{e^{3x}\sqrt{b^2-c^2}\,b^2c}{2}+\frac{e^{3x}\sqrt{b^2-c^2}\,b\,c^2}{2}+\frac{7e^{x}\sqrt{b^2-c^2}\,b^2c}{2}-\frac{7e^{x}\sqrt{b^2-c^2}\,b}{2}$

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2-c^2)^2*x+c^4*((1/4*sinh(x)^3-3/8*sinh(x))*cosh(x)+3/8*x)+4*b^3*(1/4*c*
sinh(x)^4+1/3*(b^2-c^2)^(1/2)*sinh(x)^3+1/2*c*sinh(x)^2+sinh(x)*(b^2-c^2)^(
1/2))+6*b^2*c^2*(1/4*cosh(x)^3*sinh(x)-1/8*cosh(x)*sinh(x)-1/8*x)+4*(b^2-c^
2)^(1/2)*b^2*c*cosh(x)^3+6*b^4*(1/2*cosh(x)*sinh(x)+1/2*x)-6*b^2*c^2*(1/2*c
osh(x)*sinh(x)+1/2*x)+4*b*(1/4*sinh(x)^4*c^3+((b-c)*(b+c))^(1/2)*sinh(x)^3*
c^2+3/2*sinh(x)^2*b^2*c-3/2*sinh(x)^2*c^3+((b-c)*(b+c))^(3/2)*sinh(x))+b^4*
((1/4*cosh(x)^3+3/8*cosh(x))*sinh(x)+3/8*x)+4*c*(b^2-c^2)^(3/2)*cosh(x)+6*c
^2*(b^2-c^2)*(1/2*cosh(x)*sinh(x)-1/2*x)+4*(b^2-c^2)^(1/2)*c^3*(-2/3+1/3*si
nh(x)^2)*cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(164) = 328$.

Time = 0.27 (sec) , antiderivative size = 1293, normalized size of antiderivative = 6.88

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx = \text{Too large to display}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(3*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^8 + 24*(b^4 +
4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^7 + 3*(b^4 + 4*b^3*c +
 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^8 + 168*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)
*cosh(x)^6 + 84*(2*b^4 + 4*b^3*c - 4*b*c^3 - 2*c^4 + (b^4 + 4*b^3*c + 6*b^2
*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^6 + 840*(b^4 - 2*b^2*c^2 + c^4)*x*
cosh(x)^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 6*
```

```
(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^5 + 210*((b^4 + 4*b^3*c +
6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + 12*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*c
osh(x)^2 + 4*(b^4 - 2*b^2*c^2 + c^4)*x)*sinh(x)^4 - 3*b^4 + 12*b^3*c - 18*b
^2*c^2 + 12*b*c^3 - 3*c^4 + 168*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4
)*cosh(x)^5 + 20*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + 20*(b^4 - 2*b^
2*c^2 + c^4)*x*cosh(x))*sinh(x)^3 - 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*cos
h(x)^2 + 84*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 30*(b^
4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 2*b^4 + 4*b^3*c - 4*b*c^3 + 2*c^4
+ 60*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^2)*sinh(x)^2 + 24*((b^4 + 4*b^3*c +
6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 42*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*c
osh(x)^5 + 140*(b^4 - 2*b^2*c^2 + c^4)*x*cosh(x)^3 - 14*(b^4 - 2*b^3*c + 2*
b*c^3 - c^4)*cosh(x))*sinh(x) + 32*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)
^7 + 7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^6 + (b^3 + 3*b^2*c +
 3*b*c^2 + c^3)*sinh(x)^7 + 21*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^5 + 21*(
b^3 + b^2*c - b*c^2 - c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh
(x)^5 + 35*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 3*(b^3 + b^2*c - b*
c^2 - c^3)*cosh(x))*sinh(x)^4 - 21*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^3 +
7*(5*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*b^2*c + 3*b*c^2
- 3*c^3 + 30*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^3 + 21*((b^3 +
3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 10*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)
^3 - 3*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x)^2 - (b^3 - 3*b^2*c + 3*
b*c^2 - c^3)*cosh(x) + (7*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 105*(
b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 63*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^4
+ 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(178) = 356$.

Time = 0.31 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.33

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx = \frac{3b^4x\sinh^4(x)}{8} - \frac{3b^4x\sinh^2(x)\cosh^2(x)}{4} - 3b^4x\sinh^2(x) + \frac{3b^4x\cosh^4(x)}{8} + 3b^4x\cosh^2(x) + b^4x - \frac{3b^4\sinh^3(x)\cosh(x)}{8} + \frac{5b^4\sinh(x)\cosh^3(x)}{8} + 3b^4\sinh(x)\cosh(x) + b^3c\cosh^4(x) + 6b^3c\cosh^2(x) - \frac{8b^3\sqrt{b^2-c^2}\sinh^3(x)}{3} + 4b^3\sqrt{b^2-c^2}\sinh(x)\cosh^2(x) + 4b^3\sqrt{b^2-c^2}\sinh(x) - \frac{3b^2c^2x\sinh^4(x)}{4} + \frac{3b^2c^2x\sinh^2(x)\cosh^2(x)}{2} + 6b^2c^2x\sinh^2(x) - \frac{3b^2c^2x\cosh^4(x)}{4} - 6b^2c^2x\cosh^2(x) - 2b^2c^2x + \frac{3b^2c^2\sinh^3(x)\cosh(x)}{4} + \frac{3b^2c^2\sinh(x)\cosh^3(x)}{4} + 4b^2c\sqrt{b^2-c^2}\cosh^3(x) + 4b^2c\sqrt{b^2-c^2}\cosh(x) + bc^3\sinh^4(x) - 6bc^3\cosh^2(x) + 4bc^2\sqrt{b^2-c^2}\sinh^3(x) - 4bc^2\sqrt{b^2-c^2}\sinh(x) + \frac{3c^4x\sinh^4(x)}{8} - \frac{3c^4x\sinh^2(x)\cosh^2(x)}{4} - 3c^4x\sinh^2(x) + \frac{3c^4x\cosh^4(x)}{8} + 3c^4x\cosh^2(x) + c^4x + \frac{5c^4\sinh^3(x)\cosh(x)}{8} - \frac{3c^4\sinh(x)\cosh^3(x)}{8} - 3c^4\sinh(x)\cosh(x) + 4c^3\sqrt{b^2-c^2}\sinh^2(x)\cosh(x) - \frac{8c^3\sqrt{b^2-c^2}\cosh^3(x)}{3} - 4c^3\sqrt{b^2-c^2}\cosh(x)$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

```
[Out] 3*b**4*x*sinh(x)**4/8 - 3*b**4*x*sinh(x)**2*cosh(x)**2/4 - 3*b**4*x*sinh(x)
**2 + 3*b**4*x*cosh(x)**4/8 + 3*b**4*x*cosh(x)**2 + b**4*x - 3*b**4*sinh(x)
**3*cosh(x)/8 + 5*b**4*sinh(x)*cosh(x)**3/8 + 3*b**4*sinh(x)*cosh(x) + b**3
*c*cosh(x)**4 + 6*b**3*c*cosh(x)**2 - 8*b**3*sqrt(b**2 - c**2)*sinh(x)**3/3
 + 4*b**3*sqrt(b**2 - c**2)*sinh(x)*cosh(x)**2 + 4*b**3*sqrt(b**2 - c**2)*s
inh(x) - 3*b**2*c**2*x*sinh(x)**4/4 + 3*b**2*c**2*x*sinh(x)**2*cosh(x)**2/2
 + 6*b**2*c**2*x*sinh(x)**2 - 3*b**2*c**2*x*cosh(x)**4/4 - 6*b**2*c**2*x*co
sh(x)**2 - 2*b**2*c**2*x + 3*b**2*c**2*sinh(x)**3*cosh(x)/4 + 3*b**2*c**2*s
inh(x)*cosh(x)**3/4 + 4*b**2*c*sqrt(b**2 - c**2)*cosh(x)**3 + 4*b**2*c*sqrt
(b**2 - c**2)*cosh(x) + b*c**3*sinh(x)**4 - 6*b*c**3*cosh(x)**2 + 4*b*c**2*
sqrt(b**2 - c**2)*sinh(x)**3 - 4*b*c**2*sqrt(b**2 - c**2)*sinh(x) + 3*c**4*
x*sinh(x)**4/8 - 3*c**4*x*sinh(x)**2*cosh(x)**2/4 - 3*c**4*x*sinh(x)**2 + 3
*c**4*x*cosh(x)**4/8 + 3*c**4*x*cosh(x)**2 + c**4*x + 5*c**4*sinh(x)**3*cos
h(x)/8 - 3*c**4*sinh(x)*cosh(x)**3/8 - 3*c**4*sinh(x)*cosh(x) + 4*c**3*sqrt
(b**2 - c**2)*sinh(x)**2*cosh(x) - 8*c**3*sqrt(b**2 - c**2)*cosh(x)**3/3 -
4*c**3*sqrt(b**2 - c**2)*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx$$
$$= b^3c\cosh(x)^4 + bc^3\sinh(x)^4 + \frac{1}{64}b^4\left(24x + e^{(4x)} + 8e^{(2x)} - 8e^{(-2x)} - e^{(-4x)}\right) + \frac{1}{64}c^4\left(24x + e^{(4x)} - 8e^{(2x)} + 8e^{(-2x)} - e^{(-4x)}\right) - \frac{3}{32}b^2c^2\left(8x - e^{(4x)} + e^{(-4x)}\right) + \left(b^2-c^2\right)^2x + 4\left(b^2-c^2\right)^{\frac{3}{2}}(c\cosh(x) + b\sinh(x)) + \frac{3}{4}\left(8bc\cosh(x)^2 + b^2\left(4x + e^{(2x)} - e^{(-2x)}\right) - c^2\left(4x - e^{(2x)} + e^{(-2x)}\right)\right)\left(b^2-c^2\right) + \frac{1}{6}\left(24b^2c\cosh(x)^3 + 24bc^2\sinh(x)^3 + c^3\left(e^{(3x)} - 9e^{(-x)} + e^{(-3x)} - 9e^{x}\right) + b^3\left(e^{(3x)} - 9e^{(-x)} - e^{(-3x)} + \right.\right.$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

```
[Out] b^3*c*cosh(x)^4 + b*c^3*sinh(x)^4 + 1/64*b^4*(24*x + e^(4*x) + 8*e^(2*x) -
8*e^(-2*x) - e^(-4*x)) + 1/64*c^4*(24*x + e^(4*x) - 8*e^(2*x) + 8*e^(-2*x)
- e^(-4*x)) - 3/32*b^2*c^2*(8*x - e^(4*x) + e^(-4*x)) + (b^2 - c^2)^2*x + 4
*(b^2 - c^2)^(3/2)*(c*cosh(x) + b*sinh(x)) + 3/4*(8*b*c*cosh(x)^2 + b^2*(4*
x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*(b^2 - c^2) + 1/6
*(24*b^2*c*cosh(x)^3 + 24*b*c^2*sinh(x)^3 + c^3*(e^(3*x) - 9*e^(-x) + e^(-3
*x) - 9*e^x) + b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x))*sqrt(b^2 - c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(164) = 328$.

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.07

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4 dx = \frac{7}{2}\left(b^3+b^2c-bc^2-c^3\right)\sqrt{b^2-c^2}e^x + \frac{35}{8}\left(b^4-2b^2c^2+c^4\right)x + \frac{1}{64}\left(b^4+4b^3c+6b^2c^2+4bc^3+c^4\right)e^{(4x)} + \frac{1}{6}\left(\sqrt{b^2-c^2}b^3+3\sqrt{b^2-c^2}b^2c+3\sqrt{b^2-c^2}bc^2+\sqrt{b^2-c^2}c^3\right)e^{(3x)} + \frac{7}{8}\left(b^4+2b^3c-2bc^3-c^4\right)e^{(2x)} - \frac{1}{192}\left(3b^4-12b^3c+18b^2c^2-12bc^3+3c^4+672\left(\sqrt{b^2-c^2}b^3-\sqrt{b^2-c^2}b^2c-\sqrt{b^2-c^2}bc^2+\sqrt{b^2-c^2}\right.\right.$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

```
[Out] 7/2*(b^3 + b^2*c - b*c^2 - c^3)*sqrt(b^2 - c^2)*e^x + 35/8*(b^4 - 2*b^2*c^2
 + c^4)*x + 1/64*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*e^(4*x) + 1/6*
(sqrt(b^2 - c^2)*b^3 + 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 +
sqrt(b^2 - c^2)*c^3)*e^(3*x) + 7/8*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*e^(2*x)
- 1/192*(3*b^4 - 12*b^3*c + 18*b^2*c^2 - 12*b*c^3 + 3*c^4 + 672*(sqrt(b^2 -
 c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqrt(b^2 - c^2)*b*c^2 + sqrt(b^2 - c^2)
*c^3)*e^(3*x) + 168*(b^4 - 2*b^3*c + 2*b*c^3 - c^4)*e^(2*x) + 32*(sqrt(b^2
- c^2)*b^3 - 3*sqrt(b^2 - c^2)*b^2*c + 3*sqrt(b^2 - c^2)*b*c^2 - sqrt(b^2 -
 c^2)*c^3)*e^x)*e^(-4*x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4\,dx = x\left(b^2-c^2\right)^2-\cosh(x)^2\left(6\,b\,c^3-6\,b^3\,c\right) - \cosh(x)^4\left(b\,c^3-b^3\,c\right) + \cosh(x)\,\sinh(x)^3\left(-\frac{3\,b^4}{8}+\frac{3\,b^2\,c^2}{4}+\frac{5\,c^4}{8}\right) + \cosh(x)^3\sinh(x)\,\left(\frac{5\,b^4}{8}+\frac{3\,b^2\,c^2}{4}-\frac{3\,c^4}{8}\right) + 4\,c\cosh(x)\,\left(b^2-c^2\right)^{3/2} + 4\,b\sinh(x)\,\left(b^2-c^2\right)^{3/2} + 3\,x\cosh(x)^2\left(b^2-c^2\right)^2 + \frac{3\,x\cosh(x)^4\left(b^2-c^2\right)^2}{8} - 3\,x\sinh(x)^2\left(b^2-c^2\right)^2 + \frac{3\,x\sinh(x)^4\left(b^2-c^2\right)^2}{8} + \cosh(x)\,\sinh(x)\,\left(3\,b^4-3\,c^4\right) + 2\,b\,c^3\cosh(x)^2\sinh(x)^2 + \frac{4\,c\cosh(x)^3\sqrt{b^2-c^2}\left(3\,b^2-2\,c^2\right)}{3} - \frac{4\,b\sinh(x)^3\sqrt{b^2-c^2}\left(2\,b^2-3\,c^2\right)}{3} + 4\,b^3\cosh(x)^2\sinh(x)\,\sqrt{b^2-c^2} + 4\,c^3\cosh(x)\,\sinh(x)^2\sqrt{b^2-c^2} - \frac{3\,x\cosh(x)^2\sinh(x)^2\left(b^2-c^2\right)^2}{4}$$

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4,x)
```

```
[Out] x*(b^2 - c^2)^2 - cosh(x)^2*(6*b*c^3 - 6*b^3*c) - cosh(x)^4*(b*c^3 - b^3*c)
 + cosh(x)*sinh(x)^3*((5*c^4)/8 - (3*b^4)/8 + (3*b^2*c^2)/4) + cosh(x)^3*si
nh(x)*((5*b^4)/8 - (3*c^4)/8 + (3*b^2*c^2)/4) + 4*c*cosh(x)*(b^2 - c^2)^(3/
2) + 4*b*sinh(x)*(b^2 - c^2)^(3/2) + 3*x*cosh(x)^2*(b^2 - c^2)^2 + (3*x*cos
h(x)^4*(b^2 - c^2)^2)/8 - 3*x*sinh(x)^2*(b^2 - c^2)^2 + (3*x*sinh(x)^4*(b^2
 - c^2)^2)/8 + cosh(x)*sinh(x)*(3*b^4 - 3*c^4) + 2*b*c^3*cosh(x)^2*sinh(x)^
2 + (4*c*cosh(x)^3*(b^2 - c^2)^(1/2)*(3*b^2 - 2*c^2))/3 - (4*b*sinh(x)^3*(b
^2 - c^2)^(1/2)*(2*b^2 - 3*c^2))/3 + 4*b^3*cosh(x)^2*sinh(x)*(b^2 - c^2)^(1
/2) + 4*c^3*cosh(x)*sinh(x)^2*(b^2 - c^2)^(1/2) - (3*x*cosh(x)^2*sinh(x)^2*
(b^2 - c^2)^2)/4
```

3.754 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx=\frac{5}{2}(b^2-c^2)^{3/2}x+\frac{5}{2}c(b^2-c^2)\cosh(x)+\frac{5}{2}b(b^2-c^2)\sinh(x)+\frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)+\frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2$$

```
[Out] 5/2*(b^2-c^2)^(3/2)*x+5/2*c*(b^2-c^2)*cosh(x)+5/2*b*(b^2-c^2)*sinh(x)+5/6*(
c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))+
1/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3192, 2717, 2718\}$

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx = \frac{5}{2}x\left(b^2-c^2\right)^{3/2} + \frac{5}{2}b\left(b^2-c^2\right)\sinh(x) + \frac{5}{2}c\left(b^2-c^2\right)\cosh(x) + \frac{1}{3}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2} + b\cosh(x)+c\sinh(x)\right)^2 + \frac{5}{6}\sqrt{b^2-c^2}(b\sinh(x) + c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (5*(b^2 - c^2)^(3/2)*x)/2 + (5*c*(b^2 - c^2)*Cosh[x])/2 + (5*b*(b^2 - c^2)*
Sinh[x])/2 + (5*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] +
b*Cosh[x] + c*Sinh[x]))/6 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x])^2)/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 + \frac{1}{3}\left(5\sqrt{b^2-c^2}\right)\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 dx$$

$$= \frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) + \frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 + \frac{1}{2}\left(5\left(b^2-c^2\right)\right)\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) dx$$

$$= \frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) + \frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2 + \frac{1}{2}\left(5b\left(b^2-c^2\right)\right)\int\cosh(x)\,dx+\frac{1}{2}\left(5c\left(b^2-c^2\right)\right)\int\sinh(x)\,dx$$

$$= \frac{5}{2}\left(b^2-c^2\right)^{3/2}x+\frac{5}{2}c\left(b^2-c^2\right)\cosh(x)+\frac{5}{2}b\left(b^2-c^2\right)\sinh(x) + \frac{5}{6}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) + \frac{1}{3}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx = \frac{1}{12}\Big(30(b-c)(b+c)\sqrt{b^2-c^2}x + 45c\left(b^2-c^2\right)\cosh(x)+18bc\sqrt{b^2-c^2}\cosh(2x) + c\left(3b^2+c^2\right)\cosh(3x)+45b\left(b^2-c^2\right)\sinh(x) + 9\sqrt{b^2-c^2}\left(b^2+c^2\right)\sinh(2x) + b\left(b^2+3c^2\right)\sinh(3x)\Big)$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (30*(b - c)*(b + c)*Sqrt[b^2 - c^2]*x + 45*c*(b^2 - c^2)*Cosh[x] + 18*b*c*S
qrt[b^2 - c^2]*Cosh[2*x] + c*(3*b^2 + c^2)*Cosh[3*x] + 45*b*(b^2 - c^2)*Sin
h[x] + 9*Sqrt[b^2 - c^2]*(b^2 + c^2)*Sinh[2*x] + b*(b^2 + 3*c^2)*Sinh[3*x])
/12
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

method	result
parts	$\left(b^2-c^2\right)^{\frac{3}{2}}x+c^3\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh\left(x\right)+c\,b^2\cosh\left(x\right)^3+3\sqrt{b^2-c^2}\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+3b$
default	$b^3\left(\frac{2}{3}+\frac{\cosh(x)^2}{3}\right)\sinh\left(x\right)+c\,b^2\cosh\left(x\right)^3+3\sqrt{b^2-c^2}\,b^2\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+c^2b\sinh\left(x\right)^3+3\sqrt{b}$
risch	$\frac{5\left(b^2-c^2\right)^{\frac{3}{2}}x}{2}+\frac{e^{3x}b^3}{24}+\frac{e^{3x}c\,b^2}{8}+\frac{e^{3x}c^2b}{8}+\frac{e^{3x}c^3}{24}+\frac{3\,e^{2x}\sqrt{b^2-c^2}\,b^2}{8}+\frac{3\,e^{2x}\sqrt{b^2-c^2}\,bc}{4}+\frac{3\,e^{2x}\sqrt{b^2-c^2}\,c^2}{8}+\frac{15b^3e^x}{8}+$

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2-c^2)^(3/2)*x+c^3*(-2/3+1/3*sinh(x)^2)*cosh(x)+c*b^2*cosh(x)^3+3*(b^2-c
^2)^(1/2)*b^2*(1/2*cosh(x)*sinh(x)+1/2*x)+3*b*(1/3*sinh(x)^3*c^2+((b-c)*(b+
c))^(1/2)*sinh(x)^2*c+b^2*sinh(x)-sinh(x)*c^2)+b^3*(2/3+1/3*cosh(x)^2)*sinh
(x)+3*c*(b^2-c^2)*cosh(x)+3*(b^2-c^2)^(1/2)*c^2*(1/2*cosh(x)*sinh(x)-1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.88

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3 dx = \frac{(b^3+3\,b^2c+3\,bc^2+c^3)\cosh(x)^6+6\,(b^3+3\,b^2c+3\,bc^2+c^3)\cosh(x)\sinh(x)^5+(b^3+3\,b^2c+3\,bc^2+c^3)\,\mathrm{s}}{}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*(b^3 + 3*b^2*c + 3*b*c^
2 + c^3)*cosh(x)*sinh(x)^5 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 + 45
*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 15*(3*b^3 + 3*b^2*c - 3*b*c^2 - 3*
c^3 + (b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*((b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 9*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*si
nh(x)^3 - b^3 + 3*b^2*c - 3*b*c^2 + c^3 - 45*(b^3 - b^2*c - b*c^2 + c^3)*co
sh(x)^2 + 15*((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - 3*b^3 + 3*b^2*c +
 3*b*c^2 - 3*c^3 + 18*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 6*
((b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 30*(b^3 + b^2*c - b*c^2 - c^3)
*cosh(x)^3 - 15*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) + 3*(3*(b^2 +
2*b*c + c^2)*cosh(x)^5 + 15*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + 3*(b^2
+ 2*b*c + c^2)*sinh(x)^5 + 20*(b^2 - c^2)*x*cosh(x)^3 + 10*(3*(b^2 + 2*b*c
+ c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^3 + 30*((b^2 + 2*b*c + c^2)*cos
h(x)^3 + 2*(b^2 - c^2)*x*cosh(x))*sinh(x)^2 - 3*(b^2 - 2*b*c + c^2)*cosh(x)
 + 3*(5*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 20*(b^2 - c^2)*x*cosh(x)^2 - b^2 +
2*b*c - c^2)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3
*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(128) = 256$.

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.19

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx=-\frac{2b^3\sinh^3(x)}{3}+b^3\sinh(x)\cosh^2(x)+3b^3\sinh(x)+b^2c\cosh^3(x)+3b^2c\cosh(x)-\frac{3b^2x\sqrt{b^2-c^2}\sinh^2(x)}{2}+\frac{3b^2x\sqrt{b^2-c^2}\cosh^2(x)}{2}+b^2x\sqrt{b^2-c^2}+\frac{3b^2\sqrt{b^2-c^2}\sinh(x)\cosh(x)}{2}+bc^2\sinh^3(x)-3bc^2\sinh(x)+3bc\sqrt{b^2-c^2}\cosh^2(x)+c^3\sinh^2(x)\cosh(x)-\frac{2c^3\cosh^3(x)}{3}-3c^3\cosh(x)+\frac{3c^2x\sqrt{b^2-c^2}\sinh^2(x)}{2}-\frac{3c^2x\sqrt{b^2-c^2}\cosh^2(x)}{2}-c^2x\sqrt{b^2-c^2}+\frac{3c^2\sqrt{b^2-c^2}\sinh(x)\cosh(x)}{2}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

```
[Out] -2*b**3*sinh(x)**3/3 + b**3*sinh(x)*cosh(x)**2 + 3*b**3*sinh(x) + b**2*c*co
sh(x)**3 + 3*b**2*c*cosh(x) - 3*b**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 + 3*b
**2*x*sqrt(b**2 - c**2)*cosh(x)**2/2 + b**2*x*sqrt(b**2 - c**2) + 3*b**2*sq
rt(b**2 - c**2)*sinh(x)*cosh(x)/2 + b*c**2*sinh(x)**3 - 3*b*c**2*sinh(x) +
3*b*c*sqrt(b**2 - c**2)*cosh(x)**2 + c**3*sinh(x)**2*cosh(x) - 2*c**3*cosh(
x)**3/3 - 3*c**3*cosh(x) + 3*c**2*x*sqrt(b**2 - c**2)*sinh(x)**2/2 - 3*c**2
*x*sqrt(b**2 - c**2)*cosh(x)**2/2 - c**2*x*sqrt(b**2 - c**2) + 3*c**2*sqrt(
b**2 - c**2)*sinh(x)*cosh(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$$
$$= b^2c\cosh{(x)}^3+bc^2\sinh{(x)}^3+\frac{1}{24}\,c^3\left(e^{(3\,x)}-9\,e^{(-x)}+e^{(-3\,x)}-9\,e^x\right)$$
$$+\frac{1}{24}\,b^3\left(e^{(3\,x)}-9\,e^{(-x)}-e^{(-3\,x)}+9\,e^x\right)$$
$$+\left(b^2-c^2\right)^{\frac{3}{2}}x+3\left(b^2-c^2\right)\left(c\cosh{(x)}+b\sinh{(x)}\right)$$
$$+\frac{3}{8}\left(8\,bc\cosh{(x)}^2+b^2\left(4\,x+e^{(2\,x)}-e^{(-2\,x)}\right)-c^2\left(4\,x-e^{(2\,x)}+e^{(-2\,x)}\right)\right)\sqrt{b^2-c^2}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] b^2*c*cosh(x)^3 + b*c^2*sinh(x)^3 + 1/24*c^3*(e^(3*x) - 9*e^(-x) + e^(-3*x)
 - 9*e^x) + 1/24*b^3*(e^(3*x) - 9*e^(-x) - e^(-3*x) + 9*e^x) + (b^2 - c^2)^
(3/2)*x + 3*(b^2 - c^2)*(c*cosh(x) + b*sinh(x)) + 3/8*(8*b*c*cosh(x)^2 + b^
2*(4*x + e^(2*x) - e^(-2*x)) - c^2*(4*x - e^(2*x) + e^(-2*x)))*sqrt(b^2 - c
^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$$
$$=\frac{5}{2}\left(b^2-c^2\right)^{\frac{3}{2}}x+\frac{3}{8}\left(b^2+2\,bc+c^2\right)\sqrt{b^2-c^2}e^{(2\,x)}+\frac{1}{24}\left(b^3+3\,b^2c+3\,bc^2+c^3\right)e^{(3\,x)}$$
$$-\frac{1}{24}\left(b^3-3\,b^2c+3\,bc^2-c^3+45\left(b^3-b^2c-bc^2+c^3\right)e^{(2\,x)}+9\left(\sqrt{b^2-c^2}b^2-2\sqrt{b^2-c^2}bc+\sqrt{b^2-c^2}c^2\right)\right.$$
$$+\frac{15}{8}\left(b^3+b^2c-bc^2-c^3\right)e^x$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 5/2*(b^2 - c^2)^(3/2)*x + 3/8*(b^2 + 2*b*c + c^2)*sqrt(b^2 - c^2)*e^(2*x) +
 1/24*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*e^(3*x) - 1/24*(b^3 - 3*b^2*c + 3*b*c
^2 - c^3 + 45*(b^3 - b^2*c - b*c^2 + c^3)*e^(2*x) + 9*(sqrt(b^2 - c^2)*b^2
- 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*c^2)*e^x)*e^(-3*x) + 15/8*(b^3 +
b^2*c - b*c^2 - c^3)*e^x
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3\,dx$$
$$=\frac{11\,b^3\sinh(x)}{3}+\frac{c^3\cosh(x)^3}{3}+\frac{5\,x\,\left(b^2-c^2\right)^{3/2}}{2}-4\,c^3\cosh(x)+\frac{b^3\cosh(x)^2\sinh(x)}{3}$$
$$+3\,b^2\,c\cosh(x)-4\,b\,c^2\sinh(x)+b^2\,c\cosh(x)^3+3\,b\,c\cosh(x)^2\,\sqrt{b^2-c^2}$$
$$+\frac{3\,b^2\cosh(x)\,\sinh(x)\,\sqrt{b^2-c^2}}{2}+\frac{3\,c^2\cosh(x)\,\sinh(x)\,\sqrt{b^2-c^2}}{2}+b\,c^2\cosh(x)^2\sinh(x)$$

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3,x)
```

```
[Out] (11*b^3*sinh(x))/3 + (c^3*cosh(x)^3)/3 + (5*x*(b^2 - c^2)^(3/2))/2 - 4*c^3*
cosh(x) + (b^3*cosh(x)^2*sinh(x))/3 + 3*b^2*c*cosh(x) - 4*b*c^2*sinh(x) + b
^2*c*cosh(x)^3 + 3*b*c*cosh(x)^2*(b^2 - c^2)^(1/2) + (3*b^2*cosh(x)*sinh(x)
*(b^2 - c^2)^(1/2))/2 + (3*c^2*cosh(x)*sinh(x)*(b^2 - c^2)^(1/2))/2 + b*c^2
*cosh(x)^2*sinh(x)
```

3.755 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx=\frac{3}{2}\left(b^2-c^2\right)x+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

```
[Out] 3/2*(b^2-c^2)*x+3/2*c*cosh(x)*(b^2-c^2)^(1/2)+3/2*b*sinh(x)*(b^2-c^2)^(1/2)
+1/2*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3192, 2717, 2718}

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx=\frac{3}{2}x\left(b^2-c^2\right)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x)+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{1}{2}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (3*(b^2 - c^2)*x)/2 + (3*c*Sqrt[b^2 - c^2]*Cosh[x])/2 + (3*b*Sqrt[b^2 - c^2
]*Sinh[x])/2 + ((c*Cosh[x] + b*Sinh[x])*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Si
nh[x]))/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) \\
&\quad+\frac{1}{2}\left(3\sqrt{b^2-c^2}\right)\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx \\
&= \frac{3}{2}\left(b^2-c^2\right)x+\frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) \\
&\quad+\frac{1}{2}\left(3b\sqrt{b^2-c^2}\right)\int\cosh(x)\,dx+\frac{1}{2}\left(3c\sqrt{b^2-c^2}\right)\int\sinh(x)\,dx \\
&= \frac{3}{2}\left(b^2-c^2\right)x+\frac{3}{2}c\sqrt{b^2-c^2}\cosh(x)+\frac{3}{2}b\sqrt{b^2-c^2}\sinh(x) \\
&\quad+\frac{1}{2}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\begin{aligned}
\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx = \frac{1}{4}\Big(&6(b-c)(b+c)x+8c\sqrt{b^2-c^2}\cosh(x) \\
&+2bc\cosh(2x)+8b\sqrt{b^2-c^2}\sinh(x) \\
&+\left(b^2+c^2\right)\sinh(2x)\Big)
\end{aligned}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2,x]
[Out] (6*(b - c)*(b + c)*x + 8*c*Sqrt[b^2 - c^2]*Cosh[x] + 2*b*c*Cosh[2*x] + 8*b*
Sqrt[b^2 - c^2]*Sinh[x] + (b^2 + c^2)*Sinh[2*x])/4
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

method	result
default	$c^{2}\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)+cb\cosh\left(x\right)^{2}+b^{2}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+2c\cosh\left(x\right)\sqrt{b^{2}-c^{2}}+2b\sinh\left(x\right)\sqrt{}$
parts	$b^{2}x+2b\left(\frac{c\sinh(x)^{2}}{2}+\sinh\left(x\right)\sqrt{\left(b-c\right)\left(b+c\right)}\right)+b^{2}\left(\frac{\cosh(x)\sinh(x)}{2}+\frac{x}{2}\right)+c^{2}\left(\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}\right)-c$
risch	$\frac{3b^{2}x}{2}-\frac{3c^{2}x}{2}+\frac{b^{2}e^{2x}}{8}+\frac{e^{2x}cb}{4}+\frac{e^{2x}c^{2}}{8}+\sqrt{b^{2}-c^{2}}\,e^{x}b+\sqrt{b^{2}-c^{2}}\,e^{x}c-\sqrt{b^{2}-c^{2}}\,e^{-x}b+\sqrt{b^{2}-c^{2}}\,e^{-x}c$

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*cosh(x)*sinh(x)-1/2*x)+c*b*cosh(x)^2+b^2*(1/2*cosh(x)*sinh(x)+1/2*
x)+2*c*cosh(x)*(b^2-c^2)^(1/2)+2*b*sinh(x)*(b^2-c^2)^(1/2)+b^2*x-c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(76) = 152$.

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.64

$$\int\left(\sqrt{b^{2}-c^{2}}+b\cosh(x)+c\sinh(x)\right)^{2}\,dx$$
$$=\frac{(b^{2}+2\,bc+c^{2})\cosh\left(x\right)^{4}+4\,(b^{2}+2\,bc+c^{2})\cosh\left(x\right)\sinh\left(x\right)^{3}+(b^{2}+2\,bc+c^{2})\sinh\left(x\right)^{4}+12\,(b^{2}-c^{2})x\,c}{}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^
3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 + 12*(b^2 - c^2)*x*cosh(x)^2 + 6*((b^2 +
2*b*c + c^2)*cosh(x)^2 + 2*(b^2 - c^2)*x)*sinh(x)^2 - b^2 + 2*b*c - c^2 + 4
*((b^2 + 2*b*c + c^2)*cosh(x)^3 + 6*(b^2 - c^2)*x*cosh(x))*sinh(x) + 8*((b
+ c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*
cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx = -\frac{b^2x\sinh^2{(x)}}{2}+\frac{b^2x\cosh^2{(x)}}{2}+b^2x+\frac{b^2\sinh{(x)}\cosh{(x)}}{2}+bc\cosh^2{(x)}+2b\sqrt{b^2-c^2}\sinh{(x)}+\frac{c^2x\sinh^2{(x)}}{2}-\frac{c^2x\cosh^2{(x)}}{2}-c^2x+\frac{c^2\sinh{(x)}\cosh{(x)}}{2}+2c\sqrt{b^2-c^2}\cosh{(x)}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

```
[Out] -b**2*x*sinh(x)**2/2 + b**2*x*cosh(x)**2/2 + b**2*x + b**2*sinh(x)*cosh(x)/
2 + b*c*cosh(x)**2 + 2*b*sqrt(b**2 - c**2)*sinh(x) + c**2*x*sinh(x)**2/2 -
c**2*x*cosh(x)**2/2 - c**2*x + c**2*sinh(x)*cosh(x)/2 + 2*c*sqrt(b**2 - c**
2)*cosh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx = bc\cosh{(x)}^2+\frac{1}{8}\,b^2\left(4\,x+e^{(2\,x)}-e^{(-2\,x)}\right)-\frac{1}{8}\,c^2\left(4\,x-e^{(2\,x)}+e^{(-2\,x)}\right)+b^2x-c^2x+2\,\sqrt{b^2-c^2}(c\cosh{(x)}+b\sinh{(x)})$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] b*c*cosh(x)^2 + 1/8*b^2*(4*x + e^(2*x) - e^(-2*x)) - 1/8*c^2*(4*x - e^(2*x)
 + e^(-2*x)) + b^2*x - c^2*x + 2*sqrt(b^2 - c^2)*(c*cosh(x) + b*sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx$$
$$=\sqrt{b^2-c^2}(b+c)e^x+\frac{3}{2}\left(b^2-c^2\right)x+\frac{1}{8}\left(b^2+2\,bc+c^2\right)e^{(2\,x)}$$
$$-\frac{1}{8}\left(b^2-2\,bc+c^2+8\left(\sqrt{b^2-c^2}b-\sqrt{b^2-c^2}c\right)e^x\right)e^{(-2\,x)}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(b^2 - c^2)*(b + c)*e^x + 3/2*(b^2 - c^2)*x + 1/8*(b^2 + 2*b*c + c^2)*e
^(2*x) - 1/8*(b^2 - 2*b*c + c^2 + 8*(sqrt(b^2 - c^2)*b - sqrt(b^2 - c^2)*c)
*e^x)*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2\,dx=\frac{3\,b^2\,x}{2}-\frac{3\,c^2\,x}{2}+2\,c\cosh(x)\,\sqrt{b^2-c^2}$$
$$+2\,b\sinh(x)\,\sqrt{b^2-c^2}+b\,c\cosh(x)^2$$
$$+\frac{b^2\cosh(x)\,\sinh(x)}{2}+\frac{c^2\cosh(x)\,\sinh(x)}{2}$$

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2,x)
```

```
[Out] (3*b^2*x)/2 - (3*c^2*x)/2 + 2*c*cosh(x)*(b^2 - c^2)^(1/2) + 2*b*sinh(x)*(b^
2 - c^2)^(1/2) + b*c*cosh(x)^2 + (b^2*cosh(x)*sinh(x))/2 + (c^2*cosh(x)*sin
h(x))/2
```

3.756 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) dx = \sqrt{b^2-c^2}x+c\cosh(x)+b\sinh(x)$$

```
[Out] c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2717, 2718}

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right) dx = x\sqrt{b^2-c^2}+b\sinh(x)+c\cosh(x)$$

```
[In] Int[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
```

```
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \sqrt{b^2-c^2}x + b\int \cosh(x)\,dx + c\int \sinh(x)\,dx \\
&= \sqrt{b^2-c^2}x + c\cosh(x) + b\sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx = \sqrt{b^2-c^2}x + c\cosh(x) + b\sinh(x)$$

```
[In] Integrate[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x],x]
[Out] Sqrt[b^2 - c^2]*x + c*Cosh[x] + b*Sinh[x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$c\cosh(x)+b\sinh(x)+x\sqrt{b^2-c^2}$	23
parts	$c\cosh(x)+b\sinh(x)+x\sqrt{b^2-c^2}$	23

```
[In] int(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] c*cosh(x)+b*sinh(x)+x*(b^2-c^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx = \frac{(b+c)\cosh(x)^2+2(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2+2\sqrt{b^2-c^2}(x\cosh(x)+x\sinh(x))-b+}{2(\cosh(x)+\sinh(x))}$$

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*
sqrt(b^2 - c^2)*(x*cosh(x) + x*sinh(x)) - b + c)/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=b\sinh{(x)}+c\cosh{(x)}+x\sqrt{b^2-c^2}$$

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2),x)
[Out] b*sinh(x) + c*cosh(x) + x*sqrt(b**2 - c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=c\cosh{(x)}+b\sinh{(x)}+\sqrt{b^2-c^2}x$$

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="maxima")
[Out] c*cosh(x) + b*sinh(x) + sqrt(b^2 - c^2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=\frac{1}{2}\,c\left(e^{(-x)}+e^{x}\right)-\frac{1}{2}\,b\left(e^{(-x)}-e^{x}\right)+\sqrt{b^2-c^2}x$$

```
[In] integrate(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2),x, algorithm="giac")
[Out] 1/2*c*(e^(-x) + e^x) - 1/2*b*(e^(-x) - e^x) + sqrt(b^2 - c^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)\,dx=x\,\sqrt{b^2-c^2}+c\cosh(x)+b\sinh(x)$$

```
[In] int(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x),x)
[Out] x*(b^2 - c^2)^(1/2) + c*cosh(x) + b*sinh(x)
```

3.757 $\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = -\frac{c+\sqrt{b^2-c^2}\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

```
[Out] (-c-sinh(x)*(b^2-c^2)^(1/2))/c/(c*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3193}

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = -\frac{\sqrt{b^2-c^2}\sinh(x)+c}{c(b\sinh(x)+c\cosh(x))}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
[Out] -((c + Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x])))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{c+\sqrt{b^2-c^2}\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{-c-\sqrt{b^2-c^2}\sinh(x)}{c(c\cosh(x)+b\sinh(x))}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-1),x]
[Out] (-c - Sqrt[b^2 - c^2]*Sinh[x])/(c*(c*Cosh[x] + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{2}{{\rm e}^{x} b+c\,{\rm e}^{x}+\sqrt{b^{2}-c^{2}}}$	25
default	$-\frac{2\left(\sqrt{b^{2}-c^{2}}+b\right)}{c^{2}\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{(b-c)(b+c)}}{c}+\frac{b}{c}\right)}$	46

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/(exp(x)*b+c*exp(x)+(b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.59

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2\left((b+c)\cosh\left(x\right)+(b+c)\sinh\left(x\right)-\sqrt{b^2-c^2}\right)}{\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)^2+2\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)\sinh\left(x\right)+\left(b^2+2\,bc+c^2\right)\sinh\left(x\right)^2-b^2+c^2}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="fricas")
[Out] -2*((b + c)*cosh(x) + (b + c)*sinh(x) - sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^
2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x) + (b^2 + 2*b*c + c^2)*
sinh(x)^2 - b^2 + c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[1,0]%%%}+%%%{1,[0,1]%%%},[2]%%%}+%%%{%%{[2,0]:[1,0,%%%
{-1,[2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \frac{1}{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x)),x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x)), x)
```

3.758 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{c+\sqrt{b^2-c^2}\sinh(x)}{3c\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}$$

```
[Out] 1/3*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^2+1/3*(-c-sinh(x)*(b^2-c^2)^(1/2))/c/(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3195, 3193}

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \frac{b\sinh(x)+c\cosh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{\sqrt{b^2-c^2}\sinh(x)+c}{3c\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(3*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^2) - (c + Sqrt[b^2 - c^2]*Sinh[x])/(3*c*Sqrt[b^2 - c^2]*(c*Cosh[x
] + b*Sinh[x]))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx}{3\sqrt{b^2-c^2}}\\ &= \frac{c\cosh(x)+b\sinh(x)}{3\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{c+\sqrt{b^2-c^2}\sinh(x)}{3c\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = -\frac{-2c\sqrt{b^2-c^2}+2bc\cosh^3(x)+2c^2\sinh(x)+c^2\cosh^2(x)\sinh(x)+b^2\sinh^3(x)}{3c(c\cosh(x)+b\sinh(x))^3}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-2),x]
```

```
[Out] -1/3*(-2*c*Sqrt[b^2 - c^2] + 2*b*c*Cosh[x]^3 + 2*c^2*Sinh[x] + c^2*Cosh[x]^
2*Sinh[x] + b^2*Sinh[x]^3)/(c*(c*Cosh[x] + b*Sinh[x])^3)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{2\left(3\,{\mathrm e}^{x} b+3 c\,{\mathrm e}^{x}+\sqrt{b^{2}-c^{2}}\right)}{3\left({\mathrm e}^{x} b+c\,{\mathrm e}^{x}+\sqrt{b^{2}-c^{2}}\right)^{3}}$	47
default	$\frac{2\left(\sqrt{b^{2}-c^{2}}+b\right)\left(\frac{\left(\sqrt{b^{2}-c^{2}}+b\right)\tanh\left(\frac{x}{2}\right)^{2}}{c^{2}}+\frac{\left(2b^{2}-c^{2}+2\sqrt{b^{2}-c^{2}}\,b\right)\tanh\left(\frac{x}{2}\right)}{c^{3}}+\frac{\frac{4\sqrt{b^{2}-c^{2}}\,b^{2}}{3}-\frac{2\sqrt{b^{2}-c^{2}}\,c^{2}}{3}+\frac{4b^{3}}{3}-\frac{4c^{2}b}{3}}{c^{4}}\right)}{c^{2}\left(\tanh\left(\frac{x}{2}\right)^{2}+\frac{2\sqrt{(b-c)(b+c)}\,\tanh\left(\frac{x}{2}\right)}{c}+\frac{2\tanh\left(\frac{x}{2}\right)b}{c}+\frac{2\sqrt{(b-c)(b+c)}\,b}{c^{2}}+\frac{2b^{2}}{c^{2}}-1\right)\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{(b-c)(b+c)}}{c}+\frac{b}{c}\right)}$	217

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*exp(x)*b+3*c*exp(x)+(b^2-c^2)^(1/2))/(exp(x)*b+c*exp(x)+(b^2-c^2)^(
1/2))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 660, normalized size of antiderivative = 6.60

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx =$$

$$-\frac{}{3\left(\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)\cosh\left(x\right)^6+6\left(b^4+4\,b^3c+6\,b^2c^2+4\,bc^3+c^4\right)\cosh\left(x\right)\sinh\left(x\right)^5+(b\right.}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 12*(b^2 + 2*b*c + c^2)*cosh(x)*sinh
(x)^3 + 3*(b^2 + 2*b*c + c^2)*sinh(x)^4 + 6*(b^2 - c^2)*cosh(x)^2 + 6*(3*(b
^2 + 2*b*c + c^2)*cosh(x)^2 + b^2 - c^2)*sinh(x)^2 - b^2 + 2*b*c - c^2 + 12
*((b^2 + 2*b*c + c^2)*cosh(x)^3 + (b^2 - c^2)*cosh(x))*sinh(x) - 8*((b + c)
*cosh(x)^3 + 3*(b + c)*cosh(x)^2*sinh(x) + 3*(b + c)*cosh(x)*sinh(x)^2 + (b
 + c)*sinh(x)^3)*sqrt(b^2 - c^2))/((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c
^4)*cosh(x)^6 + 6*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(
x)^5 + (b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^6 - 3*(b^4 + 2*b
^3*c - 2*b*c^3 - c^4)*cosh(x)^4 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4 - 5*(b^4
 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^4 - b^4 + 2*b^3*
c - 2*b*c^3 + c^4 + 4*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x
)^3 - 3*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x))*sinh(x)^3 + 3*(b^4 - 2*b^2
*c^2 + c^4)*cosh(x)^2 + 3*(5*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*co
sh(x)^4 + b^4 - 2*b^2*c^2 + c^4 - 6*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)
^2)*sinh(x)^2 + 6*((b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 -
```

```
2*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^3 + (b^4 - 2*b^2*c^2 + c^4)*cosh(
x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[2,0]%%%}+%%%{2,[1,1]%%%}+%%%{1,[0,2]%%%},[4]%%%}+%%%{%
%{[%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^2}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2,x)
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^2, x)
```

3.759 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\begin{aligned}\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx &= \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3} \\ &\quad+\frac{2(c\cosh(x)+b\sinh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}-\frac{2\left(c+\sqrt{b^2-c^2}\sinh(x)\right)}{15c\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))}\end{aligned}$$

```
[Out] 1/5*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^3+2/15*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^
(1/2))^2-2/15*(c+sinh(x)*(b^2-c^2)^(1/2))/c/(b^2-c^2)/(c*cosh(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3195, 3193}

$$\begin{aligned}\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx &= \frac{2(b\sinh(x)+c\cosh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} \\ &\quad+\frac{b\sinh(x)+c\cosh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3} \\ &\quad-\frac{2\left(\sqrt{b^2-c^2}\sinh(x)+c\right)}{15c\left(b^2-c^2\right)(b\sinh(x)+c\cosh(x))}\end{aligned}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(5*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(15*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c*(b^2
 - c^2)*(c*Cosh[x] + b*Sinh[x]))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{2\int\frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\,dx}{5\sqrt{b^2-c^2}}\\
&= \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\\
&\quad+\frac{2(c\cosh(x)+b\sinh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{2\int\frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx}{15\left(b^2-c^2\right)}\\
&= \frac{c\cosh(x)+b\sinh(x)}{5\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\\
&\quad+\frac{2(c\cosh(x)+b\sinh(x))}{15\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\\
&\quad-\frac{2\left(c+\sqrt{b^2-c^2}\sinh(x)\right)}{15c\left(b^2-c^2\right)(c\cosh(x)+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx$$
$$=\frac{12b\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))-\frac{b\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))^3}{(b-c)(b+c)}-\frac{2\sqrt{b^2-c^2}\sinh(x)(c\cosh(x)+b\sinh(x))^4}{(b-c)(b+c)}+(c\cosh(x)}{15c(c\cosh(x)+b\sinh(x))^5}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3),x]
```

```
[Out] (12*b*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]) - (b*Sqrt[b^2 - c^2]*(c*Cosh[
x] + b*Sinh[x])^3)/((b - c)*(b + c)) - (2*Sqrt[b^2 - c^2]*Sinh[x]*(c*Cosh[x
] + b*Sinh[x])^4)/((b - c)*(b + c)) + (c*Cosh[x] + b*Sinh[x])^2*(-5*c + Sqr
t[b^2 - c^2]*Sinh[x]) - 12*(b^2 - c^2)*(c + Sqrt[b^2 - c^2]*Sinh[x]))/(15*c
*(c*Cosh[x] + b*Sinh[x])^5)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{4\left(10b^2 e^{2x}+20\,e^{2x}cb+10\,e^{2x}c^2+5\sqrt{b^2-c^2}\,e^x b+5\sqrt{b^2-c^2}\,e^x c+b^2-c^2\right)}{15\left(e^x b+c\,e^x+\sqrt{b^2-c^2}\right)^5}$
default	$\frac{-\frac{2\left(4\sqrt{b^2-c^2}\,b^2-\sqrt{b^2-c^2}\,c^2+4b^3-3c^2b\right)\tanh\left(\frac{x}{2}\right)^4}{c^2}-\frac{4\left(8b^4-8b^2c^2+c^4+8\sqrt{b^2-c^2}\,b^3-4\sqrt{b^2-c^2}\,b\,c^2\right)\tanh\left(\frac{x}{2}\right)^3}{c^3}-\frac{8\left(24b^4\sqrt{b^2-c^2}-20b^2c^2\sqrt{b^2-c^2}\right.}{}}{c^4\left(\right.}$

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(10*b^2*exp(x)^2+20*exp(x)^2*c*b+10*c^2*exp(x)^2+5*(b^2-c^2)^(1/2)*ex
p(x)*b+5*(b^2-c^2)^(1/2)*exp(x)*c+b^2-c^2)/(exp(x)*b+c*exp(x)+(b^2-c^2)^(1/
2))^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3035 vs. $2(132) = 264$.

Time = 0.45 (sec) , antiderivative size = 3035, normalized size of antiderivative = 20.79

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="fricas")
[Out] -4/15*(10*(b^4 + 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^7 + 70*(b^4 +
 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)*sinh(x)^6 + 10*(b^4 + 4*b^3*c
 + 6*b^2*c^2 + 4*b*c^3 + c^4)*sinh(x)^7 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c^4
)*cosh(x)^5 + 2*(38*b^4 + 76*b^3*c - 76*b*c^3 - 38*c^4 + 105*(b^4 + 4*b^3*c
 + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^2)*sinh(x)^5 + 10*(35*(b^4 + 4*b^3*c
+ 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^3 + 38*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)
*cosh(x))*sinh(x)^4 + 10*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^3 + 10*(35*(b^4 +
4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^4 + b^4 - 2*b^2*c^2 + c^4 + 76
*(b^4 + 2*b^3*c - 2*b*c^3 - c^4)*cosh(x)^2)*sinh(x)^3 + 10*(21*(b^4 + 4*b^3
*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^5 + 76*(b^4 + 2*b^3*c - 2*b*c^3 - c
^4)*cosh(x)^3 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x))*sinh(x)^2 + 10*(7*(b^4 +
 4*b^3*c + 6*b^2*c^2 + 4*b*c^3 + c^4)*cosh(x)^6 + 38*(b^4 + 2*b^3*c - 2*b*c
^3 - c^4)*cosh(x)^4 + 3*(b^4 - 2*b^2*c^2 + c^4)*cosh(x)^2)*sinh(x) - (45*(b
^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 270*(b^3 + 3*b^2*c + 3*b*c^2 + c^
3)*cosh(x)*sinh(x)^5 + 45*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^6 + 55*(b
^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(11*b^3 + 11*b^2*c - 11*b*c^2 - 11*
c^3 + 135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(x)^4 + 20*(45*(b^
3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 11*(b^3 + b^2*c - b*c^2 - c^3)*cos
h(x))*sinh(x)^3 + b^3 - 3*b^2*c + 3*b*c^2 - c^3 - 5*(b^3 - b^2*c - b*c^2 +
c^3)*cosh(x)^2 + 5*(135*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^4 - b^3 + b
^2*c + b*c^2 - c^3 + 66*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 +
10*(27*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^5 + 22*(b^3 + b^2*c - b*c^2
- c^3)*cosh(x)^3 - (b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x))*sqrt(b^2 -
 c^2))/((b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5
+ 7*b*c^6 + c^7)*cosh(x)^10 + 10*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 +
 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)*sinh(x)^9 + (b^7 + 7*b^6*
c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*sinh
(x)^10 - 5*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 -
 5*b*c^6 - c^7)*cosh(x)^8 - 5*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^
3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7 - 9*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*
c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^8 + 40*(3
*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c
^6 + c^7)*cosh(x)^3 - (b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 -
9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x))*sinh(x)^7 - b^7 + 3*b^6*c - b^5*c^2 - 5
*b^4*c^3 + 5*b^3*c^4 + b^2*c^5 - 3*b*c^6 + c^7 + 10*(b^7 + 3*b^6*c + b^5*c^
```

```
2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^6 + 10*(b^7 +
3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7 + 21*(b
^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6
+ c^7)*cosh(x)^4 - 14*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 -
9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^2)*sinh(x)^6 + 4*(63*(b^7 + 7*b^6*c + 21
*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^5
- 70*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c
^6 - c^7)*cosh(x)^3 + 15*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 +
 b^2*c^5 + 3*b*c^6 + c^7)*cosh(x))*sinh(x)^5 - 10*(b^7 + b^6*c - 3*b^5*c^2
- 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x)^4 - 10*(b^7 + b^
6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 - 21*(b^7
 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 +
c^7)*cosh(x)^6 + 35*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*
b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^4 - 15*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^
3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^2)*sinh(x)^4 + 40*(3*(b^7
+ 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c
^7)*cosh(x)^7 - 7*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^
2*c^5 - 5*b*c^6 - c^7)*cosh(x)^5 + 5*(b^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 -
 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^3 - (b^7 + b^6*c - 3*b^5*c^2
- 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7)*cosh(x))*sinh(x)^3 + 5*(
b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7)*
cosh(x)^2 + 5*(9*(b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 35*b^3*c^4 + 21
*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^8 + b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 +
 3*b^3*c^4 - 3*b^2*c^5 - b*c^6 + c^7 - 28*(b^7 + 5*b^6*c + 9*b^5*c^2 + 5*b^
4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^6 + 30*(b^7 + 3*b^6*
c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*cosh(x)^4 -
12*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c
^7)*cosh(x)^2)*sinh(x)^2 + 10*((b^7 + 7*b^6*c + 21*b^5*c^2 + 35*b^4*c^3 + 3
5*b^3*c^4 + 21*b^2*c^5 + 7*b*c^6 + c^7)*cosh(x)^9 - 4*(b^7 + 5*b^6*c + 9*b^
5*c^2 + 5*b^4*c^3 - 5*b^3*c^4 - 9*b^2*c^5 - 5*b*c^6 - c^7)*cosh(x)^7 + 6*(b
^7 + 3*b^6*c + b^5*c^2 - 5*b^4*c^3 - 5*b^3*c^4 + b^2*c^5 + 3*b*c^6 + c^7)*c
osh(x)^5 - 4*(b^7 + b^6*c - 3*b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 + 3*b^2*c^5 -
 b*c^6 - c^7)*cosh(x)^3 + (b^7 - b^6*c - 3*b^5*c^2 + 3*b^4*c^3 + 3*b^3*c^4
- 3*b^2*c^5 - b*c^6 + c^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[3,0]%%%}+%%%{3,[2,1]%%%}+%%%{3,[1,2]%%%}+%%%{1,[0,3]%%
%},[6]
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^3}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3,x)
```

```
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^3, x)
```

3.760 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx &= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\\ &\quad+\frac{3(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\\ &\quad+\frac{2(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}\\ &\quad-\frac{2\left(c+\sqrt{b^2-c^2}\sinh(x)\right)}{35c\left(b^2-c^2\right)^{3/2}(c\cosh(x)+b\sinh(x))}\end{aligned}$$

```
[Out] 1/7*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^4+3/35*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^
(1/2))^3+2/35*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(3/2)/(b*cosh(x)+c*sinh(x)+(b
^2-c^2)^(1/2))^2-2/35*(c+sinh(x)*(b^2-c^2)^(1/2))/c/(b^2-c^2)^(3/2)/(c*cosh
(x)+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3195, 3193}

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx$$
$$=\frac{2(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}+\frac{3(b\sinh(x)+c\cosh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}+\frac{b\sinh(x)+c\cosh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}-\frac{2\left(\sqrt{b^2-c^2}\sinh(x)+c\right)}{35c\left(b^2-c^2\right)^{3/2}(b\sinh(x)+c\cosh(x))}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(7*Sqrt[b^2 - c^2]*(Sqrt[b^2 - c^2] + b*Cosh[x] + c
*Sinh[x])^4) + (3*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)*(Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x])^3) + (2*(c*Cosh[x] + b*Sinh[x]))/(35*(b^2 - c^2)^
(3/2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^2) - (2*(c + Sqrt[b^2 - c^2
]*Sinh[x]))/(35*c*(b^2 - c^2)^(3/2)*(c*Cosh[x] + b*Sinh[x]))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral}=\frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}+\frac{3\int\frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3}\,dx}{7\sqrt{b^2-c^2}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3} + \frac{6\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2}dx}{35\left(b^2-c^2\right)}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3} + \frac{2(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} + \frac{2\int \frac{1}{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}dx}{35\left(b^2-c^2\right)^{3/2}}$$

$$= \frac{c\cosh(x)+b\sinh(x)}{7\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4} + \frac{3(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^3} + \frac{2(c\cosh(x)+b\sinh(x))}{35\left(b^2-c^2\right)^{3/2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^2} - \frac{2\left(c+\sqrt{b^2-c^2}\sinh(x)\right)}{35c\left(b^2-c^2\right)^{3/2}(c\cosh(x)+b\sinh(x))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 425 vs. 2(198) = 396.

Time = 0.62 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.15

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}dx = -\frac{-832b^4c\sqrt{b^2-c^2}+1664b^2c^3\sqrt{b^2-c^2}-832c^5\sqrt{b^2-c^2}+1190bc(b^2-c^2)^2\cosh(x)+448c\sqrt{b^2-c^2}(-b^4}{}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-4),x]
```

```
[Out] -1/1120*(-832*b^4*c*Sqrt[b^2 - c^2] + 1664*b^2*c^3*Sqrt[b^2 - c^2] - 832*c^
5*Sqrt[b^2 - c^2] + 1190*b*c*(b^2 - c^2)^2*Cosh[x] + 448*c*Sqrt[b^2 - c^2]*
```

```
(-b^4 + c^4)*Cosh[2*x] + 112*b^5*c*Cosh[3*x] + 56*b^3*c^3*Cosh[3*x] - 168*b
*c^5*Cosh[3*x] - 28*b^5*c*Cosh[5*x] + 28*b*c^5*Cosh[5*x] + 6*b^5*c*Cosh[7*x
] + 20*b^3*c^3*Cosh[7*x] + 6*b*c^5*Cosh[7*x] - 35*b^6*Sinh[x] + 1295*b^4*c^
2*Sinh[x] - 2485*b^2*c^4*Sinh[x] + 1225*c^6*Sinh[x] - 896*b^3*c^2*Sqrt[b^2
- c^2]*Sinh[2*x] + 896*b*c^4*Sqrt[b^2 - c^2]*Sinh[2*x] + 21*b^6*Sinh[3*x] +
 189*b^4*c^2*Sinh[3*x] - 161*b^2*c^4*Sinh[3*x] - 49*c^6*Sinh[3*x] - 7*b^6*S
inh[5*x] - 35*b^4*c^2*Sinh[5*x] + 35*b^2*c^4*Sinh[5*x] + 7*c^6*Sinh[5*x] +
b^6*Sinh[7*x] + 15*b^4*c^2*Sinh[7*x] + 15*b^2*c^4*Sinh[7*x] + c^6*Sinh[7*x]
)/((b - c)*c*(b + c)*(c*Cosh[x] + b*Sinh[x])^7)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{4\left(35\,\mathrm{e}^{3x} b^3+105\,\mathrm{e}^{3x} c\,b^2+105\,\mathrm{e}^{3x} c^2 b+35\,\mathrm{e}^{3x} c^3+21\,\mathrm{e}^{2x}\sqrt{b^2-c^2}\,b^2+42\,\mathrm{e}^{2x}\sqrt{b^2-c^2}\,bc+21\,\mathrm{e}^{2x}\sqrt{b^2-c^2}\,c^2+7b^3\mathrm{e}^{x}+7\,\mathrm{e}^{x} c\,b^2-7\,\mathrm{e}^{x} c^2 b-}{35\left(\mathrm{e}^{x} b+c\,\mathrm{e}^{x}+\sqrt{b^2-c^2}\right)^7}$
default	$\frac{\frac{2\left(8b^4-8b^2c^2+c^4+8\sqrt{b^2-c^2}\,b^3-4\sqrt{b^2-c^2}\,b\,c^2\right)\tanh\left(\frac{x}{2}\right)^6}{c^2}+\frac{6\left(16b^4\sqrt{b^2-c^2}-12b^2c^2\sqrt{b^2-c^2}+c^4\sqrt{b^2-c^2}+16b^5-20b^3c^2+5b\,c^4\right)\tanh\left(\frac{x}{2}\right)^5}{c^3}+\frac{4(8}{}}{}$

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -4/35*(35*b^3*exp(x)^3+105*exp(x)^3*c*b^2+105*exp(x)^3*c^2*b+35*exp(x)^3*c^
3+21*exp(x)^2*(b^2-c^2)^(1/2)*b^2+42*exp(x)^2*(b^2-c^2)^(1/2)*b*c+21*exp(x)
^2*(b^2-c^2)^(1/2)*c^2+7*b^3*exp(x)+7*exp(x)*c*b^2-7*exp(x)*c^2*b-7*exp(x)*
c^3+(b^2-c^2)^(1/2)*b^2-(b^2-c^2)^(1/2)*c^2)/(exp(x)*b+c*exp(x)+(b^2-c^2)^(
1/2))^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6590 vs. 2(176) = 352.

Time = 1.60 (sec) , antiderivative size = 6590, normalized size of antiderivative = 33.28

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[4,0]%%%}+%%%{4,[3,1]%%%}+%%%{6,[2,2]%%%}+%%%{4,[1,3]%%
%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^4}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^4}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4,x)
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^4, x)
```

3.761 $\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 294

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx = \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} - \frac{2i(23a^2+9b^2-9c^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{15\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{16ia(a^2-b^2+c^2)\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 2/5*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(3/2)+16/15*(a*c*cosh(x)+
a*b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(1/2)-2/15*I*(23*a^2+9*b^2-9*c^2)*(cos
(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*Ellip
ticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^
(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(x))/(a+(b
^2-c^2)^(1/2)))^(1/2)+16/15*I*a*(a^2-b^2+c^2)*(cos(1/2*I*x-1/2*arctan(b,-I*
c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arct
an(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh
(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3199, 3225, 3228, 3198, 2732, 3206, 2740}

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\frac{16ia(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i(23a^2+9b^2-9c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}+\frac{2}{5}(b\sinh(x)+c\cosh(x))(a+b\cosh(x)+c\sinh(x))^{3/2}+\frac{16}{15}(ab\sinh(x)+ac\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (16*(a*c*Cosh[x] + a*b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c
*Cosh[x] + b*Sinh[x])*(a + b*Cosh[x] + c*Sinh[x])^(3/2))/5 - (((2*I)/15)*(2
3*a^2 + 9*b^2 - 9*c^2)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 -
 c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*
Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])] + (((16*I)/15)*a*(a^2 - b^2 + c
^2)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^
2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a
+ b*Cosh[x] + c*Sinh[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
```

```
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} \\
&\quad +\frac{2}{5}\int \sqrt{a+b\cosh(x)+c\sinh(x)}\left(\frac{1}{2}\left(5a^2+3b^2-3c^2\right)+4ab\cosh(x)+4ac\sinh(x)\right)dx \\
&= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} \\
&\quad +\frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} \\
&\quad +\frac{4\int \frac{\frac{1}{4}a^2\left(15a^2+17b^2-17c^2\right)+\frac{1}{4}ab\left(23a^2+9b^2-9c^2\right)\cosh(x)+\frac{1}{4}ac\left(23a^2+9b^2-9c^2\right)\sinh(x)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx}{15a} \\
&= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} \\
&\quad +\frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} \\
&\quad +\frac{1}{15}\left(23a^2+9b^2-9c^2\right)\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx \\
&\quad -\frac{1}{15}\left(8a\left(a^2-b^2+c^2\right)\right)\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}dx \\
&= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} \\
&\quad +\frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} \\
&\quad +\frac{\left(\left(23a^2+9b^2-9c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}\right)\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}dx}{15\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} \\
&\quad -\frac{\left(8a\left(a^2-b^2+c^2\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\right)\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}dx}{15\sqrt{a+b\cosh(x)+c\sinh(x)}} \\
&= \frac{16}{15}(ac\cosh(x)+ab\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} \\
&\quad +\frac{2}{5}(c\cosh(x)+b\sinh(x))(a+b\cosh(x)+c\sinh(x))^{3/2} \\
&\quad -\frac{2i\left(23a^2+9b^2-9c^2\right)E\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right)\middle|\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{15\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} \\
&\quad +\frac{16ia\left(a^2-b^2+c^2\right)\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{15\sqrt{a+b\cosh(x)+c\sinh(x)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.28 (sec) , antiderivative size = 3775, normalized size of antiderivative = 12.84

$$\int (a + b\cosh(x) + c\sinh(x))^{5/2}\, dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((2*b*(23*a^2 + 9*b^2 - 9*c^2))/(15*c) + (2
2*a*c*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 + (22*a*b*Sinh[x])/15 + ((b^2 + c^2
)*Sinh[2*x])/5) + (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b
^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 -
b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(S
qrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/
c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*
c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)
/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
+ ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sinh
[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + S
qrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(
Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b
/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + A
rcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/
c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b
^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2
]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*S
qrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*c*Sqrt
[I*(I + Sinh[x + ArcTanh[b/c]])]) - (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((
-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1
 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
 ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*
Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b
^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a +
c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c
^2]*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (23*a^2*b^2*((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1
 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x
 + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sin
h[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2]
```

)]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b
^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sq
rt[(b^2 - c^2)/b^2])]]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c
/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt
[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(15*c) - (3*b^4*((c*Appe
llF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]
)/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/
b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b
^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^
2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2
- c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt
[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])
/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x
+ ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/
b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*c) + (23*a
^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x +
ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*
Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*S
qrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*S
qrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a +
b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTan
h[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + A
rcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*
Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/
15 + (6*b^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*
Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2])))
, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1
+ a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*
Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/
b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x
+ ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*C
osh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqr
t[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[
c/b]]]))/5 - (3*c^3*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2
/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b
^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^
2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2
/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcT
anh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*
Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/
b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a +
b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + Arc
Tanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + Ar

```
cTanh[c/b]]]))/5
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(328) = 656$.

Time = 1.74 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.06

method	result
default	$-\frac{\sqrt{(b-c)(b+c)}\left(\frac{\cosh(x)^3 b^2}{3}-\frac{\cosh(x)^3 c^2}{3}+3\cosh(x)a^2-\cosh(x)b^2+\cosh(x)c^2\right)}{\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\left(a^3\right.}{}$

```
[In] int((a+b*cosh(x)+c*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)*((b
-c)*(b+c))^(1/2)*(1/3*cosh(x)^3*b^2-1/3*cosh(x)^3*c^2+3*cosh(x)*a^2-cosh(x)
*b^2+cosh(x)*c^2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(
1/2)*sinh(x)^2)^(1/2)*(a^3*ln((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
(b^2-c^2)^(1/2)*cosh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-
c^2)^(1/2))^(1/2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(
1/2)*sinh(x)^2)^(1/2))/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c
^2)^(1/2))^(1/2)+1/2*a*c^2*ln((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
(b^2-c^2)^(1/2)*cosh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-
c^2)^(1/2))^(1/2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(
1/2)*sinh(x)^2)^(1/2))/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c
^2)^(1/2))^(1/2)-1/2*a*b^2*ln((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
(b^2-c^2)^(1/2)*cosh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-
c^2)^(1/2))^(1/2)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(
1/2)*sinh(x)^2)^(1/2))/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c
^2)^(1/2))^(1/2)+1/2*a*cosh(x)/(-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))
*(b^2-c^2)^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1
/2)*sinh(x)^2)^(1/2)*b^2-1/2*a*cosh(x)/(-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2
)^(1/2))*(b^2-c^2)^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2
-c^2)^(1/2)*sinh(x)^2)^(1/2)*c^2)/sinh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2
-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 928, normalized size of antiderivative = 3.16

$$\int (a + b\cosh(x) + c\sinh(x))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/90*(4*(sqrt(2)*(a^3 - 33*a*b^2 + 33*a*c^2)*cosh(x)^2 + 2*sqrt(2)*(a^3 -
33*a*b^2 + 33*a*c^2)*cosh(x)*sinh(x) + sqrt(2)*(a^3 - 33*a*b^2 + 33*a*c^2)*
sinh(x)^2)*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2
 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2
 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c)) + 12*(s
qrt(2)*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2)*c)*cosh(x)^2
+ 2*sqrt(2)*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2)*c)*cosh(
x)*sinh(x) + sqrt(2)*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2)
*c)*sinh(x)^2)*sqrt(b + c)*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2
 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2
 + c^3), weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2
), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(
3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))) - 3*(3*(b^3 + 3*b^2*
c + 3*b*c^2 + c^3)*cosh(x)^4 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*sinh(x)^4
+ 22*(a*b^2 + 2*a*b*c + a*c^2)*cosh(x)^3 + 2*(11*a*b^2 + 22*a*b*c + 11*a*c^
2 + 6*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x))*sinh(x)^3 - 3*b^3 + 3*b^2*c
+ 3*b*c^2 - 3*c^3 - 4*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2
)*c)*cosh(x)^2 - 2*(46*a^2*b + 18*b^3 - 18*b*c^2 - 18*c^3 - 9*(b^3 + 3*b^2*
c + 3*b*c^2 + c^3)*cosh(x)^2 + 2*(23*a^2 + 9*b^2)*c - 33*(a*b^2 + 2*a*b*c +
 a*c^2)*cosh(x))*sinh(x)^2 - 22*(a*b^2 - a*c^2)*cosh(x) + 2*(6*(b^3 + 3*b^2
*c + 3*b*c^2 + c^3)*cosh(x)^3 - 11*a*b^2 + 11*a*c^2 + 33*(a*b^2 + 2*a*b*c +
 a*c^2)*cosh(x)^2 - 4*(23*a^2*b + 9*b^3 - 9*b*c^2 - 9*c^3 + (23*a^2 + 9*b^2
)*c)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a))/((b + c)*cosh(x)^2
+ 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cosh(x) + c\sinh(x))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\int (b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\int (b\cosh{(x)}+c\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx=\int (a+b\cosh(x)+c\sinh(x))^{5/2}\,dx$$

```
[In] int((a + b*cosh(x) + c*sinh(x))^(5/2),x)
[Out] int((a + b*cosh(x) + c*sinh(x))^(5/2), x)
```

3.762 $\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 249

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx = \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} - \frac{8iaE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2i(a^2-b^2+c^2)\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{3\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 2/3*(c*cosh(x)+b*sinh(x))*(a+b*cosh(x)+c*sinh(x))^(1/2)-8/3*I*a*(cos(1/2*I*
x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticE(si
n(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))
^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)
^(1/2)))^(1/2)+2/3*I*(a^2-b^2+c^2)*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2
)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arctan(b,-I*c))
,2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x)+c*sinh(
x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3199, 3228, 3198, 2732, 3206, 2740}

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx = \frac{2i(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{a+b\cosh(x)+c\sinh(x)}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/3 - (((8*I)/3)*
a*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2
- c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sinh[x])/
(a + Sqrt[b^2 - c^2])] + (((2*I)/3)*(a^2 - b^2 + c^2)*EllipticF[(I*x - ArcT
an[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Co
sh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqrt[a + b*Cosh[x] + c*Sinh[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} \\
&\quad+\frac{2}{3}\int\frac{\frac{1}{2}(3a^2+b^2-c^2)+2ab\cosh(x)+2ac\sinh(x)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx \\
&= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{a+b\cosh(x)+c\sinh(x)} \\
&\quad+\frac{1}{3}(4a)\int\sqrt{a+b\cosh(x)+c\sinh(x)}\,dx \\
&\quad+\frac{1}{3}\left(-a^2+b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx
\end{aligned}$$

$$= \frac{2}{3}(c\cosh(x) + b\sinh(x))\sqrt{a + b\cosh(x) + c\sinh(x)} + \frac{\left(4a\sqrt{a + b\cosh(x) + c\sinh(x)}\right)\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}} + \frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}dx}{3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{\left((-a^2 + b^2 - c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\right)\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}} + \frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}dx}{3\sqrt{a + b\cosh(x) + c\sinh(x)}}$$

$$= \frac{2}{3}(c\cosh(x) + b\sinh(x))\sqrt{a + b\cosh(x) + c\sinh(x)} - \frac{8iaE\left(\frac{1}{2}(ix - \tan^{-1}(b, -ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a + b\cosh(x) + c\sinh(x)}}{3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2i(a^2 - b^2 + c^2)\operatorname{EllipticF}\left(\frac{1}{2}(ix - \tan^{-1}(b, -ic)), \frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{3\sqrt{a + b\cosh(x) + c\sinh(x)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.12 (sec) , antiderivative size = 2292, normalized size of antiderivative = 9.20

$$\int (a + b\cosh(x) + c\sinh(x))^{3/2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] ((8*a*b)/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[a + b*Cosh[x] + c*
Sinh[x]] + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]
*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2
]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 -
 b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqr
t[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[
(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]
*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTa
nh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sinh[x + Ar
cTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^
2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b
^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sq
rt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c
]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/
```

```
c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x +
 ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2
+ c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*c*Sqrt[I*(I + Sin
h[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[
1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt
[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]
))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTa
nh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2]
 - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 +
 c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Si
nh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*Sqrt[I*(I +
 Sinh[x + ArcTanh[b/c]])]) - (4*a*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a
/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/
(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/
b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt
[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2
])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2)
 + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^
2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(3*c) + (4*a*c*((c*AppellF1[-1/2, -1/2, -1
/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b
^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTa
nh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + Ar
cTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b
^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[
a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2
)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2
- c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(
b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*S
qrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/3
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.29

method	result
default	$\frac{2a(-b^2+c^2)\cosh(x)}{\sqrt{b^2-c^2}\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\,a^2\ln\left(\frac{-\sinh(x)\cosh(x)b^2+\sinh(x)\cosh(x)c}{}\right.}{\left(-b^2\sinh(x)+\sinh\right.}$

```
[In] int((a+b*cosh(x)+c*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/(b^2-c^2)^(1/2)*(-b^2+c^2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2)
)/(b^2-c^2)^(1/2))^(1/2)*cosh(x)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a^2*ln((-sinh(x)*cosh(x)*b^2+sinh(x)*c
osh(x)*c^2+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+
a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(
1/2))/(b^2-c^2)^(1/2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^
2)^(1/2))^(1/2))/(-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/
2)/sinh(x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.86

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}(a^2+3\,b^2-3\,c^2)\cosh\left(x\right)+\sqrt{2}(a^2+3\,b^2-3\,c^2)\sinh\left(x\right)\right)\sqrt{b+c}\text{weierstrassPInvers}}{}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(sqrt(2)*(a^2 + 3*b^2 - 3*c^2)*cosh(x) + sqrt(2)*(a^2 + 3*b^2 - 3*c^
2)*sinh(x))*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^
2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^
2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c)) - 24*(
sqrt(2)*(a*b + a*c)*cosh(x) + sqrt(2)*(a*b + a*c)*sinh(x))*sqrt(b + c)*weie
rstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 -
 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), weierstrassPInverse(4/
3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a
*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*s
inh(x) + 2*a)/(b + c))) + 3*((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c +
 c^2)*sinh(x)^2 - b^2 + c^2 - 8*(a*b + a*c)*cosh(x) - 2*(4*a*b + 4*a*c - (b
^2 + 2*b*c + c^2)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a))/((b +
c)*cosh(x) + (b + c)*sinh(x))
```

Sympy [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (a+b\cosh(x)+c\sinh(x))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(3/2),x)
[Out] Integral((a + b*cosh(x) + c*sinh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx=\int (a+b\cosh(x)+c\sinh(x))^{3/2}\,dx$$

```
[In] int((a + b*cosh(x) + c*sinh(x))^(3/2),x)
[Out] int((a + b*cosh(x) + c*sinh(x))^(3/2), x)
```

3.763 $\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*
c))*EllipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(
b^2-c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/((a+b*cosh(x)+c*sinh(
x))/(a+(b^2-c^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3198, 2732}

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[In] Int[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/Sqrt[(a + b*Cosh[x] + c*Sin
h[x])/(a + Sqrt[b^2 - c^2])]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}\,dx}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

$$= -\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.09 (sec) , antiderivative size = 1401, normalized size of antiderivative = 13.74

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2b\sqrt{a+b\cosh(x)+c\sinh(x)}}{c}$$

$$+\frac{2a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},-\frac{i\left(a+\sqrt{1-\frac{b^2}{c^2}}c\sinh\left(x+\operatorname{arctanh}\left(\frac{b}{c}\right)\right)\right)}{\sqrt{1-\frac{b^2}{c^2}}\left(1-\frac{ia}{\sqrt{1-\frac{b^2}{c^2}}c}\right)c},-\frac{i\left(a+\sqrt{1-\frac{b^2}{c^2}}c\sinh\left(x+\operatorname{arctanh}\left(\frac{b}{c}\right)\right)\right)}{\sqrt{1-\frac{b^2}{c^2}}\left(-1-\frac{ia}{\sqrt{1-\frac{b^2}{c^2}}c}\right)c}\right)\operatorname{sech}(x+\cdots}{\cdots}$$

$$-\frac{b^2\left(\frac{c\,\mathrm{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{a+b\sqrt{1-\frac{c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{b\sqrt{1-\frac{c^2}{b^2}}\left(1+\frac{a}{b\sqrt{1-\frac{c^2}{b^2}}}\right)},\frac{a+b\sqrt{1-\frac{c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{b\sqrt{1-\frac{c^2}{b^2}}\left(-1+\frac{a}{b\sqrt{1-\frac{c^2}{b^2}}}\right)}\right)\sinh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{b\sqrt{1-\frac{c^2}{b^2}}\sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}}-b\sqrt{\frac{b^2-c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{a+b\sqrt{\frac{b^2-c^2}{b^2}}}}\sqrt{a+b\sqrt{\frac{b^2-c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}\sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}}+b\sqrt{\frac{b^2-c^2}{b^2}}\cosh\left(x+\operatorname{arctan}\cdots\right.}{-a+b\sqrt{\frac{b^2-c^2}{b^2}}}}}\cdots\right.}{c}$$

$$+c\left(\frac{c\,\mathrm{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{a+b\sqrt{1-\frac{c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{b\sqrt{1-\frac{c^2}{b^2}}\left(1+\frac{a}{b\sqrt{1-\frac{c^2}{b^2}}}\right)},\frac{a+b\sqrt{1-\frac{c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{b\sqrt{1-\frac{c^2}{b^2}}\left(-1+\frac{a}{b\sqrt{1-\frac{c^2}{b^2}}}\right)}\right)\sinh\left(x\cdots\right.}{b\sqrt{1-\frac{c^2}{b^2}}\sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}}-b\sqrt{\frac{b^2-c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{a+b\sqrt{\frac{b^2-c^2}{b^2}}}}\sqrt{a+b\sqrt{\frac{b^2-c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}\sqrt{\frac{b\sqrt{\frac{b^2-c^2}{b^2}}+b\sqrt{\frac{b^2-\cdots}{b^2}}\cdots}{-a\cdots}}}\right.$$

$$\left.-\frac{-\frac{2b\left(a+b\sqrt{1-\frac{c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)\right)}{b^2-c^2}+\frac{c\sinh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}{b\sqrt{1-\frac{c^2}{b^2}}}}{\sqrt{a+b\sqrt{1-\frac{c^2}{b^2}}\cosh\left(x+\operatorname{arctanh}\left(\frac{c}{b}\right)\right)}}\right)$$

```
[In] Integrate[Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*b*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/c + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2,
 ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]
*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[
x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c
)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[
```

```
(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a
 + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b
^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]
*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c
^2]*c*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (b^2*((c*AppellF1[-1/2, -1/2,
 -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^
2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + Ar
cTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x +
 ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt
[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sq
rt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 -
c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b
^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])
)/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a +
b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/c + c*((c*AppellF1[-1/2, -1/2
, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c
^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + A
rcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x
+ ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqr
t[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*S
qrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 -
 c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(
b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]
))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a +
 b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(125) = 250$.

Time = 1.82 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.11

method	result
default	$\frac{(-b^2+c^2)\cosh(x)}{\sqrt{b^2-c^2}\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}} + \frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\,a\ln\left(\frac{-\sinh(x)\cosh(x)b^2+\sinh(x)\cosh(x)c^2}{}\right.}{\left(-b^2\sinh(x)+\sinh(\right.}$
risch	Expression too large to display

```
[In] int((a+b*cosh(x)+c*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b^2-c^2)^(1/2)*(-b^2+c^2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/
(b^2-c^2)^(1/2))^(1/2)*cosh(x)+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2)
)/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a*ln((-sinh(x)*cosh(x)*b^2+sinh(x)*cosh(
x)*c^2+cosh(x)*(b^2-c^2)^(1/2)*a+((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*si
```

```
nh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2)
)/(b^2-c^2)^(1/2)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(
1/2))^(1/2))/(-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/s
inh(x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.08

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{2\left(\sqrt{2}a\sqrt{b+c}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2+3\,c^2)}{3\,(b^2+2\,bc+c^2)},-\frac{8\,(8\,a^3-9\,ab^2+9\,ac^2)}{27\,(b^3+3\,b^2c+3\,bc^2+c^3)},\frac{3\,(b+c)\cosh(x)+3\,(b+c)\sinh(x)+2\,a}{3\,(b+c)}\right)-3\,\sqrt{}\right.}{}$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*a*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/
(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b
*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c)) - 3
*sqrt(2)*(b + c)^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2
*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c
^3), weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -
8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*(b
 + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))) - 3*sqrt(b*cosh(x) + c*s
inh(x) + a)*(b + c))/(b + c)
```

Sympy [F]

$$\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{a+b\cosh{(x)}+c\sinh{(x)}}\,dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\cosh(x) + c\sinh(x)}\, dx = \int \sqrt{b\cosh\left(x\right) + c\sinh\left(x\right) + a}\, dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\cosh(x) + c\sinh(x)}\, dx = \int \sqrt{b\cosh\left(x\right) + c\sinh\left(x\right) + a}\, dx$$

```
[In] integrate((a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\cosh(x) + c\sinh(x)}\, dx = \int \sqrt{a + b\cosh\left(x\right) + c\sinh\left(x\right)}\, dx$$

```
[In] int((a + b*cosh(x) + c*sinh(x))^(1/2),x)
[Out] int((a + b*cosh(x) + c*sinh(x))^(1/2), x)
```

3.764 $\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{2i\operatorname{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] -2*I*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*
c))*EllipticF(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(
b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)
/(a+b*cosh(x)+c*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3206, 2740}

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\operatorname{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[In] Int[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqr
t[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/Sqr
t[a + b*Cosh[x] + c*Sinh[x]]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}\,dx}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ix-\tan^{-1}(b,-ic)\right),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx$$

$$= \frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{a+b\cosh(x)+c\sinh(x)}{a+i\sqrt{1-\frac{b^2}{c^2}}c},\frac{a+b\cosh(x)+c\sinh(x)}{a-i\sqrt{1-\frac{b^2}{c^2}}c}\right)\text{sech}\left(x+\text{arctanh}\left(\frac{b}{c}\right)\right)\sqrt{a+b\cosh(x)+c\sin}}{\sqrt{1-\frac{b^2}{c^2}}c}$$

```
[In] Integrate[1/Sqrt[a + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 -
 b^2/c^2]*c), (a + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c)]*Sech
[x + ArcTanh[b/c]]*Sqrt[a + b*Cosh[x] + c*Sinh[x]]*Sqrt[-(((-I)*Sqrt[1 - b^
2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 - b^2/c^2]*c))]*Sqrt[-((I*S
qrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*c))])/
(Sqrt[1 - b^2/c^2]*c)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.46

method	result
default	$\frac{\sqrt{\frac{\left(-b^{2} \sinh(x)+\sinh(x) c^{2}+a \sqrt{b^{2}-c^{2}}\right) \sinh(x)^{2}}{\sqrt{b^{2}-c^{2}}}}\, \ln\left(\frac{-\sinh(x) \cosh(x) b^{2}+\sinh(x) \cosh(x) c^{2}+\cosh(x) \sqrt{b^{2}-c^{2}}\, a+\sqrt{\frac{\left(-b^{2}+c^{2}\right) \sinh(x)^{3}}{\sqrt{b^{2}-c^{2}}}+a \sinh(x)^{2}}}{\sqrt{b^{2}-c^{2}}\, \sqrt{\frac{-b^{2} \sinh(x)+\sinh(x) c^{2}+a \sqrt{b^{2}-c^{2}}}{\sqrt{b^{2}-c^{2}}}}}\right.}{\left(-b^{2} \sinh(x)+\sinh(x) c^{2}+a \sqrt{b^{2}-c^{2}}\right) \sinh(x)}$

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1
/2)*ln((-sinh(x)*cosh(x)*b^2+sinh(x)*cosh(x)*c^2+cosh(x)*(b^2-c^2)^(1/2)*a+
((-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)^3+a*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*((
-b^2+c^2)/(b^2-c^2)^(1/2)*sinh(x)+a)^(1/2))/(b^2-c^2)^(1/2)/((-b^2*sinh(x)+
sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2))/(-b^2*sinh(x)+sinh(x
)*c^2+a*(b^2-c^2)^(1/2))*(b^2-c^2)^(1/2)/sinh(x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \frac{2\sqrt{2}\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^2-3b^2+3c^2\right)}{3\left(b^2+2bc+c^2\right)}, -\frac{8\left(8a^3-9ab^2+9ac^2\right)}{27\left(b^3+3b^2c+3bc^2+c^3\right)}, \frac{3(b+c)\cosh(x)+3(b+c)\sinh(x)+2a}{3(b+c)}\right)}{\sqrt{b+c}}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^
2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*
(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))/sqrt(b + c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh{(x)}+c\sinh{(x)}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*cosh(x) + c*sinh(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+a}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+a}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh{(x)}+c\sinh{(x)}}}\,dx$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(1/2),x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(1/2), x)
```

3.765 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 156

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx=-\frac{2(c\cosh(x)+b\sinh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[Out] -2*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(1/2)-2*I*(c
os(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*Ell
ipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2
)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)/((a+b*cosh(x)+
c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3207, 3198, 2732}

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx=-\frac{2(b\sinh(x)+c\cosh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}-\frac{2i\sqrt{a+b\cosh(x)+c\sinh(x)}E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh
[x]]) - ((2*I)*EllipticE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(
a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)*S
qrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(c\cosh(x)+b\sinh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}+\frac{\int\sqrt{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{2(c\cosh(x)+b\sinh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}\\
&\quad+\frac{\sqrt{a+b\cosh(x)+c\sinh(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}\,dx}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}\\
&= -\frac{2(c\cosh(x)+b\sinh(x))}{(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}\\
&\quad-\frac{2iE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{(a^2-b^2+c^2)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.17

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \frac{2(ab+(b^2-c^2)\cosh(x)) - \frac{2b^3(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{2a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{a\cdots}{}\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] (2*(a*b + (b^2 - c^2)*Cosh[x]) - (2*b^3*(a + b*Cosh[x] + c*Sinh[x]))/(b^2 -
 c^2) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cosh[x] + c*Sinh[x])/(a +
I*Sqrt[1 - b^2/c^2]*c), (a + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2
]*c)]*Sech[x + ArcTanh[b/c]]*(a + b*Cosh[x] + c*Sinh[x])*Sqrt[-(((-I)*Sqrt[
1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a + I*Sqrt[1 - b^2/c^2]*c))]*Sqrt[
-((I*Sqrt[1 - b^2/c^2]*c + b*Cosh[x] + c*Sinh[x])/(a - I*Sqrt[1 - b^2/c^2]*
c))])/Sqrt[1 - b^2/c^2] + (b*c*Sinh[x + ArcTanh[c/b]])/Sqrt[1 - c^2/b^2] -
(b*c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Cosh[x] + c*Sinh[x])/(a + b*Sqr
t[1 - c^2/b^2]), (a + b*Cosh[x] + c*Sinh[x])/(a - b*Sqrt[1 - c^2/b^2])]*Sin
h[x + ArcTanh[c/b]])/(Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[1 - c^2/b^2] - b*Cosh[
x] - c*Sinh[x])/(a + b*Sqrt[1 - c^2/b^2])]*Sqrt[(b*Sqrt[1 - c^2/b^2] + b*Co
sh[x] + c*Sinh[x])/(-a + b*Sqrt[1 - c^2/b^2])]) + (c^2*((2*b^2*(a + b*Cosh[
x] + c*Sinh[x]))/(b^2 - c^2) - (c*Sinh[x + ArcTanh[c/b]])/Sqrt[1 - c^2/b^2]
 + (c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Cosh[x] + c*Sinh[x])/(a + b*Sq
rt[1 - c^2/b^2]), (a + b*Cosh[x] + c*Sinh[x])/(a - b*Sqrt[1 - c^2/b^2])]*Si
nh[x + ArcTanh[c/b]])/(Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[1 - c^2/b^2] - b*Cosh
[x] - c*Sinh[x])/(a + b*Sqrt[1 - c^2/b^2])]*Sqrt[(b*Sqrt[1 - c^2/b^2] + b*C
osh[x] + c*Sinh[x])/(-a + b*Sqrt[1 - c^2/b^2])])))/b)/(c*(a^2 - b^2 + c^2)*
Sqrt[a + b*Cosh[x] + c*Sinh[x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(177) = 354.

Time = 1.02 (sec) , antiderivative size = 888, normalized size of antiderivative = 5.69

method	result
default	$\frac{\sqrt{b^2-c^2}\,\operatorname{arctanh}\left(\frac{\left(b^2-c^2\right)\cosh(x)}{\sqrt{\left(a^2+b^2-c^2\right)\left(b^2-c^2\right)}}\right)}{\sqrt{\frac{-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}}{\sqrt{b^2-c^2}}}\,\sqrt{(a^2+b^2-c^2)(b^2-c^2)}} - \frac{\sqrt{\frac{\left(-b^2\sinh(x)+\sinh(x)c^2+a\sqrt{b^2-c^2}\right)\sinh(x)^2}{\sqrt{b^2-c^2}}}\,a\left(-\frac{\left(-b^2+c^2\right)\ln\left(-\frac{2\left(\sinh\right.}{}\right.}{}\right.}{}$

[In] int(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)*(b^2
-c^2)^(1/2)/((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*arctanh((b^2-c^2)*cosh(x)/((a^2
+b^2-c^2)*(b^2-c^2))^(1/2))-((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(
b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*a*(-1/2*(-b^2+c^2)/((a^2+b^2-c^2)*(b-c)*(b+
c))^(1/2)/(b^2-c^2)/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*ln((
-2*(sinh(x)*(b^2-c^2)^(1/2)-a)*a^2/(b^2-c^2)+2*(b^2*sinh(x)-sinh(x)*c^2-a*(
b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh(x)+
((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(b^2-c^2))+2*((-sinh(x)*(b^2-c^2)^(1/2)+a
)*a^2/(b^2-c^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c
^2)^(1/2)*sinh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(b^2
-c^2)))+1/2*(b^2-c^2)/((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2)/((-sinh(
x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*ln((-2*(sinh(x)*(b^2-c^2)^(1/2)-
a)*a^2/(b^2-c^2)-2*(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(b^2-c^2)^(3
/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^
(1/2)/(-b^2+c^2))+2*((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*((-b
^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))
/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2)))))/sinh(x)/((-b^2*si
nh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 798, normalized size of antiderivative = 5.12

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((2*sqrt(2)*a^2*cosh(x) + sqrt(2)*(a*b + a*c)*cosh(x)^2 + sqrt(2)*(a*b
+ a*c)*sinh(x)^2 + 2*(sqrt(2)*a^2 + sqrt(2)*(a*b + a*c)*cosh(x))*sinh(x) +
sqrt(2)*(a*b - a*c))*sqrt(b + c)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3
*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c
 + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c
)) - 3*(sqrt(2)*(b^2 + 2*b*c + c^2)*cosh(x)^2 + sqrt(2)*(b^2 + 2*b*c + c^2)
*sinh(x)^2 + 2*sqrt(2)*(a*b + a*c)*cosh(x) + 2*(sqrt(2)*(b^2 + 2*b*c + c^2)
*cosh(x) + sqrt(2)*(a*b + a*c))*sinh(x) + sqrt(2)*(b^2 - c^2))*sqrt(b + c)*
weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a
^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 +
 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3*(b +
c)*sinh(x) + 2*a)/(b + c))) - 6*((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b
*c + c^2)*sinh(x)^2 + (a*b + a*c)*cosh(x) + (a*b + a*c + 2*(b^2 + 2*b*c + c
^2)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a))/(a^2*b^2 - b^4 - c^4
 - (a^2 - 2*b^2)*c^2 + (a^2*b^2 - b^4 + a^2*c^2 + 2*b*c^3 + c^4 + 2*(a^2*b
- b^3)*c)*cosh(x)^2 + (a^2*b^2 - b^4 + a^2*c^2 + 2*b*c^3 + c^4 + 2*(a^2*b -
 b^3)*c)*sinh(x)^2 + 2*(a^3*b - a*b^3 + a*b*c^2 + a*c^3 + (a^3 - a*b^2)*c)*
cosh(x) + 2*(a^3*b - a*b^3 + a*b*c^2 + a*c^3 + (a^3 - a*b^2)*c + (a^2*b^2 -
 b^4 + a^2*c^2 + 2*b*c^3 + c^4 + 2*(a^2*b - b^3)*c)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x) + c*sinh(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(3/2),x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(3/2), x)
```

3.766 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}} dx$

Optimal result

Integrand size = 14, antiderivative size = 322

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}} dx = -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(x))}{3(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{8iaE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{3(a^2-b^2+c^2)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{3(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] -2/3*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(3/2)-8/3*
(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))^(1/2)-8/3
*I*a*(cos(1/2*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*
c))*EllipticE(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(
b^2-c^2)^(1/2)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)^2/((a+b
*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)+2/3*I*(cos(1/2*I*x-1/2*arcta
n(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1
/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a
+b*cosh(x)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)/(a+b*cosh(x)
+c*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \frac{2i\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\left(a^2-b^2+c^2\right)\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{8ia\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{3\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} - \frac{8(ab\sinh(x)+ac\cosh(x))}{3\left(a^2-b^2+c^2\right)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{2(b\sinh(x)+c\cosh(x))}{3\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^{3/2}}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(3*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(3/2)) - (8*(a*c*Cosh[x] + a*b*Sinh[x]))/(3*(a^2 - b^2 + c^2)^2*Sqrt[a +
 b*Cosh[x] + c*Sinh[x]]) - (((8*I)/3)*a*EllipticE[(I*x - ArcTan[b, (-I)*c])
/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[a + b*Cosh[x] + c*Sinh[
x]])/((a^2 - b^2 + c^2)^2*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2 -
c^2])]) + (((2*I)/3)*EllipticF[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c
^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cosh[x] + c*Sinh[x])/(a + Sqrt[b^2
- c^2])])/((a^2 - b^2 + c^2)*Sqrt[a + b*Cosh[x] + c*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
```

```
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\text{integral} = -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cosh(x)+\frac{1}{2}c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx}{3(a^2-b^2+c^2)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(x))}{3(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}(3a^2+b^2-c^2)+ab\cosh(x)+ac\sinh(x)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx}{3(a^2-b^2+c^2)^2}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(x))}{3(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} + \frac{(4a)\int \sqrt{a+b\cosh(x)+c\sinh(x)}\,dx}{3(a^2-b^2+c^2)^2} - \frac{\int \frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx}{3(a^2-b^2+c^2)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(x))}{3(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} + \frac{\left(4a\sqrt{a+b\cosh(x)+c\sinh(x)}\right)\int \sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}\,dx}{3(a^2-b^2+c^2)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} - \frac{\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}\,dx}{3(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{3(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{8(ac\cosh(x)+ab\sinh(x))}{3(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{8iaE\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{3(a^2-b^2+c^2)^2\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{3(a^2-b^2+c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.23 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.74

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((8*a*(b^2 - c^2))/(3*b*c*(a^2 - b^2 + c^2)
^2) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)*(a + b*
Cosh[x] + c*Sinh[x])^2) - (2*(-3*a^2*c - b^2*c + c^3 + 4*a*b^2*Sinh[x] - 4*
a*c^2*Sinh[x]))/(3*b*(-a^2 + b^2 - c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))) + (
2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((
-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-
1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sin
h[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)
/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqr
t[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((
-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x
 + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^2*Sqrt[I*(I + Sin
h[x + ArcTanh[b/c]])]) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqr
t[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sq
rt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c
]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + Arc
Tanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^
2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2
 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*
Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqr
t[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*Sqrt[1 - b^2/c^2]*c*(a^2 -
b^2 + c^2)^2*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (2*c*AppellF1[1/2, 1/2
, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1
 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c
^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2
/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sq
rt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[
b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I
*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 +
c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(3*S
qrt[1 - b^2/c^2]*(a^2 - b^2 + c^2)^2*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])])
- (4*a*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cos
h[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 +
```

```
a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqr
t[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]
)/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x +
ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh
[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1
 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b
]]]))/(3*c*(a^2 - b^2 + c^2)^2) + (4*a*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2
, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1
+ a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]
])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^
2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*S
qrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2]
+ b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/
b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c
^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 -
 c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(3*(a^2 - b^2 + c^2)^2)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2159 vs. $2(356) = 712$.

Time = 2.51 (sec) , antiderivative size = 2160, normalized size of antiderivative = 6.71

method	result	size
default	Expression too large to display	2160

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)*a*(b
^2-c^2)^(1/2)*(-1/2*cosh(x)/(a^2+b^2-c^2)/(sinh(x)^2*(b^2-c^2)-a^2)+1/2/(a^
2+b^2-c^2)/((a^2+b^2-c^2)*(b^2-c^2))^(1/2)*arctanh((b^2-c^2)*cosh(x)/((a^2+
b^2-c^2)*(b^2-c^2))^(1/2)))+((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(
b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)*(1/(b^2-c^2)*a^2/(2*a^2+2*b^2-2*c^2)*(1/(si
nh(x)*(b^2-c^2)^(1/2)-a)/a^2*(b^2-c^2)/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))
^(1/2)/(b^2-c^2))*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(
1/2)*sinh(x)^2)^(1/2)-(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(b^2-c^2)
^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(sinh(x)*(b^2-c^2)^(1/2)-a)/a^2/((
-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*ln((-2*(sinh(x)*(b^2-c^2)^
(1/2)-a)*a^2/(b^2-c^2)+2*(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(b^2-c
^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh(x)+((a^2+b^2-c^2)*(b-c)*(
b+c))^(1/2)/(b^2-c^2))+2*((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)
*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(
1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(b^2-c^2))))+(b^2-c^2)*a^2
/(2*a^2+2*b^2-2*c^2)/(-b^2+c^2)^2*(1/(sinh(x)*(b^2-c^2)^(1/2)-a)/a^2*(b^2-c
```

```
^2)/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2))*((-b^2*sinh(x)+s
inh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2)+(b^2*sinh(x)
-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*((a^2+b^2-c^2)*(b-c)*(b+c))
^(1/2)/(sinh(x)*(b^2-c^2)^(1/2)-a)/a^2/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b
^2-c^2))^(1/2)*ln((-2*(sinh(x)*(b^2-c^2)^(1/2)-a)*a^2/(b^2-c^2)-2*(b^2*sinh
(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+
c))^(1/2)*(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2))+2*((-sinh(
x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^
2-c^2)^(1/2))/(b^2-c^2)^(1/2)*sinh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-
c)*(b+c))^(1/2)/(-b^2+c^2))))-1/2*(a^2*b^2-c^2*a^2-((a^2+b^2-c^2)*(b-c)*(b+
c))^(1/2)*((a^2+b^2-c^2)*(b^2-c^2))^(1/2))/(a^2+b^2-c^2)/((a^2+b^2-c^2)*(b^
2-c^2))^(1/2)/(b^2-c^2)/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*
ln((-2*(sinh(x)*(b^2-c^2)^(1/2)-a)*a^2/(b^2-c^2)+2*(b^2*sinh(x)-sinh(x)*c^2
-a*(b^2-c^2)^(1/2))/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh
(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(b^2-c^2))+2*((-sinh(x)*(b^2-c^2)^(1/
2)+a)*a^2/(b^2-c^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b
^2-c^2)^(1/2)*sinh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/
(b^2-c^2)))+1/2*(-a^2*b^2+c^2*a^2+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*((a^2+b
^2-c^2)*(b^2-c^2))^(1/2))/(a^2+b^2-c^2)/((a^2+b^2-c^2)*(b^2-c^2))^(1/2)/(-b
^2+c^2)/((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-c^2))^(1/2)*ln((-2*(sinh(x)*
(b^2-c^2)^(1/2)-a)*a^2/(b^2-c^2)-2*(b^2*sinh(x)-sinh(x)*c^2-a*(b^2-c^2)^(1/
2))/(b^2-c^2)^(3/2)*((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)*(cosh(x)+((a^2+b^2-c^
2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2))+2*((-sinh(x)*(b^2-c^2)^(1/2)+a)*a^2/(b^2-
c^2))^(1/2)*((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2)*s
inh(x)^2)^(1/2))/(cosh(x)+((a^2+b^2-c^2)*(b-c)*(b+c))^(1/2)/(-b^2+c^2))))/s
inh(x)/((-b^2*sinh(x)+sinh(x)*c^2+a*(b^2-c^2)^(1/2))/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 3730, normalized size of antiderivative = 11.58

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^4 + 2*(a^2*b + 3*b
^3)*c)*cosh(x)^4 + sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^4 + 2
*(a^2*b + 3*b^3)*c)*sinh(x)^4 + 4*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2 - 3*
a*c^3 + (a^3 + 3*a*b^2)*c)*cosh(x)^3 + 4*(sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^
2 - 6*b*c^3 - 3*c^4 + 2*(a^2*b + 3*b^3)*c)*cosh(x) + sqrt(2)*(a^3*b + 3*a*b
^3 - 3*a*b*c^2 - 3*a*c^3 + (a^3 + 3*a*b^2)*c))*sinh(x)^3 + 2*sqrt(2)*(2*a^4
 + 7*a^2*b^2 + 3*b^4 + 3*c^4 - (7*a^2 + 6*b^2)*c^2)*cosh(x)^2 + 2*(3*sqrt(2
)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^4 + 2*(a^2*b + 3*b^3)*c)*cosh(
```

x)^2 + 6*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2 - 3*a*c^3 + (a^3 + 3*a*b^2)*c
)*cosh(x) + sqrt(2)*(2*a^4 + 7*a^2*b^2 + 3*b^4 + 3*c^4 - (7*a^2 + 6*b^2)*c^
2))*sinh(x)^2 + 4*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2 + 3*a*c^3 - (a^3 + 3
*a*b^2)*c)*cosh(x) + 4*(sqrt(2)*(a^2*b^2 + 3*b^4 + a^2*c^2 - 6*b*c^3 - 3*c^
4 + 2*(a^2*b + 3*b^3)*c)*cosh(x)^3 + 3*sqrt(2)*(a^3*b + 3*a*b^3 - 3*a*b*c^2
 - 3*a*c^3 + (a^3 + 3*a*b^2)*c)*cosh(x)^2 + sqrt(2)*(2*a^4 + 7*a^2*b^2 + 3*
b^4 + 3*c^4 - (7*a^2 + 6*b^2)*c^2)*cosh(x) + sqrt(2)*(a^3*b + 3*a*b^3 - 3*a
*b*c^2 + 3*a*c^3 - (a^3 + 3*a*b^2)*c))*sinh(x) + sqrt(2)*(a^2*b^2 + 3*b^4 +
 a^2*c^2 + 6*b*c^3 - 3*c^4 - 2*(a^2*b + 3*b^3)*c))*sqrt(b + c)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2), -8/27*(8*a^3 - 9*a
*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3*(b + c)*cosh(x) + 3
*(b + c)*sinh(x) + 2*a)/(b + c)) - 12*(sqrt(2)*(a*b^3 + 3*a*b^2*c + 3*a*b*c
^2 + a*c^3)*cosh(x)^4 + sqrt(2)*(a*b^3 + 3*a*b^2*c + 3*a*b*c^2 + a*c^3)*sin
h(x)^4 + 4*sqrt(2)*(a^2*b^2 + 2*a^2*b*c + a^2*c^2)*cosh(x)^3 + 4*(sqrt(2)*(
a*b^3 + 3*a*b^2*c + 3*a*b*c^2 + a*c^3)*cosh(x) + sqrt(2)*(a^2*b^2 + 2*a^2*b
*c + a^2*c^2))*sinh(x)^3 + 2*sqrt(2)*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (
2*a^3 + a*b^2)*c)*cosh(x)^2 + 2*(3*sqrt(2)*(a*b^3 + 3*a*b^2*c + 3*a*b*c^2 +
 a*c^3)*cosh(x)^2 + 6*sqrt(2)*(a^2*b^2 + 2*a^2*b*c + a^2*c^2)*cosh(x) + sqr
t(2)*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c))*sinh(x)^2 + 4
sqrt(2)(a^2*b^2 - a^2*c^2)*cosh(x) + 4*(sqrt(2)*(a*b^3 + 3*a*b^2*c + 3*a*
b*c^2 + a*c^3)*cosh(x)^3 + 3*sqrt(2)*(a^2*b^2 + 2*a^2*b*c + a^2*c^2)*cosh(x
)^2 + sqrt(2)*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(
x) + sqrt(2)*(a^2*b^2 - a^2*c^2))*sinh(x) + sqrt(2)*(a*b^3 - a*b^2*c - a*b*
c^2 + a*c^3))*sqrt(b + c)*weierstrassZeta(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2
+ 2*b*c + c^2), -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2
+ c^3), weierstrassPInverse(4/3*(4*a^2 - 3*b^2 + 3*c^2)/(b^2 + 2*b*c + c^2)
, -8/27*(8*a^3 - 9*a*b^2 + 9*a*c^2)/(b^3 + 3*b^2*c + 3*b*c^2 + c^3), 1/3*(3
*(b + c)*cosh(x) + 3*(b + c)*sinh(x) + 2*a)/(b + c))) - 6*(4*(a*b^3 + 3*a*b
^2*c + 3*a*b*c^2 + a*c^3)*cosh(x)^4 + 4*(a*b^3 + 3*a*b^2*c + 3*a*b*c^2 + a*
c^3)*sinh(x)^4 + (13*a^2*b^2 - b^4 + 13*a^2*c^2 + 2*b*c^3 + c^4 + 2*(13*a^2
*b - b^3)*c)*cosh(x)^3 + (13*a^2*b^2 - b^4 + 13*a^2*c^2 + 2*b*c^3 + c^4 + 2
*(13*a^2*b - b^3)*c + 16*(a*b^3 + 3*a*b^2*c + 3*a*b*c^2 + a*c^3)*cosh(x))*s
inh(x)^3 + 4*(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh(x
)^2 + (8*a^3*b + 4*a*b^3 - 4*a*b*c^2 - 4*a*c^3 + 24*(a*b^3 + 3*a*b^2*c + 3*
a*b*c^2 + a*c^3)*cosh(x)^2 + 4*(2*a^3 + a*b^2)*c + 3*(13*a^2*b^2 - b^4 + 13
*a^2*c^2 + 2*b*c^3 + c^4 + 2*(13*a^2*b - b^3)*c)*cosh(x))*sinh(x)^2 + (3*a^
2*b^2 + b^4 + c^4 - (3*a^2 + 2*b^2)*c^2)*cosh(x) + (3*a^2*b^2 + b^4 + c^4 +
 16*(a*b^3 + 3*a*b^2*c + 3*a*b*c^2 + a*c^3)*cosh(x)^3 - (3*a^2 + 2*b^2)*c^2
 + 3*(13*a^2*b^2 - b^4 + 13*a^2*c^2 + 2*b*c^3 + c^4 + 2*(13*a^2*b - b^3)*c)
cosh(x)^2 + 8(2*a^3*b + a*b^3 - a*b*c^2 - a*c^3 + (2*a^3 + a*b^2)*c)*cosh
(x))*sinh(x))*sqrt(b*cosh(x) + c*sinh(x) + a))/(a^4*b^3 - 2*a^2*b^5 + b^7 -
 b*c^6 + c^7 + (2*a^2 - 3*b^2)*c^5 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b^3 - 2*a
^2*b^5 + b^7 + 3*b*c^6 + c^7 + (2*a^2 + b^2)*c^5 + (6*a^2*b - 5*b^3)*c^4 +
(a^4 + 4*a^2*b^2 - 5*b^4)*c^3 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^2 + 3*(a^4*b^
2 - 2*a^2*b^4 + b^6)*c)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5 + b^7 + 3*b*c^6 +

```
c^7 + (2*a^2 + b^2)*c^5 + (6*a^2*b - 5*b^3)*c^4 + (a^4 + 4*a^2*b^2 - 5*b^4)
*c^3 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^2 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c)*s
inh(x)^4 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 +
 2*a*b*c^5 + a*c^6 + (2*a^3 - a*b^2)*c^4 + 4*(a^3*b - a*b^3)*c^3 + (a^5 - a
*b^4)*c^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*c)*cosh(x)^3 + 4*(a^5*b^2 - 2*a^3
*b^4 + a*b^6 + 2*a*b*c^5 + a*c^6 + (2*a^3 - a*b^2)*c^4 + 4*(a^3*b - a*b^3)*
c^3 + (a^5 - a*b^4)*c^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*c + (a^4*b^3 - 2*a^
2*b^5 + b^7 + 3*b*c^6 + c^7 + (2*a^2 + b^2)*c^5 + (6*a^2*b - 5*b^3)*c^4 + (
a^4 + 4*a^2*b^2 - 5*b^4)*c^3 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^2 + 3*(a^4*b^2
 - 2*a^2*b^4 + b^6)*c)*cosh(x))*sinh(x)^3 - (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2
 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 + 3*(
a^4 - b^4)*c^3 + 3*(a^4*b - b^5)*c^2 + (2*a^6 - 3*a^4*b^2 + b^6)*c)*cosh(x)
^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 + 3
*(a^4 - b^4)*c^3 + 3*(a^4*b - b^5)*c^2 + 3*(a^4*b^3 - 2*a^2*b^5 + b^7 + 3*b
*c^6 + c^7 + (2*a^2 + b^2)*c^5 + (6*a^2*b - 5*b^3)*c^4 + (a^4 + 4*a^2*b^2 -
 5*b^4)*c^3 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^2 + 3*(a^4*b^2 - 2*a^2*b^4 + b^
6)*c)*cosh(x)^2 + (2*a^6 - 3*a^4*b^2 + b^6)*c + 6*(a^5*b^2 - 2*a^3*b^4 + a*
b^6 + 2*a*b*c^5 + a*c^6 + (2*a^3 - a*b^2)*c^4 + 4*(a^3*b - a*b^3)*c^3 + (a^
5 - a*b^4)*c^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*c)*cosh(x))*sinh(x)^2 - (a^4
*b^2 - 2*a^2*b^4 + b^6)*c + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 - a*c^6 - (2*a^3
 - 3*a*b^2)*c^4 - (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2)*cosh(x) + 4*(a^5*b^2 - 2
*a^3*b^4 + a*b^6 - a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^4*b^3 - 2*a^2*b^5 + b
^7 + 3*b*c^6 + c^7 + (2*a^2 + b^2)*c^5 + (6*a^2*b - 5*b^3)*c^4 + (a^4 + 4*a
^2*b^2 - 5*b^4)*c^3 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^2 + 3*(a^4*b^2 - 2*a^2*
b^4 + b^6)*c)*cosh(x)^3 - (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + 3*(a^5*b^2 - 2*
a^3*b^4 + a*b^6 + 2*a*b*c^5 + a*c^6 + (2*a^3 - a*b^2)*c^4 + 4*(a^3*b - a*b^
3)*c^3 + (a^5 - a*b^4)*c^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*c)*cosh(x)^2 + (
2*a^6*b - 3*a^4*b^3 + b^7 + 3*b^3*c^4 + 3*b^2*c^5 - b*c^6 - c^7 + 3*(a^4 -
b^4)*c^3 + 3*(a^4*b - b^5)*c^2 + (2*a^6 - 3*a^4*b^2 + b^6)*c)*cosh(x))*sinh
(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(5/2),x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(5/2), x)
```

3.767 $\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 411

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\sinh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2i(23a^2+9b^2-9c^2)\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{15(a^2-b^2+c^2)^3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{16ia\,\mathrm{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{15(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{2(c(23a^2+9b^2-9c^2)\cosh(x)+b(23a^2+9b^2-9c^2)\sinh(x))}{15(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

```
[Out] -2/5*(c*cosh(x)+b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^(5/2)-16/1
5*(a*c*cosh(x)+a*b*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))^(3/2)-2
/15*(c*(23*a^2+9*b^2-9*c^2)*cosh(x)+b*(23*a^2+9*b^2-9*c^2)*sinh(x))/(a^2-b^
2+c^2)^3/(a+b*cosh(x)+c*sinh(x))^(1/2)-2/15*I*(23*a^2+9*b^2-9*c^2)*(cos(1/2
*I*x-1/2*arctan(b,-I*c))^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticE
(sin(1/2*I*x-1/2*arctan(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2
)))^(1/2))*(a+b*cosh(x)+c*sinh(x))^(1/2)/(a^2-b^2+c^2)^3/((a+b*cosh(x)+c*si
nh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)+16/15*I*a*(cos(1/2*I*x-1/2*arctan(b,-I*c)
)^2)^(1/2)/cos(1/2*I*x-1/2*arctan(b,-I*c))*EllipticF(sin(1/2*I*x-1/2*arctan
(b,-I*c)),2^(1/2)*((b^2-c^2)^(1/2)/(a+(b^2-c^2)^(1/2)))^(1/2))*((a+b*cosh(x
)+c*sinh(x))/(a+(b^2-c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh
(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \frac{16ia\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{2i(23a^2+9b^2-9c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}\,E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)}{15(a^2-b^2+c^2)^3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} - \frac{2(b\sinh(x)(23a^2+9b^2-9c^2)+c\cosh(x)(23a^2+9b^2-9c^2))}{15(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{16(ab\sinh(x)+ac\cosh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2(b\sinh(x)+c\cosh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}}$$

```
[In] Int[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

```
[Out] (-2*(c*Cosh[x] + b*Sinh[x]))/(5*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x
])^(5/2)) - (16*(a*c*Cosh[x] + a*b*Sinh[x]))/(15*(a^2 - b^2 + c^2)^2*(a + b
*Cosh[x] + c*Sinh[x])^(3/2)) - (((2*I)/15)*(23*a^2 + 9*b^2 - 9*c^2)*Ellipti
cE[(I*x - ArcTan[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*
Sqrt[a + b*Cosh[x] + c*Sinh[x]])/((a^2 - b^2 + c^2)^3*Sqrt[(a + b*Cosh[x] +
 c*Sinh[x])/(a + Sqrt[b^2 - c^2])]) + (((16*I)/15)*a*EllipticF[(I*x - ArcTa
n[b, (-I)*c])/2, (2*Sqrt[b^2 - c^2])/(a + Sqrt[b^2 - c^2])]*Sqrt[(a + b*Cos
h[x] + c*Sinh[x])/(a + Sqrt[b^2 - c^2])])/((a^2 - b^2 + c^2)^2*Sqrt[a + b*C
osh[x] + c*Sinh[x]]) - (2*(c*(23*a^2 + 9*b^2 - 9*c^2)*Cosh[x] + b*(23*a^2 +
 9*b^2 - 9*c^2)*Sinh[x]))/(15*(a^2 - b^2 + c^2)^3*Sqrt[a + b*Cosh[x] + c*Si
nh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\text{integral} = -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cosh(x)+\frac{3}{2}c\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{5/2}}\,dx}{5(a^2-b^2+c^2)}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\sinh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} + \frac{4\int \frac{\frac{3}{4}(5a^2+3b^2-3c^2)-2ab\cosh(x)-2ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^{3/2}}\,dx}{15(a^2-b^2+c^2)^2}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\sinh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2(c(23a^2+9b^2-9c^2)\cosh(x)+b(23a^2+9b^2-9c^2)\sinh(x))}{15(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{8\int \frac{-\frac{1}{8}a(15a^2+17b^2-17c^2)-\frac{1}{8}b(23a^2+9b^2-9c^2)\cosh(x)-\frac{1}{8}c(23a^2+9b^2-9c^2)\sinh(x)}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx}{15(a^2-b^2+c^2)^3}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\sinh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2(c(23a^2+9b^2-9c^2)\cosh(x)+b(23a^2+9b^2-9c^2)\sinh(x))}{15(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}} + \frac{(23a^2+9b^2-9c^2)\int\sqrt{a+b\cosh(x)+c\sinh(x)}\,dx}{15(a^2-b^2+c^2)^3} - \frac{(8a)\int\frac{1}{\sqrt{a+b\cosh(x)+c\sinh(x)}}\,dx}{15(a^2-b^2+c^2)^2}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\sinh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2(c(23a^2+9b^2-9c^2)\cosh(x)+b(23a^2+9b^2-9c^2)\sinh(x))}{15(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}} + \frac{\left((23a^2+9b^2-9c^2)\sqrt{a+b\cosh(x)+c\sinh(x)}\right)\int\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}{a+\sqrt{b^2-c^2}}}\,dx}{15(a^2-b^2+c^2)^3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} - \frac{\left(8a\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}\right)\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2-c^2}}+\frac{\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}{a+\sqrt{b^2-c^2}}}}\,dx}{15(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

$$= -\frac{2(c\cosh(x)+b\sinh(x))}{5(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^{5/2}} - \frac{16(ac\cosh(x)+ab\sinh(x))}{15(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))^{3/2}} - \frac{2i(23a^2+9b^2-9c^2)E\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic))\,|\,\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{a+b\cosh(x)+c\sinh(x)}}{15(a^2-b^2+c^2)^3\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}} + \frac{16ia\,\text{EllipticF}\left(\frac{1}{2}(ix-\tan^{-1}(b,-ic)),\frac{2\sqrt{b^2-c^2}}{a+\sqrt{b^2-c^2}}\right)\sqrt{\frac{a+b\cosh(x)+c\sinh(x)}{a+\sqrt{b^2-c^2}}}}{15(a^2-b^2+c^2)^2\sqrt{a+b\cosh(x)+c\sinh(x)}} - \frac{2(c(23a^2+9b^2-9c^2)\cosh(x)+b(23a^2+9b^2-9c^2)\sinh(x))}{15(a^2-b^2+c^2)^3\sqrt{a+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.42 (sec) , antiderivative size = 4093, normalized size of antiderivative = 9.96

$$\int\frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Sinh[x])^(-7/2),x]
```

```
[Out] Sqrt[a + b*Cosh[x] + c*Sinh[x]]*((-2*(23*a^2 + 9*b^2 - 9*c^2)*(b^2 - c^2))/
(15*b*c*(-a^2 + b^2 - c^2)^3) - (2*(a*c - b^2*Sinh[x] + c^2*Sinh[x]))/(5*b*
(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x])^3) - (2*(-5*a^2*c - 3*b^2*c
+ 3*c^3 + 8*a*b^2*Sinh[x] - 8*a*c^2*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x])^2) + (2*(-15*a^3*c - 17*a*b^2*c + 17*a*c^3 + 23*a
^2*b^2*Sinh[x] + 9*b^4*Sinh[x] - 23*a^2*c^2*Sinh[x] - 18*b^2*c^2*Sinh[x] +
```

9*c^4*Sinh[x]))/(15*b*(-a^2 + b^2 - c^2)^3*(a + b*Cosh[x] + c*Sinh[x]))) +
(2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x +
 ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), (
(-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(
-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Si
nh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2
)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sq
rt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(
(-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[
x + ArcTanh[b/c]]])/(Sqrt[1 - b^2/c^2]*c*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Si
nh[x + ArcTanh[b/c]])]) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, ((-I)*(a +
 Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(1 - (I*a)
/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh
[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c)]*Sech[x +
 ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2
)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/(I*a + c*Sqrt[(
-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] + I*c*Sqrt[(-b^2 + c^2)/c
^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[a + c
*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]]])/(15*Sqrt[1 - b^2/c^2]*c*(a
^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[b/c]])]) - (34*a*c*AppellF1[
1/2, 1/2, 1/2, 3/2, ((-I)*(a + Sqrt[1 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))
/(Sqrt[1 - b^2/c^2]*(1 - (I*a)/(Sqrt[1 - b^2/c^2]*c))*c), ((-I)*(a + Sqrt[1
 - b^2/c^2]*c*Sinh[x + ArcTanh[b/c]]))/(Sqrt[1 - b^2/c^2]*(-1 - (I*a)/(Sqrt
[1 - b^2/c^2]*c))*c)]*Sech[x + ArcTanh[b/c]]*Sqrt[-1 + I*Sinh[x + ArcTanh[b
/c]]]*Sqrt[(c*Sqrt[(-b^2 + c^2)/c^2] - I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x +
ArcTanh[b/c]])/(I*a + c*Sqrt[(-b^2 + c^2)/c^2])]*Sqrt[(c*Sqrt[(-b^2 + c^2)/
c^2] + I*c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]])/((-I)*a + c*Sqrt[
(-b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(-b^2 + c^2)/c^2]*Sinh[x + ArcTanh[b/c]
]])/(15*Sqrt[1 - b^2/c^2]*(a^2 - b^2 + c^2)^3*Sqrt[I*(I + Sinh[x + ArcTanh[
b/c]])]) - (23*a^2*b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 -
c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^
2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2
/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 -
c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + A
rcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^
2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^
2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(
a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x +
ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x +
 ArcTanh[c/b]]]))/(15*c*(a^2 - b^2 + c^2)^3) - (3*b^4*((c*AppellF1[-1/2, -1
/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 -
 c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x +
 ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[
x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*S
qrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]

```
*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2
 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt
[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b
]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a
 + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*c*(a^2 - b^2 + c^2)^3)
+ (23*a^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Co
sh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))),
(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 +
 a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sq
rt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]
])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x +
 ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cos
h[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[
1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b
]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/
b]]]))/(15*(a^2 - b^2 + c^2)^3) + (6*b^2*c*((c*AppellF1[-1/2, -1/2, -1/2, 1
/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(
1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/
b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c^2/b^2])))]*Sinh[x + ArcTanh
[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] - b*Sqrt[(b^2 -
c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(b^2 - c^2)/b^2])]*Sqrt[a + b
*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*Sqrt[(b*Sqrt[(b^2 - c^2)/b^2
] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(-a + b*Sqrt[(b^2 - c^2
)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]))/(b^2 -
 c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]))/Sqrt[a + b*Sqrt[1
 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*(a^2 - b^2 + c^2)^3) - (3*c^3*((c*
AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c
/b]])/(b*Sqrt[1 - c^2/b^2]*(1 + a/(b*Sqrt[1 - c^2/b^2]))), (a + b*Sqrt[1 -
c^2/b^2]*Cosh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*(-1 + a/(b*Sqrt[1 - c
^2/b^2])))]*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 - c^2/b^2]*Sqrt[(b*Sqrt[(b^2
- c^2)/b^2] - b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]])/(a + b*Sqrt[(
b^2 - c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/b]]]*
Sqrt[(b*Sqrt[(b^2 - c^2)/b^2] + b*Sqrt[(b^2 - c^2)/b^2]*Cosh[x + ArcTanh[c/
b]])/(-a + b*Sqrt[(b^2 - c^2)/b^2])]) - ((-2*b*(a + b*Sqrt[1 - c^2/b^2]*Cos
h[x + ArcTanh[c/b]]))/(b^2 - c^2) + (c*Sinh[x + ArcTanh[c/b]])/(b*Sqrt[1 -
c^2/b^2]))/Sqrt[a + b*Sqrt[1 - c^2/b^2]*Cosh[x + ArcTanh[c/b]]]))/(5*(a^2 -
 b^2 + c^2)^3)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 57908 vs. 2(441) = 882.

Time = 8.22 (sec) , antiderivative size = 57909, normalized size of antiderivative = 140.90

method	result	size
default	Expression too large to display	57909

```
[In] int(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 13897, normalized size of antiderivative = 33.81

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \int \frac{1}{(b\cosh(x)+c\sinh(x)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(x)+c*sinh(x))^(7/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + c*sinh(x) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx = \int \frac{1}{(a+b\cosh(x)+c\sinh(x))^{7/2}}\,dx$$

```
[In] int(1/(a + b*cosh(x) + c*sinh(x))^(7/2),x)
[Out] int(1/(a + b*cosh(x) + c*sinh(x))^(7/2), x)
```

3.768 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 140

$$\begin{aligned}
\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2} dx &= \frac{64(b^2-c^2)\,(c\cosh(x)+b\sinh(x))}{15\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}} \\
&+ \frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} \\
&+ \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}
\end{aligned}$$

```
[Out] 2/5*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2)+64/15
*(b^2-c^2)*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2
)+16/15*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3192, 3191}

$$\begin{aligned}
&\int \left(\sqrt{b^2-c^2}+b\cosh(x)\right. \\
&\left.+c\sinh(x)\right)^{5/2} dx = \frac{2}{5}(b\sinh(x)+c\cosh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} \\
&+ \frac{16}{15}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} \\
&+ \frac{64(b^2-c^2)\,(b\sinh(x)+c\cosh(x))}{15\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}
\end{aligned}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[
x] + c*Sinh[x]]) + (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rule steps

$$\begin{aligned}
\text{integral} &= \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} \\
&\quad+\frac{1}{5}\left(8\sqrt{b^2-c^2}\right)\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}dx \\
&= \frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} \\
&\quad+\frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} \\
&\quad+\frac{1}{15}\left(32\left(b^2-c^2\right)\right)\int\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx \\
&= \frac{64(b^2-c^2)\,(c\cosh(x)+b\sinh(x))}{15\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}} \\
&\quad+\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} \\
&\quad+\frac{2}{5}(c\cosh(x)+b\sinh(x))\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 56.31 (sec) , antiderivative size = 4500, normalized size of antiderivative = 32.14

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Sqrt[b^2 - c^2]*((4*b*Sqrt[b^2 - c^2])/(3*c) + (4*c*Cosh[x])/3 + (4*b*Sinh[
x])/3)*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] + Sqrt[Sqrt[b^2 - c^2]
 + b*Cosh[x] + c*Sinh[x]]*((44*b*(b^2 - c^2))/(15*c) + (2*c*Sqrt[b^2 - c^2]
*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 + (2*b*Sqrt[b^2 - c^2]*Sinh[x])/15 + ((b
^2 + c^2)*Sinh[2*x])/5) + (256*b*(-b + c)*(b + c)^2*Sqrt[b^2 - c^2]*(Ellipt
icF[ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + S
qrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, ArcSin[Sqrt[-((
(-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1
 + Tanh[x/2])))]], 1])*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x]]*
(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b +
 c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-c + (-b + Sqrt[b^2 - c^2])*Tanh
[x/2]))/(15*(b + c - Sqrt[b^2 - c^2])^2*(b + c + Sqrt[b^2 - c^2])*(1 + Cosh
[x])*Sqrt[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*
Sqrt[(-1 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2
) - b*(1 + Tanh[x/2]^2))]) + (128*(b - c)^2*(b + c)^2*Sqrt[Sqrt[(b - c)*(b
+ c)] + b*Cosh[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 - 2*b^2*c^2*Sqr
t[b^2 - c^2] - 3*b*c^3*Sqrt[b^2 - c^2] - c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh
[x/2] + 12*b^3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*
c^5*Tanh[x/2] - 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] - 12*b^2*c^2*Sqrt[b^2 - c
^2]*Tanh[x/2] - 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] + 2*c^4*Sqrt[b^2 - c^2]*T
anh[x/2] + 8*b^5*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2
 - 11*b^2*c^3*Tanh[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 - 8*b^4*S
qrt[b^2 - c^2]*Tanh[x/2]^2 - 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 + 5*b*c^3
*Sqrt[b^2 - c^2]*Tanh[x/2]^2 + c^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 8*b^4*c*El
lipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-
b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((-b - c + Sqrt[b^
2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
+ 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + T
anh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((-b
 - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 +
Tanh[x/2])))] + 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b -
 c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Ta
nh[x/2])))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b +
 c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^3*Sqrt[b^2 - c^2]*Ellipti
cPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c
```

+ Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c
^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*
b^5*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]
))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((
-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1
+ Tanh[x/2])))] + 8*b^4*c*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2
- c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]],
1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c +
Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 20*b^3*c^2*EllipticPi[-1, ArcSin[Sq
rt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2
])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1
+ Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^2*c^3*
EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/(
(-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b -
c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Ta
nh[x/2])))] - 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^
2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*T
anh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqr
t[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, A
rcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b
^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 -
c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*
b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^
2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*T
anh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqr
t[b^2 - c^2])*(-1 + Tanh[x/2])))] - 12*b^2*c^2*Sqrt[b^2 - c^2]*EllipticPi[-
1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sq
rt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^
2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
+ 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2
- c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]],
1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c +
Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-
(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(
-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 +
Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 20*b^3*c^2*E
llipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((
-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b
- c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + T
anh[x/2])))] + 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c
^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*
Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c +
Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1
, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqr
t[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b

```
^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
 + 12*b^2*c^2*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[
b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))
]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b
 + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 2*c*EllipticE[ArcSin[Sqrt[-((
(-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1
 + Tanh[x/2])))]], 1]*(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(
1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh
[x/2] + c^3*(Sqrt[b^2 - c^2] + c*Tanh[x/2]) - b^2*c*(2*Sqrt[b^2 - c^2] + 5*
c*Tanh[x/2]) + b^3*(2*c - 4*Sqrt[b^2 - c^2]*Tanh[x/2]) + b*c^2*(-2*c + 3*Sq
rt[b^2 - c^2]*Tanh[x/2])) + 2*b*EllipticF[ArcSin[Sqrt[-(((-b - c + Sqrt[b^2
 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]],
 1]*(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((
-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-4*b^4*Tanh[x/2] - c^3*(Sqrt
[b^2 - c^2] + c*Tanh[x/2]) + b^2*c*(2*Sqrt[b^2 - c^2] + 5*c*Tanh[x/2]) + b*
c^2*(2*c - 3*Sqrt[b^2 - c^2]*Tanh[x/2]) + b^3*(-2*c + 4*Sqrt[b^2 - c^2]*Tan
h[x/2]))))/(15*c*Sqrt[b^2 - c^2]*(b + c - Sqrt[b^2 - c^2])^2*(-b + Sqrt[b^2
 - c^2])*(-b + c + Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(Sqrt[(b - c)*(b + c
)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*Sqrt[(-1 + Tanh[x/2]^2)*(-2*c*
Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) - b*(1 + Tanh[x/2]^2))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(120) = 240$.

Time = 0.60 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.06

method	result
default	$-\frac{(b-c)^2(b+c)^2\left(\frac{\cosh(x)^3}{3}+2\cosh(x)\right)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}\,\sqrt{(b-c)(b+c)}}-\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,\sqrt{b^2-c^2}\,(-b^2+c^2)\left(-\frac{\cosh(x)\sqrt{b^2-c^2}\,\sqrt{-\sqrt{b^2-}}}{2\left(\sinh(x)b^2-\sin\right.}\right.}{\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh}{\sqrt{b^2-}}}}$

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*(b-c)^2*(b+c)
^2/((b-c)*(b+c))^(1/2)*(1/3*cosh(x)^3+2*cosh(x))-(-(b^2-c^2)^(1/2)*(sinh(x)
-1)*sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*(-b^2+c^2)*(-1/2*cosh(x)/(sinh(x)*b^2-
sinh(x)*c^2-b^2+c^2)*(b^2-c^2)^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^
2)^(1/2)+1/2/((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(s
inh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)))/s
inh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. 2(120) = 240.

Time = 0.32 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.60

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*b^2
*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 + 125*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(
x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2*c - b*c^2
 + c^3)*cosh(x))*sinh(x) + 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 55*(b^2 +
2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)^5 - 150*(b^
2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15*b^2 + 15*c^2
)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2 - c^2)*cosh(x)
)*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*b*c + c^2)*cosh
(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^2)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*
sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(
x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 + (b + c)*sinh(x)^4 -
 (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*sinh(x)^2 + 2*(2*(b + c)
*cosh(x)^3 - (b - c)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1783 vs. 2(120) = 240.

Time = 2.15 (sec) , antiderivative size = 1783, normalized size of antiderivative = 12.74

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c +
 sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e
^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 -
 c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^
2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt
(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*
e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/12*sqrt(2)*(b^3 +
b^2*c - b*c^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) +
 b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b
 - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(
-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*
x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b
^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x)
+ (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(sqrt(b + c)*sqrt
(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x
) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2
+ 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^
2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*s
qrt(b - c)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqr
t(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt
(b - c)*c^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/2*sqr
t(2)*(b^3 - b^2*c - b*c^2 + c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c
)*e^(-2*x) + b + c)^(5/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(
b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c
^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c
)*c^2)*e^(-2*x) + 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*
sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c
^2)*e^(-4*x) + (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)*(sq
rt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqr
t(b - c)*c^2)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)
^(5/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*
b*c + sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) +
10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 1
```

```
0*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 - 2
*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) + (b^3
 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 1/20*sqrt(2)*(b^3 - 3*b^2*c + 3*b*c
^2 - c^3)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(5/
2)*e^(-5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c
+ sqrt(b + c)*sqrt(b - c)*c^2 + 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x) + 10*(
sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) + 10*(b
^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqr
t(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) + (b^3 - 3
*b^2*c + 3*b*c^2 - c^3)*e^(-5*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx =$$
$$-\frac{\sqrt{2}\left(150\left(b^2-c^2\right)^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)+3\left(\sqrt{b^2-c^2}b^2+2\sqrt{b^2-c^2}bc+\sqrt{b^2-c^2}c^2\right)e^{\left(\frac{5}{2}x\right)}\operatorname{sgn}(\right.}{}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(150*(b^2 - c^2)^(3/2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b
 + c) + 3*(sqrt(b^2 - c^2)*b^2 + 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*c^
2)*e^(5/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 25*(b^3 + b^2*c - b*c^2 -
c^3)*e^(3/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - (25*(b^2 - 2*b*c + c^2)*
sqrt(b^2 - c^2)*e^x*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 150*(b^3 - b^2*c -
b*c^2 + c^3)*e^(2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) + 3*(b^3 - 3*b^2*c +
 3*b*c^2 - c^3)*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-5/2*x))/sqrt(b - c)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \int \left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}\,dx$$

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

```
[Out] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.769 $\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}$$

```
[Out] 8/3*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1
/2))^(1/2)+2/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3192, 3191}

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{2}{3}\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}(b\sinh(x)+c\cosh(x))+\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[Sqrt[b^2 - c^2] + b*Cos
h[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[Sqrt[b^2 - c^2] + b*Co
sh[x] + c*Sinh[x]])/3
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} \\ &\quad+\frac{1}{3}\left(4\sqrt{b^2-c^2}\right)\int\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx \\ &= \frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}} \\ &\quad+\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.93 (sec) , antiderivative size = 4392, normalized size of antiderivative = 47.74

$$\int\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b^2 - c^2]*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c + ((2
*b*Sqrt[b^2 - c^2])/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[Sqrt[b^
2 - c^2] + b*Cosh[x] + c*Sinh[x]] + (32*b*(-b + c)*(b + c)^2*(EllipticF[Arc
Sin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2
 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c
 + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh
[x/2])))]], 1])*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x]]*(-1 + T
anh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sq
```

rt[b^2 - c^2])*(-1 + Tanh[x/2]))))]*(-c + (-b + Sqrt[b^2 - c^2])*Tanh[x/2]))
/(3*(b + c - Sqrt[b^2 - c^2])^2*(b + c + Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqr
t[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*Sqrt[(-1
 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) - b*(1
 + Tanh[x/2]^2))]) + (16*(b - c)*(b + c)*Sqrt[Sqrt[(b - c)*(b + c)] + b*Cos
h[x] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 - 2*b^2*c^2*Sqrt[b^2 - c^2]
- 3*b*c^3*Sqrt[b^2 - c^2] - c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x/2] + 12*b^
3*c^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c^5*Tanh[x/2]
 - 8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] - 12*b^2*c^2*Sqrt[b^2 - c^2]*Tanh[x/2]
 - 2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] + 2*c^4*Sqrt[b^2 - c^2]*Tanh[x/2] + 8*
b^5*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2 - 11*b^2*c^3
*Tanh[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 - 8*b^4*Sqrt[b^2 - c^2
]*Tanh[x/2]^2 - 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 + 5*b*c^3*Sqrt[b^2 - c
^2]*Tanh[x/2]^2 + c^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 8*b^4*c*EllipticPi[-1,
ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[
b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1
+ Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^2*c^3*E
llipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((
-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((-b - c + Sqrt[b
^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
 + 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2
 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]],
 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2
 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSi
n[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 -
 c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tan
h[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^5*EllipticP
i[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c +
 Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt
[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))
)] + 8*b^4*c*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 +
Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*
Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c
^2])*(-1 + Tanh[x/2])))] + 20*b^3*c^2*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c
 + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh
[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2])
)/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^2*c^3*EllipticPi[-1
, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqr
t[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2
 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] -
 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt
[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])
*(-1 + Tanh[x/2])))] + 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(
((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-

1 + Tanh[x/2]))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Ta
nh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^3*c*Sqrt[b^
2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt
[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])
*(-1 + Tanh[x/2])))] - 12*b^2*c^2*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqr
t[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2]
)*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1
+ Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 4*b*c^3*Sqr
t[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 +
Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*
Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c
^2])*(-1 + Tanh[x/2])))] + 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + S
qrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2
])))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/
((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 20*b^3*c^2*EllipticPi[-1,
 ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt
[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^
2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
+ 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tan
h[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*S
qrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^
2])*(-1 + Tanh[x/2])))] - 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt
[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])
*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1
 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 12*b^2*c^2
*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(
1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x
/2]^2*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b
^2 - c^2])*(-1 + Tanh[x/2])))] + 2*c*EllipticE[ArcSin[Sqrt[-(((-b - c + Sqr
t[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])
))]], 1]*(-1 + Tanh[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]
))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + c^3*(
Sqrt[b^2 - c^2] + c*Tanh[x/2]) - b^2*c*(2*Sqrt[b^2 - c^2] + 5*c*Tanh[x/2])
+ b^3*(2*c - 4*Sqrt[b^2 - c^2]*Tanh[x/2]) + b*c^2*(-2*c + 3*Sqrt[b^2 - c^2]
*Tanh[x/2])) + 2*b*EllipticF[ArcSin[Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 +
 Tanh[x/2]))/((-b + c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*(-1 + Tan
h[x/2])*Sqrt[-(((-b - c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((-b + c + Sqrt
[b^2 - c^2])*(-1 + Tanh[x/2])))]*(-4*b^4*Tanh[x/2] - c^3*(Sqrt[b^2 - c^2] +
 c*Tanh[x/2]) + b^2*c*(2*Sqrt[b^2 - c^2] + 5*c*Tanh[x/2]) + b*c^2*(2*c - 3*
Sqrt[b^2 - c^2]*Tanh[x/2]) + b^3*(-2*c + 4*Sqrt[b^2 - c^2]*Tanh[x/2]))))/(3
c(b + c - Sqrt[b^2 - c^2])^2*(-b + Sqrt[b^2 - c^2])*(-b + c + Sqrt[b^2 -
c^2])*(1 + Cosh[x])*Sqrt[(Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1
 + Cosh[x])^2]*Sqrt[(-1 + Tanh[x/2]^2)*(-2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-
1 + Tanh[x/2]^2) - b*(1 + Tanh[x/2]^2))])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(78) = 156$.

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{2(-b^2+c^2)\cosh(x)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}} + \frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,(b^2-c^2)\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}$	189

```
[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-b^2+c^2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)*cos
h(x)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)/((b^2-c^2)^(1
/2)*(sinh(x)-1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*cosh(x)/(
-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(
x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(78) = 156$.

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.58

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x) +c\sinh(x)\right)^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left((b^2+2\,bc+c^2)\cosh\left(x\right)^4+4\,(b^2+2\,bc+c^2)\cosh\left(x\right)\sinh\left(x\right)^3+(b^2+2\,bc+c^2)\,\mathrm{si}\right.}{}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 + 6*
((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c + c
^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x) + 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b -
 c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(
((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt
(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh
(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) +
 (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

Sympy [F]

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\int \left(b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(78) = 156$.

Time = 0.53 (sec) , antiderivative size = 640, normalized size of antiderivative = 6.96

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c\right)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-\cdots}\right.}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-\cdots}\right.\right.}+\frac{3\sqrt{2}(b^2-c^2)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}+(b^2-\cdots\right.}-\frac{3\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(-\frac{1}{2}\cdots\right.}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}+(b^2-\cdots\right.}-\frac{\sqrt{2}(b^2-2bc+c^2)\left(2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c+3(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}+(b^2-\cdots\right.}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima"
)
[Out] 1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(2*sqrt
(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(sqr
t(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) +
 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2
- 2*b*c + c^2)*e^(-3*x)) + 3/2*sqrt(2)*(b^2 - c^2)*(2*sqrt(b + c)*sqrt(b -
c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b -
 c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)*s
qrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b^2 - 2*b*c + c^2)*e^
(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*
c)*(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-
1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^
2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2
*x) + (b^2 - 2*b*c + c^2)*e^(-3*x)) - 1/6*sqrt(2)*(b^2 - 2*b*c + c^2)*(2*sq
```

```
rt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-3/2*x)/(
sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c + 3*(b^2 - c^2)*e^(-x
) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) + (b
^2 - 2*b*c + c^2)*e^(-3*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.99

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} dx = -\frac{\sqrt{2}\Big(\big(\sqrt{b^2-c^2}b+\sqrt{b^2-c^2}c\big)e^{\left(\frac{3}{2}x\right)}\mathrm{sgn}\big(-\sqrt{b^2-c^2}e^x-b+c\big)+9\left(b^2-c^2\right)e^{\left(\frac{1}{2}x\right)}\mathrm{sgn}\big(-\sqrt{b^2-c^2}e^x-b+c\big)}{6\sqrt{b}}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((sqrt(b^2 - c^2)*b + sqrt(b^2 - c^2)*c)*e^(3/2*x)*sgn(-sqrt(b
^2 - c^2)*e^x - b + c) + 9*(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x -
 b + c) - (9*sqrt(b^2 - c^2)*(b - c)*e^x*sgn(-sqrt(b^2 - c^2)*e^x - b + c)
+ (b^2 - 2*b*c + c^2)*sgn(-sqrt(b^2 - c^2)*e^x - b + c))*e^(-3/2*x))/sqrt(b
 - c)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} dx = \int \left(b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2} dx$$

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

```
[Out] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.770 $\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 2*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3191}

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[In] Int[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 31.80 (sec) , antiderivative size = 455, normalized size of antiderivative = 12.30

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{4(b-c)(b+c)^2\left(2b^3-2bc^2-2b^2\sqrt{b^2-c^2}+c^2\sqrt{b^2-c^2}+b\left(-2b^2+c^2+2b\sqrt{b^2-c^2}\right)\cosh(x)-2b^2c\sin\right.}{}$$

[In] Integrate[Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]

[Out] (4*(b - c)*(b + c)^2*(2*b^3 - 2*b*c^2 - 2*b^2*Sqrt[b^2 - c^2] + c^2*Sqrt[b^2 - c^2] + b*(-2*b^2 + c^2 + 2*b*Sqrt[b^2 - c^2])*Cosh[x] - 2*b^2*c*Sinh[x] + c^3*Sinh[x] + 2*b*c*Sqrt[b^2 - c^2]*Sinh[x] + EllipticE[ArcSin[Sqrt[((-b - c + Sqrt[b^2 - c^2])*(Cosh[x] + Sinh[x]))/(-b + c + Sqrt[b^2 - c^2])]], 1]*(-Cosh[x/2] + Sinh[x/2])*(c*(-2*b^2 + c*(c - Sqrt[b^2 - c^2]) + b*(c + 2*Sqrt[b^2 - c^2]))*Cosh[x/2] + (-4*b^3 + b*c*(3*c - 2*Sqrt[b^2 - c^2]) - c^2*(c + Sqrt[b^2 - c^2]) + 2*b^2*(c + 2*Sqrt[b^2 - c^2]))*Sinh[x/2])*Sqrt[((-b - c + Sqrt[b^2 - c^2])*(Cosh[x] + Sinh[x]))/(-b + c + Sqrt[b^2 - c^2])]))/(Sqrt[b^2 - c^2]*(b + c - Sqrt[b^2 - c^2])^2*(-b^2 + c^2 + b*Sqrt[b^2 - c^2])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(33) = 66$.

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

method	result
risch	$\frac{\sqrt{2}\,\sqrt{\left(e^{2x}b+e^{2x}c+2\sqrt{b^2-c^2}\,e^{x}+b-c\right)e^{-x}}\,\left(e^{x}b+e^{x}c-\sqrt{b^2-c^2}\right)\left(e^{x}b+e^{x}c+\sqrt{b^2-c^2}\right)}{\left(e^{2x}b+e^{2x}c+2\sqrt{b^2-c^2}\,e^{x}+b-c\right)(b+c)}$
default	$\frac{(-b^2+c^2)\cosh(x)}{\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,\sqrt{b^2-c^2}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}$

[In] int((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2^(1/2)*((exp(2*x)*b+exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)+b-c)*exp(-x))^(1/2)/(exp(2*x)*b+exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)+b-c)*(exp(x)*b+exp(x)*c-(b^2-c^2)^(1/2))*(exp(x)*b+exp(x)*c+(b^2-c^2)^(1/2))/(b+c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+c)\sinh(x)^2-2\sqrt{b^2-c^2}(\cosh(x)+\sinh(x))+\right.}{(b+c)\cosh(x)^2+2\,(b+c)\cosh(x)\sinh(x)+(b+}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x)^
2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh
(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b + c)
*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

Sympy [F]

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}}\,dx$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(33) = 66.

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.14

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2\,x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}\,x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\sqrt{b-c}} - \frac{\sqrt{2}\sqrt{2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2\,x)}+b+c}(b-c)e^{\left(-\frac{1}{2}\,x\right)}}{(b-c)e^{(-x)}+\sqrt{b+c}\sqrt{b-c}}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)*s
qrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
 - sqrt(2)*sqrt(2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c
)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) + sqrt(b + c)*sqrt(b - c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \frac{\sqrt{2}\left((b-c)e^{\left(-\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)-\sqrt{b^2-c^2}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x-b+c\right)\right)}{\sqrt{b-c}}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*((b - c)*e^(-1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c) - sqrt(b^2 -
c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x - b + c))/sqrt(b - c)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh(x)+\sqrt{b^2-c^2}+c\sinh(x)}\,dx$$

```
[In] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
```

```
[Out] int((b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.771 $\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[Out] arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)^(1/2
)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))*2^(1/2)/(b^2-c^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3194, 2728, 212}

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[In] Int[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*
Sqrt[Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(b
^2 - c^2)^(1/4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx$$

$$= 2i\text{Subst}\left(\int \frac{1}{2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)$$

$$= \frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{\sqrt[4]{b^2-c^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 49.81 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{\sqrt{b^2-c^2}-b\cosh(x)-c\sinh(x)}{\sqrt{b^2-c^2}}}}{\sqrt{2}}\right),1\right)\left(b^2-c^2+b\sqrt{b^2-c^2}\cosh(x)+c\sqrt{b^2-c^2}\sinh(x)\right)}{\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[In] Integrate[1/Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
[Out] -((Sqrt[2]*EllipticF[ArcSin[Sqrt[(Sqrt[b^2 - c^2] - b*Cosh[x] - c*Sinh[x])/
Sqrt[b^2 - c^2]]/Sqrt[2]], 1]*(b^2 - c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*Sq
rt[b^2 - c^2]*Sinh[x])*Sqrt[-((-b^2 + c^2 + b*Sqrt[b^2 - c^2]*Cosh[x] + c*S
qrt[b^2 - c^2]*Sinh[x])/(b^2 - c^2))])/(Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh
[x])*Sqrt[Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)-1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}}$	129

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)-1)
)^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2
)*(sinh(x)-1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)
/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 681, normalized size of antiderivative = 6.88

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(2)*sqrt(-1/sqrt(b^2 - c^2))*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(
b^2 + 2*b*c + c^2)*cosh(x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*sinh
(x)^2 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(
x)^4 - 2*sqrt(2)*sqrt(1/2)*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cosh(x)
*sinh(x) + 2*(b^2 - c^2)*sinh(x)^2 - ((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)
*sinh(x)^2 + (b + c)*sinh(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b
 - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)
*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b -
c)/(cosh(x) + sinh(x)))*sqrt(-1/sqrt(b^2 - c^2)) - b^2 + 2*b*c - c^2 - 2*((
b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c
)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))*sqrt(b^2 - c^2))/((b^2 +
 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 +
2*b*c + c^2)*sinh(x)^4 - 2*(b^2 - c^2)*cosh(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)
*cosh(x)^2 - b^2 + c^2)*sinh(x)^2 + b^2 - 2*b*c + c^2 + 4*((b^2 + 2*b*c + c
^2)*cosh(x)^3 - (b^2 - c^2)*cosh(x))*sinh(x))), -2*sqrt(2)*arctan(sqrt(2)*s
qrt(1/2)*(sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) - b + c)*sqrt(((b + c)*cosh(x
)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2 - c^2)*(co
```

```
sh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/(((b + c)*cosh(x)^2 + 2*(b +
 c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)*(b^2 - c^2)^(1/4)))/(b^2 -
 c^2)^(1/4)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(80)=160$.

Time = 0.49 (sec) , antiderivative size = 538, normalized size of antiderivative = 5.43

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx= -\frac{2\sqrt{2}(b^2-c^2-b+c)\sqrt{b+c}\arctan\left(\frac{}{\sqrt{\sqrt{b^2-c^2}b^5+\sqrt{b^2-c^2}b^4c-2\sqrt{b^2-c^2}b^3c^2-2\sqrt{b^2-}}}\right.}{\sqrt{\sqrt{b^2-c^2}b^5+\sqrt{b^2-c^2}b^4c-2\sqrt{b^2-c^2}b^3c^2-2\sqrt{b^2-c^2}b^2c^3+\sqrt{b^2-c^2}bc^4+\sqrt{b^2-c^2}c^5-2\sqrt{b^2-}}}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac"
)
[Out] -2*sqrt(2)*(b^2 - c^2 - b + c)*sqrt(b + c)*arctan((b^3*e^(1/2*x) + b^2*c*e^
(1/2*x) - b*c^2*e^(1/2*x) - c^3*e^(1/2*x) - b^2*e^(1/2*x) + c^2*e^(1/2*x))/
sqrt(sqrt(b^2 - c^2)*b^5 + sqrt(b^2 - c^2)*b^4*c - 2*sqrt(b^2 - c^2)*b^3*c^
```

```
2 - 2*sqrt(b^2 - c^2)*b^2*c^3 + sqrt(b^2 - c^2)*b*c^4 + sqrt(b^2 - c^2)*c^5
 - 2*sqrt(b^2 - c^2)*b^4 + 4*sqrt(b^2 - c^2)*b^2*c^2 - 2*sqrt(b^2 - c^2)*c^
4 + sqrt(b^2 - c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqrt(b^2 - c^2)*b*c^2 + s
qrt(b^2 - c^2)*c^3))/(sqrt(sqrt(b^2 - c^2)*b^5 + sqrt(b^2 - c^2)*b^4*c - 2*
sqrt(b^2 - c^2)*b^3*c^2 - 2*sqrt(b^2 - c^2)*b^2*c^3 + sqrt(b^2 - c^2)*b*c^4
 + sqrt(b^2 - c^2)*c^5 - 2*sqrt(b^2 - c^2)*b^4 + 4*sqrt(b^2 - c^2)*b^2*c^2
- 2*sqrt(b^2 - c^2)*c^4 + sqrt(b^2 - c^2)*b^3 - sqrt(b^2 - c^2)*b^2*c - sqr
t(b^2 - c^2)*b*c^2 + sqrt(b^2 - c^2)*c^3)*sgn(-sqrt(b^2 - c^2)*e^x - b + c)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)}}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.772 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}+\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

```
[Out] 1/4*arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)^
(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(3/4)*2^(1
/2)+1/2*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^2
)^(1/2))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3195, 3194, 2728, 212}

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{2\sqrt{2}\left(b^2-c^2\right)^{3/4}}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[Sqrt
[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]*(
b^2 - c^2)^(3/4)) + (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(Sqrt[b^2 -
c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int\frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx}{4\sqrt{b^2-c^2}}\\
&= \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}+\frac{\int\frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\,dx}{4\sqrt{b^2-c^2}}\\
&= \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\\
&\quad+\frac{i\text{Subst}\left(\int\frac{1}{2\sqrt{b^2-c^2}-x^2}\,dx,x,-\frac{i\sqrt{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{b^2-c^2}}
\end{aligned}$$

$$= \frac{\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\,(b^2-c^2)^{3/4}} + \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
[Out] $Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(128) = 256$.

Time = 0.24 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.69

method	result
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{2\sqrt{b^2-c^2}\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2-b^2+c^2}{\sqrt{b^2-c^2}}}} + \frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)-1)\sinh(x)^2}\,\sqrt{b^2-c^2}\,\sqrt{2}\left(\ln\left(-\frac{2\left(\cosh(x)\sqrt{b^2-c^2}\,\sqrt{2}\,\sinh(x)-\sinh\right.}{}\right.\right.}{}$

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1
/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))+1/4*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*
sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*2^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(b^2-
c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^(1/2)
*(sinh(x)-1))^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)-cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(b^2-c^2
)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^(1/2)*(s
inh(x)-1))^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2)*(sinh(x
)-1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(126) = 252$.

Time = 0.43 (sec) , antiderivative size = 1801, normalized size of antiderivative = 11.62

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="frica
s")
[Out] 1/2*((sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*sqrt(2)*(b^3 +
3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + sqrt(2)*(b^3 + 3*b^2*c + 3*b*c
^2 + c^3)*sinh(x)^6 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 3*(
5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 - sqrt(2)*(b^3 + b^2*c
- b*c^2 - c^3))*sinh(x)^4 + 4*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*co
sh(x)^3 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*sqrt
(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 3*(5*sqrt(2)*(b^3 + 3*b^2*c + 3
*b*c^2 + c^3)*cosh(x)^4 - 6*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2 +
 sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3))*sinh(x)^2 + 6*(sqrt(2)*(b^3 + 3*b^2*c
 + 3*b*c^2 + c^3)*cosh(x)^5 - 2*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)
^3 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) - sqrt(2)*(b^3 -
3*b^2*c + 3*b*c^2 - c^3))*(b^2 - c^2)^(1/4)*arctan(-sqrt(1/2)*(sqrt(2)*(b +
 c)*cosh(x) + sqrt(2)*(b + c)*sinh(x) - sqrt(2)*sqrt(b^2 - c^2))*(b^2 - c^2
)^(1/4)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(
x)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/
((b^2 + 2*b*c + c^2)*cosh(x)^2 + 2*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x) + (b
^2 + 2*b*c + c^2)*sinh(x)^2 - b^2 + c^2)) - 2*sqrt(1/2)*(4*(b^3 + b^2*c - b
*c^2 - c^3)*cosh(x)^4 + 16*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)*sinh(x)^3 +
4*(b^3 + b^2*c - b*c^2 - c^3)*sinh(x)^4 + 4*(b^3 - b^2*c - b*c^2 + c^3)*cos
h(x)^2 + 4*(b^3 - b^2*c - b*c^2 + c^3 + 6*(b^3 + b^2*c - b*c^2 - c^3)*cosh(
x)^2)*sinh(x)^2 + 8*(2*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + (b^3 - b^2*c
 - b*c^2 + c^3)*cosh(x))*sinh(x) - ((b^2 + 2*b*c + c^2)*cosh(x)^5 + 5*(b^2
+ 2*b*c + c^2)*cosh(x)*sinh(x)^4 + (b^2 + 2*b*c + c^2)*sinh(x)^5 + 6*(b^2 -
 c^2)*cosh(x)^3 + 2*(5*(b^2 + 2*b*c + c^2)*cosh(x)^2 + 3*b^2 - 3*c^2)*sinh(
x)^3 + 2*(5*(b^2 + 2*b*c + c^2)*cosh(x)^3 + 9*(b^2 - c^2)*cosh(x))*sinh(x)^
2 + (b^2 - 2*b*c + c^2)*cosh(x) + (5*(b^2 + 2*b*c + c^2)*cosh(x)^4 + 18*(b^
2 - c^2)*cosh(x)^2 + b^2 - 2*b*c + c^2)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b
+ c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 + 2*sqrt(b^2
 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x))))/((b^5 + 3*b^4*c
+ 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^6 + 6*(b^5 + 3*b^4*c + 2*b
^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)*sinh(x)^5 + (b^5 + 3*b^4*c + 2*
b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*sinh(x)^6 - b^5 + 3*b^4*c - 2*b^3*c^2
- 2*b^2*c^3 + 3*b*c^4 - c^5 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^
4 + c^5)*cosh(x)^4 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5 -
```

```
 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*sinh(
x)^4 + 4*(5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)
^3 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x))*sinh(x)
^3 + 3*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x)^2 + 3*(b
^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5 + 5*(b^5 + 3*b^4*c + 2*b^3
*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^4 - 6*(b^5 + b^4*c - 2*b^3*c^2 -
2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*sinh(x)^2 + 6*((b^5 + 3*b^4*c + 2*b^3*c
^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^5 - 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*
b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 + (b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b
*c^4 - c^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh{(x)}+c\sinh{(x)}+\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxim
a")
[Out] integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{8,[4,0]%%%}+%%%{16,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{-8,[2,1
]%%%}+
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.773 $\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx=\frac{3\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}}+\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}+\frac{3(c\cosh(x)+b\sinh(x))}{16\left(b^2-c^2\right)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

```
[Out] 3/32*arctan(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/((b^2-c^2)
^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(5/4)*2^(
1/2)+1/4*(c*cosh(x)+b*sinh(x))/(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)+(b^2-c^
2)^(1/2))^(5/2)+3/16*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)+(
b^2-c^2)^(1/2))^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

$= \{3195, 3194, 2728, 212\}$

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\,(b^2-c^2)^{5/4}} + \frac{3(b\sinh(x)+c\cosh(x))}{16\,(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} + \frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}$$

```
[In] Int[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (3*ArcTan[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[S
qrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*Sqrt
[2]*(b^2 - c^2)^(5/4)) + (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(Sqrt[b
^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))/(16
*(b^2 - c^2)*(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
&\text{integral} \\
&= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx}{8\sqrt{b^2-c^2}} \\
&= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} + \frac{3\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx}{32\,(b^2-c^2)} \\
&= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} \\
&\quad + \frac{3\int \frac{1}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\,dx}{32\,(b^2-c^2)} \\
&= \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} \\
&\quad + \frac{(3i)\text{Subst}\left(\int \frac{1}{2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\,(b^2-c^2)} \\
&= \frac{3\arctan\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\,(b^2-c^2)^{5/4}} \\
&\quad + \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}
\end{aligned}
$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] $Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(172) = 344$.

Time = 0.27 (sec) , antiderivative size = 817, normalized size of antiderivative = 3.99

method	result	size
default	Expression too large to display	817

```
[In] int(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b^2-c^2)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2+c^2)/(b^2-c^2)^(1/2))^(1/2)/(b^4
-2*b^2*c^2+c^4)*(-1/4*cosh(x)/(cosh(x)^2-2)+1/8*2^(1/2)*arctanh(1/2*cosh(x)
*2^(1/2)))+(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(b^2-c^2)*(1/4/(b
-c)^2/(b+c)^2*(1/(b^2-c^2)^(1/2)/(sinh(x)-1)/(cosh(x)-2^(1/2))*(-(b^2-c^2)^
(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)+2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(
1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)-1)-2*(sinh(x)-1)*2^(1/2)*(b^2-c^2)^(1/
2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^
2-c^2)^(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)-2^(1/2))))+1/4/(b-c)^2/(b+c)^2*(1
/(b^2-c^2)^(1/2)/(sinh(x)-1)/(cosh(x)+2^(1/2))*(-(b^2-c^2)^(1/2)*(sinh(x)-1
)*sinh(x)^2)^(1/2)-2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*ln((-2*(b^2
-c^2)^(1/2)*(sinh(x)-1)+2*(sinh(x)-1)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)+2^(1
/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^(1/2)*(si
nh(x)-1))^(1/2))/(cosh(x)+2^(1/2))))-1/8/(b-c)^2/(b+c)^2*2^(1/2)/(-(b^2-c^2
)^(1/2)*(sinh(x)-1))^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)-1)-2*(sinh(x)-1)
*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*
sinh(x)^2)^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2))/(cosh(x)-2^(1/2)))+1
/8/(b-c)^2/(b+c)^2*2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)-1))^(1/2)*ln((-2*(b^2
-c^2)^(1/2)*(sinh(x)-1)+2*(sinh(x)-1)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)+2^(1
/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)-1)*sinh(x)^2)^(1/2)*(-(b^2-c^2)^(1/2)*(si
nh(x)-1))^(1/2))/(cosh(x)+2^(1/2))))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2-b^2
+c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5297 vs. $2(170) = 340$.

Time = 1.55 (sec) , antiderivative size = 5297, normalized size of antiderivative = 25.84

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="frica
s")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b**2-c**2)**(1/2))**(5/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) + sqrt(b**2 - c**2))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh(x)+c\sinh(x)+\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxim
a")
[Out] integrate((b*cosh(x) + c*sinh(x) + sqrt(b^2 - c^2))^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)+(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac"
)
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{32,[5,0]%%%}+%%%{96,[4,1]%%%}+%%%{-32,[4,0]%%%}+%%%{64,[3
,2]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)+\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
[Out] int(1/(b*cosh(x) + (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.774 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 146

$$\begin{aligned}\int \Big(-\sqrt{b^2-c^2}+b\cosh(x) &+ c\sinh(x)\Big)^{5/2}\,dx = \frac{64(b^2-c^2)\left(c\cosh(x)+b\sinh(x)\right)}{15\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\\ &-\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\\ &+\frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\end{aligned}$$

```
[Out] 2/5*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2)+64/15
*(b^2-c^2)*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2
)-16/15*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)*(b*cosh(x)+c*sinh(x)-(b^2-c^2
)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used

$= \{3192, 3191\}$

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2} dx = \frac{2}{5}(b\sinh(x)+c\cosh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} - \frac{16}{15}\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} + \frac{64(b^2-c^2)\,(b\sinh(x)+c\cosh(x))}{15\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] (64*(b^2 - c^2)*(c*Cosh[x] + b*Sinh[x]))/(15*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh
[x] + c*Sinh[x]]) - (16*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[
b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/15 + (2*(c*Cosh[x] + b*Sinh[x])*(-Sqrt
[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))/5
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} - \frac{1}{5}\left(8\sqrt{b^2-c^2}\right)\int\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} dx \\ &= -\frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2} + \frac{1}{15}\left(32\left(b^2-c^2\right)\right)\int\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx \end{aligned}$$

$$= \frac{64(b^2-c^2)\,(c\cosh(x)+b\sinh(x))}{15\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}} - \frac{16}{15}\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} + \frac{2}{5}(c\cosh(x)+b\sinh(x))\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 56.15 (sec) , antiderivative size = 4368, normalized size of antiderivative = 29.92

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2} dx = \text{Result too large to show}$$

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2),x]
```

```
[Out] Sqrt[b^2 - c^2]*((4*b*Sqrt[b^2 - c^2])/(3*c) - (4*c*Cosh[x])/3 - (4*b*Sinh[
x])/3)*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] + Sqrt[-Sqrt[b^2 - c^
2] + b*Cosh[x] + c*Sinh[x]]*((44*b*(b^2 - c^2))/(15*c) - (2*c*Sqrt[b^2 - c^
2]*Cosh[x])/15 + (2*b*c*Cosh[2*x])/5 - (2*b*Sqrt[b^2 - c^2]*Sinh[x])/15 + (
(b^2 + c^2)*Sinh[2*x])/5) + (256*b*c*(-b + c)*(b + c)*Sqrt[b^2 - c^2]*(-b^2
 + c^2)*(EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2])
)/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, Ar
cSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2
- c^2])*(-1 + Tanh[x/2])))]], 1])*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh[x] +
 c*Sinh[x]]*(-1 + Tanh[x/2])*(-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))
/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))^(3/2)*(c + (b + Sqrt[b^2 -
c^2])*Tanh[x/2])*(-1 + Tanh[x/2]^2))/(15*(b + c + Sqrt[b^2 - c^2])^3*(-b^2
+ c^2 + b*Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(b + c)] + b*C
osh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*(1 + Tanh[x/2])^2*Sqrt[-((-1 + Tanh[x/
2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) + b*(1 + Tanh[x/2
]^2)))]) - (128*(b - c)^2*(b + c)^2*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh[x]
 + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 + 2*b^2*c^2*Sqrt[b^2 - c^2] + 3*
b*c^3*Sqrt[b^2 - c^2] + c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x/2] + 12*b^3*c^
2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c^5*Tanh[x/2] + 8
*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] + 12*b^2*c^2*Sqrt[b^2 - c^2]*Tanh[x/2] + 2
*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] - 2*c^4*Sqrt[b^2 - c^2]*Tanh[x/2] + 8*b^5*
Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2 - 11*b^2*c^3*Tan
h[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 + 8*b^4*Sqrt[b^2 - c^2]*Ta
nh[x/2]^2 + 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 5*b*c^3*Sqrt[b^2 - c^2]*
Tanh[x/2]^2 - c^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 8*b^4*c*EllipticPi[-1, ArcS
in[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 -
c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[
```

x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^4*c*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 20*b^3*c^2*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 12*b^2*c^2*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 20*b^3*c^2*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/(

```
(b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((b +
 c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tan
h[x/2])))] + 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2]
)*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh
[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b
^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcS
in[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 -
c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2]
)*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 12*b^2*
c^2*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])
*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[
x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^
2 - c^2])*(-1 + Tanh[x/2])))] + 2*c*EllipticE[ArcSin[Sqrt[-(((b + c + Sqrt[
b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
], 1]*(-1 + Tanh[x/2])*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/(
(b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + b^2*c*(2*S
qrt[b^2 - c^2] - 5*c*Tanh[x/2]) + c^3*(-Sqrt[b^2 - c^2] + c*Tanh[x/2]) + 2*
b^3*(c + 2*Sqrt[b^2 - c^2]*Tanh[x/2]) - b*c^2*(2*c + 3*Sqrt[b^2 - c^2]*Tanh
[x/2])) - 2*b*EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[
x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*(-1 + Tanh[x/2])
*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^
2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + b^2*c*(2*Sqrt[b^2 - c^2] - 5*c*T
anh[x/2]) + c^3*(-Sqrt[b^2 - c^2] + c*Tanh[x/2]) + 2*b^3*(c + 2*Sqrt[b^2 -
c^2]*Tanh[x/2]) - b*c^2*(2*c + 3*Sqrt[b^2 - c^2]*Tanh[x/2]))))/(15*c*Sqrt[b
^2 - c^2]*(b + Sqrt[b^2 - c^2])*(b - c + Sqrt[b^2 - c^2])*(b + c + Sqrt[b^2
 - c^2])^2*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[
x])/(1 + Cosh[x])^2]*Sqrt[-((-1 + Tanh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 -
c^2]*(-1 + Tanh[x/2]^2) + b*(1 + Tanh[x/2]^2)))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(126) = 252$.

Time = 0.60 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{(b-c)(b+c)}\,(b-c)(b+c)\left(\frac{\cosh(x)^3}{3}+2\cosh(x)\right)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\left(\frac{\left(b^2-c^2\right)^2\cosh(x)\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\,\mathrm{si}}}{2\sinh(x)b^2-2\sinh(x)c^2+2b^2-2c^2}\right.}{\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(}{\sqrt{b^2-}}}}$

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*((b-c)*(b+c))
^(1/2)*(b-c)*(b+c)*(1/3*cosh(x)^3+2*cosh(x))+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
```

```
sinh(x)^2)^(1/2)*(1/2*(b^2-c^2)^2*cosh(x)/(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)
*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)-1/2*(b^2-c^2)^(3/2)/((b^2-c
^2)^(1/2)*(sinh(x)+1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cos
h(x)/(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)))/sinh(x)/(-(sinh(x)*b^
2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(126) = 252$.

Time = 0.33 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.37

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*sqrt(1/2)*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 18*(b^3 + 3*b
^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + 3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*sinh(x)^6 + 125*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 5*(25*b^3 + 25*b^2
*c - 25*b*c^2 - 25*c^3 + 9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2)*sinh(
x)^4 + 20*(3*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^3 + 25*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*b^3 - 9*b^2*c + 9*b*c^2 - 3*c^3 + 125*(
b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 5*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)
*cosh(x)^4 + 25*b^3 - 25*b^2*c - 25*b*c^2 + 25*c^3 + 150*(b^3 + b^2*c - b*c
^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 2*(9*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(
x)^5 + 250*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^3 + 125*(b^3 - b^2*c - b*c^2
 + c^3)*cosh(x))*sinh(x) - 2*(11*(b^2 + 2*b*c + c^2)*cosh(x)^5 + 55*(b^2 +
2*b*c + c^2)*cosh(x)*sinh(x)^4 + 11*(b^2 + 2*b*c + c^2)*sinh(x)^5 - 150*(b^
2 - c^2)*cosh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^2 - 15*b^2 + 15*c^2
)*sinh(x)^3 + 10*(11*(b^2 + 2*b*c + c^2)*cosh(x)^3 - 45*(b^2 - c^2)*cosh(x)
)*sinh(x)^2 + 11*(b^2 - 2*b*c + c^2)*cosh(x) + (55*(b^2 + 2*b*c + c^2)*cosh
(x)^4 - 450*(b^2 - c^2)*cosh(x)^2 + 11*b^2 - 22*b*c + 11*c^2)*sinh(x))*sqrt
(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*
sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(
x)))/((b + c)*cosh(x)^4 + 4*(b + c)*cosh(x)*sinh(x)^3 + (b + c)*sinh(x)^4 -
 (b - c)*cosh(x)^2 + (6*(b + c)*cosh(x)^2 - b + c)*sinh(x)^2 + 2*(2*(b + c)
*cosh(x)^3 - (b - c)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. 2(126) = 252.

Time = 2.28 (sec) , antiderivative size = 1789, normalized size of antiderivative = 12.25

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\text{Too large to display}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxima"
)
[Out] 1/20*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c +
 sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*
e^(-2*x) + b + c)^(5/2)*e^(5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b +
 c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2
- c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c
^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqr
t(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)
*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)*(b^3 +
 b^2*c - b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x)
 + b + c)^(5/2)*e^(3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt
(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e
^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-
2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)
*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x
) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*sqrt(2)*(sqrt(b + c)*sq
rt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^
(-x) + (b - c)*e^(-2*x) + b + c)^(5/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b - c)*b
^2 + 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 +
 b^2*c - b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c
)*sqrt(b - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(
sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*s
qrt(b - c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 5/2*
sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b
 - c)*e^(-2*x) + b + c)^(5/2)*e^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*s
```

```
qrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c -
 b*c^2 - c^3)*e^(-x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b
 - c)*c^2)*e^(-2*x) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b +
 c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b -
c)*c^2)*e^(-4*x) - (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) - 5/12*sqrt(2)
*(sqrt(b + c)*sqrt(b - c)*b^2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)
*sqrt(b - c)*c^2)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
 + c)^(5/2)*e^(-3/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b
- c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-
x) + 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x
) - 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^
2 - 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) -
 (b^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x)) + 1/20*sqrt(2)*(b^3 - 3*b^2*c +
3*b*c^2 - c^3)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b +
c)^(5/2)*e^(-5/2*x)/(sqrt(b + c)*sqrt(b - c)*b^2 + 2*sqrt(b + c)*sqrt(b - c
)*b*c + sqrt(b + c)*sqrt(b - c)*c^2 - 5*(b^3 + b^2*c - b*c^2 - c^3)*e^(-x)
+ 10*(sqrt(b + c)*sqrt(b - c)*b^2 - sqrt(b + c)*sqrt(b - c)*c^2)*e^(-2*x) -
 10*(b^3 - b^2*c - b*c^2 + c^3)*e^(-3*x) + 5*(sqrt(b + c)*sqrt(b - c)*b^2 -
 2*sqrt(b + c)*sqrt(b - c)*b*c + sqrt(b + c)*sqrt(b - c)*c^2)*e^(-4*x) - (b
^3 - 3*b^2*c + 3*b*c^2 - c^3)*e^(-5*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(126) = 252$.

Time = 0.32 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.16

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx =$$

$$-\frac{\sqrt{2}\Big(150\,(b^2-c^2)^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\mathrm{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+3\left(\sqrt{b^2-c^2}b^2+2\sqrt{b^2-c^2}bc+\sqrt{b^2-c^2}c^2\right)e^{\left(\frac{5}{2}\,x\right)}\mathrm{sgn}\Big(}{}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(150*(b^2 - c^2)^(3/2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b
 - c) + 3*(sqrt(b^2 - c^2)*b^2 + 2*sqrt(b^2 - c^2)*b*c + sqrt(b^2 - c^2)*c^
2)*e^(5/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 25*(b^3 + b^2*c - b*c^2 -
c^3)*e^(3/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - (25*(b^2 - 2*b*c + c^2)*
sqrt(b^2 - c^2)*e^x*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 150*(b^3 - b^2*c -
b*c^2 + c^3)*e^(2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) - 3*(b^3 - 3*b^2*c +
 3*b*c^2 - c^3)*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-5/2*x))/sqrt(b - c)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}\,dx=\int \left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{5/2}\,dx$$

```
[In] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
[Out] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.775 $\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=-\frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}+\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}$$

```
[Out] -8/3*(c*cosh(x)+b*sinh(x))*(b^2-c^2)^(1/2)/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(
1/2))^(1/2)+2/3*(c*cosh(x)+b*sinh(x))*(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3192, 3191}

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{2}{3}(b\sinh(x)+c\cosh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}-\frac{8\sqrt{b^2-c^2}(b\sinh(x)+c\cosh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (-8*Sqrt[b^2 - c^2]*(c*Cosh[x] + b*Sinh[x]))/(3*Sqrt[-Sqrt[b^2 - c^2] + b*C
osh[x] + c*Sinh[x]]) + (2*(c*Cosh[x] + b*Sinh[x])*Sqrt[-Sqrt[b^2 - c^2] + b
*Cosh[x] + c*Sinh[x]])/3
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)} \\ &\quad -\frac{1}{3}\left(4\sqrt{b^2-c^2}\right)\int\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx \\ &= -\frac{8\sqrt{b^2-c^2}(c\cosh(x)+b\sinh(x))}{3\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}} \\ &\quad +\frac{2}{3}(c\cosh(x)+b\sinh(x))\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 31.28 (sec) , antiderivative size = 4260, normalized size of antiderivative = 44.38

$$\int\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b^2 - c^2]*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c + (
(-2*b*Sqrt[b^2 - c^2])/(3*c) + (2*c*Cosh[x])/3 + (2*b*Sinh[x])/3)*Sqrt[-Sqr
t[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]] - (32*b*c*(-b + c)*(b + c)*(-b^2 + c^
2)*(EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b
 - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*EllipticPi[-1, ArcSin[
Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2
])*(-1 + Tanh[x/2])))]], 1])*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Si
nh[x]]*(-1 + Tanh[x/2])*(-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b
```

- c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))^(3/2)*(c + (b + Sqrt[b^2 - c^2])
Tanh[x/2])(-1 + Tanh[x/2]^2))/(3*(b + c + Sqrt[b^2 - c^2])^3*(-b^2 + c^2
+ b*Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(b + c)] + b*Cosh[x]
 + c*Sinh[x])/(1 + Cosh[x])^2]*(1 + Tanh[x/2])^2*Sqrt[-((-1 + Tanh[x/2]^2)*
(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) + b*(1 + Tanh[x/2]^2)))
]) + (16*(b - c)*(b + c)*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x
]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 + 2*b^2*c^2*Sqrt[b^2 - c^2] + 3*b*c^3*Sqrt[
b^2 - c^2] + c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x/2] + 12*b^3*c^2*Tanh[x/2]
 - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c^5*Tanh[x/2] + 8*b^3*c*Sqrt
[b^2 - c^2]*Tanh[x/2] + 12*b^2*c^2*Sqrt[b^2 - c^2]*Tanh[x/2] + 2*b*c^3*Sqrt
[b^2 - c^2]*Tanh[x/2] - 2*c^4*Sqrt[b^2 - c^2]*Tanh[x/2] + 8*b^5*Tanh[x/2]^2
 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2 - 11*b^2*c^3*Tanh[x/2]^2 -
2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 + 8*b^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 +
 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 5*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2]^2
 - c^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 8*b^4*c*EllipticPi[-1, ArcSin[Sqrt[-((
(b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 +
 Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b
- c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^2*c^3*EllipticPi[-1, ArcSi
n[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c
^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x
/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^3*c*Sqrt[b^2 - c
^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2])
)/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt
[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))
] + 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2
 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]],
1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 -
c^2])*(-1 + Tanh[x/2])))] - 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + S
qrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]
)))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b
- c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^4*c*EllipticPi[-1, ArcSin[
Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2
])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1
+ Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 20*b^3*c^2*E
llipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b
 - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c +
 Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/
2])))] - 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*
(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x
/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 -
 c^2])*(-1 + Tanh[x/2])))] - 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((b + c +
 Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/
2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((
b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^4*Sqrt[b^2 - c^2]*Ellip
ticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c

```
 + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqr
t[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))
)] + 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^
2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]],
 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c +
Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 12*b^2*c^2*Sqrt[b^2 - c^2]*EllipticP
i[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + S
qrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^
2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] -
 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 -
c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*
Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt
[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-(((b
+ c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Ta
nh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/
2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 20*b^3*c^2*EllipticPi
[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sq
rt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b
^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
+ 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqr
t[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*
(-1 + Tanh[x/2])))] + 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-((
(b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 +
 Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 12*b^2*c^2*Sqrt[b^
2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[
x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Tanh[x/2]^2*Sqrt
[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(
-1 + Tanh[x/2])))] + 2*c*EllipticE[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])
*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*(-1 +
 Tanh[x/2])*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sq
rt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + b^2*c*(2*Sqrt[b^2 - c
^2] - 5*c*Tanh[x/2]) + c^3*(-Sqrt[b^2 - c^2] + c*Tanh[x/2]) + 2*b^3*(c + 2*
Sqrt[b^2 - c^2]*Tanh[x/2]) - b*c^2*(2*c + 3*Sqrt[b^2 - c^2]*Tanh[x/2])) - 2
*b*EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b
- c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*(-1 + Tanh[x/2])*Sqrt[-(((b
 + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + T
anh[x/2])))]*(4*b^4*Tanh[x/2] + b^2*c*(2*Sqrt[b^2 - c^2] - 5*c*Tanh[x/2]) +
 c^3*(-Sqrt[b^2 - c^2] + c*Tanh[x/2]) + 2*b^3*(c + 2*Sqrt[b^2 - c^2]*Tanh[x
/2]) - b*c^2*(2*c + 3*Sqrt[b^2 - c^2]*Tanh[x/2]))))/(3*c*(b + Sqrt[b^2 - c^
2])*(b - c + Sqrt[b^2 - c^2])*(b + c + Sqrt[b^2 - c^2])^2*(1 + Cosh[x])*Sqr
t[(-Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*Sqrt[-(
(-1 + Tanh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) + b*
(1 + Tanh[x/2]^2)))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(82) = 164$.

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.97

method	result	size
default	$\frac{2(b^2-c^2)\cosh(x)}{\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}+\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\,(b^2-c^2)\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$	189

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b^2-c^2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)*cosh
(x)+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*(b^2-c^2)/((b^2-c^2)^(1/
2)*(sinh(x)+1))^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-
(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x
)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.43

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)^4+4\left(b^2+2\,bc+c^2\right)\cosh\left(x\right)\sinh\left(x\right)^3+\left(b^2+2\,bc+c^2\right)\sin\right.}{}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*sqrt(1/2)*((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 18*(b^2 - c^2)*cosh(x)^2 + 6*
((b^2 + 2*b*c + c^2)*cosh(x)^2 - 3*b^2 + 3*c^2)*sinh(x)^2 + b^2 - 2*b*c + c
^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - 9*(b^2 - c^2)*cosh(x))*sinh(x) - 8*
((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 + (b -
 c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 - c^2))*sqrt(
((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt
(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh
(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) +
 (3*(b + c)*cosh(x)^2 - b + c)*sinh(x))
```

Sympy [F]

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\int \left(b\cosh\left(x\right)+c\sinh\left(x\right)-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(82) = 164$.

Time = 0.47 (sec) , antiderivative size = 644, normalized size of antiderivative = 6.71

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx=\frac{\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c\right)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)\right.}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b\right.\right.}}$$
$$-\frac{3\sqrt{2}(b^2-c^2)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}-(b^2\right.}$$
$$-\frac{3\sqrt{2}\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{(}}{2\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}-(b^2\right.}$$
$$+\frac{\sqrt{2}(b^2-2\,bc+c^2)\left(-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c\right)^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}}{6\left(\sqrt{b+c}\sqrt{b-c}b+\sqrt{b+c}\sqrt{b-c}c-3\,(b^2-c^2)e^{(-x)}+3\left(\sqrt{b+c}\sqrt{b-c}b-\sqrt{b+c}\sqrt{b-c}c\right)e^{(-2x)}-(b^2\right.}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxima"
)
[Out] 1/6*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c)*(-2*sqr
t(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(3/2*x)/(sq
rt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x)
+ 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2
 - 2*b*c + c^2)*e^(-3*x)) - 3/2*sqrt(2)*(b^2 - c^2)*(-2*sqrt(b + c)*sqrt(b
- c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(1/2*x)/(sqrt(b + c)*sqrt(b
 - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e^(-x) + 3*(sqrt(b + c)
*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x) - (b^2 - 2*b*c + c^2)*
e^(-3*x)) - 3/2*sqrt(2)*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c
)*c)*(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e
^(-1/2*x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 -
 c^2)*e^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^
(-2*x) - (b^2 - 2*b*c + c^2)*e^(-3*x)) + 1/6*sqrt(2)*(b^2 - 2*b*c + c^2)*(-
```

```
2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)^(3/2)*e^(-3/2*
x)/(sqrt(b + c)*sqrt(b - c)*b + sqrt(b + c)*sqrt(b - c)*c - 3*(b^2 - c^2)*e
^(-x) + 3*(sqrt(b + c)*sqrt(b - c)*b - sqrt(b + c)*sqrt(b - c)*c)*e^(-2*x)
- (b^2 - 2*b*c + c^2)*e^(-3*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.92

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx =$$
$$-\frac{\sqrt{2}\left(\left(\sqrt{b^2-c^2}b+\sqrt{b^2-c^2}c\right)e^{\left(\frac{3}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)-9\left(b^2-c^2\right)e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)}{6\sqrt{b-}}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((sqrt(b^2 - c^2)*b + sqrt(b^2 - c^2)*c)*e^(3/2*x)*sgn(-sqrt(b
^2 - c^2)*e^x + b - c) - 9*(b^2 - c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x +
 b - c) - (9*sqrt(b^2 - c^2)*(b - c)*e^x*sgn(-sqrt(b^2 - c^2)*e^x + b - c)
- (b^2 - 2*b*c + c^2)*sgn(-sqrt(b^2 - c^2)*e^x + b - c))*e^(-3/2*x))/sqrt(b
 - c)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}\,dx = \int \left(b\cosh(x)-\sqrt{b^2-c^2}+c\sinh(x)\right)^{3/2}\,dx$$

```
[In] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

```
[Out] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.776 $\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[Out] 2*(c*cosh(x)+b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3191}

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(b\sinh(x)+c\cosh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

```
[In] Int[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
[Out] (2*(c*Cosh[x] + b*Sinh[x]))/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]]
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\text{integral}=\frac{2(c\cosh(x)+b\sinh(x))}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.97 (sec) , antiderivative size = 4196, normalized size of antiderivative = 107.59

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] (2*b*Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]])/c - (8*b*c*Sqrt[b^2 -
c^2]*(-b^2 + c^2)*(EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 +
Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1] - 2*Ellipti
cPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c +
 Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1])*Sqrt[-Sqrt[(b - c)*(b + c)] + b
*Cosh[x] + c*Sinh[x]]*(-1 + Tanh[x/2])*(-(((b + c + Sqrt[b^2 - c^2])*(1 + T
anh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))^(3/2)*(c + (b + S
qrt[b^2 - c^2])*Tanh[x/2])*(-1 + Tanh[x/2]^2))/((b + c + Sqrt[b^2 - c^2])^3
*(-b^2 + c^2 + b*Sqrt[b^2 - c^2])*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(b + c)
] + b*Cosh[x] + c*Sinh[x])/(1 + Cosh[x])^2]*(1 + Tanh[x/2])^2*Sqrt[-((-1 +
Tanh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^2]*(-1 + Tanh[x/2]^2) + b*(1 + T
anh[x/2]^2)))]) - (4*(b - c)*(b + c)*Sqrt[-Sqrt[(b - c)*(b + c)] + b*Cosh[x
] + c*Sinh[x]]*(2*b^3*c^2 + 3*b^2*c^3 - c^5 + 2*b^2*c^2*Sqrt[b^2 - c^2] + 3
*b*c^3*Sqrt[b^2 - c^2] + c^4*Sqrt[b^2 - c^2] + 8*b^4*c*Tanh[x/2] + 12*b^3*c
^2*Tanh[x/2] - 2*b^2*c^3*Tanh[x/2] - 8*b*c^4*Tanh[x/2] - 2*c^5*Tanh[x/2] +
8*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2] + 12*b^2*c^2*Sqrt[b^2 - c^2]*Tanh[x/2] +
2*b*c^3*Sqrt[b^2 - c^2]*Tanh[x/2] - 2*c^4*Sqrt[b^2 - c^2]*Tanh[x/2] + 8*b^5
*Tanh[x/2]^2 + 12*b^4*c*Tanh[x/2]^2 - 4*b^3*c^2*Tanh[x/2]^2 - 11*b^2*c^3*Ta
nh[x/2]^2 - 2*b*c^4*Tanh[x/2]^2 + c^5*Tanh[x/2]^2 + 8*b^4*Sqrt[b^2 - c^2]*T
anh[x/2]^2 + 12*b^3*c*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 5*b*c^3*Sqrt[b^2 - c^2]
*Tanh[x/2]^2 - c^4*Sqrt[b^2 - c^2]*Tanh[x/2]^2 - 8*b^4*c*EllipticPi[-1, Arc
Sin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 -
 c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^2*c^3*Elliptic
Pi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c +
Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])
*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 8*b^3*c*
Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1
+ Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*Sqrt[-(((
b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 +
Tanh[x/2])))] + 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b +
c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh
[x/2])))]], 1]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c +
 Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^5*EllipticPi[-1, ArcSin[Sqrt[-
(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1
```

+ Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 8*b^4*c*EllipticPi
[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sq
rt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2
 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] +
20*b^3*c^2*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tan
h[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt
[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(
-1 + Tanh[x/2])))] - 8*b^2*c^3*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[
b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]
], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c
+ Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt
[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(
-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Ta
nh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 16*b^4*Sqrt[b^2
- c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/
2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-((
(b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 +
 Tanh[x/2])))] + 8*b^3*c*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b +
 c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tan
h[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2])
)/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 12*b^2*c^2*Sqrt[b^2 - c^
2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))
/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]*Sqrt[-(((b +
 c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tan
h[x/2])))] - 4*b*c^3*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c +
 Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/
2]))))]], 1]*Tanh[x/2]*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((
b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^5*EllipticPi[-1, ArcSin
[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^
2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*
(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 20*b^3*c^
2*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/
((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((b
+ c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Ta
nh[x/2])))] + 4*b*c^4*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2
])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tan
h[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[
b^2 - c^2])*(-1 + Tanh[x/2])))] + 16*b^4*Sqrt[b^2 - c^2]*EllipticPi[-1, Arc
Sin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 -
 c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2
])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))] - 12*b^2
*c^2*Sqrt[b^2 - c^2]*EllipticPi[-1, ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2]
)*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2]))))]], 1]*Tanh
[x/2]^2*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b

```
^2 - c^2])*(-1 + Tanh[x/2]))))] + 2*c*EllipticE[ArcSin[Sqrt[-(((b + c + Sqrt
[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))
]], 1]*(-1 + Tanh[x/2])*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/
((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + b^2*c*(2*
Sqrt[b^2 - c^2] - 5*c*Tanh[x/2]) + c^3*(-Sqrt[b^2 - c^2] + c*Tanh[x/2]) + 2
*b^3*(c + 2*Sqrt[b^2 - c^2]*Tanh[x/2]) - b*c^2*(2*c + 3*Sqrt[b^2 - c^2]*Tan
h[x/2])) - 2*b*EllipticF[ArcSin[Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh
[x/2]))/((b - c + Sqrt[b^2 - c^2])*(-1 + Tanh[x/2])))]], 1]*(-1 + Tanh[x/2]
)*Sqrt[-(((b + c + Sqrt[b^2 - c^2])*(1 + Tanh[x/2]))/((b - c + Sqrt[b^2 - c
^2])*(-1 + Tanh[x/2])))]*(4*b^4*Tanh[x/2] + b^2*c*(2*Sqrt[b^2 - c^2] - 5*c*
Tanh[x/2]) + c^3*(-Sqrt[b^2 - c^2] + c*Tanh[x/2]) + 2*b^3*(c + 2*Sqrt[b^2 -
 c^2]*Tanh[x/2]) - b*c^2*(2*c + 3*Sqrt[b^2 - c^2]*Tanh[x/2]))))/(c*Sqrt[b^2
 - c^2]*(b + Sqrt[b^2 - c^2])*(b - c + Sqrt[b^2 - c^2])*(b + c + Sqrt[b^2 -
 c^2])^2*(1 + Cosh[x])*Sqrt[(-Sqrt[(b - c)*(b + c)] + b*Cosh[x] + c*Sinh[x]
)/(1 + Cosh[x])^2]*Sqrt[-((-1 + Tanh[x/2]^2)*(2*c*Tanh[x/2] + Sqrt[b^2 - c^
2]*(-1 + Tanh[x/2]^2) + b*(1 + Tanh[x/2]^2)))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(35) = 70$.

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.36

method	result	si
risch	$-\frac{\sqrt{2}\,\sqrt{-\left(-{\mathrm e}^{2x} b-{\mathrm e}^{2x} c+2\sqrt{b^{2}-c^{2}}\,{\mathrm e}^{x}-b+c\right){\mathrm e}^{-x}}\,\left({\mathrm e}^{x} b+{\mathrm e}^{x} c+\sqrt{b^{2}-c^{2}}\right)\left({\mathrm e}^{x} b+{\mathrm e}^{x} c-\sqrt{b^{2}-c^{2}}\right)}{\left(-{\mathrm e}^{2x} b-{\mathrm e}^{2x} c+2\sqrt{b^{2}-c^{2}}\,{\mathrm e}^{x}-b+c\right)(b+c)}$	13
default	$\frac{\left(-b^{2}+c^{2}\right)\cosh\left(x\right)}{\sqrt{b^{2}-c^{2}}\,\sqrt{-\frac{\sinh\left(x\right) b^{2}-\sinh\left(x\right) c^{2}+b^{2}-c^{2}}{\sqrt{b^{2}-c^{2}}}}}-\frac{\sqrt{-\sqrt{b^{2}-c^{2}}\,\left(\sinh\left(x\right)+1\right)\sinh\left(x\right)^{2}}\,\sqrt{b^{2}-c^{2}}\,\arctan\left(\frac{\sqrt{\sqrt{b^{2}-c^{2}}\,\left(\sinh\left(x\right)+1\right)}\,\cosh\left(x\right)}{\sqrt{-\sqrt{b^{2}-c^{2}}\,\left(\sinh\left(x\right)+1\right)\sinh\left(x\right)^{2}}}\right)}{\sqrt{\sqrt{b^{2}-c^{2}}\,\left(\sinh\left(x\right)+1\right)}\,\sinh\left(x\right)\sqrt{-\frac{\sinh\left(x\right) b^{2}-\sinh\left(x\right) c^{2}+b^{2}-c^{2}}{\sqrt{b^{2}-c^{2}}}}}$	20

```
[In] int((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2^(1/2)*(-(-exp(2*x)*b-exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)-b+c)*exp(-x))^(
1/2)/(-exp(2*x)*b-exp(2*x)*c+2*(b^2-c^2)^(1/2)*exp(x)-b+c)*(exp(x)*b+exp(x)
*c+(b^2-c^2)^(1/2))*(exp(x)*b+exp(x)*c-(b^2-c^2)^(1/2))/(b+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(35) = 70.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.67

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\frac{2\sqrt{\frac{1}{2}}\left((b+c)\cosh\left(x\right)^2+2\left(b+c\right)\cosh\left(x\right)\sinh\left(x\right)+\left(b+c\right)\sinh\left(x\right)^2+2\sqrt{b^2-c^2}(\cosh\left(x\right)+\sinh\left(x\right))+\right.}{(b+c)\cosh\left(x\right)^2+2\left(b+c\right)\cosh\left(x\right)\sinh\left(x\right)+(b}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(1/2)*((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 + 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b + c)*cosh(x)^
2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh
(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/((b + c)*cosh(x)^2 + 2*(b + c)
*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - b + c)
```

Sympy [F]

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx=\int \sqrt{b\cosh\left(x\right)+c\sinh\left(x\right)-\sqrt{b^2-c^2}}\,dx$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(35) = 70.

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.00

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx$$
$$=-\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}\sqrt{b+c}\sqrt{b-c}e^{\left(\frac{1}{2}x\right)}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}}-\frac{\sqrt{2}\sqrt{-2\sqrt{b+c}\sqrt{b-c}e^{(-x)}+(b-c)e^{(-2x)}+b+c}(b-c)e^{\left(-\frac{1}{2}x\right)}}{(b-c)e^{(-x)}-\sqrt{b+c}\sqrt{b-c}}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] -sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b + c)
*sqrt(b + c)*sqrt(b - c)*e^(1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c
)) - sqrt(2)*sqrt(-2*sqrt(b + c)*sqrt(b - c)*e^(-x) + (b - c)*e^(-2*x) + b
+ c)*(b - c)*e^(-1/2*x)/((b - c)*e^(-x) - sqrt(b + c)*sqrt(b - c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(35) = 70$.

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.08

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = -\frac{\sqrt{2}\left((b-c)e^{\left(-\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)+\sqrt{b^2-c^2}e^{\left(\frac{1}{2}x\right)}\operatorname{sgn}\left(-\sqrt{b^2-c^2}e^x+b-c\right)\right)}{\sqrt{b-c}}$$

```
[In] integrate((b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] -sqrt(2)*((b - c)*e^(-1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c) + sqrt(b^2 -
 c^2)*e^(1/2*x)*sgn(-sqrt(b^2 - c^2)*e^x + b - c))/sqrt(b - c)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}\,dx = \int \sqrt{b\cosh\left(x\right)-\sqrt{b^2-c^2}+c\sinh\left(x\right)}\,dx$$

```
[In] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
[Out] int((b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.777 $\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 102

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[Out] -arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c^2)^(
1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))*2^(1/2)/(b^2-c^2)^(1/
4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3194, 2728, 210}

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

```
[In] Int[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[
2]*Sqrt[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]
)/(b^2 - c^2)^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx$$

$$=2i\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)$$

$$=-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{\sqrt[4]{b^2-c^2}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 54.83 (sec) , antiderivative size = 52609, normalized size of antiderivative = 515.77

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/Sqrt[-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x]],x]
[Out] Result too large to show
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\,\arctan\left(\frac{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\cosh(x)}{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}}\right)}{\sqrt{\sqrt{b^2-c^2}\,(\sinh(x)+1)}\,\sinh(x)\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}}$	129

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)/((b^2-c^2)^(1/2)*(sinh(x)+1)
)^(1/2)*arctan(((b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*cosh(x)/(-(b^2-c^2)^(1/2
)*(sinh(x)+1)*sinh(x)^2)^(1/2))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)
/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 680, normalized size of antiderivative = 6.67

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [sqrt(2)*log(-((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x
)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cosh(x)^2*sinh(x)^2 + 4*(b^2 + 2*b*c +
c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*sqrt(2)*sqrt(1/2
)*(2*(b^2 - c^2)*cosh(x)^2 + 4*(b^2 - c^2)*cosh(x)*sinh(x) + 2*(b^2 - c^2)*
sinh(x)^2 + ((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x)^2 + (b + c)*sinh
(x)^3 + (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 + b - c)*sinh(x))*sqrt(b^2 -
 c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x
)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))/(
b^2 - c^2)^(1/4) - b^2 + 2*b*c - c^2 + 2*((b + c)*cosh(x)^3 + 3*(b + c)*cos
h(x)*sinh(x)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2
 - b + c)*sinh(x))*sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2
 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*(b^2
- c^2)*cosh(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)*cosh(x)^2 - b^2 + c^2)*sinh(x)^
2 + b^2 - 2*b*c + c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - (b^2 - c^2)*cosh
(x))*sinh(x)))/(b^2 - c^2)^(1/4), 2*sqrt(2)*sqrt(-1/sqrt(b^2 - c^2))*arctan
(sqrt(2)*sqrt(1/2)*(sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)*sqrt(((b +
 c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2
```

```
- c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh(x)))*sqrt(-1/sqrt(b^2 -
 c^2))/((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)*sinh(x)^2 -
 b + c))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/sqrt(b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(83) = 166$.

Time = 0.52 (sec) , antiderivative size = 546, normalized size of antiderivative = 5.35

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx$$

$$=\frac{2\sqrt{2}(b^2-c^2-b+c)\sqrt{b+c}\arctan\left(\frac{}{\sqrt{-\sqrt{b^2-c^2}b^5-\sqrt{b^2-c^2}b^4c+2\sqrt{b^2-c^2}b^3c^2+2\sqrt{b^2-c^2}b}}\right.}{\sqrt{-\sqrt{b^2-c^2}b^5-\sqrt{b^2-c^2}b^4c+2\sqrt{b^2-c^2}b^3c^2+2\sqrt{b^2-c^2}b^2c^3-\sqrt{b^2-c^2}bc^4-\sqrt{b^2-c^2}c^5+2\sqrt{b^2-c^2}}}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(1/2),x, algorithm="giac"
)
[Out] 2*sqrt(2)*(b^2 - c^2 - b + c)*sqrt(b + c)*arctan((b^3*e^(1/2*x) + b^2*c*e^(
1/2*x) - b*c^2*e^(1/2*x) - c^3*e^(1/2*x) - b^2*e^(1/2*x) + c^2*e^(1/2*x))/s
qrt(-sqrt(b^2 - c^2)*b^5 - sqrt(b^2 - c^2)*b^4*c + 2*sqrt(b^2 - c^2)*b^3*c^
```

```
2 + 2*sqrt(b^2 - c^2)*b^2*c^3 - sqrt(b^2 - c^2)*b*c^4 - sqrt(b^2 - c^2)*c^5
 + 2*sqrt(b^2 - c^2)*b^4 - 4*sqrt(b^2 - c^2)*b^2*c^2 + 2*sqrt(b^2 - c^2)*c^
4 - sqrt(b^2 - c^2)*b^3 + sqrt(b^2 - c^2)*b^2*c + sqrt(b^2 - c^2)*b*c^2 - s
qrt(b^2 - c^2)*c^3))/(sqrt(-sqrt(b^2 - c^2)*b^5 - sqrt(b^2 - c^2)*b^4*c + 2
*sqrt(b^2 - c^2)*b^3*c^2 + 2*sqrt(b^2 - c^2)*b^2*c^3 - sqrt(b^2 - c^2)*b*c^
4 - sqrt(b^2 - c^2)*c^5 + 2*sqrt(b^2 - c^2)*b^4 - 4*sqrt(b^2 - c^2)*b^2*c^2
 + 2*sqrt(b^2 - c^2)*c^4 - sqrt(b^2 - c^2)*b^3 + sqrt(b^2 - c^2)*b^2*c + sq
rt(b^2 - c^2)*b*c^2 - sqrt(b^2 - c^2)*c^3)*sgn(-sqrt(b^2 - c^2)*e^x + b - c
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh\left(x\right)-\sqrt{b^2-c^2}+c\sinh\left(x\right)}}\,dx$$

```
[In] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2),x)
[Out] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(1/2), x)
```

3.778 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 159

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\,(b^2-c^2)^{3/4}} - \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

```
[Out] 1/4*arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c^2
)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(3/4)*2^
(1/2)+1/2*(-c*cosh(x)-b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2
)/(b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3195, 3194, 2728, 210}

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = -\frac{b\sinh(x)+c\cosh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\,(b^2-c^2)^{3/4}}$$

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt[-Sq
rt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])]/(2*Sqrt[2]
*(b^2 - c^2)^(3/4)) - (c*Cosh[x] + b*Sinh[x])/(2*Sqrt[b^2 - c^2]*(-Sqrt[b^2
 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

integral

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx}{4\sqrt{b^2-c^2}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{\int \frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\,dx}{4\sqrt{b^2-c^2}}$$

$$= -\frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{i\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2-c^2}-x^2}\,dx, x, -\frac{i\sqrt{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{b^2-c^2}}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{2\sqrt{2}\,(b^2-c^2)^{3/4}} - \frac{c\cosh(x)+b\sinh(x)}{2\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \$\text{Aborted}$$

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(134) = 268$.

Time = 0.23 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.61

method	result
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cosh(x)\sqrt{2}}{2}\right)}{2\sqrt{b^2-c^2}\,\sqrt{-\frac{\sinh(x)b^2-\sinh(x)c^2+b^2-c^2}{\sqrt{b^2-c^2}}}} - \frac{\sqrt{-\sqrt{b^2-c^2}\,(\sinh(x)+1)\sinh(x)^2}\,\sqrt{b^2-c^2}\,\sqrt{2}\,\Big(\ln\Big(-\frac{2\left(\cosh(x)\sqrt{b^2-c^2}\,\sqrt{2}\,\sinh(x)-\sinh(x\right.}{}$

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(b^2-c^2)^(1/2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1
/2)*2^(1/2)*arctanh(1/2*cosh(x)*2^(1/2))-1/4*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2)*(b^2-c^2)^(1/2)*2^(1/2)*(ln(-2*(cosh(x)*(b^2-c^2)^(1/2)*2^
(1/2)*sinh(x)-sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)-(b^2-
c^2)^(1/2)-(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+
1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2)))-ln(2*(cosh(x)*(b^2-c^2)^(1/2)*2^(1/
2)*sinh(x)+sinh(x)*(b^2-c^2)^(1/2)+cosh(x)*(b^2-c^2)^(1/2)*2^(1/2)+(b^2-c^2
)^(1/2)+(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/(b-c)/(b+c)/(-(b^2-c^2)^(1/2)*(sinh(x
)+1))^(1/2)/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. 2(130) = 260.

Time = 0.47 (sec) , antiderivative size = 2137, normalized size of antiderivative = 13.44

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/4*((sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^6 + 6*sqrt(2)*(b^3 +
 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)*sinh(x)^5 + sqrt(2)*(b^3 + 3*b^2*c + 3*b*
c^2 + c^3)*sinh(x)^6 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 3*
(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*cosh(x)^2 - sqrt(2)*(b^3 + b^2*c
 - b*c^2 - c^3))*sinh(x)^4 + 4*(5*sqrt(2)*(b^3 + 3*b^2*c + 3*b*c^2 + c^3)*c
osh(x)^3 - 3*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x))*sinh(x)^3 + 3*sqr
t(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 3*(5*sqrt(2)*(b^3 + 3*b^2*c +
3*b*c^2 + c^3)*cosh(x)^4 - 6*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2
+ sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3))*sinh(x)^2 + 6*(sqrt(2)*(b^3 + 3*b^2*
c + 3*b*c^2 + c^3)*cosh(x)^5 - 2*sqrt(2)*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x
)^3 + sqrt(2)*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) - sqrt(2)*(b^3 -
 3*b^2*c + 3*b*c^2 - c^3))*(b^2 - c^2)^(1/4)*log(-((b^2 + 2*b*c + c^2)*cosh
(x)^4 + 4*(b^2 + 2*b*c + c^2)*cosh(x)^3*sinh(x) + 6*(b^2 + 2*b*c + c^2)*cos
h(x)^2*sinh(x)^2 + 4*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c +
 c^2)*sinh(x)^4 - 2*sqrt(1/2)*(sqrt(2)*(b + c)*cosh(x)^3 + 3*sqrt(2)*(b + c
)*cosh(x)*sinh(x)^2 + sqrt(2)*(b + c)*sinh(x)^3 + sqrt(2)*(b - c)*cosh(x) +
 (3*sqrt(2)*(b + c)*cosh(x)^2 + sqrt(2)*(b - c))*sinh(x) + 2*(sqrt(2)*cosh(
x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2)*sqrt(b^2 - c^2))*(b^2
 - c^2)^(1/4)*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sinh(x) + (b + c)
*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(cosh(x) + sinh
(x))) - b^2 + 2*b*c - c^2 + 2*((b + c)*cosh(x)^3 + 3*(b + c)*cosh(x)*sinh(x
)^2 + (b + c)*sinh(x)^3 - (b - c)*cosh(x) + (3*(b + c)*cosh(x)^2 - b + c)*s
inh(x))*sqrt(b^2 - c^2))/((b^2 + 2*b*c + c^2)*cosh(x)^4 + 4*(b^2 + 2*b*c +
c^2)*cosh(x)*sinh(x)^3 + (b^2 + 2*b*c + c^2)*sinh(x)^4 - 2*(b^2 - c^2)*cosh
(x)^2 + 2*(3*(b^2 + 2*b*c + c^2)*cosh(x)^2 - b^2 + c^2)*sinh(x)^2 + b^2 - 2
*b*c + c^2 + 4*((b^2 + 2*b*c + c^2)*cosh(x)^3 - (b^2 - c^2)*cosh(x))*sinh(x
))) + 4*sqrt(1/2)*(4*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^4 + 16*(b^3 + b^2*
c - b*c^2 - c^3)*cosh(x)*sinh(x)^3 + 4*(b^3 + b^2*c - b*c^2 - c^3)*sinh(x)^
4 + 4*(b^3 - b^2*c - b*c^2 + c^3)*cosh(x)^2 + 4*(b^3 - b^2*c - b*c^2 + c^3
+ 6*(b^3 + b^2*c - b*c^2 - c^3)*cosh(x)^2)*sinh(x)^2 + 8*(2*(b^3 + b^2*c -
b*c^2 - c^3)*cosh(x)^3 + (b^3 - b^2*c - b*c^2 + c^3)*cosh(x))*sinh(x) + ((b
^2 + 2*b*c + c^2)*cosh(x)^5 + 5*(b^2 + 2*b*c + c^2)*cosh(x)*sinh(x)^4 + (b^
2 + 2*b*c + c^2)*sinh(x)^5 + 6*(b^2 - c^2)*cosh(x)^3 + 2*(5*(b^2 + 2*b*c +
c^2)*cosh(x)^2 + 3*b^2 - 3*c^2)*sinh(x)^3 + 2*(5*(b^2 + 2*b*c + c^2)*cosh(x
```

```
)^3 + 9*(b^2 - c^2)*cosh(x))*sinh(x)^2 + (b^2 - 2*b*c + c^2)*cosh(x) + (5*(
b^2 + 2*b*c + c^2)*cosh(x)^4 + 18*(b^2 - c^2)*cosh(x)^2 + b^2 - 2*b*c + c^2
)*sinh(x))*sqrt(b^2 - c^2))*sqrt(((b + c)*cosh(x)^2 + 2*(b + c)*cosh(x)*sin
h(x) + (b + c)*sinh(x)^2 - 2*sqrt(b^2 - c^2)*(cosh(x) + sinh(x)) + b - c)/(
cosh(x) + sinh(x))))/((b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^
5)*cosh(x)^6 + 6*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*co
sh(x)*sinh(x)^5 + (b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*s
inh(x)^6 - b^5 + 3*b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + 3*b*c^4 - c^5 - 3*(b^5 +
 b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^4 - 3*(b^5 + b^4*c -
2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5 - 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*
c^3 - 3*b*c^4 - c^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^5 + 3*b^4*c + 2*b^3*c^2
 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x)^3 - 3*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^
2*c^3 + b*c^4 + c^5)*cosh(x))*sinh(x)^3 + 3*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^
2*c^3 + b*c^4 - c^5)*cosh(x)^2 + 3*(b^5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b
*c^4 - c^5 + 5*(b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh
(x)^4 - 6*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^2)*si
nh(x)^2 + 6*((b^5 + 3*b^4*c + 2*b^3*c^2 - 2*b^2*c^3 - 3*b*c^4 - c^5)*cosh(x
)^5 - 2*(b^5 + b^4*c - 2*b^3*c^2 - 2*b^2*c^3 + b*c^4 + c^5)*cosh(x)^3 + (b^
5 - b^4*c - 2*b^3*c^2 + 2*b^2*c^3 + b*c^4 - c^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(3/2),x)
[Out] Integral((b*cosh(x) + c*sinh(x) - sqrt(b**2 - c**2))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cosh{(x)}+c\sinh{(x)}-\sqrt{b^2-c^2}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="maxim
a")
[Out] integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{8,[4,0]%%%}+%%%{16,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{-8,[2,1
]%%%}+
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)-\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2),x)
```

```
[Out] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(3/2), x)
```

3.779 $\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 211

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\,(b^2-c^2)^{5/4}} - \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}$$

```
[Out] -3/32*arctanh(1/2*(b^2-c^2)^(1/4)*sinh(x+I*arctan(b,-I*c))*2^(1/2)/(-(b^2-c
^2)^(1/2)+cosh(x+I*arctan(b,-I*c))*(b^2-c^2)^(1/2))^(1/2))/(b^2-c^2)^(5/4)*
2^(1/2)+3/16*(c*cosh(x)+b*sinh(x))/(b^2-c^2)/(b*cosh(x)+c*sinh(x)-(b^2-c^2)
^(1/2))^(3/2)+1/4*(-c*cosh(x)-b*sinh(x))/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/
2))^(5/2)/(b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3195, 3194, 2728, 210}

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = -\frac{3\text{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\left(b^2-c^2\right)^{5/4}} + \frac{3(b\sinh(x)+c\cosh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} - \frac{b\sinh(x)+c\cosh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}$$

```
[In] Int[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
```

```
[Out] (-3*ArcTanh[((b^2 - c^2)^(1/4)*Sinh[x + I*ArcTan[b, (-I)*c]])/(Sqrt[2]*Sqrt
[-Sqrt[b^2 - c^2] + Sqrt[b^2 - c^2]*Cosh[x + I*ArcTan[b, (-I)*c]]])])/(16*S
qrt[2]*(b^2 - c^2)^(5/4)) - (c*Cosh[x] + b*Sinh[x])/(4*Sqrt[b^2 - c^2]*(-Sq
rt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(5/2)) + (3*(c*Cosh[x] + b*Sinh[x]))
/(16*(b^2 - c^2)*(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
```

```
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad -\frac{3\int\frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}\,dx}{8\sqrt{b^2-c^2}} \\
&= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad +\frac{3(c\cosh(x)+b\sinh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} \\
&\quad +\frac{3\int\frac{1}{\sqrt{-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)}}\,dx}{32\left(b^2-c^2\right)} \\
&= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad +\frac{3(c\cosh(x)+b\sinh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} \\
&\quad +\frac{3\int\frac{1}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\,dx}{32\left(b^2-c^2\right)} \\
&= -\frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} \\
&\quad +\frac{3(c\cosh(x)+b\sinh(x))}{16\left(b^2-c^2\right)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}} \\
&\quad +\frac{(3i)\text{Subst}\left(\int\frac{1}{-2\sqrt{b^2-c^2}-x^2}\,dx,x,-\frac{i\sqrt{b^2-c^2}\sinh\left(x+i\tan^{-1}(b,-ic)\right)}{\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh\left(x+i\tan^{-1}(b,-ic)\right)}}\right)}{16\left(b^2-c^2\right)}
\end{aligned}$$

$$
= -\frac{3\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2-c^2}\sinh(x+i\tan^{-1}(b,-ic))}{\sqrt{2}\sqrt{-\sqrt{b^2-c^2}+\sqrt{b^2-c^2}\cosh(x+i\tan^{-1}(b,-ic))}}\right)}{16\sqrt{2}\,(b^2-c^2)^{5/4}} - \frac{c\cosh(x)+b\sinh(x)}{4\sqrt{b^2-c^2}\,\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}} + \frac{3(c\cosh(x)+b\sinh(x))}{16\,(b^2-c^2)\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{3/2}}
$$

Mathematica [F(-1)]

Timed out.

$$
\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \$\text{Aborted}
$$

```
[In] Integrate[(-Sqrt[b^2 - c^2] + b*Cosh[x] + c*Sinh[x])^(-5/2),x]
[Out] $Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 817, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	817

```
[In] int(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2*(-b^2+c^2)/(-(sinh(x)*b^2-sinh(x)*c^2+b^2-c^2)/(b^2-c^2)^(1/2))^(1/2)/(b^
4-2*b^2*c^2+c^4)*(-1/4*cosh(x)/(cosh(x)^2-2)+1/8*2^(1/2)*arctanh(1/2*cosh(x
)*2^(1/2)))+(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)*(b^2-c^2)*(1/4/(
b-c)^2/(b+c)^2*(1/(b^2-c^2)^(1/2)/(sinh(x)+1)/(cosh(x)-2^(1/2))*(-(b^2-c^2)
^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2)+2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)+1))^
(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)+1)-2*(sinh(x)+1)*2^(1/2)*(b^2-c^2)^(1
/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1
/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2))))+1/4/(b-c)^2/(b+c)^2*(
1/(b^2-c^2)^(1/2)/(sinh(x)+1)/(cosh(x)+2^(1/2))*(-(b^2-c^2)^(1/2)*(sinh(x)+
1)*sinh(x)^2)^(1/2)-2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*ln((-2*(b^
2-c^2)^(1/2)*(sinh(x)+1)+2*(sinh(x)+1)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)+2^(
1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))-1/8/(b-c)^2/(b+c)^2*2^(1/2)/(-(b^2-c^
2)^(1/2)*(sinh(x)+1))^(1/2)*ln((-2*(b^2-c^2)^(1/2)*(sinh(x)+1)-2*(sinh(x)+1
)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)-2^(1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1)
```

```
)^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*sinh(x)^2)^(1/2))/(cosh(x)-2^(1/2))+
1/8/(b-c)^2/(b+c)^2*2^(1/2)/(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*ln((-2*(b^
2-c^2)^(1/2)*(sinh(x)+1)+2*(sinh(x)+1)*2^(1/2)*(b^2-c^2)^(1/2)*(cosh(x)+2^(
1/2))+2*(-(b^2-c^2)^(1/2)*(sinh(x)+1))^(1/2)*(-(b^2-c^2)^(1/2)*(sinh(x)+1)*
sinh(x)^2)^(1/2))/(cosh(x)+2^(1/2))))/sinh(x)/(-(sinh(x)*b^2-sinh(x)*c^2+b^
2-c^2)/(b^2-c^2)^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5675 vs. $2(176) = 352$.

Time = 1.63 (sec) , antiderivative size = 5675, normalized size of antiderivative = 26.90

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="frica
s")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b**2-c**2)**(1/2))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)+c\sinh\left(x\right)-\sqrt{b^2-c^2}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="maxim
a")
[Out] integrate((b*cosh(x) + c*sinh(x) - sqrt(b^2 - c^2))^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*cosh(x)+c*sinh(x)-(b^2-c^2)^(1/2))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{32,[5,0]%%%}+%%%{96,[4,1]%%%}+%%%{-32,[4,0]%%%}+%%%{64,[3
,2]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2-c^2}+b\cosh(x)+c\sinh(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)-\sqrt{b^2-c^2}+c\sinh\left(x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2),x)
```

```
[Out] int(1/(b*cosh(x) - (b^2 - c^2)^(1/2) + c*sinh(x))^(5/2), x)
```

3.780 $\int \frac{1}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int \frac{1}{a+c\text{sech}(x)+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2-c^2}}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(c+a*cosh(x)+b*sinh(x))/(a^2-b^2)-2*a*c*arctan((b+(a-c)*t
anh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3238, 3217, 3203, 632, 210}

$$\int \frac{1}{a+c\text{sech}(x)+b\tanh(x)}\,dx=-\frac{2ac\arctan\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a\cosh(x)+b\sinh(x)+c)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

```
[In] Int[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 -
c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 - c^2]) - (b*Log[c + a*Cosh[x] + b*Sinh[
x]])/(a^2 - b^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3217

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[c*B*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(x)}{c+a\cosh(x)+b\sinh(x)}\,dx\\
&= \frac{ax}{a^2-b^2}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{(ac)\int\frac{1}{c+a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{ax}{a^2-b^2}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{(2ac)\text{Subst}\left(\int\frac{1}{a+c+2bx-(-a+c)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{ax}{a^2-b^2}-\frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}\\
&\quad+\frac{(4ac)\text{Subst}\left(\int\frac{1}{-4(a^2-b^2-c^2)-x^2}\,dx,x,2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}
\end{aligned}
$$

$$= \frac{ax}{a^2-b^2} - \frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2-c^2}} - \frac{b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \frac{ax - \frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}} - b\log(c+a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Integrate[(a + c*Sech[x] + b*Tanh[x])^(-1),x]
[Out] (a*x - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a
^2 - b^2 - c^2] - b*Log[c + a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

method	result
default	$\frac{2\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a-2b} - \frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a+2b} + \frac{\frac{2(-ab+bc)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-c\tanh\left(\frac{x}{2}\right)^2+2b\tanh\left(\frac{x}{2}\right)+a+c\right)}{2a-2c} + \frac{2\left(-ac-b^2-\frac{(-ab+bc)b}{a-c}\right)\arctan\left(\frac{2}{\cdots}\right.}{\sqrt{a^2-b^2-c^2}}}{(a-b)(a+b)}$
risch	$\frac{x}{a+b} + \frac{2x\,a^2b}{a^4-2a^2b^2-a^2c^2+b^4+b^2c^2} - \frac{2x\,b^3}{a^4-2a^2b^2-a^2c^2+b^4+b^2c^2} - \frac{2xb\,c^2}{a^4-2a^2b^2-a^2c^2+b^4+b^2c^2} - \frac{\ln\left(e^x-\frac{-a\,c^2+\sqrt{-a^4c^2+a^2b^2c^2\cdots}}{(a+b)ac}\right.}{a^4-2a^2b^2-a^2c^2+b^4+b\cdots}$

```
[In] int(1/(a+c*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] 2/(2*a-2*b)*ln(1+tanh(1/2*x))-2/(2*a+2*b)*ln(tanh(1/2*x)-1)+2/(a-b)/(a+b)*(
1/2*(-a*b+b*c)/(a-c)*ln(a*tanh(1/2*x)^2-c*tanh(1/2*x)^2+2*b*tanh(1/2*x)+a+c
)+(-a*c-b^2-(-a*b+b*c)*b/(a-c))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan
h(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 429, normalized size of antiderivative = 4.01

$$\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$$
$$= \left[\frac{\sqrt{-a^2+b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2-a^2+b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cosh(x)}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)}\right.}{}\right.$$

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a +
b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + (a^3 + a^2*b - a
*b^2 - b^3 - (a + b)*c^2)*x - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*si
nh(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2),
 (2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) +
c)/sqrt(a^2 - b^2 - c^2)) + (a^3 + a^2*b - a*b^2 - b^3 - (a + b)*c^2)*x - (
a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x)))
)/(a^4 - 2*a^2*b^2 + b^4 - (a^2 - b^2)*c^2)]
```

Sympy [F]

$$\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \int \frac{1}{a+b\tanh{(x)}+c\operatorname{sech}{(x)}}\,dx$$

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(1/(a + b*tanh(x) + c*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = -\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}(a^2-b^2)} - \frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x+a-b\right)}{a^2-b^2} + \frac{x}{a-b}$$

```
[In] integrate(1/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
c^2)*(a^2 - b^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x + a - b)/(a^2 - b
^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.41

$$\int \frac{1}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \frac{x}{a-b} + \frac{\ln\left(a-b+2\,c\,e^x+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(-2\,a^2\,b+2\,b^3+2\,b\,c^2\right)}{2\,\left(a^4-2\,a^2\,b^2-a^2\,c^2+b^4+b^2\,c^2\right)} - \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,a\,c}{(a+b)^2\,(a^2-b^2)\,(a-b)^2\,\sqrt{a^2\,c^2}} - \frac{2\left(a^2\,c\,\sqrt{a^2\,c^2}-b^2\,c\,\sqrt{a^2\,c^2}\right)}{a\,(a+b)^2\,(a^2-b^2)^2\,(a-b)^2\,(-a^2+b^2+c^2)}\right) - \frac{2\left(a^3\,\sqrt{a^2\,c^2}+b^3\,\sqrt{a^2\,c^2}-a\,b^2\,\sqrt{a^2\,c^2}-a^2\right.}{a\,(a+b)^2\,(a^2-b^2)^2\,(-a^2+b^2}\right.\right.\right.}{\sqrt{-(}}$$

```
[In] int(1/(a + b*tanh(x) + c/cosh(x)),x)
```

```
[Out] x/(a - b) + (log(a - b + 2*c*exp(x) + a*exp(2*x) + b*exp(2*x))*(2*b*c^2 - 2
*a^2*b + 2*b^3))/(2*(a^4 + b^4 - 2*a^2*b^2 - a^2*c^2 + b^2*c^2)) - (2*atan(
(exp(x)*((2*a*c)/((a + b)^2*(a^2 - b^2)*(a - b)^2*(a^2*c^2)^(1/2)) - (2*(a^
2*c*(a^2*c^2)^(1/2) - b^2*c*(a^2*c^2)^(1/2)))/(a*(a + b)^2*(a^2 - b^2)^2*(a
 - b)^2*(b^2 - a^2 + c^2))) - (2*(a^3*(a^2*c^2)^(1/2) + b^3*(a^2*c^2)^(1/2)
 - a*b^2*(a^2*c^2)^(1/2) - a^2*b*(a^2*c^2)^(1/2)))/(a*(a + b)^2*(a^2 - b^2)
^2*(a - b)^2*(b^2 - a^2 + c^2)))*((a^3*(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(
1/2))/2 - (b^3*(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(1/2))/2 - (a*b^2*(-(a^2
- b^2)^2*(b^2 - a^2 + c^2))^(1/2))/2 + (a^2*b*(-(a^2 - b^2)^2*(b^2 - a^2 +
c^2))^(1/2))/2))*(a^2*c^2)^(1/2))/(-(a^2 - b^2)^2*(b^2 - a^2 + c^2))^(1/2)
```

3.781 $\int \frac{1}{a+b\coth(x)+c\mathbf{csch}(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{ax}{a^2-b^2}+\frac{2ac\operatorname{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2+c^2}} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(I*c+I*b*cosh(x)+I*a*sinh(x))/(a^2-b^2)+2*a*c*arctanh((a+
(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2)/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3239, 3216, 3203, 632, 210}

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{2ac\operatorname{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2+c^2}} - \frac{b\log(ia\sinh(x)+ib\cosh(x)+ic)}{a^2-b^2}+\frac{ax}{a^2-b^2}$$

```
[In] Int[(a + b*Coth[x] + c*Csch[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 +
 c^2]])/((a^2 - b^2)*Sqrt[a^2 - b^2 + c^2]) - (b*Log[I*c + I*b*Cosh[x] + I*
a*Sinh[x]])/(a^2 - b^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= i\int \frac{\sinh(x)}{ic+ib\cosh(x)+ia\sinh(x)}\,dx \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}-\frac{(iac)\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}\,dx}{a^2-b^2} \\
&= \frac{ax}{a^2-b^2}-\frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2} \\
&\quad -\frac{(2iac)\text{Subst}\left(\int \frac{1}{ib+ic+2iax-(-ib+ic)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}
\end{aligned}
$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2} + \frac{(4iac)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2+c^2)-x^2}\,dx, x, 2ia+2(ib-ic)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} + \frac{2ac\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2)\sqrt{a^2-b^2+c^2}} - \frac{b\log(ic+ib\cosh(x)+ia\sinh(x))}{a^2-b^2}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx = \frac{ax - \frac{2ac\arctan\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} - b\log(c+b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

```
[In] Integrate[(a + b*Coth[x] + c*Csch[x])^(-1),x]
[Out] (a*x - (2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[
-a^2 + b^2 - c^2] - b*Log[c + b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.58

method	result
default	$\frac{4\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4a-4b} + \frac{\frac{2\left(-b^2+bc\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2 b-c\tanh\left(\frac{x}{2}\right)^2+2a\tanh\left(\frac{x}{2}\right)+b+c\right)}{2b-2c} + \frac{2\left(-ab-ac-\frac{\left(-b^2+bc\right)a}{b-c}\right)\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}}{(a-b)(a+b)}$
risch	$\frac{x}{a+b} + \frac{2x\,a^2b}{a^4-2a^2b^2+a^2c^2+b^4-b^2c^2} - \frac{2x\,b^3}{a^4-2a^2b^2+a^2c^2+b^4-b^2c^2} + \frac{2xb\,c^2}{a^4-2a^2b^2+a^2c^2+b^4-b^2c^2} - \frac{\ln\left(e^x+\frac{a\,c^2+\sqrt{a^4c^2-a^2b^2c^2+}}{(a+b)ac}\right.}{a^4-2a^2b^2+a^2c^2+b^4-b}$

```
[In] int(1/(a+b*coth(x)+c*csch(x)),x,method=_RETURNVERBOSE)
[Out] 4/(4*a-4*b)*ln(1+tanh(1/2*x))+2/(a-b)/(a+b)*(1/2*(-b^2+b*c)/(b-c)*ln(tanh(1
/2*x)^2*b-c*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b+c)+(-a*b-a*c-(-b^2+b*c)*a/(b-c)
)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^
(1/2)))-4/(4*a+4*b)*ln(tanh(1/2*x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.88

$$\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx$$
$$= \left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2+a^2-b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cosh\cdots}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+\cdots}\cdots\right.}{a^{\cdots}}\right.$$
$$-\frac{2\sqrt{-a^2+b^2-c^2}ac\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((a+b)\cosh(x)+(a+b)\sinh(x)+c)}{a^2-b^2+c^2}\right)-(a^3+a^2b-ab^2-b^3+(a+b)c^2)x+\cdots}{a^4-2\,a^2b^2+b^4+(a^2-b^2)c^2}$$

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) - (a^3 + a^2*b - a*
b^2 - b^3 + (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sin
h(x) + c)/(cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2),
-(2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(
x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^3 + a^2*b - a*b^2 - b^3 +
 (a + b)*c^2)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(
cosh(x) - sinh(x))))/(a^4 - 2*a^2*b^2 + b^4 + (a^2 - b^2)*c^2)]
```

Sympy [F]

$$\int \frac{1}{a+b\coth(x)+c\text{csch}(x)}\,dx = \int \frac{1}{a+b\coth{(x)}+c\,\text{csch}\,(x)}\,dx$$

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(1/(a + b*coth(x) + c*csch(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = -\frac{2\,ac\arctan\left(\frac{ae^{x}+be^{x}+c}{\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{(a^{2}-b^{2})\sqrt{-a^{2}+b^{2}-c^{2}}} - \frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^{x}-a+b\right)}{a^{2}-b^{2}} + \frac{x}{a-b}$$

```
[In] integrate(1/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^2)*sqrt
(-a^2 + b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(a^2 -
 b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

$$\int \frac{1}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{x}{a-b} - \frac{\ln\left(\frac{2\,(b+c\,e^{x})}{(a+b)^{2}}+\frac{2\,(b-a+c\,e^{x})\,\left(a^{2}\,b+b\,c^{2}-b^{3}+a\,c\,\sqrt{a^{2}-b^{2}+c^{2}}\right)}{(a+b)\,(a^{2}-b^{2})\,(a^{2}-b^{2}+c^{2})}\right)\,\left(a^{2}\,b+b\,c^{2}-b^{3}+a\,c\,\sqrt{a^{2}-b^{2}+c^{2}}\right)}{a^{4}-2\,a^{2}\,b^{2}+a^{2}\,c^{2}+b^{4}-b^{2}\,c^{2}} - \frac{\ln\left(\frac{2\,(b+c\,e^{x})}{(a+b)^{2}}+\frac{2\,(b-a+c\,e^{x})\,\left(a^{2}\,b+b\,c^{2}-b^{3}-a\,c\,\sqrt{a^{2}-b^{2}+c^{2}}\right)}{(a+b)\,(a^{2}-b^{2})\,(a^{2}-b^{2}+c^{2})}\right)\,\left(a^{2}\,b+b\,c^{2}-b^{3}-a\,c\,\sqrt{a^{2}-b^{2}+c^{2}}\right)}{a^{4}-2\,a^{2}\,b^{2}+a^{2}\,c^{2}+b^{4}-b^{2}\,c^{2}}$$

```
[In] int(1/(a + c/sinh(x) + b*coth(x)),x)
```

```
[Out] x/(a - b) - (log((2*(b + c*exp(x)))/(a + b)^2 + (2*(b - a + c*exp(x))*(a^2*
b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((a + b)*(a^2 - b^2)*(a^2 -
 b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(a^4 + b
^4 - 2*a^2*b^2 + a^2*c^2 - b^2*c^2) - (log((2*(b + c*exp(x)))/(a + b)^2 + (
2*(b - a + c*exp(x))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))/(
(a + b)*(a^2 - b^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b
^2 + c^2)^(1/2)))/(a^4 + b^4 - 2*a^2*b^2 + a^2*c^2 - b^2*c^2)
```

3.782 $\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cx}{b^2-c^2} - \frac{2ac\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} + \frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] -c*x/(b^2-c^2)+b*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*a*c*arctanh((c-(a-b)
*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3216, 3203, 632, 212}

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2ac\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} + \frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{cx}{b^2-c^2}$$

```
[In] Int[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*x)/(b^2 - c^2)) - (2*a*c*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^
2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*Log[a + b*Cosh[x] + c*S
inh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{(ac)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}\\
&= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{(2ac)\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}\\
&= -\frac{cx}{b^2-c^2}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}\\
&\quad-\frac{(4ac)\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}\\
&= -\frac{cx}{b^2-c^2}-\frac{2ac\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{-cx+\frac{2ac\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+b\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (-(c*x) + (2*a*c*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/S
qrt[-a^2 + b^2 - c^2] + b*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.74

method	result
default	$-\frac{4\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4b-4c}-\frac{4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4b+4c}+\frac{\frac{2\left(ab-b^2\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\frac{2\left(-ac-bc+\frac{\left(ab-b^2\right)c}{a-b}\right)\arctan}{\sqrt{-a^2+b^2}}}{(b-c)(b+c)}$
risch	$\frac{x}{b+c}+\frac{2x\,a^2b}{-a^2b^2+a^2c^2+b^4-2b^2c^2+c^4}-\frac{2x\,b^3}{-a^2b^2+a^2c^2+b^4-2b^2c^2+c^4}+\frac{2xb\,c^2}{-a^2b^2+a^2c^2+b^4-2b^2c^2+c^4}+\frac{\ln\left(\mathrm{e}^{x}-\frac{-a^2c+\sqrt{a^4c^2-a^2}}{(b+c)ac}\right.}{a^2b^2-a^2c^2-b^4+2}$

```
[In] int(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(4*b-4*c)*ln(1+tanh(1/2*x))-4/(4*b+4*c)*ln(tanh(1/2*x)-1)+2/(b-c)/(b+c)*
(1/2*(a*b-b^2)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b
)+(-a*c-b*c+(a*b-b^2)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan
h(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.38

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{\left(b^2+2\,bc+c^2\right)\cosh(x)^2+\left(b^2+2\,bc+c^2\right)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,\left(ab+ac+\left(b^2+2\,bc+c^2\right)\cos\right.}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(x)+2\,((b+c)\cosh(x)-}\right.}{}\right.$$

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*
b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b +
 a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b +
 c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2
+ 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + (a^2*b - b^3 +
b*c^2 + c^3 + (a^2 - b^2)*c)*x - (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c
*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^
2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*co
sh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (a^2*b - b^3 + b*c^2 + c^
3 + (a^2 - b^2)*c)*x + (a^2*b - b^3 + b*c^2)*log(2*(b*cosh(x) + c*sinh(x) +
 a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2\,ac\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)} + \frac{b\log\left(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c\right)}{b^2-c^2} - \frac{x}{b-c}$$

```
[In] integrate(sinh(x)/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*a*c*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 -
 c^2)*(b^2 - c^2)) + b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 -
c^2) - x/(b - c)
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.12

$$\int \frac{\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{x}{b-c} - \frac{\ln\left(\frac{2\,(b+a\,e^x)}{(b+c)^2}-\frac{2\,(b-c+a\,e^x)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(b+c)\,(b^2-c^2)\,(a^2-b^2+c^2)}\right)\left(a^2\,b+b\,c^2-b^3+a\,c\,\sqrt{a^2-b^2+c^2}\right)}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4} - \frac{\ln\left(\frac{2\,(b+a\,e^x)}{(b+c)^2}-\frac{2\,(b-c+a\,e^x)\left(a^2\,b+b\,c^2-b^3-a\,c\,\sqrt{a^2-b^2+c^2}\right)}{(b+c)\,(b^2-c^2)\,(a^2-b^2+c^2)}\right)\left(a^2\,b+b\,c^2-b^3-a\,c\,\sqrt{a^2-b^2+c^2}\right)}{-a^2\,b^2+a^2\,c^2+b^4-2\,b^2\,c^2+c^4}$$

```
[In] int(sinh(x)/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] - x/(b - c) - (log((2*(b + a*exp(x)))/(b + c)^2 - (2*(b - c + a*exp(x))*(a^
2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b + c)*(b^2 - c^2)*(a^2
 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 +
 c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) - (log((2*(b + a*exp(x)))/(b + c)^2 -
 (2*(b - c + a*exp(x))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))
/((b + c)*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3 - a*c*(a^2 -
 b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2)
```

3.783 $\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}+\frac{\cosh(x)}{2}-\frac{\sinh(x)}{2}$$

```
[Out] 1/2*x+1/2*cosh(x)-1/2*sinh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3210}

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}-\frac{\sinh(x)}{2}+\frac{\cosh(x)}{2}$$

```
[In] Int[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

Rule 3210

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\text{integral} = \frac{x}{2}+\frac{\cosh(x)}{2}-\frac{\sinh(x)}{2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}+\frac{\cosh(x)}{2}-\frac{\sinh(x)}{2}$$

```
[In] Integrate[Sinh[x]/(1 + Cosh[x] + Sinh[x]),x]
[Out] x/2 + Cosh[x]/2 - Sinh[x]/2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{x}{2}+\frac{{\mathrm e}^{-x}}{2}$	11
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}$	28

```
[In] int(sinh(x)/(1+cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*exp(-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x\cosh\left(x\right)+x\sinh\left(x\right)+1}{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] 1/2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x\tanh\left(\frac{x}{2}\right)}{2\tanh\left(\frac{x}{2}\right)+2} + \frac{x}{2\tanh\left(\frac{x}{2}\right)+2} + \frac{2}{2\tanh\left(\frac{x}{2}\right)+2}$$

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x)
[Out] x*tanh(x/2)/(2*tanh(x/2) + 2) + x/(2*tanh(x/2) + 2) + 2/(2*tanh(x/2) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,e^{(-x)}$$

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,e^{(-x)}$$

```
[In] integrate(sinh(x)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
[Out] 1/2*x + 1/2*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{\sinh(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{x}{2}+\frac{e^{-x}}{2}$$

```
[In] int(sinh(x)/(cosh(x) + sinh(x) + 1),x)
```

```
[Out] x/2 + exp(-x)/2
```

3.784 $\int \frac{\text{sech}(x)}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\text{sech}(x)}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx=\frac{2\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

```
[Out] 2*arctan((b+(a-c)*tanh(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3244, 3203, 632, 210}

$$\int \frac{\text{sech}(x)}{a+c\,\text{sech}(x)+b\tanh(x)}\,dx=\frac{2\arctan\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

```
[In] Int[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3244

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{c+a\cosh(x)+b\sinh(x)}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{a+c+2bx-(-a+c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= -\left(4\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2-c^2\right)-x^2}\,dx, x, 2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)\right) \\
&= \frac{2\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$
\int \frac{\operatorname{sech}(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \frac{2\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}
$$

```
[In] Integrate[Sech[x]/(a + c*Sech[x] + b*Tanh[x]),x]
[Out] (2*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b^2 - c^2]])/Sqrt[a^2 - b^2 -
c^2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$	53
risch	$-\frac{\ln\left(\mathrm{e}^x+\frac{c\sqrt{-a^2+b^2+c^2}-a^2+b^2+c^2}{(a+b)\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+\frac{\ln\left(\mathrm{e}^x+\frac{c\sqrt{-a^2+b^2+c^2}+a^2-b^2-c^2}{(a+b)\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}$	139

```
[In] int(sech(x)/(a+c*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.33

$$\int \frac{\mathrm{sech}(x)}{a+c\mathrm{sech}(x)+b\tanh(x)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2+c^2}\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2-a^2+b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cosh}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+}\right.}{a^2-b^2-c^2}\right.$$
$$\left.-\frac{2\,\arctan\left(-\frac{(a+b)\cosh(x)+(a+b)\sinh(x)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}\right]$$

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2 + c^2)*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cos
h(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b)*c +
 (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) - 2*sqrt(-a^2 + b^2 + c^2)*((a + b)*c
osh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c
*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b))/(a^2 - b^2 - c^2), -2*
arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a^2 - b^2 - c^2))/sqrt
(a^2 - b^2 - c^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a + b\tanh(x) + c\operatorname{sech}(x)}\,dx$$

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x)
[Out] Integral(sech(x)/(a + b*tanh(x) + c*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \frac{2\arctan\left(\frac{ae^x + be^x + c}{\sqrt{a^2 - b^2 - c^2}}\right)}{\sqrt{a^2 - b^2 - c^2}}$$

```
[In] integrate(sech(x)/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
[Out] 2*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/sqrt(a^2 - b^2 - c^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{sech}(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \frac{2\,\operatorname{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}} + \frac{a\,e^x}{\sqrt{a^2-b^2-c^2}} + \frac{b\,e^x}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}$$

```
[In] int(1/(cosh(x)*(a + b*tanh(x) + c/cosh(x))),x)
```

```
[Out] (2*atan(c/(a^2 - b^2 - c^2)^(1/2) + (a*exp(x))/(a^2 - b^2 - c^2)^(1/2) + (b
*exp(x))/(a^2 - b^2 - c^2)^(1/2)))/(a^2 - b^2 - c^2)^(1/2)
```

3.785 $\int \frac{\text{sech}^2(x)}{a+c\text{sech}(x)+b\tanh(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 146

$$\int \frac{\text{sech}^2(x)}{a+c\text{sech}(x)+b\tanh(x)}\,dx = \frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2}-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}-\frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}+\frac{b\log\left(a+c+2b\tanh\left(\frac{x}{2}\right)+(a-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}$$

```
[Out] 2*c*arctan(tanh(1/2*x))/(b^2+c^2)-b*ln(1+tanh(1/2*x)^2)/(b^2+c^2)+b*ln(a+c+
2*b*tanh(1/2*x)+(a-c)*tanh(1/2*x)^2)/(b^2+c^2)-2*a*c*arctan((b+(a-c)*tanh(1
/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {4482, 1089, 648, 632, 210, 642, 649, 209, 266}

$$\int \frac{\text{sech}^2(x)}{a+c\text{sech}(x)+b\tanh(x)}\,dx = -\frac{2ac\arctan\left(\frac{(a-c)\tanh\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}+\frac{b\log\left((a-c)\tanh^2\left(\frac{x}{2}\right)+a+2b\tanh\left(\frac{x}{2}\right)+c\right)}{b^2+c^2}+\frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2}-\frac{b\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2+c^2}$$

```
[In] Int[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*c*ArcTan[Tanh[x/2]])/(b^2 + c^2) - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])
/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[1 + T
anh[x/2]^2])/(b^2 + c^2) + (b*Log[a + c + 2*b*Tanh[x/2] + (a - c)*Tanh[x/2]
^2])/(b^2 + c^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1089

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*((-b)*
C*d + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*((-b)*C*d + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\operatorname{sech}(x)}{c+a\cosh(x)+b\sinh(x)}\,dx \\
&= 2\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)\,(a+c+2bx+(a-c)x^2)}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{\text{Subst}\left(\int \frac{4c-4bx}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{2\,(b^2+c^2)} + \frac{\text{Subst}\left(\int \frac{4b^2+(a-c)^2-(a+c)^2+4b(a-c)x}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{2\,(b^2+c^2)} \\
&= \frac{b\text{Subst}\left(\int \frac{2b+2(a-c)x}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{(2b)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} \\
&\quad + \frac{(2c)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{(2ac)\text{Subst}\left(\int \frac{1}{a+c+2bx+(a-c)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} \\
&= \frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} \\
&\quad + \frac{b\log\left(a+c+2b\tanh\left(\frac{x}{2}\right)+(a-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} \\
&\quad + \frac{(4ac)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2}\,dx, x, 2b+2(a-c)\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} \\
&= \frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2+c^2} - \frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)} \\
&\quad - \frac{b\log\left(1+\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2} + \frac{b\log\left(a+c+2b\tanh\left(\frac{x}{2}\right)+(a-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \frac{2c\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2ac\arctan\left(\frac{b+(a-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+b(-\log(\cosh(x))+\log(c+a\cosh(x)+b\sinh(x)))}{b^2+c^2}$$

```
[In] Integrate[Sech[x]^2/(a + c*Sech[x] + b*Tanh[x]),x]
```

```
[Out] (2*c*ArcTan[Tanh[x/2]] - (2*a*c*ArcTan[(b + (a - c)*Tanh[x/2])/Sqrt[a^2 - b
^2 - c^2]])/Sqrt[a^2 - b^2 - c^2] + b*(-Log[Cosh[x]] + Log[c + a*Cosh[x] +
b*Sinh[x]]))/(b^2 + c^2)
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{2(ab-bc)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-c\tanh\left(\frac{x}{2}\right)^2+2b\tanh\left(\frac{x}{2}\right)+a+c\right)}{2a-2c}+\frac{2\left(-ac+b^2-\frac{(ab-bc)b}{a-c}\right)\arctan\left(\frac{2(a-c)\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}+\frac{-b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+2}{b^2+c}$
risch	$\frac{2xb}{b^2+c^2}-\frac{2x\,a^2b}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}+\frac{2x\,b^3}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}+\frac{2xb\,c^2}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}-\frac{i\ln(e^x-i)c}{b^2+c^2}-\frac{\ln(e^x+i)b}{b^2+c^2}+$

```
[In] int(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b^2+c^2)*(1/2*(a*b-b*c)/(a-c)*ln(a*tanh(1/2*x)^2-c*tanh(1/2*x)^2+2*b*tan
h(1/2*x)+a+c)+(-a*c+b^2-(a*b-b*c)*b/(a-c))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(
2*(a-c)*tanh(1/2*x)+2*b)/(a^2-b^2-c^2)^(1/2)))+2/(b^2+c^2)*(-1/2*b*ln(1+tan
h(1/2*x)^2)+c*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.92 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.33

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2-a^2+b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\,c}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x}\right.}{}\right.$$

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2
)*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 - a^2 + b^2 + 2*c^2 + 2*((a + b
)*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(-a^2 + b^2 + c^2)*((a +
 b)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ 2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) + a - b)) + 2*(c^3 - (a^2 -
 b^2)*c)*arctan(cosh(x) + sinh(x)) - (a^2*b - b^3 - b*c^2)*log(2*(a*cosh(x)
 + b*sinh(x) + c)/(cosh(x) - sinh(x))) + (a^2*b - b^3 - b*c^2)*log(2*cosh(x
)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), (2*sqrt(
a^2 - b^2 - c^2)*a*c*arctan(-((a + b)*cosh(x) + (a + b)*sinh(x) + c)/sqrt(a
^2 - b^2 - c^2)) - 2*(c^3 - (a^2 - b^2)*c)*arctan(cosh(x) + sinh(x)) + (a^2
*b - b^3 - b*c^2)*log(2*(a*cosh(x) + b*sinh(x) + c)/(cosh(x) - sinh(x))) -
(a^2*b - b^3 - b*c^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 -
c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x) + c\operatorname{sech}(x)}\,dx$$

```
[In] integrate(sech(x)**2/(a+c*sech(x)+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*tanh(x) + c*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(x)}{a + c\operatorname{sech}(x) + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx=-\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}(b^2+c^2)}+\frac{2\,c\arctan\left(e^x\right)}{b^2+c^2}+\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x+a-b\right)}{b^2+c^2}-\frac{b\log\left(e^{(2\,x)}+1\right)}{b^2+c^2}$$

```
[In] integrate(sech(x)^2/(a+c*sech(x)+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(a^2 - b^2 - c^2))/(sqrt(a^2 - b^2 -
c^2)*(b^2 + c^2)) + 2*c*arctan(e^x)/(b^2 + c^2) + b*log(a*e^(2*x) + b*e^(2*
x) + 2*c*e^x + a - b)/(b^2 + c^2) - b*log(e^(2*x) + 1)/(b^2 + c^2)
```

Mupad [B] (verification not implemented)

Time = 30.90 (sec) , antiderivative size = 1069, normalized size of antiderivative = 7.32

$$\int \frac{\operatorname{sech}^2(x)}{a+c\operatorname{sech}(x)+b\tanh(x)}\,dx$$

$$=\frac{\ln\left(\frac{64\,(a-b+2\,c\,e^x)}{(a+b)^4}+\frac{\left(\frac{32\,\left(2\,a^3+3\,e^x\,a^2\,c-2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2-4\,e^x\,c^3\right)}{(a+b)^5}+\frac{\left(\frac{32\,(a-b)\,\left(-2\,b^3+6\,e^x\,b^2\,c-2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,c+3\,e^x\,c^3\right.}{(a+b)^5}\right.}{}\right.}{}\right.}{}$$

$$-\frac{\ln\left(\frac{64\,(a-b+2\,c\,e^x)}{(a+b)^4}-\frac{\left(\frac{32\,\left(2\,a^3+3\,e^x\,a^2\,c-2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2-4\,e^x\,c^3\right)}{(a+b)^5}-\frac{\left(\frac{32\,(a-b)\,\left(-2\,b^3+6\,e^x\,b^2\,c-2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,c+3\,e\right.}{(a+b)^5}\right.}{}\right.}{}\right.}{}$$

$$-\frac{\ln\left(1+e^x\,1i\right)}{b-c\,1i}-\frac{\ln\left(e^x+1i\right)\,1i}{-c+b\,1i}$$

```
[In] int(1/(cosh(x)^2*(a + b*tanh(x) + c/cosh(x))),x)
```

```
[Out] (log((64*(a - b + 2*c*exp(x)))/(a + b)^4 + (((32*(2*b*c^2 - 2*a*c^2 - 2*a*b
^2 + 2*a^3 - 4*c^3*exp(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x
```

```
)))/(a + b)^5 + (((32*(a - b)*(2*a*c^2 - 2*a*b^2 + b*c^2 - 2*b^3 + 3*c^3*ex
p(x) + 6*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(b*c^2 - a^2*b + b
^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(3*a*c^4 - 2*a*b^4 - 3*b*c^4 + 2*a^3*b^2
- 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*ex
p(x) - 3*a^2*c^3*exp(x) + 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) + 6*a*b*c^3*e
xp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b*c*exp(x)))/((a + b)^5*(b^2 + c^2)*(b^2 -
 a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 +
c^2)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)
))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 +
 c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 - a^2*c^2 + 2*b^2*c^2) - (log(exp(x) + 1
i)*1i)/(b*1i - c) - (log((64*(a - b + 2*c*exp(x)))/(a + b)^4 - (((32*(2*b*c
^2 - 2*a*c^2 - 2*a*b^2 + 2*a^3 - 4*c^3*exp(x) + 3*a^2*c*exp(x) + 3*b^2*c*ex
p(x) + 6*a*b*c*exp(x)))/(a + b)^5 - (((32*(a - b)*(2*a*c^2 - 2*a*b^2 + b*c^
2 - 2*b^3 + 3*c^3*exp(x) + 6*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (3
2*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(3*a*c^4 - 2*a*b^4 -
3*b*c^4 + 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*
a^2*b*c^2 + b^4*c*exp(x) - 3*a^2*c^3*exp(x) + 5*b^2*c^3*exp(x) + a^2*b^2*c*
exp(x) + 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b*c*exp(x)))/((a + b)^
5*(b^2 + c^2)*(b^2 - a^2 + c^2)))*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c
^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(a^2*b - b*c^2 - b^3 + a*c*(b^
2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(a^2*b - b*c^2 - b^
3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 - a^2*c^2 + 2*b^2*c^
2) - log(exp(x)*1i + 1)/(b - c*1i)
```

3.786 $\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=-\frac{2}{3}\operatorname{arctanh}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

```
[Out] 2/3*arctanh(-2/3+1/3*tanh(1/2*x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3245, 3203, 632, 210}

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=-\frac{2}{3}\operatorname{arctanh}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)$$

```
[In] Int[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(2 - Tanh[x/2])/3])/3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int \frac{1}{3i+2i\cosh(x)+2i\sinh(x)}\,dx\\ &= 2i\text{Subst}\left(\int \frac{1}{5i+4ix-ix^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\ &= -\left(4i\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,4i-2i\tanh\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2}{3}\text{arctanh}\left(\frac{1}{3}\left(2-\tanh\left(\frac{x}{2}\right)\right)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\text{csch}(x)}{2+2\coth(x)+3\text{csch}(x)}\,dx = \frac{x}{6}-\frac{1}{3}\log\left(5\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x]/(2 + 2*Coth[x] + 3*Csch[x]),x]
[Out] x/6 - Log[5*Cosh[x/2] - Sinh[x/2]]/3
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{x}{3}-\frac{\ln\left(\mathrm{e}^{x}+\frac{3}{2}\right)}{3}$	12
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-5\right)}{3}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{3}$	20

```
[In] int(csch(x)/(2+2*coth(x)+3*csch(x)),x,method=_RETURNVERBOSE)
[Out] 1/3*x-1/3*ln(exp(x)+3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=\frac{1}{3}\,x-\frac{1}{3}\,\log\left(2\,\cosh\left(x\right)+2\,\sinh\left(x\right)+3\right)$$

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="fricas")
[Out] 1/3*x - 1/3*log(2*cosh(x) + 2*sinh(x) + 3)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{csch}\left(x\right)}{2\coth\left(x\right)+3\operatorname{csch}\left(x\right)+2}\,dx$$

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x)
[Out] Integral(csch(x)/(2*coth(x) + 3*csch(x) + 2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=-\frac{1}{3}\,\log\left(3\,e^{(-x)}+2\right)$$

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="maxima")
[Out] -1/3*log(3*e^(-x) + 2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=\frac{1}{3}\,x-\frac{1}{3}\,\log\left(2\,e^{x}+3\right)$$

```
[In] integrate(csch(x)/(2+2*coth(x)+3*csch(x)),x, algorithm="giac")
[Out] 1/3*x - 1/3*log(2*e^x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\operatorname{csch}(x)}{2+2\coth(x)+3\operatorname{csch}(x)}\,dx=\frac{x}{3}-\frac{\ln\left(\mathrm{e}^{x}+\frac{3}{2}\right)}{3}$$

```
[In] int(1/(sinh(x)*(2*coth(x) + 3/sinh(x) + 2)),x)
[Out] x/3 - log(exp(x) + 3/2)/3
```

3.787 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[Out] -2*arctanh((a+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3245, 3203, 632, 210}

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}$$

```
[In] Int[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/Sqrt[a^2 - b^2
+ c^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \frac{1}{ic+ib\cosh(x)+ia\sinh(x)}\,dx\\
&= 2i\text{Subst}\left(\int \frac{1}{ib+ic+2iax-(-ib+ic)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\
&= -\left(4i\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2+c^2\right)-x^2}\,dx, x, 2ia+2(ib-ic)\tanh\left(\frac{x}{2}\right)\right)\right)\\
&= -\frac{2\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\text{csch}(x)}{a+b\coth(x)+c\text{csch}(x)}\,dx = \frac{2\arctan\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

```
[In] Integrate[Csch[x]/(a + b*Coth[x] + c*Csch[x]),x]
[Out] (2*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2
- c^2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{2\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}}$	53
risch	$\frac{\ln\left(\mathrm{e}^{x}+\frac{c\sqrt{a^{2}-b^{2}+c^{2}}-a^{2}+b^{2}-c^{2}}{(a+b)\sqrt{a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{a^{2}-b^{2}+c^{2}}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{c\sqrt{a^{2}-b^{2}+c^{2}}+a^{2}-b^{2}+c^{2}}{(a+b)\sqrt{a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{a^{2}-b^{2}+c^{2}}}$	139

```
[In] int(csch(x)/(a+b*coth(x)+c*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)/(-a^2+b^2-c^2)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$
$$=\left[\frac{\log\left(\frac{2\,(a+b)c\cosh(x)+(a^{2}+2\,ab+b^{2})\cosh(x)^{2}+(a^{2}+2\,ab+b^{2})\sinh(x)^{2}+a^{2}-b^{2}+2\,c^{2}+2\,((a+b)c+(a^{2}+2\,ab+b^{2})\cosh(x))\sinh(x)-2\,\sqrt{a^{2}-b^{2}+}}{(a+b)\cosh(x)^{2}+(a+b)\sinh(x)^{2}+2\,c\cosh(x)+2\,((a+b)\cosh(x)+c)\sinh(x)-a+b}\right.}{\sqrt{a^{2}-b^{2}+c^{2}}}\right.$$

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)*cosh(x)^2 + (a^2 + 2*a*b +
b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)*c + (a^2 + 2*a*b + b^2)*cos
h(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((a + b)*cosh(x) + (a + b)*sinh(x)
+ c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + 2*c*cosh(x) + 2*((a + b)*cos
h(x) + c)*sinh(x) - a + b))/sqrt(a^2 - b^2 + c^2), 2*sqrt(-a^2 + b^2 - c^2)
*arctan(sqrt(-a^2 + b^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2
 - b^2 + c^2))/(a^2 - b^2 + c^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x) + c*csch(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\frac{2\arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

```
[In] integrate(csch(x)/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/sqrt(-a^2 + b^2 - c^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{-a^2+b^2-c^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}+\frac{b\,\mathrm{e}^x}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}$$

```
[In] int(1/(sinh(x)*(a + c/sinh(x) + b*coth(x))),x)
```

```
[Out] (2*atan(c/(b^2 - a^2 - c^2)^(1/2) + (a*exp(x))/(b^2 - a^2 - c^2)^(1/2) + (b
*exp(x))/(b^2 - a^2 - c^2)^(1/2)))/(b^2 - a^2 - c^2)^(1/2)
```

3.788 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = -\frac{2ac\operatorname{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}$$

```
[Out] ln(tanh(1/2*x))/(b+c)-b*ln(b+c+2*a*tanh(1/2*x)+(b-c)*tanh(1/2*x)^2)/(b^2-c^
2)-2*a*c*arctanh((a+(b-c)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-
b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4482, 12, 1642, 648, 632, 212, 642}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = -\frac{2ac\operatorname{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}-\frac{b\log\left(2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$$

```
[In] Int[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(a + (b - c)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2
)*Sqrt[a^2 - b^2 + c^2]) + Log[Tanh[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tanh
[x/2] + (b - c)*Tanh[x/2]^2])/(b^2 - c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \frac{\text{csch}(x)}{ic+ib\cosh(x)+ia\sinh(x)}\,dx \\
&= -\left(2\text{Subst}\left(\int \frac{-1+x^2}{2x\left(b+c+2ax+(b-c)x^2\right)}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\right) \\
&= -\text{Subst}\left(\int \frac{-1+x^2}{x\left(b+c+2ax+(b-c)x^2\right)}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= -\text{Subst}\left(\int \left(-\frac{1}{(b+c)x}+\frac{2(a+bx)}{(b+c)\left(b+c+2ax+(b-c)x^2\right)}\right)dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{2\text{Subst}\left(\int \frac{a+bx}{b+c+2ax+(b-c)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b+c} \\
&= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\text{Subst}\left(\int \frac{2a+2(b-c)x}{b+c+2ax+(b-c)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2} \\
&\quad+\frac{(2ac)\text{Subst}\left(\int \frac{1}{b+c+2ax+(b-c)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2} \\
&= \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2-c^2} \\
&\quad-\frac{(4ac)\text{Subst}\left(\int \frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2a+2(b-c)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2} \\
&= -\frac{2ac\,\text{arctanh}\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c} \\
&\quad-\frac{b\log\left(b+c+2a\tanh\left(\frac{x}{2}\right)+(b-c)\tanh^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{\text{csch}^2(x)}{a+b\coth(x)+c\,\text{csch}(x)}\,dx = \frac{\frac{2ac\arctan\left(\frac{a+(b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+(b+c)\log\left(\cosh\left(\frac{x}{2}\right)\right)+(b-c)\log\left(\sinh\left(\frac{x}{2}\right)\right)-b\log(c+b\cosh(x)+a\sinh(x))}{(b-c)(b+c)}$$

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x] + c*Csch[x]),x]
[Out] ((2*a*c*ArcTan[(a + (b - c)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 +
 b^2 - c^2] + (b + c)*Log[Cosh[x/2]] + (b - c)*Log[Sinh[x/2]] - b*Log[c + b
*Cosh[x] + a*Sinh[x]])/((b - c)*(b + c))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

method	result
default	$\frac{-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b-c\tanh\left(\frac{x}{2}\right)^{2}+2a\tanh\left(\frac{x}{2}\right)+b+c\right)}{b-c}+\frac{\left(-2a+\frac{2ba}{b-c}\right)\arctan\left(\frac{2(b-c)\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\sqrt{-a^{2}+b^{2}-c^{2}}}}{b+c}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b+c}$
risch	$-\frac{x}{b-c}-\frac{x}{b+c}+\frac{2x\,a^{2}b}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}-\frac{2x\,b^{3}}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}+\frac{2xb\,c^{2}}{a^{2}b^{2}-a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}+\frac{\ln(1+e^{x})}{b-c}+\frac{\ln(e^{x}-}{b+c}$

```
[In] int(csch(x)^2/(a+b*coth(x)+c*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b+c)*(-b/(b-c)*ln(tanh(1/2*x)^2*b-c*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b+c)+(
-2*a+2*b*a/(b-c))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(b-c)*tanh(1/2*x)+2*a)
/(-a^2+b^2-c^2)^(1/2)))+ln(tanh(1/2*x))/(b+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(106) = 212.

Time = 0.90 (sec) , antiderivative size = 546, normalized size of antiderivative = 4.63

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$
$$=\left[-\frac{\sqrt{a^2-b^2+c^2}ac\log\left(\frac{2\,(a+b)c\cosh(x)+(a^2+2\,ab+b^2)\cosh(x)^2+(a^2+2\,ab+b^2)\sinh(x)^2+a^2-b^2+2\,c^2+2\,((a+b)c+(a^2+2\,ab+b^2)\cosh}{(a+b)\cosh(x)^2+(a+b)\sinh(x)^2+2\,c\cosh(x)+2\,((a+b)\cosh(x)+}\right.}{}\right.$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a^2 - b^2 + c^2)*a*c*log((2*(a + b)*c*cosh(x) + (a^2 + 2*a*b + b^2)
*cosh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 2*c^2 + 2*((a + b)
*c + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((a + b
)*cosh(x) + (a + b)*sinh(x) + c))/((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
2*c*cosh(x) + 2*((a + b)*cosh(x) + c)*sinh(x) - a + b)) + (a^2*b - b^3 + b*
c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) - (a^2*b - b^3
+ b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) - (a^2*b - b^3 +
b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 - b^4 - c
^4 - (a^2 - 2*b^2)*c^2), (2*sqrt(-a^2 + b^2 - c^2)*a*c*arctan(sqrt(-a^2 + b
^2 - c^2)*((a + b)*cosh(x) + (a + b)*sinh(x) + c)/(a^2 - b^2 + c^2)) - (a^2
*b - b^3 + b*c^2)*log(2*(b*cosh(x) + a*sinh(x) + c)/(cosh(x) - sinh(x))) +
(a^2*b - b^3 + b*c^2 + c^3 + (a^2 - b^2)*c)*log(cosh(x) + sinh(x) + 1) + (a
^2*b - b^3 + b*c^2 - c^3 - (a^2 - b^2)*c)*log(cosh(x) + sinh(x) - 1))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*coth(x)+c*csch(x)),x)
[Out] Integral(csch(x)**2/(a + b*coth(x) + c*csch(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx=\frac{2\,ac\arctan\left(\frac{ae^x+be^x+c}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)}-\frac{b\log\left(ae^{(2\,x)}+be^{(2\,x)}+2\,ce^x-a+b\right)}{b^2-c^2}+\frac{\log\left(e^x+1\right)}{b-c}+\frac{\log\left(|e^x-1|\right)}{b+c}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)+c*csch(x)),x, algorithm="giac")
[Out] 2*a*c*arctan((a*e^x + b*e^x + c)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 -
 c^2)*(b^2 - c^2)) - b*log(a*e^(2*x) + b*e^(2*x) + 2*c*e^x - a + b)/(b^2 -
c^2) + log(e^x + 1)/(b - c) + log(abs(e^x - 1))/(b + c)
```

Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 1069, normalized size of antiderivative = 9.06

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)+c\operatorname{csch}(x)}\,dx = \frac{\ln\left(e^x-1\right)}{b+c}+\frac{\ln\left(e^x+1\right)}{b-c}$$

$$+\frac{\ln\left(-\frac{64\,(b-a+2\,c\,e^x)}{(a+b)^4}-\frac{\left(\frac{32\,\left(-2\,a^3+3\,e^x\,a^2\,c+2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2+4\,e^x\,c^3\right)}{(a+b)^5}+\frac{\left(\frac{32\,(a-b)\,\left(2\,b^3+6\,e^x\,b^2\,c+2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,c-\right.}{(a+b)^5}\right.}{}\right.}{}\right.}{}$$

$$+\frac{\ln\left(-\frac{64\,(b-a+2\,c\,e^x)}{(a+b)^4}-\frac{\left(\frac{32\,\left(-2\,a^3+3\,e^x\,a^2\,c+2\,a\,b^2+6\,e^x\,a\,b\,c-2\,a\,c^2+3\,e^x\,b^2\,c+2\,b\,c^2+4\,e^x\,c^3\right)}{(a+b)^5}+\frac{\left(\frac{32\,(a-b)\,\left(2\,b^3+6\,e^x\,b^2\,c+2\,a\,b^2+b\,c^2+6\,a\,e^x\,b\,c-\right.}{(a+b)^5}\right.}{}\right.}{}\right.}{}$$

```
[In] int(1/(sinh(x)^2*(a + c/sinh(x) + b*coth(x))),x)
```

```
[Out] log(exp(x) - 1)/(b + c) + log(exp(x) + 1)/(b - c) + (log(- (64*(b - a + 2*c
*exp(x)))/(a + b)^4 - (((32*(2*a*b^2 - 2*a*c^2 + 2*b*c^2 - 2*a^3 + 4*c^3*ex
p(x) + 3*a^2*c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (((32
*(a - b)*(2*a*b^2 + 2*a*c^2 + b*c^2 + 2*b^3 - 3*c^3*exp(x) + 6*b^2*c*exp(x)
 + 6*a*b*c*exp(x)))/(a + b)^5 - (32*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 +
 c^2)^(1/2))*(2*a*b^4 - 3*a*c^4 + 3*b*c^4 - 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c
^2 + 4*c^5*exp(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) + 3*a^2*c^3*exp(
x) - 5*b^2*c^3*exp(x) + a^2*b^2*c*exp(x) - 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp
(x) - 4*a^3*b*c*exp(x)))/((a + b)^5*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b
+ b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 - b^2 + c^2
)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2
- b^2 + c^2)))*(a^2*b + b*c^2 - b^3 + a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 +
c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (log(- (64*(b - a + 2*c*exp(x)))/(a
+ b)^4 - (((32*(2*a*b^2 - 2*a*c^2 + 2*b*c^2 - 2*a^3 + 4*c^3*exp(x) + 3*a^2*
c*exp(x) + 3*b^2*c*exp(x) + 6*a*b*c*exp(x)))/(a + b)^5 + (((32*(a - b)*(2*a
*b^2 + 2*a*c^2 + b*c^2 + 2*b^3 - 3*c^3*exp(x) + 6*b^2*c*exp(x) + 6*a*b*c*ex
p(x)))/(a + b)^5 - (32*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2))*
(2*a*b^4 - 3*a*c^4 + 3*b*c^4 - 2*a^3*b^2 - 2*a^3*c^2 - 3*b^3*c^2 + 4*c^5*ex
p(x) + a*b^2*c^2 + 4*a^2*b*c^2 + b^4*c*exp(x) + 3*a^2*c^3*exp(x) - 5*b^2*c^
3*exp(x) + a^2*b^2*c*exp(x) - 6*a*b*c^3*exp(x) + 6*a*b^3*c*exp(x) - 4*a^3*b
*c*exp(x)))/((a + b)^5*(b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b + b*c^2 - b^3
 - a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 - b^2 + c^2)))*(a^2*b +
```

```
b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))/((b^2 - c^2)*(a^2 - b^2 + c^2))
)*(a^2*b + b*c^2 - b^3 - a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2
 + a^2*c^2 - 2*b^2*c^2)
```

3.789 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{cCx}{b^2-c^2}-\frac{2(A(b^2-c^2)+acC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] -c*C*x/(b^2-c^2)+b*C*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*(A*(b^2-c^2)+a*c
*C)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3216, 3203, 632, 212}

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2(acC+A(b^2-c^2))\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}-\frac{cCx}{b^2-c^2}$$

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] -((c*C*x)/(b^2 - c^2)) - (2*(A*(b^2 - c^2) + a*c*C)*ArcTanh[(c - (a - b)*Ta
nh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) + (b*C
*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad +\left(A+\frac{acC}{b^2-c^2}\right)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx \\
&= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad +\left(2\left(A+\frac{acC}{b^2-c^2}\right)\right)\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) \\
&= -\frac{cCx}{b^2-c^2}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad -\left(4\left(A+\frac{acC}{b^2-c^2}\right)\right)\text{Subst}\left(\int\frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx,x,2c\right. \\
&\qquad \left.+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)
\end{aligned}
$$

$$=-\frac{cCx}{b^2-c^2}-\frac{2\left(A+\frac{acC}{b^2-c^2}\right)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2-b^2+c^2}}+\frac{bC\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int\frac{A+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$=\frac{\frac{2(A(b^2-c^2)+acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}}+C(-cx+b\log(a+b\cosh(x)+c\sinh(x)))}{(b-c)(b+c)}$$

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((2*(A*(b^2 - c^2) + a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2
 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + C*(-(c*x) + b*Log[a + b*Cosh[x] + c*Sinh
[x]]))/((b - c)*(b + c))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.67

method	result
default	$-\frac{2C\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b-2c}+\frac{\frac{2\left(abC-C\,b^2\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\frac{2\left(-A\,b^2+A\,c^2-acC-Ccb+\frac{\left(abC-C\,b^2\right)c}{a-b}\right)\arctan(}{\sqrt{-a^2+b^2-c^2}}}{(b-c)(b+c)}$
risch	Expression too large to display

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*C/(2*b-2*c)*ln(1+tanh(1/2*x))+2/(b-c)/(b+c)*(1/2*(C*a*b-C*b^2)/(a-b)*ln(
a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(-A*b^2+A*c^2-a*c*C-C*
c*b+(C*a*b-C*b^2)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/
2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))-2*C/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.21

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \left[-\frac{(Ab^2 + Cac - Ac^2)\sqrt{a^2 - b^2 + c^2}\log\left(\frac{(b^2+2bc+c^2)\cosh(x)^2+(b^2+2bc+c^2)\sinh(x)^2+2a^2-b^2+c^2+2(ab+ac)\cosh(x)+2(a}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2a\cos}\right.}{}\right.$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-((A*b^2 + C*a*c - A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*c
osh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c
)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^
2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2
+ (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c
)) + (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^2 - C*b^2)*c)*x - (C*a^2*b -
 C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(
a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), (2*(A*b^2 + C*a*c - A*c^2)*sqrt(-
a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*s
inh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^2*b - C*b^3 + C*b*c^2 + C*c^3 + (C*a^
2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2)*log(2*(b*cosh(x) + c*sinh(x)
+ a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{Cb\log\left(be^{(2x)} + ce^{(2x)} + 2\,ae^{x} + b - c\right)}{b^2 - c^2} - \frac{Cx}{b - c} + \frac{2\left(Ab^2 + Cac - Ac^2\right)\arctan\left(\frac{be^{x} + ce^{x} + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}\,(b^2 - c^2)}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] C*b*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) - C*x/(b - c)
+ 2*(A*b^2 + C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^
2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.14

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{\ln\left(b\sqrt{a^2 - b^2 + c^2} - c\sqrt{a^2 - b^2 + c^2} - a^2\,e^{x} + b^2\,e^{x} - c^2\,e^{x} + a\,e^{x}\sqrt{a^2 - b^2 + c^2}\right)\left(C\,b^3 + A\,b^2\sqrt{a^2 - b^2 + \cdots}\right.}{-a^2\,b^2 + a^2\,c^2 + b^4 - 2\,b^2\,c^2 + c^4} - \frac{\ln\left(b\sqrt{a^2 - b^2 + c^2} - c\sqrt{a^2 - b^2 + c^2} + a^2\,e^{x} - b^2\,e^{x} + c^2\,e^{x} + a\,e^{x}\sqrt{a^2 - b^2 + c^2}\right)\left(A\,b^2\sqrt{a^2 - b^2 + c^2} - \cdots\right.}{-a^2\,b^2 + a^2\,c^2 + b^4 - 2\,b^2\,c^2 + c^4} - \frac{C\,x}{b - c}$$

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) + b
^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(C*b^3 + A*b^2*(
a^2 - b^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) - C*b*c^2
+ C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^
2) - (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x
) - b^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(A*b^2*(a^2
 - b^2 + c^2)^(1/2) - C*b^3 + C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) + C*b
*c^2 + C*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b
^2*c^2) - (C*x)/(b - c)
```

3.790 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA+cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} + \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*(A*a+C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^
2)^(3/2)+(b*C-(A*c-C*a)*cosh(x)-A*b*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*s
inh(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3233, 3203, 632, 212}

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{-\cosh(x)(Ac-aC)-Ab\sinh(x)+bC}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))} - \frac{2(aA+cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}$$

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) + (b*C - (A*c - a*C)*Cosh[x] - A*b*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}+\frac{(aA+cC)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}\\
&\quad+\frac{(2(aA+cC))\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}\\
&= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}\\
&\quad-\frac{(4(aA+cC))\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}\\
&= -\frac{2(aA+cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}+\frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA+cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}} + \frac{-aAc+a^2C-b^2C+(A(b^2-c^2)+acC)\sinh(x)}{b\,(-a^2+b^2-c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C - b^2*C + (A*(b^2 - c^2) + a*c*C
)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(102) = 204.

Time = 1.71 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.07

method	result
default	$-\frac{2\left(-\frac{\left(Aab-A\,b^2+A\,c^2-acC+Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}-\frac{Aac-C\,a^2+C\,b^2}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}\right)}{a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2(Aa+Cc)\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}}$
risch	$\frac{2Aab\,e^x+2Aac\,e^x-2C\,a^2e^x+2C\,b^2e^x+2Cbc\,e^x+2A\,b^2-2A\,c^2+2acC}{(b+c)(a^2-b^2+c^2)(e^{2x}b+e^{2x}c+2a\,e^x+b-c)}+\frac{\ln\left(e^x+\frac{\left(a^2-b^2+c^2\right)^{\frac{3}{2}}a-a^4+2a^2b^2-2a^2c^2-b^4+2b^2c^2-c^4}{\left(a^2-b^2+c^2\right)^{\frac{3}{2}}(b+c)}\right)Aa}{(a^2-b^2+c^2)^{\frac{3}{2}}}$

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh
(1/2*x)-(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2*(A*a+C*c)/(a^2-b^2+c^2)/(-a^2+b
^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(105) = 210.

Time = 0.31 (sec) , antiderivative size = 2211, normalized size of antiderivative = 20.47

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
[Out] [(2*A*a^2*b^2 - 2*A*b^4 + 2*C*a*c^3 - 2*A*c^4 - 2*(A*a^2 - 2*A*b^2)*c^2 + (
A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2
+ (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2 + (
2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c)*c
osh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A*a*b^2 + C*c^3 + (A*a
 + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2
)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^
2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*
cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(
x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*
cosh(x) + a)*sinh(x) + b - c)) + 2*(C*a^3 - C*a*b^2)*c - 2*(C*a^4 - A*a^3*b
 - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^2
)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - 2*(C*a^4 - A*a^3*b
 - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^2
)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^
4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*
b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)
*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^
2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c
^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b
- 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*
b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*
b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5
- 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^
2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3
 + b^5)*c)*cosh(x))*sinh(x)), 2*(A*a^2*b^2 - A*b^4 + C*a*c^3 - A*c^4 - (A*a
^2 - 2*A*b^2)*c^2 + (A*a*b^2 + C*b^2*c - A*a*c^2 - C*c^3 + (A*a*b^2 + C*c^3
 + (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x)^2 + (A*a*b^2 + C*c^3 +
(A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b + C*a*c^2 +
 (A*a^2 + C*a*b)*c)*cosh(x) + 2*(A*a^2*b + C*a*c^2 + (A*a^2 + C*a*b)*c + (A
*a*b^2 + C*c^3 + (A*a + 2*C*b)*c^2 + (2*A*a*b + C*b^2)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (C*a^3 - C*a*b^2)*c - (C*a^4 - A*a^
3*b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*
b^2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*cosh(x) - (C*a^4 - A*a^3*
```

```
b - 2*C*a^2*b^2 + A*a*b^3 + C*b^4 - (A*a + C*b)*c^3 + (C*a^2 - A*a*b - C*b^
2)*c^2 - (A*a^3 + C*a^2*b - A*a*b^2 - C*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b
^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4
*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3
)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b
^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*
c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b
 - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a
*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a
*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5
 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a
^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^
3 + b^5)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2} \, dx = \text{Timed out}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$$
$$=\frac{2\,(Aa+Cc)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}}-\frac{2\,(Ca^2e^x-Aabe^x-Cb^2e^x-Aace^x-Cbce^x-Ab^2-Cac+Ac^2)}{(a^2b-b^3+a^2c-b^2c+bc^2+c^3)(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c)}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^
2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(C*a^2*e^x - A*a*b*e^x - C*b^2*e^x - A
*a*c*e^x - C*b*c*e^x - A*b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c
 + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\int \frac{A+C\sinh(x)}{{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)}^2}\,dx$$

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.791 $\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\begin{aligned}\int \frac{A+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx &= -\frac{(2a^2A+A(b^2-c^2)+3acC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}\\ &\quad+\frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}\\ &\quad+\frac{abC-(3aAc-a^2C+2c^2C)\cosh(x)-b(3aA+2cC)\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}\end{aligned}$$

```
[Out] -(2*a^2*A+A*(b^2-c^2)+3*a*c*C)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^
(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(b*C-(A*c-C*a)*cosh(x)-A*b*sinh(x))/(a^2-b^2
+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(a*b*C-(3*A*a*c-C*a^2+2*C*c^2)*cosh(x)-
b*(3*A*a+2*C*c)*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{3236, 3232, 3203, 632, 212\}$

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$
$$= -\frac{(2a^2A + 3acC + A(b^2 - c^2))\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2 - b^2 + c^2)^{5/2}}$$
$$+ \frac{-\cosh(x)\left(a^2(-C) + 3aAc + 2c^2C\right) - b\sinh(x)(3aA + 2cC) + abC}{2\left(a^2 - b^2 + c^2\right)^2\left(a + b\cosh(x) + c\sinh(x)\right)}$$
$$+ \frac{-\cosh(x)(Ac - aC) - Ab\sinh(x) + bC}{2\left(a^2 - b^2 + c^2\right)\left(a + b\cosh(x) + c\sinh(x)\right)^2}$$

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) + (b*C - (A*c - a*C)*Cosh[x] -
 A*b*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) + (a*b*C
- (3*a*A*c - a^2*C + 2*c^2*C)*Cosh[x] - b*(3*a*A + 2*c*C)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
```

```
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3236

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

integral

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2}-\frac{\int\frac{-2(aA+cC)+Ab\cosh(x)+(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2(a^2-b^2+c^2)}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-(3aAc-a^2C+2c^2C)\cosh(x)-b(3aA+2cC)\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))} + \frac{(2a^2A+A(b^2-c^2)+3acC)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{2(a^2-b^2+c^2)^2}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-(3aAc-a^2C+2c^2C)\cosh(x)-b(3aA+2cC)\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))} + \frac{(2a^2A+A(b^2-c^2)+3acC)\operatorname{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2+c^2)^2}$$

$$= \frac{bC-(Ac-aC)\cosh(x)-Ab\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} + \frac{abC-(3aAc-a^2C+2c^2C)\cosh(x)-b(3aA+2cC)\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))} - \frac{(2(2a^2A+A(b^2-c^2)+3acC))\operatorname{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2+c^2)^2}$$

$$= -\frac{(2a^2A + A(b^2 - c^2) + 3acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2 - b^2 + c^2)^{5/2}} + \frac{bC - (Ac - aC)\cosh(x) - Ab\sinh(x)}{2\,(a^2 - b^2 + c^2)\,(a + b\cosh(x) + c\sinh(x))^2} + \frac{abC - (3aAc - a^2C + 2c^2C)\cosh(x) - b(3aA + 2cC)\sinh(x)}{2\,(a^2 - b^2 + c^2)^2\,(a + b\cosh(x) + c\sinh(x))}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.88

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \frac{(2a^2A + A(b^2 - c^2) + 3acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2 + b^2 - c^2)^{5/2}} + \frac{6a^3Ac + 3aAb^2c - 3aAc^3 - 2a^4C + 4a^2b^2C - 2b^4C + 5a^2c^2C + 4b^2c^2C - 2c^4C + 2bc(2a^2A + A(b^2 - c}{}$$

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-
a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c - 3*
a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c
^4*C + 2*b*c*(2*a^2*A + A*(b^2 - c^2) + 3*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 +
 c^2) - a^2*c*C + 2*c*(-b^2 + c^2)*C)*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A
*b^4*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 4*a^3*c*C*Sinh[
x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Sinh[x] - 3*a*A*b^3*Sinh[2*x] + 3*a*A*
b*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sin
h[2*x]])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(188) = 376$.

Time = 11.62 (sec) , antiderivative size = 836, normalized size of antiderivative = 4.22

method	result
default	$-\frac{2\left(-\frac{\left(4A\,a^3b-7A\,a^2b^2+5A\,a^2c^2+2Aa\,b^3-2Aab\,c^2+A\,b^4-3A\,b^2c^2+2A\,c^4-3C\,a^3c+6C\,a^2bc-3Ca\,b^2c\right)\tanh\left(\frac{x}{2}\right)^3}{2\left(a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4\right)(a-b)}-\frac{\left(4A\,a^4c-12A\,a^3bc+13A\,a^{}\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^
2+b^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*a
^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5-2*C*
a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+
13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^
2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a
^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a
*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c+4*C*a^2*
c^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a
^2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3-A*b^4*c+A*
b^2*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2
+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*A*a^2+A*b^2-A*c^2+3*C*a*c)/(a^4-2*a^2*b^2+2*
a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/
2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6084 vs. $2(188) = 376$.

Time = 0.59 (sec) , antiderivative size = 12285, normalized size of antiderivative = 62.05

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(188) = 376$.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.16

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \frac{(2\,Aa^2 + Ab^2 + 3\,Cac - Ac^2)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4 + 2\,a^2c^2 - 2\,b^2c^2 + c^4)\sqrt{-a^2 + b^2 - c^2}} + \frac{2\,Aa^2b^2e^{(3\,x)} + Ab^4e^{(3\,x)} + 4\,Aa^2bce^{(3\,x)} + 3\,Cab^2ce^{(3\,x)} + 2\,Ab^3ce^{(3\,x)} + 2\,Aa^2c^2e^{(3\,x)} + 6\,Cabc^2e^{(3\,x)} + 3\,C}{}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^
2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) + A*b^4*e^(3*x) + 4*A*a^2*b*c*e^(3*x
) + 3*C*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) + 6*C*a*b
*c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*C*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 4*C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) -
 2*C*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b^2*c*e^
(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3
*A*a*c^3*e^(2*x) - 2*C*c^4*e^(2*x) + 10*A*a^2*b^2*e^x - A*b^4*e^x + 4*C*a^3
*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e
^x - A*c^4*e^x + 3*A*a*b^3 + C*a^2*b*c - 3*A*a*b^2*c + 2*C*b^3*c - C*a^2*c^
2 - 3*A*a*b*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b -
2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2
*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \int \frac{A + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
[Out] int((A + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.792 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx=\frac{bBx}{b^2-c^2}+\frac{2(abB-A(b^2-c^2))\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] b*B*x/(b^2-c^2)-B*c*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)+2*(a*b*B-A*(b^2-c^2
))*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^2+c^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3217, 3203, 632, 212}

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx=\frac{2(abB-A(b^2-c^2))\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}}-\frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{bBx}{b^2-c^2}$$

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] (b*B*x)/(b^2 - c^2) + (2*(a*b*B - A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - (B*c*Lo
g[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3217

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[c*B*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bBx}{b^2-c^2} - \frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad + \left(A-\frac{abB}{b^2-c^2}\right)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx \\
&= \frac{bBx}{b^2-c^2} - \frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad + \left(2\left(A-\frac{abB}{b^2-c^2}\right)\right)\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{bBx}{b^2-c^2} - \frac{Bc\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad - \left(4\left(A-\frac{abB}{b^2-c^2}\right)\right)\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx, x, 2c\right. \\
&\qquad \left. + 2(-a+b)\tanh\left(\frac{x}{2}\right)\right)
\end{aligned}$$

$$= \frac{bBx}{b^2 - c^2} - \frac{2\left(A - \frac{abB}{b^2-c^2}\right) \operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\sqrt{a^2 - b^2 + c^2}} - \frac{Bc\log(a + b\cosh(x) + c\sinh(x))}{b^2 - c^2}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx$$
$$= \frac{-\frac{2(abB+A(-b^2+c^2))\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} + B(bx - c\log(a + b\cosh(x) + c\sinh(x)))}{(b - c)(b + c)}$$

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((-2*(a*b*B + A*(-b^2 + c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + B*(b*x - c*Log[a + b*Cosh[x] + c*Sinh[
x]]))/((b - c)*(b + c))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c} + \frac{\frac{2(-aBc+bBc)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2 b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b} + \frac{2\left(-A\,b^2+A\,c^2+abB+B\,c^2+\frac{(-aBc+bBc)c}{a-b}\right)\operatorname{arct}}{\sqrt{-a^2+b^2-c^2}}}{(b-c)(b+c)}$
risch	Expression too large to display

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*B/(2*b+2*c)*ln(tanh(1/2*x)-1)+2/(b-c)/(b+c)*(1/2*(-B*a*c+B*b*c)/(a-b)*ln
(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(-A*b^2+A*c^2+a*b*B+B
*c^2+(-B*a*c+B*b*c)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(
1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))+2*B/(2*b-2*c)*ln(1+tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 508, normalized size of antiderivative = 4.20

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \left[-\frac{(Bab-Ab^2+Ac^2)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)+2\,(ab+\cdots}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2\,a\cosh(\cdots}\right)\cdots}{\cdots}\right.$$
$$\left.-\frac{2\,(Bab-Ab^2+Ac^2)\sqrt{-a^2+b^2-c^2}\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((b+c)\cosh(x)+(b+c)\sinh(x)+a)}{a^2-b^2+c^2}\right)-(Ba^2b-Bb^3+Bb\cdots}{a^2b^2-b^4-c^4-(a^2-2\,b^2)\cdots}\right.$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fricas")
```

```
[Out] [-((B*a*b - A*b^2 + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*c
osh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c
)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^
2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2
+ (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c
)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a^2 - B*b^2)*c)*x + (B*c^3 + (
B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(
a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 + A*c^2)*sqrt(
-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*
sinh(x) + a)/(a^2 - b^2 + c^2)) - (B*a^2*b - B*b^3 + B*b*c^2 + B*c^3 + (B*a
^2 - B*b^2)*c)*x + (B*c^3 + (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x)
 + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = -\frac{Bc\log\left(be^{(2x)} + ce^{(2x)} + 2\,ae^{x} + b - c\right)}{b^2 - c^2} + \frac{Bx}{b - c} - \frac{2\,(Bab - Ab^2 + Ac^2)\arctan\left(\frac{be^{x} + ce^{x} + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}(b^2 - c^2)}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*c*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)/(b^2 - c^2) + B*x/(b - c)
 - 2*(B*a*b - A*b^2 + A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c
^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.10

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{\ln\left(b\sqrt{a^2 - b^2 + c^2} - c\sqrt{a^2 - b^2 + c^2} + a^2\,\mathrm{e}^{x} - b^2\,\mathrm{e}^{x} + c^2\,\mathrm{e}^{x} + a\,\mathrm{e}^{x}\sqrt{a^2 - b^2 + c^2}\right)\left(B\,c^3 - A\,b^2\sqrt{a^2 - b^2}\right.}{-a^2\,b^2 + a^2\,c^2 + b^4 - 2\,b^2\,c^2 + c^4} + \frac{\ln\left(b\sqrt{a^2 - b^2 + c^2} - c\sqrt{a^2 - b^2 + c^2} - a^2\,\mathrm{e}^{x} + b^2\,\mathrm{e}^{x} - c^2\,\mathrm{e}^{x} + a\,\mathrm{e}^{x}\sqrt{a^2 - b^2 + c^2}\right)\left(B\,c^3 + A\,b^2\sqrt{a^2 -}\right.}{-a^2\,b^2 + a^2\,c^2 + b^4 - 2\,b^2\,c^2 + c^4} + \frac{B\,x}{b - c}$$

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) - b
^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 - A*b^2*(
a^2 - b^2 + c^2)^(1/2) + B*a^2*c + A*c^2*(a^2 - b^2 + c^2)^(1/2) - B*b^2*c
+ B*a*b*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^
2) + (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x
) + b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + A*
b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - A*c^2*(a^2 - b^2 + c^2)^(1/2) - B*b
^2*c - B*a*b*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b
^2*c^2) + (B*x)/(b - c)
```

3.793 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*(A*a-B*b)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^
2)^(3/2)+(-B*c-A*c*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*
sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3234, 3203, 632, 212}

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^
2 - b^2 + c^2)^(3/2) - (B*c + A*c*Cosh[x] + (A*b - a*B)*Sinh[x])/((a^2 - b^
2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}+\frac{(aA-bB)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}\\
&\quad+\frac{(2(aA-bB))\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}\\
&= -\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}\\
&\quad-\frac{(4(aA-bB))\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}\\
&= -\frac{2(aA-bB)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=-\frac{2(aA-bB)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}+\frac{-aAc+bBc+\left(-abB+A\left(b^2-c^2\right)\right)\sinh(x)}{b\left(-a^2+b^2-c^2\right)\left(a+b\cosh(x)+c\sinh(x)\right)}$$

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-
a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + b*B*c + (-(a*b*B) + A*(b^2 - c^2))*Sin
h[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(104) = 208.

Time = 1.70 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

method	result
default	$-\frac{2\left(-\frac{\left(Aab-A\,b^{2}+A\,c^{2}-B\,a^{2}+abB-B\,c^{2}\right)\tanh\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}-\frac{(Aa-Bb)c}{a^{3}-a^{2}b-a\,b^{2}+a\,c^{2}+b^{3}-b\,c^{2}}\right)}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}-\frac{2(Aa-Bb)\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^{2}+b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}-c^{2}}}$
risch	$\frac{2Aab\,e^{x}+2Aac\,e^{x}-2B\,a^{2}e^{x}-2Bbc\,e^{x}-2B\,c^{2}e^{x}+2A\,b^{2}-2A\,c^{2}-2abB}{(b+c)(a^{2}-b^{2}+c^{2})(e^{2x}b+e^{2x}c+2a\,e^{x}+b-c)}+\frac{\ln\left(e^{x}+\frac{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}a-a^{4}+2a^{2}b^{2}-2a^{2}c^{2}-b^{4}+2b^{2}c^{2}-c^{4}}{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}(b+c)}\right)}{\left(a^{2}-b^{2}+c^{2}\right)^{\frac{3}{2}}}$

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2
)*tanh(1/2*x)-(A*a-B*b)*c/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x)
^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2*(A*a-B*b)/(a^2-b^2+c^2)/(-a^2+b^2
-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(105) = 210$.

Time = 0.31 (sec) , antiderivative size = 2228, normalized size of antiderivative = 20.63

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 + 2*A*c^4 + 2*(A*a^2 + B*a
*b - 2*A*b^2)*c^2 + (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3 +
 (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + (A*a
 - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + (A*a^
2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c + (A*a*b^2
 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^
2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sin
h(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 +
2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) +
(b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x)
+ 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^
2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*
a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + 2*(B*a^4 - A*a^3*b - B*a^2*b^
2 + A*a*b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*
a^3 - B*a^2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c
^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2
*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4
 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b
^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 -
 b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3
 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 +
(a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c
^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2
 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^
4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*
cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4 + A*c^4 + (A*a
^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 - (A*a - B*b)*c^2 + (A*a*b^2 -
 B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^
3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
 + (A*a^2 - B*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a^2 - B*a*b)*c +
(A*a*b^2 - B*b^3 + (A*a - B*b)*c^2 + 2*(A*a*b - B*b^2)*c)*cosh(x))*sinh(x))
*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b
 + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*
b^3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*a^3 - B*
a^2*b - A*a*b^2 + B*b^3)*c)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^
```

```
3 + B*c^4 - (A*a - B*b)*c^3 + (2*B*a^2 - A*a*b - B*b^2)*c^2 - (A*a^3 - B*a^
2*b - A*a*b^2 + B*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^
2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6
 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^
2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 +
 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2
+ 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 +
 a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a
^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5
 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*
c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2
*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*s
inh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,(Aa-Bb)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}} - \frac{2\,(Ba^2e^x-Aabe^x-Aace^x+Bbce^x+Bc^2e^x+Bab-Ab^2+Ac^2)}{(a^2b-b^3+a^2c-b^2c+bc^2+c^3)(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c)}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b^
2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x - A*a*b*e^x - A*a*c*e^x + B
*b*c*e^x + B*c^2*e^x + B*a*b - A*b^2 + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c
 + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^2}\,dx$$

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.794 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 194

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx &= -\frac{(2a^2A-3abB+A(b^2-c^2))\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}\\ &\quad-\frac{Bc+Ac\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}\\ &\quad-\frac{aBc+(3aA-2bB)c\cosh(x)+(3aAb-a^2B-2b^2B)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}\end{aligned}$$

```
[Out] -(2*a^2*A-3*a*b*B+A*(b^2-c^2))*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^
(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(-B*c-A*c*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^
2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(-a*B*c-(3*A*a-2*B*b)*c*cosh(x)-(3*A*a
*b-B*a^2-2*B*b^2)*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

$= \{3237, 3232, 3203, 632, 212\}$

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$
$$= -\frac{(2a^2A-3abB+A(b^2-c^2))\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}$$
$$-\frac{\sinh(x)\,(a^2(-B)+3aAb-2b^2B)+c\cosh(x)(3aA-2bB)+aBc}{2\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))}$$
$$-\frac{\sinh(x)(Ab-aB)+Ac\cosh(x)+Bc}{2\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^2}$$

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt
[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c + A*c*Cosh[x] + (A*b -
a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*B*c
+ (3*a*A - 2*b*B)*c*Cosh[x] + (3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2
- b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
```

```
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3237

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2
- b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x
], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 -
 b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\text{integral} = -\frac{Bc + Ac\cosh(x) + (Ab - aB)\sinh(x)}{2(a^2 - b^2 + c^2)(a + b\cosh(x) + c\sinh(x))^2} - \frac{\int \frac{-2(aA - bB) + (Ab - aB)\cosh(x) + Ac\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx}{2(a^2 - b^2 + c^2)}$$

$$= -\frac{Bc + Ac\cosh(x) + (Ab - aB)\sinh(x)}{2(a^2 - b^2 + c^2)(a + b\cosh(x) + c\sinh(x))^2} - \frac{aBc + (3aA - 2bB)c\cosh(x) + (3aAb - a^2B - 2b^2B)\sinh(x)}{2(a^2 - b^2 + c^2)^2(a + b\cosh(x) + c\sinh(x))} + \frac{(2a^2A - 3abB + A(b^2 - c^2))\int \frac{1}{a + b\cosh(x) + c\sinh(x)}\,dx}{2(a^2 - b^2 + c^2)^2}$$

$$= -\frac{Bc + Ac\cosh(x) + (Ab - aB)\sinh(x)}{2(a^2 - b^2 + c^2)(a + b\cosh(x) + c\sinh(x))^2} - \frac{aBc + (3aA - 2bB)c\cosh(x) + (3aAb - a^2B - 2b^2B)\sinh(x)}{2(a^2 - b^2 + c^2)^2(a + b\cosh(x) + c\sinh(x))} + \frac{(2a^2A - 3abB + A(b^2 - c^2))\,\text{Subst}\left(\int \frac{1}{a + b + 2cx - (a - b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2 - b^2 + c^2)^2}$$

$$= -\frac{Bc + Ac\cosh(x) + (Ab - aB)\sinh(x)}{2(a^2 - b^2 + c^2)(a + b\cosh(x) + c\sinh(x))^2} - \frac{aBc + (3aA - 2bB)c\cosh(x) + (3aAb - a^2B - 2b^2B)\sinh(x)}{2(a^2 - b^2 + c^2)^2(a + b\cosh(x) + c\sinh(x))} - \frac{(2(2a^2A - 3abB + A(b^2 - c^2)))\,\text{Subst}\left(\int \frac{1}{4(a^2 - b^2 + c^2) - x^2}\,dx, x, 2c + 2(-a + b)\tanh\left(\frac{x}{2}\right)\right)}{(a^2 - b^2 + c^2)^2}$$

$$= -\frac{(2a^2A - 3abB + A(b^2 - c^2))\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2 - b^2 + c^2)^{5/2}} - \frac{Bc + Ac\cosh(x) + (Ab - aB)\sinh(x)}{2(a^2 - b^2 + c^2)(a + b\cosh(x) + c\sinh(x))^2} - \frac{aBc + (3aA - 2bB)c\cosh(x) + (3aAb - a^2B - 2b^2B)\sinh(x)}{2(a^2 - b^2 + c^2)^2(a + b\cosh(x) + c\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.73

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \frac{(2a^2A - 3abB + A(b^2 - c^2))\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2 + b^2 - c^2)^{5/2}} + \frac{6a^3Ac + 3aAb^2c - 9a^2bBc - 3aAc^3 + 2bc(2a^2A - 3abB + A(b^2 - c^2))\cosh(x) + c(a^2bB + 2bB(b^2 - c^2)}{}$$

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-
a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*b^2*c - 9*
a^2*b*B*c - 3*a*A*c^3 + 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 - c^2))*Cosh[x] +
 c*(a^2*b*B + 2*b*B*(b^2 - c^2) + 3*a*A*(-b^2 + c^2))*Cosh[2*x] - 8*a^2*A*b
^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Sinh[x] + 12*a
^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8*a*b*B*c^2*Sinh[x] - 3*a*A*b^3*Si
nh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a*A*b*c^2*Sinh[2*x] -
 2*b^2*B*c^2*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x]
)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(190) = 380.

Time = 11.59 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.42

method	result
default	$-\frac{2\left(-\frac{\left(4A\,a^3b-7A\,a^2b^2+5A\,a^2c^2+2Aa\,b^3-2Aab\,c^2+A\,b^4-3A\,b^2c^2+2A\,c^4-2B\,a^4+3B\,a^3b-2B\,a^2b^2-4B\,a^2c^2+3Ba\,b^3-2B\,b^4+4B\,b^2c^2-2B\,c^4\right)}{2\left(a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4\right)(a-b)}\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*b^
4+4*B*b^2*c^2-2*B*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a-b)*ta
nh(1/2*x)^3-1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2-7*A*a^2*c^2-6*A*a*b^3+6*
A*a*b*c^2+A*b^4+A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2+4*B*a^2*c^
2-9*B*a*b^3+2*B*b^4-4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2
*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3
*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^
3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b
*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5+4*B*b^3*c^2-2*B*b*c^4)/(a^4-
2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)+1/2*c*(4
*A*a^4-3*A*a^2*b^2+A*a^2*c^2-A*b^4+A*b^2*c^2-5*B*a^3*b+5*B*a*b^3-2*B*a*b*c^
2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*A*a^2+A*b^2-A*c^2-3*B*a*b)/
(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2
*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6126 vs. $2(187) = 374$.

Time = 0.60 (sec) , antiderivative size = 12366, normalized size of antiderivative = 63.74

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(187) = 374$.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.22

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \frac{(2\,Aa^2 - 3\,Bab + Ab^2 - Ac^2)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4 + 2\,a^2c^2 - 2\,b^2c^2 + c^4)\sqrt{-a^2 + b^2 - c^2}} + \frac{2\,Aa^2b^2e^{(3\,x)} - 3\,Bab^3e^{(3\,x)} + Ab^4e^{(3\,x)} + 4\,Aa^2bce^{(3\,x)} - 6\,Bab^2ce^{(3\,x)} + 2\,Ab^3ce^{(3\,x)} + 2\,Aa^2c^2e^{(3\,x)} - 3\,Ba}{}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2 - A*c^2)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 +
b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^
2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^4*e^(3*x)
+ 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c
^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 3*A*a*b^2*c*e^
(2*x) - 4*B*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x) + 4*B*b^2*c^2*e^(2*x) - 3
*A*a*c^3*e^(2*x) - 2*B*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x - 5*B
*a*b^3*e^x - A*b^4*e^x - 10*A*a^2*c^2*e^x + 5*B*a*b*c^2*e^x + 2*A*b^2*c^2*e
^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4 + B*a^2*b*c - 3*A*a*b^2*c
+ 2*B*b^3*c - 3*A*a*b*c^2 + 2*B*b^2*c^2 + 3*A*a*c^3 - 2*B*b*c^3)/((a^4*b -
2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2
*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b -
c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx=\int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
[Out] int((A + B*cosh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.795 $\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2}+\frac{2a(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)+2*a*(B*
b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^2-c^2)/(a^2-b^
2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3215, 3203, 632, 212}

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{2a(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}+\frac{x(bB-cC)}{b^2-c^2}$$

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) + (2*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/
2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 + c^2]) - ((B*c - b
*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3215

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[(c*B - b*C)*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 +
c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad - \frac{(a(bB-cC))\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2} \\
&= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad - \frac{(2a(bB-cC))\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2} \\
&= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} \\
&\quad + \frac{(4a(bB-cC))\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}
\end{aligned}$$

$$= \frac{(bB-cC)x}{b^2-c^2} + \frac{2a(bB-cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x - \frac{2a(bB-cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} + (-Bc+bC)\log(a+b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x - (2*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2
 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*Log[a + b*Cosh[x] +
 c*Sinh[x]])/((b - c)*(b + c))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.81

method	result
default	$\frac{2(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b+2c} + \frac{2(B-C)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b-2c} + \frac{\frac{2\left(-aBc+bBc+abC-C\,b^2\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b}+\cdots}{(b-\cdots}$
risch	Expression too large to display

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-B-C)/(2*b+2*c)*ln(tanh(1/2*x)-1)+2*(B-C)/(2*b-2*c)*ln(1+tanh(1/2*x))+2/
(b-c)/(b+c)*(1/2*(-B*a*c+B*b*c+C*a*b-C*b^2)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1
/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(a*b*B+B*c^2-a*c*C-C*c*b+(-B*a*c+B*b*c+C*a*b
-C*b^2)*c/(a-b))/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/
(-a^2+b^2-c^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.66

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \left[\frac{(Bab-Cac)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2bc+c^2)\cosh(x)^2+(b^2+2bc+c^2)\sinh(x)^2+2a^2-b^2+c^2+2(ab+ac)\cosh(x)+2(ab+ac+(b^2+\ldots}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2a\cosh(x)+2((b\ldots}\right.}{}\right.$$
$$-\frac{2(Bab-Cac)\sqrt{-a^2+b^2-c^2}\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((b+c)\cosh(x)+(b+c)\sinh(x)+a)}{a^2-b^2+c^2}\right)-((B-C)a^2b-(B-C\ldots}{}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="frica
s")
```

```
[Out] [((B*a*b - C*a*c)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2
+ (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x)
 + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*sqrt(a^2 - b^2 +
 c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)
*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) + ((B
- C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B
- C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*lo
g(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4
- (a^2 - 2*b^2)*c^2), -(2*(B*a*b - C*a*c)*sqrt(-a^2 + b^2 - c^2)*arctan(sqr
t(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^
2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((B - C)
*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^2 - B*
b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b^2 -
b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{(B-C)x}{b-c} + \frac{(Cb-Bc)\log\left(be^{(2x)} + ce^{(2x)} + 2ae^{x} + b - c\right)}{b^2-c^2} - \frac{2(Bab-Cac)\arctan\left(\frac{be^{x}+ce^{x}+a}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}(b^2-c^2)}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="giac"
)
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c
)/(b^2 - c^2) - 2*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^
2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.01

$$\int \frac{B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{\ln\left(b\sqrt{a^2-b^2+c^2} - c\sqrt{a^2-b^2+c^2} + a^2\,e^{x} - b^2\,e^{x} + c^2\,e^{x} + a\,e^{x}\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3 + C\,b^3 + B\,a^2\,c - \right.}{-a^2\,b^2 + a^2\,c^2 + b^4 - 2\,b^2\,c^2 + c^4} + \frac{\ln\left(b\sqrt{a^2-b^2+c^2} - c\sqrt{a^2-b^2+c^2} - a^2\,e^{x} + b^2\,e^{x} - c^2\,e^{x} + a\,e^{x}\sqrt{a^2-b^2+c^2}\right)\left(B\,c^3 + C\,b^3 + B\,a^2\,c\right.}{-a^2\,b^2 + a^2\,c^2 + b^4 - 2\,b^2\,c^2 + c^4} + \frac{x\,(B-C)}{b-c}$$

```
[In] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) - b
^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 +
 B*a^2*c - C*a^2*b - B*b^2*c - C*b*c^2 + B*a*b*(a^2 - b^2 + c^2)^(1/2) - C*
a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) +
 (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) +
b^2*exp(x) - c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3
+ B*a^2*c - C*a^2*b - B*b^2*c - C*b*c^2 - B*a*b*(a^2 - b^2 + c^2)^(1/2) + C
*a*c*(a^2 - b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2)
+ (x*(B - C))/(b - c)
```

3.796 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2(bB-cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] 2*(B*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c^2
)^(3/2)+(-B*c+b*C+a*C*cosh(x)+a*B*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sin
h(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3232, 3203, 632, 212}

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2(bB-cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{\left(a^2-b^2+c^2\right)^{3/2}} - \frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2
 - b^2 + c^2)^(3/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/((a^2 - b^2 +
 c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}-\frac{(bB-cC)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2}\\
&= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}\\
&\quad-\frac{(2(bB-cC))\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}\\
&= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}\\
&\quad+\frac{(4(bB-cC))\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}\\
&= \frac{2(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}}-\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{2(bB-cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\left(-a^2+b^2-c^2\right)^{3/2}}+\frac{bBc+a^2C-b^2C+a(-bB+cC)\sinh(x)}{b\left(-a^2+b^2-c^2\right)(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (2*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/(-a
^2 + b^2 - c^2)^(3/2) + (b*B*c + a^2*C - b^2*C + a*(-(b*B) + c*C)*Sinh[x])/
(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(99) = 198.

Time = 1.72 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

method	result
default	$\frac{-\frac{2\left(B\,a^2-abB+B\,c^2+acC-Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}-\frac{2\left(bBc+C\,a^2-C\,b^2\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}}{a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b}+\frac{2(Bb-Cc)\arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right)-2c}{2\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}}$
risch	$-\frac{2\left(B\,a^2\mathrm{e}^x+Bbc\,\mathrm{e}^x+B\,c^2\mathrm{e}^x+C\,a^2\mathrm{e}^x-C\,b^2\mathrm{e}^x-Cbc\,\mathrm{e}^x+abB-acC\right)}{(b+c)(a^2-b^2+c^2)(\mathrm{e}^{2x}b+\mathrm{e}^{2x}c+2a\,\mathrm{e}^x+b-c)}+\frac{\ln\left(\mathrm{e}^x+\frac{\left(a^2-b^2+c^2\right)^{\frac{3}{2}}a+a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4}{\left(a^2-b^2+c^2\right)^{\frac{3}{2}}(b+c)}\right)Bb}{(a^2-b^2+c^2)^{\frac{3}{2}}}$

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 2*(-(B*a^2-B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2)*tanh(
1/2*x)-(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+b^3-b*c^2))/(a*tanh(1/2*x
)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+2*(B*b-C*c)/(a^2-b^2+c^2)/(-a^2+b^
2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(101) = 202$.

Time = 0.31 (sec) , antiderivative size = 2119, normalized size of antiderivative = 19.62

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="fri
cas")
```

```
[Out] [-(2*B*a^3*b - 2*B*a*b^3 + 2*B*a*b*c^2 - 2*C*a*c^3 + (B*b^3 - C*b^2*c - B*b
*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x)^
2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B*a*
b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(x) + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c
^2 + (B*b^3 + (2*B - C)*b^2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh(x))*
sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c
^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c +
(b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cos
h(x) + (b + c)*sinh(x) + a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*c
osh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c
 + 2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B*c^4 + ((2*
B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c)*cosh(x) +
2*((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c^3 + B*c^4 + ((2*B
 + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b^3)*c)*sinh(x))/(a^
4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*
b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 +
 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cos
h(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4
*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(
x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3
 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) + 2*(a^5*b
- 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*
b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c
^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a
^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x)), -2*(B*a^3*b - B*a*b^3 + B*a*b
*c^2 - C*a*c^3 + (B*b^3 - C*b^2*c - B*b*c^2 + C*c^3 + (B*b^3 + (2*B - C)*b^
2*c + (B - 2*C)*b*c^2 - C*c^3)*cosh(x)^2 + (B*b^3 + (2*B - C)*b^2*c + (B -
2*C)*b*c^2 - C*c^3)*sinh(x)^2 + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c^2)*cosh(
x) + 2*(B*a*b^2 + (B - C)*a*b*c - C*a*c^2 + (B*b^3 + (2*B - C)*b^2*c + (B -
 2*C)*b*c^2 - C*c^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-
a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2))
 - (C*a^3 - C*a*b^2)*c + ((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)
*b*c^3 + B*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B -
C)*b^3)*c)*cosh(x) + ((B + C)*a^4 - (B + 2*C)*a^2*b^2 + C*b^4 + (B - C)*b*c
```

```
^3 + B*c^4 + ((2*B + C)*a^2 - (B + C)*b^2)*c^2 + ((B - C)*a^2*b - (B - C)*b
^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a
^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 +
(2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^
2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2
*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*
b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 +
2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*
cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*
c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^
2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^
4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$
$$= -\frac{2\,(Bb - Cc)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}}$$
$$- \frac{2\,(Ba^2e^x + Ca^2e^x - Cb^2e^x + Bbce^x - Cbce^x + Bc^2e^x + Bab - Cac)}{(a^2b - b^3 + a^2c - b^2c + bc^2 + c^3)(be^{(2\,x)} + ce^{(2\,x)} + 2\,ae^x + b - c)}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="gia
c")
```

```
[Out] -2*(B*b - C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^2 - b
^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - C*b^2*e^x +
B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - C*a*c)/((a^2*b - b^3 + a^2*c -
b^2*c + b*c^2 + c^3)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx$$

```
[In] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.797 $\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{3a(bB-cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-(2bBc+(a^2-2c^2)C)\cosh(x)-(a^2B+2b(bB-cC))\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] 3*a*(B*b-C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^2+c
^2)^(5/2)+1/2*(-B*c+b*C+a*C*cosh(x)+a*B*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)
+c*sinh(x))^2+1/2*(-a*(B*c-C*b)+(2*b*B*c+(a^2-2*c^2)*C)*cosh(x)+(B*a^2+2*b*
(B*b-C*c))*sinh(x))/(a^2-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3235, 3232, 3203, 632, 212}

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$$
$$=\frac{3a(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}}-\frac{-\cosh(x)\left(C(a^2-2c^2)+2bBc\right)-\sinh(x)\left(a^2B+2b(bB-cC)\right)+a(Bc-bC)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}-\frac{-aB\sinh(x)-aC\cosh(x)-bC+Bc}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}$$

```
[In] Int[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (3*a*(b*B - c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/(a
^2 - b^2 + c^2)^(5/2) - (B*c - b*C - a*C*Cosh[x] - a*B*Sinh[x])/(2*(a^2 - b
^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x])^2) - (a*(B*c - b*C) - (2*b*B*c + (a^2
 - 2*c^2)*C)*Cosh[x] - (a^2*B + 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c
^2)^2*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
```

```
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}-\frac{\int \frac{2(bB-cC)-aB\cosh(x)-aC\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(a^2-b^2+c^2\right)}\\
&= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}\\
&\quad-\frac{a(Bc-bC)-\left(2bBc+\left(a^2-2c^2\right)C\right)\cosh(x)-\left(a^2B+2b(bB-cC)\right)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}\\
&\quad-\frac{(3a(bB-cC))\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{2\left(a^2-b^2+c^2\right)^2}\\
&= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}\\
&\quad-\frac{a(Bc-bC)-\left(2bBc+\left(a^2-2c^2\right)C\right)\cosh(x)-\left(a^2B+2b(bB-cC)\right)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}\\
&\quad-\frac{(3a(bB-cC))\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^2}\\
&= -\frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2}\\
&\quad-\frac{a(Bc-bC)-\left(2bBc+\left(a^2-2c^2\right)C\right)\cosh(x)-\left(a^2B+2b(bB-cC)\right)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))}\\
&\quad+\frac{(6a(bB-cC))\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^2}
\end{aligned}$$

$$= \frac{3a(bB-cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{Bc-bC-aC\cosh(x)-aB\sinh(x)}{2(a^2-b^2+c^2)(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)-(2bBc+(a^2-2c^2)C)\cosh(x)-(a^2B+2b(bB-cC))\sinh(x)}{2(a^2-b^2+c^2)^2(a+b\cosh(x)+c\sinh(x))}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.64

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{3a(bB-cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}} + \frac{-9a^2bBc-2a^4C+4a^2b^2C-2b^4C+5a^2c^2C+4b^2c^2C-2c^4C-6abc(bB-cC)\cosh(x)+c(a^2+2b^2}{}$$

```
[In] Integrate[(B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] (-3*a*(b*B - c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/
(-a^2 + b^2 - c^2)^(5/2) + (-9*a^2*b*B*c - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C
+ 5*a^2*c^2*C + 4*b^2*c^2*C - 2*c^4*C - 6*a*b*c*(b*B - c*C)*Cosh[x] + c*(a^
2 + 2*b^2 - 2*c^2)*(b*B - c*C)*Cosh[2*x] + 4*a^3*b*B*Sinh[x] + 2*a*b^3*B*Si
nh[x] - 8*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a
*c^3*C*Sinh[x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] - 2*b^2*B*c^2*Sinh
[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4
*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x] + c*Sinh[x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(181) = 362.

Time = 11.48 (sec) , antiderivative size = 800, normalized size of antiderivative = 4.12

method	result
default	$-\frac{\frac{\left(2B a^4-3B a^3 b+2B a^2 b^2+4B a^2 c^2-3B a b^3+2B b^4-4B b^2 c^2+2B c^4+3C a^3 c-6C a^2 bc+3C a b^2 c\right)\tanh\left(\frac{x}{2}\right)^3}{\left(a^4-2a^2b^2+2a^2c^2+b^4-2b^2c^2+c^4\right)(a-b)}+\frac{\left(2B a^4 c-9B a^3 bc+14B a^2 b^2 c+4\right.}{}}{}$
risch	$-\frac{-C a^2bc-B a^2bc+2Cb c^3-2C c^4+2Bb c^3-2C b^3c+C a^2c^2+2C b^2c^2+B a^2b^2-2B b^3c-2B b^2c^2+2B b^4-4C a^2b^2 e^{2x}-5C a^2c^2 e^{2x}}{}$

```
[In] int((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2+4*B*a^2*c^2-3*B*a*b^3+2*B*b^4-4*B*b^
2*c^2+2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b
^4-2*b^2*c^2+c^4)/(a-b)*tanh(1/2*x)^3+1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b
^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*
b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c
^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2+1/2*(2*B*a^5-3*B*a^4*b+B*a^3*b^2
+4*B*a^3*c^2+B*a^2*b^3+8*B*a^2*b*c^2-3*B*a*b^4-8*B*a*b^2*c^2+2*B*a*c^4+2*B*
b^5-4*B*b^3*c^2+2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c-4*C*a^2*c^3+5
*C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*
a*b+b^2)*tanh(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c+2*B*b*c^3+2*C*a^4-4*C*a^2
*b^2-C*a^2*c^2+2*C*b^4+C*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^
4)/(a^2-2*a*b+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2
+3*a*(B*b-C*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2+b^2-c^2)^(
1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4996 vs. $2(182) = 364$.

Time = 0.59 (sec) , antiderivative size = 10107, normalized size of antiderivative = 52.10

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3} \, dx = \text{Too large to display}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="fri
cas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3} \, dx = \text{Timed out}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(182) = 364$.

Time = 0.27 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.97

$$\int \frac{B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$
$$= -\frac{3\,(Bab - Cac)\arctan\left(\frac{be^x + ce^x + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4 + 2\,a^2c^2 - 2\,b^2c^2 + c^4)\sqrt{-a^2 + b^2 - c^2}} - \frac{3\,Bab^3e^{(3\,x)} + 6\,Bab^2ce^{(3\,x)} - 3\,Cab^2ce^{(3\,x)} + 3\,Babc^2e^{(3\,x)} - 6\,Cabc^2e^{(3\,x)} - 3\,Cac^3e^{(3\,x)} + 2\,Ba^4e^{(2\,x)} +}{}$$

```
[In] integrate((B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="gia
c")
```

```
[Out] -3*(B*a*b - C*a*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^4
 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4)*sqrt(-a^2 + b^2 - c^2)) -
 (3*B*a*b^3*e^(3*x) + 6*B*a*b^2*c*e^(3*x) - 3*C*a*b^2*c*e^(3*x) + 3*B*a*b*c
^2*e^(3*x) - 6*C*a*b*c^2*e^(3*x) - 3*C*a*c^3*e^(3*x) + 2*B*a^4*e^(2*x) + 2*
C*a^4*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 4*C*a^2*b^2*e^(2*x) + 2*B*b^4*e^(2*x)
 + 2*C*b^4*e^(2*x) + 9*B*a^2*b*c*e^(2*x) - 9*C*a^2*b*c*e^(2*x) + 4*B*a^2*c^
2*e^(2*x) - 5*C*a^2*c^2*e^(2*x) - 4*B*b^2*c^2*e^(2*x) - 4*C*b^2*c^2*e^(2*x)
 + 2*B*c^4*e^(2*x) + 2*C*c^4*e^(2*x) + 4*B*a^3*b*e^x + 5*B*a*b^3*e^x - 4*C*
a^3*c*e^x - 5*C*a*b^2*c*e^x - 5*B*a*b*c^2*e^x + 5*C*a*c^3*e^x + B*a^2*b^2 +
 2*B*b^4 - B*a^2*b*c - C*a^2*b*c - 2*B*b^3*c - 2*C*b^3*c + C*a^2*c^2 - 2*B*
b^2*c^2 + 2*C*b^2*c^2 + 2*B*b*c^3 + 2*C*b*c^3 - 2*C*c^4)/((a^4*b - 2*a^2*b^
3 + b^5 + a^4*c - 2*a^2*b^2*c + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3
 - 2*b^2*c^3 + b*c^4 + c^5)*(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx=\int \frac{B\cosh(x)+C\sinh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

```
[In] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
[Out] int((B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.798 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x}{b^2-c^2} - \frac{2(Ab^2-abB-Ac^2+acC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

```
[Out] (B*b-C*c)*x/(b^2-c^2)-(B*c-C*b)*ln(a+b*cosh(x)+c*sinh(x))/(b^2-c^2)-2*(A*b^
2-A*c^2-B*a*b+C*a*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(b^
2-c^2)/(a^2-b^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 3203, 632, 212}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = -\frac{2(-abB+acC+Ab^2-Ac^2)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} + \frac{x(bB-cC)}{b^2-c^2}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x)/(b^2 - c^2) - (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTanh[(c
 - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]])/((b^2 - c^2)*Sqrt[a^2 - b^2 +
 c^2]) - ((B*c - b*C)*Log[a + b*Cosh[x] + c*Sinh[x]])/(b^2 - c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3215

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[(c*B - b*C)*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 +
c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}\\
&\quad+\frac{\left(Ab^2-abB-Ac^2+acC\right)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{b^2-c^2}\\
&=\frac{(bB-cC)x}{b^2-c^2}-\frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}\\
&\quad+\frac{\left(2\left(Ab^2-abB-Ac^2+acC\right)\right)\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}
\end{aligned}
$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2} - \frac{(4(Ab^2-abB-Ac^2+acC))\,\text{Subst}\left(\int \frac{1}{4(a^2-b^2+c^2)-x^2}\,dx, x, 2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{b^2-c^2}$$

$$= \frac{(bB-cC)x}{b^2-c^2} - \frac{2(Ab^2-abB-Ac^2+acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(b^2-c^2)\sqrt{a^2-b^2+c^2}} - \frac{(Bc-bC)\log(a+b\cosh(x)+c\sinh(x))}{b^2-c^2}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx = \frac{(bB-cC)x + \frac{2(Ab^2-abB-Ac^2+acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{\sqrt{-a^2+b^2-c^2}} + (-Bc+bC)\log(a+b\cosh(x)+c\sinh(x))}{(b-c)(b+c)}$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x]),x]
```

```
[Out] ((b*B - c*C)*x + (2*(A*b^2 - a*b*B - A*c^2 + a*c*C)*ArcTan[(c + (-a + b)*Ta
nh[x/2])/Sqrt[-a^2 + b^2 - c^2]])/Sqrt[-a^2 + b^2 - c^2] + (-(B*c) + b*C)*L
og[a + b*Cosh[x] + c*Sinh[x]])/((b - c)*(b + c))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.73

method	result
default	$\frac{2(B-C)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b-2c} + \frac{\frac{2\left(-aBc+bBc+abC-Cb^2\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a-2b} + \frac{2\left(-Ab^2+Ac^2+abB+Bc^2-acC\cdots\right.}{\cdots}}{(b-c)(b+c)}$
risch	Expression too large to display

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*(B-C)/(2*b-2*c)*ln(1+tanh(1/2*x))+2/(b-c)/(b+c)*(1/2*(-B*a*c+B*b*c+C*a*b-
C*b^2)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)+(-A*b^
2+A*c^2+a*b*B+B*c^2-a*c*C-C*c*b+(-B*a*c+B*b*c+C*a*b-C*b^2)*c/(a-b))/(-a^2+b
```

```
^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2)))+2
*(-B-C)/(2*b+2*c)*ln(tanh(1/2*x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.42

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \left[\frac{(Bab-Ab^2-Cac+Ac^2)\sqrt{a^2-b^2+c^2}\log\left(\frac{(b^2+2\,bc+c^2)\cosh(x)^2+(b^2+2\,bc+c^2)\sinh(x)^2+2\,a^2-b^2+c^2+2\,(ab+ac)\cosh(x)}{(b+c)\cosh(x)^2+(b+c)\sinh(x)^2+2}\right.}{}\right.$$
$$-\frac{2\,(Bab-Ab^2-Cac+Ac^2)\sqrt{-a^2+b^2-c^2}\arctan\left(\frac{\sqrt{-a^2+b^2-c^2}((b+c)\cosh(x)+(b+c)\sinh(x)+a)}{a^2-b^2+c^2}\right)-((B-C)a^2}{}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] [((B*a*b - A*b^2 - C*a*c + A*c^2)*sqrt(a^2 - b^2 + c^2)*log(((b^2 + 2*b*c +
 c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 - b^2 + c^2 + 2*(a*
b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cosh(x))*sinh(x) + 2*
sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x) + a))/((b + c)*cos
h(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x)
+ b - c)) + ((B - C)*a^2*b - (B - C)*b^3 + (B - C)*b*c^2 + (B - C)*c^3 + ((
B - C)*a^2 - (B - C)*b^2)*c)*x + (C*a^2*b - C*b^3 + C*b*c^2 - B*c^3 - (B*a^
2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(cosh(x) - sinh(x))))/(a^2*
b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2), -(2*(B*a*b - A*b^2 - C*a*c + A*c^2)*s
qrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((b + c)*cosh(x) + (b +
 c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - ((B - C)*a^2*b - (B - C)*b^3 + (B - C
)*b*c^2 + (B - C)*c^3 + ((B - C)*a^2 - (B - C)*b^2)*c)*x - (C*a^2*b - C*b^3
 + C*b*c^2 - B*c^3 - (B*a^2 - B*b^2)*c)*log(2*(b*cosh(x) + c*sinh(x) + a)/(
cosh(x) - sinh(x))))/(a^2*b^2 - b^4 - c^4 - (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{a + b\cosh(x) + c\sinh(x)}\,dx = \frac{(B - C)x}{b - c} + \frac{(Cb - Bc)\log\left(be^{(2x)} + ce^{(2x)} + 2\,ae^{x} + b - c\right)}{b^2 - c^2} - \frac{2\,(Bab - Ab^2 - Cac + Ac^2)\arctan\left(\frac{be^{x} + ce^{x} + a}{\sqrt{-a^2 + b^2 - c^2}}\right)}{\sqrt{-a^2 + b^2 - c^2}\,(b^2 - c^2)}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x)),x, algorithm="gia
c")
```

```
[Out] (B - C)*x/(b - c) + (C*b - B*c)*log(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c
)/(b^2 - c^2) - 2*(B*a*b - A*b^2 - C*a*c + A*c^2)*arctan((b*e^x + c*e^x + a
)/sqrt(-a^2 + b^2 - c^2))/(sqrt(-a^2 + b^2 - c^2)*(b^2 - c^2))
```

Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.31

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+c\sinh(x)}\,dx$$
$$= \frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}+a^2\,e^x-b^2\,e^x+c^2\,e^x+a\,e^x\sqrt{a^2-b^2+c^2}\right)\,\left(B\,c^3+C\,b^3-A\,b^2\sqrt{a^2\cdots}\right.}{-a^2\,b^2+a^2\,c^2\cdots}$$
$$+\frac{\ln\left(b\sqrt{a^2-b^2+c^2}-c\sqrt{a^2-b^2+c^2}-a^2\,e^x+b^2\,e^x-c^2\,e^x+a\,e^x\sqrt{a^2-b^2+c^2}\right)\,\left(B\,c^3+C\,b^3+A\,b^2\sqrt{\cdots}\right.}{-a^2\,b^2+a^2\cdots}$$
$$+\frac{x\,(B-C)}{b-c}$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x)),x)
```

```
[Out] (log(b*(a^2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) + a^2*exp(x) - b
^2*exp(x) + c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 -
 A*b^2*(a^2 - b^2 + c^2)^(1/2) + B*a^2*c - C*a^2*b + A*c^2*(a^2 - b^2 + c^2
)^(1/2) - B*b^2*c - C*b*c^2 + B*a*b*(a^2 - b^2 + c^2)^(1/2) - C*a*c*(a^2 -
b^2 + c^2)^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (log(b*(a^
2 - b^2 + c^2)^(1/2) - c*(a^2 - b^2 + c^2)^(1/2) - a^2*exp(x) + b^2*exp(x)
- c^2*exp(x) + a*exp(x)*(a^2 - b^2 + c^2)^(1/2))*(B*c^3 + C*b^3 + A*b^2*(a^
2 - b^2 + c^2)^(1/2) + B*a^2*c - C*a^2*b - A*c^2*(a^2 - b^2 + c^2)^(1/2) -
B*b^2*c - C*b*c^2 - B*a*b*(a^2 - b^2 + c^2)^(1/2) + C*a*c*(a^2 - b^2 + c^2)
^(1/2)))/(b^4 + c^4 - a^2*b^2 + a^2*c^2 - 2*b^2*c^2) + (x*(B - C))/(b - c)
```

3.799 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB+cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -2*(A*a-B*b+C*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b^2+c^2)^(1/2))/(a^2-b^
2+c^2)^(3/2)+(-B*c+b*C-(A*c-C*a)*cosh(x)-(A*b-B*a)*sinh(x))/(a^2-b^2+c^2)/(
a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3232, 3203, 632, 212}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = -\frac{2(aA-bB+cC)\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{\sinh(x)(Ab-aB)+\cosh(x)(Ac-aC)-bC+Bc}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B + c*C)*ArcTanh[(c - (a - b)*Tanh[x/2])/Sqrt[a^2 - b^2 + c^2]
])/(a^2 - b^2 + c^2)^(3/2) - (B*c - b*C + (A*c - a*C)*Cosh[x] + (A*b - a*B)
*Sinh[x])/((a^2 - b^2 + c^2)*(a + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))} \\
&\quad+\frac{(aA-bB+cC)\int\frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{a^2-b^2+c^2} \\
&= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))} \\
&\quad+\frac{(2(aA-bB+cC))\text{Subst}\left(\int\frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2} \\
&= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))} \\
&\quad-\frac{(4(aA-bB+cC))\text{Subst}\left(\int\frac{1}{4(a^2-b^2+c^2)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2+c^2}
\end{aligned}$$

$$= -\frac{2(aA - bB + cC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{3/2}} - \frac{Bc - bC + (Ac - aC)\cosh(x) + (Ab - aB)\sinh(x)}{(a^2-b^2+c^2)\,(a + b\cosh(x) + c\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = -\frac{2(aA - bB + cC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{3/2}} + \frac{-aAc + a^2C + b(Bc - bC) + (-abB + A(b^2-c^2) + acC)\sinh(x)}{b\,(-a^2+b^2-c^2)\,(a + b\cosh(x) + c\sinh(x))}$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A - b*B + c*C)*ArcTan[(c + (-a + b)*Tanh[x/2])/Sqrt[-a^2 + b^2 - c^2
]])/(-a^2 + b^2 - c^2)^(3/2) + (-(a*A*c) + a^2*C + b*(B*c - b*C) + (-(a*b*B
) + A*(b^2 - c^2) + a*c*C)*Sinh[x])/(b*(-a^2 + b^2 - c^2)*(a + b*Cosh[x] +
c*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(116) = 232$.

Time = 1.74 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\left(-\frac{\left(Aab-A\,b^2+A\,c^2-B\,a^2+abB-B\,c^2-acC+Ccb\right)\tanh\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}-\frac{Aac-bBc-C\,a^2+C\,b^2}{a^3-a^2b-a\,b^2+a\,c^2+b^3-b\,c^2}\right)}{a\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)^2b-2c\tanh\left(\frac{x}{2}\right)-a-b} - \frac{2(Aa-Bb+Cc)\arctan\left(\frac{2(a-b)\tanh(}{2\sqrt{-a^2+b^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}}$
risch	$\frac{2Aab\,\mathrm{e}^x+2Aac\,\mathrm{e}^x-2B\,a^2\mathrm{e}^x-2Bbc\,\mathrm{e}^x-2B\,c^2\mathrm{e}^x-2C\,a^2\mathrm{e}^x+2C\,b^2\mathrm{e}^x+2Cbc\,\mathrm{e}^x+2A\,b^2-2A\,c^2-2abB+2acC}{(b+c)(a^2-b^2+c^2)(\mathrm{e}^{2x}b+\mathrm{e}^{2x}c+2a\,\mathrm{e}^x+b-c)} + \frac{\ln\left(\mathrm{e}^x+\frac{\left(a^2-b^2+c^2\right)^{\frac{3}{2}}}{}\right)}{}$

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -2*(-(A*a*b-A*b^2+A*c^2-B*a^2+B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2+a*c
^2+b^3-b*c^2)*tanh(1/2*x)-(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2+a*c^2+
b^3-b*c^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)-2*(A*a-B*
```

```
b+C*c)/(a^2-b^2+c^2)/(-a^2+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2
*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. 2(116) = 232.

Time = 0.31 (sec) , antiderivative size = 2541, normalized size of antiderivative = 21.00

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="f
ricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - 2*C*a*c^3 + 2*A*c^4 + 2*
(A*a^2 + B*a*b - 2*A*b^2)*c^2 - (A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a -
 B*b)*c^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b -
 (2*B - C)*b^2)*c)*cosh(x)^2 + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*
b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + C*
a*c^2 + (A*a^2 - (B - C)*a*b)*c)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 +
 (A*a^2 - (B - C)*a*b)*c + (A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c
^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x))*sinh(x))*sqrt(a^2 - b^2 + c^2)*l
og(((b^2 + 2*b*c + c^2)*cosh(x)^2 + (b^2 + 2*b*c + c^2)*sinh(x)^2 + 2*a^2 -
 b^2 + c^2 + 2*(a*b + a*c)*cosh(x) + 2*(a*b + a*c + (b^2 + 2*b*c + c^2)*cos
h(x))*sinh(x) - 2*sqrt(a^2 - b^2 + c^2)*((b + c)*cosh(x) + (b + c)*sinh(x)
+ a))/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 + 2*a*cosh(x) + 2*((b + c)*cos
h(x) + a)*sinh(x) + b - c)) - 2*(C*a^3 - C*a*b^2)*c + 2*((B + C)*a^4 - A*a^
3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 +
 ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b
^2 + (B - C)*b^3)*c)*cosh(x) + 2*((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2
 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b
 - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*si
nh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3*b^2)*c^4 - (a^4 - 4*a^
2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 -
b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b
^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^
2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5
)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 -
a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c)*cosh(x) +
 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(
a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (a^4*b^2 - 2*a^2*b^4 + b
^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*
c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2
*b^2 - B*a*b^3 + A*b^4 - C*a*c^3 + A*c^4 + (A*a^2 + B*a*b - 2*A*b^2)*c^2 -
(A*a*b^2 - B*b^3 + C*b^2*c - C*c^3 - (A*a - B*b)*c^2 + (A*a*b^2 - B*b^3 + C
```

```
*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c)*cosh(x)^2 + (
A*a*b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^
2)*c)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c)
*cosh(x) + 2*(A*a^2*b - B*a*b^2 + C*a*c^2 + (A*a^2 - (B - C)*a*b)*c + (A*a*
b^2 - B*b^3 + C*c^3 + (A*a - (B - 2*C)*b)*c^2 + (2*A*a*b - (2*B - C)*b^2)*c
)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2 - c^2)*arctan(sqrt(-a^2 + b^2 - c^2)*((
b + c)*cosh(x) + (b + c)*sinh(x) + a)/(a^2 - b^2 + c^2)) - (C*a^3 - C*a*b^2
)*c + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4
- (A*a - (B - C)*b)*c^3 + ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^
3 - (B - C)*a^2*b - A*a*b^2 + (B - C)*b^3)*c)*cosh(x) + ((B + C)*a^4 - A*a^
3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4 + B*c^4 - (A*a - (B - C)*b)*c^3 +
 ((2*B + C)*a^2 - A*a*b - (B + C)*b^2)*c^2 - (A*a^3 - (B - C)*a^2*b - A*a*b
^2 + (B - C)*b^3)*c)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 - c^6 - (2*a^2 - 3
*b^2)*c^4 - (a^4 - 4*a^2*b^2 + 3*b^4)*c^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*
b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2
*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*
c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b - b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(
a^4*b - 2*a^2*b^3 + b^5)*c)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*
c^4 + a*c^5 + 2*(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^
2 + a*b^4)*c)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + a*b*c^4 + a*c^5 + 2*
(a^3 - a*b^2)*c^3 + 2*(a^3*b - a*b^3)*c^2 + (a^5 - 2*a^3*b^2 + a*b^4)*c + (
a^4*b^2 - 2*a^2*b^4 + b^6 + 2*b*c^5 + c^6 + (2*a^2 - b^2)*c^4 + 4*(a^2*b -
b^3)*c^3 + (a^4 - b^4)*c^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*c)*cosh(x))*sinh(x
))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.71

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \frac{2\,(Aa-Bb+Cc)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^2-b^2+c^2)\sqrt{-a^2+b^2-c^2}} - \frac{2\,(Ba^2e^x+Ca^2e^x-Aabe^x-Cb^2e^x-Aace^x+Bbce^x-Cbce^x+Bc^2e^x+Bab-Ab^2-Cac+Ac^2)}{(a^2b-b^3+a^2c-b^2c+bc^2+c^3)(be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c)}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, algorithm="g
iac")
```

```
[Out] 2*(A*a - B*b + C*c)*arctan((b*e^x + c*e^x + a)/sqrt(-a^2 + b^2 - c^2))/((a^
2 - b^2 + c^2)*sqrt(-a^2 + b^2 - c^2)) - 2*(B*a^2*e^x + C*a^2*e^x - A*a*b*e
^x - C*b^2*e^x - A*a*c*e^x + B*b*c*e^x - C*b*c*e^x + B*c^2*e^x + B*a*b - A*
b^2 - C*a*c + A*c^2)/((a^2*b - b^3 + a^2*c - b^2*c + b*c^2 + c^3)*(b*e^(2*x
) + c*e^(2*x) + 2*a*e^x + b - c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx = \int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2,x)
```

```
[Out] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^2, x)
```

3.800 $\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 233

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = -\frac{(2a^2A+Ab^2-3abB-Ac^2+3acC)\,\text{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\,(a^2-b^2+c^2)\,(a+b\cosh(x)+c\sinh(x))^2} - \frac{a(Bc-bC)+(3aAc-a^2C-2c(bB-cC))\cosh(x)+(3aAb-a^2B-2b(bB-cC))\sinh(x)}{2\,(a^2-b^2+c^2)^2\,(a+b\cosh(x)+c\sinh(x))}$$

```
[Out] -(2*A*a^2+A*b^2-A*c^2-3*B*a*b+3*C*a*c)*arctanh((c-(a-b)*tanh(1/2*x))/(a^2-b
^2+c^2)^(1/2))/(a^2-b^2+c^2)^(5/2)+1/2*(-B*c+b*C-(A*c-C*a)*cosh(x)-(A*b-B*a
)*sinh(x))/(a^2-b^2+c^2)/(a+b*cosh(x)+c*sinh(x))^2+1/2*(-a*(B*c-C*b)-(3*A*a
*c-C*a^2-2*c*(B*b-C*c))*cosh(x)-(3*A*a*b-B*a^2-2*b*(B*b-C*c))*sinh(x))/(a^2
-b^2+c^2)^2/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3235, 3232, 3203, 632, 212\}$

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx$$
$$= -\frac{(2a^2A - 3abB + 3acC + Ab^2 - Ac^2)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2 - b^2 + c^2)^{5/2}}$$
$$- \frac{\sinh(x)\left(a^2(-B) + 3aAb - 2b(bB - cC)\right) + \cosh(x)\left(a^2(-C) + 3aAc - 2c(bB - cC)\right) + a(Bc - bC)}{2\left(a^2 - b^2 + c^2\right)^2 (a + b\cosh(x) + c\sinh(x))}$$
$$- \frac{\sinh(x)(Ab - aB) + \cosh(x)(Ac - aC) - bC + Bc}{2\left(a^2 - b^2 + c^2\right)(a + b\cosh(x) + c\sinh(x))^2}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTanh[(c - (a - b)*Tanh[
x/2])/Sqrt[a^2 - b^2 + c^2]])/(a^2 - b^2 + c^2)^(5/2)) - (B*c - b*C + (A*c
- a*C)*Cosh[x] + (A*b - a*B)*Sinh[x])/(2*(a^2 - b^2 + c^2)*(a + b*Cosh[x] +
 c*Sinh[x])^2) - (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B - c*C))*Cosh[
x] + (3*a*A*b - a^2*B - 2*b*(b*B - c*C))*Sinh[x])/(2*(a^2 - b^2 + c^2)^2*(a
 + b*Cosh[x] + c*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
```

```
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} \\
&\quad -\frac{\int \frac{-2(aA-bB+cC)+(Ab-aB)\cosh(x)+(Ac-aC)\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx}{2\left(a^2-b^2+c^2\right)} \\
&= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} \\
&\quad -\frac{a(Bc-bC)+\left(3aAc-a^2C-2c(bB-cC)\right)\cosh(x)+\left(3aAb-a^2B-2b(bB-cC)\right)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))} \\
&\quad +\frac{\left(2a^2A+Ab^2-3abB-Ac^2+3acC\right)\int \frac{1}{a+b\cosh(x)+c\sinh(x)}\,dx}{2\left(a^2-b^2+c^2\right)^2} \\
&= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} \\
&\quad -\frac{a(Bc-bC)+\left(3aAc-a^2C-2c(bB-cC)\right)\cosh(x)+\left(3aAb-a^2B-2b(bB-cC)\right)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))} \\
&\quad +\frac{\left(2a^2A+Ab^2-3abB-Ac^2+3acC\right)\text{Subst}\left(\int \frac{1}{a+b+2cx-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^2} \\
&= -\frac{Bc-bC+(Ac-aC)\cosh(x)+(Ab-aB)\sinh(x)}{2\left(a^2-b^2+c^2\right)(a+b\cosh(x)+c\sinh(x))^2} \\
&\quad -\frac{a(Bc-bC)+\left(3aAc-a^2C-2c(bB-cC)\right)\cosh(x)+\left(3aAb-a^2B-2b(bB-cC)\right)\sinh(x)}{2\left(a^2-b^2+c^2\right)^2(a+b\cosh(x)+c\sinh(x))} \\
&\quad -\frac{\left(2\left(2a^2A+Ab^2-3abB-Ac^2+3acC\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2-b^2+c^2\right)-x^2}\,dx,x,2c+2(-a+b)\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2-b^2+c^2\right)^2}
\end{aligned}$$

$$= -\frac{(2a^2A + Ab^2 - 3abB - Ac^2 + 3acC)\operatorname{arctanh}\left(\frac{c-(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2+c^2}}\right)}{(a^2-b^2+c^2)^{5/2}} - \frac{Bc - bC + (Ac - aC)\cosh(x) + (Ab - aB)\sinh(x)}{2(a^2-b^2+c^2)(a + b\cosh(x) + c\sinh(x))^2} - \frac{a(Bc - bC) + (3aAc - a^2C - 2c(bB - cC))\cosh(x) + (3aAb - a^2B - 2b(bB - cC))\sinh(x)}{2(a^2-b^2+c^2)^2(a + b\cosh(x) + c\sinh(x))}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.00

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \frac{(2a^2A + Ab^2 - 3abB - Ac^2 + 3acC)\arctan\left(\frac{c+(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2-c^2}}\right)}{(-a^2+b^2-c^2)^{5/2}} + \frac{6a^3Ac + 3aAb^2c - 9a^2bBc - 3aAc^3 - 2a^4C + 4a^2b^2C - 2b^4C + 5a^2c^2C + 4b^2c^2C - 2c^4C + 2bc(2a^2A +}{}$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3*a*c*C)*ArcTan[(c + (-a + b)*Tanh[x/
2])/Sqrt[-a^2 + b^2 - c^2]])/(-a^2 + b^2 - c^2)^(5/2) + (6*a^3*A*c + 3*a*A*
b^2*c - 9*a^2*b*B*c - 3*a*A*c^3 - 2*a^4*C + 4*a^2*b^2*C - 2*b^4*C + 5*a^2*c
^2*C + 4*b^2*c^2*C - 2*c^4*C + 2*b*c*(2*a^2*A + A*b^2 - 3*a*b*B - A*c^2 + 3
*a*c*C)*Cosh[x] + c*(3*a*A*(-b^2 + c^2) + a^2*(b*B - c*C) + 2*(b^2 - c^2)*(
b*B - c*C))*Cosh[2*x] - 8*a^2*A*b^2*Sinh[x] + 2*A*b^4*Sinh[x] + 4*a^3*b*B*S
inh[x] + 2*a*b^3*B*Sinh[x] + 12*a^2*A*c^2*Sinh[x] - 2*A*b^2*c^2*Sinh[x] - 8
*a*b*B*c^2*Sinh[x] - 4*a^3*c*C*Sinh[x] - 2*a*b^2*c*C*Sinh[x] + 8*a*c^3*C*Si
nh[x] - 3*a*A*b^3*Sinh[2*x] + a^2*b^2*B*Sinh[2*x] + 2*b^4*B*Sinh[2*x] + 3*a
*A*b*c^2*Sinh[2*x] - 2*b^2*B*c^2*Sinh[2*x] - a^2*b*c*C*Sinh[2*x] - 2*b^3*c*
C*Sinh[2*x] + 2*b*c^3*C*Sinh[2*x])/(4*b*(a^2 - b^2 + c^2)^2*(a + b*Cosh[x]
+ c*Sinh[x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(228) = 456.

Time = 11.80 (sec) , antiderivative size = 1084, normalized size of antiderivative = 4.65

method	result	size
default	Expression too large to display	1084
risch	Expression too large to display	1966

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x,method=_RETURNVERBO
SE)
```

```
[Out] -2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2+5*A*a^2*c^2+2*A*a*b^3-2*A*a*b*c^2+A*b^4-3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2-4*B*a^2*c^2+3*B*a*b^3-2*B*b^
4+4*B*b^2*c^2-2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a-b)/(a^4-2*a^2*b
^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)*tanh(1/2*x)^3-1/2*(4*A*a^4*c-12*A*a^3*b*c+1
3*A*a^2*b^2*c-7*A*a^2*c^3-6*A*a*b^3*c+6*A*a*b*c^3+A*b^4*c+A*b^2*c^3-2*A*c^5
+2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c+4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c-4*B
*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2+5*C*a^3*c^2-4*C*a^2*b^3-14*C
*a^2*b*c^2-2*C*a*b^4+13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5-4*C*b^3*c^2+2*C*b*c^4
)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tanh(1/2*x)^2
+1/2*(4*A*a^4*b-5*A*a^3*b^2+11*A*a^3*c^2-3*A*a^2*b^3-3*A*a^2*b*c^2+5*A*a*b^
4-7*A*a*b^2*c^2+2*A*a*c^4-A*b^5-A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3
*b^2-4*B*a^3*c^2-B*a^2*b^3-8*B*a^2*b*c^2+3*B*a*b^4+8*B*a*b^2*c^2-2*B*a*c^4-
2*B*b^5+4*B*b^3*c^2-2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c+4*C*a^2*c
^3-5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^
2-2*a*b+b^2)*tanh(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c+A*a^2*c^3-A*b^4*c+A*b
^2*c^3-5*B*a^3*b*c+5*B*a*b^3*c-2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2+C*a^3*c^2-2*
C*a*b^4-C*a*b^2*c^2)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(a^2-2*a*b
+b^2))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-2*c*tanh(1/2*x)-a-b)^2-(2*A*a^2+A*b
^2-A*c^2-3*B*a*b+3*C*a*c)/(a^4-2*a^2*b^2+2*a^2*c^2+b^4-2*b^2*c^2+c^4)/(-a^2
+b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tanh(1/2*x)-2*c)/(-a^2+b^2-c^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6850 vs. $2(223) = 446$.

Time = 0.73 (sec) , antiderivative size = 13813, normalized size of antiderivative = 59.28

$$\int \frac{A + B\cosh(x) + C\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="f
ricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(223) = 446$.

Time = 0.30 (sec) , antiderivative size = 819, normalized size of antiderivative = 3.52

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx = \frac{(2\,Aa^2-3\,Bab+Ab^2+3\,Cac-Ac^2)\arctan\left(\frac{be^x+ce^x+a}{\sqrt{-a^2+b^2-c^2}}\right)}{(a^4-2\,a^2b^2+b^4+2\,a^2c^2-2\,b^2c^2+c^4)\sqrt{-a^2+b^2-c^2}} + \frac{2\,Aa^2b^2e^{(3\,x)}-3\,Bab^3e^{(3\,x)}+Ab^4e^{(3\,x)}+4\,Aa^2bce^{(3\,x)}-6\,Bab^2ce^{(3\,x)}+3\,Cab^2ce^{(3\,x)}+2\,Ab^3ce^{(3\,x)}+2\,Aa}{}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+c*sinh(x))^3,x, algorithm="g
iac")
[Out] (2*A*a^2 - 3*B*a*b + A*b^2 + 3*C*a*c - A*c^2)*arctan((b*e^x + c*e^x + a)/sq
rt(-a^2 + b^2 - c^2))/((a^4 - 2*a^2*b^2 + b^4 + 2*a^2*c^2 - 2*b^2*c^2 + c^4
)*sqrt(-a^2 + b^2 - c^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x) + A*b^
4*e^(3*x) + 4*A*a^2*b*c*e^(3*x) - 6*B*a*b^2*c*e^(3*x) + 3*C*a*b^2*c*e^(3*x)
```

```
 + 2*A*b^3*c*e^(3*x) + 2*A*a^2*c^2*e^(3*x) - 3*B*a*b*c^2*e^(3*x) + 6*C*a*b*
c^2*e^(3*x) + 3*C*a*c^3*e^(3*x) - 2*A*b*c^3*e^(3*x) - A*c^4*e^(3*x) - 2*B*a
^4*e^(2*x) - 2*C*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x) + 4*
C*a^2*b^2*e^(2*x) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 2*C*b^4*e^(2*x) +
 6*A*a^3*c*e^(2*x) - 9*B*a^2*b*c*e^(2*x) + 9*C*a^2*b*c*e^(2*x) + 3*A*a*b^2*
c*e^(2*x) - 4*B*a^2*c^2*e^(2*x) + 5*C*a^2*c^2*e^(2*x) - 3*A*a*b*c^2*e^(2*x)
 + 4*B*b^2*c^2*e^(2*x) + 4*C*b^2*c^2*e^(2*x) - 3*A*a*c^3*e^(2*x) - 2*B*c^4*
e^(2*x) - 2*C*c^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^
x - A*b^4*e^x + 4*C*a^3*c*e^x + 5*C*a*b^2*c*e^x - 10*A*a^2*c^2*e^x + 5*B*a*
b*c^2*e^x + 2*A*b^2*c^2*e^x - 5*C*a*c^3*e^x - A*c^4*e^x - B*a^2*b^2 + 3*A*a
*b^3 - 2*B*b^4 + B*a^2*b*c + C*a^2*b*c - 3*A*a*b^2*c + 2*B*b^3*c + 2*C*b^3*
c - C*a^2*c^2 - 3*A*a*b*c^2 + 2*B*b^2*c^2 - 2*C*b^2*c^2 + 3*A*a*c^3 - 2*B*b
*c^3 - 2*C*b*c^3 + 2*C*c^4)/((a^4*b - 2*a^2*b^3 + b^5 + a^4*c - 2*a^2*b^2*c
 + b^4*c + 2*a^2*b*c^2 - 2*b^3*c^2 + 2*a^2*c^3 - 2*b^2*c^3 + b*c^4 + c^5)*(
b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^3}\,dx=\int \frac{A+B\cosh(x)+C\sinh(x)}{\left(a+b\cosh\left(x\right)+c\sinh\left(x\right)\right)^3}\,dx$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3,x)
[Out] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + c*sinh(x))^3, x)
```

3.801 $\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 22

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{c\cosh(x)+b\sinh(x)}{a+b\cosh(x)+c\sinh(x)}$$

```
[Out] (c*cosh(x)+b*sinh(x))/(a+b*cosh(x)+c*sinh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3229}

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\frac{b\sinh(x)+c\cosh(x)}{a+b\cosh(x)+c\sinh(x)}$$

```
[In] Int[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])^2,x
]
```

```
[Out] (c*Cosh[x] + b*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x])
```

Rule 3229

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

Rubi steps

$$\text{integral} = \frac{c\cosh(x) + b\sinh(x)}{a + b\cosh(x) + c\sinh(x)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{b^2 - c^2 + ab\cosh(x) + ac\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \frac{-ac + b^2\sinh(x) - c^2\sinh(x)}{b(a + b\cosh(x) + c\sinh(x))}$$

```
[In] Integrate[(b^2 - c^2 + a*b*Cosh[x] + a*c*Sinh[x])/(a + b*Cosh[x] + c*Sinh[x
])^2,x]
```

```
[Out] (-(a*c) + b^2*Sinh[x] - c^2*Sinh[x])/(b*(a + b*Cosh[x] + c*Sinh[x]))
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

method	result	size
risch	$-\frac{2(a\,\mathrm{e}^{x}+b-c)}{\mathrm{e}^{2x}b+\mathrm{e}^{2x}c+2a\,\mathrm{e}^{x}+b-c}$	36
default	$\frac{-\frac{2\left(ab-b^{2}+c^{2}\right)\tanh\left(\frac{x}{2}\right)}{a-b}-\frac{2ac}{a-b}}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b-2c\tanh\left(\frac{x}{2}\right)-a-b}$	73

```
[In] int((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] -2*(a*exp(x)+b-c)/(exp(2*x)*b+exp(2*x)*c+2*a*exp(x)+b-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(22) = 44$.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \frac{b^2 - c^2 + ab\cosh(x) + ac\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = -\frac{2\,(a\cosh{(x)} + a\sinh{(x)} + b - c)}{(b + c)\cosh{(x)}^2 + (b + c)\sinh{(x)}^2 + 2\,a\cosh{(x)} + 2\,((b + c)\cosh{(x)} + a)\sinh{(x)} + b - c}$$

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="fricas")
```

```
[Out] -2*(a*cosh(x) + a*sinh(x) + b - c)/((b + c)*cosh(x)^2 + (b + c)*sinh(x)^2 +
 2*a*cosh(x) + 2*((b + c)*cosh(x) + a)*sinh(x) + b - c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\text{Timed out}$$

```
[In] integrate((b**2-c**2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2+a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{b^2-c^2+ab\cosh(x)+ac\sinh(x)}{(a+b\cosh(x)+c\sinh(x))^2}\,dx=-\frac{2\,(ae^x+b-c)}{be^{(2\,x)}+ce^{(2\,x)}+2\,ae^x+b-c}$$

```
[In] integrate((b^2-c^2+a*b*cosh(x)+a*c*sinh(x))/(a+b*cosh(x)+c*sinh(x))^2,x, al
gorithm="giac")
```

```
[Out] -2*(a*e^x + b - c)/(b*e^(2*x) + c*e^(2*x) + 2*a*e^x + b - c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{b^2 - c^2 + ab\cosh(x) + ac\sinh(x)}{(a + b\cosh(x) + c\sinh(x))^2}\,dx = \int \frac{b^2 + a\cosh(x)\ b - c^2 + a\sinh(x)\ c}{\left(a + b\cosh\left(x\right) + c\sinh\left(x\right)\right)^2}\,dx$$

```
[In] int((b^2 - c^2 + a*c*sinh(x) + a*b*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,x
)
```

```
[Out] int((b^2 - c^2 + a*c*sinh(x) + a*b*cosh(x))/(a + b*cosh(x) + c*sinh(x))^2,
x)
```

3.802 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aA+bC)x}{2a^2}+\frac{C\cosh(x)}{2a} - \frac{1}{2}\left(\frac{2A}{a}-\frac{C}{b}+\frac{bC}{a^2}\right)\log(a+b\cosh(x)+b\sinh(x)) - \frac{C\sinh(x)}{2a}$$

```
[Out] 1/2*(2*A*a+C*b)*x/a^2+1/2*C*cosh(x)/a-1/2*(2*A*a*b-C*a^2+C*b^2)*ln(a+b*cosh
(x)+b*sinh(x))/a^2/b-1/2*C*sinh(x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3210}

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{x(2aA+bC)}{2a^2} - \frac{1}{2}\left(\frac{bC}{a^2}+\frac{2A}{a}-\frac{C}{b}\right)\log(a+b\sinh(x)+b\cosh(x)) - \frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) - (((2*A)/a - C/b + (b*C)/a^2
)*Log[a + b*Cosh[x] + b*Sinh[x]])/2 - (C*Sinh[x])/(2*a)
```

Rule 3210

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2aA+bC)x}{2a^2}+\frac{C\cosh(x)}{2a}\\ &\quad -\frac{1}{2}\left(\frac{2A}{a}-\frac{C}{b}+\frac{bC}{a^2}\right)\log(a+b\cosh(x)+b\sinh(x))-\frac{C\sinh(x)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aAb+a^2C+b^2C)\,x+2abC\cosh(x)+2(-2aAb+a^2C-b^2C)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(-a+b)\sinh\left(\frac{x}{2}\right.\right.}{4a^2b}$$

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
[Out] ((2*a*A*b + a^2*C + b^2*C)*x + 2*a*b*C*Cosh[x] + 2*(-2*a*A*b + a^2*C - b^2*
C)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]] - 2*a*b*C*Sinh[x])/(4*a^2*b)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

method	result
risch	$\frac{C\,\mathrm{e}^{-x}}{2a}+\frac{xA}{a}+\frac{bxC}{2a^2}-\frac{\ln(\mathrm{e}^x+\frac{a}{b})A}{a}+\frac{\ln(\mathrm{e}^x+\frac{a}{b})C}{2b}-\frac{b\ln(\mathrm{e}^x+\frac{a}{b})C}{2a^2}$
default	$-\frac{(2Aab-C\,a^2+C\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a^2b}+\frac{C}{a\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{(2Aa+bC)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a^2}-\frac{C\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}$

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*C/a/exp(x)+1/a*x*A+1/2/a^2*b*x*C-1/a*ln(exp(x)+1/b*a)*A+1/2/b*ln(exp(x)
+1/b*a)*C-1/2/a^2*b*ln(exp(x)+1/b*a)*C
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{Cab+(2\,Aab+Cb^2)x\cosh{(x)}+(2\,Aab+Cb^2)x\sinh{(x)}+((Ca^2-2\,Aab-Cb^2)\cosh{(x)}+(Ca^2-2\,Aa}{2\,(a^2b\cosh{(x)}+a^2b\sinh{(x)})}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(C*a*b + (2*A*a*b + C*b^2)*x*cosh(x) + (2*A*a*b + C*b^2)*x*sinh(x) + ((
C*a^2 - 2*A*a*b - C*b^2)*cosh(x) + (C*a^2 - 2*A*a*b - C*b^2)*sinh(x))*log(b
*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(66) = 132$.

Time = 2.21 (sec) , antiderivative size = 753, normalized size of antiderivative = 10.61

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*log(tanh(x/2)
+ 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2
) + 2*b) + C*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + C*x/(2*b*tanh(x/2) + 2*b)
+ 2*C/(2*b*tanh(x/2) + 2*b), Eq(a, b)), (-2*A/(2*b*sinh(x) + 2*b*cosh(x)) +
 C*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*x*cosh(x)/(2*b*sinh(x) + 2*b*c
osh(x)) + C*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + C*cosh(
x))/a, Eq(b, 0)), (2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2)
 + 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) -
 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x
/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b
*tanh(x/2) + 2*a**2*b) + C*a**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b)
 + C*a**2*x/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*a**2*log(tanh(x/2) + 1)*tan
h(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*a**2*log(tanh(x/2) + 1)/(2*a**2*
b*tanh(x/2) + 2*a**2*b) + C*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*ta
nh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*log(-a/(a - b) - b/(a - b)
 + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*C*a*b/(2*a**2*b*tanh(x/2)
 + 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*
a**2*b) + C*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*b**
2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2
*a**2*b) - C*b**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/
2) + 2*a**2*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{1}{2}C\left(\frac{x}{b}+\frac{e^{(-x)}}{a}+\frac{(a^2-b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right) - \frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^
(-x) + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{Ce^{(-x)}}{2\,a}+\frac{(2\,Aa+Cb)x}{2\,a^2}+\frac{(Ca^2-2\,Aab-Cb^2)\log\left(|be^x+a|\right)}{2\,a^2b}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*C*e^(-x)/a + 1/2*(2*A*a + C*b)*x/a^2 + 1/2*(C*a^2 - 2*A*a*b - C*b^2)*lo
g(abs(b*e^x + a))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{C\,\mathrm{e}^{-x}}{2\,a}+\frac{x\,(2\,A\,a+C\,b)}{2\,a^2} - \frac{\ln\left(a+b\,\mathrm{e}^x\right)\,\left(-C\,a^2+2\,A\,a\,b+C\,b^2\right)}{2\,a^2\,b}$$

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

```
[Out] (C*exp(-x))/(2*a) + (x*(2*A*a + C*b))/(2*a^2) - (log(a + b*exp(x))*(C*b^2 -
 C*a^2 + 2*A*a*b))/(2*a^2*b)
```

3.803 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aA-bB)x}{2a^2} - \frac{B\cosh(x)}{2a} - \frac{(2aAb-a^2B-b^2B)\log(a+b\cosh(x)+b\sinh(x))}{2a^2b} + \frac{B\sinh(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2-1/2*B*cosh(x)/a-1/2*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cosh
(x)+b*sinh(x))/a^2/b+1/2*B*sinh(x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3211}

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{(a^2(-B)+2aAb-b^2B)\log(a+b\sinh(x)+b\cosh(x))}{2a^2b} + \frac{x(2aA-bB)}{2a^2} + \frac{B\sinh(x)}{2a} - \frac{B\cosh(x)}{2a}$$

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - (B*Cosh[x])/(2*a) - ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rule 3211

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
```

```
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2aA-bB)x}{2a^2} - \frac{B\cosh(x)}{2a} - \frac{(2aAb-a^2B-b^2B)\log(a+b\cosh(x)+b\sinh(x))}{2a^2b} + \frac{B\sinh(x)}{2a}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{\left(2aA+\frac{a^2B}{b}-bB\right)x-2aB\cosh(x)+\frac{2(-2aAb+a^2B+b^2B)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(-a+b)\sinh\left(\frac{x}{2}\right)\right)}{b}+2aB\sinh(x)}{4a^2}$$

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
[Out] ((2*a*A + (a^2*B)/b - b*B)*x - 2*a*B*Cosh[x] + (2*(-2*a*A*b + a^2*B + b^2*B
)*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]])/b + 2*a*B*Sinh[x])/(4*a^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{B\,e^{-x}}{2a}+\frac{xA}{a}-\frac{bxB}{2a^2}-\frac{\ln\left(e^x+\frac{a}{b}\right)A}{a}+\frac{\ln\left(e^x+\frac{a}{b}\right)B}{2b}+\frac{b\ln\left(e^x+\frac{a}{b}\right)B}{2a^2}$
default	$-\frac{B}{a\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{(2Aa-Bb)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a^2}-\frac{(2Aab-B\,a^2-B\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)-a-b\right)}{2a^2b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}$

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*B/a/exp(x)+1/a*x*A-1/2/a^2*b*x*B-1/a*ln(exp(x)+1/b*a)*A+1/2/b*ln(exp(x
)+1/b*a)*B+1/2/a^2*b*ln(exp(x)+1/b*a)*B
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{Bab-(2\,Aab-Bb^2)x\cosh{(x)}-(2\,Aab-Bb^2)x\sinh{(x)}-((Ba^2-2\,Aab+Bb^2)\cosh{(x)}+(Ba^2-2}{2\,(a^2b\cosh{(x)}+a^2b\sinh{(x)})}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fricas")
[Out] -1/2*(B*a*b - (2*A*a*b - B*b^2)*x*cosh(x) - (2*A*a*b - B*b^2)*x*sinh(x) - (
(B*a^2 - 2*A*a*b + B*b^2)*cosh(x) + (B*a^2 - 2*A*a*b + B*b^2)*sinh(x))*log(
b*cosh(x) + b*sinh(x) + a))/(a^2*b*cosh(x) + a^2*b*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(66) = 132.

Time = 2.26 (sec) , antiderivative size = 806, normalized size of antiderivative = 10.47

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x)
[Out] Piecewise((zoo*(A*x + B*sinh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*log(tanh(x/2)
+ 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2
) + 2*b) + B*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + B*x/(2*b*tanh(x/2) + 2*b)
- 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) - 2*B*log(tanh(x/2
) + 1)/(2*b*tanh(x/2) + 2*b) - 2*B/(2*b*tanh(x/2) + 2*b), Eq(a, b)), (-2*A/
(2*b*sinh(x) + 2*b*cosh(x)) + B*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + B*x
*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - B*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)
), Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (2*A*a*b*log(tanh(x/2) + 1)*
tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a
**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(a - b) + tanh(x/2
))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(
a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x*tanh(x/2)/(2
*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x/(2*a**2*b*tanh(x/2) + 2*a**2*b) -
B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*a**
2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*log(-a/(a - b
) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a*
*2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b)
- 2*B*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x/2) + 1)*tanh(
```

```
x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x/2) + 1)/(2*a**2*b*
tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh
(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a - b) +
 tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{1}{2}\,B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{(a^2+b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right)-\frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*B*(x/b - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^
(-x) + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=-\frac{Be^{(-x)}}{2\,a}+\frac{(2\,Aa-Bb)x}{2\,a^2}+\frac{(Ba^2-2\,Aab+Bb^2)\log\left(|be^x+a|\right)}{2\,a^2b}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*B*e^(-x)/a + 1/2*(2*A*a - B*b)*x/a^2 + 1/2*(B*a^2 - 2*A*a*b + B*b^2)*l
og(abs(b*e^x + a))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx=\frac{x\,(2\,A\,a-B\,b)}{2\,a^2}-\frac{B\,\mathrm{e}^{-x}}{2\,a}+\frac{\ln\left(a+b\,\mathrm{e}^x\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{2\,a^2\,b}$$

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

```
[Out] (x*(2*A*a - B*b))/(2*a^2) - (B*exp(-x))/(2*a) + (log(a + b*exp(x))*(B*a^2 +
 B*b^2 - 2*A*a*b))/(2*a^2*b)
```

3.804 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2aA-b(B-C))x}{2a^2} - \frac{(2aAb-b^2(B-C)-a^2(B+C))\log(a+b\cosh(x)+b\sinh(x))}{2a^2b} - \frac{(B-C)(\cosh(x)-\sinh(x))}{2a}$$

```
[Out] 1/2*(2*A*a-b*(B-C))*x/a^2-1/2*(2*A*a*b-b^2*(B-C)-a^2*(B+C))*ln(a+b*cosh(x)+
b*sinh(x))/a^2/b-1/2*(B-C)*(cosh(x)-sinh(x))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3209}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{(-(a^2(B+C))+2aAb-b^2(B-C))\log(a+b\sinh(x)+b\cosh(x))}{2a^2b} + \frac{x(2aA-b(B-C))}{2a^2} - \frac{(B-C)(\cosh(x)-\sinh(x))}{2a}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*(B - C))*x)/(2*a^2) - ((2*a*A*b - b^2*(B - C) - a^2*(B + C))*Lo
g[a + b*Cosh[x] + b*Sinh[x]])/(2*a^2*b) - ((B - C)*(Cosh[x] - Sinh[x]))/(2*
a)
```

Rule 3209

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2aA-b(B-C))x}{2a^2}\\ &\quad-\frac{(2aAb-b^2(B-C)-a^2(B+C))\log(a+b\cosh(x)+b\sinh(x))}{2a^2b}\\ &\quad-\frac{(B-C)(\cosh(x)-\sinh(x))}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.20

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{\left(2aA+b(-B+C)+\frac{a^2(B+C)}{b}\right)x-2a(B-C)\cosh(x)+\frac{2(-2aAb+b^2(B-C)+a^2(B+C))\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(-a+b)\sinh\left(\frac{x}{2}\right)\right)}{b}}{4a^2}$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] + b*Sinh[x]),x]
```

```
[Out] ((2*a*A + b*(-B + C) + (a^2*(B + C))/b)*x - 2*a*(B - C)*Cosh[x] + (2*(-2*a*
A*b + b^2*(B - C) + a^2*(B + C))*Log[(a + b)*Cosh[x/2] + (-a + b)*Sinh[x/2]
])/b + 2*a*(B - C)*Sinh[x])/(4*a^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

method	result
default	$\frac{(-B-C)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{B-C}{a\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{(2Aa-Bb+bC)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2a^2}-\frac{(2Aab-B\,a^2-B\,b^2-C\,a^2+C\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right)-\right.}{2a^2b}$
risch	$-\frac{B\,\mathrm{e}^{-x}}{2a}+\frac{C\,\mathrm{e}^{-x}}{2a}+\frac{xA}{a}-\frac{bxB}{2a^2}+\frac{bxC}{2a^2}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)A}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)B}{2b}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)B}{2a^2}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)C}{2b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)C}{2a^2}$

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*(-B-C)/b*ln(tanh(1/2*x)-1)-(B-C)/a/(1+tanh(1/2*x))+1/2*(2*A*a-B*b+C*b)/
a^2*ln(1+tanh(1/2*x))-1/2*(2*A*a*b-B*a^2-B*b^2-C*a^2+C*b^2)/a^2/b*ln(a*tanh
(1/2*x)-b*tanh(1/2*x)-a-b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = -\frac{(B-C)ab-(2\,Aab-(B-C)b^2)x\cosh\left(x\right)-(2\,Aab-(B-C)b^2)x\sinh\left(x\right)-(((B+C)a^2-2\,Aab}{2\,(a^2b\cosh\left(x\right)+}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] -1/2*((B - C)*a*b - (2*A*a*b - (B - C)*b^2)*x*cosh(x) - (2*A*a*b - (B - C)*
b^2)*x*sinh(x) - (((B + C)*a^2 - 2*A*a*b + (B - C)*b^2)*cosh(x) + ((B + C)*
a^2 - 2*A*a*b + (B - C)*b^2)*sinh(x))*log(b*cosh(x) + b*sinh(x) + a))/(a^2*
b*cosh(x) + a^2*b*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(70) = 140.

Time = 2.60 (sec) , antiderivative size = 1321, normalized size of antiderivative = 15.36

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + B*sinh(x) + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*lo
g(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + 2*A*log(tanh(x/2) + 1)/(
2*b*tanh(x/2) + 2*b) + B*x*tanh(x/2)/(2*b*tanh(x/2) + 2*b) + B*x/(2*b*tanh(
x/2) + 2*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) + 2*b) - 2*B*
log(tanh(x/2) + 1)/(2*b*tanh(x/2) + 2*b) - 2*B/(2*b*tanh(x/2) + 2*b) + C*x*
tanh(x/2)/(2*b*tanh(x/2) + 2*b) + C*x/(2*b*tanh(x/2) + 2*b) + 2*C/(2*b*tanh
(x/2) + 2*b), Eq(a, b)), (-2*A/(2*b*sinh(x) + 2*b*cosh(x)) + B*x*sinh(x)/(2
*b*sinh(x) + 2*b*cosh(x)) + B*x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) - B*cos
h(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*x*sinh(x)/(2*b*sinh(x) + 2*b*cosh(x))
```

```
+ C*x*cosh(x)/(2*b*sinh(x) + 2*b*cosh(x)) + C*cosh(x)/(2*b*sinh(x) + 2*b*co
sh(x)), Eq(a, 0)), ((A*x + B*sinh(x) + C*cosh(x))/a, Eq(b, 0)), (2*A*a*b*lo
g(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + 2*A*a*b*log(ta
nh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log(-a/(a - b) - b/(
a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - 2*A*a*b*log
(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a*
*2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a**2*x/(2*a**2*b*tanh(x/
2) + 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) +
2*a**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*a
**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) +
 2*a**2*b) + B*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(
x/2) + 2*a**2*b) - 2*B*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tan
h(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - B*b**2*log(tanh(x/2
) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a - b) - b/(a - b)
+ tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + B*b**2*log(-a/(a -
 b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*x*tan
h(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*x/(2*a**2*b*tanh(x/2) + 2*a
**2*b) - C*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b
) - C*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*a**2*log(
-a/(a - b) - b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*
b) + C*a**2*log(-a/(a - b) - b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2
*a**2*b) + 2*C*a*b/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*b**2*log(tanh(x/2) +
 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)/(
2*a**2*b*tanh(x/2) + 2*a**2*b) - C*b**2*log(-a/(a - b) - b/(a - b) + tanh(x
/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) + 2*a**2*b) - C*b**2*log(-a/(a - b) - b/
(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) + 2*a**2*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{1}{2}\,C\left(\frac{x}{b}+\frac{e^{(-x)}}{a}+\frac{(a^2-b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right) + \frac{1}{2}\,B\left(\frac{x}{b}-\frac{e^{(-x)}}{a}+\frac{(a^2+b^2)\log\left(ae^{(-x)}+b\right)}{a^2b}\right) - \frac{A\log\left(ae^{(-x)}+b\right)}{a}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="max
ima")
```

```
[Out] 1/2*C*(x/b + e^(-x)/a + (a^2 - b^2)*log(a*e^(-x) + b)/(a^2*b)) + 1/2*B*(x/b
 - e^(-x)/a + (a^2 + b^2)*log(a*e^(-x) + b)/(a^2*b)) - A*log(a*e^(-x) + b)/
a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{(2\,Aa-Bb+Cb)x}{2\,a^2} - \frac{(Ba-Ca)e^{(-x)}}{2\,a^2} + \frac{(Ba^2+Ca^2-2\,Aab+Bb^2-Cb^2)\log\left(|be^x+a|\right)}{2\,a^2b}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)+b*sinh(x)),x, algorithm="gia
c")
```

```
[Out] 1/2*(2*A*a - B*b + C*b)*x/a^2 - 1/2*(B*a - C*a)*e^(-x)/a^2 + 1/2*(B*a^2 + C
*a^2 - 2*A*a*b + B*b^2 - C*b^2)*log(abs(b*e^x + a))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)+b\sinh(x)}\,dx = \frac{x\,(2\,A\,a-B\,b+C\,b)}{2\,a^2} - \frac{\mathrm{e}^{-x}\,(B-C)}{2\,a} + \frac{\ln\left(a+b\,\mathrm{e}^x\right)\,\left(B\,a^2+B\,b^2+C\,a^2-C\,b^2-2\,A\,a\,b\right)}{2\,a^2\,b}$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) + b*sinh(x)),x)
```

```
[Out] (x*(2*A*a - B*b + C*b))/(2*a^2) - (exp(-x)*(B - C))/(2*a) + (log(a + b*exp(
x))*(B*a^2 + B*b^2 + C*a^2 - C*b^2 - 2*A*a*b))/(2*a^2*b)
```

3.805 $\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aA-bC)x}{2a^2}+\frac{C\cosh(x)}{2a}+\frac{(2aAb+a^2C-b^2C)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}+\frac{C\sinh(x)}{2a}$$

```
[Out] 1/2*(2*A*a-C*b)*x/a^2+1/2*C*cosh(x)/a+1/2*(2*A*a*b+C*a^2-C*b^2)*ln(a+b*cosh
(x)-b*sinh(x))/a^2/b+1/2*C*sinh(x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3210}

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(a^2C+2aAb-b^2C)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bC)}{2a^2}+\frac{C\sinh(x)}{2a}+\frac{C\cosh(x)}{2a}$$

```
[In] Int[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*C)*x)/(2*a^2) + (C*Cosh[x])/(2*a) + ((2*a*A*b + a^2*C - b^2*C)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (C*Sinh[x])/(2*a)
```

Rule 3210

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
```

```
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2aA - bC)x}{2a^2} + \frac{C\cosh(x)}{2a} + \frac{(2aAb + a^2C - b^2C)\log(a + b\cosh(x) - b\sinh(x))}{2a^2b} + \frac{C\sinh(x)}{2a}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = \frac{\left(2aA - \frac{a^2C}{b} - bC\right)x + 2aC\cosh(x) + \frac{2(2aAb+a^2C-b^2C)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(a-b)\sinh\left(\frac{x}{2}\right)\right)}{b} + 2aC\sinh(x)}{4a^2}$$

```
[In] Integrate[(A + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
[Out] ((2*a*A - (a^2*C)/b - b*C)*x + 2*a*C*Cosh[x] + (2*(2*a*A*b + a^2*C - b^2*C)
*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]])/b + 2*a*C*Sinh[x])/(4*a^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result
risch	$\frac{C\,\mathrm{e}^{x}}{2a} - \frac{Cx}{2b} + \frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)A}{a} + \frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)C}{2b} - \frac{b\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)C}{2a^2}$
default	$\frac{(2Aab+C\,a^2-C\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)+a+b\right)}{2a^2b} - \frac{C}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{(-2Aa+bC)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^2} - \frac{C\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b}$

```
[In] int((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*C/a*exp(x)-1/2*C*x/b+1/a*ln(exp(x)+1/a*b)*A+1/2/b*ln(exp(x)+1/a*b)*C-1/
2/a^2*b*ln(exp(x)+1/a*b)*C
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = -\frac{Ca^2x - Cab\cosh(x) - Cab\sinh(x) - (Ca^2 + 2\,Aab - Cb^2)\log(a\cosh(x) + a\sinh(x) + b)}{2\,a^2b}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
[Out] -1/2*(C*a^2*x - C*a*b*cosh(x) - C*a*b*sinh(x) - (C*a^2 + 2*A*a*b - C*b^2)*l
og(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(66) = 132.

Time = 2.28 (sec) , antiderivative size = 852, normalized size of antiderivative = 11.06

$$\int \frac{A + C\sinh(x)}{a + b\cosh(x) - b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
[Out] Piecewise((zoo*(A*x + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*x*tanh(x/2)/(2
*b*tanh(x/2) - 2*b) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1)*
tanh(x/2)/(2*b*tanh(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) - 2
*b) - C*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + C*x/(2*b*tanh(x/2) - 2*b) - 2*C
/(2*b*tanh(x/2) - 2*b), Eq(a, b)), (2*A/(-2*b*sinh(x) + 2*b*cosh(x)) + C*x*
sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) - C*x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh
(x)) + C*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)), Eq(a, 0)), ((A*x + C*cosh(x)
)/a, Eq(b, 0)), (2*A*a*b*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*
a*b*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2
)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*ta
nh(x/2) - 2*a**2*b) + 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x
/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*log(a/(a - b) + b/(a - b) + t
anh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*a**2*log(tanh(x/2) + 1)*tanh(
x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*a**2*log(tanh(x/2) + 1)/(2*a**2*b*
tanh(x/2) - 2*a**2*b) + C*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(
x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*a**2*log(a/(a - b) + b/(a - b) + t
anh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*C*a*b/(2*a**2*b*tanh(x/2) - 2
*a**2*b) - C*b**2*x*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*x/(2
*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2
```

```
*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) -
2*a**2*b) - C*b**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2
*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2
*a**2*b*tanh(x/2) - 2*a**2*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right) - \frac{1}{2}\,C\left(\frac{bx}{a^2}-\frac{e^x}{a}-\frac{(a^2-b^2)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*
e^(-x) + a)/(a^2*b))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = -\frac{Cx}{2\,b}+\frac{Ce^x}{2\,a}+\frac{(Ca^2+2\,Aab-Cb^2)\log\left(|ae^x+b|\right)}{2\,a^2b}$$

```
[In] integrate((A+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*C*x/b + 1/2*C*e^x/a + 1/2*(C*a^2 + 2*A*a*b - C*b^2)*log(abs(a*e^x + b)
)/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{A+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{C\,\mathrm{e}^x}{2\,a}-\frac{C\,x}{2\,b}+\frac{\ln\left(b+a\,\mathrm{e}^x\right)\,\left(C\,a^2+2\,A\,a\,b-C\,b^2\right)}{2\,a^2\,b}$$

```
[In] int((A + C*sinh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

```
[Out] (C*exp(x))/(2*a) - (C*x)/(2*b) + (log(b + a*exp(x))*(C*a^2 - C*b^2 + 2*A*a*
b))/(2*a^2*b)
```

3.806 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aA-bB)x}{2a^2}+\frac{B\cosh(x)}{2a}+\frac{(2aAb-a^2B-b^2B)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}+\frac{B\sinh(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2+1/2*B*cosh(x)/a+1/2*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cosh
(x)-b*sinh(x))/a^2/b+1/2*B*sinh(x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3211}

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(a^2(-B)+2aAb-b^2B)\log(a-b\sinh(x)+b\cosh(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sinh(x)}{2a}+\frac{B\cosh(x)}{2a}$$

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + (B*Cosh[x])/(2*a) + ((2*a*A*b - a^2*B - b^2*B)*
Log[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + (B*Sinh[x])/(2*a)
```

Rule 3211

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
```

```
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2aA-bB)x}{2a^2} + \frac{B\cosh(x)}{2a} + \frac{(2aAb-a^2B-b^2B)\log(a+b\cosh(x)-b\sinh(x))}{2a^2b} + \frac{B\sinh(x)}{2a}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aAb+a^2B-b^2B)\,x+2abB\cosh(x)-2(-2aAb+a^2B+b^2B)\log\left((a+b)\cosh\left(\frac{x}{2}\right)+(a-b)\sinh\left(\frac{x}{2}\right)\right)}{4a^2b}$$

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
[Out] ((2*a*A*b + a^2*B - b^2*B)*x + 2*a*b*B*Cosh[x] - 2*(-2*a*A*b + a^2*B + b^2*
B)*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]] + 2*a*b*B*Sinh[x])/(4*a^2*b)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

method	result
risch	$\frac{B\,e^{x}}{2a}+\frac{Bx}{2b}+\frac{\ln\left(e^{x}+\frac{b}{a}\right)A}{a}-\frac{\ln\left(e^{x}+\frac{b}{a}\right)B}{2b}-\frac{b\ln\left(e^{x}+\frac{b}{a}\right)B}{2a^2}$
default	$\frac{B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b}-\frac{B}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-2Aa+Bb)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^2}+\frac{\left(2Aab-B\,a^2-B\,b^2\right)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)+a+b\right)}{2a^2b}$

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*B/a*exp(x)+1/2*B*x/b+1/a*ln(exp(x)+1/a*b)*A-1/2/b*ln(exp(x)+1/a*b)*B-1/
2/a^2*b*ln(exp(x)+1/a*b)*B
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{Ba^2x+Bab\cosh\left(x\right)+Bab\sinh\left(x\right)-\left(Ba^2-2\,Aab+Bb^2\right)\log\left(a\cosh\left(x\right)+a\sinh\left(x\right)+b\right)}{2\,a^2b}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fricas")
[Out] 1/2*(B*a^2*x + B*a*b*cosh(x) + B*a*b*sinh(x) - (B*a^2 - 2*A*a*b + B*b^2)*lo
g(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. 2(66) = 132.

Time = 2.35 (sec) , antiderivative size = 904, normalized size of antiderivative = 11.59

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x)
[Out] Piecewise((zoo*(A*x + B*sinh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*x*tanh(x/2)/(2
*b*tanh(x/2) - 2*b) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tanh(x/2) + 1)*
tanh(x/2)/(2*b*tanh(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*tanh(x/2) - 2
*b) - B*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + B*x/(2*b*tanh(x/2) - 2*b) + 2*B
*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*B*log(tanh(x/2) + 1
)/(2*b*tanh(x/2) - 2*b) - 2*B/(2*b*tanh(x/2) - 2*b), Eq(a, b)), (2*A/(-2*b*
sinh(x) + 2*b*cosh(x)) - B*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*x*cos
h(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)),
 Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (2*A*a*b*x*tanh(x/2)/(2*a**2*b
*tanh(x/2) - 2*a**2*b) - 2*A*a*b*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*
b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*lo
g(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*a*b*log(a/(a - b) +
b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*
log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*
a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*a**2*
log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*a**2*log(a/(a - b) +
 b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*a**2*
log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*
B*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*x*tanh(x/2)/(2*a**2*b*tanh(x
```

```
/2) - 2*a**2*b) + B*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*log(tan
h(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*log(tanh(x/2
) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*log(a/(a - b) + b/(a - b) +
 tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*log(a/(a - b
) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right) - \frac{1}{2}B\left(\frac{bx}{a^2}-\frac{e^x}{a}+\frac{(a^2+b^2)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="maxima")
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b*
e^(-x) + a)/(a^2*b))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{Bx}{2b}+\frac{Be^x}{2a}-\frac{(Ba^2-2Aab+Bb^2)\log(|ae^x+b|)}{2a^2b}$$

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="giac")
[Out] 1/2*B*x/b + 1/2*B*e^x/a - 1/2*(B*a^2 - 2*A*a*b + B*b^2)*log(abs(a*e^x + b))
/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{B\,e^x}{2\,a}+\frac{B\,x}{2\,b}-\frac{\ln\left(b+a\,e^x\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{2\,a^2\,b}$$

```
[In] int((A + B*cosh(x))/(a + b*cosh(x) - b*sinh(x)),x)
[Out] (B*exp(x))/(2*a) + (B*x)/(2*b) - (log(b + a*exp(x))*(B*a^2 + B*b^2 - 2*A*a*
b))/(2*a^2*b)
```

3.807 $\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aA-b(B+C))x}{2a^2}+\frac{(2aAb-a^2(B-C)-b^2(B+C))\log(a+b\cosh(x)-b\sinh(x))}{2a^2b} + \frac{(B+C)(\cosh(x)+\sinh(x))}{2a}$$

```
[Out] 1/2*(2*A*a-b*(B+C))*x/a^2+1/2*(2*A*a*b-a^2*(B-C)-b^2*(B+C))*ln(a+b*cosh(x)-
b*sinh(x))/a^2/b+1/2*(B+C)*(cosh(x)+sinh(x))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3209}

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(-(a^2(B-C))+2aAb-b^2(B+C))\log(a-b\sinh(x)+b\cosh(x))}{2a^2b} + \frac{x(2aA-b(B+C))}{2a^2}+\frac{(B+C)(\sinh(x)+\cosh(x))}{2a}$$

```
[In] Int[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A - b*(B + C))*x)/(2*a^2) + ((2*a*A*b - a^2*(B - C) - b^2*(B + C))*Lo
g[a + b*Cosh[x] - b*Sinh[x]])/(2*a^2*b) + ((B + C)*(Cosh[x] + Sinh[x]))/(2*
a)
```

Rule 3209

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2aA-b(B+C))x}{2a^2}\\ &\quad+\frac{(2aAb-a^2(B-C)-b^2(B+C))\log(a+b\cosh(x)-b\sinh(x))}{2a^2b}\\ &\quad+\frac{(B+C)(\cosh(x)+\sinh(x))}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(2aAb+a^2(B-C)-b^2(B+C))\,x+2ab(B+C)\cosh(x)-2(-2aAb+a^2(B-C)+b^2(B+C))\log\left((\right.}{4a^2b}$$

```
[In] Integrate[(A + B*Cosh[x] + C*Sinh[x])/(a + b*Cosh[x] - b*Sinh[x]),x]
```

```
[Out] ((2*a*A*b + a^2*(B - C) - b^2*(B + C))*x + 2*a*b*(B + C)*Cosh[x] - 2*(-2*a*
A*b + a^2*(B - C) + b^2*(B + C))*Log[(a + b)*Cosh[x/2] + (a - b)*Sinh[x/2]]
 + 2*a*b*(B + C)*Sinh[x])/(4*a^2*b)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result
risch	$\frac{B\,\mathrm{e}^{x}}{2a}+\frac{C\,\mathrm{e}^{x}}{2a}+\frac{Bx}{2b}-\frac{Cx}{2b}+\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)A}{a}-\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)B}{2b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)B}{2a^2}+\frac{\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)C}{2b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{b}{a}\right)C}{2a^2}$
default	$-\frac{B+C}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-2Aa+Bb+bC)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^2}+\frac{(2Aab-B\,a^2-B\,b^2+C\,a^2-C\,b^2)\ln\left(a\tanh\left(\frac{x}{2}\right)-b\tanh\left(\frac{x}{2}\right)+a+b\right)}{2a^2b}+$

```
[In] int((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*B/a*exp(x)+1/2*C/a*exp(x)+1/2*B*x/b-1/2*C*x/b+1/a*ln(exp(x)+1/a*b)*A-1/
2/b*ln(exp(x)+1/a*b)*B-1/2/a^2*b*ln(exp(x)+1/a*b)*B+1/2/b*ln(exp(x)+1/a*b)*
C-1/2/a^2*b*ln(exp(x)+1/a*b)*C
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(B-C)a^2x+(B+C)ab\cosh(x)+(B+C)ab\sinh(x)-((B-C)a^2-2\,Aab+(B+C)b^2)\log(a\cosh(x$$

$$2\,a^2b$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="fri
cas")
```

```
[Out] 1/2*((B - C)*a^2*x + (B + C)*a*b*cosh(x) + (B + C)*a*b*sinh(x) - ((B - C)*a
^2 - 2*A*a*b + (B + C)*b^2)*log(a*cosh(x) + a*sinh(x) + b))/(a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. 2(70) = 140.

Time = 2.72 (sec) , antiderivative size = 1420, normalized size of antiderivative = 17.53

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*x + B*sinh(x) + C*cosh(x)), Eq(a, 0) & Eq(b, 0)), (2*A*x*
tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*A*x/(2*b*tanh(x/2) - 2*b) - 2*A*log(tan
h(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + 2*A*log(tanh(x/2) + 1)/(2*b*t
anh(x/2) - 2*b) - B*x*tanh(x/2)/(2*b*tanh(x/2) - 2*b) + B*x/(2*b*tanh(x/2)
- 2*b) + 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(2*b*tanh(x/2) - 2*b) - 2*B*log(t
anh(x/2) + 1)/(2*b*tanh(x/2) - 2*b) - 2*B/(2*b*tanh(x/2) - 2*b) - C*x*tanh(
x/2)/(2*b*tanh(x/2) - 2*b) + C*x/(2*b*tanh(x/2) - 2*b) - 2*C/(2*b*tanh(x/2)
 - 2*b), Eq(a, b)), (2*A/(-2*b*sinh(x) + 2*b*cosh(x)) - B*x*sinh(x)/(-2*b*s
inh(x) + 2*b*cosh(x)) + B*x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + B*cosh(x
)/(-2*b*sinh(x) + 2*b*cosh(x)) + C*x*sinh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) -
 C*x*cosh(x)/(-2*b*sinh(x) + 2*b*cosh(x)) + C*cosh(x)/(-2*b*sinh(x) + 2*b*c
osh(x)), Eq(a, 0)), ((A*x + B*sinh(x) + C*cosh(x))/a, Eq(b, 0)), (2*A*a*b*x
```

```
*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - 2*A*a*b*x/(2*a**2*b*tanh(x/2)
- 2*a**2*b) - 2*A*a*b*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*
a**2*b) + 2*A*a*b*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + 2*A*
a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) -
2*a**2*b) - 2*A*a*b*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x
/2) - 2*a**2*b) + B*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) -
 2*a**2*b) - B*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*
a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) -
 2*a**2*b) + B*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x
/2) - 2*a**2*b) - 2*B*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*x*tanh(x
/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + B*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2
*b) + B*b**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) -
 B*b**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - B*b**2*log(a/(
a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) +
 B*b**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2
*b) - C*a**2*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) +
 C*a**2*log(tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*a**2*log(a/(
a - b) + b/(a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) -
 C*a**2*log(a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2
*b) - 2*C*a*b/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*x*tanh(x/2)/(2*a**2*
b*tanh(x/2) - 2*a**2*b) + C*b**2*x/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2
*log(tanh(x/2) + 1)*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(
tanh(x/2) + 1)/(2*a**2*b*tanh(x/2) - 2*a**2*b) - C*b**2*log(a/(a - b) + b/(
a - b) + tanh(x/2))*tanh(x/2)/(2*a**2*b*tanh(x/2) - 2*a**2*b) + C*b**2*log(
a/(a - b) + b/(a - b) + tanh(x/2))/(2*a**2*b*tanh(x/2) - 2*a**2*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = A\left(\frac{x}{a}+\frac{\log\left(be^{(-x)}+a\right)}{a}\right) - \frac{1}{2}B\left(\frac{bx}{a^2}-\frac{e^x}{a}+\frac{(a^2+b^2)\log\left(be^{(-x)}+a\right)}{a^2b}\right) - \frac{1}{2}C\left(\frac{bx}{a^2}-\frac{e^x}{a}-\frac{(a^2-b^2)\log\left(be^{(-x)}+a\right)}{a^2b}\right)$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="max
ima")
```

```
[Out] A*(x/a + log(b*e^(-x) + a)/a) - 1/2*B*(b*x/a^2 - e^x/a + (a^2 + b^2)*log(b*
e^(-x) + a)/(a^2*b)) - 1/2*C*(b*x/a^2 - e^x/a - (a^2 - b^2)*log(b*e^(-x) +
a)/(a^2*b))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{(B-C)x}{2\,b}+\frac{Be^{x}+Ce^{x}}{2\,a} - \frac{(Ba^{2}-Ca^{2}-2\,Aab+Bb^{2}+Cb^{2})\log\left(|ae^{x}+b|\right)}{2\,a^{2}b}$$

```
[In] integrate((A+B*cosh(x)+C*sinh(x))/(a+b*cosh(x)-b*sinh(x)),x, algorithm="gia
c")
```

```
[Out] 1/2*(B - C)*x/b + 1/2*(B*e^x + C*e^x)/a - 1/2*(B*a^2 - C*a^2 - 2*A*a*b + B*
b^2 + C*b^2)*log(abs(a*e^x + b))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{A+B\cosh(x)+C\sinh(x)}{a+b\cosh(x)-b\sinh(x)}\,dx = \frac{x\,(B-C)}{2\,b}+\frac{\mathrm{e}^{x}\,(B+C)}{2\,a} - \frac{\ln\left(b+a\,\mathrm{e}^{x}\right)\,\left(B\,a^{2}+B\,b^{2}-C\,a^{2}+C\,b^{2}-2\,A\,a\,b\right)}{2\,a^{2}\,b}$$

```
[In] int((A + B*cosh(x) + C*sinh(x))/(a + b*cosh(x) - b*sinh(x)),x)
```

```
[Out] (x*(B - C))/(2*b) + (exp(x)*(B + C))/(2*a) - (log(b + a*exp(x))*(B*a^2 + B*
b^2 - C*a^2 + C*b^2 - 2*A*a*b))/(2*a^2*b)
```

3.808 $\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 3

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \arctan(\tanh(x))$$

```
[Out] arctan(tanh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {209}

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \arctan(\tanh(x))$$

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right)\\ &= \arctan(\tanh(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$.

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{1}{\cosh^2(x) + \sinh^2(x)} \, dx = \frac{1}{2} \arctan(\sinh(2x))$$

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-1),x]
[Out] ArcTan[Sinh[2*x]]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 8.00

method	result	size
risch	$\frac{i \ln(e^{2x}+i)}{2} - \frac{i \ln(e^{2x}-i)}{2}$	24
default	$-\frac{\sqrt{2}\left(2+\sqrt{2}\right) \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}} - \frac{\left(-2+\sqrt{2}\right)\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}$	72

```
[In] int(1/(cosh(x)^2+sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*ln(exp(2*x)+I)-1/2*I*ln(exp(2*x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(3) = 6$.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int \frac{1}{\cosh^2(x) + \sinh^2(x)} \, dx = -\arctan\left(-\frac{\cosh(x) + \sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="fricas")
[Out] -arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(3) = 6$.

Time = 3.77 (sec) , antiderivative size = 172, normalized size of antiderivative = 57.33

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \frac{47321\sqrt{3-2\sqrt{2}}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{13860\sqrt{2}+19601} + \frac{33461\sqrt{2}\sqrt{3-2\sqrt{2}}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)}{13860\sqrt{2}+19601} - \frac{5741\sqrt{2}\sqrt{2\sqrt{2}+3}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)}{13860\sqrt{2}+19601} - \frac{8119\sqrt{2\sqrt{2}+3}\operatorname{atan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)}{13860\sqrt{2}+19601}$$

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2),x)
```

```
[Out] 47321*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(13860*sqrt(2
) + 19601) + 33461*sqrt(2)*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sq
rt(2)))/(13860*sqrt(2) + 19601) - 5741*sqrt(2)*sqrt(2*sqrt(2) + 3)*atan(tan
h(x/2)/sqrt(2*sqrt(2) + 3))/(13860*sqrt(2) + 19601) - 8119*sqrt(2*sqrt(2) +
 3)*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(13860*sqrt(2) + 19601)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(3) = 6$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 11.67

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) - 2
*e^(-x)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \arctan\left(e^{(2x)}\right)$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2),x, algorithm="giac")
[Out] arctan(e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{1}{\cosh^2(x)+\sinh^2(x)}\,dx = \mathrm{atan}\left(\mathrm{e}^{2x}\right)$$

```
[In] int(1/(cosh(x)^2 + sinh(x)^2),x)
[Out] atan(exp(2*x))
```

3.809 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} \, dx=\frac{\tanh(x)}{1+\tanh^2(x)}$$

```
[Out] tanh(x)/(1+tanh(x)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {391}

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2} \, dx=\frac{\tanh(x)}{\tanh^2(x)+1}$$

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
```

```
[Out] Tanh[x]/(1 + Tanh[x]^2)
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx,x,\tanh(x)\right) \\ &= \frac{\tanh(x)}{1+\tanh^2(x)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx=\frac{1}{2}\tanh(2x)$$

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-2),x]
[Out] Tanh[2*x]/2
```

Maple [A] (verified)

Time = 30.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{1}{1+e^{4x}}$	11
default	$-\frac{2\left(-\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)\right)}{\tanh\left(\frac{x}{2}\right)^{4}+6\tanh\left(\frac{x}{2}\right)^{2}+1}$	36

```
[In] int(1/(cosh(x)^2+sinh(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/(1+exp(4*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx = -\frac{1}{\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cosh(x)\sinh(x)^3+\sinh(x)^4+1}$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="fricas")
[Out] -1/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(8) = 16$.

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.36

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx = \frac{2\tanh^3\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1} + \frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^4\left(\frac{x}{2}\right)+6\tanh^2\left(\frac{x}{2}\right)+1}$$

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2)**2,x)
[Out] 2*tanh(x/2)**3/(tanh(x/2)**4 + 6*tanh(x/2)**2 + 1) + 2*tanh(x/2)/(tanh(x/2)
**4 + 6*tanh(x/2)**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx = \frac{1}{e^{(-4\,x)}+1}$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="maxima")
[Out] 1/(e^(-4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^2}\,dx = -\frac{1}{e^{(4\,x)}+1}$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^2,x, algorithm="giac")
[Out] -1/(e^(4*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(\cosh^2(x) + \sinh^2(x)\right)^2} \, dx = -\frac{1}{\mathrm{e}^{4\,x} + 1}$$

```
[In] int(1/(cosh(x)^2 + sinh(x)^2)^2,x)
[Out] -1/(exp(4*x) + 1)
```

3.810 $\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3} dx = \frac{1}{2}\arctan(\tanh(x)) + \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}$$

```
[Out] 1/2*arctan(tanh(x))+1/2*sech(x)^2*tanh(x)/(1+tanh(x)^2)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {424, 21, 209}

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3} dx = \frac{1}{2}\arctan(\tanh(x)) + \frac{\tanh(x)\operatorname{sech}^2(x)}{2\left(\tanh^2(x)+1\right)^2}$$

```
[In] Int[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
```

```
[Out] ArcTan[Tanh[x]]/2 + (Sech[x]^2*Tanh[x])/(2*(1 + Tanh[x]^2)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^2)^2}{(1+x^2)^3}\,dx, x, \tanh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{2+2x^2}{(1+x^2)^2}\,dx, x, \tanh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\arctan(\tanh(x))+\frac{\operatorname{sech}^2(x)\tanh(x)}{2\left(1+\tanh^2(x)\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx=\frac{1}{4}\arctan(\sinh(2x))+\frac{1}{4}\operatorname{sech}(2x)\tanh(2x)$$

```
[In] Integrate[(Cosh[x]^2 + Sinh[x]^2)^(-3),x]
[Out] ArcTan[Sinh[2*x]]/4 + (Sech[2*x]*Tanh[2*x])/4
```

Maple [A] (verified)

Time = 29.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.08

method	result
parallelrisch	0
risch	$\frac{e^{2x}(e^{4x}-1)}{2(1+e^{4x})^2}+\frac{i\ln(e^{2x}+i)}{4}-\frac{i\ln(e^{2x}-i)}{4}$
default	$-\frac{2\left(-\frac{\tanh(\frac{x}{2})^7}{2}+\frac{\tanh(\frac{x}{2})^5}{2}+\frac{\tanh(\frac{x}{2})^3}{2}-\frac{\tanh(\frac{x}{2})}{2}\right)}{\left(\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1\right)^2}-\frac{\sqrt{2}\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh(\frac{x}{2})}{2+2\sqrt{2}}\right)}{2\left(2+2\sqrt{2}\right)}-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\arctan\left(\frac{2\tanh}{2\sqrt{2}}\right.}{2\left(2\sqrt{2}-2\right)}$

[In] int(1/(cosh(x)^2+sinh(x)^2)^3,x,method=_RETURNVERBOSE)

[Out] 0

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 11.69

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx$$

$$=\frac{\cosh(x)^6+20\cosh(x)^3\sinh(x)^3+15\cosh(x)^2\sinh(x)^4+6\cosh(x)\sinh(x)^5+\sinh(x)^6+\left(15\cosh(x\right.}{2\left(\cosh\right.}$$

[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="fricas")

[Out] 1/2*(cosh(x)^6 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x
)*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^4 - 1)*sinh(x)^2 - (cosh(x)^8 + 56*co
sh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x)
)*sinh(x)^3 + 4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh
(x)^3)*sinh(x) + 1)*arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x))) - cosh
(x)^2 + 2*(3*cosh(x)^5 - cosh(x))*sinh(x))/(cosh(x)^8 + 56*cosh(x)^3*sinh(x
)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh
(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x))*sinh(x)^3 +
4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh(x)^3)*sinh(x)
 + 1)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3602 vs. 2(24) = 48.

Time = 131.07 (sec) , antiderivative size = 3602, normalized size of antiderivative = 138.54

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(cosh(x)**2+sinh(x)**2)**3,x)
```

```
[Out] 1939450125521*sqrt(3 - 2*sqrt(2))*tanh(x/2)**8*atan(tanh(x/2)/sqrt(3 - 2*sq
rt(2)))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 +
13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 4317193
6039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808
*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2)
 + 1606693091074) + 1371398335529*sqrt(2)*sqrt(3 - 2*sqrt(2))*tanh(x/2)**8*
atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1
606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 192803170
92888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*t
anh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2
)**2 + 1136103579984*sqrt(2) + 1606693091074) - 235294755545*sqrt(2)*sqrt(2
*sqrt(2) + 3)*tanh(x/2)**8*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(11361035799
84*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(
2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh
(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**
2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) -
332757034447*sqrt(2*sqrt(2) + 3)*tanh(x/2)**8*atan(tanh(x/2)/sqrt(2*sqrt(2)
 + 3))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1
3633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936
039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*
sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2)
+ 1606693091074) + 1136103579984*sqrt(2)*tanh(x/2)**7/(1136103579984*sqrt(2
)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x
/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4
+ 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280
317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) + 160669309
1074*tanh(x/2)**7/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(
x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6
 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 1363
3242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 11361035799
84*sqrt(2) + 1606693091074) + 23273401506252*sqrt(3 - 2*sqrt(2))*tanh(x/2)*
*6*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(1136103579984*sqrt(2)*tanh(x/2)**8
+ 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 192803
17092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 6105433746081
2*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(
```

x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) + 16456780026348*sqrt(2)*s
qrt(3 - 2*sqrt(2))*tanh(x/2)**6*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(113610
3579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*
sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)
*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x
/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 160669309107
4) - 2823537066540*sqrt(2)*sqrt(2*sqrt(2) + 3)*tanh(x/2)**6*atan(tanh(x/2)/
sqrt(2*sqrt(2) + 3))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*ta
nh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)
6 + 43171936039392*sqrt(2)*tanh(x/2)4 + 61054337460812*tanh(x/2)**4 + 1
3633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 11361035
79984*sqrt(2) + 1606693091074) - 3993084413364*sqrt(2*sqrt(2) + 3)*tanh(x/2
)**6*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(1136103579984*sqrt(2)*tanh(x/2)**
8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 1928
0317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460
812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tan
h(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) - 1606693091074*tanh(x/2
)**5/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 136
33242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 4317193603
9392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sq
rt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) +
1606693091074) - 1136103579984*sqrt(2)*tanh(x/2)**5/(1136103579984*sqrt(2)*
tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2
)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 +
61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 1928031
7092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) + 73699104769
798*sqrt(3 - 2*sqrt(2))*tanh(x/2)**4*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(1
136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1363324295
9808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sq
rt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*t
anh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693
091074) + 52113136750102*sqrt(2)*sqrt(3 - 2*sqrt(2))*tanh(x/2)**4*atan(tanh
(x/2)/sqrt(3 - 2*sqrt(2)))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091
074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tan
h(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)*
*4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 11
36103579984*sqrt(2) + 1606693091074) - 8941200710710*sqrt(2)*sqrt(2*sqrt(2)
 + 3)*tanh(x/2)**4*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(1136103579984*sqrt(
2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(
x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4
 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 1928
0317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) - 12644767
308986*sqrt(2*sqrt(2) + 3)*tanh(x/2)**4*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))
/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1363324
2959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392

*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2
)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606
693091074) - 1606693091074*tanh(x/2)**3/(1136103579984*sqrt(2)*tanh(x/2)**8
 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280
317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 610543374608
12*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh
(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) - 1136103579984*sqrt(2)*t
anh(x/2)**3/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**
8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 431
71936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 1363324295
9808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqr
t(2) + 1606693091074) + 23273401506252*sqrt(3 - 2*sqrt(2))*tanh(x/2)**2*ata
n(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606
693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 192803170928
88*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh
(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**
2 + 1136103579984*sqrt(2) + 1606693091074) + 16456780026348*sqrt(2)*sqrt(3
 - 2*sqrt(2))*tanh(x/2)**2*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(113610357998
4*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2
)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(
x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2
 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) - 2
823537066540*sqrt(2)*sqrt(2*sqrt(2) + 3)*tanh(x/2)**2*atan(tanh(x/2)/sqrt(2
*sqrt(2) + 3))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2
)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 +
43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 1363324
2959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*
sqrt(2) + 1606693091074) - 3993084413364*sqrt(2*sqrt(2) + 3)*tanh(x/2)**2*a
tan(tanh(x/2)/sqrt(2*sqrt(2) + 3))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 16
06693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 1928031709
2888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*ta
nh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)
**2 + 1136103579984*sqrt(2) + 1606693091074) + 1136103579984*sqrt(2)*tanh(x
/2)/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1363
3242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039
392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqr
t(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1
606693091074) + 1606693091074*tanh(x/2)/(1136103579984*sqrt(2)*tanh(x/2)**8
 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6 + 19280
317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 610543374608
12*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh
(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) + 1939450125521*sqrt(3 -
2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(1136103579984*sqrt(2)*tanh(
x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2)*tanh(x/2)**6
+ 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(x/2)**4 + 61054

```
337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2 + 192803170928
88*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) + 1371398335529*sq
rt(2)*sqrt(3 - 2*sqrt(2))*atan(tanh(x/2)/sqrt(3 - 2*sqrt(2)))/(113610357998
4*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 13633242959808*sqrt(2
)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392*sqrt(2)*tanh(
x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2)*tanh(x/2)**2
 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606693091074) - 2
35294755545*sqrt(2)*sqrt(2*sqrt(2) + 3)*atan(tanh(x/2)/sqrt(2*sqrt(2) + 3))
/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1363324
2959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936039392
*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*sqrt(2
)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2) + 1606
693091074) - 332757034447*sqrt(2*sqrt(2) + 3)*atan(tanh(x/2)/sqrt(2*sqrt(2)
 + 3))/(1136103579984*sqrt(2)*tanh(x/2)**8 + 1606693091074*tanh(x/2)**8 + 1
3633242959808*sqrt(2)*tanh(x/2)**6 + 19280317092888*tanh(x/2)**6 + 43171936
039392*sqrt(2)*tanh(x/2)**4 + 61054337460812*tanh(x/2)**4 + 13633242959808*
sqrt(2)*tanh(x/2)**2 + 19280317092888*tanh(x/2)**2 + 1136103579984*sqrt(2)
+ 1606693091074)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{e^{(-2x)}-e^{(-6x)}}{2\left(2e^{(-4x)}+e^{(-8x)}+1\right)}+\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right)-\frac{1}{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="maxima")
[Out] 1/2*(e^(-2*x) - e^(-6*x))/(2*e^(-4*x) + e^(-8*x) + 1) + 1/2*arctan(1/2*sqrt
(2)*(sqrt(2) + 2*e^(-x))) - 1/2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^(-x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{e^{(2x)}-e^{(-2x)}}{2\left(\left(e^{(2x)}-e^{(-2x)}\right)^2+4\right)}+\frac{1}{4}\arctan\left(\frac{1}{2}\left(e^{(4x)}-1\right)e^{(-2x)}\right)$$

```
[In] integrate(1/(cosh(x)^2+sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*(e^(2*x) - e^(-2*x))/((e^(2*x) - e^(-2*x))^2 + 4) + 1/4*arctan(1/2*(e^(
4*x) - 1)*e^(-2*x))
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(\cosh^2(x)+\sinh^2(x)\right)^3}\,dx = \frac{\mathrm{atan}(\mathrm{e}^{2\,x})}{2} - \frac{\mathrm{e}^{-2\,x}}{4\cosh\left(2\,x\right)^2} + \frac{1}{4\cosh\left(2\,x\right)}$$

```
[In] int(1/(cosh(x)^2 + sinh(x)^2)^3,x)
```

```
[Out] atan(exp(2*x))/2 - exp(-2*x)/(4*cosh(2*x)^2) + 1/(4*cosh(2*x))
```

3.811 $\int \frac{1}{\cosh^2(x)-\sinh^2(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)} \, dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4465, 8}

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)} \, dx = x$$

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\text{integral} = \int 1\,dx$$
$$= x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\,dx = x$$

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-1),x]
[Out] x
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(cosh(x)^2-sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\,dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="fricas")
[Out] x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(0) = 0$.

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 10.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\, dx = \frac{x}{-\sinh^2(x) + \cosh^2(x)}$$

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2),x)
[Out] x/(-sinh(x)**2 + cosh(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\, dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)}\, dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2),x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^2(x) - \sinh^2(x)} \, dx = x$$

```
[In] int(1/(cosh(x)^2 - sinh(x)^2),x)
[Out] x
```

3.812 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^2} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4465, 8}

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^2} dx = x$$

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\text{integral} = \int 1\,dx$$
$$= x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

```
[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-2),x]
[Out] x
```

Maple [A] (verified)

Time = 11.54 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(cosh(x)^2-sinh(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="fricas")
[Out] x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(0) = 0$.

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 22.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = \frac{x}{\sinh^4(x)-2\sinh^2(x)\cosh^2(x)+\cosh^4(x)}$$

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2)**2,x)
[Out] x/(sinh(x)**4 - 2*sinh(x)**2*cosh(x)**2 + cosh(x)**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^2,x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x) - \sinh^2(x)\right)^2}\,dx = x$$

```
[In] int(1/(cosh(x)^2 - sinh(x)^2)^2,x)
[Out] x
```

3.813 $\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4465, 8}

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3} dx = x$$

```
[In] Int[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = x$$

[In] Integrate[(Cosh[x]^2 - Sinh[x]^2)^(-3),x]

[Out] x

Maple [A] (verified)

Time = 98.96 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

[In] int(1/(cosh(x)^2-sinh(x)^2)^3,x,method=_RETURNVERBOSE)

[Out] x

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = x$$

[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="fricas")

[Out] x

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(0) = 0$.

Time = 0.74 (sec) , antiderivative size = 34, normalized size of antiderivative = 34.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = \frac{x}{-\sinh^6(x)+3\sinh^4(x)\cosh^2(x)-3\sinh^2(x)\cosh^4(x)+\cosh^6(x)}$$

```
[In] integrate(1/(cosh(x)**2-sinh(x)**2)**3,x)
[Out] x/(-sinh(x)**6 + 3*sinh(x)**4*cosh(x)**2 - 3*sinh(x)**2*cosh(x)**4 + cosh(x
)**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx = x$$

```
[In] integrate(1/(cosh(x)^2-sinh(x)^2)^3,x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cosh^2(x)-\sinh^2(x)\right)^3}\,dx=x$$

```
[In] int(1/(cosh(x)^2 - sinh(x)^2)^3,x)
[Out] x
```

3.814 $\int \frac{1}{\mathrm{sech}^2(x)+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\mathrm{sech}^2(x)+\tanh^2(x)}\,dx=x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4466, 8}

$$\int \frac{1}{\mathrm{sech}^2(x)+\tanh^2(x)}\,dx=x$$

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4466

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\text{integral} = \int 1\,dx$$
$$= x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x)+\tanh^2(x)}\,dx = x$$

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-1),x]
[Out] x
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(sech(x)^2+tanh(x)^2),x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x)+\tanh^2(x)}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="fricas")
[Out] x
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\,dx = \int \frac{1}{\tanh^2{(x)} + \operatorname{sech}^2{(x)}}\,dx$$

```
[In] integrate(1/(sech(x)**2+tanh(x)**2),x)
[Out] Integral(1/(tanh(x)**2 + sech(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2),x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{sech}^2(x) + \tanh^2(x)}\,dx = x$$

```
[In] int(1/(1/cosh(x)^2 + tanh(x)^2),x)
[Out] x
```

3.815 $\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4466, 8}

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2} dx = x$$

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-2),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4466

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\text{integral} = \int 1\,dx$$
$$= x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-2),x]
[Out] x
```

Maple [A] (verified)

Time = 38.34 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(sech(x)^2+tanh(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="fricas")
[Out] x
```

Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = \int \frac{1}{\left(\tanh^2(x)+\operatorname{sech}^2(x)\right)^2}\,dx$$

```
[In] integrate(1/(sech(x)**2+tanh(x)**2)**2,x)
[Out] Integral((tanh(x)**2 + sech(x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^2,x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^2}\,dx = x$$

```
[In] int(1/(1/cosh(x)^2 + tanh(x)^2)^2,x)
[Out] x
```

3.816 $\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4466, 8}

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3} dx = x$$

```
[In] Int[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4466

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int 1\,dx \\
&= x
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3}\,dx=x$$

```
[In] Integrate[(Sech[x]^2 + Tanh[x]^2)^(-3),x]
[Out] x
```

Maple [A] (verified)

Time = 50.52 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
parallelrisch	0	2
default	$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(sech(x)^2+tanh(x)^2)^3,x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x)+\tanh^2(x)\right)^3}\,dx=x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="fricas")
[Out] x
```

Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x) + \tanh^2(x)\right)^3}\,dx = \int \frac{1}{\left(\tanh^2(x) + \operatorname{sech}^2(x)\right)^3}\,dx$$

```
[In] integrate(1/(sech(x)**2+tanh(x)**2)**3,x)
[Out] Integral((tanh(x)**2 + sech(x)**2)**(-3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x) + \tanh^2(x)\right)^3}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x) + \tanh^2(x)\right)^3}\,dx = x$$

```
[In] integrate(1/(sech(x)^2+tanh(x)^2)^3,x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\operatorname{sech}^2(x) + \tanh^2(x)\right)^3}\,dx = x$$

```
[In] int(1/(1/cosh(x)^2 + tanh(x)^2)^3,x)
[Out] x
```

3.817 $\int \frac{1}{\mathbf{sech}^2(x)-\tanh^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx=-x+\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)$$

```
[Out] -x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1107, 213}

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx=\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)-x$$

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \tanh(x)\right) \\ &= 2\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \tanh(x)\right) - 2\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tanh(x)\right) \\ &= -x+\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right) \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 529, normalized size of antiderivative = 27.84

$$\int \frac{1}{\text{sech}^2(x)-\tanh^2(x)}\,dx = \frac{\text{sech}(x)\left(\sqrt{2+2i}\arcsin\left(\frac{1}{2}\sqrt{1+i}\sqrt{(-1-i)(i+\sinh(x))}\right)\sqrt{(-1-i)(i+\sinh(x))} - 2\text{arctanh}\left(\frac{\sqrt{(-1-i)}}{\sqrt{(1+i)-(}}\right.\right.}{}$$

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-1),x]
```

```
[Out] (Sech[x]*(Sqrt[2 + 2*I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2
]*Sqrt[(-1 - I)*(I + Sinh[x])] - 2*ArcTanh[Sqrt[(-1 - I)*(I + Sinh[x])]/Sqr
t[(1 + I) - (1 - I)*Sinh[x]]]*Sqrt[(1 + I) - (1 - I)*Sinh[x]]*Sqrt[1 + I*Si
nh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + 2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*Sqrt
[(-1 - I)*(I + Sinh[x])])/Sqrt[(1 + I) - (1 - I)*Sinh[x]]]*Sqrt[(1 + I) - (
1 - I)*Sinh[x]]*Sqrt[1 + I*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + I*Sqrt[2
 + 2*I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2]*Sinh[x]*Sqrt[(
-1 - I)*(I + Sinh[x])] + Sqrt[-2 + 2*I]*ArcSinh[(Sqrt[-1 + I]*Sqrt[(1 - I)*
(I + Sinh[x])])/2]*Sqrt[(1 - I)*(I + Sinh[x])] + I*Sqrt[-2 + 2*I]*ArcSinh[(
Sqrt[-1 + I]*Sqrt[(1 - I)*(I + Sinh[x])])/2]*Sinh[x]*Sqrt[(1 - I)*(I + Sinh
[x])] + 2*(-1)^(3/4)*ArcTan[((-1)^(3/4)*Sqrt[(1 - I)*(I + Sinh[x])])/Sqrt[(
1 + I)*(-I + Sinh[x])]]*Sqrt[1 + I*Sinh[x]]*Sqrt[(1 + I)*(-I + Sinh[x])]*Sq
rt[(1 - I)*(I + Sinh[x])] + 2*ArcTanh[Sqrt[(1 - I)*(I + Sinh[x])]/Sqrt[(1 +
 I)*(-I + Sinh[x])]]*Sqrt[1 + I*Sinh[x]]*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[
(1 - I)*(I + Sinh[x])]))/(2*Sqrt[1 + I*Sinh[x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2x}+2\sqrt{2}-3\right)}{2}-\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{2}$
default	$-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\, \operatorname{arctanh}\left(\frac{(2\tanh\left(\frac{x}{2}\right)}{4}\right.$

```
[In] int(1/(sech(x)^2-tanh(x)^2),x,method=_RETURNVERBOSE)
[Out] -x+1/2*2^(1/2)*ln(exp(2*x)+2*2^(1/2)-3)-1/2*2^(1/2)*ln(exp(2*x)-3-2*2^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right) - x$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^2(x)-\tanh^2(x)}\,dx = \int \frac{1}{(-\tanh(x)+\operatorname{sech}(x))\,(\tanh(x)+\operatorname{sech}(x))}\,dx$$

```
[In] integrate(1/(sech(x)**2-tanh(x)**2),x)
[Out] Integral(1/((-tanh(x) + sech(x))*(tanh(x) + sech(x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1}\right) - \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1}\right) - x$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} + 2e^{(2x)} - 6\right|}{\left|4\sqrt{2} + 2e^{(2x)} - 6\right|}\right) - x$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1}{\operatorname{sech}^2(x) - \tanh^2(x)}\,dx = \frac{\sqrt{2}\ln\left(8e^{2x} + \frac{\sqrt{2}(12e^{2x}-4)}{2}\right)}{2} - \frac{\sqrt{2}\ln\left(8e^{2x} - \frac{\sqrt{2}(12e^{2x}-4)}{2}\right)}{2} - x$$

```
[In] int(1/(1/cosh(x)^2 - tanh(x)^2),x)
[Out] (2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log(
8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

3.818 $\int \frac{1}{\left(\mathrm{sech}^2(x)-\tanh^2(x)\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{\left(\mathrm{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = x-\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

[Out] x-1/2*arctanh(2^(1/2)*tanh(x))*2^(1/2)+tanh(x)/(1-2*tanh(x)^2)

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {425, 12, 492, 212}

$$\int \frac{1}{\left(\mathrm{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}+x+\frac{\tanh(x)}{1-2\tanh^2(x)}$$

[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-2),x]

[Out] x - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2] + Tanh[x]/(1 - 2*Tanh[x]^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-2x^2)^2(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{1-2\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int -\frac{2x^2}{(1-2x^2)(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{1-2\tanh^2(x)} - \text{Subst}\left(\int \frac{x^2}{(1-2x^2)(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{1-2\tanh^2(x)} - \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\
&= x - \frac{\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}} + \frac{\tanh(x)}{1-2\tanh^2(x)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 17.71

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx$$

$$= \frac{\operatorname{sech}(x)\left(-4\sinh(x)-4\sinh^3(x)-(1+i)\sqrt{2}\left(\arctan\left(\frac{(1+i)\sqrt{(-1-i)(i+\sinh(x))}}{\sqrt{(2+2i)-(2-2i)\sinh(x)}}\right)\sqrt{(1+i)-(1-i)\sinh(x)}+\right.\right.}{}$$

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-2),x]
```

```
[Out] (Sech[x]*(-4*Sinh[x] - 4*Sinh[x]^3 - (1 + I)*Sqrt[2]*(ArcTan[((1 + I)*Sqrt[
(-1 - I)*(I + Sinh[x])])/Sqrt[(2 + 2*I) - (2 - 2*I)*Sinh[x]]]*Sqrt[(1 + I)
- (1 - I)*Sinh[x]] + I*Sqrt[1 + I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + S
inh[x])])/2]*Sqrt[1 + I*Sinh[x]])*Sqrt[(-1 - I)*(I + Sinh[x])] + ArcTanh[Sq
rt[(-1 - I)*(I + Sinh[x])]/Sqrt[(1 + I) - (1 - I)*Sinh[x]]]*(-3 + Cosh[2*x]
)*Sqrt[(1 + I) - (1 - I)*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + (1 + I)*Sq
rt[2]*(ArcTan[((1 + I)*Sqrt[(-1 - I)*(I + Sinh[x])])/Sqrt[(2 + 2*I) - (2 -
2*I)*Sinh[x]]]*Sqrt[(1 + I) - (1 - I)*Sinh[x]] + I*Sqrt[1 + I]*ArcSin[(Sqrt
[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]])*Sinh[x]^2*Sqr
t[(-1 - I)*(I + Sinh[x])] + (ArcSinh[(Sqrt[-1 + I]*Sqrt[(1 - I)*(I + Sinh[x
])])/2]*(-3 + Cosh[2*x])*Sqrt[2 + (2*I)*Sinh[x]]*Sqrt[(1 - I)*(I + Sinh[x])
])/Sqrt[-1 + I] - ArcTanh[Sqrt[(1 - I)*(I + Sinh[x])]/Sqrt[(1 + I)*(-I + Si
nh[x])]]*(-3 + Cosh[2*x])*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1 - I)*(I + Si
nh[x])] + ((1 + I)*ArcTanh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x])])/(Sqrt[2]*S
qrt[(1 + I)*(-I + Sinh[x])])]*(-3 + Cosh[2*x])*Sqrt[(1 + I)*(-I + Sinh[x])]
*Sqrt[(1 - I)*(I + Sinh[x])])/Sqrt[2]))/(2*(-3 + Cosh[2*x]))
```

Maple [A] (verified)

Time = 16.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.06

method	result
parallelrisch	0
risch	$x-\frac{2\left(3\,e^{2x}-1\right)}{e^{4x}-6\,e^{2x}+1}+\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2x}+2\sqrt{2}-3\right)}{4}$
default	$-\frac{-2\tanh\left(\frac{x}{2}\right)+2}{2\left(\tanh\left(\frac{x}{2}\right)^2+2\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{2}+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{2\tanh\left(\frac{x}{2}\right)+2}{2\tanh\left(\frac{x}{2}\right)^2-4\tanh\left(\frac{x}{2}\right)-}$

```
[In] int(1/(sech(x)^2-tanh(x)^2)^2,x,method=_RETURNVERBOSE)
```

[Out] 0

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(28) = 56.

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx$$

$$= \frac{4\,x\cosh\left(x\right)^4+16\,x\cosh\left(x\right)\sinh\left(x\right)^3+4\,x\sinh\left(x\right)^4-24\left(x+1\right)\cosh\left(x\right)^2+24\left(x\cosh\left(x\right)^2-x-1\right)\sinh}{}$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 - 24*(x + 1)*co
sh(x)^2 + 24*(x*cosh(x)^2 - x - 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqrt(
2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2))*
sinh(x)^2 - 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))
*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*co
sh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 - 2*sqrt(2) - 3)/(cosh(x)^2 + s
inh(x)^2 - 3)) + 16*(x*cosh(x)^3 - 3*(x + 1)*cosh(x))*sinh(x) + 4*x + 8)/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 -
6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx = \int \frac{1}{\left(-\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^2\left(\tanh\left(x\right)+\operatorname{sech}\left(x\right)\right)^2}\,dx$$

```
[In] integrate(1/(sech(x)**2-tanh(x)**2)**2,x)
```

```
[Out] Integral(1/((-tanh(x) + sech(x))**2*(tanh(x) + sech(x))**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(28) = 56.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx=-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+x-\frac{2\left(3e^{(-2x)}-1\right)}{6e^{(-2x)}-e^{(-4x)}-1}$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="maxima")
[Out] -1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) + 1/4*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + x - 2*(3*e^(-2*x)
 - 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx=\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)+x-\frac{2\left(3e^{(2x)}-1\right)}{e^{(4x)}-6e^{(2x)}+1}$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^2,x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) -
 6)) + x - 2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^2}\,dx=x-\frac{\sqrt{2}\,\ln\left(-4\,e^{2x}-\frac{\sqrt{2}\,\left(12\,e^{2x}-4\right)}{4}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,e^{2x}-4\right)}{4}-4\,e^{2x}\right)}{4}-\frac{6\,e^{2x}-2}{e^{4x}-6\,e^{2x}+1}$$

```
[In] int(1/(1/cosh(x)^2 - tanh(x)^2)^2,x)
[Out] x - (2^(1/2)*log(- 4*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/4))/4 + (2^(1/2
)*log((2^(1/2)*(12*exp(2*x) - 4))/4 - 4*exp(2*x)))/4 - (6*exp(2*x) - 2)/(ex
p(4*x) - 6*exp(2*x) + 1)
```

3.819 $\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} \, dx=-x+\frac{7\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}$$

```
[Out] -x+7/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)/(1-2*tanh(x)^2)^2-1/4*t
anh(x)/(1-2*tanh(x)^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {425, 541, 536, 212}

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3} \, dx=\frac{7\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}-x-\frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}+\frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2}$$

```
[In] Int[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

```
[Out] -x + (7*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + Tanh[x]/(2*(1 - 2*Tanh[x]^2
)^2) - Tanh[x]/(4*(1 - 2*Tanh[x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-2x^2)^3(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2} + \frac{1}{4}\text{Subst}\left(\int \frac{2-6x^2}{(1-2x^2)^2(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2} - \frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)} + \frac{1}{8}\text{Subst}\left(\int \frac{6+2x^2}{(1-2x^2)(1-x^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2} - \frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}\\
&\quad + \frac{7}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)
\end{aligned}$$

$$= -x + \frac{7\operatorname{arctanh}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} + \frac{\tanh(x)}{2\left(1-2\tanh^2(x)\right)^2} - \frac{\tanh(x)}{4\left(1-2\tanh^2(x)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 765, normalized size of antiderivative = 14.17

$$\int \frac{1}{\left(\operatorname{sech}^2(x) - \tanh^2(x)\right)^3}\,dx$$

$$= \frac{\operatorname{sech}(x)\left(-7\operatorname{arctanh}\left(\frac{\sqrt{(-1-i)(i+\sinh(x))}}{\sqrt{(1+i)-(1-i)\sinh(x)}}\right)(-3+\cosh(2x))^2\sqrt{(2+2i)-(2-2i)\sinh(x)}\sqrt{(-1-i)(i+\sinh(x))}\right.}{}$$

```
[In] Integrate[(Sech[x]^2 - Tanh[x]^2)^(-3),x]
```

```
[Out] (Sech[x]*(-7*ArcTanh[Sqrt[(-1 - I)*(I + Sinh[x])]/Sqrt[(1 + I) - (1 - I)*Si
nh[x]]]*(-3 + Cosh[2*x])^2*Sqrt[(2 + 2*I) - (2 - 2*I)*Sinh[x]]*Sqrt[(-1 - I
)*(I + Sinh[x])] + 7*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*Sqrt[(-1 - I)*(I + Sinh
[x])])/Sqrt[(1 + I) - (1 - I)*Sinh[x]]]*(-3 + Cosh[2*x])^2*Sqrt[(2 + 2*I) -
 (2 - 2*I)*Sinh[x]]*Sqrt[(-1 - I)*(I + Sinh[x])] + 4*(2*Sqrt[2]*Sinh[x] + 8
*Sqrt[2]*Sinh[x]^3 + 6*Sqrt[2]*Sinh[x]^5 + (7 - I)*Sqrt[1 + I]*ArcSin[(Sqrt
[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]]*Sqrt[(-1 - I)*
(I + Sinh[x])] - (14 - 2*I)*Sqrt[1 + I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(
I + Sinh[x])])/2]*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[(-1 - I)*(I + Sinh[x])
] + (7 - I)*Sqrt[1 + I]*ArcSin[(Sqrt[1 + I]*Sqrt[(-1 - I)*(I + Sinh[x])])/2
]*Sqrt[1 + I*Sinh[x]]*Sinh[x]^4*Sqrt[(-1 - I)*(I + Sinh[x])] + (7/4 + I/4)*
Sqrt[-1 + I]*ArcSinh[(Sqrt[-1 + I]*Sqrt[(1 - I)*(I + Sinh[x])])/2]*(-3 + Co
sh[2*x])^2*Sqrt[1 + I*Sinh[x]]*Sqrt[(1 - I)*(I + Sinh[x])] - (7 + 7*I)*ArcT
anh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x])])/(Sqrt[2]*Sqrt[(1 + I)*(-I + Sinh[
x])])]*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1 - I)*(I + Sinh[x])] + (7*ArcTan
h[Sqrt[(1 - I)*(I + Sinh[x])]/Sqrt[(1 + I)*(-I + Sinh[x])]]*(-3 + Cosh[2*x]
)^2*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1 - I)*(I + Sinh[x])])/(2*Sqrt[2]) +
 (14 + 14*I)*ArcTanh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x])])/(Sqrt[2]*Sqrt[(1
 + I)*(-I + Sinh[x])])]*Sinh[x]^2*Sqrt[(1 + I)*(-I + Sinh[x])]*Sqrt[(1 - I)
*(I + Sinh[x])] - (7 + 7*I)*ArcTanh[((1 + I)*Sqrt[(1 - I)*(I + Sinh[x])])/(
Sqrt[2]*Sqrt[(1 + I)*(-I + Sinh[x])])]*Sinh[x]^4*Sqrt[(1 + I)*(-I + Sinh[x]
)]*Sqrt[(1 - I)*(I + Sinh[x])])))/(8*Sqrt[2]*(-3 + Cosh[2*x])^2)
```

Maple [A] (verified)

Time = 210.08 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

method	result
parallelrisch	0
risch	$-x+\frac{17\,\mathrm{e}^{6x}-57\,\mathrm{e}^{4x}+19\,\mathrm{e}^{2x}-3}{2(\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1)^2}+\frac{7\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+2\sqrt{2}-3\right)}{16}-\frac{7\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{16}$
default	$-\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^3}{8}+\frac{\tanh\left(\frac{x}{2}\right)^2}{8}-\frac{5\tanh\left(\frac{x}{2}\right)}{8}+\frac{1}{8}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+2\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{7\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{8}-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\frac{2\left(-\tanh\cdots\right.}{(\cdots}$

[In] int(1/(sech(x)^2-tanh(x)^2)^3,x,method=_RETURNVERBOSE)

[Out] 0

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 717, normalized size of antiderivative = 13.28

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx=\text{Too large to display}$$

[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="fricas")

[Out] -1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 - 8*(24*x + 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 - 24*x - 17)*sinh(x)^6 + 16*(56*x*cosh(x)^3 - 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(140*x*cosh(x)^4 - 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x*cosh(x)^5 - 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 - 8*(24*x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 - 15*(24*x + 17)*cosh(x)^4 + 114*(4*x + 3)*cosh(x)^2 - 24*x - 19)*sinh(x)^2 - 7*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 - 9*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 - 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 - 30*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 - 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^2 - 12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16*(8*x*cosh(x)^7 - 3*(24*x + 17)*cosh(x)^5 + 38*(4*x + 3)*cosh(x)^3 - (24*x + 19)*cosh(x))*sinh(x) + 16*x + 24

```
)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sinh(x
)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^
4 - 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(
x)^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 + 57*cosh(x)^2
 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 8*(cosh(x)^7 - 9*cosh(x)^5 + 19*cosh(x)^3
- 3*cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx=\int \frac{1}{\left(-\tanh(x)+\operatorname{sech}(x)\right)^3\left(\tanh(x)+\operatorname{sech}(x)\right)^3}\,dx$$

```
[In] integrate(1/(sech(x)**2-tanh(x)**2)**3,x)
[Out] Integral(1/((-tanh(x) + sech(x))**3*(tanh(x) + sech(x))**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\begin{aligned}\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx &= \frac{7}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)\\ &\quad-\frac{7}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x\\ &\quad+\frac{19\,e^{(-2\,x)}-57\,e^{(-4\,x)}+17\,e^{(-6\,x)}-3}{2\left(12\,e^{(-2\,x)}-38\,e^{(-4\,x)}+12\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}\end{aligned}$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="maxima")
[Out] 7/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 7/16*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x + 1/2*(19*e^(-
2*x) - 57*e^(-4*x) + 17*e^(-6*x) - 3)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(-6
*x) - e^(-8*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx = -\frac{7}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right) - x + \frac{17e^{(6x)}-57e^{(4x)}+19e^{(2x)}-3}{2\left(e^{(4x)}-6e^{(2x)}+1\right)^2}$$

```
[In] integrate(1/(sech(x)^2-tanh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - x + 1/2*(17*e^(6*x) - 57*e^(4*x) + 19*e^(2*x) - 3)/(e^(4*x) - 6*e^
(2*x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int \frac{1}{\left(\operatorname{sech}^2(x)-\tanh^2(x)\right)^3}\,dx = \frac{136\,\mathrm{e}^{2x}-24}{38\,\mathrm{e}^{4x}-12\,\mathrm{e}^{2x}-12\,\mathrm{e}^{6x}+\mathrm{e}^{8x}+1} - x - \frac{7\sqrt{2}\,\ln\left(7\,\mathrm{e}^{2x}-\frac{7\sqrt{2}\,(12\,\mathrm{e}^{2x}-4)}{16}\right)}{16} + \frac{7\sqrt{2}\,\ln\left(7\,\mathrm{e}^{2x}+\frac{7\sqrt{2}\,(12\,\mathrm{e}^{2x}-4)}{16}\right)}{16} + \frac{\frac{17\,\mathrm{e}^{2x}}{2}+\frac{45}{2}}{\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1}$$

```
[In] int(1/(1/cosh(x)^2 - tanh(x)^2)^3,x)
```

```
[Out] (136*exp(2*x) - 24)/(38*exp(4*x) - 12*exp(2*x) - 12*exp(6*x) + exp(8*x) + 1
) - x - (7*2^(1/2)*log(7*exp(2*x) - (7*2^(1/2)*(12*exp(2*x) - 4))/16))/16 +
 (7*2^(1/2)*log(7*exp(2*x) + (7*2^(1/2)*(12*exp(2*x) - 4))/16))/16 + ((17*e
xp(2*x))/2 + 45/2)/(exp(4*x) - 6*exp(2*x) + 1)
```

3.820 $\int \frac{1}{\coth^2(x)+\mathbf{csch}^2(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx = x-\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

```
[Out] x-arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1144, 213}

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx = x-\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-1),x]
```

```
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
```

```
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x^2}{2-3x^2+x^4}\,dx, x, \tanh(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\ &= x-\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x)+\text{csch}^2(x)}\,dx = x-\sqrt{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)$$

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-1),x]
[Out] x - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(15) = 30.

Time = 0.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

method	result
risch	$x+\frac{\sqrt{2}\,\ln\left(e^{2x}+3+2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{2x}-2\sqrt{2}+3\right)}{2}$
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\cdots\right.\right.}{4}$

```
[In] int(1/(coth(x)^2+csch(x)^2),x,method=_RETURNVERBOSE)
[Out] x+1/2*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)-2*2^(1/2)+3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{1}{\coth^2(x) + \operatorname{csch}^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh\left(x\right)^2 - 4\left(3\sqrt{2}+4\right)\cosh\left(x\right)\sinh\left(x\right) + 3\left(2\sqrt{2}+3\right)\sinh\left(x\right)^2 + 2\sqrt{2} + 3}{\cosh\left(x\right)^2 + \sinh\left(x\right)^2 + 3}\right) + x$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*cosh(x)*si
nh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2
 + 3)) + x
```

Sympy [F]

$$\int \frac{1}{\coth^2(x) + \operatorname{csch}^2(x)}\,dx = \int \frac{1}{\coth^2\left(x\right) + \operatorname{csch}^2\left(x\right)}\,dx$$

```
[In] integrate(1/(coth(x)**2+csch(x)**2),x)
[Out] Integral(1/(coth(x)**2 + csch(x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{\coth^2(x) + \operatorname{csch}^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2} - e^{(-2x)} - 3}{2\sqrt{2} + e^{(-2x)} + 3}\right) + x$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) + x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx = -\frac{1}{2}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right)+x$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \frac{1}{\coth^2(x)+\operatorname{csch}^2(x)}\,dx = x+\frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{2}\right)}{2}$$

```
[In] int(1/(coth(x)^2 + 1/sinh(x)^2),x)
[Out] x + (2^(1/2)*log(8*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/2))/2 - (2^(1/2)*
log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) + 4))/2))/2
```

3.821 $\int \frac{1}{\left(\coth^2(x)+\mathbf{csch}^2(x)\right)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2} dx = x-\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh(x)}{2-\tanh^2(x)}$$

```
[Out] x-1/2*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)-tanh(x)/(2-tanh(x)^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {481, 12, 400, 212}

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2} dx = -\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}+x-\frac{\tanh(x)}{2-\tanh^2(x)}$$

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-2),x]
```

```
[Out] x - ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2] - Tanh[x]/(2 - Tanh[x]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^4}{(1-x^2)\,(2-x^2)^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{2}{(1-x^2)\,(2-x^2)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)} + \text{Subst}\left(\int \frac{1}{(1-x^2)\,(2-x^2)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{2-\tanh^2(x)} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \tanh(x)\right)\\
&= x - \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tanh(x)}{2-\tanh^2(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = \frac{(3+\cosh(2x))\operatorname{csch}^4(x)\left(6x+2x\cosh(2x)-\sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)(3+\cosh(2x))-2\sinh(2x)\right)}{8\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}$$

[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-2),x]

[Out] ((3 + Cosh[2*x])*Csch[x]^4*(6*x + 2*x*Cosh[2*x] - Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]*(3 + Cosh[2*x]) - 2*Sinh[2*x]))/(8*(Coth[x]^2 + Csch[x]^2)^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 26.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

method	result
risch	$x+\frac{6\,\mathrm{e}^{2x}+2}{\mathrm{e}^{4x}+6\,\mathrm{e}^{2x}+1}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-2\sqrt{2}+3\right)}{4}$
default	$\frac{-\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{4}+1}-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}-1\right)\right)}{8}+\frac{\sqrt{2}\,(\ln}{}$

[In] int(1/(coth(x)^2+csch(x)^2)^2,x,method=_RETURNVERBOSE)

[Out] x+2*(3*exp(2*x)+1)/(exp(4*x)+6*exp(2*x)+1)+1/4*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2))-1/4*2^(1/2)*ln(exp(2*x)-2*2^(1/2)+3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 8.19

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx$$

$$=\frac{4\,x\cosh\left(x\right)^{4}+16\,x\cosh\left(x\right)\sinh\left(x\right)^{3}+4\,x\sinh\left(x\right)^{4}+24\,(x+1)\cosh\left(x\right)^{2}+24\,\left(x\cosh\left(x\right)^{2}+x+1\right)\sinh}{}$$

[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="fricas")

[Out] 1/4*(4*x*cosh(x)^4 + 16*x*cosh(x)*sinh(x)^3 + 4*x*sinh(x)^4 + 24*(x + 1)*cosh(x)^2 + 24*(x*cosh(x)^2 + x + 1)*sinh(x)^2 + (sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 6*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 - 4*(3*sqrt(2) + 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 16*(x*cosh(x)^3 + 3*(x + 1)*cosh(x))*sinh(x) + 4*x + 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*cosh(x))*sinh(x) + 1)

Sympy [F]

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = \int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx$$

```
[In] integrate(1/(coth(x)**2+csch(x)**2)**2,x)
```

```
[Out] Integral((coth(x)**2 + csch(x)**2)**(-2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(26) = 52.

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x-\frac{2\left(3e^{(-2x)}+1\right)}{6e^{(-2x)}+e^{(-4x)}+1}$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) + x
 - 2*(3*e^(-2*x) + 1)/(6*e^(-2*x) + e^(-4*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = -\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+x+\frac{2\left(3e^{(2x)}+1\right)}{e^{(4x)}+6e^{(2x)}+1}$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
+ 2*(3*e^(2*x) + 1)/(e^(4*x) + 6*e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.41

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^2}\,dx = x+\frac{\sqrt{2}\,\ln\left(4\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{4}\right)}{4} - \frac{\sqrt{2}\,\ln\left(4\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{4}\right)}{4}+\frac{6\,e^{2\,x}+2}{6\,e^{2\,x}+e^{4\,x}+1}$$

```
[In] int(1/(coth(x)^2 + 1/sinh(x)^2)^2,x)
```

```
[Out] x + (2^(1/2)*log(4*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/4))/4 - (2^(1/2)*
log(4*exp(2*x) + (2^(1/2)*(12*exp(2*x) + 4))/4))/4 + (6*exp(2*x) + 2)/(6*ex
p(2*x) + exp(4*x) + 1)
```

3.822 $\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3} dx = x-\frac{7\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}$$

```
[Out] x-7/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)-1/2*tanh(x)^3/(2-tanh(x)^2)^2-1/
4*tanh(x)/(2-tanh(x)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {481, 592, 536, 212}

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3} dx = -\frac{7\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}+x-\frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}-\frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2}$$

```
[In] Int[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

```
[Out] x - (7*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - Tanh[x]^3/(2*(2 - Tanh[x]^2)
^2) - Tanh[x]/(4*(2 - Tanh[x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^6}{(1-x^2)\,(2-x^2)^3}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh^3(x)}{2\,(2-\tanh^2(x))^2} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2(6-2x^2)}{(1-x^2)\,(2-x^2)^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh^3(x)}{2\,(2-\tanh^2(x))^2} - \frac{\tanh(x)}{4\,(2-\tanh^2(x))} - \frac{1}{8}\text{Subst}\left(\int \frac{-2-6x^2}{(1-x^2)\,(2-x^2)}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh^3(x)}{2\,(2-\tanh^2(x))^2} - \frac{\tanh(x)}{4\,(2-\tanh^2(x))}\\
&\quad - \frac{7}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)
\end{aligned}$$

$$= x - \frac{7\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\tanh^3(x)}{2\left(2-\tanh^2(x)\right)^2} - \frac{\tanh(x)}{4\left(2-\tanh^2(x)\right)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx$$
$$= \frac{76x + 48x\cosh(2x) - 7\sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)(3+\cosh(2x))^2 + 4x\cosh(4x) - 2\sinh(2x) - 3\sinh(4x)}{8(3+\cosh(2x))^2}$$

```
[In] Integrate[(Coth[x]^2 + Csch[x]^2)^(-3),x]
```

```
[Out] (76*x + 48*x*Cosh[2*x] - 7*Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]]*(3 + Cosh[2*x])
^2 + 4*x*Cosh[4*x] - 2*Sinh[2*x] - 3*Sinh[4*x])/(8*(3 + Cosh[2*x])^2)
```

Maple [A] (verified)

Time = 37.53 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

method	result
parallelrisch	0
risch	$x+\frac{17\,\mathrm{e}^{6x}+57\,\mathrm{e}^{4x}+19\,\mathrm{e}^{2x}+3}{2\left(\mathrm{e}^{4x}+6\,\mathrm{e}^{2x}+1\right)^2}+\frac{7\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{16}-\frac{7\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-2\sqrt{2}+3\right)}{16}$
default	$\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{-\frac{\tanh\left(\frac{x}{2}\right)^7}{4}-\frac{5\tanh\left(\frac{x}{2}\right)^5}{4}-\frac{5\tanh\left(\frac{x}{2}\right)^3}{4}-\frac{\tanh\left(\frac{x}{2}\right)}{4}}{\left(\tanh\left(\frac{x}{2}\right)^4+1\right)^2}-\frac{7\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\,\mathrm{arcta}\right.}{}$

```
[In] int(1/(coth(x)^2+csch(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 715, normalized size of antiderivative = 13.24

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*x*cosh(x)^8 + 128*x*cosh(x)*sinh(x)^7 + 16*x*sinh(x)^8 + 8*(24*x +
 17)*cosh(x)^6 + 8*(56*x*cosh(x)^2 + 24*x + 17)*sinh(x)^6 + 16*(56*x*cosh(x
)^3 + 3*(24*x + 17)*cosh(x))*sinh(x)^5 + 152*(4*x + 3)*cosh(x)^4 + 8*(140*x
*cosh(x)^4 + 15*(24*x + 17)*cosh(x)^2 + 76*x + 57)*sinh(x)^4 + 32*(28*x*cos
h(x)^5 + 5*(24*x + 17)*cosh(x)^3 + 19*(4*x + 3)*cosh(x))*sinh(x)^3 + 8*(24*
x + 19)*cosh(x)^2 + 8*(56*x*cosh(x)^6 + 15*(24*x + 17)*cosh(x)^4 + 114*(4*x
 + 3)*cosh(x)^2 + 24*x + 19)*sinh(x)^2 + 7*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*c
osh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4*(7*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*
sinh(x)^6 + 12*sqrt(2)*cosh(x)^6 + 8*(7*sqrt(2)*cosh(x)^3 + 9*sqrt(2)*cosh(
x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4 + 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2)
)*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + 30*sqrt(2)*co
sh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 45*sqrt(
2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^2 + 12*sqrt(2)*cos
h(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3
+ 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log((3*(2*sqrt(2) + 3)*cosh(x)^2 -
4*(3*sqrt(2) + 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) + 3)*sinh(x)^2 + 2*sqrt(2)
 + 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 16*(8*x*cosh(x)^7 + 3*(24*x + 17)*cosh
(x)^5 + 38*(4*x + 3)*cosh(x)^3 + (24*x + 19)*cosh(x))*sinh(x) + 16*x + 24)/
(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 3)*sinh(x)^
6 + 12*cosh(x)^6 + 8*(7*cosh(x)^3 + 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4
+ 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 + 30*cosh(x)
^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 45*cosh(x)^4 + 57*cosh(x)^2 +
 3)*sinh(x)^2 + 12*cosh(x)^2 + 8*(cosh(x)^7 + 9*cosh(x)^5 + 19*cosh(x)^3 +
3*cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = \int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx$$

```
[In] integrate(1/(coth(x)**2+csch(x)**2)**3,x)
```

```
[Out] Integral((coth(x)**2 + csch(x)**2)**(-3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = \frac{7}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)+x-\frac{19\,e^{(-2x)}+57\,e^{(-4x)}+17\,e^{(-6x)}+3}{2\left(12\,e^{(-2x)}+38\,e^{(-4x)}+12\,e^{(-6x)}+e^{(-8x)}+1\right)}$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="maxima")
[Out] 7/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) +
x - 1/2*(19*e^(-2*x) + 57*e^(-4*x) + 17*e^(-6*x) + 3)/(12*e^(-2*x) + 38*e^(
-4*x) + 12*e^(-6*x) + e^(-8*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = -\frac{7}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+x+\frac{17\,e^{(6x)}+57\,e^{(4x)}+19\,e^{(2x)}+3}{2\left(e^{(4x)}+6\,e^{(2x)}+1\right)^2}$$

```
[In] integrate(1/(coth(x)^2+csch(x)^2)^3,x, algorithm="giac")
[Out] -7/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + x
 + 1/2*(17*e^(6*x) + 57*e^(4*x) + 19*e^(2*x) + 3)/(e^(4*x) + 6*e^(2*x) + 1)
^2
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.07

$$\int \frac{1}{\left(\coth^2(x)+\operatorname{csch}^2(x)\right)^3}\,dx = x+\frac{136\,\mathrm{e}^{2\,x}+24}{12\,\mathrm{e}^{2\,x}+38\,\mathrm{e}^{4\,x}+12\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}+\frac{7\sqrt{2}\,\ln\left(7\,\mathrm{e}^{2\,x}-\frac{7\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}+4\right)}{16}\right)}{16}-\frac{7\sqrt{2}\,\ln\left(7\,\mathrm{e}^{2\,x}+\frac{7\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}+4\right)}{16}\right)}{16}+\frac{\frac{17\,\mathrm{e}^{2\,x}}{2}-\frac{45}{2}}{6\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

```
[In] int(1/(coth(x)^2 + 1/sinh(x)^2)^3,x)
```

```
[Out] x + (136*exp(2*x) + 24)/(12*exp(2*x) + 38*exp(4*x) + 12*exp(6*x) + exp(8*x)
 + 1) + (7*2^(1/2)*log(7*exp(2*x) - (7*2^(1/2)*(12*exp(2*x) + 4))/16))/16 -
 (7*2^(1/2)*log(7*exp(2*x) + (7*2^(1/2)*(12*exp(2*x) + 4))/16))/16 + ((17*e
xp(2*x))/2 - 45/2)/(6*exp(2*x) + exp(4*x) + 1)
```

3.823 $\int \frac{1}{\coth^2(x)-\mathbf{csch}^2(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4467, 8}

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)} dx = x$$

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4467

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int 1\,dx \\ &= x \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)}\,dx = x$$

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-1),x]
[Out] x
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(coth(x)^2-csch(x)^2),x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x)-\operatorname{csch}^2(x)}\,dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="fricas")
[Out] x
```

Sympy [F]

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)}\, dx = \int \frac{1}{(\coth(x) - \operatorname{csch}(x))(\coth(x) + \operatorname{csch}(x))}\, dx$$

```
[In] integrate(1/(coth(x)**2-csch(x)**2),x)
[Out] Integral(1/((coth(x) - csch(x))*(coth(x) + csch(x))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)}\, dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)}\, dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2),x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\coth^2(x) - \operatorname{csch}^2(x)}\, dx = x$$

```
[In] int(1/(coth(x)^2 - 1/sinh(x)^2),x)
[Out] x
```

3.824 $\int \frac{1}{\left(\coth^2(x)-\textbf{csch}^2(x)\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4467, 8}

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2} dx = x$$

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4467

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\text{integral} = \int 1\,dx$$
$$= x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx = x$$

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-2),x]
[Out] x
```

Maple [A] (verified)

Time = 10.52 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$2\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(coth(x)^2-csch(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="fricas")
[Out] x
```

Sympy [F]

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\,dx = \int \frac{1}{\left(\coth(x) - \operatorname{csch}(x)\right)^2 \left(\coth(x) + \operatorname{csch}(x)\right)^2}\,dx$$

```
[In] integrate(1/(coth(x)**2-csch(x)**2)**2,x)
[Out] Integral(1/((coth(x) - csch(x))**2*(coth(x) + csch(x))**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\,dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^2,x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^2}\,dx = x$$

```
[In] int(1/(coth(x)^2 - 1/sinh(x)^2)^2,x)
[Out] x
```

3.825 $\int \frac{1}{\left(\coth^2(x)-\mathbf{csch}^2(x)\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3} dx = x$$

```
[Out] x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4467, 8}

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3} dx = x$$

```
[In] Int[(Coth[x]^2 - Csch[x]^2)^(-3),x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4467

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\text{integral} = \int 1\,dx$$
$$= x$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3}\,dx = x$$

```
[In] Integrate[(Coth[x]^2 - Csch[x]^2)^(-3),x]
[Out] x
```

Maple [A] (verified)

Time = 116.97 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
parallelrisch	0	2
default	$2\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$	8

```
[In] int(1/(coth(x)^2-csch(x)^2)^3,x,method=_RETURNVERBOSE)
[Out] x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x)-\operatorname{csch}^2(x)\right)^3}\,dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="fricas")
[Out] x
```

Sympy [F]

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3}\, dx = \int \frac{1}{\left(\coth(x) - \operatorname{csch}(x)\right)^3 \left(\coth(x) + \operatorname{csch}(x)\right)^3}\, dx$$

```
[In] integrate(1/(coth(x)**2-csch(x)**2)**3,x)
[Out] Integral(1/((coth(x) - csch(x))**3*(coth(x) + csch(x))**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3}\, dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="maxima")
[Out] x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3}\, dx = x$$

```
[In] integrate(1/(coth(x)^2-csch(x)^2)^3,x, algorithm="giac")
[Out] x
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\coth^2(x) - \operatorname{csch}^2(x)\right)^3}\, dx = x$$

```
[In] int(1/(coth(x)^2 - 1/sinh(x)^2)^3,x)
[Out] x
```

3.826 $\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 271

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{2\sqrt{2}c\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{2\sqrt{2}c\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

```
[Out] 2*c*arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(
a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)/(4*a*c-b^2)^(1/2)/(b^2-2*(a-c)
*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)-2*c*arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*
c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/
2))*2^(1/2)/(4*a*c-b^2)^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3329, 2739, 632, 210}

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\frac{2\sqrt{2}c\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}-\frac{2\sqrt{2}c\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{4ac-b^2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[In] Int[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

```
[Out] (-2*Sqrt[2]*c*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2
])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2
+ 4*a*c]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) + (2*Sqrt[2]*c*A
rcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 -
2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(Sqrt[-b^2 + 4*a*c]*Sqrt[b^2 - 2*(
a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3329

```
Int[((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*
(x_)]^(n2_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c
```

```
/q), Int[1/(b - q + 2*c*Sin[d + e*x]^n), x], x] - Dist[2*(c/q), Int[1/(b +
q + 2*c*Sin[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2c)\int \frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{\sqrt{-b^2+4ac}} + \frac{(2c)\int \frac{1}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{\sqrt{-b^2+4ac}}$$

$$= -\frac{(4c)\text{Subst}\left(\int \frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-b^2+4ac}} + \frac{(4c)\text{Subst}\left(\int \frac{1}{-ib+\sqrt{-b^2+4ac}-4icx-\left(-ib+\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-b^2+4ac}}$$

$$= \frac{(8c)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, -4ic+2\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-b^2+4ac}} - \frac{(8c)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, -4ic+2\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-b^2+4ac}}$$

$$= -\frac{2\sqrt{2}c\arctan\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{2\sqrt{2}c\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{-b^2+4ac}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{2\sqrt{2}c\left(\frac{\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[(a + b*Sinh[x] + c*Sinh[x]^2)^(-1),x]
```

```
[Out] (2*Sqrt[2]*c*(ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b
^2 - 4*a*c]] - ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sq
rt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])]/Sqrt[-b^2 + 2*(a - c)*c - b*
Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.27

method	result
default	$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(-\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(16a^4c^2-8a^3b^2c-32a^3c^3+a^2b^4+32a^2b^2c^2+16a^2c^4-10a\,b^4c-8a\,b^2c^3+b^6+b^4c^2\right)\_Z^4+\left(-8a^2c^2+6a\,b^2c+8a\,c^3-b^4-2b^2c\right.\right.}$

```
[In] int(1/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] sum((-_R^2+1)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=Roo
tOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3313 vs. $2(219) = 438$.

Time = 0.37 (sec) , antiderivative size = 3313, normalized size of antiderivative = 12.23

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 +
 b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*
a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 +
 (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*
sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 + b^6 - 8*a*c^5 + 2
*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*
a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2
+ (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqr
```

t(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2 + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) + 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 + b^6 - 8*a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2 - (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 -

```
4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*b*c^2*cosh(x) +
 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 + b^6 - 8*
a*c^5 + 2*(12*a^2 + b^2)*c^4 - 6*(4*a^3 + 3*a*b^2)*c^3 + (8*a^4 + 22*a^2*b^
2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 + 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 +
 b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c
 + 2*c^2 - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^
2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 -
12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*
b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3
 + 3*a*b^2)*c)) - 2*(4*a*c^4 - (8*a^2 + b^2)*c^3 + 2*(2*a^3 + 3*a*b^2)*c^2
- (a^2*b^2 + b^4)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^
2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{1}{c\sinh{(x)}^2+b\sinh{(x)}+a}\,dx$$

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
[Out] integrate(1/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 62.36 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = 0$$

```
[In] integrate(1/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
[Out] 0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Hanged}$$

```
[In] int(1/(a + c*sinh(x)^2 + b*sinh(x)),x)
[Out] \text{Hanged}
```

3.827 $\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 280

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

```
[Out] arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)
*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I-b/(4*a*c-b^2)^(1/2))/(b^2-2*(a-
c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*c-
b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
)*2^(1/2)*(I+b/(4*a*c-b^2)^(1/2))/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used

$= \{3337, 2739, 632, 210\}$

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(\frac{b}{\sqrt{4ac-b^2}}+i\right)\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}} + \frac{\sqrt{2}\left(-\frac{b}{\sqrt{4ac-b^2}}+i\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[In] Int[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(I + b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[
-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 +
4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] + (Sqrt[2]*(I -
 b/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/
2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 -
 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3337

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rubi steps

integral

$$= -\left(i\int\left(\frac{1+\frac{ib}{\sqrt{-b^2+4ac}}}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}+\frac{1-\frac{ib}{\sqrt{-b^2+4ac}}}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\right)dx\right)$$

$$= -\left(\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx\right) - \left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\int\frac{1}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx$$

$$= -\left(\left(2\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\right) - \left(2\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib+\sqrt{-b^2+4ac}-4icx-\left(-ib+\sqrt{-b^2+4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)$$

$$= \left(4\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx,x,-4ic+2\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right) + \left(4\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2}\,dx,x,-4ic+2\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{\sqrt{2}\left(i+\frac{b}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(i-\frac{b}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(\frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}+\frac{\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*
Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 +
 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((b + Sqrt[b^2 - 4*a*c])*ArcTan[(2*c
- (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*S
qrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/Sqrt[
b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.25

method	result
default	$2\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\_R\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}\right)$
risch	$\sum_{\_R=\operatorname{RootOf}\left(\left(16a^4c^2-8a^3b^2c-32a^3c^3+a^2b^4+32a^2b^2c^2+16a^2c^4-10a\,b^4c-8a\,b^2c^3+b^6+b^4c^2\right)\_Z^4+\left(8c\,a^3-2a^2b^2-8a^2c^2+6a\,b^2c-b\right.\right.}$

```
[In] int(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_R),_R=RootOf(a
*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3309 vs. $2(224) = 448$.

Time = 0.36 (sec) , antiderivative size = 3309, normalized size of antiderivative = 11.82

$$\int \frac{\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2
+ b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4
*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3
+ (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c
*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c + (a^3*
b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (
4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5
 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c + (a^
2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2
/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*
b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)
) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2
*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c + (a^2*b^2 +
 b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b
^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^
3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(
a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4
*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 + 4*a*b*c^2 - (
4*a^2*b + b^3)*c + (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*
a^3*b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2
*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*
(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2
*a^2 + b^2 - 2*a*c + (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^
3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b
^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2
 - 2*(2*a^3 + 3*a*b^2)*c)) + 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2
)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((2*a^2 +
```

```
b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a
*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4
 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a
^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*
a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 + sqrt(2
)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c - (a^3*b^3 + a*b^5 - 4*a*b*c^4 + (
4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (4*a^4*b + 5*a^2*b^3 - b^5)*
c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*
(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2
 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*
a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6
 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*
c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) - 2*(a^3*b^2 + a*b^4 - 4*
a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2
+ 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 +
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c))) + 1/
2*sqrt(2)*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b
^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*
c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 + b^4 - 4*a*c^3 + (
8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*si
nh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 + 4*a*b*c^2 - (4*a^2*b + b^3)*c - (a^3*b^3
 + a*b^5 - 4*a*b*c^4 + (4*a^2*b + b^3)*c^3 + (4*a^3*b - 5*a*b^3)*c^2 - (4*a
^4*b + 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 +
(16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*
c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 + b^2 - 2*a*c - (a^2*b
^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)*sqrt(b^2/(a
^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 - 12*(2*a^3 + a*b^2
)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c)
))/(a^2*b^2 + b^4 - 4*a*c^3 + (8*a^2 + b^2)*c^2 - 2*(2*a^3 + 3*a*b^2)*c)) -
 2*(a^3*b^2 + a*b^4 - 4*a^2*c^3 + (8*a^3 + a*b^2)*c^2 - 2*(2*a^4 + 3*a^2*b^
2)*c)*sqrt(b^2/(a^4*b^2 + 2*a^2*b^4 + b^6 - 4*a*c^5 + (16*a^2 + b^2)*c^4 -
12*(2*a^3 + a*b^2)*c^3 + 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 + 3*a^3*
b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{\sinh\left(x\right)}{c\sinh\left(x\right)^2+b\sinh\left(x\right)+a}\,dx$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
[Out] integrate(sinh(x)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Hanged}$$

```
[In] int(sinh(x)/(a + c*sinh(x)^2 + b*sinh(x)),x)
[Out] \text{Hanged}
```

3.828 $\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 309

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{x}{c} - \frac{\sqrt{2}\left(ib+\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} - \frac{\sqrt{2}\left(ib-\frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

```
[Out] x/c-arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(
a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I*b+(2*a*c-b^2)/(4*a*c-b^2)^(
1/2))/c/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)-arctan(1/2*(2*I*c-(I*b-
(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2
))^(1/2))*2^(1/2)*(I*b+(-2*a*c+b^2)/(4*a*c-b^2)^(1/2))/c/(b^2-2*(a-c)*c+I*b
*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

$= \{3337, 3373, 2739, 632, 210\}$

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(-\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}} - \frac{\sqrt{2}\left(-\frac{b^2-2ac}{\sqrt{4ac-b^2}}+ib\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}} + \frac{x}{c}$$

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] x/c - (Sqrt[2]*(I*b + (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (
I*b - Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*
Sqrt[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])
 - (Sqrt[2]*(I*b - (b^2 - 2*a*c)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b
 + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqr
t[-b^2 + 4*a*c]])])/(c*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3337

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
```

```
ersQ[m, n, p]
```

Rule 3373

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\int \left(-\frac{1}{c} + \frac{-a - b\sinh(x)}{c\left(-a - b\sinh(x) - c\sinh^2(x)\right)}\right) dx$$

$$= \frac{x}{c} - \frac{\int \frac{-a-b\sinh(x)}{-a-b\sinh(x)-c\sinh^2(x)}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(ib - \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{ib+\sqrt{-b^2+4ac}+2ic\sinh(x)}\,dx}{c} - \frac{\left(ib + \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\int \frac{1}{ib-\sqrt{-b^2+4ac}+2ic\sinh(x)}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(2\left(ib - \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{ib+\sqrt{-b^2+4ac}+4icx-\left(ib+\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c}$$
$$- \frac{\left(2\left(ib + \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{ib-\sqrt{-b^2+4ac}+4icx-\left(ib-\sqrt{-b^2+4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c}$$

$$= \frac{x}{c}$$
$$+ \frac{\left(4\left(ib - \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, 4ic + 2\left(-ib - \sqrt{-b^2+4ac}\right)\text{tan}\right.}{c}$$
$$+ \frac{\left(4\left(ib + \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2}\,dx, x, 4ic + 2\left(-ib + \sqrt{-b^2+4ac}\right)\text{tan}\right.}{c}$$

$$= \frac{x}{c} - \frac{\sqrt{2}\left(ib + \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}$$
$$- \frac{\sqrt{2}\left(ib - \frac{b^2-2ac}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{c\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{x-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}}{c}$$

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (x - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (-b + Sqrt
[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]]) - (S
qrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*
a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])
/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]))/c
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.35

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(\_R^2a-2\_Rb-a\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}}{c}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{c}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right.}{c}$
risch	Expression too large to display

```
[In] int(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*sum((_R^2*a-2*_R*b-a)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x
)-_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))-1/c*ln(tanh(1/2*
x)-1)+1/c*ln(1+tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4943 vs. $2(253) = 506$.

Time = 0.60 (sec) , antiderivative size = 4943, normalized size of antiderivative = 16.00

$$\int \frac{\sinh^2(x)}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*
a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*
sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(
4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b
^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 +
b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b
^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*a^2*b
^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c + (8*a^2*c^7 - 6*(
4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2
 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2
*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^
2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5
 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2
*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4
 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^
4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*
c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 +
3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8
*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^
3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)
*c)*sinh(x) - 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*
c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2
*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a
*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4
)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2 + b^4
 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^
3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 +
4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(
2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2
+ 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2
)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*
a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^
2 + (a^2*b^4 + b^6)*c + (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a
^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^
2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3
```

```
*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2
*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*
b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2
*a*b^2)*c + (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b
^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^
2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8
*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2
 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^
2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh
(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) - 2*(4*a^3*c^5 - (8*a^4
 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt
(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*
c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 +
 b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)
*c^4))) + sqrt(2)*c*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c
 - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*
c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)
*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*
a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b
^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (
a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2 - 2*a^2*b^4 + 4*a^3*b^2*c + sqrt(2)*(8*
a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^4)*c^2 + (a^2*b^4 + b^6)*c - (8*a^2*c^7
 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 + 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^
3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 +
2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2
 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4
*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(
-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2
)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a
^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 +
b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a
^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5
 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*
(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2
*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 - (8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*
b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^
2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^
3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*
a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4))) - sqrt(2)*c*sqrt(-(a^2*b^2
+ b^4 + 2*a^2*c^2 - 2*(a^3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*
(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b
^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 +
 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3
*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2
+ b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2))*log(-2*a^4*b^2
```

```
 - 2*a^2*b^4 + 4*a^3*b^2*c - sqrt(2)*(8*a^2*b^2*c^3 - 2*(2*a^3*b^2 + 3*a*b^
4)*c^2 + (a^2*b^4 + b^6)*c - (8*a^2*c^7 - 6*(4*a^3 + a*b^2)*c^6 + (24*a^4 +
 22*a^2*b^2 + b^4)*c^5 - 2*(4*a^5 + 9*a^3*b^2 + 4*a*b^4)*c^4 + (2*a^4*b^2 +
 3*a^2*b^4 + b^6)*c^3)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4
*(a^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^
7 - 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 -
(a^4*b^2 + 2*a^2*b^4 + b^6)*c^4)))*sqrt(-(a^2*b^2 + b^4 + 2*a^2*c^2 - 2*(a^
3 + 2*a*b^2)*c - (4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3*a*b^2)*c^3 - (
a^2*b^2 + b^4)*c^2)*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a
^3*b^2 + a*b^4)*c)/(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 -
 2*(8*a^4 + 11*a^2*b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^
4*b^2 + 2*a^2*b^4 + b^6)*c^4)))/(4*a*c^5 - (8*a^2 + b^2)*c^4 + 2*(2*a^3 + 3
*a*b^2)*c^3 - (a^2*b^2 + b^4)*c^2)) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)
*cosh(x) + 4*(2*a^3*b*c^2 - (a^4*b + a^2*b^3)*c)*sinh(x) + 2*(4*a^3*c^5 - (
8*a^4 + a^2*b^2)*c^4 + 2*(2*a^5 + 3*a^3*b^2)*c^3 - (a^4*b^2 + a^2*b^4)*c^2)
*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6 + 4*a^2*b^2*c^2 - 4*(a^3*b^2 + a*b^4)*c)/
(4*a*c^9 - (16*a^2 + b^2)*c^8 + 12*(2*a^3 + a*b^2)*c^7 - 2*(8*a^4 + 11*a^2*
b^2 + b^4)*c^6 + 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^4*b^2 + 2*a^2*b^4 +
 b^6)*c^4))) + 2*x)/c
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(x)}{a + b\sinh(x) + c\sinh^2(x)}\,dx = \int \frac{\sinh\left(x\right)^2}{c\sinh\left(x\right)^2 + b\sinh\left(x\right) + a}\,dx$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) - b*e^x)/(c^2*e^(4*x) + 2*b*
c*e^(3*x) - 2*b*c*e^x + c^2 + 2*(2*a*c - c^2)*e^(2*x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\frac{x}{c}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
[Out] x/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx=\text{Hanged}$$

```
[In] int(sinh(x)^2/(a + c*sinh(x)^2 + b*sinh(x)),x)
[Out] \text{Hanged}
```

3.829 $\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 363

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{bx}{c^2}+\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}}+i\left(b^2-ac+\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} - \frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}}-i\left(b^2-ac-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}+\frac{\cosh(x)}{c}$$

```
[Out] -b*x/c^2+cosh(x)/c-arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2
^(1/2)/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(-I*(b^2-a*c-3*
I*a*b*c/(4*a*c-b^2)^(1/2))+b^3/(4*a*c-b^2)^(1/2))/c^2/(b^2-2*(a-c)*c-I*b*(4
*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1/2*x)+(4*a*c-b^2)^(1/2)*
tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(
I*(b^2-a*c+3*I*a*b*c/(4*a*c-b^2)^(1/2))+b^3/(4*a*c-b^2)^(1/2))/c^2/(b^2-2*(
a-c)*c+I*b*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.24 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3337, 2718, 3373, 2739, 632, 210}

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$$
$$=\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}+i\left(\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$
$$-\frac{\sqrt{2}\left(\frac{b^3}{\sqrt{4ac-b^2}}-i\left(-\frac{3iabc}{\sqrt{4ac-b^2}}-ac+b^2\right)\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{c^2\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$
$$-\frac{bx}{c^2}+\frac{\cosh(x)}{c}$$

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] -((b*x)/c^2) + (Sqrt[2]*(b^3/Sqrt[-b^2 + 4*a*c] + I*(b^2 - a*c + ((3*I)*a*b
*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[((2*I)*c - I*b*Tanh[x/2] + Sqrt[-b^2 + 4*a*
c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]])])/
(c^2*Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]]) - (Sqrt[2]*(b^3/Sqrt
[-b^2 + 4*a*c] - I*(b^2 - a*c - ((3*I)*a*b*c)/Sqrt[-b^2 + 4*a*c]))*ArcTan[(
(2*I)*c - (I*b + Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a -
c)*c - I*b*Sqrt[-b^2 + 4*a*c]])])/(c^2*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b
^2 + 4*a*c]]) + Cosh[x]/c
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3337

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3373

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = i\int\left(\frac{ib}{c^2}-\frac{i\sinh(x)}{c}+\frac{-iab-ib^2\left(1-\frac{ac}{b^2}\right)\sinh(x)}{c^2\left(a+b\sinh(x)+c\sinh^2(x)\right)}\right)dx$$

$$=-\frac{bx}{c^2}+\frac{i\int\frac{-iab-ib^2\left(1-\frac{ac}{b^2}\right)\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx}{c^2}+\frac{\int\sinh(x)\,dx}{c}$$

$$=-\frac{bx}{c^2}+\frac{\cosh(x)}{c}-\frac{\left(i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{c^2}$$
$$+\frac{\left(i\left(-b^2+ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\int\frac{1}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx}{c^2}$$

$$=-\frac{bx}{c^2}+\frac{\cosh(x)}{c}$$
$$-\frac{\left(2i\left(b^2-ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)x^2}\,dx,x,\tanh\left(\right.\right.}{c^2}$$
$$+\frac{\left(2i\left(-b^2+ac+\frac{ib^3}{\sqrt{-b^2+4ac}}-\frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib+\sqrt{-b^2+4ac}-4icx-\left(-ib+\sqrt{-b^2+4ac}\right)x^2}\,dx,x,\tanh\right.}{c^2}$$

$$= -\frac{bx}{c^2} + \frac{\cosh(x)}{c} + \frac{\left(4i\left(b^2 - ac + \frac{ib^3}{\sqrt{-b^2+4ac}} - \frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2} dx, x, -4ic + 2(ib +\right.}{c^2} - \frac{\left(4i\left(-b^2 + ac + \frac{ib^3}{\sqrt{-b^2+4ac}} - \frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2} dx, x, -4ic + 2(ib\right.}{c^2}$$

$$= -\frac{bx}{c^2} + \frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}} + i\left(b^2 - ac + \frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right) \arctan\left(\frac{2ic - \left(ib - \sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2 - 2(a-c)c + ib\sqrt{-b^2+4ac}}} - \frac{\sqrt{2}\left(\frac{b^3}{\sqrt{-b^2+4ac}} - i\left(b^2 - ac - \frac{3iabc}{\sqrt{-b^2+4ac}}\right)\right) \arctan\left(\frac{2ic - \left(ib + \sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{c^2\sqrt{b^2 - 2(a-c)c - ib\sqrt{-b^2+4ac}}} + \frac{\cosh(x)}{c}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^3(x)}{a + b\sinh(x) + c\sinh^2(x)} dx$$

$$= \frac{-bx + \frac{\sqrt{2}\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\right.}{\sqrt{b^2-4ac}\sqrt{-b^2+2(a-c)c-b}}}{c^2}$$

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (-(b*x) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 -
 4*a*c])*ArcTan[(2*c + (-b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*
(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*(a -
c)*c + b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2
])/(Sqrt[2]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/(Sqrt[b^2 - 4
*a*c]*Sqrt[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]]) + c*Cosh[x])/c^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.40

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)} \frac{\left(-ab\_R^2+2\left(-ac+b^2\right)\_R+ab\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3a-3\_R^2b-2\_Ra+4\_Rc+b}}{c^2}-\frac{1}{c\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b}{\cdots}$
risch	Expression too large to display

```
[In] int(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*sum((-a*b*_R^2+2*(-a*c+b^2)*_R+a*b)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+
b)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))-
1/c/(tanh(1/2*x)-1)+b/c^2*ln(tanh(1/2*x)-1)+1/c/(1+tanh(1/2*x))-b/c^2*ln(1+
tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6680 vs. 2(297) = 594.

Time = 1.11 (sec) , antiderivative size = 6680, normalized size of antiderivative = 18.40

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)+c*sinh(x)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \int \frac{\sinh(x)^3}{c\sinh(x)^2+b\sinh(x)+a}\,dx$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*e^x - c*e^(2*x) - c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(2
*x) + (b^2 - a*c)*e^(3*x) - (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*x)
 - 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 - c^3)*e^(2*x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = -\frac{bx}{c^2}+\frac{e^{(-x)}}{2\,c}+\frac{e^x}{2\,c}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/c^2 + 1/2*e^(-x)/c + 1/2*e^x/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Hanged}$$

```
[In] int(sinh(x)^3/(a + c*sinh(x)^2 + b*sinh(x)),x)
```

```
[Out] \text{Hanged}
```

3.830 $\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 12

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx=\frac{\cosh(x)}{b-a\sinh(x)}$$

```
[Out] cosh(x)/(b-a*sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3369, 2833, 8}

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx=\frac{\cosh(x)}{b-a\sinh(x)}$$

```
[In] Int[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
```

```
[Out] Cosh[x]/(b - a*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3369

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(4a^2\right)\int \frac{a+b\sinh(x)}{(2iab-2ia^2\sinh(x))^2}\,dx\right)\\ &= \frac{\cosh(x)}{b-a\sinh(x)} - \frac{\int 0\,dx}{a^2+b^2}\\ &= \frac{\cosh(x)}{b-a\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx = -\frac{\cosh(x)}{-b+a\sinh(x)}$$

```
[In] Integrate[(a + b*Sinh[x])/(b^2 - 2*a*b*Sinh[x] + a^2*Sinh[x]^2),x]
[Out] -(Cosh[x]/(-b + a*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

method	result	size
parallelrisch	$\frac{a\sinh(x)-b\cosh(x)-b}{(a\sinh(x)-b)b}$	28
risch	$-\frac{2(e^{x}b+a)}{a(e^{2x}a-2\,e^{x}b-a)}$	29
default	$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{2b}+\frac{1}{2}\right)}{\frac{\tanh\left(\frac{x}{2}\right)^{2}b}{2}+a\tanh\left(\frac{x}{2}\right)-\frac{b}{2}}$	36

```
[In] int((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x,method=_RETURNVERBOSE
)
```

```
[Out] (a*sinh(x)-b*cosh(x)-b)/(a*sinh(x)-b)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.75

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx = -\frac{2\,(b\cosh(x)+b\sinh(x)+a)}{a^2\cosh(x)^2+a^2\sinh(x)^2-2\,ab\cosh(x)-a^2+2\,(a^2\cosh(x)-ab)\sinh(x)}$$

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="fri
cas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 - 2*a*b*cosh(
x) - a^2 + 2*(a^2*cosh(x) - a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*sinh(x))/(b**2-2*a*b*sinh(x)+a**2*sinh(x)**2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 18.75

$$\begin{aligned}\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx &= b\left(\frac{a\log\left(\frac{ae^{(-x)}+b-\sqrt{a^2+b^2}}{ae^{(-x)}+b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,(b^2e^{(-x)}-ab)}{a^4+a^2b^2-2\,(a^3b+ab^3)e^{(-x)}-(a^4+a^2b^2)e^{(-2\,x)}}\right)\\ &\quad -a\left(\frac{b\log\left(\frac{ae^{(-x)}+b-\sqrt{a^2+b^2}}{ae^{(-x)}+b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,(be^{(-x)}-a)}{a^3+ab^2-2\,(a^2b+b^3)e^{(-x)}-(a^3+ab^2)e^{(-2\,x)}}\right)\end{aligned}$$

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="max
ima")
```

```
[Out] b*(a*log((a*e^(-x) + b - sqrt(a^2 + b^2))/(a*e^(-x) + b + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) - a*b)/(a^4 + a^2*b^2 - 2*(a^3*b + a*b^3
)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) - a*(b*log((a*e^(-x) + b - sqrt(a^2 +
 b^2))/(a*e^(-x) + b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) -
a)/(a^3 + a*b^2 - 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx=-\frac{2\,(be^x+a)}{(ae^{(2\,x)}-2\,be^x-a)a}$$

```
[In] integrate((a+b*sinh(x))/(b^2-2*a*b*sinh(x)+a^2*sinh(x)^2),x, algorithm="gia
c")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) - 2*b*e^x - a)*a)
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \frac{a+b\sinh(x)}{b^2-2ab\sinh(x)+a^2\sinh^2(x)}\,dx=\frac{\frac{2\,\mathrm{e}^x\,\left(a^3\,b+a\,b^3\right)}{a\,\left(a^3+a\,b^2\right)}+2}{a+2\,b\,\mathrm{e}^x-a\,\mathrm{e}^{2\,x}}$$

```
[In] int((a + b*sinh(x))/(a^2*sinh(x)^2 + b^2 - 2*a*b*sinh(x)),x)
```

```
[Out] ((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) + 2)/(a + 2*b*exp(x) - a*exp(
2*x))
```

3.831 $\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 300

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-ib\tanh\left(\frac{x}{2}\right)+\sqrt{-b^2+4ac}\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

```
[Out] arctan(1/2*(2*I*c-(I*b+(4*a*c-b^2)^(1/2))*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)
*c-I*b*(4*a*c-b^2)^(1/2))^(1/2))*2^(1/2)*(I*e+(-b*e+2*c*d)/(4*a*c-b^2)^(1/2
))/(b^2-2*(a-c)*c-I*b*(4*a*c-b^2)^(1/2))^(1/2)+arctan(1/2*(2*I*c-I*b*tanh(1
/2*x)+(4*a*c-b^2)^(1/2)*tanh(1/2*x))*2^(1/2)/(b^2-2*(a-c)*c+I*b*(4*a*c-b^2)
^(1/2))^(1/2))*2^(1/2)*(I*e+(b*e-2*c*d)/(4*a*c-b^2)^(1/2))/(b^2-2*(a-c)*c+I
*b*(4*a*c-b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{3373, 2739, 632, 210\}$

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(-\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{4ac-b^2}-ib\tanh\left(\frac{x}{2}\right)+2ic}{\sqrt{2}\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{ib\sqrt{4ac-b^2}-2c(a-c)+b^2}} + \frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{4ac-b^2}}+ie\right)\arctan\left(\frac{2ic-\tanh\left(\frac{x}{2}\right)\left(\sqrt{4ac-b^2}+ib\right)}{\sqrt{2}\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}\right)}{\sqrt{-ib\sqrt{4ac-b^2}-2c(a-c)+b^2}}$$

```
[In] Int[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(I*e - (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - I*b*Tan
h[x/2] + Sqrt[-b^2 + 4*a*c]*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c + I*
b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c + I*b*Sqrt[-b^2 + 4*a*c]] +
 (Sqrt[2]*(I*e + (2*c*d - b*e)/Sqrt[-b^2 + 4*a*c])*ArcTan[((2*I)*c - (I*b +
 Sqrt[-b^2 + 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[
-b^2 + 4*a*c]])])/Sqrt[b^2 - 2*(a - c)*c - I*b*Sqrt[-b^2 + 4*a*c]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3373

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \left(-ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\int\frac{1}{-ib-\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx + \left(-ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\int\frac{1}{-ib+\sqrt{-b^2+4ac}-2ic\sinh(x)}\,dx$$

$$= -\left(\left(2\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib+\sqrt{-b^2+4ac}-4icx-\left(-ib+\sqrt{-b^2+4ac}\right)x^2}\,dx,\right.\right.$$
$$-\left(2\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-ib-\sqrt{-b^2+4ac}-4icx-\left(-ib-\sqrt{-b^2+4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right.\right.$$

$$= \left(4\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c+ib\sqrt{-b^2+4ac}\right)-x^2}\,dx,x,-4ic+2\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right) + \left(4\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-8\left(b^2-2(a-c)c-ib\sqrt{-b^2+4ac}\right)-x^2}\,dx,x,-4ic+2\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{\sqrt{2}\left(ie-\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib-\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c+ib\sqrt{-b^2+4ac}}} + \frac{\sqrt{2}\left(ie+\frac{2cd-be}{\sqrt{-b^2+4ac}}\right)\arctan\left(\frac{2ic-\left(ib+\sqrt{-b^2+4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2(a-c)c-ib\sqrt{-b^2+4ac}}}$$

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.86

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \frac{\sqrt{2}\left(\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{2c+\left(-b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4(a-c)c+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c+b\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{2c-\left(b+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2(a-c)c-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[(d + e*Sinh[x])/(a + b*Sinh[x] + c*Sinh[x]^2),x]
```

```
[Out] (Sqrt[2]*(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2*c + (-b + Sqrt[b^
2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*(a - c)*c + 2*b*Sqrt[b^2 - 4*a*c]]])
/Sqrt[-b^2 + 2*(a - c)*c + b*Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
- 4*a*c])*e)*ArcTan[(2*c - (b + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/(Sqrt[2]*Sqrt
[-b^2 + 2*(a - c)*c - b*Sqrt[b^2 - 4*a*c]])])/Sqrt[-b^2 + 2*(a - c)*c - b*S
qrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.26

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-2b\_Z^3+(-2a+4c)\_Z^2+2b\_Z+a\right)}\frac{\left(-\_R^2 d+2\_Re+d\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3 a-3\_R^2 b-2\_Ra+4\_Rc+b}$	79
risch	Expression too large to display	8284

```
[In] int((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] sum((-_R^2*d+2*_R*e+d)/(2*_R^3*a-3*_R^2*b-2*_R*a+4*_R*c+b)*ln(tanh(1/2*x)-_
R),_R=RootOf(a*_Z^4-2*b*_Z^3+(-2*a+4*c)*_Z^2+2*b*_Z+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6841 vs. $2(244) = 488$.

Time = 2.97 (sec) , antiderivative size = 6841, normalized size of antiderivative = 22.80

$$\int \frac{d + e\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\, dx = \text{Too large to display}$$

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + e\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\, dx = \text{Timed out}$$

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d + e\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\, dx = \int \frac{e\sinh(x) + d}{c\sinh(x)^2 + b\sinh(x) + a}\, dx$$

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="maxima")
[Out] integrate((e*sinh(x) + d)/(c*sinh(x)^2 + b*sinh(x) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 1.55 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{d + e\sinh(x)}{a + b\sinh(x) + c\sinh^2(x)}\, dx = 0$$

```
[In] integrate((d+e*sinh(x))/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")
[Out] 0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\sinh(x)}{a+b\sinh(x)+c\sinh^2(x)}\,dx = \text{Hanged}$$

```
[In] int((d + e*sinh(x))/(a + c*sinh(x)^2 + b*sinh(x)),x)
[Out] \text{Hanged}
```

3.832 $\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 223

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{4c\,\text{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{4c\,\text{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] 4*c*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2
)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-
(-4*a*c+b^2)^(1/2))^(1/2)-4*c*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tanh
(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b-2*c+(-4*a*c
+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3330, 2738, 214}

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{4c\,\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{4c\,\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[In] Int[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

```
[Out] (4*c*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - S
qrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (4*c*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 -
4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3330

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b_.) + cos[(d_.) + (e_.)*(x_)]^
(n2_.)*(c_.))^(-1), x_Symbol] :> Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c
/q), Int[1/(b - q + 2*c*Cos[d + e*x]^n), x], x] - Dist[2*(c/q), Int[1/(b +
q + 2*c*Cos[d + e*x]^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{(4c)\text{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{b^2-4ac}} \\
&\quad - \frac{(4c)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{b^2-4ac}} \\
&= \frac{4c\,\text{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} \\
&\quad - \frac{4c\,\text{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\sqrt{2}c\left(\frac{\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[(a + b*Cosh[x] + c*Cosh[x]^2)^(-1),x]
```

```
[Out] (2*Sqrt[2]*c*(ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
+ 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^
2 - 4*a*c]] + ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2
 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]]/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b
^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.93

method	result
default	$2(a-b+c)\left(\frac{\left(-b+2c-\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(b-2c-\sqrt{-4ac+b^2}\right)\arctan\left(\frac{(a-b+c)}{\sqrt{\left(\sqrt{-4ac+b^2}\right.}}\right.}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a\right.}}\right.$
risch	$\sum_{\_R=\text{RootOf}\left(\left(16a^4c^2-8a^3b^2c+32a^3c^3+a^2b^4-32a^2b^2c^2+16a^2c^4+10a\,b^4c-8a\,b^2c^3-b^6+b^4c^2\right)\_Z^4+\left(-8a^2c^2+6a\,b^2c-8a\,c^3-b^4+2b\right.\right.}$

```
[In] int(1/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a-b+c)*(1/2*(-b+2*c-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4
*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+
b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*(b-2*c-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh
(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3485 vs. $2(183) = 366$.

Time = 0.38 (sec) , antiderivative size = 3485, normalized size of antiderivative = 15.63

$$\int \frac{1}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
 b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*
sinh(x) + 2*b^2*c + sqrt(2)*(b^4 - 4*a*b^2*c - (a^2*b^4 - b^6 + 8*a*c^5 + 2
*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4
)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*
a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2
 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c - 2*c^2
+ (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqr
t(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3
 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a
*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^
2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^
2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*
c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 -
3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^
2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*lo
g(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c -
(a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 +
 (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^
4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)
*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))
)*sqrt((b^2 - 2*a*c - 2*c^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (1
6*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^
2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 -
b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*
a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((b
```

```
^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
- 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) + 4*b*c^2*sinh(x) + 2*b^2*c +
 sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4
 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^
2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 -
4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*
a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(
8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2
 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(4*a*c^4
 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*sqrt(b^
2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a
*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4
)*c))) - 1/2*sqrt(2)*sqrt((b^2 - 2*a*c - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 -
 (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 -
4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*cosh(x) +
 4*b*c^2*sinh(x) + 2*b^2*c - sqrt(2)*(b^4 - 4*a*b^2*c + (a^2*b^4 - b^6 + 8*
a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^
2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 +
 b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 1
1*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((b^2 - 2*a*c
 - 2*c^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^
2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 -
12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*
b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)) - 2*(4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2
- (a^2*b^2 - b^4)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(x)+c*cosh(x)**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int \frac{1}{c\cosh{(x)}^2+b\cosh{(x)}+a}\,dx$$

[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")

[Out] integrate(1/(c*cosh(x)^2 + b*cosh(x) + a), x)

Giac [A] (verification not implemented)

none

Time = 61.08 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = 0$$

[In] integrate(1/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")

[Out] 0

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Hanged}$$

[In] int(1/(a + b*cosh(x) + c*cosh(x)^2),x)

[Out] \text{Hanged}

3.833 $\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 230

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] 2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3338, 2738, 214}

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[In] Int[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tan
h[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c + Sqrt[b^2 -
4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3338

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{b-\sqrt{b^2-4ac}+2c\cosh(x)}+\frac{1+\frac{b}{\sqrt{b^2-4ac}}}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\right)dx\\
&= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx\\
&\quad+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx
\end{aligned}$$

$$= \left(2\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + \left(2\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)$$

$$= \frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{\sqrt{2}\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}} + \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((b + Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*
Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 +
 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[((-
b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqr
t[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2
 - 4*a*c]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

method	result
default	$2(a-b+c)\left(\frac{\left(\sqrt{-4ac+b^2}+2a-b\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(-2a+b+\sqrt{-4ac+b^2}\right)\arctan\left(\frac{(a-b+c)\,\mathrm{ta}}{\sqrt{\left(\sqrt{-4ac+b^2}-\right.}}\right.}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right.}}\right.$
risch	$\sum_{\_R=\mathrm{RootOf}\left((16a^4c^2-8a^3b^2c+32a^3c^3+a^2b^4-32a^2b^2c^2+16a^2c^4+10a\,b^4c-8a\,b^2c^3-b^6+b^4c^2)\_Z^4+(8c\,a^3-2a^2b^2+8a^2c^2-6a\,b^2c+b^4)\right.}$

```
[In] int(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a-b+c)*(1/2*((-4*a*c+b^2)^(1/2)+2*a-b)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*
a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b
^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*(-2*a+b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh
(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3505 vs. $2(189) = 378$.

Time = 0.38 (sec) , antiderivative size = 3505, normalized size of antiderivative = 15.24

$$\int \frac{\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2
- b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4
*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^
2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3
- (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c
*sinh(x) + 2*a*b^2 + sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c - (a^3*
b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (
4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5
 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^
4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c + (a^
2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2
/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*
b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)
*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)
```

) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2
*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a
^3*b^2 + 2*a*b^4)*c))) + 1/2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c + (a^2*b^2 -
 b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b
^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^
3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4
*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 - 4*a*b*c^2 - (
4*a^2*b - b^3)*c - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*
a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2
*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*
(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2
*a^2 - b^2 + 2*a*c + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^
3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b
^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^
5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2
 - 2*(2*a^3 - 3*a*b^2)*c)) - 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2
)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*
c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) - 1/2*sqrt(2)*sqrt((2*a^2 -
b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a
*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4
 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a
^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*
a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*sinh(x) + 2*a*b^2 + sqrt(2
)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c + (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (
4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*
c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*
(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2
 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*
a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6
 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^
2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*
c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(a^3*b^2 - a*b^4 - 4*
a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*sqrt(b^2/(a^4*b^2
 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 -
 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))) + 1/
2*sqrt(2)*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b
^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*
c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 +
 b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (
8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*a*b*c*cosh(x) + 4*a*b*c*si
nh(x) + 2*a*b^2 - sqrt(2)*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c + (a^3*b^3
 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a

```
^4*b - 5*a^2*b^3 - b^5)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 -
(16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*
c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))*sqrt((2*a^2 - b^2 + 2*a*c - (a^2*b
^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt(b^2/(a
^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2
)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)
))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) +
 2*(a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^
2)*c)*sqrt(b^2/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 -
12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*
b^2 + 2*a*b^4)*c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \text{Timed out}$$

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \int \frac{\cosh(x)}{c\cosh(x)^2 + b\cosh(x) + a}\, dx$$

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
[Out] integrate(cosh(x)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\, dx = \text{Timed out}$$

```
[In] integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Hanged}$$

```
[In] int(cosh(x)/(a + b*cosh(x) + c*cosh(x)^2),x)
[Out] \text{Hanged}
```

3.834 $\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 255

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{x}{c} - \frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b
^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(b-2*c-(-4*a*c+b^2)^
(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctanh((b-2*c+(-4*a*c+b^2)
^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))/c/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3338, 3374, 2738, 214}

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = -\frac{2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}} - \frac{2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}} + \frac{x}{c}$$

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] x/c - (2*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt
[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2
*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (2*(b + (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[
x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*
c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3338

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3374

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{c} + \frac{-a - b\cosh(x)}{c\left(a + b\cosh(x) + c\cosh^2(x)\right)}\right) dx$$

$$= \frac{x}{c} + \frac{\int \frac{-a-b\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\, dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\, dx}{c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\, dx}{c}$$

$$= \frac{x}{c} - \frac{\left(2\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2} dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c} - \frac{\left(2\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)x^2} dx, x, \tanh\left(\frac{x}{2}\right)\right)}{c}$$

$$= \frac{x}{c} - \frac{2\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{2\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)} dx = \frac{x + \frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}}{c}$$

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (x + (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*
a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/c
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{c}+\frac{2(a-b+c)\left(\frac{\left(a\sqrt{-4ac+b^2}-b\sqrt{-4ac+b^2}-ab-2ac+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(a\sqrt{-4ac+b^2}-b\sqrt{-\right.}{2\sqrt{}}$
risch	Expression too large to display

[In] int(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)

[Out] -1/c*ln(tanh(1/2*x)-1)+2/c*(a-b+c)*(1/2*(a*(-4*a*c+b^2)^(1/2)-b*(-4*a*c+b^2
)^(1/2)-a*b-2*a*c+b^2)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)
*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b
+c))^(1/2))+1/2*(a*(-4*a*c+b^2)^(1/2)-b*(-4*a*c+b^2)^(1/2)+a*b+2*a*c-b^2)/(
-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((
a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))+1/c*ln(1+tanh
(1/2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5079 vs. 2(215) = 430.

Time = 0.58 (sec) , antiderivative size = 5079, normalized size of antiderivative = 19.92

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Too large to display}$$

[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Timed out}$$

[In] integrate(cosh(x)**2/(a+b*cosh(x)+c*cosh(x)**2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int \frac{\cosh(x)^2}{c\cosh(x)^2+b\cosh(x)+a}\,dx$$

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/4*integrate(8*(b*e^(3*x) + 2*a*e^(2*x) + b*e^x)/(c^2*e^(4*x) + 2*b*
c*e^(3*x) + 2*b*c*e^x + c^2 + 2*(2*a*c + c^2)*e^(2*x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.02

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{x}{c}$$

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

```
[Out] x/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Hanged}$$

```
[In] int(cosh(x)^2/(a + b*cosh(x) + c*cosh(x)^2),x)
```

```
[Out] \text{Hanged}
```

3.835 $\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = -\frac{bx}{c^2}+\frac{2\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}+\frac{\sinh(x)}{c}$$

```
[Out] -b*x/c^2+sinh(x)/c+2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(
b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b^3/(-4*a*c+b^2)^(1/2)+3*a*b*c/(-
4*a*c+b^2)^(1/2))/c^2/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^
(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c+
(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b^3/(-4*a*c+b^2)^(1/2)-3*a*b*c/(-4*a*c+
b^2)^(1/2))/c^2/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A] (verified)

Time = 4.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3338, 2717, 3374, 2738, 214}

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(\frac{3abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}-ac+b^2\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{2\left(-\frac{3abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}-ac+b^2\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{bx}{c^2}+\frac{\sinh(x)}{c}$$

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] -((b*x)/c^2) + (2*(b^2 - a*c - b^3/Sqrt[b^2 - 4*a*c] + (3*a*b*c)/Sqrt[b^2 -
 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c
 - Sqrt[b^2 - 4*a*c]]) + (2*(b^2 - a*c + b^3/Sqrt[b^2 - 4*a*c] - (3*a*b*c)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(c^2*Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]) + Sinh[x]/c
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3338

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^(n_.)*(b
_.) + cos[(d_.) + (e_.)*(x_)]^(n2_.)*(c_.))^(p_), x_Symbol] :> Int[ExpandTr
```

```
ig[cos[d + e*x]^m*(a + b*cos[d + e*x]^n + c*cos[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 3374

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{b}{c^2}+\frac{\cosh(x)}{c}+\frac{ab+b^2\left(1-\frac{ac}{b^2}\right)\cosh(x)}{c^2\left(a+b\cosh(x)+c\cosh^2(x)\right)}\right)dx$$

$$= -\frac{bx}{c^2}+\frac{\int \frac{ab+b^2\left(1-\frac{ac}{b^2}\right)\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx}{c^2}+\frac{\int \cosh(x)\,dx}{c}$$

$$= -\frac{bx}{c^2}+\frac{\sinh(x)}{c}+\frac{\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c^2}+\frac{\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{\sinh(x)}{c}+\frac{\left(2\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c^2}+\frac{\left(2\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{c^2}$$

$$= -\frac{bx}{c^2}+\frac{2\left(b^2-ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{3abc}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(b^2-ac+\frac{b^3}{\sqrt{b^2-4ac}}-\frac{3abc}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}+\frac{\sinh(x)}{c}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$$
$$= \frac{-bx-\frac{\sqrt{2}\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\right.}{\sqrt{b^2-4ac}\sqrt{-b^2+2c(a+c)+b\sqrt{}}}}{c^2}$$

[In] Integrate[Cosh[x]^3/(a + b*Cosh[x] + c*Cosh[x]^2),x]

[Out] (-(b*x) - (Sqrt[2]*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]) + c*Sinh[x])/c^2

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.18

method	result
default	$-\frac{1}{c\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{b\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{c^2}-\frac{1}{c\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{c^2}+\frac{2(a-b+c)\left(\frac{\left(-ab\sqrt{-4ac+b^2}-ac\sqrt{-4ac+b^2}+b^2\sqrt{-}\right.}{2\sqrt{-4ac-}}\right.}{}$
risch	Expression too large to display

[In] int(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)

[Out] -1/c/(1+tanh(1/2*x))-b/c^2*ln(1+tanh(1/2*x))-1/c/(tanh(1/2*x)-1)+b/c^2*ln(tanh(1/2*x)-1)+2/c^2*(a-b+c)*(1/2*(-a*b*(-4*a*c+b^2)^(1/2)-a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)-2*a^2*c+a*b^2+3*b*c*a-b^3)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*(-a*b*(-4*a*c+b^2)^(1/2)-a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)+2*a^2*c-a*b^2-3*b*c*a+b^3)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6794 vs. 2(255) = 510.

Time = 1.08 (sec) , antiderivative size = 6794, normalized size of antiderivative = 22.72

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)+c*cosh(x)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \int \frac{\cosh(x)^3}{c\cosh(x)^2+b\cosh(x)+a}\,dx$$

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
[Out] -1/2*(2*b*x*e^x - c*e^(2*x) + c)*e^(-x)/c^2 - 1/8*integrate(-16*(2*a*b*e^(2
*x) + (b^2 - a*c)*e^(3*x) + (b^2 - a*c)*e^x)/(c^3*e^(4*x) + 2*b*c^2*e^(3*x)
 + 2*b*c^2*e^x + c^3 + 2*(2*a*c^2 + c^3)*e^(2*x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.08

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = -\frac{bx}{c^2}-\frac{e^{(-x)}}{2\,c}+\frac{e^x}{2\,c}$$

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
[Out] -b*x/c^2 - 1/2*e^(-x)/c + 1/2*e^x/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Hanged}$$

```
[In] int(cosh(x)^3/(a + b*cosh(x) + c*cosh(x)^2),x)
[Out] \text{Hanged}
```

3.836 $\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 11

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx=\frac{\sinh(x)}{b+a\cosh(x)}$$

```
[Out] sinh(x)/(b+a*cosh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3370, 2833, 8}

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx=\frac{\sinh(x)}{a\cosh(x)+b}$$

```
[In] Int[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3370

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (4a^2)\int \frac{a+b\cosh(x)}{(2ab+2a^2\cosh(x))^2}\,dx\\ &= \frac{\sinh(x)}{b+a\cosh(x)}+\frac{\int 0\,dx}{a^2-b^2}\\ &= \frac{\sinh(x)}{b+a\cosh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx=\frac{\sinh(x)}{b+a\cosh(x)}$$

```
[In] Integrate[(a + b*Cosh[x])/(b^2 + 2*a*b*Cosh[x] + a^2*Cosh[x]^2),x]
[Out] Sinh[x]/(b + a*Cosh[x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{\sinh(x)}{b+a\cosh(x)}$	12
risch	$-\frac{2(e^{x}b+a)}{a(e^{2x}a+2\,e^{x}b+a)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{2}b+a+b}$	29

```
[In] int((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x,method=_RETURNVERBOSE
)
[Out] sinh(x)/(b+a*cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = -\frac{2\,(b\cosh\left(x\right)+b\sinh\left(x\right)+a)}{a^2\cosh\left(x\right)^2+a^2\sinh\left(x\right)^2+2\,ab\cosh\left(x\right)+a^2+2\,(a^2\cosh\left(x\right)+ab)\sinh\left(x\right)}$$

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="fri
cas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cosh(x))/(b**2+2*a*b*cosh(x)+a**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = -\frac{2\,(be^x+a)}{(ae^{(2\,x)}+2\,be^x+a)a}$$

```
[In] integrate((a+b*cosh(x))/(b^2+2*a*b*cosh(x)+a^2*cosh(x)^2),x, algorithm="gia
c")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.64

$$\int \frac{a+b\cosh(x)}{b^2+2ab\cosh(x)+a^2\cosh^2(x)}\,dx = -\frac{\frac{2\,\mathrm{e}^x\,(a\,b^3-a^3\,b)}{a\,(a\,b^2-a^3)}+2}{a+2\,b\,\mathrm{e}^x+a\,\mathrm{e}^{2\,x}}$$

```
[In] int((a + b*cosh(x))/(a^2*cosh(x)^2 + b^2 + 2*a*b*cosh(x)),x)
```

```
[Out] -((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*exp
(2*x))
```

3.837 $\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 246

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

```
[Out] 2*arctanh((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tanh(1/2*x)/(b+2*c-(-4*a*c+b^2)^
(1/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2)
)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctanh((b-2*c+(-4*a*c+b^2)^(1/2
))^(1/2)*tanh(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a
*c+b^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3374, 2738, 214}

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[In] Int[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c - Sqrt[b^2 -
 4*a*c]]*Tanh[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqr
t[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tanh[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3374

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{b+\sqrt{b^2-4ac}+2c\cosh(x)}\,dx\\ &\quad+\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{b-\sqrt{b^2-4ac}+2c\cosh(x)}\,dx\\ &= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b+2c+\sqrt{b^2-4ac}-\left(b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\ &\quad+\left(2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{b+2c-\sqrt{b^2-4ac}-\left(b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\end{aligned}$$

$$= \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx = \frac{\sqrt{2}\left(-\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}} + \frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}}$$

```
[In] Integrate[(d + e*Cosh[x])/(a + b*Cosh[x] + c*Cosh[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[((b - 2*c + Sqrt[b
^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*ArcTan[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tanh[x/2])/Sqrt[-2*b^
2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt
[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

method	result
default	$2(a-b+c)\left(\frac{\left(-d\sqrt{-4ac+b^2}+e\sqrt{-4ac+b^2}-2ae+bd+be-2cd\right)\arctan\left(\frac{(a-b+c)\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right)(a-b+c)}}+\frac{\left(-d\sqrt{-4ac+b^2}+e\right.}{}\right.$
risch	Expression too large to display

```
[In] int((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a-b+c)*(1/2*(-d*(-4*a*c+b^2)^(1/2)+e*(-4*a*c+b^2)^(1/2)-2*a*e+b*d+b*e-2*
c*d)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*ar
ctan((a-b+c)*tanh(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+1/2*(-d*
```

```
(-4*a*c+b^2)^(1/2)+e*(-4*a*c+b^2)^(1/2)+2*a*e-b*d-b*e+2*c*d)/(-4*a*c+b^2)^(
1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tanh
(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6997 vs. $2(206) = 412$.

Time = 3.06 (sec) , antiderivative size = 6997, normalized size of antiderivative = 28.44

$$\int \frac{d + e\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + e\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)**2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d + e\cosh(x)}{a + b\cosh(x) + c\cosh^2(x)}\,dx = \int \frac{e\cosh\left(x\right) + d}{c\cosh\left(x\right)^2 + b\cosh\left(x\right) + a}\,dx$$

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="maxima")
[Out] integrate((e*cosh(x) + d)/(c*cosh(x)^2 + b*cosh(x) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 1.56 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx=0$$

```
[In] integrate((d+e*cosh(x))/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
[Out] 0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\cosh(x)}{a+b\cosh(x)+c\cosh^2(x)}\,dx=\text{Hanged}$$

```
[In] int((d + e*cosh(x))/(a + b*cosh(x) + c*cosh(x)^2),x)
[Out] \text{Hanged}
```

3.838 $\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

```
[Out] x/(a+b)-arctan(b^(1/2)*tanh(x)/a^(1/2))*a^(1/2)/(a+b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {492, 213, 211}

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}$$

```
[In] Int[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[b]*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2}{(-1+x^2)\,(a+bx^2)}\,dx, x, \tanh(x)\right) \\
&= -\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)}{a+b} - \frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(x)\right)}{a+b} \\
&= \frac{x}{a+b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$
\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \frac{x-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a+b}
$$

```
[In] Integrate[Sinh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[b])/(a + b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(31) = 62.

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.36

method	result
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2x}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2b(a+b)}-\frac{\sqrt{-ab}\,\ln\left(e^{2x}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2b(a+b)}$
default	$\frac{8a^2\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{4a+4b}+\frac{8\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{8a+8b}-$

```
[In] int(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/(a+b)+1/2/b*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)-(-a+2*(-a*b)^(1/2)+b)/(a+b))-1
/2/b*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)+(a+2*(-a*b)^(1/2)-b)/(a+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 9.41

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$$
$$=\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{\left(a^2+2\,ab+b^2\right)\cosh(x)^4+4\left(a^2+2\,ab+b^2\right)\cosh(x)\sinh(x)^3+\left(a^2+2\,ab+b^2\right)\sinh(x)^4+2\left(a^2-b^2\right)\cosh(x)^2+2\left(3\left(a^2+2\,ab+b^2\right)\cosh\right.}{(a+b)\cosh(x)^4+4\,(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x)^4}\right.}{}\right.$$
$$\left.-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{\left((a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b\right)\sqrt{\frac{a}{b}}}{2\,a}\right)-x}{a+b}\right]$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)
*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^
2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*a*b
 + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x) -
4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh
(x)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(
x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a
 - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))
 + 2*x)/(a + b), -(sqrt(a/b)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh
(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(a/b)/a) - x)/(a + b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(32) = 64$.

Time = 0.59 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.18

$$\int \frac{\sinh^2(x)}{a\cosh^2(x) + b\sinh^2(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ -\frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} + \frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} - \frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a = -b \\ \frac{x-\frac{\sinh(x)}{\cosh(x)}}{a} & \text{for } b = 0 \\ \frac{2x\sqrt{-\frac{b}{a}}}{2a\sqrt{-\frac{b}{a}}+2b\sqrt{-\frac{b}{a}}} + \frac{\log\left(-\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a\sqrt{-\frac{b}{a}}+2b\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a\sqrt{-\frac{b}{a}}+2b\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (-x*sinh(x)**2/(-2
*b*sinh(x)**2 + 2*b*cosh(x)**2) + x*cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(
x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2), Eq(a, -b)), ((
x - sinh(x)/cosh(x))/a, Eq(b, 0)), (2*x*sqrt(-b/a)/(2*a*sqrt(-b/a) + 2*b*sq
rt(-b/a)) + log(-sqrt(-b/a)*sinh(x) + cosh(x))/(2*a*sqrt(-b/a) + 2*b*sqrt(-
b/a)) - log(sqrt(-b/a)*sinh(x) + cosh(x))/(2*a*sqrt(-b/a) + 2*b*sqrt(-b/a))
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^2(x)}{a\cosh^2(x) + b\sinh^2(x)}\,dx = -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)} + \frac{\arctan\left(\frac{(a+b)e^{(-2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{x}{a+b}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a - b)*arctan(1/2*((a + b)*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
 + b)) + 1/2*arctan(1/2*((a + b)*e^(-2*x) + a - b)/sqrt(a*b))/sqrt(a*b) + x/
(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=-\frac{a\arctan\left(\frac{ae^{(2\,x)}+be^{(2\,x)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{x}{a+b}$$

```
[In] integrate(sinh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)
) + x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.36

$$\int \frac{\sinh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{2\,x}\,\left(\frac{4\,a}{{\left(a+b\right)}^4}+\frac{\left(a^2-b^2\right)\,\left(a-b\right)}{{\left(a+b\right)}^3\,\sqrt{b\,{\left(a+b\right)}^2}\,\sqrt{a^2\,b+2\,a\,b^2+b^3}}\right)+\frac{\left(a-b\right)\,\left(a^2+2\,a\,b+b^2\right)}{{\left(a+b\right)}^3\,\sqrt{b\,{\left(a+b\right)}^2}\,\sqrt{a^2\,b+2\,a\,b^2+b^3}}\right)\,\left(a^2\,\sqrt{a^2\,b+2\,a\,b^2+b^3}+b^2\,\sqrt{a^2\,b+2\,a\,b^2+b}\right.}{2\,\sqrt{a}}\right.}{\sqrt{a^2\,b+2\,a\,b^2+b^3}}$$

```
[In] int(sinh(x)^2/(b*sinh(x)^2 + a*cosh(x)^2),x)
```

```
[Out] x/(a + b) - (a^(1/2)*atan(((exp(2*x)*((4*a)/(a + b)^4 + ((a^2 - b^2)*(a - b
))/((a + b)^3*(b*(a + b)^2)^(1/2)*(2*a*b^2 + a^2*b + b^3)^(1/2))) + ((a - b
)*(2*a*b + a^2 + b^2))/((a + b)^3*(b*(a + b)^2)^(1/2)*(2*a*b^2 + a^2*b + b^
3)^(1/2)))*(a^2*(2*a*b^2 + a^2*b + b^3)^(1/2) + b^2*(2*a*b^2 + a^2*b + b^3)
^(1/2) + 2*a*b*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(2*a^(1/2))))/(2*a*b^2 + a^2
*b + b^3)^(1/2)
```

3.839 $\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

```
[Out] x/(a+b)+arctan(b^(1/2)*tanh(x)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {400, 212, 211}

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{x}{a+b}$$

```
[In] Int[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
```

```
[Out] x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/(Sqrt[a]*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)}\,dx, x, \tanh(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)}{a+b} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tanh(x)\right)}{a+b}\\ &= \frac{x}{a+b} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \frac{x+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a+b}$$

```
[In] Integrate[Cosh[x]^2/(a*Cosh[x]^2 + b*Sinh[x]^2),x]
[Out] (x + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a]])/Sqrt[a])/(a + b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(30) = 60$.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.42

method	result
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2x}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a(a+b)}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2x}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2a(a+b)}$
default	$-\frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a+2b}-\frac{2ba\left(-\frac{\left(-a+\sqrt{b(a+b)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}-a-2b\right)a}}+\frac{\left(a+\sqrt{b(a+b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{x}{2}\right)}{\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{2a\sqrt{b(a+b)}\sqrt{\left(2\sqrt{b(a+b)}+a+2b\right)a}}\right)}{a+b}$

[In] int(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x,method=_RETURNVERBOSE)

[Out] x/(a+b)+1/2/a*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)+(a+2*(-a*b)^(1/2)-b)/(a+b))-1/2/a*(-a*b)^(1/2)/(a+b)*ln(exp(2*x)-(-a+2*(-a*b)^(1/2)+b)/(a+b))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 9.55

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+2ab+b^2\right)\cosh(x)^4+4\left(a^2+2ab+b^2\right)\cosh(x)\sinh(x)^3+\left(a^2+2ab+b^2\right)\sinh(x)^4+2\left(a^2-b^2\right)\cosh(x)^2+2\left(3\left(a^2+2ab+b^2\right)\cosh(x)^2\right.}{(a+b)\cosh(x)^4+4(a+b)\cosh(x)\sinh(x)^3+(a+b)\sinh(x)^4}\right.}{}\right.$$

[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="fricas")

[Out] [1/2*(sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x) + 4*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*x)/(a + b), (sqrt(b/a)*arctan(1/2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)*sqrt(b/a)/b) + x)/(a + b)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(32) = 64$.

Time = 0.58 (sec) , antiderivative size = 224, normalized size of antiderivative = 5.89

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = \begin{cases} \tilde{\infty}\left(x-\frac{\cosh(x)}{\sinh(x)}\right) & \text{for } a=0\wedge b=0 \\ \frac{x-\frac{\cosh(x)}{\sinh(x)}}{b} & \text{for } a=0 \\ \frac{x\sinh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{x\cosh^2(x)}{-2b\sinh^2(x)+2b\cosh^2(x)}-\frac{\sinh(x)\cosh(x)}{-2b\sinh^2(x)+2b\cosh^2(x)} & \text{for } a=-b \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2ax\sqrt{-\frac{b}{a}}}{2a^2\sqrt{-\frac{b}{a}}+2ab\sqrt{-\frac{b}{a}}}-\frac{b\log\left(-\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a^2\sqrt{-\frac{b}{a}}+2ab\sqrt{-\frac{b}{a}}}+\frac{b\log\left(\sqrt{-\frac{b}{a}}\sinh(x)+\cosh(x)\right)}{2a^2\sqrt{-\frac{b}{a}}+2ab\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)**2/(a*cosh(x)**2+b*sinh(x)**2),x)
```

```
[Out] Piecewise((zoo*(x - cosh(x)/sinh(x)), Eq(a, 0) & Eq(b, 0)), ((x - cosh(x)/s
inh(x))/b, Eq(a, 0)), (x*sinh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - x*
cosh(x)**2/(-2*b*sinh(x)**2 + 2*b*cosh(x)**2) - sinh(x)*cosh(x)/(-2*b*sinh(
x)**2 + 2*b*cosh(x)**2), Eq(a, -b)), (x/a, Eq(b, 0)), (2*a*x*sqrt(-b/a)/(2*
a**2*sqrt(-b/a) + 2*a*b*sqrt(-b/a)) - b*log(-sqrt(-b/a)*sinh(x) + cosh(x))/
(2*a**2*sqrt(-b/a) + 2*a*b*sqrt(-b/a)) + b*log(sqrt(-b/a)*sinh(x) + cosh(x)
)/(2*a**2*sqrt(-b/a) + 2*a*b*sqrt(-b/a)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.95

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx = -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)} - \frac{\arctan\left(\frac{(a+b)e^{(-2x)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{x}{a+b}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a - b)*arctan(1/2*((a + b)*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
+ b)) - 1/2*arctan(1/2*((a + b)*e^(-2*x) + a - b)/sqrt(a*b))/sqrt(a*b) + x/
(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{b\arctan\left(\frac{ae^{(2x)}+be^{(2x)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{x}{a+b}$$

```
[In] integrate(cosh(x)^2/(a*cosh(x)^2+b*sinh(x)^2),x, algorithm="giac")
[Out] b*arctan(1/2*(a*e^(2*x) + b*e^(2*x) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b))
 + x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 208, normalized size of antiderivative = 5.47

$$\int \frac{\cosh^2(x)}{a\cosh^2(x)+b\sinh^2(x)}\,dx=\frac{x}{a+b}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(e^{2x}\left(\frac{4b}{(a+b)^4}+\frac{(a^2-b^2)(a-b)}{(a+b)^3\sqrt{a(a+b)^2}\sqrt{a^3+2a^2b+ab^2}}\right)+\frac{(a-b)(a^2+2ab+b^2)}{(a+b)^3\sqrt{a(a+b)^2}\sqrt{a^3+2a^2b+ab^2}}\right)\left(a^2\sqrt{a^3+2a^2b+ab^2}+b^2\sqrt{a^3+2a^2b+\cdots}\right.}{2\sqrt{b}}\right.}{\sqrt{a^3+2a^2b+ab^2}}$$

```
[In] int(cosh(x)^2/(b*sinh(x)^2 + a*cosh(x)^2),x)
[Out] x/(a + b) + (b^(1/2)*atan(((exp(2*x)*((4*b)/(a + b)^4 + ((a^2 - b^2)*(a - b
))/((a + b)^3*(a*(a + b)^2)^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))) + ((a - b
)*(2*a*b + a^2 + b^2))/((a + b)^3*(a*(a + b)^2)^(1/2)*(a*b^2 + 2*a^2*b + a^
3)^(1/2)))*(a^2*(a*b^2 + 2*a^2*b + a^3)^(1/2) + b^2*(a*b^2 + 2*a^2*b + a^3)
^(1/2) + 2*a*b*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(2*b^(1/2))))/(a*b^2 + 2*a^2
*b + a^3)^(1/2)
```

3.840 $\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx = \frac{x}{2}+\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6(1+\tanh(x))}$$

```
[Out] 1/2*x+2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)+1/6/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2099, 213, 632, 210}

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)} \, dx = \frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x}{2}+\frac{1}{6(\tanh(x)+1)}$$

```
[In] Int[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] x/2 + (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + 1/(6*(1 + Tanh[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3}{-1+x^2-x^3+x^5}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{6(1+x)^2}+\frac{1}{2\,(-1+x^2)}+\frac{1}{3\,(1-x+x^2)}\right)\,dx, x, \tanh(x)\right)\\
&= \frac{1}{6(1+\tanh(x))}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{x}{2}+\frac{1}{6(1+\tanh(x))}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2}+\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{1}{36}\left(18x-8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)+3\cosh(2x)-3\sinh(2x)\right)$$

```
[In] Integrate[Sinh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
[Out] (18*x - 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] + 3*Cosh[2*x] - 3*Sinh[2
*x])/36
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
risch	$\frac{x}{2}+\frac{{\mathrm e}^{-2 x}}{12}+\frac{i \sqrt{3}\, \ln \left({\mathrm e}^{2 x}-i \sqrt{3}\right)}{9}-\frac{i \sqrt{3}\, \ln \left({\mathrm e}^{2 x}+i \sqrt{3}\right)}{9}$
default	$\frac{i \sqrt{3}\, \ln \left(\tanh \left(\frac{x}{2}\right)^{2}+\left(i \sqrt{3}-1\right) \tanh \left(\frac{x}{2}\right)+1\right)}{9}-\frac{i \sqrt{3}\, \ln \left(\tanh \left(\frac{x}{2}\right)^{2}+\left(-i \sqrt{3}-1\right) \tanh \left(\frac{x}{2}\right)+1\right)}{9}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{3 \left(1+\tanh \left(\frac{x}{2}\right)\right)^{2}}$

```
[In] int(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/12*exp(-2*x)+1/9*I*3^(1/2)*ln(exp(2*x)-I*3^(1/2))-1/9*I*3^(1/2)*ln(
exp(2*x)+I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$$
$$= \frac{18\,x\cosh\left(x\right)^{2}+36\,x\cosh\left(x\right)\sinh\left(x\right)+18\,x\sinh\left(x\right)^{2}+8\,\left(\sqrt{3}\cosh\left(x\right)^{2}+2\,\sqrt{3}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{3}\sinh\right.}{36\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 + 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.58

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)} + \frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)} - \frac{4\sqrt{3}\sinh(x)\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)} - \frac{4\sqrt{3}\cosh(x)\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)} + \frac{3\cosh(x)}{18\sinh(x)+18\cosh(x)}$$

```
[In] integrate(sinh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(x
)) - 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(18*
sinh(x) + 18*cosh(x)) - 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x
)/(3*sinh(x)))/(18*sinh(x) + 18*cosh(x)) + 3*cosh(x)/(18*sinh(x) + 18*cosh(
x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{2}x + \frac{1}{12}e^{(-2x)}$$

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) + 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqr
t(2))) + 1/2*x + 1/12*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{1}{12}\left(3\,e^{(2\,x)}-1\right)e^{(-2\,x)}-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}e^{(2\,x)}\right)+\frac{1}{2}\,x$$

```
[In] integrate(sinh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*(3*e^(2*x) - 1)*e^(-2*x) - 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
1/2*x
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{x}{2}+\frac{\mathrm{e}^{-2\,x}}{12}-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,x}}{3}\right)}{9}$$

```
[In] int(sinh(x)^3/(cosh(x)^3 + sinh(x)^3),x)
```

```
[Out] x/2 + exp(-2*x)/12 - (2*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9
```

3.841 $\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)} dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)} dx = \frac{x}{2} - \frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6(1+\tanh(x))}$$

```
[Out] 1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2083, 213, 632, 210}

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)} dx = -\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{x}{2} - \frac{1}{6(\tanh(x)+1)}$$

```
[In] Int[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{1-x^2+x^3-x^5}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}+\frac{1}{3\left(1-x+x^2\right)}\right)\,dx, x, \tanh(x)\right)\\
&= -\frac{1}{6(1+\tanh(x))}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{x}{2}-\frac{1}{6(1+\tanh(x))}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= \frac{x}{2}-\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{1}{36}\left(18x+8\sqrt{3}\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)-3\cosh(2x)+3\sinh(2x)\right)$$

```
[In] Integrate[Cosh[x]^3/(Cosh[x]^3 + Sinh[x]^3),x]
[Out] (18*x + 8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2
*x])/36
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
risch	$\frac{x}{2}-\frac{{\mathrm e}^{-2x}}{12}+\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{2x}+i\sqrt{3}\right)}{9}-\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{2x}-i\sqrt{3}\right)}{9}$
default	$\frac{i\sqrt{3}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{i\sqrt{3}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+\left(i\sqrt{3}-1\right)\tanh\left(\frac{x}{2}\right)+1\right)}{9}-\frac{1}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{3+3\tanh\left(\frac{x}{2}\right)}$

```
[In] int(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x-1/12*exp(-2*x)+1/9*I*3^(1/2)*ln(exp(2*x)+I*3^(1/2))-1/9*I*3^(1/2)*ln(
exp(2*x)-I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$$
$$=\frac{18\,x\cosh\left(x\right)^{2}+36\,x\cosh\left(x\right)\sinh\left(x\right)+18\,x\sinh\left(x\right)^{2}-8\left(\sqrt{3}\cosh\left(x\right)^{2}+2\sqrt{3}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{3}\sin\right.}{36\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*c
osh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt
(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.58

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{9x\sinh(x)}{18\sinh(x)+18\cosh(x)} + \frac{9x\cosh(x)}{18\sinh(x)+18\cosh(x)} + \frac{4\sqrt{3}\sinh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)} + \frac{4\sqrt{3}\cosh(x)\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\cosh(x)}{3\sinh(x)}\right)}{18\sinh(x)+18\cosh(x)} - \frac{3\cosh(x)}{18\sinh(x)+18\cosh(x)}$$

```
[In] integrate(cosh(x)**3/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 9*x*sinh(x)/(18*sinh(x) + 18*cosh(x)) + 9*x*cosh(x)/(18*sinh(x) + 18*cosh(x
)) + 4*sqrt(3)*sinh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x)/(3*sinh(x)))/(18*
sinh(x) + 18*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(sqrt(3)/3 - 2*sqrt(3)*cosh(x
)/(3*sinh(x)))/(18*sinh(x) + 18*cosh(x)) - 3*cosh(x)/(18*sinh(x) + 18*cosh(
x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{2}x - \frac{1}{12}e^{(-2x)}$$

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/2*x - 1/12*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = -\frac{1}{12}\left(3\,e^{(2\,x)}+1\right)e^{(-2\,x)}+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}e^{(2\,x)}\right)+\frac{1}{2}\,x$$

```
[In] integrate(cosh(x)^3/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
[Out] -1/12*(3*e^(2*x) + 1)*e^(-2*x) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) +
1/2*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \frac{x}{2}-\frac{\mathrm{e}^{-2\,x}}{12}+\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,x}}{3}\right)}{9}$$

```
[In] int(cosh(x)^3/(cosh(x)^3 + sinh(x)^3),x)
[Out] x/2 - exp(-2*x)/12 + (2*3^(1/2)*atan((3^(1/2)*exp(2*x))/3))/9
```

3.842 $\int \frac{x\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^2(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{2x\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[Out] -2*x*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-polylog(2,-exp(x))*sech(x)
/(a*sech(x)^2)^(1/2)+polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 4267, 2317, 2438}

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{2x\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (PolyLog[2, -E^x]*Sech[x])/
Sqrt[a*Sech[x]^2] + (PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}(x)\int x\operatorname{csch}(x)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&= -\frac{2x\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{\operatorname{sech}(x)\int \log\left(1-e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{\operatorname{sech}(x)\int \log\left(1+e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&= -\frac{2x\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{\operatorname{sech}(x)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^x\right)}{\sqrt{a\operatorname{sech}^2(x)}}\\
&\quad + \frac{\operatorname{sech}(x)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^x\right)}{\sqrt{a\operatorname{sech}^2(x)}}\\
&= -\frac{2x\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} - \frac{\operatorname{PolyLog}\left(2,-e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}} + \frac{\operatorname{PolyLog}\left(2,e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{(x(\log(1-e^x)-\log(1+e^x))-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x))\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
[Out] ((x*(Log[1 - E^x] - Log[1 + E^x]) - PolyLog[2, -E^x] + PolyLog[2, E^x])*Sec
h[x])/Sqrt[a*Sech[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(50) = 100.

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.31

method	result	size
risch	$-\frac{e^x x\ln(1+e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{e^x\,\text{polylog}(2,-e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{e^x x\ln(1-e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{e^x\,\text{polylog}(2,e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}$	136

```
[In] int(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*ln(1+exp(x))-1/(
a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(2,-exp(x))+1/(
a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*ln(1-exp(x))+1/(a*ex
p(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(2,exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\left(\left(e^{(2x)}+1\right)\text{Li}_2(\cosh(x)+\sinh(x))-\left(e^{(2x)}+1\right)\text{Li}_2(-\cosh(x)-\sinh(x))-\left(xe^{(2x)}+x\right)\log(\cosh(x)+\right.}{a}$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) + 1)*dilog(cosh(x) + sinh(x)) - (e^(2*x) + 1)*dilog(-cosh(x) - si
nh(x)) - (x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) + (x*e^(2*x) + x)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))/a
```

Sympy [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
[Out] Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{x\log(e^x+1)+\text{Li}_2(-e^x)}{\sqrt{a}} + \frac{x\log(-e^x+1)+\text{Li}_2(e^x)}{\sqrt{a}}$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] -(x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))/sq
rt(a)
```

Giac [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}(x)^2}}\,dx$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

```
[In] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
[Out] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

3.843 $\int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 104

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{2x^2\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\,\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\,\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2\,\text{PolyLog}\,(3,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2\,\text{PolyLog}\,(3,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[Out] -2*x^2*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-2*x*polylog(2,-exp(x))*s
ech(x)/(a*sech(x)^2)^(1/2)+2*x*polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^(1/2
)+2*polylog(3,-exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-2*polylog(3,exp(x))*sech
(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6852, 4267, 2611, 2320, 6724}

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{2x^2\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\,\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\,\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2\,\text{PolyLog}\,(3,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2\,\text{PolyLog}\,(3,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x^2*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*x*PolyLog[2, -E^x]*Sec
h[x])/Sqrt[a*Sech[x]^2] + (2*x*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^2] +
 (2*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (2*PolyLog[3, E^x]*Sech[x
])/Sqrt[a*Sech[x]^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \frac{\operatorname{sech}(x) \int x^2 \operatorname{csch}(x)\, dx}{\sqrt{a \operatorname{sech}^2(x)}}$$

$$= -\frac{2x^2\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{(2\text{sech}(x))\int x\log(1-e^x)\,dx}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\int x\log(1+e^x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\,\text{PolyLog}(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\,\text{PolyLog}(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\int \text{PolyLog}(2,-e^x)\,dx}{\sqrt{a\text{sech}^2(x)}} - \frac{(2\text{sech}(x))\int \text{PolyLog}(2,e^x)\,dx}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\,\text{PolyLog}(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\,\text{PolyLog}(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{(2\text{sech}(x))\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^x\right)}{\sqrt{a\text{sech}^2(x)}} - \frac{(2\text{sech}(x))\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^x\right)}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^2\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2x\,\text{PolyLog}(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2x\,\text{PolyLog}(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{2\,\text{PolyLog}(3,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{2\,\text{PolyLog}(3,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{(x^2\log(1-e^x) - x^2\log(1+e^x) - 2x\,\text{PolyLog}(2,-e^x) + 2x\,\text{PolyLog}(2,e^x) + 2\,\text{PolyLog}(3,-e^x) - 2\,\text{Po}}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2, -E^x] + 2*x*PolyLog[
2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])*Sech[x])/Sqrt[a*Sech[x]^2
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(89) = 178.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.01

method	result
risch	$-\frac{e^x x^2 \ln(1+e^x)}{\sqrt{\frac{a e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{2 e^x x\,\mathrm{polylog}(2,-e^x)}{\sqrt{\frac{a e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{2 e^x\,\mathrm{polylog}(3,-e^x)}{\sqrt{\frac{a e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{e^x x^2 \ln(1-e^x)}{\sqrt{\frac{a e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{2 e^x x\,\mathrm{polylog}(2,e^x)}{\sqrt{\frac{a e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{2}{\sqrt{}}$

```
[In] int(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*ln(1+exp(x))-2
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(2,-exp(x))
+2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(3,-exp(x))
+1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*ln(1-exp(x))+2
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(2,exp(x))-
2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(3,exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.81

$$\int \frac{x^2 \mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^2(x)}}\,dx = -\frac{\left(2\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\mathrm{polylog}(3,\cosh(x)+\sinh(x))-2\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x\mathrm{polylog}(3\right.}{}$$

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(3, cosh(x)
+ sinh(x)) - 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(
3, -cosh(x) - sinh(x)) - (2*(x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 2*(x
*e^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - (x^2*e^(2*x) + x^2)*log(cosh(x) +
 sinh(x) + 1) + (x^2*e^(2*x) + x^2)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^
(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

Sympy [F]

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^2\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}^2\,(x)}}\,dx$$

[In] integrate(x**2*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)

[Out] Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{x^2\log\left(e^x+1\right)+2\,x\text{Li}_2(-e^x)-2\,\text{Li}_3(-e^x)}{\sqrt{a}} + \frac{x^2\log\left(-e^x+1\right)+2\,x\text{Li}_2(e^x)-2\,\text{Li}_3(e^x)}{\sqrt{a}}$$

[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] -(x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))/sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a)

Giac [F]

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^2\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}\,(x)^2}}\,dx$$

[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \int \frac{x^2}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

```
[In] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
[Out] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

3.844 $\int \frac{x^3 \mathrm{csch}(x) \mathrm{sech}(x)}{\sqrt{a \mathrm{sech}^2(x)}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x^3 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} \, dx = -\frac{2x^3 \operatorname{arctanh}(e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} - \frac{3x^2 \operatorname{PolyLog}(2,-e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} + \frac{3x^2 \operatorname{PolyLog}(2,e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} + \frac{6x \operatorname{PolyLog}(3,-e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} - \frac{6x \operatorname{PolyLog}(3,e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} - \frac{6 \operatorname{PolyLog}(4,-e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}} + \frac{6 \operatorname{PolyLog}(4,e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^2(x)}}$$

```
[Out] -2*x^3*arctanh(exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-3*x^2*polylog(2,-exp(x))
*sech(x)/(a*sech(x)^2)^(1/2)+3*x^2*polylog(2,exp(x))*sech(x)/(a*sech(x)^2)^
(1/2)+6*x*polylog(3,-exp(x))*sech(x)/(a*sech(x)^2)^(1/2)-6*x*polylog(3,exp(
x))*sech(x)/(a*sech(x)^2)^(1/2)-6*polylog(4,-exp(x))*sech(x)/(a*sech(x)^2)^
(1/2)+6*polylog(4,exp(x))*sech(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{6852, 4267, 2611, 6744, 2320, 6724\}$

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{2x^3\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\,\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\,\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\,\text{PolyLog}\,(3,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{6x\,\text{PolyLog}\,(3,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{6\,\text{PolyLog}\,(4,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6\,\text{PolyLog}\,(4,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] (-2*x^3*ArcTanh[E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (3*x^2*PolyLog[2, -E^x]*S
ech[x])/Sqrt[a*Sech[x]^2] + (3*x^2*PolyLog[2, E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*x*PolyLog[3, -E^x]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*x*PolyLog[3, E^x
]*Sech[x])/Sqrt[a*Sech[x]^2] - (6*PolyLog[4, -E^x]*Sech[x])/Sqrt[a*Sech[x]^
2] + (6*PolyLog[4, E^x]*Sech[x])/Sqrt[a*Sech[x]^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}(x)\int x^3\operatorname{csch}(x)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&= -\frac{2x^3\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{(3\operatorname{sech}(x))\int x^2\log\left(1-e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{(3\operatorname{sech}(x))\int x^2\log\left(1+e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&= -\frac{2x^3\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\\
&\quad+\frac{3x^2\operatorname{PolyLog}\left(2,e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{(6\operatorname{sech}(x))\int x\operatorname{PolyLog}\left(2,-e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&\quad-\frac{(6\operatorname{sech}(x))\int x\operatorname{PolyLog}\left(2,e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&= -\frac{2x^3\operatorname{arctanh}(e^x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{3x^2\operatorname{PolyLog}\left(2,-e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\\
&\quad+\frac{3x^2\operatorname{PolyLog}\left(2,e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}+\frac{6x\operatorname{PolyLog}\left(3,-e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\\
&\quad-\frac{6x\operatorname{PolyLog}\left(3,e^x\right)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}-\frac{(6\operatorname{sech}(x))\int\operatorname{PolyLog}\left(3,-e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}\\
&\quad+\frac{(6\operatorname{sech}(x))\int\operatorname{PolyLog}\left(3,e^x\right)\,dx}{\sqrt{a\operatorname{sech}^2(x)}}
\end{aligned}$$

$$= -\frac{2x^3\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\,\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\,\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\,\text{PolyLog}\,(3,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{6x\,\text{PolyLog}\,(3,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{(6\text{sech}(x))\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^x\right)}{\sqrt{a\text{sech}^2(x)}} + \frac{(6\text{sech}(x))\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx,x,e^x\right)}{\sqrt{a\text{sech}^2(x)}}$$

$$= -\frac{2x^3\text{arctanh}(e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{3x^2\,\text{PolyLog}\,(2,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{3x^2\,\text{PolyLog}\,(2,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6x\,\text{PolyLog}\,(3,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{6x\,\text{PolyLog}\,(3,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} - \frac{6\,\text{PolyLog}\,(4,-e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}} + \frac{6\,\text{PolyLog}\,(4,e^x)\,\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{(x^3\log(1-e^x) - x^3\log(1+e^x) - 3x^2\,\text{PolyLog}\,(2,-e^x) + 3x^2\,\text{PolyLog}\,(2,e^x) + 6x\,\text{PolyLog}\,(3,-e^x) - 6x}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^2],x]
```

```
[Out] ((x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2, -E^x] + 3*x^2*Poly
Log[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x] - 6*PolyLog[4, -E^
x] + 6*PolyLog[4, E^x])*Sech[x])/Sqrt[a*Sech[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(129) = 258.

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{e^x x^3 \ln(1+e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} - \frac{3\,e^x x^2 \operatorname{polylog}(2,-e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} + \frac{6\,e^x x \operatorname{polylog}(3,-e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} - \frac{6\,e^x \operatorname{polylog}(4,-e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} + \frac{e^x x^3 \ln(1-e^x)}{\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} +$

```
[In] int(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^3*ln(1+exp(x))-3
/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*polylog(2,-exp(x
))+6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(3,-exp
(x))-6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(4,-exp
(x))+1/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^3*ln(1-exp(x
))+3/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x^2*polylog(2,ex
p(x))-6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*x*polylog(3,e
xp(x))+6/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(x)*polylog(4,ex
p(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(127) = 254.

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.81

$$\int \frac{x^3 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \frac{\left(6\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x \operatorname{polylog}(4,\cosh(x)+\sinh(x)) - 6\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^x \operatorname{polylog}(4,\right.}{}$$

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4, cosh(x) +
 sinh(x)) - 6*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^x*polylog(4
, -cosh(x) - sinh(x)) - 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))
*e^x*polylog(3, cosh(x) + sinh(x)) + 6*(x*e^(2*x) + x)*sqrt(a/(e^(4*x) + 2*
e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) + (3*(x^2*e^(2*x) + x^2)*d
ilog(cosh(x) + sinh(x)) - 3*(x^2*e^(2*x) + x^2)*dilog(-cosh(x) - sinh(x)) -
 (x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(x) + 1) + (x^3*e^(2*x) + x^3)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)*e^(-x)/a
```

Sympy [F]

$$\int \frac{x^3\mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^2(x)}}\,dx = \int \frac{x^3\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)}{\sqrt{a\,\mathrm{sech}^2\,(x)}}\,dx$$

```
[In] integrate(x**3*csch(x)*sech(x)/(a*sech(x)**2)**(1/2),x)
[Out] Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{x^3\mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^2(x)}}\,dx = -\frac{x^3\log\left(e^x+1\right)+3\,x^2\mathrm{Li}_2(-e^x)-6\,x\mathrm{Li}_3(-e^x)+6\,\mathrm{Li}_4(-e^x)}{\sqrt{a}} + \frac{x^3\log\left(-e^x+1\right)+3\,x^2\mathrm{Li}_2(e^x)-6\,x\mathrm{Li}_3(e^x)+6\,\mathrm{Li}_4(e^x)}{\sqrt{a}}$$

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] -(x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4
, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e
^x) + 6*polylog(4, e^x))/sqrt(a)
```

Giac [F]

$$\int \frac{x^3\mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^2(x)}}\,dx = \int \frac{x^3\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)}{\sqrt{a\,\mathrm{sech}\,(x)^2}}\,dx$$

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^2(x)}}\,dx = \int \frac{x^3}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^2}}}\,dx$$

```
[In] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)),x)
[Out] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^2)^(1/2)), x)
```

3.845 $\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}+\frac{\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] -1/2*x^2*sech(x)^2/(a*sech(x)^4)^(1/2)+x*ln(1-exp(2*x))*sech(x)^2/(a*sech(x
)^4)^(1/2)+1/2*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6852, 3797, 2221, 2317, 2438}

$$\int \frac{x\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}-\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}$$

```
[In] Int[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -1/2*(x^2*Sech[x]^2)/Sqrt[a*Sech[x]^4] + (x*Log[1 - E^(2*x)]*Sech[x]^2)/Sqr
t[a*Sech[x]^4] + (PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^2(x)\int x\coth(x)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} - \frac{\left(2\operatorname{sech}^2(x)\right)\int\frac{e^{2x}x}{1-e^{2x}}\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\operatorname{sech}^2(x)\int\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\operatorname{sech}^2(x)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)}{2\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^2\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} + \frac{\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx=\frac{\left(-x\left(x-2\log\left(1-e^{2x}\right)\right)+\text{PolyLog}\left(2,e^{2x}\right)\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

```
[In] Integrate[(x*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] ((-(x*(x - 2*Log[1 - E^(2*x)])) + PolyLog[2, E^(2*x)])*Sech[x]^2)/(2*Sqrt[a
*Sech[x]^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(61) = 122$.

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.40

method	result
risch	$-\frac{e^{2x}x^{2}}{2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^{4}}}\,(1+e^{2x})^{2}}+\frac{e^{2x}x\ln(1+e^{x})}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^{4}}}\,(1+e^{2x})^{2}}+\frac{e^{2x}\,\text{polylog}(2,-e^{x})}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^{4}}}\,(1+e^{2x})^{2}}+\frac{e^{2x}x\ln(1-e^{x})}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^{4}}}\,(1+e^{2x})^{2}}+\frac{e^{2x}\,\text{polylog}(2,e^{x})}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^{4}}}\,(1+e^{2x})^{2}}$

```
[In] int(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*ln(1+exp(x))+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(2,-exp(x))+1/(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*ln(1-exp(x))+1/(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(2,exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.08

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(x^2e^{(4\,x)}+2\,x^2e^{(2\,x)}+x^2-2\left(e^{(4\,x)}+2\,e^{(2\,x)}+1\right)\text{Li}_2(\cosh(x)+\sinh(x))-2\left(e^{(4\,x)}+2\,e^{(2\,x)}+1\right)\text{Li}_2(-\cdots}{}$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2 - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog
(cosh(x) + sinh(x)) - 2*(e^(4*x) + 2*e^(2*x) + 1)*dilog(-cosh(x) - sinh(x))
```

```
 - 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*log(cosh(x) + sinh(x) + 1) - 2*(x*e^(4*x
) + 2*x*e^(2*x) + x)*log(-cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*
x) + 6*e^(4*x) + 4*e^(2*x) + 1))/a
```

Sympy [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}^4(x)}}\,dx$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
[Out] Integral(x*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^2}{2\sqrt{a}} + \frac{x\log(e^x+1) + \text{Li}_2(-e^x)}{\sqrt{a}} + \frac{x\log(-e^x+1) + \text{Li}_2(e^x)}{\sqrt{a}}$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
[Out] -1/2*x^2/sqrt(a) + (x*log(e^x + 1) + dilog(-e^x))/sqrt(a) + (x*log(-e^x + 1
) + dilog(e^x))/sqrt(a)
```

Giac [F]

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^4}}\,dx$$

```
[In] integrate(x*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(x*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

```
[In] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
[Out] int(x/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

3.846 $\int \frac{x^2\mathbf{csch}(x)\mathbf{sech}(x)}{\sqrt{a\mathbf{sech}^4(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} + \frac{x\,\text{PolyLog}\left(2,e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{PolyLog}\left(3,e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

```
[Out] -1/3*x^3*sech(x)^2/(a*sech(x)^4)^(1/2)+x^2*ln(1-exp(2*x))*sech(x)^2/(a*sech
(x)^4)^(1/2)+x*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)-1/2*polylo
g(3,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6852, 3797, 2221, 2611, 2320, 6724}

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{x\,\text{PolyLog}\left(2,e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}} - \frac{\text{PolyLog}\left(3,e^{2x}\right)\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} - \frac{x^3\text{sech}^2(x)}{3\sqrt{a\text{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

```
[In] Int[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -1/3*(x^3*Sech[x]^2)/Sqrt[a*Sech[x]^4] + (x^2*Log[1 - E^(2*x)]*Sech[x]^2)/S
qrt[a*Sech[x]^4] + (x*PolyLog[2, E^(2*x)]*Sech[x]^2)/Sqrt[a*Sech[x]^4] - (P
olyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \frac{\operatorname{sech}^2(x)\int x^2\coth(x)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}$$

$$= -\frac{x^3\operatorname{sech}^2(x)}{3\sqrt{a\operatorname{sech}^4(x)}} - \frac{\left(2\operatorname{sech}^2(x)\right)\int\frac{e^{2x}x^2}{1-e^{2x}}\,dx}{\sqrt{a\operatorname{sech}^4(x)}}$$

$$= -\frac{x^3\operatorname{sech}^2(x)}{3\sqrt{a\operatorname{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\left(2\operatorname{sech}^2(x)\right)\int x\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}$$

$$= -\frac{x^3\operatorname{sech}^2(x)}{3\sqrt{a\operatorname{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\,\text{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\operatorname{sech}^2(x)\int\text{PolyLog}\left(2,e^{2x}\right)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}$$

$$= -\frac{x^3\operatorname{sech}^2(x)}{3\sqrt{a\operatorname{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\,\text{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\operatorname{sech}^2(x)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2x}\right)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

$$= -\frac{x^3\operatorname{sech}^2(x)}{3\sqrt{a\operatorname{sech}^4(x)}} + \frac{x^2\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} + \frac{x\,\text{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}} - \frac{\text{PolyLog}\left(3,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int\frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = \frac{\left(-2x^2\left(x-3\log\left(1-e^{2x}\right)\right)+6x\,\text{PolyLog}\left(2,e^{2x}\right)-3\,\text{PolyLog}\left(3,e^{2x}\right)\right)\operatorname{sech}^2(x)}{6\sqrt{a\operatorname{sech}^4(x)}}$$

```
[In] Integrate[(x^2*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
[Out] ((-2*x^2*(x - 3*Log[1 - E^(2*x)]) + 6*x*PolyLog[2, E^(2*x)] - 3*PolyLog[3,
E^(2*x)])*Sech[x]^2)/(6*Sqrt[a*Sech[x]^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(83) = 166.

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.58

method	result
risch	$-\frac{e^{2x}x^3}{3\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{2x}x^2\ln(1+e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{2\,e^{2x}x\,\mathrm{polylog}(2,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}-\frac{2\,e^{2x}\,\mathrm{polylog}(3,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{2x}x^2\ln(1-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}$

```
[In] int(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*ln(1+exp(x))+2/(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog(2,-exp(x))-
2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog(3,-exp(
x))+1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*ln(1-ex
p(x))+2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog
(2,exp(x))-2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*poly
log(3,exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(82) = 164.

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.00

$$\int \frac{x^2\mathrm{csch}(x)\mathrm{sech}(x)}{\sqrt{a\mathrm{sech}^4(x)}}\,dx = -\frac{\left(6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\mathrm{polylog}(3,\cosh(x)+\sinh(x))+6\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}\cdots}}\right.}{}$$

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
+ 2*e^(2*x) + 1)*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*sqrt(a/(e^(8*x)
+ 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2*x)
*polylog(3, -cosh(x) - sinh(x)) + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3 - 6*(x
*e^(4*x) + 2*x*e^(2*x) + x)*dilog(cosh(x) + sinh(x)) - 6*(x*e^(4*x) + 2*x*e
^(2*x) + x)*dilog(-cosh(x) - sinh(x)) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^
2)*log(cosh(x) + sinh(x) + 1) - 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*log(-
cosh(x) - sinh(x) + 1))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x)
 + 1))*e^(2*x))*e^(-2*x)/a
```

Sympy [F]

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$$

```
[In] integrate(x**2*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
[Out] Integral(x**2*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^3}{3\sqrt{a}} + \frac{x^2\log(e^x+1) + 2\,x\text{Li}_2(-e^x) - 2\,\text{Li}_3(-e^x)}{\sqrt{a}} + \frac{x^2\log(-e^x+1) + 2\,x\text{Li}_2(e^x) - 2\,\text{Li}_3(e^x)}{\sqrt{a}}$$

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
[Out] -1/3*x^3/sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x)
)/sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))/sqrt(a
)
```

Giac [F]

$$\int \frac{x^2\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^2\operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}(x)^4}}\,dx$$

```
[In] integrate(x^2*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx = \int \frac{x^2}{\cosh(x)\,\sinh(x)\,\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

```
[In] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
[Out] int(x^2/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

3.847 $\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log(1-e^{2x})\,\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}+\frac{3x^2\,\text{PolyLog}(2,e^{2x})\,\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\,\text{PolyLog}(3,e^{2x})\,\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\,\text{PolyLog}(4,e^{2x})\,\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}$$

```
[Out] -1/4*x^4*sech(x)^2/(a*sech(x)^4)^(1/2)+x^3*ln(1-exp(2*x))*sech(x)^2/(a*sech
(x)^4)^(1/2)+3/2*x^2*polylog(2,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)-3/2*
x*polylog(3,exp(2*x))*sech(x)^2/(a*sech(x)^4)^(1/2)+3/4*polylog(4,exp(2*x))
*sech(x)^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6852, 3797, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{3x^2\,\text{PolyLog}(2,e^{2x})\,\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}-\frac{3x\,\text{PolyLog}(3,e^{2x})\,\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{3\,\text{PolyLog}(4,e^{2x})\,\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}-\frac{x^4\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}+\frac{x^3\log(1-e^{2x})\,\text{sech}^2(x)}{\sqrt{a\text{sech}^4(x)}}$$

[In] Int[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]

[Out] -1/4*(x^4*Sech[x]^2)/Sqrt[a*Sech[x]^4] + (x^3*Log[1 - E^(2*x)]*Sech[x]^2)/Sqrt[a*Sech[x]^4] + (3*x^2*PolyLog[2, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) - (3*x*PolyLog[3, E^(2*x)]*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + (3*PolyLog[4, E^(2*x)]*Sech[x]^2)/(4*Sqrt[a*Sech[x]^4])

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^2(x)\int x^3\coth(x)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^4\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}-\frac{\left(2\operatorname{sech}^2(x)\right)\int\frac{e^{2x}x^3}{1-e^{2x}}\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^4\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}-\frac{\left(3\operatorname{sech}^2(x)\right)\int x^2\log\left(1-e^{2x}\right)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^4\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}\\
&\quad+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}-\frac{\left(3\operatorname{sech}^2(x)\right)\int x\operatorname{PolyLog}\left(2,e^{2x}\right)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^4\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}\\
&\quad-\frac{3x\operatorname{PolyLog}\left(3,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\left(3\operatorname{sech}^2(x)\right)\int\operatorname{PolyLog}\left(3,e^{2x}\right)\,dx}{2\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^4\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}\\
&\quad-\frac{3x\operatorname{PolyLog}\left(3,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\left(3\operatorname{sech}^2(x)\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{2x}\right)}{4\sqrt{a\operatorname{sech}^4(x)}}\\
&= -\frac{x^4\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}+\frac{x^3\log\left(1-e^{2x}\right)\operatorname{sech}^2(x)}{\sqrt{a\operatorname{sech}^4(x)}}+\frac{3x^2\operatorname{PolyLog}\left(2,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}\\
&\quad-\frac{3x\operatorname{PolyLog}\left(3,e^{2x}\right)\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{3\operatorname{PolyLog}\left(4,e^{2x}\right)\operatorname{sech}^2(x)}{4\sqrt{a\operatorname{sech}^4(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{(x^4-4x^3\log(1-e^{2x})-6x^2\,\text{PolyLog}(2,e^{2x})+6x\,\text{PolyLog}(3,e^{2x})-3\,\text{PolyLog}(4,e^{2x}))\,\text{sech}^2(x)}{4\sqrt{a\text{sech}^4(x)}}$$

```
[In] Integrate[(x^3*Csch[x]*Sech[x])/Sqrt[a*Sech[x]^4],x]
```

```
[Out] -1/4*((x^4 - 4*x^3*Log[1 - E^(2*x)] - 6*x^2*PolyLog[2, E^(2*x)] + 6*x*PolyL
og[3, E^(2*x)] - 3*PolyLog[4, E^(2*x)])*Sech[x]^2)/Sqrt[a*Sech[x]^4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(107) = 214.

Time = 0.11 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.55

method	result
risch	$-\frac{e^{2x}x^4}{4\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{2x}x^3\ln(1+e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{3\,e^{2x}x^2\,\text{polylog}(2,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}-\frac{6\,e^{2x}x\,\text{polylog}(3,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{6\,e^{2x}\,\text{polylog}(4,-e^x)}{\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2}$

```
[In] int(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^4+1/(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3*ln(1+exp(x))+3/(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^2*polylog(2,-exp(x)
)-6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x*polylog(3,-
exp(x))+6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*polylog
(4,-exp(x))+1/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x^3
*ln(1-exp(x))+3/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x
^2*polylog(2,exp(x))-6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp
(2*x)*x*polylog(3,exp(x))+6/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^
2*exp(2*x)*polylog(4,exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(106) = 212$.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.31

$$\int \frac{x^3 \text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx$$
$$= \frac{\left(24\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}\text{polylog}(4,\cosh(x)+\sinh(x))+24\sqrt{\frac{}{e^{(8x)}+4e^{(6x}}}\right.}{}$$

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(24*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x)
+ 2*e^(2*x) + 1)*e^(2*x)*polylog(4, cosh(x) + sinh(x)) + 24*sqrt(a/(e^(8*x)
 + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e^(2*x) + 1)*e^(2*x
)*polylog(4, -cosh(x) - sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/
(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(
x) + sinh(x)) - 24*(x*e^(4*x) + 2*x*e^(2*x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x
) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) - (x
^4*e^(4*x) + 2*x^4*e^(2*x) + x^4 - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*d
ilog(cosh(x) + sinh(x)) - 12*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*dilog(-cos
h(x) - sinh(x)) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(cosh(x) + sinh(
x) + 1) - 4*(x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*log(-cosh(x) - sinh(x) + 1)
)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x))*e^(-2*
x)/a
```

Sympy [F]

$$\int \frac{x^3 \text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^3 \operatorname{csch}(x)\operatorname{sech}(x)}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$$

```
[In] integrate(x**3*csch(x)*sech(x)/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*csch(x)*sech(x)/sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{x^4}{4\sqrt{a}} + \frac{x^3\log\left(e^x+1\right)+3\,x^2\text{Li}_2(-e^x)-6\,x\text{Li}_3(-e^x)+6\,\text{Li}_4(-e^x)}{\sqrt{a}} + \frac{x^3\log\left(-e^x+1\right)+3\,x^2\text{Li}_2(e^x)-6\,x\text{Li}_3(e^x)+6\,\text{Li}_4(e^x)}{\sqrt{a}}$$

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
[Out] -1/4*x^4/sqrt(a) + (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6*x*polylog(3, -
e^x) + 6*polylog(4, -e^x))/sqrt(a) + (x^3*log(-e^x + 1) + 3*x^2*dilog(e^x)
- 6*x*polylog(3, e^x) + 6*polylog(4, e^x))/sqrt(a)
```

Giac [F]

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^3\,\text{csch}\,(x)\,\text{sech}\,(x)}{\sqrt{a\,\text{sech}\,(x)^4}}\,dx$$

```
[In] integrate(x^3*csch(x)*sech(x)/(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(x^3*csch(x)*sech(x)/sqrt(a*sech(x)^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\text{csch}(x)\text{sech}(x)}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{x^3}{\cosh\left(x\right)\,\sinh\left(x\right)\,\sqrt{\frac{a}{{\cosh\left(x\right)}^4}}}\,dx$$

```
[In] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)),x)
[Out] int(x^3/(cosh(x)*sinh(x)*(a/cosh(x)^4)^(1/2)), x)
```

3.848 $\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\begin{aligned}\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx &= x\sqrt{a\text{sech}^2(x)} - \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &\quad - 2x\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &\quad - \cosh(x)\,\text{PolyLog}\left(2,-e^x\right)\sqrt{a\text{sech}^2(x)} \\ &\quad + \cosh(x)\,\text{PolyLog}\left(2,e^x\right)\sqrt{a\text{sech}^2(x)}\end{aligned}$$

```
[Out] x*(a*sech(x)^2)^(1/2)-arctan(sinh(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x*arcta
nh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-cosh(x)*polylog(2,-exp(x))*(a*sech(x
)^2)^(1/2)+cosh(x)*polylog(2,exp(x))*(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6852, 2702, 327, 213, 5570, 6406, 4267, 2317, 2438, 3855}

$$\begin{aligned}\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx &= -\cosh(x)\sqrt{a\text{sech}^2(x)}\arctan(\sinh(x)) \\ &\quad - 2x\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &\quad - \text{PolyLog}\left(2,-e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &\quad + \text{PolyLog}\left(2,e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} + x\sqrt{a\text{sech}^2(x)}\end{aligned}$$

```
[In] Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x*Sqrt[a*Sech[x]^2] - ArcTan[Sinh[x]]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*ArcTa
nh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - Cosh[x]*PolyLog[2, -E^x]*Sqrt[a*Sech[x]
^2] + Cosh[x]*PolyLog[2, E^x]*Sqrt[a*Sech[x]^2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\text{csch}(x)\text{sech}^2(x)\,dx \\
&= x\sqrt{a\text{sech}^2(x)} - x\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} \\
&\quad - \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int(-\text{arctanh}(\cosh(x)) + \text{sech}(x))\,dx \\
&= x\sqrt{a\text{sech}^2(x)} - x\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} \\
&\quad + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\text{arctanh}(\cosh(x))\,dx \\
&\quad - \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\text{sech}(x)\,dx \\
&= x\sqrt{a\text{sech}^2(x)} - \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\text{csch}(x)\,dx \\
&= x\sqrt{a\text{sech}^2(x)} - \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} \\
&\quad - \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\log(1-e^x)\,dx + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\log(1+e^x)\,dx
\end{aligned}$$

$$= x\sqrt{a\text{sech}^2(x)} - \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)}$$
$$- 2x\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}$$
$$- \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^x\right)$$
$$+ \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^x\right)$$
$$= x\sqrt{a\text{sech}^2(x)} - \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}$$
$$- \cosh(x)\,\text{PolyLog}\left(2, -e^x\right)\sqrt{a\text{sech}^2(x)} + \cosh(x)\,\text{PolyLog}\left(2, e^x\right)\sqrt{a\text{sech}^2(x)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \Big(x - 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x) + x\cosh(x)\log\left(1-e^x\right) - x\cosh(x)\log\left(1+e^x\right) - \cosh(x)\,\text{PolyLog}\left(2,-e^x\right) + \cosh(x)\,\text{PolyLog}\left(2,e^x\right)\Big)\sqrt{a\text{sech}^2(x)}$$

```
[In] Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x - 2*ArcTan[Tanh[x/2]]*Cosh[x] + x*Cosh[x]*Log[1 - E^x] - x*Cosh[x]*Log[1
 + E^x] - Cosh[x]*PolyLog[2, -E^x] + Cosh[x]*PolyLog[2, E^x])*Sqrt[a*Sech[x
]^2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

method	result
risch	$2\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,x - 2\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\arctan\left(e^x\right) - \sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\,\text{dilog}\left(1+e^x\right) - \sqrt{\frac{a}{(1}}$

```
[In] int(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*x-2*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*e
xp(-x)*(1+exp(2*x))*arctan(exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x
)*(1+exp(2*x))*dilog(1+exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1
+exp(2*x))*x*ln(1+exp(x))-(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(
2*x))*dilog(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.99

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \frac{\left(2\,x\cosh\left(x\right)e^{(2\,x)} - 2\left(\left(e^{(2\,x)}+1\right)\sinh\left(x\right)^{2}+\cosh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}+1\right)e^{(2\,x)}+2\left(\cosh\left(x\right)e^{(2\,x)}+\cosh\left(\right.\right.\right.\right.}{}$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)*e^(2*x) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
+ 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*arctan(cosh(x) +
sinh(x)) + 2*x*cosh(x) + ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2
+ 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + s
inh(x)) - ((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) +
2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x) - sinh(x)) - (x*c
osh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cos
h(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(cosh(x) + sinh(x) + 1) + (x*cosh
(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x
)*e^(2*x) + x*cosh(x))*sinh(x) + x)*log(-cosh(x) - sinh(x) + 1) + 2*(x*e^(2
*x) + x)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(2*cosh(x)*e^x*sinh
(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \int x\sqrt{a\,\text{sech}^{2}\left(x\right)}\,\text{csch}\left(x\right)\text{sech}\left(x\right)\,dx$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = -(x\log\left(e^{x}+1\right)+\text{Li}_2(-e^{x}))\sqrt{a} + (x\log\left(-e^{x}+1\right)+\text{Li}_2(e^{x}))\sqrt{a} - 2\sqrt{a}\arctan\left(e^{x}\right)+\frac{2\sqrt{a}xe^{x}}{e^{(2\,x)}+1}$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x*log(e^x + 1) + dilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sq
rt(a) - 2*sqrt(a)*arctan(e^x) + 2*sqrt(a)*x*e^x/(e^(2*x) + 1)
```

Giac [F]

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx=\int\sqrt{a\,\text{sech}\,(x)^2}x\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^2)*x*csch(x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx=\int\frac{x\sqrt{\frac{a}{\cosh(x)^2}}}{\cosh\,(x)\,\sinh\,(x)}\,dx$$

```
[In] int((x*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
```

```
[Out] int((x*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.849 $\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 187

$$\begin{aligned}\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx &= x^2\sqrt{a\text{sech}^2(x)} - 4x\arctan\left(e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &\quad - 2x^2\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &\quad - 2x\cosh(x)\,\text{PolyLog}\left(2,-e^x\right)\sqrt{a\text{sech}^2(x)} \\ &\quad + 2i\cosh(x)\,\text{PolyLog}\left(2,-ie^x\right)\sqrt{a\text{sech}^2(x)} \\ &\quad - 2i\cosh(x)\,\text{PolyLog}\left(2,ie^x\right)\sqrt{a\text{sech}^2(x)} \\ &\quad + 2x\cosh(x)\,\text{PolyLog}\left(2,e^x\right)\sqrt{a\text{sech}^2(x)} \\ &\quad + 2\cosh(x)\,\text{PolyLog}\left(3,-e^x\right)\sqrt{a\text{sech}^2(x)} \\ &\quad - 2\cosh(x)\,\text{PolyLog}\left(3,e^x\right)\sqrt{a\text{sech}^2(x)}\end{aligned}$$

```
[Out] x^2*(a*sech(x)^2)^(1/2)-4*x*arctan(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x^
2*arctanh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*x*cosh(x)*polylog(2,-exp(x)
)*(a*sech(x)^2)^(1/2)+2*I*cosh(x)*polylog(2,-I*exp(x))*(a*sech(x)^2)^(1/2)-
2*I*cosh(x)*polylog(2,I*exp(x))*(a*sech(x)^2)^(1/2)+2*x*cosh(x)*polylog(2,e
xp(x))*(a*sech(x)^2)^(1/2)+2*cosh(x)*polylog(3,-exp(x))*(a*sech(x)^2)^(1/2)
-2*cosh(x)*polylog(3,exp(x))*(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6852, 2702, 327, 213, 5570, 14, 6408, 4267, 2611, 2320, 6724, 4265, 2317, 2438}

$$\begin{aligned}\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = &-4x\arctan\left(e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &-2x^2\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &-2x\,\text{PolyLog}\left(2,-e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &+2x\,\text{PolyLog}\left(2,e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &+2i\,\text{PolyLog}\left(2,-ie^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &-2i\,\text{PolyLog}\left(2,ie^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &+2\,\text{PolyLog}\left(3,-e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &-2\,\text{PolyLog}\left(3,e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)} \\ &+x^2\sqrt{a\text{sech}^2(x)}\end{aligned}$$

```
[In] Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x^2*Sqrt[a*Sech[x]^2] - 4*x*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^2*A
rcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x*Cosh[x]*PolyLog[2, -E^x]*Sqrt[a
*Sech[x]^2] + (2*I)*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] - (2*I)*
Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 2*x*Cosh[x]*PolyLog[2, E^x]*S
qrt[a*Sech[x]^2] + 2*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2] - 2*Cosh[x]
*PolyLog[3, E^x]*Sqrt[a*Sech[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2\text{csch}(x)\text{sech}^2(x)\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x(-\text{arctanh}(\cosh(x)) + \text{sech}(x))\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int(-x\text{arctanh}(\cosh(x)) + x\text{sech}(x))\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - x^2\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\text{arctanh}(\cosh(x))\,dx - \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\text{sech}(x)\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(2i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\log(1-ie^x)\,dx - \left(2i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\log(1+ie^x)\,dx + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2\text{csch}(x)\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(2i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^x\right) - \left(2i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^x\right) - \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\log(1-e^x)\,dx + \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\log(1+e^x)\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)} + 2i\cosh(x)\text{PolyLog}(2,-ie^x)\sqrt{a\text{sech}^2(x)} - 2i\cosh(x)\text{PolyLog}(2,ie^x)\sqrt{a\text{sech}^2(x)} + 2x\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)} + \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\text{PolyLog}(2,-e^x)\,dx - \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\text{PolyLog}(2,e^x)\,dx$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)} + 2i\cosh(x)\text{PolyLog}(2,-ie^x)\sqrt{a\text{sech}^2(x)} - 2i\cosh(x)\text{PolyLog}(2,ie^x)\sqrt{a\text{sech}^2(x)} + 2x\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)} + \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^x\right) - \left(2\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^x\right)$$

$$= x^2\sqrt{a\text{sech}^2(x)} - 4x\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^2\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)} + 2i\cosh(x)\text{PolyLog}(2,-ie^x)\sqrt{a\text{sech}^2(x)} - 2i\cosh(x)\text{PolyLog}(2,ie^x)\sqrt{a\text{sech}^2(x)} + 2x\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)} + 2\cosh(x)\text{PolyLog}(3,-e^x)\sqrt{a\text{sech}^2(x)} - 2\cosh(x)\text{PolyLog}(3,e^x)\sqrt{a\text{sech}^2(x)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(x^2 - 2i\cosh(x)\left(x(\log(1-ie^x) - \log(1+ie^x)) - \text{PolyLog}(2,-ie^x) + \text{PolyLog}(2,ie^x)\right) + \cosh(x)\left(x^2\log(1-e^x) - x^2\log(1+e^x) - 2x\,\text{PolyLog}(2,-e^x) + 2x\,\text{PolyLog}(2,e^x) + 2\,\text{PolyLog}(3,-e^x) - 2\,\text{PolyLog}(3,e^x)\right)\right)\sqrt{a\text{sech}^2(x)}$$

```
[In] Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x^2 - (2*I)*Cosh[x]*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I)
*E^x] + PolyLog[2, I*E^x]) + Cosh[x]*(x^2*Log[1 - E^x] - x^2*Log[1 + E^x] -
 2*x*PolyLog[2, -E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLo
g[3, E^x]))*Sqrt[a*Sech[x]^2]
```

Maple [F]

$$\int x^2 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}(x)^2} dx$$

```
[In] int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
[Out] int(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(149) = 298$.

Time $= 0.28$ (sec) , antiderivative size $= 786$, normalized size of antiderivative $= 4.20$

$$\int x^2 \operatorname{csch}(x) \operatorname{sech}(x) \sqrt{a \operatorname{sech}^2(x)} \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] -(2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*
polylog(3, cosh(x) + sinh(x)) - 2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (c
osh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(
e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - sinh(x)) - (2*x^2*cosh(
x)*e^(2*x) + 2*x^2*cosh(x) + 2*(x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2 + (
x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x)*d
ilog(cosh(x) + sinh(x)) - 2*((I*e^(2*x) + I)*sinh(x)^2 + I*cosh(x)^2 + (I*c
osh(x)^2 + I)*e^(2*x) + 2*(I*cosh(x)*e^(2*x) + I*cosh(x))*sinh(x) + I)*dilo
g(I*cosh(x) + I*sinh(x)) - 2*((-I*e^(2*x) - I)*sinh(x)^2 - I*cosh(x)^2 + (-
I*cosh(x)^2 - I)*e^(2*x) + 2*(-I*cosh(x)*e^(2*x) - I*cosh(x))*sinh(x) - I)*
dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^2 + (x*e^(2*x) + x)*sinh(x)^2
+ (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x
)*dilog(-cosh(x) - sinh(x)) - (x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^
2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*(-I*x*cosh(x)^2 + (-I*x*e^(2*x
) - I*x)*sinh(x)^2 + (-I*x*cosh(x)^2 - I*x)*e^(2*x) + 2*(-I*x*cosh(x)*e^(2*
x) - I*x*cosh(x))*sinh(x) - I*x)*log(I*cosh(x) + I*sinh(x) + 1) - 2*(I*x*co
sh(x)^2 + (I*x*e^(2*x) + I*x)*sinh(x)^2 + (I*x*cosh(x)^2 + I*x)*e^(2*x) + 2
*(I*x*cosh(x)*e^(2*x) + I*x*cosh(x))*sinh(x) + I*x)*log(-I*cosh(x) - I*sinh
(x) + 1) + (x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh
(x)^2 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*log(-
cosh(x) - sinh(x) + 1) + 2*(x^2*e^(2*x) + x^2)*sinh(x))*sqrt(a/(e^(4*x) + 2
*e^(2*x) + 1))*e^x)/(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1
)*e^x)
```

Sympy [F]

$$\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = \int x^2\sqrt{a\,\mathrm{sech}^2\,(x)}\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)\,dx$$

```
[In] integrate(x**2*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
[Out] Integral(x**2*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

Maxima [F]

$$\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = \int \sqrt{a\,\mathrm{sech}\,(x)^2}x^2\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)\,dx$$

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)*x^2*e^x/(e^(2*x) + 1) - (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*p
olylog(3, -e^x))*sqrt(a) + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(
3, e^x))*sqrt(a) - 4*sqrt(a)*integrate(x*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = \int \sqrt{a\,\mathrm{sech}\,(x)^2}x^2\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)\,dx$$

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^2)*x^2*csch(x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^2(x)}\,dx = \int \frac{x^2\sqrt{\frac{a}{\cosh(x)^2}}}{\cosh\,(x)\,\sinh\,(x)}\,dx$$

```
[In] int((x^2*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x^2*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.850 $\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 287

$$\begin{aligned}\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx &= x^3\sqrt{a\text{sech}^2(x)}-6x^2\arctan\left(e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&\quad-2x^3\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&\quad-3x^2\cosh(x)\,\text{PolyLog}\left(2,-e^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad+6ix\cosh(x)\,\text{PolyLog}\left(2,-ie^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad-6ix\cosh(x)\,\text{PolyLog}\left(2,ie^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad+3x^2\cosh(x)\,\text{PolyLog}\left(2,e^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad+6x\cosh(x)\,\text{PolyLog}\left(3,-e^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad-6i\cosh(x)\,\text{PolyLog}\left(3,-ie^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad+6i\cosh(x)\,\text{PolyLog}\left(3,ie^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad-6x\cosh(x)\,\text{PolyLog}\left(3,e^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad-6\cosh(x)\,\text{PolyLog}\left(4,-e^x\right)\sqrt{a\text{sech}^2(x)}\\&\quad+6\cosh(x)\,\text{PolyLog}\left(4,e^x\right)\sqrt{a\text{sech}^2(x)}\end{aligned}$$

```
[Out] x^3*(a*sech(x)^2)^(1/2)-6*x^2*arctan(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-2*
x^3*arctanh(exp(x))*cosh(x)*(a*sech(x)^2)^(1/2)-3*x^2*cosh(x)*polylog(2,-ex
p(x))*(a*sech(x)^2)^(1/2)+6*I*x*cosh(x)*polylog(2,-I*exp(x))*(a*sech(x)^2)^
(1/2)-6*I*x*cosh(x)*polylog(2,I*exp(x))*(a*sech(x)^2)^(1/2)+3*x^2*cosh(x)*p
olylog(2,exp(x))*(a*sech(x)^2)^(1/2)+6*x*cosh(x)*polylog(3,-exp(x))*(a*sech
```

```
(x)^2)^(1/2)-6*I*cosh(x)*polylog(3,-I*exp(x))*(a*sech(x)^2)^(1/2)+6*I*cosh(
x)*polylog(3,I*exp(x))*(a*sech(x)^2)^(1/2)-6*x*cosh(x)*polylog(3,exp(x))*(a
*sech(x)^2)^(1/2)-6*cosh(x)*polylog(4,-exp(x))*(a*sech(x)^2)^(1/2)+6*cosh(x
)*polylog(4,exp(x))*(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6852, 2702, 327, 213, 5570, 14, 6408, 4267, 2611, 6744, 2320, 6724, 4265}

$$\begin{aligned}\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = &-6x^2\arctan\left(e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&-2x^3\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&-3x^2\,\text{PolyLog}\left(2,-e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&+3x^2\,\text{PolyLog}\left(2,e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&+6ix\,\text{PolyLog}\left(2,-ie^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&-6ix\,\text{PolyLog}\left(2,ie^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&+6x\,\text{PolyLog}\left(3,-e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&-6x\,\text{PolyLog}\left(3,e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&-6i\,\text{PolyLog}\left(3,-ie^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&+6i\,\text{PolyLog}\left(3,ie^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&-6\,\text{PolyLog}\left(4,-e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&+6\,\text{PolyLog}\left(4,e^x\right)\cosh(x)\sqrt{a\text{sech}^2(x)}\\&+x^3\sqrt{a\text{sech}^2(x)}\end{aligned}$$

```
[In] Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] x^3*Sqrt[a*Sech[x]^2] - 6*x^2*ArcTan[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 2*x^3
*ArcTanh[E^x]*Cosh[x]*Sqrt[a*Sech[x]^2] - 3*x^2*Cosh[x]*PolyLog[2, -E^x]*Sq
rt[a*Sech[x]^2] + (6*I)*x*Cosh[x]*PolyLog[2, (-I)*E^x]*Sqrt[a*Sech[x]^2] -
(6*I)*x*Cosh[x]*PolyLog[2, I*E^x]*Sqrt[a*Sech[x]^2] + 3*x^2*Cosh[x]*PolyLog
[2, E^x]*Sqrt[a*Sech[x]^2] + 6*x*Cosh[x]*PolyLog[3, -E^x]*Sqrt[a*Sech[x]^2]
 - (6*I)*Cosh[x]*PolyLog[3, (-I)*E^x]*Sqrt[a*Sech[x]^2] + (6*I)*Cosh[x]*Pol
yLog[3, I*E^x]*Sqrt[a*Sech[x]^2] - 6*x*Cosh[x]*PolyLog[3, E^x]*Sqrt[a*Sech[
x]^2] - 6*Cosh[x]*PolyLog[4, -E^x]*Sqrt[a*Sech[x]^2] + 6*Cosh[x]*PolyLog[4,
 E^x]*Sqrt[a*Sech[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^3\text{csch}(x)\text{sech}^2(x)\,dx$$

$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(3\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2(-\text{arctanh}(\cosh(x)) + \text{sech}(x))\,dx$$

$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} - \left(3\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int \left(-x^2\text{arctanh}(\cosh(x)) + x^2\text{sech}(x)\right)\,dx$$

$$= x^3\sqrt{a\text{sech}^2(x)} - x^3\text{arctanh}(\cosh(x))\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(3\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2\text{arctanh}(\cosh(x))\,dx - \left(3\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^2\text{sech}(x)\,dx$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} + \left(6i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\log(1-ie^x)\,dx - \left(6i\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x\log(1+ie^x)\,dx + \left(\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int x^3\text{csch}(x)\,dx$$

$$
\begin{aligned}
&= x^3\sqrt{a\operatorname{sech}^2(x)} - 6x^2\arctan\left(e^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad - 2x^3\operatorname{arctanh}(e^x)\cosh(x)\sqrt{a\operatorname{sech}^2(x)} + 6ix\cosh(x)\operatorname{PolyLog}\left(2,-ie^x\right)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad - 6ix\cosh(x)\operatorname{PolyLog}\left(2,ie^x\right)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad - \left(6i\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\int \operatorname{PolyLog}\left(2,-ie^x\right)\,dx \\
&\quad + \left(6i\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\int \operatorname{PolyLog}\left(2,ie^x\right)\,dx \\
&\quad - \left(3\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\int x^2\log\left(1-e^x\right)\,dx \\
&\quad + \left(3\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\int x^2\log\left(1+e^x\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= x^3\sqrt{a\operatorname{sech}^2(x)} - 6x^2\arctan\left(e^x\right)\cosh(x)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad - 2x^3\operatorname{arctanh}(e^x)\cosh(x)\sqrt{a\operatorname{sech}^2(x)} - 3x^2\cosh(x)\operatorname{PolyLog}\left(2,-e^x\right)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad + 6ix\cosh(x)\operatorname{PolyLog}\left(2,-ie^x\right)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad - 6ix\cosh(x)\operatorname{PolyLog}\left(2,ie^x\right)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad + 3x^2\cosh(x)\operatorname{PolyLog}\left(2,e^x\right)\sqrt{a\operatorname{sech}^2(x)} \\
&\quad - \left(6i\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x}\,dx, x, e^x\right) \\
&\quad + \left(6i\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x}\,dx, x, e^x\right) \\
&\quad + \left(6\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\int x\operatorname{PolyLog}\left(2,-e^x\right)\,dx \\
&\quad - \left(6\cosh(x)\sqrt{a\operatorname{sech}^2(x)}\right)\int x\operatorname{PolyLog}\left(2,e^x\right)\,dx
\end{aligned}
$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}$$
$$- 2x^3\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 3x^2\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 6ix\cosh(x)\text{PolyLog}(2,-ie^x)\sqrt{a\text{sech}^2(x)}$$
$$- 6ix\cosh(x)\text{PolyLog}(2,ie^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 3x^2\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 6x\cosh(x)\text{PolyLog}(3,-e^x)\sqrt{a\text{sech}^2(x)}$$
$$- 6i\cosh(x)\text{PolyLog}(3,-ie^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 6i\cosh(x)\text{PolyLog}(3,ie^x)\sqrt{a\text{sech}^2(x)} - 6x\cosh(x)\text{PolyLog}(3,e^x)\sqrt{a\text{sech}^2(x)}$$
$$- \left(6\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\text{PolyLog}(3,-e^x)\,dx$$
$$+ \left(6\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\int\text{PolyLog}(3,e^x)\,dx$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)}$$
$$- 2x^3\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 3x^2\cosh(x)\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 6ix\cosh(x)\text{PolyLog}(2,-ie^x)\sqrt{a\text{sech}^2(x)}$$
$$- 6ix\cosh(x)\text{PolyLog}(2,ie^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 3x^2\cosh(x)\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 6x\cosh(x)\text{PolyLog}(3,-e^x)\sqrt{a\text{sech}^2(x)}$$
$$- 6i\cosh(x)\text{PolyLog}(3,-ie^x)\sqrt{a\text{sech}^2(x)}$$
$$+ 6i\cosh(x)\text{PolyLog}(3,ie^x)\sqrt{a\text{sech}^2(x)} - 6x\cosh(x)\text{PolyLog}(3,e^x)\sqrt{a\text{sech}^2(x)}$$
$$- \left(6\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(3,-x)}{x}dx,x,e^x\right)$$
$$+ \left(6\cosh(x)\sqrt{a\text{sech}^2(x)}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(3,x)}{x}dx,x,e^x\right)$$

$$= x^3\sqrt{a\text{sech}^2(x)} - 6x^2\arctan(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 2x^3\text{arctanh}(e^x)\cosh(x)\sqrt{a\text{sech}^2(x)} - 3x^2\cosh(x)\,\text{PolyLog}(2,-e^x)\sqrt{a\text{sech}^2(x)} + 6ix\cosh(x)\,\text{PolyLog}(2,-ie^x)\sqrt{a\text{sech}^2(x)} - 6ix\cosh(x)\,\text{PolyLog}(2,ie^x)\sqrt{a\text{sech}^2(x)} + 3x^2\cosh(x)\,\text{PolyLog}(2,e^x)\sqrt{a\text{sech}^2(x)} + 6x\cosh(x)\,\text{PolyLog}(3,-e^x)\sqrt{a\text{sech}^2(x)} - 6i\cosh(x)\,\text{PolyLog}(3,-ie^x)\sqrt{a\text{sech}^2(x)} + 6i\cosh(x)\,\text{PolyLog}(3,ie^x)\sqrt{a\text{sech}^2(x)} - 6x\cosh(x)\,\text{PolyLog}(3,e^x)\sqrt{a\text{sech}^2(x)} - 6\cosh(x)\,\text{PolyLog}(4,-e^x)\sqrt{a\text{sech}^2(x)} + 6\cosh(x)\,\text{PolyLog}(4,e^x)\sqrt{a\text{sech}^2(x)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.63

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \left(x^3 - 3i\cosh(x)\left(x^2\log(1-ie^x) - x^2\log(1+ie^x) - 2x\,\text{PolyLog}(2,-ie^x) + 2x\,\text{PolyLog}(2,ie^x) + 2\,\text{PolyLog}(3,-ie^x) - 2\,\text{PolyLog}(3,ie^x)\right) + \cosh(x)\left(x^3\log(1-e^x) - x^3\log(1+e^x) - 3x^2\,\text{PolyLog}(2,-e^x) + 3x^2\,\text{PolyLog}(2,e^x) + 6x\,\text{PolyLog}(3,-e^x) - 6x\,\text{PolyLog}(3,e^x) - 6\,\text{PolyLog}(4,-e^x) + 6\,\text{PolyLog}(4,e^x)\right)\right)\sqrt{a\text{sech}^2(x)}$$

```
[In] Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^2],x]
```

```
[Out] (x^3 - (3*I)*Cosh[x]*(x^2*Log[1 - I*E^x] - x^2*Log[1 + I*E^x] - 2*x*PolyLog
[2, (-I)*E^x] + 2*x*PolyLog[2, I*E^x] + 2*PolyLog[3, (-I)*E^x] - 2*PolyLog[
3, I*E^x]) + Cosh[x]*(x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2
, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x
] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x]))*Sqrt[a*Sech[x]^2]
```

Maple [F]

$$\int x^3\,\text{csch}(x)\,\text{sech}(x)\,\sqrt{a\,\text{sech}(x)^2}\,dx$$

```
[In] int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

```
[Out] int(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(229) = 458.

Time = 0.29 (sec) , antiderivative size = 1202, normalized size of antiderivative = 4.19

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh
(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*p
olylog(4, cosh(x) + sinh(x)) - 6*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (co
sh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e
^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(4, -cosh(x) - sinh(x)) - 6*(x*cosh(x)^
2 + (x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^
(2*x) + x*cosh(x))*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polyl
og(3, cosh(x) + sinh(x)) - 6*((-I*e^(2*x) - I)*sinh(x)^2 - I*cosh(x)^2 + (-
I*cosh(x)^2 - I)*e^(2*x) + 2*(-I*cosh(x)*e^(2*x) - I*cosh(x))*sinh(x) - I)*
sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, I*cosh(x) + I*sinh(x)) - 6
*((I*e^(2*x) + I)*sinh(x)^2 + I*cosh(x)^2 + (I*cosh(x)^2 + I)*e^(2*x) + 2*(
I*cosh(x)*e^(2*x) + I*cosh(x))*sinh(x) + I)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1
))*e^x*polylog(3, -I*cosh(x) - I*sinh(x)) + 6*(x*cosh(x)^2 + (x*e^(2*x) + x
)*sinh(x)^2 + (x*cosh(x)^2 + x)*e^(2*x) + 2*(x*cosh(x)*e^(2*x) + x*cosh(x))
*sinh(x) + x)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x*polylog(3, -cosh(x) - s
inh(x)) + (2*x^3*cosh(x)*e^(2*x) + 2*x^3*cosh(x) + 3*(x^2*cosh(x)^2 + (x^2*
e^(2*x) + x^2)*sinh(x)^2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x^2*cos
h(x)*e^(2*x) + x^2*cosh(x))*sinh(x))*dilog(cosh(x) + sinh(x)) - 6*(I*x*cosh
(x)^2 + (I*x*e^(2*x) + I*x)*sinh(x)^2 + (I*x*cosh(x)^2 + I*x)*e^(2*x) + 2*(
I*x*cosh(x)*e^(2*x) + I*x*cosh(x))*sinh(x) + I*x)*dilog(I*cosh(x) + I*sinh(
x)) - 6*(-I*x*cosh(x)^2 + (-I*x*e^(2*x) - I*x)*sinh(x)^2 + (-I*x*cosh(x)^2
- I*x)*e^(2*x) + 2*(-I*x*cosh(x)*e^(2*x) - I*x*cosh(x))*sinh(x) - I*x)*dilo
g(-I*cosh(x) - I*sinh(x)) - 3*(x^2*cosh(x)^2 + (x^2*e^(2*x) + x^2)*sinh(x)^
2 + x^2 + (x^2*cosh(x)^2 + x^2)*e^(2*x) + 2*(x^2*cosh(x)*e^(2*x) + x^2*cosh
(x))*sinh(x))*dilog(-cosh(x) - sinh(x)) - (x^3*cosh(x)^2 + x^3 + (x^3*e^(2*
x) + x^3)*sinh(x)^2 + (x^3*cosh(x)^2 + x^3)*e^(2*x) + 2*(x^3*cosh(x)*e^(2*x
) + x^3*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*(-I*x^2*cosh(x)^2
+ (-I*x^2*e^(2*x) - I*x^2)*sinh(x)^2 - I*x^2 + (-I*x^2*cosh(x)^2 - I*x^2)*e
^(2*x) + 2*(-I*x^2*cosh(x)*e^(2*x) - I*x^2*cosh(x))*sinh(x))*log(I*cosh(x)
+ I*sinh(x) + 1) - 3*(I*x^2*cosh(x)^2 + (I*x^2*e^(2*x) + I*x^2)*sinh(x)^2 +
 I*x^2 + (I*x^2*cosh(x)^2 + I*x^2)*e^(2*x) + 2*(I*x^2*cosh(x)*e^(2*x) + I*x
^2*cosh(x))*sinh(x))*log(-I*cosh(x) - I*sinh(x) + 1) + (x^3*cosh(x)^2 + x^3
 + (x^3*e^(2*x) + x^3)*sinh(x)^2 + (x^3*cosh(x)^2 + x^3)*e^(2*x) + 2*(x^3*c
osh(x)*e^(2*x) + x^3*cosh(x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 2*(x^3
```

```
*e^(2*x) + x^3)*sinh(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x)/(2*cosh(x)*
e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx = \int x^3\sqrt{a\operatorname{sech}^2(x)}\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

```
[In] integrate(x**3*csch(x)*sech(x)*(a*sech(x)**2)**(1/2),x)
[Out] Integral(x**3*sqrt(a*sech(x)**2)*csch(x)*sech(x), x)
```

Maxima [F]

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx = \int \sqrt{a\operatorname{sech}(x)^2}x^3\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)*x^3*e^x/(e^(2*x) + 1) - (x^3*log(e^x + 1) + 3*x^2*dilog(-e^x) - 6
*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) + (x^3*log(-e^x + 1) + 3*
x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4, e^x))*sqrt(a) - 12*sqrt
(a)*integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx = \int \sqrt{a\operatorname{sech}(x)^2}x^3\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^2)*x^3*csch(x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^2(x)}\,dx = \int \frac{x^3\sqrt{\frac{a}{\cosh(x)^2}}}{\cosh(x)\sinh(x)}\,dx$$

```
[In] int((x^3*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x^3*(a/cosh(x)^2)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.851 $\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\begin{aligned}\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx &= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\ &\quad - 2x\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\ &\quad - \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\ &\quad + \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\ &\quad - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)\\ &\quad - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x)\end{aligned}$$

```
[Out] 1/2*x*cosh(x)^2*(a*sech(x)^4)^(1/2)-2*x*arctanh(exp(2*x))*cosh(x)^2*(a*sech
(x)^4)^(1/2)-1/2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+1/2*cos
h(x)^2*polylog(2,exp(2*x))*(a*sech(x)^4)^(1/2)-1/2*cosh(x)*sinh(x)*(a*sech(
x)^4)^(1/2)-1/2*x*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules

used = {6852, 2700, 14, 5570, 2628, 5569, 4267, 2317, 2438, 3554, 8}

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = -2x\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sinh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}$$

```
[In] Int[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*S
ech[x]^4] - (Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (Cosh[x]
^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[x]*Sqrt[a*Sech[x]^4]*Si
nh[x])/2 - (x*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\text{csch}(x)\text{sech}^3(x)\,dx\\
&= x\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x)\\
&\quad - \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int\left(\log(\tanh(x)) - \frac{\tanh^2(x)}{2}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= x\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad + \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \tanh^2(x)\,dx \\
&\quad - \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \log(\tanh(x))\,dx \\
&= -\frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad + \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int 1\,dx \\
&\quad + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\text{csch}(x)\text{sech}(x)\,dx \\
&= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) \\
&\quad - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\text{csch}(2x)\,dx \\
&= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\
&\quad - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad - \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \log\left(1-e^{2x}\right)\,dx \\
&\quad + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \log\left(1+e^{2x}\right)\,dx \\
&= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\
&\quad - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad - \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&\quad + \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{1}{2}x\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\
&\quad - \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)} \\
&\quad + \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)} \\
&\quad - \frac{1}{2}\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x\sqrt{a\text{sech}^4(x)}\sinh^2(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \frac{1}{2}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(2x\log\left(1-e^{-2x}\right) - 2x\log\left(1+e^{-2x}\right) + \text{PolyLog}\left(2,-e^{-2x}\right) - \text{PolyLog}\left(2,e^{-2x}\right) + x\text{sech}^2(x) - \tanh(x)\right)$$

```
[In] Integrate[x*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2*x*Log[1 - E^(-2*x)] - 2*x*Log[1 + E^(-2*x)]
 + PolyLog[2, -E^(-2*x)] - PolyLog[2, E^(-2*x)] + x*Sech[x]^2 - Tanh[x]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(107) = 214.

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.91

method	result
risch	$\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(2x\,\mathrm{e}^{2x}+\mathrm{e}^{2x}+1)+\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(1+\mathrm{e}^{2x})^2\,x\ln(1+\mathrm{e}^{x})+\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\mathrm{e}^{-2x}(1+\mathrm{e}^2$

```
[In] int(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(2*x*exp(2*x)+exp(2*x)+1)+(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(x))+(a*exp
(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp(x))-(a*
exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(2*x))-1/
2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp
(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1-ex
p(x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,
exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1757, normalized size of antiderivative = 13.31

$$\int x\operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] ((2*x + 1)*cosh(x)^2 + ((2*x + 1)*e^(4*x) + 2*(2*x + 1)*e^(2*x) + 2*x + 1)*
sinh(x)^2 + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e
^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(
x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^
2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)
*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + co
sh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(cosh(x) + sinh(x)) - ((e^(4*x)
 + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^
(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2
*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*co
sh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)
^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh
(x))*sinh(x) + 1)*dilog(I*cosh(x) + I*sinh(x)) - ((e^(4*x) + 2*e^(2*x) + 1)
*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*
sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1
)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4
*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 +
 cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)
*dilog(-I*cosh(x) - I*sinh(x)) + ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cos
h(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3
*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*s
inh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)
^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x
) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*dilog(-cosh(x)
- sinh(x)) + ((2*x + 1)*cosh(x)^2 + 1)*e^(4*x) + 2*((2*x + 1)*cosh(x)^2 + 1
)*e^(2*x) + (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*c
osh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2
 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e
^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*co
sh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh
(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) +
 x)*log(cosh(x) + sinh(x) + 1) - (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) +
x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh
(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*
(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 +
```

```
x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3
+ x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x
))*e^(2*x))*sinh(x) + x)*log(I*cosh(x) + I*sinh(x) + 1) - (x*cosh(x)^4 + (x
*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*
e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*co
sh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*co
sh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*
e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) +
2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*log(-I*cosh(x) - I*sinh(x
) + 1) + (x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh
(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 +
2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2
*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(
x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)
^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)
*log(-cosh(x) - sinh(x) + 1) + 2*((2*x + 1)*cosh(x)*e^(4*x) + 2*(2*x + 1)*c
osh(x)*e^(2*x) + (2*x + 1)*cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x
) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2
*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh
(x))*e^(2*x)*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \int x\sqrt{a\,\text{sech}^4\,(x)}\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
[Out] Integral(x*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\begin{aligned}\int x\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = & -\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)+\text{Li}_2(-e^{(2\,x)})\right)\sqrt{a} \\ & +\left(x\log\left(e^{x}+1\right)+\text{Li}_2(-e^{x})\right)\sqrt{a} \\ & +\left(x\log\left(-e^{x}+1\right)+\text{Li}_2(e^{x})\right)\sqrt{a} \\ & +\frac{(2\sqrt{a}x+\sqrt{a})e^{(2\,x)}+\sqrt{a}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}\end{aligned}$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (x*log(e^x + 1) + d
ilog(-e^x))*sqrt(a) + (x*log(-e^x + 1) + dilog(e^x))*sqrt(a) + ((2*sqrt(a)*
x + sqrt(a))*e^(2*x) + sqrt(a))/(e^(4*x) + 2*e^(2*x) + 1)
```

Giac [F]

$$\int x\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^4(x)}\,dx = \int \sqrt{a\,\mathrm{sech}\,(x)^4}x\,\mathrm{csch}\,(x)\,\mathrm{sech}\,(x)\,dx$$

```
[In] integrate(x*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^4)*x*csch(x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^4(x)}\,dx = \int \frac{x\sqrt{\frac{a}{\cosh(x)^4}}}{\cosh\,(x)\,\sinh\,(x)}\,dx$$

```
[In] int((x*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.852 $\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^4(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 204

$$\begin{aligned}\int x^2\mathrm{csch}(x)\mathrm{sech}(x)\sqrt{a\mathrm{sech}^4(x)}\,dx &= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\mathrm{sech}^4(x)} \\ &- 2x^2\mathrm{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\mathrm{sech}^4(x)} \\ &+ \cosh^2(x)\log(\cosh(x))\sqrt{a\mathrm{sech}^4(x)} \\ &- x\cosh^2(x)\,\mathrm{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)} \\ &+ x\cosh^2(x)\,\mathrm{PolyLog}\left(2,e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)} \\ &+ \frac{1}{2}\cosh^2(x)\,\mathrm{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)} \\ &- \frac{1}{2}\cosh^2(x)\,\mathrm{PolyLog}\left(3,e^{2x}\right)\sqrt{a\mathrm{sech}^4(x)} \\ &- x\cosh(x)\sqrt{a\mathrm{sech}^4(x)}\sinh(x) \\ &- \frac{1}{2}x^2\sqrt{a\mathrm{sech}^4(x)}\sinh^2(x)\end{aligned}$$

```
[Out] 1/2*x^2*cosh(x)^2*(a*sech(x)^4)^(1/2)-2*x^2*arctanh(exp(2*x))*cosh(x)^2*(a*
sech(x)^4)^(1/2)+cosh(x)^2*ln(cosh(x))*(a*sech(x)^4)^(1/2)-x*cosh(x)^2*poly
log(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+x*cosh(x)^2*polylog(2,exp(2*x))*(a*sec
h(x)^4)^(1/2)+1/2*cosh(x)^2*polylog(3,-exp(2*x))*(a*sech(x)^4)^(1/2)-1/2*co
sh(x)^2*polylog(3,exp(2*x))*(a*sech(x)^4)^(1/2)-x*cosh(x)*sinh(x)*(a*sech(x
)^4)^(1/2)-1/2*x^2*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6852, 2700, 14, 5570, 2631, 5569, 4267, 2611, 2320, 6724, 3801, 3556, 30}

$$\begin{aligned}\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = & -2x^2\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\ & - x\,\text{PolyLog}\left(2,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\ & + x\,\text{PolyLog}\left(2,e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\ & + \frac{1}{2}\text{PolyLog}\left(3,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\ & - \frac{1}{2}\text{PolyLog}\left(3,e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} \\ & + \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sinh^2(x)\sqrt{a\text{sech}^4(x)} \\ & + \cosh^2(x)\sqrt{a\text{sech}^4(x)}\log(\cosh(x)) \\ & - x\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}\end{aligned}$$

```
[In] Int[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 - 2*x^2*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt
[a*Sech[x]^4] + Cosh[x]^2*Log[Cosh[x]]*Sqrt[a*Sech[x]^4] - x*Cosh[x]^2*Poly
Log[2, -E^(2*x)]*Sqrt[a*Sech[x]^4] + x*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a
*Sech[x]^4] + (Cosh[x]^2*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (Cosh[
x]^2*PolyLog[3, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - x*Cosh[x]*Sqrt[a*Sech[x]^4]
*Sinh[x] - (x^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\text{csch}(x)\text{sech}^3(x)\,dx\\
&= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\\
&\quad - \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\left(\log(\tanh(x)) - \frac{\tanh^2(x)}{2}\right)dx\\
&= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\\
&\quad - \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int\left(x\log(\tanh(x)) - \frac{1}{2}x\tanh^2(x)\right)dx\\
&= x^2\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\\
&\quad + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\tanh^2(x)\,dx\\
&\quad - \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\log(\tanh(x))\,dx
\end{aligned}$$

$$= -x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\,dx + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\text{csch}(x)\text{sech}(x)\,dx + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \tanh(x)\,dx$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\text{csch}(2x)\,dx$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\text{arctanh}(e^{2x})\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) - \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\log\left(1-e^{2x}\right)\,dx + \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\log\left(1+e^{2x}\right)\,dx$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\text{arctanh}(e^{2x})\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)} + x\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)} - x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{PolyLog}\left(2,-e^{2x}\right)\,dx - \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{PolyLog}\left(2,e^{2x}\right)\,dx$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\text{arctanh}(e^{2x})\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)} + x\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)} - x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x) + \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2x}\right) - \frac{1}{2}\left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2x}\right)$$

$$= \frac{1}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^2\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \cosh^2(x)\log(\cosh(x))\sqrt{a\text{sech}^4(x)} - x\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)} + x\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\text{sech}^4(x)} - \frac{1}{2}\cosh^2(x)\,\text{PolyLog}\left(3,e^{2x}\right)\sqrt{a\text{sech}^4(x)} - x\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \frac{1}{2}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\left(-2x + 2x^2\log\left(1-e^{-2x}\right) - 2x^2\log\left(1+e^{-2x}\right) + 2\log\left(1+e^{2x}\right) + 2x\,\text{PolyLog}\left(2,-e^{-2x}\right) - 2x\,\text{PolyLog}\left(2,e^{-2x}\right) + \text{PolyLog}\left(3,-e^{-2x}\right) - \text{PolyLog}\left(3,e^{-2x}\right) + x^2\text{sech}^2(x) - 2x\tanh(x)\right)$$

```
[In] Integrate[x^2*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(-2*x + 2*x^2*Log[1 - E^(-2*x)] - 2*x^2*Log[1
+ E^(-2*x)] + 2*Log[1 + E^(2*x)] + 2*x*PolyLog[2, -E^(-2*x)] - 2*x*PolyLog[
2, E^(-2*x)] + PolyLog[3, -E^(-2*x)] - PolyLog[3, E^(-2*x)] + x^2*Sech[x]^2
 - 2*x*Tanh[x]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(173) = 346.

Time = 0.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.16

method	result
risch	$2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}x(x\,e^{2x}+e^{2x}+1) - 2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2x})^2\ln(e^x) + \sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2x})^2\ln$

```
[In] int(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*x*(x*exp(2*x)+exp(2*x)+1)-2*(
a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*ln(exp(x))+(a*exp
```

```
(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*ln(1+exp(2*x))+(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*ln(1+exp(x))+2*(a*e
xp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(2,-exp(x))
-2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(3,-ex
p(x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*ln(1+e
xp(2*x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polyl
og(2,-exp(2*x))+1/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x)
)^2*polylog(3,-exp(2*x))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp
(2*x))^2*x^2*ln(1-exp(x))+2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+
exp(2*x))^2*x*polylog(2,exp(x))-2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*
x)*(1+exp(2*x))^2*polylog(3,exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 3431, normalized size of antiderivative = 16.82

$$\int x^2 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1
)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cos
h(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x)
 + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e
^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*
e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) - 2*((e^(4*x) + 2*e^(2*
x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x
)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 +
1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh
(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(
x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*p
olylog(3, I*cosh(x) + I*sinh(x)) - 2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 +
 cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 +
2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) +
1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cos
h(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^
(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(
8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -I*cosh(x
) - I*sinh(x)) + 2*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(co
sh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (
3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*c
```

```
osh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)
^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)
^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) +
 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) + (2*x*
cosh(x)^4 + 2*(x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 8*(x*cosh(x)*e^(4*x
) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 - 2*(x^2 - x)*cosh(x)^2 + 2*
(6*x*cosh(x)^2 - x^2 + (6*x*cosh(x)^2 - x^2 + x)*e^(4*x) + 2*(6*x*cosh(x)^2
 - x^2 + x)*e^(2*x) + x)*sinh(x)^2 - 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2
*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x)
)*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x
) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(
x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh
(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*
cosh(x))*e^(2*x))*sinh(x) + x)*dilog(cosh(x) + sinh(x)) + 2*(x*cosh(x)^4 +
(x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x
)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*
cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*
cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x
)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x)
+ 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*dilog(I*cosh(x) + I*sin
h(x)) + 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cos
h(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 +
 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(
2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh
(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x
)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x
)*dilog(-I*cosh(x) - I*sinh(x)) - 2*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x)
 + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*s
inh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) +
 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^
2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)
^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cos
h(x))*e^(2*x))*sinh(x) + x)*dilog(-cosh(x) - sinh(x)) + 2*(x*cosh(x)^4 - (x
^2 - x)*cosh(x)^2)*e^(4*x) + 4*(x*cosh(x)^4 - (x^2 - x)*cosh(x)^2)*e^(2*x)
- (x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*co
sh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sin
h(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*
x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh
(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) +
4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*
(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*log(cosh(x) + sinh(x) + 1)
- ((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2
*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*
e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) +
```

```
4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(
2*x) + cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + I) - ((e^(4*x) + 2*e^(
2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) +
cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2
+ 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (co
sh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sin
h(x) + 1)*log(cosh(x) + sinh(x) - I) + (x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^
2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x
^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3
*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^
2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^
4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*
cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))
*sinh(x))*log(I*cosh(x) + I*sinh(x) + 1) + (x^2*cosh(x)^4 + (x^2*e^(4*x) +
2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) +
 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2
+ (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh
(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh
(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (
x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2
*x))*sinh(x))*log(-I*cosh(x) - I*sinh(x) + 1) - (x^2*cosh(x)^4 + (x^2*e^(4*
x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4
*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 +
 x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))
*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2
*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x
) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))
*e^(2*x))*sinh(x))*log(-cosh(x) - sinh(x) + 1) + 4*(2*x*cosh(x)^3 - (x^2 -
x)*cosh(x) + (2*x*cosh(x)^3 - (x^2 - x)*cosh(x))*e^(4*x) + 2*(2*x*cosh(x)^3
 - (x^2 - x)*cosh(x))*e^(2*x))*sinh(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(
4*x) + 4*e^(2*x) + 1))*e^(2*x))/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh
(x)^4 + 2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^(
2*x)*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int x^2\sqrt{a\,\text{sech}^4\,(x)}\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

```
[In] integrate(x**2*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
[Out] Integral(x**2*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\begin{aligned}
&\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx \\
&= -\frac{1}{2}\left(2\,x^2\log\left(e^{(2\,x)}+1\right)+2\,x\text{Li}_2\left(-e^{(2\,x)}\right)-\text{Li}_3\left(-e^{(2\,x)}\right)\right)\sqrt{a} \\
&\quad+\left(x^2\log\left(e^x+1\right)+2\,x\text{Li}_2(-e^x)-2\,\text{Li}_3(-e^x)\right)\sqrt{a} \\
&\quad+\left(x^2\log\left(-e^x+1\right)+2\,x\text{Li}_2(e^x)-2\,\text{Li}_3(e^x)\right)\sqrt{a}-2\,\sqrt{a}x \\
&\quad+\sqrt{a}\log\left(e^{(2\,x)}+1\right)+\frac{2\left(\left(\sqrt{a}x^2+\sqrt{a}x\right)e^{(2\,x)}+\sqrt{a}x\right)}{e^{(4\,x)}+2\,e^{(2\,x)}+1}
\end{aligned}$$

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
[Out] -1/2*(2*x^2*log(e^(2*x) + 1) + 2*x*dilog(-e^(2*x)) - polylog(3, -e^(2*x)))*
sqrt(a) + (x^2*log(e^x + 1) + 2*x*dilog(-e^x) - 2*polylog(3, -e^x))*sqrt(a)
 + (x^2*log(-e^x + 1) + 2*x*dilog(e^x) - 2*polylog(3, e^x))*sqrt(a) - 2*sqr
t(a)*x + sqrt(a)*log(e^(2*x) + 1) + 2*((sqrt(a)*x^2 + sqrt(a)*x)*e^(2*x) +
sqrt(a)*x)/(e^(4*x) + 2*e^(2*x) + 1)
```

Giac [F]

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int\sqrt{a\,\text{sech}\,(x)^4}x^2\,\text{csch}\,(x)\,\text{sech}\,(x)\,\,dx$$

```
[In] integrate(x^2*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^4)*x^2*csch(x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx=\int\frac{x^2\sqrt{\frac{a}{\cosh(x)^4}}}{\cosh\,(x)\,\sinh\,(x)}\,dx$$

```
[In] int((x^2*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x^2*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.853 $\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 326

$$\begin{aligned}\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx &= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}+\frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\
&\quad-2x^3\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\
&\quad+3x\cosh^2(x)\log\left(1+e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad+\frac{3}{2}\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad-\frac{3}{2}x^2\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad+\frac{3}{2}x^2\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad+\frac{3}{2}x\cosh^2(x)\,\text{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad-\frac{3}{2}x\cosh^2(x)\,\text{PolyLog}\left(3,e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad-\frac{3}{4}\cosh^2(x)\,\text{PolyLog}\left(4,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad+\frac{3}{4}\cosh^2(x)\,\text{PolyLog}\left(4,e^{2x}\right)\sqrt{a\text{sech}^4(x)}\\
&\quad-\frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)\\
&\quad-\frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\end{aligned}$$

```
[Out] -3/2*x^2*cosh(x)^2*(a*sech(x)^4)^(1/2)+1/2*x^3*cosh(x)^2*(a*sech(x)^4)^(1/2
)-2*x^3*arctanh(exp(2*x))*cosh(x)^2*(a*sech(x)^4)^(1/2)+3*x*cosh(x)^2*ln(1+
exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)
^4)^(1/2)-3/2*x^2*cosh(x)^2*polylog(2,-exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*x^
```

```
2*cosh(x)^2*polylog(2,exp(2*x))*(a*sech(x)^4)^(1/2)+3/2*x*cosh(x)^2*polylog
(3,-exp(2*x))*(a*sech(x)^4)^(1/2)-3/2*x*cosh(x)^2*polylog(3,exp(2*x))*(a*se
ch(x)^4)^(1/2)-3/4*cosh(x)^2*polylog(4,-exp(2*x))*(a*sech(x)^4)^(1/2)+3/4*c
osh(x)^2*polylog(4,exp(2*x))*(a*sech(x)^4)^(1/2)-3/2*x^2*cosh(x)*sinh(x)*(a
*sech(x)^4)^(1/2)-1/2*x^3*sinh(x)^2*(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {6852, 2700, 14, 5570, 2631, 5569, 4267, 2611, 6744, 2320, 6724, 3801, 3799, 2221, 2317, 2438, 30}

$$\begin{aligned}\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = &-2x^3\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&-\frac{3}{2}x^2\operatorname{PolyLog}\left(2,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&+\frac{3}{2}x^2\operatorname{PolyLog}\left(2,e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&+\frac{3}{2}x\operatorname{PolyLog}\left(3,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&-\frac{3}{2}x\operatorname{PolyLog}\left(3,e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&+\frac{3}{2}\operatorname{PolyLog}\left(2,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&-\frac{3}{4}\operatorname{PolyLog}\left(4,-e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&+\frac{3}{4}\operatorname{PolyLog}\left(4,e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&+\frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}-\frac{1}{2}x^3\sinh^2(x)\sqrt{a\text{sech}^4(x)}\\&-\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\\&-\frac{3}{2}x^2\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}\\&+3x\log\left(e^{2x}+1\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)}\end{aligned}$$

```
[In] Int[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (-3*x^2*Cosh[x]^2*Sqrt[a*Sech[x]^4])/2 + (x^3*Cosh[x]^2*Sqrt[a*Sech[x]^4])/
2 - 2*x^3*ArcTanh[E^(2*x)]*Cosh[x]^2*Sqrt[a*Sech[x]^4] + 3*x*Cosh[x]^2*Log[
1 + E^(2*x)]*Sqrt[a*Sech[x]^4] + (3*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*S
ech[x]^4])/2 - (3*x^2*Cosh[x]^2*PolyLog[2, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 +
 (3*x^2*Cosh[x]^2*PolyLog[2, E^(2*x)]*Sqrt[a*Sech[x]^4])/2 + (3*x*Cosh[x]^2
*PolyLog[3, -E^(2*x)]*Sqrt[a*Sech[x]^4])/2 - (3*x*Cosh[x]^2*PolyLog[3, E^(2
```

```
*x)]*Sqrt[a*Sech[x]^4])/2 - (3*Cosh[x]^2*PolyLog[4, -E^(2*x)]*Sqrt[a*Sech[x
]^4])/4 + (3*Cosh[x]^2*PolyLog[4, E^(2*x)]*Sqrt[a*Sech[x]^4])/4 - (3*x^2*Co
sh[x]*Sqrt[a*Sech[x]^4]*Sinh[x])/2 - (x^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^3\text{csch}(x)\text{sech}^3(x)\,dx \\
&= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad - \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\left(\log(\tanh(x)) - \frac{\tanh^2(x)}{2}\right)dx \\
&= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad - \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int\left(x^2\log(\tanh(x)) - \frac{1}{2}x^2\tanh^2(x)\right)dx \\
&= x^3\cosh^2(x)\log(\tanh(x))\sqrt{a\text{sech}^4(x)} - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x) \\
&\quad + \frac{1}{2}\left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\tanh^2(x)\,dx \\
&\quad - \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\log(\tanh(x))\,dx
\end{aligned}$$

$$= -\frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)$$
$$+ \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^3\text{csch}(x)\text{sech}(x)\,dx$$
$$+ \frac{1}{2}\left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\,dx + \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\tanh(x)\,dx$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}$$
$$- \frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)$$
$$+ \left(2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^3\text{csch}(2x)\,dx$$
$$+ \left(6\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \frac{e^{2x}x}{1+e^{2x}}\,dx$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}$$
$$- 2x^3\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + 3x\cosh^2(x)\log\left(1+e^{2x}\right)\sqrt{a\text{sech}^4(x)}$$
$$- \frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)$$
$$- \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\log\left(1-e^{2x}\right)\,dx$$
$$- \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \log\left(1+e^{2x}\right)\,dx$$
$$+ \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x^2\log\left(1+e^{2x}\right)\,dx$$
$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}$$
$$- 2x^3\text{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\text{sech}^4(x)} + 3x\cosh^2(x)\log\left(1+e^{2x}\right)\sqrt{a\text{sech}^4(x)}$$
$$- \frac{3}{2}x^2\cosh^2(x)\,\text{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\text{sech}^4(x)}$$
$$+ \frac{3}{2}x^2\cosh^2(x)\,\text{PolyLog}\left(2,e^{2x}\right)\sqrt{a\text{sech}^4(x)}$$
$$- \frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)$$
$$- \frac{1}{2}\left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)$$
$$+ \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\,\text{PolyLog}\left(2,-e^{2x}\right)\,dx$$
$$- \left(3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int x\,\text{PolyLog}\left(2,e^{2x}\right)\,dx$$

$$
\begin{aligned}
&= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - 2x^3\operatorname{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} + 3x\cosh^2(x)\log\left(1+e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad + \frac{3}{2}\cosh^2(x)\operatorname{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - \frac{3}{2}x^2\cosh^2(x)\operatorname{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad + \frac{3}{2}x^2\cosh^2(x)\operatorname{PolyLog}\left(2,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad + \frac{3}{2}x\cosh^2(x)\operatorname{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - \frac{3}{2}x\cosh^2(x)\operatorname{PolyLog}\left(3,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} - \frac{3}{2}x^2\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\sinh(x) \\
&\quad - \frac{1}{2}x^3\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x) - \frac{1}{2}\left(3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\int \operatorname{PolyLog}\left(3,-e^{2x}\right)\,dx \\
&\quad + \frac{1}{2}\left(3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\int \operatorname{PolyLog}\left(3,e^{2x}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - 2x^3\operatorname{arctanh}\left(e^{2x}\right)\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)} + 3x\cosh^2(x)\log\left(1+e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad + \frac{3}{2}\cosh^2(x)\operatorname{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - \frac{3}{2}x^2\cosh^2(x)\operatorname{PolyLog}\left(2,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad + \frac{3}{2}x^2\cosh^2(x)\operatorname{PolyLog}\left(2,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad + \frac{3}{2}x\cosh^2(x)\operatorname{PolyLog}\left(3,-e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - \frac{3}{2}x\cosh^2(x)\operatorname{PolyLog}\left(3,e^{2x}\right)\sqrt{a\operatorname{sech}^4(x)} \\
&\quad - \frac{3}{2}x^2\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x) \\
&\quad - \frac{1}{4}\left(3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,-x)}{x}\,dx,x,e^{2x}\right) \\
&\quad + \frac{1}{4}\left(3\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{2x}\right)
\end{aligned}
$$

$$= -\frac{3}{2}x^2\cosh^2(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{2}x^3\cosh^2(x)\sqrt{a\text{sech}^4(x)} - 2x^3\text{arctanh}(e^{2x})\cosh^2(x)\sqrt{a\text{sech}^4(x)} + 3x\cosh^2(x)\log(1+e^{2x})\sqrt{a\text{sech}^4(x)} + \frac{3}{2}\cosh^2(x)\,\text{PolyLog}(2,-e^{2x})\sqrt{a\text{sech}^4(x)} - \frac{3}{2}x^2\cosh^2(x)\,\text{PolyLog}(2,-e^{2x})\sqrt{a\text{sech}^4(x)} + \frac{3}{2}x^2\cosh^2(x)\,\text{PolyLog}(2,e^{2x})\sqrt{a\text{sech}^4(x)} + \frac{3}{2}x\cosh^2(x)\,\text{PolyLog}(3,-e^{2x})\sqrt{a\text{sech}^4(x)} - \frac{3}{2}x\cosh^2(x)\,\text{PolyLog}(3,e^{2x})\sqrt{a\text{sech}^4(x)} - \frac{3}{4}\cosh^2(x)\,\text{PolyLog}(4,-e^{2x})\sqrt{a\text{sech}^4(x)} + \frac{3}{4}\cosh^2(x)\,\text{PolyLog}(4,e^{2x})\sqrt{a\text{sech}^4(x)} - \frac{3}{2}x^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{1}{2}x^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.46

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \frac{1}{4}\cosh^2(x)\sqrt{a\text{sech}^4(x)}\big(6x^2 + 4x^3\log(1-e^{-2x}) + 12x\log(1+e^{-2x}) - 4x^3\log(1+e^{-2x}) + 6(-1+x^2)\,\text{PolyLog}(2,-e^{-2x}) - 6x^2\,\text{PolyLog}(2,e^{-2x}) + 6x\,\text{PolyLog}(3,-e^{-2x}) - 6x\,\text{PolyLog}(3,e^{-2x}) + 3\,\text{PolyLog}(4,-e^{-2x}) - 3\,\text{PolyLog}(4,e^{-2x}) + 2x^3\text{sech}^2(x) - 6x^2\tanh(x)\big)$$

```
[In] Integrate[x^3*Csch[x]*Sech[x]*Sqrt[a*Sech[x]^4],x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(6*x^2 + 4*x^3*Log[1 - E^(-2*x)] + 12*x*Log[1
+ E^(-2*x)] - 4*x^3*Log[1 + E^(-2*x)] + 6*(-1 + x^2)*PolyLog[2, -E^(-2*x)]
- 6*x^2*PolyLog[2, E^(-2*x)] + 6*x*PolyLog[3, -E^(-2*x)] - 6*x*PolyLog[3, E
^(-2*x)] + 3*PolyLog[4, -E^(-2*x)] - 3*PolyLog[4, E^(-2*x)] + 2*x^3*Sech[x]
^2 - 6*x^2*Tanh[x]))/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(269) = 538.

Time = 0.12 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.85

method	result
risch	$\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}x^2(2x\,e^{2x}+3\,e^{2x}+3)-3\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2x})^2\,x^2+3\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2x})^2$

```
[In] int(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*x^2*(2*x*exp(2*x)+3*exp(2*x)+3)
-3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2+3*(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*ln(1+exp(2*x))+3/2*(a
*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(2,-exp(2*x
))+(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^3*ln(1+exp(
x))+3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*polylo
g(2,-exp(x))-6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x
*polylog(3,-exp(x))+6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*
x))^2*polylog(4,-exp(x))-(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp
(2*x))^2*x^3*ln(1+exp(2*x))-3/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)
*(1+exp(2*x))^2*x^2*polylog(2,-exp(2*x))+3/2*(a*exp(4*x)/(1+exp(2*x))^4)^(1
/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(3,-exp(2*x))-3/4*(a*exp(4*x)/(1+exp(
2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(4,-exp(2*x))+(a*exp(4*x)/(1
+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^3*ln(1-exp(x))+3*(a*exp(4*x)
/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x^2*polylog(2,exp(x))-6*(a*
exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*x*polylog(3,exp(x))
+6*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))^2*polylog(4,exp
(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 4629, normalized size of antiderivative = 14.20

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)
*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh
(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x)
```

+ 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, cosh(x) + sinh(x)) - 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, I*cosh(x) + I*sinh(x)) - 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, -I*cosh(x) - I*sinh(x)) + 6*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 2*(3*cosh(x)^2 + (3*cosh(x)^2 + 1)*e^(4*x) + 2*(3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 + 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(4*x) + 2*(cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) + 4*(cosh(x)^3 + (cosh(x)^3 + cosh(x))*e^(4*x) + 2*(cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(4, -cosh(x) - sinh(x)) - 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, cosh(x) + sinh(x)) + 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, I*cosh(x) + I*sinh(x)) + 6*(x*cosh(x)^4 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -I*cosh(x) - I*sinh(x)) - 6*(x*cosh(x)^4 + (x*e

^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^4 + 4*(x*cosh(x)*e^(4*x) + 2*x*cosh(x)*e^
(2*x) + x*cosh(x))*sinh(x)^3 + 2*x*cosh(x)^2 + 2*(3*x*cosh(x)^2 + (3*x*cosh
(x)^2 + x)*e^(4*x) + 2*(3*x*cosh(x)^2 + x)*e^(2*x) + x)*sinh(x)^2 + (x*cosh
(x)^4 + 2*x*cosh(x)^2 + x)*e^(4*x) + 2*(x*cosh(x)^4 + 2*x*cosh(x)^2 + x)*e^
(2*x) + 4*(x*cosh(x)^3 + x*cosh(x) + (x*cosh(x)^3 + x*cosh(x))*e^(4*x) + 2*
(x*cosh(x)^3 + x*cosh(x))*e^(2*x))*sinh(x) + x)*sqrt(a/(e^(8*x) + 4*e^(6*x)
 + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)*polylog(3, -cosh(x) - sinh(x)) - (3*
x^2*cosh(x)^4 + 3*(x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 + 12*(x^2*c
osh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh(x))*sinh(x)^3 - (2*x^3 -
3*x^2)*cosh(x)^2 + (18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2 + (18*x^2*cosh(x)^2 -
2*x^3 + 3*x^2)*e^(4*x) + 2*(18*x^2*cosh(x)^2 - 2*x^3 + 3*x^2)*e^(2*x))*sinh
(x)^2 - 3*(x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)^4 +
2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2*cosh
(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^(4*x)
 + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4 + 2*
x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^2)*e^
(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))*e^(4
x) + 2(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*dilog(cosh(x) + sin
h(x)) + 3*((x^2 - 1)*cosh(x)^4 + (x^2 + (x^2 - 1)*e^(4*x) + 2*(x^2 - 1)*e^(
2*x) - 1)*sinh(x)^4 + 4*((x^2 - 1)*cosh(x)*e^(4*x) + 2*(x^2 - 1)*cosh(x)*e^
(2*x) + (x^2 - 1)*cosh(x))*sinh(x)^3 + 2*(x^2 - 1)*cosh(x)^2 + 2*(3*(x^2 -
1)*cosh(x)^2 + x^2 + (3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*(3*(x^2
- 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) - 1)*sinh(x)^2 + x^2 + ((x^2 - 1)*cosh(x)
^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*((x^2 - 1)*cosh(x)^4 + 2*
(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) + 4*((x^2 - 1)*cosh(x)^3 + (x^2 - 1)
*cosh(x) + ((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(4*x) + 2*((x^2 - 1)
*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(2*x))*sinh(x) - 1)*dilog(I*cosh(x) + I*s
inh(x)) + 3*((x^2 - 1)*cosh(x)^4 + (x^2 + (x^2 - 1)*e^(4*x) + 2*(x^2 - 1)*e
^(2*x) - 1)*sinh(x)^4 + 4*((x^2 - 1)*cosh(x)*e^(4*x) + 2*(x^2 - 1)*cosh(x)*
e^(2*x) + (x^2 - 1)*cosh(x))*sinh(x)^3 + 2*(x^2 - 1)*cosh(x)^2 + 2*(3*(x^2
- 1)*cosh(x)^2 + x^2 + (3*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*(3*(x^
2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) - 1)*sinh(x)^2 + x^2 + ((x^2 - 1)*cosh(
x)^4 + 2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(4*x) + 2*((x^2 - 1)*cosh(x)^4 +
2*(x^2 - 1)*cosh(x)^2 + x^2 - 1)*e^(2*x) + 4*((x^2 - 1)*cosh(x)^3 + (x^2 -
1)*cosh(x) + ((x^2 - 1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(4*x) + 2*((x^2 -
1)*cosh(x)^3 + (x^2 - 1)*cosh(x))*e^(2*x))*sinh(x) - 1)*dilog(-I*cosh(x) -
I*sinh(x)) - 3*(x^2*cosh(x)^4 + (x^2*e^(4*x) + 2*x^2*e^(2*x) + x^2)*sinh(x)
^4 + 2*x^2*cosh(x)^2 + 4*(x^2*cosh(x)*e^(4*x) + 2*x^2*cosh(x)*e^(2*x) + x^2
*cosh(x))*sinh(x)^3 + 2*(3*x^2*cosh(x)^2 + x^2 + (3*x^2*cosh(x)^2 + x^2)*e^
(4*x) + 2*(3*x^2*cosh(x)^2 + x^2)*e^(2*x))*sinh(x)^2 + x^2 + (x^2*cosh(x)^4
 + 2*x^2*cosh(x)^2 + x^2)*e^(4*x) + 2*(x^2*cosh(x)^4 + 2*x^2*cosh(x)^2 + x^
2)*e^(2*x) + 4*(x^2*cosh(x)^3 + x^2*cosh(x) + (x^2*cosh(x)^3 + x^2*cosh(x))
*e^(4*x) + 2*(x^2*cosh(x)^3 + x^2*cosh(x))*e^(2*x))*sinh(x))*dilog(-cosh(x)
 - sinh(x)) + (3*x^2*cosh(x)^4 - (2*x^3 - 3*x^2)*cosh(x)^2)*e^(4*x) + 2*(3*
x^2*cosh(x)^4 - (2*x^3 - 3*x^2)*cosh(x)^2)*e^(2*x) - (x^3*cosh(x)^4 + 2*x^3

```
*cosh(x)^2 + (x^3*e^(4*x) + 2*x^3*e^(2*x) + x^3)*sinh(x)^4 + 4*(x^3*cosh(x)
*e^(4*x) + 2*x^3*cosh(x)*e^(2*x) + x^3*cosh(x))*sinh(x)^3 + x^3 + 2*(3*x^3*
cosh(x)^2 + x^3 + (3*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2*(3*x^3*cosh(x)^2 + x^
3)*e^(2*x))*sinh(x)^2 + (x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2
*(x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x^3)*e^(2*x) + 4*(x^3*cosh(x)^3 + x^3*c
osh(x) + (x^3*cosh(x)^3 + x^3*cosh(x))*e^(4*x) + 2*(x^3*cosh(x)^3 + x^3*cos
h(x))*e^(2*x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((x^3 - 3*x)*cosh(x)^4
 + (x^3 + (x^3 - 3*x)*e^(4*x) + 2*(x^3 - 3*x)*e^(2*x) - 3*x)*sinh(x)^4 + 4*
((x^3 - 3*x)*cosh(x)*e^(4*x) + 2*(x^3 - 3*x)*cosh(x)*e^(2*x) + (x^3 - 3*x)*
cosh(x))*sinh(x)^3 + x^3 + 2*(x^3 - 3*x)*cosh(x)^2 + 2*(x^3 + 3*(x^3 - 3*x)
*cosh(x)^2 + (x^3 + 3*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*(x^3 + 3*(x^
3 - 3*x)*cosh(x)^2 - 3*x)*e^(2*x) - 3*x)*sinh(x)^2 + ((x^3 - 3*x)*cosh(x)^4
 + x^3 + 2*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*((x^3 - 3*x)*cosh(x)^4
+ x^3 + 2*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(2*x) + 4*((x^3 - 3*x)*cosh(x)^3 +
 (x^3 - 3*x)*cosh(x) + ((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(4*x
) + 2*((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(2*x))*sinh(x) - 3*x)
*log(I*cosh(x) + I*sinh(x) + 1) + ((x^3 - 3*x)*cosh(x)^4 + (x^3 + (x^3 - 3*
x)*e^(4*x) + 2*(x^3 - 3*x)*e^(2*x) - 3*x)*sinh(x)^4 + 4*((x^3 - 3*x)*cosh(x
)*e^(4*x) + 2*(x^3 - 3*x)*cosh(x)*e^(2*x) + (x^3 - 3*x)*cosh(x))*sinh(x)^3
+ x^3 + 2*(x^3 - 3*x)*cosh(x)^2 + 2*(x^3 + 3*(x^3 - 3*x)*cosh(x)^2 + (x^3 +
 3*(x^3 - 3*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*(x^3 + 3*(x^3 - 3*x)*cosh(x)^2
- 3*x)*e^(2*x) - 3*x)*sinh(x)^2 + ((x^3 - 3*x)*cosh(x)^4 + x^3 + 2*(x^3 - 3
*x)*cosh(x)^2 - 3*x)*e^(4*x) + 2*((x^3 - 3*x)*cosh(x)^4 + x^3 + 2*(x^3 - 3*
x)*cosh(x)^2 - 3*x)*e^(2*x) + 4*((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x
) + ((x^3 - 3*x)*cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(4*x) + 2*((x^3 - 3*x)*
cosh(x)^3 + (x^3 - 3*x)*cosh(x))*e^(2*x))*sinh(x) - 3*x)*log(-I*cosh(x) - I
*sinh(x) + 1) - (x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + (x^3*e^(4*x) + 2*x^3*e^(
2*x) + x^3)*sinh(x)^4 + 4*(x^3*cosh(x)*e^(4*x) + 2*x^3*cosh(x)*e^(2*x) + x^
3*cosh(x))*sinh(x)^3 + x^3 + 2*(3*x^3*cosh(x)^2 + x^3 + (3*x^3*cosh(x)^2 +
x^3)*e^(4*x) + 2*(3*x^3*cosh(x)^2 + x^3)*e^(2*x))*sinh(x)^2 + (x^3*cosh(x)^
4 + 2*x^3*cosh(x)^2 + x^3)*e^(4*x) + 2*(x^3*cosh(x)^4 + 2*x^3*cosh(x)^2 + x
^3)*e^(2*x) + 4*(x^3*cosh(x)^3 + x^3*cosh(x) + (x^3*cosh(x)^3 + x^3*cosh(x)
)*e^(4*x) + 2*(x^3*cosh(x)^3 + x^3*cosh(x))*e^(2*x))*sinh(x))*log(-cosh(x)
- sinh(x) + 1) + 2*(6*x^2*cosh(x)^3 - (2*x^3 - 3*x^2)*cosh(x) + (6*x^2*cosh
(x)^3 - (2*x^3 - 3*x^2)*cosh(x))*e^(4*x) + 2*(6*x^2*cosh(x)^3 - (2*x^3 - 3*
x^2)*cosh(x))*e^(2*x))*sinh(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4
*e^(2*x) + 1))*e^(2*x))/(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^(2*x)*sin
h(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \int x^3\sqrt{a\,\text{sech}^4\,(x)}\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

```
[In] integrate(x**3*csch(x)*sech(x)*(a*sech(x)**4)**(1/2),x)
[Out] Integral(x**3*sqrt(a*sech(x)**4)*csch(x)*sech(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.63

$$\begin{aligned}
&\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx \\
&= -3\sqrt{a}x^2 \\
&\quad -\frac{1}{3}\left(4\,x^3\log\left(e^{(2\,x)}+1\right)+6\,x^2\text{Li}_2\left(-e^{(2\,x)}\right)-6\,x\text{Li}_3(-e^{(2\,x)})+3\,\text{Li}_4(-e^{(2\,x)})\right)\sqrt{a} \\
&\quad +\left(x^3\log\left(e^x+1\right)+3\,x^2\text{Li}_2(-e^x)-6\,x\text{Li}_3(-e^x)+6\,\text{Li}_4(-e^x)\right)\sqrt{a} \\
&\quad +\left(x^3\log\left(-e^x+1\right)+3\,x^2\text{Li}_2(e^x)-6\,x\text{Li}_3(e^x)+6\,\text{Li}_4(e^x)\right)\sqrt{a} \\
&\quad +\frac{3}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)+\text{Li}_2\left(-e^{(2\,x)}\right)\right)\sqrt{a}+\frac{3\,\sqrt{a}x^2+\left(2\,\sqrt{a}x^3+3\,\sqrt{a}x^2\right)e^{(2\,x)}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}
\end{aligned}$$

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="maxima")
[Out] -3*sqrt(a)*x^2 - 1/3*(4*x^3*log(e^(2*x) + 1) + 6*x^2*dilog(-e^(2*x)) - 6*x*
polylog(3, -e^(2*x)) + 3*polylog(4, -e^(2*x)))*sqrt(a) + (x^3*log(e^x + 1)
+ 3*x^2*dilog(-e^x) - 6*x*polylog(3, -e^x) + 6*polylog(4, -e^x))*sqrt(a) +
(x^3*log(-e^x + 1) + 3*x^2*dilog(e^x) - 6*x*polylog(3, e^x) + 6*polylog(4,
e^x))*sqrt(a) + 3/2*(2*x*log(e^(2*x) + 1) + dilog(-e^(2*x)))*sqrt(a) + (3*s
qrt(a)*x^2 + (2*sqrt(a)*x^3 + 3*sqrt(a)*x^2)*e^(2*x))/(e^(4*x) + 2*e^(2*x)
+ 1)
```

Giac [F]

$$\int x^3\text{csch}(x)\text{sech}(x)\sqrt{a\text{sech}^4(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^4}x^3\,\text{csch}\,(x)\,\text{sech}\,(x)\,dx$$

```
[In] integrate(x^3*csch(x)*sech(x)*(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^4)*x^3*csch(x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}(x)\operatorname{sech}(x)\sqrt{a\operatorname{sech}^4(x)}\,dx = \int \frac{x^3\sqrt{\frac{a}{\cosh(x)^4}}}{\cosh(x)\,\sinh(x)}\,dx$$

```
[In] int((x^3*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)),x)
[Out] int((x^3*(a/cosh(x)^4)^(1/2))/(cosh(x)*sinh(x)), x)
```

3.854 $\int (a+b\cosh (c+dx)\sinh (c+dx))^m\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int (a+b\cosh (c+dx)\sinh (c+dx))^m\,dx = \frac{i\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-i\sinh (2c+2dx)),\frac{b(1-i\sinh (2c+2dx))}{2ia+b}\right)\cosh (2c+2dx)\left(a+\frac{1}{2}b\sinh (2c+2dx)\right.}{\sqrt{2}d\sqrt{1+i\sinh (2c+2dx)}}$$

```
[Out] 1/2*I*AppellF1(1/2,-m,1/2,3/2,b*(1-I*sinh(2*d*x+2*c))/(2*I*a+b),1/2-1/2*I*s
inh(2*d*x+2*c))*cosh(2*d*x+2*c)*(a+1/2*b*sinh(2*d*x+2*c))^m/d/(((2*a+b*sinh
(2*d*x+2*c))/(2*a-I*b))^m)*2^(1/2)/(1+I*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2745, 2744, 144, 143}

$$\int (a+b\cosh (c+dx)\sinh (c+dx))^m\,dx = \frac{i\cosh (2c+2dx)\left(a+\frac{1}{2}b\sinh (2c+2dx)\right)^m\left(\frac{2a+b\sinh (2c+2dx)}{2a-ib}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-i\sinh (2c+\right.}{\sqrt{2}d\sqrt{1+i\sinh (2c+2dx)}}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
```

```
[Out] (I*AppellF1[1/2, 1/2, -m, 3/2, (1 - I*Sinh[2*c + 2*d*x])/2, (b*(1 - I*Sinh[
2*c + 2*d*x]))/((2*I)*a + b)]*Cosh[2*c + 2*d*x]*(a + (b*Sinh[2*c + 2*d*x])/
2)^m)/(Sqrt[2]*d*Sqrt[1 + I*Sinh[2*c + 2*d*x]]*((2*a + b*Sinh[2*c + 2*d*x])
/(2*a - I*b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\text{integral} = \int \left(a + \frac{1}{2} b \sinh(2c + 2dx)\right)^m dx$$

$$= -\frac{(i \cosh(2c + 2dx))\text{Subst}\left(\int \frac{\left(a - \frac{ibx}{2}\right)^m}{\sqrt{1-x}\sqrt{1+x}} dx, x, i \sinh(2c + 2dx)\right)}{2d\sqrt{1 - i \sinh(2c + 2dx)}\sqrt{1 + i \sinh(2c + 2dx)}}$$

$$= -\frac{\left(i \cosh(2c + 2dx) \left(a + \frac{1}{2} b \sinh(2c + 2dx)\right)^m \left(-\frac{a + \frac{1}{2} b \sinh(2c+2dx)}{-a + \frac{ib}{2}}\right)^{-m}\right) \text{Subst}\left(\int \frac{\left(-\frac{a}{-a + \frac{ib}{2}} + \frac{ibx}{2\left(-a + \frac{ib}{2}\right)}\right.}{\sqrt{1-x}\sqrt{1+x}}\right.}{2d\sqrt{1 - i \sinh(2c + 2dx)}\sqrt{1 + i \sinh(2c + 2dx)}}$$

$$= \frac{i\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-i\sinh(2c+2dx)),\frac{b(1-i\sinh(2c+2dx))}{2ia+b}\right)\cosh(2c+2dx)\left(a+\frac{1}{2}b\sinh(2c\right.}{\sqrt{2}d\sqrt{1+i\sinh(2c+2dx)}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx$$
$$= \frac{\mathrm{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2a+b\sinh(2(c+dx))}{2a+ib},\frac{2a+b\sinh(2(c+dx))}{2a-ib}\right)\mathrm{sech}(2(c+dx))\sqrt{\frac{b(1-i\sinh(2(c+dx)))}{2ia+b}}\sqrt{\frac{b(1+i\mathrm{s}}{}}}{bd(1+m)}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^m,x]
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sinh[2*(c + d*x)])/(2*a + I*b),
(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)]*Sech[2*(c + d*x)]*Sqrt[(b*(1 - I*S
inh[2*(c + d*x)]))/((2*I)*a + b)]*Sqrt[(b*(1 + I*Sinh[2*(c + d*x)]))/((-2*I
)*a + b)]*(a + (b*Sinh[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

Maple [F]

$$\int (a+b\cosh(dx+c)\sinh(dx+c))^m\,dx$$

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
[Out] int((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \text{Timed out}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

Giac [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (b\cosh(dx+c)\sinh(dx+c)+a)^m\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^m\,dx = \int (a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^m\,dx$$

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^m,x)
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^m, x)
```

3.855 $\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{1}{8}a\left(8a^2-3b^2\right)x+\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{5ab^2\cosh(2c+2dx)\sinh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}$$

```
[Out] 1/8*a*(8*a^2-3*b^2)*x+1/24*b*(16*a^2-b^2)*cosh(2*d*x+2*c)/d+5/48*a*b^2*cosh
(2*d*x+2*c)*sinh(2*d*x+2*c)/d+1/48*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c)
)^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2745, 2735, 2813}

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}+\frac{1}{8}ax\left(8a^2-3b^2\right)+\frac{5ab^2\sinh(2c+2dx)\cosh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 - 3*b^2)*x)/8 + (b*(16*a^2 - b^2)*Cosh[2*c + 2*d*x])/(24*d) + (5*
a*b^2*Cosh[2*c + 2*d*x]*Sinh[2*c + 2*d*x])/(48*d) + (b*Cosh[2*c + 2*d*x]*(2
*a + b*Sinh[2*c + 2*d*x])^2)/(48*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3\,dx\\&= \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}\\&\quad+\frac{1}{3}\int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)\left(\frac{1}{2}(6a^2-b^2)+\frac{5}{2}ab\sinh(2c+2dx)\right)\,dx\\&= \frac{1}{8}a(8a^2-3b^2)\,x+\frac{b(16a^2-b^2)\cosh(2c+2dx)}{24d}\\&\quad+\frac{5ab^2\cosh(2c+2dx)\sinh(2c+2dx)}{48d}+\frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^2}{48d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{9(16a^2b-b^3)\cosh(2(c+dx))+b^3\cosh(6(c+dx))+6a(4(8a^2-3b^2)(c+dx)+3b^2\sinh(4(c+dx)))}{192d}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^3,x]
[Out] (9*(16*a^2*b - b^3)*Cosh[2*(c + d*x)] + b^3*Cosh[6*(c + d*x)] + 6*a*(4*(8*a
^2 - 3*b^2)*(c + d*x) + 3*b^2*Sinh[4*(c + d*x)]))/(192*d)
```

Maple [A] (verified)

Time = 88.70 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

method	result
parts	$a^3x+\frac{b^3\left(\frac{\cosh(dx+c)^6}{6}-\frac{\cosh(dx+c)^4}{4}\right)}{d}+\frac{3a^2b\cosh(dx+c)^2}{2d}+\frac{3a\,b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}\right.}{d}$
derivativedivides	$\frac{a^3(dx+c)+\frac{3a^2b\cosh(dx+c)^2}{2}+3a\,b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+b^3\left(\frac{\sinh(dx+c)^2\cosh(dx+c}{6}\right.}{d}$
default	$\frac{a^3(dx+c)+\frac{3a^2b\cosh(dx+c)^2}{2}+3a\,b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+b^3\left(\frac{\sinh(dx+c)^2\cosh(dx+c}{6}\right.}{d}$
risch	$a^3x-\frac{3a\,b^2x}{8}+\frac{b^3e^{6dx+6c}}{384d}+\frac{3a\,b^2e^{4dx+4c}}{64d}+\frac{3b\,e^{2dx+2c}a^2}{8d}-\frac{3b^3e^{2dx+2c}}{128d}+\frac{3b\,e^{-2dx-2c}a^2}{8d}-\frac{3b^3e^{-2dx-2c}}{128d}$

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+b^3/d*(1/6*cosh(d*x+c)^6-1/4*cosh(d*x+c)^4)+3/2*a^2*b/d*cosh(d*x+c)^2
+3*a*b^2/d*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d
*x-1/8*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx$$
$$=\frac{b^3\cosh(dx+c)^6+15\,b^3\cosh(dx+c)^2\sinh(dx+c)^4+b^3\sinh(dx+c)^6+72\,ab^2\cosh(dx+c)^3\sinh(dx+c)}{}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(b^3*cosh(d*x + c)^6 + 15*b^3*cosh(d*x + c)^2*sinh(d*x + c)^4 + b^3*s
inh(d*x + c)^6 + 72*a*b^2*cosh(d*x + c)^3*sinh(d*x + c) + 72*a*b^2*cosh(d*x
 + c)*sinh(d*x + c)^3 + 24*(8*a^3 - 3*a*b^2)*d*x + 9*(16*a^2*b - b^3)*cosh(
d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 48*a^2*b - 3*b^3)*sinh(d*x + c)^2)/
d
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.74

$$\int (a + b\cosh(c + dx)\sinh(c + dx))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2b\sinh^2(c+dx)}{2d} - \frac{3ab^2x\sinh^4(c+dx)}{8} + \frac{3ab^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{3ab^2x\cosh^4(c+dx)}{8} + \frac{3ab^2\sinh^3(c+dx)\cosh(c+}{8d} \\ x(a + b\sinh(c)\cosh(c))^3 \end{cases}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2/(2*d) - 3*a*b**2*x*sinh(c + d
*x)**4/8 + 3*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a*b**2*x*cosh
(c + d*x)**4/8 + 3*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**3/(8*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)**
4/(4*d) - b**3*cosh(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c
))**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int (a + b\cosh(c + dx)\sinh(c + dx))^3\,dx$$
$$= a^3x - \frac{1}{384}b^3\left(\frac{(9e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{9e^{(-2dx-2c)} - e^{(-6dx-6c)}}{d}\right) - \frac{3}{64}ab^2\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right) + \frac{3a^2b\cosh(dx+c)^2}{2d}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/384*b^3*((9*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + (9*e^(-2*d*
x - 2*c) - e^(-6*d*x - 6*c))/d) - 3/64*a*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*
c)/d + e^(-4*d*x - 4*c)/d) + 3/2*a^2*b*cosh(d*x + c)^2/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{b^3e^{(6\,dx+6\,c)}}{384\,d} + \frac{3\,ab^2e^{(4\,dx+4\,c)}}{64\,d} - \frac{3\,ab^2e^{(-4\,dx-4\,c)}}{64\,d} + \frac{b^3e^{(-6\,dx-6\,c)}}{384\,d} + \frac{1}{8}\left(8\,a^3-3\,ab^2\right)x + \frac{3\left(16\,a^2b-b^3\right)e^{(2\,dx+2\,c)}}{128\,d} + \frac{3\left(16\,a^2b-b^3\right)e^{(-2\,dx-2\,c)}}{128\,d}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/384*b^3*e^(6*d*x + 6*c)/d + 3/64*a*b^2*e^(4*d*x + 4*c)/d - 3/64*a*b^2*e^(
-4*d*x - 4*c)/d + 1/384*b^3*e^(-6*d*x - 6*c)/d + 1/8*(8*a^3 - 3*a*b^2)*x +
3/128*(16*a^2*b - b^3)*e^(2*d*x + 2*c)/d + 3/128*(16*a^2*b - b^3)*e^(-2*d*x
 - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^3\,dx = \frac{\frac{b^3\cosh(6\,c+6\,d\,x)}{8} - \frac{9\,b^3\cosh(2\,c+2\,d\,x)}{8} + 18\,a^2\,b\cosh(2\,c+2\,d\,x) + \frac{9\,a\,b^2\sinh(4\,c+4\,d\,x)}{4} + 24\,a^3\,d\,x - 9\,a\,b^2\,d\,x}{24\,d}$$

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^3,x)
```

```
[Out] ((b^3*cosh(6*c + 6*d*x))/8 - (9*b^3*cosh(2*c + 2*d*x))/8 + 18*a^2*b*cosh(2*
c + 2*d*x) + (9*a*b^2*sinh(4*c + 4*d*x))/4 + 24*a^3*d*x - 9*a*b^2*d*x)/(24*
d)
```

3.856 $\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx=\frac{1}{8}\left(8a^2-b^2\right)x+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\cosh(2c+2dx)\sinh(2c+2dx)}{16d}$$

```
[Out] 1/8*(8*a^2-b^2)*x+1/2*a*b*cosh(2*d*x+2*c)/d+1/16*b^2*cosh(2*d*x+2*c)*sinh(2
*d*x+2*c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2745, 2723}

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx=\frac{1}{8}x\left(8a^2-b^2\right)+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\sinh(2c+2dx)\cosh(2c+2dx)}{16d}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
```

```
[Out] ((8*a^2 - b^2)*x)/8 + (a*b*Cosh[2*c + 2*d*x])/(2*d) + (b^2*Cosh[2*c + 2*d*x
]*Sinh[2*c + 2*d*x])/(16*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
```

```
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2\,dx\\ &= \frac{1}{8}\left(8a^2-b^2\right)x+\frac{ab\cosh(2c+2dx)}{2d}+\frac{b^2\cosh(2c+2dx)\sinh(2c+2dx)}{16d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx = \frac{4(8a^2-b^2)(c+dx)+16ab\cosh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^2,x]
[Out] (4*(8*a^2 - b^2)*(c + d*x) + 16*a*b*Cosh[2*(c + d*x)] + b^2*Sinh[4*(c + d*x
)])/(32*d)
```

Maple [A] (verified)

Time = 7.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

method	result	size
parts	$a^2x+\frac{b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}+\frac{ab\cosh(dx+c)^2}{d}$	66
derivativedivides	$\frac{a^2(dx+c)+ab\cosh(dx+c)^2+b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$	68
default	$\frac{a^2(dx+c)+ab\cosh(dx+c)^2+b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$	68
risch	$a^2x-\frac{b^2x}{8}+\frac{b^2e^{4dx+4c}}{64d}+\frac{ab\,e^{2dx+2c}}{4d}+\frac{ab\,e^{-2dx-2c}}{4d}-\frac{b^2e^{-4dx-4c}}{64d}$	79

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+b^2/d*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*
d*x-1/8*c)+a*b/d*cosh(d*x+c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int (a + b\cosh(c + dx)\sinh(c + dx))^2 \, dx$$
$$= \frac{b^2 \cosh{(dx + c)}^3 \sinh{(dx + c)} + b^2 \cosh{(dx + c)} \sinh{(dx + c)}^3 + 4\,ab\cosh{(dx + c)}^2 + 4\,ab\sinh{(dx + c)}^2 +}{8\,d}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^3*sinh(d*x + c) + b^2*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^2 + (8*a^2 - b^2)*d*x)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int (a + b\cosh(c + dx)\sinh(c + dx))^2 \, dx$$
$$= \begin{cases} a^2x + \frac{ab\sinh^2{(c+dx)}}{d} - \frac{b^2x\sinh^4{(c+dx)}}{8} + \frac{b^2x\sinh^2{(c+dx)}\cosh^2{(c+dx)}}{4} - \frac{b^2x\cosh^4{(c+dx)}}{8} + \frac{b^2\sinh^3{(c+dx)}\cosh{(c+dx)}}{8d} + \\ x(a + b\sinh{(c)}\cosh{(c)})^2 \end{cases}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*sinh(c + d*x)**2/d - b**2*x*sinh(c + d*x)**4/8 + b*
*2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - b**2*x*cosh(c + d*x)**4/8 + b**2
*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + b**2*sinh(c + d*x)*cosh(c + d*x)**3
/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)*cosh(c))**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx = a^2x-\frac{1}{64}\,b^2\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{ab\cosh\left(dx+c\right)^2}{d}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/64*b^2*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d) +
 a*b*cosh(d*x + c)^2/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.29

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx = \frac{1}{8}\left(8\,a^2-b^2\right)x+\frac{b^2e^{(4\,dx+4\,c)}}{64\,d}+\frac{abe^{(2\,dx+2\,c)}}{4\,d}+\frac{abe^{(-2\,dx-2\,c)}}{4\,d}-\frac{b^2e^{(-4\,dx-4\,c)}}{64\,d}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - b^2)*x + 1/64*b^2*e^(4*d*x + 4*c)/d + 1/4*a*b*e^(2*d*x + 2*c)/
d + 1/4*a*b*e^(-2*d*x - 2*c)/d - 1/64*b^2*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^2\,dx = \frac{\frac{\sinh(4\,c+4\,d\,x)\,b^2}{32}+\frac{a\cosh(2\,c+2\,d\,x)\,b}{2}}{d}+a^2\,x-\frac{b^2\,x}{8}$$

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^2,x)
```

```
[Out] ((b^2*sinh(4*c + 4*d*x))/32 + (a*b*cosh(2*c + 2*d*x))/2)/d + a^2*x - (b^2*x
)/8
```

3.857 $\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx = ax + \frac{b\sinh^2(c + dx)}{2d}$$

```
[Out] a*x+1/2*b*sinh(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2644, 30}

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx = ax + \frac{b\sinh^2(c + dx)}{2d}$$

```
[In] Int[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
[Out] a*x + (b*Sinh[c + d*x]^2)/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \cosh(c+dx)\sinh(c+dx)\,dx \\ &= ax - \frac{b\text{Subst}(\int x\,dx, x, i\sinh(c+dx))}{d} \\ &= ax + \frac{b\sinh^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = ax + \frac{b\cosh(2c)\cosh(2dx)}{4d} + \frac{b\sinh(2c)\sinh(2dx)}{4d}$$

```
[In] Integrate[a + b*Cosh[c + d*x]*Sinh[c + d*x],x]
[Out] a*x + (b*Cosh[2*c]*Cosh[2*d*x])/(4*d) + (b*Sinh[2*c]*Sinh[2*d*x])/(4*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$ax + \frac{b\cosh(dx+c)^2}{2d}$	19
parts	$ax + \frac{b\cosh(dx+c)^2}{2d}$	19
derivativedivides	$\frac{(dx+c)a+\frac{b\cosh(dx+c)^2}{2}}{d}$	24
risch	$ax + \frac{b\,e^{2dx+2c}}{8d} + \frac{b\,e^{-2dx-2c}}{8d}$	35

```
[In] int(a+b*cosh(d*x+c)*sinh(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b/d*cosh(d*x+c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx = \frac{4\,adx + b\cosh\left(dx + c\right)^{2} + b\sinh\left(dx + c\right)^{2}}{4\,d}$$

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="fricas")
[Out] 1/4*(4*a*d*x + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2)/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx = ax + b\left(\begin{cases} \frac{\sinh^{2}\left(c+dx\right)}{2d} & \text{for}\: d \neq 0 \\ x\sinh\left(c\right)\cosh\left(c\right) & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x)
[Out] a*x + b*Piecewise((sinh(c + d*x)**2/(2*d), Ne(d, 0)), (x*sinh(c)*cosh(c), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a + b\cosh(c + dx)\sinh(c + dx))\,dx = ax + \frac{b\cosh\left(dx + c\right)^{2}}{2\,d}$$

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="maxima")
[Out] a*x + 1/2*b*cosh(d*x + c)^2/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = ax+\frac{1}{8}\,b\left(\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

```
[In] integrate(a+b*cosh(d*x+c)*sinh(d*x+c),x, algorithm="giac")
[Out] a*x + 1/8*b*(e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\cosh(c+dx)\sinh(c+dx))\,dx = a\,x+\frac{b\cosh(c+d\,x)^2}{2\,d}$$

```
[In] int(a + b*cosh(c + d*x)*sinh(c + d*x),x)
[Out] a*x + (b*cosh(c + d*x)^2)/(2*d)
```

3.858 $\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)} dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)} dx = -\frac{2\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}d}$$

```
[Out] -2*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/d/(4*a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2745, 2739, 632, 210}

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)} dx = -\frac{2\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d\sqrt{4a^2+b^2}}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/(Sqrt[4*a^2 + b^2]*
d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx\\ &= -\frac{i\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{d}\\ &= \frac{(2i)\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx, x, -ib+2a\tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{d}\\ &= -\frac{2\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}\,d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \frac{2\arctan\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\sqrt{-4a^2-b^2}\,d}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-1),x]
[Out] (2*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(Sqrt[-4*a^2 - b^2]*
d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(40) = 80.

Time = 1.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.02

method	result
risch	$\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{4a^2+b^2}-4a^2-b^2}{\sqrt{4a^2+b^2}\,b}\right)}{\sqrt{4a^2+b^2}\,d}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{4a^2+b^2}+4a^2+b^2}{\sqrt{4a^2+b^2}\,b}\right)}{\sqrt{4a^2+b^2}\,d}$
derivativedivides	$\frac{2a\left(\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{2\sqrt{4a^2+b^2}\,a}+\frac{\left(-4a^2-b^2\right)\ln\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2\left(4a^2+b^2\right)^{\frac{3}{2}}a}\right.}{d}$
default	$\frac{2a\left(\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{2\sqrt{4a^2+b^2}\,a}+\frac{\left(-4a^2-b^2\right)\ln\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+\sqrt{4a^2+b^2}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2\left(4a^2+b^2\right)^{\frac{3}{2}}a}\right.}{d}$

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4*a^2+b^2)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*a*(4*a^2+b^2)^(1/2)-4*a^2-b^2)/(
4*a^2+b^2)^(1/2)/b)-1/(4*a^2+b^2)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*a*(4*a^2+b^2
)^(1/2)+4*a^2+b^2)/(4*a^2+b^2)^(1/2)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 6.80

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$$
$$=\frac{\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab\right)\sinh(dx+c)^2+8\,a^2+b^2+4\,(b}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+4\,a\cosh(dx+c)^2+2\,(3\,b\cosh(d}\right.}{\sqrt{4\,a^2+b^2}d}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d
*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sinh(
d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))*si
nh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*b*cosh
(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh(d*x +
c))*sinh(d*x + c) - b))/(sqrt(4*a^2 + b^2)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh(c + dx)\sinh(c + dx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\int \frac{1}{a + b\cosh(c + dx)\sinh(c + dx)}\,dx = \frac{\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{\sqrt{4\,a^2+b^2}d}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="maxima")
[Out] log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2*d*x - 2*c) - 2*
a + sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{1}{a + b\cosh(c + dx)\sinh(c + dx)}\,dx = \frac{\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{\sqrt{4\,a^2+b^2}d}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c)),x, algorithm="giac")
[Out] log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2*d*x +
 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(sqrt(4*a^2 + b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 343, normalized size of antiderivative = 7.80

$$\int \frac{1}{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$$
$$= \frac{2\,\mathrm{atan}\left(\left(\frac{b^4\sqrt{-4a^2d^2-b^2d^2}}{16}+\frac{a^2b^2\sqrt{-4a^2d^2-b^2d^2}}{4}\right)\left(\frac{32a\,(8a^2+b^2)}{b^4d\,(4a^2+b^2)^2}-e^{2c}\,e^{2dx}\left(\frac{64a\,(16da^3+4dab^2)}{b^5\sqrt{-4a^2d^2-b^2d^2}\,(4a^2+b^2)\sqrt{-d^2\,(4a^2+b^2)}}\right.\right.\right.}{\sqrt{-4a^2d^2-b^2d^2}}$$

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x)),x)
```

```
[Out] (2*atan(((b^4*(- 4*a^2*d^2 - b^2*d^2)^(1/2))/16 + (a^2*b^2*(- 4*a^2*d^2 - b
^2*d^2)^(1/2))/4)*((32*a*(8*a^2 + b^2))/(b^4*d*(4*a^2 + b^2)^2) - exp(2*c)*
exp(2*d*x)*((64*a*(16*a^3*d + 4*a*b^2*d))/(b^5*(- 4*a^2*d^2 - b^2*d^2)^(1/2
)*(4*a^2 + b^2)*(-d^2*(4*a^2 + b^2))^(1/2)) + (16*(8*a^2 + b^2)*(8*a^2*(- 4
*a^2*d^2 - b^2*d^2)^(1/2) + b^2*(- 4*a^2*d^2 - b^2*d^2)^(1/2)))/(b^5*d*(- 4
*a^2*d^2 - b^2*d^2)^(1/2)*(4*a^2 + b^2)^2)) + (64*a*(b^3*d + 4*a^2*b*d))/(b
^5*(- 4*a^2*d^2 - b^2*d^2)^(1/2)*(4*a^2 + b^2)*(-d^2*(4*a^2 + b^2))^(1/2)))
))/(- 4*a^2*d^2 - b^2*d^2)^(1/2)
```

3.859 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2} dx = -\frac{8a\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{(4a^2+b^2)^{3/2} d} - \frac{2b\cosh(2c+2dx)}{(4a^2+b^2) d(2a+b\sinh(2c+2dx))}$$

```
[Out] -8*a*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/(4*a^2+b^2)^(3/2)/d-2*b
*cosh(2*d*x+2*c)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2745, 2743, 12, 2739, 632, 210}

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2} dx = -\frac{8a\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d(4a^2+b^2)^{3/2}} - \frac{2b\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-8*a*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a^2 + b^2)^(3
/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2}\,dx\\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))}+\frac{4\int \frac{a}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}\\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))}+\frac{(4a)\int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}
\end{aligned}
$$

$$= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))} - \frac{(4ia)\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{(4a^2+b^2)\,d}$$

$$= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))} + \frac{(8ia)\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx, x, -ib+2a\tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{(4a^2+b^2)\,d}$$

$$= -\frac{8a\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{(4a^2+b^2)^{3/2}\,d} - \frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{2\left(-\frac{4a\arctan\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{(-4a^2-b^2)^{3/2}} - \frac{b\cosh(2(c+dx))}{(4a^2+b^2)(2a+b\sinh(2(c+dx)))}\right)}{d}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-2),x]
```

```
[Out] (2*((-4*a*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/(-4*a^2 - b^2
)^(3/2) - (b*Cosh[2*(c + d*x)])/((4*a^2 + b^2)*(2*a + b*Sinh[2*(c + d*x)]))
))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(85) = 170.

Time = 15.76 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.39

method	result
risch	$\frac{8a\,e^{2dx+2c}-4b}{(4a^2+b^2)d(4a\,e^{2dx+2c}+e^{4dx+4c}b-b)}+\frac{4a\ln\left(e^{2dx+2c}+\frac{2(4a^2+b^2)^{\frac{3}{2}}a-16a^4-8a^2b^2-b^4}{b(4a^2+b^2)^{\frac{3}{2}}}\right)}{(4a^2+b^2)^{\frac{3}{2}}d}-\frac{4a\ln\left(e^{2dx+2c}+\frac{2(4a^2+b^2}{}\right.}{(4a^2+b}$
derivativedivides	$-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a(4a^2+b^2)}+\frac{4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a^2+b^2}-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(4a^2+b^2)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a}-8a^2\left(\frac{(-4a^2-b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\,\tan\right.}{2(4a^2+b^2)^{\frac{3}{2}}}\right.$ over d
default	$-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a(4a^2+b^2)}+\frac{4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a^2+b^2}-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(4a^2+b^2)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+a}-8a^2\left(\frac{(-4a^2-b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{4a^2+b^2}\,\tan\right.}{2(4a^2+b^2)^{\frac{3}{2}}}\right.$ over d

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(2*a*exp(2*d*x+2*c)-b)/(4*a^2+b^2)/d/(4*a*exp(2*d*x+2*c)+exp(4*d*x+4*c)*b
-b)+4/(4*a^2+b^2)^(3/2)*a/d*ln(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(3/2)*a-16*a^4
-8*a^2*b^2-b^4)/b/(4*a^2+b^2)^(3/2))-4/(4*a^2+b^2)^(3/2)*a/d*ln(exp(2*d*x+2
*c)+(2*(4*a^2+b^2)^(3/2)*a+16*a^4+8*a^2*b^2+b^4)/b/(4*a^2+b^2)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 765, normalized size of antiderivative = 8.60

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx =$$
$$-\frac{4\left(4\,a^2b+b^3-2\,(4\,a^3+ab^2)\cosh(dx+c)^2-4\,(4\,a^3+ab^2)\cosh(dx+c)\sinh(dx+c)-2\,(4\,a^3+ab^2)\right.}{(16\,a^4b+8\,a^2b^3+b^5)d\cosh(}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -4*(4*a^2*b + b^3 - 2*(4*a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(4*a^3 + a*b^2)*c
osh(d*x + c)*sinh(d*x + c) - 2*(4*a^3 + a*b^2)*sinh(d*x + c)^2 - (a*b*cosh(
d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 4*
a^2*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2)*sinh(d*x + c)^2 - a
*b + 4*(a*b*cosh(d*x + c)^3 + 2*a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(4*a^
2 + b^2)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b
```

```
^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b)*sinh(d*x + c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x
 + c))*sinh(d*x + c) - 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*c
osh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*co
sh(d*x + c))*sinh(d*x + c) - b)))/((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x
+ c)^4 + 4*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
16*a^4*b + 8*a^2*b^3 + b^5)*d*sinh(d*x + c)^4 + 4*(16*a^5 + 8*a^3*b^2 + a*b
^4)*d*cosh(d*x + c)^2 + 2*(3*(16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)^2
 + 2*(16*a^5 + 8*a^3*b^2 + a*b^4)*d)*sinh(d*x + c)^2 - (16*a^4*b + 8*a^2*b^
3 + b^5)*d + 4*((16*a^4*b + 8*a^2*b^3 + b^5)*d*cosh(d*x + c)^3 + 2*(16*a^5
+ 8*a^3*b^2 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx &= \frac{4\,a\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{(4\,a^2+b^2)^{\frac{3}{2}}d}\\&\quad-\frac{4\left(2\,ae^{(-2\,dx-2\,c)}+b\right)}{(4\,a^2b+b^3+4\,(4\,a^3+ab^2)e^{(-2\,dx-2\,c)}-(4\,a^2b+b^3)e^{(-4\,dx-4\,c)})d}\end{aligned}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 4*a*log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2*d*x - 2*c)
- 2*a + sqrt(4*a^2 + b^2)))/((4*a^2 + b^2)^(3/2)*d) - 4*(2*a*e^(-2*d*x - 2*
c) + b)/((4*a^2*b + b^3 + 4*(4*a^3 + a*b^2)*e^(-2*d*x - 2*c) - (4*a^2*b + b
^3)*e^(-4*d*x - 4*c))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{4\left(\frac{a\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{(4\,a^2+b^2)^{\frac{3}{2}}}+\frac{2\,ae^{(2\,dx+2\,c)}-b}{(4\,a^2+b^2)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-b\right)}\right)}{d}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 4*(a*log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/abs(2*b*e^(2*
d*x + 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/(4*a^2 + b^2)^(3/2) + (2*a*e^(2*d*
x + 2*c) - b)/((4*a^2 + b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b))
)/d
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.57

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^2}\,dx = \frac{4\,a\,\ln\left(\frac{16\,a\,\left(b-2\,a\,e^{2\,c+2\,d\,x}\right)}{b\,{\left(4\,a^2+b^2\right)}^{3/2}}-\frac{16\,a\,e^{2\,c+2\,d\,x}}{4\,a^2\,b+b^3}\right)}{d\,{\left(4\,a^2+b^2\right)}^{3/2}} - \frac{4\,a\,\ln\left(-\frac{16\,a\,e^{2\,c+2\,d\,x}}{4\,a^2\,b+b^3}-\frac{16\,a\,\left(b-2\,a\,e^{2\,c+2\,d\,x}\right)}{b\,{\left(4\,a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(4\,a^2+b^2\right)}^{3/2}} - \frac{\frac{4\,b^2}{d\,\left(4\,a^2\,b+b^3\right)}-\frac{8\,a\,b\,e^{2\,c+2\,d\,x}}{d\,\left(4\,a^2\,b+b^3\right)}}{4\,a\,e^{2\,c+2\,d\,x}-b+b\,e^{4\,c+4\,d\,x}}$$

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^2,x)
```

```
[Out] (4*a*log((16*a*(b - 2*a*exp(2*c + 2*d*x)))/(b*(4*a^2 + b^2)^(3/2)) - (16*a*
exp(2*c + 2*d*x))/(4*a^2*b + b^3)))/(d*(4*a^2 + b^2)^(3/2)) - (4*a*log(- (1
6*a*exp(2*c + 2*d*x))/(4*a^2*b + b^3) - (16*a*(b - 2*a*exp(2*c + 2*d*x)))/(
b*(4*a^2 + b^2)^(3/2))))/(d*(4*a^2 + b^2)^(3/2)) - ((4*b^2)/(d*(4*a^2*b + b
^3)) - (8*a*b*exp(2*c + 2*d*x))/(d*(4*a^2*b + b^3)))/(4*a*exp(2*c + 2*d*x)
- b + b*exp(4*c + 4*d*x))
```

3.860 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3} dx = -\frac{4(8a^2-b^2)\operatorname{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{(4a^2+b^2)^{5/2} d} - \frac{2b\cosh(2c+2dx)}{(4a^2+b^2) d(2a+b\sinh(2c+2dx))^2} - \frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2 d(2a+b\sinh(2c+2dx))}$$

```
[Out] -4*(8*a^2-b^2)*arctanh((b-2*a*tanh(d*x+c))/(4*a^2+b^2)^(1/2))/(4*a^2+b^2)^(
5/2)/d-2*b*cosh(2*d*x+2*c)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^2-12*a*b*c
osh(2*d*x+2*c)/(4*a^2+b^2)^2/d/(2*a+b*sinh(2*d*x+2*c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2745, 2743, 2833, 12, 2739, 632, 210}

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3} dx = -\frac{4(8a^2-b^2)\operatorname{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{d(4a^2+b^2)^{5/2}} - \frac{12ab\cosh(2c+2dx)}{d(4a^2+b^2)^2(2a+b\sinh(2c+2dx))} - \frac{2b\cosh(2c+2dx)}{d(4a^2+b^2)(2a+b\sinh(2c+2dx))^2}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]
```

```
[Out] (-4*(8*a^2 - b^2)*ArcTanh[(b - 2*a*Tanh[c + d*x])/Sqrt[4*a^2 + b^2]])/((4*a
^2 + b^2)^(5/2)*d) - (2*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*(2*a + b*Sinh
[2*c + 2*d*x])^2) - (12*a*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)^2*d*(2*a + b*
Sinh[2*c + 2*d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^3}\,dx \\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2}-\frac{2\int \frac{-2a+\frac{1}{2}b\sinh(2c+2dx)}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^2}\,dx}{4a^2+b^2} \\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2} \\
&\quad -\frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2\,d(2a+b\sinh(2c+2dx))}+\frac{8\int \frac{2a^2-\frac{b^2}{4}}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{(4a^2+b^2)^2} \\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2} \\
&\quad -\frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2\,d(2a+b\sinh(2c+2dx))}+\frac{(2(8a^2-b^2))\int \frac{1}{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{(4a^2+b^2)^2} \\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2}-\frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2\,d(2a+b\sinh(2c+2dx))} \\
&\quad -\frac{(2i(8a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a-ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{(4a^2+b^2)^2\,d} \\
&= -\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2}-\frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2\,d(2a+b\sinh(2c+2dx))} \\
&\quad +\frac{(4i(8a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4a^2-b^2-x^2}\,dx,x,-ib+2a\tan\left(\frac{1}{2}(2ic+2idx)\right)\right)}{(4a^2+b^2)^2\,d} \\
&= -\frac{4(8a^2-b^2)\,\text{arctanh}\left(\frac{b-2a\tanh(c+dx)}{\sqrt{4a^2+b^2}}\right)}{(4a^2+b^2)^{5/2}\,d}-\frac{2b\cosh(2c+2dx)}{(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^2} \\
&\quad -\frac{12ab\cosh(2c+2dx)}{(4a^2+b^2)^2\,d(2a+b\sinh(2c+2dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \frac{2\left(\frac{2(8a^2-b^2)\arctan\left(\frac{b-2a\tanh(c+dx)}{\sqrt{-4a^2-b^2}}\right)}{\sqrt{-4a^2-b^2}} - \frac{b\cosh(2(c+dx))(16a^2+b^2+6ab\sinh(2(c+dx)))}{(2a+b\sinh(2(c+dx)))^2}\right)}{(4a^2+b^2)^2\,d}$$

[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3),x]

[Out] (2*((2*(8*a^2 - b^2)*ArcTan[(b - 2*a*Tanh[c + d*x])/Sqrt[-4*a^2 - b^2]])/Sq
rt[-4*a^2 - b^2] - (b*Cosh[2*(c + d*x)]*(16*a^2 + b^2 + 6*a*b*Sinh[2*(c + d
*x)]))/(2*a + b*Sinh[2*(c + d*x)])^2))/((4*a^2 + b^2)^2*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(139) = 278.

Time = 211.64 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.36

method	result
risch	$\frac{32a^2 b\,e^{6dx+6c}-4b^3 e^{6dx+6c}+192a^3 e^{4dx+4c}-24a\,b^2 e^{4dx+4c}-160a^2 b\,e^{2dx+2c}-4b^3 e^{2dx+2c}+24a\,b^2}{d(4a^2+b^2)^2\left(4a\,e^{2dx+2c}+e^{4dx+4c}b-b\right)^2} + \frac{16\ln\left(e^{2dx+2c}+\frac{2(\cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{b^2(10a^2+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a(16a^4+8a^2b^2+b^4)}+\frac{(32a^4-14a^2b^2-b^4)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a^2(16a^4+8a^2b^2+b^4)}+\frac{(58a^2+b^2)b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a(16a^4+8a^2b^2+b^4)}-\frac{2b(32a^4+18a^2b^2+b^4)\tanh\cdots}{a^2(16a^4+8a^2b^2\cdots}\cdots\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh(\cdots\right.}$
default	$-\frac{2\left(-\frac{b^2(10a^2+b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a(16a^4+8a^2b^2+b^4)}+\frac{(32a^4-14a^2b^2-b^4)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{a^2(16a^4+8a^2b^2+b^4)}+\frac{(58a^2+b^2)b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a(16a^4+8a^2b^2+b^4)}-\frac{2b(32a^4+18a^2b^2+b^4)\tanh\cdots}{a^2(16a^4+8a^2b^2\cdots}\cdots\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tanh(\cdots\right.}$

[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 4*(8*a^2*b*exp(6*d*x+6*c)-b^3*exp(6*d*x+6*c)+48*a^3*exp(4*d*x+4*c)-6*a*b^2*
exp(4*d*x+4*c)-40*a^2*b*exp(2*d*x+2*c)-b^3*exp(2*d*x+2*c)+6*a*b^2)/d/(4*a^2
+b^2)^2/(4*a*exp(2*d*x+2*c)+exp(4*d*x+4*c)*b-b)^2+16/(4*a^2+b^2)^(5/2)/d*ln
(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(5/2)*a-64*a^6-48*a^4*b^2-12*a^2*b^4-b^6)/(4
*a^2+b^2)^(5/2)/b)*a^2-2/(4*a^2+b^2)^(5/2)/d*ln(exp(2*d*x+2*c)+(2*(4*a^2+b^
2)^(5/2)*a-64*a^6-48*a^4*b^2-12*a^2*b^4-b^6)/(4*a^2+b^2)^(5/2)/b)*b^2-16/(4
*a^2+b^2)^(5/2)/d*ln(exp(2*d*x+2*c)+(2*(4*a^2+b^2)^(5/2)*a+64*a^6+48*a^4*b^

```
2+12*a^2*b^4+b^6)/(4*a^2+b^2)^(5/2)/b)*a^2+2/(4*a^2+b^2)^(5/2)/d*ln(exp(2*d
*x+2*c)+(2*(4*a^2+b^2)^(5/2)*a+64*a^6+48*a^4*b^2+12*a^2*b^4+b^6)/(4*a^2+b^2
)^(5/2)/b)*b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2439 vs. $2(142) = 284$.

Time = 0.29 (sec) , antiderivative size = 2439, normalized size of antiderivative = 17.06

$$\int \frac{1}{(a + b\cosh(c + dx)\sinh(c + dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(2*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^6 + 12*(32*a^4*b + 4*a^2*b^
3 - b^5)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(32*a^4*b + 4*a^2*b^3 - b^5)*sin
h(d*x + c)^6 + 48*a^3*b^2 + 12*a*b^4 + 12*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh
(d*x + c)^4 + 6*(64*a^5 + 8*a^3*b^2 - 2*a*b^4 + 5*(32*a^4*b + 4*a^2*b^3 - b
^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(32*a^4*b + 4*a^2*b^3 - b^5)*co
sh(d*x + c)^3 + 6*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(160*a^4*b + 44*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 2*(160*a^4*b + 44*a
^2*b^3 + b^5 - 15*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^4 - 36*(32*a^5
 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((8*a^2*b^2 - b^4)
*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a
^2*b^2 - b^4)*sinh(d*x + c)^8 + 8*(8*a^3*b - a*b^3)*cosh(d*x + c)^6 + 4*(16
*a^3*b - 2*a*b^3 + 7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8
*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^3 + 6*(8*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*(35*(8*
a^2*b^2 - b^4)*cosh(d*x + c)^4 + 64*a^4 - 16*a^2*b^2 + b^4 + 60*(8*a^3*b -
a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - b^4 + 8*(7*(8*a^2*b^2
 - b^4)*cosh(d*x + c)^5 + 20*(8*a^3*b - a*b^3)*cosh(d*x + c)^3 + (64*a^4 -
16*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(8*a^3*b - a*b^3)*cosh
(d*x + c)^2 + 4*(7*(8*a^2*b^2 - b^4)*cosh(d*x + c)^6 + 30*(8*a^3*b - a*b^3)
*cosh(d*x + c)^4 - 16*a^3*b + 2*a*b^3 + 3*(64*a^4 - 16*a^2*b^2 + b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - b^4)*cosh(d*x + c)^7 + 6*(8*a
^3*b - a*b^3)*cosh(d*x + c)^5 + (64*a^4 - 16*a^2*b^2 + b^4)*cosh(d*x + c)^3
 - 2*(8*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(4*a^2 + b^2)*log(
(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
 c)^4 + 4*a*b*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b)*sinh(d*x
+ c)^2 + 8*a^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + 2*a*b*cosh(d*x + c))*sinh(d
*x + c) + 2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d
*x + c)^2 + 2*a)*sqrt(4*a^2 + b^2))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 4*a*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
 + c)^2 + 2*a)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + 2*a*cosh(d*x + c))*
sinh(d*x + c) - b)) + 4*(3*(32*a^4*b + 4*a^2*b^3 - b^5)*cosh(d*x + c)^5 + 1
```

```
2*(32*a^5 + 4*a^3*b^2 - a*b^4)*cosh(d*x + c)^3 - (160*a^4*b + 44*a^2*b^3 +
b^5)*cosh(d*x + c))*sinh(d*x + c))/((64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 +
 b^8)*d*cosh(d*x + c)^8 + 8*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8
)*d*sinh(d*x + c)^8 + 8*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh
(d*x + c)^6 + 4*(7*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x
+ c)^2 + 2*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^6
+ 2*(512*a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c)^
4 + 8*(7*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^3 + 6
*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^4
+ 60*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + (512*
a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d)*sinh(d*x + c)^4 - 8*(6
4*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(64*a^6
*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^5 + 20*(64*a^7*b + 48
*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + (512*a^8 + 320*a^6*b^2 +
 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(64*
a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*cosh(d*x + c)^6 + 30*(64*a^7*b +
 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^4 + 3*(512*a^8 + 320*a^6*
b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 - 2*(64*a^7*b + 48*a^
5*b^3 + 12*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^2 + (64*a^6*b^2 + 48*a^4*b^4 +
 12*a^2*b^6 + b^8)*d + 8*((64*a^6*b^2 + 48*a^4*b^4 + 12*a^2*b^6 + b^8)*d*co
sh(d*x + c)^7 + 6*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x +
 c)^5 + (512*a^8 + 320*a^6*b^2 + 48*a^4*b^4 - 4*a^2*b^6 - b^8)*d*cosh(d*x +
 c)^3 - 2*(64*a^7*b + 48*a^5*b^3 + 12*a^3*b^5 + a*b^7)*d*cosh(d*x + c))*sin
h(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(142) = 284$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \frac{2\,(8\,a^2-b^2)\log\left(\frac{be^{(-2\,dx-2\,c)}-2\,a-\sqrt{4\,a^2+b^2}}{be^{(-2\,dx-2\,c)}-2\,a+\sqrt{4\,a^2+b^2}}\right)}{(16\,a^4+8\,a^2b^2+b^4)\sqrt{4\,a^2+b^2}d} - \frac{4\,(6\,ab^2+(40\,a^2b+b^3)e^{(-2\,dx-2\,c)}+6\,(8\,a^3-ab^2)e^{(-4\,dx-4\,c)}}{(16\,a^4b^2+8\,a^2b^4+b^6+8\,(16\,a^5b+8\,a^3b^3+ab^5)e^{(-2\,dx-2\,c)}+2\,(128\,a^6+48\,a^4b^2-b^6)e^{(-4\,dx-4\,c)}-8\,(16}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2*(8*a^2 - b^2)*log((b*e^(-2*d*x - 2*c) - 2*a - sqrt(4*a^2 + b^2))/(b*e^(-2
*d*x - 2*c) - 2*a + sqrt(4*a^2 + b^2)))/((16*a^4 + 8*a^2*b^2 + b^4)*sqrt(4*
a^2 + b^2)*d) - 4*(6*a*b^2 + (40*a^2*b + b^3)*e^(-2*d*x - 2*c) + 6*(8*a^3 -
 a*b^2)*e^(-4*d*x - 4*c) - (8*a^2*b - b^3)*e^(-6*d*x - 6*c))/((16*a^4*b^2 +
 8*a^2*b^4 + b^6 + 8*(16*a^5*b + 8*a^3*b^3 + a*b^5)*e^(-2*d*x - 2*c) + 2*(1
28*a^6 + 48*a^4*b^2 - b^6)*e^(-4*d*x - 4*c) - 8*(16*a^5*b + 8*a^3*b^3 + a*b
^5)*e^(-6*d*x - 6*c) + (16*a^4*b^2 + 8*a^2*b^4 + b^6)*e^(-8*d*x - 8*c))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \frac{2\left(\frac{(8\,a^2-b^2)\log\left(\frac{\left|2\,be^{(2\,dx+2\,c)}+4\,a-2\,\sqrt{4\,a^2+b^2}\right|}{\left|2\,be^{(2\,dx+2\,c)}+4\,a+2\,\sqrt{4\,a^2+b^2}\right|}\right)}{(16\,a^4+8\,a^2b^2+b^4)\sqrt{4\,a^2+b^2}} + \frac{2\,(8\,a^2be^{(6\,dx+6\,c)}-b^3e^{(6\,dx+6\,c)}+48\,a^3e^{(4\,dx+4\,c)}-6\,ab^2e^{(4\,dx+4\,c)}-40\,a^2be^{(2\,dx+2\,c)}-}{(16\,a^4+8\,a^2b^2+b^4)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-b\right)^2}\right.}{d}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*((8*a^2 - b^2)*log(abs(2*b*e^(2*d*x + 2*c) + 4*a - 2*sqrt(4*a^2 + b^2))/a
bs(2*b*e^(2*d*x + 2*c) + 4*a + 2*sqrt(4*a^2 + b^2)))/((16*a^4 + 8*a^2*b^2 +
 b^4)*sqrt(4*a^2 + b^2)) + 2*(8*a^2*b*e^(6*d*x + 6*c) - b^3*e^(6*d*x + 6*c)
 + 48*a^3*e^(4*d*x + 4*c) - 6*a*b^2*e^(4*d*x + 4*c) - 40*a^2*b*e^(2*d*x + 2
*c) - b^3*e^(2*d*x + 2*c) + 6*a*b^2)/((16*a^4 + 8*a^2*b^2 + b^4)*(b*e^(4*d*
x + 4*c) + 4*a*e^(2*d*x + 2*c) - b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\right)^3}\,dx$$

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^3,x)
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^3, x)
```

3.861 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} - \frac{i(92a^2-9b^2)\,E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{60\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{2i\sqrt{2}a(4a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{15d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[Out] 1/40*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c))^(3/2)/d*2^(1/2)+2/15*a*b*cos
h(2*d*x+2*c)*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d+1/120*I*(92*a^2-9*b^2)
*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1/
4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d*2^
(1/2)/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-2/15*I*a*(4*a^2+b^2)*(sin(I
*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1/4*Pi+I*
d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b
))^(1/2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2745, 2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \frac{2i\sqrt{2}a(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{15d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{i(92a^2-9b^2)\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{60\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[2]*a*b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(15*d) +
(b*Cosh[2*c + 2*d*x]*(2*a + b*Sinh[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) - ((
I/60)*(92*a^2 - 9*b^2)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*
I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2
*c + 2*d*x])/(2*a - I*b)]) + (((2*I)/15)*Sqrt[2]*a*(4*a^2 + b^2)*EllipticF[
((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c
 + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
```

```
x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}\,dx \\
&= \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} \\
&\quad+\frac{2}{5}\int\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\left(\frac{1}{8}\left(20a^2-3b^2\right)+2ab\sinh(2c+2dx)\right)\,dx
\end{aligned}$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{4}{15}\int \frac{\frac{1}{16}a(60a^2-17b^2)+\frac{1}{32}b(92a^2-9b^2)\sinh(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{1}{60}(92a^2-9b^2)\int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx - \frac{1}{15}\left(2a\left(4a^2+b^2\right)\right)\int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{\left((92a^2-9b^2)\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\right)\int \sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}\,dx}{60\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}} - \frac{\left(2a(4a^2+b^2)\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}\right)\int \frac{1}{\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}}\,dx}{15\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}$$

$$= \frac{2\sqrt{2}ab\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{15d} + \frac{b\cosh(2c+2dx)(2a+b\sinh(2c+2dx))^{3/2}}{20\sqrt{2}d} - \frac{i(92a^2-9b^2)\,E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{60\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{2i\sqrt{2}a(4a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{15d\sqrt{2a+b\sinh(2c+2dx)}}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx = \frac{2(184ia^3+92a^2b-18iab^2-9b^3)\,E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}-32ia(4a$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*((184*I)*a^3 + 92*a^2*b - (18*I)*a*b^2 - 9*b^3)*EllipticE[((-4*I)*c + Pi
 - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(
2*a - I*b)] - (32*I)*a*(4*a^2 + b^2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/
4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] +
b*(88*a^2*Cosh[2*(c + d*x)] + b*(28*a + 3*b*Sinh[2*(c + d*x)])*Sinh[4*(c +
d*x)]))/(120*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1259 vs. 2(331) = 662.

Time = 6.30 (sec) , antiderivative size = 1260, normalized size of antiderivative = 4.19

method	result	size
default	Expression too large to display	1260

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(64*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)
*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*
a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3*b+1
6*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b
+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^3+240*(-(2
*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(
1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+
2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^4+24*(-(2*a+b*sinh(2
*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh
(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-
2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b^2-9*(-(2*a+b*sinh(2*d*x+2*c
))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2
*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1
/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^4-368*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
```

```
a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(
I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))*a^4-56*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*(
(-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(
1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+
2*a))^(1/2))*a^2*b^2+9*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2
*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*Ell
ipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1
/2))*b^4+3*b^4*sinh(2*d*x+2*c)^4+28*a*b^3*sinh(2*d*x+2*c)^3+44*a^2*b^2*sinh
(2*d*x+2*c)^2+3*b^4*sinh(2*d*x+2*c)^2+28*a*b^3*sinh(2*d*x+2*c)+44*a^2*b^2)/
b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Fricas [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx=\int (b\cosh{(dx+c)}\sinh{(dx+c)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*
x + c) + a^2)*sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{5/2}\,dx=\int (b\cosh{(dx+c)}\sinh{(dx+c)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a + b\cosh(c + dx)\sinh(c + dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\cosh(c + dx)\sinh(c + dx))^{5/2}\,dx = \int (a + b\cosh(c + d\,x)\,\sinh(c + d\,x))^{5/2}\,dx$$

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2), x)
```

3.862 $\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 248

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} - \frac{2i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{3d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{i(4a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{6\sqrt{2}d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[Out] 1/12*b*cosh(2*d*x+2*c)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d*2^(1/2)+2/3*I*a*(sin
(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1/4*Pi+
I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(1/2)/d
/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-1/12*I*(4*a^2+b^2)*(sin(I*c+1/4*
Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1/4*Pi+I*d*x),2^
(1/2)*(b/(2*I*a+b))^(1/2))*((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)/d*2^(1
/2)/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used

$= \{2745, 2735, 2831, 2742, 2740, 2734, 2732\}$

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \frac{i(4a^2+b^2)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{6\sqrt{2}d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} - \frac{2i\sqrt{2}a\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{3d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (b*Cosh[2*c + 2*d*x]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(6*Sqrt[2]*d) - (((2*
I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a +
b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2
*a - I*b)]) + ((I/6)*(4*a^2 + b^2)*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2
, (2*b)/((2*I)*a + b)]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(Sqrt
[2]*d*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}\,dx\\
&= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{2}{3}\int \frac{\frac{1}{8}(12a^2-b^2)+ab\sinh(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx\\
&= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}+\frac{1}{3}(4a)\int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx\\
&\quad+\frac{1}{12}(-4a^2-b^2)\int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx\\
&= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d}\\
&\quad+\frac{\left(4a\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\right)\int \sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}\,dx}{3\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}}\\
&\quad+\frac{\left((-4a^2-b^2)\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}\right)\int \frac{1}{\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}}\,dx}{12\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}
\end{aligned}$$

$$= \frac{b\cosh(2c+2dx)\sqrt{2a+b\sinh(2c+2dx)}}{6\sqrt{2}d} - \frac{2i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{3d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{i(4a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{6\sqrt{2}d\sqrt{2a+b\sinh(2c+2dx)}}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.81

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \frac{16a(2ia+b)E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}} - 2i(4a^2+b^2)\,\text{EllipticF}\left(\frac{1}{4}(-4\right.}{12d\sqrt{4a+2b\sinh(}}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (16*a*((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*
a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] - (2*I)*(4*a^2 + b^
2)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2
*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)] + b*(4*a*Cosh[2*(c + d*x)] + b*Sinh[
4*(c + d*x)]))/(12*d*Sqrt[4*a + 2*b*Sinh[2*(c + d*x)]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 934 vs. 2(284) = 568.

Time = 0.64 (sec) , antiderivative size = 935, normalized size of antiderivative = 3.77

method	result
default	$\frac{4i\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\,\text{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)a^2b+i\sqrt{-\frac{2a+b\sinh(}{ib-}}}{}$

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(4*I*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b
/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+
b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2*b+I*(
-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a)
)^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d
```

```
*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^3+24*(-(2*a+b*sin
h(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((s
inh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I
*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^3+6*(-(2*a+b*sinh(2*d*x+2*c)
)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*
c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/
2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a*b^2-32*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*
a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(
I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))*a^3-8*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((
-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1
/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2
*a))^(1/2))*a*b^2+sinh(2*d*x+2*c)^3*b^3+2*sinh(2*d*x+2*c)^2*a*b^2+b^3*sinh(
2*d*x+2*c)+2*a*b^2)/b/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Fricas [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx=\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

Sympy [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx=\int (a+b\sinh(c+dx)\cosh(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
[Out] Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx=\int (b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(c+dx)\sinh(c+dx))^{3/2}\,dx = \int (a+b\cosh(c+d\,x)\,\sinh(c+d\,x))^{3/2}\,dx$$

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2), x)
```

3.863 $\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = -\frac{iE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] 1/2*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I
*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*(2*a+b*sinh(2*d*x+2*c))^(1/2)
/d*2^(1/2)/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2745, 2734, 2732}

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = -\frac{i\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[In] Int[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] ((-I)*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2
*a + b*Sinh[2*c + 2*d*x]])/(Sqrt[2]*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a
 - I*b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx\\ &= \frac{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\int\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}\,dx}{\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}}\\ &= -\frac{iE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{\sqrt{2}d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \frac{(2ia+b)E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}}{d\sqrt{4a+2b\sinh(2(c+dx))}}$$

```
[In] Integrate[Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
[Out] (((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I
*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[4*a + 2*b*Sinh[
2*(c + d*x)]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(119) = 238$.

Time = 0.46 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.67

method	result
default	$-\frac{(ib-2a)\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\,\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\,\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)b-i\,\mathrm{Ellipti}}{b\cosh(2dx+2c)}$

```
[In] int((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)
*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)/b*(I*EllipticF(
(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b-
I*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a
))^(1/2))*b+2*a*EllipticF((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b
-2*a)/(I*b+2*a))^(1/2))-2*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1
/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a)/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*
c))^(1/2)/d
```

Fricas [F]

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{a+b\sinh\left(c+dx\right)\cosh\left(c+dx\right)}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(c+dx)\sinh(c+dx)}\,dx = \int \sqrt{a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)}\,dx$$

```
[In] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2),x)
[Out] int((a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2), x)
```

3.864 $\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = -\frac{i\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[Out] I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*c+1
/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c))/
(2*a-I*b))^(1/2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2745, 2742, 2740}

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[In] Int[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)
]*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/(d*Sqrt[2*a + b*Sinh[2*c +
 2*d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx$$

$$= \frac{\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}\int \frac{1}{\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}}\,dx}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}$$

$$= -\frac{i\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{d\sqrt{2a+b\sinh(2c+2dx)}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx$$
$$= \frac{i\,\text{EllipticF}\left(\frac{1}{4}(-4ic+\pi-4idx),-\frac{2ib}{2a-ib}\right)\sqrt{\frac{2a+b\sinh(2(c+dx))}{2a-ib}}}{d\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

```
[In] Integrate[1/Sqrt[a + b*Cosh[c + d*x]*Sinh[c + d*x]],x]
[Out] (I*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2
*a + b*Sinh[2*(c + d*x)])/(2*a - I*b)])/(d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2
])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{2(ib-2a)\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\,\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\,\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)}{b\cosh(2dx+2c)\sqrt{4a+2b\sinh(2dx+2c)}\,d}$	181

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(I*b-2*a)*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+
I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(
2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))/b/cos
h(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = -\frac{2\left(\sqrt{-b}b\sqrt{\frac{4\,a^2+b^2}{b^2}}-2\,a\sqrt{-b}\right)\sqrt{\frac{b\sqrt{\frac{4\,a^2+b^2}{b^2}}+2\,a}{b}}F(\arcsin\left(\sqrt{\frac{b\sqrt{\frac{4\,a^2+b^2}{b^2}}+2\,a}{b}}(\cosh\left(dx+c\right)+\sinh\left(dx+c\right))\right)}{b^2d}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(-b)*b*sqrt((4*a^2 + b^2)/b^2) - 2*a*sqrt(-b))*sqrt((b*sqrt((4*a^2
+ b^2)/b^2) + 2*a)/b)*elliptic_f(arcsin(sqrt((b*sqrt((4*a^2 + b^2)/b^2) + 2
*a)/b)*(cosh(d*x + c) + sinh(d*x + c))), (4*a*b*sqrt((4*a^2 + b^2)/b^2) - 8
*a^2 - b^2)/b^2)/(b^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh\left(c+dx\right)\cosh\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(c + d*x)*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*cosh(d*x + c)*sinh(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh(c+dx)\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)}}\,dx$$

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2),x)
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(1/2), x)
```

3.865 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 158

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{2}b\cosh(2c+2dx)}{(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{2i\sqrt{2}E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)\middle|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{(4a^2+b^2)\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[Out] -2*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)+2*
I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticE(cos(I*c+1
/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*d*x+2*c))^(
1/2)/(4*a^2+b^2)/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2745, 2743, 21, 2734, 2732}

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{2}b\cosh(2c+2dx)}{d\,(4a^2+b^2)\,\sqrt{2a+b\sinh(2c+2dx)}} - \frac{2i\sqrt{2}\sqrt{2a+b\sinh(2c+2dx)}E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)\middle|\frac{2b}{2ia+b}\right)}{d\,(4a^2+b^2)\,\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[2]*b*Cosh[2*c + 2*d*x])/((4*a^2 + b^2)*d*Sqrt[2*a + b*Sinh[2*c + 2
*d*x]]) - ((2*I)*Sqrt[2]*EllipticE[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((
```

```
2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((4*a^2 + b^2)*d*Sqrt[(2*a +
b*Sinh[2*c + 2*d*x]])/(2*a - I*b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}\,dx$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{8\int \frac{-\frac{a}{2}-\frac{1}{4}b\sinh(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}\,dx}{4a^2+b^2}$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{4\int\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{4a^2+b^2}$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{\left(4\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\right)\int\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}\,dx}{(4a^2+b^2)\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}}$$

$$= -\frac{2\sqrt{2}b\cosh(2c+2dx)}{(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{2i\sqrt{2}E\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{(4a^2+b^2)\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

$$\int\frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \frac{2\left(-b\cosh(2(c+dx))+(2ia+b)E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-ib}\right.\right.}{(4a^2+b^2)\,d\sqrt{a+\frac{1}{2}b\sinh(2(c+dx))}}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*(-(b*Cosh[2*(c + d*x)]) + ((2*I)*a + b)*EllipticE[((-4*I)*c + Pi - (4*I)
*d*x)/4, ((-2*I)*b)/(2*a - I*b)]*Sqrt[(2*a + b*Sinh[2*(c + d*x)])/(2*a - I*
b)]))/((4*a^2 + b^2)*d*Sqrt[a + (b*Sinh[2*(c + d*x)])/2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(177) = 354.

Time = 0.41 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.99

method	result
default	$\frac{16\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}}\sqrt{\frac{(-\sinh(2dx+2c)+i)b}{ib+2a}}\sqrt{\frac{(\sinh(2dx+2c)+i)b}{ib-2a}}\operatorname{EllipticF}\left(\sqrt{-\frac{2a+b\sinh(2dx+2c)}{ib-2a}},\sqrt{-\frac{ib-2a}{ib+2a}}\right)a^2+4\sqrt{-\frac{2a+b\sinh(2d}{ib-2a}}}{}$

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(4*(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*
b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*si
nh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2+(-(2*a+b*
sinh(2*d*x+2*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*
((sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticF((-(2*a+b*sinh(2*d*x+2*c))
/(I*b-2*a))^(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*b^2-4*(-(2*a+b*sinh(2*d*x+2
*c))/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x
+2*c)+I)*b/(I*b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^
(1/2),(-(I*b-2*a)/(I*b+2*a))^(1/2))*a^2-(-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a)
)^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x+2*c)+I)*b/(I*
b-2*a))^(1/2)*EllipticE((-(2*a+b*sinh(2*d*x+2*c))/(I*b-2*a))^(1/2),(-(I*b-2
*a)/(I*b+2*a))^(1/2))*b^2-b^2*sinh(2*d*x+2*c)^2-b^2)/(4*a^2+b^2)/b/cosh(2*d
*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(168) = 336$.

Time = 0.11 (sec) , antiderivative size = 1283, normalized size of antiderivative = 8.12

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -4*(((b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(
d*x + c)^4 + 4*a*b^2*cosh(d*x + c)^2 - b^3 + 2*(3*b^3*cosh(d*x + c)^2 + 2*a
*b^2)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + 2*a*b^2*cosh(d*x + c))*sin
h(d*x + c))*sqrt(-b)*sqrt((4*a^2 + b^2)/b^2) + 2*(a*b^2*cosh(d*x + c)^4 + 4
*a*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 4*a^2*b*cosh
(d*x + c)^2 - a*b^2 + 2*(3*a*b^2*cosh(d*x + c)^2 + 2*a^2*b)*sinh(d*x + c)^2
 + 4*(a*b^2*cosh(d*x + c)^3 + 2*a^2*b*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b
))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*elliptic_e(arcsin(sqrt((b*sqrt
((4*a^2 + b^2)/b^2) + 2*a)/b)*(cosh(d*x + c) + sinh(d*x + c))), (4*a*b*sqrt
((4*a^2 + b^2)/b^2) - 8*a^2 - b^2)/b^2) + (((2*a*b^2 - b^3)*cosh(d*x + c)^4
 + 4*(2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2 - b^3)*sinh(d
*x + c)^4 - 2*a*b^2 + b^3 + 4*(2*a^2*b - a*b^2)*cosh(d*x + c)^2 + 2*(4*a^2*
b - 2*a*b^2 + 3*(2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2*a*
b^2 - b^3)*cosh(d*x + c)^3 + 2*(2*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-b)*sqrt((4*a^2 + b^2)/b^2) - 2*((2*a^2*b + a*b^2)*cosh(d*x + c)^4
 + 4*(2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b + a*b^2)*si
nh(d*x + c)^4 - 2*a^2*b - a*b^2 + 4*(2*a^3 + a^2*b)*cosh(d*x + c)^2 + 2*(4*
a^3 + 2*a^2*b + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((
2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 2*(2*a^3 + a^2*b)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*elliptic_f(arcs
in(sqrt((b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*(cosh(d*x + c) + sinh(d*x + c)
```

```
)), (4*a*b*sqrt((4*a^2 + b^2)/b^2) - 8*a^2 - b^2)/b^2) + 2*(b^3*cosh(d*x +
c)^3 + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*sinh(d*x + c)^3 + 2*a*b^2*
cosh(d*x + c) + (3*b^3*cosh(d*x + c)^2 + 2*a*b^2)*sinh(d*x + c))*sqrt((b*co
sh(d*x + c)*sinh(d*x + c) + a)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2)))/((4*a^2*b^3 + b^5)*d*cosh(d*x + c)^4 + 4*(4*a^2*b
^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^2*b^3 + b^5)*d*sinh(d*x +
c)^4 + 4*(4*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(4*a^2*b^3 + b^5)*d*c
osh(d*x + c)^2 + 2*(4*a^3*b^2 + a*b^4)*d)*sinh(d*x + c)^2 - (4*a^2*b^3 + b^
5)*d + 4*((4*a^2*b^3 + b^5)*d*cosh(d*x + c)^3 + 2*(4*a^3*b^2 + a*b^4)*d*cos
h(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sinh{(c+dx)}\cosh{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(3/2),x)
[Out] Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cosh{(dx+c)}\sinh{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2),x)
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(3/2), x)
```

3.866 $\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 325

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3(4a^2+b^2)\,d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3(4a^2+b^2)^2\,d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{32i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)\middle|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{3(4a^2+b^2)^2\,d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{4i\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{3(4a^2+b^2)\,d\sqrt{2a+b\sinh(2c+2dx)}}$$

```
[Out] -4/3*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(3/2)-
32/3*a*b*cosh(2*d*x+2*c)*2^(1/2)/(4*a^2+b^2)^2/d/(2*a+b*sinh(2*d*x+2*c))^(1
/2)+32/3*I*a*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*Elliptic
E(cos(I*c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*(2*a+b*sinh(2*
d*x+2*c))^(1/2)/(4*a^2+b^2)^2/d/((2*a+b*sinh(2*d*x+2*c))/(2*a-I*b))^(1/2)-4
/3*I*(sin(I*c+1/4*Pi+I*d*x)^2)^(1/2)/sin(I*c+1/4*Pi+I*d*x)*EllipticF(cos(I*
c+1/4*Pi+I*d*x),2^(1/2)*(b/(2*I*a+b))^(1/2))*2^(1/2)*((2*a+b*sinh(2*d*x+2*c
))/(2*a-I*b))^(1/2)/(4*a^2+b^2)/d/(2*a+b*sinh(2*d*x+2*c))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2745, 2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = -\frac{32\sqrt{2}ab\cosh(2c+2dx)}{3d\,(4a^2+b^2)^2\,\sqrt{2a+b\sinh(2c+2dx)}} - \frac{4\sqrt{2}b\cosh(2c+2dx)}{3d\,(4a^2+b^2)\,(2a+b\sinh(2c+2dx))^{3/2}} + \frac{4i\sqrt{2}\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right),\frac{2b}{2ia+b}\right)}{3d\,(4a^2+b^2)\,\sqrt{2a+b\sinh(2c+2dx)}} - \frac{32i\sqrt{2}a\sqrt{2a+b\sinh(2c+2dx)}\,E\left(\frac{1}{2}\left(2ic+2idx-\frac{\pi}{2}\right)|\frac{2b}{2ia+b}\right)}{3d\,(4a^2+b^2)^2\,\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}$$

```
[In] Int[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-4*Sqrt[2]*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)*d*(2*a + b*Sinh[2*c + 2*d
*x])^(3/2)) - (32*Sqrt[2]*a*b*Cosh[2*c + 2*d*x])/(3*(4*a^2 + b^2)^2*d*Sqrt[
2*a + b*Sinh[2*c + 2*d*x]]) - (((32*I)/3)*Sqrt[2]*a*EllipticE[((2*I)*c - Pi
/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sqrt[2*a + b*Sinh[2*c + 2*d*x]])/((
4*a^2 + b^2)^2*d*Sqrt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)]) + (((4*I)/3
)*Sqrt[2]*EllipticF[((2*I)*c - Pi/2 + (2*I)*d*x)/2, (2*b)/((2*I)*a + b)]*Sq
rt[(2*a + b*Sinh[2*c + 2*d*x])/(2*a - I*b)])/((4*a^2 + b^2)*d*Sqrt[2*a + b*
Sinh[2*c + 2*d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{5/2}}\,dx$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3\left(4a^2+b^2\right)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{8\int \frac{-\frac{3a}{2}+\frac{1}{4}b\sinh(2c+2dx)}{\left(a+\frac{1}{2}b\sinh(2c+2dx)\right)^{3/2}}\,dx}{3\left(4a^2+b^2\right)}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3(4a^2+b^2)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3(4a^2+b^2)^2 d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{64\int \frac{\frac{1}{16}(12a^2-b^2)+\frac{1}{2}ab\sinh(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}dx}{3(4a^2+b^2)^2}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3(4a^2+b^2)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3(4a^2+b^2)^2 d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{(64a)\int \sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\,dx}{3(4a^2+b^2)^2} - \frac{4\int \frac{1}{\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}dx}{3(4a^2+b^2)}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3(4a^2+b^2)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3(4a^2+b^2)^2 d\sqrt{2a+b\sinh(2c+2dx)}} + \frac{\left(64a\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}\right)\int \sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}dx}{3(4a^2+b^2)^2\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}} - \frac{\left(4\sqrt{\frac{a+\frac{1}{2}b\sinh(2c+2dx)}{a-\frac{ib}{2}}}\right)\int \frac{1}{\sqrt{\frac{a}{a-\frac{ib}{2}}+\frac{b\sinh(2c+2dx)}{2\left(a-\frac{ib}{2}\right)}}}dx}{3(4a^2+b^2)\sqrt{a+\frac{1}{2}b\sinh(2c+2dx)}}$$

$$= -\frac{4\sqrt{2}b\cosh(2c+2dx)}{3(4a^2+b^2)d(2a+b\sinh(2c+2dx))^{3/2}} - \frac{32\sqrt{2}ab\cosh(2c+2dx)}{3(4a^2+b^2)^2 d\sqrt{2a+b\sinh(2c+2dx)}} - \frac{32i\sqrt{2}aE\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right)|\frac{2b}{2ia+b}\right)\sqrt{2a+b\sinh(2c+2dx)}}{3(4a^2+b^2)^2 d\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}} + \frac{4i\sqrt{2}\,\text{EllipticF}\left(\frac{1}{2}\left(2ic-\frac{\pi}{2}+2idx\right),\frac{2b}{2ia+b}\right)\sqrt{\frac{2a+b\sinh(2c+2dx)}{2a-ib}}}{3(4a^2+b^2)d\sqrt{2a+b\sinh(2c+2dx)}}$$

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}dx = \frac{4\sqrt{2}\left(8ia(2a-ib)^2E\left(\frac{1}{4}(-4ic+\pi-4idx)|-\frac{2ib}{2a-ib}\right)\left(\frac{2a+b\sinh(2(\cdots}{2a-ib}\right.\right.}{}$$

```
[In] Integrate[(a + b*Cosh[c + d*x]*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2]*((8*I)*a*(2*a - I*b)^2*EllipticE[((-4*I)*c + Pi - (4*I)*d*x)/4,
((-2*I)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) + (
```

```
2*a - I*b)^2*((-2*I)*a + b)*EllipticF[((-4*I)*c + Pi - (4*I)*d*x)/4, ((-2*I
)*b)/(2*a - I*b)]*((2*a + b*Sinh[2*(c + d*x)])/(2*a - I*b))^(3/2) - b*Cosh[
2*(c + d*x)]*(20*a^2 + b^2 + 8*a*b*Sinh[2*(c + d*x)])))/(3*(4*a^2 + b^2)^2*
d*(2*a + b*Sinh[2*(c + d*x)])^(3/2))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.97

method	result
default	$\frac{4\sqrt{\cosh(2dx+2c)^2(2a+b\sinh(2dx+2c))}\left(-\frac{2\sqrt{\cosh(2dx+2c)^2(2a+b\sinh(2dx+2c))}}{3b(4a^2+b^2)\left(\sinh(2dx+2c)+\frac{2a}{b}\right)^2}-\frac{16b\cosh(2dx+2c)^2a}{3(4a^2+b^2)^2\sqrt{\cosh(2dx+2c)^2(2a+b\sinh(2dx+2c))}}+\frac{2(12\cdots}{}\right.}{}$
risch	Expression too large to display

```
[In] int(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x+2*c)))^(1/2)*(-2/3/b/(4*a^2+b^2)*(co
sh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x+2*c)))^(1/2)/(sinh(2*d*x+2*c)+2/b*a)^2-16
/3*b*cosh(2*d*x+2*c)^2/(4*a^2+b^2)^2*a/(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x
+2*c)))^(1/2)+2*(12*a^2-b^2)/(48*a^4+24*a^2*b^2+3*b^4)*(2/b*a-I)*((-b*sinh(
2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*(
(sinh(2*d*x+2*c)+I)*b/(I*b-2*a))^(1/2)/(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x
+2*c)))^(1/2)*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),((2*a-I*
b)/(I*b+2*a))^(1/2))+16/3*a*b/(4*a^2+b^2)^2*(2/b*a-I)*((-b*sinh(2*d*x+2*c)-
2*a)/(I*b-2*a))^(1/2)*((-sinh(2*d*x+2*c)+I)*b/(I*b+2*a))^(1/2)*((sinh(2*d*x
+2*c)+I)*b/(I*b-2*a))^(1/2)/(cosh(2*d*x+2*c)^2*(2*a+b*sinh(2*d*x+2*c)))^(1/
2)*((-2/b*a-I)*EllipticE(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2),((2*a-I
*b)/(I*b+2*a))^(1/2))+I*EllipticF(((-b*sinh(2*d*x+2*c)-2*a)/(I*b-2*a))^(1/2
),((2*a-I*b)/(I*b+2*a))^(1/2))))/cosh(2*d*x+2*c)/(4*a+2*b*sinh(2*d*x+2*c))^
(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4231 vs. $2(337) = 674$.

Time = 0.19 (sec) , antiderivative size = 4231, normalized size of antiderivative = 13.02

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -8/3*(8*((a*b^4*cosh(d*x + c)^8 + 8*a*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + a
*b^4*sinh(d*x + c)^8 + 8*a^2*b^3*cosh(d*x + c)^6 - 8*a^2*b^3*cosh(d*x + c)^
```

2 + 4*(7*a*b^4*cosh(d*x + c)^2 + 2*a^2*b^3)*sinh(d*x + c)^6 + 8*(7*a*b^4*co
sh(d*x + c)^3 + 6*a^2*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + a*b^4 + 2*(8*a^3
*b^2 - a*b^4)*cosh(d*x + c)^4 + 2*(35*a*b^4*cosh(d*x + c)^4 + 60*a^2*b^3*co
sh(d*x + c)^2 + 8*a^3*b^2 - a*b^4)*sinh(d*x + c)^4 + 8*(7*a*b^4*cosh(d*x +
c)^5 + 20*a^2*b^3*cosh(d*x + c)^3 + (8*a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh
(d*x + c)^3 + 4*(7*a*b^4*cosh(d*x + c)^6 + 30*a^2*b^3*cosh(d*x + c)^4 - 2*a
^2*b^3 + 3*(8*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*b^4*
cosh(d*x + c)^7 + 6*a^2*b^3*cosh(d*x + c)^5 - 2*a^2*b^3*cosh(d*x + c) + (8*
a^3*b^2 - a*b^4)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt(-b)*sqrt((4*a^2 + b^2
)/b^2) + 2*(a^2*b^3*cosh(d*x + c)^8 + 8*a^2*b^3*cosh(d*x + c)*sinh(d*x + c)
^7 + a^2*b^3*sinh(d*x + c)^8 + 8*a^3*b^2*cosh(d*x + c)^6 - 8*a^3*b^2*cosh(d
x + c)^2 + 4(7*a^2*b^3*cosh(d*x + c)^2 + 2*a^3*b^2)*sinh(d*x + c)^6 + 8*(
7*a^2*b^3*cosh(d*x + c)^3 + 6*a^3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + a^2*
b^3 + 2*(8*a^4*b - a^2*b^3)*cosh(d*x + c)^4 + 2*(35*a^2*b^3*cosh(d*x + c)^4
 + 60*a^3*b^2*cosh(d*x + c)^2 + 8*a^4*b - a^2*b^3)*sinh(d*x + c)^4 + 8*(7*a
^2*b^3*cosh(d*x + c)^5 + 20*a^3*b^2*cosh(d*x + c)^3 + (8*a^4*b - a^2*b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^3*cosh(d*x + c)^6 + 30*a^3*b^2*c
osh(d*x + c)^4 - 2*a^3*b^2 + 3*(8*a^4*b - a^2*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 8*(a^2*b^3*cosh(d*x + c)^7 + 6*a^3*b^2*cosh(d*x + c)^5 - 2*a^3*b
^2*cosh(d*x + c) + (8*a^4*b - a^2*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*sqrt
(-b))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*elliptic_e(arcsin(sqrt((b*s
qrt((4*a^2 + b^2)/b^2) + 2*a)/b)*(cosh(d*x + c) + sinh(d*x + c))), (4*a*b*s
qrt((4*a^2 + b^2)/b^2) - 8*a^2 - b^2)/b^2) + (((12*a^2*b^3 - 8*a*b^4 - b^5)
*cosh(d*x + c)^8 + 8*(12*a^2*b^3 - 8*a*b^4 - b^5)*cosh(d*x + c)*sinh(d*x +
c)^7 + (12*a^2*b^3 - 8*a*b^4 - b^5)*sinh(d*x + c)^8 + 8*(12*a^3*b^2 - 8*a^2
*b^3 - a*b^4)*cosh(d*x + c)^6 + 4*(24*a^3*b^2 - 16*a^2*b^3 - 2*a*b^4 + 7*(1
2*a^2*b^3 - 8*a*b^4 - b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(12*a^2*
b^3 - 8*a*b^4 - b^5)*cosh(d*x + c)^3 + 6*(12*a^3*b^2 - 8*a^2*b^3 - a*b^4)*c
osh(d*x + c))*sinh(d*x + c)^5 + 12*a^2*b^3 - 8*a*b^4 - b^5 + 2*(96*a^4*b -
64*a^3*b^2 - 20*a^2*b^3 + 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 2*(96*a^4*b - 64
*a^3*b^2 - 20*a^2*b^3 + 8*a*b^4 + b^5 + 35*(12*a^2*b^3 - 8*a*b^4 - b^5)*cos
h(d*x + c)^4 + 60*(12*a^3*b^2 - 8*a^2*b^3 - a*b^4)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(12*a^2*b^3 - 8*a*b^4 - b^5)*cosh(d*x + c)^5 + 20*(12*a^3*b
^2 - 8*a^2*b^3 - a*b^4)*cosh(d*x + c)^3 + (96*a^4*b - 64*a^3*b^2 - 20*a^2*b
^3 + 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(12*a^3*b^2 - 8*a^2*
b^3 - a*b^4)*cosh(d*x + c)^2 + 4*(7*(12*a^2*b^3 - 8*a*b^4 - b^5)*cosh(d*x +
 c)^6 - 24*a^3*b^2 + 16*a^2*b^3 + 2*a*b^4 + 30*(12*a^3*b^2 - 8*a^2*b^3 - a*
b^4)*cosh(d*x + c)^4 + 3*(96*a^4*b - 64*a^3*b^2 - 20*a^2*b^3 + 8*a*b^4 + b^
5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((12*a^2*b^3 - 8*a*b^4 - b^5)*cosh(
d*x + c)^7 + 6*(12*a^3*b^2 - 8*a^2*b^3 - a*b^4)*cosh(d*x + c)^5 + (96*a^4*b
 - 64*a^3*b^2 - 20*a^2*b^3 + 8*a*b^4 + b^5)*cosh(d*x + c)^3 - 2*(12*a^3*b^2
 - 8*a^2*b^3 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)*sqrt((4*a^2 +
b^2)/b^2) - 2*((12*a^3*b^2 + 8*a^2*b^3 - a*b^4)*cosh(d*x + c)^8 + 8*(12*a^3
*b^2 + 8*a^2*b^3 - a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (12*a^3*b^2 + 8*a
^2*b^3 - a*b^4)*sinh(d*x + c)^8 + 8*(12*a^4*b + 8*a^3*b^2 - a^2*b^3)*cosh(d

x + c)^6 + 4(24*a^4*b + 16*a^3*b^2 - 2*a^2*b^3 + 7*(12*a^3*b^2 + 8*a^2*b^
3 - a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(12*a^3*b^2 + 8*a^2*b^3
- a*b^4)*cosh(d*x + c)^3 + 6*(12*a^4*b + 8*a^3*b^2 - a^2*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 12*a^3*b^2 + 8*a^2*b^3 - a*b^4 + 2*(96*a^5 + 64*a^4*b -
20*a^3*b^2 - 8*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 + 2*(96*a^5 + 64*a^4*b - 2
0*a^3*b^2 - 8*a^2*b^3 + a*b^4 + 35*(12*a^3*b^2 + 8*a^2*b^3 - a*b^4)*cosh(d*
x + c)^4 + 60*(12*a^4*b + 8*a^3*b^2 - a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*(12*a^3*b^2 + 8*a^2*b^3 - a*b^4)*cosh(d*x + c)^5 + 20*(12*a^4*b
+ 8*a^3*b^2 - a^2*b^3)*cosh(d*x + c)^3 + (96*a^5 + 64*a^4*b - 20*a^3*b^2 -
8*a^2*b^3 + a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(12*a^4*b + 8*a^3*b^
2 - a^2*b^3)*cosh(d*x + c)^2 + 4*(7*(12*a^3*b^2 + 8*a^2*b^3 - a*b^4)*cosh(d
*x + c)^6 - 24*a^4*b - 16*a^3*b^2 + 2*a^2*b^3 + 30*(12*a^4*b + 8*a^3*b^2 -
a^2*b^3)*cosh(d*x + c)^4 + 3*(96*a^5 + 64*a^4*b - 20*a^3*b^2 - 8*a^2*b^3 +
a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((12*a^3*b^2 + 8*a^2*b^3 - a*b^
4)*cosh(d*x + c)^7 + 6*(12*a^4*b + 8*a^3*b^2 - a^2*b^3)*cosh(d*x + c)^5 + (
96*a^5 + 64*a^4*b - 20*a^3*b^2 - 8*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 - 2*(12
*a^4*b + 8*a^3*b^2 - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b))*sqrt(
(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)*elliptic_f(arcsin(sqrt((b*sqrt((4*a^2
+ b^2)/b^2) + 2*a)/b)*(cosh(d*x + c) + sinh(d*x + c))), (4*a*b*sqrt((4*a^2
+ b^2)/b^2) - 8*a^2 - b^2)/b^2) + 2*(8*a*b^4*cosh(d*x + c)^7 + 56*a*b^4*cos
h(d*x + c)*sinh(d*x + c)^6 + 8*a*b^4*sinh(d*x + c)^7 + (52*a^2*b^3 + b^5)*c
osh(d*x + c)^5 + (168*a*b^4*cosh(d*x + c)^2 + 52*a^2*b^3 + b^5)*sinh(d*x +
c)^5 + 5*(56*a*b^4*cosh(d*x + c)^3 + (52*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c)^4 + 8*(8*a^3*b^2 - a*b^4)*cosh(d*x + c)^3 + 2*(140*a*b^4*cosh(d*x
+ c)^4 + 32*a^3*b^2 - 4*a*b^4 + 5*(52*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh
(d*x + c)^3 + 2*(84*a*b^4*cosh(d*x + c)^5 + 5*(52*a^2*b^3 + b^5)*cosh(d*x +
c)^3 + 12*(8*a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - (12*a^2*b^3
- b^5)*cosh(d*x + c) + (56*a*b^4*cosh(d*x + c)^6 - 12*a^2*b^3 + b^5 + 5*(5
2*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 24*(8*a^3*b^2 - a*b^4)*cosh(d*x + c)^2)*
sinh(d*x + c))*sqrt((b*cosh(d*x + c)*sinh(d*x + c) + a)/(cosh(d*x + c)^2 -
2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((16*a^4*b^4 + 8*a^2*b^6
+ b^8)*d*cosh(d*x + c)^8 + 8*(16*a^4*b^4 + 8*a^2*b^6 + b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^7 + (16*a^4*b^4 + 8*a^2*b^6 + b^8)*d*sinh(d*x + c)^8 + 8*(1
6*a^5*b^3 + 8*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^6 + 4*(7*(16*a^4*b^4 + 8*a^2
*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(16*a^5*b^3 + 8*a^3*b^5 + a*b^7)*d)*sinh(
d*x + c)^6 + 2*(128*a^6*b^2 + 48*a^4*b^4 - b^8)*d*cosh(d*x + c)^4 + 8*(7*(1
6*a^4*b^4 + 8*a^2*b^6 + b^8)*d*cosh(d*x + c)^3 + 6*(16*a^5*b^3 + 8*a^3*b^5
+ a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(16*a^4*b^4 + 8*a^2*b^6 +
b^8)*d*cosh(d*x + c)^4 + 60*(16*a^5*b^3 + 8*a^3*b^5 + a*b^7)*d*cosh(d*x +
c)^2 + (128*a^6*b^2 + 48*a^4*b^4 - b^8)*d)*sinh(d*x + c)^4 - 8*(16*a^5*b^3
+ 8*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(16*a^4*b^4 + 8*a^2*b^6 + b^8
)*d*cosh(d*x + c)^5 + 20*(16*a^5*b^3 + 8*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3
+ (128*a^6*b^2 + 48*a^4*b^4 - b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(16*a^4*b^4 + 8*a^2*b^6 + b^8)*d*cosh(d*x + c)^6 + 30*(16*a^5*b^3 + 8*a^3*
b^5 + a*b^7)*d*cosh(d*x + c)^4 + 3*(128*a^6*b^2 + 48*a^4*b^4 - b^8)*d*cosh(

```
d*x + c)^2 - 2*(16*a^5*b^3 + 8*a^3*b^5 + a*b^7)*d)*sinh(d*x + c)^2 + (16*a^
4*b^4 + 8*a^2*b^6 + b^8)*d + 8*((16*a^4*b^4 + 8*a^2*b^6 + b^8)*d*cosh(d*x +
 c)^7 + 6*(16*a^5*b^3 + 8*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^5 + (128*a^6*b^2
 + 48*a^4*b^4 - b^8)*d*cosh(d*x + c)^3 - 2*(16*a^5*b^3 + 8*a^3*b^5 + a*b^7)
*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\sinh(c+dx)\cosh(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))**(5/2),x)
[Out] Integral((a + b*sinh(c + d*x)*cosh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cosh(dx+c)\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cosh(d*x + c)*sinh(d*x + c) + a)^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+b*cosh(d*x+c)*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx)\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*cosh(c + d*x)*sinh(c + d*x))^(5/2), x)
```

3.867 $\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 386

$$\begin{aligned}\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx &= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} \\ &+ \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} \\ &- \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} \\ &- \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} \\ &+ \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} \\ &+ \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}} - \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}}\end{aligned}$$

```
[Out] x^3*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x^3*ln(1+b*e
xp(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/2*x^2*polylog(2,-b*exp
(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/2*x^2*polylog(2,-b*exp(2
*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/2*x*polylog(3,-b*exp(2*x)/
(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/2*x*polylog(3,-b*exp(2*x)/(2*a
+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+3/4*polylog(4,-b*exp(2*x)/(2*a-(4*a^
2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-3/4*polylog(4,-b*exp(2*x)/(2*a+(4*a^2+b^2)
^(1/2)))/(4*a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5747, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}} - \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}} + \frac{x^3\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

```
[In] Int[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^3*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (3*x
^2*PolyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^
2]) - (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[
4*a^2 + b^2]) - (3*x*PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/
(2*Sqrt[4*a^2 + b^2]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(2*Sqrt[4*a^2 + b^2]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - Sqrt[4*a
^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a + Sq
rt[4*a^2 + b^2]))])/(4*Sqrt[4*a^2 + b^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5747

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{a+\frac{1}{2}b\sinh(2x)}\,dx$$

$$= 2\int \frac{e^{2x}x^3}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} - \frac{(2b)\int \frac{e^{2x}x^3}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{3\int x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}} + \frac{3\int x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3\int x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}} + \frac{3\int x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3x\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3\int \text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)dx}{2\sqrt{4a^2+b^2}} - \frac{3\int \text{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)dx}{2\sqrt{4a^2+b^2}}$$

$$= \frac{x^3 \log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3 \log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x \operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3x \operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,\frac{bx}{-2a+\sqrt{4a^2+b^2}}\right)}{x}dx, x, e^{2x}\right)}{4\sqrt{4a^2+b^2}} - \frac{3\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3,-\frac{bx}{2a+\sqrt{4a^2+b^2}}\right)}{x}dx, x, e^{2x}\right)}{4\sqrt{4a^2+b^2}}$$

$$= \frac{x^3 \log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^3 \log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{3x \operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3x \operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{3 \operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}} - \frac{3 \operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{4\sqrt{4a^2+b^2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \frac{4x^3 \log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right) - 4x^3 \log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right) + 6x^2 \operatorname{PolyLog}\left(2,\frac{be^{2x}}{-2a+\sqrt{4a^2+b^2}}\right) - 6x^2 \operatorname{PolyLog}\left(2,-\right.}{}$$

```
[In] Integrate[x^3/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (4*x^3*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 4*x^3*Log[1 + (b*E^
(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 6*x^2*PolyLog[2, (b*E^(2*x))/(-2*a + Sq
rt[4*a^2 + b^2])] - 6*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]
))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + 6*x*PolyLog[
3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))] + 3*PolyLog[4, (b*E^(2*x))/(-2
*a + Sqrt[4*a^2 + b^2])] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b
^2]))])/(4*Sqrt[4*a^2 + b^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(334) = 668.

Time = 1.55 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.78

method	result
risch	$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)x^3}{-2a-\sqrt{4a^2+b^2}}+\frac{2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)a\,x^3}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{x^4}{2\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{a\,x^4}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{3\,\mathrm{polylog}\left(2,\right.}{2\left(-2a\right.}$

```
[In] int(x^3/(a+b*cosh(x)*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x^3+2/
(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2
)^(1/2)))*a*x^3-1/2/(-2*a-(4*a^2+b^2)^(1/2))*x^4-1/(4*a^2+b^2)^(1/2)/(-2*a-
(4*a^2+b^2)^(1/2))*a*x^4+3/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/
(-2*a-(4*a^2+b^2)^(1/2)))*x^2+3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*
polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x^2-3/2/(-2*a-(4*a^2+b^2)^
(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-3/(4*a^2+b^2)^(1/2)
/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*
x+3/4/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)
))+3/2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(4,b*exp(2*x)/(-2*
a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x^3*ln(1-b*exp(2*x)/((4*a^2+b^2
)^(1/2)-2*a))-1/2/(4*a^2+b^2)^(1/2)*x^4+3/2/(4*a^2+b^2)^(1/2)*x^2*polylog(2
,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))-3/2/(4*a^2+b^2)^(1/2)*x*polylog(3,b*ex
p(2*x)/((4*a^2+b^2)^(1/2)-2*a))+3/4/(4*a^2+b^2)^(1/2)*polylog(4,b*exp(2*x)/
((4*a^2+b^2)^(1/2)-2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. 2(332) = 664.

Time = 0.29 (sec) , antiderivative size = 1488, normalized size of antiderivative = 3.85

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*si
nh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^3*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*
cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^3*sqrt((4*a^2 + b^2)/
```

```
b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + 3*b*x^2*s
qrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*
sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b
) + b)/b + 1) + 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*s
inh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*
a^2 + b^2)/b^2) + 2*a)/b) - b)/b + 1) - 3*b*x^2*sqrt((4*a^2 + b^2)/b^2)*dil
og(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2
)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 3*b*x^2*sqr
t((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sin
h(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) -
b)/b + 1) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x) + 2*a*sin
h(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^
2 + b^2)/b^2) + 2*a)/b)/b) - 6*b*x*sqrt((4*a^2 + b^2)/b^2)*polylog(3, -(2*a
*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*s
qrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)/b) + 6*b*x*sqrt((4*a^2 + b^2)/b^2
)*polylog(3, (2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a
^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) + 6*b*x*sqrt((
4*a^2 + b^2)/b^2)*polylog(3, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*s
inh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/
b) + 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4, (2*a*cosh(x) + 2*a*sinh(x) - (b
*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/
b^2) + 2*a)/b)/b) + 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4, -(2*a*cosh(x) +
2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqr
t((4*a^2 + b^2)/b^2) + 2*a)/b)/b) - 6*b*sqrt((4*a^2 + b^2)/b^2)*polylog(4,
(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2
))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) - 6*b*sqrt((4*a^2 + b^2)/b^
2)*polylog(4, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4
*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b))/(4*a^2 + b^
2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\cosh(x)\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(a+b*cosh(x)*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{b\cosh\left(x\right)\sinh\left(x\right)+a}\,dx$$

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
[Out] integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{b\cosh\left(x\right)\sinh\left(x\right)+a}\,dx$$

```
[In] integrate(x^3/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
[Out] integrate(x^3/(b*cosh(x)*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^3}{a+b\cosh\left(x\right)\,\sinh\left(x\right)}\,dx$$

```
[In] int(x^3/(a + b*cosh(x)*sinh(x)),x)
[Out] int(x^3/(a + b*cosh(x)*sinh(x)), x)
```

3.868 $\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 281

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

```
[Out] x^2*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x^2*ln(1+b*e
xp(2*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+x*polylog(2,-b*exp(2*x)/
(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x*polylog(2,-b*exp(2*x)/(2*a+(4*
a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-1/2*polylog(3,-b*exp(2*x)/(2*a-(4*a^2+b^
2)^(1/2)))/(4*a^2+b^2)^(1/2)+1/2*polylog(3,-b*exp(2*x)/(2*a+(4*a^2+b^2)^(1/
2)))/(4*a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5747, 3403, 2296, 2221, 2611, 2320, 6724\}$

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \frac{x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{x^2\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}} - \frac{x^2\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

```
[In] Int[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x
^2*Log[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + (x*P
olyLog[2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] - (x
*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/Sqrt[4*a^2 + b^2] -
PolyLog[3, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
+ PolyLog[3, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5747

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2}{a+\frac{1}{2}b\sinh(2x)}\,dx \\
&= 2\int \frac{e^{2x}x^2}{-\frac{b}{2}+2ae^{2x}+\frac{1}{2}be^{4x}}\,dx \\
&= \frac{(2b)\int \frac{e^{2x}x^2}{2a-\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}}-\frac{(2b)\int \frac{e^{2x}x^2}{2a+\sqrt{4a^2+b^2}+be^{2x}}\,dx}{\sqrt{4a^2+b^2}} \\
&= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}}-\frac{x^2\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} \\
&\quad -\frac{2\int x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)\,dx}{\sqrt{4a^2+b^2}}+\frac{2\int x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)\,dx}{\sqrt{4a^2+b^2}}
\end{aligned}
$$

$$= \frac{x^2 \log \left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2 \log \left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\int \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right) dx}{\sqrt{4a^2+b^2}} + \frac{\int \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right) dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x^2 \log \left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2 \log \left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{bx}{-2a+\sqrt{4a^2+b^2}}\right)}{x} dx, x, e^{2x}\right)}{2\sqrt{4a^2+b^2}} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{bx}{2a+\sqrt{4a^2+b^2}}\right)}{x} dx, x, e^{2x}\right)}{2\sqrt{4a^2+b^2}}$$

$$= \frac{x^2 \log \left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x^2 \log \left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \frac{2x^2 \log \left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right) - 2x^2 \log \left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right) + 2x \operatorname{PolyLog}\left(2,\frac{be^{2x}}{-2a+\sqrt{4a^2+b^2}}\right) - 2x \operatorname{PolyLog}\left(2,-\frac{}{2a}\right.}{2\sqrt{4a^2+b^2}}$$

```
[In] Integrate[x^2/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (2*x^2*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - 2*x^2*Log[1 + (b*E^
(2*x))/(2*a + Sqrt[4*a^2 + b^2])] + 2*x*PolyLog[2, (b*E^(2*x))/(-2*a + Sqrt
[4*a^2 + b^2])] - 2*x*PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]
- PolyLog[3, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b^2])] + PolyLog[3, -((b*E^(2
*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2 + b^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(247) = 494$.

Time = 0.94 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{2x^3}{3\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{-2a-\sqrt{4a^2+b^2}}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{-2a-\sqrt{4a^2+b^2}}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)}{2\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{}{3\sqrt{4a^2+}}$

```
[In] int(x^2/(a+b*cosh(x)*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-2*a-(4*a^2+b^2)^(1/2))*x^3+1/(-2*a-(4*a^2+b^2)^(1/2))*x^2*ln(1-b*exp
(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))+1/(-2*a-(4*a^2+b^2)^(1/2))*x*polylog(2,b*ex
p(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(3,b*e
xp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-4/3/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(
1/2))*a*x^3+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*a*x^2*ln(1-b*exp(2
*x)/(-2*a-(4*a^2+b^2)^(1/2)))+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*
a*x*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-1/(4*a^2+b^2)^(1/2)/(-2*
a-(4*a^2+b^2)^(1/2))*a*polylog(3,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))-2/3/(
4*a^2+b^2)^(1/2)*x^3+1/(4*a^2+b^2)^(1/2)*x^2*ln(1-b*exp(2*x)/((4*a^2+b^2)^(
1/2)-2*a))+1/(4*a^2+b^2)^(1/2)*x*polylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*
a))-1/2/(4*a^2+b^2)^(1/2)*polylog(3,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(245) = 490$.

Time = 0.28 (sec) , antiderivative size = 1122, normalized size of antiderivative = 3.99

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x
) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) +
2*a)/b) + b)/b) + b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*si
nh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a
^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x^2*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*
cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sq
rt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x^2*sqrt((4*a^2 + b^2)/
b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2
 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + 2*b*x*sqr
t((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*si
nh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b)
```

```
+ b)/b + 1) + 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(
x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2
+ b^2)/b^2) + 2*a)/b) - b)/b + 1) - 2*b*x*sqrt((4*a^2 + b^2)/b^2)*dilog(-((
2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2)
)*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - 2*b*x*sqrt((4*a^2
 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sq
rt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1
) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (2*a*cosh(x) + 2*a*sinh(x) - (b*
cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b
^2) + 2*a)/b)/b) - 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, -(2*a*cosh(x) + 2
*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt
((4*a^2 + b^2)/b^2) + 2*a)/b)/b) + 2*b*sqrt((4*a^2 + b^2)/b^2)*polylog(3, (
2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2)
)*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b) + 2*b*sqrt((4*a^2 + b^2)/b^2
)*polylog(3, -(2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*
a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b)/b))/(4*a^2 + b^2
)
```

Sympy [F]

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{a+b\sinh{(x)}\cosh{(x)}}\,dx$$

```
[In] integrate(x**2/(a+b*cosh(x)*sinh(x)),x)
[Out] Integral(x**2/(a + b*sinh(x)*cosh(x)), x)
```

Maxima [F]

$$\int \frac{x^2}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{b\cosh{(x)}\sinh{(x)}+a}\,dx$$

```
[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
[Out] integrate(x^2/(b*cosh(x)*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{x^2}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{b\cosh\left(x\right)\sinh\left(x\right) + a}\,dx$$

[In] integrate(x^2/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")

[Out] integrate(x^2/(b*cosh(x)*sinh(x) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x^2}{a + b\cosh\left(x\right)\,\sinh\left(x\right)}\,dx$$

[In] int(x^2/(a + b*cosh(x)*sinh(x)),x)

[Out] int(x^2/(a + b*cosh(x)*sinh(x)), x)

3.869 $\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 186

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \frac{x\log\left(1+\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

```
[Out] x*ln(1+b*exp(2*x)/(2*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-x*ln(1+b*exp(2
*x)/(2*a+(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)+1/2*polylog(2,-b*exp(2*x)/(2
*a-(4*a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)-1/2*polylog(2,-b*exp(2*x)/(2*a+(4*
a^2+b^2)^(1/2)))/(4*a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5747, 3403, 2296, 2221, 2317, 2438}

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+\sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} + \frac{x\log\left(\frac{be^{2x}}{2a-\sqrt{4a^2+b^2}}+1\right)}{\sqrt{4a^2+b^2}} - \frac{x\log\left(\frac{be^{2x}}{\sqrt{4a^2+b^2}+2a}+1\right)}{\sqrt{4a^2+b^2}}$$

```
[In] Int[x/(a + b*Cosh[x]*Sinh[x]),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] - (x*L
og[1 + (b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2])])/Sqrt[4*a^2 + b^2] + PolyLog[
```

```
2, -((b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2]) - PolyLo
g[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))]/(2*Sqrt[4*a^2 + b^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5747

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cosh[(c_.) + (d_.)*(x_)]*(b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sinh[2*c +
 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{a + \frac{1}{2}b\sinh(2x)}\,dx$$

$$= 2 \int \frac{e^{2x} x}{-\frac{b}{2} + 2ae^{2x} + \frac{1}{2} be^{4x}} \, dx$$

$$= \frac{(2b) \int \frac{e^{2x} x}{2a - \sqrt{4a^2+b^2} + be^{2x}} \, dx}{\sqrt{4a^2+b^2}} - \frac{(2b) \int \frac{e^{2x} x}{2a + \sqrt{4a^2+b^2} + be^{2x}} \, dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x \log \left(1 + \frac{be^{2x}}{2a - \sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x \log \left(1 + \frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\int \log \left(1 + \frac{be^{2x}}{2a - \sqrt{4a^2+b^2}}\right) dx}{\sqrt{4a^2+b^2}} + \frac{\int \log \left(1 + \frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right) dx}{\sqrt{4a^2+b^2}}$$

$$= \frac{x \log \left(1 + \frac{be^{2x}}{2a - \sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x \log \left(1 + \frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{\text{Subst}\left(\int \frac{\log \left(1 + \frac{bx}{2a - \sqrt{4a^2+b^2}}\right)}{x} dx, x, e^{2x}\right)}{2\sqrt{4a^2+b^2}} + \frac{\text{Subst}\left(\int \frac{\log \left(1 + \frac{bx}{2a + \sqrt{4a^2+b^2}}\right)}{x} dx, x, e^{2x}\right)}{2\sqrt{4a^2+b^2}}$$

$$= \frac{x \log \left(1 + \frac{be^{2x}}{2a - \sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} - \frac{x \log \left(1 + \frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right)}{\sqrt{4a^2+b^2}} + \frac{\text{PolyLog}\left(2, -\frac{be^{2x}}{2a - \sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}} - \frac{\text{PolyLog}\left(2, -\frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.77

$$\int \frac{x}{a + b \cosh(x) \sinh(x)} \, dx = \frac{2x\left(\log \left(1 + \frac{be^{2x}}{2a - \sqrt{4a^2+b^2}}\right) - \log \left(1 + \frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right)\right) + \text{PolyLog}\left(2, \frac{be^{2x}}{-2a + \sqrt{4a^2+b^2}}\right) - \text{PolyLog}\left(2, -\frac{be^{2x}}{2a + \sqrt{4a^2+b^2}}\right)}{2\sqrt{4a^2+b^2}}$$

```
[In] Integrate[x/(a + b*Cosh[x]*Sinh[x]),x]
[Out] (2*x*(Log[1 + (b*E^(2*x))/(2*a - Sqrt[4*a^2 + b^2])] - Log[1 + (b*E^(2*x))/
(2*a + Sqrt[4*a^2 + b^2])]) + PolyLog[2, (b*E^(2*x))/(-2*a + Sqrt[4*a^2 + b
^2])] - PolyLog[2, -((b*E^(2*x))/(2*a + Sqrt[4*a^2 + b^2]))])/(2*Sqrt[4*a^2
 + b^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(162) = 324.

Time = 0.91 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.02

method	result
risch	$\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)x}{-2a-\sqrt{4a^2+b^2}}-\frac{x^2}{-2a-\sqrt{4a^2+b^2}}+\frac{2\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2a-\sqrt{4a^2+b^2}}\right)ax}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}-\frac{2a\,x^2}{\sqrt{4a^2+b^2}\left(-2a-\sqrt{4a^2+b^2}\right)}+\frac{\mathrm{polylog}\left(2,\frac{\ldots}{-2a-\ldots}\right)}{-4a-2\sqrt{4\ldots}}$

[In] int(x/(a+b*cosh(x)*sinh(x)),x,method=_RETURNVERBOSE)

[Out] 1/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*x-1/(-2*a-(4*a^2+b^2)^(1/2))*x^2+2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*ln(1-b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a*x-2/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*a*x^2+1/2/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))+1/(4*a^2+b^2)^(1/2)/(-2*a-(4*a^2+b^2)^(1/2))*polylog(2,b*exp(2*x)/(-2*a-(4*a^2+b^2)^(1/2)))*a+1/(4*a^2+b^2)^(1/2)*x*ln(1-b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))-1/(4*a^2+b^2)^(1/2)*x^2+1/2/(4*a^2+b^2)^(1/2)*polylog(2,b*exp(2*x)/((4*a^2+b^2)^(1/2)-2*a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(160) = 320.

Time = 0.28 (sec) , antiderivative size = 754, normalized size of antiderivative = 4.05

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = -\frac{bx\sqrt{\frac{4a^2+b^2}{b^2}}\log\left(\frac{\left(2a\cosh(x)+2a\sinh(x)-(b\cosh(x)+b\sinh(x))\sqrt{\frac{4a^2+b^2}{b^2}}\right)\sqrt{-\frac{b\sqrt{\frac{4a^2+b^2}{b^2}}+2a}{b}}+b}{b}\right)+bx\sqrt{\frac{4a^2+b^2}{b^2}}\log\left(-\ldots\right)\ldots}{\ldots}$$

[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")

[Out] -(b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) + b)/b) + b*x*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) - b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log(((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) + b)/b) - b*x*sqrt((4*a^2 + b^2)/b^2)*log(-((2*a*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b) + b*sqrt((4*a^2 + b

```
^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x) + b*sinh(x))*sqrt(
(4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*a)/b) + b)/b + 1)
+ b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a*cosh(x) + 2*a*sinh(x) - (b*cosh(x)
+ b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt(-(b*sqrt((4*a^2 + b^2)/b^2) + 2*
a)/b) - b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(-((2*a*cosh(x) + 2*a*si
nh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*sqrt((b*sqrt((4*a^
2 + b^2)/b^2) - 2*a)/b) + b)/b + 1) - b*sqrt((4*a^2 + b^2)/b^2)*dilog(((2*a
*cosh(x) + 2*a*sinh(x) + (b*cosh(x) + b*sinh(x))*sqrt((4*a^2 + b^2)/b^2))*s
qrt((b*sqrt((4*a^2 + b^2)/b^2) - 2*a)/b) - b)/b + 1))/(4*a^2 + b^2)
```

Sympy [F]

$$\int \frac{x}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x}{a + b\sinh{(x)}\cosh{(x)}}\,dx$$

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x)
[Out] Integral(x/(a + b*sinh(x)*cosh(x)), x)
```

Maxima [F]

$$\int \frac{x}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x}{b\cosh{(x)}\sinh{(x)} + a}\,dx$$

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
[Out] integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{x}{a + b\cosh(x)\sinh(x)}\,dx = \int \frac{x}{b\cosh{(x)}\sinh{(x)} + a}\,dx$$

```
[In] integrate(x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
[Out] integrate(x/(b*cosh(x)*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\cosh(x)\sinh(x)}\,dx = \int \frac{x}{a+b\cosh\left(x\right)\,\sinh\left(x\right)}\,dx$$

```
[In] int(x/(a + b*cosh(x)*sinh(x)),x)
[Out] int(x/(a + b*cosh(x)*sinh(x)), x)
```

3.870 $\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \operatorname{Int}\left(\frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+1/2*b*sinh(2*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

```
[In] Int[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
```

```
[Out] Defer[Int][1/(x*(a + (b*Sinh[2*x])/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+\frac{1}{2}b\sinh(2x)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx$$

```
[In] Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])),x]
[Out] Integrate[1/(x*(a + b*Cosh[x]*Sinh[x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\cosh\left(x\right)\sinh\left(x\right)\right)}dx$$

```
[In] int(1/x/(a+b*cosh(x)*sinh(x)),x)
[Out] int(1/x/(a+b*cosh(x)*sinh(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{(b\cosh\left(x\right)\sinh\left(x\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="fricas")
[Out] integral(1/(b*x*cosh(x)*sinh(x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 66.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx=\int \frac{1}{x\left(a+b\sinh\left(x\right)\cosh\left(x\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x)
[Out] Integral(1/(x*(a + b*sinh(x)*cosh(x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \int \frac{1}{(b\cosh{(x)}\sinh{(x)}+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="maxima")
[Out] integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \int \frac{1}{(b\cosh{(x)}\sinh{(x)}+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*cosh(x)*sinh(x)),x, algorithm="giac")
[Out] integrate(1/((b*cosh(x)*sinh(x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cosh(x)\sinh(x))}\,dx = \int \frac{1}{x\,\left(a+b\cosh{(x)}\,\sinh{(x)}\right)}\,dx$$

```
[In] int(1/(x*(a + b*cosh(x)*sinh(x))),x)
[Out] int(1/(x*(a + b*cosh(x)*sinh(x))), x)
```

3.871 $\int F^{c(a+bx)}\sinh^n(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx = -\frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)\sinh^n(d}{en-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n+b*c*ln(F))/e],[1-1/2*n+1/2*b*c*ln(F
)/e],exp(2*e*x+2*d))*sinh(e*x+d)^n/((1-exp(2*e*x+2*d))^n)/(e*n-b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5590, 2291}

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx = -\frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\sinh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right),e}{en-bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n - b*c*Log[F])/e, (2 - n
+ (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d + e*x)
))^n*(e*n - b*c*Log[F])))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5590

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Dist[E^(n*(d + e*x))*(Sinh[d + e*x]^n/(-1 + E^(2*(d + e*x)))^n), Int
[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; Free
Q[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{n(d+ex)}\left(-1+e^{2(d+ex)}\right)^{-n}\sinh^n(d+ex)\right)\int e^{-n(d+ex)}\left(-1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,dx$$
$$= -\frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)\text{si}}{en-bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx = \frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,\frac{-en+bc\log(F)}{2e},1+\frac{-en+bc\log(F)}{2e},e^{2(d+ex)}\right)\sinh^n(d+ex)}{-en+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^n,x]
[Out] (F^(c*(a + b*x))*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F])/(2*e), 1 + (-(
e*n) + b*c*Log[F])/(2*e), E^(2*(d + e*x))]*Sinh[d + e*x]^n)/((1 - E^(2*(d +
 e*x)))^n*(-(e*n) + b*c*Log[F]))
```

Maple [F]

$$\int F^{c(bx+a)} \sinh (ex+d)^n \, dx$$

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*sinh(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sinh^n(d+ex) \, dx = \int F^{(bx+a)c} \sinh (ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sinh(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sinh^n(d+ex) \, dx = \int F^{c(a+bx)} \sinh^n (d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*sinh(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sinh^n(d+ex) \, dx = \int F^{(bx+a)c} \sinh (ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx = \int F^{(bx+a)c}\sinh(ex+d)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sinh(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sinh^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\sinh(d+e\,x)^n\,dx$$

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^n,x)
[Out] int(F^(c*(a + b*x))*sinh(d + e*x)^n, x)
```

3.872 $\int e^{2(a+bx)} \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int e^{2(a+bx)} \sinh^3(a+bx) \, dx = \frac{e^{-a-bx}}{8b} + \frac{3e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{8b} + \frac{e^{5a+5bx}}{40b}$$

```
[Out] 1/8*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/8*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a)
/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 12, 276}

$$\int e^{2(a+bx)} \sinh^3(a+bx) \, dx = \frac{e^{-a-bx}}{8b} + \frac{3e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{8b} + \frac{e^{5a+5bx}}{40b}$$

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-a - b*x)/(8*b) + (3*E^(a + b*x))/(8*b) - E^(3*a + 3*b*x)/(8*b) + E^(5*a
 + 5*b*x)/(40*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{8x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{x^2}\,dx, x, e^{a+bx}\right)}{8b} \\
&= \frac{\text{Subst}\left(\int \left(3-\frac{1}{x^2}-3x^2+x^4\right)\,dx, x, e^{a+bx}\right)}{8b} \\
&= \frac{e^{-a-bx}}{8b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{8b}+\frac{e^{5a+5bx}}{40b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{e^{-a-bx}\left(5+15e^{2(a+bx)}-5e^{4(a+bx)}+e^{6(a+bx)}\right)}{40b}$$

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^3,x]
[Out] (E^(-a - b*x)*(5 + 15*E^(2*(a + b*x)) - 5*E^(4*(a + b*x)) + E^(6*(a + b*x))
))/(40*b)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{-bx-a}}{8b}+\frac{3\,e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{8b}+\frac{e^{5bx+5a}}{40b}$	55
parallelrisch	$-\frac{e^{2bx+2a}(8\sinh(2bx+2a)+2\sinh(3bx+3a)+10\sinh(bx+a)-8\cosh(2bx+2a)-3\cosh(3bx+3a)-5\cosh(bx+a))}{20b}$	76
default	$\frac{\sinh(bx+a)}{4b}-\frac{\sinh(3bx+3a)}{8b}+\frac{\sinh(5bx+5a)}{40b}+\frac{\cosh(bx+a)}{2b}-\frac{\cosh(3bx+3a)}{8b}+\frac{\cosh(5bx+5a)}{40b}$	80

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/8*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a)
/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-2\sinh(bx+a)^3-2\left(3\cosh(bx+a)^2+5\right)\sinh(bx+}{20\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - 2*sinh(b*x + a)
^3 - 2*(3*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x
 + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(49) = 98.

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \begin{cases} \frac{2e^{2a}e^{2bx}\sinh^3(a+bx)}{5b}+\frac{e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{5b}-\frac{4e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{5b}+\frac{2e^{2a}e^{2bx}\cosh^3(a+bx)}{5b} & \text{for } b\neq 0 \\ xe^{2a}\sinh^3(a) & \text{otherwis} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(5*b) + exp(2*a)*exp(2*b*
x)*sinh(a + b*x)**2*cosh(a + b*x)/(5*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*
x)*cosh(a + b*x)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(5*b), N
e(b, 0)), (x*exp(2*a)*sinh(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,a)} - 15\,e^{(-4\,bx-4\,a)} - 1\right)e^{(5\,bx+5\,a)}}{40\,b} + \frac{e^{(-bx-a)}}{8\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) - 1)*e^(5*b*x + 5*a)/b + 1/
8*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{40\,b} - \frac{e^{(3\,bx+3\,a)}}{8\,b} + \frac{3\,e^{(bx+a)}}{8\,b} + \frac{e^{(-bx-a)}}{8\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/40*e^(5*b*x + 5*a)/b - 1/8*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/8*e^
(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int e^{2(a+bx)}\sinh^3(a+bx)\,dx = \frac{15\,\mathrm{e}^{a+b\,x} + 5\,\mathrm{e}^{-a-b\,x} - 5\,\mathrm{e}^{3\,a+3\,b\,x} + \mathrm{e}^{5\,a+5\,b\,x}}{40\,b}$$

```
[In] int(exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (15*exp(a + b*x) + 5*exp(- a - b*x) - 5*exp(3*a + 3*b*x) + exp(5*a + 5*b*x)
)/(40*b)
```

3.873 $\int e^{2(a+bx)} \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int e^{2(a+bx)} \sinh^2(a+bx) \, dx = -\frac{e^{2a+2bx}}{4b} + \frac{e^{4a+4bx}}{16b} + \frac{x}{4}$$

```
[Out] -1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b+1/4*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2320, 12, 272, 45}

$$\int e^{2(a+bx)} \sinh^2(a+bx) \, dx = -\frac{e^{2a+2bx}}{4b} + \frac{e^{4a+4bx}}{16b} + \frac{x}{4}$$

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*E^(2*a + 2*b*x)/b + E^(4*a + 4*b*x)/(16*b) + x/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{4x}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= -\frac{e^{2a+2bx}}{4b}+\frac{e^{4a+4bx}}{16b}+\frac{x}{4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{-4e^{2(a+bx)}+e^{4(a+bx)}+4bx}{16b}$$

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x]^2,x]
[Out] (-4*E^(2*(a + b*x)) + E^(4*(a + b*x)) + 4*b*x)/(16*b)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{{e}^{2bx+2a}}{4b}+\frac{{e}^{4bx+4a}}{16b}+\frac{x}{4}$	33
default	$\frac{x}{4}-\frac{\sinh(2bx+2a)}{4b}+\frac{\sinh(4bx+4a)}{16b}-\frac{\cosh(2bx+2a)}{4b}+\frac{\cosh(4bx+4a)}{16b}$	61

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b+1/4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{(4\,bx+1)\cosh\left(bx+a\right)^2-2\,(4\,bx-1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+(4\,bx+1)\sinh\left(bx+a\right)^2-4}{16\,\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/16*((4*b*x + 1)*cosh(b*x + a)^2 - 2*(4*b*x - 1)*cosh(b*x + a)*sinh(b*x +
a) + (4*b*x + 1)*sinh(b*x + a)^2 - 4)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(29) = 58.

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.48

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^2{(a+bx)}}{4}-\frac{xe^{2a}e^{2bx}\sinh{(a+bx)}\cosh{(a+bx)}}{2}+\frac{xe^{2a}e^{2bx}\cosh^2{(a+bx)}}{4}+\frac{e^{2a}e^{2bx}\sinh^2{(a+bx)}}{2b}-\frac{e^{2a}e^{2bx}\sinh{(a+bx)}\cos}{4b} \\ xe^{2a}\sinh^2{(a)} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)**2,x)
[Out] Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 - x*exp(2*a)*exp(2*b*x)
*sinh(a + b*x)*cosh(a + b*x)/2 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4 +
 exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/(2*b) - exp(2*a)*exp(2*b*x)*sinh(a +
b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{1}{4}\,x - \frac{\left(4\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{a}{4\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*x - 1/16*(4*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + 1/4*a/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{1}{4}\,x + \frac{e^{(4\,bx+4\,a)}}{16\,b} - \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/4*x + 1/16*e^(4*b*x + 4*a)/b - 1/4*e^(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\sinh^2(a+bx)\,dx = \frac{x}{4} - \frac{\frac{e^{2\,a+2\,b\,x}}{4} - \frac{e^{4\,a+4\,b\,x}}{16}}{b}$$

```
[In] int(exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
[Out] x/4 - (exp(2*a + 2*b*x)/4 - exp(4*a + 4*b*x)/16)/b
```

3.874 $\int e^{2(a+bx)} \sinh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int e^{2(a+bx)} \sinh(a+bx)\,dx = -\frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b}$$

```
[Out] -1/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2320, 12}

$$\int e^{2(a+bx)} \sinh(a+bx)\,dx = \frac{e^{3a+3bx}}{6b} - \frac{e^{a+bx}}{2b}$$

```
[In] Int[E^(2*(a + b*x))*Sinh[a + b*x],x]
```

```
[Out] -1/2*E^(a + b*x)/b + E^(3*a + 3*b*x)/(6*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{2}(-1+x^2)\ dx, x, e^{a+bx}\right)}{b} \\ &= \frac{\text{Subst}\left(\int (-1+x^2)\ dx, x, e^{a+bx}\right)}{2b} \\ &= -\frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)} \sinh(a+bx)\, dx = \frac{e^{a+bx}\left(-3+e^{2(a+bx)}\right)}{6b}$$

```
[In] Integrate[E^(2*(a + b*x))*Sinh[a + b*x],x]
[Out] (E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(6*b)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{6b}$	27
parallelrisch	$-\frac{e^{2bx+2a}(\cosh(bx+a)-2\sinh(bx+a))}{3b}$	30
default	$-\frac{\sinh(bx+a)}{2b}+\frac{\sinh(3bx+3a)}{6b}-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(3bx+3a)}{6b}$	52

```
[In] int(exp(2*b*x+2*a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 3}{6\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3)
/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(22) = 44.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = \begin{cases} \frac{2e^{2a}e^{2bx}\sinh(a+bx)}{3b} - \frac{e^{2a}e^{2bx}\cosh(a+bx)}{3b} & \text{for } b \neq 0 \\ xe^{2a}\sinh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x)
[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)/(3*b) - exp(2*a)*exp(2*b*x)*
cosh(a + b*x)/(3*b), Ne(b, 0)), (x*exp(2*a)*sinh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{6\,b} - \frac{e^{(bx+a)}}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/6*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx=\frac{e^{(3\,bx+3\,a)}}{6\,b}-\frac{e^{(bx+a)}}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/6*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\sinh(a+bx)\,dx=-\frac{3\,\mathrm{e}^{a+b\,x}-\mathrm{e}^{3\,a+3\,b\,x}}{6\,b}$$

```
[In] int(exp(2*a + 2*b*x)*sinh(a + b*x),x)
[Out] -(3*exp(a + b*x) - exp(3*a + 3*b*x))/(6*b)
```

3.875 $\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx=\frac{2e^{a+bx}}{b}-\frac{2\mathrm{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 327, 213}

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx=\frac{2e^{a+bx}}{b}-\frac{2\mathrm{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{2x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b} + \frac{2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\text{csch}(a+bx)\,dx = \frac{2e^{a+bx} - 2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x],x]
[Out] (2*E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	40

[In] int(exp(2*b*x+2*a)*csch(b*x+a),x,method=_RETURNVERBOSE)

[Out] 2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(exp(b*x+a)+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx = \frac{2\,\cosh\left(bx+a\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+2\,\sin}{b}$$

[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="fricas")

[Out] (2*cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*sinh(b*x + a))/b

Sympy [F]

$$\int e^{2(a+bx)}\mathrm{csch}(a+bx)\,dx = e^{2a}\int e^{2bx}\,\mathrm{csch}\left(a+bx\right)dx$$

[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x)

[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int e^{2(a+bx)}\operatorname{csch}(a+bx)\,dx = \frac{2\,e^{(bx+a)}}{b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="maxima")
[Out] 2*e^(b*x + a)/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int e^{2(a+bx)}\operatorname{csch}(a+bx)\,dx = \frac{2\,e^{(bx+a)} - \log\left(e^{(bx+a)}+1\right) + \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a),x, algorithm="giac")
[Out] (2*e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int e^{2(a+bx)}\operatorname{csch}(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int(exp(2*a + 2*b*x)/sinh(a + b*x),x)
[Out] (2*exp(a + b*x))/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
)
```

3.876 $\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx=\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 2/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2320, 12, 272, 45}

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx=\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
```

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2*b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{4x^3}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\text{Subst}\left(\int \frac{x^3}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \frac{x}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx=\frac{2\left(\frac{1}{1-e^{2a+2bx}}+\log\left(1-e^{2a+2bx}\right)\right)}{b}$$

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x]^2,x]
[Out] (2*((1 - E^(2*a + 2*b*x))^(-1) + Log[1 - E^(2*a + 2*b*x)]))/b
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{4a}{b}-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	43

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/b*a-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.48

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = \frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-1\right)}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2-b}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 1)/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^2(a+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^2\,(a+bx)\,dx$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)**2,x)
```

```
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx = 4\,x + \frac{4\,a}{b} + \frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2}{b(e^{(-2\,bx-2\,a)}-1)}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] 4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 2/(b*
(e^(-2*b*x - 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx = -\frac{2\left(\frac{e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1} - \log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] -2*(e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1) - log(abs(e^(2*b*x + 2*a) - 1)))/
b
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\operatorname{csch}^2(a+bx)\,dx = \frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(exp(2*a + 2*b*x)/sinh(a + b*x)^2,x)
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2/(b*(exp(2*a + 2*b*x) - 1))
```

3.877 $\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx=-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] -2*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-
3*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2320, 12, 294, 213}

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx=-\frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[In] Int[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 -
E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{8x^4}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{3a+3bx}}{b\,(1-e^{2a+2bx})^2} + \frac{6\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{3a+3bx}}{b\,(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b\,(1-e^{2a+2bx})} + \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{3a+3bx}}{b\,(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = \frac{3e^{a+bx} - 5e^{3(a+bx)} - 3\left(-1+e^{2(a+bx)}\right)^2\text{arctanh}\left(e^{a+bx}\right)}{b\left(-1+e^{2(a+bx)}\right)^2}$$

```
[In] Integrate[E^(2*(a + b*x))*Csch[a + b*x]^3,x]
[Out] (3*E^(a + b*x) - 5*E^(3*(a + b*x)) - 3*(-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(
a + b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{e^{bx+a}\left(5\,e^{2bx+2a}-3\right)}{b\left(e^{2bx+2a}-1\right)^{2}}-\frac{3\ln\left(e^{bx+a}+1\right)}{2b}+\frac{3\ln\left(e^{bx+a}-1\right)}{2b}$	67

```
[In] int(exp(2*b*x+2*a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2-3/2/b*ln(exp(b*x+a)
+1)+3/2/b*ln(exp(b*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(64) = 128.

Time = 0.25 (sec) , antiderivative size = 388, normalized size of antiderivative = 5.32

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx =$$

$$-\frac{10\cosh\left(bx+a\right)^{3}+30\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+10\sinh\left(bx+a\right)^{3}+3\left(\cosh\left(bx+a\right)^{4}+4\cosh\left(b\right.\right.}{}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(10*cosh(b*x + a)^3 + 30*cosh(b*x + a)*sinh(b*x + a)^2 + 10*sinh(b*x +
 a)^3 + 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a
)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(co
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(
b*x + a) + 1) - 3*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh
(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 6*cosh(b*
x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x
 + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2
 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^3\,(a+bx)\,dx$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)**3,x)
[Out] exp(2*a)*Integral(exp(2*b*x)*csch(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = -\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{5\,e^{(-bx-a)} - 3\,e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) - 1)/b + (5*e^(-b*x - a
) - 3*e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\text{csch}^3(a+bx)\,dx = -\frac{\frac{2\left(5\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2} + 3\,\log\left(e^{(bx+a)}+1\right) - 3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*(5*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + 3*log
(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int e^{2(a+bx)}\operatorname{csch}^3(a+bx)\,dx=-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{3\,a+3\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{3\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(exp(2*a + 2*b*x)/sinh(a + b*x)^3,x)
[Out] - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(3*a + 3*
b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) - 1))
```

3.878 $\int e^{a+bx} \sinh^3(c+dx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int e^{a+bx} \sinh^3(c+dx) \, dx = -\frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4} + \frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2} + \frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}$$

```
[Out] -6*d^3*exp(b*x+a)*cosh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)+6*b*d^2*exp(b*x+a)*sin
h(d*x+c)/(b^4-10*b^2*d^2+9*d^4)-3*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)^2/(b
^2-9*d^2)+b*exp(b*x+a)*sinh(d*x+c)^3/(b^2-9*d^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5584, 5582}

$$\int e^{a+bx} \sinh^3(c+dx) \, dx = \frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2} - \frac{3de^{a+bx}\sinh^2(c+dx)\cosh(c+dx)}{b^2-9d^2} + \frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}$$

```
[In] Int[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

```
[Out] (-6*d^3*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (6*b*d^2*E^
(a + b*x)*Sinh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh
[c + d*x]*Sinh[c + d*x]^2)/(b^2 - 9*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^3)/
(b^2 - 9*d^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2} \\ &\quad+\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}+\frac{(6d^2)\int e^{a+bx}\sinh(c+dx)\,dx}{b^2-9d^2} \\ &= -\frac{6d^3e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}+\frac{6bd^2e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} \\ &\quad-\frac{3de^{a+bx}\cosh(c+dx)\sinh^2(c+dx)}{b^2-9d^2}+\frac{be^{a+bx}\sinh^3(c+dx)}{b^2-9d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\sinh^3(c+dx)\,dx$$
$$=\frac{e^{a+bx}(3d(b^2-9d^2)\cosh(c+dx)+(-3b^2d+3d^3)\cosh(3(c+dx))+2b(-b^2+13d^2+(b^2-d^2)\cosh(2(c+}{4\left(b^4-10b^2d^2+9d^4\right)}$$

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(3*d*(b^2 - 9*d^2)*Cosh[c + d*x] + (-3*b^2*d + 3*d^3)*Cosh[3*(
c + d*x)] + 2*b*(-b^2 + 13*d^2 + (b^2 - d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*
x]))/(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

method	result
risch	$\frac{e^{bx+3dx+a+3c}}{8b+24d}-\frac{3\,e^{bx+dx+a+c}}{8(b+d)}+\frac{3\,e^{bx-dx+a-c}}{8(b-d)}-\frac{e^{bx-3dx+a-3c}}{8(b-3d)}$
parallelrisch	$\frac{e^{bx+a}\left(\left(-3b^2d+3d^3\right)\cosh(3dx+3c)+\left(b^3-b\,d^2\right)\sinh(3dx+3c)-3(b-3d)(b+3d)(b\sinh(dx+c)-d\cosh(dx+c))\right)}{4b^4-40b^2d^2+36d^4}$
default	$-\frac{\sinh(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}-\frac{\cosh(a-3c+(b-3d)x)}{8(b-3d)}$

[In] int(exp(b*x+a)*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/8/(b+3*d)*exp(b*x+3*d*x+a+3*c)-3/8/(b+d)*exp(b*x+d*x+a+c)+3/8/(b-d)*exp(b*x-d*x+a-c)-1/8/(b-3*d)*exp(b*x-3*d*x+a-3*c)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(135) = 270.

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.27

$$\int e^{a+bx}\sinh^3(c+dx)\,dx =$$
$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)^3-\left(\left(b^3-bd^2\right)\cosh\left(bx+a\right)+\left(b^3-bd^2\right)\sinh\left(bx+a\right)\right)\sinh\left(dx+\right.}{}$$

[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="fricas")

[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^3 - ((b^3 - b*d^2)*cosh(b*x + a) + (b^3 - b*d^2)*sinh(b*x + a))*sinh(d*x + c)^3 - 3*(b^2*d - 9*d^3)*cosh(b*x + a)*cosh(d*x + c) + 9*((b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c) + (b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + 3*((b^2*d - d^3)*cosh(d*x + c)^3 - (b^2*d - 9*d^3)*cosh(d*x + c))*sinh(b*x + a) - 3*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^2 - (b^3 - 9*b*d^2)*cosh(b*x + a) - (b^3 - 9*b*d^2 - (b^3 - b*d^2)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(131) = 262$.

Time = 2.35 (sec) , antiderivative size = 976, normalized size of antiderivative = 7.02

$$\int e^{a+bx}\sinh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-3*d*x)
*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)
/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + x*exp(a)*exp
(-3*d*x)*cosh(c + d*x)**3/8 - 3*exp(a)*exp(-3*d*x)*sinh(c + d*x)**3/(8*d) -
 exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(-3*d*
x)*cosh(c + d*x)**3/(24*d), Eq(b, -3*d)), (3*x*exp(a)*exp(-d*x)*sinh(c + d*
x)**3/8 + 3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*x*exp(a
)*exp(-d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - 3*x*exp(a)*exp(-d*x)*cosh(c
+ d*x)**3/8 + exp(a)*exp(-d*x)*sinh(c + d*x)**3/(8*d) + 3*exp(a)*exp(-d*x)*
sinh(c + d*x)**2*cosh(c + d*x)/(4*d) - 3*exp(a)*exp(-d*x)*cosh(c + d*x)**3/
(8*d), Eq(b, -d)), (3*x*exp(a)*exp(d*x)*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp
(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 - 3*x*exp(a)*exp(d*x)*sinh(c + d*x)*
cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(d*x)*cosh(c + d*x)**3/8 - exp(a)*exp(d*
x)*sinh(c + d*x)**3/(8*d) + 3*exp(a)*exp(d*x)*sinh(c + d*x)**2*cosh(c + d*x
)/(4*d) - 3*exp(a)*exp(d*x)*cosh(c + d*x)**3/(8*d), Eq(b, d)), (x*exp(a)*ex
p(3*d*x)*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c
 + d*x)/8 + 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 - x*exp(
a)*exp(3*d*x)*cosh(c + d*x)**3/8 + 3*exp(a)*exp(3*d*x)*sinh(c + d*x)**3/(8*
d) - exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(3*
d*x)*cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*exp(a)*exp(b*x)*sinh(c + d
*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*exp(a)*exp(b*x)*sinh(c + d
*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*exp(a)*exp(b
*x)*sinh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*exp(a)*exp(b
*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*
exp(a)*exp(b*x)*sinh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**
4) - 6*d**3*exp(a)*exp(b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4)
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\sinh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

$$\int e^{a+bx}\sinh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)} - \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)} - \frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*e^(b*x + d*x + a + c)/(b + d)
 + 3/8*e^(b*x - d*x + a - c)/(b - d) - 1/8*e^(b*x - 3*d*x + a - 3*c)/(b - 3
*d)
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\sinh^3(c+dx)\,dx =$$
$$-\frac{e^{a+b\,x}\,\left(-b^3\,\sinh(c+d\,x)^3+3\,b^2\,d\,\cosh(c+d\,x)\,\sinh(c+d\,x)^2-6\,b\,d^2\,\cosh(c+d\,x)^2\,\sinh(c+d\,x)+7\,b\,d\right.}{b^4-10\,b^2\,d^2+9\,d^4}$$

```
[In] int(exp(a + b*x)*sinh(c + d*x)^3,x)
```

```
[Out] -(exp(a + b*x)*(6*d^3*cosh(c + d*x)^3 - b^3*sinh(c + d*x)^3 - 9*d^3*cosh(c
+ d*x)*sinh(c + d*x)^2 + 7*b*d^2*sinh(c + d*x)^3 - 6*b*d^2*cosh(c + d*x)^2*
sinh(c + d*x) + 3*b^2*d*cosh(c + d*x)*sinh(c + d*x)^2))/(b^4 + 9*d^4 - 10*b
^2*d^2)
```

3.879 $\int e^{a+bx} \sinh^2(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = \frac{2d^2e^{a+bx}}{b\,(b^2-4d^2)} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2} + \frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}$$

```
[Out] 2*d^2*exp(b*x+a)/b/(b^2-4*d^2)-2*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)/(b^2-
4*d^2)+b*exp(b*x+a)*sinh(d*x+c)^2/(b^2-4*d^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5584, 2225}

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = \frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\sinh(c+dx)\cosh(c+dx)}{b^2-4d^2} + \frac{2d^2e^{a+bx}}{b\,(b^2-4d^2)}$$

```
[In] Int[E^(a + b*x)*Sinh[c + d*x]^2,x]
```

```
[Out] (2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh
[c + d*x])/(b^2 - 4*d^2) + (b*E^(a + b*x)*Sinh[c + d*x]^2)/(b^2 - 4*d^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}+\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}+\frac{(2d^2)\int e^{a+bx}\,dx}{b^2-4d^2}\\ &= \frac{2d^2e^{a+bx}}{b\,(b^2-4d^2)}-\frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}+\frac{be^{a+bx}\sinh^2(c+dx)}{b^2-4d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int e^{a+bx}\sinh^2(c+dx)\,dx=\frac{e^{a+bx}(-b^2+4d^2+b^2\cosh(2(c+dx))-2bd\sinh(2(c+dx)))}{2\,(b^3-4bd^2)}$$

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x]^2,x]
[Out] (E^(a + b*x)*(-b^2 + 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)
]))/(2*(b^3 - 4*b*d^2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{e^{bx+a}}{2b}+\frac{e^{bx+2dx+a+2c}}{4b+8d}+\frac{e^{bx-2dx+a-2c}}{4b-8d}$
parallelrisch	$\frac{e^{bx+a}(\cosh(2dx+2c)b^2-2bd\sinh(2dx+2c)-b^2+4d^2)}{2b^3-8b\,d^2}$
default	$-\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}-\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(}{4b+8c}$

```
[In] int(exp(b*x+a)*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*exp(b*x+a)/b+1/4/(b+2*d)*exp(b*x+2*d*x+a+2*c)+1/4/(b-2*d)*exp(b*x-2*d*
x+a-2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.69

$$\int e^{a+bx}\sinh^2(c+dx)\,dx$$
$$=\frac{b^2\cosh\left(bx+a\right)\cosh\left(dx+c\right)^2+\left(b^2\cosh\left(bx+a\right)+b^2\sinh\left(bx+a\right)\right)\sinh\left(dx+c\right)^2-\left(b^2-4\,d^2\right)\cosh\left(bx\right.}{}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))*sinh(d*x + c)^2 - (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^2
- b^2 + 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*cos
h(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(78) = 156.

Time = 0.84 (sec) , antiderivative size = 428, normalized size of antiderivative = 4.86

$$\int e^{a+bx}\sinh^2(c+dx)\,dx$$
$$=\begin{cases} xe^{a}\sinh^{2}(c) \\ \left(\frac{x\sinh^{2}(c+dx)}{2}-\frac{x\cosh^{2}(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^{a} \\ \frac{xe^{a}e^{-2dx}\sinh^{2}(c+dx)}{4}+\frac{xe^{a}e^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^{a}e^{-2dx}\cosh^{2}(c+dx)}{4}-\frac{e^{a}e^{-2dx}\sinh^{2}(c+dx)}{2d}-\frac{e^{a}e^{-2dx}\sinh(c+dx)}{4d} \\ \frac{xe^{a}e^{2dx}\sinh^{2}(c+dx)}{4}-\frac{xe^{a}e^{2dx}\sinh(c+dx)\cosh(c+dx)}{2}+\frac{xe^{a}e^{2dx}\cosh^{2}(c+dx)}{4}+\frac{e^{a}e^{2dx}\sinh^{2}(c+dx)}{2d}-\frac{e^{a}e^{2dx}\sinh(c+dx)\cosh(c-}{4d} \\ \frac{b^{2}e^{a}e^{bx}\sinh^{2}(c+dx)}{b^{3}-4bd^{2}}-\frac{2bde^{a}e^{bx}\sinh(c+dx)\cosh(c+dx)}{b^{3}-4bd^{2}}-\frac{2d^{2}e^{a}e^{bx}\sinh^{2}(c+dx)}{b^{3}-4bd^{2}}+\frac{2d^{2}e^{a}e^{bx}\cosh^{2}(c+dx)}{b^{3}-4bd^{2}} \end{cases}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sinh(c + d*x)**2/
2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(b,
 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*sinh(
c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 - exp(a)
*exp(-2*d*x)*sinh(c + d*x)**2/(2*d) - exp(a)*exp(-2*d*x)*sinh(c + d*x)*cosh
(c + d*x)/(4*d), Eq(b, -2*d)), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/4 - x*
exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x)*cosh(
c + d*x)**2/4 + exp(a)*exp(2*d*x)*sinh(c + d*x)**2/(2*d) - exp(a)*exp(2*d*x
)*sinh(c + d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*exp(a)*exp(b*x)*sin
```

```
h(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sinh(c + d*x)*cosh(
c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*sinh(c + d*x)**2/(b**3
- 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e detai
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)} - \frac{e^{(bx+a)}}{2\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) - 1/2*e^(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int e^{a+bx}\sinh^2(c+dx)\,dx = -\frac{2\,d^2\,\mathrm{e}^{a+b\,x}-b^2\,\left(\frac{\mathrm{e}^{a+b\,x}}{2}-\mathrm{e}^{a+b\,x}\,\left(\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{4}+\frac{\mathrm{e}^{2\,c+2\,d\,x}}{4}\right)\right)+b\,d\,\mathrm{e}^{a+b\,x}\,\left(\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{2}-\frac{\mathrm{e}^{2\,c+2\,d\,x}}{2}\right)}{4\,b\,d^2-b^3}$$

```
[In] int(exp(a + b*x)*sinh(c + d*x)^2,x)
```

```
[Out] -(2*d^2*exp(a + b*x) - b^2*(exp(a + b*x)/2 - exp(a + b*x)*(exp(- 2*c - 2*d*
x)/4 + exp(2*c + 2*d*x)/4)) + b*d*exp(a + b*x)*(exp(- 2*c - 2*d*x)/2 - exp(
2*c + 2*d*x)/2))/(4*b*d^2 - b^3)
```

3.880 $\int e^{a+bx} \sinh(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int e^{a+bx} \sinh(c+dx)\,dx = -\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2} + \frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

```
[Out] -d*exp(b*x+a)*cosh(d*x+c)/(b^2-d^2)+b*exp(b*x+a)*sinh(d*x+c)/(b^2-d^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5582}

$$\int e^{a+bx} \sinh(c+dx)\,dx = \frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2} - \frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2}$$

```
[In] Int[E^(a + b*x)*Sinh[c + d*x],x]
```

```
[Out] -((d*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2)) + (b*E^(a + b*x)*Sinh[c + d*x]
)/(b^2 - d^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{de^{a+bx}\cosh(c+dx)}{b^2-d^2} + \frac{be^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\sinh(c+dx)\,dx=\frac{e^{a+bx}(-d\cosh(c+dx)+b\sinh(c+dx))}{(b-d)(b+d)}$$

```
[In] Integrate[E^(a + b*x)*Sinh[c + d*x],x]
```

```
[Out] (E^(a + b*x)*(-(d*Cosh[c + d*x]) + b*Sinh[c + d*x]))/((b - d)*(b + d))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{bx+a}(b\sinh(dx+c)-d\cosh(dx+c))}{b^2-d^2}$	37
risch	$\frac{e^{bx+dx+a+c}}{2b+2d}-\frac{e^{bx-dx+a-c}}{2(b-d)}$	41
default	$-\frac{\sinh(a-c+(b-d)x)}{2(b-d)}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}-\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	78

```
[In] int(exp(b*x+a)*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] exp(b*x+a)/(b^2-d^2)*(b*sinh(d*x+c)-d*cosh(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\sinh(c+dx)\,dx = -\frac{d\cosh(bx+a)\cosh(dx+c)+d\cosh(dx+c)\sinh(bx+a)-(b\cosh(bx+a)+b\sinh(bx+a))\sinh(dx}{b^2-d^2}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a)*cosh(d*x + c) + d*cosh(d*x + c)*sinh(b*x + a) - (b*cosh(b
*x + a) + b*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(42) = 84$.

Time = 0.38 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.72

$$\int e^{a+bx}\sinh(c+dx)\,dx$$
$$= \begin{cases} xe^a\sinh{(c)} & \text{for}\ b=0\wedge d=0 \\ \frac{xe^a e^{-dx}\sinh{(c+dx)}}{2}+\frac{xe^a e^{-dx}\cosh{(c+dx)}}{2}+\frac{e^a e^{-dx}\sinh{(c+dx)}}{2d}+\frac{e^a e^{-dx}\cosh{(c+dx)}}{d} & \text{for}\ b=-d \\ \frac{xe^a e^{dx}\sinh{(c+dx)}}{2}-\frac{xe^a e^{dx}\cosh{(c+dx)}}{2}-\frac{e^a e^{dx}\sinh{(c+dx)}}{2d}+\frac{e^a e^{dx}\cosh{(c+dx)}}{d} & \text{for}\ b=d \\ \frac{be^a e^{bx}\sinh{(c+dx)}}{b^2-d^2}-\frac{de^a e^{bx}\cosh{(c+dx)}}{b^2-d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sinh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sinh
(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(c
+ d*x)/(2*d) + exp(a)*exp(-d*x)*cosh(c + d*x)/d, Eq(b, -d)), (x*exp(a)*exp(
d*x)*sinh(c + d*x)/2 - x*exp(a)*exp(d*x)*cosh(c + d*x)/2 - exp(a)*exp(d*x)*
sinh(c + d*x)/(2*d) + exp(a)*exp(d*x)*cosh(c + d*x)/d, Eq(b, d)), (b*exp(a)
*exp(b*x)*sinh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*cosh(c + d*x)/(b*
*2 - d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\sinh(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\sinh(c+dx)\,dx=\frac{e^{(bx+dx+a+c)}}{2\,(b+d)}-\frac{e^{(bx-dx+a-c)}}{2\,(b-d)}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c),x, algorithm="giac")
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(b*x - d*x + a - c)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\sinh(c+dx)\,dx=-\frac{e^{a-c+b\,x-d\,x}\,\left(b+d-b\,e^{2\,c+2\,d\,x}+d\,e^{2\,c+2\,d\,x}\right)}{2\,\left(b^2-d^2\right)}$$

```
[In] int(exp(a + b*x)*sinh(c + d*x),x)
[Out] -(exp(a - c + b*x - d*x)*(b + d - b*exp(2*c + 2*d*x) + d*exp(2*c + 2*d*x)))
/(2*(b^2 - d^2))
```

3.881 $\int e^{a+bx}\text{csch}(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int e^{a+bx}\text{csch}(c+dx)\,dx=-\frac{2e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2(c+dx)}\right)}{b+d}$$

```
[Out] -2*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],exp(2*d*x+2*c)
)/(b+d)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5601}

$$\int e^{a+bx}\text{csch}(c+dx)\,dx=-\frac{2e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(\frac{b}{d}+3\right),e^{2(c+dx)}\right)}{b+d}$$

```
[In] Int[E^(a + b*x)*Csch[c + d*x],x]
```

```
[Out] (-2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
E^(2*(c + d*x))])/(b + d)
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral}=-\frac{2e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2(c+dx)}\right)}{b+d}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\text{csch}(c+dx)\,dx=-\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),e^{2(c+dx)}\right)}{b+d}$$

```
[In] Integrate[E^(a + b*x)*Csch[c + d*x],x]
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2,
E^(2*(c + d*x))])/(b + d)
```

Maple [F]

$$\int e^{bx+a}\,\text{csch}\,(dx+c)\,dx$$

```
[In] int(exp(b*x+a)*csch(d*x+c),x)
[Out] int(exp(b*x+a)*csch(d*x+c),x)
```

Fricas [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx=\int \text{csch}\,(dx+c)\,e^{(bx+a)}\,dx$$

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="fricas")
[Out] integral(csch(d*x + c)*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx=e^{a}\int e^{bx}\,\text{csch}\,(c+dx)\,dx$$

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x)
[Out] exp(a)*Integral(exp(b*x)*csch(c + d*x), x)
```

Maxima [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \text{csch}\left(dx+c\right)e^{(bx+a)}\,dx$$

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="maxima")
[Out] integrate(csch(d*x + c)*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \text{csch}\left(dx+c\right)e^{(bx+a)}\,dx$$

```
[In] integrate(exp(b*x+a)*csch(d*x+c),x, algorithm="giac")
[Out] integrate(csch(d*x + c)*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}(c+dx)\,dx = \int \frac{e^{a+b\,x}}{\sinh\left(c+d\,x\right)}\,dx$$

```
[In] int(exp(a + b*x)/sinh(c + d*x),x)
[Out] int(exp(a + b*x)/sinh(c + d*x), x)
```

3.882 $\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx=\frac{4e^{c+dx+2(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}$$

```
[Out] 4*exp(2*b*x+d*x+2*a+c)*hypergeom([2, 1+1/2*d/b],[2+1/2*d/b],exp(2*b*x+2*a))
/(2*b+d)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5601}

$$\int e^{c+dx}\operatorname{csch}^2(a+bx)\,dx=\frac{4e^{2(a+bx)+c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b}+1,\frac{d}{2b}+2,e^{2(a+bx)}\right)}{2b+d}$$

```
[In] Int[E^(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] (4*E^(c + d*x + 2*(a + b*x))*Hypergeometric2F1[2, 1 + d/(2*b), 2 + d/(2*b),
 E^(2*(a + b*x))])/(2*b + d)
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral}=\frac{4e^{c+dx+2(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(54) = 108.

Time = 1.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.54

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx = -\frac{2d\left(\frac{e^{2a+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{b\left(-1+e^{2a}\right)} + \frac{e^{c+dx}\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

```
[In] Integrate[E^(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] (-2*d*((E^(2*a + c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^(2*(
a + b*x))])/d - (E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b)
, 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(b*(-1 + E^(2*a))) + (E^(c + d
*x)*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [F]

$$\int e^{dx+c}\,\text{csch}\,(bx+a)^2\,dx$$

```
[In] int(exp(d*x+c)*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*csch(b*x+a)^2,x)
```

Fricas [F]

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx = \int \text{csch}\,(bx+a)^2\,e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F]

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=e^c\int e^{dx}\,\text{csch}^2\,(a+bx)\,dx$$

[In] integrate(exp(d*x+c)*csch(b*x+a)**2,x)

[Out] exp(c)*Integral(exp(d*x)*csch(a + b*x)**2, x)

Maxima [F]

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=\int \text{csch}\,(bx+a)^2\,e^{(dx+c)}\,dx$$

[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")

[Out] 16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)*
e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*b*
d + d^2)*e^(2*b*x + 2*a)), x) - 4*((4*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 4*b*
e^c)*e^(d*x)/(8*b^2 - 6*b*d + d^2 + (8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) -
2*(8*b^2 - 6*b*d + d^2)*e^(2*b*x + 2*a))

Giac [F]

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=\int \text{csch}\,(bx+a)^2\,e^{(dx+c)}\,dx$$

[In] integrate(exp(d*x+c)*csch(b*x+a)^2,x, algorithm="giac")

[Out] integrate(csch(b*x + a)^2*e^(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\text{csch}^2(a+bx)\,dx=\int \frac{e^{c+d\,x}}{\sinh\left(a+b\,x\right)^2}\,dx$$

[In] int(exp(c + d*x)/sinh(a + b*x)^2,x)

[Out] int(exp(c + d*x)/sinh(a + b*x)^2, x)

3.883 $\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = -\frac{de^{c+dx}\operatorname{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)}{2b}+\frac{(b-d)e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(3+\frac{d}{b}\right),e^{2(a+bx)}\right)}{b^2}$$

```
[Out] -1/2*d*exp(d*x+c)*csch(b*x+a)/b^2-1/2*exp(d*x+c)*coth(b*x+a)*csch(b*x+a)/b+
(b-d)*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/b],[3/2+1/2*d/b],exp(2*b*x+2
*a))/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5599, 5601}

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx = \frac{(b-d)e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(\frac{d}{b}+3\right),e^{2(a+bx)}\right)}{b^2}-\frac{de^{c+dx}\operatorname{csch}(a+bx)}{2b^2}-\frac{e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

```
[In] Int[E^(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] -1/2*(d*E^(c + d*x)*Csch[a + b*x])/b^2 - (E^(c + d*x)*Coth[a + b*x]*Csch[a
+ b*x])/(2*b) + ((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)
/(2*b), (3 + d/b)/2, E^(2*(a + b*x))])/b^2
```

Rule 5599

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(
n - 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a +
 b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F,
a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
 && NeQ[n, 2]
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2} - \frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b} \\ &\quad - \frac{1}{2}\left(1-\frac{d^2}{b^2}\right)\int e^{c+dx}\text{csch}(a+bx)\,dx \\ &= -\frac{de^{c+dx}\text{csch}(a+bx)}{2b^2} - \frac{e^{c+dx}\coth(a+bx)\text{csch}(a+bx)}{2b} \\ &\quad + \frac{(b-d)e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(3+\frac{d}{b}\right),e^{2(a+bx)}\right)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{c+dx}\text{csch}^3(a+bx)\,dx = \frac{-4e^{c+dx}(d+b\coth(a+bx))\text{csch}(a+bx)+8(b-d)e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{1}{2}\left(3+\frac{d}{b}\right),e^{2(a+bx)}\right)}{8b^2}$$

```
[In] Integrate[E^(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-4*E^(c + d*x)*(d + b*Coth[a + b*x])*Csch[a + b*x] + 8*(b - d)*E^(a + c +
(b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3 + d/b)/2, E^(2*(a + b*x))
])/(8*b^2)
```

Maple [F]

$$\int e^{dx+c} \operatorname{csch}(bx+a)^3 \, dx$$

```
[In] int(exp(d*x+c)*csch(b*x+a)^3,x)
[Out] int(exp(d*x+c)*csch(b*x+a)^3,x)
```

Fricas [F]

$$\int e^{c+dx} \operatorname{csch}^3(a+bx) \, dx = \int \operatorname{csch}(bx+a)^3 \, e^{(dx+c)} \, dx$$

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F]

$$\int e^{c+dx} \operatorname{csch}^3(a+bx) \, dx = e^c \int e^{dx} \operatorname{csch}^3(a+bx) \, dx$$

```
[In] integrate(exp(d*x+c)*csch(b*x+a)**3,x)
[Out] exp(c)*Integral(exp(d*x)*csch(a + b*x)**3, x)
```

Maxima [F]

$$\int e^{c+dx} \operatorname{csch}^3(a+bx) \, dx = \int \operatorname{csch}(bx+a)^3 \, e^{(dx+c)} \, dx$$

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] 48*(b^2*e^c + b*d*e^c)*integrate(e^(b*x + d*x + a)/(15*b^2 - 8*b*d + d^2 +
(15*b^2 - 8*b*d + d^2)*e^(8*b*x + 8*a) - 4*(15*b^2 - 8*b*d + d^2)*e^(6*b*x
+ 6*a) + 6*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 4*(15*b^2 - 8*b*d + d^2
)*e^(2*b*x + 2*a)), x) + 8*((5*b*e^c - d*e^c)*e^(3*b*x + 3*a) - 6*b*e^(b*x
+ a + c))*e^(d*x)/(15*b^2 - 8*b*d + d^2 - (15*b^2 - 8*b*d + d^2)*e^(6*b*x +
 6*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(4*b*x + 4*a) - 3*(15*b^2 - 8*b*d + d^2)
*e^(2*b*x + 2*a))
```

Giac [F]

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx=\int \operatorname{csch}\,(bx+a)^3\,e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(csch(b*x + a)^3*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\operatorname{csch}^3(a+bx)\,dx=\int \frac{e^{c+d\,x}}{\sinh\,(a+b\,x)^3}\,dx$$

```
[In] int(exp(c + d*x)/sinh(a + b*x)^3,x)
[Out] int(exp(c + d*x)/sinh(a + b*x)^3, x)
```

3.884 $\int F^{c(a+bx)} \cosh^n(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int F^{c(a+bx)} \cosh^n(d+ex)\,dx = -\frac{\left(1+e^{2(d+ex)}\right)^{-n} F^{c(a+bx)} \cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right),-e\right.}{en-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*cosh(e*x+d)^n*hypergeom([-n, 1/2*(-e*n+b*c*ln(F))/e],[1-1/2*
n+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/((1+exp(2*e*x+2*d))^n)/(e*n-b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5591, 2291}

$$\int F^{c(a+bx)} \cosh^n(d+ex)\,dx = -\frac{\left(e^{2(d+ex)}+1\right)^{-n} F^{c(a+bx)} \cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right),\right.}{en-bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, -1/2*(e*n - b*c*Lo
g[F])/e, (2 - n + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/((1 + E^(2*(d + e*x
)))^n*(e*n - b*c*Log[F])))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5591

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[E^(n*(d + e*x))*(Cosh[d + e*x]^n/(1 + E^(2*(d + e*x)))^n), Int[
F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; FreeQ[
{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{n(d+ex)}\left(1+e^{2(d+ex)}\right)^{-n}\cosh^n(d+ex)\right)\int e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,dx$$

$$= -\frac{\left(1+e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2-n+\frac{bc\log(F}{e}\right.\right.}{en-bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx$$

$$= \frac{\left(1+e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\cosh^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,\frac{-en+bc\log(F)}{2e},1+\frac{-en+bc\log(F)}{2e},-e^{2(d+ex)}\right)}{-en+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Cosh[d + e*x]^n*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F]
)/(2*e), 1 + (-(e*n) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/((1 + E^(2*(d
+ e*x)))^n*(-(e*n) + b*c*Log[F]))
```

Maple [F]

$$\int F^{c(bx+a)} \cosh\left(ex+d\right)^{n} dx$$

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*cosh(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \cosh^{n}(d+ex)\,dx = \int F^{(bx+a)c} \cosh\left(ex+d\right)^{n}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*cosh(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \cosh^{n}(d+ex)\,dx = \int F^{c(a+bx)} \cosh^{n}\left(d+ex\right)dx$$

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*cosh(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \cosh^{n}(d+ex)\,dx = \int F^{(bx+a)c} \cosh\left(ex+d\right)^{n}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \int F^{(bx+a)c}\cosh{(ex+d)}^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*cosh(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\cosh(d+e\,x)^n\,dx$$

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x)^n,x)
[Out] int(F^(c*(a + b*x))*cosh(d + e*x)^n, x)
```

3.885 $\int e^{a+bx}\cosh^3(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int e^{a+bx}\cosh^3(c+dx)\,dx=-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}+\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}+\frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4}-\frac{3de^{a+bx}\cosh^2(c+dx)\sinh(c+dx)}{b^2-9d^2}$$

```
[Out] -6*b*d^2*exp(b*x+a)*cosh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)+b*exp(b*x+a)*cosh(d*
x+c)^3/(b^2-9*d^2)+6*d^3*exp(b*x+a)*sinh(d*x+c)/(b^4-10*b^2*d^2+9*d^4)-3*d*
exp(b*x+a)*cosh(d*x+c)^2*sinh(d*x+c)/(b^2-9*d^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 5583}

$$\int e^{a+bx}\cosh^3(c+dx)\,dx=\frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2}-\frac{3de^{a+bx}\sinh(c+dx)\cosh^2(c+dx)}{b^2-9d^2}-\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4}+\frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4}$$

```
[In] Int[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

```
[Out] (-6*b*d^2*E^(a + b*x)*Cosh[c + d*x])/(b^4 - 10*b^2*d^2 + 9*d^4) + (b*E^(a +
 b*x)*Cosh[c + d*x]^3)/(b^2 - 9*d^2) + (6*d^3*E^(a + b*x)*Sinh[c + d*x])/(b
^4 - 10*b^2*d^2 + 9*d^4) - (3*d*E^(a + b*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/
(b^2 - 9*d^2)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2} - \frac{3de^{a+bx}\cosh^2(c+dx)\sinh(c+dx)}{b^2-9d^2} \\
&\quad - \frac{(6d^2)\int e^{a+bx}\cosh(c+dx)\,dx}{b^2-9d^2} \\
&= -\frac{6bd^2e^{a+bx}\cosh(c+dx)}{b^4-10b^2d^2+9d^4} + \frac{be^{a+bx}\cosh^3(c+dx)}{b^2-9d^2} \\
&\quad + \frac{6d^3e^{a+bx}\sinh(c+dx)}{b^4-10b^2d^2+9d^4} - \frac{3de^{a+bx}\cosh^2(c+dx)\sinh(c+dx)}{b^2-9d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int e^{a+bx}\cosh^3(c+dx)\,dx$$

$$= \frac{e^{a+bx}(3b(b^2-9d^2)\cosh(c+dx) + (b^3-bd^2)\cosh(3(c+dx)) + 6d(-b^2+5d^2+(-b^2+d^2)\cosh(2(c+dx))}{4\,(b^4-10b^2d^2+9d^4)}$$

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(3*b*(b^2 - 9*d^2)*Cosh[c + d*x] + (b^3 - b*d^2)*Cosh[3*(c + d
*x)] + 6*d*(-b^2 + 5*d^2 + (-b^2 + d^2)*Cosh[2*(c + d*x)])*Sinh[c + d*x]))/
(4*(b^4 - 10*b^2*d^2 + 9*d^4))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\mathrm{e}^{bx+3dx+a+3c}}{8b+24d}+\frac{3\,\mathrm{e}^{bx+dx+a+c}}{8(b+d)}+\frac{3\,\mathrm{e}^{bx-dx+a-c}}{8(b-d)}+\frac{\mathrm{e}^{bx-3dx+a-3c}}{8b-24d}$
parallelrisch	$-\frac{3\,\mathrm{e}^{bx+a}\left(\left(-\frac{1}{3}b^{3}+\frac{1}{3}b\,d^{2}\right)\cosh(3dx+3c)+\left(b^{2}d-d^{3}\right)\sinh(3dx+3c)-(b-3d)(b+3d)(b\cosh(dx+c)-d\sinh(dx+c))\right)}{4b^{4}-40b^{2}d^{2}+36d^{4}}$
default	$\frac{\sinh(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{3\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(a+3c+(b+3d)x)}{8b+24d}+\frac{\cosh(a-3c+(b-3d)x)}{8b-24d}$

```
[In] int(exp(b*x+a)*cosh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(b+3*d)*exp(b*x+3*d*x+a+3*c)+3/8/(b+d)*exp(b*x+d*x+a+c)+3/8/(b-d)*exp(b
*x-d*x+a-c)+1/8/(b-3*d)*exp(b*x-3*d*x+a-3*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(135) = 270.

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.25

$$\int e^{a+bx}\cosh^3(c+dx)\,dx$$
$$=\frac{(b^3-bd^2)\cosh(bx+a)\cosh(dx+c)^3-3\left((b^2d-d^3)\cosh(bx+a)+(b^2d-d^3)\sinh(bx+a)\right)\sinh(dx+}{}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)^3 - 3*((b^2*d - d^3)*cosh(b*
x + a) + (b^2*d - d^3)*sinh(b*x + a))*sinh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*c
osh(b*x + a)*cosh(d*x + c) + 3*((b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c) +
 (b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a))*sinh(d*x + c)^2 + ((b^3 - b*d^2
)*cosh(d*x + c)^3 + 3*(b^3 - 9*b*d^2)*cosh(d*x + c))*sinh(b*x + a) - 3*(3*(
b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*d - 9*d^3)*cosh(b*x + a)
+ (b^2*d - 9*d^3 + 3*(b^2*d - d^3)*cosh(d*x + c)^2)*sinh(b*x + a))*sinh(d*x
 + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(131) = 262.

Time = 2.29 (sec) , antiderivative size = 1085, normalized size of antiderivative = 7.81

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c)**3, Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-3*d*x)
*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)
/8 + 3*x*exp(a)*exp(-3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + x*exp(a)*exp
(-3*d*x)*cosh(c + d*x)**3/8 + 11*exp(a)*exp(-3*d*x)*sinh(c + d*x)**3/(24*d)
 + 5*exp(a)*exp(-3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(-
3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d + exp(a)*exp(-3*d*x)*cosh(c + d*x)*
*3/(24*d), Eq(b, -3*d)), (-3*x*exp(a)*exp(-d*x)*sinh(c + d*x)**3/8 - 3*x*ex
p(a)*exp(-d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8 + 3*x*exp(a)*exp(-d*x)*sinh
(c + d*x)*cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(-d*x)*cosh(c + d*x)**3/8 - 5*
exp(a)*exp(-d*x)*sinh(c + d*x)**3/(8*d) - exp(a)*exp(-d*x)*sinh(c + d*x)**2
*cosh(c + d*x)/(4*d) + exp(a)*exp(-d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d +
3*exp(a)*exp(-d*x)*cosh(c + d*x)**3/(8*d), Eq(b, -d)), (3*x*exp(a)*exp(d*x)
*sinh(c + d*x)**3/8 - 3*x*exp(a)*exp(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/8
- 3*x*exp(a)*exp(d*x)*sinh(c + d*x)*cosh(c + d*x)**2/8 + 3*x*exp(a)*exp(d*x
)*cosh(c + d*x)**3/8 - 5*exp(a)*exp(d*x)*sinh(c + d*x)**3/(8*d) + exp(a)*ex
p(d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d) + exp(a)*exp(d*x)*sinh(c + d*x)
*cosh(c + d*x)**2/d - 3*exp(a)*exp(d*x)*cosh(c + d*x)**3/(8*d), Eq(b, d)),
(-x*exp(a)*exp(3*d*x)*sinh(c + d*x)**3/8 + 3*x*exp(a)*exp(3*d*x)*sinh(c + d
*x)**2*cosh(c + d*x)/8 - 3*x*exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d*x)*
*2/8 + x*exp(a)*exp(3*d*x)*cosh(c + d*x)**3/8 + 11*exp(a)*exp(3*d*x)*sinh(c
 + d*x)**3/(24*d) - 5*exp(a)*exp(3*d*x)*sinh(c + d*x)**2*cosh(c + d*x)/(4*d
) + exp(a)*exp(3*d*x)*sinh(c + d*x)*cosh(c + d*x)**2/d - exp(a)*exp(3*d*x)*
cosh(c + d*x)**3/(24*d), Eq(b, 3*d)), (b**3*exp(a)*exp(b*x)*cosh(c + d*x)**
3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*exp(a)*exp(b*x)*sinh(c + d*x)*c
osh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*exp(a)*exp(b*x)*s
inh(c + d*x)**2*cosh(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*d**2*exp
(a)*exp(b*x)*cosh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d**3*exp(a
)*exp(b*x)*sinh(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*exp(a)*
exp(b*x)*sinh(c + d*x)*cosh(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), Tru
e))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \text{Exception raised: ValueError}$$

[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*d)/b>0)', see `assume?` for mor
e detai

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \frac{e^{(bx+3\,dx+a+3\,c)}}{8\,(b+3\,d)} + \frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{3\,e^{(bx-dx+a-c)}}{8\,(b-d)} + \frac{e^{(bx-3\,dx+a-3\,c)}}{8\,(b-3\,d)}$$

[In] integrate(exp(b*x+a)*cosh(d*x+c)^3,x, algorithm="giac")

[Out] 1/8*e^(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*e^(b*x + d*x + a + c)/(b + d)
 + 3/8*e^(b*x - d*x + a - c)/(b - d) + 1/8*e^(b*x - 3*d*x + a - 3*c)/(b - 3
*d)

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int e^{a+bx}\cosh^3(c+dx)\,dx = \frac{e^{a+b\,x}\,\left(b^3\cosh(c+d\,x)^3 - 3\,b^2\,d\cosh(c+d\,x)^2\sinh(c+d\,x) - 7\,b\,d^2\cosh(c+d\,x)^3 + 6\,b\,d^2\cosh(c+d\,x)\,\mathrm{s}\right.}{b^4 - 10\,b^2\,d^2 + 9\,d^4}$$

[In] int(cosh(c + d*x)^3*exp(a + b*x),x)

[Out] (exp(a + b*x)*(b^3*cosh(c + d*x)^3 - 6*d^3*sinh(c + d*x)^3 - 7*b*d^2*cosh(c
 + d*x)^3 + 9*d^3*cosh(c + d*x)^2*sinh(c + d*x) + 6*b*d^2*cosh(c + d*x)*sin
h(c + d*x)^2 - 3*b^2*d*cosh(c + d*x)^2*sinh(c + d*x)))/(b^4 + 9*d^4 - 10*b^
2*d^2)

3.886 $\int e^{a+bx} \cosh^2(c+dx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int e^{a+bx} \cosh^2(c+dx) \, dx = -\frac{2d^2 e^{a+bx}}{b(b^2-4d^2)} + \frac{b e^{a+bx} \cosh^2(c+dx)}{b^2-4d^2} - \frac{2d e^{a+bx} \cosh(c+dx) \sinh(c+dx)}{b^2-4d^2}$$

```
[Out] -2*d^2*exp(b*x+a)/b/(b^2-4*d^2)+b*exp(b*x+a)*cosh(d*x+c)^2/(b^2-4*d^2)-2*d*
exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)/(b^2-4*d^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 2225}

$$\int e^{a+bx} \cosh^2(c+dx) \, dx = \frac{b e^{a+bx} \cosh^2(c+dx)}{b^2-4d^2} - \frac{2d e^{a+bx} \sinh(c+dx) \cosh(c+dx)}{b^2-4d^2} - \frac{2d^2 e^{a+bx}}{b(b^2-4d^2)}$$

```
[In] Int[E^(a + b*x)*Cosh[c + d*x]^2,x]
```

```
[Out] (-2*d^2*E^(a + b*x))/(b*(b^2 - 4*d^2)) + (b*E^(a + b*x)*Cosh[c + d*x]^2)/(b
^2 - 4*d^2) - (2*d*E^(a + b*x)*Cosh[c + d*x]*Sinh[c + d*x])/(b^2 - 4*d^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2} - \frac{(2d^2)\int e^{a+bx}\,dx}{b^2-4d^2} \\ &= -\frac{2d^2e^{a+bx}}{b(b^2-4d^2)} + \frac{be^{a+bx}\cosh^2(c+dx)}{b^2-4d^2} - \frac{2de^{a+bx}\cosh(c+dx)\sinh(c+dx)}{b^2-4d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{e^{a+bx}(b^2-4d^2+b^2\cosh(2(c+dx))-2bd\sinh(2(c+dx)))}{2(b^3-4bd^2)}$$

```
[In] Integrate[E^(a + b*x)*Cosh[c + d*x]^2,x]
[Out] (E^(a + b*x)*(b^2 - 4*d^2 + b^2*Cosh[2*(c + d*x)] - 2*b*d*Sinh[2*(c + d*x)]
))/(2*(b^3 - 4*b*d^2))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{bx+2dx+a+2c}}{4b+8d}+\frac{e^{bx-2dx+a-2c}}{4b-8d}$
parallelrisch	$\frac{e^{bx+a}\left(\cosh(2dx+2c)b^2-2bd\sinh(2dx+2c)+b^2-4d^2\right)}{2b^3-8b\,d^2}$
default	$\frac{\sinh(bx+a)}{2b}+\frac{\sinh(a-2c+(b-2d)x)}{4b-8d}+\frac{\sinh(a+2c+(b+2d)x)}{4b+8d}+\frac{\cosh(bx+a)}{2b}+\frac{\cosh(a-2c+(b-2d)x)}{4b-8d}+\frac{\cosh(a+2c+(b}{4b+8d}$

```
[In] int(exp(b*x+a)*cosh(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(b*x+a)/b+1/4/(b+2*d)*exp(b*x+2*d*x+a+2*c)+1/4/(b-2*d)*exp(b*x-2*d*x
+a-2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int e^{a+bx}\cosh^2(c+dx)\,dx$$
$$= \frac{b^2\cosh(bx+a)\cosh(dx+c)^2 + (b^2\cosh(bx+a) + b^2\sinh(bx+a))\sinh(dx+c)^2 + (b^2-4d^2)\cosh(bx+\cdots}{}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(b*x + a)*cosh(d*x + c)^2 + (b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))*sinh(d*x + c)^2 + (b^2 - 4*d^2)*cosh(b*x + a) + (b^2*cosh(d*x + c)^2
+ b^2 - 4*d^2)*sinh(b*x + a) - 4*(b*d*cosh(b*x + a)*cosh(d*x + c) + b*d*cos
h(d*x + c)*sinh(b*x + a))*sinh(d*x + c))/(b^3 - 4*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(78) = 156.

Time = 0.84 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.91

$$\int e^{a+bx}\cosh^2(c+dx)\,dx$$
$$= \begin{cases} xe^a\cosh^2(c) \\ \left(-\frac{x\sinh^2(c+dx)}{2} + \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)e^a \\ \frac{xe^ae^{-2dx}\sinh^2(c+dx)}{4} + \frac{xe^ae^{-2dx}\sinh(c+dx)\cosh(c+dx)}{2} + \frac{xe^ae^{-2dx}\cosh^2(c+dx)}{4} + \frac{e^ae^{-2dx}\sinh^2(c+dx)}{2d} + \frac{3e^ae^{-2dx}\sinh(c+dx)\cdots}{4d} \\ \frac{xe^ae^{2dx}\sinh^2(c+dx)}{4} - \frac{xe^ae^{2dx}\sinh(c+dx)\cosh(c+dx)}{2} + \frac{xe^ae^{2dx}\cosh^2(c+dx)}{4} - \frac{e^ae^{2dx}\sinh^2(c+dx)}{2d} + \frac{3e^ae^{2dx}\sinh(c+dx)\cosh(c+\cdots}{4d} \\ \frac{b^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2} - \frac{2bde^ae^{bx}\sinh(c+dx)\cosh(c+dx)}{b^3-4bd^2} + \frac{2d^2e^ae^{bx}\sinh^2(c+dx)}{b^3-4bd^2} - \frac{2d^2e^ae^{bx}\cosh^2(c+dx)}{b^3-4bd^2} \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c)**2, Eq(b, 0) & Eq(d, 0)), ((-x*sinh(c + d*x)**2
/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))*exp(a), Eq(b
, 0)), (x*exp(a)*exp(-2*d*x)*sinh(c + d*x)**2/4 + x*exp(a)*exp(-2*d*x)*sinh
(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(-2*d*x)*cosh(c + d*x)**2/4 + exp(a
)*exp(-2*d*x)*sinh(c + d*x)**2/(2*d) + 3*exp(a)*exp(-2*d*x)*sinh(c + d*x)*c
osh(c + d*x)/(4*d), Eq(b, -2*d)), (x*exp(a)*exp(2*d*x)*sinh(c + d*x)**2/4 -
 x*exp(a)*exp(2*d*x)*sinh(c + d*x)*cosh(c + d*x)/2 + x*exp(a)*exp(2*d*x)*co
sh(c + d*x)**2/4 - exp(a)*exp(2*d*x)*sinh(c + d*x)**2/(2*d) + 3*exp(a)*exp(
2*d*x)*sinh(c + d*x)*cosh(c + d*x)/(4*d), Eq(b, 2*d)), (b**2*exp(a)*exp(b*x
```

```
)*cosh(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sinh(c + d*x)*
cosh(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sinh(c + d*x)**2/(
b**3 - 4*b*d**2) - 2*d**2*exp(a)*exp(b*x)*cosh(c + d*x)**2/(b**3 - 4*b*d**2
), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*d)/b>0)', see `assume?` for mor
e detai
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{e^{(bx+2\,dx+a+2\,c)}}{4\,(b+2\,d)} + \frac{e^{(bx-2\,dx+a-2\,c)}}{4\,(b-2\,d)} + \frac{e^{(bx+a)}}{2\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*e^(b*x - 2*d*x + a - 2*c)/(b
- 2*d) + 1/2*e^(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\cosh^2(c+dx)\,dx = \frac{2\,d^2\,\mathrm{e}^{a+b\,x}-b^2\,{\cosh\left(c+d\,x\right)}^2\,\mathrm{e}^{a+b\,x}+2\,b\,d\,\cosh\left(c+d\,x\right)\,\mathrm{e}^{a+b\,x}\,\sinh\left(c+d\,x\right)}{4\,b\,d^2-b^3}$$

```
[In] int(cosh(c + d*x)^2*exp(a + b*x),x)
```

```
[Out] (2*d^2*exp(a + b*x) - b^2*cosh(c + d*x)^2*exp(a + b*x) + 2*b*d*cosh(c + d*x
)*exp(a + b*x)*sinh(c + d*x))/(4*b*d^2 - b^3)
```

3.887 $\int e^{a+bx}\cosh(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

```
[Out] b*exp(b*x+a)*cosh(d*x+c)/(b^2-d^2)-d*exp(b*x+a)*sinh(d*x+c)/(b^2-d^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5583}

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

```
[In] Int[E^(a + b*x)*Cosh[c + d*x],x]
[Out] (b*E^(a + b*x)*Cosh[c + d*x])/(b^2 - d^2) - (d*E^(a + b*x)*Sinh[c + d*x])/(
b^2 - d^2)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral}=\frac{be^{a+bx}\cosh(c+dx)}{b^2-d^2}-\frac{de^{a+bx}\sinh(c+dx)}{b^2-d^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cosh(c+dx)\,dx=\frac{e^{a+bx}(b\cosh(c+dx)-d\sinh(c+dx))}{(b-d)(b+d)}$$

[In] Integrate[E^(a + b*x)*Cosh[c + d*x],x]

[Out] (E^(a + b*x)*(b*Cosh[c + d*x] - d*Sinh[c + d*x]))/((b - d)*(b + d))

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{bx+a}(b\cosh(dx+c)-d\sinh(dx+c))}{b^2-d^2}$	37
risch	$\frac{e^{bx+dx+a+c}}{2b+2d}+\frac{e^{bx-dx+a-c}}{2b-2d}$	41
default	$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}+\frac{\cosh(a-c+(b-d)x)}{2b-2d}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	78

[In] int(exp(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)

[Out] exp(b*x+a)*(b*cosh(d*x+c)-d*sinh(d*x+c))/(b^2-d^2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int e^{a+bx}\cosh(c+dx)\,dx$$
$$=\frac{b\cosh(bx+a)\cosh(dx+c)+b\cosh(dx+c)\sinh(bx+a)-(d\cosh(bx+a)+d\sinh(bx+a))\sinh(dx}{b^2-d^2}$$

[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="fricas")

[Out] (b*cosh(b*x + a)*cosh(d*x + c) + b*cosh(d*x + c)*sinh(b*x + a) - (d*cosh(b*x + a) + d*sinh(b*x + a))*sinh(d*x + c))/(b^2 - d^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(42) = 84$.

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int e^{a+bx}\cosh(c+dx)\,dx = \begin{cases} xe^a\cosh{(c)} & \text{for}\ b=0\wedge d=0 \\ \frac{xe^ae^{-dx}\sinh{(c+dx)}}{2}+\frac{xe^ae^{-dx}\cosh{(c+dx)}}{2}+\frac{e^ae^{-dx}\sinh{(c+dx)}}{2d} & \text{for}\ b=-d \\ -\frac{xe^ae^{dx}\sinh{(c+dx)}}{2}+\frac{xe^ae^{dx}\cosh{(c+dx)}}{2}+\frac{e^ae^{dx}\sinh{(c+dx)}}{2d} & \text{for}\ b=d \\ \frac{be^ae^{bx}\cosh{(c+dx)}}{b^2-d^2}-\frac{de^ae^{bx}\sinh{(c+dx)}}{b^2-d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*cosh(c), Eq(b, 0) & Eq(d, 0)), (x*exp(a)*exp(-d*x)*sinh
(c + d*x)/2 + x*exp(a)*exp(-d*x)*cosh(c + d*x)/2 + exp(a)*exp(-d*x)*sinh(c
+ d*x)/(2*d), Eq(b, -d)), (-x*exp(a)*exp(d*x)*sinh(c + d*x)/2 + x*exp(a)*ex
p(d*x)*cosh(c + d*x)/2 + exp(a)*exp(d*x)*sinh(c + d*x)/(2*d), Eq(b, d)), (b
*exp(a)*exp(b*x)*cosh(c + d*x)/(b**2 - d**2) - d*exp(a)*exp(b*x)*sinh(c + d
*x)/(b**2 - d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{a+bx}\cosh(c+dx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(c+dx)\,dx = \frac{e^{(bx+dx+a+c)}}{2\,(b+d)} + \frac{e^{(bx-dx+a-c)}}{2\,(b-d)}$$

```
[In] integrate(exp(b*x+a)*cosh(d*x+c),x, algorithm="giac")
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) + 1/2*e^(b*x - d*x + a - c)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh(c+dx)\,dx = \frac{\mathrm{e}^{a-c+b\,x-d\,x}\,\left(b+d+b\,\mathrm{e}^{2\,c+2\,d\,x}-d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{2\,\left(b^2-d^2\right)}$$

```
[In] int(cosh(c + d*x)*exp(a + b*x),x)
[Out] (exp(a - c + b*x - d*x)*(b + d + b*exp(2*c + 2*d*x) - d*exp(2*c + 2*d*x)))/
(2*(b^2 - d^2))
```

3.888 $\int e^{a+bx}\mathbf{sech}(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int e^{a+bx}\operatorname{sech}(c+dx)\,dx=\frac{2e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{b+d}$$

```
[Out] 2*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*d*x+2*c)
)/(b+d)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5600}

$$\int e^{a+bx}\operatorname{sech}(c+dx)\,dx=\frac{2e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(\frac{b}{d}+3\right),-e^{2(c+dx)}\right)}{b+d}$$

```
[In] Int[E^(a + b*x)*Sech[c + d*x],x]
```

```
[Out] (2*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral}=\frac{2e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{b+d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\mathrm{sech}(c+dx)\,dx=\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{b+d}$$

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x],x]
[Out] (2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b/d)/2, -
E^(2*(c + d*x))])/(b + d)
```

Maple [F]

$$\int e^{bx+a}\,\mathrm{sech}\,(dx+c)\,dx$$

```
[In] int(exp(b*x+a)*sech(d*x+c),x)
[Out] int(exp(b*x+a)*sech(d*x+c),x)
```

Fricas [F]

$$\int e^{a+bx}\mathrm{sech}(c+dx)\,dx=\int e^{(bx+a)}\,\mathrm{sech}\,(dx+c)\,\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="fricas")
[Out] integral(e^(b*x + a)*sech(d*x + c), x)
```

Sympy [F]

$$\int e^{a+bx}\mathrm{sech}(c+dx)\,dx=e^{a}\int e^{bx}\,\mathrm{sech}\,(c+dx)\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x)
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x), x)
```

Maxima [F]

$$\int e^{a+bx}\text{sech}(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)\,\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="maxima")
[Out] integrate(e^(b*x + a)*sech(d*x + c), x)
```

Giac [F]

$$\int e^{a+bx}\text{sech}(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)\,\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c),x, algorithm="giac")
[Out] integrate(e^(b*x + a)*sech(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}(c+dx)\,dx = \int \frac{\mathrm{e}^{a+b\,x}}{\cosh\left(c+d\,x\right)}\,dx$$

```
[In] int(exp(a + b*x)/cosh(c + d*x),x)
[Out] int(exp(a + b*x)/cosh(c + d*x), x)
```

3.889 $\int e^{a+bx}\text{sech}^2(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 56

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2(c+dx)}\right)}{b+2d}$$

```
[Out] 4*exp(b*x+2*d*x+a+2*c)*hypergeom([2, 1+1/2*b/d],[2+1/2*b/d],-exp(2*d*x+2*c)
)/(b+2*d)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5600}

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{b}{2d}+1,\frac{b}{2d}+2,-e^{2(c+dx)}\right)}{b+2d}$$

```
[In] Int[E^(a + b*x)*Sech[c + d*x]^2,x]
```

```
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral}=\frac{4e^{a+bx+2(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2(c+dx)}\right)}{b+2d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=\frac{4e^{a+bx+2(c+dx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{b}{2d},2+\frac{b}{2d},-e^{2(c+dx)}\right)}{b+2d}$$

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x]^2,x]
[Out] (4*E^(a + b*x + 2*(c + d*x))*Hypergeometric2F1[2, 1 + b/(2*d), 2 + b/(2*d),
 -E^(2*(c + d*x))])/(b + 2*d)
```

Maple [F]

$$\int \mathrm{e}^{bx+a}\operatorname{sech}(dx+c)^2\,dx$$

```
[In] int(exp(b*x+a)*sech(d*x+c)^2,x)
[Out] int(exp(b*x+a)*sech(d*x+c)^2,x)
```

Fricas [F]

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=\int e^{(bx+a)}\operatorname{sech}(dx+c)^2\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="fricas")
[Out] integral(e^(b*x + a)*sech(d*x + c)^2, x)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^2(c+dx)\,dx=e^a\int e^{bx}\operatorname{sech}^2(c+dx)\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)**2,x)
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x)**2, x)
```

Maxima [F]

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)^2\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="maxima")
[Out] 4*b*integrate(1/2*e^(b*x + a)/(d*e^(2*d*x + 2*c) + d), x) - 2*e^(b*x + a)/(
d*e^(2*d*x + 2*c) + d)
```

Giac [F]

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx = \int e^{(bx+a)}\,\text{sech}\,(dx+c)^2\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^2,x, algorithm="giac")
[Out] integrate(e^(b*x + a)*sech(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}^2(c+dx)\,dx = \int \frac{e^{a+b\,x}}{\cosh\left(c+d\,x\right)^2}\,dx$$

```
[In] int(exp(a + b*x)/cosh(c + d*x)^2,x)
[Out] int(exp(a + b*x)/cosh(c + d*x)^2, x)
```

3.890 $\int e^{a+bx}\mathbf{sech}^3(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx = -\frac{(b-d)e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{d^2} + \frac{be^{a+bx}\operatorname{sech}(c+dx)}{2d^2} + \frac{e^{a+bx}\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] -(b-d)*exp(b*x+d*x+a+c)*hypergeom([1, 1/2*(b+d)/d],[3/2+1/2*b/d],-exp(2*d*x
+2*c))/d^2+1/2*b*exp(b*x+a)*sech(d*x+c)/d^2+1/2*exp(b*x+a)*sech(d*x+c)*tanh
(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5598, 5600}

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx = -\frac{(b-d)e^{a+bx+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(\frac{b}{d}+3\right),-e^{2(c+dx)}\right)}{d^2} + \frac{be^{a+bx}\operatorname{sech}(c+dx)}{2d^2} + \frac{e^{a+bx}\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[In] Int[E^(a + b*x)*Sech[c + d*x]^3,x]
```

```
[Out] -(((b - d)*E^(a + c + b*x + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3 + b
/d)/2, -E^(2*(c + d*x))])/d^2) + (b*E^(a + b*x)*Sech[c + d*x])/(2*d^2) + (E
^(a + b*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 5598

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{be^{a+bx}\operatorname{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\\&\quad+\frac{1}{2}\left(1-\frac{b^2}{d^2}\right)\int e^{a+bx}\operatorname{sech}(c+dx)\,dx\\&=-\frac{(b-d)e^{a+c+bx+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)}{d^2}\\&\quad+\frac{be^{a+bx}\operatorname{sech}(c+dx)}{2d^2}+\frac{e^{a+bx}\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx =\frac{e^{a+bx}\left(-2(b-d)e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2d},\frac{1}{2}\left(3+\frac{b}{d}\right),-e^{2(c+dx)}\right)+\operatorname{sech}(c+dx)(b+d\tanh(c+dx\right.}{2d^2}$$

```
[In] Integrate[E^(a + b*x)*Sech[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(-2*(b - d)*E^(c + d*x)*Hypergeometric2F1[1, (b + d)/(2*d), (3
 + b/d)/2, -E^(2*(c + d*x))] + Sech[c + d*x]*(b + d*Tanh[c + d*x])))/(2*d^2
)
```

Maple [F]

$$\int e^{bx+a}\operatorname{sech}\left(dx+c\right)^{3}dx$$

```
[In] int(exp(b*x+a)*sech(d*x+c)^3,x)
[Out] int(exp(b*x+a)*sech(d*x+c)^3,x)
```

Fricas [F]

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx=\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^{3}\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="fricas")
[Out] integral(e^(b*x + a)*sech(d*x + c)^3, x)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx=e^{a}\int e^{bx}\operatorname{sech}^{3}\left(c+dx\right)dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)**3,x)
[Out] exp(a)*Integral(exp(b*x)*sech(c + d*x)**3, x)
```

Maxima [F]

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx=\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^{3}\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="maxima")
[Out] -8*(b^2*e^c - d^2*e^c)*integrate(1/8*e^(b*x + d*x + a)/(d^2*e^(2*d*x + 2*c)
 + d^2), x) + ((b*e^(3*c) + d*e^(3*c))*e^(b*x + 3*d*x + a) + (b*e^c - d*e^c
)*e^(b*x + d*x + a))/(d^2*e^(4*d*x + 4*c) + 2*d^2*e^(2*d*x + 2*c) + d^2)
```

Giac [F]

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx=\int e^{(bx+a)}\operatorname{sech}\left(dx+c\right)^3\,dx$$

```
[In] integrate(exp(b*x+a)*sech(d*x+c)^3,x, algorithm="giac")
[Out] integrate(e^(b*x + a)*sech(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{sech}^3(c+dx)\,dx=\int \frac{\mathrm{e}^{a+b\,x}}{\cosh\left(c+d\,x\right)^3}\,dx$$

```
[In] int(exp(a + b*x)/cosh(c + d*x)^3,x)
[Out] int(exp(a + b*x)/cosh(c + d*x)^3, x)
```

3.891 $\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \frac{\left(1+e^{2(d+ex)}\right)^n F^{ac+bcx}\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},1+\frac{en+bc\log(F)}{2e},-e^{2(d+ex)}\right)\text{sech}^n(d+ex)}{en+bc\log(F)}$$

```
[Out] (1+exp(2*e*x+2*d))^n*F^(b*c*x+a*c)*hypergeom([n, 1/2*(e*n+b*c*ln(F))/e],[1+
1/2*(e*n+b*c*ln(F))/e],-exp(2*e*x+2*d))*sech(e*x+d)^n/(e*n+b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5602, 2291}

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \frac{\left(e^{2(d+ex)}+1\right)^n F^{ac+bcx}\text{sech}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},\frac{en+bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)+en}$$

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] && !IntegerQ[p]
```

Rule 5602

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Dist[(1 + E^(2*(d + e*x)))^n*(Sech[d + e*x]^n/E^(n*(d + e*x))), Int
[SimplifyIntegrand[F^(c*(a + b*x))*(E^(n*(d + e*x))/(1 + E^(2*(d + e*x)))^n
), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^n \operatorname{sech}^n(d+ex)\right)\int e^{dn+enx}\left(1+e^{2(d+ex)}\right)^{-n} F^{ac+bcx}\,dx$$

$$= \frac{\left(1+e^{2(d+ex)}\right)^n F^{ac+bcx}\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},1+\frac{en+bc\log(F)}{2e},-e^{2(d+ex)}\right)\operatorname{sech}^n(d+e}{en+bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx$$
$$= \frac{\left(1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},1+\frac{en+bc\log(F)}{2e},-e^{2(d+ex)}\right)\operatorname{sech}^n(d+ex)}{en+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^n,x]
```

```
[Out] ((1 + E^(2*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n + b*c*Lo
g[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*Sech[d + e*x]^
n)/(e*n + b*c*Log[F])
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)^n\,dx$$

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx$$

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}^n(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\left(\frac{1}{\cosh\left(d+e\,x\right)}\right)^n\,dx$$

```
[In] int(F^(c*(a + b*x))*(1/cosh(d + e*x))^n,x)
```

```
[Out] int(F^(c*(a + b*x))*(1/cosh(d + e*x))^n, x)
```

3.892 $\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\text{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right),e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

```
[Out] -(1-1/exp(2*e*x+2*d))^n*F^(b*c*x+a*c)*csch(e*x+d)^n*hypergeom([n, 1/2*(e*n-
b*c*ln(F))/e],[1+1/2*n-1/2*b*c*ln(F)/e],exp(-2*e*x-2*d))/(e*n-b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5603, 2291}

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\text{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(n-\frac{bc\log(F)}{e}+2\right),e^{-2(d+ex)}\right)}{en-bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

```
[Out] -(((1 - E^(-2*(d + e*x)))^n*F^(a*c + b*c*x)*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 5603

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Dist[(1 - E^(-2*(d + e*x)))^n*(Csch[d + e*x]^n/E^((-n)*(d + e*x))),
 Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(n*(d + e*x))*(1 - E^(-2*(d +
e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{n(d+ex)}\left(1-e^{-2(d+ex)}\right)^n \operatorname{csch}^n(d+ex)\right)\int e^{-dn-enx}\left(1-e^{-2(d+ex)}\right)^{-n} F^{ac+bcx}\,dx$$

$$= -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}\operatorname{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right),\right.}{en-bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,dx = -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{c(a+bx)}\operatorname{csch}^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right),e^{-2(d+e}\right.}{en-bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^n,x]
```

```
[Out] -(((1 - E^(-2*(d + e*x)))^n*F^(c*(a + b*x))*Csch[d + e*x]^n*Hypergeometric2
F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x)
)])/(e*n - b*c*Log[F]))
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^n \, dx$$

[In] int(F^(c*(b*x+a))*csch(e*x+d)^n,x)

[Out] int(F^(c*(b*x+a))*csch(e*x+d)^n,x)

Fricas [F]

$$\int F^{c(a+bx)} \operatorname{csch}^n(d+ex) \, dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^n \, dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="fricas")

[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^n, x)

Sympy [F]

$$\int F^{c(a+bx)} \operatorname{csch}^n(d+ex) \, dx = \int F^{c(a+bx)} \operatorname{csch}^n(d+ex) \, dx$$

[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**n,x)

[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**n, x)

Maxima [F]

$$\int F^{c(a+bx)} \operatorname{csch}^n(d+ex) \, dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^n \, dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="maxima")

[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^n\;dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}^n(d+ex)\,dx = \int F^{c\,(a+b\,x)}\left(\frac{1}{\sinh\left(d+e\,x\right)}\right)^n\,dx$$

```
[In] int(F^(c*(a + b*x))*(1/sinh(d + e*x))^n,x)
[Out] int(F^(c*(a + b*x))*(1/sinh(d + e*x))^n, x)
```

3.893 $\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 254

$$\begin{aligned}\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx=&\frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}\\&+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}\\&-\frac{2ibcf^2F^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}\\&-\frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}\\&+\frac{bcf^2F^{ac+bcx}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}\end{aligned}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)+2*I*e*f^2*F^(b*c*x+a*c)*cosh(e*x+d)/(e^2-b^2*c^
2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-2*I*b*
c*f^2*F^(b*c*x+a*c)*ln(F)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-2*e*f^2*F^(b*c*
x+a*c)*cosh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)+b*c*f^2*F^(b*c*x+a*c
)*ln(F)*sinh(e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6873, 12, 6874, 2225, 5582, 5584}

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx=\frac{bcf^2\log(F)\sinh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ief^2\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{2ef^2\sinh(d+ex)\cosh(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{f^2F^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + ((2*I)*e*f^2*F^(a*c + b*c*x)*Cosh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4
*e^2 - b^2*c^2*Log[F]^2)) - ((2*I)*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sinh[d +
e*x])/(e^2 - b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sin
h[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Si
nh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int f^2 F^{ac+bcx}(1+i\sinh(d+ex))^2\,dx \\
&= f^2\int F^{ac+bcx}(1+i\sinh(d+ex))^2\,dx \\
&= f^2\int \left(F^{ac+bcx}+2iF^{ac+bcx}\sinh(d+ex)-F^{ac+bcx}\sinh^2(d+ex)\right)\,dx \\
&= \left(2if^2\right)\int F^{ac+bcx}\sinh(d+ex)\,dx+f^2\int F^{ac+bcx}\,dx-f^2\int F^{ac+bcx}\sinh^2(d+ex)\,dx \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{2ibcf^2F^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)} \\
&\quad-\frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} \\
&\quad+\frac{bcf^2F^{ac+bcx}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}+\frac{\left(2e^2f^2\right)\int F^{ac+bcx}\,dx}{4e^2-b^2c^2\log^2(F)} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2ief^2F^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)} \\
&\quad+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{2ibcf^2F^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)} \\
&\quad-\frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}+\frac{bcf^2F^{ac+bcx}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx = \frac{F^{c(a+bx)}(f+if\sinh(d+ex))^2\left(\frac{3}{bc\log(F)}+\frac{4ie\cosh(d+ex)}{(e-bc\log(F))(e+bc\log(F))}-\frac{bc\cosh(2(d+ex))\log(F)}{-4e^2+b^2c^2\log^2(F)}+\frac{4ibc\log(F)\sinh(d+ex)}{(-e+bc\log(F))(e+bc\log(F))}\right.}{2\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x])^2*(3/(b*c*Log[F]) + ((4*I)*e*Cosh[
d + e*x])/((e - b*c*Log[F])*(e + b*c*Log[F])) - (b*c*Cosh[2*(d + e*x)]*Log[
F])/(-4*e^2 + b^2*c^2*Log[F]^2) + ((4*I)*b*c*Log[F]*Sinh[d + e*x])/((-e + b
*c*Log[F])*(e + b*c*Log[F])) - (2*e*Sinh[2*(d + e*x)])/(4*e^2 - b^2*c^2*Log
[F]^2)))/(2*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])^4)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.77

method	result
parallelrisch	$-\frac{2F^{c(bx+a)}\left(\frac{\left(\ln(F)^4b^4c^4-\ln(F)^2b^2c^2e^2\right)\cosh(2ex+2d)}{4}+\frac{\left(-\ln(F)^3b^3c^3e+\ln(F)bce^3\right)\sinh(2ex+2d)}{2}+(bc\ln(F)+2e)(bc\ln(F)-2e)\right.}{c^5b^5\ln(F)^5-5c^3b^3\ln(F)^3e^2+4\ln(F)bce^4}$
risch	$\frac{f^2\left(16i\ln(F)bce^3e^{3ex+3d}-\ln(F)^4b^4c^4e^{4ex+4d}-4i\ln(F)^3b^3c^3e\,e^{ex+d}+6\ln(F)^4b^4c^4e^{2ex+2d}-16i\ln(F)^2b^2c^2e^2e^{3ex+3d}+2\ln(\right.}{}$

```
[In] int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*F^(c*(b*x+a))*(1/4*(ln(F)^4*b^4*c^4-ln(F)^2*b^2*c^2*e^2)*cosh(2*e*x+2*d)
+1/2*(-ln(F)^3*b^3*c^3*e+ln(F)*b*c*e^3)*sinh(2*e*x+2*d)+(b*c*ln(F)+2*e)*(b*
c*ln(F)-2*e)*(-I*sinh(e*x+d)*b^2*c^2*ln(F)^2+I*cosh(e*x+d)*b*c*e*ln(F)-3/4*
b^2*c^2*ln(F)^2+3/4*e^2))*f^2/(c^5*b^5*ln(F)^5-5*c^3*b^3*ln(F)^3*e^2+4*ln(F
)*b*c*e^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.74

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$$
$$=\frac{\left(24\,e^4f^2e^{(2\,ex+2\,d)}-\left(b^4c^4f^2e^{(4\,ex+4\,d)}-4i\,b^4c^4f^2e^{(3\,ex+3\,d)}-6\,b^4c^4f^2e^{(2\,ex+2\,d)}+4i\,b^4c^4f^2e^{(ex+d)}+b^4c^4f^2\right)\mathrm{lo}\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(24*e^4*f^2*e^(2*e*x + 2*d) - (b^4*c^4*f^2*e^(4*e*x + 4*d) - 4*I*b^4*c^
4*f^2*e^(3*e*x + 3*d) - 6*b^4*c^4*f^2*e^(2*e*x + 2*d) + 4*I*b^4*c^4*f^2*e^(
e*x + d) + b^4*c^4*f^2)*log(F)^4 + 2*(b^3*c^3*e*f^2*e^(4*e*x + 4*d) - 2*I*b
^3*c^3*e*f^2*e^(3*e*x + 3*d) - 2*I*b^3*c^3*e*f^2*e^(e*x + d) - b^3*c^3*e*f^
2)*log(F)^3 + (b^2*c^2*e^2*f^2*e^(4*e*x + 4*d) - 16*I*b^2*c^2*e^2*f^2*e^(3*
e*x + 3*d) - 30*b^2*c^2*e^2*f^2*e^(2*e*x + 2*d) + 16*I*b^2*c^2*e^2*f^2*e^(e
*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 - 2*(b*c*e^3*f^2*e^(4*e*x + 4*d) - 8*I*
b*c*e^3*f^2*e^(3*e*x + 3*d) - 8*I*b*c*e^3*f^2*e^(e*x + d) - b*c*e^3*f^2)*lo
g(F))*F^(b*c*x + a*c)/(b^5*c^5*e^(2*e*x + 2*d)*log(F)^5 - 5*b^3*c^3*e^2*e^(
2*e*x + 2*d)*log(F)^3 + 4*b*c*e^4*e^(2*e*x + 2*d)*log(F))
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2377 vs. 2(241) = 482.

Time = 35.87 (sec) , antiderivative size = 2377, normalized size of antiderivative = 9.36

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx=\text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d))**2,x)
```

```
[Out] Piecewise((x*(I*f*sinh(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0
)), (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f**2
*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(F, 1)), (
F**(a*c)*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x -
 f**2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e), Eq(b,
0)), (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x - f**
2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*I*f**2*cosh(d + e*x)/e, Eq(c, 0)),
(-I*F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F) - d) + I*F**(a*c + b*c*x)*f**
2*x*cosh(b*c*x*log(F) - d) - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)**
2/(3*b*c*log(F)) - 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)*cosh(b*c*
```

x*log(F) - d)/(3*b*c*log(F)) + I*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)/(b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) - d)**2/(3*b*c*log(F)) - 2*I*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 - d)**2/4 + F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*f**2*x*cosh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 - d)**2/(b*c*log(F)) + F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/(2*b*c*log(F)) - 8*I*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 - d)/(3*b*c*log(F)) + 4*I*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F)/2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*log(F)/2)), (-F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 + d)**2/4 + F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/2 - F**(a*c + b*c*x)*f**2*x*cosh(b*c*x*log(F)/2 + d)**2/4 - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)) + F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)) + 8*I*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 + d)/(3*b*c*log(F)) - 4*I*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F)/2 + d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, b*c*log(F)/2)), (I*F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F) + d) - I*F**(a*c + b*c*x)*f**2*x*cosh(b*c*x*log(F) + d) - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) + d)**2/(3*b*c*log(F)) - 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) + d)*cosh(b*c*x*log(F) + d)/(3*b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) + d)**2/(3*b*c*log(F)) + I*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) + d)/(b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, b*c*log(F))), (-F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*I*F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4*sinh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b**3*c**3*e*f**2*log(F)**3*sinh(d + e*x)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*I*F**(a*c + b*c*x)*b**3*c**3*e*f**2*log(F)**3*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 3*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 8*I*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sinh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 5*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c + b*c*x)*b*c*e**3*f**2*log(F)*sinh(d + e*x)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 8*I*F**(a*c + b*c*x)*b*c*e**3*f**2*log(F)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c + b*c*x)*e**4*f**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b

```
*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*e**4*f**2*cosh(d + e*x)**2/(b**5*c**5*
log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 4*F**(a*c + b
*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e
**4*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx = -\frac{1}{4}f^2\left(\frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{bc\log(F)+2\,e}+\frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}}-\frac{2\,F^{bcx+ac}}{bc\log(F)}\right) + i\,f^2\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}-\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f^2}{bc\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*
c)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 2*F^(b*c*x
 + a*c)/(b*c*log(F))) + I*f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(
F) + e) - F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c
*x + a*c)*f^2/(b*c*log(F))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(250) = 500.

Time = 0.33 (sec) , antiderivative size = 1546, normalized size of antiderivative = 6.09

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*f^2*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2
*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I*pi*b*c
*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b
```

*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) - 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*
c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F)
 - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(
abs(F)) + 2*e)*x + 2*d) + I*(-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c
*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c +
8*b*c*log(abs(F)) + 16*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*
x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c +
8*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x +
 2*d) - 2*((pi*b*c*sgn(F) - pi*b*c)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b
*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
log(abs(F)) + e)^2) + 2(b*c*log(abs(F)) + e)*f^2*sin(-1/2*pi*b*c*x*sgn(F)
 + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)
^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)
*x + d) - (-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*
e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
 + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e
^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 2*((pi*b*c*sgn(F) - pi*b
*c)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) + 2*(b*c*lo
g(abs(F)) - e)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn
(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))
*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - (I*f^2*e^(1/2*I*pi*b*c
*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*
sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F
) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(
F)) - e)*x - d) - 1/2*(2*(b*c*log(abs(F)) - 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn(
F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*
c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2
*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*
sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2))*e^(a*c*log(abs(F)) + (b*
c*log(abs(F)) - 2*e)*x - 2*d) + I*(-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*
pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*
b*c + 8*b*c*log(abs(F)) - 16*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*p
i*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*
b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*
e)*x - 2*d)

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))^2\,dx$$
$$=\frac{F^{c\,(a+b\,x)}\,f^2\,\left(12\,e^4+3\,b^4\,c^4\ln\left(F\right)^4+b^4\,c^4\sinh(d+e\,x)\,\ln\left(F\right)^4\,4\mathrm{i}-b^4\,c^4\ln\left(F\right)^4\cosh(2\,d+2\,e\,x)-15\,b^2\,c^2\right.}{}$$

```
[In] int(F^(c*(a + b*x))*(f + f*sinh(d + e*x)*1i)^2,x)
```

```
[Out] (F^(c*(a + b*x))*f^2*(12*e^4 + 3*b^4*c^4*log(F)^4 + b^4*c^4*sinh(d + e*x)*l
og(F)^4*4i - b^4*c^4*log(F)^4*cosh(2*d + 2*e*x) - 15*b^2*c^2*e^2*log(F)^2 +
 2*b^3*c^3*e*log(F)^3*sinh(2*d + 2*e*x) - b^2*c^2*e^2*sinh(d + e*x)*log(F)^
2*16i - 2*b*c*e^3*log(F)*sinh(2*d + 2*e*x) + b^2*c^2*e^2*log(F)^2*cosh(2*d
+ 2*e*x) - b^3*c^3*e*cosh(d + e*x)*log(F)^3*4i + b*c*e^3*cosh(d + e*x)*log(
F)*16i))/(2*b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 - 5*b^2*c^2*e^2*log(F)^2))
```

3.894 $\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}+\frac{iefF^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{ibcfF^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)+I*e*f*F^(b*c*x+a*c)*cosh(e*x+d)/(e^2-b^2*c^2*ln(F
)^2)-I*b*c*f*F^(b*c*x+a*c)*ln(F)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6873, 12, 6874, 2225, 5582}

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx=-\frac{ibcf\log(F)\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{ief\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (I*e*f*F^(a*c + b*c*x)*Cosh[d + e*x])/(e
^2 - b^2*c^2*Log[F]^2) - (I*b*c*f*F^(a*c + b*c*x)*Log[F]*Sinh[d + e*x])/(e^
2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int fF^{ac+bcx}(1+i\sinh(d+ex))\,dx \\
&= f\int F^{ac+bcx}(1+i\sinh(d+ex))\,dx \\
&= f\int \left(F^{ac+bcx}+iF^{ac+bcx}\sinh(d+ex)\right)\,dx \\
&= (if)\int F^{ac+bcx}\sinh(d+ex)\,dx+f\int F^{ac+bcx}\,dx \\
&= \frac{fF^{ac+bcx}}{bc\log(F)}+\frac{iefF^{ac+bcx}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{ibcfF^{ac+bcx}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = \frac{fF^{c(a+bx)}\left(-e^2-ibce\cosh(d+ex)\log(F)+b^2c^2\log^2(F)+ib^2c^2\log^2(F)\sinh(d+ex)\right)}{bc\log(F)(-e+bc\log(F))(e+bc\log(F))}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(-e^2 - I*b*c*e*Cosh[d + e*x]*Log[F] + b^2*c^2*Log[F]^2
+ I*b^2*c^2*Log[F]^2*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c
*Log[F]))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result	s
parallelrisch	$\frac{f\,F^{c(bx+a)}\left(i\sinh(ex+d)b^2c^2\ln(F)^2+b^2c^2\ln(F)^2-i\cosh(ex+d)bce\ln(F)-e^2\right)}{bc\ln(F)\left(b^2c^2\ln(F)^2-e^2\right)}$	9
risch	$\frac{f\left(-i\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}+i\ln(F)^2b^2c^2-2\ln(F)^2b^2c^2\mathrm{e}^{ex+d}+i\ln(F)bce\,\mathrm{e}^{2ex+2d}+i\ln(F)bce+2e^2\mathrm{e}^{ex+d}\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2bc\ln(F)(e-bc\ln(F))(e+bc\ln(F))}$	1

```
[In] int(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] f*F^(c*(b*x+a))/b/c/ln(F)/(b^2*c^2*ln(F)^2-e^2)*(I*ln(F)^2*b^2*c^2*sinh(e*x
+d)+b^2*c^2*ln(F)^2-I*cosh(e*x+d)*b*c*e*ln(F)-e^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = -\frac{\left(2\,e^2fe^{(ex+d)}-\left(i\,b^2c^2fe^{(2\,ex+2\,d)}+2\,b^2c^2fe^{(ex+d)}-i\,b^2c^2f\right)\log\left(F\right)^2-\left(-i\,bcefe^{(2\,ex+2\,d)}-i\,bcef\right)\log\left(\right.}{2\left(b^3c^3e^{(ex+d)}\log\left(F\right)^3-bce^2e^{(ex+d)}\log\left(F\right)\right)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*e^2*f*e^(e*x + d) - (I*b^2*c^2*f*e^(2*e*x + 2*d) + 2*b^2*c^2*f*e^(e
*x + d) - I*b^2*c^2*f)*log(F)^2 - (-I*b*c*e*f*e^(2*e*x + 2*d) - I*b*c*e*f)*
```

```
log(F))*F^(b*c*x + a*c)/(b^3*c^3*e^(e*x + d)*log(F)^3 - b*c*e^2*e^(e*x + d)
*log(F))
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(97) = 194.

Time = 1.19 (sec) , antiderivative size = 510, normalized size of antiderivative = 4.81

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$$
$$= \begin{cases} x(if\sinh(d)+f) \\ fx+\frac{if\cosh(d+ex)}{e} \\ F^{ac}\left(fx+\frac{if\cosh(d+ex)}{e}\right) \\ fx+\frac{if\cosh(d+ex)}{e} \\ -\frac{iF^{ac+bcx}fx\sinh(bcx\log(F)-d)}{2}+\frac{iF^{ac+bcx}fx\cosh(bcx\log(F)-d)}{2}+\frac{iF^{ac+bcx}f\sinh(bcx\log(F)-d)}{2bc\log(F)}-\frac{iF^{ac+bcx}f\cosh(bcx\log(F)-d)}{bc\log(F)} \\ \frac{iF^{ac+bcx}fx\sinh(bcx\log(F)+d)}{2}-\frac{iF^{ac+bcx}fx\cosh(bcx\log(F)+d)}{2}+\frac{iF^{ac+bcx}f\cosh(bcx\log(F)+d)}{2bc\log(F)}+\frac{F^{ac+bcx}f}{bc\log(F)} \\ \frac{iF^{ac+bcx}b^2c^2f\log(F)^2\sinh(d+ex)}{b^3c^3\log(F)^3-bce^2\log(F)}+\frac{F^{ac+bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3-bce^2\log(F)}-\frac{iF^{ac+bcx}bcef\log(F)\cosh(d+ex)}{b^3c^3\log(F)^3-bce^2\log(F)}-\frac{F^{ac+bcx}e^2f}{b^3c^3\log(F)^3-bce^2\log(F)} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x)
```

```
[Out] Piecewise((x*(I*f*sinh(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)),
 (f*x + I*f*cosh(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x + I*f*cosh(d + e*x)/
e), Eq(b, 0)), (f*x + I*f*cosh(d + e*x)/e, Eq(c, 0)), (-I*F**(a*c + b*c*x)*
f*x*sinh(b*c*x*log(F) - d)/2 + I*F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) - d
)/2 + I*F**(a*c + b*c*x)*f*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) - I*F**(a*
c + b*c*x)*f*cosh(b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*
log(F)), Eq(e, -b*c*log(F))), (I*F**(a*c + b*c*x)*f*x*sinh(b*c*x*log(F) + d
)/2 - I*F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) + d)/2 + I*F**(a*c + b*c*x)*
f*cosh(b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)),
Eq(e, b*c*log(F))), (I*F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2*sinh(d + e*x)
/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) + F**(a*c + b*c*x)*b**2*c**2*f*log
(F)**2/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - I*F**(a*c + b*c*x)*b*c*e*f
*log(F)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c + b
*c*x)*e**2*f/(b**3*c**3*log(F)**3 - b*c*e**2*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = \frac{1}{2}i\,f\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e} - \frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right) + \frac{F^{bcx+ac}f}{bc\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*I*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F
))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 885 vs. 2(104) = 208.

Time = 0.29 (sec) , antiderivative size = 885, normalized size of antiderivative = 8.35

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+I*f*sinh(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/2*I*pi*b*c
*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*
sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/
2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi
*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - ((pi*b
*c*sgn(F) - pi*b*c)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*
sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) + 2*(b*c*log(abs(F)) + e)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - (-I*f*e^(1
/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)
/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e) - I*f*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
```

```
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e))*e^(a*c*log(abs(F)
) + (b*c*log(abs(F)) + e)*x + d) + ((pi*b*c*sgn(F) - pi*b*c)*f*cos(-1/2*pi*
b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(
F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) + 2*(b*c*log(abs(F)) - e)*f*sin
(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi
*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) +
(b*c*log(abs(F)) - e)*x - d) - (I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c
*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c +
4*b*c*log(abs(F)) - 4*e) + I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b
*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int F^{c(a+bx)}(f+if\sinh(d+ex))\,dx$$
$$=\frac{F^{c\,(a+b\,x)}\,f\,\left(e^2-b^2\,c^2\ln\left(F\right)^2-b^2\,c^2\sinh(d+e\,x)\,\ln\left(F\right)^2\,1\mathrm{i}+b\,c\,e\cosh(d+e\,x)\,\ln\left(F\right)\,1\mathrm{i}\right)}{b\,c\,\ln\left(F\right)\,\left(e^2-b^2\,c^2\ln\left(F\right)^2\right)}$$

```
[In] int(F^(c*(a + b*x))*(f + f*sinh(d + e*x)*1i),x)
```

```
[Out] (F^(c*(a + b*x))*f*(e^2 - b^2*c^2*log(F)^2 - b^2*c^2*sinh(d + e*x)*log(F)^2
*1i + b*c*e*cosh(d + e*x)*log(F)*1i))/(b*c*log(F)*(e^2 - b^2*c^2*log(F)^2))
```

3.895 $\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)}{f(e+bc\log(F))}$$

```
[Out] 2*exp(d+1/2*I*Pi+e*x)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(
F)/e],-exp(d+1/2*I*Pi+e*x))/f/(e+b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5604, 5600}

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{f(bc\log(F)+e)}$$

```
[In] Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
```

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)])/(f*(e + b*c*L
og[F]))
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5604

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cosh[d/2 - f*(Pi/(
4*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ
[f^2 + g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\,dx}{2f}$$

$$= \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)}{f(e+bc\log(F))}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx$$
$$= \frac{2F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},-ie^{d+ex}\right)+\frac{\cosh\left(\frac{ex}{2}\right)-\sinh\left(\frac{ex}{2}\right)}{(-1-ie^d)\cosh\left(\frac{ex}{2}\right)+(1-ie^d)\sinh\left(\frac{ex}{2}\right)}\right)}{ef}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x]),x]
[Out] (2*F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e
, (-I)*E^(d + e*x)] + (Cosh[(e*x)/2] - Sinh[(e*x)/2])/((-1 - I*E^d)*Cosh[(e
*x)/2] + (1 - I*E^d)*Sinh[(e*x)/2])))/(e*f)
```

Maple [F]

$$\int \frac{F^{c(bx+a)}}{f+if\sinh(ex+d)}dx$$

```
[In] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
[Out] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{i\,f\sinh\left(ex+d\right)+f}\,dx$$

[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="fricas")

[Out] ((e*f*e^(e*x + d) - I*e*f)*integral(-2*I*F^(b*c*x + a*c)*b*c*log(F)/(e*f*e^(e*x + d) - I*e*f), x) + 2*I*F^(b*c*x + a*c))/(e*f*e^(e*x + d) - I*e*f)

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = -\frac{i\int \frac{F^{ac+bcx}}{\sinh(d+ex)-i}\,dx}{f}$$

[In] integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x)

[Out] -I*Integral(F**(a*c + b*c*x)/(sinh(d + e*x) - I), x)/f

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{i\,f\sinh\left(ex+d\right)+f}\,dx$$

[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="maxima")

[Out] -4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(I*b^2*c^2*f*log(F)^2 - 3*I*b*c*e*f*log(F) + 2*I*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) + 2*e^2*f*e^(3*d))*e^(3*e*x) - 3*(I*b^2*c^2*f*e^(2*d)*log(F)^2 - 3*I*b*c*e*f*e^(2*d)*log(F) + 2*I*e^2*f*e^(2*d))*e^(2*e*x) - 3*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(-2*I*F^(a*c)*e - (F^(a*c)*b*c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(-I*b^2*c^2*f*log(F)^2 + 3*I*b*c*e*f*log(F) - 2*I*e^2*f + (I*b^2*c^2*f*e^(2*d)*log(F)^2 - 3*I*b*c*e*f*e^(2*d)*log(F) + 2*I*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))

Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{i\,f\sinh\left(ex+d\right)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d)),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+if\sinh(d+ex)}\,dx = \int \frac{F^{c\,(a+b\,x)}}{f+f\sinh\left(d+e\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i),x)
[Out] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i), x)
```

3.896 $\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6ef^2}$$

```
[Out] 2/3*exp(d+1/2*I*Pi+e*x)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*l
n(F)/e],-exp(d+1/2*I*Pi+e*x))*(e-b*c*ln(F))/e^2/f^2+1/6*b*c*F^(c*(b*x+a))*l
n(F)*sech(1/2*d+1/4*I*Pi+1/2*e*x)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sech(1/2*d+1/
4*I*Pi+1/2*e*x)^2*tanh(1/2*d+1/4*I*Pi+1/2*e*x)/e/f^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5604, 5598, 5600}

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \frac{2e^{\frac{1}{2}(2d+2ex+i\pi)}F^{c(a+bx)}(e-bc\log(F))\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{\frac{1}{2}(2d+2ex+i\pi)}\right)}{3e^2f^2} + \frac{bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)F^{c(a+bx)}}{6e^2f^2} + \frac{\tanh\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{i\pi}{4}\right)F^{c(a+bx)}}{6ef^2}$$

```
[In] Int[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (2*E^((2*d + I*Pi + 2*e*x)/2)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c
*Log[F])/e, 2 + (b*c*Log[F])/e, -E^((2*d + I*Pi + 2*e*x)/2)]*(e - b*c*Log[F
]))/(3*e^2*f^2) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d/2 + (I/4)*Pi + (e*x)/2
]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[d/2 + (I/4)*Pi + (e*x)/2]^2*Tanh[d
/2 + (I/4)*Pi + (e*x)/2])/(6*e*f^2)
```

Rule 5598

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5604

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sinh[(d_.) + (e_.)*(x_)
])^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cosh[d/2 - f*(Pi/(
4*g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ
[f^2 + g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int F^{c(a+bx)}\operatorname{sech}^4\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\,dx}{4f^2}\\
&= \frac{bcF^{c(a+bx)}\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6e^2f^2}\\
&\quad+\frac{F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6ef^2}\\
&\quad+\frac{\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\,dx}{6f^2}
\end{aligned}
$$

$$= \frac{2e^{\frac{1}{2}(2d+i\pi+2ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{\frac{1}{2}(2d+i\pi+2ex)}\right)(e-bc\log}{3e^2f^2}$$

$$+\frac{bcF^{c(a+bx)}\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6e^2f^2}$$

$$+\frac{F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{i\pi}{4}+\frac{ex}{2}\right)}{6ef^2}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.28

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx$$

$$= \frac{F^{c(a+bx)}\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right)\left(e(ie+bc\log(F))\left(\cosh\left(\frac{1}{2}(d+ex)\right)+i\sinh\left(\frac{1}{2}(d+ex)\right)\right.\right.}{}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + I*f*Sinh[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])*(e*(I*e + b*c*Lo
g[F])*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2]) - (1 - I)*(-1 + (1 + I)*Hyp
ergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, (-I)*E^(d + e*x)])*(-
e^2 + b^2*c^2*Log[F]^2)*(Cosh[(d + e*x)/2] + I*Sinh[(d + e*x)/2])^3 + 2*e^2
*Sinh[(d + e*x)/2] + 2*(e^2 - b^2*c^2*Log[F]^2)*(Cosh[(d + e*x)/2] + I*Sinh
[(d + e*x)/2])^2*Sinh[(d + e*x)/2]))/(3*e^3*(f + I*f*Sinh[d + e*x])^2)
```

Maple [F]

$$\int \frac{F^{c(bx+a)}}{(f+if\sinh(ex+d))^2}dx$$

```
[In] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(i\,f\sinh{(ex+d)}+f)^2}\,dx$$

[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="fricas")

[Out] 1/3*(2*(3*e^2*e^(e*x + d) - (I*b^2*c^2*e^(2*e*x + 2*d) + 2*b^2*c^2*e^(e*x + d) - I*b^2*c^2)*log(F)^2 - I*e^2 - (I*b*c*e*e^(2*e*x + 2*d) + b*c*e*e^(e*x + d))*log(F))*F^(b*c*x + a*c) + 3*(e^3*f^2*e^(3*e*x + 3*d) - 3*I*e^3*f^2*e^(2*e*x + 2*d) - 3*e^3*f^2*e^(e*x + d) + I*e^3*f^2)*integral(-2/3*(-I*b^3*c^3*log(F)^3 + I*b*c*e^2*log(F))*F^(b*c*x + a*c)/(e^3*f^2*e^(e*x + d) - I*e^3*f^2), x))/(e^3*f^2*e^(3*e*x + 3*d) - 3*I*e^3*f^2*e^(2*e*x + 2*d) - 3*e^3*f^2*e^(e*x + d) + I*e^3*f^2)

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = -\frac{\int \frac{F^{ac+bcx}}{\sinh^2{(d+ex)}-2i\sinh{(d+ex)}-1}\,dx}{f^2}$$

[In] integrate(F**(c*(b*x+a))/(f+I*f*sinh(e*x+d))**2,x)

[Out] -Integral(F**(a*c + b*c*x)/(sinh(d + e*x)**2 - 2*I*sinh(d + e*x) - 1), x)/f**2

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(i\,f\sinh{(ex+d)}+f)^2}\,dx$$

[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="maxima")

[Out] -16*(-I*F^(a*c)*b^2*c^2*e*log(F)^2 - I*F^(a*c)*b*c*e^2*log(F))*integrate(F^(b*c*x)/(-I*b^3*c^3*f^2*log(F)^3 + 9*I*b^2*c^2*e*f^2*log(F)^2 - 26*I*b*c*e^2*f^2*log(F) + 24*I*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(5*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*e^(5*e*x) - 5*(I*b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*I*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*I*b*c*e^2*f^2*e^(4*d)*log(F) - 24*I*e^3*f^2*e^(4*d))*e^(4*e*x) - 10*(b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) - 10*(-I*b^3*c^3*f^2*e^(2*d)*log(F)^3 + 9*I*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 - 26*I*b*c*e^2*f

```
^2*e^(2*d)*log(F) + 24*I*e^3*f^2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2*e^d*lo
g(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*
f^2*e^d)*e^(e*x)), x) + 4*(4*F^(a*c)*b*c*e*log(F) + 4*F^(a*c)*e^2 - (F^(a*c
)*b^2*c^2*e^(2*d)*log(F)^2 - 7*F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F^(a*c)*e^
2*e^(2*d))*e^(2*e*x) + 4*(-I*F^(a*c)*b*c*e*e^d*log(F) + 4*I*F^(a*c)*e^2*e^d
)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*
b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2
*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d
))*e^(4*e*x) - 4*(I*b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*I*b^2*c^2*e*f^2*e^(3*d
)*log(F)^2 + 26*I*b*c*e^2*f^2*e^(3*d)*log(F) - 24*I*e^3*f^2*e^(3*d))*e^(3*e
*x) - 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 +
26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e^(2*d))*e^(2*e*x) - 4*(-I*b^3*c
^3*f^2*e^d*log(F)^3 + 9*I*b^2*c^2*e*f^2*e^d*log(F)^2 - 26*I*b*c*e^2*f^2*e^d
*log(F) + 24*I*e^3*f^2*e^d)*e^(e*x))
```

Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx=\int \frac{F^{(bx+a)c}}{\left(i\,f\sinh\left(ex+d\right)+f\right)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+I*f*sinh(e*x+d))^2,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(I*f*sinh(e*x + d) + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+if\sinh(d+ex))^2}\,dx=\int \frac{F^{c\,(a+b\,x)}}{\left(f+f\sinh\left(d+e\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i)^2,x)
[Out] int(F^(c*(a + b*x))/(f + f*sinh(d + e*x)*1i)^2, x)
```

3.897 $\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 251

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \frac{f^2F^{ac+bcx}}{bc\log(F)} - \frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)} + \frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} - \frac{bcf^2F^{ac+bcx}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2ef^2F^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)} + \frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)-2*b*c*f^2*F^(b*c*x+a*c)*cosh(e*x+d)*ln(F)/(e^2-
b^2*c^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-
b*c*f^2*F^(b*c*x+a*c)*cosh(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*f^2*F
^(b*c*x+a*c)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)+2*e*f^2*F^(b*c*x+a*c)*cosh(e
*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6873, 12, 6874, 2225, 5583, 5585}

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx=\frac{2ef^2\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf^2\log(F)\cosh^2(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}-\frac{2bcf^2\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{2ef^2\sinh(d+ex)\cosh(d+ex)F^{ac+bcx}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{f^2F^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*
x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[
F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]^2*L
og[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sinh[d + e*x])
/(e^2 - b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cosh[d + e*x]*Sinh[d +
 e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int f^2 F^{ac+bcx}(1+\cosh(d+ex))^2\,dx \\
&= f^2 \int F^{ac+bcx}(1+\cosh(d+ex))^2\,dx \\
&= f^2 \int \left(F^{ac+bcx}+2F^{ac+bcx}\cosh(d+ex)+F^{ac+bcx}\cosh^2(d+ex)\right)\,dx \\
&= f^2 \int F^{ac+bcx}\,dx + f^2 \int F^{ac+bcx}\cosh^2(d+ex)\,dx + \left(2f^2\right)\int F^{ac+bcx}\cosh(d+ex)\,dx \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)} - \frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)} \\
&\quad - \frac{bcf^2F^{ac+bcx}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2ef^2F^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)} \\
&\quad + \frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} + \frac{\left(2e^2f^2\right)\int F^{ac+bcx}\,dx}{4e^2-b^2c^2\log^2(F)} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)} - \frac{2bcf^2F^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)} \\
&\quad + \frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} - \frac{bcf^2F^{ac+bcx}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} \\
&\quad + \frac{2ef^2F^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)} + \frac{2ef^2F^{ac+bcx}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.92

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx$$
$$=\frac{f^2F^{c(a+bx)}\left(12e^4-15b^2c^2e^2\log^2(F)+3b^4c^4\log^4(F)+4\cosh(d+ex)\left(-4b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)+\right.}{}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(12*e^4 - 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4
 + 4*Cosh[d + e*x]*(-4*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + Cosh[2*(d
 + e*x)]*(-(b^2*c^2*e^2*Log[F]^2) + b^4*c^4*Log[F]^4) + 16*b*c*e^3*Log[F]*S
inh[d + e*x] - 4*b^3*c^3*e*Log[F]^3*Sinh[d + e*x] + 2*b*c*e^3*Log[F]*Sinh[2
*(d + e*x)] - 2*b^3*c^3*e*Log[F]^3*Sinh[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F]
 - 5*b^3*c^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{2F^{c(bx+a)}\left(\left(\frac{\left(\ln(F)^4b^4c^4-\ln(F)^2b^2c^2e^2\right)\cosh(2ex+2d)}{4}+\frac{\left(-\ln(F)^3b^3c^3e+\ln(F)bc\,e^3\right)\sinh(2ex+2d)}{2}+(bc\ln(F)+2e)\left(\cosh(ex+d)b^2c\right.\right.\right.}{c^5b^5\ln(F)^5-5c^3b^3\ln(F)^3e^2+4\ln(F)bc\,e^4}$
risch	$\frac{f^2\left(\ln(F)^4b^4c^4e^{4ex+4d}+4\ln(F)^4b^4c^4e^{3ex+3d}+6\ln(F)^4b^4c^4e^{2ex+2d}-2\ln(F)^3b^3c^3e\,e^{4ex+4d}+4\ln(F)^4b^4c^4e^{ex+d}-4\ln(F)^3b^3c\right.}{}$

```
[In] int(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*F^(c*(b*x+a))*(1/4*(ln(F)^4*b^4*c^4-ln(F)^2*b^2*c^2*e^2)*cosh(2*e*x+2*d)+
1/2*(-ln(F)^3*b^3*c^3*e+ln(F)*b*c*e^3)*sinh(2*e*x+2*d)+(b*c*ln(F)+2*e)*(cos
h(e*x+d)*b^2*c^2*ln(F)^2+3/4*b^2*c^2*ln(F)^2-sinh(e*x+d)*b*c*e*ln(F)-3/4*e^
2)*(b*c*ln(F)-2*e))*f^2/(c^5*b^5*ln(F)^5-5*c^3*b^3*ln(F)^3*e^2+4*ln(F)*b*c*
e^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2340 vs. $2(249) = 498$.

Time = 0.34 (sec) , antiderivative size = 2340, normalized size of antiderivative = 9.32

$$\int F^{c(a+bx)}(f + f\cosh(d + ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((24*e^4*f^2*cosh(e*x + d)^2 + (b^4*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^4
*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*cosh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh(e
*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^2*l
og(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*b*c*e^3*f^2*log(F))*sinh(e*x + d)^4
- 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*b^3*c^3*e*f^2*cosh(e*x + d)^3 - 2*b^
3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f^2)*log(F)^3 + 4*((b^4*c^4*f^2*cosh(
e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d) + b^3*c^3
*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(
F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2*b*c*e^3*f^2)*log(F))*sinh(e*x + d)^
3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 + 16*b^2*c^2*e^2*f^2*cosh(e*x + d)^3 +
 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 16*b^2*c^2*e^2*f^2*cosh(e*x + d) + b^
2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2 + (b^4*c^4*f^2*cosh(e*x + d)^2 + 2*b
^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - 2*(b^3*c^3*e*f^2*cosh(e*
x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*
x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x + d) + 5*b^2*c^2*e^2*f^2)*log(F)^2 +
2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*e^3*f^2*cosh(e*x + d))*log(F))*sinh(
e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^4 + 8*b*c*e^3*f^2*cosh(e*x + d)^3
 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^3*f^2)*log(F) + 4*(12*e^4*f^2*cosh(e
*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3 + 3*b^4*c^4*f^2*cosh(e*x + d)^2 + 3*
b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e
*x + d)^3 + 3*b^3*c^3*e*f^2*cosh(e*x + d)^2 - b^3*c^3*e*f^2)*log(F)^3 - (b^
2*c^2*e^2*f^2*cosh(e*x + d)^3 + 12*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 15*b^2
*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*c
osh(e*x + d)^3 + 6*b*c*e^3*f^2*cosh(e*x + d)^2 - 2*b*c*e^3*f^2)*log(F))*sin
h(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (24*e^4*f^2*cosh(e*x + d)^2 + (b^4
*c^4*f^2*cosh(e*x + d)^4 + 4*b^4*c^4*f^2*cosh(e*x + d)^3 + 6*b^4*c^4*f^2*co
sh(e*x + d)^2 + 4*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 + (b^4*
c^4*f^2*log(F)^4 - 2*b^3*c^3*e*f^2*log(F)^3 - b^2*c^2*e^2*f^2*log(F)^2 + 2*
b*c*e^3*f^2*log(F))*sinh(e*x + d)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^4 + 2*
b^3*c^3*e*f^2*cosh(e*x + d)^3 - 2*b^3*c^3*e*f^2*cosh(e*x + d) - b^3*c^3*e*f
^2)*log(F)^3 + 4*((b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)*log(F)^4 - (2*b
^3*c^3*e*f^2*cosh(e*x + d) + b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cos
h(e*x + d) + 4*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d) + 2
*b*c*e^3*f^2)*log(F))*sinh(e*x + d)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^4 +
16*b^2*c^2*e^2*f^2*cosh(e*x + d)^3 + 30*b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 1
```

```
6*b^2*c^2*e^2*f^2*cosh(e*x + d) + b^2*c^2*e^2*f^2)*log(F)^2 + 6*(4*e^4*f^2
+ (b^4*c^4*f^2*cosh(e*x + d)^2 + 2*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2)
*log(F)^4 - 2*(b^3*c^3*e*f^2*cosh(e*x + d)^2 + b^3*c^3*e*f^2*cosh(e*x + d))
*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*cosh(e*x +
 d) + 5*b^2*c^2*e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^2 + 4*b*c*
e^3*f^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^2 + 2*(b*c*e^3*f^2*cosh(e*x +
d)^4 + 8*b*c*e^3*f^2*cosh(e*x + d)^3 - 8*b*c*e^3*f^2*cosh(e*x + d) - b*c*e^
3*f^2)*log(F) + 4*(12*e^4*f^2*cosh(e*x + d) + (b^4*c^4*f^2*cosh(e*x + d)^3
+ 3*b^4*c^4*f^2*cosh(e*x + d)^2 + 3*b^4*c^4*f^2*cosh(e*x + d) + b^4*c^4*f^2
)*log(F)^4 - (2*b^3*c^3*e*f^2*cosh(e*x + d)^3 + 3*b^3*c^3*e*f^2*cosh(e*x +
d)^2 - b^3*c^3*e*f^2)*log(F)^3 - (b^2*c^2*e^2*f^2*cosh(e*x + d)^3 + 12*b^2*
c^2*e^2*f^2*cosh(e*x + d)^2 + 15*b^2*c^2*e^2*f^2*cosh(e*x + d) + 4*b^2*c^2*
e^2*f^2)*log(F)^2 + 2*(b*c*e^3*f^2*cosh(e*x + d)^3 + 6*b*c*e^3*f^2*cosh(e*x
 + d)^2 - 2*b*c*e^3*f^2)*log(F))*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))
/(b^5*c^5*cosh(e*x + d)^2*log(F)^5 - 5*b^3*c^3*e^2*cosh(e*x + d)^2*log(F)^3
 + 4*b*c*e^4*cosh(e*x + d)^2*log(F) + (b^5*c^5*log(F)^5 - 5*b^3*c^3*e^2*log
(F)^3 + 4*b*c*e^4*log(F))*sinh(e*x + d)^2 + 2*(b^5*c^5*cosh(e*x + d)*log(F)
^5 - 5*b^3*c^3*e^2*cosh(e*x + d)*log(F)^3 + 4*b*c*e^4*cosh(e*x + d)*log(F))
*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2346 vs. $2(238) = 476$.

Time = 2.14 (sec) , antiderivative size = 2346, normalized size of antiderivative = 9.35

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d))**2,x)
```

```
[Out] Piecewise((x*(f*cosh(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0))
, (-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2*s
inh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(F, 1)), (F**(
a*c)*(-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**
2*sinh(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e), Eq(b, 0)), (
-f**2*x*sinh(d + e*x)**2/2 + f**2*x*cosh(d + e*x)**2/2 + f**2*x + f**2*sinh
(d + e*x)*cosh(d + e*x)/(2*e) + 2*f**2*sinh(d + e*x)/e, Eq(c, 0)), (-F**(a*
c + b*c*x)*f**2*x*sinh(b*c*x*log(F) - d) + F**(a*c + b*c*x)*f**2*x*cosh(b*c
*x*log(F) - d) - 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)**2/(3*b*c*l
og(F)) + 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) -
 d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) - d)/(b*c*log(
F)) + F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) - d)**2/(3*b*c*log(F)) + F**(
a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*log(F))), (F**(a*c + b*c*x)*f**2
*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F
)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*f**2*x*cosh(b*c*x*lo
```

```
g(F)/2 - d)**2/4 - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*
x*log(F)/2 - d)/(2*b*c*log(F)) - 4*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/
2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F)/2 - d)**2/(
b*c*log(F)) + 8*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F)/2 - d)/(3*b*c*log(F
)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c +
b*c*x)*f**2*x*sinh(b*c*x*log(F)/2 + d)**2/4 - F**(a*c + b*c*x)*f**2*x*sinh(
b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/2 + F**(a*c + b*c*x)*f**2*x*co
sh(b*c*x*log(F)/2 + d)**2/4 - F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 + d
)**2/(b*c*log(F)) + 3*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F)/2 + d)*cosh(b
*c*x*log(F)/2 + d)/(2*b*c*log(F)) - 4*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(
F)/2 + d)/(3*b*c*log(F)) + 8*F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F)/2 + d)
/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, b*c*log(F)/2)),
 (-F**(a*c + b*c*x)*f**2*x*sinh(b*c*x*log(F) + d) + F**(a*c + b*c*x)*f**2*x
*cosh(b*c*x*log(F) + d) - 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) + d)**2
/(3*b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) + d)*cosh(b*c*x
*log(F) + d)/(3*b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*sinh(b*c*x*log(F) + d
)/(b*c*log(F)) + F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) + d)**2/(3*b*c*log
(F)) - F**(a*c + b*c*x)*f**2*cosh(b*c*x*log(F) + d)/(b*c*log(F)) + F**(a*c
+ b*c*x)*f**2/(b*c*log(F)), Eq(e, b*c*log(F))), (F**(a*c + b*c*x)*b**4*c**4
*f**2*log(F)**4*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*lo
g(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4*
cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**
4*log(F)) + F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4/(b**5*c**5*log(F)**5
- 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c + b*c*x)*b**3
*c**3*e*f**2*log(F)**3*sinh(d + e*x)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5
*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a*c + b*c*x)*b**3*c*
*3*e*f**2*log(F)**3*sinh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*l
og(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(
F)**2*sinh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 +
4*b*c*e**4*log(F)) - 3*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*cosh(
d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) - 8*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*cosh(d + e*x)/(b
**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 5*F*
*(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 - 5*b**3*
c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b*c*e**3*f**2
*log(F)*sinh(d + e*x)*cosh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2
*log(F)**3 + 4*b*c*e**4*log(F)) + 8*F**(a*c + b*c*x)*b*c*e**3*f**2*log(F)*s
inh(d + e*x)/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4
*log(F)) - 2*F**(a*c + b*c*x)*e**4*f**2*sinh(d + e*x)**2/(b**5*c**5*log(F)*
*5 - 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*e
**4*f**2*cosh(d + e*x)**2/(b**5*c**5*log(F)**5 - 5*b**3*c**3*e**2*log(F)**3
 + 4*b*c*e**4*log(F)) + 4*F**(a*c + b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 -
 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.75

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx$$
$$=\frac{1}{4}f^2\left(\frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{bc\log(F)+2\,e}+\frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}}+\frac{2\,F^{bcx+ac}}{bc\log(F)}\right)$$
$$+f^2\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log(F)+e}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log(F)-ee^{d}}\right)+\frac{F^{bcx+ac}f^2}{bc\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*f^2*(F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 2*F^(b*c*x
+ a*c)/(b*c*log(F))) + f^2*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F)
+ e) + F^(a*c)*e^(b*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x
+ a*c)*f^2/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1548, normalized size of antiderivative = 6.17

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx=\text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
 - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*f^2*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2
*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I*pi*b*c
*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b
*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) + 1/2*(2*(b*c*log(abs(F)) + 2*e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
 4*(b*c*log(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*
c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F)
 - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(
```

```
abs(F)) + 2*e)*x + 2*d) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*
x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8
*b*c*log(abs(F)) + 16*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
 - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8
*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x +
2*d) + 2*(2*(b*c*log(abs(F)) + e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c
*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*l
og(abs(F)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F)
+ 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^
2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*
x + d) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*
e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
 + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e
^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 2*(2*(b*c*log(abs(F)) -
e)*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi
*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sg
n(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn
(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))
*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + I*(I*f^2*e^(1/2*I*pi*b
*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*
c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*e) - I*f^2*e^(-1/2*I*pi*b*c*x*s
gn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn
(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(ab
s(F)) - e)*x - d) + 1/2*(2*(b*c*log(abs(F)) - 2*e)*f^2*cos(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*
b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1
/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*
c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2))*e^(a*c*log(abs(F)) + (
b*c*log(abs(F)) - 2*e)*x - 2*d) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I
*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi
*b*c + 8*b*c*log(abs(F)) - 16*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*
pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi
*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2
*e)*x - 2*d)
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))^2\,dx = \frac{2\,F^{b\,c\,x}\,F^{a\,c}\,e\,f^2\sinh(d+e\,x)}{e^2-b^2\,c^2\ln\left(F\right)^2}+\frac{F^{b\,c\,x}\,F^{a\,c}\,f^2}{b\,c\,\ln\left(F\right)}+\frac{2\,F^{b\,c\,x}\,F^{a\,c}\,e\,f^2\cosh(d+e\,x)\,\sinh(d+e\,x)}{4\,e^2-b^2\,c^2\ln\left(F\right)^2}-\frac{2\,F^{b\,c\,x}\,F^{a\,c}\,b\,c\,f^2\cosh(d+e\,x)\,\ln\left(F\right)}{e^2-b^2\,c^2\ln\left(F\right)^2}-\frac{2\,F^{b\,c\,x}\,F^{a\,c}\,e^2\,f^2\sinh(d+e\,x)^2}{b\,c\,\ln\left(F\right)\,\left(4\,e^2-b^2\,c^2\ln\left(F\right)^2\right)}+\frac{F^{b\,c\,x}\,F^{a\,c}\,f^2\cosh(d+e\,x)^2\,\left(2\,e^2-b^2\,c^2\ln\left(F\right)^2\right)}{b\,c\,\ln\left(F\right)\,\left(4\,e^2-b^2\,c^2\ln\left(F\right)^2\right)}$$

```
[In] int(F^(c*(a + b*x))*(f + f*cosh(d + e*x))^2,x)
```

```
[Out] (2*F^(b*c*x)*F^(a*c)*e*f^2*sinh(d + e*x))/(e^2 - b^2*c^2*log(F)^2) + (F^(b*
c*x)*F^(a*c)*f^2)/(b*c*log(F)) + (2*F^(b*c*x)*F^(a*c)*e*f^2*cosh(d + e*x)*s
inh(d + e*x))/(4*e^2 - b^2*c^2*log(F)^2) - (2*F^(b*c*x)*F^(a*c)*b*c*f^2*cos
h(d + e*x)*log(F))/(e^2 - b^2*c^2*log(F)^2) - (2*F^(b*c*x)*F^(a*c)*e^2*f^2*
sinh(d + e*x)^2)/(b*c*log(F)*(4*e^2 - b^2*c^2*log(F)^2)) + (F^(b*c*x)*F^(a*
c)*f^2*cosh(d + e*x)^2*(2*e^2 - b^2*c^2*log(F)^2))/(b*c*log(F)*(4*e^2 - b^2
*c^2*log(F)^2))
```

3.898 $\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}-\frac{bcfF^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)-b*c*f*F^(b*c*x+a*c)*cosh(e*x+d)*ln(F)/(e^2-b^2*c^2*ln(F)^2)+e*f*F^(b*c*x+a*c)*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6873, 12, 6874, 2225, 5583}

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx=\frac{ef\sinh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}-\frac{bcf\log(F)\cosh(d+ex)F^{ac+bcx}}{e^2-b^2c^2\log^2(F)}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (b*c*f*F^(a*c + b*c*x)*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int fF^{ac+bcx}(1+\cosh(d+ex))\,dx \\
&= f\int F^{ac+bcx}(1+\cosh(d+ex))\,dx \\
&= f\int \left(F^{ac+bcx}+F^{ac+bcx}\cosh(d+ex)\right)\,dx \\
&= f\int F^{ac+bcx}\,dx + f\int F^{ac+bcx}\cosh(d+ex)\,dx \\
&= \frac{fF^{ac+bcx}}{bc\log(F)} - \frac{bcfF^{ac+bcx}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)} + \frac{efF^{ac+bcx}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx = \frac{fF^{c(a+bx)}\left(-e^2+b^2c^2\log^2(F)+b^2c^2\cosh(d+ex)\log^2(F)-bce\log(F)\sinh(d+ex)\right)}{bc\log(F)(-e+bc\log(F))(e+bc\log(F))}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cosh[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(-e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[d + e*x]*Log[F]^
2 - b*c*e*Log[F]*Sinh[d + e*x]))/(b*c*Log[F]*(-e + b*c*Log[F])*(e + b*c*Log
[F]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{f\,F^{c(bx+a)}\left(\cosh(ex+d)b^2c^2\ln(F)^2+b^2c^2\ln(F)^2-\sinh(ex+d)bce\ln(F)-e^2\right)}{\left(b^2c^2\ln(F)^2-e^2\right)bc\ln(F)}$	88
risch	$\frac{f\left(\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}+2\ln(F)^2b^2c^2\mathrm{e}^{ex+d}+b^2c^2\ln(F)^2-\ln(F)bce\,\mathrm{e}^{2ex+2d}+\ln(F)bce-2e^2\mathrm{e}^{ex+d}\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2bc\ln(F)(bc\ln(F)-e)(e+bc\ln(F))}$	135

```
[In] int(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] f*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)/b/c/ln(F)*(cosh(e*x+d)*b^2*c^2*ln(F)^
2+b^2*c^2*ln(F)^2-sinh(e*x+d)*b*c*e*ln(F)-e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(103) = 206$.

Time = 0.26 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.26

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx = -\frac{\left(2\,e^2f\cosh\left(ex+d\right)-\left(b^2c^2f\cosh\left(ex+d\right)^2+2\,b^2c^2f\cosh\left(ex+d\right)+b^2c^2f\right)\log\left(F\right)^2-\left(b^2c^2f\log\left(F\right)^2\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/2*((2*e^2*f*cosh(e*x + d) - (b^2*c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cos
h(e*x + d) + b^2*c^2*f)*log(F)^2 - (b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*si
```

```
nh(e*x + d)^2 + (b*c*e*f*cosh(e*x + d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cos
h(e*x + d)*log(F) + e^2*f - (b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)
*sinh(e*x + d))*cosh((b*c*x + a*c)*log(F)) + (2*e^2*f*cosh(e*x + d) - (b^2*
c^2*f*cosh(e*x + d)^2 + 2*b^2*c^2*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2 - (
b^2*c^2*f*log(F)^2 - b*c*e*f*log(F))*sinh(e*x + d)^2 + (b*c*e*f*cosh(e*x +
d)^2 - b*c*e*f)*log(F) + 2*(b*c*e*f*cosh(e*x + d)*log(F) + e^2*f - (b^2*c^2
*f*cosh(e*x + d) + b^2*c^2*f)*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*l
og(F)))/(b^3*c^3*cosh(e*x + d)*log(F)^3 - b*c*e^2*cosh(e*x + d)*log(F) + (b
^3*c^3*log(F)^3 - b*c*e^2*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(94) = 188$.

Time = 0.65 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.83

$$\int F^{c(a+bx)}(f + f\cosh(d+ex))\,dx$$

$$= \begin{cases} x(f\cosh(d) + f) \\ fx + \frac{f\sinh(d+ex)}{e} \\ F^{ac}\left(fx + \frac{f\sinh(d+ex)}{e}\right) \\ fx + \frac{f\sinh(d+ex)}{e} \\ -\frac{F^{ac+bcx}fx\sinh(bcx\log(F)-d)}{2} + \frac{F^{ac+bcx}fx\cosh(bcx\log(F)-d)}{2} + \frac{F^{ac+bcx}f\sinh(bcx\log(F)-d)}{2bc\log(F)} + \frac{F^{ac+bcx}f}{bc\log(F)} \\ -\frac{F^{ac+bcx}fx\sinh(bcx\log(F)+d)}{2} + \frac{F^{ac+bcx}fx\cosh(bcx\log(F)+d)}{2} + \frac{F^{ac+bcx}f\sinh(bcx\log(F)+d)}{bc\log(F)} - \frac{F^{ac+bcx}f\cosh(bcx\log(F)+d)}{2bc\log(F)} \\ \frac{F^{ac+bcx}b^2c^2f\log(F)^2\cosh(d+ex)}{b^3c^3\log(F)^3-bce^2\log(F)} + \frac{F^{ac+bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3-bce^2\log(F)} - \frac{F^{ac+bcx}bcef\log(F)\sinh(d+ex)}{b^3c^3\log(F)^3-bce^2\log(F)} - \frac{F^{ac+bcx}e^2f}{b^3c^3\log(F)^3-bce^2\log(F)} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*(f+f*cosh(e*x+d)),x)
```

```
[Out] Piecewise((x*(f*cosh(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (
f*x + f*sinh(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x + f*sinh(d + e*x)/e), Eq
(b, 0)), (f*x + f*sinh(d + e*x)/e, Eq(c, 0)), (-F**(a*c + b*c*x)*f*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) - d)/2 + F**(a*
c + b*c*x)*f*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(b*
c*log(F)), Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*f*x*sinh(b*c*x*log(F) +
d)/2 + F**(a*c + b*c*x)*f*x*cosh(b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*f*s
inh(b*c*x*log(F) + d)/(b*c*log(F)) - F**(a*c + b*c*x)*f*cosh(b*c*x*log(F) +
 d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)), Eq(e, b*c*log(F))), (
F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2*cosh(d + e*x)/(b**3*c**3*log(F)**3 -
 b*c*e**2*log(F)) + F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2/(b**3*c**3*log(F
)**3 - b*c*e**2*log(F)) - F**(a*c + b*c*x)*b*c*e*f*log(F)*sinh(d + e*x)/(b*
*3*c**3*log(F)**3 - b*c*e**2*log(F)) - F**(a*c + b*c*x)*e**2*f/(b**3*c**3*l
og(F)**3 - b*c*e**2*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx = \frac{1}{2}\,f\left(\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{bc\log\left(F\right)+e}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{bce^{d}\log\left(F\right)-ee^{d}}\right)+\frac{F^{bcx+ac}f}{bc\log\left(F\right)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + F^(a*c)*e^(b*c
*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)) + F^(b*c*x + a*c)*f/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 886, normalized size of antiderivative = 8.77

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cosh(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/2*I*pi*b*c
*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*
sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/
2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi
*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*(b*
c*log(abs(F)) + e)*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + I*(I*f*e^(
1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c
)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e) - I*f*e^(-1/2*
I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-
2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*e))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*f*cos(-1/2*pi*
```

```
b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(
F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*si
n(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((p
i*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) +
 (b*c*log(abs(F)) - e)*x - d) + I*(I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*
b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
 + 4*b*c*log(abs(F)) - 4*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*
x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c +
4*b*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int F^{c(a+bx)}(f+f\cosh(d+ex))\,dx = -\frac{F^{b\,c\,x}\,F^{a\,c}\,f\,\mathrm{e}^{-d-e\,x}\,\left(b^2\,c^2\ln\left(F\right)^2-2\,e^2\,\mathrm{e}^{d+e\,x}+b\,c\,e\,\ln\left(F\right)+2\,b^2\,c^2\,\mathrm{e}^{d+e\,x}\ln\left(F\right)^2+b^2\,c^2\,\mathrm{e}^{2\,d+2\,e\,x}\ln\left(F\right)^2}{2\,b\,c\,\ln\left(F\right)\,\left(e^2-b^2\,c^2\ln\left(F\right)^2\right)}$$

```
[In] int(F^(c*(a + b*x))*(f + f*cosh(d + e*x)),x)
```

```
[Out] -(F^(b*c*x)*F^(a*c)*f*exp(- d - e*x)*(b^2*c^2*log(F)^2 - 2*e^2*exp(d + e*x)
 + b*c*e*log(F) + 2*b^2*c^2*exp(d + e*x)*log(F)^2 + b^2*c^2*exp(2*d + 2*e*x
)*log(F)^2 - b*c*e*exp(2*d + 2*e*x)*log(F)))/(2*b*c*log(F)*(e^2 - b^2*c^2*l
og(F)^2))
```

3.899 $\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)}{f(e+bc\log(F))}$$

```
[Out] 2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(F)/e],-ex
p(e*x+d))/f/(e+b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5605, 5600}

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{d+ex}\right)}{f(bc\log(F)+e)}$$

```
[In] Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(f*(e + b*c*Log[F]))
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5605

```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Dist[2^n*g^n, Int[F^(c*(a + b*x))*Cosh[d/2 + e*(x/2)
]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && I
LtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{2f}$$

$$= \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)}{f(e+bc\log(F))}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)}{ef+bcf\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x]),x]
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)])/(e*f + b*c*f*Log[F])
```

Maple [F]

$$\int \frac{F^{c(bx+a)}}{f+f\cosh(ex+d)}dx$$

```
[In] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
[Out] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cosh\left(ex+d\right)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)/(f*cosh(e*x + d) + f), x)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \frac{\int \frac{F^{ac+bcx}}{\cosh\left(d+ex\right)+1}\,dx}{f}$$

```
[In] integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d)),x)
[Out] Integral(F**(a*c + b*c*x)/(cosh(d + e*x) + 1), x)/f
```

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cosh\left(ex+d\right)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="maxima")
[Out] 4*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*b*c*e*f*log(F)
+ 2*e^2*f + (b^2*c^2*f*e^(3*d)*log(F)^2 - 3*b*c*e*f*e^(3*d)*log(F) + 2*e^2*
f*e^(3*d))*e^(3*e*x) + 3*(b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)*lo
g(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 3*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f*e
^d*log(F) + 2*e^2*f*e^d)*e^(e*x)), x)*log(F) - 2*(2*F^(a*c)*e - (F^(a*c)*b*
c*e^d*log(F) - 2*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*f*log(F)^2 - 3*
b*c*e*f*log(F) + 2*e^2*f + (b^2*c^2*f*e^(2*d)*log(F)^2 - 3*b*c*e*f*e^(2*d)*
log(F) + 2*e^2*f*e^(2*d))*e^(2*e*x) + 2*(b^2*c^2*f*e^d*log(F)^2 - 3*b*c*e*f
*e^d*log(F) + 2*e^2*f*e^d)*e^(e*x))
```

Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cosh{(ex+d)}+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d)),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+f\cosh(d+ex)}\,dx = \int \frac{F^{c\,(a+b\,x)}}{f+f\cosh{(d+e\,x)}}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x)),x)
[Out] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x)), x)
```

3.900 $\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)(e-bc\log(F))}{3e^2f^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}$$

```
[Out] 2/3*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([2, 1+b*c*ln(F)/e],[2+b*c*ln(F)/e],-
exp(e*x+d))*(e-b*c*ln(F))/e^2/f^2+1/6*b*c*F^(c*(b*x+a))*ln(F)*sech(1/2*e*x+
1/2*d)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sech(1/2*e*x+1/2*d)^2*tanh(1/2*e*x+1/2*d
)/e/f^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5605, 5598, 5600}

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \frac{2e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+1,\frac{bc\log(F)}{e}+2,-e^{d+ex}\right)}{3e^2f^2} + \frac{bc\log(F)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2} + \frac{\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\text{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6ef^2}$$

```
[In] Int[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 +
 (b*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]))/(3*e^2*f^2) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sech[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a + b*x))*Sech[
d/2 + (e*x)/2]^2*Tanh[d/2 + (e*x)/2])/(6*e*f^2)
```

Rule 5598

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 5605

```
Int[(Cosh[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.
)*(x_))), x_Symbol] :> Dist[2^n*g^n, Int[F^(c*(a + b*x))*Cosh[d/2 + e*(x/2)
]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && I
LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int F^{c(a+bx)}\operatorname{sech}^4\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{4f^2}\\
&= \frac{bcF^{c(a+bx)}\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2}+\frac{F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}\\
&\quad+\frac{\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{6f^2}\\
&= \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{e},2+\frac{bc\log(F)}{e},-e^{d+ex}\right)(e-bc\log(F))}{3e^2f^2}\\
&\quad+\frac{bcF^{c(a+bx)}\log(F)\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2}+\frac{F^{c(a+bx)}\operatorname{sech}^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx$$
$$= \frac{2F^{c(a+bx)}\cosh\left(\frac{1}{2}(d+ex)\right)\left(bc\cosh\left(\frac{1}{2}(d+ex)\right)\log(F)+4e^{d+ex}\cosh^3\left(\frac{1}{2}(d+ex)\right)\text{Hypergeometric2F1}\left(2,\right.\right.}{3e^2f^2(1+\cosh(d+ex))^2}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cosh[d + e*x])^2,x]
[Out] (2*F^(c*(a + b*x))*Cosh[(d + e*x)/2]*(b*c*Cosh[(d + e*x)/2]*Log[F] + 4*E^(d
 + e*x)*Cosh[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 + (b*c*Log[F])/e, 2 + (b
*c*Log[F])/e, -E^(d + e*x)]*(e - b*c*Log[F]) + e*Sinh[(d + e*x)/2]))/(3*e^2
*f^2*(1 + Cosh[d + e*x])^2)
```

Maple [F]

$$\int \frac{F^{c(bx+a)}}{(f+f\cosh(ex+d))^2}dx$$

```
[In] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
[Out] int(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cosh(ex+d)+f)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)/(f^2*cosh(e*x + d)^2 + 2*f^2*cosh(e*x + d) + f^2),
 x)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx=\frac{\int \frac{F^{ac+bcx}}{\cosh^2(d+ex)+2\cosh(d+ex)+1}\,dx}{f^2}$$

[In] integrate(F**(c*(b*x+a))/(f+f*cosh(e*x+d))**2,x)

[Out] Integral(F**(a*c + b*c*x)/(cosh(d + e*x)**2 + 2*cosh(d + e*x) + 1), x)/f**2

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx=\int \frac{F^{(bx+a)c}}{(f\cosh(ex+d)+f)^2}\,dx$$

[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="maxima")

[Out] -16*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*b*c*e^2*log(F))*integrate(F^(b*c*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(5*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(5*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(5*d)*log(F) - 24*e^3*f^2*e^(5*d))*e^(5*e*x) + 5*(b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 10*(b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 10*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e^(2*d))*e^(2*e*x) + 5*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x)), x) + 4*(4*F^(a*c)*b*c*e*log(F) + 4*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 7*F^(a*c)*b*c*e*e^(2*d)*log(F) + 12*F^(a*c)*e^2*e^(2*d))*e^(2*e*x) - 4*(F^(a*c)*b*c*e*e^d*log(F) - 4*F^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*f^2*log(F)^3 - 9*b^2*c^2*e*f^2*log(F)^2 + 26*b*c*e^2*f^2*log(F) - 24*e^3*f^2 + (b^3*c^3*f^2*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(4*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(4*d)*log(F) - 24*e^3*f^2*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*f^2*e^(3*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(3*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(3*d)*log(F) - 24*e^3*f^2*e^(3*d))*e^(3*e*x) + 6*(b^3*c^3*f^2*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*f^2*e^(2*d)*log(F)^2 + 26*b*c*e^2*f^2*e^(2*d)*log(F) - 24*e^3*f^2*e^(2*d))*e^(2*e*x) + 4*(b^3*c^3*f^2*e^d*log(F)^3 - 9*b^2*c^2*e*f^2*e^d*log(F)^2 + 26*b*c*e^2*f^2*e^d*log(F) - 24*e^3*f^2*e^d)*e^(e*x))

Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cosh(ex+d)+f)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cosh(e*x+d))^2,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*cosh(e*x + d) + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+f\cosh(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{(f+f\cosh(d+e\,x))^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x))^2,x)
[Out] int(F^(c*(a + b*x))/(f + f*cosh(d + e*x))^2, x)
```

3.901 $\int e^{a+bx} \cosh(a+bx) \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{a+bx} \cosh(a+bx) \sinh^3(a+bx)\,dx = \frac{e^{-3a-3bx}}{48b} - \frac{e^{-a-bx}}{8b} - \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{80b}$$

```
[Out] 1/48*exp(-3*b*x-3*a)/b-1/8*exp(-b*x-a)/b-1/24*exp(3*b*x+3*a)/b+1/80*exp(5*b
*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2320, 12, 459}

$$\int e^{a+bx} \cosh(a+bx) \sinh^3(a+bx)\,dx = \frac{e^{-3a-3bx}}{48b} - \frac{e^{-a-bx}}{8b} - \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{80b}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{16x^4}\,dx, x, e^{a+bx}\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{16b} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{2}{x^2}-2x^2+x^4\right)\,dx, x, e^{a+bx}\right)}{16b} \\ &= \frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(5-30e^{2(a+bx)}-10e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
[Out] (5 - 30*E^(2*(a + b*x)) - 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^
(3*(a + b*x)))
```

Maple [A] (verified)

Time = 284.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}}{b}$	44
default	$\frac{\frac{\sinh(bx+a)^5}{5}+\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}}{b}$	44
risch	$\frac{\mathrm{e}^{-3bx-3a}}{48b}-\frac{\mathrm{e}^{-bx-a}}{8b}-\frac{\mathrm{e}^{3bx+3a}}{24b}+\frac{\mathrm{e}^{5bx+5a}}{80b}$	58

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*sinh(b*x+a)^5+1/5*cosh(b*x+a)^3*sinh(b*x+a)^2-2/15*cosh(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^4-\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2-5\right)\sinh(bx+a)^2-5}{30\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + (
6*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^2 - 5*cosh(b*x + a)^2 - (cosh(b*x + a)
^3 - 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(53) = 106.

Time = 2.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.01

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{e^ae^{bx}\sinh^4(a+bx)}{5b}-\frac{e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{5b}+\frac{e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}+\frac{2e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{15b}-\frac{2e^ae^{bx}}{} \\ xe^a\sinh^3(a)\cosh(a)\end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) - exp(a)*exp(b*x)*sinh(a
+ b*x)**3*cosh(a + b*x)/(5*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b
*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b) - 2*
exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3*co
sh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}}{48\,b} + \frac{3\,e^{(5\,bx+5\,a)}-10\,e^{(3\,bx+3\,a)}}{240\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
 - 10*e^(3*b*x + 3*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx = -\frac{5\left(6\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}-3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) + 10
*e^(3*b*x + 3*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh(a+bx)\sinh^3(a+bx)\,dx=-\frac{30\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}+10\,\mathrm{e}^{3\,a+3\,b\,x}-3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

```
[In] int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x)^3,x)
[Out] -(30*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 10*exp(3*a + 3*b*x) - 3*exp(5*
a + 5*b*x))/(240*b)
```

3.902 $\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

```
[Out] -1/16*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 12, 457, 76}

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/16*E^(-2*a - 2*b*x)/b - E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b)
- x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
```

```
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2(1+x)}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}-\frac{1}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= -\frac{e^{-2a-2bx}}{16b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{2e^{-2(a+bx)}+2e^{2(a+bx)}-e^{4(a+bx)}+4bx}{32b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
[Out] -1/32*(2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) - E^(4*(a + b*x)) + 4*b*x)/b
```

Maple [A] (verified)

Time = 19.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{{\mathrm e}^{-2 b x -2 a}}{16 b}-\frac{{\mathrm e}^{2 b x +2 a}}{16 b}+\frac{{\mathrm e}^{4 b x +4 a}}{32 b}-\frac{x}{8}$	47
derivativedivides	$\frac{\frac{\sinh \left(b x +a \right)^{4}}{4}+\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	53
default	$\frac{\frac{\sinh \left(b x +a \right)^{4}}{4}+\frac{\cosh \left(b x +a \right)^{3} \sinh \left(b x +a \right)}{4}-\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$	53

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/16*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-3\,\sinh\left(bx+a\right)^{3}+2\,\left(2\,bx+1\right)\cosh\left(bx+a\right)-\left(4\,bx+\right.}{32\,\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)^
3 + 2*(2*b*x + 1)*cosh(b*x + a) - (4*b*x + 9*cosh(b*x + a)^2 - 2)*sinh(b*x
+ a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(44) = 88.

Time = 0.93 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.11

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{xe^{a}e^{bx}\sinh^{3}\left(a+bx\right)}{8}+\frac{xe^{a}e^{bx}\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{8}+\frac{xe^{a}e^{bx}\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{8}-\frac{xe^{a}e^{bx}\cosh^{3}\left(a+bx\right)}{8}+\frac{3e^{a}e^{bx}\sinh^{3}\left(a-\right.}{8b} \\ xe^{a}\sinh^{2}\left(a\right)\cosh\left(a\right) \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(a
 + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)*
*2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 + 3*exp(a)*exp(b*x)*sinh(a + b*
x)**3/(8*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(a)
*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh(a),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x - \frac{a}{8\,b} + \frac{e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b} - \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a))/b - 1/16*e^(-
2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{4\,bx - 2\,\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*b*x - 2*(e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
4*a) + 2*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{8} - \frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{16} - \frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

```
[In] int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] - x/8 - (exp(- 2*a - 2*b*x)/16 + exp(2*a + 2*b*x)/16 - exp(4*a + 4*b*x)/32)
/b
```

3.903 $\int e^{a+bx} \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int e^{a+bx} \cosh(a+bx) \sinh(a+bx) \, dx = \frac{e^{-a-bx}}{4b} + \frac{e^{3a+3bx}}{12b}$$

```
[Out] 1/4*exp(-b*x-a)/b+1/12*exp(3*b*x+3*a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2320, 12, 14}

$$\int e^{a+bx} \cosh(a+bx) \sinh(a+bx) \, dx = \frac{e^{-a-bx}}{4b} + \frac{e^{3a+3bx}}{12b}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(4*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^4}{4x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx, x, e^{a+bx}\right)}{4b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)\,dx, x, e^{a+bx}\right)}{4b} \\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{3a+3bx}}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{e^{-a-bx}\left(3+e^{4(a+bx)}\right)}{12b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] (E^(-a - b*x)*(3 + E^(4*(a + b*x))))/(12*b)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{3}+\frac{\cosh(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\sinh(bx+a)^3}{3}+\frac{\cosh(bx+a)^3}{3}}{b}$	26
risch	$\frac{\mathrm{e}^{-bx-a}}{4b}+\frac{\mathrm{e}^{3bx+3a}}{12b}$	30

```
[In] int(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*sinh(b*x+a)^3+1/3*cosh(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^2-\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2}{3\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/3*(cosh(b*x + a)^2 - cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*co
sh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{e^a e^{bx}\sinh^2(a+bx)}{3b} - \frac{e^a e^{bx}\sinh(a+bx)\cosh(a+bx)}{3b} + \frac{e^a e^{bx}\cosh^2(a+bx)}{3b} & \text{for } b\neq 0 \\ xe^a\sinh(a)\cosh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a
+ b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne(b,
0)), (x*exp(a)*sinh(a)*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{e^{(-bx-a)}}{4\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/12*e^(3*b*x + 3*a)/b + 1/4*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}+3\,e^{(-bx-a)}}{12\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/12*(e^(3*b*x + 3*a) + 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{3\,\mathrm{e}^{-a-b\,x}+\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

```
[In] int(cosh(a + b*x)*exp(a + b*x)*sinh(a + b*x),x)
[Out] (3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

3.904 $\int e^{a+bx} \coth(a+bx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int e^{a+bx} \coth(a+bx) \, dx = \frac{e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2320, 396, 212}

$$\int e^{a+bx} \coth(a+bx) \, dx = \frac{e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{e^{a+bx}}{b} - \frac{2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\coth(a+bx)\,dx = \frac{e^{a+bx}-2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x],x]
[Out] (E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\sinh(bx+a)+\cosh(bx+a)-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	27
default	$\frac{\sinh(bx+a)+\cosh(bx+a)-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	27
risch	$\frac{\mathrm{e}^{bx+a}}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	39

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int e^{a+bx} \coth(a+bx)\,dx = \frac{\cosh(bx+a) - \log(\cosh(bx+a) + \sinh(bx+a) + 1) + \log(\cosh(bx+a) + \sinh(bx+a) - 1) + \sinh(bx+a)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] (cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + sinh(b*x + a))/b
```

Sympy [F]

$$\int e^{a+bx} \coth(a+bx)\,dx = e^{a} \int e^{bx} \cosh(a+bx) \operatorname{csch}(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx} \coth(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(e^{(bx+a)} - 1\right)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{e^{(bx+a)}-\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] (e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{e^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x),x)
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.905 $\int e^{a+bx} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx=\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 2/b/(1-exp(2*b*x+2*a))+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2320, 12, 455, 45}

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx=\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] 2/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{2x(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \frac{x(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{-\frac{2}{-1+e^{2(a+bx)}}+\log\left(1-e^{2(a+bx)}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x],x]
[Out] (-2/(-1 + E^(2*(a + b*x))) + Log[1 - E^(2*(a + b*x))])/b
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{\ln(\sinh(bx+a))+bx+a-\coth(bx+a)}{b}$	25
default	$\frac{\ln(\sinh(bx+a))+bx+a-\coth(bx+a)}{b}$	25
risch	$-\frac{2a}{b}-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	42

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(sinh(b*x+a))+b*x+a-coth(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-2}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] ((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*lo
g(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - 2)/(b*cosh(b*x + a)^2
+ 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1
))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - log(abs(e^(2*b*x + 2*a) - 1
)))/b
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - 2/(b*(exp(2*a + 2*b*x) - 1))
```

3.906 $\int e^{a+bx} \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] -2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-arct
anh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2320, 12, 466, 393, 212}

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x))/(b*(1 - E^(2
*a + 2*b*x))) - ArcTanh[E^(a + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{4x^2(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\text{Subst}\left(\int \frac{x^2(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} - \frac{\text{Subst}\left(\int \frac{-2-4x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=\frac{e^{a+bx}-3e^{3(a+bx)}-\left(-1+e^{2(a+bx)}\right)^2\text{arctanh}\left(e^{a+bx}\right)}{b\left(-1+e^{2(a+bx)}\right)^2}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] (E^(a + b*x) - 3*E^(3*(a + b*x)) - (-1 + E^(2*(a + b*x)))^2*ArcTanh[E^(a +
b*x)])/(b*(-1 + E^(2*(a + b*x)))^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\,\text{csch}(bx+a)}{2}-\text{arctanh}\left(e^{bx+a}\right)}{b}$	55
default	$\frac{-\frac{1}{\sinh(bx+a)}-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\,\text{csch}(bx+a)}{2}-\text{arctanh}\left(e^{bx+a}\right)}{b}$	55
risch	$-\frac{e^{bx+a}\left(3\,e^{2bx+2a}-1\right)}{b\left(e^{2bx+2a}-1\right)^2}-\frac{\ln\left(e^{bx+a}+1\right)}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	67

```
[In] int(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sinh(b*x+a)-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-a
rctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(61) = 122.

Time = 0.25 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.53

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=$$

$$-\frac{6\,\cosh\left(bx+a\right)^3+18\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+6\,\sinh\left(bx+a\right)^3+\left(\cosh\left(bx+a\right)^4+4\,\cosh\left(bx+a\right.\right.}{}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 + 6*sinh(b*x + a
)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
+ 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b
```

```
*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x
+ a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
 a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sin
h(b*x + a) - 1) + 2*(9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 2*cosh(b*x + a)
)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^
4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(
b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int e^{a+bx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}+\frac{\log\left(e^{(bx+a)}-1\right)}{2\,b}-\frac{3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*log(e^(b*x + a) + 1)/b + 1/2*log(e^(b*x + a) - 1)/b - (3*e^(3*b*x + 3*
a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\frac{2\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 + log(e^(
b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.46

$$\int e^{a+bx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\text{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{\frac{e^{a+b\,x}}{b}+\frac{e^{3\,a+3\,b\,x}}{b}}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)*exp(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] - atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (exp(a + b*x)/b + e
xp(3*a + 3*b*x)/b)/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) - (2*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.907 $\int e^{a+bx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int e^{a+bx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{e^{-4a-4bx}}{128b} - \frac{e^{-2a-2bx}}{64b} - \frac{e^{2a+2bx}}{32b} - \frac{e^{4a+4bx}}{128b} + \frac{e^{6a+6bx}}{192b} + \frac{x}{16}$$

```
[Out] 1/128*exp(-4*b*x-4*a)/b-1/64*exp(-2*b*x-2*a)/b-1/32*exp(2*b*x+2*a)/b-1/128*
exp(4*b*x+4*a)/b+1/192*exp(6*b*x+6*a)/b+1/16*x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 12, 457, 90}

$$\int e^{a+bx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{e^{-4a-4bx}}{128b} - \frac{e^{-2a-2bx}}{64b} - \frac{e^{2a+2bx}}{32b} - \frac{e^{4a+4bx}}{128b} + \frac{e^{6a+6bx}}{192b} + \frac{x}{16}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-4*a - 4*b*x)/(128*b) - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b)
 - E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) + x/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3(1+x^2)^2}{32x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3(1+x^2)^2}{x^5}\,dx, x, e^{a+bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3(1+x)^2}{x^3}\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(-2-\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-x+x^2\right)\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}-\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}+\frac{x}{16}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{3e^{-4(a+bx)}-6e^{-2(a+bx)}-12e^{2(a+bx)}-3e^{4(a+bx)}+2e^{6(a+bx)}+24bx}{384b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (3/E^(4*(a + b*x)) - 6/E^(2*(a + b*x)) - 12*E^(2*(a + b*x)) - 3*E^(4*(a + b
*x)) + 2*E^(6*(a + b*x)) + 24*b*x)/(384*b)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^3}{6}-\frac{\cosh(bx+a)^3\sinh(bx+a)}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}+\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}}{b}$$

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^3*sinh(b*x+a)^3-1/8*cosh(b*x+a)^3*sinh(b*x+a)+1/16*cos
h(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a+1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*c
osh(b*x+a)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(74) = 148.

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.84

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{5\cosh(bx+a)^5+25\cosh(bx+a)\sinh(bx+a)^4-\sinh(bx+a)^5-\left(10\cosh(bx+a)^2-3\right)\sinh(bx+a}{}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*x + a)^5 + 25*cosh(b*x + a)*sinh(b*x + a)^4 - sinh(b*x + a)
^5 - (10*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^3 - 9*cosh(b*x + a)^3 + (50*cos
h(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x - 1)*cosh(b*x
+ a) - (5*cosh(b*x + a)^4 + 24*b*x - 9*cosh(b*x + a)^2 + 12)*sinh(b*x + a))
/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(73) = 146$.

Time = 5.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.23

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{xe^ae^{bx}\sinh^5(a+bx)}{16} + \frac{xe^ae^{bx}\sinh^4(a+bx)\cosh(a+bx)}{16} + \frac{xe^ae^{bx}\sinh^3(a+bx)\cosh^2(a+bx)}{8} - \frac{xe^ae^{bx}\sinh^2(a+bx)\cosh^3(a+bx)}{8} - x \\ xe^a\sinh^3(a)\cosh^2(a) \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**5/16 + x*exp(a)*exp(b*x)*sinh(
a + b*x)**4*cosh(a + b*x)/16 + x*exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a +
b*x)**2/8 - x*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/8 - x*exp(a
)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/16 + x*exp(a)*exp(b*x)*cosh(a + b
*x)**5/16 - exp(a)*exp(b*x)*sinh(a + b*x)**5/(32*b) + 3*exp(a)*exp(b*x)*sin
h(a + b*x)**4*cosh(a + b*x)/(32*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(
a + b*x)**3/(6*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(96*b) -
 5*exp(a)*exp(b*x)*cosh(a + b*x)**5/(96*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3
*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{(2\,e^{(2\,bx+2\,a)}-1)e^{(-4\,bx-4\,a)}}{128\,b} + \frac{bx+a}{16\,b} + \frac{2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/128*(2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4*a)/b + 1/16*(b*x + a)/b + 1/38
4*(2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{24\,bx-3\left(6\,e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}+24\,a+2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x - 4
*a) + 24*a + 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{6\,\mathrm{e}^{-2\,a-2\,b\,x}+12\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{-4\,a-4\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{6\,a+6\,b\,x}-24\,b\,x}{384\,b}$$

```
[In] int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(6*exp(- 2*a - 2*b*x) + 12*exp(2*a + 2*b*x) - 3*exp(- 4*a - 4*b*x) + 3*exp
(4*a + 4*b*x) - 2*exp(6*a + 6*b*x) - 24*b*x)/(384*b)
```

3.908 $\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2320, 12, 276}

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx=-\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/48*E^(-3*a - 3*b*x)/b - E^(a + b*x)/(8*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{16x^4}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{x^4}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)\,dx, x, e^{a+bx}\right)}{16b}\\ &= -\frac{e^{-3a-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{5a+5bx}}{80b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(-5-30e^{4(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-5 - 30*E^(4*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [A] (verified)

Time = 96.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{e^{-3bx-3a}}{48b}-\frac{e^{bx+a}}{8b}+\frac{e^{5bx+5a}}{80b}$	41
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}+\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}}{b}$	70
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)^2}{5}-\frac{2\cosh(bx+a)^3}{15}+\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}}{b}$	70

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/48*exp(-3*b*x-3*a)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+5*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^4 - 16\cosh(bx+a)^3\sinh(bx+a) + 6\cosh(bx+a)^2\sinh(bx+a)^2 - 16\cosh(bx+a)\sinh}{120\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a
)^2*sinh(b*x + a)^2 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
15)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(37) = 74$.

Time = 2.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.94

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{2e^ae^{bx}\sinh^4(a+bx)}{15b} + \frac{2e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b} + \frac{e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b} + \frac{2e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{15b} - \frac{2e^ae}{} \\ xe^a\sinh^2(a)\cosh^2(a) \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cos
h(a + b*x)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*
b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a
)**2*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}-10\,e^{(bx+a)}}{80\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/80*(e^(5*b*x + 5*a) - 10*e^(b*x + a))/b - 1/48*e^(-3*b*x - 3*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{3\,e^{(5\,bx+5\,a)}-30\,e^{(bx+a)}-5\,e^{(-3\,bx-3\,a)}}{240\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/240*(3*e^(5*b*x + 5*a) - 30*e^(b*x + a) - 5*e^(-3*b*x - 3*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{30\,\mathrm{e}^{a+b\,x}+5\,\mathrm{e}^{-3\,a-3\,b\,x}-3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

```
[In] int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x)^2,x)
[Out] -(30*exp(a + b*x) + 5*exp(- 3*a - 3*b*x) - 3*exp(5*a + 5*b*x))/(240*b)
```

3.909 $\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{e^{-2a-2bx}}{16b} + \frac{e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} - \frac{x}{8}$$

```
[Out] 1/16*exp(-2*b*x-2*a)/b+1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 12, 457, 76}

$$\int e^{a+bx} \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{e^{-2a-2bx}}{16b} + \frac{e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} - \frac{x}{8}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) + E^(2*a + 2*b*x)/(16*b) + E^(4*a + 4*b*x)/(32*b) -
 x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
```

```
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)^2}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^2}-\frac{1}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{e^{-2a-2bx}}{16b}+\frac{e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}-\frac{x}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{2e^{-2(a+bx)}+2e^{2(a+bx)}+e^{4(a+bx)}-4bx}{32b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
[Out] (2/E^(2*(a + b*x)) + 2*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x)/(32*b)
```

Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{e^{-2bx-2a}}{16b}+\frac{e^{2bx+2a}}{16b}+\frac{e^{4bx+4a}}{32b}-\frac{x}{8}$	47
derivativedivides	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}+\frac{\cosh(bx+a)^4}{4}}{b}$	53
default	$\frac{\frac{\cosh(bx+a)^3\sinh(bx+a)}{4}-\frac{\cosh(bx+a)\sinh(bx+a)}{8}-\frac{bx}{8}-\frac{a}{8}+\frac{\cosh(bx+a)^4}{4}}{b}$	53

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/16*exp(-2*b*x-2*a)/b+1/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b-1/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(46) = 92.

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3-2(2bx-1)\cosh(bx+a)+(4bx-3}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 - 2*(2*b*x - 1)*cosh(b*x + a) + (4*b*x - 3*cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(44) = 88.

Time = 0.88 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.07

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{xe^ae^{bx}\sinh^3(a+bx)}{8}+\frac{xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8}+\frac{xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8}-\frac{xe^ae^{bx}\cosh^3(a+bx)}{8}-\frac{e^ae^{bx}\sinh^3(a+}{8b} \\ xe^a\sinh(a)\cosh^2(a) \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 + x*exp(a)*exp(b*x)*sinh(a
 + b*x)**2*cosh(a + b*x)/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)*
*2/8 - x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - exp(a)*exp(b*x)*sinh(a + b*x)
**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) + exp(a)*e
xp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh(a)**2, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = -\frac{1}{8}\,x - \frac{a}{8\,b} + \frac{e^{(4\,bx+4\,a)} + 2\,e^{(2\,bx+2\,a)}}{32\,b} + \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*x - 1/8*a/b + 1/32*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a))/b + 1/16*e^(-
2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$= -\frac{4\,bx - 2\,\left(e^{(2\,bx+2\,a)} + 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)}}{32\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(4*b*x - 2*(e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 4*a - e^(4*b*x +
4*a) - 2*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{16} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b} - \frac{x}{8}$$

```
[In] int(cosh(a + b*x)^2*exp(a + b*x)*sinh(a + b*x),x)
```

```
[Out] (exp(- 2*a - 2*b*x)/16 + exp(2*a + 2*b*x)/16 + exp(4*a + 4*b*x)/32)/b - x/8
```

3.910 $\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx=\frac{e^{2a+2bx}}{4b}-\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b-1/2*x+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2320, 12, 457, 84}

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx=\frac{e^{2a+2bx}}{4b}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}-\frac{x}{2}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2 + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{2x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(-1+x)x}\,dx, x, e^{2a+2bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{4}{-1+x}-\frac{1}{x}\right)\,dx, x, e^{2a+2bx}\right)}{4b}\\
&= \frac{e^{2a+2bx}}{4b}-\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx=\frac{e^{2(a+bx)}-2bx+4\log\left(1-e^{2(a+bx)}\right)}{4b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] (E^(2*(a + b*x)) - 2*b*x + 4*Log[1 - E^(2*(a + b*x))])/(4*b)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{x}{2}+\frac{e^{2bx+2a}}{4b}-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	41
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	44
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	44

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/4*exp(2*b*x+2*a)/b-2/b*a+1/b*ln(exp(2*b*x+2*a)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.71

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2\,bx-\cosh\left(bx+a\right)^2-2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\sinh\left(bx+a\right)^2-4\,\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{4\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
[Out] -1/4*(2*b*x - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x +
a)^2 - 4*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{1}{2}\,x - \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b} + \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(
b*x + a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{2\,bx + 2\,a - e^{(2\,bx+2\,a)} - 4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{4\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] -1/4*(2*b*x + 2*a - e^(2*b*x + 2*a) - 4*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{x}{2} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

```
[In] int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x),x)
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x/2 + exp(2*a + 2*b*x)/(4*b)
```

3.911 $\int e^{a+bx}\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int e^{a+bx}\coth^2(a+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 398, 294, 212}

$$\int e^{a+bx}\coth^2(a+bx)\,dx=-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{4x^2}{(1-x^2)^2}\right)\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} + \frac{4\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\coth^2(a+bx)\,dx$$
$$=\frac{e^{a+bx}\left(\frac{1}{48}e^{-4(a+bx)}\left(-375-713e^{2(a+bx)}-181e^{4(a+bx)}+61e^{6(a+bx)}+\frac{3\left(125+196e^{2(a+bx)}-14e^{4(a+bx)}-52e^{6(a+bx)}+e^{8(a+bx)}\right.}{\sqrt{e^{2(a+bx)}}}\right.\right.}{b}$$

[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2,x]

[Out] (E^(a + b*x)*((-375 - 713*E^(2*(a + b*x)) - 181*E^(4*(a + b*x)) + 61*E^(6*(a + b*x)) + (3*(125 + 196*E^(2*(a + b*x)) - 14*E^(4*(a + b*x)) - 52*E^(6*(a + b*x)) + E^(8*(a + b*x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))])/(48*E^(4*(a + b*x))) + (4*(E^(a + b*x) + E^(3*(a + b*x)))^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*(a + b*x))])/105))/b

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)+\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	48
default	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)+\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	48
risch	$\frac{e^{bx+a}}{b}-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}-\frac{\ln\left(e^{bx+a}+1\right)}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}$	63

[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a))+cosh(b*x+a)^2/sinh(b*x+a)-2/sinh(b*x+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.74

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - (\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a)\ldots}{\ldots}$$

[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")

[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \text{Timed out}$$

[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}-1)}$$

[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")

[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) - 1))

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\coth^2(a+bx)\,dx = -\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - e^{(bx+a)} + \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{e^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.912 $\int e^{a+bx}\coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{4}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] -2/b/(1-exp(2*b*x+2*a))^2+4/b/(1-exp(2*b*x+2*a))+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 12, 455, 45}

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx=\frac{4}{b\left(1-e^{2a+2bx}\right)}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 4/(b*(1 - E^(2*a + 2*b*x))) + Log[1 - E^(2
*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{2x(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2\text{Subst}\left(\int \frac{x(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(-1+x)^3}\,dx, x, e^{2a+2bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{4}{(-1+x)^3}+\frac{4}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, e^{2a+2bx}\right)}{b} \\
&= -\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{4}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \frac{\frac{2-4e^{2(a+bx)}}{\left(-1+e^{2(a+bx)}\right)^2}+\log\left(1-e^{2(a+bx)}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] ((2 - 4*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + Log[1 - E^(2*(a + b*x))
])/b
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$\frac{bx+a-\coth(bx+a)+\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	35
default	$\frac{bx+a-\coth(bx+a)+\ln(\sinh(bx+a))-\frac{\coth(bx+a)^2}{2}}{b}$	35
risch	$-\frac{2a}{b}-\frac{2(2\,e^{2bx+2a}-1)}{b(e^{2bx+2a}-1)^2}+\frac{\ln(e^{2bx+2a}-1)}{b}$	55

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(b*x+a-coth(b*x+a)+ln(sinh(b*x+a))-1/2*coth(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.23

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{4\cosh(bx+a)^2-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+\right.\right.}{b\cosh(bx+a)^4+4\,b\cosh(bx+a)\sinh(bx+a)^3+b\sin}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
[Out] -(4*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x +
a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 8*cosh(b*x + a)*sinh(b*x + a) + 4
*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^
3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\left(2e^{(2bx+2a)}-1\right)}{b(e^{(4bx+4a)}-2e^{(2bx+2a)}+1)}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*(2*e^(2*b*x + 2*a) - 1)
/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int e^{a+bx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{3e^{(4bx+4a)}+2e^{(2bx+2a)}-1}{\left(e^{(2bx+2a)}-1\right)^2} - 2\log\left(\left|e^{(2bx+2a)}-1\right|\right)}{2b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*((3*e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) - 1)^2 -
 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx=\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{4}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int((cosh(a + b*x)^2*exp(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - 4/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(4
*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.913 $\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

```
[Out] 1/320*exp(-5*b*x-5*a)/b-3/64*exp(-b*x-a)/b-1/64*exp(3*b*x+3*a)/b+1/448*exp(
7*b*x+7*a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2320, 12, 276}

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-5*a - 5*b*x)/(320*b) - (3*E^(-a - b*x))/(64*b) - E^(3*a + 3*b*x)/(64*b)
 + E^(7*a + 7*b*x)/(448*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{64x^6}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{x^6}\,dx, x, e^{a+bx}\right)}{64b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^6}+\frac{3}{x^2}-3x^2+x^6\right)\,dx, x, e^{a+bx}\right)}{64b}\\ &= \frac{e^{-5a-5bx}}{320b}-\frac{3e^{-a-bx}}{64b}-\frac{e^{3a+3bx}}{64b}+\frac{e^{7a+7bx}}{448b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{-5(a+bx)}\left(7-105e^{4(a+bx)}-35e^{8(a+bx)}+5e^{12(a+bx)}\right)}{2240b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
[Out] (7 - 105*E^(4*(a + b*x)) - 35*E^(8*(a + b*x)) + 5*E^(12*(a + b*x)))/(2240*b
*E^(5*(a + b*x)))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\frac{\frac{\cosh(bx+a)^4\sinh(bx+a)^3}{7}-\frac{3\sinh(bx+a)\cosh(bx+a)^4}{35}+\frac{3\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{35}+\frac{\sinh(bx+a)^2\cosh(bx+a)^5}{7}-\frac{2\cosh(bx+a)^5}{35}}{b}$$

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
[Out] 1/b*(1/7*cosh(b*x+a)^4*sinh(b*x+a)^3-3/35*sinh(b*x+a)*cosh(b*x+a)^4+3/35*(2
/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/7*sinh(b*x+a)^2*cosh(b*x+a)^5-2/35*cosh
(b*x+a)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(57) = 114$.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.23

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{3\cosh(bx+a)^6 - 10\cosh(bx+a)^3\sinh(bx+a)^3 + 45\cosh(bx+a)^2\sinh(bx+a)^4 - 3\cosh(bx+a)\sin}{}$$

[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/560*(3*cosh(b*x + a)^6 - 10*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*x + a)^2*sinh(b*x + a)^4 - 3*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + 5*(9*cosh(b*x + a)^4 - 7)*sinh(b*x + a)^2 - 35*cosh(b*x + a)^2 - (3*cosh(b*x + a)^5 - 35*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(54) = 108$.

Time = 11.96 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.93

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{2e^ae^{bx}\sinh^6(a+bx)}{35b} + \frac{2e^ae^{bx}\sinh^5(a+bx)\cosh(a+bx)}{35b} + \frac{e^ae^{bx}\sinh^4(a+bx)\cosh^2(a+bx)}{7b} - \frac{e^ae^{bx}\sinh^3(a+bx)\cosh^3(a+bx)}{7b} + \frac{e^ae^{b}}{} \\ xe^a\sinh^3(a)\cosh^3(a) \end{cases}$$

[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)

[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**6/(35*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)**5*cosh(a + b*x)/(35*b) + exp(a)*exp(b*x)*sinh(a + b*x)**4*cosh(a + b*x)**2/(7*b) - exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a + b*x)**3/(7*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**4/(7*b) + 2*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**5/(35*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**6/(35*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3*cosh(a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}}{320\,b} + \frac{e^{(7\,bx+7\,a)}-7\,e^{(3\,bx+3\,a)}}{448\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/320*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a)/b + 1/448*(e^(7*b*x + 7*a)
 - 7*e^(3*b*x + 3*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{7\left(15\,e^{(4\,bx+4\,a)}-1\right)e^{(-5\,bx-5\,a)}-5\,e^{(7\,bx+7\,a)}+35\,e^{(3\,bx+3\,a)}}{2240\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2240*(7*(15*e^(4*b*x + 4*a) - 1)*e^(-5*b*x - 5*a) - 5*e^(7*b*x + 7*a) +
35*e^(3*b*x + 3*a))/b
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{105\,\mathrm{e}^{-a-b\,x}+35\,\mathrm{e}^{3\,a+3\,b\,x}-7\,\mathrm{e}^{-5\,a-5\,b\,x}-5\,\mathrm{e}^{7\,a+7\,b\,x}}{2240\,b}$$

```
[In] int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(105*exp(- a - b*x) + 35*exp(3*a + 3*b*x) - 7*exp(- 5*a - 5*b*x) - 5*exp(7
*a + 7*b*x))/(2240*b)
```

3.914 $\int e^{a+bx} \cosh^3(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int e^{a+bx} \cosh^3(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-4a-4bx}}{128b} - \frac{e^{-2a-2bx}}{64b} - \frac{e^{2a+2bx}}{32b} + \frac{e^{4a+4bx}}{128b} + \frac{e^{6a+6bx}}{192b} - \frac{x}{16}$$

```
[Out] -1/128*exp(-4*b*x-4*a)/b-1/64*exp(-2*b*x-2*a)/b-1/32*exp(2*b*x+2*a)/b+1/128
*exp(4*b*x+4*a)/b+1/192*exp(6*b*x+6*a)/b-1/16*x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 12, 457, 90}

$$\int e^{a+bx} \cosh^3(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-4a-4bx}}{128b} - \frac{e^{-2a-2bx}}{64b} - \frac{e^{2a+2bx}}{32b} + \frac{e^{4a+4bx}}{128b} + \frac{e^{6a+6bx}}{192b} - \frac{x}{16}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -1/128*E^(-4*a - 4*b*x)/b - E^(-2*a - 2*b*x)/(64*b) - E^(2*a + 2*b*x)/(32*b
) + E^(4*a + 4*b*x)/(128*b) + E^(6*a + 6*b*x)/(192*b) - x/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)^3}{32x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)^3}{x^5}\,dx, x, e^{a+bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2(1+x)^3}{x^3}\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^3}+\frac{1}{x^2}-\frac{2}{x}+x+x^2\right)\,dx, x, e^{2a+2bx}\right)}{64b}\\
&= -\frac{e^{-4a-4bx}}{128b}-\frac{e^{-2a-2bx}}{64b}-\frac{e^{2a+2bx}}{32b}+\frac{e^{4a+4bx}}{128b}+\frac{e^{6a+6bx}}{192b}-\frac{x}{16}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{3e^{-4(a+bx)}+6e^{-2(a+bx)}+12e^{2(a+bx)}-3e^{4(a+bx)}-2e^{6(a+bx)}+24bx}{384b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -1/384*(3/E^(4*(a + b*x)) + 6/E^(2*(a + b*x)) + 12*E^(2*(a + b*x)) - 3*E^(4
*(a + b*x)) - 2*E^(6*(a + b*x)) + 24*b*x)/b
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\frac{\frac{\cosh(bx+a)^4\sinh(bx+a)^2}{6}-\frac{\cosh(bx+a)^4}{12}+\frac{\cosh(bx+a)^5\sinh(bx+a)}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$$

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/6*cosh(b*x+a)^4*sinh(b*x+a)^2-1/12*cosh(b*x+a)^4+1/6*cosh(b*x+a)^5*s
inh(b*x+a)-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)-1/16*b*x-1/1
6*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(74) = 148.

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^5+5\cosh(bx+a)\sinh(bx+a)^4-5\sinh(bx+a)^5-\left(50\cosh(bx+a)^2+9\right)\sinh(bx+a}{}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 - 5*sinh(b*x + a)
^5 - (50*cosh(b*x + a)^2 + 9)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + (10*cos
h(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^2 + 12*(2*b*x + 1)*cosh(b*x +
 a) - (25*cosh(b*x + a)^4 + 24*b*x + 27*cosh(b*x + a)^2 - 12)*sinh(b*x + a)
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(73) = 146$.

Time = 5.27 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.57

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{xe^ae^{bx}\sinh^5(a+bx)}{16} - \frac{xe^ae^{bx}\sinh^4(a+bx)\cosh(a+bx)}{16} - \frac{xe^ae^{bx}\sinh^3(a+bx)\cosh^2(a+bx)}{8} + \frac{xe^ae^{bx}\sinh^2(a+bx)\cosh^3(a+bx)}{8} + xe \\ xe^a\sinh^2(a)\cosh^3(a) \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)**5/16 - x*exp(a)*exp(b*x)*sinh(a
 + b*x)**4*cosh(a + b*x)/16 - x*exp(a)*exp(b*x)*sinh(a + b*x)**3*cosh(a + b
*x)**2/8 + x*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/8 + x*exp(a)
*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/16 - x*exp(a)*exp(b*x)*cosh(a + b*
x)**5/16 - 13*exp(a)*exp(b*x)*sinh(a + b*x)**5/(96*b) + 7*exp(a)*exp(b*x)*s
inh(a + b*x)**4*cosh(a + b*x)/(96*b) + exp(a)*exp(b*x)*sinh(a + b*x)**3*cos
h(a + b*x)**2/(3*b) - exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(6*
b) + exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(96*b) + 5*exp(a)*exp(b
*x)*cosh(a + b*x)**5/(96*b), Ne(b, 0)), (x*exp(a)*sinh(a)**2*cosh(a)**3, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{\left(2\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}}{128\,b} - \frac{bx+a}{16\,b} + \frac{2\,e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}}{384\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/128*(2*e^(2*b*x + 2*a) + 1)*e^(-4*b*x - 4*a)/b - 1/16*(b*x + a)/b + 1/38
4*(2*e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{24\,bx-3\left(6\,e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}-1\right)e^{(-4\,bx-4\,a)}+24\,a-2\,e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}}{384\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/384*(24*b*x - 3*(6*e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) - 1)*e^(-4*b*x -
4*a) + 24*a - 2*e^(6*b*x + 6*a) - 3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int e^{a+bx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{6\,\mathrm{e}^{-2\,a-2\,b\,x}+12\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{-4\,a-4\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{6\,a+6\,b\,x}+24\,b\,x}{384\,b}$$

```
[In] int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(6*exp(- 2*a - 2*b*x) + 12*exp(2*a + 2*b*x) + 3*exp(- 4*a - 4*b*x) - 3*exp
(4*a + 4*b*x) - 2*exp(6*a + 6*b*x) + 24*b*x)/(384*b)
```

3.915 $\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] 1/48*exp(-3*b*x-3*a)/b+1/8*exp(-b*x-a)/b+1/24*exp(3*b*x+3*a)/b+1/80*exp(5*b
*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2320, 12, 459}

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(
5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{16x^4}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}-\frac{2}{x^2}+2x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{80b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(5+30e^{2(a+bx)}+10e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
[Out] (5 + 30*E^(2*(a + b*x)) + 10*E^(6*(a + b*x)) + 3*E^(8*(a + b*x)))/(240*b*E^
(3*(a + b*x)))
```

Maple [A] (verified)

Time = 48.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}+\frac{\cosh(bx+a)^5}{5}}{b}$	52
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^4}{5}-\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{5}+\frac{\cosh(bx+a)^5}{5}}{b}$	52
risch	$\frac{e^{-3bx-3a}}{48b}+\frac{e^{-bx-a}}{8b}+\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{80b}$	58

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/5*sinh(b*x+a)*cosh(b*x+a)^4-1/5*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+
1/5*cosh(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^4-\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2+5\right)\sinh(bx+a)^2+5}{30\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(b*x + a)^4 - cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + (
6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^2 + 5*cosh(b*x + a)^2 - (cosh(b*x + a)
^3 + 5*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(53) = 106$.

Time = 2.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.01

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{2e^ae^{bx}\sinh^4(a+bx)}{15b}+\frac{2e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b}+\frac{e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}-\frac{e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{5b}+\frac{e^ae \\ xe^a\sinh(a)\cosh^3(a) \end{cases}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cos
h(a + b*x)**2/(5*b) - exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*b)
+ exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*sinh(a)*cosh
(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}}{48\,b}+\frac{3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/48*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a)/b + 1/240*(3*e^(5*b*x + 5*a)
+ 10*e^(3*b*x + 3*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{5\left(6\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}+10\,e^{(3\,bx+3\,a)}}{240\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/240*(5*(6*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) + 10*
e^(3*b*x + 3*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{30\,\mathrm{e}^{-a-b\,x}+5\,\mathrm{e}^{-3\,a-3\,b\,x}+10\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

```
[In] int(cosh(a + b*x)^3*exp(a + b*x)*sinh(a + b*x),x)
```

```
[Out] (30*exp(- a - b*x) + 5*exp(- 3*a - 3*b*x) + 10*exp(3*a + 3*b*x) + 3*exp(5*a
 + 5*b*x))/(240*b)
```

3.916 $\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 1/4*exp(-b*x-a)/b+exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b-2*arctanh(exp(b*x+a))/
b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 12, 472, 213}

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx=-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(4*b) + E^(a + b*x)/b + E^(3*a + 3*b*x)/(12*b) - (2*ArcTanh[E^
(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{4x^2(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^2(-1+x^2)}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \left(4-\frac{1}{x^2}+x^2+\frac{8}{-1+x^2}\right)\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}+\frac{2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{3a+3bx}}{12b}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{-a-bx}\left(3+12e^{2(a+bx)}+e^{4(a+bx)}-24\sqrt{e^{2(a+bx)}}\text{arctanh}\left(\sqrt{e^{2(a+bx)}}\right)\right)}{12b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] (E^(-a - b*x)*(3 + 12*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 24*Sqrt[E^(2*(a +
 b*x))]*ArcTanh[Sqrt[E^(2*(a + b*x))]]))/(12*b)
```

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)+\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	50
default	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)+\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	50
risch	$\frac{\mathrm{e}^{3bx+3a}}{12b}+\frac{\mathrm{e}^{bx+a}}{b}+\frac{\mathrm{e}^{-bx-a}}{4b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	67

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)+1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*ar
ctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(51) = 102.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.88

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+6\left(\cosh(bx+a)^2+2\right)\sinh(bx+a)^2+}{}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 - 12*(cosh(b*
x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 12*(cosh(b
*x + a) + sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 4*(cosh(b
*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) + 3)/(b*cosh(b*x + a) + b*sinh(b
*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} + 12\,e^{(bx+a)}}{12\,b} + \frac{e^{(-bx-a)}}{4\,b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] 1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a))/b + 1/4*e^(-b*x - a)/b - log(e^(b*x
 + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} + 12\,e^{(bx+a)} + 3\,e^{(-bx-a)} - 12\,\log\left(e^{(bx+a)}+1\right) + 12\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{12\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] 1/12*(e^(3*b*x + 3*a) + 12*e^(b*x + a) + 3*e^(-b*x - a) - 12*log(e^(b*x + a
) + 1) + 12*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int e^{a+bx}\cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}+\frac{\mathrm{e}^{-a-b\,x}}{4\,b}+\frac{\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

```
[In] int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x),x)
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) +
exp(- a - b*x)/(4*b) + exp(3*a + 3*b*x)/(12*b)
```

3.917 $\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx=\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b+2/b/(1-exp(2*b*x+2*a))+1/2*x+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2320, 12, 457, 90}

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx=\frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}+\frac{x}{2}$$

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + x/2 + Log[1 - E^(2*a
+ 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{2x(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(1-x)^2x}\,dx, x, e^{2a+2bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{8}{(-1+x)^2}+\frac{4}{-1+x}+\frac{1}{x}\right)dx, x, e^{2a+2bx}\right)}{4b}\\
&= \frac{e^{2a+2bx}}{4b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{x}{2}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx=\frac{\frac{1}{4}\left(e^{2(a+bx)}-\frac{8}{-1+e^{2(a+bx)}}+2bx\right)+\log\left(1-e^{2(a+bx)}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] ((E^(2*(a + b*x)) - 8/(-1 + E^(2*(a + b*x))) + 2*b*x)/4 + Log[1 - E^(2*(a +
 b*x))])/b
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))+\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	56
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))+\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	56
risch	$\frac{x}{2}+\frac{e^{2bx+2a}}{4b}-\frac{2a}{b}-\frac{2}{b(e^{2bx+2a}-1)}+\frac{\ln(e^{2bx+2a}-1)}{b}$	59

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a))+1/2*cosh(b*x+a)^3/sinh(b*x+a)+3/2*b*
x+3/2*a-3/2*coth(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(54) = 108.

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+(2bx-1)\cosh(bx+a)^2+(2bx+6\cos}{}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(2*b*x - 1)*cosh(b*x + a)^2 + (2*b*x + 6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 2*b*x + 4*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cos
h(b*x + a)^3 + (2*b*x - 1)*cosh(b*x + a))*sinh(b*x + a) - 8)/(b*cosh(b*x +
a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b} + \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b + log(e^(b*x + a) + 1)/b + log(e^(b
*x + a) - 1)/b - 2/(b*(e^(2*b*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{2\,bx + 2\,a - \frac{4\,\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,bx+2\,a)}-1} + e^{(2\,bx+2\,a)} + 4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{4\,b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
[Out] 1/4*(2*b*x + 2*a - 4*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + e^(2*b*x
 + 2*a) + 4*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{x}{2}+\frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

```
[In] int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x)^2,x)
```

```
[Out] x/2 + log(exp(2*a)*exp(2*b*x) - 1)/b - 2/(b*(exp(2*a + 2*b*x) - 1)) + exp(2
*a + 2*b*x)/(4*b)
```

3.918 $\int e^{a+bx} \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int e^{a+bx} \coth^3(a+bx)\,dx = \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*
x+2*a))-3*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2320, 398, 1172, 12, 294, 213}

$$\int e^{a+bx} \coth^3(a+bx)\,dx = -\frac{3\operatorname{arctanh}\left(e^{a+bx}\right)}{b} + \frac{e^{a+bx}}{b} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{2\text{Subst}\left(\int \frac{1+3x^4}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}
\end{aligned}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{\text{Subst}\left(\int \frac{12x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{6\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.53

$$\int e^{a+bx}\coth^3(a+bx)\,dx = -\frac{e^{-5(a+bx)}\left(-21\left(252105+507305e^{2(a+bx)}+173916e^{4(a+bx)}-154296e^{6(a+bx)}-73885e^{8(a+bx)}+4887e^{10(a+bx)}\right)\right.}{}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] -1/60480*(-21*(252105 + 507305*E^(2*(a + b*x)) + 173916*E^(4*(a + b*x)) - 1
54296*E^(6*(a + b*x)) - 73885*E^(8*(a + b*x)) + 4887*E^(10*(a + b*x))) - (3
15*(-16807 - 28218*E^(2*(a + b*x)) + 1173*E^(4*(a + b*x)) + 17748*E^(6*(a +
 b*x)) + 4299*E^(8*(a + b*x)) - 1434*E^(10*(a + b*x)) + 7*E^(12*(a + b*x)))
*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 384*E^(8*(a + b*x)
)*(1 + E^(2*(a + b*x)))^2*(7 + 5*E^(2*(a + b*x)))*HypergeometricPFQ[{3/2, 2
, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^
(2*(a + b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}
, E^(2*(a + b*x))])/(b*E^(5*(a + b*x)))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{e^{bx+a}}{b}-\frac{e^{bx+a}(3\,e^{2bx+2a}-1)}{b(e^{2bx+2a}-1)^2}+\frac{3\ln(e^{bx+a}-1)}{2b}-\frac{3\ln(e^{bx+a}+1)}{2b}$	77
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\operatorname{arctanh}(e^{bx+a})}{b}$	89
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\operatorname{arctanh}(e^{bx+a})}{b}$	89

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] exp(b*x+a)/b-exp(b*x+a)*(3*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^2+3/2/b*l
n(exp(b*x+a)-1)-3/2/b*ln(exp(b*x+a)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.67

$$\int e^{a+bx}\coth^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^5+10\cosh(bx+a)\sinh(bx+a)^4+2\sinh(bx+a)^5+10\left(2\cosh(bx+a)^2-1\right)\sinh(bx+}{}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*sinh(b*x + a)
^5 + 10*(2*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 10*(
2*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 +
 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(5
*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 2)*sinh(b*x + a) + 4*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{3\log\left(e^{(bx+a)}+1\right)}{2b} + \frac{3\log\left(e^{(bx+a)}-1\right)}{2b} - \frac{3e^{(3bx+3a)}-e^{(bx+a)}}{b\left(e^{(4bx+4a)}-2e^{(2bx+2a)}+1\right)}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - (
3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) +
1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\coth^3(a+bx)\,dx = -\frac{\frac{2\left(3e^{(3bx+3a)}-e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^2} - 2e^{(bx+a)} + 3\log\left(e^{(bx+a)}+1\right) - 3\log\left(\left|e^{(bx+a)}-1\right|\right)}{2b}$$

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 2*e^(b*
x + a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \frac{e^{a+b\,x}}{b}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{3\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)^3*exp(a + b*x))/sinh(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
(2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp(a
 + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.919 $\int e^{2(a+bx)} \cosh(a+bx) \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh^3(a+bx)\,dx = \frac{e^{-2a-2bx}}{32b} - \frac{e^{4a+4bx}}{32b} + \frac{e^{6a+6bx}}{96b} + \frac{x}{8}$$

```
[Out] 1/32*exp(-2*b*x-2*a)/b-1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b+1/8*x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 12, 457, 76}

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh^3(a+bx)\,dx = \frac{e^{-2a-2bx}}{32b} - \frac{e^{4a+4bx}}{32b} + \frac{e^{6a+6bx}}{96b} + \frac{x}{8}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(32*b) - E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) +
 x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
```

```
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{16x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1-x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1-x)(1-x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{32b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{2}{x}-2x+x^2\right)\,dx, x, e^{2a+2bx}\right)}{32b}\\
&= \frac{e^{-2a-2bx}}{32b}-\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}+\frac{x}{8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{3e^{-2(a+bx)}-3e^{4(a+bx)}+e^{6(a+bx)}+12bx}{96b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
[Out] (3/E^(2*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) + 12*b*x)/(96*b)
```

Maple [A] (verified)

Time = 290.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{e^{-2bx-2a}}{32b}-\frac{e^{4bx+4a}}{32b}+\frac{e^{6bx+6a}}{96b}+\frac{x}{8}$	47
default	$\frac{x}{8}-\frac{\sinh(2bx+2a)}{32b}-\frac{\sinh(4bx+4a)}{32b}+\frac{\sinh(6bx+6a)}{96b}+\frac{\cosh(2bx+2a)}{32b}-\frac{\cosh(4bx+4a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$	89

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*exp(-2*b*x-2*a)/b-1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b+1/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.67

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{4\cosh(bx+a)^4-8\cosh(bx+a)\sinh(bx+a)^3+4\sinh(bx+a)^4+3(4bx-1)\cosh(bx+a)^2+3(4bx}{96(b\cosh(bx+a)^2-2b\cosh(bx+}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a)
^4 + 3*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(4*b*x + 8*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 + 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(44) = 88.

Time = 2.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 4.12

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}-\frac{xe^{2a}e^{2bx}\sinh^4(a+bx)}{8}+\frac{xe^{2a}e^{2bx}\sinh^3(a+bx)\cosh(a+bx)}{4}-\frac{xe^{2a}e^{2bx}\sinh(a+bx)\cosh^3(a+bx)}{4}+\frac{xe^{2a}e^{2bx}\cosh^4(a+bx)}{8}+\frac{7e^{2a}e}{} \\ xe^{2a}\sinh^3(a)\cosh(a)\end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 + x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)**3*cosh(a + b*x)/4 - x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*co
sh(a + b*x)**3/4 + x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + 7*exp(2*a)*ex
p(2*b*x)*sinh(a + b*x)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cos
h(a + b*x)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(4
*b) - exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b), Ne(b, 0)), (x*exp(2*a)*s
inh(a)**3*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx=-\frac{\left(3\,e^{(-2\,bx-2\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b}+\frac{bx+a}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*e^(-2*b*x - 2*a) - 1)*e^(6*b*x + 6*a)/b + 1/8*(b*x + a)/b + 1/32*e
^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{1}{8}\,x+\frac{e^{(6\,bx+6\,a)}}{96\,b}-\frac{e^{(4\,bx+4\,a)}}{32\,b}+\frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*x + 1/96*e^(6*b*x + 6*a)/b - 1/32*e^(4*b*x + 4*a)/b + 1/32*e^(-2*b*x -
2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{x}{8}+\frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{32}-\frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}+\frac{\mathrm{e}^{6\,a+6\,b\,x}}{96}}{b}$$

```
[In] int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] x/8 + (exp(- 2*a - 2*b*x)/32 - exp(4*a + 4*b*x)/32 + exp(6*a + 6*b*x)/96)/b
```

3.920 $\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

```
[Out] -1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b-1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*
a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2320, 12, 459}

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/8*E^(-a - b*x)/b - E^(a + b*x)/(8*b) - E^(3*a + 3*b*x)/(24*b) + E^(5*a +
 5*b*x)/(40*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)}{8x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)}{x^2}\,dx, x, e^{a+bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}-x^2+x^4\right)\,dx, x, e^{a+bx}\right)}{8b}\\
&= -\frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$
\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{-5e^{a+bx}\left(3+e^{2(a+bx)}\right)+3e^{-a-bx}\left(-5+e^{6(a+bx)}\right)}{120b}
$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*E^(a + b*x)*(3 + E^(2*(a + b*x))) + 3*E^(-a - b*x)*(-5 + E^(6*(a + b*x)
)))/(120*b)
```

Maple [A] (verified)

Time = 21.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{e^{-bx-a}}{8b}-\frac{e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{40b}$	55
default	$-\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}-\frac{\cosh(bx+a)}{4b}-\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$	69

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b-1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*
a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{6\cosh(bx+a)^3+18\cosh(bx+a)\sinh(bx+a)^2-9\sinh(bx+a)^3-(27\cosh(bx+a)^2+5)\sinh(bx+a)+10\cosh(bx+a)}{60\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/60*(6*cosh(b*x + a)^3 + 18*cosh(b*x + a)*sinh(b*x + a)^2 - 9*sinh(b*x +
a)^3 - (27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x + a))/(b*cosh(b
*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(48) = 96.

Time = 0.87 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{e^{2a}e^{2bx}\sinh^3(a+bx)}{15b}-\frac{2e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b}+\frac{8e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b}-\frac{4e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b\neq 0 \\ xe^{2a}\sinh^2(a)\cosh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)**2,x)
[Out] Piecewise((exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) - 2*exp(2*a)*exp(2*b
*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a +
b*x)*cosh(a + b*x)**2/(15*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(15*b
), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}-3\right)e^{(5\,bx+5\,a)}}{120\,b} - \frac{e^{(-bx-a)}}{8\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/120*(5*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 3)*e^(5*b*x + 5*a)/b - 1
/8*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{40\,b} - \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{e^{(bx+a)}}{8\,b} - \frac{e^{(-bx-a)}}{8\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/40*e^(5*b*x + 5*a)/b - 1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b - 1/8*e
^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh^2(a+bx)\,dx = -\frac{15\,e^{a+b\,x}+15\,e^{-a-b\,x}+5\,e^{3\,a+3\,b\,x}-3\,e^{5\,a+5\,b\,x}}{120\,b}$$

```
[In] int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
[Out] -(15*exp(a + b*x) + 15*exp(- a - b*x) + 5*exp(3*a + 3*b*x) - 3*exp(5*a + 5*
b*x))/(120*b)
```

3.921 $\int e^{2(a+bx)} \cosh(a+bx) \sinh(a+bx) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh(a+bx) \, dx = \frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

```
[Out] 1/16*exp(4*b*x+4*a)/b-1/4*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2320, 12, 14}

$$\int e^{2(a+bx)} \cosh(a+bx) \sinh(a+bx) \, dx = \frac{e^{4a+4bx}}{16b} - \frac{x}{4}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] E^(4*a + 4*b*x)/(16*b) - x/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^4}{4x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{-1+x^4}{x}\,dx, x, e^{a+bx}\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x^3\right)\,dx, x, e^{a+bx}\right)}{4b}\\ &= \frac{e^{4a+4bx}}{16b}-\frac{x}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{1}{4}\left(\frac{e^{4a+4bx}}{4b}-x\right)$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] (E^(4*a + 4*b*x)/(4*b) - x)/4
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{{\mathrm e}^{4bx+4a}}{16b}-\frac{x}{4}$	19
default	$-\frac{x}{4}+\frac{\sinh(4bx+4a)}{16b}+\frac{\cosh(4bx+4a)}{16b}$	33

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/16*exp(4*b*x+4*a)/b-1/4*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{(4\,bx-1)\cosh\left(bx+a\right)^2-2\,(4\,bx+1)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+(4\,bx-1)\sinh\left(bx+a\right)^2}{16\,\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] -1/16*((4*b*x - 1)*cosh(b*x + a)^2 - 2*(4*b*x + 1)*cosh(b*x + a)*sinh(b*x +
 a) + (4*b*x - 1)*sinh(b*x + a)^2)/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*s
inh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(15) = 30$.

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^2\left(a+bx\right)}{4}+\frac{xe^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2}-\frac{xe^{2a}e^{2bx}\cosh^2\left(a+bx\right)}{4}+\frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b} & \text{for } b \neq \\ xe^{2a}\sinh\left(a\right)\cosh\left(a\right) & \text{otherwi} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2/4 + x*exp(2*a)*exp(2*b*x
)*sinh(a + b*x)*cosh(a + b*x)/2 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**2/4
+ exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)/(4*b), Ne(b, 0)), (x*exp(
2*a)*sinh(a)*cosh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{1}{4}\,x-\frac{a}{4\,b}+\frac{e^{(4\,bx+4\,a)}}{16\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/4*x - 1/4*a/b + 1/16*e^(4*b*x + 4*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = -\frac{1}{4}\,x + \frac{e^{(4\,bx+4\,a)}}{16\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] -1/4*x + 1/16*e^(4*b*x + 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\mathrm{e}^{4\,a+4\,b\,x}}{16\,b} - \frac{x}{4}$$

```
[In] int(cosh(a + b*x)*exp(2*a + 2*b*x)*sinh(a + b*x),x)
[Out] exp(4*a + 4*b*x)/(16*b) - x/4
```

3.922 $\int e^{2(a+bx)} \coth(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int e^{2(a+bx)} \coth(a+bx) \, dx = \frac{e^{2a+2bx}}{2b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 455, 45}

$$\int e^{2(a+bx)} \coth(a+bx) \, dx = \frac{e^{2a+2bx}}{2b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(-1-x^2)}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{-1-x}{1-x}\,dx, x, e^{2a+2bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{2}{-1+x}\right)\,dx, x, e^{2a+2bx}\right)}{2b}\\
&= \frac{e^{2a+2bx}}{2b} + \frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{\frac{1}{2}e^{2a+2bx} + \log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x],x]
[Out] (E^(2*a + 2*b*x)/2 + Log[1 - E^(2*a + 2*b*x)])/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{e^{2bx+2a}}{2b} - \frac{2a}{b} + \frac{\ln(e^{2bx+2a}-1)}{b}$	38

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*b*x+2*a)/b-2/b*a+1/b*ln(exp(2*b*x+2*a)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+2\,\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 2*
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = e^{2a}\int e^{2bx}\cosh\left(a+bx\right)\operatorname{csch}\left(a+bx\right)dx$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x)
[Out] exp(2*a)*Integral(exp(2*b*x)*cosh(a + b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{2\left(bx+a\right)}{b}+\frac{e^{(2\,bx+2\,a)}}{2\,b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] 2*(b*x + a)/b + 1/2*e^(2*b*x + 2*a)/b + log(e^(-b*x - a) + 1)/b + log(e^(-b
*x - a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{e^{(2\,bx+2\,a)} + 2\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] 1/2*(e^(2*b*x + 2*a) + 2*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\coth(a+bx)\,dx = \frac{e^{2\,a+2\,b\,x} + 2\,\ln\left(e^{2\,a}\,e^{2\,b\,x} - 1\right)}{2\,b}$$

```
[In] int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x),x)
[Out] (exp(2*a + 2*b*x) + 2*log(exp(2*a)*exp(2*b*x) - 1))/(2*b)
```

3.923 $\int e^{2(a+bx)} \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}(a+bx)\,dx = \frac{2e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{4\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-4*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2320, 12, 466, 396, 212}

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{4\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{2e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})}$$

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (4*ArcTanh[
E^(a + b*x)])/b
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{2x^2(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2\text{Subst}\left(\int \frac{x^2(1+x^2)}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{\text{Subst}\left(\int \frac{2+2x^2}{1-x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{2e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{4\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{2e^{a+bx}+\frac{1}{1-e^{a+bx}}-\frac{1}{1+e^{a+bx}}+2\log\left(1-e^{a+bx}\right)-2\log\left(1+e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x) + (1 - E^(a + b*x))^(-1) - (1 + E^(a + b*x))^(-1) + 2*Log[1
- E^(a + b*x)] - 2*Log[1 + E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b}-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{2\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	65

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/b*ln(exp(b*x+a)+1)+2/b*l
n(exp(b*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(48) = 96.

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.70

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx = \frac{2\left(\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sin\right.\right.}{}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (
3*cosh(b*x + a)^2 - 2)*sinh(b*x + a) - 2*cosh(b*x + a))/(b*cosh(b*x + a)^2
+ 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2\left(2\,e^{(-2\,bx-2\,a)}-1\right)}{b(e^{(-bx-a)}-e^{(-3\,bx-3\,a)})}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(2*e^(-2*b*x - 2
*a) - 1)/(b*(e^(-b*x - a) - e^(-3*b*x - 3*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\left(\frac{e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}-e^{(bx+a)}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] -2*(e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}(a+bx)\,dx=\frac{2\,e^{a+b\,x}}{b}-\frac{4\,\text{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
[Out] (2*exp(a + b*x))/b - (4*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.924 $\int e^{2(a+bx)} \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int e^{2(a+bx)} \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{2}{b(1-e^{2a+2bx})^2}+\frac{6}{b(1-e^{2a+2bx})}+\frac{2\log(1-e^{2a+2bx})}{b}$$

```
[Out] -2/b/(1-exp(2*b*x+2*a))^2+6/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 12, 457, 78}

$$\int e^{2(a+bx)} \coth(a+bx)\mathrm{csch}^2(a+bx)\,dx = \frac{6}{b(1-e^{2a+2bx})}-\frac{2}{b(1-e^{2a+2bx})^2}+\frac{2\log(1-e^{2a+2bx})}{b}$$

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E
^(2*a + 2*b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{4x^3(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\text{Subst}\left(\int \frac{x^3(-1-x^2)}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \frac{(-1-x)x}{(1-x)^3}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{2}{(-1+x)^3}+\frac{3}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=\frac{\frac{4-6e^{2(a+bx)}}{\left(-1+e^{2(a+bx)}\right)^2}+2\log\left(1-e^{2(a+bx)}\right)}{b}$$

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]*Csch[a + b*x]^2,x]
[Out] ((4 - 6*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + 2*Log[1 - E^(2*(a + b*x
))])/b
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{4a}{b}-\frac{2\left(3\,\mathrm{e}^{2bx+2a}-2\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	56

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -4/b*a-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+2/b*ln(exp(2*b*x+2*a)-
1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(56) = 112.

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.16

$$\int e^{2(a+bx)}\coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=$$

$$-\frac{2\left(3\cosh\left(bx+a\right)^2-\left(\cosh\left(bx+a\right)^4+4\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sinh\left(bx+a\right)^4+2\left(3\cosh\left(bx+\right.\right.\right.}{b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sin}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] -2*(3*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 6*cosh(b*x + a)*sinh(b*x + a) +
 3*sinh(b*x + a)^2 - 2)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a
)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) +
b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx = 4\,x + \frac{4\,a}{b} + \frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2\left(e^{(-2\,bx-2\,a)}-2\right)}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] 4*x + 4*a/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 2*(e^
(-2*b*x - 2*a) - 2)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\coth(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\frac{3\,e^{(4\,bx+4\,a)}-1}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - 2\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] -((3*e^(4*b*x + 4*a) - 1)/(e^(2*b*x + 2*a) - 1)^2 - 2*log(abs(e^(2*b*x + 2*
a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int e^{2(a+bx)} \coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{2\,\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b} - \frac{6}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int((cosh(a + b*x)*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

```
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 6/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(e
xp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.925 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 100

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} + \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} - \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

```
[Out] 1/96*exp(-3*b*x-3*a)/b-1/32*exp(-b*x-a)/b+1/16*exp(b*x+a)/b-1/48*exp(3*b*x+
3*a)/b-1/160*exp(5*b*x+5*a)/b+1/224*exp(7*b*x+7*a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2320, 12, 459}

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} + \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} - \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-3*a - 3*b*x)/(96*b) - E^(-a - b*x)/(32*b) + E^(a + b*x)/(16*b) - E^(3*a
 + 3*b*x)/(48*b) - E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3(1+x^2)^2}{32x^4}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3(1+x^2)^2}{x^4}\,dx, x, e^{a+bx}\right)}{32b} \\
&= \frac{\text{Subst}\left(\int \left(2-\frac{1}{x^4}+\frac{1}{x^2}-2x^2-x^4+x^6\right)\,dx, x, e^{a+bx}\right)}{32b} \\
&= \frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}+\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}-\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(35-105e^{2(a+bx)}+210e^{4(a+bx)}-70e^{6(a+bx)}-21e^{8(a+bx)}+15e^{10(a+bx)}\right)}{3360b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
[Out] (35 - 105*E^(2*(a + b*x)) + 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) - 21*E
^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\frac{3\sinh\left(bx+a\right)}{32b}-\frac{\sinh\left(3bx+3a\right)}{32b}-\frac{\sinh\left(5bx+5a\right)}{160b}+\frac{\sinh\left(7bx+7a\right)}{224b}+\frac{\cosh\left(bx+a\right)}{32b}-\frac{\cosh\left(3bx+3a\right)}{96b}-\cosh$$

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 3/32*sinh(b*x+a)/b-1/32/b*sinh(3*b*x+3*a)-1/160/b*sinh(5*b*x+5*a)+1/224/b*s
inh(7*b*x+7*a)+1/32*cosh(b*x+a)/b-1/96*cosh(3*b*x+3*a)/b-1/160*cosh(5*b*x+5
*a)/b+1/224*cosh(7*b*x+7*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{25\,\cosh\left(bx+a\right)^5+125\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4-10\,\sinh\left(bx+a\right)^5-2\left(50\,\cosh\left(bx+a\right)^2-21\right)\sinh}{168}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1680*(25*cosh(b*x + a)^5 + 125*cosh(b*x + a)*sinh(b*x + a)^4 - 10*sinh(b*
x + a)^5 - 2*(50*cosh(b*x + a)^2 - 21)*sinh(b*x + a)^3 - 63*cosh(b*x + a)^3
 + (250*cosh(b*x + a)^3 - 189*cosh(b*x + a))*sinh(b*x + a)^2 - 2*(25*cosh(b
*x + a)^4 - 63*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 70*cosh(b*x + a))/(b*c
osh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(76) = 152.

Time = 5.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}-\frac{4e^{2a}e^{2bx}\sinh^5\left(a+bx\right)}{35b}+\frac{8e^{2a}e^{2bx}\sinh^4\left(a+bx\right)\cosh\left(a+bx\right)}{35b}+\frac{2e^{2a}e^{2bx}\sinh^3\left(a+bx\right)\cosh^2\left(a+bx\right)}{35b}-\frac{e^{2a}e^{2bx}\sinh^2\left(a+bx\right)\cosh^3\left(a+b\right.}{105b} \\ xe^{2a}\sinh^3\left(a\right)\cosh^2\left(a\right)\end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5/(35*b) + 8*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**4*cosh(a + b*x)/(35*b) + 2*exp(2*a)*exp(2*b*x)*sinh(a
 + b*x)**3*cosh(a + b*x)**2/(35*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*c
osh(a + b*x)**3/(105*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)
**4/(105*b) + 2*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**5/(105*b), Ne(b, 0)), (x
*exp(2*a)*sinh(a)**3*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(21\,e^{(-2\,bx-2\,a)}+70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}-15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{96\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3360*(21*e^(-2*b*x - 2*a) + 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) -
 15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) - e^(-3*b*x - 3*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(7\,bx+7\,a)}}{224\,b}-\frac{e^{(5\,bx+5\,a)}}{160\,b}-\frac{e^{(3\,bx+3\,a)}}{48\,b}+\frac{e^{(bx+a)}}{16\,b}-\frac{e^{(-bx-a)}}{32\,b}+\frac{e^{(-3\,bx-3\,a)}}{96\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/224*e^(7*b*x + 7*a)/b - 1/160*e^(5*b*x + 5*a)/b - 1/48*e^(3*b*x + 3*a)/b
+ 1/16*e^(b*x + a)/b - 1/32*e^(-b*x - a)/b + 1/96*e^(-3*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{210\,\mathrm{e}^{a+b\,x}-105\,\mathrm{e}^{-a-b\,x}+35\,\mathrm{e}^{-3\,a-3\,b\,x}-70\,\mathrm{e}^{3\,a+3\,b\,x}-21\,\mathrm{e}^{5\,a+5\,b\,x}+15\,\mathrm{e}^{7\,a+7\,b\,x}}{3360\,b}$$

```
[In] int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
[Out] (210*exp(a + b*x) - 105*exp(- a - b*x) + 35*exp(- 3*a - 3*b*x) - 70*exp(3*a
 + 3*b*x) - 21*exp(5*a + 5*b*x) + 15*exp(7*a + 7*b*x))/(3360*b)
```

3.926 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 52

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-2a-2bx}}{32b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{6a+6bx}}{96b}$$

```
[Out] -1/32*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+6*a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2320, 12, 276}

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh^2(a+bx) \, dx = -\frac{e^{-2a-2bx}}{32b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{6a+6bx}}{96b}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/32*E^(-2*a - 2*b*x)/b - E^(2*a + 2*b*x)/(16*b) + E^(6*a + 6*b*x)/(96*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{16x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x^4)^2}{x^3}\,dx, x, e^{a+bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}-2x+x^5\right)\,dx, x, e^{a+bx}\right)}{16b}\\ &= -\frac{e^{-2a-2bx}}{32b}-\frac{e^{2a+2bx}}{16b}+\frac{e^{6a+6bx}}{96b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{-2(a+bx)}\left(-3-6e^{4(a+bx)}+e^{8(a+bx)}\right)}{96b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
[Out] (-3 - 6*E^(4*(a + b*x)) + E^(8*(a + b*x)))/(96*b*E^(2*(a + b*x)))
```

Maple [A] (verified)

Time = 107.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{e^{-2bx-2a}}{32b}-\frac{e^{2bx+2a}}{16b}+\frac{e^{6bx+6a}}{96b}$	44
default	$-\frac{\sinh(2bx+2a)}{32b}+\frac{\sinh(6bx+6a)}{96b}-\frac{3\cosh(2bx+2a)}{32b}+\frac{\cosh(6bx+6a)}{96b}$	58

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/32*exp(-2*b*x-2*a)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+6*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\cosh\left(bx+a\right)^4 - 8\,\cosh\left(bx+a\right)^3\sinh\left(bx+a\right) + 6\,\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2 - 8\,\cosh\left(bx+a\right)\sinh\left(\right.}{48\,\left(b\cosh\left(bx+a\right)^2 - 2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right) + b\sinh\left(bx+a\right)^2\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/48*(cosh(b*x + a)^4 - 8*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^
2*sinh(b*x + a)^2 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 3)/
(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(41) = 82.

Time = 2.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.46

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{5e^{2a}e^{2bx}\sinh^4\left(a+bx\right)}{48b} + \frac{5e^{2a}e^{2bx}\sinh^3\left(a+bx\right)\cosh\left(a+bx\right)}{24b} + \frac{e^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{8b} - \frac{e^{2a}e^{2bx}\cosh^4\left(a+bx\right)}{16b} & \text{for } b \neq 0 \\ xe^{2a}\sinh^2\left(a\right)\cosh^2\left(a\right) & \text{otherwis} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
[Out] Piecewise((-5*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/(48*b) + 5*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)/(24*b) + exp(2*a)*exp(2*b*x)*sinh(a +
 b*x)*cosh(a + b*x)**3/(8*b) - exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b),
 Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{\left(6\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{96\,b} - \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/96*(6*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 1/32*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(6\,bx+6\,a)}}{96\,b} - \frac{e^{(2\,bx+2\,a)}}{16\,b} - \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/96*e^(6*b*x + 6*a)/b - 1/16*e^(2*b*x + 2*a)/b - 1/32*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}+6\,\mathrm{e}^{2\,a+2\,b\,x}-\mathrm{e}^{6\,a+6\,b\,x}}{96\,b}$$

```
[In] int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
[Out] -(3*exp(- 2*a - 2*b*x) + 6*exp(2*a + 2*b*x) - exp(6*a + 6*b*x))/(96*b)
```

3.927 $\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

```
[Out] 1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b+1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a
)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2320, 12, 459}

$$\int e^{2(a+bx)} \cosh^2(a+bx) \sinh(a+bx)\, dx = \frac{e^{-a-bx}}{8b} - \frac{e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{24b} + \frac{e^{5a+5bx}}{40b}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] E^(-a - b*x)/(8*b) - E^(a + b*x)/(8*b) + E^(3*a + 3*b*x)/(24*b) + E^(5*a +
5*b*x)/(40*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{8x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \left(-1-\frac{1}{x^2}+x^2+x^4\right)\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{e^{-a-bx}}{8b}-\frac{e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{24b}+\frac{e^{5a+5bx}}{40b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{a+bx}\left(-3+e^{2(a+bx)}\right)}{24b}+\frac{e^{-a-bx}\left(5+e^{6(a+bx)}\right)}{40b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Sinh[a + b*x],x]
[Out] (E^(a + b*x)*(-3 + E^(2*(a + b*x))))/(24*b) + (E^(-a - b*x)*(5 + E^(6*(a +
b*x))))/(40*b)
```

Maple [A] (verified)

Time = 10.72 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{-bx-a}}{8b}-\frac{e^{bx+a}}{8b}+\frac{e^{3bx+3a}}{24b}+\frac{e^{5bx+5a}}{40b}$	55
default	$-\frac{\sinh(bx+a)}{4b}+\frac{\sinh(3bx+3a)}{24b}+\frac{\sinh(5bx+5a)}{40b}+\frac{\cosh(3bx+3a)}{24b}+\frac{\cosh(5bx+5a)}{40b}$	69

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/8*exp(-b*x-a)/b-1/8*exp(b*x+a)/b+1/24*exp(3*b*x+3*a)/b+1/40*exp(5*b*x+5*a
)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{9\cosh(bx+a)^3+27\cosh(bx+a)\sinh(bx+a)^2-6\sinh(bx+a)^3-2\left(9\cosh(bx+a)^2-5\right)\sinh(bx+a)-5\cosh(bx+a)}{60\left(b\cosh(bx+a)^2-2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/60*(9*cosh(b*x + a)^3 + 27*cosh(b*x + a)*sinh(b*x + a)^2 - 6*sinh(b*x + a
)^3 - 2*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) - 5*cosh(b*x + a))/(b*cosh(b*
x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(48) = 96.

Time = 1.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.94

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^3(a+bx)}{15b}+\frac{8e^{2a}e^{2bx}\sinh^2(a+bx)\cosh(a+bx)}{15b}-\frac{2e^{2a}e^{2bx}\sinh(a+bx)\cosh^2(a+bx)}{15b}+\frac{e^{2a}e^{2bx}\cosh^3(a+bx)}{15b} & \text{for } b\neq 0 \\ xe^{2a}\sinh(a)\cosh^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*sinh(b*x+a),x)
[Out] Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3/(15*b) + 8*exp(2*a)*exp(
2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(15*b) - 2*exp(2*a)*exp(2*b*x)*sinh(a
 + b*x)*cosh(a + b*x)**2/(15*b) + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**3/(15*
b), Ne(b, 0)), (x*exp(2*a)*sinh(a)*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,a)}-15\,e^{(-4\,bx-4\,a)}+3\right)e^{(5\,bx+5\,a)}}{120\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/120*(5*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 3)*e^(5*b*x + 5*a)/b + 1/
8*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{40\,b}+\frac{e^{(3\,bx+3\,a)}}{24\,b}-\frac{e^{(bx+a)}}{8\,b}+\frac{e^{(-bx-a)}}{8\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/40*e^(5*b*x + 5*a)/b + 1/24*e^(3*b*x + 3*a)/b - 1/8*e^(b*x + a)/b + 1/8*e
^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{15\,\mathrm{e}^{-a-b\,x}-15\,\mathrm{e}^{a+b\,x}+5\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{120\,b}$$

```
[In] int(cosh(a + b*x)^2*exp(2*a + 2*b*x)*sinh(a + b*x),x)
```

```
[Out] (15*exp(- a - b*x) - 15*exp(a + b*x) + 5*exp(3*a + 3*b*x) + 3*exp(5*a + 5*b
*x))/(120*b)
```

3.928 $\int e^{2(a+bx)} \cosh(a+bx) \coth(a+bx) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int e^{2(a+bx)} \cosh(a+bx) \coth(a+bx) \, dx = \frac{3e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 3/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 12, 398, 213}

$$\int e^{2(a+bx)} \cosh(a+bx) \coth(a+bx) \, dx = -\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{3e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (3*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{2(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+x^2+\frac{4}{-1+x^2}\right)\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{3e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{e^{a+bx}\left(-3-\frac{1}{3}e^{2(a+bx)}+\frac{4\text{arctanh}\left(\sqrt{e^{2(a+bx)}}\right)}{\sqrt{e^{2(a+bx)}}}\right)}{2b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] -1/2*(E^(a + b*x)*(-3 - E^(2*(a + b*x))/3 + (4*ArcTanh[Sqrt[E^(2*(a + b*x))
]])/Sqrt[E^(2*(a + b*x))]))/b
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{e^{3bx+3a}}{6b}+\frac{3\,e^{bx+a}}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	54

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*exp(3*b*x+3*a)/b+3/2*exp(b*x+a)/b+1/b*ln(exp(b*x+a)-1)-1/b*ln(exp(b*x+a
)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(38) = 76.

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = \frac{\cosh\left(bx+a\right)^3+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+\sinh\left(bx+a\right)^3+3\,\left(\cosh\left(bx+a\right)^2+3\right)\sinh\left(bx+a\right)+9}{6\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 9*cosh(b*x + a) - 6*log(cosh(b*x +
a) + sinh(b*x + a) + 1) + 6*log(cosh(b*x + a) + sinh(b*x + a) - 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = \frac{\left(9\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{6\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
[Out] 1/6*(9*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b - log(e^(-b*x - a) + 1)/b +
log(e^(-b*x - a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}-6\,\log\left(e^{(bx+a)}+1\right)+6\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
[Out] 1/6*(e^(3*b*x + 3*a) + 9*e^(b*x + a) - 6*log(e^(b*x + a) + 1) + 6*log(abs(e
^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int e^{2(a+bx)}\cosh(a+bx)\coth(a+bx)\,dx = \frac{3\,\mathrm{e}^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{6\,b}$$

```
[In] int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x),x)
[Out] (3*exp(a + b*x))/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + exp(3*a + 3*b*x)/(6*b)
```

3.929 $\int e^{2(a+bx)} \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int e^{2(a+bx)} \coth^2(a+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{2}{b\left(1-e^{2a+2bx}\right)} + \frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b+2/b/(1-exp(2*b*x+2*a))+2*ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 455, 45}

$$\int e^{2(a+bx)} \coth^2(a+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{2}{b\left(1-e^{2a+2bx}\right)} + \frac{2\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + 2/(b*(1 - E^(2*a + 2*b*x))) + (2*Log[1 - E^(2*a + 2
*b*x)])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(1+x^2)^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^2}\,dx, x, e^{2a+2bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{4}{(-1+x)^2}+\frac{4}{-1+x}\right)\,dx, x, e^{2a+2bx}\right)}{2b}\\
&= \frac{e^{2a+2bx}}{2b}+\frac{2}{b\left(1-e^{2a+2bx}\right)}+\frac{2\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{\frac{1}{2}e^{2(a+bx)}-\frac{2}{-1+e^{2(a+bx)}}+2\log\left(1-e^{2(a+bx)}\right)}{b}$$

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2,x]
[Out] (E^(2*(a + b*x))/2 - 2/(-1 + E^(2*(a + b*x))) + 2*Log[1 - E^(2*(a + b*x))])
/b
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{4a}{b}-\frac{2}{b\left(e^{2bx+2a}-1\right)}+\frac{2\ln\left(e^{2bx+2a}-1\right)}{b}$	57

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*b*x+2*a)/b-4/b*a-2/b/(exp(2*b*x+2*a)-1)+2/b*ln(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.31

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+\left(6\cosh(bx+a)^2-1\right)\sinh(bx+a)^2-c}{2\left(b\,\mathrm{co}\right.}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
(6*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 4*(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*x +
 a)/(cosh(b*x + a) - sinh(b*x + a))) + 2*(2*cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) +
 b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{4\,(bx+a)}{b} + \frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
[Out] 4*(b*x + a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b - 1/2
*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = -\frac{\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1} - e^{(2\,bx+2\,a)} - 4\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1) - e^(2*b*x + 2*a) - 4*log(abs
(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\coth^2(a+bx)\,dx = \frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}$$

```
[In] int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
[Out] (2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2/(b*(exp(2*a + 2*b*x) - 1)) + exp(2*a
 + 2*b*x)/(2*b)
```

3.930 $\int e^{2(a+bx)} \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int e^{2(a+bx)} \coth^2(a+bx)\text{csch}(a+bx)\,dx = \frac{2e^{a+bx}}{b} - \frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{5\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b-2*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-ex
p(2*b*x+2*a))-5*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 474, 466, 396, 213}

$$\int e^{2(a+bx)} \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{5\text{arctanh}\left(e^{a+bx}\right)}{b} + \frac{2e^{a+bx}}{b} + \frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2e^{3a+3bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (2*E^(a + b*x))/b - (2*E^(3*a + 3*b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^
(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (5*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{2x^2(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{x^2(1+x^2)^2}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$

$$= -\frac{2e^{3a+3bx}}{b(1-e^{2a+2bx})^2} + \frac{\text{Subst}\left(\int \frac{x^2(8+4x^2)}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}$$

$$= -\frac{2e^{3a+3bx}}{b(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b(1-e^{2a+2bx})} - \frac{\text{Subst}\left(\int \frac{-12-8x^2}{-1+x^2}\,dx, x, e^{a+bx}\right)}{4b}$$

$$= \frac{2e^{a+bx}}{b} - \frac{2e^{3a+3bx}}{b(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b(1-e^{2a+2bx})} + \frac{5\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{2e^{a+bx}}{b} - \frac{2e^{3a+3bx}}{b(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b(1-e^{2a+2bx})} - \frac{5\text{arctanh}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.91

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx =$$

$$-\frac{e^{-3(a+bx)}\left(-21\left(56595+62725e^{2(a+bx)}-12071e^{4(a+bx)}-19353e^{6(a+bx)}+768e^{8(a+bx)}\right)+\frac{315(3773+2924e^{2(a+b}}{}\right.}{}$$

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] -1/10080*(-21*(56595 + 62725*E^(2*(a + b*x)) - 12071*E^(4*(a + b*x)) - 1935
3*E^(6*(a + b*x)) + 768*E^(8*(a + b*x))) + (315*(3773 + 2924*E^(2*(a + b*x)
) - 2534*E^(4*(a + b*x)) - 1548*E^(6*(a + b*x)) + 297*E^(8*(a + b*x)))*ArcT
anh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(9
+ 16*E^(2*(a + b*x)) + 7*E^(4*(a + b*x)))*HypergeometricPFQ[{2, 2, 2, 5/2},
 {1, 1, 11/2}, E^(2*(a + b*x))] + 128*E^(8*(a + b*x))*(1 + E^(2*(a + b*x)))
^2*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))])/
(b*E^(3*(a + b*x)))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b}-\frac{\mathrm{e}^{bx+a}\left(5\,\mathrm{e}^{2bx+2a}-3\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}}+\frac{5\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}-\frac{5\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}$	78

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(b*x+a)/b-exp(b*x+a)*(5*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^2+5/2/b
*ln(exp(b*x+a)-1)-5/2/b*ln(exp(b*x+a)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(75) = 150.

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.40

$$\int e^{2(a+bx)}\coth^2(a+bx)\mathrm{csch}(a+bx)\,dx = \frac{4\cosh\left(bx+a\right)^5+20\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+4\sinh\left(bx+a\right)^5+2\left(20\cosh\left(bx+a\right)^2-9\right)\sinh\left(bx\right.}{}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(b*x + a)^5 + 20*cosh(b*x + a)*sinh(b*x + a)^4 + 4*sinh(b*x + a)
^5 + 2*(20*cosh(b*x + a)^2 - 9)*sinh(b*x + a)^3 - 18*cosh(b*x + a)^3 + 2*(2
0*cosh(b*x + a)^3 - 27*cosh(b*x + a))*sinh(b*x + a)^2 - 5*(cosh(b*x + a)^4
+ 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 5*(cosh(b*x
 + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cos
h(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(
10*cosh(b*x + a)^4 - 27*cosh(b*x + a)^2 + 5)*sinh(b*x + a) + 10*cosh(b*x +
a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a
)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4
*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b}+\frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b}-\frac{9\,e^{(-2\,bx-2\,a)}-5\,e^{(-4\,bx-4\,a)}-2}{b(e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)})}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -5/2*log(e^(-b*x - a) + 1)/b + 5/2*log(e^(-b*x - a) - 1)/b - (9*e^(-2*b*x -
 2*a) - 5*e^(-4*b*x - 4*a) - 2)/(b*(e^(-b*x - a) - 2*e^(-3*b*x - 3*a) + e^(
-5*b*x - 5*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{2\,\left(5\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2}-4\,e^{(bx+a)}+5\,\log\left(e^{(bx+a)}+1\right)-5\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*(5*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 4*e^(
b*x + a) + 5*log(e^(b*x + a) + 1) - 5*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int e^{2(a+bx)}\coth^2(a+bx)\text{csch}(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b}-\frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{5\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)^2*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

```
[Out] (2*exp(a + b*x))/b - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2
) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (5*e
xp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.931 $\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx = \frac{e^{-4a-4bx}}{256b} - \frac{3e^{4a+4bx}}{256b} + \frac{e^{8a+8bx}}{512b} + \frac{3x}{64}$$

```
[Out] 1/256*exp(-4*b*x-4*a)/b-3/256*exp(4*b*x+4*a)/b+1/512*exp(8*b*x+8*a)/b+3/64*
x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2320, 12, 272, 45}

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx = \frac{e^{-4a-4bx}}{256b} - \frac{3e^{4a+4bx}}{256b} + \frac{e^{8a+8bx}}{512b} + \frac{3x}{64}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-4*a - 4*b*x)/(256*b) - (3*E^(4*a + 4*b*x))/(256*b) + E^(8*a + 8*b*x)/(5
12*b) + (3*x)/64
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{64x^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^4)^3}{x^5}\,dx, x, e^{a+bx}\right)}{64b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{4a+4bx}\right)}{256b}\\
&= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{4a+4bx}\right)}{256b}\\
&= \frac{e^{-4a-4bx}}{256b}-\frac{3e^{4a+4bx}}{256b}+\frac{e^{8a+8bx}}{512b}+\frac{3x}{64}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{-4(a+bx)}-3e^{4(a+bx)}+\frac{1}{2}e^{8(a+bx)}+12bx}{256b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
[Out] (E^(-4*(a + b*x)) - 3*E^(4*(a + b*x)) + E^(8*(a + b*x))/2 + 12*b*x)/(256*b)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\frac{3x}{64}-\frac{\sinh\left(4bx+4a\right)}{64b}+\frac{\sinh\left(8bx+8a\right)}{512b}-\frac{\cosh\left(4bx+4a\right)}{128b}+\frac{\cosh\left(8bx+8a\right)}{512b}$$

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)
[Out] 3/64*x-1/64/b*sinh(4*b*x+4*a)+1/512/b*sinh(8*b*x+8*a)-1/128*cosh(4*b*x+4*a)
/b+1/512*cosh(8*b*x+8*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.26

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{3\cosh\left(bx+a\right)^6-20\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)^3+45\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^4-6\cosh\left(bx+a\right)\sin}{512}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/512*(3*cosh(b*x + a)^6 - 20*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*cosh(b*x
 + a)^2*sinh(b*x + a)^4 - 6*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)
^6 + 6*(4*b*x - 1)*cosh(b*x + a)^2 + 3*(15*cosh(b*x + a)^4 + 8*b*x - 2)*sin
h(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 2*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x +
 a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(48) = 96.

Time = 12.49 (sec) , antiderivative size = 382, normalized size of antiderivative = 6.70

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{3xe^{2a}e^{2bx}\sinh^6\left(a+bx\right)}{64}-\frac{3xe^{2a}e^{2bx}\sinh^5\left(a+bx\right)\cosh\left(a+bx\right)}{32}-\frac{3xe^{2a}e^{2bx}\sinh^4\left(a+bx\right)\cosh^2\left(a+bx\right)}{64}+\frac{3xe^{2a}e^{2bx}\sinh^3\left(a+bx\right)\cosh^3\left(a-\right.}{16}\\ xe^{2a}\sinh^3\left(a\right)\cosh^3\left(a\right)\end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
[Out] Piecewise((3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**6/64 - 3*x*exp(2*a)*exp(2
*b*x)*sinh(a + b*x)**5*cosh(a + b*x)/32 - 3*x*exp(2*a)*exp(2*b*x)*sinh(a +
```

```
b*x)**4*cosh(a + b*x)**2/64 + 3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cosh
(a + b*x)**3/16 - 3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**4
/64 - 3*x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)**5/32 + 3*x*exp(2
*a)*exp(2*b*x)*cosh(a + b*x)**6/64 + 3*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**6
/(32*b) - 15*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5*cosh(a + b*x)/(64*b) + 13
*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)**3/(32*b) - 15*exp(2*a)
*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)**5/(64*b) + 3*exp(2*a)*exp(2*b*x)*c
osh(a + b*x)**6/(32*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**3*cosh(a)**3, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{\left(6\,e^{(-4\,bx-4\,a)}-1\right)e^{(8\,bx+8\,a)}}{512\,b} + \frac{3\,(bx+a)}{64\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/512*(6*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/64*(b*x + a)/b + 1/25
6*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{3}{64}\,x + \frac{e^{(8\,bx+8\,a)}}{512\,b} - \frac{3\,e^{(4\,bx+4\,a)}}{256\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*x + 1/512*e^(8*b*x + 8*a)/b - 3/256*e^(4*b*x + 4*a)/b + 1/256*e^(-4*b*
x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^3(a+bx)\, dx = \frac{3\,x}{64} + \frac{\mathrm{e}^{-4\,a-4\,b\,x}}{256\,b} - \frac{3\,\mathrm{e}^{4\,a+4\,b\,x}}{256\,b} + \frac{\mathrm{e}^{8\,a+8\,b\,x}}{512\,b}$$

```
[In] int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*x)/64 + exp(- 4*a - 4*b*x)/(256*b) - (3*exp(4*a + 4*b*x))/(256*b) + exp(
8*a + 8*b*x)/(512*b)
```

3.932 $\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 100

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^2(a+bx)\,dx = -\frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} - \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} + \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

```
[Out] -1/96*exp(-3*b*x-3*a)/b-1/32*exp(-b*x-a)/b-1/16*exp(b*x+a)/b-1/48*exp(3*b*x
+3*a)/b+1/160*exp(5*b*x+5*a)/b+1/224*exp(7*b*x+7*a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2320, 12, 459}

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^2(a+bx)\,dx = -\frac{e^{-3a-3bx}}{96b} - \frac{e^{-a-bx}}{32b} - \frac{e^{a+bx}}{16b} - \frac{e^{3a+3bx}}{48b} + \frac{e^{5a+5bx}}{160b} + \frac{e^{7a+7bx}}{224b}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] -1/96*E^(-3*a - 3*b*x)/b - E^(-a - b*x)/(32*b) - E^(a + b*x)/(16*b) - E^(3*
a + 3*b*x)/(48*b) + E^(5*a + 5*b*x)/(160*b) + E^(7*a + 7*b*x)/(224*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)^3}{32x^4}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2(1+x^2)^3}{x^4}\,dx, x, e^{a+bx}\right)}{32b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+\frac{1}{x^2}-2x^2+x^4+x^6\right)\,dx, x, e^{a+bx}\right)}{32b}\\ &= -\frac{e^{-3a-3bx}}{96b}-\frac{e^{-a-bx}}{32b}-\frac{e^{a+bx}}{16b}-\frac{e^{3a+3bx}}{48b}+\frac{e^{5a+5bx}}{160b}+\frac{e^{7a+7bx}}{224b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(-35-105e^{2(a+bx)}-210e^{4(a+bx)}-70e^{6(a+bx)}+21e^{8(a+bx)}+15e^{10(a+bx)}\right)}{3360b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-35 - 105*E^(2*(a + b*x)) - 210*E^(4*(a + b*x)) - 70*E^(6*(a + b*x)) + 21*
E^(8*(a + b*x)) + 15*E^(10*(a + b*x)))/(3360*b*E^(3*(a + b*x)))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$-\frac{\sinh(bx+a)}{32b}-\frac{\sinh(3bx+3a)}{96b}+\frac{\sinh(5bx+5a)}{160b}+\frac{\sinh(7bx+7a)}{224b}-\frac{3\cosh(bx+a)}{32b}-\frac{\cosh(3bx+3a)}{32b}+\frac{\mathrm{co}}{}$$

[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)

[Out] -1/32*sinh(b*x+a)/b-1/96/b*sinh(3*b*x+3*a)+1/160/b*sinh(5*b*x+5*a)+1/224/b*sinh(7*b*x+7*a)-3/32*cosh(b*x+a)/b-1/32*cosh(3*b*x+3*a)/b+1/160*cosh(5*b*x+5*a)/b+1/224*cosh(7*b*x+7*a)/b

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.76

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = -\frac{10\cosh(bx+a)^5+50\cosh(bx+a)\sinh(bx+a)^4-25\sinh(bx+a)^5-\left(250\cosh(bx+a)^2+63\right)\sin}{1}$$

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")

[Out] -1/1680*(10*cosh(b*x + a)^5 + 50*cosh(b*x + a)*sinh(b*x + a)^4 - 25*sinh(b*x + a)^5 - (250*cosh(b*x + a)^2 + 63)*sinh(b*x + a)^3 + 42*cosh(b*x + a)^3 + 2*(50*cosh(b*x + a)^3 + 63*cosh(b*x + a))*sinh(b*x + a)^2 - (125*cosh(b*x + a)^4 + 189*cosh(b*x + a)^2 + 70)*sinh(b*x + a) + 140*cosh(b*x + a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(76) = 152.

Time = 5.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} \frac{2e^{2a}e^{2bx}\sinh^5(a+bx)}{105b}-\frac{4e^{2a}e^{2bx}\sinh^4(a+bx)\cosh(a+bx)}{105b}-\frac{e^{2a}e^{2bx}\sinh^3(a+bx)\cosh^2(a+bx)}{105b}+\frac{2e^{2a}e^{2bx}\sinh^2(a+bx)\cosh^3(a+bx)}{35b} \\ xe^{2a}\sinh^2(a)\cosh^3(a) \end{cases}$$

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)

[Out] Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**5/(105*b) - 4*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4*cosh(a + b*x)/(105*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cosh(a + b*x)**2/(105*b) + 2*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(35*b) + 8*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cosh(a + b*x)**4/(35*b) - 4*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**5/(35*b), Ne(b, 0)), (x*exp(2*a)*sinh(a)**2*cosh(a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(21\,e^{(-2\,bx-2\,a)}-70\,e^{(-4\,bx-4\,a)}-210\,e^{(-6\,bx-6\,a)}+15\right)e^{(7\,bx+7\,a)}}{3360\,b}-\frac{3\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{96\,b}$$

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")

[Out] 1/3360*(21*e^(-2*b*x - 2*a) - 70*e^(-4*b*x - 4*a) - 210*e^(-6*b*x - 6*a) + 15)*e^(7*b*x + 7*a)/b - 1/96*(3*e^(-b*x - a) + e^(-3*b*x - 3*a))/b

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(7\,bx+7\,a)}}{224\,b}+\frac{e^{(5\,bx+5\,a)}}{160\,b}-\frac{e^{(3\,bx+3\,a)}}{48\,b}-\frac{e^{(bx+a)}}{16\,b}-\frac{e^{(-bx-a)}}{32\,b}-\frac{e^{(-3\,bx-3\,a)}}{96\,b}$$

[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")

[Out] 1/224*e^(7*b*x + 7*a)/b + 1/160*e^(5*b*x + 5*a)/b - 1/48*e^(3*b*x + 3*a)/b - 1/16*e^(b*x + a)/b - 1/32*e^(-b*x - a)/b - 1/96*e^(-3*b*x - 3*a)/b

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int e^{2(a+bx)} \cosh^3(a+bx) \sinh^2(a+bx)\,dx = -\frac{210\,\mathrm{e}^{a+b\,x}+105\,\mathrm{e}^{-a-b\,x}+35\,\mathrm{e}^{-3\,a-3\,b\,x}+70\,\mathrm{e}^{3\,a+3\,b\,x}-21\,\mathrm{e}^{5\,a+5\,b\,x}-15\,\mathrm{e}^{7\,a+7\,b\,x}}{3360\,b}$$

```
[In] int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(210*exp(a + b*x) + 105*exp(- a - b*x) + 35*exp(- 3*a - 3*b*x) + 70*exp(3*
a + 3*b*x) - 21*exp(5*a + 5*b*x) - 15*exp(7*a + 7*b*x))/(3360*b)
```

3.933 $\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

```
[Out] 1/32*exp(-2*b*x-2*a)/b+1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b-1/8*x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 12, 457, 76}

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] E^(-2*a - 2*b*x)/(32*b) + E^(4*a + 4*b*x)/(32*b) + E^(6*a + 6*b*x)/(96*b) -
 x/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
```

```
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{16x^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{16b} \\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{32b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2}{x}+2x+x^2\right)\,dx, x, e^{2a+2bx}\right)}{32b} \\
&= \frac{e^{-2a-2bx}}{32b}+\frac{e^{4a+4bx}}{32b}+\frac{e^{6a+6bx}}{96b}-\frac{x}{8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{3e^{-2(a+bx)}+3e^{4(a+bx)}+e^{6(a+bx)}-12bx}{96b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^3*Sinh[a + b*x],x]
[Out] (3/E^(2*(a + b*x)) + 3*E^(4*(a + b*x)) + E^(6*(a + b*x)) - 12*b*x)/(96*b)
```

Maple [A] (verified)

Time = 48.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{{\mathrm e}^{-2 b x -2 a}}{32 b}+\frac{{\mathrm e}^{4 b x +4 a}}{32 b}+\frac{{\mathrm e}^{6 b x +6 a}}{96 b}-\frac{x}{8}$	47
default	$-\frac{x}{8}-\frac{\sinh \left(2 b x +2 a \right)}{32 b}+\frac{\sinh \left(4 b x +4 a \right)}{32 b}+\frac{\sinh \left(6 b x +6 a \right)}{96 b}+\frac{\cosh \left(2 b x +2 a \right)}{32 b}+\frac{\cosh \left(4 b x +4 a \right)}{32 b}+\frac{\cosh \left(6 b x +6 a \right)}{96 b}$	89

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/32*exp(-2*b*x-2*a)/b+1/32*exp(4*b*x+4*a)/b+1/96*exp(6*b*x+6*a)/b-1/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.67

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{4\,\cosh\left(bx+a\right)^4-8\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+4\,\sinh\left(bx+a\right)^4-3\left(4\,bx-1\right)\cosh\left(bx+a\right)^2-3\left(4\,bx\right.}{96\left(b\cosh\left(bx+a\right)^2-2\,b\cosh\left(bx+a\right.\right.}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/96*(4*cosh(b*x + a)^4 - 8*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x + a)
^4 - 3*(4*b*x - 1)*cosh(b*x + a)^2 - 3*(4*b*x - 8*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^2 - 2*(4*cosh(b*x + a)^3 - 3*(4*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a))/(b*cosh(b*x + a)^2 - 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a
)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(44) = 88.

Time = 2.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.09

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^4\left(a+bx\right)}{8}-\frac{xe^{2a}e^{2bx}\sinh^3\left(a+bx\right)\cosh\left(a+bx\right)}{4}+\frac{xe^{2a}e^{2bx}\sinh\left(a+bx\right)\cosh^3\left(a+bx\right)}{4}-\frac{xe^{2a}e^{2bx}\cosh^4\left(a+bx\right)}{8}+\frac{e^{2a}e^{2bx}\sin}{4} \\ xe^{2a}\sinh\left(a\right)\cosh^3\left(a\right) \end{cases}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)**4/8 - x*exp(2*a)*exp(2*b*x)
*sinh(a + b*x)**3*cosh(a + b*x)/4 + x*exp(2*a)*exp(2*b*x)*sinh(a + b*x)*cos
h(a + b*x)**3/4 - x*exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/8 + exp(2*a)*exp(2
*b*x)*sinh(a + b*x)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(a + b*x)**3*cosh(a
 + b*x)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(4*b)
 + exp(2*a)*exp(2*b*x)*cosh(a + b*x)**4/(16*b), Ne(b, 0)), (x*exp(2*a)*sinh
(a)*cosh(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\left(3\,e^{(-2\,bx-2\,a)}+1\right)e^{(6\,bx+6\,a)}}{96\,b} - \frac{bx+a}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
[Out] 1/96*(3*e^(-2*b*x - 2*a) + 1)*e^(6*b*x + 6*a)/b - 1/8*(b*x + a)/b + 1/32*e^
(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(6\,bx+6\,a)}}{96\,b} + \frac{e^{(4\,bx+4\,a)}}{32\,b} + \frac{e^{(-2\,bx-2\,a)}}{32\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
[Out] -1/8*x + 1/96*e^(6*b*x + 6*a)/b + 1/32*e^(4*b*x + 4*a)/b + 1/32*e^(-2*b*x -
 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{\frac{e^{-2\,a-2\,b\,x}}{32}+\frac{e^{4\,a+4\,b\,x}}{32}+\frac{e^{6\,a+6\,b\,x}}{96}}{b} - \frac{x}{8}$$

```
[In] int(cosh(a + b*x)^3*exp(2*a + 2*b*x)*sinh(a + b*x),x)
[Out] (exp(- 2*a - 2*b*x)/32 + exp(4*a + 4*b*x)/32 + exp(6*a + 6*b*x)/96)/b - x/8
```

3.934 $\int e^{2(a+bx)} \cosh^2(a+bx) \coth(a+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int e^{2(a+bx)} \cosh^2(a+bx) \coth(a+bx) \, dx = \frac{e^{2a+2bx}}{2b} + \frac{e^{4a+4bx}}{16b} - \frac{x}{4} + \frac{\log(1-e^{2a+2bx})}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+4*a)/b-1/4*x+ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 12, 457, 84}

$$\int e^{2(a+bx)} \cosh^2(a+bx) \coth(a+bx) \, dx = \frac{e^{2a+2bx}}{2b} + \frac{e^{4a+4bx}}{16b} + \frac{\log(1-e^{2a+2bx})}{b} - \frac{x}{4}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) + E^(4*a + 4*b*x)/(16*b) - x/4 + Log[1 - E^(2*a + 2*b
*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{4x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x(-1+x^2)}\,dx, x, e^{a+bx}\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(-1+x)x}\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{8}{-1+x}-\frac{1}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{8b}\\
&= \frac{e^{2a+2bx}}{2b}+\frac{e^{4a+4bx}}{16b}-\frac{x}{4}+\frac{\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{8e^{2(a+bx)}+e^{4(a+bx)}-4bx+16\log\left(1-e^{2(a+bx)}\right)}{16b}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]^2*Coth[a + b*x],x]
[Out] (8*E^(2*(a + b*x)) + E^(4*(a + b*x)) - 4*b*x + 16*Log[1 - E^(2*(a + b*x))])
/(16*b)
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{x}{4}+\frac{e^{4bx+4a}}{16b}+\frac{e^{2bx+2a}}{2b}-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	55

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/4*x+1/16*exp(4*b*x+4*a)/b+1/2*exp(2*b*x+2*a)/b-2/b*a+1/b*ln(exp(2*b*x+2*
a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+4\right)\sinh(bx+a)^2-\cdots}{1\cdots}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
[Out] 1/16*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^2 - 4*b*x + 8*cosh(b*x + a)^2 + 4*
(cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a) + 16*log(2*sinh(b*x + a)/
(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\left(8\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{16\,b} + \frac{7\,(bx+a)}{4\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] 1/16*(8*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + 7/4*(b*x + a)/b + log(e^(
-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{4\,bx+4\,a-e^{(4\,bx+4\,a)}-8\,e^{(2\,bx+2\,a)}-16\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{16\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] -1/16*(4*b*x + 4*a - e^(4*b*x + 4*a) - 8*e^(2*b*x + 2*a) - 16*log(abs(e^(2*
b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - \frac{x}{4} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{16\,b}$$

```
[In] int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x),x)
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x/4 + exp(2*a + 2*b*x)/(2*b) + exp(4*a + 4
*b*x)/(16*b)
```

3.935 $\int e^{2(a+bx)} \cosh(a+bx) \coth^2(a+bx)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int e^{2(a+bx)} \cosh(a+bx) \coth^2(a+bx)\, dx = \frac{5e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} + \frac{2e^{a+bx}}{b(1-e^{2a+2bx})} - \frac{4\operatorname{arctanh}(e^{a+bx})}{b}$$

```
[Out] 5/2*exp(b*x+a)/b+1/6*exp(3*b*x+3*a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-4*a
rctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2320, 12, 398, 393, 212}

$$\int e^{2(a+bx)} \cosh(a+bx) \coth^2(a+bx)\, dx = -\frac{4\operatorname{arctanh}(e^{a+bx})}{b} + \frac{5e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} + \frac{2e^{a+bx}}{b(1-e^{2a+2bx})}$$

```
[In] Int[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (5*E^(a + b*x))/(2*b) + E^(3*a + 3*b*x)/(6*b) + (2*E^(a + b*x))/(b*(1 - E^(
2*a + 2*b*x))) - (4*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{2(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(5+x^2-\frac{4(1-3x^2)}{(1-x^2)^2}\right)\,dx, x, e^{a+bx}\right)}{2b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}-\frac{2\text{Subst}\left(\int \frac{1-3x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{5e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{6b}+\frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})}-\frac{4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}
\end{aligned}
$$

$$= \frac{5e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{6b} + \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{4\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.01 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.01

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx$$

$$= \frac{e^{-5(a+bx)}\left(-21\left(36015+91925e^{2(a+bx)}+61158e^{4(a+bx)}-20166e^{6(a+bx)}-15061e^{8(a+bx)}+753e^{10(a+bx)}\right)-\frac{31}{}\right.}{}$$

```
[In] Integrate[E^(2*(a + b*x))*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-21*(36015 + 91925*E^(2*(a + b*x)) + 61158*E^(4*(a + b*x)) - 20166*E^(6*(a
 + b*x)) - 15061*E^(8*(a + b*x)) + 753*E^(10*(a + b*x))) - (315*(-2401 - 53
28*E^(2*(a + b*x)) - 1821*E^(4*(a + b*x)) + 3264*E^(6*(a + b*x)) + 1149*E^(
8*(a + b*x)) - 240*E^(10*(a + b*x)) + E^(12*(a + b*x)))*ArcTanh[Sqrt[E^(2*(
a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^(2*(a + b*x
)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))
])/(60480*b*E^(5*(a + b*x)))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{e^{3bx+3a}}{6b}+\frac{5\,e^{bx+a}}{2b}-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}-\frac{2\ln\left(e^{bx+a}+1\right)}{b}+\frac{2\ln\left(e^{bx+a}-1\right)}{b}$	79

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*exp(3*b*x+3*a)/b+5/2*exp(b*x+a)/b-2/b*exp(b*x+a)/(exp(2*b*x+2*a)-1)-2/b
*ln(exp(b*x+a)+1)+2/b*ln(exp(b*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.73

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^5 + 5\cosh(bx+a)\sinh(bx+a)^4 + \sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^2+7\right)\sinh(bx+a)^3}{}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
[Out] 1/6*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 +
2*(5*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 2*(5*cosh(
b*x + a)^3 + 21*cosh(b*x + a))*sinh(b*x + a)^2 - 12*(cosh(b*x + a)^2 + 2*co
sh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) + 12*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b
*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (5*cosh(b*x + a)^4
+ 42*cosh(b*x + a)^2 - 27)*sinh(b*x + a) - 27*cosh(b*x + a))/(b*cosh(b*x +
a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{2\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{14\,e^{(-2\,bx-2\,a)} - 27\,e^{(-4\,bx-4\,a)} + 1}{6\,b\left(e^{(-3\,bx-3\,a)} - e^{(-5\,bx-5\,a)}\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
[Out] -2*log(e^(-b*x - a) + 1)/b + 2*log(e^(-b*x - a) - 1)/b + 1/6*(14*e^(-2*b*x
 - 2*a) - 27*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{12\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - e^{(3\,bx+3\,a)} - 15\,e^{(bx+a)} + 12\,\log\left(e^{(bx+a)}+1\right) - 12\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(3*b*x + 3*a) - 15*e^(b*x +
a) + 12*log(e^(b*x + a) + 1) - 12*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int e^{2(a+bx)}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{5\,e^{a+b\,x}}{2\,b} - \frac{4\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{3\,a+3\,b\,x}}{6\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x)^2,x)
```

```
[Out] (5*exp(a + b*x))/(2*b) - (4*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^
(1/2) + exp(3*a + 3*b*x)/(6*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1)
)
```

3.936 $\int e^{2(a+bx)} \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int e^{2(a+bx)} \coth^3(a+bx)\,dx = \frac{e^{2a+2bx}}{2b} - \frac{2}{b\left(1-e^{2a+2bx}\right)^2} + \frac{6}{b\left(1-e^{2a+2bx}\right)} + \frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] 1/2*exp(2*b*x+2*a)/b-2/b/(1-exp(2*b*x+2*a))^2+6/b/(1-exp(2*b*x+2*a))+3*ln(1
-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 455, 45}

$$\int e^{2(a+bx)} \coth^3(a+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{6}{b\left(1-e^{2a+2bx}\right)} - \frac{2}{b\left(1-e^{2a+2bx}\right)^2} + \frac{3\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
```

```
[Out] E^(2*a + 2*b*x)/(2*b) - 2/(b*(1 - E^(2*a + 2*b*x))^2) + 6/(b*(1 - E^(2*a +
2*b*x))) + (3*Log[1 - E^(2*a + 2*b*x)])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{(-1+x)^3}\,dx, x, e^{2a+2bx}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{8}{(-1+x)^3}+\frac{12}{(-1+x)^2}+\frac{6}{-1+x}\right)\,dx, x, e^{2a+2bx}\right)}{2b}\\
&= \frac{e^{2a+2bx}}{2b}-\frac{2}{b\left(1-e^{2a+2bx}\right)^2}+\frac{6}{b\left(1-e^{2a+2bx}\right)}+\frac{3\log\left(1-e^{2a+2bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$
\int e^{2(a+bx)}\coth^3(a+bx)\,dx=\frac{\frac{1}{2}e^{2(a+bx)}+\frac{4-6e^{2(a+bx)}}{\left(-1+e^{2(a+bx)}\right)^2}+3\log\left(1-e^{2(a+bx)}\right)}{b}
$$

```
[In] Integrate[E^(2*(a + b*x))*Coth[a + b*x]^3,x]
[Out] (E^(2*(a + b*x))/2 + (4 - 6*E^(2*(a + b*x)))/(-1 + E^(2*(a + b*x)))^2 + 3*L
og[1 - E^(2*(a + b*x))])/b
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{6a}{b}-\frac{2(3\,e^{2bx+2a}-2)}{b(e^{2bx+2a}-1)^2}+\frac{3\ln(e^{2bx+2a}-1)}{b}$	70

```
[In] int(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(2*b*x+2*a)/b-6/b*a-2*(3*exp(2*b*x+2*a)-2)/b/(exp(2*b*x+2*a)-1)^2+3/
b*ln(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.98

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \frac{\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+\left(15\cosh(bx+a)^2-2\right)\sinh(bx+a)^4}{}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
(15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 4*(5*cosh(b*
x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 12*cosh
(b*x + a)^2 - 11)*sinh(b*x + a)^2 - 11*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^4
 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2
 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x +
 a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))
) + 2*(3*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x +
 a) + 8)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*
x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^
2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \frac{6\,(bx+a)}{b} + \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{10\,e^{(-2\,bx-2\,a)} - 5\,e^{(-4\,bx-4\,a)} - 1}{2\,b\left(e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}\right)}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
[Out] 6*(b*x + a)/b + 3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 1/2
*(10*e^(-2*b*x - 2*a) - 5*e^(-4*b*x - 4*a) - 1)/(b*(e^(-2*b*x - 2*a) - 2*e^
(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = -\frac{\frac{9\,e^{(4\,bx+4\,a)} - 6\,e^{(2\,bx+2\,a)} + 1}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - e^{(2\,bx+2\,a)} - 6\,\log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{2\,b}$$

```
[In] integrate(exp(2*b*x+2*a)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*((9*e^(4*b*x + 4*a) - 6*e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)^2 -
 e^(2*b*x + 2*a) - 6*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\coth^3(a+bx)\,dx = \frac{3\,\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b} - \frac{6}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} + \frac{e^{2\,a+2\,b\,x}}{2\,b}$$

```
[In] int((cosh(a + b*x)^3*exp(2*a + 2*b*x))/sinh(a + b*x)^3,x)
```

```
[Out] (3*log(exp(2*a)*exp(2*b*x) - 1))/b - 6/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(e
xp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(2*a + 2*b*x)/(2*b)
```

3.937 $\int e^x \textbf{sech}(2x)\tanh(2x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int e^x \text{sech}(2x)\tanh(2x)\,dx = -\frac{e^{3x}}{1+e^{4x}} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

```
[Out] -exp(3*x)/(1+exp(4*x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/4*arctan(1+e
xp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+e
xp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2320, 12, 468, 303, 1176, 631, 210, 1179, 642}

$$\int e^x \text{sech}(2x)\tanh(2x)\,dx = -\frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}} - \frac{e^{3x}}{e^{4x}+1} + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}$$

```
[In] Int[E^x*Sech[2*x]*Tanh[2*x],x]
```

```
[Out] -(E^(3*x)/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Lo
g[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^2(-1+x^4)}{(1+x^4)^2}\,dx, x, e^x\right)\\
&= 2\text{Subst}\left(\int \frac{x^2(-1+x^4)}{(1+x^4)^2}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{1+e^{4x}} + \text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{1+e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)\\
&= -\frac{e^{3x}}{1+e^{4x}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}}\\
&= -\frac{e^{3x}}{1+e^{4x}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{2\sqrt{2}}\\
&= -\frac{e^{3x}}{1+e^{4x}} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}}\\
&\quad + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.37

$$\int e^x \text{sech}(2x)\tanh(2x)\,dx = \frac{2}{3}e^{3x}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{4x}\right) - 2\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{4x}\right)\right)$$

```
[In] Integrate[E^x*Sech[2*x]*Tanh[2*x],x]
[Out] (2*E^(3*x)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)] - 2*Hypergeometric2F1[
3/4, 2, 7/4, -E^(4*x)]))/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{e^{3x}}{1+e^{4x}}+2\left(\sum_{\_R=\text{RootOf}(4096\_Z^4+1)}\_R\ln\left(512\_R^3+e^x\right)\right)$
default	$\frac{\tanh(\frac{x}{2})^3-3\tanh(\frac{x}{2})^2-\tanh(\frac{x}{2})-1}{\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1}-\frac{\sqrt{2}\ln\left(\tanh(\frac{x}{2})^2+3+2\sqrt{2}\right)}{8}+\frac{\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh(\frac{x}{2})}{2+2\sqrt{2}}\right)}{4+4\sqrt{2}}+\frac{\sqrt{2}\ln\left(\tanh(\frac{x}{2})^2+3-2\right.}{8}$

```
[In] int(exp(x)*sech(2*x)*tanh(2*x),x,method=_RETURNVERBOSE)
[Out] -exp(x)^3/(exp(x)^4+1)+2*sum(_R*ln(512*_R^3+exp(x)),_R=RootOf(4096*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.30

$$\int e^x \text{sech}(2x)\tanh(2x)\,dx = -\frac{8\cosh(x)^3+24\cosh(x)^2\sinh(x)+24\cosh(x)\sinh(x)^2+8\sinh(x)^3-\left((i-1)\sqrt{2}\cosh(x)^4+(4i-\right.}{}$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/8*(8*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 24*cosh(x)*sinh(x)^2 + 8*sinh(x)
^3 - ((I - 1)*sqrt(2)*cosh(x)^4 + (4*I - 4)*sqrt(2)*cosh(x)^3*sinh(x) + (6*
I - 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 + (4*I - 4)*sqrt(2)*cosh(x)*sinh(x)^3 +
(I - 1)*sqrt(2)*sinh(x)^4 + (I - 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cosh(x
) + 2*sinh(x)) - (-(I + 1)*sqrt(2)*cosh(x)^4 - (4*I + 4)*sqrt(2)*cosh(x)^3*
sinh(x) - (6*I + 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 - (4*I + 4)*sqrt(2)*cosh(x)
*sinh(x)^3 - (I + 1)*sqrt(2)*sinh(x)^4 - (I + 1)*sqrt(2))*log(-(I - 1)*sqrt
(2) + 2*cosh(x) + 2*sinh(x)) - ((I + 1)*sqrt(2)*cosh(x)^4 + (4*I + 4)*sqrt(
2)*cosh(x)^3*sinh(x) + (6*I + 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 + (4*I + 4)*sq
rt(2)*cosh(x)*sinh(x)^3 + (I + 1)*sqrt(2)*sinh(x)^4 + (I + 1)*sqrt(2))*log(
(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (-(I - 1)*sqrt(2)*cosh(x)^4 - (4
*I - 4)*sqrt(2)*cosh(x)^3*sinh(x) - (6*I - 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 -
 (4*I - 4)*sqrt(2)*cosh(x)*sinh(x)^3 - (I - 1)*sqrt(2)*sinh(x)^4 - (I - 1)*
sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)))/(cosh(x)^4 + 4*cosh
(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 1
)
```

Sympy [F]

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \int e^x \tanh(2x)\operatorname{sech}(2x)\,dx$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x)
```

```
[Out] Integral(exp(x)*tanh(2*x)*sech(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\begin{aligned}\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx =& \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\&+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\&-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)\\&+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{e^{(3x)}}{e^{(4x)}+1}\end{aligned}$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int e^x \text{sech}(2x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^{(3x)}}{e^{(4x)}+1}$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int e^x \text{sech}(2x)\tanh(2x)\,dx = -\frac{e^{3x}}{e^{4x}+1} + \sqrt{2}\,\ln\left(1+\sqrt{2}\,e^x\left(-\frac{1}{2}-\frac{1}{2}i\right)\right)\left(\frac{1}{8}+\frac{1}{8}i\right) + \sqrt{2}\,\ln\left(1+\sqrt{2}\,e^x\left(-\frac{1}{2}+\frac{1}{2}i\right)\right)\left(\frac{1}{8}-\frac{1}{8}i\right) + \sqrt{2}\,\ln\left(1+\sqrt{2}\,e^x\left(\frac{1}{2}-\frac{1}{2}i\right)\right)\left(-\frac{1}{8}+\frac{1}{8}i\right) + \sqrt{2}\,\ln\left(1+\sqrt{2}\,e^x\left(\frac{1}{2}+\frac{1}{2}i\right)\right)\left(-\frac{1}{8}-\frac{1}{8}i\right)$$

```
[In] int((tanh(2*x)*exp(x))/cosh(2*x),x)
```

```
[Out] 2^(1/2)*log(1 - 2^(1/2)*exp(x)*(1/2 + 1i/2))*(1/8 + 1i/8) + 2^(1/2)*log(1 -
 2^(1/2)*exp(x)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(1/
2 - 1i/2) + 1)*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(1/2 + 1i/2) + 1)*
(1/8 + 1i/8) - exp(3*x)/(exp(4*x) + 1)
```

3.938 $\int e^x \mathbf{sech}^2(2x)\tanh(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 129

$$\int e^x \text{sech}^2(2x)\tanh(2x)\,dx = -\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

```
[Out] -exp(5*x)/(1+exp(4*x))^2-1/4*exp(x)/(1+exp(4*x))+1/16*arctan(-1+exp(x)*2^(1
/2))*2^(1/2)+1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)-exp(x
)*2^(1/2))*2^(1/2)+1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2320, 12, 468, 294, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \text{sech}^2(2x)\tanh(2x)\,dx = -\frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\arctan\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}} - \frac{e^x}{4(e^{4x}+1)} - \frac{e^{5x}}{(e^{4x}+1)^2} - \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}} + \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}$$

```
[In] Int[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

```
[Out] -(E^(5*x)/(1 + E^(4*x))^2) - E^x/(4*(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x
]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*E^x +
 E^(2*x)]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{4x^4(-1+x^4)}{(1+x^4)^3}\,dx, x, e^x\right)\\
&= 4\text{Subst}\left(\int \frac{x^4(-1+x^4)}{(1+x^4)^3}\,dx, x, e^x\right)\\
&= -\frac{e^{5x}}{(1+e^{4x})^2}+\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2}\,dx, x, e^x\right)\\
&= -\frac{e^{5x}}{(1+e^{4x})^2}-\frac{e^x}{4\,(1+e^{4x})}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= -\frac{e^{5x}}{(1+e^{4x})^2}-\frac{e^x}{4\,(1+e^{4x})}+\frac{1}{8}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} + \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) \\
&\quad + \frac{1}{16}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}} \\
&= -\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{8\sqrt{2}} \\
&= -\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} \\
&\quad - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx &= \frac{1}{32}\left(\frac{32e^x}{(1+e^{4x})^2} - \frac{40e^x}{1+e^{4x}} - 2\sqrt{2}\arctan\left(1-\sqrt{2}e^x\right)\right. \\
&\quad + 2\sqrt{2}\arctan\left(1+\sqrt{2}e^x\right) - \sqrt{2}\log\left(1-\sqrt{2}e^x+e^{2x}\right) \\
&\quad \left. + \sqrt{2}\log\left(1+\sqrt{2}e^x+e^{2x}\right)\right)
\end{aligned}
$$

```
[In] Integrate[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

```
[Out] ((32*E^x)/(1 + E^(4*x))^2 - (40*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E
^x + E^(2*x)] + Sqrt[2]*Log[1 + Sqrt[2]*E^x + E^(2*x)])/32
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

method	result
risch	$-\frac{e^{x}(5\,e^{4x}+1)}{4(1+e^{4x})^{2}}+4\left(\sum_{\_R=\operatorname{RootOf}(16777216\_Z^{4}+1)}\_R\ln\left(e^{x}+64\_R\right)\right)$
default	$\frac{\frac{\tanh(\frac{x}{2})^{7}}{4}-\frac{17\tanh(\frac{x}{2})^{6}}{4}-\frac{11\tanh(\frac{x}{2})^{5}}{4}-\frac{57\tanh(\frac{x}{2})^{4}}{4}+\frac{11\tanh(\frac{x}{2})^{3}}{4}-\frac{19\tanh(\frac{x}{2})^{2}}{4}-\frac{\tanh(\frac{x}{2})}{4}-\frac{3}{4}}{\left(\tanh(\frac{x}{2})^{4}+6\tanh(\frac{x}{2})^{2}+1\right)^{2}}+\frac{\sqrt{2}\,\ln\left(\tanh(\frac{x}{2})^{2}+3+2\sqrt{2}\right)}{32}+$

```
[In] int(exp(x)*sech(2*x)^2*tanh(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(x)*(5*exp(4*x)+1)/(1+exp(4*x))^2+4*sum(_R*ln(exp(x)+64*_R),_R=Root
Of(16777216*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 882, normalized size of antiderivative = 6.84

$$\int e^x\text{sech}^2(2x)\tanh(2x)\,dx=\text{Too large to display}$$

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="fricas")
```

```
[Out] -1/32*(40*cosh(x)^5 + 400*cosh(x)^3*sinh(x)^2 + 400*cosh(x)^2*sinh(x)^3 + 2
00*cosh(x)*sinh(x)^4 + 40*sinh(x)^5 - ((I + 1)*sqrt(2)*cosh(x)^8 + (56*I +
56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 +
 (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*sinh(x)^8 - 2*(-(35*
I + 35)*sqrt(2)*cosh(x)^4 - (I + 1)*sqrt(2))*sinh(x)^4 + (2*I + 2)*sqrt(2)*
cosh(x)^4 - 8*(-(7*I + 7)*sqrt(2)*cosh(x)^5 - (I + 1)*sqrt(2)*cosh(x))*sinh
(x)^3 - 4*(-(7*I + 7)*sqrt(2)*cosh(x)^6 - (3*I + 3)*sqrt(2)*cosh(x)^2)*sinh
(x)^2 - 8*(-(I + 1)*sqrt(2)*cosh(x)^7 - (I + 1)*sqrt(2)*cosh(x)^3)*sinh(x)
+ (I + 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (-(I - 1)
*sqrt(2)*cosh(x)^8 - (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (28*I - 28)*
sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 - (I - 1)
*sqrt(2)*sinh(x)^8 - 2*((35*I - 35)*sqrt(2)*cosh(x)^4 + (I - 1)*sqrt(2))*si
nh(x)^4 - (2*I - 2)*sqrt(2)*cosh(x)^4 - 8*((7*I - 7)*sqrt(2)*cosh(x)^5 + (I
 - 1)*sqrt(2)*cosh(x))*sinh(x)^3 - 4*((7*I - 7)*sqrt(2)*cosh(x)^6 + (3*I -
3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 - 8*((I - 1)*sqrt(2)*cosh(x)^7 + (I - 1)*sq
rt(2)*cosh(x)^3)*sinh(x) - (I - 1)*sqrt(2))*log(-(I - 1)*sqrt(2) + 2*cosh(x
) + 2*sinh(x)) - ((I - 1)*sqrt(2)*cosh(x)^8 + (56*I - 56)*sqrt(2)*cosh(x)^3
*sinh(x)^5 + (28*I - 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 + (8*I - 8)*sqrt(2)*co
sh(x)*sinh(x)^7 + (I - 1)*sqrt(2)*sinh(x)^8 - 2*(-(35*I - 35)*sqrt(2)*cosh(
x)^4 - (I - 1)*sqrt(2))*sinh(x)^4 + (2*I - 2)*sqrt(2)*cosh(x)^4 - 8*(-(7*I
- 7)*sqrt(2)*cosh(x)^5 - (I - 1)*sqrt(2)*cosh(x))*sinh(x)^3 - 4*(-(7*I - 7)
```

```
*sqrt(2)*cosh(x)^6 - (3*I - 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 - 8*(-(I - 1)*s
qrt(2)*cosh(x)^7 - (I - 1)*sqrt(2)*cosh(x)^3)*sinh(x) + (I - 1)*sqrt(2))*lo
g((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (-(I + 1)*sqrt(2)*cosh(x)^8 -
(56*I + 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (28*I + 28)*sqrt(2)*cosh(x)^2*sin
h(x)^6 - (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 - (I + 1)*sqrt(2)*sinh(x)^8 -
2*((35*I + 35)*sqrt(2)*cosh(x)^4 + (I + 1)*sqrt(2))*sinh(x)^4 - (2*I + 2)*s
qrt(2)*cosh(x)^4 - 8*((7*I + 7)*sqrt(2)*cosh(x)^5 + (I + 1)*sqrt(2)*cosh(x)
)*sinh(x)^3 - 4*((7*I + 7)*sqrt(2)*cosh(x)^6 + (3*I + 3)*sqrt(2)*cosh(x)^2)
*sinh(x)^2 - 8*((I + 1)*sqrt(2)*cosh(x)^7 + (I + 1)*sqrt(2)*cosh(x)^3)*sinh
(x) - (I + 1)*sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 8*(2
5*cosh(x)^4 + 1)*sinh(x) + 8*cosh(x))/(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 +
 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4
 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x))*sinh(x)^3 + 4*(7*
cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh(x)^3)*sinh(x) + 1)
```

Sympy [F]

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = \int e^x \tanh(2x)\operatorname{sech}^2(2x)\,dx$$

```
[In] integrate(exp(x)*sech(2*x)**2*tanh(2*x),x)
[Out] Integral(exp(x)*tanh(2*x)*sech(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\begin{aligned}\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx &= \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) \\ &\quad+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) \\ &\quad+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) \\ &\quad-\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{5e^{(5x)}+e^x}{4\left(e^{(8x)}+2e^{(4x)}+1\right)}\end{aligned}$$

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="maxima")
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/(e^
(8*x) + 2*e^(4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int e^x \text{sech}^2(2x)\tanh(2x)\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{5e^{(5x)}+e^x}{4\left(e^{(4x)}+1\right)^2}$$

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/(e^
(4*x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int e^x \text{sech}^2(2x)\tanh(2x)\,dx = -\frac{\frac{e^{5x}}{2}-\frac{e^x}{2}}{2e^{4x}+e^{8x}+1} - \frac{3e^x}{4\left(e^{4x}+1\right)} + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(-\frac{1}{8}-\frac{1}{8}i\right)\right)\left(\frac{1}{32}+\frac{1}{32}i\right) + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(-\frac{1}{8}+\frac{1}{8}i\right)\right)\left(\frac{1}{32}-\frac{1}{32}i\right) + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(\frac{1}{8}-\frac{1}{8}i\right)\right)\left(-\frac{1}{32}+\frac{1}{32}i\right) + \sqrt{2}\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(\frac{1}{8}+\frac{1}{8}i\right)\right)\left(-\frac{1}{32}-\frac{1}{32}i\right)$$

```
[In] int((tanh(2*x)*exp(x))/cosh(2*x)^2,x)
```

```
[Out] 2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 + 1i/8))*(1/32 + 1i/32) - (3*exp(x))/
(4*(exp(4*x) + 1)) - (exp(5*x)/2 - exp(x)/2)/(2*exp(4*x) + exp(8*x) + 1) +
2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 - 1i/8))*(1/32 - 1i/32) - 2^(1/2)*log
(2^(1/2)*(1/8 - 1i/8) - exp(x)/4)*(1/32 - 1i/32) - 2^(1/2)*log(2^(1/2)*(1/8
 + 1i/8) - exp(x)/4)*(1/32 + 1i/32)
```

3.939 $\int e^x \textbf{sech}(2x)\tanh^2(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 130

$$\int e^x\text{sech}(2x)\tanh^2(2x)\,dx=\frac{e^{3x}}{(1+e^{4x})^2}-\frac{3e^{3x}}{4(1+e^{4x})}-\frac{5\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{5\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

```
[Out] exp(3*x)/(1+exp(4*x))^2-3/4*exp(3*x)/(1+exp(4*x))+5/16*arctan(-1+exp(x)*2^(
1/2))*2^(1/2)+5/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+5/32*ln(1+exp(2*x)-exp(
x)*2^(1/2))*2^(1/2)-5/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2320, 12, 474, 468, 303, 1176, 631, 210, 1179, 642}

$$\int e^x\text{sech}(2x)\tanh^2(2x)\,dx=-\frac{5\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}+\frac{5\arctan\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}-\frac{3e^{3x}}{4(e^{4x}+1)}+\frac{e^{3x}}{(e^{4x}+1)^2}+\frac{5\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{5\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}$$

```
[In] Int[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

```
[Out] E^(3*x)/(1 + E^(4*x))^2 - (3*E^(3*x))/(4*(1 + E^(4*x))) - (5*ArcTan[1 - Sqr
t[2]*E^x])/(8*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*E^x])/(8*Sqrt[2]) + (5*Log[1
 - Sqrt[2]*E^x + E^(2*x)])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*E^x + E^(2*x)]
)/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^2(1-x^4)^2}{(1+x^4)^3}\,dx, x, e^x\right)\\
&= 2\text{Subst}\left(\int \frac{x^2(1-x^4)^2}{(1+x^4)^3}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{(1+e^{4x})^2} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2(4-8x^4)}{(1+x^4)^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} + \frac{5}{4}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)
\end{aligned}$$

$$= \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} - \frac{5}{8}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{5}{8}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} + \frac{5}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{5}{16}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{5\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}} + \frac{5\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}}$$

$$= \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} + \frac{5\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} + \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{5\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{8\sqrt{2}}$$

$$= \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} - \frac{5\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

$$\int e^x \text{sech}(2x)\tanh^2(2x)\,dx = \frac{e^{3x}-3e^{7x}}{4(1+e^{4x})^2} - \frac{5}{16}\text{RootSum}\left[1+\#1^4\&, \frac{x-\log\left(e^x-\#1\right)}{\#1}\&\right]$$

```
[In] Integrate[E^x*Sech[2*x]*Tanh[2*x]^2,x]
[Out] (E^(3*x) - 3*E^(7*x))/(4*(1 + E^(4*x))^2) - (5*RootSum[1 + #1^4 & , (x - Lo
g[E^x - #1])/#1 & ])/16
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.37

method	result
risch	$-\frac{e^{3x}(3\,e^{4x}-1)}{4(1+e^{4x})^2}+2\left(\sum_{\_R=\mathrm{RootOf}(1048576\_Z^4+625)}\_R\ln\left(e^x+\frac{32768\_R^3}{125}\right)\right)$
default	$\frac{\frac{5\tanh(\frac{x}{2})^7}{4}+\frac{5\tanh(\frac{x}{2})^6}{4}+\frac{9\tanh(\frac{x}{2})^5}{4}-\frac{19\tanh(\frac{x}{2})^4}{4}-\frac{9\tanh(\frac{x}{2})^3}{4}-\frac{17\tanh(\frac{x}{2})^2}{4}-\frac{5\tanh(\frac{x}{2})}{4}-\frac{1}{4}}{\left(\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1\right)^2}+\frac{5\sqrt{2}\ln\left(\tanh(\frac{x}{2})^2+3-2\sqrt{2}\right)}{32}-$

[In] int(exp(x)*sech(2*x)*tanh(2*x)^2,x,method=_RETURNVERBOSE)

[Out] -1/4*exp(x)^3*(3*exp(x)^4-1)/(exp(x)^4+1)^2+2*sum(_R*ln(exp(x)+32768/125*_R^3),_R=RootOf(1048576*_Z^4+625))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 920, normalized size of antiderivative = 7.08

$$\int e^x \mathrm{sech}(2x)\tanh^2(2x)\,dx = \text{Too large to display}$$

[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="fricas")

[Out] -1/32*(24*cosh(x)^7 + 840*cosh(x)^3*sinh(x)^4 + 504*cosh(x)^2*sinh(x)^5 + 168*cosh(x)*sinh(x)^6 + 24*sinh(x)^7 + 8*(105*cosh(x)^4 - 1)*sinh(x)^3 - 8*cosh(x)^3 + 24*(21*cosh(x)^5 - cosh(x))*sinh(x)^2 + 5*(-(I - 1)*sqrt(2)*cosh(x)^8 - (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (28*I - 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 - (I - 1)*sqrt(2)*sinh(x)^8 + 2*(-(35*I - 35)*sqrt(2)*cosh(x)^4 - (I - 1)*sqrt(2))*sinh(x)^4 - (2*I - 2)*sqrt(2)*cosh(x)^4 + 8*(-(7*I - 7)*sqrt(2)*cosh(x)^5 - (I - 1)*sqrt(2)*cosh(x))*sinh(x)^3 + 4*(-(7*I - 7)*sqrt(2)*cosh(x)^6 - (3*I - 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(-(I - 1)*sqrt(2)*cosh(x)^7 - (I - 1)*sqrt(2)*cosh(x)^3)*sinh(x) - (I - 1)*sqrt(2))*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 5*((I + 1)*sqrt(2)*cosh(x)^8 + (56*I + 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 + (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*sinh(x)^8 + 2*((35*I + 35)*sqrt(2)*cosh(x)^4 + (I + 1)*sqrt(2))*sinh(x)^4 + (2*I + 2)*sqrt(2)*cosh(x)^4 + 8*((7*I + 7)*sqrt(2)*cosh(x)^5 + (I + 1)*sqrt(2)*cosh(x))*sinh(x)^3 + 4*((7*I + 7)*sqrt(2)*cosh(x)^6 + (3*I + 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*((I + 1)*sqrt(2)*cosh(x)^7 + (I + 1)*sqrt(2)*cosh(x)^3)*sinh(x) + (I + 1)*sqrt(2))*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 5*(-(I + 1)*sqrt(2)*cosh(x)^8 - (56*I + 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 - (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 - (I + 1)*sqrt(2)*sinh(x)^8 + 2*(-(35*I + 35)*sqrt(2)*cosh(x)^4 - (I + 1)*sqrt(2))*sinh(x)^4 - (2*I + 2)*sqrt(2)*c

```
osh(x)^4 + 8*(-(7*I + 7)*sqrt(2)*cosh(x)^5 - (I + 1)*sqrt(2)*cosh(x))*sinh(
x)^3 + 4*(-(7*I + 7)*sqrt(2)*cosh(x)^6 - (3*I + 3)*sqrt(2)*cosh(x)^2)*sinh(
x)^2 + 8*(-(I + 1)*sqrt(2)*cosh(x)^7 - (I + 1)*sqrt(2)*cosh(x)^3)*sinh(x) -
 (I + 1)*sqrt(2))*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 5*((I - 1)
*sqrt(2)*cosh(x)^8 + (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I - 28)*
sqrt(2)*cosh(x)^2*sinh(x)^6 + (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I - 1)
*sqrt(2)*sinh(x)^8 + 2*((35*I - 35)*sqrt(2)*cosh(x)^4 + (I - 1)*sqrt(2))*si
nh(x)^4 + (2*I - 2)*sqrt(2)*cosh(x)^4 + 8*((7*I - 7)*sqrt(2)*cosh(x)^5 + (I
 - 1)*sqrt(2)*cosh(x))*sinh(x)^3 + 4*((7*I - 7)*sqrt(2)*cosh(x)^6 + (3*I -
3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*((I - 1)*sqrt(2)*cosh(x)^7 + (I - 1)*sq
rt(2)*cosh(x)^3)*sinh(x) + (I - 1)*sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x
) + 2*sinh(x)) + 24*(7*cosh(x)^6 - cosh(x)^2)*sinh(x))/(cosh(x)^8 + 56*cosh
(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
+ 2*(35*cosh(x)^4 + 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(7*cosh(x)^5 + cosh(x))*
sinh(x)^3 + 4*(7*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 + cosh(x
)^3)*sinh(x) + 1)
```

Sympy [F]

$$\int e^x \text{sech}(2x)\tanh^2(2x)\,dx = \int e^x \tanh^2(2x)\operatorname{sech}(2x)\,dx$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)**2,x)
[Out] Integral(exp(x)*tanh(2*x)**2*sech(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\begin{aligned}\int e^x \text{sech}(2x)\tanh^2(2x)\,dx =& \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) \\ &+ \frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) \\ &- \frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) \\ &+ \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3e^{(7x)}-e^{(3x)}}{4\left(e^{(8x)}+2e^{(4x)}+1\right)}\end{aligned}$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="maxima")
[Out] 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*x))
/(e^(8*x) + 2*e^(4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3e^{(7x)}-e^{(3x)}}{4\left(e^{(4x)}+1\right)^2}$$

```
[In] integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*x))
/(e^(4*x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \frac{e^{3x}}{2e^{4x}+e^{8x}+1} - \frac{3e^{3x}}{4\left(e^{4x}+1\right)} + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(-\frac{25}{32}-\frac{25}{32}i\right)\right)\left(\frac{5}{32}+\frac{5}{32}i\right) + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(-\frac{25}{32}+\frac{25}{32}i\right)\right)\left(\frac{5}{32}-\frac{5}{32}i\right) + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(\frac{25}{32}-\frac{25}{32}i\right)\right)\left(-\frac{5}{32}+\frac{5}{32}i\right) + \sqrt{2}\ln\left(\frac{25}{16}+\sqrt{2}e^x\left(\frac{25}{32}+\frac{25}{32}i\right)\right)\left(-\frac{5}{32}-\frac{5}{32}i\right)$$

```
[In] int((tanh(2*x)^2*exp(x))/cosh(2*x),x)
```

```
[Out] 2^(1/2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 + 25i/32))*(5/32 + 5i/32) + 2^(1/
2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 - 25i/32))*(5/32 - 5i/32) - 2^(1/2)*lo
g(2^(1/2)*exp(x)*(25/32 - 25i/32) + 25/16)*(5/32 - 5i/32) - 2^(1/2)*log(2^(
1/2)*exp(x)*(25/32 + 25i/32) + 25/16)*(5/32 + 5i/32) + exp(3*x)/(2*exp(4*x)
 + exp(8*x) + 1) - (3*exp(3*x))/(4*(exp(4*x) + 1))
```

3.940 $\int e^x \mathbf{sech}^2(2x) \tanh^2(2x) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int e^x \operatorname{sech}^2(2x) \tanh^2(2x) \, dx = \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} - \frac{3\arctan\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}e^x\right)}{16\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}}$$

```
[Out] 4/3*exp(5*x)/(1+exp(4*x))^3-5/6*exp(5*x)/(1+exp(4*x))^2-3/8*exp(x)/(1+exp(4
*x))+3/32*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+3/32*arctan(1+exp(x)*2^(1/2))*2
^(1/2)-3/64*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)+3/64*ln(1+exp(2*x)+exp(x)
*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {2320, 12, 474, 468, 294, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \operatorname{sech}^2(2x) \tanh^2(2x) \, dx = -\frac{3\arctan\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}} + \frac{3\arctan\left(\sqrt{2}e^x+1\right)}{16\sqrt{2}} - \frac{3e^x}{8(e^{4x}+1)} - \frac{5e^{5x}}{6(e^{4x}+1)^2} + \frac{4e^{5x}}{3(e^{4x}+1)^3} - \frac{3\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{32\sqrt{2}} + \frac{3\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{32\sqrt{2}}$$

```
[In] Int[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

```
[Out] (4*E^(5*x))/(3*(1 + E^(4*x))^3) - (5*E^(5*x))/(6*(1 + E^(4*x))^2) - (3*E^x)
/(8*(1 + E^(4*x))) - (3*ArcTan[1 - Sqrt[2]*E^x])/(16*Sqrt[2]) + (3*ArcTan[1
 + Sqrt[2]*E^x])/(16*Sqrt[2]) - (3*Log[1 - Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt
[2]) + (3*Log[1 + Sqrt[2]*E^x + E^(2*x)])/(32*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{4x^4(1-x^4)^2}{(1+x^4)^4}\,dx, x, e^x\right)$$

$$= 4\text{Subst}\left(\int \frac{x^4(1-x^4)^2}{(1+x^4)^4}\,dx, x, e^x\right)$$

$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{1}{3}\text{Subst}\left(\int \frac{x^4(8-12x^4)}{(1+x^4)^3}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} + \frac{3}{16}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{3}{16}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} + \frac{3}{32}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{3}{32}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{32\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{32\sqrt{2}}$$
$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} - \frac{3\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{16\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{16\sqrt{2}}$$
$$= \frac{4e^{5x}}{3(1+e^{4x})^3} - \frac{5e^{5x}}{6(1+e^{4x})^2} - \frac{3e^x}{8(1+e^{4x})} - \frac{3\arctan\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}e^x\right)}{16\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{32\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.43

$$\int e^x \text{sech}^2(2x)\tanh^2(2x)\,dx = \frac{1}{96}\left(-\frac{4e^x(9+6e^{4x}+29e^{8x})}{(1+e^{4x})^3} - 9\text{RootSum}\left[1+\#1^4\&, \frac{x-\log(e^x-\#1)}{\#1^3}\&\right]\right)$$

```
[In] Integrate[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

```
[Out] ((-4*E^x*(9 + 6*E^(4*x) + 29*E^(8*x)))/(1 + E^(4*x))^3 - 9*RootSum[1 + #1^4
 & , (x - Log[E^x - #1])/#1^3 & ])/96
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

method	result
risch	$-\frac{e^{x} \left(29 e^{8 x}+6 e^{4 x}+9\right)}{24 \left(1+e^{4 x}\right)^{3}}+4 \left(\sum_{\_R =\operatorname{RootOf}\left(268435456 \_Z^{4}+81\right)}\_R \ln \left(e^{x}+\frac{128 \_R}{3}\right)\right)$
default	$\frac{\frac{3 \tanh \left(\frac{x}{2}\right)^{11}}{8}-\frac{3 \tanh \left(\frac{x}{2}\right)^{10}}{8}-\frac{109 \tanh \left(\frac{x}{2}\right)^{9}}{24}-\frac{173 \tanh \left(\frac{x}{2}\right)^{8}}{8}-\frac{49 \tanh \left(\frac{x}{2}\right)^{7}}{4}-\frac{231 \tanh \left(\frac{x}{2}\right)^{6}}{4}+\frac{49 \tanh \left(\frac{x}{2}\right)^{5}}{4}-\frac{117 \tanh \left(\frac{x}{2}\right)^{4}}{4}+\frac{109 \tanh \left(\frac{x}{2}\right)^{3}}{24}-}{\left(\tanh \left(\frac{x}{2}\right)^{4}+6 \tanh \left(\frac{x}{2}\right)^{2}+1\right)^{3}}$

```
[In] int(exp(x)*sech(2*x)^2*tanh(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*exp(x)*(29*exp(8*x)+6*exp(4*x)+9)/(1+exp(4*x))^3+4*sum(_R*ln(exp(x)+1
28/3*_R),_R=RootOf(268435456*_Z^4+81))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1616, normalized size of antiderivative = 10.85

$$\int e^{x} \operatorname{sech}^{2}(2x) \tanh^{2}(2x)\, dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(232*cosh(x)^9 + 19488*cosh(x)^3*sinh(x)^6 + 8352*cosh(x)^2*sinh(x)^
7 + 2088*cosh(x)*sinh(x)^8 + 232*sinh(x)^9 + 48*(609*cosh(x)^4 + 1)*sinh(x)
^5 + 48*cosh(x)^5 + 48*(609*cosh(x)^5 + 5*cosh(x))*sinh(x)^4 + 96*(203*cosh
(x)^6 + 5*cosh(x)^2)*sinh(x)^3 + 96*(87*cosh(x)^7 + 5*cosh(x)^3)*sinh(x)^2
+ 9*(-(I + 1)*sqrt(2)*cosh(x)^12 - (220*I + 220)*sqrt(2)*cosh(x)^3*sinh(x)^
9 - (66*I + 66)*sqrt(2)*cosh(x)^2*sinh(x)^10 - (12*I + 12)*sqrt(2)*cosh(x)*
sinh(x)^11 - (I + 1)*sqrt(2)*sinh(x)^12 + 3*(-(165*I + 165)*sqrt(2)*cosh(x)
^4 - (I + 1)*sqrt(2))*sinh(x)^8 - (3*I + 3)*sqrt(2)*cosh(x)^8 + 24*(-(33*I
+ 33)*sqrt(2)*cosh(x)^5 - (I + 1)*sqrt(2)*cosh(x))*sinh(x)^7 + 84*(-(11*I +
 11)*sqrt(2)*cosh(x)^6 - (I + 1)*sqrt(2)*cosh(x)^2)*sinh(x)^6 + 24*(-(33*I
+ 33)*sqrt(2)*cosh(x)^7 - (7*I + 7)*sqrt(2)*cosh(x)^3)*sinh(x)^5 + 3*(-(165
*I + 165)*sqrt(2)*cosh(x)^8 - (70*I + 70)*sqrt(2)*cosh(x)^4 - (I + 1)*sqrt(
2))*sinh(x)^4 - (3*I + 3)*sqrt(2)*cosh(x)^4 + 4*(-(55*I + 55)*sqrt(2)*cosh(
```

x)^9 - (42*I + 42)*sqrt(2)*cosh(x)^5 - (3*I + 3)*sqrt(2)*cosh(x))*sinh(x)^3
 + 6*(-(11*I + 11)*sqrt(2)*cosh(x)^10 - (14*I + 14)*sqrt(2)*cosh(x)^6 - (3*
I + 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 12*(-(I + 1)*sqrt(2)*cosh(x)^11 - (2*
I + 2)*sqrt(2)*cosh(x)^7 - (I + 1)*sqrt(2)*cosh(x)^3)*sinh(x) - (I + 1)*sqr
t(2))*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 9*((I - 1)*sqrt(2)*cos
h(x)^12 + (220*I - 220)*sqrt(2)*cosh(x)^3*sinh(x)^9 + (66*I - 66)*sqrt(2)*c
osh(x)^2*sinh(x)^10 + (12*I - 12)*sqrt(2)*cosh(x)*sinh(x)^11 + (I - 1)*sqrt
(2)*sinh(x)^12 + 3*((165*I - 165)*sqrt(2)*cosh(x)^4 + (I - 1)*sqrt(2))*sinh
(x)^8 + (3*I - 3)*sqrt(2)*cosh(x)^8 + 24*((33*I - 33)*sqrt(2)*cosh(x)^5 + (
I - 1)*sqrt(2)*cosh(x))*sinh(x)^7 + 84*((11*I - 11)*sqrt(2)*cosh(x)^6 + (I
 - 1)*sqrt(2)*cosh(x)^2)*sinh(x)^6 + 24*((33*I - 33)*sqrt(2)*cosh(x)^7 + (7*
I - 7)*sqrt(2)*cosh(x)^3)*sinh(x)^5 + 3*((165*I - 165)*sqrt(2)*cosh(x)^8 +
(70*I - 70)*sqrt(2)*cosh(x)^4 + (I - 1)*sqrt(2))*sinh(x)^4 + (3*I - 3)*sqrt
(2)*cosh(x)^4 + 4*((55*I - 55)*sqrt(2)*cosh(x)^9 + (42*I - 42)*sqrt(2)*cosh
(x)^5 + (3*I - 3)*sqrt(2)*cosh(x))*sinh(x)^3 + 6*((11*I - 11)*sqrt(2)*cosh(
x)^10 + (14*I - 14)*sqrt(2)*cosh(x)^6 + (3*I - 3)*sqrt(2)*cosh(x)^2)*sinh(x
)^2 + 12*((I - 1)*sqrt(2)*cosh(x)^11 + (2*I - 2)*sqrt(2)*cosh(x)^7 + (I - 1
)*sqrt(2)*cosh(x)^3)*sinh(x) + (I - 1)*sqrt(2))*log(-(I - 1)*sqrt(2) + 2*co
sh(x) + 2*sinh(x)) + 9*(-(I - 1)*sqrt(2)*cosh(x)^12 - (220*I - 220)*sqrt(2)
*cosh(x)^3*sinh(x)^9 - (66*I - 66)*sqrt(2)*cosh(x)^2*sinh(x)^10 - (12*I - 1
2)*sqrt(2)*cosh(x)*sinh(x)^11 - (I - 1)*sqrt(2)*sinh(x)^12 + 3*(-(165*I - 1
65)*sqrt(2)*cosh(x)^4 - (I - 1)*sqrt(2))*sinh(x)^8 - (3*I - 3)*sqrt(2)*cosh
(x)^8 + 24*(-(33*I - 33)*sqrt(2)*cosh(x)^5 - (I - 1)*sqrt(2)*cosh(x))*sinh(
x)^7 + 84*(-(11*I - 11)*sqrt(2)*cosh(x)^6 - (I - 1)*sqrt(2)*cosh(x)^2)*sinh
(x)^6 + 24*(-(33*I - 33)*sqrt(2)*cosh(x)^7 - (7*I - 7)*sqrt(2)*cosh(x)^3)*s
inh(x)^5 + 3*(-(165*I - 165)*sqrt(2)*cosh(x)^8 - (70*I - 70)*sqrt(2)*cosh(x
)^4 - (I - 1)*sqrt(2))*sinh(x)^4 - (3*I - 3)*sqrt(2)*cosh(x)^4 + 4*(-(55*I
 - 55)*sqrt(2)*cosh(x)^9 - (42*I - 42)*sqrt(2)*cosh(x)^5 - (3*I - 3)*sqrt(2)
*cosh(x))*sinh(x)^3 + 6*(-(11*I - 11)*sqrt(2)*cosh(x)^10 - (14*I - 14)*sqrt
(2)*cosh(x)^6 - (3*I - 3)*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 12*(-(I - 1)*sqrt(
2)*cosh(x)^11 - (2*I - 2)*sqrt(2)*cosh(x)^7 - (I - 1)*sqrt(2)*cosh(x)^3)*si
nh(x) - (I - 1)*sqrt(2))*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 9*(
(I + 1)*sqrt(2)*cosh(x)^12 + (220*I + 220)*sqrt(2)*cosh(x)^3*sinh(x)^9 + (6
6*I + 66)*sqrt(2)*cosh(x)^2*sinh(x)^10 + (12*I + 12)*sqrt(2)*cosh(x)*sinh(x
)^11 + (I + 1)*sqrt(2)*sinh(x)^12 + 3*((165*I + 165)*sqrt(2)*cosh(x)^4 + (I
 + 1)*sqrt(2))*sinh(x)^8 + (3*I + 3)*sqrt(2)*cosh(x)^8 + 24*((33*I + 33)*sq
rt(2)*cosh(x)^5 + (I + 1)*sqrt(2)*cosh(x))*sinh(x)^7 + 84*((11*I + 11)*sqrt
(2)*cosh(x)^6 + (I + 1)*sqrt(2)*cosh(x)^2)*sinh(x)^6 + 24*((33*I + 33)*sqrt
(2)*cosh(x)^7 + (7*I + 7)*sqrt(2)*cosh(x)^3)*sinh(x)^5 + 3*((165*I + 165)*s
qrt(2)*cosh(x)^8 + (70*I + 70)*sqrt(2)*cosh(x)^4 + (I + 1)*sqrt(2))*sinh(x)
^4 + (3*I + 3)*sqrt(2)*cosh(x)^4 + 4*((55*I + 55)*sqrt(2)*cosh(x)^9 + (42*I
 + 42)*sqrt(2)*cosh(x)^5 + (3*I + 3)*sqrt(2)*cosh(x))*sinh(x)^3 + 6*((11*I
+ 11)*sqrt(2)*cosh(x)^10 + (14*I + 14)*sqrt(2)*cosh(x)^6 + (3*I + 3)*sqrt(2
)*cosh(x)^2)*sinh(x)^2 + 12*((I + 1)*sqrt(2)*cosh(x)^11 + (2*I + 2)*sqrt(2)
*cosh(x)^7 + (I + 1)*sqrt(2)*cosh(x)^3)*sinh(x) + (I + 1)*sqrt(2))*log(-(I

```
+ 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 24*(87*cosh(x)^8 + 10*cosh(x)^4 + 3
)*sinh(x) + 72*cosh(x))/(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^
2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 + 1)*s
inh(x)^8 + 3*cosh(x)^8 + 24*(33*cosh(x)^5 + cosh(x))*sinh(x)^7 + 84*(11*cos
h(x)^6 + cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 + 7*cosh(x)^3)*sinh(x)^5 +
 3*(165*cosh(x)^8 + 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(
x)^9 + 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 14*cosh(x)^
6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 + 2*cosh(x)^7 + cosh(x)^3)*sinh
(x) + 1)
```

Sympy [F]

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \int e^x \tanh^2(2x)\operatorname{sech}^2(2x)\,dx$$

```
[In] integrate(exp(x)*sech(2*x)**2*tanh(2*x)**2,x)
[Out] Integral(exp(x)*tanh(2*x)**2*sech(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\begin{aligned}\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = {} & \frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) \\ & + \frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) \\ & + \frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) \\ & - \frac{3}{64}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) \\ & - \frac{29e^{(9x)} + 6e^{(5x)} + 9e^x}{24\left(e^{(12x)} + 3e^{(8x)} + 3e^{(4x)} + 1\right)}\end{aligned}$$

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="maxima")
[Out] 3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e^(5
*x) + 9*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.69

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3}{64}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{29e^{(9x)}+6e^{(5x)}+9e^x}{24\left(e^{(4x)}+1\right)^3}$$

```
[In] integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
 - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e^(5
*x) + 9*e^x)/(e^(4*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{5e^x}{6\left(2e^{4x}+e^{8x}+1\right)} - \frac{\frac{e^{9x}}{3}-\frac{2e^{5x}}{3}+\frac{e^x}{3}}{3e^{4x}+3e^{8x}+e^{12x}+1} - \frac{7e^x}{8\left(e^{4x}+1\right)} + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(-\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(-\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)\right)\left(-\frac{3}{64}+\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)\right)\left(-\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$

```
[In] int((tanh(2*x)^2*exp(x))/cosh(2*x)^2,x)
```

```
[Out] 2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2)*(3/16 + 3i/16))*(3/64 + 3i/64) - (exp(
9*x)/3 - (2*exp(5*x))/3 + exp(x)/3)/(3*exp(4*x) + 3*exp(8*x) + exp(12*x) +
1) - (7*exp(x))/(8*(exp(4*x) + 1)) + 2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2)*(
3/16 - 3i/16))*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 - 3i/16) - (3*exp
(x))/8)*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 + 3i/16) - (3*exp(x))/8)
*(3/64 + 3i/64) + (5*exp(x))/(6*(2*exp(4*x) + exp(8*x) + 1))
```

3.941 $\int e^x \coth(2x)\mathbf{csch}(2x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{e^{3x}}{1-e^{4x}} + \frac{\arctan(e^x)}{2} - \frac{\text{arctanh}(e^x)}{2}$$

```
[Out] exp(3*x)/(1-exp(4*x))+1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 12, 468, 304, 209, 212}

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{\arctan(e^x)}{2} - \frac{\text{arctanh}(e^x)}{2} + \frac{e^{3x}}{1-e^{4x}}$$

```
[In] Int[E^x*Coth[2*x]*Csch[2*x],x]
```

```
[Out] E^(3*x)/(1 - E^(4*x)) + ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^2(1+x^4)}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= 2\text{Subst}\left(\int \frac{x^2(1+x^4)}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} - \text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^{3x}}{1-e^{4x}} + \frac{\arctan(e^x)}{2} - \frac{\text{arctanh}(e^x)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{1}{2}\left(-\frac{2e^{3x}}{-1+e^{4x}} + \arctan\left(e^x\right) - \text{arctanh}(e^x)\right)$$

```
[In] Integrate[E^x*Coth[2*x]*Csch[2*x],x]
[Out] ((-2*E^(3*x))/(-1 + E^(4*x)) + ArcTan[E^x] - ArcTanh[E^x])/2
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{1}{4\sinh(x)}+\frac{\arctan(e^x)}{2}-\frac{\text{arctanh}(e^x)}{2}-\frac{1}{4\cosh(x)}$	24
risch	$-\frac{e^{3x}}{e^{4x}-1}+\frac{\ln(e^x-1)}{4}+\frac{i\ln(e^x+i)}{4}-\frac{i\ln(e^x-i)}{4}-\frac{\ln(1+e^x)}{4}$	48

```
[In] int(exp(x)*coth(2*x)*csch(2*x),x,method=_RETURNVERBOSE)
[Out] -1/4/sinh(x)+1/2*arctan(exp(x))-1/2*arctanh(exp(x))-1/4/cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = -\frac{4\cosh(x)^3 + 12\cosh(x)^2\sinh(x) + 12\cosh(x)\sinh(x)^2 + 4\sinh(x)^3 - 2\left(\cosh(x)^4 + 4\cosh(x)^3\sinh\right.}{}$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="fricas")
[Out] -1/4*(4*cosh(x)^3 + 12*cosh(x)^2*sinh(x) + 12*cosh(x)*sinh(x)^2 + 4*sinh(x)
^3 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cosh
(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1
)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x) -
 1))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*s
inh(x)^3 + sinh(x)^4 - 1)
```

Sympy [F]

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \int e^x \coth(2x)\,\text{csch}(2x)\,dx$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x)
[Out] Integral(exp(x)*coth(2*x)*csch(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = -\frac{e^{(3x)}}{e^{(4x)}-1} + \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(e^x-1)$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="maxima")
[Out] -e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x - 
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = -\frac{e^{(3x)}}{e^{(4x)}-1} + \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(|e^x-1|)$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="giac")
[Out] -e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e
^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int e^x \coth(2x)\text{csch}(2x)\,dx = \frac{\ln(e^x-1)}{4} - \frac{\text{atan}(e^{-x})}{2} - \frac{\ln(-e^x-1)}{4} - \frac{e^{3x}}{e^{4x}-1}$$

```
[In] int((coth(2*x)*exp(x))/sinh(2*x),x)
[Out] log(exp(x) - 1)/4 - atan(exp(-x))/2 - log(- exp(x) - 1)/4 - exp(3*x)/(exp(4
*x) - 1)
```

3.942 $\int e^x \coth(2x)\mathbf{csch}^2(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{e^{5x}}{(1-e^{4x})^2}+\frac{e^x}{4(1-e^{4x})}-\frac{\arctan(e^x)}{8}-\frac{\text{arctanh}(e^x)}{8}$$

```
[Out] -exp(5*x)/(1-exp(4*x))^2+1/4*exp(x)/(1-exp(4*x))-1/8*arctan(exp(x))-1/8*arc
tanh(exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 12, 468, 294, 218, 212, 209}

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{1}{8}\arctan(e^x)-\frac{\text{arctanh}(e^x)}{8}+\frac{e^x}{4(1-e^{4x})}-\frac{e^{5x}}{(1-e^{4x})^2}$$

```
[In] Int[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

```
[Out] -(E^(5*x)/(1 - E^(4*x))^2) + E^x/(4*(1 - E^(4*x))) - ArcTan[E^x]/8 - ArcTan
h[E^x]/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{4x^4(-1-x^4)}{(1-x^4)^3}\,dx, x, e^x\right)$$

$$= 4\text{Subst}\left(\int \frac{x^4(-1-x^4)}{(1-x^4)^3}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4\,(1-e^{4x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4\,(1-e^{4x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)$$

$$= -\frac{e^{5x}}{(1-e^{4x})^2} + \frac{e^x}{4\,(1-e^{4x})} - \frac{\arctan(e^x)}{8} - \frac{\text{arctanh}(e^x)}{8}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{-2e^x + 10e^{5x} + (-1+e^{4x})^2 \arctan(e^x) + (-1+e^{4x})^2\,\text{arctanh}(e^x)}{8\,(-1+e^{4x})^2}$$

```
[In] Integrate[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

```
[Out] -1/8*(-2*E^x + 10*E^(5*x) + (-1 + E^(4*x))^2*ArcTan[E^x] + (-1 + E^(4*x))^2
*ArcTanh[E^x])/(-1 + E^(4*x))^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{e^x(5\,e^{4x}-1)}{4(e^{4x}-1)^2} + \frac{i\ln(e^x-i)}{16} - \frac{i\ln(e^x+i)}{16} - \frac{\ln(1+e^x)}{16} + \frac{\ln(e^x-1)}{16}$
default	$-\frac{\coth(x)\,\text{csch}(x)}{8} - \frac{\text{arctanh}(e^x)}{8} + \frac{1}{16\sinh(x)^2\cosh(x)} + \frac{3}{16\cosh(x)} - \frac{1}{4\sinh(x)} - \frac{\arctan(e^x)}{8} + \frac{1}{8\sinh(x)\cosh(x)^2} + \frac{3\ \text{se}}{}$

```
[In] int(exp(x)*coth(2*x)*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(x)*(5*exp(4*x)-1)/(exp(4*x)-1)^2+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp
(x)+I)-1/16*ln(1+exp(x))+1/16*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(37) = 74$.

Time = 0.27 (sec) , antiderivative size = 522, normalized size of antiderivative = 9.85

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(20*cosh(x)^5 + 200*cosh(x)^3*sinh(x)^2 + 200*cosh(x)^2*sinh(x)^3 + 1
00*cosh(x)*sinh(x)^4 + 20*sinh(x)^5 + 2*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5
 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)
^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(
7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cos
h(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*s
inh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*log(co
sh(x) + sinh(x) + 1) - (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*s
inh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4
 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*c
osh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*log(cosh(x) +
sinh(x) - 1) + 4*(25*cosh(x)^4 - 1)*sinh(x) - 4*cosh(x))/(cosh(x)^8 + 56*co
sh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x)
)*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh
(x)^3)*sinh(x) + 1)
```

Sympy [F]

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = \int e^x \coth(2x)\,\text{csch}^2(2x)\,dx$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(2*x)*csch(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{5\,e^{(5\,x)} - e^x}{4\,(e^{(8\,x)} - 2\,e^{(4\,x)} + 1)} - \frac{1}{8}\,\arctan\left(e^x\right) - \frac{1}{16}\,\log\left(e^x + 1\right) + \frac{1}{16}\,\log\left(e^x - 1\right)$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*e^(5*x) - e^x)/(e^(8*x) - 2*e^(4*x) + 1) - 1/8*arctan(e^x) - 1/16*l
og(e^x + 1) + 1/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{5\,e^{(5\,x)} - e^x}{4\,(e^{(4\,x)} - 1)^2} - \frac{1}{8}\,\arctan\left(e^x\right) - \frac{1}{16}\,\log\left(e^x + 1\right) + \frac{1}{16}\,\log\left(|e^x - 1|\right)$$

```
[In] integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*e^(5*x) - e^x)/(e^(4*x) - 1)^2 - 1/8*arctan(e^x) - 1/16*log(e^x + 1
) + 1/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = \frac{\ln\left(\frac{1}{4} - \frac{\mathrm{e}^x}{4}\right)}{16} - \frac{\ln\left(\frac{\mathrm{e}^x}{4} + \frac{1}{4}\right)}{16} - \frac{\mathrm{atan}(\mathrm{e}^x)}{8} - \frac{\mathrm{e}^{5\,x}}{2\,\left(\mathrm{e}^{8\,x} - 2\,\mathrm{e}^{4\,x} + 1\right)} - \frac{3\,\mathrm{e}^x}{4\,\left(\mathrm{e}^{4\,x} - 1\right)} - \frac{\mathrm{e}^x}{2\,\left(\mathrm{e}^{8\,x} - 2\,\mathrm{e}^{4\,x} + 1\right)}$$

```
[In] int((coth(2*x)*exp(x))/sinh(2*x)^2,x)
```

```
[Out] log(1/4 - exp(x)/4)/16 - log(exp(x)/4 + 1/4)/16 - atan(exp(x))/8 - exp(5*x)
/(2*(exp(8*x) - 2*exp(4*x) + 1)) - (3*exp(x))/(4*(exp(4*x) - 1)) - exp(x)/(
2*(exp(8*x) - 2*exp(4*x) + 1))
```

3.943 $\int e^x \coth^2(2x)\mathbf{csch}(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx = -\frac{e^{3x}}{(1-e^{4x})^2}+\frac{3e^{3x}}{4(1-e^{4x})}+\frac{5\arctan(e^x)}{8}-\frac{5\operatorname{arctanh}(e^x)}{8}$$

```
[Out] -exp(3*x)/(1-exp(4*x))^2+3/4*exp(3*x)/(1-exp(4*x))+5/8*arctan(exp(x))-5/8*a
rctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 12, 474, 468, 304, 209, 212}

$$\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx = \frac{5\arctan(e^x)}{8}-\frac{5\operatorname{arctanh}(e^x)}{8}+\frac{3e^{3x}}{4(1-e^{4x})}-\frac{e^{3x}}{(1-e^{4x})^2}$$

```
[In] Int[E^x*Coth[2*x]^2*Csch[2*x],x]
```

```
[Out] -(E^(3*x)/(1 - E^(4*x))^2) + (3*E^(3*x))/(4*(1 - E^(4*x))) + (5*ArcTan[E^x]
)/8 - (5*ArcTanh[E^x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{2x^2(1+x^4)^2}{(-1+x^4)^3}\,dx, x, e^x\right)$$

$$= 2\text{Subst}\left(\int \frac{x^2(1+x^4)^2}{(-1+x^4)^3}\,dx, x, e^x\right)$$

$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2(4+8x^4)}{(-1+x^4)^2}\,dx, x, e^x\right)$$

$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4\,(1-e^{4x})} + \frac{5}{4}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, e^x\right)$$

$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4\,(1-e^{4x})} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \frac{5}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)$$

$$= -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4\,(1-e^{4x})} + \frac{5\arctan(e^x)}{8} - \frac{5\text{arctanh}(e^x)}{8}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.62 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.93

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx$$

$$= \frac{e^{-5x}\left(177023 + 244931e^{4x} + 43161e^{8x} - 26091e^{12x} - 7(25289 + 24152e^{4x} - 10058e^{8x} - 9048e^{12x} + 513e^{16}\right.}{10752}$$

$$- \frac{8e^{7x}(15+26e^{4x}+11e^{8x})\ {}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{4x}\right)}{1155}$$

$$- \frac{16e^{7x}(1+e^{4x})^2\ {}_5F_4\left(\frac{7}{4},2,2,2,2;1,1,1,\frac{19}{4};e^{4x}\right)}{1155}$$

```
[In] Integrate[E^x*Coth[2*x]^2*Csch[2*x],x]
```

```
[Out] (177023 + 244931*E^(4*x) + 43161*E^(8*x) - 26091*E^(12*x) - 7*(25289 + 2415
2*E^(4*x) - 10058*E^(8*x) - 9048*E^(12*x) + 513*E^(16*x))*Hypergeometric2F1
[3/4, 1, 7/4, E^(4*x)])/(10752*E^(5*x)) - (8*E^(7*x)*(15 + 26*E^(4*x) + 11*
E^(8*x))*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4}, E^(4*x)])/1155 - (
16*E^(7*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1, 1, 1
9/4}, E^(4*x)])/1155
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{e^{3x}(3\,e^{4x}+1)}{4(e^{4x}-1)^2}+\frac{5i\ln(e^x+i)}{16}-\frac{5i\ln(e^x-i)}{16}+\frac{5\ln(e^x-1)}{16}-\frac{5\ln(1+e^x)}{16}$
default	$-\frac{\cosh(x)}{2\sinh(x)^2}+\frac{\coth(x)\,\mathrm{csch}(x)}{2}-\frac{5\,\mathrm{arctanh}(e^x)}{8}+\frac{5\arctan(e^x)}{8}-\frac{1}{8\sinh(x)\cosh(x)^2}-\frac{3\,\mathrm{sech}(x)\tanh(x)}{16}-\frac{1}{16\sinh(x)^2\cosh}$

[In] int(exp(x)*coth(2*x)^2*csch(2*x),x,method=_RETURNVERBOSE)

[Out] -1/4*exp(x)^3*(3*exp(x)^4+1)/(exp(x)^4-1)^2+5/16*I*ln(exp(x)+I)-5/16*I*ln(e
xp(x)-I)+5/16*ln(exp(x)-1)-5/16*ln(1+exp(x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 557, normalized size of antiderivative = 10.13

$$\int e^x \coth^2(2x)\mathrm{csch}(2x)\,dx = \text{Too large to display}$$

[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="fricas")

[Out] -1/16*(12*cosh(x)^7 + 420*cosh(x)^3*sinh(x)^4 + 252*cosh(x)^2*sinh(x)^5 + 8
4*cosh(x)*sinh(x)^6 + 12*sinh(x)^7 + 4*(105*cosh(x)^4 + 1)*sinh(x)^3 + 4*co
sh(x)^3 + 12*(21*cosh(x)^5 + cosh(x))*sinh(x)^2 - 10*(cosh(x)^8 + 56*cosh(x
)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 +
2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*si
nh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^
3)*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(cosh(x)^8 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*c
osh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3
 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh
(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5
 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)
^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(
7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*log(cosh(x) + sinh(x) - 1) + 12*(7*cosh(x)^6 + cosh(x)^2)*sinh(x))/(cosh
(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)
^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x
)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cos
h(x)^7 - cosh(x)^3)*sinh(x) + 1)

Sympy [F]

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = \int e^x \coth^2(2x)\,\text{csch}\,(2x)\,dx$$

```
[In] integrate(exp(x)*coth(2*x)**2*csch(2*x),x)
[Out] Integral(exp(x)*coth(2*x)**2*csch(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\,(e^{(8\,x)} - 2\,e^{(4\,x)} + 1)} + \frac{5}{8}\,\arctan\left(e^x\right) - \frac{5}{16}\,\log\left(e^x + 1\right) + \frac{5}{16}\,\log\left(e^x - 1\right)$$

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="maxima")
[Out] -1/4*(3*e^(7*x) + e^(3*x))/(e^(8*x) - 2*e^(4*x) + 1) + 5/8*arctan(e^x) - 5/
16*log(e^x + 1) + 5/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\,(e^{(4\,x)} - 1)^2} + \frac{5}{8}\,\arctan\left(e^x\right) - \frac{5}{16}\,\log\left(e^x + 1\right) + \frac{5}{16}\,\log\left(|e^x - 1|\right)$$

```
[In] integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="giac")
[Out] -1/4*(3*e^(7*x) + e^(3*x))/(e^(4*x) - 1)^2 + 5/8*arctan(e^x) - 5/16*log(e^x
 + 1) + 5/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx = \frac{5\,\ln\left(\frac{25\,e^x}{16}-\frac{25}{16}\right)}{16}-\frac{5\,\ln\left(\frac{25\,e^x}{16}+\frac{25}{16}\right)}{16}-\frac{5\,\mathrm{atan}\left(e^{-x}\right)}{8}-\frac{e^{3\,x}}{e^{8\,x}-2\,e^{4\,x}+1}-\frac{3\,e^{3\,x}}{4\,\left(e^{4\,x}-1\right)}$$

```
[In] int((coth(2*x)^2*exp(x))/sinh(2*x),x)
```

```
[Out] (5*log((25*exp(x))/16 - 25/16))/16 - (5*log((25*exp(x))/16 + 25/16))/16 - (
5*atan(exp(-x)))/8 - exp(3*x)/(exp(8*x) - 2*exp(4*x) + 1) - (3*exp(3*x))/(4
*(exp(4*x) - 1))
```

3.944 $\int e^x \coth^2(2x)\mathbf{csch}^2(2x)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\, dx = \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{3e^x}{8(1-e^{4x})} - \frac{3\arctan(e^x)}{16} - \frac{3\text{arctanh}(e^x)}{16}$$

```
[Out] 4/3*exp(5*x)/(1-exp(4*x))^3-5/6*exp(5*x)/(1-exp(4*x))^2+3/8*exp(x)/(1-exp(4
*x))-3/16*arctan(exp(x))-3/16*arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 12, 474, 468, 294, 218, 212, 209}

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\, dx = -\frac{3}{16}\arctan(e^x) - \frac{3\text{arctanh}(e^x)}{16} + \frac{3e^x}{8(1-e^{4x})} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{4e^{5x}}{3(1-e^{4x})^3}$$

```
[In] Int[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

```
[Out] (4*E^(5*x))/(3*(1 - E^(4*x))^3) - (5*E^(5*x))/(6*(1 - E^(4*x))^2) + (3*E^x)
/(8*(1 - E^(4*x))) - (3*ArcTan[E^x])/16 - (3*ArcTanh[E^x])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{4x^4(1+x^4)^2}{(1-x^4)^4}\,dx, x, e^x\right)\\
&= 4\text{Subst}\left(\int \frac{x^4(1+x^4)^2}{(1-x^4)^4}\,dx, x, e^x\right)\\
&= \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{1}{3}\text{Subst}\left(\int \frac{x^4(8+12x^4)}{(1-x^4)^3}\,dx, x, e^x\right)\\
&= \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{3e^x}{8(1-e^{4x})} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{3e^x}{8(1-e^{4x})}\\
&\quad - \frac{3}{16}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{3}{16}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{4e^{5x}}{3(1-e^{4x})^3} - \frac{5e^{5x}}{6(1-e^{4x})^2} + \frac{3e^x}{8(1-e^{4x})} - \frac{3\arctan(e^x)}{16} - \frac{3\text{arctanh}(e^x)}{16}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.13

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx$$
$$= \frac{e^{-7x}\left(-1070609085 - 946471617e^{4x} + 369641285e^{8x} + 351173641e^{12x} - 23818496e^{16x} + 1070609085\,\text{Hyp}\right.}{}$$

```
[In] Integrate[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

[Out] (-1070609085 - 946471617*E^(4*x) + 369641285*E^(8*x) + 351173641*E^(12*x) -
 23818496*E^(16*x) + 1070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] + 7
32349800*E^(4*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 635067810*E^(8*x
)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 384831720*E^(12*x)*Hypergeometr
ic2F1[1/4, 1, 5/4, E^(4*x)] + 60913125*E^(16*x)*Hypergeometric2F1[1/4, 1, 5
/4, E^(4*x)] + 1280*E^(16*x)*(821 + 1346*E^(4*x) + 557*E^(8*x))*Hypergeomet
ricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(4*x)] + 10240*E^(16*x)*(23 + 42*E^(
4*x) + 19*E^(8*x))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^
(4*x)] + 20480*E^(16*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1
, 21/4}, E^(4*x)] + 40960*E^(20*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4},
{1, 1, 1, 1, 21/4}, E^(4*x)] + 20480*E^(24*x)*HypergeometricPFQ[{2, 2, 2, 2
, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)])/(3818880*E^(7*x))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{e^{x}(29\,e^{8x}-6\,e^{4x}+9)}{24(e^{4x}-1)^{3}}+\frac{3i\ln(e^{x}-i)}{32}-\frac{3i\ln(e^{x}+i)}{32}-\frac{3\ln(1+e^{x})}{32}+\frac{3\ln(e^{x}-1)}{32}$
default	$-\frac{\coth(x)\operatorname{csch}(x)}{8}-\frac{3\,\operatorname{arctanh}(e^{x})}{16}+\frac{1}{8\sinh(x)^{2}\cosh(x)}+\frac{7}{32\cosh(x)}-\frac{1}{4\sinh(x)}-\frac{3\arctan(e^{x})}{16}-\frac{1}{32\sinh(x)^{2}\cosh(x)^{3}}-$

[In] int(exp(x)*coth(2*x)^2*csch(2*x)^2,x,method=_RETURNVERBOSE)

[Out] -1/24*exp(x)*(29*exp(8*x)-6*exp(4*x)+9)/(exp(4*x)-1)^3+3/32*I*ln(exp(x)-I)-
3/32*I*ln(exp(x)+I)-3/32*ln(1+exp(x))+3/32*ln(exp(x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 992, normalized size of antiderivative = 13.23

$$\int e^{x}\coth^{2}(2x)\operatorname{csch}^{2}(2x)\,dx=\text{Too large to display}$$

[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="fricas")

[Out] -1/96*(116*cosh(x)^9 + 9744*cosh(x)^3*sinh(x)^6 + 4176*cosh(x)^2*sinh(x)^7
+ 1044*cosh(x)*sinh(x)^8 + 116*sinh(x)^9 + 24*(609*cosh(x)^4 - 1)*sinh(x)^5
 - 24*cosh(x)^5 + 24*(609*cosh(x)^5 - 5*cosh(x))*sinh(x)^4 + 48*(203*cosh(x
)^6 - 5*cosh(x)^2)*sinh(x)^3 + 48*(87*cosh(x)^7 - 5*cosh(x)^3)*sinh(x)^2 +
18*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 12*cos
h(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)^8 - 3*cosh(x)^

```
8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6 - cosh(x)^2)*s
inh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(165*cosh(x)^8 - 7
0*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 - 42*cosh(x)^5 +
 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*cosh(x)^2)*sinh
(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) - 1)*arctan(cosh(
x) + sinh(x)) + 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh
(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)
^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6
 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(16
5*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 -
 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*
cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) -
1)*log(cosh(x) + sinh(x) + 1) - 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 6
6*cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x
)^4 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 +
84*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*s
inh(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4
*(55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 1
4*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(
x)^3)*sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) + 12*(87*cosh(x)^8 - 10*cosh(
x)^4 + 3)*sinh(x) + 36*cosh(x))/(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*
cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^
4 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84
*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sin
h(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(
55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*
cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)
^3)*sinh(x) - 1)
```

Sympy [F]

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = \int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx$$

```
[In] integrate(exp(x)*coth(2*x)**2*csch(2*x)**2,x)
[Out] Integral(exp(x)*coth(2*x)**2*csch(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = -\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^x}{24\,\left(e^{(12\,x)} - 3\,e^{(8\,x)} + 3\,e^{(4\,x)} - 1\right)} - \frac{3}{16}\,\arctan\left(e^x\right) - \frac{3}{32}\,\log\left(e^x + 1\right) + \frac{3}{32}\,\log\left(e^x - 1\right)$$

[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="maxima")

[Out] -1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(12*x) - 3*e^(8*x) + 3*e^(4*x) - 1) - 3/16*arctan(e^x) - 3/32*log(e^x + 1) + 3/32*log(e^x - 1)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = -\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^x}{24\,\left(e^{(4\,x)} - 1\right)^3} - \frac{3}{16}\,\arctan\left(e^x\right) - \frac{3}{32}\,\log\left(e^x + 1\right) + \frac{3}{32}\,\log\left(|e^x - 1|\right)$$

[In] integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="giac")

[Out] -1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(4*x) - 1)^3 - 3/16*arctan(e^x) - 3/32*log(e^x + 1) + 3/32*log(abs(e^x - 1))

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = \frac{3\,\ln\left(\frac{3}{8} - \frac{3\,\mathrm{e}^x}{8}\right)}{32} - \frac{3\,\ln\left(-\frac{3\,\mathrm{e}^x}{8} - \frac{3}{8}\right)}{32} - \frac{7\,\mathrm{e}^x}{8\,\left(\mathrm{e}^{4\,x} - 1\right)} - \frac{\frac{2\,\mathrm{e}^{5\,x}}{3} + \frac{\mathrm{e}^{9\,x}}{3} + \frac{\mathrm{e}^x}{3}}{3\,\mathrm{e}^{4\,x} - 3\,\mathrm{e}^{8\,x} + \mathrm{e}^{12\,x} - 1} - \frac{5\,\mathrm{e}^x}{6\,\left(\mathrm{e}^{8\,x} - 2\,\mathrm{e}^{4\,x} + 1\right)} - \frac{\ln\left(-\frac{3\,\mathrm{e}^x}{8} - \frac{3}{8}\mathrm{i}\right)\,3\mathrm{i}}{32} + \frac{\ln\left(-\frac{3\,\mathrm{e}^x}{8} + \frac{3}{8}\mathrm{i}\right)\,3\mathrm{i}}{32}$$

[In] int((coth(2*x)^2*exp(x))/sinh(2*x)^2,x)

[Out] (3*log(3/8 - (3*exp(x))/8))/32 - (3*log(- (3*exp(x))/8 - 3/8))/32 - (log(-(3*exp(x))/8 - 3i/8)*3i)/32 + (log(3i/8 - (3*exp(x))/8)*3i)/32 - (7*exp(x))/(8*(exp(4*x) - 1)) - ((2*exp(5*x))/3 + exp(9*x)/3 + exp(x)/3)/(3*exp(4*x) - 3*exp(8*x) + exp(12*x) - 1) - (5*exp(x))/(6*(exp(8*x) - 2*exp(4*x) + 1))

3.945 $\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx=-\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}+\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}$$

```
[Out] -1/2*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+1/2*b*exp(d*x+c)*cosh(4*b*x+4
*a)/(16*b^2-d^2)+1/4*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/8*d*exp(d*x
+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5620, 5582}

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}-\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
```

```
[Out] -1/2*(b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) + (b*E^(c + d*x)*Cosh[
4*a + 4*b*x])/(2*(16*b^2 - d^2)) + (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*
b^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx\\
&= \frac{1}{8}\int e^{c+dx}\sinh(4a+4bx)\,dx-\frac{1}{4}\int e^{c+dx}\sinh(2a+2bx)\,dx\\
&= -\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}\\
&\quad+\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx=\frac{1}{8}e^{c+dx}\left(\frac{-4b\cosh(2(a+bx))+2d\sinh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^3,x]
[Out] (E^(c + d*x)*((-4*b*Cosh[2*(a + b*x)] + 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2
) + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\frac{\sinh\left(2a-c+(2b-d)\,x\right)}{16b-8d}-\frac{\sinh\left(2a+c+(2b+d)\,x\right)}{8\,(2b+d)}-\frac{\sinh\left((4b-d)\,x+4a-c\right)}{16\,(4b-d)}+\frac{\sinh\left((4b+d)\,x+4a+\right.}{64b+16d}$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(4
*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)-1/8*cosh(2*a
-c+(2*b-d)*x)/(2*b-d)-1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d)*x
+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(125) = 250.

Time = 0.27 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.69

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{(4\,b^2d-d^3)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-(4\,b^3-bd^2)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^4+(16}{}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 + (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2
)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh(b*
x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - ((
4*b^3 - b*d^2)*cosh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x
 + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x + a)*si
nh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 - (16*b^3 - b*d^2)*cosh(b*x
 + a)^2 - (16*b^3 - b*d^2 - 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 - (16*b^2*d - d^3)*cosh(b*x + a))*si
nh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^4 -
2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (64*b^4 - 2
0*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1295 vs. 2(114) = 228.

Time = 7.77 (sec) , antiderivative size = 1295, normalized size of antiderivative = 9.45

$$\int e^{c+dx} \cosh(a+bx) \sinh^3(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**3*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp
(d*x)*sinh(a - d*x/2)**4/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a -
d*x/2)/4 - x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/4 - x*exp(c
)*exp(d*x)*cosh(a - d*x/2)**4/8 - 7*exp(c)*exp(d*x)*sinh(a - d*x/2)**4/(24*
d) - exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a - d*x/2)/(3*d) - exp(c)*exp(
d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**2/(2*d) + exp(c)*exp(d*x)*cosh(a -
 d*x/2)**4/(8*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/4)**4/16 +
x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/4 + 3*x*exp(c)*exp(d*x
)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/
4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a - d*x/4)**4/16 + exp(c)*
exp(d*x)*sinh(a - d*x/4)**4/(6*d) + 11*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*c
osh(a - d*x/4)/(12*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**
3/(12*d) - exp(c)*exp(d*x)*cosh(a - d*x/4)**4/(6*d), Eq(b, -d/4)), (-x*exp(
c)*exp(d*x)*sinh(a + d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*co
sh(a + d*x/4)/4 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cosh(a + d*x/4)**2
/8 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4 - x*exp(c)*exp(
d*x)*cosh(a + d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a + d*x/4)**4/(6*d) + 11*
exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(12*d) - 5*exp(c)*exp(d*
x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d) + exp(c)*exp(d*x)*cosh(a + d*x
/4)**4/(6*d), Eq(b, d/4)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/2)**4/8 + x*exp
(c)*exp(d*x)*sinh(a + d*x/2)**3*cosh(a + d*x/2)/4 - x*exp(c)*exp(d*x)*sinh(
a + d*x/2)*cosh(a + d*x/2)**3/4 + x*exp(c)*exp(d*x)*cosh(a + d*x/2)**4/8 +
7*exp(c)*exp(d*x)*sinh(a + d*x/2)**4/(24*d) - exp(c)*exp(d*x)*sinh(a + d*x/
2)**3*cosh(a + d*x/2)/(3*d) + exp(c)*exp(d*x)*sinh(a + d*x/2)**2*cosh(a + d
*x/2)**2/(2*d) - exp(c)*exp(d*x)*cosh(a + d*x/2)**4/(8*d), Eq(b, d/2)), (10
*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) + 12
*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*
d**2 + d**4) - 6*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**4/(64*b**4 - 20*b**2*d
**2 + d**4) - 10*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(64*
b**4 - 20*b**2*d**2 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a
 + b*x)**3/(64*b**4 - 20*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d*x)*sinh(a
+ b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a
 + b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*d**2 + d**4) + d**3*exp(c)*e
xp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**4 - 20*b**2*d**2 + d**4), Tru
e))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} - \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} - \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} + \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/8*e^(2*b*x + d*x + 2*a + c)/(2
*b + d) - 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x -
 4*a + c)/(4*b - d)
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.66

$$\int e^{c+dx}\cosh(a+bx)\sinh^3(a+bx)\,dx = \frac{b\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)^4\,(10\,b^2-d^2)}{64\,b^4-20\,b^2\,d^2+d^4} - \frac{6\,b^3\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}}{64\,b^4-20\,b^2\,d^2+d^4} + \frac{3\,b\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)^2}{16\,b^2-d^2} - \frac{d\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)^3\,(10\,b^2-d^2)}{64\,b^4-20\,b^2\,d^2+d^4} + \frac{6\,b^2\,d\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}\,\sinh(a+b\,x)}{(4\,b^2-d^2)\,(16\,b^2-d^2)}$$

```
[In] int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] (b*exp(c + d*x)*sinh(a + b*x)^4*(10*b^2 - d^2))/(64*b^4 + d^4 - 20*b^2*d^2)
 - (6*b^3*cosh(a + b*x)^4*exp(c + d*x))/(64*b^4 + d^4 - 20*b^2*d^2) + (3*b*
cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^2)/(16*b^2 - d^2) - (d*cosh(a +
b*x)*exp(c + d*x)*sinh(a + b*x)^3*(10*b^2 - d^2))/(64*b^4 + d^4 - 20*b^2*d^
2) + (6*b^2*d*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x))/((4*b^2 - d^2)*(1
6*b^2 - d^2))
```

3.946 $\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx=\frac{de^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}-\frac{be^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}$$

```
[Out] 1/4*d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-1/4*d*exp(d*x+c)*cosh(3*b*x+3*a)/(9*
b^2-d^2)-1/4*b*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)+3/4*b*exp(d*x+c)*sinh(3*b*x
+3*a)/(9*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5620, 5583}

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx=-\frac{be^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}+\frac{de^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(4*(9*b^2 - d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2)) +
(3*b*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{4}e^{c+dx}\cosh(a+bx)+\frac{1}{4}e^{c+dx}\cosh(3a+3bx)\right)\,dx \\
&= -\left(\frac{1}{4}\int e^{c+dx}\cosh(a+bx)\,dx\right)+\frac{1}{4}\int e^{c+dx}\cosh(3a+3bx)\,dx \\
&= \frac{de^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}-\frac{be^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{1}{4}e^{c+dx}\left(\frac{d\cosh(a+bx)-b\sinh(a+bx)}{(b-d)(b+d)} + \frac{-d\cosh(3(a+bx))+3b\sinh(3(a+bx))}{9b^2-d^2}\right)$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2,x]
[Out] (E^(c + d*x)*((d*Cosh[a + b*x] - b*Sinh[a + b*x])/((b - d)*(b + d)) + (-(d*
Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

Maple [A] (verified)

Time = 24.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sinh(a-c+(b-d)x)}{8(b-d)}-\frac{\sinh(a+c+(b+d)x)}{8(b+d)}+\frac{\sinh(3a-c+(3b-d)x)}{24b-8d}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh(a-c+(b-d)x)}{8b-8d}-\frac{\cosh(}{}$
risch	$\frac{(3b^3 e^{6bx+6a}-b^2 d\, e^{6bx+6a}-3b\, d^2 e^{6bx+6a}+d^3 e^{6bx+6a}-9b^3 e^{4bx+4a}+9b^2 d\, e^{4bx+4a}+b\, d^2 e^{4bx+4a}-d^3 e^{4bx+4a}+9b^3 e^{2bx+2a}+9b^2 d\, e^{2bx+}}{8(3b+d)(b+d)(3b-d)(b-d)}$

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sinh(a-c+(b-d)*x)/(b-d)-1/8*sinh(a+c+(b+d)*x)/(b+d)+1/8*sinh(3*a-c+(3*
b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(b-
d)-1/8*cosh(a+c+(b+d)*x)/(b+d)-1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3
*a+c+(3*b+d)*x)/(3*b+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(115) = 230$.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.98

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx =$$
$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3+\left(9\,b^{3}\right.}{}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (9*b^3 - b*d^2 - 9*(b^3 - b*d^2)*c
osh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x + a)^
3 - (9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(b*x
+ a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2)*si
nh(b*x + a)^3 - (9*b^2*d - d^3)*cosh(b*x + a) + (9*b^3 - b*d^2 - 9*(b^3 - b
*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 +
 d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b
*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. 2(109) = 218.

Time = 2.74 (sec) , antiderivative size = 972, normalized size of antiderivative = 7.65

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp
(d*x)*sinh(a - d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)
/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 - x*exp(c)*exp(d*x)
*cosh(a - d*x)**3/8 - 3*exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) - exp(c)*exp
(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) + exp(c)*exp(d*x)*cosh(a - d*x)*
*3/(8*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*
exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a
- d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 + ex
p(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*
*2*cosh(a - d*x/3)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b,
-d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh
(a + d*x/3)**2*cosh(a + d*x/3)/8 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh
(a + d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 + exp(c)*exp(d*x)
*sinh(a + d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/
3)**2/(4*d) - 3*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq(b, d/3)), (-x*
exp(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cos
h(a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/8 - x*exp(c
)*exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) + ex
p(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(4*d) - exp(c)*exp(d*x)*cosh(a
 + d*x)**3/(8*d), Eq(b, d)), (3*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4) - 3*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(
a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 2*b**2*d*exp(c)*exp(d*x)*cosh(a +
 b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d*x)*sinh(a + b
*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) - 2*b*d**2*exp(c)*exp(d*x)*sinh(a + b
*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*
sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)} - \frac{e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{e^{(-bx+dx-a+c)}}{8\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*e^(b*x + d*x + a + c)/(b + d)
 + 1/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int e^{c+dx}\cosh(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{c+d\,x}\,\left(3\,b^3\,\sinh(a+b\,x)^3+2\,b^2\,d\cosh(a+b\,x)^3-3\,b^2\,d\cosh(a+b\,x)\,\sinh(a+b\,x)^2-2\,b\,d^2\cosh(a+b\,x)}{9\,b^4-10\,b^2\,d^2+d^4}$$

```
[In] int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] (exp(c + d*x)*(3*b^3*sinh(a + b*x)^3 + 2*b^2*d*cosh(a + b*x)^3 + d^3*cosh(a
 + b*x)*sinh(a + b*x)^2 - b*d^2*sinh(a + b*x)^3 - 2*b*d^2*cosh(a + b*x)^2*s
inh(a + b*x) - 3*b^2*d*cosh(a + b*x)*sinh(a + b*x)^2))/(9*b^4 + d^4 - 10*b^
2*d^2)
```

3.947 $\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2(4b^2-d^2)}$$

```
[Out] b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)-1/2*d*exp(d*x+c)*sinh(2*b*x+2*a)/(
4*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5620, 12, 5582}

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2(4b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(4*b^2 - d^2) - (d*E^(c + d*x)*Sinh[2*a +
 2*b*x])/(2*(4*b^2 - d^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
```

```
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{2}e^{c+dx}\sinh(2a+2bx)\,dx\\ &= \frac{1}{2}\int e^{c+dx}\sinh(2a+2bx)\,dx\\ &= \frac{be^{c+dx}\cosh(2a+2bx)}{4b^2-d^2}-\frac{de^{c+dx}\sinh(2a+2bx)}{2\,(4b^2-d^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx=\frac{e^{c+dx}(2b\cosh(2(a+bx))-d\sinh(2(a+bx)))}{2\,(4b^2-d^2)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x],x]
[Out] (E^(c + d*x)*(2*b*Cosh[2*(a + b*x)] - d*Sinh[2*(a + b*x)]))/(2*(4*b^2 - d^2
))
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{(2\,\mathrm{e}^{4bx+4a}b-d\,\mathrm{e}^{4bx+4a}+2b+d)\,\mathrm{e}^{-2bx+dx-2a+c}}{4(2b+d)(2b-d)}$	61
default	$-\frac{\sinh(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(2a-c+(2b-d)x)}{8b-4d}+\frac{\cosh(2a+c+(2b+d)x)}{8b+4d}$	102

```
[In] int(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/4/(2*b+d)/(2*b-d)*(2*exp(4*b*x+4*a)*b-d*exp(4*b*x+4*a)+2*b+d)*exp(-2*b*x+
d*x-2*a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.15

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{b\cosh\left(bx+a\right)^{2}\cosh\left(dx+c\right) - d\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right) + b\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{2}}{\left(4\,b^{2}-d^{2}\right)\cosh\left(bx+a\right)^{2} - \left(4\,b^{2}-d^{2}\right)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)^2*cosh(d*x + c) - d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x +
 a) + b*cosh(d*x + c)*sinh(b*x + a)^2 + (b*cosh(b*x + a)^2 - d*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)*sinh(d*x + c))/((4*b^2 - d^2)*cosh(b*x
 + a)^2 - (4*b^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(54) = 108$.

Time = 0.97 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.61

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx$$
$$= \begin{cases} xe^{c}\sinh\left(a\right)\cosh\left(a\right) & \text{for } b = 0 \\ \frac{xe^{c}e^{dx}\sinh^{2}\left(a-\frac{dx}{2}\right)}{4} + \frac{xe^{c}e^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2} + \frac{xe^{c}e^{dx}\cosh^{2}\left(a-\frac{dx}{2}\right)}{4} + \frac{e^{c}e^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2d} & \text{for } b = - \\ -\frac{xe^{c}e^{dx}\sinh^{2}\left(a+\frac{dx}{2}\right)}{4} + \frac{xe^{c}e^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2} - \frac{xe^{c}e^{dx}\cosh^{2}\left(a+\frac{dx}{2}\right)}{4} + \frac{e^{c}e^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2d} & \text{for } b = \frac{d}{2} \\ \frac{be^{c}e^{dx}\sinh^{2}\left(a+bx\right)}{4b^{2}-d^{2}} + \frac{be^{c}e^{dx}\cosh^{2}\left(a+bx\right)}{4b^{2}-d^{2}} - \frac{de^{c}e^{dx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^{2}-d^{2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*
x)*sinh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)
/2 + x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a - d*x/
2)*cosh(a - d*x/2)/(2*d), Eq(b, -d/2)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/2)
**2/4 + x*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)/2 - x*exp(c)*exp(
d*x)*cosh(a + d*x/2)**2/4 + exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)
/(2*d), Eq(b, d/2)), (b*exp(c)*exp(d*x)*sinh(a + b*x)**2/(4*b**2 - d**2) +
b*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2 - d**2) - d*exp(c)*exp(d*x)*sinh
(a + b*x)*cosh(a + b*x)/(4*b**2 - d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{e^{(2\,bx+dx+2\,a+c)}}{4\,(2\,b+d)} + \frac{e^{(-2\,bx+dx-2\,a+c)}}{4\,(2\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) + 1/4*e^(-2*b*x + d*x - 2*a + c)/(2
*b - d)
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int e^{c+dx}\cosh(a+bx)\sinh(a+bx)\,dx = \frac{\mathrm{e}^{c+d\,x}\,\mathrm{e}^{-2\,a-2\,b\,x}\,\left(2\,b+d+2\,b\,\mathrm{e}^{4\,a+4\,b\,x}-d\,\mathrm{e}^{4\,a+4\,b\,x}\right)}{4\,\left(4\,b^2-d^2\right)}$$

```
[In] int(cosh(a + b*x)*exp(c + d*x)*sinh(a + b*x),x)
```

```
[Out] (exp(c + d*x)*exp(- 2*a - 2*b*x)*(2*b + d + 2*b*exp(4*a + 4*b*x) - d*exp(4*
a + 4*b*x)))/(4*(4*b^2 - d^2))
```

3.948 $\int e^{c+dx}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int e^{c+dx}\cosh(a+bx)\,dx=-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}+\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}$$

```
[Out] -d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)+b*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5583}

$$\int e^{c+dx}\cosh(a+bx)\,dx=\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] -((d*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2)) + (b*E^(c + d*x)*Sinh[a + b*x]
)/(b^2 - d^2)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral}=-\frac{de^{c+dx}\cosh(a+bx)}{b^2-d^2}+\frac{be^{c+dx}\sinh(a+bx)}{b^2-d^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{c+dx}\cosh(a+bx)\,dx=\frac{e^{c+dx}(-d\cosh(a+bx)+b\sinh(a+bx))}{(b-d)(b+d)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x],x]
[Out] (E^(c + d*x)*(-(d*Cosh[a + b*x]) + b*Sinh[a + b*x]))/((b - d)*(b + d))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{e^{dx+c}(-\cosh(bx+a)d+b\sinh(bx+a))}{b^2-d^2}$	37
risch	$\frac{(b\,e^{2bx+2a}-d\,e^{2bx+2a}-b-d)\,e^{-bx+dx-a+c}}{2(b+d)(b-d)}$	58
default	$\frac{\sinh(a-c+(b-d)x)}{2b-2d}+\frac{\sinh(a+c+(b+d)x)}{2b+2d}-\frac{\cosh(a-c+(b-d)x)}{2(b-d)}+\frac{\cosh(a+c+(b+d)x)}{2b+2d}$	78

```
[In] int(exp(d*x+c)*cosh(b*x+a),x,method=_RETURNVERBOSE)
[Out] exp(d*x+c)/(b^2-d^2)*(-cosh(b*x+a)*d+b*sinh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int e^{c+dx}\cosh(a+bx)\,dx = -\frac{d\cosh(bx+a)\cosh(dx+c)-b\cosh(dx+c)\sinh(bx+a)+(d\cosh(bx+a)-b\sinh(bx+a))\sinh(}{(b^2-d^2)\cosh(bx+a)^2-(b^2-d^2)\sinh(bx+a)^2}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="fricas")
[Out] -(d*cosh(b*x + a)*cosh(d*x + c) - b*cosh(d*x + c)*sinh(b*x + a) + (d*cosh(b
*x + a) - b*sinh(b*x + a))*sinh(d*x + c))/((b^2 - d^2)*cosh(b*x + a)^2 - (b
^2 - d^2)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(42) = 84$.

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.41

$$\int e^{c+dx}\cosh(a+bx)\,dx = \begin{cases} xe^c\cosh(a) & \text{for } b=0\wedge d=0 \\ \frac{xe^ce^{dx}\sinh(a-dx)}{2}+\frac{xe^ce^{dx}\cosh(a-dx)}{2}-\frac{e^ce^{dx}\sinh(a-dx)}{2d} & \text{for } b=-d \\ -\frac{xe^ce^{dx}\sinh(a+dx)}{2}+\frac{xe^ce^{dx}\cosh(a+dx)}{2}+\frac{e^ce^{dx}\sinh(a+dx)}{d}-\frac{e^ce^{dx}\cosh(a+dx)}{2d} & \text{for } b=d \\ \frac{be^ce^{dx}\sinh(a+bx)}{b^2-d^2}-\frac{de^ce^{dx}\cosh(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a), Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*sinh(
a - d*x)/2 + x*exp(c)*exp(d*x)*cosh(a - d*x)/2 - exp(c)*exp(d*x)*sinh(a - d
*x)/(2*d), Eq(b, -d)), (-x*exp(c)*exp(d*x)*sinh(a + d*x)/2 + x*exp(c)*exp(d
*x)*cosh(a + d*x)/2 + exp(c)*exp(d*x)*sinh(a + d*x)/d - exp(c)*exp(d*x)*cos
h(a + d*x)/(2*d), Eq(b, d)), (b*exp(c)*exp(d*x)*sinh(a + b*x)/(b**2 - d**2)
 - d*exp(c)*exp(d*x)*cosh(a + b*x)/(b**2 - d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int e^{c+dx}\cosh(a+bx)\,dx = \frac{e^{(bx+dx+a+c)}}{2\,(b+d)} - \frac{e^{(-bx+dx-a+c)}}{2\,(b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a),x, algorithm="giac")
[Out] 1/2*e^(b*x + d*x + a + c)/(b + d) - 1/2*e^(-b*x + d*x - a + c)/(b - d)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int e^{c+dx}\cosh(a+bx)\,dx = -\frac{\mathrm{e}^{c-a-b\,x+d\,x}\,\left(b+d-b\,\mathrm{e}^{2\,a+2\,b\,x}+d\,\mathrm{e}^{2\,a+2\,b\,x}\right)}{2\,\left(b^2-d^2\right)}$$

```
[In] int(cosh(a + b*x)*exp(c + d*x),x)
[Out] -(exp(c - a - b*x + d*x)*(b + d - b*exp(2*a + 2*b*x) + d*exp(2*a + 2*b*x)))
/(2*(b^2 - d^2))
```

3.949 $\int e^{c+dx} \coth(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int e^{c+dx} \coth(a+bx)\,dx = \frac{e^{c+dx}}{d} - \frac{2e^{c+dx}\,\text{Hypergeometric2F1}\left(1, \frac{d}{2b}, 1+\frac{d}{2b}, e^{2(a+bx)}\right)}{d}$$

```
[Out] exp(d*x+c)/d-2*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a)
)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5593, 2225, 2283}

$$\int e^{c+dx} \coth(a+bx)\,dx = \frac{e^{c+dx}}{d} - \frac{2e^{c+dx}\,\text{Hypergeometric2F1}\left(1, \frac{d}{2b}, \frac{d}{2b}+1, e^{2(a+bx)}\right)}{d}$$

```
[In] Int[E^(c + d*x)*Coth[a + b*x],x]
```

```
[Out] E^(c + d*x)/d - (2*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5593

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(e^{c+dx}+\frac{2e^{c+dx}}{-1+e^{2(a+bx)}}\right)dx\\ &= 2\int \frac{e^{c+dx}}{-1+e^{2(a+bx)}}\,dx+\int e^{c+dx}\,dx\\ &= \frac{e^{c+dx}}{d}-\frac{2e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(53) = 106.

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int e^{c+dx}\coth(a+bx)\,dx = \frac{e^{c+dx}\coth(a)}{d} - \frac{2e^{2a+c}\left(\frac{e^{dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{-1+e^{2a}}$$

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x],x]
```

```
[Out] (E^(c + d*x)*Coth[a])/d - (2*E^(2*a + c)*((E^(d*x)*Hypergeometric2F1[1, d/(
2*b), 1 + d/(2*b), E^(2*(a + b*x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1
[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(-1 + E^(2*a))
```

Maple [F]

$$\int e^{dx+c}\cosh(bx+a)\operatorname{csch}(bx+a)\,dx$$

[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)

[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)

Fricas [F]

$$\int e^{c+dx}\coth(a+bx)\,dx = \int \cosh(bx+a)\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="fricas")

[Out] integral(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)

Sympy [F]

$$\int e^{c+dx}\coth(a+bx)\,dx = e^{c}\int e^{dx}\cosh(a+bx)\operatorname{csch}(a+bx)\,dx$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x)

[Out] exp(c)*Integral(exp(d*x)*cosh(a + b*x)*csch(a + b*x), x)

Maxima [F]

$$\int e^{c+dx}\coth(a+bx)\,dx = \int \cosh(bx+a)\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="maxima")

[Out] -4*b*integrate(e^(d*x + c)/((2*b - d)*e^(4*b*x + 4*a) - 2*(2*b - d)*e^(2*b*x + 2*a) + 2*b - d), x) - ((2*b*e^c - d*e^c)*e^(2*b*x + 2*a) - 2*b*e^c - d*e^c)*e^(d*x)/(2*b*d - d^2 - (2*b*d - d^2)*e^(2*b*x + 2*a))

Giac [F]

$$\int e^{c+dx}\coth(a+bx)\,dx=\int \cosh\left(bx+a\right)\operatorname{csch}\left(bx+a\right)e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*csch(b*x + a)*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth(a+bx)\,dx=\int \frac{\cosh(a+b\,x)\;\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x),x)
[Out] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x), x)
```

3.950 $\int e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} + \frac{4e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

```
[Out] -2*exp(a+c+(b+d)*x)*hypergeom([1, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp(2*b*x+2*
a))/(b+d)+4*exp(a+c+(b+d)*x)*hypergeom([2, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp
(2*b*x+2*a))/(b+d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5622, 2283}

$$\int e^{c+dx}\coth(a+bx)\operatorname{csch}(a+bx)\,dx = \frac{4e^{a+x(b+d)+c}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} - \frac{2e^{a+x(b+d)+c}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

```
[In] Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (4*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d)
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5622

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{4e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{2e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\right)dx\\
&= 2\int \frac{e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\,dx+4\int \frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx\\
&= -\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}\\
&\quad+\frac{4e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{e^c\text{csch}\left(\frac{1}{2}(a+bx)\right)\text{sech}\left(\frac{1}{2}(a+bx)\right)(\cosh(a)+\sinh(a))\left((b+d)e^{dx}(\cosh(a)-\sinh(a))+2de^{(b+d)x}\,\text{Hyp}\right.}{2b(b+d)}$$

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x],x]
```

```
[Out] -1/2*(E^c*Csch[(a + b*x)/2]*Sech[(a + b*x)/2]*(Cosh[a] + Sinh[a])*((b + d)*
E^(d*x)*(Cosh[a] - Sinh[a]) + 2*d*E^((b + d)*x)*Hypergeometric2F1[1, (b + d
)/(2*b), (3 + d/b)/2, E^(2*(a + b*x))]*Sinh[a + b*x]))/(b*(b + d))
```

Maple [F]

$$\int e^{dx+c} \cosh (bx + a) \operatorname{csch} (bx + a)^2 \, dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x)
```

Fricas [F]

$$\int e^{c+dx} \coth(a + bx)\operatorname{csch}(a + bx) \, dx = \int \cosh (bx + a) \operatorname{csch} (bx + a)^2 e^{(dx+c)} \, dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \coth(a + bx)\operatorname{csch}(a + bx) \, dx = \text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx} \coth(a + bx)\operatorname{csch}(a + bx) \, dx = \int \cosh (bx + a) \operatorname{csch} (bx + a)^2 e^{(dx+c)} \, dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
[Out] 16*b*d*integrate(-e^(b*x + d*x + a + c)/(3*b^2 - 4*b*d + d^2 - (3*b^2 - 4*b
*d + d^2)*e^(6*b*x + 6*a) + 3*(3*b^2 - 4*b*d + d^2)*e^(4*b*x + 4*a) - 3*(3*
b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a)), x) - 2*((3*b*e^c - d*e^c)*e^(3*b*x + 3
*a) - (3*b*e^c + d*e^c)*e^(b*x + a))*e^(d*x)/(3*b^2 - 4*b*d + d^2 + (3*b^2
- 4*b*d + d^2)*e^(4*b*x + 4*a) - 2*(3*b^2 - 4*b*d + d^2)*e^(2*b*x + 2*a))
```

Giac [F]

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^{2}e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^2*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth(a+bx)\text{csch}(a+bx)\,dx=\int \frac{\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^{2}}\,dx$$

```
[In] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^2,x)
[Out] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^2, x)
```

3.951 $\int e^{c+dx} \coth(a+bx)\text{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{4e^{2a+c+(2b+d)x}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{d}{b}\right),\frac{1}{2}\left(4+\frac{d}{b}\right),e^{2(a+bx)}\right)}{2b+d} - \frac{8e^{2a+c+(2b+d)x}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(2+\frac{d}{b}\right),\frac{1}{2}\left(4+\frac{d}{b}\right),e^{2(a+bx)}\right)}{2b+d}$$

```
[Out] 4*exp(2*a+c+(2*b+d)*x)*hypergeom([2, 1+1/2*d/b],[2+1/2*d/b],exp(2*b*x+2*a))
/(2*b+d)-8*exp(2*a+c+(2*b+d)*x)*hypergeom([3, 1+1/2*d/b],[2+1/2*d/b],exp(2*
b*x+2*a))/(2*b+d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5622, 2283}

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx = \frac{4e^{2a+x(2b+d)+c}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{d}{b}+2\right),\frac{1}{2}\left(\frac{d}{b}+4\right),e^{2(a+bx)}\right)}{2b+d} - \frac{8e^{2a+x(2b+d)+c}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{d}{b}+2\right),\frac{1}{2}\left(\frac{d}{b}+4\right),e^{2(a+bx)}\right)}{2b+d}$$

```
[In] Int[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] (4*E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1[2, (2 + d/b)/2, (4 + d/b)/2,
 E^(2*(a + b*x))])/(2*b + d) - (8*E^(2*a + c + (2*b + d)*x)*Hypergeometric2
F1[3, (2 + d/b)/2, (4 + d/b)/2, E^(2*(a + b*x))])/(2*b + d)
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5622

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{8e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3} + \frac{4e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2} \right) dx \\
&= 4 \int \frac{e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\, dx + 8 \int \frac{e^{2a+c+(2b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}\, dx \\
&= \frac{4e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2+\frac{d}{b}\right), \frac{1}{2}\left(4+\frac{d}{b}\right), e^{2(a+bx)}\right)}{2b+d} \\
&\quad - \frac{8e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(3, \frac{1}{2}\left(2+\frac{d}{b}\right), \frac{1}{2}\left(4+\frac{d}{b}\right), e^{2(a+bx)}\right)}{2b+d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx =$$

$$-\frac{e^{c-\frac{ad}{b}}\left((2b+d)e^{d\left(\frac{a}{b}+x\right)}\left(d\coth(a+bx)+b\text{csch}^2(a+bx)\right)+d(2b+d)e^{d\left(\frac{a}{b}+x\right)}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b}\right.\right.}{2b^2(2b+d)}$$

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]*Csch[a + b*x]^2,x]
```

```
[Out] -1/2*(E^(c - (a*d)/b)*((2*b + d)*E^(d*(a/b + x))*(d*Coth[a + b*x] + b*Csch[
a + b*x]^2) + d*(2*b + d)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))] + d^2*E^((2 + d/b)*(a + b*x))*Hypergeometric2F1[
1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))]))/(b^2*(2*b + d))
```

Maple [F]

$$\int \mathrm{e}^{dx+c} \cosh\left(bx+a\right) \operatorname{csch}\left(bx+a\right)^{3} dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
[Out] int(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x)
```

Fricas [F]

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \cosh\left(bx+a\right) \operatorname{csch}\left(bx+a\right)^{3} e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx} \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \cosh\left(bx+a\right) \operatorname{csch}\left(bx+a\right)^{3} e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -48*b*d^2*integrate(e^(d*x + c)/(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3 + (48*b
^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 - 44*b^2*d + 12
*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4
*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a)), x) +
 4*(12*b*d*e^c + (24*b^2*e^c - 10*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - (24*
b^2*e^c + 2*b*d*e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3 - 44*b^2*d
+ 12*b*d^2 - d^3 - (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3
*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(48*b^3 - 44*b^2*
d + 12*b*d^2 - d^3)*e^(2*b*x + 2*a))
```

Giac [F]

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=\int\cosh\left(bx+a\right)\text{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)*csch(b*x + a)^3*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth(a+bx)\text{csch}^2(a+bx)\,dx=\int\frac{\cosh(a+b\,x)\;e^{c+d\,x}}{\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^3,x)
[Out] int((cosh(a + b*x)*exp(c + d*x))/sinh(a + b*x)^3, x)
```

3.952 $\int e^{c+dx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\int e^{c+dx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx = -\frac{be^{c+dx} \cosh(a+bx)}{8(b^2-d^2)} - \frac{3be^{c+dx} \cosh(3a+3bx)}{16(9b^2-d^2)} + \frac{5be^{c+dx} \cosh(5a+5bx)}{16(25b^2-d^2)} + \frac{de^{c+dx} \sinh(a+bx)}{8(b^2-d^2)} + \frac{de^{c+dx} \sinh(3a+3bx)}{16(9b^2-d^2)} - \frac{de^{c+dx} \sinh(5a+5bx)}{16(25b^2-d^2)}$$

```
[Out] -1/8*b*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-3/16*b*exp(d*x+c)*cosh(3*b*x+3*a)/(
9*b^2-d^2)+5/16*b*exp(d*x+c)*cosh(5*b*x+5*a)/(25*b^2-d^2)+1/8*d*exp(d*x+c)*
sinh(b*x+a)/(b^2-d^2)+1/16*d*exp(d*x+c)*sinh(3*b*x+3*a)/(9*b^2-d^2)-1/16*d*
exp(d*x+c)*sinh(5*b*x+5*a)/(25*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5620, 5582}

$$\int e^{c+dx} \cosh^2(a+bx) \sinh^3(a+bx) \, dx = \frac{de^{c+dx} \sinh(a+bx)}{8(b^2-d^2)} + \frac{de^{c+dx} \sinh(3a+3bx)}{16(9b^2-d^2)} - \frac{de^{c+dx} \sinh(5a+5bx)}{16(25b^2-d^2)} - \frac{be^{c+dx} \cosh(a+bx)}{8(b^2-d^2)} - \frac{3be^{c+dx} \cosh(3a+3bx)}{16(9b^2-d^2)} + \frac{5be^{c+dx} \cosh(5a+5bx)}{16(25b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] -1/8*(b*E^(c + d*x)*Cosh[a + b*x])/(b^2 - d^2) - (3*b*E^(c + d*x)*Cosh[3*a
+ 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*
b^2 - d^2)) + (d*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (d*E^(c + d*x
)*Sinh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[5*a + 5*b*x])
/(16*(25*b^2 - d^2))
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{c+dx}\sinh(a+bx)-\frac{1}{16}e^{c+dx}\sinh(3a+3bx)+\frac{1}{16}e^{c+dx}\sinh(5a+5bx)\right)dx\\
&= -\left(\frac{1}{16}\int e^{c+dx}\sinh(3a+3bx)\,dx\right)\\
&\quad+\frac{1}{16}\int e^{c+dx}\sinh(5a+5bx)\,dx-\frac{1}{8}\int e^{c+dx}\sinh(a+bx)\,dx\\
&= -\frac{be^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{3be^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}\\
&\quad+\frac{de^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{de^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\begin{aligned}
\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx &= \frac{1}{16}e^{c+dx}\left(\frac{-2b\cosh(a+bx)+2d\sinh(a+bx)}{(b-d)(b+d)}\right.\\
&\quad+\frac{-3b\cosh(3(a+bx))+d\sinh(3(a+bx))}{9b^2-d^2}\\
&\quad\left.+\frac{5b\cosh(5(a+bx))-d\sinh(5(a+bx))}{25b^2-d^2}\right)
\end{aligned}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*((-2*b*Cosh[a + b*x] + 2*d*Sinh[a + b*x])/((b - d)*(b + d)) +
(-3*b*Cosh[3*(a + b*x)] + d*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (5*b*Cosh[5*
(a + b*x)] - d*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.43

$$\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{16b-16d}-\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{16\left(b+d\right)}+\frac{\sinh\left(3a-c+\left(3b-d\right)x\right)}{96b-32d}-\frac{\sinh\left(3a+c+\left(3b+d\right)x\right)}{32\left(3b+d\right)}-\text{si}$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+(
3*b-d)*x)/(3*b-d)-1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)-1/32/(5*b-d)*sinh((5*b
-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)-1/16*cosh(a-c+(b-d)*x)/(b-d
)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)-1/32*cosh
(3*a+c+(3*b+d)*x)/(3*b+d)+1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh((5*b
+d)*x+5*a+c)/(5*b+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(177) = 354.

Time = 0.28 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.71

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx=\text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
+ a)^4 - (9*b^4*d - 10*b^2*d^3 + d^5)*cosh(d*x + c)*sinh(b*x + a)^5 + (25*b
^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^2)*
cosh(d*x + c)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x +
 a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*cosh(d*x + c)*sinh(b
*x + a)^2 + (450*b^4*d - 68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5
)*cosh(b*x + a)^4 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*cosh(d
*x + c)*sinh(b*x + a) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^5 - 3
*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^3 - 2*(225*b^5 - 34*b^3*d^2 +
b*d^4)*cosh(b*x + a))*cosh(d*x + c) + (5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(
b*x + a)^5 + 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)*sinh(b*x + a)^4
- (9*b^4*d - 10*b^2*d^3 + d^5)*sinh(b*x + a)^5 - 3*(25*b^5 - 26*b^3*d^2 + b
*d^4)*cosh(b*x + a)^3 + (25*b^4*d - 26*b^2*d^3 + d^5 - 10*(9*b^4*d - 10*b^2
*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (50*(9*b^5 - 10*b^3*d^2 + b*
```

```
d^4)*cosh(b*x + a)^3 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a))*sinh(
b*x + a)^2 - 2*(225*b^5 - 34*b^3*d^2 + b*d^4)*cosh(b*x + a) + (450*b^4*d -
68*b^2*d^3 + 2*d^5 - 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^4 + 3*(25
*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/(
(225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^6 - 3*(225*b^6 - 2
59*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(225*b^6
 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (225*b
^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*sinh(b*x + a)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. $2(168) = 336$.

Time = 26.30 (sec) , antiderivative size = 2693, normalized size of antiderivative = 13.81

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**3*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)
*exp(d*x)*sinh(a - d*x)**5/16 - x*exp(c)*exp(d*x)*sinh(a - d*x)**4*cosh(a -
 d*x)/16 + x*exp(c)*exp(d*x)*sinh(a - d*x)**3*cosh(a - d*x)**2/8 + x*exp(c)
*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a -
d*x)*cosh(a - d*x)**4/16 - x*exp(c)*exp(d*x)*cosh(a - d*x)**5/16 - exp(c)*e
xp(d*x)*sinh(a - d*x)**5/(32*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x)**4*cosh(a
 - d*x)/(32*d) - exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/(6*d) -
exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**4/(96*d) + 5*exp(c)*exp(d*x)*c
osh(a - d*x)**5/(96*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**5/3
2 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)**4*cosh(a - d*x/3)/32 + x*exp(c)*ex
p(d*x)*sinh(a - d*x/3)**3*cosh(a - d*x/3)**2/16 - x*exp(c)*exp(d*x)*sinh(a
- d*x/3)**2*cosh(a - d*x/3)**3/16 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cos
h(a - d*x/3)**4/32 - x*exp(c)*exp(d*x)*cosh(a - d*x/3)**5/32 - 9*exp(c)*exp
(d*x)*sinh(a - d*x/3)**5/(64*d) - 21*exp(c)*exp(d*x)*sinh(a - d*x/3)**4*cos
h(a - d*x/3)/(64*d) - exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)**3
/(2*d) - 27*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**4/(64*d) - 7*e
xp(c)*exp(d*x)*cosh(a - d*x/3)**5/(64*d), Eq(b, -d/3)), (x*exp(c)*exp(d*x)*
sinh(a - d*x/5)**5/32 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/5)**4*cosh(a - d*x
/5)/32 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/5)**3*cosh(a - d*x/5)**2/16 + 5*x
*exp(c)*exp(d*x)*sinh(a - d*x/5)**2*cosh(a - d*x/5)**3/16 + 5*x*exp(c)*exp(
d*x)*sinh(a - d*x/5)*cosh(a - d*x/5)**4/32 + x*exp(c)*exp(d*x)*cosh(a - d*x
/5)**5/32 - 5*exp(c)*exp(d*x)*sinh(a - d*x/5)**5/(64*d) - 15*exp(c)*exp(d*x
)*sinh(a - d*x/5)**4*cosh(a - d*x/5)/(64*d) - 5*exp(c)*exp(d*x)*sinh(a - d*
x/5)**2*cosh(a - d*x/5)**3/(6*d) - 125*exp(c)*exp(d*x)*sinh(a - d*x/5)*cosh
(a - d*x/5)**4/(192*d) - 31*exp(c)*exp(d*x)*cosh(a - d*x/5)**5/(192*d), Eq(
b, -d/5)), (x*exp(c)*exp(d*x)*sinh(a + d*x/5)**5/32 - 5*x*exp(c)*exp(d*x)*s
```

inh(a + d*x/5)**4*cosh(a + d*x/5)/32 + 5*x*exp(c)*exp(d*x)*sinh(a + d*x/5)*
*3*cosh(a + d*x/5)**2/16 - 5*x*exp(c)*exp(d*x)*sinh(a + d*x/5)**2*cosh(a +
d*x/5)**3/16 + 5*x*exp(c)*exp(d*x)*sinh(a + d*x/5)*cosh(a + d*x/5)**4/32 -
x*exp(c)*exp(d*x)*cosh(a + d*x/5)**5/32 - 5*exp(c)*exp(d*x)*sinh(a + d*x/5)
5/(64*d) + 15*exp(c)*exp(d*x)*sinh(a + d*x/5)4*cosh(a + d*x/5)/(64*d) +
5*exp(c)*exp(d*x)*sinh(a + d*x/5)**2*cosh(a + d*x/5)**3/(6*d) - 125*exp(c)
*exp(d*x)*sinh(a + d*x/5)*cosh(a + d*x/5)**4/(192*d) + 31*exp(c)*exp(d*x)*c
osh(a + d*x/5)**5/(192*d), Eq(b, d/5)), (x*exp(c)*exp(d*x)*sinh(a + d*x/3)*
*5/32 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)**4*cosh(a + d*x/3)/32 + x*exp(c
)*exp(d*x)*sinh(a + d*x/3)**3*cosh(a + d*x/3)**2/16 + x*exp(c)*exp(d*x)*sin
h(a + d*x/3)**2*cosh(a + d*x/3)**3/16 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)
*cosh(a + d*x/3)**4/32 + x*exp(c)*exp(d*x)*cosh(a + d*x/3)**5/32 + 7*exp(c)
*exp(d*x)*sinh(a + d*x/3)**5/(64*d) - 27*exp(c)*exp(d*x)*sinh(a + d*x/3)**4
*cosh(a + d*x/3)/(64*d) + exp(c)*exp(d*x)*sinh(a + d*x/3)**3*cosh(a + d*x/3
)**2/(2*d) + exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)**3/d - 75*e
xp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**4/(64*d) + 23*exp(c)*exp(d*
x)*cosh(a + d*x/3)**5/(64*d), Eq(b, d/3)), (-x*exp(c)*exp(d*x)*sinh(a + d*x
)**5/16 + x*exp(c)*exp(d*x)*sinh(a + d*x)**4*cosh(a + d*x)/16 + x*exp(c)*ex
p(d*x)*sinh(a + d*x)**3*cosh(a + d*x)**2/8 - x*exp(c)*exp(d*x)*sinh(a + d*x
)**2*cosh(a + d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**4/
16 + x*exp(c)*exp(d*x)*cosh(a + d*x)**5/16 + 5*exp(c)*exp(d*x)*sinh(a + d*x
)**5/(96*d) + exp(c)*exp(d*x)*sinh(a + d*x)**4*cosh(a + d*x)/(96*d) - exp(c
)*exp(d*x)*sinh(a + d*x)**3*cosh(a + d*x)**2/(6*d) + exp(c)*exp(d*x)*sinh(a
+ d*x)**2*cosh(a + d*x)**3/(3*d) + 7*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a
+ d*x)**4/(96*d) - 13*exp(c)*exp(d*x)*cosh(a + d*x)**5/(96*d), Eq(b, d)), (
75*b**5*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(225*b**6 - 259*b
4*d2 + 35*b**2*d**4 - d**6) - 30*b**5*exp(c)*exp(d*x)*cosh(a + b*x)**5/
(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) + 26*b**4*d*exp(c)*exp(d*x
)*sinh(a + b*x)**5/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 65*b*
*4*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)**2/(225*b**6 - 259*b**4
*d**2 + 35*b**2*d**4 - d**6) + 30*b**4*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh
(a + b*x)**4/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 26*b**3*d**
2*exp(c)*exp(d*x)*sinh(a + b*x)**4*cosh(a + b*x)/(225*b**6 - 259*b**4*d**2
+ 35*b**2*d**4 - d**6) - 30*b**3*d**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh
(a + b*x)**3/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) + 6*b**3*d**2
*exp(c)*exp(d*x)*cosh(a + b*x)**5/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4
- d**6) - 2*b**2*d**3*exp(c)*exp(d*x)*sinh(a + b*x)**5/(225*b**6 - 259*b**4
*d**2 + 35*b**2*d**4 - d**6) + 18*b**2*d**3*exp(c)*exp(d*x)*sinh(a + b*x)**
3*cosh(a + b*x)**2/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 6*b**
2*d**3*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**4/(225*b**6 - 259*b**4*
d**2 + 35*b**2*d**4 - d**6) + 2*b*d**4*exp(c)*exp(d*x)*sinh(a + b*x)**4*cos
h(a + b*x)/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) + 3*b*d**4*exp(
c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(225*b**6 - 259*b**4*d**2 + 3
5*b**2*d**4 - d**6) - d**5*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)**
2/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6), True))

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)} - \frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)} - \frac{e^{(bx+dx+a+c)}}{16\,(b+d)} - \frac{e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)} + \frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) - 1/32*e^(3*b*x + d*x + 3*a + c)/(
3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) - 1/16*e^(-b*x + d*x - a + c)
/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) + 1/32*e^(-5*b*x + d*x
 - 5*a + c)/(5*b - d)
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.03

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{3\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^2\,(25\,b^5-10\,b^3\,d^2+b\,d^4)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$
$$-\frac{\cosh(a+b\,x)^5\,\mathrm{e}^{c+d\,x}\,(30\,b^5-6\,b^3\,d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$
$$+\frac{6\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)\,\left(5\,b^4\,d-b^2\,d^3\right)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$
$$-\frac{\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^3\,(65\,b^4\,d-18\,b^2\,d^3+d^5)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$
$$+\frac{2\,b^2\,d\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^5\,(13\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$
$$-\frac{2\,b\,d^2\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^4\,(13\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$

```
[In] int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^2*(b*d^4 + 25*b^5 - 10*b^3*d^
2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^5*exp(c + d
*x)*(30*b^5 - 6*b^3*d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) + (6*c
osh(a + b*x)^4*exp(c + d*x)*sinh(a + b*x)*(5*b^4*d - b^2*d^3))/(225*b^6 - d
^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x
)^3*(65*b^4*d + d^5 - 18*b^2*d^3))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^
2) + (2*b^2*d*exp(c + d*x)*sinh(a + b*x)^5*(13*b^2 - d^2))/(225*b^6 - d^6 +
 35*b^2*d^4 - 259*b^4*d^2) - (2*b*d^2*cosh(a + b*x)*exp(c + d*x)*sinh(a + b
*x)^4*(13*b^2 - d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2)
```

3.953 $\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{e^{c+dx}}{8d}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}+\frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}$$

```
[Out] -1/8*exp(d*x+c)/d-1/8*d*exp(d*x+c)*cosh(4*b*x+4*a)/(16*b^2-d^2)+1/2*b*exp(d
*x+c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5620, 2225, 5583}

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{be^{c+dx}\sinh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\cosh(4a+4bx)}{8(16b^2-d^2)}-\frac{e^{c+dx}}{8d}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/8*E^(c + d*x)/d - (d*E^(c + d*x)*Cosh[4*a + 4*b*x])/(8*(16*b^2 - d^2)) +
 (b*E^(c + d*x)*Sinh[4*a + 4*b*x])/(2*(16*b^2 - d^2))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{8}e^{c+dx}+\frac{1}{8}e^{c+dx}\cosh(4a+4bx)\right)dx\\ &= -\left(\frac{1}{8}\int e^{c+dx}\,dx\right)+\frac{1}{8}\int e^{c+dx}\cosh(4a+4bx)\,dx\\ &= -\frac{e^{c+dx}}{8d}-\frac{de^{c+dx}\cosh(4a+4bx)}{8\left(16b^2-d^2\right)}+\frac{be^{c+dx}\sinh(4a+4bx)}{2\left(16b^2-d^2\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{c+dx}\left(16b^2-d^2+d^2\cosh(4(a+bx))-4bd\sinh(4(a+bx))\right)}{8\left(-16b^2d+d^3\right)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*(16*b^2 - d^2 + d^2*Cosh[4*(a + b*x)] - 4*b*d*Sinh[4*(a + b*x)
]))/(8*(-16*b^2*d + d^3))
```

Maple [A] (verified)

Time = 112.68 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{\left(-4d\,\mathrm{e}^{8bx+8a}b+d^{2}\mathrm{e}^{8bx+8a}+32\,\mathrm{e}^{4bx+4a}b^{2}-2d^{2}\mathrm{e}^{4bx+4a}+4bd+d^{2}\right)\mathrm{e}^{-4bx+dx-4a+c}}{16(4b+d)(4b-d)d}$
default	$-\frac{\sinh(dx+c)}{8d}+\frac{\sinh((4b-d)x+4a-c)}{64b-16d}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}-\frac{\cosh(dx+c)}{8d}-\frac{\cosh((4b-d)x+4a-c)}{16(4b-d)}+\frac{\cosh((4b+d)x+4a}{64b+16d}$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(4*b+d)/(4*b-d)/d*(-4*d*exp(8*b*x+8*a)*b+d^2*exp(8*b*x+8*a)+32*exp(4*
b*x+4*a)*b^2-2*d^2*exp(4*b*x+4*a)+4*b*d+d^2)*exp(-4*b*x+d*x-4*a+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(74) = 148$.

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.65

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{16\,bd\cosh\left(bx+a\right)^{3}\cosh\left(dx+c\right)\sinh\left(bx+a\right)-6\,d^{2}\cosh\left(bx+a\right)^{2}\cosh\left(dx+c\right)\sinh\left(bx+a\right)^{2}+16\,bd\,c}{}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(16*b*d*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a) - 6*d^2*cosh(b*x +
a)^2*cosh(d*x + c)*sinh(b*x + a)^2 + 16*b*d*cosh(b*x + a)*cosh(d*x + c)*sin
h(b*x + a)^3 - d^2*cosh(d*x + c)*sinh(b*x + a)^4 - (d^2*cosh(b*x + a)^4 + 1
6*b^2 - d^2)*cosh(d*x + c) - (d^2*cosh(b*x + a)^4 - 16*b*d*cosh(b*x + a)^3*
sinh(b*x + a) + 6*d^2*cosh(b*x + a)^2*sinh(b*x + a)^2 - 16*b*d*cosh(b*x + a
)*sinh(b*x + a)^3 + d^2*sinh(b*x + a)^4 + 16*b^2 - d^2)*sinh(d*x + c))/((16
*b^2*d - d^3)*cosh(b*x + a)^4 - 2*(16*b^2*d - d^3)*cosh(b*x + a)^2*sinh(b*x
 + a)^2 + (16*b^2*d - d^3)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(66) = 132$.

Time = 6.14 (sec) , antiderivative size = 819, normalized size of antiderivative = 9.87

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$

$$= \begin{cases} xe^c\sinh^2(a)\cosh^2(a) \\ \frac{xe^ce^{dx}\sinh^4\left(a-\frac{dx}{4}\right)}{16} + \frac{xe^ce^{dx}\sinh^3\left(a-\frac{dx}{4}\right)\cosh\left(a-\frac{dx}{4}\right)}{4} + \frac{3xe^ce^{dx}\sinh^2\left(a-\frac{dx}{4}\right)\cosh^2\left(a-\frac{dx}{4}\right)}{8} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{4}\right)\cosh^3\left(a-\frac{dx}{4}\right)}{4} \\ \frac{xe^ce^{dx}\sinh^4\left(a+\frac{dx}{4}\right)}{16} - \frac{xe^ce^{dx}\sinh^3\left(a+\frac{dx}{4}\right)\cosh\left(a+\frac{dx}{4}\right)}{4} + \frac{3xe^ce^{dx}\sinh^2\left(a+\frac{dx}{4}\right)\cosh^2\left(a+\frac{dx}{4}\right)}{8} - \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{4}\right)\cosh^3\left(a+\frac{dx}{4}\right)}{4} \\ \left(-\frac{x\sinh^4(a+bx)}{8} + \frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4} - \frac{x\cosh^4(a+bx)}{8} + \frac{\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{\sinh(a+bx)\cosh^3(a+bx)}{8b}\right)e^c \\ -\frac{2b^2e^ce^{dx}\sinh^4(a+bx)}{16b^2d-d^3} + \frac{4b^2e^ce^{dx}\sinh^2(a+bx)\cosh^2(a+bx)}{16b^2d-d^3} - \frac{2b^2e^ce^{dx}\cosh^4(a+bx)}{16b^2d-d^3} + \frac{2bde^ce^{dx}\sinh^3(a+bx)\cosh(a+bx)}{16b^2d-d^3} + \frac{2bde^ce}{} \end{cases}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*
exp(d*x)*sinh(a - d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(
a - d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8
+ x*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x
)*cosh(a - d*x/4)**4/16 - exp(c)*exp(d*x)*sinh(a - d*x/4)**4/(6*d) - 5*exp(
c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/(12*d) - 5*exp(c)*exp(d*x)*s
inh(a - d*x/4)*cosh(a - d*x/4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a - d*x/4)*
*4/(6*d), Eq(b, -d/4)), (x*exp(c)*exp(d*x)*sinh(a + d*x/4)**4/16 - x*exp(c)
*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/4 + 3*x*exp(c)*exp(d*x)*sinh(a
 + d*x/4)**2*cosh(a + d*x/4)**2/8 - x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(
a + d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a + d*x/4)**4/16 - exp(c)*exp(d*x)
*sinh(a + d*x/4)**4/(6*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d
*x/4)/(12*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d)
- exp(c)*exp(d*x)*cosh(a + d*x/4)**4/(6*d), Eq(b, d/4)), ((-x*sinh(a + b*x)
**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 - x*cosh(a + b*x)**4/8 + sinh
(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*x)*cosh(a + b*x)**3/(8*b))*ex
p(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**4/(16*b**2*d - d**
3) + 4*b**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(16*b**2*d -
d**3) - 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**4/(16*b**2*d - d**3) + 2*b*d*
exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(16*b**2*d - d**3) + 2*b*d*e
xp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**3/(16*b**2*d - d**3) - d**2*exp
(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(16*b**2*d - d**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(3-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} - \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)} - \frac{e^{(dx+c)}}{8\,d}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) - 1/16*e^(-4*b*x + d*x - 4*a + c)/
(4*b - d) - 1/8*e^(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int e^{c+dx}\cosh^2(a+bx)\sinh^2(a+bx)\,dx$$

$$= -\frac{\frac{d^2\,\mathrm{e}^{c+d\,x}\,\left(\frac{\mathrm{e}^{-4\,a-4\,b\,x}}{2}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{2}\right)}{8}+\frac{b\,d\,\mathrm{e}^{c+d\,x}\,\left(\frac{\mathrm{e}^{-4\,a-4\,b\,x}}{2}-\frac{\mathrm{e}^{4\,a+4\,b\,x}}{2}\right)}{2}}{16\,b^2\,d-d^3} - \frac{\mathrm{e}^{c+d\,x}}{8\,d}$$

```
[In] int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] - ((d^2*exp(c + d*x)*(exp(- 4*a - 4*b*x)/2 + exp(4*a + 4*b*x)/2))/8 + (b*d*
exp(c + d*x)*(exp(- 4*a - 4*b*x)/2 - exp(4*a + 4*b*x)/2))/2)/(16*b^2*d - d^
3) - exp(c + d*x)/(8*d)
```

3.954 $\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}-\frac{de^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}$$

```
[Out] 1/4*b*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)+3/4*b*exp(d*x+c)*cosh(3*b*x+3*a)/(9*
b^2-d^2)-1/4*d*exp(d*x+c)*sinh(b*x+a)/(b^2-d^2)-1/4*d*exp(d*x+c)*sinh(3*b*x
+3*a)/(9*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5620, 5582}

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx=-\frac{de^{c+dx}\sinh(a+bx)}{4(b^2-d^2)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4(9b^2-d^2)}+\frac{be^{c+dx}\cosh(a+bx)}{4(b^2-d^2)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4(9b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[a + b*x])/(4*(b^2 - d^2)) + (3*b*E^(c + d*x)*Cosh[3*a +
 3*b*x])/(4*(9*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[a + b*x])/(4*(b^2 - d^2))
- (d*E^(c + d*x)*Sinh[3*a + 3*b*x])/(4*(9*b^2 - d^2))
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}e^{c+dx}\sinh(a+bx)+\frac{1}{4}e^{c+dx}\sinh(3a+3bx)\right)\,dx\\
&= \frac{1}{4}\int e^{c+dx}\sinh(a+bx)\,dx+\frac{1}{4}\int e^{c+dx}\sinh(3a+3bx)\,dx\\
&= \frac{be^{c+dx}\cosh(a+bx)}{4\left(b^2-d^2\right)}+\frac{3be^{c+dx}\cosh(3a+3bx)}{4\left(9b^2-d^2\right)}-\frac{de^{c+dx}\sinh(a+bx)}{4\left(b^2-d^2\right)}-\frac{de^{c+dx}\sinh(3a+3bx)}{4\left(9b^2-d^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx=\frac{1}{4}e^{c+dx}\left(\frac{b\cosh(a+bx)-d\sinh(a+bx)}{(b-d)(b+d)}+\frac{3b\cosh(3(a+bx))-d\sinh(3(a+bx))}{9b^2-d^2}\right)$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x],x]
```

```
[Out] (E^(c + d*x)*((b*Cosh[a + b*x] - d*Sinh[a + b*x])/((b - d)*(b + d)) + (3*b*
Cosh[3*(a + b*x)] - d*Sinh[3*(a + b*x)])/(9*b^2 - d^2)))/4
```

Maple [A] (verified)

Time = 11.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sinh(a-c+(b-d)x)}{8(b-d)}+\frac{\sinh(a+c+(b+d)x)}{8b+8d}-\frac{\sinh(3a-c+(3b-d)x)}{8(3b-d)}+\frac{\sinh(3a+c+(3b+d)x)}{24b+8d}+\frac{\cosh(a-c+(b-d)x)}{8b-8d}+\frac{\cosh(a\cdots}{8\cdots}$
risch	$\frac{(3b^3 e^{6bx+6a}-b^2 d\, e^{6bx+6a}-3b\, d^2 e^{6bx+6a}+d^3 e^{6bx+6a}+9b^3 e^{4bx+4a}-9b^2 d\, e^{4bx+4a}-b\, d^2 e^{4bx+4a}+d^3 e^{4bx+4a}+9b^3 e^{2bx+2a}+9b^2 d\, e^{2bx+2a}\cdots}{8(3b+d)(b+d)(3b-d)(b-d)}$

[In] int(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)

[Out] -1/8*sinh(a-c+(b-d)*x)/(b-d)+1/8*sinh(a+c+(b+d)*x)/(b+d)-1/8*sinh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/8*cosh(a-c+(b-d)*x)/(b-d)+1/8*cosh(a+c+(b+d)*x)/(b+d)+1/8*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/8*cosh(3*a+c+(3*b+d)*x)/(3*b+d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(115) = 230.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.00

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{9\left(b^3-bd^2\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-\left(b^2d-d^3\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-\left(9\,b^2d-d\cdots\right.}{\cdots}$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="fricas")

[Out] 1/4*(9*(b^3 - b*d^2)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - (b^2*d - d^3)*cosh(d*x + c)*sinh(b*x + a)^3 - (9*b^2*d - d^3 + 3*(b^2*d - d^3)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + (3*(b^3 - b*d^2)*cosh(b*x + a)^3 + (9*b^3 - b*d^2)*cosh(b*x + a))*cosh(d*x + c) + (3*(b^3 - b*d^2)*cosh(b*x + a)^3 + 9*(b^3 - b*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 - (b^2*d - d^3)*sinh(b*x + a)^3 + (9*b^3 - b*d^2)*cosh(b*x + a) - (9*b^2*d - d^3 + 3*(b^2*d - d^3)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(109) = 218$.

Time = 2.60 (sec) , antiderivative size = 972, normalized size of antiderivative = 7.65

$$\int e^{c+dx} \cosh^2(a+bx) \sinh(a+bx) \, dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*ex
p(d*x)*sinh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x
)/8 + x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + x*exp(c)*exp(d*x
)*cosh(a - d*x)**3/8 - exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) - exp(c)*exp(
d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x)**
3/(8*d), Eq(b, -d)), (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*e
xp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a -
 d*x/3)*cosh(a - d*x/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - 3*e
xp(c)*exp(d*x)*sinh(a - d*x/3)**3/(8*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)
**2*cosh(a - d*x/3)/(4*d) - exp(c)*exp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b,
 -d/3)), (x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3/8 - 3*x*exp(c)*exp(d*x)*sinh
(a + d*x/3)**2*cosh(a + d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh
(a + d*x/3)**2/8 - x*exp(c)*exp(d*x)*cosh(a + d*x/3)**3/8 - exp(c)*exp(d*x)
*sinh(a + d*x/3)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/
3)**2/(4*d) - exp(c)*exp(d*x)*cosh(a + d*x/3)**3/(8*d), Eq(b, d/3)), (-x*ex
p(c)*exp(d*x)*sinh(a + d*x)**3/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(
a + d*x)/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/8 - x*exp(c)*
exp(d*x)*cosh(a + d*x)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) - exp(
c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**2/(4*d) + 3*exp(c)*exp(d*x)*cosh(a
 + d*x)**3/(8*d), Eq(b, d)), (3*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4) + 2*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b*
*4 - 10*b**2*d**2 + d**4) - 3*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a +
 b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) - 2*b*d**2*exp(c)*exp(d*x)*sinh(a +
 b*x)**2*cosh(a + b*x)/(9*b**4 - 10*b**2*d**2 + d**4) - b*d**2*exp(c)*exp(d
*x)*cosh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*
sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)} + \frac{e^{(bx+dx+a+c)}}{8\,(b+d)} + \frac{e^{(-bx+dx-a+c)}}{8\,(b-d)} + \frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*e^(b*x + d*x + a + c)/(b + d)
 + 1/8*e^(-b*x + d*x - a + c)/(b - d) + 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int e^{c+dx}\cosh^2(a+bx)\sinh(a+bx)\,dx$$
$$= \frac{e^{c+d\,x}\,\left(3\,b^3\cosh(a+b\,x)^3 - 3\,b^2\,d\cosh(a+b\,x)^2\sinh(a+b\,x) + 2\,b^2\,d\sinh(a+b\,x)^3 - b\,d^2\cosh(a+b\,x)^3 - \right.}{9\,b^4 - 10\,b^2\,d^2 + d^4}$$

```
[In] int(cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x),x)
```

```
[Out] (exp(c + d*x)*(3*b^3*cosh(a + b*x)^3 - b*d^2*cosh(a + b*x)^3 + d^3*cosh(a +
 b*x)^2*sinh(a + b*x) + 2*b^2*d*sinh(a + b*x)^3 - 2*b*d^2*cosh(a + b*x)*sin
h(a + b*x)^2 - 3*b^2*d*cosh(a + b*x)^2*sinh(a + b*x)))/(9*b^4 + d^4 - 10*b^
2*d^2)
```

3.955 $\int e^{c+dx}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \frac{2b^2e^{c+dx}}{d(4b^2-d^2)} - \frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2} + \frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}$$

```
[Out] 2*b^2*exp(d*x+c)/d/(4*b^2-d^2)-d*exp(d*x+c)*cosh(b*x+a)^2/(4*b^2-d^2)+2*b*e
xp(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/(4*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 2225}

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = -\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2} + \frac{2be^{c+dx}\sinh(a+bx)\cosh(a+bx)}{4b^2-d^2} + \frac{2b^2e^{c+dx}}{d(4b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (2*b^2*E^(c + d*x))/(d*(4*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[a + b*x]^2)/(4*
b^2 - d^2) + (2*b*E^(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^2 - d^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}+\frac{(2b^2)\int e^{c+dx}\,dx}{4b^2-d^2}\\
&= \frac{2b^2e^{c+dx}}{d(4b^2-d^2)}-\frac{de^{c+dx}\cosh^2(a+bx)}{4b^2-d^2}+\frac{2be^{c+dx}\cosh(a+bx)\sinh(a+bx)}{4b^2-d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

$$
\int e^{c+dx}\cosh^2(a+bx)\,dx=\frac{e^{c+dx}(-4b^2+d^2+d^2\cosh(2(a+bx))-2bd\sinh(2(a+bx)))}{-8b^2d+2d^3}
$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*(-4*b^2 + d^2 + d^2*Cosh[2*(a + b*x)] - 2*b*d*Sinh[2*(a + b*x)
]))/(-8*b^2*d + 2*d^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{e^{dx+c}\left(-d^2\cosh(2bx+2a)+2bd\sinh(2bx+2a)+4b^2-d^2\right)}{8b^2d-2d^3}$
risch	$\frac{\left(2d\,e^{4bx+4a}b-d^2e^{4bx+4a}+8b^2e^{2bx+2a}-2d^2e^{2bx+2a}-2bd-d^2\right)e^{-2bx+dx-2a+c}}{4(2b+d)(2b-d)d}$
default	$\frac{\sinh(dx+c)}{2d}+\frac{\sinh(2a-c+(2b-d)x)}{8b-4d}+\frac{\sinh(2a+c+(2b+d)x)}{8b+4d}+\frac{\cosh(dx+c)}{2d}-\frac{\cosh(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\cosh(2a+c+(2b-}{8b+4d}$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(d*x+c)*(-d^2*cosh(2*b*x+2*a)+2*b*d*sinh(2*b*x+2*a)+4*b^2-d^2)/(8*b^2*d-
2*d^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.85

$$\int e^{c+dx}\cosh^2(a+bx)\,dx$$
$$= \frac{4\,bd\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right) - d^2\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2 - \left(d^2\cosh\left(bx+a\right)^2 - 4\,b^2\right.}{2\left(\left(4\,b^2d - d^3\right)\cosh\left(bx\right.\right.}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*d*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a) - d^2*cosh(d*x + c)*si
nh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 - 4*b^2 + d^2)*cosh(d*x + c) - (d^2*co
sh(b*x + a)^2 - 4*b*d*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - 4
*b^2 + d^2)*sinh(d*x + c))/((4*b^2*d - d^3)*cosh(b*x + a)^2 - (4*b^2*d - d^
3)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(78) = 156.

Time = 1.05 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.58

$$\int e^{c+dx}\cosh^2(a+bx)\,dx$$
$$= \begin{cases} xe^c\cosh^2\left(a\right) \\ \frac{xe^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{4} + \frac{xe^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cosh\left(a-\frac{dx}{2}\right)}{2} + \frac{xe^ce^{dx}\cosh^2\left(a-\frac{dx}{2}\right)}{4} - \frac{e^ce^{dx}\sinh^2\left(a-\frac{dx}{2}\right)}{d} - \frac{3e^ce^{dx}\sinh\left(a-\frac{dx}{2}\right)\cos}{2d} \\ \frac{xe^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{4} - \frac{xe^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cosh\left(a+\frac{dx}{2}\right)}{2} + \frac{xe^ce^{dx}\cosh^2\left(a+\frac{dx}{2}\right)}{4} - \frac{e^ce^{dx}\sinh^2\left(a+\frac{dx}{2}\right)}{d} + \frac{3e^ce^{dx}\sinh\left(a+\frac{dx}{2}\right)\cos}{2d} \\ \left(-\frac{x\sinh^2\left(a+bx\right)}{2} + \frac{x\cosh^2\left(a+bx\right)}{2} + \frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}\right)e^c \\ -\frac{2b^2e^ce^{dx}\sinh^2\left(a+bx\right)}{4b^2d-d^3} + \frac{2b^2e^ce^{dx}\cosh^2\left(a+bx\right)}{4b^2d-d^3} + \frac{2bde^ce^{dx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^2d-d^3} - \frac{d^2e^ce^{dx}\cosh^2\left(a+bx\right)}{4b^2d-d^3} \end{cases}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a)**2, Eq(b, 0) & Eq(d, 0)), (x*exp(c)*exp(d*x)*si
nh(a - d*x/2)**2/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)/2 +
x*exp(c)*exp(d*x)*cosh(a - d*x/2)**2/4 - exp(c)*exp(d*x)*sinh(a - d*x/2)**2
/d - 3*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)/(2*d), Eq(b, -d/2)),
 (x*exp(c)*exp(d*x)*sinh(a + d*x/2)**2/4 - x*exp(c)*exp(d*x)*sinh(a + d*x/2
)*cosh(a + d*x/2)/2 + x*exp(c)*exp(d*x)*cosh(a + d*x/2)**2/4 - exp(c)*exp(d
*x)*sinh(a + d*x/2)**2/d + 3*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2
```

```
)/(2*d), Eq(b, d/2)), ((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh
(a + b*x)*cosh(a + b*x)/(2*b))*exp(c), Eq(d, 0)), (-2*b**2*exp(c)*exp(d*x)*
sinh(a + b*x)**2/(4*b**2*d - d**3) + 2*b**2*exp(c)*exp(d*x)*cosh(a + b*x)**
2/(4*b**2*d - d**3) + 2*b*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)/(4*
b**2*d - d**3) - d**2*exp(c)*exp(d*x)*cosh(a + b*x)**2/(4*b**2*d - d**3), T
rue))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \frac{e^{(2bx+dx+2a+c)}}{4(2b+d)} - \frac{e^{(-2bx+dx-2a+c)}}{4(2b-d)} + \frac{e^{(dx+c)}}{2d}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*e^(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*e^(-2*b*x + d*x - 2*a + c)/(2
*b - d) + 1/2*e^(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int e^{c+dx}\cosh^2(a+bx)\,dx = \frac{2b^2 e^{c+dx} - d^2\cosh(a+bx)^2 e^{c+dx} + 2bd\cosh(a+bx)\, e^{c+dx}\sinh(a+bx)}{4b^2 d - d^3}$$

```
[In] int(cosh(a + b*x)^2*exp(c + d*x),x)
```

```
[Out] (2*b^2*exp(c + d*x) - d^2*cosh(a + b*x)^2*exp(c + d*x) + 2*b*d*cosh(a + b*x
)*exp(c + d*x)*sinh(a + b*x))/(4*b^2*d - d^3)
```

3.956 $\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = -\frac{3e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{2e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}$$

```
[Out] -3/2*exp(-a+c-(b-d)*x)/(b-d)+1/2*exp(a+c+(b+d)*x)/(b+d)+2*exp(-a+c-(b-d)*x)
*hypergeom([1, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5622, 2225, 2259, 2283}

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx = \frac{2e^{-a-x(b-d)+c}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}-\frac{3e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
```

```
[Out] (-3*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (2*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -1/2*(b - d)/b, (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2259

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5622

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{2}e^{-a+c-(b-d)x} + \frac{1}{2}e^{-a+c-(b-d)x+2(a+bx)} + \frac{2e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\right) dx \\
&= \frac{1}{2}\int e^{-a+c-(b-d)x+2(a+bx)}\,dx + \frac{3}{2}\int e^{-a+c-(b-d)x}\,dx + 2\int \frac{e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\,dx \\
&= -\frac{3e^{-a+c-(b-d)x}}{2(b-d)} + \frac{2e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d} \\
&\quad + \frac{1}{2}\int e^{a+c+(b+d)x}\,dx \\
&= -\frac{3e^{-a+c-(b-d)x}}{2(b-d)} + \frac{e^{a+c+(b+d)x}}{2(b+d)} + \frac{2e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx$$
$$=\frac{e^{c+dx}\left(b\cosh(a+bx)-2(b-d)e^{a+bx}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)-d\sinh(a+bx)\right)}{(b-d)(b+d)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x],x]
[Out] (E^(c + d*x)*(b*Cosh[a + b*x] - 2*(b - d)*E^(a + b*x)*Hypergeometric2F1[1,
(b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))] - d*Sinh[a + b*x]))/((b -
d)*(b + d))
```

Maple [F]

$$\int \mathrm{e}^{dx+c}\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x)
```

Fricas [F]

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx=\int\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx=\int \cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -4*b*integrate(e^(d*x + c)/((3*b - d)*e^(5*b*x + 5*a) - 2*(3*b - d)*e^(3*b*
x + 3*a) + (3*b - d)*e^(b*x + a)), x) + 1/2*(5*b^2*e^c + 6*b*d*e^c + d^2*e^
c + (3*b^2*e^c - 4*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(6*b^2*e^c + b*d*
e^c - d^2*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^
(3*b*x + 3*a) - (3*b^3 - b^2*d - 3*b*d^2 + d^3)*e^(b*x + a))
```

Giac [F]

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx=\int \cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh(a+bx)\coth(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^{2}\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x),x)
```

```
[Out] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x), x)
```

3.957 $\int e^{c+dx} \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\frac{e^{c+dx}}{d}-\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}$$

```
[Out] exp(d*x+c)/d-4*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a)
)/d+4*exp(d*x+c)*hypergeom([2, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5593, 2225, 2283}

$$\int e^{c+dx}\coth^2(a+bx)\,dx=-\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(c + d*x)/d - (4*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (4*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2*
b), E^(2*(a + b*x))])/d
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5593

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(e^{c+dx} + \frac{4e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2} + \frac{4e^{c+dx}}{-1+e^{2(a+bx)}}\right) dx \\ &= 4\int \frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2} dx + 4\int \frac{e^{c+dx}}{-1+e^{2(a+bx)}} dx + \int e^{c+dx} dx \\ &= \frac{e^{c+dx}}{d} - \frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d} \\ &\quad + \frac{4e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.57

$$\int e^{c+dx}\coth^2(a+bx)\,dx = \frac{e^{c+dx}}{d} - \frac{2d\left(\frac{e^{2a+c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d} - \frac{e^{2a+c+(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{b\left(-1+e^{2a}\right)} + \frac{e^{c+dx}\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(c + d*x)/d - (2*d*((E^(2*a + c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 +
d/(2*b), E^(2*(a + b*x))])/d - (E^(2*a + c + (2*b + d)*x)*Hypergeometric2F1
[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))])/(2*b + d)))/(b*(-1 + E^(2*a
))) + (E^(c + d*x)*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [F]

$$\int e^{dx+c}\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{2}dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x)
```

Fricas [F]

$$\int e^{c+dx}\coth^{2}(a+bx)\,dx=\int\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{2}e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^{2}(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx}\coth^{2}(a+bx)\,dx=\int\cosh\left(bx+a\right)^{2}\operatorname{csch}\left(bx+a\right)^{2}e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
[Out] 16*b*d*integrate(-e^(d*x + c)/(8*b^2 - 6*b*d + d^2 - (8*b^2 - 6*b*d + d^2)*
e^(6*b*x + 6*a) + 3*(8*b^2 - 6*b*d + d^2)*e^(4*b*x + 4*a) - 3*(8*b^2 - 6*b*
d + d^2)*e^(2*b*x + 2*a)), x) + (8*b^2*e^c + 10*b*d*e^c + d^2*e^c + (8*b^2*
e^c - 6*b*d*e^c + d^2*e^c)*e^(4*b*x + 4*a) - 2*(8*b^2*e^c + 2*b*d*e^c - d^2
*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(8*b^2*d - 6*b*d^2 + d^3 + (8*b^2*d - 6*b*d^
2 + d^3)*e^(4*b*x + 4*a) - 2*(8*b^2*d - 6*b*d^2 + d^3)*e^(2*b*x + 2*a))
```

Giac [F]

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^2*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^2(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^2,x)
[Out] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^2, x)
```

3.958 $\int e^{c+dx} \coth^2(a+bx)\text{csch}(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} + \frac{8e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} - \frac{8e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(3,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

```
[Out] -2*exp(a+c+(b+d)*x)*hypergeom([1, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp(2*b*x+2*
a))/(b+d)+8*exp(a+c+(b+d)*x)*hypergeom([2, 1/2*(b+d)/b],[1/2*(3*b+d)/b],exp
(2*b*x+2*a))/(b+d)-8*exp(a+c+(b+d)*x)*hypergeom([3, 1/2*(b+d)/b],[1/2*(3*b+
d)/b],exp(2*b*x+2*a))/(b+d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5622, 2283}

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{2e^{a+x(b+d)+c}\,\text{Hypergeometric2F1}\left(1,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} + \frac{8e^{a+x(b+d)+c}\,\text{Hypergeometric2F1}\left(2,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d} - \frac{8e^{a+x(b+d)+c}\,\text{Hypergeometric2F1}\left(3,\frac{b+d}{2b},\frac{3b+d}{2b},e^{2(a+bx)}\right)}{b+d}$$

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
```

```
[Out] (-2*E^(a + c + (b + d)*x)*Hypergeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*
b), E^(2*(a + b*x))])/(b + d) + (8*E^(a + c + (b + d)*x)*Hypergeometric2F1[
2, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))])/(b + d) - (8*E^(a + c
+ (b + d)*x)*Hypergeometric2F1[3, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a +
 b*x))])/(b + d)
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5622

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{8e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3} + \frac{8e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2} + \frac{2e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}} \right) dx \\
&= 2\int \frac{e^{a+c+(b+d)x}}{-1+e^{2(a+bx)}}\, dx + 8\int \frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^3}\, dx + 8\int \frac{e^{a+c+(b+d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\, dx \\
&= -\frac{2e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(1, \frac{b+d}{2b}, \frac{3b+d}{2b}, e^{2(a+bx)}\right)}{b+d} \\
&\quad + \frac{8e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(2, \frac{b+d}{2b}, \frac{3b+d}{2b}, e^{2(a+bx)}\right)}{b+d} \\
&\quad - \frac{8e^{a+c+(b+d)x}\,\text{Hypergeometric2F1}\left(3, \frac{b+d}{2b}, \frac{3b+d}{2b}, e^{2(a+bx)}\right)}{b+d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{e^{c-\frac{ad}{b}}\left((b+d)e^{d\left(\frac{a}{b}+x\right)}(d+b\coth(a+bx))\operatorname{csch}(a+bx)+2(b^2+d^2)\,e^{\frac{(b+d)(a+bx)}{b}}\,\text{Hypergeometric2F1}\left(1,\frac{b}{\ldots}\right.\right.}{2b^2(b+d)}$$

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^2*Csch[a + b*x],x]
[Out] -1/2*(E^(c - (a*d)/b)*((b + d)*E^(d*(a/b + x))*(d + b*Coth[a + b*x])*Csch[a
 + b*x] + 2*(b^2 + d^2)*E^(((b + d)*(a + b*x))/b)*Hypergeometric2F1[1, (b +
 d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]))/(b^2*(b + d))
```

Maple [F]

$$\int e^{dx+c}\cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
[Out] int(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x)
```

Fricas [F]

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \int \cosh\left(bx+a\right)^2\operatorname{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
[Out] integral(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**2*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\text{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")

[Out] -48*(b^3*e^c + b*d^2*e^c)*integrate(e^(b*x + d*x + a)/(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3 + (15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(4*b*x + 4*a) - 4*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(2*b*x + 2*a)), x) + 2*((15*b^2*e^c - 8*b*d*e^c + d^2*e^c)*e^(5*b*x + 5*a) - 2*(10*b^2*e^c + 3*b*d*e^c - d^2*e^c)*e^(3*b*x + 3*a) + (9*b^2*e^c + 14*b*d*e^c + d^2*e^c)*e^(b*x + a))*e^(d*x)/(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3 - (15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(6*b*x + 6*a) + 3*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(4*b*x + 4*a) - 3*(15*b^3 - 23*b^2*d + 9*b*d^2 - d^3)*e^(2*b*x + 2*a))

Giac [F]

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int \cosh\left(bx+a\right)^2\text{csch}\left(bx+a\right)^3e^{(dx+c)}\,dx$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")

[Out] integrate(cosh(b*x + a)^2*csch(b*x + a)^3*e^(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^2(a+bx)\text{csch}(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^3}\,dx$$

[In] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^3,x)

[Out] int((cosh(a + b*x)^2*exp(c + d*x))/sinh(a + b*x)^3, x)

3.959 $\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 137

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=-\frac{3be^{c+dx}\cosh(2a+2bx)}{16(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16(36b^2-d^2)}+\frac{3de^{c+dx}\sinh(2a+2bx)}{32(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32(36b^2-d^2)}$$

```
[Out] -3/16*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+3/16*b*exp(d*x+c)*cosh(6*b*x
+6*a)/(36*b^2-d^2)+3/32*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/32*d*exp
(d*x+c)*sinh(6*b*x+6*a)/(36*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5620, 5582}

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx=\frac{3de^{c+dx}\sinh(2a+2bx)}{32(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32(36b^2-d^2)}-\frac{3be^{c+dx}\cosh(2a+2bx)}{16(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16(36b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(16*(4*b^2 - d^2)) + (3*b*E^(c + d*x)*
Cosh[6*a + 6*b*x])/(16*(36*b^2 - d^2)) + (3*d*E^(c + d*x)*Sinh[2*a + 2*b*x]
)/(32*(4*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[6*a + 6*b*x])/(32*(36*b^2 - d^2)
)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{3}{32}e^{c+dx}\sinh(2a+2bx)+\frac{1}{32}e^{c+dx}\sinh(6a+6bx)\right)dx\\ &= \frac{1}{32}\int e^{c+dx}\sinh(6a+6bx)\,dx-\frac{3}{32}\int e^{c+dx}\sinh(2a+2bx)\,dx\\ &= -\frac{3be^{c+dx}\cosh(2a+2bx)}{16\,(4b^2-d^2)}+\frac{3be^{c+dx}\cosh(6a+6bx)}{16\,(36b^2-d^2)}\\ &\quad+\frac{3de^{c+dx}\sinh(2a+2bx)}{32\,(4b^2-d^2)}-\frac{de^{c+dx}\sinh(6a+6bx)}{32\,(36b^2-d^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx$$
$$=\frac{e^{c+dx}(6b(-36b^2+d^2)\cosh(2(a+bx))+6(4b^3-bd^2)\cosh(6(a+bx))+2d(52b^2-d^2+(-4b^2+d^2)\cosh(4}{32\,(144b^4-40b^2d^2+d^4)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*(6*b*(-36*b^2 + d^2)*Cosh[2*(a + b*x)] + 6*(4*b^3 - b*d^2)*Cos
h[6*(a + b*x)] + 2*d*(52*b^2 - d^2 + (-4*b^2 + d^2)*Cosh[4*(a + b*x)])*Sinh
[2*(a + b*x)]))/(32*(144*b^4 - 40*b^2*d^2 + d^4))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\frac{3\sinh\left(2a-c+\left(2b-d\right)x\right)}{64\left(2b-d\right)}-\frac{3\sinh\left(2a+c+\left(2b+d\right)x\right)}{64\left(2b+d\right)}-\frac{\sinh\left(\left(6b-d\right)x+6a-c\right)}{64\left(6b-d\right)}+\frac{\sinh\left(\left(6b+d\right)x+6a\right.}{384b+64d}$$

[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x)

[Out] 3/64*sinh(2*a-c+(2*b-d)*x)/(2*b-d)-3/64*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/64/(6*b-d)*sinh((6*b-d)*x+6*a-c)+1/64/(6*b+d)*sinh((6*b+d)*x+6*a+c)-3/64*cosh(2*a-c+(2*b-d)*x)/(2*b-d)-3/64*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/64*cosh((6*b-d)*x+6*a-c)/(6*b-d)+1/64*cosh((6*b+d)*x+6*a+c)/(6*b+d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(125) = 250.

Time = 0.27 (sec) , antiderivative size = 676, normalized size of antiderivative = 4.93

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx =$$
$$-\frac{10\left(4\,b^2d-d^3\right)\cosh\left(bx+a\right)^3\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-45\left(4\,b^3-bd^2\right)\cosh\left(bx+a\right)^2\cosh\left(dx+c\right)}{}$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="fricas")

[Out] -1/16*(10*(4*b^2*d - d^3)*cosh(b*x + a)^3*cosh(d*x + c)*sinh(b*x + a)^3 - 45*(4*b^3 - b*d^2)*cosh(b*x + a)^2*cosh(d*x + c)*sinh(b*x + a)^4 + 3*(4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^5 - 3*(4*b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)^6 - 3*(15*(4*b^3 - b*d^2)*cosh(b*x + a)^4 - 36*b^3 + b*d^2)*cosh(d*x + c)*sinh(b*x + a)^2 + 3*((4*b^2*d - d^3)*cosh(b*x + a)^5 - (36*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - 3*((4*b^3 - b*d^2)*cosh(b*x + a)^6 - (36*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x + c) - (3*(4*b^3 - b*d^2)*cosh(b*x + a)^6 - 10*(4*b^2*d - d^3)*cosh(b*x + a)^3*sinh(b*x + a)^3 + 45*(4*b^3 - b*d^2)*cosh(b*x + a)^2*sinh(b*x + a)^4 - 3*(4*b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^5 + 3*(4*b^3 - b*d^2)*sinh(b*x + a)^6 - 3*(36*b^3 - b*d^2)*cosh(b*x + a)^2 + 3*(15*(4*b^3 - b*d^2)*cosh(b*x + a)^4 - 36*b^3 + b*d^2)*sinh(b*x + a)^2 - 3*((4*b^2*d - d^3)*cosh(b*x + a)^5 - (36*b^2*d - d^3)*cosh(b*x + a))*sinh(b*x + a))*sinh(d*x + c))/((144*b^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)^6 - 3*(144*b^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(144*b^4 - 40*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (144*b^4 - 40*b^2*d^2 + d^4)*sinh(b*x + a)^6)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(119) = 238$.

Time = 68.04 (sec) , antiderivative size = 1916, normalized size of antiderivative = 13.99

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)**3*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*exp(
c)*exp(d*x)*sinh(a - d*x/2)**6/64 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**5*
cosh(a - d*x/2)/32 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**4*cosh(a - d*x/2)
**2/64 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a - d*x/2)**3/16 + 3*x
*exp(c)*exp(d*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**4/64 - 3*x*exp(c)*exp(
d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**5/32 - 3*x*exp(c)*exp(d*x)*cosh(a - d
*x/2)**6/64 - 3*exp(c)*exp(d*x)*sinh(a - d*x/2)**6/(16*d) - 15*exp(c)*exp(d
*x)*sinh(a - d*x/2)**5*cosh(a - d*x/2)/(32*d) + 13*exp(c)*exp(d*x)*sinh(a -
 d*x/2)**3*cosh(a - d*x/2)**3/(16*d) - 15*exp(c)*exp(d*x)*sinh(a - d*x/2)*c
osh(a - d*x/2)**5/(32*d) - 3*exp(c)*exp(d*x)*cosh(a - d*x/2)**6/(16*d), Eq(
b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/6)**6/64 + 3*x*exp(c)*exp(d*x)*s
inh(a - d*x/6)**5*cosh(a - d*x/6)/32 + 15*x*exp(c)*exp(d*x)*sinh(a - d*x/6)
**4*cosh(a - d*x/6)**2/64 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/6)**3*cosh(a -
 d*x/6)**3/16 + 15*x*exp(c)*exp(d*x)*sinh(a - d*x/6)**2*cosh(a - d*x/6)**4/
64 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/6)*cosh(a - d*x/6)**5/32 + x*exp(c)*e
xp(d*x)*cosh(a - d*x/6)**6/64 - 3*exp(c)*exp(d*x)*sinh(a - d*x/6)**6/(80*d)
 - 21*exp(c)*exp(d*x)*sinh(a - d*x/6)**5*cosh(a - d*x/6)/(160*d) + 11*exp(c
)*exp(d*x)*sinh(a - d*x/6)**3*cosh(a - d*x/6)**3/(16*d) - 21*exp(c)*exp(d*x
)*sinh(a - d*x/6)*cosh(a - d*x/6)**5/(160*d) - 3*exp(c)*exp(d*x)*cosh(a - d
*x/6)**6/(80*d), Eq(b, -d/6)), (-x*exp(c)*exp(d*x)*sinh(a + d*x/6)**6/64 +
3*x*exp(c)*exp(d*x)*sinh(a + d*x/6)**5*cosh(a + d*x/6)/32 - 15*x*exp(c)*exp
(d*x)*sinh(a + d*x/6)**4*cosh(a + d*x/6)**2/64 + 5*x*exp(c)*exp(d*x)*sinh(a
 + d*x/6)**3*cosh(a + d*x/6)**3/16 - 15*x*exp(c)*exp(d*x)*sinh(a + d*x/6)**
2*cosh(a + d*x/6)**4/64 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/6)*cosh(a + d*x/
6)**5/32 - x*exp(c)*exp(d*x)*cosh(a + d*x/6)**6/64 + 3*exp(c)*exp(d*x)*sinh
(a + d*x/6)**6/(80*d) - 21*exp(c)*exp(d*x)*sinh(a + d*x/6)**5*cosh(a + d*x/
6)/(160*d) + 11*exp(c)*exp(d*x)*sinh(a + d*x/6)**3*cosh(a + d*x/6)**3/(16*d
) - 21*exp(c)*exp(d*x)*sinh(a + d*x/6)*cosh(a + d*x/6)**5/(160*d) + 3*exp(c
)*exp(d*x)*cosh(a + d*x/6)**6/(80*d), Eq(b, d/6)), (3*x*exp(c)*exp(d*x)*sin
h(a + d*x/2)**6/64 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/2)**5*cosh(a + d*x/2)
/32 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/2)**4*cosh(a + d*x/2)**2/64 + 3*x*ex
p(c)*exp(d*x)*sinh(a + d*x/2)**3*cosh(a + d*x/2)**3/16 - 3*x*exp(c)*exp(d*x
)*sinh(a + d*x/2)**2*cosh(a + d*x/2)**4/64 - 3*x*exp(c)*exp(d*x)*sinh(a + d
*x/2)*cosh(a + d*x/2)**5/32 + 3*x*exp(c)*exp(d*x)*cosh(a + d*x/2)**6/64 + 3
*exp(c)*exp(d*x)*sinh(a + d*x/2)**6/(16*d) - 15*exp(c)*exp(d*x)*sinh(a + d*
```

```
x/2)**5*cosh(a + d*x/2)/(32*d) + 13*exp(c)*exp(d*x)*sinh(a + d*x/2)**3*cosh
(a + d*x/2)**3/(16*d) - 15*exp(c)*exp(d*x)*sinh(a + d*x/2)*cosh(a + d*x/2)*
*5/(32*d) + 3*exp(c)*exp(d*x)*cosh(a + d*x/2)**6/(16*d), Eq(b, d/2)), (-6*b
**3*exp(c)*exp(d*x)*sinh(a + b*x)**6/(144*b**4 - 40*b**2*d**2 + d**4) + 18*
b**3*exp(c)*exp(d*x)*sinh(a + b*x)**4*cosh(a + b*x)**2/(144*b**4 - 40*b**2*
d**2 + d**4) + 18*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**4/(1
44*b**4 - 40*b**2*d**2 + d**4) - 6*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**6/(1
44*b**4 - 40*b**2*d**2 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**5*
cosh(a + b*x)/(144*b**4 - 40*b**2*d**2 + d**4) - 16*b**2*d*exp(c)*exp(d*x)*
sinh(a + b*x)**3*cosh(a + b*x)**3/(144*b**4 - 40*b**2*d**2 + d**4) + 6*b**2
*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**5/(144*b**4 - 40*b**2*d**2
+ d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a + b*x)**4*cosh(a + b*x)**2/(144*b
**4 - 40*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh
(a + b*x)**4/(144*b**4 - 40*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*sinh(a
 + b*x)**3*cosh(a + b*x)**3/(144*b**4 - 40*b**2*d**2 + d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = \frac{e^{(6\,bx+dx+6\,a+c)}}{64\,(6\,b+d)} - \frac{3\,e^{(2\,bx+dx+2\,a+c)}}{64\,(2\,b+d)} - \frac{3\,e^{(-2\,bx+dx-2\,a+c)}}{64\,(2\,b-d)} + \frac{e^{(-6\,bx+dx-6\,a+c)}}{64\,(6\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*e^(6*b*x + d*x + 6*a + c)/(6*b + d) - 3/64*e^(2*b*x + d*x + 2*a + c)/(
2*b + d) - 3/64*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/64*e^(-6*b*x + d*x
 - 6*a + c)/(6*b - d)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^3(a+bx)\,dx = -\frac{b^3\left(\frac{27\,e^{c+d\,x}\cosh(2\,a+2\,b\,x)}{4}-\frac{3\,e^{c+d\,x}\cosh(6\,a+6\,b\,x)}{4}\right)+d^3\left(\frac{3\,e^{c+d\,x}\sinh(2\,a+2\,b\,x)}{32}-\frac{e^{c+d\,x}\sinh(6\,a+6\,b\,x)}{32}\right)-b^2\,d\left(\frac{27\,e^{c+d\,x}}{}\right.}{144\,b^4-40\,b^2\,d^2+d^4}$$

```
[In] int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^3,x)
```

```
[Out] -(b^3*((27*exp(c + d*x)*cosh(2*a + 2*b*x))/4 - (3*exp(c + d*x)*cosh(6*a + 6
*b*x))/4) + d^3*((3*exp(c + d*x)*sinh(2*a + 2*b*x))/32 - (exp(c + d*x)*sinh
(6*a + 6*b*x))/32) - b^2*d*((27*exp(c + d*x)*sinh(2*a + 2*b*x))/8 - (exp(c
+ d*x)*sinh(6*a + 6*b*x))/8) - b*d^2*((3*exp(c + d*x)*cosh(2*a + 2*b*x))/16
 - (3*exp(c + d*x)*cosh(6*a + 6*b*x))/16))/(144*b^4 + d^4 - 40*b^2*d^2)
```

3.960 $\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=\frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}-\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}$$

```
[Out] 1/8*d*exp(d*x+c)*cosh(b*x+a)/(b^2-d^2)-1/16*d*exp(d*x+c)*cosh(3*b*x+3*a)/(9
*b^2-d^2)-1/16*d*exp(d*x+c)*cosh(5*b*x+5*a)/(25*b^2-d^2)-1/8*b*exp(d*x+c)*s
inh(b*x+a)/(b^2-d^2)+3/16*b*exp(d*x+c)*sinh(3*b*x+3*a)/(9*b^2-d^2)+5/16*b*e
xp(d*x+c)*sinh(5*b*x+5*a)/(25*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5620, 5583}

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx=-\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}+\frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (d*E^(c + d*x)*Cosh[a + b*x])/(8*(b^2 - d^2)) - (d*E^(c + d*x)*Cosh[3*a + 3
*b*x])/(16*(9*b^2 - d^2)) - (d*E^(c + d*x)*Cosh[5*a + 5*b*x])/(16*(25*b^2 -
 d^2)) - (b*E^(c + d*x)*Sinh[a + b*x])/(8*(b^2 - d^2)) + (3*b*E^(c + d*x)*S
inh[3*a + 3*b*x])/(16*(9*b^2 - d^2)) + (5*b*E^(c + d*x)*Sinh[5*a + 5*b*x])/
(16*(25*b^2 - d^2))
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{c+dx}\cosh(a+bx)+\frac{1}{16}e^{c+dx}\cosh(3a+3bx)+\frac{1}{16}e^{c+dx}\cosh(5a+5bx)\right)\,dx \\
&= \frac{1}{16}\int e^{c+dx}\cosh(3a+3bx)\,dx+\frac{1}{16}\int e^{c+dx}\cosh(5a+5bx)\,dx-\frac{1}{8}\int e^{c+dx}\cosh(a+bx)\,dx \\
&= \frac{de^{c+dx}\cosh(a+bx)}{8(b^2-d^2)}-\frac{de^{c+dx}\cosh(3a+3bx)}{16(9b^2-d^2)}-\frac{de^{c+dx}\cosh(5a+5bx)}{16(25b^2-d^2)} \\
&\quad -\frac{be^{c+dx}\sinh(a+bx)}{8(b^2-d^2)}+\frac{3be^{c+dx}\sinh(3a+3bx)}{16(9b^2-d^2)}+\frac{5be^{c+dx}\sinh(5a+5bx)}{16(25b^2-d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.61

$$\begin{aligned}
\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{1}{16}e^{c+dx}\Bigg(&\frac{2d\cosh(a+bx)-2b\sinh(a+bx)}{(b-d)(b+d)} \\
&+\frac{-d\cosh(3(a+bx))+3b\sinh(3(a+bx))}{9b^2-d^2} \\
&+\frac{-d\cosh(5(a+bx))+5b\sinh(5(a+bx))}{25b^2-d^2}\Bigg)
\end{aligned}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(c + d*x)*((2*d*Cosh[a + b*x] - 2*b*Sinh[a + b*x])/((b - d)*(b + d)) + (
-(d*Cosh[3*(a + b*x)]) + 3*b*Sinh[3*(a + b*x)])/(9*b^2 - d^2) + (-(d*Cosh[5
*(a + b*x)]) + 5*b*Sinh[5*(a + b*x)])/(25*b^2 - d^2)))/16
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.43

$$-\frac{\sinh\left(a-c+\left(b-d\right)x\right)}{16\left(b-d\right)}-\frac{\sinh\left(a+c+\left(b+d\right)x\right)}{16\left(b+d\right)}+\frac{\sinh\left(3a-c+\left(3b-d\right)x\right)}{96b-32d}+\frac{\sinh\left(3a+c+\left(3b+d\right)x\right)}{96b+32d}$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x)
```

```
[Out] -1/16*sinh(a-c+(b-d)*x)/(b-d)-1/16*sinh(a+c+(b+d)*x)/(b+d)+1/32*sinh(3*a-c+
(3*b-d)*x)/(3*b-d)+1/32*sinh(3*a+c+(3*b+d)*x)/(3*b+d)+1/32/(5*b-d)*sinh((5*
b-d)*x+5*a-c)+1/32/(5*b+d)*sinh((5*b+d)*x+5*a+c)+1/16*cosh(a-c+(b-d)*x)/(b-
d)-1/16*cosh(a+c+(b+d)*x)/(b+d)-1/32*cosh(3*a-c+(3*b-d)*x)/(3*b-d)+1/32*cos
h(3*a+c+(3*b+d)*x)/(3*b+d)-1/32*cosh((5*b-d)*x+5*a-c)/(5*b-d)+1/32*cosh((5*
b+d)*x+5*a+c)/(5*b+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(177) = 354$.

Time = 0.26 (sec) , antiderivative size = 917, normalized size of antiderivative = 4.70

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x
+ a)^4 - 5*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(d*x + c)*sinh(b*x + a)^5 - (75
*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)
^2)*cosh(d*x + c)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*cosh(d*x + c)*si
nh(b*x + a)^2 + (450*b^5 - 68*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 +
b*d^4)*cosh(b*x + a)^4 - 9*(25*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*c
osh(d*x + c)*sinh(b*x + a) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)^5
+ (25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a)^3 - 2*(225*b^4*d - 34*b^2*d^3
 + d^5)*cosh(b*x + a))*cosh(d*x + c) + ((9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b
*x + a)^5 + 5*(9*b^4*d - 10*b^2*d^3 + d^5)*cosh(b*x + a)*sinh(b*x + a)^4 -
5*(9*b^5 - 10*b^3*d^2 + b*d^4)*sinh(b*x + a)^5 + (25*b^4*d - 26*b^2*d^3 + d
^5)*cosh(b*x + a)^3 - (75*b^5 - 78*b^3*d^2 + 3*b*d^4 + 50*(9*b^5 - 10*b^3*d
^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a)^3 + (10*(9*b^4*d - 10*b^2*d^3 +
d^5)*cosh(b*x + a)^3 + 3*(25*b^4*d - 26*b^2*d^3 + d^5)*cosh(b*x + a))*sinh(
b*x + a)^2 - 2*(225*b^4*d - 34*b^2*d^3 + d^5)*cosh(b*x + a) + (450*b^5 - 68
*b^3*d^2 + 2*b*d^4 - 25*(9*b^5 - 10*b^3*d^2 + b*d^4)*cosh(b*x + a)^4 - 9*(2
5*b^5 - 26*b^3*d^2 + b*d^4)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/
((225*b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^6 - 3*(225*b^6 -
259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(225*b^
```

6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*cosh(b*x + a)^2*sinh(b*x + a)^4 - (225*
b^6 - 259*b^4*d^2 + 35*b^2*d^4 - d^6)*sinh(b*x + a)^6)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2761 vs. $2(168) = 336$.

Time = 25.24 (sec) , antiderivative size = 2761, normalized size of antiderivative = 14.16

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a)**2,x)

[Out] Piecewise((x*exp(c)*sinh(a)**2*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)
*exp(d*x)*sinh(a - d*x)**5/16 - x*exp(c)*exp(d*x)*sinh(a - d*x)**4*cosh(a -
d*x)/16 + x*exp(c)*exp(d*x)*sinh(a - d*x)**3*cosh(a - d*x)**2/8 + x*exp(c)
*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/8 - x*exp(c)*exp(d*x)*sinh(a -
d*x)*cosh(a - d*x)**4/16 - x*exp(c)*exp(d*x)*cosh(a - d*x)**5/16 + 13*exp(c
)*exp(d*x)*sinh(a - d*x)**5/(96*d) + 7*exp(c)*exp(d*x)*sinh(a - d*x)**4*cos
h(a - d*x)/(96*d) - exp(c)*exp(d*x)*sinh(a - d*x)**3*cosh(a - d*x)**2/(3*d)
- exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)**3/(6*d) - exp(c)*exp(d*x
)*sinh(a - d*x)*cosh(a - d*x)**4/(96*d) + 5*exp(c)*exp(d*x)*cosh(a - d*x)**
5/(96*d), Eq(b, -d)), (-x*exp(c)*exp(d*x)*sinh(a - d*x/3)**5/32 - 3*x*exp(c
)*exp(d*x)*sinh(a - d*x/3)**4*cosh(a - d*x/3)/32 - x*exp(c)*exp(d*x)*sinh(a
- d*x/3)**3*cosh(a - d*x/3)**2/16 + x*exp(c)*exp(d*x)*sinh(a - d*x/3)**2*c
osh(a - d*x/3)**3/16 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)*
*4/32 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**5/32 - 23*exp(c)*exp(d*x)*sinh(a
- d*x/3)**5/(64*d) - 75*exp(c)*exp(d*x)*sinh(a - d*x/3)**4*cosh(a - d*x/3)
/(64*d) - exp(c)*exp(d*x)*sinh(a - d*x/3)**3*cosh(a - d*x/3)**2/d + exp(c)*
exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3)**3/(2*d) + 27*exp(c)*exp(d*x)*s
inh(a - d*x/3)*cosh(a - d*x/3)**4/(64*d) + 7*exp(c)*exp(d*x)*cosh(a - d*x/3
)**5/(64*d), Eq(b, -d/3)), (x*exp(c)*exp(d*x)*sinh(a - d*x/5)**5/32 + 5*x*e
xp(c)*exp(d*x)*sinh(a - d*x/5)**4*cosh(a - d*x/5)/32 + 5*x*exp(c)*exp(d*x)*
sinh(a - d*x/5)**3*cosh(a - d*x/5)**2/16 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x
/5)**2*cosh(a - d*x/5)**3/16 + 5*x*exp(c)*exp(d*x)*sinh(a - d*x/5)*cosh(a -
d*x/5)**4/32 + x*exp(c)*exp(d*x)*cosh(a - d*x/5)**5/32 - 47*exp(c)*exp(d*x
)*sinh(a - d*x/5)**5/(192*d) - 205*exp(c)*exp(d*x)*sinh(a - d*x/5)**4*cosh(
a - d*x/5)/(192*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/5)**3*cosh(a - d*x/5)**
2/(3*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/5)**2*cosh(a - d*x/5)**3/(6*d) - 1
25*exp(c)*exp(d*x)*sinh(a - d*x/5)*cosh(a - d*x/5)**4/(192*d) - 31*exp(c)*e
xp(d*x)*cosh(a - d*x/5)**5/(192*d), Eq(b, -d/5)), (-x*exp(c)*exp(d*x)*sinh(
a + d*x/5)**5/32 + 5*x*exp(c)*exp(d*x)*sinh(a + d*x/5)**4*cosh(a + d*x/5)/3
2 - 5*x*exp(c)*exp(d*x)*sinh(a + d*x/5)**3*cosh(a + d*x/5)**2/16 + 5*x*exp(
c)*exp(d*x)*sinh(a + d*x/5)**2*cosh(a + d*x/5)**3/16 - 5*x*exp(c)*exp(d*x)*
sinh(a + d*x/5)*cosh(a + d*x/5)**4/32 + x*exp(c)*exp(d*x)*cosh(a + d*x/5)**

```
5/32 + 47*exp(c)*exp(d*x)*sinh(a + d*x/5)**5/(192*d) - 205*exp(c)*exp(d*x)*
sinh(a + d*x/5)**4*cosh(a + d*x/5)/(192*d) + 5*exp(c)*exp(d*x)*sinh(a + d*x
/5)**3*cosh(a + d*x/5)**2/(3*d) - 5*exp(c)*exp(d*x)*sinh(a + d*x/5)**2*cosh
(a + d*x/5)**3/(6*d) + 125*exp(c)*exp(d*x)*sinh(a + d*x/5)*cosh(a + d*x/5)*
*4/(192*d) - 31*exp(c)*exp(d*x)*cosh(a + d*x/5)**5/(192*d), Eq(b, d/5)), (x
*exp(c)*exp(d*x)*sinh(a + d*x/3)**5/32 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3
)**4*cosh(a + d*x/3)/32 + x*exp(c)*exp(d*x)*sinh(a + d*x/3)**3*cosh(a + d*x
/3)**2/16 + x*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)**3/16 - 3*
x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**4/32 + x*exp(c)*exp(d*x)
*cosh(a + d*x/3)**5/32 + 7*exp(c)*exp(d*x)*sinh(a + d*x/3)**5/(64*d) - 27*e
xp(c)*exp(d*x)*sinh(a + d*x/3)**4*cosh(a + d*x/3)/(64*d) + exp(c)*exp(d*x)*
sinh(a + d*x/3)**3*cosh(a + d*x/3)**2/(2*d) + 21*exp(c)*exp(d*x)*sinh(a + d
*x/3)*cosh(a + d*x/3)**4/(64*d) - 9*exp(c)*exp(d*x)*cosh(a + d*x/3)**5/(64*
d), Eq(b, d/3)), (x*exp(c)*exp(d*x)*sinh(a + d*x)**5/16 - x*exp(c)*exp(d*x)
*sinh(a + d*x)**4*cosh(a + d*x)/16 - x*exp(c)*exp(d*x)*sinh(a + d*x)**3*cos
h(a + d*x)**2/8 + x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)**3/8 + x
*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**4/16 - x*exp(c)*exp(d*x)*cosh
(a + d*x)**5/16 - 5*exp(c)*exp(d*x)*sinh(a + d*x)**5/(96*d) - exp(c)*exp(d*
x)*sinh(a + d*x)**4*cosh(a + d*x)/(96*d) + exp(c)*exp(d*x)*sinh(a + d*x)**3
*cosh(a + d*x)**2/(6*d) + 3*exp(c)*exp(d*x)*sinh(a + d*x)*cosh(a + d*x)**4/
(32*d) - exp(c)*exp(d*x)*cosh(a + d*x)**5/(32*d), Eq(b, d)), (-30*b**5*exp(
c)*exp(d*x)*sinh(a + b*x)**5/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**
6) + 75*b**5*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)**2/(225*b**6 -
259*b**4*d**2 + 35*b**2*d**4 - d**6) + 30*b**4*d*exp(c)*exp(d*x)*sinh(a + b
*x)**4*cosh(a + b*x)/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 65*
b**4*d*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(225*b**6 - 259*b*
*4*d**2 + 35*b**2*d**4 - d**6) + 26*b**4*d*exp(c)*exp(d*x)*cosh(a + b*x)**5
/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) + 6*b**3*d**2*exp(c)*exp(
d*x)*sinh(a + b*x)**5/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 30
*b**3*d**2*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)**2/(225*b**6 - 25
9*b**4*d**2 + 35*b**2*d**4 - d**6) - 26*b**3*d**2*exp(c)*exp(d*x)*sinh(a +
b*x)*cosh(a + b*x)**4/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 6*
b**2*d**3*exp(c)*exp(d*x)*sinh(a + b*x)**4*cosh(a + b*x)/(225*b**6 - 259*b*
*4*d**2 + 35*b**2*d**4 - d**6) + 18*b**2*d**3*exp(c)*exp(d*x)*sinh(a + b*x)
**2*cosh(a + b*x)**3/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) - 2*b
**2*d**3*exp(c)*exp(d*x)*cosh(a + b*x)**5/(225*b**6 - 259*b**4*d**2 + 35*b*
*2*d**4 - d**6) + 3*b*d**4*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)**
2/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4 - d**6) + 2*b*d**4*exp(c)*exp(d*
x)*sinh(a + b*x)*cosh(a + b*x)**4/(225*b**6 - 259*b**4*d**2 + 35*b**2*d**4
- d**6) - d**5*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**3/(225*b**6
- 259*b**4*d**2 + 35*b**2*d**4 - d**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{(5\,bx+dx+5\,a+c)}}{32\,(5\,b+d)} + \frac{e^{(3\,bx+dx+3\,a+c)}}{32\,(3\,b+d)} - \frac{e^{(bx+dx+a+c)}}{16\,(b+d)} + \frac{e^{(-bx+dx-a+c)}}{16\,(b-d)} - \frac{e^{(-3\,bx+dx-3\,a+c)}}{32\,(3\,b-d)} - \frac{e^{(-5\,bx+dx-5\,a+c)}}{32\,(5\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*e^(5*b*x + d*x + 5*a + c)/(5*b + d) + 1/32*e^(3*b*x + d*x + 3*a + c)/(
3*b + d) - 1/16*e^(b*x + d*x + a + c)/(b + d) + 1/16*e^(-b*x + d*x - a + c)
/(b - d) - 1/32*e^(-3*b*x + d*x - 3*a + c)/(3*b - d) - 1/32*e^(-5*b*x + d*x
 - 5*a + c)/(5*b - d)
```

Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.02

$$\int e^{c+dx}\cosh^3(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{\cosh(a+b\,x)^5\,\mathrm{e}^{c+d\,x}\,(26\,b^4\,d-2\,b^2\,d^3)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{3\cosh(a+b\,x)^2\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^3\,(25\,b^5-10\,b^3\,d^2+b\,d^4)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{2\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)\,(b\,d^4-13\,b^3\,d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}-\frac{\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^2\,(65\,b^4\,d-18\,b^2\,d^3+d^5)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}-\frac{6\,b^3\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^5\,(5\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}+\frac{6\,b^2\,d\cosh(a+b\,x)\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)^4\,(5\,b^2-d^2)}{225\,b^6-259\,b^4\,d^2+35\,b^2\,d^4-d^6}$$

```
[In] int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x)^2,x)
```

```
[Out] (cosh(a + b*x)^5*exp(c + d*x)*(26*b^4*d - 2*b^2*d^3))/(225*b^6 - d^6 + 35*b
^2*d^4 - 259*b^4*d^2) + (3*cosh(a + b*x)^2*exp(c + d*x)*sinh(a + b*x)^3*(b*
d^4 + 25*b^5 - 10*b^3*d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2) + (2
*cosh(a + b*x)^4*exp(c + d*x)*sinh(a + b*x)*(b*d^4 - 13*b^3*d^2))/(225*b^6
- d^6 + 35*b^2*d^4 - 259*b^4*d^2) - (cosh(a + b*x)^3*exp(c + d*x)*sinh(a +
b*x)^2*(65*b^4*d + d^5 - 18*b^2*d^3))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4
*d^2) - (6*b^3*exp(c + d*x)*sinh(a + b*x)^5*(5*b^2 - d^2))/(225*b^6 - d^6 +
 35*b^2*d^4 - 259*b^4*d^2) + (6*b^2*d*cosh(a + b*x)*exp(c + d*x)*sinh(a + b
*x)^4*(5*b^2 - d^2))/(225*b^6 - d^6 + 35*b^2*d^4 - 259*b^4*d^2)
```

3.961 $\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}$$

```
[Out] 1/2*b*exp(d*x+c)*cosh(2*b*x+2*a)/(4*b^2-d^2)+1/2*b*exp(d*x+c)*cosh(4*b*x+4*
a)/(16*b^2-d^2)-1/4*d*exp(d*x+c)*sinh(2*b*x+2*a)/(4*b^2-d^2)-1/8*d*exp(d*x+
c)*sinh(4*b*x+4*a)/(16*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5620, 5582}

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx=-\frac{de^{c+dx}\sinh(2a+2bx)}{4(4b^2-d^2)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8(16b^2-d^2)}+\frac{be^{c+dx}\cosh(2a+2bx)}{2(4b^2-d^2)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2(16b^2-d^2)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
```

```
[Out] (b*E^(c + d*x)*Cosh[2*a + 2*b*x])/(2*(4*b^2 - d^2)) + (b*E^(c + d*x)*Cosh[4
*a + 4*b*x])/(2*(16*b^2 - d^2)) - (d*E^(c + d*x)*Sinh[2*a + 2*b*x])/(4*(4*b
^2 - d^2)) - (d*E^(c + d*x)*Sinh[4*a + 4*b*x])/(8*(16*b^2 - d^2))
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5620

```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}e^{c+dx}\sinh(2a+2bx)+\frac{1}{8}e^{c+dx}\sinh(4a+4bx)\right)dx\\
&= \frac{1}{8}\int e^{c+dx}\sinh(4a+4bx)\,dx+\frac{1}{4}\int e^{c+dx}\sinh(2a+2bx)\,dx\\
&= \frac{be^{c+dx}\cosh(2a+2bx)}{2\left(4b^2-d^2\right)}+\frac{be^{c+dx}\cosh(4a+4bx)}{2\left(16b^2-d^2\right)}\\
&\quad -\frac{de^{c+dx}\sinh(2a+2bx)}{4\left(4b^2-d^2\right)}-\frac{de^{c+dx}\sinh(4a+4bx)}{8\left(16b^2-d^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx=\frac{1}{8}e^{c+dx}\left(\frac{4b\cosh(2(a+bx))-2d\sinh(2(a+bx))}{4b^2-d^2}+\frac{4b\cosh(4(a+bx))-d\sinh(4(a+bx))}{16b^2-d^2}\right)$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3*Sinh[a + b*x],x]
[Out] (E^(c + d*x)*((4*b*Cosh[2*(a + b*x)] - 2*d*Sinh[2*(a + b*x)])/(4*b^2 - d^2)
 + (4*b*Cosh[4*(a + b*x)] - d*Sinh[4*(a + b*x)])/(16*b^2 - d^2)))/8
```

Maple [A] (verified)

Time = 63.86 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\sinh(2a-c+(2b-d)x)}{8(2b-d)}+\frac{\sinh(2a+c+(2b+d)x)}{16b+8d}-\frac{\sinh((4b-d)x+4a-c)}{16(4b-d)}+\frac{\sinh((4b+d)x+4a+c)}{64b+16d}+\frac{\cosh(2a-c+(2b-d)x)}{16b-8d}+$
risch	$\frac{(16b^3 e^{8bx+8a}-4d\,e^{8bx+8a}b^2-4d^2 e^{8bx+8a}b+d^3 e^{8bx+8a}+64b^3 e^{6bx+6a}-32b^2 d\,e^{6bx+6a}-4b\,d^2 e^{6bx+6a}+2d^3 e^{6bx+6a}+64b^3 e^{2bx+2a}+32b^2}{16(4b+d)(2b+d)(4b-d)(2b-d)}$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sinh(2*a-c+(2*b-d)*x)/(2*b-d)+1/8*sinh(2*a+c+(2*b+d)*x)/(2*b+d)-1/16/(
4*b-d)*sinh((4*b-d)*x+4*a-c)+1/16/(4*b+d)*sinh((4*b+d)*x+4*a+c)+1/8*cosh(2*
a-c+(2*b-d)*x)/(2*b-d)+1/8*cosh(2*a+c+(2*b+d)*x)/(2*b+d)+1/16*cosh((4*b-d)*
x+4*a-c)/(4*b-d)+1/16*cosh((4*b+d)*x+4*a+c)/(4*b+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(125) = 250.

Time = 0.27 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.66

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{(4\,b^2d-d^3)\cosh{(bx+a)}\cosh{(dx+c)}\sinh{(bx+a)}^3-(4\,b^3-bd^2)\cosh{(dx+c)}\sinh{(bx+a)}^4-(16\,b^3}{}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^3 - (4*b^3
- b*d^2)*cosh(d*x + c)*sinh(b*x + a)^4 - (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2
)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a)^2 + ((4*b^2*d - d^3)*cosh(b*
x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c)*sinh(b*x + a) - ((
4*b^3 - b*d^2)*cosh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x
 + c) - ((4*b^3 - b*d^2)*cosh(b*x + a)^4 - (4*b^2*d - d^3)*cosh(b*x + a)*si
nh(b*x + a)^3 + (4*b^3 - b*d^2)*sinh(b*x + a)^4 + (16*b^3 - b*d^2)*cosh(b*x
 + a)^2 + (16*b^3 - b*d^2 + 6*(4*b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 - ((4*b^2*d - d^3)*cosh(b*x + a)^3 + (16*b^2*d - d^3)*cosh(b*x + a))*si
nh(b*x + a))*sinh(d*x + c))/((64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^4 -
2*(64*b^4 - 20*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (64*b^4 - 2
0*b^2*d^2 + d^4)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. 2(114) = 228.

Time = 7.79 (sec) , antiderivative size = 1292, normalized size of antiderivative = 9.43

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(c)*sinh(a)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-x*exp(c)*ex
p(d*x)*sinh(a - d*x/2)**4/8 - x*exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a -
 d*x/2)/4 + x*exp(c)*exp(d*x)*sinh(a - d*x/2)*cosh(a - d*x/2)**3/4 + x*exp(
c)*exp(d*x)*cosh(a - d*x/2)**4/8 - exp(c)*exp(d*x)*sinh(a - d*x/2)**4/(24*d
) - exp(c)*exp(d*x)*sinh(a - d*x/2)**3*cosh(a - d*x/2)/(3*d) - exp(c)*exp(d
*x)*sinh(a - d*x/2)**2*cosh(a - d*x/2)**2/(2*d) - exp(c)*exp(d*x)*cosh(a -
d*x/2)**4/(8*d), Eq(b, -d/2)), (x*exp(c)*exp(d*x)*sinh(a - d*x/4)**4/16 + x
*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cosh(a - d*x/4)/4 + 3*x*exp(c)*exp(d*x)
*sinh(a - d*x/4)**2*cosh(a - d*x/4)**2/8 + x*exp(c)*exp(d*x)*sinh(a - d*x/4
)*cosh(a - d*x/4)**3/4 + x*exp(c)*exp(d*x)*cosh(a - d*x/4)**4/16 - exp(c)*e
xp(d*x)*sinh(a - d*x/4)**4/(6*d) - 5*exp(c)*exp(d*x)*sinh(a - d*x/4)**3*cos
h(a - d*x/4)/(12*d) + 11*exp(c)*exp(d*x)*sinh(a - d*x/4)*cosh(a - d*x/4)**3
/(12*d) + exp(c)*exp(d*x)*cosh(a - d*x/4)**4/(6*d), Eq(b, -d/4)), (-x*exp(c
)*exp(d*x)*sinh(a + d*x/4)**4/16 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)**3*cos
h(a + d*x/4)/4 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x/4)**2*cosh(a + d*x/4)**2/
8 + x*exp(c)*exp(d*x)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/4 - x*exp(c)*exp(d
*x)*cosh(a + d*x/4)**4/16 + exp(c)*exp(d*x)*sinh(a + d*x/4)**4/(6*d) - 5*ex
p(c)*exp(d*x)*sinh(a + d*x/4)**3*cosh(a + d*x/4)/(12*d) + 11*exp(c)*exp(d*x
)*sinh(a + d*x/4)*cosh(a + d*x/4)**3/(12*d) - exp(c)*exp(d*x)*cosh(a + d*x/
4)**4/(6*d), Eq(b, d/4)), (x*exp(c)*exp(d*x)*sinh(a + d*x/2)**4/8 - x*exp(c
)*exp(d*x)*sinh(a + d*x/2)**3*cosh(a + d*x/2)/4 + x*exp(c)*exp(d*x)*sinh(a
 + d*x/2)*cosh(a + d*x/2)**3/4 - x*exp(c)*exp(d*x)*cosh(a + d*x/2)**4/8 + ex
p(c)*exp(d*x)*sinh(a + d*x/2)**4/(24*d) - exp(c)*exp(d*x)*sinh(a + d*x/2)**
3*cosh(a + d*x/2)/(3*d) + exp(c)*exp(d*x)*sinh(a + d*x/2)**2*cosh(a + d*x/2
)**2/(2*d) + exp(c)*exp(d*x)*cosh(a + d*x/2)**4/(8*d), Eq(b, d/2)), (-6*b**
3*exp(c)*exp(d*x)*sinh(a + b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) + 12*b**
3*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*d**2
 + d**4) + 10*b**3*exp(c)*exp(d*x)*cosh(a + b*x)**4/(64*b**4 - 20*b**2*d**2
 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**4
 - 20*b**2*d**2 + d**4) - 10*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a +
b*x)**3/(64*b**4 - 20*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a +
 b*x)**2*cosh(a + b*x)**2/(64*b**4 - 20*b**2*d**2 + d**4) - b*d**2*exp(c)*e
xp(d*x)*cosh(a + b*x)**4/(64*b**4 - 20*b**2*d**2 + d**4) + d**3*exp(c)*exp(
d*x)*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**4 - 20*b**2*d**2 + d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-d/b>0)', see `assume?` for more d
etails)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = \frac{e^{(4\,bx+dx+4\,a+c)}}{16\,(4\,b+d)} + \frac{e^{(2\,bx+dx+2\,a+c)}}{8\,(2\,b+d)} + \frac{e^{(-2\,bx+dx-2\,a+c)}}{8\,(2\,b-d)} + \frac{e^{(-4\,bx+dx-4\,a+c)}}{16\,(4\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*e^(4*b*x + d*x + 4*a + c)/(4*b + d) + 1/8*e^(2*b*x + d*x + 2*a + c)/(2
*b + d) + 1/8*e^(-2*b*x + d*x - 2*a + c)/(2*b - d) + 1/16*e^(-4*b*x + d*x -
 4*a + c)/(4*b - d)
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int e^{c+dx}\cosh^3(a+bx)\sinh(a+bx)\,dx = -\frac{b^3\left(6\,\mathrm{e}^{c+d\,x}-16\cosh(a+b\,x)^4\,\mathrm{e}^{c+d\,x}\right)+b^2\,d\left(4\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)\,\cosh(a+b\,x)^3+6\,\mathrm{e}^{c+d\,x}\sinh(a+b\,x)\right.}{64\,b^4-}$$

```
[In] int(cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x),x)
```

```
[Out] -(b^3*(6*exp(c + d*x) - 16*cosh(a + b*x)^4*exp(c + d*x)) + b^2*d*(6*cosh(a
+ b*x)*exp(c + d*x)*sinh(a + b*x) + 4*cosh(a + b*x)^3*exp(c + d*x)*sinh(a +
 b*x)) - b*d^2*(3*cosh(a + b*x)^2*exp(c + d*x) - 4*cosh(a + b*x)^4*exp(c +
d*x)) - d^3*cosh(a + b*x)^3*exp(c + d*x)*sinh(a + b*x))/(64*b^4 + d^4 - 20*
b^2*d^2)
```

3.962 $\int e^{c+dx}\cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 144

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = -\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4}-\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}+\frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4}+\frac{3be^{c+dx}\cosh^2(a+bx)\sinh(a+bx)}{9b^2-d^2}$$

```
[Out] -6*b^2*d*exp(d*x+c)*cosh(b*x+a)/(9*b^4-10*b^2*d^2+d^4)-d*exp(d*x+c)*cosh(b*
x+a)^3/(9*b^2-d^2)+6*b^3*exp(d*x+c)*sinh(b*x+a)/(9*b^4-10*b^2*d^2+d^4)+3*b*
exp(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/(9*b^2-d^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 5583}

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = -\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2}+\frac{3be^{c+dx}\sinh(a+bx)\cosh^2(a+bx)}{9b^2-d^2}-\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4}+\frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (-6*b^2*d*E^(c + d*x)*Cosh[a + b*x])/(9*b^4 - 10*b^2*d^2 + d^4) - (d*E^(c +
 d*x)*Cosh[a + b*x]^3)/(9*b^2 - d^2) + (6*b^3*E^(c + d*x)*Sinh[a + b*x])/(9
*b^4 - 10*b^2*d^2 + d^4) + (3*b*E^(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/
(9*b^2 - d^2)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2} + \frac{3be^{c+dx}\cosh^2(a+bx)\sinh(a+bx)}{9b^2-d^2} \\ &\quad + \frac{(6b^2)\int e^{c+dx}\cosh(a+bx)\,dx}{9b^2-d^2} \\ &= -\frac{6b^2de^{c+dx}\cosh(a+bx)}{9b^4-10b^2d^2+d^4} - \frac{de^{c+dx}\cosh^3(a+bx)}{9b^2-d^2} \\ &\quad + \frac{6b^3e^{c+dx}\sinh(a+bx)}{9b^4-10b^2d^2+d^4} + \frac{3be^{c+dx}\cosh^2(a+bx)\sinh(a+bx)}{9b^2-d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int e^{c+dx}\cosh^3(a+bx)\,dx$$
$$= \frac{e^{c+dx}(3d(-9b^2+d^2)\cosh(a+bx)+(-b^2d+d^3)\cosh(3(a+bx))+6b(5b^2-d^2+(b^2-d^2)\cosh(2(a+bx)}{4\left(9b^4-10b^2d^2+d^4\right)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (E^(c + d*x)*(3*d*(-9*b^2 + d^2)*Cosh[a + b*x] + (-(b^2*d) + d^3)*Cosh[3*(a
 + b*x)] + 6*b*(5*b^2 - d^2 + (b^2 - d^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])
)/(4*(9*b^4 - 10*b^2*d^2 + d^4))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{e^{dx+c}\left(\left(-b^{2}d+d^{3}\right)\cosh\left(3bx+3a\right)+3\left(b^{3}-b\,d^{2}\right)\sinh\left(3bx+3a\right)+27\left(b+\frac{d}{3}\right)\left(-\cosh\left(bx+a\right)d+b\sinh\left(bx+a\right)\right)\left(b-\frac{d}{3}\right)\right)}{36b^{4}-40b^{2}d^{2}+4d^{4}}$
default	$\frac{3\sinh\left(a-c+\left(b-d\right)x\right)}{8\left(b-d\right)}+\frac{3\sinh\left(a+c+\left(b+d\right)x\right)}{8\left(b+d\right)}+\frac{\sinh\left(3a-c+\left(3b-d\right)x\right)}{24b-8d}+\frac{\sinh\left(3a+c+\left(3b+d\right)x\right)}{24b+8d}-\frac{3\cosh\left(a-c+\left(b-d\right)x\right)}{8\left(b-d\right)}$
risch	$\frac{\left(3b^{3}e^{6bx+6a}-b^{2}d\,e^{6bx+6a}-3b\,d^{2}e^{6bx+6a}+d^{3}e^{6bx+6a}+27b^{3}e^{4bx+4a}-27b^{2}d\,e^{4bx+4a}-3b\,d^{2}e^{4bx+4a}+3d^{3}e^{4bx+4a}-27b^{3}e^{2bx+2a}\right.}{8\left(3b+d\right)\left(b+d\right)\left(3b-d\right)\left(b-d\right)}$

[In] int(exp(d*x+c)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*exp(d*x+c)*((-b^2*d+d^3)*cosh(3*b*x+3*a)+3*(b^3-b*d^2)*sinh(3*b*x+3*a)+27*(b+1/3*d)*(-cosh(b*x+a)*d+b*sinh(b*x+a))*(b-1/3*d))/(9*b^4-10*b^2*d^2+d^4)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(140) = 280.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.65

$$\int e^{c+dx}\cosh^3(a+bx)\,dx =$$
$$-\frac{3\left(b^2d-d^3\right)\cosh\left(bx+a\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^2-3\left(b^3-bd^2\right)\cosh\left(dx+c\right)\sinh\left(bx+a\right)^3-3\left(9\right.}{}$$

[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")

[Out] -1/4*(3*(b^2*d - d^3)*cosh(b*x + a)*cosh(d*x + c)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2)*cosh(d*x + c)*sinh(b*x + a)^3 - 3*(9*b^3 - b*d^2 + 3*(b^3 - b*d^2)*cosh(b*x + a)^2)*cosh(d*x + c)*sinh(b*x + a) + ((b^2*d - d^3)*cosh(b*x + a)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a))*cosh(d*x + c) + ((b^2*d - d^3)*cosh(b*x + a)^3 + 3*(b^2*d - d^3)*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b^3 - b*d^2)*sinh(b*x + a)^3 + 3*(9*b^2*d - d^3)*cosh(b*x + a) - 3*(9*b^3 - b*d^2 + 3*(b^3 - b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a))*sinh(d*x + c))/((9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^4 - 2*(9*b^4 - 10*b^2*d^2 + d^4)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (9*b^4 - 10*b^2*d^2 + d^4)*sinh(b*x + a)^4)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(131) = 262$.

Time = 2.95 (sec) , antiderivative size = 1046, normalized size of antiderivative = 7.26

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((x*exp(c)*cosh(a)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*exp(c)*exp(d*x)
*sinh(a - d*x)**3/8 - 3*x*exp(c)*exp(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/8
+ 3*x*exp(c)*exp(d*x)*sinh(a - d*x)*cosh(a - d*x)**2/8 + 3*x*exp(c)*exp(d*x
)*cosh(a - d*x)**3/8 + 5*exp(c)*exp(d*x)*sinh(a - d*x)**3/(8*d) + exp(c)*ex
p(d*x)*sinh(a - d*x)**2*cosh(a - d*x)/(4*d) - exp(c)*exp(d*x)*sinh(a - d*x)
*cosh(a - d*x)**2/d - 3*exp(c)*exp(d*x)*cosh(a - d*x)**3/(8*d), Eq(b, -d)),
 (x*exp(c)*exp(d*x)*sinh(a - d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x
/3)**2*cosh(a - d*x/3)/8 + 3*x*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x
/3)**2/8 + x*exp(c)*exp(d*x)*cosh(a - d*x/3)**3/8 - 11*exp(c)*exp(d*x)*sinh
(a - d*x/3)**3/(8*d) - 15*exp(c)*exp(d*x)*sinh(a - d*x/3)**2*cosh(a - d*x/3
)/(4*d) - 3*exp(c)*exp(d*x)*sinh(a - d*x/3)*cosh(a - d*x/3)**2/d - exp(c)*e
xp(d*x)*cosh(a - d*x/3)**3/(8*d), Eq(b, -d/3)), (-x*exp(c)*exp(d*x)*sinh(a
+ d*x/3)**3/8 + 3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)**2*cosh(a + d*x/3)/8 -
3*x*exp(c)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/8 + x*exp(c)*exp(d*x
)*cosh(a + d*x/3)**3/8 + exp(c)*exp(d*x)*sinh(a + d*x/3)**3/(8*d) - 3*exp(c
)*exp(d*x)*sinh(a + d*x/3)*cosh(a + d*x/3)**2/(4*d) + 9*exp(c)*exp(d*x)*cos
h(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*exp(c)*exp(d*x)*sinh(a + d*x)**3/8
 - 3*x*exp(c)*exp(d*x)*sinh(a + d*x)**2*cosh(a + d*x)/8 - 3*x*exp(c)*exp(d*
x)*sinh(a + d*x)*cosh(a + d*x)**2/8 + 3*x*exp(c)*exp(d*x)*cosh(a + d*x)**3/
8 - 3*exp(c)*exp(d*x)*sinh(a + d*x)**3/(8*d) + 3*exp(c)*exp(d*x)*sinh(a + d
*x)*cosh(a + d*x)**2/(4*d) - exp(c)*exp(d*x)*cosh(a + d*x)**3/(8*d), Eq(b,
d)), (-6*b**3*exp(c)*exp(d*x)*sinh(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**
4) + 9*b**3*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4 - 10*b**
2*d**2 + d**4) + 6*b**2*d*exp(c)*exp(d*x)*sinh(a + b*x)**2*cosh(a + b*x)/(9
*b**4 - 10*b**2*d**2 + d**4) - 7*b**2*d*exp(c)*exp(d*x)*cosh(a + b*x)**3/(9
*b**4 - 10*b**2*d**2 + d**4) - 3*b*d**2*exp(c)*exp(d*x)*sinh(a + b*x)*cosh(
a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d**4) + d**3*exp(c)*exp(d*x)*cosh(a +
b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-d/b>0)', see `assume?` for more de
tails)I
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int e^{c+dx}\cosh^3(a+bx)\,dx = \frac{e^{(3\,bx+dx+3\,a+c)}}{8\,(3\,b+d)}+\frac{3\,e^{(bx+dx+a+c)}}{8\,(b+d)}-\frac{3\,e^{(-bx+dx-a+c)}}{8\,(b-d)}-\frac{e^{(-3\,bx+dx-3\,a+c)}}{8\,(3\,b-d)}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*e^(3*b*x + d*x + 3*a + c)/(3*b + d) + 3/8*e^(b*x + d*x + a + c)/(b + d)
 - 3/8*e^(-b*x + d*x - a + c)/(b - d) - 1/8*e^(-3*b*x + d*x - 3*a + c)/(3*b
 - d)
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int e^{c+dx}\cosh^3(a+bx)\,dx$$
$$= \frac{e^{c+d\,x}\,\left(9\,b^3\cosh(a+b\,x)^2\sinh(a+b\,x)-6\,b^3\sinh(a+b\,x)^3-7\,b^2\,d\cosh(a+b\,x)^3+6\,b^2\,d\cosh(a+b\,x)\,\mathrm{s}\right.}{9\,b^4-10\,b^2\,d^2+d^4}$$

```
[In] int(cosh(a + b*x)^3*exp(c + d*x),x)
```

```
[Out] (exp(c + d*x)*(d^3*cosh(a + b*x)^3 - 6*b^3*sinh(a + b*x)^3 - 7*b^2*d*cosh(a
 + b*x)^3 + 9*b^3*cosh(a + b*x)^2*sinh(a + b*x) - 3*b*d^2*cosh(a + b*x)^2*s
inh(a + b*x) + 6*b^2*d*cosh(a + b*x)*sinh(a + b*x)^2))/(9*b^4 + d^4 - 10*b^
2*d^2)
```

3.963 $\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)}+\frac{e^{c+dx}}{d}+\frac{e^{2a+c+(2b+d)x}}{4(2b+d)} + \frac{2e^{-2a+c-(2b-d)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-2+\frac{d}{b}\right),\frac{d}{2b},e^{2(a+bx)}\right)}{2b-d}$$

```
[Out] -7/4*exp(-2*a+c-(2*b-d)*x)/(2*b-d)+exp(d*x+c)/d+1/4*exp(2*a+c+(2*b+d)*x)/(2
*b+d)+2*exp(-2*a+c-(2*b-d)*x)*hypergeom([1, -1+1/2*d/b],[1/2*d/b],exp(2*b*x
+2*a))/(2*b-d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5622, 2225, 2259, 2283}

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \frac{2e^{-2a-x(2b-d)+c}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{d}{b}-2\right),\frac{d}{2b},e^{2(a+bx)}\right)}{2b-d} - \frac{7e^{-2a-x(2b-d)+c}}{4(2b-d)}+\frac{e^{2a+x(2b+d)+c}}{4(2b+d)}+\frac{e^{c+dx}}{d}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] (-7*E^(-2*a + c - (2*b - d)*x))/(4*(2*b - d)) + E^(c + d*x)/d + E^(2*a + c
+ (2*b + d)*x)/(4*(2*b + d)) + (2*E^(-2*a + c - (2*b - d)*x)*Hypergeometric
2F1[1, (-2 + d/b)/2, d/(2*b), E^(2*(a + b*x))])/(2*b - d)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2259

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5622

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{7}{4}e^{-2a+c-(2b-d)x}+e^{-2a+c-(2b-d)x+2(a+bx)}+\frac{1}{4}e^{-2a+c-(2b-d)x+4(a+bx)}+\frac{2e^{-2a+c-(2b-d)x}}{-1+e^{2(a+bx)}}\right)dx\\
&= \frac{1}{4}\int e^{-2a+c-(2b-d)x+4(a+bx)}\,dx+\frac{7}{4}\int e^{-2a+c-(2b-d)x}\,dx\\
&\quad +2\int \frac{e^{-2a+c-(2b-d)x}}{-1+e^{2(a+bx)}}\,dx+\int e^{-2a+c-(2b-d)x+2(a+bx)}\,dx\\
&= -\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)}+\frac{2e^{-2a+c-(2b-d)x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-2+\frac{d}{b}\right),\frac{d}{2b},e^{2(a+bx)}\right)}{2b-d}\\
&\quad +\frac{1}{4}\int e^{2a+c+(2b+d)x}\,dx+\int e^{c+dx}\,dx
\end{aligned}$$

$$= -\frac{7e^{-2a+c-(2b-d)x}}{4(2b-d)} + \frac{e^{c+dx}}{d} + \frac{e^{2a+c+(2b+d)x}}{4(2b+d)} + \frac{2e^{-2a+c-(2b-d)x}\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-2+\frac{d}{b}\right), \frac{d}{2b}, e^{2(a+bx)}\right)}{2b-d}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{e^{c-\frac{ad}{b}}\left(2(4b^2-d^2)\,e^{d\left(\frac{a}{b}+x\right)}\,\text{Hypergeometric2F1}\left(1, \frac{d}{2b}, 1+\frac{d}{2b}, e^{2(a+bx)}\right) + 2(2b-d)de^{\left(2+\frac{d}{b}\right)(a+bx)}\,\text{Hyperge}\right.}{8b^2d-2d^3}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]^2*Coth[a + b*x],x]
```

```
[Out] -((E^(c - (a*d)/b)*(2*(4*b^2 - d^2)*E^(d*(a/b + x))*Hypergeometric2F1[1, d/
(2*b), 1 + d/(2*b), E^(2*(a + b*x))] + 2*(2*b - d)*d*E^((2 + d/b)*(a + b*x)
)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E^(2*(a + b*x))] + d*E^(d*
(a/b + x))*(-2*b*Cosh[2*(a + b*x)] + d*Sinh[2*(a + b*x)])))/(8*b^2*d - 2*d^
3))
```

Maple [F]

$$\int e^{dx+c}\cosh(bx+a)^3\operatorname{csch}(bx+a)\,dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x)
```

Fricas [F]

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \int \cosh(bx+a)^3\operatorname{csch}(bx+a)\,e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a),x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \int \cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}\,e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
[Out] -4*b*integrate(e^(d*x + c)/((4*b - d)*e^(6*b*x + 6*a) - 2*(4*b - d)*e^(4*b*
x + 4*a) + (4*b - d)*e^(2*b*x + 2*a)), x) + 1/4*(24*b^2*d*e^c + 14*b*d^2*e^
c + d^3*e^c + (8*b^2*d*e^c - 6*b*d^2*e^c + d^3*e^c)*e^(6*b*x + 6*a) + (64*b
^3*e^c - 24*b^2*d*e^c - 10*b*d^2*e^c + 3*d^3*e^c)*e^(4*b*x + 4*a) - (64*b^3
*e^c + 40*b^2*d*e^c - 2*b*d^2*e^c - 3*d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/((1
6*b^3*d - 4*b^2*d^2 - 4*b*d^3 + d^4)*e^(4*b*x + 4*a) - (16*b^3*d - 4*b^2*d^
2 - 4*b*d^3 + d^4)*e^(2*b*x + 2*a))
```

Giac [F]

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \int \cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}\,e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh^2(a+bx)\coth(a+bx)\,dx = \int \frac{\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x),x)
[Out] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x), x)
```

3.964 $\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{5e^{-a+c-(b-d)x}}{2(b-d)}+\frac{e^{a+c+(b+d)x}}{2(b+d)}+\frac{6e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d} - \frac{4e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(2,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}$$

```
[Out] -5/2*exp(-a+c-(b-d)*x)/(b-d)+1/2*exp(a+c+(b+d)*x)/(b+d)+6*exp(-a+c-(b-d)*x)
*hypergeom([1, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)-4*exp(-a+c
-(b-d)*x)*hypergeom([2, 1/2*(-b+d)/b],[1/2*(b+d)/b],exp(2*b*x+2*a))/(b-d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5622, 2225, 2259, 2283}

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{6e^{-a-x(b-d)+c}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d} - \frac{4e^{-a-x(b-d)+c}\,\text{Hypergeometric2F1}\left(2,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}-\frac{5e^{-a-x(b-d)+c}}{2(b-d)}+\frac{e^{a+x(b+d)+c}}{2(b+d)}$$

```
[In] Int[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (-5*E^(-a + c - (b - d)*x))/(2*(b - d)) + E^(a + c + (b + d)*x)/(2*(b + d))
 + (6*E^(-a + c - (b - d)*x)*Hypergeometric2F1[1, -1/2*(b - d)/b, (b + d)/(
2*b), E^(2*(a + b*x))])/(b - d) - (4*E^(-a + c - (b - d)*x)*Hypergeometric2
F1[2, -1/2*(b - d)/b, (b + d)/(2*b), E^(2*(a + b*x))])/(b - d)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2259

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5622

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{5}{2}e^{-a+c-(b-d)x}+\frac{1}{2}e^{-a+c-(b-d)x+2(a+bx)}+\frac{4e^{-a+c-(b-d)x}}{\left(-1+e^{2(a+bx)}\right)^2}+\frac{6e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\right)dx\\
&= \frac{1}{2}\int e^{-a+c-(b-d)x+2(a+bx)}\,dx+\frac{5}{2}\int e^{-a+c-(b-d)x}\,dx\\
&\quad +4\int \frac{e^{-a+c-(b-d)x}}{\left(-1+e^{2(a+bx)}\right)^2}\,dx+6\int \frac{e^{-a+c-(b-d)x}}{-1+e^{2(a+bx)}}\,dx\\
&= -\frac{5e^{-a+c-(b-d)x}}{2(b-d)}+\frac{6e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(1,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}\\
&\quad -\frac{4e^{-a+c-(b-d)x}\,\text{Hypergeometric2F1}\left(2,-\frac{b-d}{2b},\frac{b+d}{2b},e^{2(a+bx)}\right)}{b-d}+\frac{1}{2}\int e^{a+c+(b+d)x}\,dx
\end{aligned}
$$

$$= -\frac{5e^{-a+c-(b-d)x}}{2(b-d)} + \frac{e^{a+c+(b+d)x}}{2(b+d)} + \frac{6e^{-a+c-(b-d)x}\text{Hypergeometric2F1}\left(1, -\frac{b-d}{2b}, \frac{b+d}{2b}, e^{2(a+bx)}\right)}{b-d} - \frac{4e^{-a+c-(b-d)x}\text{Hypergeometric2F1}\left(2, -\frac{b-d}{2b}, \frac{b+d}{2b}, e^{2(a+bx)}\right)}{b-d}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^{c-\frac{ad}{b}}\operatorname{csch}(a+bx)\left(-4(b-d)de^{\frac{(b+d)(a+bx)}{b}}\text{Hypergeometric2F1}\left(1, \frac{b+d}{2b}, \frac{3b+d}{2b}, e^{2(a+bx)}\right)\sinh(a+bx) + e^{d\left(\frac{a}{b}+x\right)}\right.}{2b(b-d)(b+d)}$$

```
[In] Integrate[E^(c + d*x)*Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

```
[Out] (E^(c - (a*d)/b)*Csch[a + b*x]*(-4*(b - d)*d*E^(((b + d)*(a + b*x))/b)*Hype
rgeometric2F1[1, (b + d)/(2*b), (3*b + d)/(2*b), E^(2*(a + b*x))]*Sinh[a +
b*x] + E^(d*(a/b + x))*(-3*b^2 + 2*d^2 + b^2*Cosh[2*(a + b*x)] - b*d*Sinh[2
*(a + b*x)])))/(2*b*(b - d)*(b + d))
```

Maple [F]

$$\int e^{dx+c}\cosh(bx+a)^3\operatorname{csch}(bx+a)^2\,dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x)
```

Fricas [F]

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh(bx+a)^3\operatorname{csch}(bx+a)^2 e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx=\int\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 16*b*d*integrate(e^(d*x + c)/((15*b^2 - 8*b*d + d^2)*e^(7*b*x + 7*a) - 3*(1
5*b^2 - 8*b*d + d^2)*e^(5*b*x + 5*a) + 3*(15*b^2 - 8*b*d + d^2)*e^(3*b*x +
3*a) - (15*b^2 - 8*b*d + d^2)*e^(b*x + a)), x) - 1/2*(15*b^3*e^c + 39*b^2*d
*e^c + 25*b*d^2*e^c + d^3*e^c - (15*b^3*e^c - 23*b^2*d*e^c + 9*b*d^2*e^c -
d^3*e^c)*e^(6*b*x + 6*a) + (105*b^3*e^c - 11*b^2*d*e^c - 17*b*d^2*e^c + 3*d
^3*e^c)*e^(4*b*x + 4*a) - (105*b^3*e^c + 59*b^2*d*e^c - b*d^2*e^c - 3*d^3*e
^c)*e^(2*b*x + 2*a))*e^(d*x)/((15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^
4)*e^(5*b*x + 5*a) - 2*(15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(3
*b*x + 3*a) + (15*b^4 - 8*b^3*d - 14*b^2*d^2 + 8*b*d^3 - d^4)*e^(b*x + a))
```

Giac [F]

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx=\int\cosh\left(bx+a\right)^3\operatorname{csch}\left(bx+a\right)^2e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^2*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\cosh(a+bx)\coth^2(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^3\,\mathrm{e}^{c+d\,x}}{\sinh\left(a+b\,x\right)^2}\,dx$$

```
[In] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^2,x)
[Out] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^2, x)
```

3.965 $\int e^{c+dx}\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\frac{e^{c+dx}}{d}-\frac{6e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,\text{Hypergeometric2F1}\left(3,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}$$

```
[Out] exp(d*x+c)/d-6*exp(d*x+c)*hypergeom([1, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a)
)/d+12*exp(d*x+c)*hypergeom([2, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d-8*ex
p(d*x+c)*hypergeom([3, 1/2*d/b],[1+1/2*d/b],exp(2*b*x+2*a))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5593, 2225, 2283}

$$\int e^{c+dx}\coth^3(a+bx)\,dx=-\frac{6e^{c+dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{12e^{c+dx}\,\text{Hypergeometric2F1}\left(2,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}-\frac{8e^{c+dx}\,\text{Hypergeometric2F1}\left(3,\frac{d}{2b},\frac{d}{2b}+1,e^{2(a+bx)}\right)}{d}+\frac{e^{c+dx}}{d}$$

```
[In] Int[E^(c + d*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(c + d*x)/d - (6*E^(c + d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E
^(2*(a + b*x))])/d + (12*E^(c + d*x)*Hypergeometric2F1[2, d/(2*b), 1 + d/(2
```

```
*b), E^(2*(a + b*x))])/d - (8*E^(c + d*x)*Hypergeometric2F1[3, d/(2*b), 1 +
 d/(2*b), E^(2*(a + b*x))])/d
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5593

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{c+dx} + \frac{8e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^3} + \frac{12e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2} + \frac{6e^{c+dx}}{-1+e^{2(a+bx)}}\right) dx \\
&= 6\int \frac{e^{c+dx}}{-1+e^{2(a+bx)}}\, dx + 8\int \frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^3}\, dx + 12\int \frac{e^{c+dx}}{\left(-1+e^{2(a+bx)}\right)^2}\, dx + \int e^{c+dx}\, dx \\
&= \frac{e^{c+dx}}{d} - \frac{6e^{c+dx}\,\text{Hypergeometric2F1}\left(1, \frac{d}{2b}, 1+\frac{d}{2b}, e^{2(a+bx)}\right)}{d} \\
&\quad + \frac{12e^{c+dx}\,\text{Hypergeometric2F1}\left(2, \frac{d}{2b}, 1+\frac{d}{2b}, e^{2(a+bx)}\right)}{d} \\
&\quad - \frac{8e^{c+dx}\,\text{Hypergeometric2F1}\left(3, \frac{d}{2b}, 1+\frac{d}{2b}, e^{2(a+bx)}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\frac{1}{2}e^c\left(\frac{2e^{dx}\coth(a)}{d}-\frac{e^{dx}\text{csch}^2(a+bx)}{b}-\frac{2(2b^2+d^2)\,e^{2a}\left(\frac{e^{dx}\,\text{Hypergeometric2F1}\left(1,\frac{d}{2b},1+\frac{d}{2b},e^{2(a+bx)}\right)}{d}-\frac{e^{(2b+d)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{d}{2b},2+\frac{d}{2b},e^{2(a+bx)}\right)}{2b+d}\right)}{b^2\left(-1+e^{2a}\right)}+\frac{de^{dx}\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

```
[In] Integrate[E^(c + d*x)*Coth[a + b*x]^3,x]
```

```
[Out] (E^c*((2*E^(d*x)*Coth[a])/d - (E^(d*x)*Csch[a + b*x]^2)/b - (2*(2*b^2 + d^2
)*E^(2*a)*((E^(d*x)*Hypergeometric2F1[1, d/(2*b), 1 + d/(2*b), E^(2*(a + b*
x))])/d - (E^((2*b + d)*x)*Hypergeometric2F1[1, 1 + d/(2*b), 2 + d/(2*b), E
^(2*(a + b*x))])/(2*b + d)))/(b^2*(-1 + E^(2*a))) + (d*E^(d*x)*Csch[a]*Csch
[a + b*x]*Sinh[b*x])/b^2))/2
```

Maple [F]

$$\int e^{dx+c}\cosh(bx+a)^3\operatorname{csch}(bx+a)^3\,dx$$

```
[In] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

```
[Out] int(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x)
```

Fricas [F]

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int\cosh(bx+a)^3\operatorname{csch}(bx+a)^3\,e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\text{Timed out}$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)**3*csch(b*x+a)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int \cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}^3e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -48*(2*b^3*e^c + b*d^2*e^c)*integrate(e^(d*x)/(48*b^3 - 44*b^2*d + 12*b*d^2
 - d^3 + (48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(8*b*x + 8*a) - 4*(48*b^3 -
 44*b^2*d + 12*b*d^2 - d^3)*e^(6*b*x + 6*a) + 6*(48*b^3 - 44*b^2*d + 12*b*d
^2 - d^3)*e^(4*b*x + 4*a) - 4*(48*b^3 - 44*b^2*d + 12*b*d^2 - d^3)*e^(2*b*x
 + 2*a)), x) + (48*b^3*e^c + 44*b^2*d*e^c + 36*b*d^2*e^c + d^3*e^c - (48*b^
3*e^c - 44*b^2*d*e^c + 12*b*d^2*e^c - d^3*e^c)*e^(6*b*x + 6*a) + 3*(48*b^3*
e^c + 4*b^2*d*e^c - 8*b*d^2*e^c + d^3*e^c)*e^(4*b*x + 4*a) - 3*(48*b^3*e^c
+ 28*b^2*d*e^c - d^3*e^c)*e^(2*b*x + 2*a))*e^(d*x)/(48*b^3*d - 44*b^2*d^2 +
 12*b*d^3 - d^4 - (48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(6*b*x + 6*a)
+ 3*(48*b^3*d - 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(4*b*x + 4*a) - 3*(48*b^3*d
- 44*b^2*d^2 + 12*b*d^3 - d^4)*e^(2*b*x + 2*a))
```

Giac [F]

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int \cosh{(bx+a)}^3\operatorname{csch}{(bx+a)}^3e^{(dx+c)}\,dx$$

```
[In] integrate(exp(d*x+c)*cosh(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3*csch(b*x + a)^3*e^(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+dx}\coth^3(a+bx)\,dx=\int \frac{\cosh(a+b\,x)^3\,e^{c+d\,x}}{\sinh\left(a+b\,x\right)^3}\,dx$$

```
[In] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^3,x)
[Out] int((cosh(a + b*x)^3*exp(c + d*x))/sinh(a + b*x)^3, x)
```

3.966 $\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx$

Optimal result

Integrand size = 56, antiderivative size = 73

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx = -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}$$

```
[Out] 4*b*exp(b*x+a)*sinh(d*x+c)^(3/2)/(4*b^2-9*d^2)-6*d*exp(b*x+a)*cosh(d*x+c)*s
inh(d*x+c)^(1/2)/(4*b^2-9*d^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {5590, 2285, 2284, 2283, 5584}

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx = \frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

```
[In] Int[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) + E^(a +
 b*x)*Sinh[c + d*x]^(3/2),x]
```

```
[Out] (-6*d*E^(a + b*x)*Cosh[c + d*x]*Sqrt[Sinh[c + d*x]])/(4*b^2 - 9*d^2) + (4*b
*E^(a + b*x)*Sinh[c + d*x]^(3/2))/(4*b^2 - 9*d^2)
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2284

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b/a)*F^(e*(c + d*x)))^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2285

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rule 5590

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Dist[E^(n*(d + e*x))*(Sinh[d + e*x]^n/(-1 + E^(2*(d + e*x)))^n), Int
[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; Free
Q[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{(3d^2)\int \frac{e^{a+bx}}{\sqrt{\sinh(c+dx)}}\,dx}{4b^2-9d^2} + \int e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\,dx$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2} + \frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2} + \frac{(3d^2)\int\frac{e^{a+bx}}{\sqrt{\sinh(c+dx)}}dx}{4b^2-9d^2} - \frac{\left(3d^2e^{\frac{1}{2}(-c-dx)}\sqrt{-1+e^{2(c+dx)}}\right)\int\frac{e^{a+bx+\frac{1}{2}(c+dx)}}{\sqrt{-1+e^{2(c+dx)}}}dx}{(4b^2-9d^2)\sqrt{\sinh(c+dx)}}$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2} + \frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2} - \frac{\left(3d^2e^{\frac{1}{2}(-c-dx)}\sqrt{-1+e^{2(c+dx)}}\right)\int\frac{e^{\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x}}{\sqrt{-1+e^{2(c+dx)}}}dx}{(4b^2-9d^2)\sqrt{\sinh(c+dx)}} + \frac{\left(3d^2e^{\frac{1}{2}(-c-dx)}\sqrt{-1+e^{2(c+dx)}}\right)\int\frac{e^{a+bx+\frac{1}{2}(c+dx)}}{\sqrt{-1+e^{2(c+dx)}}}dx}{(4b^2-9d^2)\sqrt{\sinh(c+dx)}}$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2} + \frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2} - \frac{\left(3d^2e^{\frac{1}{2}(-c-dx)}\sqrt{1-e^{2(c+dx)}}\right)\int\frac{e^{\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x}}{\sqrt{1-e^{2(c+dx)}}}dx}{(4b^2-9d^2)\sqrt{\sinh(c+dx)}} + \frac{\left(3d^2e^{\frac{1}{2}(-c-dx)}\sqrt{-1+e^{2(c+dx)}}\right)\int\frac{e^{\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x}}{\sqrt{-1+e^{2(c+dx)}}}dx}{(4b^2-9d^2)\sqrt{\sinh(c+dx)}}$$

$$= -\frac{6d^2\exp\left(\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x+\frac{1}{2}(-c-dx)\right)\sqrt{1-e^{2(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2b+d}{4d},\frac{1}{4}(5-\right.}{(2b+d)(4b^2-9d^2)\sqrt{\sinh(c+dx)}} - \frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2} + \frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2} + \frac{\left(3d^2e^{\frac{1}{2}(-c-dx)}\sqrt{1-e^{2(c+dx)}}\right)\int\frac{e^{\frac{1}{2}(2a+c)+\frac{1}{2}(2b+d)x}}{\sqrt{1-e^{2(c+dx)}}}dx}{(4b^2-9d^2)\sqrt{\sinh(c+dx)}}$$

$$= -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2} + \frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}$$

Mathematica [A] (verified)

Time = 4.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int\left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx = \frac{2e^{a+bx}\sqrt{\sinh(c+dx)}(-3d\cosh(c+dx)+2b\sinh(c+dx))}{4b^2-9d^2}$$

```
[In] Integrate[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) +
E^(a + b*x)*Sinh[c + d*x]^(3/2),x]
```

```
[Out] (2*E^(a + b*x)*Sqrt[Sinh[c + d*x]]*(-3*d*Cosh[c + d*x] + 2*b*Sinh[c + d*x])
)/(4*b^2 - 9*d^2)
```

Maple [F]

$$\int \left(\mathrm{e}^{bx+a} \sinh (dx+c)^{\frac{3}{2}} - \frac{3d^2 \mathrm{e}^{bx+a}}{4 \left(b^2 - \frac{9d^2}{4}\right) \sqrt{\sinh (dx+c)}}\right) dx$$

```
[In] int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+
c)^(1/2),x)
```

```
[Out] int(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+
c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{3d^2 e^{a+bx}}{4 \left(b^2 - \frac{9d^2}{4}\right) \sqrt{\sinh(c+dx)}} + e^{a+bx} \sinh^{\frac{3}{2}}(c+dx)\right) dx = \text{Exception raised: TypeError}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(-\frac{3d^2 e^{a+bx}}{4 \left(b^2 - \frac{9d^2}{4}\right) \sqrt{\sinh(c+dx)}} + e^{a+bx} \sinh^{\frac{3}{2}}(c+dx)\right) dx = \frac{\left(\int 4b^2 e^{bx} \sinh^{\frac{3}{2}}(c+dx)\, dx + \int \left(-\frac{3d^2 e^{bx}}{\sqrt{\sinh(c+dx)}}\right) dx + \int \left(-9d^2 e^{bx} \sinh^{\frac{3}{2}}(c+dx)\right) dx\right) e^a}{(2b-3d)(2b+3d)}$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)**(3/2)-3/4*d**2*exp(b*x+a)/(b**2-9/4*d**2)
/sinh(d*x+c)**(1/2),x)
```

```
[Out] (Integral(4*b**2*exp(b*x)*sinh(c + d*x)**(3/2), x) + Integral(-3*d**2*exp(b
*x)/sqrt(sinh(c + d*x)), x) + Integral(-9*d**2*exp(b*x)*sinh(c + d*x)**(3/2
), x))*exp(a)/((2*b - 3*d)*(2*b + 3*d))
```

Maxima [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int e^{(bx+a)}\sinh\left(dx+c\right)^{\frac{3}{2}}-\frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh\left(dx+c\right)}}\,dx$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*d
^2)*sqrt(sinh(d*x + c))), x)
```

Giac [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int e^{(bx+a)}\sinh\left(dx+c\right)^{\frac{3}{2}}-\frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh\left(dx+c\right)}}\,dx$$

```
[In] integrate(exp(b*x+a)*sinh(d*x+c)^(3/2)-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sin
h(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*d
^2)*sqrt(sinh(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int \mathrm{e}^{a+b\,x}\,\sinh(c+d\,x)^{3/2}-\frac{3\,d^2\,\mathrm{e}^{a+b\,x}}{4\,\sqrt{\sinh\left(c+d\,x\right)}\,\left(b^2-\frac{9\,d^2}{4}\right)}\,dx$$

```
[In] int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*x
)^(1/2)*(b^2 - (9*d^2)/4)),x)
```

```
[Out] int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*x
)^(1/2)*(b^2 - (9*d^2)/4)), x)
```

3.967 $\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{e^{n\cosh(a+bx)}}{bn}$$

```
[Out] exp(n*cosh(b*x+a))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4422, 2225}

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{e^{n\cosh(a+bx)}}{bn}$$

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4422

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{e^{n\cosh(a+bx)}}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \frac{e^{n\cosh(a+bx)}}{bn}$$

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[a + b*x],x]
[Out] E^(n*Cosh[a + b*x])/(b*n)
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{e^{n\cosh(bx+a)}}{bn}$	17
default	$\frac{e^{n\cosh(bx+a)}}{bn}$	17
risch	$\frac{e^{\frac{n\left(e^{bx+a}+e^{-bx-a}\right)}{2}}}{nb}$	28

```
[In] int(exp(n*cosh(b*x+a))*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] exp(n*cosh(b*x+a))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \frac{\cosh\left(n\cosh\left(bx+a\right)\right)+\sinh\left(n\cosh\left(bx+a\right)\right)}{bn}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="fricas")
[Out] (cosh(n*cosh(b*x + a)) + sinh(n*cosh(b*x + a)))/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \begin{cases} x\sinh(a) & \text{for } b=0 \wedge n=0 \\ xe^{n\cosh(a)}\sinh(a) & \text{for } b=0 \\ \frac{\cosh(a+bx)}{b} & \text{for } n=0 \\ \frac{e^{n\cosh(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cosh(a))*sinh(a), Eq(b
, 0)), (cosh(a + b*x)/b, Eq(n, 0)), (exp(n*cosh(a + b*x))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \frac{e^{(n\cosh(bx+a))}}{bn}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*cosh(b*x + a))/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx = \frac{e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] e^(1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\cosh(a+bx)}\sinh(a+bx)\,dx=\frac{\mathrm{e}^{n\cosh(a+b\,x)}}{b\,n}$$

```
[In] int(exp(n*cosh(a + b*x))*sinh(a + b*x),x)
[Out] exp(n*cosh(a + b*x))/(b*n)
```

3.968 $\int e^{n \cosh(ac+bcx)} \sinh(c(a+bx)) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int e^{n \cosh(ac+bcx)} \sinh(c(a+bx)) \, dx = \frac{e^{n \cosh(c(a+bx))}}{bcn}$$

```
[Out] exp(n*cosh(c*(b*x+a)))/b/c/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4422, 2225}

$$\int e^{n \cosh(ac+bcx)} \sinh(c(a+bx)) \, dx = \frac{e^{n \cosh(c(a+bx))}}{bcn}$$

```
[In] Int[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4422

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(c(a+bx))\right)}{bc}$$
$$= \frac{e^{n\cosh(c(a+bx))}}{bcn}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx = \frac{e^{n\cosh(c(a+bx))}}{bcn}$$

```
[In] Integrate[E^(n*Cosh[a*c + b*c*x])*Sinh[c*(a + b*x)],x]
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
derivativedivides	$\frac{e^{n\cosh(bcx+ac)}}{bcn}$	23
default	$\frac{e^{n\cosh(bcx+ac)}}{bcn}$	23
risch	$\frac{e^{\frac{n\left(e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	33

```
[In] int(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x,method=_RETURNVERBOSE)
[Out] exp(n*cosh(c*(b*x+a)))/b/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="fricas")
[Out] (cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(15) = 30.

Time = 1.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\begin{cases} xe^{n\cosh\left(ac\right)}\sinh\left(ac\right) & \text{for } b=0\\ 0 & \text{for } c=0\\ \begin{cases} x\sinh\left(ac\right) & \text{for } b=0\\ 0 & \text{for } c=0\\ \frac{\cosh\left(c(a+bx)\right)}{bc} & \text{otherwise}\end{cases} & \text{for } n=0\\ \frac{e^{n\cosh\left(ac+bcx\right)}}{bcn} & \text{otherwise}\end{cases}$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x)
[Out] Piecewise((x*exp(n*cosh(a*c))*sinh(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecewi
se((x*sinh(a*c), Eq(b, 0)), (0, Eq(c, 0)), (cosh(c*(a + b*x))/(b*c), True))
, Eq(n, 0)), (exp(n*cosh(a*c + b*c*x))/(b*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{e^{\left(n\cosh\left(bcx+ac\right)\right)}}{bcn}$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="maxima")
[Out] e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}+\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*sinh(c*(b*x+a)),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*c*x + a*c) + 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n\cosh(ac+bcx)}\sinh(c(a+bx))\,dx=\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

```
[In] int(sinh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp((n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.969 $\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

```
[Out] exp(n*cosh(b*c*x+a*c))/b/c/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4422, 2225}

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{n\cosh(ac+bcx)}}{bcn}$$

```
[In] Int[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
[Out] E^(n*Cosh[a*c + b*c*x])/(b*c*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4422

```
Int[(u_)*Sinh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Cosh[c*(a +
b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(ac+bcx)\right)}{bc}$$
$$= \frac{e^{n\cosh(ac+bcx)}}{bcn}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx = \frac{e^{n\cosh(c(a+bx))}}{bcn}$$

```
[In] Integrate[E^(n*Cosh[c*(a + b*x)])*Sinh[a*c + b*c*x],x]
[Out] E^(n*Cosh[c*(a + b*x)])/(b*c*n)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
default	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
parallelrisch	$\frac{e^{n\cosh(c(bx+a))}}{bcn}$	22
risch	$\frac{e^{\frac{n\left(e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	33

```
[In] int(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] exp(n*cosh(c*(b*x+a)))/b/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{\cosh\left(n\cosh\left(bcx+ac\right)\right)+\sinh\left(n\cosh\left(bcx+ac\right)\right)}{bcn}$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="fricas")
[Out] (cosh(n*cosh(b*c*x + a*c)) + sinh(n*cosh(b*c*x + a*c)))/(b*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(17) = 34.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\begin{cases}0 & \text{for}\ b=0\wedge c=0\wedge n=0\\ xe^{n\cosh\left(ac\right)}\sinh\left(ac\right) & \text{for}\ b=0\\ 0 & \text{for}\ c=0\\ \frac{\cosh\left(ac+bcx\right)}{bc} & \text{for}\ n=0\\ \frac{e^{n\cosh\left(ac+bcx\right)}}{bcn} & \text{otherwise}\end{cases}$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x)
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*cosh(a*c))*sinh(a*c
), Eq(b, 0)), (0, Eq(c, 0)), (cosh(a*c + b*c*x)/(b*c), Eq(n, 0)), (exp(n*co
sh(a*c + b*c*x))/(b*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{\left(n\cosh\left(bcx+ac\right)\right)}}{bcn}$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="maxima")
[Out] e^(n*cosh(b*c*x + a*c))/(b*c*n)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}+\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*sinh(b*c*x+a*c),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*c*x + a*c) + 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\cosh(c(a+bx))}\sinh(ac+bcx)\,dx=\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}}{b\,c\,n}$$

```
[In] int(exp(n*cosh(c*(a + b*x)))*sinh(a*c + b*c*x),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp((n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.970 $\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

```
[Out] Ei(n*cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4426, 2209}

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

```
[In] Int[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 4426

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(a+bx)\right)}{b}$$
$$= \frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \frac{\text{ExpIntegralEi}(n\cosh(a+bx))}{b}$$

```
[In] Integrate[E^(n*Cosh[a + b*x])*Tanh[a + b*x],x]
[Out] ExpIntegralEi[n*Cosh[a + b*x]]/b
```

Maple [F]

$$\int e^{n\cosh(bx+a)}\tanh(bx+a)\,dx$$

```
[In] int(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
[Out] int(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \frac{\text{Ei}(n\cosh(bx+a))}{b}$$

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="fricas")
[Out] Ei(n*cosh(b*x + a))/b
```

Sympy [F]

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx$$

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x)
[Out] Integral(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

Maxima [F]

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{(n\cosh(bx+a))}\tanh(bx+a)\,dx$$

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="maxima")
[Out] integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

Giac [F]

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{(n\cosh(bx+a))}\tanh(bx+a)\,dx$$

```
[In] integrate(exp(n*cosh(b*x+a))*tanh(b*x+a),x, algorithm="giac")
[Out] integrate(e^(n*cosh(b*x + a))*tanh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\cosh(a+bx)}\tanh(a+bx)\,dx = \int e^{n\cosh(a+b\,x)}\tanh(a+b\,x)\,dx$$

```
[In] int(exp(n*cosh(a + b*x))*tanh(a + b*x),x)
[Out] int(exp(n*cosh(a + b*x))*tanh(a + b*x), x)
```

3.971 $\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

```
[Out] Ei(n*cosh(c*(b*x+a)))/b/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4426, 2209}

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

```
[In] Int[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 4426

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(c(a+bx))\right)}{bc}\\ &= \frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

```
[In] Integrate[E^(n*Cosh[a*c + b*c*x])*Tanh[c*(a + b*x)],x]
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Maple [F]

$$\int e^{n\cosh(bcx+ac)}\tanh\left(c(bx+a)\right)dx$$

```
[In] int(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
[Out] int(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \frac{\text{Ei}\left(n\cosh\left(bcx+ac\right)\right)}{bc}$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="fricas")
[Out] Ei(n*cosh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int e^{n\cosh{(ac+bcx)}}\tanh{(ac+bcx)}\,dx$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x)
[Out] Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

Maxima [F]

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="maxima")
[Out] integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

Giac [F]

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int e^{(n\cosh(bcx+ac))}\tanh\left((bx+a)c\right)\,dx$$

```
[In] integrate(exp(n*cosh(b*c*x+a*c))*tanh(c*(b*x+a)),x, algorithm="giac")
[Out] integrate(e^(n*cosh(b*c*x + a*c))*tanh((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\cosh(ac+bcx)}\tanh(c(a+bx))\,dx = \int \tanh(c\,(a+b\,x))\,\mathrm{e}^{n\cosh(a\,c+b\,c\,x)}\,dx$$

```
[In] int(tanh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)),x)
[Out] int(tanh(c*(a + b*x))*exp(n*cosh(a*c + b*c*x)), x)
```

3.972 $\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(ac+bcx))}{bc}$$

```
[Out] Ei(n*cosh(b*c*x+a*c))/b/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4426, 2209}

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx=\frac{\text{ExpIntegralEi}(n\cosh(ac+bxc))}{bc}$$

```
[In] Int[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Cosh[a*c + b*c*x]]/(b*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4426

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cosh(ac+bcx)\right)}{bc}\\ &= \frac{\text{ExpIntegralEi}(n\cosh(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \frac{\text{ExpIntegralEi}(n\cosh(c(a+bx)))}{bc}$$

```
[In] Integrate[E^(n*Cosh[c*(a + b*x)])*Tanh[a*c + b*c*x],x]
[Out] ExpIntegralEi[n*Cosh[c*(a + b*x)]]/(b*c)
```

Maple [F]

$$\int \mathrm{e}^{n\cosh(c(bx+a))}\tanh(bcx+ac)\,dx$$

```
[In] int(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
[Out] int(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \frac{\text{Ei}(n\cosh(bcx+ac))}{bc}$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="fricas")
[Out] Ei(n*cosh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \int e^{n\cosh(ac+bcx)}\tanh(ac+bcx)\,dx$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x)
[Out] Integral(exp(n*cosh(a*c + b*c*x))*tanh(a*c + b*c*x), x)
```

Maxima [F]

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \int e^{(n\cosh((bx+a)c))}\tanh(bcx+ac)\,dx$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="maxima")
[Out] integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

Giac [F]

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \int e^{(n\cosh((bx+a)c))}\tanh(bcx+ac)\,dx$$

```
[In] integrate(exp(n*cosh(c*(b*x+a)))*tanh(b*c*x+a*c),x, algorithm="giac")
[Out] integrate(e^(n*cosh((b*x + a)*c))*tanh(b*c*x + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\cosh(c(a+bx))}\tanh(ac+bcx)\,dx = \int e^{n\cosh(c\,(a+b\,x))}\tanh(a\,c+b\,c\,x)\,dx$$

```
[In] int(exp(n*cosh(c*(a + b*x)))*tanh(a*c + b*c*x),x)
[Out] int(exp(n*cosh(c*(a + b*x)))*tanh(a*c + b*c*x), x)
```

3.973 $\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx=\frac{e^{n\sinh(a+bx)}}{bn}$$

```
[Out] exp(n*sinh(b*x+a))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4421, 2225}

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx=\frac{e^{n\sinh(a+bx)}}{bn}$$

```
[In] Int[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
```

```
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4421

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{e^{n\sinh(a+bx)}}{bn}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{e^{n\sinh(a+bx)}}{bn}$$

```
[In] Integrate[E^(n*Sinh[a + b*x])*Cosh[a + b*x],x]
[Out] E^(n*Sinh[a + b*x])/(b*n)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{e^{n\sinh(bx+a)}}{bn}$	17
default	$\frac{e^{n\sinh(bx+a)}}{bn}$	17
risch	$\frac{e^{-\frac{n\left(-e^{bx+a}+e^{-bx-a}\right)}{2}}}{nb}$	30

```
[In] int(exp(n*sinh(b*x+a))*cosh(b*x+a),x,method=_RETURNVERBOSE)
[Out] exp(n*sinh(b*x+a))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{\cosh\left(n\sinh\left(bx+a\right)\right)+\sinh\left(n\sinh\left(bx+a\right)\right)}{bn}$$

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="fricas")
[Out] (cosh(n*sinh(b*x + a)) + sinh(n*sinh(b*x + a)))/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \begin{cases} x\cosh(a) & \text{for } b=0\wedge n=0 \\ xe^{n\sinh(a)}\cosh(a) & \text{for } b=0 \\ \frac{\sinh(a+bx)}{b} & \text{for } n=0 \\ \frac{e^{n\sinh(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x)
[Out] Piecewise((x*cosh(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sinh(a))*cosh(a), Eq(b
, 0)), (sinh(a + b*x)/b, Eq(n, 0)), (exp(n*sinh(a + b*x))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{e^{(n\sinh(bx+a))}}{bn}$$

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="maxima")
[Out] e^(n*sinh(b*x + a))/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{e^{\left(\frac{1}{2}\,ne^{(bx+a)}-\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn}$$

```
[In] integrate(exp(n*sinh(b*x+a))*cosh(b*x+a),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sinh(a+bx)}\cosh(a+bx)\,dx = \frac{e^{n\sinh(a+b\,x)}}{b\,n}$$

[In] `int(cosh(a + b*x)*exp(n*sinh(a + b*x)),x)`

[Out] `exp(n*sinh(a + b*x))/(b*n)`

3.974 $\int e^{n \sinh(ac+bcx)} \cosh(c(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int e^{n \sinh(ac+bcx)} \cosh(c(a+bx))\,dx = \frac{e^{n \sinh(c(a+bx))}}{bcn}$$

```
[Out] exp(n*sinh(c*(b*x+a)))/b/c/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4421, 2225}

$$\int e^{n \sinh(ac+bcx)} \cosh(c(a+bx))\,dx = \frac{e^{n \sinh(c(a+bx))}}{bcn}$$

```
[In] Int[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
```

```
[Out] E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4421

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(c(a+bx))\right)}{bc}$$
$$= \frac{e^{n\sinh(c(a+bx))}}{bcn}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \frac{e^{n\sinh(c(a+bx))}}{bcn}$$

```
[In] Integrate[E^(n*Sinh[a*c + b*c*x])*Cosh[c*(a + b*x)],x]
[Out] E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
derivativedivides	$\frac{e^{n\sinh(bcx+ac)}}{bcn}$	23
default	$\frac{e^{n\sinh(bcx+ac)}}{bcn}$	23
risch	$\frac{e^{-\frac{n\left(-e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	35

```
[In] int(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x,method=_RETURNVERBOSE)
[Out] exp(n*sinh(c*(b*x+a)))/b/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="fricas")
[Out] (cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 1.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \begin{cases} xe^{n\sinh\left(ac\right)}\cosh\left(ac\right) & \text{for}\ b=0 \\ x & \text{for}\ c=0 \\ \begin{cases} x\cosh\left(ac\right) & \text{for}\ b=0 \\ x & \text{for}\ c=0 \\ \frac{\sinh\left(c(a+bx)\right)}{bc} & \text{otherwise} \end{cases} & \text{for}\ n=0 \\ \frac{e^{n\sinh\left(ac+bcx\right)}}{bcn} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x)
[Out] Piecewise((x*exp(n*sinh(a*c))*cosh(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecewi
se((x*cosh(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sinh(c*(a + b*x))/(b*c), True))
, Eq(n, 0)), (exp(n*sinh(a*c + b*c*x))/(b*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(ac+bcx)}\cosh(c(a+bx))\,dx = \frac{e^{\left(n\sinh(bcx+ac)\right)}}{bcn}$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="maxima")
[Out] e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n \sinh(ac+bcx)} \cosh(c(a+bx)) \, dx = \frac{e^{\left(\frac{1}{2} ne^{(bcx+ac)} - \frac{1}{2} ne^{(-bcx-ac)}\right)}}{bcn}$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*cosh(c*(b*x+a)),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*c*x + a*c) - 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int e^{n \sinh(ac+bcx)} \cosh(c(a+bx)) \, dx = \frac{e^{\frac{n\,e^{b\,c\,x}\,e^{a\,c}}{2}}\,e^{-\frac{n\,e^{-b\,c\,x}\,e^{-a\,c}}{2}}}{b\,c\,n}$$

```
[In] int(cosh(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp(-(n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.975 $\int e^{n \sinh(c(a+bx))} \cosh(ac + bcx)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{n \sinh(c(a+bx))} \cosh(ac + bcx)\, dx = \frac{e^{n \sinh(ac+bcx)}}{bcn}$$

```
[Out] exp(n*sinh(b*c*x+a*c))/b/c/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4421, 2225}

$$\int e^{n \sinh(c(a+bx))} \cosh(ac + bcx)\, dx = \frac{e^{n \sinh(ac+bcx)}}{bcn}$$

```
[In] Int[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
```

```
[Out] E^(n*Sinh[a*c + b*c*x])/(b*c*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4421

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(ac + bcx)\right)}{bc}$$
$$= \frac{e^{n\sinh(ac+bcx)}}{bcn}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sinh(c(a+bx))}\cosh(ac + bcx)\,dx = \frac{e^{n\sinh(c(a+bx))}}{bcn}$$

```
[In] Integrate[E^(n*Sinh[c*(a + b*x)])*Cosh[a*c + b*c*x],x]
[Out] E^(n*Sinh[c*(a + b*x)])/(b*c*n)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
default	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
parallelrisch	$\frac{e^{n\sinh(c(bx+a))}}{bcn}$	22
risch	$\frac{e^{-\frac{n\left(-e^{c(bx+a)}+e^{-c(bx+a)}\right)}{2}}}{nbc}$	35

```
[In] int(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] exp(n*sinh(c*(b*x+a)))/b/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \frac{\cosh\left(n\sinh\left(bcx+ac\right)\right)+\sinh\left(n\sinh\left(bcx+ac\right)\right)}{bcn}$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="fricas")
[Out] (cosh(n*sinh(b*c*x + a*c)) + sinh(n*sinh(b*c*x + a*c)))/(b*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(17) = 34.

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \begin{cases} x & \text{for } b=0 \wedge c=0 \wedge n=0 \\ xe^{n\sinh(ac)}\cosh(ac) & \text{for } b=0 \\ x & \text{for } c=0 \\ \frac{\sinh(ac+bcx)}{bc} & \text{for } n=0 \\ \frac{e^{n\sinh(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x)
[Out] Piecewise((x, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*sinh(a*c))*cosh(a*c
), Eq(b, 0)), (x, Eq(c, 0)), (sinh(a*c + b*c*x)/(b*c), Eq(n, 0)), (exp(n*si
nh(a*c + b*c*x))/(b*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx = \frac{e^{(n\sinh(bcx+ac))}}{bcn}$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="maxima")
[Out] e^(n*sinh(b*c*x + a*c))/(b*c*n)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx=\frac{e^{\left(\frac{1}{2}\,ne^{(bcx+ac)}-\frac{1}{2}\,ne^{(-bcx-ac)}\right)}}{bcn}$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*cosh(b*c*x+a*c),x, algorithm="giac")
[Out] e^(1/2*n*e^(b*c*x + a*c) - 1/2*n*e^(-b*c*x - a*c))/(b*c*n)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{n\sinh(c(a+bx))}\cosh(ac+bcx)\,dx=\frac{e^{\frac{n\,e^{b\,c\,x}\,e^{a\,c}}{2}}\,e^{-\frac{n\,e^{-b\,c\,x}\,e^{-a\,c}}{2}}}{b\,c\,n}$$

```
[In] int(exp(n*sinh(c*(a + b*x)))*cosh(a*c + b*c*x),x)
[Out] (exp((n*exp(b*c*x)*exp(a*c))/2)*exp(-(n*exp(-b*c*x)*exp(-a*c))/2))/(b*c*n)
```

3.976 $\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

```
[Out] Ei(n*sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4425, 2209}

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx=\frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

```
[In] Int[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 4425

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(a+bx)\right)}{b}$$
$$= \frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \frac{\text{ExpIntegralEi}(n\sinh(a+bx))}{b}$$

```
[In] Integrate[E^(n*Sinh[a + b*x])*Coth[a + b*x],x]
[Out] ExpIntegralEi[n*Sinh[a + b*x]]/b
```

Maple [F]

$$\int e^{n\sinh(bx+a)}\coth(bx+a)\,dx$$

```
[In] int(exp(n*sinh(b*x+a))*coth(b*x+a),x)
[Out] int(exp(n*sinh(b*x+a))*coth(b*x+a),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \frac{\text{Ei}(n\sinh(bx+a))}{b}$$

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="fricas")
[Out] Ei(n*sinh(b*x + a))/b
```

Sympy [F]

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int e^{n\sinh(a+bx)}\coth(a+bx)\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x)
[Out] Integral(exp(n*sinh(a + b*x))*coth(a + b*x), x)
```

Maxima [F]

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int \coth(bx+a)\,e^{(n\sinh(bx+a))}\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="maxima")
[Out] integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

Giac [F]

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int \coth(bx+a)\,e^{(n\sinh(bx+a))}\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*coth(b*x+a),x, algorithm="giac")
[Out] integrate(coth(b*x + a)*e^(n*sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\sinh(a+bx)}\coth(a+bx)\,dx = \int \coth(a+b\,x)\,\mathrm{e}^{n\sinh(a+b\,x)}\,dx$$

```
[In] int(coth(a + b*x)*exp(n*sinh(a + b*x)),x)
[Out] int(coth(a + b*x)*exp(n*sinh(a + b*x)), x)
```

3.977 $\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\sinh(c(a+bx)))}{bc}$$

```
[Out] Ei(n*sinh(c*(b*x+a)))/b/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4425, 2209}

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\sinh(c(a+bx)))}{bc}$$

```
[In] Int[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4425

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(c(a+bx))\right)}{bc}$$
$$= \frac{\text{ExpIntegralEi}(n\sinh(c(a+bx)))}{bc}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx = \frac{\text{ExpIntegralEi}(n\sinh(c(a+bx)))}{bc}$$

```
[In] Integrate[E^(n*Sinh[a*c + b*c*x])*Coth[c*(a + b*x)],x]
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Maple [F]

$$\int \mathrm{e}^{n\sinh(bcx+ac)}\coth\left(c(bx+a)\right)dx$$

```
[In] int(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
[Out] int(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx = \frac{\text{Ei}(n\sinh\left(bcx+ac\right))}{bc}$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="fricas")
[Out] Ei(n*sinh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx=\int e^{n\sinh{(ac+bcx)}}\coth{(ac+bcx)}\,dx$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x)
[Out] Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

Maxima [F]

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx=\int \coth\left((bx+a)c\right)e^{(n\sinh(bcx+ac))}\,dx$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="maxima")
[Out] integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

Giac [F]

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx=\int \coth\left((bx+a)c\right)e^{(n\sinh(bcx+ac))}\,dx$$

```
[In] integrate(exp(n*sinh(b*c*x+a*c))*coth(c*(b*x+a)),x, algorithm="giac")
[Out] integrate(coth((b*x + a)*c)*e^(n*sinh(b*c*x + a*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\sinh(ac+bcx)}\coth(c(a+bx))\,dx=\int \coth(c\,(a+b\,x))\,\mathrm{e}^{n\sinh(a\,c+b\,c\,x)}\,dx$$

```
[In] int(coth(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)),x)
[Out] int(coth(c*(a + b*x))*exp(n*sinh(a*c + b*c*x)), x)
```

3.978 $\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx=\frac{\text{ExpIntegralEi}(n\sinh(ac+bcx))}{bc}$$

```
[Out] Ei(n*sinh(b*c*x+a*c))/b/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4425, 2209}

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx=\frac{\text{ExpIntegralEi}(n\sinh(ac+bxc))}{bc}$$

```
[In] Int[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sinh[a*c + b*c*x]]/(b*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4425

```
Int[Coth[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sinh[c*(a
+ b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sinh(ac+bcx)\right)}{bc}$$
$$= \frac{\text{ExpIntegralEi}(n\sinh(ac+bcx))}{bc}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \frac{\text{ExpIntegralEi}(n\sinh(c(a+bx)))}{bc}$$

```
[In] Integrate[E^(n*Sinh[c*(a + b*x)])*Coth[a*c + b*c*x],x]
[Out] ExpIntegralEi[n*Sinh[c*(a + b*x)]]/(b*c)
```

Maple [F]

$$\int e^{n\sinh(c(bx+a))}\coth(bcx+ac)\,dx$$

```
[In] int(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
[Out] int(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \frac{\text{Ei}(n\sinh(bcx+ac))}{bc}$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="fricas")
[Out] Ei(n*sinh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int e^{n\sinh\,(ac+bcx)}\coth\,(ac+bcx)\,dx$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x)
[Out] Integral(exp(n*sinh(a*c + b*c*x))*coth(a*c + b*c*x), x)
```

Maxima [F]

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int \coth\,(bcx+ac)\,e^{(n\sinh((bx+a)c))}\,dx$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="maxima")
[Out] integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

Giac [F]

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int \coth\,(bcx+ac)\,e^{(n\sinh((bx+a)c))}\,dx$$

```
[In] integrate(exp(n*sinh(c*(b*x+a)))*coth(b*c*x+a*c),x, algorithm="giac")
[Out] integrate(coth(b*c*x + a*c)*e^(n*sinh((b*x + a)*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\sinh(c(a+bx))}\coth(ac+bcx)\,dx = \int \mathrm{e}^{n\sinh(c\,(a+b\,x))}\coth(a\,c+b\,c\,x)\,dx$$

```
[In] int(exp(n*sinh(c*(a + b*x)))*coth(a*c + b*c*x),x)
[Out] int(exp(n*sinh(c*(a + b*x)))*coth(a*c + b*c*x), x)
```

3.979 $\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx=\frac{\log(a+b\tanh(x))}{b}$$

```
[Out] ln(a+b*tanh(x))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 31}

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx=\frac{\log(a+b\tanh(x))}{b}$$

```
[In] Int[Sech[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] Log[a + b*Tanh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b}\\ &= \frac{\log(a+b\tanh(x))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log(a+b\tanh(x))}{b}$$

```
[In] Integrate[Sech[x]^2/(a + b*Tanh[x]),x]
[Out] Log[a + b*Tanh[x]]/b
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\tanh(x))}{b}$	12
default	$\frac{\ln(a+b\tanh(x))}{b}$	12
risch	$\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{b}-\frac{\ln(1+e^{2x})}{b}$	35

```
[In] int(sech(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*tanh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x)}\,dx = \frac{\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
[Out] (log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x
) - sinh(x))))/b
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*tanh(x)),x)
[Out] Integral(sech(x)**2/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x)}\,dx = \frac{\log(b\tanh(x) + a)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
[Out] log(b*tanh(x) + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{\operatorname{sech}^2(x)}{a + b\tanh(x)}\,dx = \frac{(a+b)\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{ab + b^2} - \frac{\log\left(e^{(2x)} + 1\right)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="giac")
[Out] (a + b)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b + b^2) - log(e^(2*x) +
 1)/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^2}+a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(cosh(x)^2*(a + b*tanh(x))),x)
```

```
[Out] -(2*atan((a*(-b^2)^(1/2) + a*exp(2*x)*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

3.980 $\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan(\tanh(x))$$

```
[Out] arctan(tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3756, 209}

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan(\tanh(x))$$

```
[In] Int[Sech[x]^2/(1 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(x)\right) \\ &= \arctan(\tanh(x)) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \frac{1}{2}\arctan(\sinh(2x))$$

```
[In] Integrate[Sech[x]^2/(1 + Tanh[x]^2),x]
[Out] ArcTan[Sinh[2*x]]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 8.00

method	result	size
risch	$\frac{i\ln\left(\mathrm{e}^{2x}+i\right)}{2}-\frac{i\ln\left(\mathrm{e}^{2x}-i\right)}{2}$	24
default	$-\frac{\left(-2+\sqrt{2}\right)\sqrt{2}\,\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}-\frac{\sqrt{2}\,\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}$	72

```
[In] int(sech(x)^2/(1+tanh(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*ln(exp(2*x)+I)-1/2*I*ln(exp(2*x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(3) = 6$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = -\arctan\left(-\frac{\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="fricas")
[Out] -arctan(-(cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+1}\,dx$$

```
[In] integrate(sech(x)**2/(1+tanh(x)**2),x)
[Out] Integral(sech(x)**2/(tanh(x)**2 + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(3) = 6$.

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 11.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-x)}\right)\right) - \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-x)}\right)\right)$$

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="maxima")
[Out] arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-x))) - arctan(-1/2*sqrt(2)*(sqrt(2) - 2
*e^(-x)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \arctan\left(e^{(2x)}\right)$$

```
[In] integrate(sech(x)^2/(1+tanh(x)^2),x, algorithm="giac")
[Out] arctan(e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh^2(x)}\,dx = \operatorname{atan}\left(e^{2x}\right)$$

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 + 1)),x)
[Out] atan(exp(2*x))
```

3.981 $\int \frac{\text{sech}^2(x)}{9+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\text{sech}^2(x)}{9+\tanh^2(x)}\,dx=\frac{1}{3}\arctan\left(\frac{\tanh(x)}{3}\right)$$

```
[Out] 1/3*arctan(1/3*tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3756, 209}

$$\int \frac{\text{sech}^2(x)}{9+\tanh^2(x)}\,dx=\frac{1}{3}\arctan\left(\frac{\tanh(x)}{3}\right)$$

```
[In] Int[Sech[x]^2/(9 + Tanh[x]^2),x]
```

```
[Out] ArcTan[Tanh[x]/3]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{3}\arctan\left(\frac{\tanh(x)}{3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = -\frac{1}{3}\arctan(3\coth(x))$$

```
[In] Integrate[Sech[x]^2/(9 + Tanh[x]^2),x]
[Out] -1/3*ArcTan[3*Coth[x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

method	result	size
risch	$\frac{i\ln\left(\mathrm{e}^{2x}+\frac{4}{5}+\frac{3i}{5}\right)}{6}-\frac{i\ln\left(\mathrm{e}^{2x}+\frac{4}{5}-\frac{3i}{5}\right)}{6}$	26
default	$-\frac{\sqrt{10}\left(-10+\sqrt{10}\right)\arctan\left(\frac{18\tanh\left(\frac{x}{2}\right)}{6\sqrt{10}-6}\right)}{5\left(6\sqrt{10}-6\right)}-\frac{\left(10+\sqrt{10}\right)\sqrt{10}\arctan\left(\frac{18\tanh\left(\frac{x}{2}\right)}{6\sqrt{10}+6}\right)}{5\left(6\sqrt{10}+6\right)}$	72

```
[In] int(sech(x)^2/(9+tanh(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*I*ln(exp(2*x)+4/5+3/5*I)-1/6*I*ln(exp(2*x)+4/5-3/5*I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = -\frac{1}{3}\arctan\left(-\frac{9\cosh(x)+\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)$$

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="fricas")
[Out] -1/3*arctan(-1/3*(9*cosh(x) + sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+9}\,dx$$

```
[In] integrate(sech(x)**2/(9+tanh(x)**2),x)
[Out] Integral(sech(x)**2/(tanh(x)**2 + 9), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = -\frac{1}{3}\arctan\left(\frac{5}{3}e^{(-2x)}+\frac{4}{3}\right)$$

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="maxima")
[Out] -1/3*arctan(5/3*e^(-2*x) + 4/3)
```

Giac [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx = \frac{1}{3}\arctan\left(\frac{5}{3}e^{(2x)}+\frac{4}{3}\right)$$

```
[In] integrate(sech(x)^2/(9+tanh(x)^2),x, algorithm="giac")
[Out] 1/3*arctan(5/3*e^(2*x) + 4/3)
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{9+\tanh^2(x)}\,dx=\frac{\operatorname{atan}\left(\frac{5\,e^{2\,x}}{3}+\frac{4}{3}\right)}{3}$$

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 + 9)),x)
[Out] atan((5*exp(2*x))/3 + 4/3)/3
```

3.982 $\int \text{sech}^2(x)(a+b\tanh(x))^n\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \text{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$$

```
[Out] (a+b*tanh(x))^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 32}

$$\int \text{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{(a+b\tanh(x))^{n+1}}{b(n+1)}$$

```
[In] Int[Sech[x]^2*(a + b*Tanh[x])^n,x]
[Out] (a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^n \, dx, x, b\tanh(x)\right)}{b} \\ &= \frac{(a+b\tanh(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n \, dx = \frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$$

```
[In] Integrate[Sech[x]^2*(a + b*Tanh[x])^n,x]
[Out] (a + b*Tanh[x])^(1 + n)/(b*(1 + n))
```

Maple [A] (verified)

Time = 30.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$
default	$\frac{(a+b\tanh(x))^{1+n}}{b(1+n)}$
risch	$\frac{\left(e^{2x}a+e^{2x}b+a-b\right)\left(\left(1+e^{2x}\right)a+\left(e^{2x}-1\right)b\right)^n\left(1+e^{2x}\right)^{-n} e^{-\frac{i\pi \operatorname{csgn}\left(\frac{i\left(\left(1+e^{2x}\right)a+\left(e^{2x}-1\right)b\right)}{1+e^{2x}}\right) n \left(-\operatorname{csgn}\left(\frac{i\left(\left(1+e^{2x}\right)a+\left(e^{2x}-\cdots\right)\right)}{1+e^{2x}}\right)\right)}{}}}{b(1+n)\left(1+e^{2x}\right)}$

```
[In] int(sech(x)^2*(a+b*tanh(x))^n,x,method=_RETURNVERBOSE)
[Out] (a+b*tanh(x))^(1+n)/b/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx$$
$$=\frac{(a\cosh(x)+b\sinh(x))\cosh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)+(a\cosh(x)+b\sinh(x))\sinh\left(n\log\left(\frac{a\cosh(x)+b\sinh(x)}{\cosh(x)}\right)\right)}{(bn+b)\cosh(x)}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="fricas")
[Out] ((a*cosh(x) + b*sinh(x))*cosh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))) + (a*
cosh(x) + b*sinh(x))*sinh(n*log((a*cosh(x) + b*sinh(x))/cosh(x))))/((b*n +
b)*cosh(x))
```

Sympy [F]

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\int (a+b\tanh(x))^n\operatorname{sech}^2(x)\,dx$$

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**n,x)
[Out] Integral((a + b*tanh(x))**n*sech(x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{(b\tanh(x)+a)^{n+1}}{b(n+1)}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="maxima")
[Out] (b*tanh(x) + a)^(n + 1)/(b*(n + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{\left(\frac{ae^{(2\,x)}+be^{(2\,x)}+a-b}{e^{(2\,x)}+1}\right)^{n+1}}{b(n+1)}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^n,x, algorithm="giac")
[Out] ((a*e^(2*x) + b*e^(2*x) + a - b)/(e^(2*x) + 1))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \operatorname{sech}^2(x)(a+b\tanh(x))^n\,dx=\frac{\left(a+\frac{b\,\left(\mathrm{e}^{2\,x}-1\right)}{\mathrm{e}^{2\,x}+1}\right)^n\,\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{b\,\left(\mathrm{e}^{2\,x}+1\right)\,\left(n+1\right)}$$

```
[In] int((a + b*tanh(x))^n/cosh(x)^2,x)
[Out] ((a + (b*(exp(2*x) - 1))/(exp(2*x) + 1))^n*(a - b + a*exp(2*x) + b*exp(2*x)
))/(b*(exp(2*x) + 1)*(n + 1))
```

3.983 $\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) \, dx = x+\tanh(x)$$

```
[Out] x+tanh(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {212}

$$\int \operatorname{sech}^2(x) \left(1+\frac{1}{1-\tanh^2(x)}\right) \, dx = x+\tanh(x)$$

```
[In] Int[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
```

```
[Out] x + Tanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \left(1 + \frac{1}{1-x^2}\right) dx, x, \tanh(x)\right)$$
$$= \tanh(x) + \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(x)\right)$$
$$= x + \tanh(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \text{sech}^2(x) \left(1 + \frac{1}{1-\tanh^2(x)}\right) dx = 2x - \text{arctanh}(\tanh(x)) + \tanh(x)$$

```
[In] Integrate[Sech[x]^2*(1 + (1 - Tanh[x]^2)^(-1)),x]
[Out] 2*x - ArcTanh[Tanh[x]] + Tanh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 1.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

method	result	size
risch	$x - \frac{2}{1+e^{2x}}$	13
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2} + \ln\left(1+\tanh\left(\frac{x}{2}\right)\right) - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	34

```
[In] int(sech(x)^2*(1+1/(1-tanh(x)^2)),x,method=_RETURNVERBOSE)
[Out] x-2/(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx=\frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="fricas")
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(3) = 6.

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx=-\frac{x\operatorname{sech}^2(x)}{\tanh^2(x)-1}-\frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x)-1}$$

```
[In] integrate(sech(x)**2*(1+1/(1-tanh(x)**2)),x)
[Out] -x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx=x+\frac{2}{e^{(-2x)}+1}$$

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="maxima")
[Out] x + 2/(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx = x - \frac{2}{e^{(2\,x)}+1}$$

```
[In] integrate(sech(x)^2*(1+1/(1-tanh(x)^2)),x, algorithm="giac")
[Out] x - 2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{sech}^2(x)\left(1+\frac{1}{1-\tanh^2(x)}\right)\,dx = x - \frac{2}{\mathrm{e}^{2\,x}+1}$$

```
[In] int(-(1/(tanh(x)^2 - 1) - 1)/cosh(x)^2,x)
[Out] x - 2/(exp(2*x) + 1)
```

3.984 $\int \frac{\operatorname{sech}^2(x)(2-\tanh^2(x))}{1-\tanh^2(x)} dx$

Optimal result

Integrand size = 23, antiderivative size = 4

$$\int \frac{\operatorname{sech}^2(x) \left(2-\tanh^2(x)\right)}{1-\tanh^2(x)} \, dx=x+\tanh(x)$$

```
[Out] x+tanh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3738, 3554, 8}

$$\int \frac{\operatorname{sech}^2(x) \left(2-\tanh^2(x)\right)}{1-\tanh^2(x)} \, dx=x+\tanh(x)$$

```
[In] Int[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
[Out] x + Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(2-\tanh^2(x)\right)\,dx \\
&= 2x - \int \tanh^2(x)\,dx \\
&= 2x + \tanh(x) - \int 1\,dx \\
&= x + \tanh(x)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(4) = 8.

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = 2x - \operatorname{arctanh}(\tanh(x)) + \tanh(x)$$

```
[In] Integrate[(Sech[x]^2*(2 - Tanh[x]^2))/(1 - Tanh[x]^2),x]
[Out] 2*x - ArcTanh[Tanh[x]] + Tanh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.90 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

method	result	size
risch	$x-\frac{2}{1+e^{2x}}$	13
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2}+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	34

```
[In] int(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x,method=_RETURNVERBOSE)
[Out] x-2/(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = \frac{(x-1)\cosh(x)+\sinh(x)}{\cosh(x)}$$

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="fricas")
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(3) = 6$.

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = -\frac{x\operatorname{sech}^2(x)}{\tanh^2(x)-1} - \frac{\tanh(x)\operatorname{sech}^2(x)}{\tanh^2(x)-1}$$

```
[In] integrate(sech(x)**2*(2-tanh(x)**2)/(1-tanh(x)**2),x)
[Out] -x*sech(x)**2/(tanh(x)**2 - 1) - tanh(x)*sech(x)**2/(tanh(x)**2 - 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x + \frac{2}{e^{(-2x)}+1}$$

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="maxima")
[Out] x + 2/(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x - \frac{2}{e^{(2x)}+1}$$

```
[In] integrate(sech(x)^2*(2-tanh(x)^2)/(1-tanh(x)^2),x, algorithm="giac")
[Out] x - 2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^2(x)\left(2-\tanh^2(x)\right)}{1-\tanh^2(x)}\,dx = x - \frac{2}{e^{2x}+1}$$

```
[In] int((tanh(x)^2 - 2)/(cosh(x)^2*(tanh(x)^2 - 1)),x)
[Out] x - 2/(exp(2*x) + 1)
```

3.985 $\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 5

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \arctan(1+\tanh(x))$$

```
[Out] arctan(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4427, 631, 210}

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \arctan(\tanh(x)+1)$$

```
[In] Int[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
[Out] ArcTan[1 + Tanh[x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{2+2x+x^2}\,dx, x, \tanh(x)\right) \\ &= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\tanh(x)\right) \\ &= \arctan(1+\tanh(x)) \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 5.40

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = -\frac{1}{2}i\log((1-i)+\tanh(x))+\frac{1}{2}i\log((1+i)+\tanh(x))$$

```
[In] Integrate[Sech[x]^2/(2 + 2*Tanh[x] + Tanh[x]^2),x]
[Out] (-1/2*I)*Log[(1 - I) + Tanh[x]] + (I/2)*Log[(1 + I) + Tanh[x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 5.20

method	result	size
risch	$\frac{i\ln\left(e^{2x}+\frac{1}{5}+\frac{2i}{5}\right)}{2}-\frac{i\ln\left(e^{2x}+\frac{1}{5}-\frac{2i}{5}\right)}{2}$	26
default	$\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)^2+(1-i)\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)^2+(1+i)\tanh\left(\frac{x}{2}\right)+1\right)}{2}$	42

```
[In] int(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*ln(exp(2*x)+1/5+2/5*I)-1/2*I*ln(exp(2*x)+1/5-2/5*I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(5) = 10$.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 4.60

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = -\arctan\left(-\frac{3\cosh(x)+2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="fricas")
[Out] -arctan(-(3*cosh(x) + 2*sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+2\tanh(x)+2}\,dx$$

```
[In] integrate(sech(x)**2/(2+2*tanh(x)+tanh(x)**2),x)
[Out] Integral(sech(x)**2/(tanh(x)**2 + 2*tanh(x) + 2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{\tanh(x)^2+2\tanh(x)+2}\,dx$$

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="maxima")
[Out] integrate(sech(x)^2/(tanh(x)^2 + 2*tanh(x) + 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \arctan\left(\frac{5}{2}e^{(2x)}+\frac{1}{2}\right)$$

```
[In] integrate(sech(x)^2/(2+2*tanh(x)+tanh(x)^2),x, algorithm="giac")
[Out] arctan(5/2*e^(2*x) + 1/2)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{2+2\tanh(x)+\tanh^2(x)}\,dx = \operatorname{atan}\left(\frac{5\,e^{2\,x}}{2}+\frac{1}{2}\right)$$

```
[In] int(1/(cosh(x)^2*(2*tanh(x) + tanh(x)^2 + 2)),x)
[Out] atan((5*exp(2*x))/2 + 1/2)
```

3.986 $\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)} \, dx=-\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))$$

```
[Out] -coth(x)-ln(tanh(x))+ln(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4427, 46}

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)} \, dx=-\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

```
[In] Int[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
```

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
```

```
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx, x, \tanh(x)\right)\\ &= -\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = x-\coth(x)-\log(\sinh(x))$$

```
[In] Integrate[Sech[x]^2/(Tanh[x]^2 + Tanh[x]^3),x]
[Out] x - Coth[x] - Log[Sinh[x]]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

method	result	size
risch	$2x-\frac{2}{e^{2x}-1}-\ln\left(e^{2x}-1\right)$	24
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)$	32

```
[In] int(sech(x)^2/(tanh(x)^2+tanh(x)^3),x,method=_RETURNVERBOSE)
[Out] 2*x-2/(exp(2*x)-1)-ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 5.13

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = \frac{2\,x\cosh\left(x\right)^2+4\,x\cosh\left(x\right)\sinh\left(x\right)+2\,x\sinh\left(x\right)^2-\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1\right)\log}{\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1}$$

[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="fricas")

[Out] (2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*x - 2)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = \int \frac{\operatorname{sech}^2\left(x\right)}{\left(\tanh\left(x\right)+1\right)\tanh^2\left(x\right)}\,dx$$

[In] integrate(sech(x)**2/(tanh(x)**2+tanh(x)**3),x)

[Out] Integral(sech(x)**2/((tanh(x) + 1)*tanh(x)**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = \frac{2}{e^{(-2\,x)}-1} - \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="maxima")

[Out] 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = 2\,x+\frac{e^{(2\,x)}-3}{e^{(2\,x)}-1}-\log\left(\left|e^{(2\,x)}-1\right|\right)$$

```
[In] integrate(sech(x)^2/(tanh(x)^2+tanh(x)^3),x, algorithm="giac")
[Out] 2*x + (e^(2*x) - 3)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{sech}^2(x)}{\tanh^2(x)+\tanh^3(x)}\,dx = 2\,x-\ln\left(\mathrm{e}^{2\,x}-1\right)-\frac{2}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 + tanh(x)^3)),x)
[Out] 2*x - log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

3.987 $\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

```
[Out] coth(x)+ln(1-tanh(x))-ln(tanh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4427, 46}

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \coth(x)+\log(1-\tanh(x))-\log(\tanh(x))$$

```
[In] Int[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
[Out] Coth[x] + Log[1 - Tanh[x]] - Log[Tanh[x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
```

```
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(-1+x)x^2}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{-1+x} - \frac{1}{x^2} - \frac{1}{x}\right)\,dx, x, \tanh(x)\right)\\ &= \coth(x) + \log(1-\tanh(x)) - \log(\tanh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = -x + \coth(x) - \log(\sinh(x))$$

```
[In] Integrate[Sech[x]^2/(-Tanh[x]^2 + Tanh[x]^3),x]
[Out] -x + Coth[x] - Log[Sinh[x]]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result	size
risch	$\frac{2}{e^{2x}-1} - \ln\left(e^{2x}-1\right)$	21
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2} + \frac{1}{2\tanh\left(\frac{x}{2}\right)} - \ln\left(\tanh\left(\frac{x}{2}\right)\right) + 2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	32

```
[In] int(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x,method=_RETURNVERBOSE)
[Out] 2/(exp(2*x)-1)-ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.53

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = -\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)-2}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="fricas")
[Out] -((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
sinh(x))) - 2)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{(\tanh(x)-1)\tanh^2(x)}\,dx$$

```
[In] integrate(sech(x)**2/(-tanh(x)**2+tanh(x)**3),x)
[Out] Integral(sech(x)**2/((tanh(x) - 1)*tanh(x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx = -2x-\frac{2}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="maxima")
[Out] -2*x - 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx=\frac{e^{(2x)}+1}{e^{(2x)}-1}-\log\left(\left|e^{(2x)}-1\right|\right)$$

```
[In] integrate(sech(x)^2/(-tanh(x)^2+tanh(x)^3),x, algorithm="giac")
[Out] (e^(2*x) + 1)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^2(x)}{-\tanh^2(x)+\tanh^3(x)}\,dx=\frac{2}{e^{2x}-1}-\ln\left(e^{2x}-1\right)$$

```
[In] int(-1/(cosh(x)^2*(tanh(x)^2 - tanh(x)^3)),x)
[Out] 2/(exp(2*x) - 1) - log(exp(2*x) - 1)
```

3.988 $\int \frac{\text{sech}^2(x)}{3-4\tanh^3(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{\text{sech}^2(x)}{3-4\tanh^3(x)}\,dx=\frac{\arctan\left(\frac{\sqrt[3]{3}+2\ 2^{2/3}\tanh(x)}{3^{5/6}}\right)}{3\ 2^{2/3}\sqrt[6]{3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}+\frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}}$$

```
[Out] 1/18*arctan(1/3*(3^(1/3)+2*2^(2/3)*tanh(x))*3^(1/6))*2^(1/3)*3^(5/6)-1/18*l
n(3^(1/3)-2^(2/3)*tanh(x))*6^(1/3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tanh(x)+
2*2^(1/3)*tanh(x)^2)*6^(1/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3756, 206, 31, 648, 631, 210, 642}

$$\int \frac{\text{sech}^2(x)}{3-4\tanh^3(x)}\,dx=\frac{\arctan\left(\frac{2\ 2^{2/3}\tanh(x)+\sqrt[3]{3}}{3^{5/6}}\right)}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{\log\left(2\sqrt[3]{2}\tanh^2(x)+2^{2/3}\sqrt[3]{3}\tanh(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}}$$

```
[In] Int[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
```

```
[Out] ArcTan[(3^(1/3) + 2*2^(2/3)*Tanh[x])/3^(5/6)]/(3*2^(2/3)*3^(1/6)) - Log[3^(
1/3) - 2^(2/3)*Tanh[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tanh[x]
 + 2*2^(1/3)*Tanh[x]^2]/(6*6^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{3-4x^3}\,dx, x, \tanh(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{3}-2^{2/3}x}\,dx, x, \tanh(x)\right)}{3\ 3^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{3}+2^{2/3}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{3\ 3^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{2\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{3}+4\sqrt[3]{2}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}x+2\sqrt[3]{2}x^2}\,dx, x, \tanh(x)\right)}{6\ 6^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\ 2^{2/3}\tanh(x)}{\sqrt[3]{3}}\right)}{6^{2/3}}$$

$$= \frac{\arctan\left(\frac{3+2\ 6^{2/3}\tanh(x)}{3\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}-2^{2/3}\tanh(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tanh(x)+2\sqrt[3]{2}\tanh^2(x)\right)}{6\ 6^{2/3}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{\text{sech}^2(x)}{3-4\tanh^3(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{3+2\ 6^{2/3}\tanh(x)}{3\sqrt{3}}\right) - 2\log\left(3-6^{2/3}\tanh(x)\right) + \log\left(3+6^{2/3}\tanh(x)+2\sqrt[3]{6}\tanh^2(x)\right)}{6\ 6^{2/3}}$$

```
[In] Integrate[Sech[x]^2/(3 - 4*Tanh[x]^3),x]
[Out] (2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tanh[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*
Tanh[x]] + Log[3 + 6^(2/3)*Tanh[x] + 2*6^(1/3)*Tanh[x]^2])/(6*6^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

method	result
risch	$4\left(\sum_{\_R=\mathrm{RootOf}(62208\_Z^3+1)} \_R\ln\left(-10368\_R^2+\mathrm{e}^{2x}+288\_R-7\right)\right)$
derivativedivides	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)^2+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tanh(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tanh(x)}{3}+\right.}{3}\right.}{36}$
default	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tanh(x)^2+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tanh(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tanh(x)}{3}+\right.}{3}\right.}{36}$

```
[In] int(sech(x)^2/(3-4*tanh(x)^3),x,method=_RETURNVERBOSE)
[Out] 4*sum(_R*ln(-10368*_R^2+exp(2*x)+288*_R-7),_R=RootOf(62208*_Z^3+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(71) = 142.

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.03

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = -\frac{1}{18}\cdot 36^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{54}\right.$$
$$\cdot 36^{\frac{1}{6}}\left(\left(36^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}+3\cdot 36^{\frac{1}{3}}\sqrt{3}-9\sqrt{3}(-1)^{\frac{1}{3}}\right)\cosh(x)^2+2\left(36^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}+3\cdot 36^{\frac{1}{3}}\sqrt{3}-9\sqrt{3}(-1)^{\frac{1}{3}}\right)c\right.$$
$$-\frac{1}{216}$$
$$\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\left(\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}+3\right)\cosh(x)^2-2\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)\cosh(x)\sin\right.}{\cosh(x)^2-2\cosh(x)s}\right.$$
$$+\frac{1}{108}$$
$$\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\left(\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}-9\right)\cosh(x)-\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-3\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}-12\right)\sinh(x)\right)}{\cosh(x)-\sinh(x)}\right.$$

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/18*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/54*36^(1/6)*((36^(2/3)*sqrt(3)*(
-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)^2 + 2*(36^(2
/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-1)^(1/3))*cosh(x)
*sinh(x) + (36^(2/3)*sqrt(3)*(-1)^(2/3) + 3*36^(1/3)*sqrt(3) - 9*sqrt(3)*(-
1)^(1/3))*sinh(x)^2 - 36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3)))
- 1/216*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(
2/3) + 3)*cosh(x)^2 - 2*(36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3))*cosh(
x)*sinh(x) + (36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) + 3)*sinh(x)^2 -
36^(2/3)*(-1)^(1/3) + 3*36^(1/3)*(-1)^(2/3) - 21)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 1/108*36^(2/3)*(-1)^(1/3)*log(2*((36^(2/3)*(-1)^(1/3)
 - 3*36^(1/3)*(-1)^(2/3) - 9)*cosh(x) - (36^(2/3)*(-1)^(1/3) - 3*36^(1/3)*(
-1)^(2/3) - 12)*sinh(x))/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx=-\int \frac{\operatorname{sech}^2(x)}{4\tanh^3(x)-3}\,dx$$

```
[In] integrate(sech(x)**2/(3-4*tanh(x)**3),x)
[Out] -Integral(sech(x)**2/(4*tanh(x)**3 - 3), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx=\int -\frac{\operatorname{sech}(x)^2}{4\tanh(x)^3-3}\,dx$$

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="maxima")
[Out] -integrate(sech(x)^2/(4*tanh(x)^3 - 3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx=0$$

```
[In] integrate(sech(x)^2/(3-4*tanh(x)^3),x, algorithm="giac")
[Out] 0
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{sech}^2(x)}{3-4\tanh^3(x)}\,dx = -\frac{6^{1/3}\ln\left(\frac{6^{1/3}\left(29856\,e^{2x}-\frac{6^{1/3}\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}+\frac{4480}{3}\right)}{18} - \frac{6^{1/3}\ln\left(\frac{4480}{3}+\frac{6^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(29856\,e^{2x}-\frac{6^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{18} + \frac{6^{1/3}\ln\left(\frac{4480}{3}-\frac{6^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(29856\,e^{2x}+\frac{6^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(109440\,e^{2x}+153216\right)}{18}+672\right)}{18}-\frac{5696\,e^{2x}}{3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{18}$$

```
[In] int(-1/(cosh(x)^2*(4*tanh(x)^3 - 3)),x)
```

```
[Out] (6^(1/3)*log(4480/3 - (6^(1/3)*((3^(1/2)*1i)/2 + 1/2)*(29856*exp(2*x) + (6^
(1/3)*((3^(1/2)*1i)/2 + 1/2)*(109440*exp(2*x) + 153216))/18 + 672))/18 - (5
696*exp(2*x))/3)*((3^(1/2)*1i)/2 + 1/2))/18 - (6^(1/3)*log((6^(1/3)*((3^(1/
2)*1i)/2 - 1/2)*(29856*exp(2*x) - (6^(1/3)*((3^(1/2)*1i)/2 - 1/2)*(109440*e
xp(2*x) + 153216))/18 + 672))/18 - (5696*exp(2*x))/3 + 4480/3)*((3^(1/2)*1i
)/2 - 1/2))/18 - (6^(1/3)*log((6^(1/3)*(29856*exp(2*x) - (6^(1/3)*(109440*e
xp(2*x) + 153216))/18 + 672))/18 - (5696*exp(2*x))/3 + 4480/3))/18
```

3.989 $\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

```
[Out] -2/195*arctan(1/39*195^(1/2)*(1-2*tanh(x)))*195^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4427, 632, 210}

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

```
[In] Int[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tanh[x])])/Sqrt[195]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{11-5x+5x^2}\,dx, x, \tanh(x)\right)\\ &= -\left(2\text{Subst}\left(\int \frac{1}{-195-x^2}\,dx, x, -5+10\tanh(x)\right)\right)\\ &= -\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tanh(x))\right)}{\sqrt{195}}$$

```
[In] Integrate[Sech[x]^2/(11 - 5*Tanh[x] + 5*Tanh[x]^2),x]
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tanh[x])])/Sqrt[195]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{i\sqrt{195}\,\ln\left(\mathrm{e}^{2x}+\frac{i\sqrt{195}}{11}+\frac{6}{11}\right)}{195}-\frac{i\sqrt{195}\,\ln\left(\mathrm{e}^{2x}-\frac{i\sqrt{195}}{11}+\frac{6}{11}\right)}{195}$	40
default	$\frac{i\sqrt{195}\,\ln\left(11\tanh\left(\frac{x}{2}\right)^{2}+\left(-i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+11\right)}{195}-\frac{i\sqrt{195}\,\ln\left(11\tanh\left(\frac{x}{2}\right)^{2}+\left(i\sqrt{195}-5\right)\tanh\left(\frac{x}{2}\right)+11\right)}{195}$	62

```
[In] int(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/195*I*195^(1/2)*ln(exp(2*x)+1/11*I*195^(1/2)+6/11)-1/195*I*195^(1/2)*ln(e
xp(2*x)-1/11*I*195^(1/2)+6/11)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = -\frac{2}{195}\,\sqrt{195}\arctan\left(-\frac{17\,\sqrt{195}\cosh\left(x\right)+5\,\sqrt{195}\sinh\left(x\right)}{195\,(\cosh\left(x\right)-\sinh\left(x\right))}\right)$$

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="fricas")
[Out] -2/195*sqrt(195)*arctan(-1/195*(17*sqrt(195)*cosh(x) + 5*sqrt(195)*sinh(x))
/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx = \int \frac{\operatorname{sech}^{2}\left(x\right)}{5\tanh^{2}\left(x\right)-5\tanh\left(x\right)+11}\,dx$$

```
[In] integrate(sech(x)**2/(11-5*tanh(x)+5*tanh(x)**2),x)
[Out] Integral(sech(x)**2/(5*tanh(x)**2 - 5*tanh(x) + 11), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx=\int \frac{\operatorname{sech}(x)^2}{5\,\tanh(x)^2-5\,\tanh(x)+11}\,dx$$

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="maxima")
[Out] integrate(sech(x)^2/(5*tanh(x)^2 - 5*tanh(x) + 11), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx=\frac{2}{195}\,\sqrt{195}\arctan\left(\frac{1}{195}\,\sqrt{195}(11\,e^{(2\,x)}+6)\right)$$

```
[In] integrate(sech(x)^2/(11-5*tanh(x)+5*tanh(x)^2),x, algorithm="giac")
[Out] 2/195*sqrt(195)*arctan(1/195*sqrt(195)*(11*e^(2*x) + 6))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2(x)}{11-5\tanh(x)+5\tanh^2(x)}\,dx=\frac{2\,\sqrt{195}\,\mathrm{atan}\left(\frac{\sqrt{195}\,(11\,\mathrm{e}^{2\,x}+6)}{195}\right)}{195}$$

```
[In] int(1/(cosh(x)^2*(5*tanh(x)^2 - 5*tanh(x) + 11)),x)
[Out] (2*195^(1/2)*atan((195^(1/2)*(11*exp(2*x) + 6))/195))/195
```

3.990 $\int \frac{\mathrm{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\mathrm{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx=-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}+\frac{b\tanh(x)}{d}$$

```
[Out] -(-a*d+b*c)*ln(c+d*tanh(x))/d^2+b*tanh(x)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4427, 45}

$$\int \frac{\mathrm{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx=\frac{b\tanh(x)}{d}-\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}$$

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*Tanh[x]])/d^2) + (b*Tanh[x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
```

```
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx, x, \tanh(x)\right)\\ &= -\frac{(bc-ad)\log(c+d\tanh(x))}{d^2}+\frac{b\tanh(x)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = \frac{(-bc+ad)\log(c+d\tanh(x))+bd\tanh(x)}{d^2}$$

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x]))/(c + d*Tanh[x]),x]
[Out] ((-(b*c) + a*d)*Log[c + d*Tanh[x]] + b*d*Tanh[x])/d^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(28) = 56.

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

method	result	size
default	$-\frac{2\left(-\frac{bd\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2}+\frac{(ad-bc)\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{2}\right)}{d^2}+\frac{(ad-bc)\ln\left(c\tanh\left(\frac{x}{2}\right)^2+2d\tanh\left(\frac{x}{2}\right)+c\right)}{d^2}$	75
risch	$-\frac{2b}{d(1+\mathrm{e}^{2x})}-\frac{\ln(1+\mathrm{e}^{2x})a}{d}+\frac{\ln(1+\mathrm{e}^{2x})bc}{d^2}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{c-d}{c+d}\right)a}{d}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{c-d}{c+d}\right)bc}{d^2}$	88

```
[In] int(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -2/d^2*(-b*d*tanh(1/2*x)/(1+tanh(1/2*x)^2)+1/2*(a*d-b*c)*ln(1+tanh(1/2*x)^2
))+(a*d-b*c)/d^2*ln(c*tanh(1/2*x)^2+2*d*tanh(1/2*x)+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.14

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = -\frac{2\,bd + \left((bc-ad)\cosh(x)^2 + 2\,(bc-ad)\cosh(x)\sinh(x) + (bc-ad)\sinh(x)^2 + bc - ad\right)\log\left(\frac{2\,(c\cosh(x}{\cosh(x}\right.}{d^2\cosh(x)^2 + 2\,d^2\,c}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(2*b*d + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a
*d)*sinh(x)^2 + b*c - a*d)*log(2*(c*cosh(x) + d*sinh(x))/(cosh(x) - sinh(x)
)) - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*s
inh(x)^2 + b*c - a*d)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^2 +
2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 + d^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = \int \frac{(a+b\tanh(x))\operatorname{sech}^2(x)}{c+d\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**2*(a+b*tanh(x))/(c+d*tanh(x)),x)
```

```
[Out] Integral((a + b*tanh(x))*sech(x)**2/(c + d*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = -b\left(\frac{c\log\left(-(c-d)e^{(-2\,x)} - c - d\right)}{d^2} - \frac{c\log\left(e^{(-2\,x)} + 1\right)}{d^2} - \frac{2}{de^{(-2\,x)} + d}\right) + \frac{a\log\left(d\tanh(x) + c\right)}{d}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*(c*log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e
^(-2*x) + d)) + a*log(d*tanh(x) + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.04

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = -\frac{(bc^2-acd+bcd-ad^2)\log\left(\left|ce^{(2x)}+de^{(2x)}+c-d\right|\right)}{cd^2+d^3} + \frac{(bc-ad)\log\left(e^{(2x)}+1\right)}{d^2} - \frac{bce^{(2x)}-ade^{(2x)}+bc-ad+2bd}{d^2(e^{(2x)}+1)}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c
*d^2 + d^3) + (b*c - a*d)*log(e^(2*x) + 1)/d^2 - (b*c*e^(2*x) - a*d*e^(2*x)
 + b*c - a*d + 2*b*d)/(d^2*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.61

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))}{c+d\tanh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\mathrm{e}^{2x}\left(\frac{4\left(a\,d\,\sqrt{-d^4}-b\,c\,\sqrt{-d^4}\right)}{d^2\,\sqrt{(a\,d-b\,c)^2}\,(c+d)\,(c-d)\,\sqrt{-d^4}}-\frac{4\,c^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{d^4\,(c+d)\,(c-d)\,(a\,d-b\,c)}\right)\left(\frac{d^2\,\sqrt{-d^4}}{4}+\frac{c\,d\,\sqrt{-d^4}}{4}\right)+\frac{4\,c\left(d^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b\right.}{}}{}\right.}{\sqrt{-d^4}} - \frac{2\,b}{d\,\left(\mathrm{e}^{2x}+1\right)}$$

```
[In] int((a + b*tanh(x))/(cosh(x)^2*(c + d*tanh(x))),x)
```

```
[Out] (2*atan(exp(2*x)*((4*(a*d*(-d^4)^(1/2) - b*c*(-d^4)^(1/2)))/(d^2*((a*d - b*
c)^2)^(1/2)*(c + d)*(c - d)*(-d^4)^(1/2)) - (4*c^2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)^(1/2))/(d^4*(c + d)*(c - d)*(a*d - b*c)))*((d^2*(-d^4)^(1/2))/4 + (
c*d*(-d^4)^(1/2))/4) + (4*c*(d^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2) - c*
d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2))*((d^2*(-d^4)^(1/2))/4 + (c*d*(-d^4
)^(1/2))/4))/(d^5*(c + d)*(c - d)*(a*d - b*c)))*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)^(1/2))/(-d^4)^(1/2) - (2*b)/(d*(exp(2*x) + 1))
```

3.991 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3} - \frac{b(bc-ad)\tanh(x)}{d^2} + \frac{(a+b\tanh(x))^2}{2d}$$

```
[Out] (-a*d+b*c)^2*ln(c+d*tanh(x))/d^3-b*(-a*d+b*c)*tanh(x)/d^2+1/2*(a+b*tanh(x))
^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4427, 45}

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3} - \frac{b\tanh(x)(bc-ad)}{d^2} + \frac{(a+b\tanh(x))^2}{2d}$$

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
```

```
[Out] ((b*c - a*d)^2*Log[c + d*Tanh[x]])/d^3 - (b*(b*c - a*d)*Tanh[x])/d^2 + (a +
 b*Tanh[x])^2/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)\,dx, x, \tanh(x)\right)\\
&= \frac{(bc-ad)^2\log(c+d\tanh(x))}{d^3}-\frac{b(bc-ad)\tanh(x)}{d^2}+\frac{(a+b\tanh(x))^2}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx$$
$$= -\frac{-2(bc-ad)^2\log(c+d\tanh(x))+b^2d^2\text{sech}^2(x)+2bd(bc-2ad)\tanh(x)}{2d^3}$$

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x])^2)/(c + d*Tanh[x]),x]
[Out] -1/2*(-2*(b*c - a*d)^2*Log[c + d*Tanh[x]] + b^2*d^2*Sech[x]^2 + 2*b*d*(b*c
- 2*a*d)*Tanh[x])/d^3
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b\left(\frac{b\tanh(x)^2 d}{2}+2\tanh(x)ad-\tanh(x)bc\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\tanh(x))}{d^3}$
default	$\frac{b\left(\frac{b\tanh(x)^2 d}{2}+2\tanh(x)ad-\tanh(x)bc\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\tanh(x))}{d^3}$
risch	$-\frac{2b(2ad\,e^{2x}-bc\,e^{2x}+bd\,e^{2x}+2ad-bc)}{(1+e^{2x})^2 d^2}-\frac{\ln(1+e^{2x})a^2}{d}+\frac{2\ln(1+e^{2x})abc}{d^2}-\frac{\ln(1+e^{2x})b^2c^2}{d^3}+\frac{\ln\left(e^{2x}+\frac{c-d}{c+d}\right)a^2}{d}$

[In] int(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x,method=_RETURNVERBOSE)

[Out] b/d^2*(1/2*b*tanh(x)^2*d+2*tanh(x)*a*d-tanh(x)*b*c)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*ln(c+d*tanh(x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 688, normalized size of antiderivative = 12.98

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{2b^2cd-4abd^2+2(b^2cd-(2ab+b^2)d^2)\cosh(x)^2+4(b^2cd-(2ab+b^2)d^2)\cosh(x)\sinh(x)+2(b^2cd-}{}$$

[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="fricas")

[Out] (2*b^2*c*d - 4*a*b*d^2 + 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 + 4*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) + 2*(b^2*c*d - (2*a*b + b^2)*d^2)*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(c*cosh(x) + d*sinh(x))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)^3 + d^3*sinh(x)^4 + 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 + d^3)*sinh(x)^2 + 4*(d^3*cosh(x)^3 + d^3*cosh(x))*sinh(x))

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \int \frac{(a+b\tanh(x))^2\operatorname{sech}^2(x)}{c+d\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**2/(c+d*tanh(x)),x)
[Out] Integral((a + b*tanh(x))**2*sech(x)**2/(c + d*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.85

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = -b^2\left(\frac{2\left((c+d)e^{(-2x)}+c\right)}{2\,d^2e^{(-2x)}+d^2e^{(-4x)}+d^2} - \frac{c^2\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^3} + \frac{c^2\log\left(e^{(-2x)}+1\right)}{d^3}\right) - 2\,ab\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2} - \frac{c\log\left(e^{(-2x)}+1\right)}{d^2} - \frac{2}{de^{(-2x)}+d}\right) + \frac{a^2\log\left(d\tanh(x)+c\right)}{d}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="maxima")
[Out] -b^2*(2*((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*
log(-(c - d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 2*a*b*(c*
log(-(c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x
) + d)) + a^2*log(d*tanh(x) + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.98

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{(b^2c^3-2\,abc^2d+b^2c^2d+a^2cd^2-2\,abcd^2+a^2d^3)\log\left(\left|ce^{(2x)}+de^{(2x)}+c-d\right|\right)}{cd^3+d^4} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(e^{(2x)}+1\right)}{d^3} + \frac{3\,b^2c^2e^{(4x)}-6\,abcde^{(4x)}+3\,a^2d^2e^{(4x)}+6\,b^2c^2e^{(2x)}-12\,abcde^{(2x)}+4\,b^2cde^{(2x)}+6\,a^2d^2e^{(2x)}-8\,abd^2e^{(2x)}}{2\,d^3(e^{(2x)}+1)^2}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^2/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] (b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*log
(abs(c*e^(2*x) + d*e^(2*x) + c - d))/(c*d^3 + d^4) - (b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*log(e^(2*x) + 1)/d^3 + 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e^(4*x)
 + 3*a^2*d^2*e^(4*x) + 6*b^2*c^2*e^(2*x) - 12*a*b*c*d*e^(2*x) + 4*b^2*c*d*e
^(2*x) + 6*a^2*d^2*e^(2*x) - 8*a*b*d^2*e^(2*x) - 4*b^2*d^2*e^(2*x) + 3*b^2*
c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^2}{c+d\tanh(x)}\,dx = \frac{\ln\left(c-d+d\,e^{2\,x}+c\,e^{2\,x}\right)\,\left(a\,d-b\,c\right)^2}{d^3} - \frac{2\,\left(b^2\,d-b^2\,c+2\,a\,b\,d\right)}{d^2\,\left(e^{2\,x}+1\right)} - \frac{\ln\left(e^{2\,x}+1\right)\,\left(a\,d-b\,c\right)^2}{d^3} + \frac{2\,b^2}{d\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

```
[In] int((a + b*tanh(x))^2/(cosh(x)^2*(c + d*tanh(x))),x)
```

```
[Out] (log(c - d + d*exp(2*x) + c*exp(2*x))*(a*d - b*c)^2)/d^3 - (2*(b^2*d - b^2*
c + 2*a*b*d))/(d^2*(exp(2*x) + 1)) - (log(exp(2*x) + 1)*(a*d - b*c)^2)/d^3
+ (2*b^2)/(d*(2*exp(2*x) + exp(4*x) + 1))
```

3.992 $\int \frac{\text{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = -\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4}+\frac{b(bc-ad)^2\tanh(x)}{d^3} - \frac{(bc-ad)(a+b\tanh(x))^2}{2d^2}+\frac{(a+b\tanh(x))^3}{3d}$$

```
[Out] -(-a*d+b*c)^3*ln(c+d*tanh(x))/d^4+b*(-a*d+b*c)^2*tanh(x)/d^3-1/2*(-a*d+b*c)
*(a+b*tanh(x))^2/d^2+1/3*(a+b*tanh(x))^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4427, 45}

$$\int \frac{\text{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = -\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4}+\frac{b\tanh(x)(bc-ad)^2}{d^3} - \frac{(bc-ad)(a+b\tanh(x))^2}{2d^2}+\frac{(a+b\tanh(x))^3}{3d}$$

```
[In] Int[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tanh[x]])/d^4) + (b*(b*c - a*d)^2*Tanh[x])/d^3 -
 ((b*c - a*d)*(a + b*Tanh[x])^2)/(2*d^2) + (a + b*Tanh[x])^3/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)\,dx, x, \tanh(x)\right)\\ &= -\frac{(bc-ad)^3\log(c+d\tanh(x))}{d^4}+\frac{b(bc-ad)^2\tanh(x)}{d^3}\\ &\quad -\frac{(bc-ad)(a+b\tanh(x))^2}{2d^2}+\frac{(a+b\tanh(x))^3}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$$
$$= \frac{(c\cosh(x)+d\sinh(x))(a+b\tanh(x))^3\left(-6(bc-ad)^3\cosh^2(x)\log(c+d\tanh(x))+6b^3c^2d\cosh(x)\sinh(\right.}{6d^4(a\cosh(x)+b\sinh(x))^3(c+d\tan}$$

```
[In] Integrate[(Sech[x]^2*(a + b*Tanh[x])^3)/(c + d*Tanh[x]),x]
```

```
[Out] ((c*Cosh[x] + d*Sinh[x])*(a + b*Tanh[x])^3*(-6*(b*c - a*d)^3*Cosh[x]^2*Log[
c + d*Tanh[x]] + 6*b^3*c^2*d*Cosh[x]*Sinh[x] + b*d^2*(9*a*(-(b*c) + a*d)*Si
nh[2*x] + b*(3*b*c - 9*a*d + 2*b*d*Sinh[x]^2*Tanh[x]))))/(6*d^4*(a*Cosh[x]
+ b*Sinh[x])^3*(c + d*Tanh[x]))
```

Maple [A] (verified)

Time = 11.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{b\left(\frac{b^2\tanh(x)^3d^2}{3}+\frac{3ab\,d^2\tanh(x)^2}{2}-\frac{b^2cd\tanh(x)^2}{2}+3\tanh(x)a^2d^2-3\tanh(x)abcd+\tanh(x)b^2c^2\right)}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b}{}$
default	$\frac{b\left(\frac{b^2\tanh(x)^3d^2}{3}+\frac{3ab\,d^2\tanh(x)^2}{2}-\frac{b^2cd\tanh(x)^2}{2}+3\tanh(x)a^2d^2-3\tanh(x)abcd+\tanh(x)b^2c^2\right)}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b}{}$
risch	$-\frac{2b(9a^2d^2e^{4x}-9abcd\,e^{4x}+9ab\,d^2e^{4x}+3b^2c^2e^{4x}-3b^2cd\,e^{4x}+3b^2d^2e^{4x}+18a^2d^2e^{2x}-18abcd\,e^{2x}+9ab\,d^2e^{2x}+6b^2c^2e^{2x}-3b}{3d^3(1+e^{2x})^3}$

[In] int(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x,method=_RETURNVERBOSE)

[Out] b/d^3*(1/3*b^2*tanh(x)^3*d^2+3/2*a*b*d^2*tanh(x)^2-1/2*b^2*c*d*tanh(x)^2+3*tanh(x)*a^2*d^2-3*tanh(x)*a*b*c*d+tanh(x)*b^2*c^2)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(c+d*tanh(x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1975 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 1975, normalized size of antiderivative = 25.32

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \text{Too large to display}$$

[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="fricas")

[Out] -1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9*a^2*b + b^3)*d^3 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)*cosh(x)^2 + 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^2)*sinh(x)^2 + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5

```
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 6*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 + 2*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*(c*cosh(x) + d*sinh(x))/
(cosh(x) - sinh(x))) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x
)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6
 + b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 3*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*cosh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 + 6*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 + 2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 12*(2*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*
cosh(x)^3 + (2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)
*cosh(x))*sinh(x))/(d^4*cosh(x)^6 + 6*d^4*cosh(x)*sinh(x)^5 + d^4*sinh(x)^6
 + 3*d^4*cosh(x)^4 + 3*d^4*cosh(x)^2 + 3*(5*d^4*cosh(x)^2 + d^4)*sinh(x)^4
+ d^4 + 4*(5*d^4*cosh(x)^3 + 3*d^4*cosh(x))*sinh(x)^3 + 3*(5*d^4*cosh(x)^4
+ 6*d^4*cosh(x)^2 + d^4)*sinh(x)^2 + 6*(d^4*cosh(x)^5 + 2*d^4*cosh(x)^3 + d
^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx=\int \frac{(a+b\tanh{(x)})^3\operatorname{sech}^2{(x)}}{c+d\tanh{(x)}}\,dx$$

```
[In] integrate(sech(x)**2*(a+b*tanh(x))**3/(c+d*tanh(x)),x)
[Out] Integral((a + b*tanh(x))**3*sech(x)**2/(c + d*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.54

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx$$
$$=\frac{1}{3}b^3\left(\frac{2\left(3c^2+d^2+3\left(2c^2+cd\right)e^{(-2x)}+3\left(c^2+cd+d^2\right)e^{(-4x)}\right)}{3d^3e^{(-2x)}+3d^3e^{(-4x)}+d^3e^{(-6x)}+d^3}-\frac{3c^3\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^4}+\frac{3c^3}{}\right.$$
$$-3ab^2\left(\frac{2\left((c+d)e^{(-2x)}+c\right)}{2d^2e^{(-2x)}+d^2e^{(-4x)}+d^2}-\frac{c^2\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^3}+\frac{c^2\log\left(e^{(-2x)}+1\right)}{d^3}\right)$$
$$-3a^2b\left(\frac{c\log\left(-(c-d)e^{(-2x)}-c-d\right)}{d^2}-\frac{c\log\left(e^{(-2x)}+1\right)}{d^2}-\frac{2}{de^{(-2x)}+d}\right)$$
$$+\frac{a^3\log\left(d\tanh\left(x\right)+c\right)}{d}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(2*(3*c^2 + d^2 + 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e^
(-4*x))/(3*d^3*e^(-2*x) + 3*d^3*e^(-4*x) + d^3*e^(-6*x) + d^3) - 3*c^3*log(
-(c - d)*e^(-2*x) - c - d)/d^4 + 3*c^3*log(e^(-2*x) + 1)/d^4) - 3*a*b^2*(2*
((c + d)*e^(-2*x) + c)/(2*d^2*e^(-2*x) + d^2*e^(-4*x) + d^2) - c^2*log(-(c
- d)*e^(-2*x) - c - d)/d^3 + c^2*log(e^(-2*x) + 1)/d^3) - 3*a^2*b*(c*log(-(
c - d)*e^(-2*x) - c - d)/d^2 - c*log(e^(-2*x) + 1)/d^2 - 2/(d*e^(-2*x) + d)
) + a^3*log(d*tanh(x) + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(74) = 148$.

Time = 0.30 (sec) , antiderivative size = 543, normalized size of antiderivative = 6.96

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx =$$
$$-\frac{\left(b^3c^4-3ab^2c^3d+b^3c^3d+3a^2bc^2d^2-3ab^2c^2d^2-a^3cd^3+3a^2bcd^3-a^3d^4\right)\log\left(\left|ce^{(2x)}+de^{(2x)}+c-d\right|\right.}{cd^4+d^5}$$
$$+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(e^{(2x)}+1\right)}{d^4}$$
$$-\frac{11b^3c^3e^{(6x)}-33ab^2c^2de^{(6x)}+33a^2bcd^2e^{(6x)}-11a^3d^3e^{(6x)}+33b^3c^3e^{(4x)}-99ab^2c^2de^{(4x)}+12b^3c^2de^{(4}}{}$$

```
[In] integrate(sech(x)^2*(a+b*tanh(x))^3/(c+d*tanh(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
 a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) + c - d
))/(c*d^4 + d^5) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
e^(2*x) + 1)/d^4 - 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 33*a^
2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) + 33*b^3*c^3*e^(4*x) - 99*a*b^2*c^2*
d*e^(4*x) + 12*b^3*c^2*d*e^(4*x) + 99*a^2*b*c*d^2*e^(4*x) - 36*a*b^2*c*d^2*
e^(4*x) - 12*b^3*c*d^2*e^(4*x) - 33*a^3*d^3*e^(4*x) + 36*a^2*b*d^3*e^(4*x)
+ 36*a*b^2*d^3*e^(4*x) + 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) - 99*a*b^2
*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 72*a*b^2*c
*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2*b*d^3*e^(
2*x) + 36*a*b^2*d^3*e^(2*x) + 11*b^3*c^3 - 33*a*b^2*c^2*d + 12*b^3*c^2*d +
33*a^2*b*c*d^2 - 36*a*b^2*c*d^2 - 11*a^3*d^3 + 36*a^2*b*d^3 + 4*b^3*d^3)/(d
^4*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 1347, normalized size of antiderivative = 17.27

$$\int \frac{\operatorname{sech}^2(x)(a+b\tanh(x))^3}{c+d\tanh(x)}\,dx = \frac{2\,(2\,b^3\,d-b^3\,c+3\,a\,b^2\,d)}{d^2\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)} - \frac{2\,(3\,a^2\,b\,d^2-3\,a\,b^2\,c\,d+3\,a\,b^2\,d^2+b^3\,c^2-b^3\,c\,d+b^3\,d^2)}{d^3\,\left(\mathrm{e}^{2\,x}+1\right)} - \frac{8\,b^3}{3\,d\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\left(\frac{32\,c\,\sqrt{a^6\,d^6-6\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4-20\,a^3\,b^3\,c^3\,d^3+15\,a^2\,b^4\,c^4\,d^2-6\,a\,b^5\,c^5\,d+b^6\,c^6}\,\left(-a^3\,c\,d^8+a^3\,d^9+3\,a^2\,b\,c^2\,d^7-3\,a^2\,b\,c\,d^8-3\,a\,b^2\,c^3\,d^6 \cdots\right.}{d^{16}\,\sqrt{(a\,d-b\,c)^6}\,(c+d)\,(c-d)^2\,\left(c^2+2\,c\,d+d^2\right)} \cdots\right.\right.}{\cdots}$$

```
[In] int((a + b*tanh(x))^3/(cosh(x)^2*(c + d*tanh(x))),x)
```

```
[Out] (2*(2*b^3*d - b^3*c + 3*a*b^2*d))/(d^2*(2*exp(2*x) + exp(4*x) + 1)) - (2*(b
^3*c^2 + b^3*d^2 + 3*a*b^2*d^2 + 3*a^2*b*d^2 - b^3*c*d - 3*a*b^2*c*d))/(d^3
*(exp(2*x) + 1)) - (8*b^3)/(3*d*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) -
 (2*atan((((32*c*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)*(a^3*d^9 - a
^3*c*d^8 - b^3*c^3*d^6 + b^3*c^4*d^5 + 3*a*b^2*c^2*d^7 - 3*a*b^2*c^3*d^6 +
3*a^2*b*c^2*d^7 - 3*a^2*b*c*d^8))/(d^16*((a*d - b*c)^6)^(1/2)*(c + d)*(c -
d)^2*(2*c*d + c^2 + d^2)) - exp(2*x)*((32*c*(2*a^3*c*d^8 - 2*b^3*c^4*d^5 +
6*a*b^2*c^3*d^6 - 6*a^2*b*c^2*d^7)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^
(1/2))/(d^16*((a*d - b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) -
 (16*(c^2*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) + d^2*
(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^
```

```
6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*
d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d -
b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2 + d^2))) + (16*(c^2*(-d^8)^(1/2)*(a^6*d^6
+ b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) - c*d*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6
+ 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^
5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*
d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2
 + d^2)))*(d^10*(-d^8)^(1/2) + 2*c*d^9*(-d^8)^(1/2) + c^2*d^8*(-d^8)^(1/2))
)/(16*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)))*(a^6*d^6 + b^6*c^6 +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5)^(1/2))/(-d^8)^(1/2)
```

3.993 $\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = -\frac{1}{3(2+\tanh^3(x))}$$

```
[Out] -1/3/(2+tanh(x)^3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4427, 267}

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{(2+\tanh^3(x))^2}\,dx = -\frac{1}{3(\tanh^3(x)+2)}$$

```
[In] Int[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
```

```
[Out] -1/3*1/(2 + Tanh[x]^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
```

```
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{x^2}{(2+x^3)^2}\,dx, x, \tanh(x)\right)\\ &= -\frac{1}{3\left(2+\tanh^3(x)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{1}{3\left(2+\tanh^3(x)\right)}$$

```
[In] Integrate[(Sech[x]^2*Tanh[x]^2)/(2 + Tanh[x]^3)^2,x]
[Out] -1/3*1/(2 + Tanh[x]^3)
```

Maple [A] (verified)

Time = 11.84 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{3\left(2+\tanh(x)^3\right)}$	11
default	$-\frac{1}{3\left(2+\tanh(x)^3\right)}$	11
risch	$-\frac{2(3\,e^{4x}+1)}{9(3\,e^{6x}+3\,e^{4x}+9\,e^{2x}+1)}$	33

```
[In] int(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x,method=_RETURNVERBOSE)
[Out] -1/3/(2+tanh(x)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(10) = 20.

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 6.08

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{8\left(\cosh(x)^2+\cosh(x)\sinh(x)+\sinh(x)^2\right)}{9\left(3\cosh(x)^4+12\cosh(x)\sinh(x)^3+3\sinh(x)^4+2\left(9\cosh(x)^2+2\right)\sinh(x)^2+4\cosh(x)^2+4\left(3\right.\right.}$$

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="fricas")
[Out] -8/9*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(3*cosh(x)^4 + 12*cosh(x)*si
nh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(3*
cosh(x)^3 + cosh(x))*sinh(x) + 9)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = \int \frac{\tanh^2(x)\operatorname{sech}^2(x)}{\left(\tanh^3(x)+2\right)^2}\,dx$$

```
[In] integrate(sech(x)**2*tanh(x)**2/(2+tanh(x)**3)**2,x)
[Out] Integral(tanh(x)**2*sech(x)**2/(tanh(x)**3 + 2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{1}{3\left(\tanh(x)^3+2\right)}$$

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="maxima")
[Out] -1/3/(tanh(x)^3 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{2\left(3\,e^{(4\,x)}+1\right)}{9\left(3\,e^{(6\,x)}+3\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}$$

```
[In] integrate(sech(x)^2*tanh(x)^2/(2+tanh(x)^3)^2,x, algorithm="giac")
[Out] -2/9*(3*e^(4*x) + 1)/(3*e^(6*x) + 3*e^(4*x) + 9*e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{\operatorname{sech}^2(x)\tanh^2(x)}{\left(2+\tanh^3(x)\right)^2}\,dx = -\frac{\frac{2\,\mathrm{e}^{4\,x}}{3}+\frac{2}{9}}{9\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+3\,\mathrm{e}^{6\,x}+1}$$

```
[In] int(tanh(x)^2/(cosh(x)^2*(tanh(x)^3 + 2)^2),x)
[Out] -((2*exp(4*x))/3 + 2/9)/(9*exp(2*x) + 3*exp(4*x) + 3*exp(6*x) + 1)
```

3.994 $\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx=\frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}$$

```
[Out] 1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3738, 2687, 276}

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx=-\frac{1}{13}\tanh^{13}(x)+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^9(x)}{3}+\frac{\tanh^7(x)}{7}$$

```
[In] Int[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
[Out] Tanh[x]^7/7 - Tanh[x]^9/3 + (3*Tanh[x]^11)/11 - Tanh[x]^13/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \operatorname{sech}^8(x)\tanh^6(x)\,dx\\ &= i\text{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx, x, i\tanh(x)\right)\\ &= i\text{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)\,dx, x, i\tanh(x)\right)\\ &= \frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. $2(33) = 66$.

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = \frac{16\tanh(x)}{3003}+\frac{8\operatorname{sech}^2(x)\tanh(x)}{3003}+\frac{2\operatorname{sech}^4(x)\tanh(x)}{1001}+\frac{5\operatorname{sech}^6(x)\tanh(x)}{3003}-\frac{53}{429}\operatorname{sech}^8(x)\tanh(x)+\frac{27}{143}\operatorname{sech}^{10}(x)\tanh(x)-\frac{1}{13}\operatorname{sech}^{12}(x)\tanh(x)$$

```
[In] Integrate[Sech[x]^2*Tanh[x]^6*(1 - Tanh[x]^2)^3,x]
```

```
[Out] (16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/1001
 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^10
*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\frac{\tanh(x)^7}{7}-\frac{\tanh(x)^9}{3}+\frac{3\tanh(x)^{11}}{11}-\frac{\tanh(x)^{13}}{13}$$

```
[In] int(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x)
[Out] 1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 778, normalized size of antiderivative = 23.58

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="fricas")
[Out] -64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (5403
6*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 2249*
cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x)^5
 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x))*s
inh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 8294)*s
inh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 15080*cos
h(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^6 + 448
50*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cosh(x)^17
+ 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x)^15 + 13
*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*cosh(x)^4 +
 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(x)^5 + 455*
cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*cosh(x)^4 + 4
68*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh(x)^7 + 273*c
osh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*cosh(x)^8 + 6506
5*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(x)^9 + 716*cosh
(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)^5 + 47190*cosh(
x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655*cosh(x)^8 + 1338
48*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*sinh(x)^7 + 1300*c
osh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*cosh(x)^7 + 132132*
cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^6 + (6188*cosh(x)^12 +
39039*cosh(x)^10 + 100386*cosh(x)^8 + 132132*cosh(x)^6 + 89964*cosh(x)^4 +
26754*cosh(x)^2 + 1638)*sinh(x)^5 + 1794*cosh(x)^5 + (2380*cosh(x)^13 + 177
45*cosh(x)^11 + 55770*cosh(x)^9 + 94380*cosh(x)^7 + 90216*cosh(x)^5 + 45500
*cosh(x)^3 + 8970*cosh(x))*sinh(x)^4 + (680*cosh(x)^14 + 5915*cosh(x)^12 +
22308*cosh(x)^10 + 47190*cosh(x)^8 + 59976*cosh(x)^6 + 44590*cosh(x)^4 + 16
```

```
380*cosh(x)^2 + 1430)*sinh(x)^3 + 2002*cosh(x)^3 + (136*cosh(x)^15 + 1365*c
osh(x)^13 + 6084*cosh(x)^11 + 15730*cosh(x)^9 + 25776*cosh(x)^7 + 27300*cos
h(x)^5 + 17940*cosh(x)^3 + 6006*cosh(x))*sinh(x)^2 + (17*cosh(x)^16 + 195*c
osh(x)^14 + 1014*cosh(x)^12 + 3146*cosh(x)^10 + 6426*cosh(x)^8 + 8918*cosh(
x)^6 + 8190*cosh(x)^4 + 4290*cosh(x)^2 + 572)*sinh(x) + 2002*cosh(x))
```

Sympy [F]

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = -\int\left(-\tanh^6(x)\operatorname{sech}^2(x)\right)dx - \int 3\tanh^8(x)\operatorname{sech}^2(x)\,dx - \int\left(-3\tanh^{10}(x)\operatorname{sech}^2(x)\right)dx - \int \tanh^{12}(x)\operatorname{sech}^2(x)\,dx$$

```
[In] integrate(sech(x)**2*tanh(x)**6*(1-tanh(x)**2)**3,x)
```

```
[Out] -Integral(-tanh(x)**6*sech(x)**2, x) - Integral(3*tanh(x)**8*sech(x)**2, x)
 - Integral(-3*tanh(x)**10*sech(x)**2, x) - Integral(tanh(x)**12*sech(x)**2
, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = -\frac{1}{13}\tanh(x)^{13} + \frac{3}{11}\tanh(x)^{11} - \frac{1}{3}\tanh(x)^9 + \frac{1}{7}\tanh(x)^7$$

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/13*tanh(x)^13 + 3/11*tanh(x)^11 - 1/3*tanh(x)^9 + 1/7*tanh(x)^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = -\frac{32\left(3003\,e^{(18\,x)} - 9009\,e^{(16\,x)} + 18018\,e^{(14\,x)} - 16302\,e^{(12\,x)} + 10296\,e^{(10\,x)} - 2288\,e^{(8\,x)} + 286\,e^{(6\,x)} + 78\,e^{(4\,x)} + 13\,e^{(2\,x)} + 1\right)}{3003\left(e^{(2\,x)}+1\right)^{13}}$$

```
[In] integrate(sech(x)^2*tanh(x)^6*(1-tanh(x)^2)^3,x, algorithm="giac")
[Out] -32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x) +
 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) + 1)
/(e^(2*x) + 1)^13
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 820, normalized size of antiderivative = 24.85

$$\int \operatorname{sech}^2(x)\tanh^6(x)\left(1-\tanh^2(x)\right)^3\,dx = \text{Too large to display}$$

```
[In] int(-(tanh(x)^6*(tanh(x)^2 - 1)^3)/cosh(x)^2,x)
[Out] - ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4*
x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(2*
x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143 +
(6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*exp(
2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(
12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(2
2*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x))/143 -
 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(1
0*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*exp(8*x))/1
3 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/13 - (768*
exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) + 286*exp(6*
x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(14*x) + 1287
*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13*exp(24*x) +
exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (1792*exp(6*x))
/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x)
 + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1)
 - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6*x))/429 - (3584*
exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2*x) + 36*exp(4*x)
+ 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12*x) + 36*exp(14*x)
+ 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 - (256*exp(6*x))/13 + (8
```

```
00*exp(8*x))/13 - (1280*exp(10*x))/13 + (1120*exp(12*x))/13 - (512*exp(14*x
))/13 + (96*exp(16*x))/13)/(12*exp(2*x) + 66*exp(4*x) + 220*exp(6*x) + 495*
exp(8*x) + 792*exp(10*x) + 924*exp(12*x) + 792*exp(14*x) + 495*exp(16*x) +
220*exp(18*x) + 66*exp(20*x) + 12*exp(22*x) + exp(24*x) + 1) - ((128*exp(2*
x))/715 - 256/2145)/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) +
exp(10*x) + 1) - 32/(715*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) +
 1)) - ((960*exp(4*x))/143 - (768*exp(2*x))/715 - (2560*exp(6*x))/143 + (33
60*exp(8*x))/143 - (10752*exp(10*x))/715 + (2688*exp(12*x))/715 + 32/715)/(
10*exp(2*x) + 45*exp(4*x) + 120*exp(6*x) + 210*exp(8*x) + 252*exp(10*x) + 2
10*exp(12*x) + 120*exp(14*x) + 45*exp(16*x) + 10*exp(18*x) + exp(20*x) + 1)
```

3.995 $\int \frac{\operatorname{sech}^2(x)(2+\tanh^2(x))}{1+\tanh^3(x)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{\operatorname{sech}^2(x) \left(2+\tanh^2(x)\right)}{1+\tanh^3(x)} \, dx = -\frac{2 \arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+\tanh(x))$$

```
[Out] ln(1+tanh(x))-2/3*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4427, 1877, 31, 632, 210}

$$\int \frac{\operatorname{sech}^2(x) \left(2+\tanh^2(x)\right)}{1+\tanh^3(x)} \, dx = \log(\tanh(x)+1)-\frac{2 \arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[In] Int[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]
[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/Sqrt[3] + Log[1 + Tanh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2+x^2}{1+x^3}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)\\
&= \log(1+\tanh(x)) - 2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)\\
&= -\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \log(1+\tanh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx=x+\frac{2\arctan\left(\frac{-1+2\tanh(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cosh(x))$$

[In] Integrate[(Sech[x]^2*(2 + Tanh[x]^2))/(1 + Tanh[x]^3),x]

[Out] x + (2*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]])/Sqrt[3] - Log[Cosh[x]]

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\ln\left(1+\tanh\left(x\right)\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2\tanh\left(x\right)-1\right)\sqrt{3}}{3}\right)}{3}$	24
default	$\ln\left(1+\tanh\left(x\right)\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2\tanh\left(x\right)-1\right)\sqrt{3}}{3}\right)}{3}$	24
risch	$2x+\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{2x}+i\sqrt{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{2x}-i\sqrt{3}\right)}{3}-\ln\left(1+{\mathrm e}^{2x}\right)$	50

[In] int(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x,method=_RETURNVERBOSE)

[Out] ln(1+tanh(x))+2/3*3^(1/2)*arctan(1/3*(2*tanh(x)-1)*3^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx=-\frac{2}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}\right)+2\,x-\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="fricas")

[Out] -2/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sin
h(x))) + 2*x - log(2*cosh(x)/(cosh(x) - sinh(x)))

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx = \int \frac{\left(\tanh^2(x)+2\right)\operatorname{sech}^2(x)}{\left(\tanh(x)+1\right)\left(\tanh^2(x)-\tanh(x)+1\right)}\,dx$$

```
[In] integrate(sech(x)**2*(2+tanh(x)**2)/(1+tanh(x)**3),x)
[Out] Integral((tanh(x)**2 + 2)*sech(x)**2/((tanh(x) + 1)*(tanh(x)**2 - tanh(x) +
 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.69

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx &= \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) \\ &\quad -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right) \\ &\quad +\frac{1}{3}\log\left(\tanh(x)^3+1\right)-\frac{1}{3}\log\left(3^{\frac{1}{4}}\sqrt{2}e^{(-x)}+\sqrt{3}e^{(-2x)}+1\right) \\ &\quad -\frac{1}{3}\log\left(-3^{\frac{1}{4}}\sqrt{2}e^{(-x)}+\sqrt{3}e^{(-2x)}+1\right)\end{aligned}$$

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 2/3*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
(2))) + 1/3*log(tanh(x)^3 + 1) - 1/3*log(3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e
^(-2*x) + 1) - 1/3*log(-3^(1/4)*sqrt(2)*e^(-x) + sqrt(3)*e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+2x-\log\left(e^{(2x)}+1\right)$$

```
[In] integrate(sech(x)^2*(2+tanh(x)^2)/(1+tanh(x)^3),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 2*x - log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}^2(x)\left(2+\tanh^2(x)\right)}{1+\tanh^3(x)}\,dx = 2\,x-\ln\left(768\,\mathrm{e}^{2\,x}+768\right)-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\frac{640\,\sqrt{3}}{3}-\frac{128\,\sqrt{3}\,\mathrm{e}^{2\,x}}{3}}{\frac{640\,\mathrm{e}^{2\,x}}{3}+128}\right)}{3}$$

```
[In] int((tanh(x)^2 + 2)/(cosh(x)^2*(tanh(x)^3 + 1)),x)
```

```
[Out] 2*x - log(768*exp(2*x) + 768) - (2*3^(1/2)*atan(((640*3^(1/2))/3 - (128*3^(
1/2)*exp(2*x))/3)/((640*exp(2*x))/3 + 128)))/3
```

3.996 $\int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx$

Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx = x+\tanh(x)$$

```
[Out] x+tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3091, 8}

$$\int \left(1+\cosh^2(x)\right) \operatorname{sech}^2(x)\, dx = x+\tanh(x)$$

```
[In] Int[(1 + Cosh[x]^2)*Sech[x]^2,x]
```

```
[Out] x + Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \tanh(x) + \int 1\,dx\\ &= x + \tanh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \left(1+\cosh^2(x)\right)\operatorname{sech}^2(x)\,dx = x + \tanh(x)$$

```
[In] Integrate[(1 + Cosh[x]^2)*Sech[x]^2,x]
[Out] x + Tanh[x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\tanh\left(x\right)$	5
parallelrisch	$x+\tanh\left(x\right)$	5
parts	$x+\tanh\left(x\right)$	5
risch	$x-\frac{2}{1+\mathrm{e}^{2x}}$	13

```
[In] int((1+cosh(x)^2)*sech(x)^2,x,method=_RETURNVERBOSE)
[Out] x+tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \left(1+\cosh^2(x)\right)\operatorname{sech}^2(x)\,dx = \frac{(x-1)\cosh\left(x\right)+\sinh\left(x\right)}{\cosh\left(x\right)}$$

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="fricas")
[Out] ((x - 1)*cosh(x) + sinh(x))/cosh(x)
```

Sympy [F]

$$\int \left(1+\cosh^2(x)\right)\operatorname{sech}^2(x)\,dx = \int \left(\cosh^2{(x)}+1\right)\operatorname{sech}^2{(x)}\,dx$$

```
[In] integrate((1+cosh(x)**2)*sech(x)**2,x)
[Out] Integral((cosh(x)**2 + 1)*sech(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(1+\cosh^2(x)\right)\operatorname{sech}^2(x)\,dx = x + \frac{2}{e^{(-2x)}+1}$$

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="maxima")
[Out] x + 2/(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(1+\cosh^2(x)\right)\operatorname{sech}^2(x)\,dx = x - \frac{2}{e^{(2x)}+1}$$

```
[In] integrate((1+cosh(x)^2)*sech(x)^2,x, algorithm="giac")
[Out] x - 2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(1+\cosh^2(x)\right)\operatorname{sech}^2(x)\,dx = x - \frac{2}{\mathrm{e}^{2x}+1}$$

```
[In] int((cosh(x)^2 + 1)/cosh(x)^2,x)
[Out] x - 2/(exp(2*x) + 1)
```

3.997 $\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)} dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)} dx = \frac{2\operatorname{arctanh}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}}$$

```
[Out] 2/17*arctanh(1/17*(3+2*tanh(x))*17^(1/2))*17^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {632, 212}

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)} dx = \frac{2\operatorname{arctanh}\left(\frac{2\tanh(x)+3}{\sqrt{17}}\right)}{\sqrt{17}}$$

```
[In] Int[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
```

```
[Out] (2*ArcTanh[(3 + 2*Tanh[x])/Sqrt[17]])/Sqrt[17]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{2-3x-x^2}\,dx, x, \tanh(x)\right)\\ &= -\left(2\text{Subst}\left(\int \frac{1}{17-x^2}\,dx, x, -3-2\tanh(x)\right)\right)\\ &= \frac{2\text{arctanh}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \frac{2\text{arctanh}\left(\frac{3+2\tanh(x)}{\sqrt{17}}\right)}{\sqrt{17}}$$

```
[In] Integrate[Sech[x]^2/(1 + Sech[x]^2 - 3*Tanh[x]),x]
[Out] (2*ArcTanh[(3 + 2*Tanh[x])/Sqrt[17]])/Sqrt[17]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

method	result	size
risch	$\frac{\sqrt{17}\,\ln\left(e^{2x}+\frac{\sqrt{17}}{2}-\frac{3}{2}\right)}{17}-\frac{\sqrt{17}\,\ln\left(e^{2x}-\frac{\sqrt{17}}{2}-\frac{3}{2}\right)}{17}$	36
default	$-\frac{\sqrt{17}\,\ln\left(-\sqrt{17}\,\tanh\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)^2-3\tanh\left(\frac{x}{2}\right)+2\right)}{17}+\frac{\sqrt{17}\,\ln\left(\sqrt{17}\,\tanh\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)^2-3\tanh\left(\frac{x}{2}\right)+2\right)}{17}$	63

```
[In] int(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/17*17^(1/2)*ln(exp(2*x)+1/2*17^(1/2)-3/2)-1/17*17^(1/2)*ln(exp(2*x)-1/2*1
7^(1/2)-3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx$$
$$= \frac{1}{17}\sqrt{17}\log\left(\frac{3\left(\sqrt{17}-5\right)\cosh(x)^2-2\left(3\sqrt{17}-11\right)\cosh(x)\sinh(x)+3\left(\sqrt{17}-5\right)\sinh(x)^2-2\sqrt{17}+}{\cosh(x)^2-6\cosh(x)\sinh(x)+\sinh(x)^2+3}\right.$$

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="fricas")
[Out] 1/17*sqrt(17)*log((3*(sqrt(17) - 5)*cosh(x)^2 - 2*(3*sqrt(17) - 11)*cosh(x)
*sinh(x) + 3*(sqrt(17) - 5)*sinh(x)^2 - 2*sqrt(17) + 6)/(cosh(x)^2 - 6*cosh
(x)*sinh(x) + sinh(x)^2 + 3))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{-3\tanh(x)+\operatorname{sech}^2(x)+1}\,dx$$

```
[In] integrate(sech(x)**2/(1+sech(x)**2-3*tanh(x)),x)
[Out] Integral(sech(x)**2/(-3*tanh(x) + sech(x)**2 + 1), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{\operatorname{sech}(x)^2-3\tanh(x)+1}\,dx$$

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="maxima")
[Out] integrate(sech(x)^2/(sech(x)^2 - 3*tanh(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = -\frac{1}{17}\sqrt{17}\log\left(\frac{\left|-\sqrt{17}+2\,e^{(2\,x)}-3\right|}{\left|\sqrt{17}+2\,e^{(2\,x)}-3\right|}\right)$$

```
[In] integrate(sech(x)^2/(1+sech(x)^2-3*tanh(x)),x, algorithm="giac")
```

```
[Out] -1/17*sqrt(17)*log(abs(-sqrt(17) + 2*e^(2*x) - 3)/abs(sqrt(17) + 2*e^(2*x)
- 3))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{sech}^2(x)}{1+\operatorname{sech}^2(x)-3\tanh(x)}\,dx = -\frac{\sqrt{17}\left(\ln\left(2\,e^{2\,x}-\frac{\sqrt{17}\,(6\,e^{2\,x}+8)}{17}\right)-\ln\left(2\,e^{2\,x}+\frac{\sqrt{17}\,(6\,e^{2\,x}+8)}{17}\right)\right)}{17}$$

```
[In] int(1/(cosh(x)^2*(1/cosh(x)^2 - 3*tanh(x) + 1)),x)
```

```
[Out] -(17^(1/2)*(log(2*exp(2*x) - (17^(1/2)*(6*exp(2*x) + 8))/17) - log(2*exp(2*
x) + (17^(1/2)*(6*exp(2*x) + 8))/17)))/17
```

3.998 $\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}} \, dx = \text{arcsinh}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

```
[Out] arcsinh(1/3*tanh(x)*3^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4231, 221}

$$\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}} \, dx = \text{arcsinh}\left(\frac{\tanh(x)}{\sqrt{3}}\right)$$

```
[In] Int[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
```

```
[Out] ArcSinh[Tanh[x]/Sqrt[3]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \tanh(x)\right)\\ &= \text{arcsinh}\left(\frac{\tanh(x)}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(9) = 18.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 4.78

$$\int \frac{\text{sech}^2(x)}{\sqrt{4-\text{sech}^2(x)}}\,dx = \frac{\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{3+4\sinh^2(x)}}\right)\sqrt{1+2\cosh(2x)}\text{sech}(x)}{\sqrt{4-\text{sech}^2(x)}}$$

```
[In] Integrate[Sech[x]^2/Sqrt[4 - Sech[x]^2],x]
[Out] (ArcTanh[Sinh[x]/Sqrt[3 + 4*Sinh[x]^2]]*Sqrt[1 + 2*Cosh[2*x]]*Sech[x])/Sqrt
[4 - Sech[x]^2]
```

Maple [F]

$$\int \frac{\text{sech}\left(x\right)^2}{\sqrt{4-\text{sech}\left(x\right)^2}}dx$$

```
[In] int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
[Out] int(sech(x)^2/(4-sech(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(8) = 16.

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 12.44

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = -\log\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \sqrt{\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 1}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}}\right) + \log\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \sqrt{\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 1}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}} - 2\right)$$

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sin
h(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))) + log(-cosh(x)^2
- 2*cosh(x)*sinh(x) - sinh(x)^2 + sqrt((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sqrt{-(\operatorname{sech}(x)-2)(\operatorname{sech}(x)+2)}}\,dx$$

```
[In] integrate(sech(x)**2/(4-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sech(x)**2/sqrt(-(sech(x) - 2)*(sech(x) + 2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sqrt{-\operatorname{sech}(x)^2+4}}\,dx$$

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^2/sqrt(-sech(x)^2 + 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.89

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = -\log\left(\sqrt{e^{(4x)}+e^{(2x)}+1}-e^{(2x)}\right) + \log\left(-\sqrt{e^{(4x)}+e^{(2x)}+1}+e^{(2x)}+2\right)$$

```
[In] integrate(sech(x)^2/(4-sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(e^(4*x) + e^(2*x) + 1) - e^(2*x)) + log(-sqrt(e^(4*x) + e^(2*x) +
 1) + e^(2*x) + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{4-\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\cosh(x)^2\sqrt{4-\frac{1}{\cosh(x)^2}}}\,dx$$

```
[In] int(1/(cosh(x)^2*(4 - 1/cosh(x)^2)^(1/2)),x)
[Out] int(1/(cosh(x)^2*(4 - 1/cosh(x)^2)^(1/2)), x)
```

3.999 $\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx=\frac{1}{2}\arcsin(2\tanh(x))$$

```
[Out] 1/2*arcsin(2*tanh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3756, 222}

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx=\frac{1}{2}\arcsin(2\tanh(x))$$

```
[In] Int[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
```

```
[Out] ArcSin[2*Tanh[x]]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{1-4x^2}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\arcsin(2\tanh(x))\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(9) = 18.

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 5.22

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sinh(x)}{\sqrt{-1+3\sinh^2(x)}}\right)\sqrt{-5+3\cosh(2x)}\operatorname{sech}(x)}{2\sqrt{2-8\tanh^2(x)}}$$

```
[In] Integrate[Sech[x]^2/Sqrt[1 - 4*Tanh[x]^2],x]
[Out] (ArcTanh[(2*Sinh[x])/Sqrt[-1 + 3*Sinh[x]^2]]*Sqrt[-5 + 3*Cosh[2*x]]*Sech[x]
)/(2*Sqrt[2 - 8*Tanh[x]^2])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\arcsin(2\tanh(x))}{2}$	8
default	$\frac{\arcsin(2\tanh(x))}{2}$	8

```
[In] int(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(2*tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(7) = 14$.

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 13.11

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx =$$

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\sqrt{-\frac{3\cos}{\cosh(x)^2-}}}{3\cosh(x)^4+12\cosh(x)\sinh(x)^3+3\sinh(x)^4+2\left(9\cosh(x)^2-5\right)\sinh(x)^2-10\cosh(}\right.$$

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*arctan(2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-(3*cosh(x)^2 + 3*sinh(x)^2 - 5)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
))/(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 - 5)*
sinh(x)^2 - 10*cosh(x)^2 + 4*(3*cosh(x)^3 - 5*cosh(x))*sinh(x) + 3))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sqrt{-\left(2\tanh(x)-1\right)\left(2\tanh(x)+1\right)}}\,dx$$

```
[In] integrate(sech(x)**2/(1-4*tanh(x)**2)**(1/2),x)
[Out] Integral(sech(x)**2/sqrt(-(2*tanh(x) - 1)*(2*tanh(x) + 1)), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sqrt{-4\tanh(x)^2+1}}\,dx$$

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sech(x)^2/sqrt(-4*tanh(x)^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.89

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = -\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{3}\sqrt{-3\,e^{(4\,x)}+10\,e^{(2\,x)}-3}-4\right)}{3\,e^{(2\,x)}-5}-1\right)\right)$$

```
[In] integrate(sech(x)^2/(1-4*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -arctan(1/3*sqrt(3)*(2*(sqrt(3)*sqrt(-3*e^(4*x) + 10*e^(2*x) - 3) - 4)/(3*e
^(2*x) - 5) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{1-4\tanh^2(x)}}\,dx = \int \frac{1}{\cosh(x)^2\,\sqrt{1-4\tanh(x)^2}}\,dx$$

```
[In] int(1/(cosh(x)^2*(1 - 4*tanh(x)^2)^(1/2)),x)
[Out] int(1/(cosh(x)^2*(1 - 4*tanh(x)^2)^(1/2)), x)
```

3.1000 $\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx = \operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)$$

```
[Out] arctanh(tanh(x)/(-4+tanh(x)^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3756, 223, 212}

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}} \, dx = \operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-4}}\right)$$

```
[In] Int[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[-4 + Tanh[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right)\\ &= \operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-4+\tanh^2(x)}}\right) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \frac{\arctan\left(\frac{\sinh(x)}{\sqrt{4+3\sinh^2(x)}}\right)\sqrt{5+3\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{-4+\tanh^2(x)}}$$

```
[In] Integrate[Sech[x]^2/Sqrt[-4 + Tanh[x]^2],x]
[Out] (ArcTan[Sinh[x]/Sqrt[4 + 3*Sinh[x]^2]]*Sqrt[5 + 3*Cosh[2*x]]*Sech[x])/(Sqrt
[2]*Sqrt[-4 + Tanh[x]^2])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\ln\left(\tanh\left(x\right)+\sqrt{-4+\tanh\left(x\right)^{2}}\right)$	13
default	$\ln\left(\tanh\left(x\right)+\sqrt{-4+\tanh\left(x\right)^{2}}\right)$	13

```
[In] int(sech(x)^2/(-4+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(tanh(x)+(-4+tanh(x)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx$$
$$=\frac{1}{2}\log\left(\frac{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+\sqrt{2}\sqrt{-\frac{3\cosh\left(x\right)^{2}+3\sinh\left(x\right)^{2}+5}{\cosh\left(x\right)^{2}-2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}}-1}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}\right)$$
$$-\frac{1}{2}\log\left(\frac{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-\sqrt{2}\sqrt{-\frac{3\cosh\left(x\right)^{2}+3\sinh\left(x\right)^{2}+5}{\cosh\left(x\right)^{2}-2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}}-1}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}\right)$$

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + sqrt(2)*sqrt(-(3*cosh(
x)^2 + 3*sinh(x)^2 + 5)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - sqrt(2)*sqrt(-(3*cosh(x)^2 + 3*sinh(x)^2 + 5)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sqrt{(\tanh(x)-2)(\tanh(x)+2)}}\,dx$$

```
[In] integrate(sech(x)**2/(-4+tanh(x)**2)**(1/2),x)
[Out] Integral(sech(x)**2/sqrt((tanh(x) - 2)*(tanh(x) + 2)), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sqrt{\tanh(x)^2-4}}\,dx$$

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sech(x)^2/sqrt(tanh(x)^2 - 4), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 6.07

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx = \log\left(\frac{8}{3}\sqrt{3}\left(2i\sqrt{3}-3\right)-8\sqrt{3}e^{(2x)}+8\sqrt{3e^{(4x)}+10e^{(2x)}+3}\right) - \log\left(\frac{8}{3}\sqrt{3}\left(-2i\sqrt{3}-3\right)-8\sqrt{3}e^{(2x)}+8\sqrt{3e^{(4x)}+10e^{(2x)}+3}\right)$$

```
[In] integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] log(8/3*sqrt(3)*(2*I*sqrt(3) - 3) - 8*sqrt(3)*e^(2*x) + 8*sqrt(3*e^(4*x) +
10*e^(2*x) + 3)) - log(8/3*sqrt(3)*(-2*I*sqrt(3) - 3) - 8*sqrt(3)*e^(2*x) +
 8*sqrt(3*e^(4*x) + 10*e^(2*x) + 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4+\tanh^2(x)}}\,dx=\int \frac{1}{\cosh{(x)}^2\,\sqrt{\tanh{(x)}^2-4}}\,dx$$

```
[In] int(1/(cosh(x)^2*(tanh(x)^2 - 4)^(1/2)),x)
[Out] int(1/(cosh(x)^2*(tanh(x)^2 - 4)^(1/2)), x)
```

3.1001 $\int \sqrt{1+\coth^2(x)}\,\mathrm{sech}^2(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \sqrt{1+\coth^2(x)}\,\mathrm{sech}^2(x)\,dx = -\operatorname{arcsinh}(\coth(x))+\sqrt{1+\coth^2(x)}\tanh(x)$$

```
[Out] -arcsinh(coth(x))+(1+coth(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3744, 283, 221}

$$\int \sqrt{1+\coth^2(x)}\,\mathrm{sech}^2(x)\,dx = \tanh(x)\sqrt{\coth^2(x)+1}-\operatorname{arcsinh}(\coth(x))$$

```
[In] Int[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
[Out] -ArcSinh[Coth[x]] + Sqrt[1 + Coth[x]^2]*Tanh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \coth(x)\right)\\
&= \sqrt{1+\coth^2(x)}\tanh(x) - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \coth(x)\right)\\
&= -\text{arcsinh}(\coth(x)) + \sqrt{1+\coth^2(x)}\tanh(x)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(19) = 38.

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx = \sqrt{1+\coth^2(x)}\text{sech}(2x)\sinh(x)\left(\cosh(x) - \arctan\left(\frac{\cosh(x)}{\sqrt{-\cosh(2x)}}\right)\sqrt{-\cosh(2x)} + \sinh(x)\tanh(x)\right)$$

```
[In] Integrate[Sqrt[1 + Coth[x]^2]*Sech[x]^2,x]
[Out] Sqrt[1 + Coth[x]^2]*Sech[2*x]*Sinh[x]*(Cosh[x] - ArcTan[Cosh[x]/Sqrt[-Cosh[
2*x]]]*Sqrt[-Cosh[2*x]] + Sinh[x]*Tanh[x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$\frac{\left(1+\coth(x)^2\right)^{\frac{3}{2}}}{\coth(x)}-\coth\left(x\right)\sqrt{1+\coth\left(x\right)^{2}}-\operatorname{arcsinh}\left(\coth\left(x\right)\right)$	32
default	$\frac{\left(1+\coth(x)^2\right)^{\frac{3}{2}}}{\coth(x)}-\coth\left(x\right)\sqrt{1+\coth\left(x\right)^{2}}-\operatorname{arcsinh}\left(\coth\left(x\right)\right)$	32

```
[In] int(sech(x)^2*(1+coth(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/coth(x)*(1+coth(x)^2)^(3/2)-coth(x)*(1+coth(x)^2)^(1/2)-arcsinh(coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 11.53

$$\int \sqrt{1+\coth^2(x)}\operatorname{sech}^2(x)\,dx =$$

$$-\frac{\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\log\left(\frac{\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+2\,\sqrt{\frac{\cosh(x)^2+\sinh(x)}{\cosh(x)^2-2\,\cosh(x)\sinh(x}}}{\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2}\right.}{}$$

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) - 4*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx = \int \sqrt{\coth^2{(x)}+1}\,\text{sech}^2{(x)}\,dx$$

```
[In] integrate(sech(x)**2*(1+coth(x)**2)**(1/2),x)
[Out] Integral(sqrt(coth(x)**2 + 1)*sech(x)**2, x)
```

Maxima [F]

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx = \int \sqrt{\coth{(x)}^2+1}\,\text{sech}{(x)}^2\,dx$$

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(coth(x)^2 + 1)*sech(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 6.32

$$\int \sqrt{1+\coth^2(x)}\text{sech}^2(x)\,dx = \frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{e^{(4x)}+1}-2e^{(2x)}+2\right|}{2\left(\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}\right) - \frac{4\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2e^{(2x)}-1}\right)$$

```
[In] integrate(sech(x)^2*(1+coth(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*(sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(e^(4*x) + 1) - 2*e^(2*
x) + 2)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) + 1)) - 4*(sqrt(e^(4*x) + 1)
 - e^(2*x) + 1)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*
e^(2*x) - 1))*sgn(e^(2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\coth^2(x)}\mathrm{sech}^2(x)\,dx = \int \frac{\sqrt{\coth(x)^2+1}}{\cosh(x)^2}\,dx$$

```
[In] int((coth(x)^2 + 1)^(1/2)/cosh(x)^2,x)
[Out] int((coth(x)^2 + 1)^(1/2)/cosh(x)^2, x)
```

3.1002 $\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx=\frac{1}{2}\operatorname{arcsinh}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

```
[Out] 1/2*arcsinh(tanh(x))+1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3756, 201, 221}

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx=\frac{1}{2}\operatorname{arcsinh}(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

```
[In] Int[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] ArcSinh[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}\text{arcsinh}(\tanh(x)) + \frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$
\begin{aligned}
&\int \text{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx\\
&= \frac{1}{4}\text{sech}(x)\text{sech}(2x)\left(2\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^2(x)\sqrt{\cosh(2x)} - \sinh(x)\right.\\
&\qquad \left. + \sinh(3x)\right)\sqrt{1+\tanh^2(x)}
\end{aligned}
$$

```
[In] Integrate[Sech[x]^2*Sqrt[1 + Tanh[x]^2],x]
```

```
[Out] (Sech[x]*Sech[2*x]*(2*ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]]*Cosh[x]^2*Sqrt[Cosh[
2*x]] - Sinh[x] + Sinh[3*x])*Sqrt[1 + Tanh[x]^2])/4
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(\tanh(x))}{2}+\frac{\sqrt{1+\tanh(x)^{2}}\,\tanh(x)}{2}$	19
default	$\frac{\operatorname{arcsinh}(\tanh(x))}{2}+\frac{\sqrt{1+\tanh(x)^{2}}\,\tanh(x)}{2}$	19

```
[In] int(sech(x)^2*(1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(tanh(x))+1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 13.92

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$$

$$=\frac{\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+1\right)\sinh(x)^2+2\cosh(x)^2+4\left(\cosh(x)^3+\right.\right.}{}$$

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sin
h(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 +
 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt((
cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(
x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*
cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2(x)+1}\operatorname{sech}^2(x)\,dx$$

```
[In] integrate(sech(x)**2*(1+tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(tanh(x)**2 + 1)*sech(x)**2, x)
```

Maxima [F]

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2+1}\operatorname{sech}(x)^2\,dx$$

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(tanh(x)^2 + 1)*sech(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.04

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right) - \frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-\cdots}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+2\cdots}\right.$$

```
[In] integrate(sech(x)^2*(1+tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(
2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*x))
^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/((s
qrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(x)\sqrt{1+\tanh^2(x)}\,dx = \int \frac{\sqrt{\tanh(x)^2+1}}{\cosh(x)^2}\,dx$$

```
[In] int((tanh(x)^2 + 1)^(1/2)/cosh(x)^2,x)
[Out] int((tanh(x)^2 + 1)^(1/2)/cosh(x)^2, x)
```

3.1003 $\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x) \, dx = \frac{\tanh^6(x)}{6} - \frac{\tanh^8(x)}{8}$$

```
[Out] 1/6*tanh(x)^6-1/8*tanh(x)^8
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 2687, 14}

$$\int \operatorname{sech}^4(x) \left(-1+\operatorname{sech}^2(x)\right)^2 \tanh(x) \, dx = \frac{\tanh^6(x)}{6} - \frac{\tanh^8(x)}{8}$$

```
[In] Int[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
```

```
[Out] Tanh[x]^6/6 - Tanh[x]^8/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \operatorname{sech}^4(x)\tanh^5(x)\,dx\\ &= -\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, i\tanh(x)\right)\\ &= -\text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, i\tanh(x)\right)\\ &= \frac{\tanh^6(x)}{6}-\frac{\tanh^8(x)}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx = -\frac{1}{4}\operatorname{sech}^4(x)+\frac{\operatorname{sech}^6(x)}{3}-\frac{\operatorname{sech}^8(x)}{8}$$

```
[In] Integrate[Sech[x]^4*(-1 + Sech[x]^2)^2*Tanh[x],x]
[Out] -1/4*Sech[x]^4 + Sech[x]^6/3 - Sech[x]^8/8
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\frac{\tanh\left(x\right)^{6}}{6}-\frac{\tanh\left(x\right)^{8}}{8}$$

```
[In] int(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x)
[Out] 1/6*tanh(x)^6-1/8*tanh(x)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 340, normalized size of antiderivative = 20.00

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx =$$

$$-\frac{}{3\left(\cosh(x)^{10}+10\cosh(x)\sinh(x)^9+\sinh(x)^{10}+\left(45\cosh(x)^2+8\right)\sinh(x)^8+8\cosh(x)^8+8\left(15\cos\right.\right.}$$

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="fricas")
```

```
[Out] -4/3*(3*cosh(x)^6 + 18*cosh(x)*sinh(x)^5 + 3*sinh(x)^6 + (45*cosh(x)^2 - 4)
*sinh(x)^4 - 4*cosh(x)^4 + 4*(15*cosh(x)^3 - 4*cosh(x))*sinh(x)^3 + (45*cos
h(x)^4 - 24*cosh(x)^2 + 13)*sinh(x)^2 + 13*cosh(x)^2 + 2*(9*cosh(x)^5 - 8*c
osh(x)^3 + 7*cosh(x))*sinh(x) - 4)/(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sin
h(x)^10 + (45*cosh(x)^2 + 8)*sinh(x)^8 + 8*cosh(x)^8 + 8*(15*cosh(x)^3 + 8*
cosh(x))*sinh(x)^7 + (210*cosh(x)^4 + 224*cosh(x)^2 + 29)*sinh(x)^6 + 29*co
sh(x)^6 + 2*(126*cosh(x)^5 + 224*cosh(x)^3 + 81*cosh(x))*sinh(x)^5 + (210*c
osh(x)^6 + 560*cosh(x)^4 + 435*cosh(x)^2 + 64)*sinh(x)^4 + 64*cosh(x)^4 + 4
*(30*cosh(x)^7 + 112*cosh(x)^5 + 135*cosh(x)^3 + 48*cosh(x))*sinh(x)^3 + (4
5*cosh(x)^8 + 224*cosh(x)^6 + 435*cosh(x)^4 + 384*cosh(x)^2 + 98)*sinh(x)^2
 + 98*cosh(x)^2 + 2*(5*cosh(x)^9 + 32*cosh(x)^7 + 81*cosh(x)^5 + 96*cosh(x)
^3 + 42*cosh(x))*sinh(x) + 56)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx = -\frac{\operatorname{sech}^8(x)}{8}+\frac{\operatorname{sech}^6(x)}{3}-\frac{\operatorname{sech}^4(x)}{4}$$

```
[In] integrate(sech(x)**4*(-1+sech(x)**2)**2*tanh(x),x)
```

```
[Out] -sech(x)**8/8 + sech(x)**6/3 - sech(x)**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx = -\frac{4}{\left(e^{(-x)}+e^x\right)^4}+\frac{64}{3\left(e^{(-x)}+e^x\right)^6}-\frac{32}{\left(e^{(-x)}+e^x\right)^8}$$

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="maxima")
[Out] -4/(e^(-x) + e^x)^4 + 64/3/(e^(-x) + e^x)^6 - 32/(e^(-x) + e^x)^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx = -\frac{4\left(3e^{(12x)}-4e^{(10x)}+10e^{(8x)}-4e^{(6x)}+3e^{(4x)}\right)}{3\left(e^{(2x)}+1\right)^8}$$

```
[In] integrate(sech(x)^4*(-1+sech(x)^2)^2*tanh(x),x, algorithm="giac")
[Out] -4/3*(3*e^(12*x) - 4*e^(10*x) + 10*e^(8*x) - 4*e^(6*x) + 3*e^(4*x))/(e^(2*x
) + 1)^8
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 375, normalized size of antiderivative = 22.06

$$\int \operatorname{sech}^4(x)\left(-1+\operatorname{sech}^2(x)\right)^2\tanh(x)\,dx = \frac{e^{2x}-5e^{4x}+10e^{6x}-10e^{8x}+5e^{10x}-e^{12x}}{8e^{2x}+28e^{4x}+56e^{6x}+70e^{8x}+56e^{10x}+28e^{12x}+8e^{14x}+e^{16x}+1} - \frac{\frac{20e^{4x}}{7}-\frac{10e^{2x}}{7}-\frac{50e^{6x}}{21}+\frac{5e^{8x}}{7}+\frac{5}{21}}{6e^{2x}+15e^{4x}+20e^{6x}+15e^{8x}+6e^{10x}+e^{12x}+1} - \frac{\frac{8e^{2x}}{7}-\frac{10e^{4x}}{7}+\frac{4e^{6x}}{7}-\frac{2}{7}}{5e^{2x}+10e^{4x}+10e^{6x}+5e^{8x}+e^{10x}+1} - \frac{\frac{10e^{2x}}{7}-\frac{30e^{4x}}{7}+\frac{40e^{6x}}{7}-\frac{25e^{8x}}{7}+\frac{6e^{10x}}{7}-\frac{1}{7}}{7e^{2x}+21e^{4x}+35e^{6x}+35e^{8x}+21e^{10x}+7e^{12x}+e^{14x}+1} - \frac{\frac{2e^{2x}}{7}-\frac{5}{21}}{3e^{2x}+3e^{4x}+e^{6x}+1} - \frac{1}{7\left(2e^{2x}+e^{4x}+1\right)} - \frac{\frac{3e^{4x}}{7}-\frac{5e^{2x}}{7}+\frac{2}{7}}{4e^{2x}+6e^{4x}+4e^{6x}+e^{8x}+1}$$

```
[In] int((tanh(x)*(1/cosh(x)^2 - 1)^2)/cosh(x)^4,x)
```

```
[Out] (exp(2*x) - 5*exp(4*x) + 10*exp(6*x) - 10*exp(8*x) + 5*exp(10*x) - exp(12*x
))/(8*exp(2*x) + 28*exp(4*x) + 56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 2
8*exp(12*x) + 8*exp(14*x) + exp(16*x) + 1) - ((20*exp(4*x))/7 - (10*exp(2*x
))/7 - (50*exp(6*x))/21 + (5*exp(8*x))/7 + 5/21)/(6*exp(2*x) + 15*exp(4*x)
+ 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2*x))/
7 - (10*exp(4*x))/7 + (4*exp(6*x))/7 - 2/7)/(5*exp(2*x) + 10*exp(4*x) + 10*
exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) - ((10*exp(2*x))/7 - (30*exp(4*x))/7
 + (40*exp(6*x))/7 - (25*exp(8*x))/7 + (6*exp(10*x))/7 - 1/7)/(7*exp(2*x) +
 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12*x) + exp
(14*x) + 1) - ((2*exp(2*x))/7 - 5/21)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) +
 1) - 1/(7*(2*exp(2*x) + exp(4*x) + 1)) - ((3*exp(4*x))/7 - (5*exp(2*x))/7
+ 2/7)/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
```

3.1004 $\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = -\frac{2e^{n\sinh(a+bx)}}{bn^2}+\frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}$$

```
[Out] -2*exp(n*sinh(b*x+a))/b/n^2+2*exp(n*sinh(b*x+a))*sinh(b*x+a)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2207, 2225}

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

```
[In] Int[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
```

```
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b}\\ &= \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn} - \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{bn}\\ &= -\frac{2e^{n\sinh(a+bx)}}{bn^2} + \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{2e^{n\sinh(a+bx)}(-1+n\sinh(a+bx))}{bn^2}$$

```
[In] Integrate[E^(n*Sinh[a + b*x])*Sinh[2*a + 2*b*x],x]
[Out] (2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{\left(n\,{\mathrm e}^{2bx+2a}-n-2\,{\mathrm e}^{bx+a}\right){\mathrm e}^{-bx-a-\frac{n\,{\mathrm e}^{-bx-a}}{2}+\frac{n\,{\mathrm e}^{bx+a}}{2}}}{n^{2}b}$	61

```
[In] int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/n^2/b*(n*exp(2*b*x+2*a)-n-2*exp(b*x+a))*exp(-b*x-a-1/2*n*exp(-b*x-a)+1/2*
n*exp(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{2\left((n\sinh(bx+a)-1)\cosh(n\sinh(bx+a))+(n\sinh(bx+a)-1)\sinh(n\sinh(bx+a))\right)}{bn^2\cosh(bx+a)^2-bn^2\sinh(bx+a)^2}$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sinh
(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F]

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
[Out] Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{e^{\left(bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}+a\right)}}{bn} - \frac{e^{\left(-bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}-a\right)}}{bn} - \frac{2e^{\left(\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
[Out] e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2*
n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) - 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Giac [F]

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \int e^{(n\sinh(bx+a))}\sinh(2\,bx+2\,a)\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.51

$$\int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx = \frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n} - \frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2}$$

```
[In] int(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x),x)
[Out] (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp(-(n*exp(-a)*exp(-b*x))/2))/
(b*n) - (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp(-(n*exp(-a)*exp(-
b*x))/2))/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp(-(n*exp(-a)*exp(-b*x))/
2))/(b*n^2)
```

3.1005 $\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = -\frac{2e^{n\sinh(a+bx)}}{bn^2}+\frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}$$

```
[Out] -2*exp(n*sinh(b*x+a))/b/n^2+2*exp(n*sinh(b*x+a))*sinh(b*x+a)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2207, 2225}

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{2\sinh(a+bx)e^{n\sinh(a+bx)}}{bn}-\frac{2e^{n\sinh(a+bx)}}{bn^2}$$

```
[In] Int[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Sinh[a + b*x]))/(b*n^2) + (2*E^(n*Sinh[a + b*x])*Sinh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
```

```
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b} \\
&= \frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh(a+bx)\right)}{b} \\
&= \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn} - \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \sinh(a+bx)\right)}{bn} \\
&= -\frac{2e^{n\sinh(a+bx)}}{bn^2} + \frac{2e^{n\sinh(a+bx)}\sinh(a+bx)}{bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{2e^{n\sinh(a+bx)}(-1+n\sinh(a+bx))}{bn^2}$$

```
[In] Integrate[E^(n*Sinh[a + b*x])*Sinh[2*(a + b*x)],x]
[Out] (2*E^(n*Sinh[a + b*x])*(-1 + n*Sinh[a + b*x]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{\left(n\,{\mathrm e}^{2bx+2a}-n-2\,{\mathrm e}^{bx+a}\right){\mathrm e}^{-bx-a-\frac{n\,{\mathrm e}^{-bx-a}}{2}+\frac{n\,{\mathrm e}^{bx+a}}{2}}}{n^{2}b}$	61

```
[In] int(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/n^2/b*(n*exp(2*b*x+2*a)-n-2*exp(b*x+a))*exp(-b*x-a-1/2*n*exp(-b*x-a)+1/2*
n*exp(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{2\left((n\sinh(bx+a)-1)\cosh(n\sinh(bx+a))+(n\sinh(bx+a)-1)\sinh(n\sinh(bx+a))\right)}{bn^2\cosh(bx+a)^2-bn^2\sinh(bx+a)^2}$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*((n*sinh(b*x + a) - 1)*cosh(n*sinh(b*x + a)) + (n*sinh(b*x + a) - 1)*sinh
(n*sinh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F]

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{n\sinh(a+bx)}\sinh(2a+2bx)\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x)
[Out] Integral(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx = \frac{e^{\left(bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}+a\right)}}{bn} - \frac{e^{\left(-bx+\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}-a\right)}}{bn} - \frac{2e^{\left(\frac{1}{2}ne^{(bx+a)}-\frac{1}{2}ne^{(-bx-a)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
[Out] e^(b*x + 1/2*n*e^(b*x + a) - 1/2*n*e^(-b*x - a) + a)/(b*n) - e^(-b*x + 1/2*
n*e^(b*x + a) - 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) - 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Giac [F]

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx=\int e^{(n\sinh(bx+a))}\sinh\left(2\,bx+2\,a\right)\,dx$$

```
[In] integrate(exp(n*sinh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*sinh(b*x + a))*sinh(2*b*x + 2*a), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.51

$$\int e^{n\sinh(a+bx)}\sinh(2(a+bx))\,dx=\frac{e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{b\,x}\,e^{a}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}-\frac{e^{-a}\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-b\,x}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n}-\frac{2\,e^{\frac{n\,e^{b\,x}\,e^{a}}{2}}\,e^{-\frac{n\,e^{-a}\,e^{-b\,x}}{2}}}{b\,n^2}$$

```
[In] int(exp(n*sinh(a + b*x))*sinh(2*a + 2*b*x),x)
```

```
[Out] (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp(-(n*exp(-a)*exp(-b*x))/2))/
(b*n) - (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp(-(n*exp(-a)*exp(-
b*x))/2))/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp(-(n*exp(-a)*exp(-b*x))/
2))/(b*n^2)
```

3.1006 $\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] -4*exp(n*sinh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sinh(1/2*a+1/2*b*x))*sinh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2207, 2225}

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=\frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[In] Int[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\text{Subst}\left(\int e^{nx}\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\
&= -\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

```
[In] Integrate[E^(n*Sinh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
[Out] (4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{2\left(n\,\mathrm{e}^{bx+a}-n-2\,\mathrm{e}^{\frac{a}{2}+\frac{bx}{2}}\right)\mathrm{e}^{-\frac{a}{2}-\frac{bx}{2}-\frac{n\,\mathrm{e}^{-\frac{a}{2}-\frac{bx}{2}}}{2}+\frac{n\,\mathrm{e}^{\frac{a}{2}+\frac{bx}{2}}}{2}}}{n^2 b}$	65

```
[In] int(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)-n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x-1/2*n*exp(
-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{4\left(\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right.\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1/
2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn} - \frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn} - \frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(50) = 100$.

Time = 0.33 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.98

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}-\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}+a\right)}-ne^{\left(\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}\right.\right.\right.}\right.}{}$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,\mathrm{e}^{-\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^2}$$

```
[In] int(exp(n*sinh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp(-(n*exp(-a/2)*exp(-(b*x)/2))
/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (2*exp(-a/2)*exp(-(b*x)/2)*ex
```

```
p(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) -
 (4*exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(
b*n^2)
```

3.1007 $\int e^{n \sinh\left(\frac{1}{2}(a+bx)\right)} \sinh(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int e^{n \sinh\left(\frac{1}{2}(a+bx)\right)} \sinh(a+bx)\,dx = -\frac{4e^{n \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)} \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] -4*exp(n*sinh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sinh(1/2*a+1/2*b*x))*sinh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2207, 2225}

$$\int e^{n \sinh\left(\frac{1}{2}(a+bx)\right)} \sinh(a+bx)\,dx = \frac{4\sinh\left(\frac{a}{2}+\frac{bx}{2}\right) e^{n \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[In] Int[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Sinh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sinh[a/2 + (b*x)/2])*Sinh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} \\
&= \frac{4\text{Subst}\left(\int e^{nx}x\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} \\
&= \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\text{Subst}\left(\int e^{nx}\,dx, x, \sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn} \\
&= -\frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{4e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

```
[In] Integrate[E^(n*Sinh[(a + b*x)/2])*Sinh[a + b*x],x]
[Out] (4*E^(n*Sinh[(a + b*x)/2])*(-1 + n*Sinh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{2\left(n\,e^{bx+a}-n-2\,e^{\frac{a}{2}+\frac{bx}{2}}\right)e^{-\frac{a}{2}-\frac{bx}{2}-\frac{n\,e^{-\frac{a}{2}-\frac{bx}{2}}}{2}+\frac{n\,e^{\frac{a}{2}+\frac{bx}{2}}}{2}}}{n^2 b}$	65

```
[In] int(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)-n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x-1/2*n*exp(
-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\sinh(\frac{1}{2}(a+bx))}\sinh(a+bx)\,dx$$
$$= \frac{4\,\big(\big(n\sinh\big(\frac{1}{2}\,bx+\frac{1}{2}\,a\big)-1\big)\cosh\big(n\sinh\big(\frac{1}{2}\,bx+\frac{1}{2}\,a\big)\big)+\big(n\sinh\big(\frac{1}{2}\,bx+\frac{1}{2}\,a\big)-1\big)\sinh\big(n\sinh\big(\frac{1}{2}\,bx+\frac{1}{2}\,a\big)}{bn^2\cosh\big(\frac{1}{2}\,bx+\frac{1}{2}\,a\big)^2-bn^2\sinh\big(\frac{1}{2}\,bx+\frac{1}{2}\,a\big)^2}$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*sinh(1/2*b*x + 1/2*a) - 1)*cosh(n*sinh(1/2*b*x + 1/2*a)) + (n*sinh(1/
2*b*x + 1/2*a) - 1)*sinh(n*sinh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]

$$\int e^{n\sinh(\frac{1}{2}(a+bx))}\sinh(a+bx)\,dx=\int e^{n\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*sinh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\sinh(\frac{1}{2}(a+bx))}\sinh(a+bx)\,dx=\frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn}-\frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn}-\frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) - 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) - 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.98

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}+ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}-\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}-ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}+a\right)} - ne^{\left(\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\right.}\right.}\right.}{}$$

```
[In] integrate(exp(n*sinh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) + a) - n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\sinh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,e^{-\frac{a}{2}}\,e^{b\,x}\,e^{-\frac{b\,x}{2}}\,e^{a}\,e^{-\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{2\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}\,e^{-\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,e^{-\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n^2}$$

```
[In] int(exp(n*sinh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp(a)*exp(-(n*exp(-a/2)*exp(-(b*x)/2))
/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) - (2*exp(-a/2)*exp(-(b*x)/2)*ex
```

```
p(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b*n) -
 (4*exp(-(n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(
b*n^2)
```

3.1008 $\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=-\frac{2e^{n\cosh(a+bx)}}{bn^2}+\frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

```
[Out] -2*exp(n*cosh(b*x+a))/b/n^2+2*exp(n*cosh(b*x+a))*cosh(b*x+a)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2207, 2225}

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
```

```
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn} - \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{bn}\\ &= -\frac{2e^{n\cosh(a+bx)}}{bn^2} + \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \frac{2e^{n\cosh(a+bx)}(-1+n\cosh(a+bx))}{bn^2}$$

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[2*a + 2*b*x],x]
[Out] (2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{\left(n\,{\mathrm e}^{2bx+2a}+n-2\,{\mathrm e}^{bx+a}\right){\mathrm e}^{-bx-a+\frac{n\,{\mathrm e}^{-bx-a}}{2}+\frac{n\,{\mathrm e}^{bx+a}}{2}}}{n^{2} b}$	59

```
[In] int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/n^2/b*(n*exp(2*b*x+2*a)+n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(-b*x-a)+1/2*
n*exp(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \frac{2\left((n\cosh\left(bx+a\right)-1)\cosh\left(n\cosh\left(bx+a\right)\right)+(n\cosh\left(bx+a\right)-1)\sinh\left(n\cosh\left(bx+a\right)\right)\right)}{bn^2\cosh\left(bx+a\right)^2-bn^2\sinh\left(bx+a\right)^2}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sinh
(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F]

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \int e^{n\cosh\left(a+bx\right)}\sinh\left(2a+2bx\right)\,dx$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
[Out] Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx = \frac{e^{\left(bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}+a\right)}}{bn} + \frac{e^{\left(-bx+\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}-a\right)}}{bn} - \frac{2\,e^{\left(\frac{1}{2}\,ne^{(bx+a)}+\frac{1}{2}\,ne^{(-bx-a)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
[Out] e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2*
n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) + 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Giac [F]

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\int e^{(n\cosh(bx+a))}\sinh\left(2\,bx+2\,a\right)\,dx$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int e^{n\cosh(a+bx)}\sinh(2a+2bx)\,dx=\frac{\mathrm{e}^{-a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-b\,x}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}-\frac{2\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n^2}+\frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}$$

```
[In] int(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x),x)
[Out] (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp((n*exp(-a)*exp(-b*x))/2))
/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp((n*exp(-a)*exp(-b*x))/2))/(b*n^2
) + (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp((n*exp(-a)*exp(-b*x))/2
))/(b*n)
```

3.1009 $\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx=-\frac{2e^{n\cosh(a+bx)}}{bn^2}+\frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}$$

```
[Out] -2*exp(n*cosh(b*x+a))/b/n^2+2*exp(n*cosh(b*x+a))*cosh(b*x+a)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {12, 2207, 2225}

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx=\frac{2\cosh(a+bx)e^{n\cosh(a+bx)}}{bn}-\frac{2e^{n\cosh(a+bx)}}{bn^2}$$

```
[In] Int[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Cosh[a + b*x]))/(b*n^2) + (2*E^(n*Cosh[a + b*x])*Cosh[a + b*x])/(b
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
```

```
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh(a+bx)\right)}{b} \\ &= \frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh(a+bx)\right)}{b} \\ &= \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn} - \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \cosh(a+bx)\right)}{bn} \\ &= -\frac{2e^{n\cosh(a+bx)}}{bn^2} + \frac{2e^{n\cosh(a+bx)}\cosh(a+bx)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{2e^{n\cosh(a+bx)}(-1+n\cosh(a+bx))}{bn^2}$$

```
[In] Integrate[E^(n*Cosh[a + b*x])*Sinh[2*(a + b*x)],x]
[Out] (2*E^(n*Cosh[a + b*x])*(-1 + n*Cosh[a + b*x]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{\left(n\,e^{2bx+2a}+n-2\,e^{bx+a}\right)e^{-bx-a+\frac{n\,e^{-bx-a}}{2}+\frac{n\,e^{bx+a}}{2}}}{n^2 b}$	59

```
[In] int(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/n^2/b*(n*exp(2*b*x+2*a)+n-2*exp(b*x+a))*exp(-b*x-a+1/2*n*exp(-b*x-a)+1/2*
n*exp(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{2\left((n\cosh\left(bx+a\right)-1)\cosh\left(n\cosh\left(bx+a\right)\right)+(n\cosh\left(bx+a\right)-1)\sinh\left(n\cosh\left(bx+a\right)\right)\right)}{bn^2\cosh\left(bx+a\right)^2-bn^2\sinh\left(bx+a\right)^2}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*((n*cosh(b*x + a) - 1)*cosh(n*cosh(b*x + a)) + (n*cosh(b*x + a) - 1)*sinh
(n*cosh(b*x + a)))/(b*n^2*cosh(b*x + a)^2 - b*n^2*sinh(b*x + a)^2)
```

Sympy [F]

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{n\cosh\left(a+bx\right)}\sinh\left(2a+2bx\right)dx$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x)
[Out] Integral(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{e^{\left(bx+\frac{1}{2}ne^{(bx+a)}+\frac{1}{2}ne^{(-bx-a)}+a\right)}}{bn} + \frac{e^{\left(-bx+\frac{1}{2}ne^{(bx+a)}+\frac{1}{2}ne^{(-bx-a)}-a\right)}}{bn} - \frac{2e^{\left(\frac{1}{2}ne^{(bx+a)}+\frac{1}{2}ne^{(-bx-a)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="maxima")
[Out] e^(b*x + 1/2*n*e^(b*x + a) + 1/2*n*e^(-b*x - a) + a)/(b*n) + e^(-b*x + 1/2*
n*e^(b*x + a) + 1/2*n*e^(-b*x - a) - a)/(b*n) - 2*e^(1/2*n*e^(b*x + a) + 1/
2*n*e^(-b*x - a))/(b*n^2)
```

Giac [F]

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \int e^{(n\cosh(bx+a))}\sinh(2\,bx+2\,a)\,dx$$

[In] integrate(exp(n*cosh(b*x+a))*sinh(2*b*x+2*a),x, algorithm="giac")

[Out] integrate(e^(n*cosh(b*x + a))*sinh(2*b*x + 2*a), x)

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int e^{n\cosh(a+bx)}\sinh(2(a+bx))\,dx = \frac{\mathrm{e}^{-a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{-b\,x}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n} - \frac{2\,\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n^2} + \frac{\mathrm{e}^{\frac{n\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-a}\,\mathrm{e}^{-b\,x}}{2}}}{b\,n}$$

[In] int(exp(n*cosh(a + b*x))*sinh(2*a + 2*b*x),x)

[Out] (exp(-a)*exp((n*exp(b*x)*exp(a))/2)*exp(-b*x)*exp((n*exp(-a)*exp(-b*x))/2))
/(b*n) - (2*exp((n*exp(b*x)*exp(a))/2)*exp((n*exp(-a)*exp(-b*x))/2))/(b*n^2
) + (exp((n*exp(b*x)*exp(a))/2)*exp(b*x)*exp(a)*exp((n*exp(-a)*exp(-b*x))/2
))/(b*n)

3.1010 $\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] -4*exp(n*cosh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*cosh(1/2*a+1/2*b*x))*cosh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2207, 2225}

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx=\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[In] Int[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2i)\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\
&= \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\text{Subst}\left(\int e^{nx}\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\
&= -\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$
\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}
$$

```
[In] Integrate[E^(n*Cosh[a/2 + (b*x)/2])*Sinh[a + b*x],x]
[Out] (4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{2\left(n\,\mathrm{e}^{bx+a}+n-2\,\mathrm{e}^{\frac{a}{2}+\frac{bx}{2}}\right)\mathrm{e}^{-\frac{a}{2}-\frac{bx}{2}+\frac{n\,\mathrm{e}^{-\frac{a}{2}-\frac{bx}{2}}}{2}+\frac{n\,\mathrm{e}^{\frac{a}{2}+\frac{bx}{2}}}{2}}}{n^2 b}$	63

```
[In] int(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n^2/b*(n*exp(b*x+a)+n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x+1/2*n*exp(
-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1/
2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
```

```
[Out] Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn} + \frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn} - \frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(50) = 100$.

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.97

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2\,bxe^{\left(\frac{1}{2}\,bx+\frac{1}{2}\right.}\right.\right.}\right.}{}$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh(a+bx)\,dx = \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n^2} + \frac{2\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{b\,x}\,\mathrm{e}^{-\frac{b\,x}{2}}\,\mathrm{e}^{a}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{-\frac{a}{2}}\,\mathrm{e}^{-\frac{b\,x}{2}}}{2}}\,\mathrm{e}^{\frac{n\,\mathrm{e}^{a/2}\,\mathrm{e}^{\frac{b\,x}{2}}}{2}}}{b\,n}$$

```
[In] int(exp(n*cosh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(-(b*x)/2)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/
2)*exp((b*x)/2))/2))/(b*n) - (4*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*e
xp(a/2)*exp((b*x)/2))/2))/(b*n^2) + (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp
(a)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b
*n)
```

3.1011 $\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] -4*exp(n*cosh(1/2*a+1/2*b*x))/b/n^2+4*exp(n*cosh(1/2*a+1/2*b*x))*cosh(1/2*a
+1/2*b*x)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2207, 2225}

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx=\frac{4\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

```
[In] Int[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
```

```
[Out] (-4*E^(n*Cosh[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Cosh[a/2 + (b*x)/2])*Cosh[
a/2 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2i)\text{Subst}\left(\int -2ie^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} \\
&= \frac{4\text{Subst}\left(\int e^{nx}x\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} \\
&= \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\text{Subst}\left(\int e^{nx}\,dx, x, \cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn} \\
&= -\frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$
\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{4e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}
$$

```
[In] Integrate[E^(n*Cosh[(a + b*x)/2])*Sinh[a + b*x],x]
[Out] (4*E^(n*Cosh[(a + b*x)/2])*(-1 + n*Cosh[(a + b*x)/2]))/(b*n^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{2\left(n\,e^{bx+a}+n-2\,e^{\frac{a}{2}+\frac{bx}{2}}\right)e^{-\frac{a}{2}-\frac{bx}{2}+\frac{n\,e^{-\frac{a}{2}-\frac{bx}{2}}}{2}+\frac{n\,e^{\frac{a}{2}+\frac{bx}{2}}}{2}}}{n^2 b}$	63

```
[In] int(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2/n^2/b*(n*exp(b*x+a)+n-2*exp(1/2*a+1/2*b*x))*exp(-1/2*a-1/2*b*x+1/2*n*exp(
-1/2*a-1/2*b*x)+1/2*n*exp(1/2*a+1/2*b*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{4\left(\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\cosh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)+\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)\sinh\left(n\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\right.\right.}{bn^2\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-bn^2\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2}$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="fricas")
[Out] 4*((n*cosh(1/2*b*x + 1/2*a) - 1)*cosh(n*cosh(1/2*b*x + 1/2*a)) + (n*cosh(1/
2*b*x + 1/2*a) - 1)*sinh(n*cosh(1/2*b*x + 1/2*a)))/(b*n^2*cosh(1/2*b*x + 1/
2*a)^2 - b*n^2*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \int e^{n\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)}\sinh\left(a+bx\right)dx$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x)
[Out] Integral(exp(n*cosh(a/2 + b*x/2))*sinh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}+\frac{1}{2}\,a\right)}}{bn} + \frac{2\,e^{\left(-\frac{1}{2}\,bx+\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}-\frac{1}{2}\,a\right)}}{bn} - \frac{4\,e^{\left(\frac{1}{2}\,ne^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+\frac{1}{2}\,ne^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)}}{bn^2}$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x - 1/2*a) + 1/2
*a)/(b*n) + 2*e^(-1/2*b*x + 1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x -
 1/2*a) - 1/2*a)/(b*n) - 4*e^(1/2*n*e^(1/2*b*x + 1/2*a) + 1/2*n*e^(-1/2*b*x
 - 1/2*a))/(b*n^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.97

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\left(ne^{\left(bx+\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}+ne^{(bx+a)}+n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}-\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}-ne^{(bx+a)}-n\right)e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}+a\right)}+ne^{\left(\frac{1}{4}\left(2bxe^{\left(\frac{1}{2}bx+\frac{1}{2}\right.}\right.\right.}\right.}{}$$

```
[In] integrate(exp(n*cosh(1/2*a+1/2*b*x))*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(n*e^(b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)*e^(-1/2*b
*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)*e^(-1/2*b
*x - 1/2*a) + a) + n*e^(1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*e^(b*x + a) + n)
*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*e^(b*x + a) - n)
*e^(-1/2*b*x - 1/2*a)) - 2*e^(1/2*b*x + 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) + n*
e^(b*x + a) + n)*e^(-1/2*b*x - 1/2*a) - 1/4*(2*b*x*e^(1/2*b*x + 1/2*a) - n*
e^(b*x + a) - n)*e^(-1/2*b*x - 1/2*a) + 1/2*a))*e^(-1/2*b*x - 1/2*a)/(b*n^2
)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int e^{n\cosh\left(\frac{1}{2}(a+bx)\right)}\sinh(a+bx)\,dx = \frac{2\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}\,e^{\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n} - \frac{4\,e^{\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n^2} + \frac{2\,e^{-\frac{a}{2}}\,e^{b\,x}\,e^{-\frac{b\,x}{2}}\,e^{a}\,e^{\frac{n\,e^{-\frac{a}{2}}\,e^{-\frac{b\,x}{2}}}{2}}\,e^{\frac{n\,e^{a/2}\,e^{\frac{b\,x}{2}}}{2}}}{b\,n}$$

```
[In] int(exp(n*cosh(a/2 + (b*x)/2))*sinh(a + b*x),x)
```

```
[Out] (2*exp(-a/2)*exp(-(b*x)/2)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/
2)*exp((b*x)/2))/2))/(b*n) - (4*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*e
xp(a/2)*exp((b*x)/2))/2))/(b*n^2) + (2*exp(-a/2)*exp(b*x)*exp(-(b*x)/2)*exp
(a)*exp((n*exp(-a/2)*exp(-(b*x)/2))/2)*exp((n*exp(a/2)*exp((b*x)/2))/2))/(b
*n)
```

3.1012 $\int \mathbf{csch}(x) \log(\tanh(x))\mathbf{sech}(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \operatorname{csch}(x) \log(\tanh(x))\operatorname{sech}(x) \, dx = \frac{1}{2} \log^2(\tanh(x))$$

```
[Out] 1/2*ln(tanh(x))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2700, 29, 6818}

$$\int \operatorname{csch}(x) \log(\tanh(x))\operatorname{sech}(x) \, dx = \frac{1}{2} \log^2(\tanh(x))$$

```
[In] Int[Csch[x]*Log[Tanh[x]]*Sech[x],x]
```

```
[Out] Log[Tanh[x]]^2/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log^2(\tanh(x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{1}{2}\log^2(\tanh(x))$$

```
[In] Integrate[Csch[x]*Log[Tanh[x]]*Sech[x],x]
[Out] Log[Tanh[x]]^2/2
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\tanh(x))^2}{2}$
default	$\frac{\ln(\tanh(x))^2}{2}$
risch	$\frac{\ln(1+e^{2x})^2}{2} - \ln(e^{2x}-1)\ln(1+e^{2x}) + \frac{\ln(e^{2x}-1)^2}{2} + \frac{i\ln(1+e^{2x})\pi\operatorname{csgn}\left(\frac{i}{1+e^{2x}}\right)\operatorname{csgn}\left(i(e^{2x}-1)\right)\operatorname{csgn}\left(\frac{i(}{}\right.}{2}$

```
[In] int(csch(x)*ln(tanh(x))*sech(x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(tanh(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{1}{2}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="fricas")
[Out] 1/2*log(sinh(x)/cosh(x))^2
```

Sympy [F]

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \int \log(\tanh(x))\operatorname{csch}(x)\operatorname{sech}(x)\,dx$$

```
[In] integrate(csch(x)*ln(tanh(x))*sech(x),x)
[Out] Integral(log(tanh(x))*csch(x)*sech(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(7) = 14.

Time = 0.96 (sec) , antiderivative size = 95, normalized size of antiderivative = 10.56

$$\begin{aligned}&\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx\\&= (\log(e^x+1)+\log(-e^x+1))\log\left(e^{(2x)}+1\right)-\frac{1}{2}\log\left(e^{(2x)}+1\right)^2\\&\quad-\frac{1}{2}\log(e^x+1)^2-\log(e^x+1)\log(-e^x+1)-\frac{1}{2}\log(-e^x+1)^2\\&\quad+\left(\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)-\log\left(e^{(-2x)}+1\right)\right)\log(\tanh(x))\end{aligned}$$

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="maxima")
[Out] (log(e^x + 1) + log(-e^x + 1))*log(e^(2*x) + 1) - 1/2*log(e^(2*x) + 1)^2 -
1/2*log(e^x + 1)^2 - log(e^x + 1)*log(-e^x + 1) - 1/2*log(-e^x + 1)^2 + (lo
g(e^(-x) + 1) + log(e^(-x) - 1) - log(e^(-2*x) + 1))*log(tanh(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{1}{2}\log\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)^2$$

```
[In] integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="giac")
[Out] 1/2*log((e^(2*x) - 1)/(e^(2*x) + 1))^2
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(x)\log(\tanh(x))\operatorname{sech}(x)\,dx = \frac{\left(\ln\left(e^{2x}-1\right)-\ln\left(e^{2x}+1\right)\right)^2}{2}$$

```
[In] int(log(tanh(x))/(cosh(x)*sinh(x)),x)
[Out] (log(exp(2*x) - 1) - log(exp(2*x) + 1))^2/2
```

3.1013 $\int \operatorname{csch}(2x)\log(\tanh(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log^2(\tanh(x))$$

```
[Out] 1/4*ln(tanh(x))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3855, 6818}

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log^2(\tanh(x))$$

```
[In] Int[Csch[2*x]*Log[Tanh[x]],x]
[Out] Log[Tanh[x]]^2/4
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\log^2(\tanh(x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log^2(\tanh(x))$$

```
[In] Integrate[Csch[2*x]*Log[Tanh[x]],x]
[Out] Log[Tanh[x]]^2/4
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\ln(\tanh(x))^2}{4}$
default	$\frac{\ln(\tanh(x))^2}{4}$
parallelrisch	$\frac{\ln(\tanh(x))^2}{4}$
risch	$\frac{\ln(1+e^{2x})^2}{4}-\frac{\ln(e^{2x}-1)\ln(1+e^{2x})}{2}+\frac{\ln(e^{2x}-1)^2}{4}+\frac{i\ln(1+e^{2x})\pi\operatorname{csgn}\left(\frac{i}{1+e^{2x}}\right)\operatorname{csgn}\left(i(e^{2x}-1)\right)\operatorname{csgn}\left(\frac{i(e^{2x}-1)}{1+e^{2x}}\right)}{4}$

```
[In] int(csch(2*x)*ln(tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(tanh(x))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log\left(\frac{\sinh(x)}{\cosh(x)}\right)^2$$

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="fricas")
[Out] 1/4*log(sinh(x)/cosh(x))^2
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \text{Timed out}$$

```
[In] integrate(csch(2*x)*ln(tanh(x)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log\left(\tanh\left(x\right)\right)^2$$

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="maxima")
[Out] 1/4*log(tanh(x))^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{1}{4}\log\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)^2$$

```
[In] integrate(csch(2*x)*log(tanh(x)),x, algorithm="giac")
[Out] 1/4*log((e^(2*x) - 1)/(e^(2*x) + 1))^2
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(2x)\log(\tanh(x))\,dx = \frac{\left(\ln\left(e^{2x}-1\right)-\ln\left(e^{2x}+1\right)\right)^2}{4}$$

```
[In] int(log(tanh(x))/sinh(2*x),x)
[Out] (log(exp(2*x) - 1) - log(exp(2*x) + 1))^2/4
```

3.1014 $\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \text{Int}(\cosh(a+bx)F(c,d,\sinh(a+bx),r,s),x)$$

```
[Out] CannotIntegrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

```
[In] Int[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sinh[a + b*x]]/b
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int F(c,d,x,r,s)\,dx, x, \sinh(a+bx))}{b}$$

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx=\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx$$

```
[In] Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s],x]
[Out] Integrate[Cosh[a + b*x]*F[c, d, Sinh[a + b*x], r, s], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cosh\left(bx+a\right)F\left(c,d,\sinh\left(bx+a\right),r,s\right)dx$$

```
[In] int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
[Out] int(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx=\int F(c,d,\sinh\left(bx+a\right),r,s)\cosh\left(bx+a\right)\,dx$$

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="fricas")
[Out] integral(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx=\int F(c,d,\sinh\left(a+bx\right),r,s)\cosh\left(a+bx\right)dx$$

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x)
[Out] Integral(F(c, d, sinh(a + b*x), r, s)*cosh(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int F(c,d,\sinh{(bx+a)}\,,r,s)\cosh{(bx+a)}\,\,dx$$

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int F(c,d,\sinh{(bx+a)}\,,r,s)\cosh{(bx+a)}\,\,dx$$

```
[In] integrate(cosh(b*x+a)*F(c,d,sinh(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, sinh(b*x + a), r, s)*cosh(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cosh(a+bx)F(c,d,\sinh(a+bx),r,s)\,dx = \int \cosh(a+b\,x)\;F(c,d,\sinh(a+b\,x)\,,r,s)\;dx$$

```
[In] int(cosh(a + b*x)*F(c, d, sinh(a + b*x), r, s),x)
[Out] int(cosh(a + b*x)*F(c, d, sinh(a + b*x), r, s), x)
```

3.1015 $\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\operatorname{Int}(F(c,d,\cosh(a+bx),r,s)\sinh(a+bx),x)$$

```
[Out] CannotIntegrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

```
[In] Int[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cosh[a + b*x]]/b
```

Rubi steps

$$\text{integral}=\frac{\operatorname{Subst}(\int F(c,d,x,r,s)\,dx,x,\cosh(a+bx))}{b}$$

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx$$

```
[In] Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x],x]
[Out] Integrate[F[c, d, Cosh[a + b*x], r, s]*Sinh[a + b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int F(c,d,\cosh\left(bx+a\right),r,s)\sinh\left(bx+a\right)dx$$

```
[In] int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
[Out] int(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\int F(c,d,\cosh\left(bx+a\right),r,s)\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="fricas")
[Out] integral(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx=\int F(c,d,\cosh\left(a+bx\right),r,s)\sinh\left(a+bx\right)dx$$

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x)
[Out] Integral(F(c, d, cosh(a + b*x), r, s)*sinh(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int F(c,d,\cosh\left(bx+a\right),r,s)\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="maxima")
[Out] integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int F(c,d,\cosh\left(bx+a\right),r,s)\sinh\left(bx+a\right)\,dx$$

```
[In] integrate(F(c,d,cosh(b*x+a),r,s)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(F(c, d, cosh(b*x + a), r, s)*sinh(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cosh(a+bx),r,s)\sinh(a+bx)\,dx = \int \sinh(a+b\,x)\ F\left(c,d,\cosh(a+b\,x)\,,r,s\right)\,dx$$

```
[In] int(sinh(a + b*x)*F(c, d, cosh(a + b*x), r, s),x)
[Out] int(sinh(a + b*x)*F(c, d, cosh(a + b*x), r, s), x)
```

3.1016 $\int F(c,d,\tanh(a+bx),r,s)\mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\text{Int}\big(F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx),x\big)$$

```
[Out] CannotIntegrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

```
[In] Int[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tanh[a + b*x]]/b
```

Rubi steps

$$\text{integral}=\frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\tanh(a+bx)\right)}{b}$$

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx$$

```
[In] Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2,x]
[Out] Integrate[F[c, d, Tanh[a + b*x], r, s]*Sech[a + b*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int F(c,d,\tanh(bx+a),r,s)\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
[Out] int(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(bx+a),r,s)\,\text{sech}\,(bx+a)^2\,dx$$

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx=\int F(c,d,\tanh(a+bx),r,s)\,\text{sech}^2(a+bx)\,dx$$

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)**2,x)
[Out] Integral(F(c, d, tanh(a + b*x), r, s)*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx = \int F(c,d,\tanh(bx+a),r,s)\,\text{sech}(bx+a)^2\,dx$$

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx = \int F(c,d,\tanh(bx+a),r,s)\,\text{sech}(bx+a)^2\,dx$$

```
[In] integrate(F(c,d,tanh(b*x+a),r,s)*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(F(c, d, tanh(b*x + a), r, s)*sech(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tanh(a+bx),r,s)\text{sech}^2(a+bx)\,dx = \int \frac{F(c,d,\tanh(a+b\,x)\,,r,s)}{\cosh\left(a+b\,x\right)^2}\,dx$$

```
[In] int(F(c, d, tanh(a + b*x), r, s)/cosh(a + b*x)^2,x)
[Out] int(F(c, d, tanh(a + b*x), r, s)/cosh(a + b*x)^2, x)
```

3.1017 $\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \operatorname{Int}\left(\operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

```
[In] Int[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Coth[a + b*x]]/b)
```

Rubi steps

$$\text{integral} = -\frac{\operatorname{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\coth(a+bx)\right)}{b}$$

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx=\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx$$

```
[In] Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s],x]
[Out] Integrate[Csch[a + b*x]^2*F[c, d, Coth[a + b*x], r, s], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}\left(bx+a\right)^{2} F\left(c,d,\coth\left(bx+a\right),r,s\right)dx$$

```
[In] int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
[Out] int(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx=\int F\left(c,d,\coth\left(bx+a\right),r,s\right)\operatorname{csch}\left(bx+a\right)^{2}\,dx$$

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="fricas")
[Out] integral(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx=\int F\left(c,d,\coth\left(a+bx\right),r,s\right)\operatorname{csch}^{2}\left(a+bx\right)dx$$

```
[In] integrate(csch(b*x+a)**2*F(c,d,coth(b*x+a),r,s),x)
[Out] Integral(F(c, d, coth(a + b*x), r, s)*csch(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int F(c,d,\coth(bx+a),r,s)\operatorname{csch}(bx+a)^2\,dx$$

```
[In] integrate(csch(b*x+a)^2*F(c,d,coth(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, coth(b*x + a), r, s)*csch(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^2(a+bx)F(c,d,\coth(a+bx),r,s)\,dx = \int \frac{F(c,d,\coth(a+b\,x),r,s)}{\sinh(a+b\,x)^2}\,dx$$

```
[In] int(F(c, d, coth(a + b*x), r, s)/sinh(a + b*x)^2,x)
[Out] int(F(c, d, coth(a + b*x), r, s)/sinh(a + b*x)^2, x)
```

3.1018 $\int \mathbf{sech}(x)\left(5-11\mathbf{sech}^2(x)\right)\tanh(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx=-5\operatorname{sech}(x)+\frac{11\operatorname{sech}^3(x)}{3}$$

```
[Out] -5*sech(x)+11/3*sech(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4424, 14}

$$\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx=\frac{11\operatorname{sech}^3(x)}{3}-5\operatorname{sech}(x)$$

```
[In] Int[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
```

```
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
```

```
n])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{-11+5x^2}{x^4}\,dx, x, \cosh(x)\right)$$

$$= \text{Subst}\left(\int \left(-\frac{11}{x^4}+\frac{5}{x^2}\right)\,dx, x, \cosh(x)\right)$$

$$= -5\text{sech}(x)+\frac{11\text{sech}^3(x)}{3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{sech}(x)\left(5-11\text{sech}^2(x)\right)\tanh(x)\,dx = -5\text{sech}(x)+\frac{11\text{sech}^3(x)}{3}$$

```
[In] Integrate[Sech[x]*(5 - 11*Sech[x]^2)*Tanh[x],x]
[Out] -5*Sech[x] + (11*Sech[x]^3)/3
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-5\,\text{sech}\left(x\right)+\frac{11\,\text{sech}\left(x\right)^{3}}{3}$	12
default	$-5\,\text{sech}\left(x\right)+\frac{11\,\text{sech}\left(x\right)^{3}}{3}$	12
parts	$-5\,\text{sech}\left(x\right)+\frac{11\,\text{sech}\left(x\right)^{3}}{3}$	12
risch	$-\frac{2\,{\mathrm e}^{x}\left(15\,{\mathrm e}^{4x}-14\,{\mathrm e}^{2x}+15\right)}{3\left(1+{\mathrm e}^{2x}\right)^{3}}$	27

```
[In] int(sech(x)*(5-11*sech(x)^2)*tanh(x),x,method=_RETURNVERBOSE)
[Out] -5*sech(x)+11/3*sech(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.69

$$\int \operatorname{sech}(x) \left(5-11\operatorname{sech}^2(x)\right) \tanh(x)\, dx = -\frac{2 \left(15 \cosh(x)^3+45 \cosh(x)\sinh(x)^2+15 \sinh(x)^3+\left(45 \cosh(x)^2-29\right)\sinh(x)+\right.}{3 \left(\cosh(x)^4+4 \cosh(x)\sinh(x)^3+\sinh(x)^4+2 \left(3 \cosh(x)^2+2\right)\sinh(x)^2+4 \cosh(x)^2+4 \left(\cosh(x)\right.\right.}$$

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="fricas")
[Out] -2/3*(15*cosh(x)^3 + 45*cosh(x)*sinh(x)^2 + 15*sinh(x)^3 + (45*cosh(x)^2 -
29)*sinh(x) + cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*
cosh(x)^2 + 2)*sinh(x)^2 + 4*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}(x) \left(5-11\operatorname{sech}^2(x)\right) \tanh(x)\, dx = \frac{11 \operatorname{sech}^3(x)}{3} - 5 \operatorname{sech}(x)$$

```
[In] integrate(sech(x)*(5-11*sech(x)**2)*tanh(x),x)
[Out] 11*sech(x)**3/3 - 5*sech(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \operatorname{sech}(x) \left(5-11\operatorname{sech}^2(x)\right) \tanh(x)\, dx = -\frac{10}{e^{(-x)}+e^x} + \frac{88}{3 \left(e^{(-x)}+e^x\right)^3}$$

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="maxima")
[Out] -10/(e^(-x) + e^x) + 88/3/(e^(-x) + e^x)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx = -\frac{2\left(15\left(e^{(-x)}+e^x\right)^2-44\right)}{3\left(e^{(-x)}+e^x\right)^3}$$

```
[In] integrate(sech(x)*(5-11*sech(x)^2)*tanh(x),x, algorithm="giac")
[Out] -2/3*(15*(e^(-x) + e^x)^2 - 44)/(e^(-x) + e^x)^3
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(x)\left(5-11\operatorname{sech}^2(x)\right)\tanh(x)\,dx = -\frac{2\,\mathrm{e}^x\left(15\,\mathrm{e}^{4\,x}-14\,\mathrm{e}^{2\,x}+15\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^3}$$

```
[In] int(-(tanh(x)*(11/cosh(x)^2 - 5))/cosh(x),x)
[Out] -(2*exp(x)*(15*exp(4*x) - 14*exp(2*x) + 15))/(3*(exp(2*x) + 1)^3)
```

3.1019 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(a+b\coth(x))}{b}$$

```
[Out] -ln(a+b*coth(x))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 31}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(a+b\coth(x))}{b}$$

```
[In] Int[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \frac{\log(\sinh(x)) - \log(b\cosh(x) + a\sinh(x))}{b}$$

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]
[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\coth(x))}{b}$	13
default	$-\frac{\ln(a+b\coth(x))}{b}$	13
risch	$-\frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b}+\frac{\ln(e^{2x}-1)}{b}$	36

```
[In] int(csch(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -ln(a+b*coth(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(b\coth(x)+a\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -log(b*coth(x) + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.83

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(sinh(x)^2*(a + b*coth(x))),x)
[Out] -(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

3.1020 $\int (a+b\coth(x))^n \mathbf{csch}^2(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx=-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$$

```
[Out] -(a+b*coth(x))^(1+n)/b/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 32}

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx=-\frac{(a+b\coth(x))^{n+1}}{b(n+1)}$$

```
[In] Int[(a + b*Coth[x])^n*Csch[x]^2,x]
```

```
[Out] -((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), 
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\coth(x)\right)}{b} \\
&= -\frac{(a+b\coth(x))^{1+n}}{b(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$
\int (a+b\coth(x))^n \text{csch}^2(x)\,dx = -\frac{(a+b\coth(x))^{1+n}}{b(1+n)}
$$

```
[In] Integrate[(a + b*Coth[x])^n*Csch[x]^2,x]
[Out] -((a + b*Coth[x])^(1 + n)/(b*(1 + n)))
```

Maple [A] (verified)

Time = 19.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$
default	$-\frac{(a+b\coth(x))^{1+n}}{b(1+n)}$
risch	$-\frac{(e^{2x}a+e^{2x}b-a+b)\left(a\left(e^{2x}-1\right)+b\left(1+e^{2x}\right)\right)^n\left(e^{2x}-1\right)^{-n} e^{-\frac{i\,\text{csgn}\left(\frac{i\left(a\left(e^{2x}-1\right)+b\left(1+e^{2x}\right)\right)}{e^{2x}-1}\right)\pi n\left(-\text{csgn}\left(\frac{i\left(a\left(e^{2x}-1\right)+b(\cdots}{e^{2x}-1 \cdots}\right.\right.}{\cdots}}}{b(1+n)(e^{2x}-1)}$

```
[In] int((a+b*coth(x))^n*csch(x)^2,x,method=_RETURNVERBOSE)
[Out] -(a+b*coth(x))^(1+n)/b/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx =$$
$$-\frac{(b\cosh(x)+a\sinh(x))\cosh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)+(b\cosh(x)+a\sinh(x))\sinh\left(n\log\left(\frac{b\cosh(x)+a\sinh(x)}{\sinh(x)}\right)\right)}{(bn+b)\sinh(x)}$$

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="fricas")
[Out] -((b*cosh(x) + a*sinh(x))*cosh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))) + (b
*cosh(x) + a*sinh(x))*sinh(n*log((b*cosh(x) + a*sinh(x))/sinh(x))))/((b*n +
 b)*sinh(x))
```

Sympy [F]

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx = \int (a+b\coth(x))^n\operatorname{csch}^2(x)\,dx$$

```
[In] integrate((a+b*coth(x))**n*csch(x)**2,x)
[Out] Integral((a + b*coth(x))**n*csch(x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx = -\frac{(b\coth(x)+a)^{n+1}}{b(n+1)}$$

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="maxima")
[Out] -(b*coth(x) + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx=-\frac{\left(\frac{ae^{(2\,x)}+be^{(2\,x)}-a+b}{e^{(2\,x)}-1}\right)^{n+1}}{b(n+1)}$$

```
[In] integrate((a+b*coth(x))^n*csch(x)^2,x, algorithm="giac")
[Out] -((a*e^(2*x) + b*e^(2*x) - a + b)/(e^(2*x) - 1))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int (a+b\coth(x))^n\text{csch}^2(x)\,dx=-\frac{\left(a+\frac{b\,\left(\mathrm{e}^{2\,x}+1\right)}{\mathrm{e}^{2\,x}-1}\right)^n\,\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{b\,\left(\mathrm{e}^{2\,x}-1\right)\,\left(n+1\right)}$$

```
[In] int((a + b*coth(x))^n/sinh(x)^2,x)
[Out] -((a + (b*(exp(2*x) + 1))/(exp(2*x) - 1))^n*(b - a + a*exp(2*x) + b*exp(2*x
)))/(b*(exp(2*x) - 1)*(n + 1))
```

3.1021 $\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx = x+\coth(x)$$

```
[Out] x+coth(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3091, 8}

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx = x+\coth(x)$$

```
[In] Int[Csch[x]^2*(-1 + Sinh[x]^2),x]
```

```
[Out] x + Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \coth(x) + \int 1\,dx$$
$$= x + \coth(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx = x + \coth(x)$$

```
[In] Integrate[Csch[x]^2*(-1 + Sinh[x]^2),x]
[Out] x + Coth[x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\coth\left(x\right)$	5
parallelrisch	$x+\coth\left(x\right)$	5
risch	$x+\frac{2}{\mathrm{e}^{2x}-1}$	13

```
[In] int(csch(x)^2*(-1+sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] x+coth(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx = \frac{(x-1)\sinh\left(x\right)+\cosh\left(x\right)}{\sinh\left(x\right)}$$

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="fricas")
[Out] ((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

Sympy [F]

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=\int (\sinh(x)-1)(\sinh(x)+1)\operatorname{csch}^2(x)\,dx$$

```
[In] integrate(csch(x)**2*(-1+sinh(x)**2),x)
[Out] Integral((sinh(x) - 1)*(sinh(x) + 1)*csch(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=x-\frac{2}{e^{(-2x)}-1}$$

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="maxima")
[Out] x - 2/(e^(-2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=x+\frac{2}{e^{(2x)}-1}$$

```
[In] integrate(csch(x)^2*(-1+sinh(x)^2),x, algorithm="giac")
[Out] x + 2/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^2(x)\left(-1+\sinh^2(x)\right)\,dx=x+\frac{2}{e^{2x}-1}$$

```
[In] int((sinh(x)^2 - 1)/sinh(x)^2,x)
[Out] x + 2/(exp(2*x) - 1)
```

3.1022 $\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 4

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx=x+\coth(x)$$

```
[Out] x+coth(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {464, 212}

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right) \operatorname{csch}^2(x) \, dx=x+\coth(x)$$

```
[In] Int[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
```

```
[Out] x + Coth[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1-2x^2}{x^2(1-x^2)}\,dx, x, \tanh(x)\right)\\ &= \coth(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= x + \coth(x)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 4.75

$$\begin{aligned}&\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx\\ &= 2x + \coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \tanh^2(x)\right)\end{aligned}$$

```
[In] Integrate[(-1 - (1 - Coth[x]^2)^(-1))*Csch[x]^2,x]
[Out] 2*x + Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

method	result	size
risch	$x+\frac{2}{{\mathrm e}^{2 x}-1}$	13
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	32

```
[In] int((-1-1/(1-coth(x)^2))*csch(x)^2,x,method=_RETURNVERBOSE)
[Out] x+2/(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx=\frac{(x-1)\sinh(x)+\cosh(x)}{\sinh(x)}$$

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="fricas")
[Out] ((x - 1)*sinh(x) + cosh(x))/sinh(x)
```

Sympy [F]

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx=-\int \left(-\frac{2\operatorname{csch}^2(x)}{\coth^2(x)-1}\right)dx-\int \frac{\coth^2(x)\operatorname{csch}^2(x)}{\coth^2(x)-1}\,dx$$

```
[In] integrate((-1-1/(1-coth(x)**2))*csch(x)**2,x)
[Out] -Integral(-2*csch(x)**2/(coth(x)**2 - 1), x) - Integral(coth(x)**2*csch(x)*
*2/(coth(x)**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx=x-\frac{2}{e^{(-2x)}-1}$$

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="maxima")
[Out] x - 2/(e^(-2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx=x+\frac{2}{e^{(2x)}-1}$$

```
[In] integrate((-1-1/(1-coth(x)^2))*csch(x)^2,x, algorithm="giac")
[Out] x + 2/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \left(-1-\frac{1}{1-\coth^2(x)}\right)\operatorname{csch}^2(x)\,dx=x+\frac{2}{e^{2x}-1}$$

```
[In] int((1/(coth(x)^2 - 1) - 1)/sinh(x)^2,x)
[Out] x + 2/(exp(2*x) - 1)
```

3.1023 $\int \frac{(a+b\coth(x))\mathbf{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx=-\frac{b\coth(x)}{d}+\frac{(bc-ad)\log(c+d\coth(x))}{d^2}$$

```
[Out] -b*coth(x)/d+(-a*d+b*c)*ln(c+d*coth(x))/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4429, 45}

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx=\frac{(bc-ad)\log(c+d\coth(x))}{d^2}-\frac{b\coth(x)}{d}$$

```
[In] Int[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] -((b*Coth[x])/d) + ((b*c - a*d)*Log[c + d*Coth[x]])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
```

```
b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx, x, \coth(x)\right)\\
&= -\frac{b\coth(x)}{d}+\frac{(bc-ad)\log(c+d\coth(x))}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\coth(x))\text{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{(a+b\coth(x))(-bd\coth(x)-(bc-ad)(\log(\sinh(x))-\log(d\cosh(x)+c\sinh(x))))\sinh(x)}{d^2(b\cosh(x)+a\sinh(x))}$$

```
[In] Integrate[((a + b*Coth[x])*Csch[x]^2)/(c + d*Coth[x]),x]
[Out] ((a + b*Coth[x])*(-(b*d*Coth[x]) - (b*c - a*d)*(Log[Sinh[x]] - Log[d*Cosh[x
] + c*Sinh[x]]))*Sinh[x])/(d^2*(b*Cosh[x] + a*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(28) = 56.

Time = 0.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

method	result	size
default	$\frac{(-2ad+2bc)\ln\left(\tanh\left(\frac{x}{2}\right)^2 d+2c\tanh\left(\frac{x}{2}\right)+d\right)}{2d^2}-\frac{\tanh\left(\frac{x}{2}\right)b}{2d}-\frac{b}{2d\tanh\left(\frac{x}{2}\right)}+\frac{(2ad-2bc)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2d^2}$	75
risch	$-\frac{2b}{d(e^{2x}-1)}-\frac{\ln\left(e^{2x}-\frac{c-d}{c+d}\right)a}{d}+\frac{\ln\left(e^{2x}-\frac{c-d}{c+d}\right)bc}{d^2}+\frac{\ln(e^{2x}-1)a}{d}-\frac{\ln(e^{2x}-1)bc}{d^2}$	90

```
[In] int((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/d^2*(-2*a*d+2*b*c)*ln(tanh(1/2*x)^2*d+2*c*tanh(1/2*x)+d)-1/2*tanh(1/2*x
)*b/d-1/2*b/d/tanh(1/2*x)+1/2/d^2*(2*a*d-2*b*c)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 6.21

$$\int \frac{(a + b\coth(x))\text{csch}^2(x)}{c + d\coth(x)}\,dx = -\frac{2\,bd - \left((bc - ad)\cosh(x)^2 + 2\,(bc - ad)\cosh(x)\sinh(x) + (bc - ad)\sinh(x)^2 - bc + ad\right)\log\left(\frac{2\,(d\cosh(x)}{\cosh(x}\right.}{d^2\cosh(x)^2 + 2\,d^2\,\mathrm{co}}$$

[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")

[Out] -(2*b*d - ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*sinh(x)^2 - b*c + a*d)*log(2*(d*cosh(x) + c*sinh(x))/(cosh(x) - sinh(x))) + ((b*c - a*d)*cosh(x)^2 + 2*(b*c - a*d)*cosh(x)*sinh(x) + (b*c - a*d)*sinh(x)^2 - b*c + a*d)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(d^2*cosh(x)^2 + 2*d^2*cosh(x)*sinh(x) + d^2*sinh(x)^2 - d^2)

Sympy [F]

$$\int \frac{(a + b\coth(x))\text{csch}^2(x)}{c + d\coth(x)}\,dx = \int \frac{(a + b\coth(x))\,\text{csch}^2(x)}{c + d\coth(x)}\,dx$$

[In] integrate((a+b*coth(x))*csch(x)**2/(c+d*coth(x)),x)

[Out] Integral((a + b*coth(x))*csch(x)**2/(c + d*coth(x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(28) = 56$.

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.75

$$\int \frac{(a + b\coth(x))\text{csch}^2(x)}{c + d\coth(x)}\,dx = b\left(\frac{c\log\left(-(c-d)e^{(-2\,x)} + c + d\right)}{d^2} - \frac{c\log\left(e^{(-x)} + 1\right)}{d^2} - \frac{c\log\left(e^{(-x)} - 1\right)}{d^2} + \frac{2}{de^{(-2\,x)} - d}\right) - \frac{a\log\left(d\coth(x) + c\right)}{d}$$

[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")

[Out] b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a*log(d*coth(x) + c)/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.04

$$\int \frac{(a+b\coth(x))\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{(bc^2-acd+bcd-ad^2)\log\left(\left|ce^{(2x)}+de^{(2x)}-c+d\right|\right)}{cd^2+d^3} - \frac{(bc-ad)\log\left(\left|e^{(2x)}-1\right|\right)}{d^2} + \frac{bce^{(2x)}-ade^{(2x)}-bc+ad-2bd}{d^2(e^{(2x)}-1)}$$

```
[In] integrate((a+b*coth(x))*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] (b*c^2 - a*c*d + b*c*d - a*d^2)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*
d^2 + d^3) - (b*c - a*d)*log(abs(e^(2*x) - 1))/d^2 + (b*c*e^(2*x) - a*d*e^(
2*x) - b*c + a*d - 2*b*d)/(d^2*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.61

$$\int \frac{(a+b\coth(x))\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{2\,\mathrm{atan}\left(\mathrm{e}^{2x}\left(\frac{4\left(a\,d\,\sqrt{-d^4}-b\,c\,\sqrt{-d^4}\right)}{d^2\,\sqrt{(a\,d-b\,c)^2}\,(c+d)\,(c-d)\,\sqrt{-d^4}}-\frac{4\,c^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{d^4\,(c+d)\,(c-d)\,(a\,d-b\,c)}\right)\left(\frac{d^2\,\sqrt{-d^4}}{4}+\frac{c\,d\,\sqrt{-d^4}}{4}\right)-\frac{4\,c\left(d^2\,\sqrt{a^2\,d^2-2\,a\,b\,c\,d+b}\right)}{}\right)}{\sqrt{-d^4}} - \frac{2\,b}{d\,\left(\mathrm{e}^{2x}-1\right)}$$

```
[In] int((a + b*coth(x))/(sinh(x)^2*(c + d*coth(x))),x)
```

```
[Out] (2*atan(exp(2*x)*((4*(a*d*(-d^4)^(1/2) - b*c*(-d^4)^(1/2)))/(d^2*((a*d - b*
c)^2)^(1/2)*(c + d)*(c - d)*(-d^4)^(1/2)) - (4*c^2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)^(1/2))/(d^4*(c + d)*(c - d)*(a*d - b*c)))*((d^2*(-d^4)^(1/2))/4 + (
c*d*(-d^4)^(1/2))/4) - (4*c*(d^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2) - c*
d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^(1/2))*((d^2*(-d^4)^(1/2))/4 + (c*d*(-d^4
)^(1/2))/4))/(d^5*(c + d)*(c - d)*(a*d - b*c)))*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)^(1/2))/(-d^4)^(1/2) - (2*b)/(d*(exp(2*x) - 1))
```

3.1024 $\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx=\frac{b(bc-ad)\coth(x)}{d^2}-\frac{(a+b\coth(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}$$

```
[Out] b*(-a*d+b*c)*coth(x)/d^2-1/2*(a+b*coth(x))^2/d-(-a*d+b*c)^2*ln(c+d*coth(x))
/d^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4429, 45}

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx=-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}+\frac{b\coth(x)(bc-ad)}{d^2}-\frac{(a+b\coth(x))^2}{2d}$$

```
[In] Int[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] (b*(b*c - a*d)*Coth[x])/d^2 - (a + b*Coth[x])^2/(2*d) - ((b*c - a*d)^2*Log[
c + d*Coth[x]])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx, x, \coth(x)\right) \\
&= -\text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)\,dx, x, \coth(x)\right) \\
&= \frac{b(bc-ad)\coth(x)}{d^2}-\frac{(a+b\coth(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\coth(x))}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{2bd(bc-2ad)\coth(x)-b^2d^2\text{csch}^2(x)+2(bc-ad)^2(\log(\sinh(x))-\log(d\cosh(x)+c\sinh(x)))}{2d^3}$$

```
[In] Integrate[((a + b*Coth[x])^2*Csch[x]^2)/(c + d*Coth[x]),x]
[Out] (2*b*d*(b*c - 2*a*d)*Coth[x] - b^2*d^2*Csch[x]^2 + 2*(b*c - a*d)^2*(Log[Sin
h[x]] - Log[d*Cosh[x] + c*Sinh[x]]))/(2*d^3)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{b\left(\frac{b\coth(x)^2 d}{2}+2\coth(x)ad-\coth(x)bc\right)}{d^2}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\coth(x))}{d^3}$
default	$-\frac{b\left(\frac{b\coth(x)^2 d}{2}+2\coth(x)ad-\coth(x)bc\right)}{d^2}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(c+d\coth(x))}{d^3}$
risch	$-\frac{2b(2ad\,e^{2x}-bc\,e^{2x}+bd\,e^{2x}-2ad+bc)}{(e^{2x}-1)^2 d^2}+\frac{\ln(e^{2x}-1)a^2}{d}-\frac{2\ln(e^{2x}-1)abc}{d^2}+\frac{\ln(e^{2x}-1)b^2c^2}{d^3}-\frac{\ln\left(e^{2x}-\frac{c-d}{c+d}\right)a^2}{d}$

```
[In] int((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/d^2*(1/2*b*coth(x)^2*d+2*coth(x)*a*d-coth(x)*b*c)-(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^3*ln(c+d*coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 694, normalized size of antiderivative = 13.09

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = -\frac{2\,b^2cd-4\,abd^2-2\,(b^2cd-(2\,ab+b^2)d^2)\cosh{(x)}^2-4\,(b^2cd-(2\,ab+b^2)d^2)\cosh{(x)}\sinh{(x)}-2\,(b^2ca}{}$$

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*b^2*c*d - 4*a*b*d^2 - 2*(b^2*c*d - (2*a*b + b^2)*d^2)*cosh(x)^2 - 4*(b^
2*c*d - (2*a*b + b^2)*d^2)*cosh(x)*sinh(x) - 2*(b^2*c*d - (2*a*b + b^2)*d^2
)*sinh(x)^2 + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sinh(
x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*co
sh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^3 - (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*(d*cosh(x) + c*sinh(
x))/(cosh(x) - sinh(x))) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)^4 + 4*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x)*sinh(x)^3 + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*sinh(x)^4 + b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*cosh(x)^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 3*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cosh(x)^2)*sinh(x)^2 + 4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
cosh(x)^3 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cosh(x))*sinh(x))*log(2*sinh(x)
/(cosh(x) - sinh(x))))/(d^3*cosh(x)^4 + 4*d^3*cosh(x)*sinh(x)^3 + d^3*sinh(
x)^4 - 2*d^3*cosh(x)^2 + d^3 + 2*(3*d^3*cosh(x)^2 - d^3)*sinh(x)^2 + 4*(d^3
*cosh(x)^3 - d^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx = \int \frac{(a+b\coth(x))^2\,\text{csch}^2(x)}{c+d\coth(x)}\,dx$$

```
[In] integrate((a+b*coth(x))**2*csch(x)**2/(c+d*coth(x)),x)
[Out] Integral((a + b*coth(x))**2*csch(x)**2/(c + d*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(51) = 102$.

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.34

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx = b^2\left(\frac{2\left((c+d)e^{(-2x)}-c\right)}{2d^2e^{(-2x)}-d^2e^{(-4x)}-d^2} - \frac{c^2\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^3} + \frac{c^2\log\left(e^{(-x)}+1\right)}{d^3} + \frac{c^2\log\left(e^{(-x)}-1\right)}{d^3}\right) + 2ab\left(\frac{c\log\left(-(c-d)e^{(-2x)}+c+d\right)}{d^2} - \frac{c\log\left(e^{(-x)}+1\right)}{d^2} - \frac{c\log\left(e^{(-x)}-1\right)}{d^2} + \frac{2}{de^{(-2x)}-d}\right) - \frac{a^2\log\left(d\coth(x)+c\right)}{d}$$

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
[Out] b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-4*x) - d^2) - c^2*l
og(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/d^3 + c^2*log(e^(-x
) - 1)/d^3) + 2*a*b*(c*log(-(c - d)*e^(-2*x) + c + d)/d^2 - c*log(e^(-x) +
1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) - a^2*log(d*coth(x) +
c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 5.00

$$\int \frac{(a+b\coth(x))^2\text{csch}^2(x)}{c+d\coth(x)}\,dx = -\frac{(b^2c^3-2abc^2d+b^2c^2d+a^2cd^2-2abcd^2+a^2d^3)\log\left(\left|ce^{(2x)}+de^{(2x)}-c+d\right|\right)}{cd^3+d^4} + \frac{(b^2c^2-2abcd+a^2d^2)\log\left(\left|e^{(2x)}-1\right|\right)}{d^3} - \frac{3b^2c^2e^{(4x)}-6abcde^{(4x)}+3a^2d^2e^{(4x)}-6b^2c^2e^{(2x)}+12abcde^{(2x)}-4b^2cde^{(2x)}-6a^2d^2e^{(2x)}+8abd^2e^{(2x)}}{2d^3\left(e^{(2x)}-1\right)^2}$$

```
[In] integrate((a+b*coth(x))^2*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + b^2*c^2*d + a^2*c*d^2 - 2*a*b*c*d^2 + a^2*d^3)*lo
g(abs(c*e^(2*x) + d*e^(2*x) - c + d))/(c*d^3 + d^4) + (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(e^(2*x) - 1))/d^3 - 1/2*(3*b^2*c^2*e^(4*x) - 6*a*b*c*d*e
^(4*x) + 3*a^2*d^2*e^(4*x) - 6*b^2*c^2*e^(2*x) + 12*a*b*c*d*e^(2*x) - 4*b^2
*c*d*e^(2*x) - 6*a^2*d^2*e^(2*x) + 8*a*b*d^2*e^(2*x) + 4*b^2*d^2*e^(2*x) +
3*b^2*c^2 - 6*a*b*c*d + 4*b^2*c*d + 3*a^2*d^2 - 8*a*b*d^2)/(d^3*(e^(2*x) -
1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{(a+b\coth(x))^2\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{\ln\left(e^{2x}-1\right)\,(a\,d-b\,c)^2}{d^3} - \frac{\ln\left(d-c+d\,e^{2x}+c\,e^{2x}\right)\,(a\,d-b\,c)^2}{d^3} - \frac{2\,(b^2\,d-b^2\,c+2\,a\,b\,d)}{d^2\,\left(e^{2x}-1\right)} - \frac{2\,b^2}{d\,\left(e^{4x}-2\,e^{2x}+1\right)}$$

```
[In] int((a + b*coth(x))^2/(sinh(x)^2*(c + d*coth(x))),x)
```

```
[Out] (log(exp(2*x) - 1)*(a*d - b*c)^2)/d^3 - (log(d - c + d*exp(2*x) + c*exp(2*x
))*(a*d - b*c)^2)/d^3 - (2*(b^2*d - b^2*c + 2*a*b*d))/(d^2*(exp(2*x) - 1))
- (2*b^2)/(d*(exp(4*x) - 2*exp(2*x) + 1))
```

3.1025 $\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = -\frac{b(bc-ad)^2\coth(x)}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2} -\frac{(a+b\coth(x))^3}{3d}+\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}$$

```
[Out] -b*(-a*d+b*c)^2*coth(x)/d^3+1/2*(-a*d+b*c)*(a+b*coth(x))^2/d^2-1/3*(a+b*cot
h(x))^3/d+(-a*d+b*c)^3*ln(c+d*coth(x))/d^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4429, 45}

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}-\frac{b\coth(x)(bc-ad)^2}{d^3} +\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}-\frac{(a+b\coth(x))^3}{3d}$$

```
[In] Int[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] -((b*(b*c - a*d)^2*Coth[x])/d^3) + ((b*c - a*d)*(a + b*Coth[x])^2)/(2*d^2)
- (a + b*Coth[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Coth[x]])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)\,dx, x, \coth(x)\right)\\
&= -\frac{b(bc-ad)^2\coth(x)}{d^3}+\frac{(bc-ad)(a+b\coth(x))^2}{2d^2}\\
&\quad-\frac{(a+b\coth(x))^3}{3d}+\frac{(bc-ad)^3\log(c+d\coth(x))}{d^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{(a+b\coth(x))^3(d\cosh(x)+c\sinh(x))\left(-2b^3d^3\coth(x)-6(bc-ad)^3(\log(\sinh(x))-\log(d\cosh(x)+c\sinh$$
$$6d^4(c+d\coth(x))(b\cosh(x)+$$

```
[In] Integrate[((a + b*Coth[x])^3*Csch[x]^2)/(c + d*Coth[x]),x]
```

```
[Out] ((a + b*Coth[x])^3*(d*Cosh[x] + c*Sinh[x])*(-2*b^3*d^3*Coth[x] - 6*(b*c - a
*d)^3*(Log[Sinh[x]] - Log[d*Cosh[x] + c*Sinh[x]])*Sinh[x]^2 - b*d*(-3*b*d*(
b*c - 3*a*d) + (-9*a*b*c*d + 9*a^2*d^2 + b^2*(3*c^2 + d^2))*Sinh[2*x])))/(6
*d^4*(c + d*Coth[x])*(b*Cosh[x] + a*Sinh[x])^3)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{b\left(\frac{b^{2}\coth(x)^{3}d^{2}}{3}+\frac{3ab\,d^{2}\coth(x)^{2}}{2}-\frac{b^{2}cd\coth(x)^{2}}{2}+3\coth(x)a^{2}d^{2}-3\coth(x)abcd+\coth(x)b^{2}c^{2}\right)}{d^{3}}-\frac{(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a$
default	$-\frac{b\left(\frac{b^{2}\coth(x)^{3}d^{2}}{3}+\frac{3ab\,d^{2}\coth(x)^{2}}{2}-\frac{b^{2}cd\coth(x)^{2}}{2}+3\coth(x)a^{2}d^{2}-3\coth(x)abcd+\coth(x)b^{2}c^{2}\right)}{d^{3}}-\frac{(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a$
risch	$-\frac{2b(9a^{2}d^{2}e^{4x}-9abcd\,e^{4x}+9ab\,d^{2}e^{4x}+3b^{2}c^{2}e^{4x}-3b^{2}cd\,e^{4x}+3b^{2}d^{2}e^{4x}-18a^{2}d^{2}e^{2x}+18abcd\,e^{2x}-9ab\,d^{2}e^{2x}-6b^{2}c^{2}e^{2x}+3b}{3d^{3}(e^{2x}-1)^{3}}$

```
[In] int((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/d^3*(1/3*b^2*coth(x)^3*d^2+3/2*a*b*d^2*coth(x)^2-1/2*b^2*c*d*coth(x)^2+3
*coth(x)*a^2*d^2-3*coth(x)*a*b*c*d+coth(x)*b^2*c^2)-(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(c+d*coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1980 vs. 2(74) = 148.

Time = 0.31 (sec) , antiderivative size = 1980, normalized size of antiderivative = 25.38

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*b^3*c^2*d - 18*a*b^2*c*d^2 + 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 +
 (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)^4 + 24*(b^3*c^2*d - (3*a*b^2 + b^3)
*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*cosh(x)*sinh(x)^3 + 6*(b^3*c^2*d -
(3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*sinh(x)^4 + 2*(9*a^2
*b + b^3)*d^3 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2
)*d^3)*cosh(x)^2 - 6*(2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*
b^2)*d^3 - 6*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)
*d^3)*cosh(x)^2)*sinh(x)^2 - 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
cosh(x)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sin
h(x)^6 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5
```

```
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 6*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 - 2*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*(d*cosh(x) + c*sinh(x))/
(cosh(x) - sinh(x))) + 3*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*cosh(x)^6 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x
)*sinh(x)^5 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sinh(x)^6
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - 3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 3*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3 - 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*cosh(x)^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x))*sinh(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*cosh(x)^2 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 + 5*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^4 - 6*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^2)*sinh(x)^2 + 6*((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^5 - 2*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cosh(x)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) +
 12*(2*(b^3*c^2*d - (3*a*b^2 + b^3)*c*d^2 + (3*a^2*b + 3*a*b^2 + b^3)*d^3)*
cosh(x)^3 - (2*b^3*c^2*d - (6*a*b^2 + b^3)*c*d^2 + 3*(2*a^2*b + a*b^2)*d^3)
*cosh(x))*sinh(x))/(d^4*cosh(x)^6 + 6*d^4*cosh(x)*sinh(x)^5 + d^4*sinh(x)^6
 - 3*d^4*cosh(x)^4 + 3*d^4*cosh(x)^2 + 3*(5*d^4*cosh(x)^2 - d^4)*sinh(x)^4
- d^4 + 4*(5*d^4*cosh(x)^3 - 3*d^4*cosh(x))*sinh(x)^3 + 3*(5*d^4*cosh(x)^4
- 6*d^4*cosh(x)^2 + d^4)*sinh(x)^2 + 6*(d^4*cosh(x)^5 - 2*d^4*cosh(x)^3 + d
^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{(a+b\coth(x))^3\operatorname{csch}^2(x)}{c+d\coth(x)}\,dx=\int \frac{(a+b\coth{(x)})^3\operatorname{csch}^2{(x)}}{c+d\coth{(x)}}\,dx$$

```
[In] integrate((a+b*coth(x))**3*csch(x)**2/(c+d*coth(x)),x)
[Out] Integral((a + b*coth(x))**3*csch(x)**2/(c + d*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(74) = 148$.

Time = 0.22 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.05

$$\int \frac{(a + b\coth(x))^3 \mathrm{csch}^2(x)}{c + d\coth(x)}\,dx$$
$$= \frac{1}{3}b^3\left(\frac{2\left(3c^2 + d^2 - 3(2c^2 + cd)e^{(-2x)} + 3(c^2 + cd + d^2)e^{(-4x)}\right)}{3d^3e^{(-2x)} - 3d^3e^{(-4x)} + d^3e^{(-6x)} - d^3} + \frac{3c^3\log\left(-(c-d)e^{(-2x)} + c + d\right)}{d^4} - \frac{3c^3}{}\right.$$
$$+3ab^2\left(\frac{2\left((c+d)e^{(-2x)} - c\right)}{2d^2e^{(-2x)} - d^2e^{(-4x)} - d^2} - \frac{c^2\log\left(-(c-d)e^{(-2x)} + c + d\right)}{d^3} + \frac{c^2\log\left(e^{(-x)} + 1\right)}{d^3} + \frac{c^2\log\left(e^{(-x)} -\right.}{d^3}\right.$$
$$+3a^2b\left(\frac{c\log\left(-(c-d)e^{(-2x)} + c + d\right)}{d^2} - \frac{c\log\left(e^{(-x)} + 1\right)}{d^2} - \frac{c\log\left(e^{(-x)} - 1\right)}{d^2} + \frac{2}{de^{(-2x)} - d}\right)$$
$$- \frac{a^3\log\left(d\coth(x) + c\right)}{d}$$

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="maxima")
```

```
[Out] 1/3*b^3*(2*(3*c^2 + d^2 - 3*(2*c^2 + c*d)*e^(-2*x) + 3*(c^2 + c*d + d^2)*e^
(-4*x))/(3*d^3*e^(-2*x) - 3*d^3*e^(-4*x) + d^3*e^(-6*x) - d^3) + 3*c^3*log(
-(c - d)*e^(-2*x) + c + d)/d^4 - 3*c^3*log(e^(-x) + 1)/d^4 - 3*c^3*log(e^(-
x) - 1)/d^4) + 3*a*b^2*(2*((c + d)*e^(-2*x) - c)/(2*d^2*e^(-2*x) - d^2*e^(-
4*x) - d^2) - c^2*log(-(c - d)*e^(-2*x) + c + d)/d^3 + c^2*log(e^(-x) + 1)/
d^3 + c^2*log(e^(-x) - 1)/d^3) + 3*a^2*b*(c*log(-(c - d)*e^(-2*x) + c + d)/
d^2 - c*log(e^(-x) + 1)/d^2 - c*log(e^(-x) - 1)/d^2 + 2/(d*e^(-2*x) - d)) -
 a^3*log(d*coth(x) + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.97

$$\int \frac{(a + b\coth(x))^3 \mathrm{csch}^2(x)}{c + d\coth(x)}\,dx$$
$$= \frac{(b^3c^4 - 3ab^2c^3d + b^3c^3d + 3a^2bc^2d^2 - 3ab^2c^2d^2 - a^3cd^3 + 3a^2bcd^3 - a^3d^4)\log\left(\left|ce^{(2x)} + de^{(2x)} - c + d\right|\right)}{cd^4 + d^5}$$
$$- \frac{(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\log\left(\left|e^{(2x)} - 1\right|\right)}{d^4}$$
$$+ \frac{11b^3c^3e^{(6x)} - 33ab^2c^2de^{(6x)} + 33a^2bcd^2e^{(6x)} - 11a^3d^3e^{(6x)} - 33b^3c^3e^{(4x)} + 99ab^2c^2de^{(4x)} - 12b^3c^2de^{(4}}{}$$

```
[In] integrate((a+b*coth(x))^3*csch(x)^2/(c+d*coth(x)),x, algorithm="giac")
```

```
[Out] (b^3*c^4 - 3*a*b^2*c^3*d + b^3*c^3*d + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^2*d^2 -
a^3*c*d^3 + 3*a^2*b*c*d^3 - a^3*d^4)*log(abs(c*e^(2*x) + d*e^(2*x) - c + d)
)/(c*d^4 + d^5) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(a
bs(e^(2*x) - 1))/d^4 + 1/6*(11*b^3*c^3*e^(6*x) - 33*a*b^2*c^2*d*e^(6*x) + 3
3*a^2*b*c*d^2*e^(6*x) - 11*a^3*d^3*e^(6*x) - 33*b^3*c^3*e^(4*x) + 99*a*b^2*
c^2*d*e^(4*x) - 12*b^3*c^2*d*e^(4*x) - 99*a^2*b*c*d^2*e^(4*x) + 36*a*b^2*c*
d^2*e^(4*x) + 12*b^3*c*d^2*e^(4*x) + 33*a^3*d^3*e^(4*x) - 36*a^2*b*d^3*e^(4
*x) - 36*a*b^2*d^3*e^(4*x) - 12*b^3*d^3*e^(4*x) + 33*b^3*c^3*e^(2*x) - 99*a
*b^2*c^2*d*e^(2*x) + 24*b^3*c^2*d*e^(2*x) + 99*a^2*b*c*d^2*e^(2*x) - 72*a*b
^2*c*d^2*e^(2*x) - 12*b^3*c*d^2*e^(2*x) - 33*a^3*d^3*e^(2*x) + 72*a^2*b*d^3
*e^(2*x) + 36*a*b^2*d^3*e^(2*x) - 11*b^3*c^3 + 33*a*b^2*c^2*d - 12*b^3*c^2*
d - 33*a^2*b*c*d^2 + 36*a*b^2*c*d^2 + 11*a^3*d^3 - 36*a^2*b*d^3 - 4*b^3*d^3
)/(d^4*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 1346, normalized size of antiderivative = 17.26

$$\int \frac{(a+b\coth(x))^3\text{csch}^2(x)}{c+d\coth(x)}\,dx = \frac{2\,\text{atan}\left(\frac{\left(e^{2x}\left(\frac{32\,c\,\left(2\,a^3\,c\,d^8-6\,a^2\,b\,c^2\,d^7+6\,a\,b^2\,c^3\,d^6-2\,b^3\,c^4\,d^5\right)\sqrt{a^6\,d^6-6\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4-20\,a^3\,b^3\,c^3\,d^3+15\,a^2\,b^4\,c^4\,d^2-6\,a\,b^5\,c^5\,d+b^6\,c^6}}{d^{16}\,\sqrt{(a\,d-b\,c)^6}\,(c+d)\,(c-d)^2\,\left(c^2+2\,c\,d+d^2\right)}-\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

$$-\frac{2\,(2\,b^3\,d-b^3\,c+3\,a\,b^2\,d)}{d^2\,\left(e^{4x}-2\,e^{2x}+1\right)}-\frac{8\,b^3}{3\,d\,\left(3\,e^{2x}-3\,e^{4x}+e^{6x}-1\right)}$$
$$-\frac{2\,(3\,a^2\,b\,d^2-3\,a\,b^2\,c\,d+3\,a\,b^2\,d^2+b^3\,c^2-b^3\,c\,d+b^3\,d^2)}{d^3\,\left(e^{2x}-1\right)}$$

```
[In] int((a + b*coth(x))^3/(sinh(x)^2*(c + d*coth(x))),x)
```

```
[Out] (2*atan(((exp(2*x)*((32*c*(2*a^3*c*d^8 - 2*b^3*c^4*d^5 + 6*a*b^2*c^3*d^6 -
6*a^2*b*c^2*d^7)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))/(d^16*((a*d
 - b*c)^6)^(1/2)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) - (16*(c^2*(-d^8)^(
1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) + d^2*(-d^8)^(1/2)*(a^6*
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^6 +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2
)*(2*c*d + c^2 + d^2))) + (32*c*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/
2)*(a^3*d^9 - a^3*c*d^8 - b^3*c^3*d^6 + b^3*c^4*d^5 + 3*a*b^2*c^2*d^7 - 3*a
```

*b^2*c^3*d^6 + 3*a^2*b*c^2*d^7 - 3*a^2*b*c*d^8))/(d^16*((a*d - b*c)^6)^(1/2
)*(c + d)*(c - d)^2*(2*c*d + c^2 + d^2)) + (16*(c^2*(-d^8)^(1/2)*(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2) - c*d*(-d^8)^(1/2)*(a^6*d^6 + b^6*c^6 +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5
*d - 6*a^5*b*c*d^5)^(1/2))*(c^2 + d^2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d
^5)^(1/2))/(d^13*(c + d)*(c - d)^2*(a*d - b*c)^3*(-d^8)^(1/2)*(2*c*d + c^2
+ d^2)))*(d^10*(-d^8)^(1/2) + 2*c*d^9*(-d^8)^(1/2) + c^2*d^8*(-d^8)^(1/2)))
/(16*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)^(1/2)))*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5)^(1/2))/(-d^8)^(1/2) - (2*(2*b^3*d - b^3*c + 3*a*b^2*d))/(
d^2*(exp(4*x) - 2*exp(2*x) + 1)) - (8*b^3)/(3*d*(3*exp(2*x) - 3*exp(4*x) +
exp(6*x) - 1)) - (2*(b^3*c^2 + b^3*d^2 + 3*a*b^2*d^2 + 3*a^2*b*d^2 - b^3*c*
d - 3*a*b^2*c*d))/(d^3*(exp(2*x) - 1))

3.1026 $\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx = -\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}$$

```
[Out] -1/8*a*(a+b*cosh(x)^2)^4/b^2+1/10*(a+b*cosh(x)^2)^5/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4420, 272, 45}

$$\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx = \frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}$$

```
[In] Int[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

```
[Out] -1/8*(a*(a + b*Cosh[x]^2)^4)/b^2 + (a + b*Cosh[x]^2)^5/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx, x, \cosh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx, x, \cosh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)\,dx, x, \cosh^2(x)\right)\\ &= -\frac{a\left(a+b\cosh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\cosh^2(x)\right)^5}{10b^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.78

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \frac{1}{32}\Big(12a^2b\cosh^4(x)+8ab^2\cosh^6(x)+2b^3\cosh^8(x)+4a^3\cosh(2x)+4a^2b\cosh^3(x)\cosh(3x)+a^3\cosh(4x)+\frac{1}{32}ab^2(48\cosh(2x)+36\cosh(4x)+16\cosh(6x)+3\cosh(8x))+\frac{1}{320}b^3(140\cosh(2x)+100\cosh(4x)+50\cosh(6x)+15\cosh(8x)+2\cosh(10x))\Big)$$

```
[In] Integrate[Cosh[x]^3*(a + b*Cosh[x]^2)^3*Sinh[x],x]
```

```
[Out] (12*a^2*b*Cosh[x]^4 + 8*a*b^2*Cosh[x]^6 + 2*b^3*Cosh[x]^8 + 4*a^3*Cosh[2*x]
 + 4*a^2*b*Cosh[x]^3*Cosh[3*x] + a^3*Cosh[4*x] + (a*b^2*(48*Cosh[2*x] + 36*
Cosh[4*x] + 16*Cosh[6*x] + 3*Cosh[8*x]))/32 + (b^3*(140*Cosh[2*x] + 100*Cos
h[4*x] + 50*Cosh[6*x] + 15*Cosh[8*x] + 2*Cosh[10*x]))/320)/32
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\frac{b^{3} \cosh \left(x\right)^{10}}{10}+\frac{3 a \,b^{2} \cosh \left(x\right)^{8}}{8}+\frac{a^{2} b \cosh \left(x\right)^{6}}{2}+\frac{a^{3} \cosh \left(x\right)^{4}}{4}$$

```
[In] int(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x)
```

```
[Out] 1/10*b^3*cosh(x)^10+3/8*a*b^2*cosh(x)^8+1/2*a^2*b*cosh(x)^6+1/4*a^3*cosh(x)
^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 10.72

$$\begin{aligned}&\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx\\&= \frac{1}{5120}\,b^{3} \cosh \left(x\right)^{10}+\frac{1}{5120}\,b^{3} \sinh \left(x\right)^{10}+\frac{1}{1024}\,\left(3\,ab^{2}+2\,b^{3}\right) \cosh \left(x\right)^{8}\\&+\frac{1}{1024}\,\left(9\,b^{3} \cosh \left(x\right)^{2}+3\,ab^{2}+2\,b^{3}\right) \sinh \left(x\right)^{8}+\frac{1}{1024}\,\left(16\,a^{2}b+24\,ab^{2}+9\,b^{3}\right) \cosh \left(x\right)^{6}\\&+\frac{1}{1024}\,\left(42\,b^{3} \cosh \left(x\right)^{4}+16\,a^{2}b+24\,ab^{2}+9\,b^{3}+28\,\left(3\,ab^{2}+2\,b^{3}\right) \cosh \left(x\right)^{2}\right) \sinh \left(x\right)^{6}\\&+\frac{1}{256}\,\left(8\,a^{3}+24\,a^{2}b+21\,ab^{2}+6\,b^{3}\right) \cosh \left(x\right)^{4}\\&+\frac{1}{1024}\,\left(42\,b^{3} \cosh \left(x\right)^{6}+70\,\left(3\,ab^{2}+2\,b^{3}\right) \cosh \left(x\right)^{4}+32\,a^{3}+96\,a^{2}b+84\,ab^{2}+24\,b^{3}+15\,\left(16\,a^{2}b+24\,ab\right.\right.\\&+\frac{1}{512}\,\left(64\,a^{3}+120\,a^{2}b+84\,ab^{2}+21\,b^{3}\right) \cosh \left(x\right)^{2}\\&+\frac{1}{1024}\,\left(9\,b^{3} \cosh \left(x\right)^{8}+28\,\left(3\,ab^{2}+2\,b^{3}\right) \cosh \left(x\right)^{6}+15\,\left(16\,a^{2}b+24\,ab^{2}+9\,b^{3}\right) \cosh \left(x\right)^{4}+128\,a^{3}+240\right.\end{aligned}$$

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 + 2*b^3)*co
sh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 + 2*b^3)*sinh(x)^8 + 1/1024*(16
*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2*b
+ 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*(8*a
^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6 + 70
*(3*a*b^2 + 2*b^3)*cosh(x)^4 + 32*a^3 + 96*a^2*b + 84*a*b^2 + 24*b^3 + 15*(
16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 + 1/512*(64*a^3 + 120*a^2
*b + 84*a*b^2 + 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3*a*b^2 +
 2*b^3)*cosh(x)^6 + 15*(16*a^2*b + 24*a*b^2 + 9*b^3)*cosh(x)^4 + 128*a^3 +
240*a^2*b + 168*a*b^2 + 42*b^3 + 24*(8*a^3 + 24*a^2*b + 21*a*b^2 + 6*b^3)*c
osh(x)^2)*sinh(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx = \frac{a^3\cosh^4(x)}{4} + \frac{a^2b\cosh^6(x)}{2} + \frac{3ab^2\cosh^8(x)}{8} + \frac{b^3\cosh^{10}(x)}{10}$$

```
[In] integrate(cosh(x)**3*(a+b*cosh(x)**2)**3*sinh(x),x)
```

```
[Out] a**3*cosh(x)**4/4 + a**2*b*cosh(x)**6/2 + 3*a*b**2*cosh(x)**8/8 + b**3*cosh
(x)**10/10
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh^3(x) \left(a+b\cosh^2(x)\right)^3 \sinh(x)\,dx = \frac{1}{10}\,b^3\cosh(x)^{10} + \frac{3}{8}\,ab^2\cosh(x)^8 + \frac{1}{2}\,a^2b\cosh(x)^6 + \frac{1}{4}\,a^3\cosh(x)^4$$

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*cosh(x)^10 + 3/8*a*b^2*cosh(x)^8 + 1/2*a^2*b*cosh(x)^6 + 1/4*a^3*c
osh(x)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 6.22

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \frac{1}{10240}b^3\left(e^{(2x)}+e^{(-2x)}\right)^5 + \frac{3}{2048}ab^2\left(e^{(2x)}+e^{(-2x)}\right)^4 + \frac{1}{1024}b^3\left(e^{(2x)}+e^{(-2x)}\right)^4 + \frac{1}{128}a^2b\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{3}{256}ab^2\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{1}{256}b^3\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{1}{64}a^3\left(e^{(2x)}+e^{(-2x)}\right)^2 + \frac{3}{64}a^2b\left(e^{(2x)}+e^{(-2x)}\right)^2 + \frac{9}{256}ab^2\left(e^{(2x)}+e^{(-2x)}\right)^2 + \frac{1}{128}b^3\left(e^{(2x)}+e^{(-2x)}\right)^2 + \frac{1}{16}a^3\left(e^{(2x)}+e^{(-2x)}\right) + \frac{3}{32}a^2b\left(e^{(2x)}+e^{(-2x)}\right) + \frac{3}{64}ab^2\left(e^{(2x)}+e^{(-2x)}\right) + \frac{1}{128}b^3\left(e^{(2x)}+e^{(-2x)}\right)$$

```
[In] integrate(cosh(x)^3*(a+b*cosh(x)^2)^3*sinh(x),x, algorithm="giac")
```

```
[Out] 1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 +
1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 + 3/
256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/64*
a^3*(e^(2*x) + e^(-2*x))^2 + 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*a*b^
2*(e^(2*x) + e^(-2*x))^2 + 1/128*b^3*(e^(2*x) + e^(-2*x))^2 + 1/16*a^3*(e^(
2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) + 3/64*a*b^2*(e^(2*x) +
e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh^3(x)\left(a+b\cosh^2(x)\right)^3\sinh(x)\,dx = \frac{a^3\cosh(x)^4}{4} + \frac{a^2\,b\cosh(x)^6}{2} + \frac{3\,a\,b^2\cosh(x)^8}{8} + \frac{b^3\cosh(x)^{10}}{10}$$

```
[In] int(cosh(x)^3*sinh(x)*(a + b*cosh(x)^2)^3,x)
```

```
[Out] (a^3*cosh(x)^4)/4 + (b^3*cosh(x)^10)/10 + (a^2*b*cosh(x)^6)/2 + (3*a*b^2*co
sh(x)^8)/8
```

3.1027 $\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=-\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}$$

```
[Out] -1/8*a*(a+b*sinh(x)^2)^4/b^2+1/10*(a+b*sinh(x)^2)^5/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3277, 272, 45}

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}$$

```
[In] Int[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

```
[Out] -1/8*(a*(a + b*Sinh[x]^2)^4)/b^2 + (a + b*Sinh[x]^2)^5/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3277

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx, x, \sinh(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)\,dx, x, \sinh^2(x)\right)\\ &= -\frac{a\left(a+b\sinh^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sinh^2(x)\right)^5}{10b^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(36) = 72.

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.17

$$\int\cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx = \frac{-20(64a^3+24ab^2-7b^3)\cosh(2x)+20(16a^3+18ab^2-5b^3)\cosh(4x)+b\big(-10(16a-5b)b\cosh(6x)+15}{10240}$$

```
[In] Integrate[Cosh[x]*Sinh[x]^3*(a + b*Sinh[x]^2)^3,x]
```

```
[Out] (-20*(64*a^3 + 24*a*b^2 - 7*b^3)*Cosh[2*x] + 20*(16*a^3 + 18*a*b^2 - 5*b^3)
*Cosh[4*x] + b*(-10*(16*a - 5*b)*b*Cosh[6*x] + 15*(2*a - b)*b*Cosh[8*x] + 2
*b^2*Cosh[10*x] + 320*((-4*a + b)^2 - b^2*Cosh[2*x])*Sinh[x]^6))/10240
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\frac{b^3 \sinh(x)^{10}}{10} + \frac{3a\, b^2 \sinh(x)^8}{8} + \frac{a^2 b \sinh(x)^6}{2} + \frac{a^3 \sinh(x)^4}{4}$$

```
[In] int(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x)
```

```
[Out] 1/10*b^3*sinh(x)^10+3/8*a*b^2*sinh(x)^8+1/2*a^2*b*sinh(x)^6+1/4*a^3*sinh(x)
^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(32) = 64.

Time = 0.25 (sec) , antiderivative size = 386, normalized size of antiderivative = 10.72

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx$$
$$= \frac{1}{5120}b^3\cosh(x)^{10} + \frac{1}{5120}b^3\sinh(x)^{10} + \frac{1}{1024}\left(3ab^2-2b^3\right)\cosh(x)^8$$
$$+ \frac{1}{1024}\left(9b^3\cosh(x)^2+3ab^2-2b^3\right)\sinh(x)^8 + \frac{1}{1024}\left(16a^2b-24ab^2+9b^3\right)\cosh(x)^6$$
$$+ \frac{1}{1024}\left(42b^3\cosh(x)^4+16a^2b-24ab^2+9b^3+28\left(3ab^2-2b^3\right)\cosh(x)^2\right)\sinh(x)^6$$
$$+ \frac{1}{256}\left(8a^3-24a^2b+21ab^2-6b^3\right)\cosh(x)^4$$
$$+ \frac{1}{1024}\left(42b^3\cosh(x)^6+70\left(3ab^2-2b^3\right)\cosh(x)^4+32a^3-96a^2b+84ab^2-24b^3+15\left(16a^2b-24ab^2\right.\right.$$
$$- \frac{1}{512}\left(64a^3-120a^2b+84ab^2-21b^3\right)\cosh(x)^2$$
$$+ \frac{1}{1024}\left(9b^3\cosh(x)^8+28\left(3ab^2-2b^3\right)\cosh(x)^6+15\left(16a^2b-24ab^2+9b^3\right)\cosh(x)^4-128a^3+240a\right.$$

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/5120*b^3*cosh(x)^10 + 1/5120*b^3*sinh(x)^10 + 1/1024*(3*a*b^2 - 2*b^3)*co
sh(x)^8 + 1/1024*(9*b^3*cosh(x)^2 + 3*a*b^2 - 2*b^3)*sinh(x)^8 + 1/1024*(16
*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^6 + 1/1024*(42*b^3*cosh(x)^4 + 16*a^2*b
- 24*a*b^2 + 9*b^3 + 28*(3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^6 + 1/256*(8*a
^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*cosh(x)^4 + 1/1024*(42*b^3*cosh(x)^6 + 70
*(3*a*b^2 - 2*b^3)*cosh(x)^4 + 32*a^3 - 96*a^2*b + 84*a*b^2 - 24*b^3 + 15*(
16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^2)*sinh(x)^4 - 1/512*(64*a^3 - 120*a^2
*b + 84*a*b^2 - 21*b^3)*cosh(x)^2 + 1/1024*(9*b^3*cosh(x)^8 + 28*(3*a*b^2 -
 2*b^3)*cosh(x)^6 + 15*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(x)^4 - 128*a^3 +
240*a^2*b - 168*a*b^2 + 42*b^3 + 24*(8*a^3 - 24*a^2*b + 21*a*b^2 - 6*b^3)*c
osh(x)^2)*sinh(x)^2
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=\frac{a^3\sinh^4(x)}{4}+\frac{a^2b\sinh^6(x)}{2}+\frac{3ab^2\sinh^8(x)}{8}+\frac{b^3\sinh^{10}(x)}{10}$$

```
[In] integrate(cosh(x)*sinh(x)**3*(a+b*sinh(x)**2)**3,x)
```

```
[Out] a**3*sinh(x)**4/4 + a**2*b*sinh(x)**6/2 + 3*a*b**2*sinh(x)**8/8 + b**3*sinh
(x)**10/10
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx=\frac{1}{10}\,b^3\sinh(x)^{10}+\frac{3}{8}\,ab^2\sinh(x)^8+\frac{1}{2}\,a^2b\sinh(x)^6+\frac{1}{4}\,a^3\sinh(x)^4$$

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sinh(x)^10 + 3/8*a*b^2*sinh(x)^8 + 1/2*a^2*b*sinh(x)^6 + 1/4*a^3*s
inh(x)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 6.22

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx = \frac{1}{10240}\,b^3\left(e^{(2x)}+e^{(-2x)}\right)^5 + \frac{3}{2048}\,ab^2\left(e^{(2x)}+e^{(-2x)}\right)^4 - \frac{1}{1024}\,b^3\left(e^{(2x)}+e^{(-2x)}\right)^4 + \frac{1}{128}\,a^2b\left(e^{(2x)}+e^{(-2x)}\right)^3 - \frac{3}{256}\,ab^2\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{1}{256}\,b^3\left(e^{(2x)}+e^{(-2x)}\right)^3 + \frac{1}{64}\,a^3\left(e^{(2x)}+e^{(-2x)}\right)^2 - \frac{3}{64}\,a^2b\left(e^{(2x)}+e^{(-2x)}\right)^2 + \frac{9}{256}\,ab^2\left(e^{(2x)}+e^{(-2x)}\right)^2 - \frac{1}{128}\,b^3\left(e^{(2x)}+e^{(-2x)}\right)^2 - \frac{1}{16}\,a^3\left(e^{(2x)}+e^{(-2x)}\right) + \frac{3}{32}\,a^2b\left(e^{(2x)}+e^{(-2x)}\right) - \frac{3}{64}\,ab^2\left(e^{(2x)}+e^{(-2x)}\right) + \frac{1}{128}\,b^3\left(e^{(2x)}+e^{(-2x)}\right)$$

```
[In] integrate(cosh(x)*sinh(x)^3*(a+b*sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*(e^(2*x) + e^(-2*x))^5 + 3/2048*a*b^2*(e^(2*x) + e^(-2*x))^4 -
1/1024*b^3*(e^(2*x) + e^(-2*x))^4 + 1/128*a^2*b*(e^(2*x) + e^(-2*x))^3 - 3/
256*a*b^2*(e^(2*x) + e^(-2*x))^3 + 1/256*b^3*(e^(2*x) + e^(-2*x))^3 + 1/64*
a^3*(e^(2*x) + e^(-2*x))^2 - 3/64*a^2*b*(e^(2*x) + e^(-2*x))^2 + 9/256*a*b^
2*(e^(2*x) + e^(-2*x))^2 - 1/128*b^3*(e^(2*x) + e^(-2*x))^2 - 1/16*a^3*(e^(
2*x) + e^(-2*x)) + 3/32*a^2*b*(e^(2*x) + e^(-2*x)) - 3/64*a*b^2*(e^(2*x) +
e^(-2*x)) + 1/128*b^3*(e^(2*x) + e^(-2*x))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cosh(x)\sinh^3(x)\left(a+b\sinh^2(x)\right)^3\,dx = \frac{a^3\sinh(x)^4}{4} + \frac{a^2\,b\sinh(x)^6}{2} + \frac{3\,a\,b^2\sinh(x)^8}{8} + \frac{b^3\sinh(x)^{10}}{10}$$

```
[In] int(cosh(x)*sinh(x)^3*(a + b*sinh(x)^2)^3,x)
```

```
[Out] (a^3*sinh(x)^4)/4 + (b^3*sinh(x)^10)/10 + (a^2*b*sinh(x)^6)/2 + (3*a*b^2*si
nh(x)^8)/8
```

3.1028 $\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx=\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

```
[Out] 1/3*(a+b*sinh(x)^2)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3277, 267}

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx=\frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

```
[In] Int[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 3277

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
```

```
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int x\sqrt{a+bx^2}\,dx, x, \sinh(x)\right)$$

$$= \frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(a+b\sinh^2(x)\right)^{3/2}}{3b}$$

```
[In] Integrate[Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2],x]
[Out] (a + b*Sinh[x]^2)^(3/2)/(3*b)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\left(a+b\sinh(x)^2\right)^{\frac{3}{2}}}{3b}$	16
default	$\frac{\left(a+b\sinh(x)^2\right)^{\frac{3}{2}}}{3b}$	16

```
[In] int(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(a+b*sinh(x)^2)^(3/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 8.11

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx$$
$$= \frac{\sqrt{2}\big(b\cosh{(x)}^4 + 4\,b\cosh{(x)}\sinh{(x)}^3 + b\sinh{(x)}^4 + 2\,(2\,a-b)\cosh{(x)}^2 + 2\,\big(3\,b\cosh{(x)}^2 + 2\,a - b\big)\sin}{24\,\big(b\cosh{(x)}^3 + 3\,b\cosh{(x)}^2\sinh{(x)} + 3\,b\cos}$$

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a -
b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a - b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*
a - b)*cosh(x))*sinh(x) + b)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a - b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(b*cosh(x)^3 + 3*b*cosh(x)^2*sinh
(x) + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \begin{cases} \frac{a\sqrt{a+b\sinh^2{(x)}}}{3b} + \frac{\sqrt{a+b\sinh^2{(x)}}\sinh^2{(x)}}{3} & \text{for}\ b \neq 0 \\ \frac{\sqrt{a}\cosh^2{(x)}}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Piecewise((a*sqrt(a + b*sinh(x)**2)/(3*b) + sqrt(a + b*sinh(x)**2)*sinh(x)*
*2/3, Ne(b, 0)), (sqrt(a)*cosh(x)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(b\sinh{(x)}^2 + a\right)^{\frac{3}{2}}}{3\,b}$$

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*sinh(x)^2 + a)^(3/2)/b
```

Giac [F(-2)]

Exception generated.

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}\,dx = \frac{\left(b\sinh(x)^2+a\right)^{3/2}}{3\,b}$$

```
[In] int(cosh(x)*sinh(x)*(a + b*sinh(x)^2)^(1/2),x)
[Out] (a + b*sinh(x)^2)^(3/2)/(3*b)
```

3.1029 $\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{1}{2}\operatorname{arcsinh}(\log(\coth(x))) - \frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))}$$

```
[Out] -1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6828, 201, 221}

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{1}{2}\operatorname{arcsinh}(\log(\coth(x))) - \frac{1}{2}\log(\coth(x))\sqrt{\log^2(\coth(x))+1}$$

```
[In] Int[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
```

```
[Out] -1/2*ArcSinh[Log[Coth[x]]] - (Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6828

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \log(\coth(x))\right)\\
&= -\frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \log(\coth(x))\right)\\
&= -\frac{1}{2}\text{arcsinh}(\log(\coth(x))) - \frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$
\int \text{csch}(x)\sqrt{1+\log^2(\coth(x))}\text{sech}(x)\,dx = -\frac{1}{2}\log(\coth(x))\sqrt{1+\log^2(\coth(x))} + \frac{1}{2}\log\left(-\log(\coth(x)) + \sqrt{1+\log^2(\coth(x))}\right)
$$

```
[In] Integrate[Csch[x]*Sqrt[1 + Log[Coth[x]]^2]*Sech[x],x]
[Out] -1/2*(Log[Coth[x]]*Sqrt[1 + Log[Coth[x]]^2]) + Log[-Log[Coth[x]] + Sqrt[1 +
 Log[Coth[x]]^2]]/2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}(\ln(\coth(x)))}{2}-\frac{\ln(\coth(x))\sqrt{1+\ln(\coth(x))^{2}}}{2}$	22
default	$-\frac{\operatorname{arcsinh}(\ln(\coth(x)))}{2}-\frac{\ln(\coth(x))\sqrt{1+\ln(\coth(x))^{2}}}{2}$	22

```
[In] int(csch(x)*sech(x)*(1+ln(coth(x))^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arcsinh(ln(coth(x)))-1/2*ln(coth(x))*(1+ln(coth(x))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{1}{2}\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1}\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + \frac{1}{2}\log\left(\sqrt{\log\left(\frac{\cosh(x)}{\sinh(x)}\right)^2+1} - \log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right)$$

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(log(cosh(x)/sinh(x))^2 + 1)*log(cosh(x)/sinh(x)) + 1/2*log(sqrt(l
og(cosh(x)/sinh(x))^2 + 1) - log(cosh(x)/sinh(x)))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = \text{Timed out}$$

```
[In] integrate(csch(x)*sech(x)*(1+ln(coth(x))**2)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = \int \sqrt{\log\left(\coth\left(x\right)\right)^2+1}\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)\,dx$$

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

Giac [F]

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = \int \sqrt{\log\left(\coth\left(x\right)\right)^2+1}\operatorname{csch}\left(x\right)\operatorname{sech}\left(x\right)\,dx$$

```
[In] integrate(csch(x)*sech(x)*(1+log(coth(x))^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(log(coth(x))^2 + 1)*csch(x)*sech(x), x)
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}(x)\sqrt{1+\log^2(\coth(x))}\operatorname{sech}(x)\,dx = -\frac{\operatorname{asinh}(\ln\left(\coth(x)\right))}{2} - \frac{\ln\left(\coth(x)\right)\,\sqrt{\ln\left(\coth\left(x\right)\right)^2+1}}{2}$$

```
[In] int((log(coth(x))^2 + 1)^(1/2)/(cosh(x)*sinh(x)),x)
[Out] - asinh(log(coth(x)))/2 - (log(coth(x))*(log(coth(x))^2 + 1)^(1/2))/2
```

3.1030 $\int \frac{\coth(\sqrt{x})\mathbf{csch}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\operatorname{csch}\left(\sqrt{x}\right)$$

```
[Out] -2*csch(x^(1/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 2686, 8}

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\operatorname{csch}\left(\sqrt{x}\right)$$

```
[In] Int[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csch[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \coth(x)\text{csch}(x)\,dx, x, \sqrt{x}\right)\\ &= -\left(2i\text{Subst}\left(\int 1\,dx, x, -i\text{csch}\left(\sqrt{x}\right)\right)\right)\\ &= -2\text{csch}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(\sqrt{x}\right)\text{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\text{csch}\left(\sqrt{x}\right)$$

```
[In] Integrate[(Coth[Sqrt[x]]*Csch[Sqrt[x]])/Sqrt[x],x]
[Out] -2*Csch[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\,\text{csch}\left(\sqrt{x}\right)$	7
default	$-2\,\text{csch}\left(\sqrt{x}\right)$	7

```
[In] int(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*csch(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(6) = 12.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.62

$$\int \frac{\coth(\sqrt{x})\operatorname{csch}(\sqrt{x})}{\sqrt{x}}\,dx = -\frac{4\left(\cosh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)\right)}{\cosh\left(\sqrt{x}\right)^2+2\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)+\sinh\left(\sqrt{x}\right)^2-1}$$

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh(
sqrt(x)) + sinh(sqrt(x))^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth(\sqrt{x})\operatorname{csch}(\sqrt{x})}{\sqrt{x}}\,dx = -2\operatorname{csch}\left(\sqrt{x}\right)$$

```
[In] integrate(coth(x**(1/2))*csch(x**(1/2))/x**(1/2),x)
[Out] -2*csch(sqrt(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\coth(\sqrt{x})\operatorname{csch}(\sqrt{x})}{\sqrt{x}}\,dx = \frac{4}{e^{(-\sqrt{x})}-e^{\sqrt{x}}}$$

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 4/(e^(-sqrt(x)) - e^sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{4}{e^{\left(-\sqrt{x}\right)} - e^{\sqrt{x}}}$$

```
[In] integrate(coth(x^(1/2))*csch(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 4/(e^(-sqrt(x)) - e^sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(\sqrt{x}\right)\operatorname{csch}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\sinh\left(\sqrt{x}\right)}$$

```
[In] int(coth(x^(1/2))/(x^(1/2)*sinh(x^(1/2))),x)
[Out] -2/sinh(x^(1/2))
```

3.1031 $\int \frac{\cosh(\sqrt{x})\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^2\left(\sqrt{x}\right)$$

```
[Out] sinh(x^(1/2))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {5478}

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^2\left(\sqrt{x}\right)$$

```
[In] Int[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sinh[Sqrt[x]]^2
```

Rule 5478

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]
^(p_.), x_Symbol] :> Simp[Sinh[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; Free
Q[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \sinh^2\left(\sqrt{x}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{1}{2}\cosh\left(2\sqrt{x}\right)$$

```
[In] Integrate[(Cosh[Sqrt[x]]*Sinh[Sqrt[x]])/Sqrt[x],x]
[Out] Cosh[2*Sqrt[x]]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\cosh\left(\sqrt{x}\right)^2$	7
default	$\cosh\left(\sqrt{x}\right)^2$	7
meijerg	$-\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(2\sqrt{x})}{\sqrt{\pi}}\right)}{2}$	21

```
[In] int(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] cosh(x^(1/2))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{1}{2}\cosh\left(\sqrt{x}\right)^2 + \frac{1}{2}\sinh\left(\sqrt{x}\right)^2$$

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 1/2*cosh(sqrt(x))^2 + 1/2*sinh(sqrt(x))^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sinh^{2}\left(\sqrt{x}\right)$$

```
[In] integrate(cosh(x**(1/2))*sinh(x**(1/2))/x**(1/2),x)
[Out] sinh(sqrt(x))**2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \cosh\left(\sqrt{x}\right)^{2}$$

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] cosh(sqrt(x))^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{1}{4}\,e^{\left(2\,\sqrt{x}\right)} + \frac{1}{4}\,e^{\left(-2\,\sqrt{x}\right)}$$

```
[In] integrate(cosh(x^(1/2))*sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 1/4*e^(2*sqrt(x)) + 1/4*e^(-2*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\sqrt{x}\right)\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = {\cosh\left(\sqrt{x}\right)}^2$$

```
[In] int((cosh(x^(1/2))*sinh(x^(1/2)))/x^(1/2),x)
[Out] cosh(x^(1/2))^2
```

3.1032 $\int \frac{\text{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\text{sech}(\sqrt{x})\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\text{sech}(\sqrt{x})$$

```
[Out] -2*sech(x^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 2686, 8}

$$\int \frac{\text{sech}(\sqrt{x})\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\text{sech}(\sqrt{x})$$

```
[In] Int[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Sech[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \text{sech}(x)\tanh(x)\,dx, x, \sqrt{x}\right)\\ &= -\left(2\text{Subst}\left(\int 1\,dx, x, \text{sech}\left(\sqrt{x}\right)\right)\right)\\ &= -2\text{sech}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\text{sech}\left(\sqrt{x}\right)$$

```
[In] Integrate[(Sech[Sqrt[x]]*Tanh[Sqrt[x]])/Sqrt[x],x]
[Out] -2*Sech[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\ \text{sech}\left(\sqrt{x}\right)$	7
default	$-2\ \text{sech}\left(\sqrt{x}\right)$	7

```
[In] int(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*sech(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.62

$$\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -\frac{4\left(\cosh(\sqrt{x}) + \sinh(\sqrt{x})\right)}{\cosh(\sqrt{x})^2 + 2\cosh(\sqrt{x})\sinh(\sqrt{x}) + \sinh(\sqrt{x})^2 + 1}$$

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -4*(cosh(sqrt(x)) + sinh(sqrt(x)))/(cosh(sqrt(x))^2 + 2*cosh(sqrt(x))*sinh(
sqrt(x)) + sinh(sqrt(x))^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -2\operatorname{sech}(\sqrt{x})$$

```
[In] integrate(sech(x**(1/2))*tanh(x**(1/2))/x**(1/2),x)
[Out] -2*sech(sqrt(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{sech}(\sqrt{x})\tanh(\sqrt{x})}{\sqrt{x}}\,dx = -\frac{4}{e^{(-\sqrt{x})} + e^{\sqrt{x}}}$$

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -4/(e^(-sqrt(x)) + e^sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{4}{e^{\left(-\sqrt{x}\right)} + e^{\sqrt{x}}}$$

```
[In] integrate(sech(x^(1/2))*tanh(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] -4/(e^(-sqrt(x)) + e^sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(\sqrt{x}\right)\tanh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\cosh\left(\sqrt{x}\right)}$$

```
[In] int(tanh(x^(1/2))/(x^(1/2)*cosh(x^(1/2))),x)
[Out] -2/cosh(x^(1/2))
```

3.1033 $\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log(a+b\sinh(2x))}{4b}$$

```
[Out] 1/4*ln(a+b*sinh(2*x))/b+1/2*arctanh((b-a*tanh(x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1089, 12, 648, 632, 212, 642, 266}

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}+\frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b}+\frac{\log(\cosh(x))}{2b}$$

```
[In] Int[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/(2*Sqrt[a^2 + b^2]) + Log[Cosh[x]]
/(2*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1089

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*((-b)*
C*d + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*((-b)*C*d + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{x^2}{(-1+x^2)\,(a+2bx-ax^2)}\,dx, x, \tanh(x)\right)\\ &= \frac{\text{Subst}\left(\int -\frac{2bx}{-1+x^2}\,dx, x, \tanh(x)\right)}{4b^2} + \frac{\text{Subst}\left(\int -\frac{2abx}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b^2}\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, \tanh(x)\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{x}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{2b}$$

$$= \frac{\log(\cosh(x))}{2b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh(x)\right) + \frac{\text{Subst}\left(\int \frac{2b-2ax}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b}$$

$$= \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b} + \text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh(x)\right)$$

$$= \frac{\text{arctanh}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = \frac{1}{4}\left(-\frac{2\arctan\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{\log(a+b\sinh(2x))}{b}\right)$$

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] ((-2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b
*Sinh[2*x]]/b)/4
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.65

method	result
default	$\frac{a\left(\frac{\ln\left(a\tanh(x)^2-2b\tanh(x)-a\right)}{2a}-\frac{b\,\text{arctanh}\left(\frac{2a\tanh(x)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{2b}-\frac{\ln(1+\tanh(x))}{4b}-\frac{\ln(\tanh(x)-1)}{4b}$
risch	$\frac{x}{2b}-\frac{x\,a^2b}{a^2b^2+b^4}-\frac{x\,b^3}{a^2b^2+b^4}+\frac{\ln\left(e^{2x}+\frac{ab+\sqrt{a^2b^2+b^4}}{b^2}\right)a^2}{4(a^2+b^2)b}+\frac{b\ln\left(e^{2x}+\frac{ab+\sqrt{a^2b^2+b^4}}{b^2}\right)}{4a^2+4b^2}+\frac{\ln\left(e^{2x}+\frac{ab+\sqrt{a^2b^2+b^4}}{b^2}\right)\sqrt{a^2b^2+b^4}}{4(a^2+b^2)b}+$

```
[In] int(sinh(x)^2/(a+b*sinh(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*a*(1/2/a*ln(a*tanh(x)^2-2*b*tanh(x)-a)-1/a*b/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2)))-1/4/b*ln(1+tanh(x))-1/4/b*ln(tanh(x
)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.83

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2ab\cosh(x)^2+2\left(3b^2\cosh(x)^2+ab\right)\sinh(x)^2+2a^2+b^2+4\left(b^2\cosh(x)^3+a\right.}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2a\cosh(x)^2+2\left(3b\cosh(x)^2+a\right)\sinh(x}\right.}{}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2 +
b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4
*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b^2)*lo
g(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^2*b + b^3)
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = \int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx$$

```
[In] integrate(sinh(x)**2/(a+b*sinh(2*x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sinh(2*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = -\frac{\log\left(\frac{be^{(-2x)}-a-\sqrt{a^2+b^2}}{be^{(-2x)}-a+\sqrt{a^2+b^2}}\right)}{4\sqrt{a^2+b^2}} - \frac{x}{2b} + \frac{\log\left(be^{(4x)}+2ae^{(2x)}-b\right)}{4b}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

```
[Out] -1/4*log((b*e^(-2*x) - a - sqrt(a^2 + b^2))/(b*e^(-2*x) - a + sqrt(a^2 + b^
2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(b*e^(4*x) + 2*a*e^(2*x) - b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = -\frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}} - \frac{x}{2\,b} + \frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2*
a*e^(2*x) - b))/b
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.25

$$\int \frac{\sinh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\operatorname{atan}\left(\frac{a^7}{(-a^2-b^2)^{7/2}} + \frac{b^7\,e^{2\,x}}{(-a^2-b^2)^{7/2}} + \frac{a\,b^6}{(-a^2-b^2)^{7/2}} + \frac{3\,a^3\,b^4}{(-a^2-b^2)^{7/2}} + \frac{3\,a^5\,b^2}{(-a^2-b^2)^{7/2}} + \frac{3\,a^2\,b^5\,e^{2\,x}}{(-a^2-b^2)^{7/2}} + \frac{3\,a^4\,b^3\,e^{2\,x}}{(-a^2-b^2)^{7/2}} + \frac{a^6\,b\,e^{2\,x}}{(-a^2-b^2)^{7/}}\right.}{2\,\sqrt{-a^2-b^2}} - \frac{x}{2\,b} + \frac{4\,b^3\,\ln\left(2\,a\,e^{2\,x}-b+b\,e^{4\,x}\right)}{16\,a^2\,b^2+16\,b^4} + \frac{4\,a^2\,b\,\ln\left(2\,a\,e^{2\,x}-b+b\,e^{4\,x}\right)}{16\,a^2\,b^2+16\,b^4}$$

```
[In] int(sinh(x)^2/(a + b*sinh(2*x)),x)
```

```
[Out] atan(a^7/(- a^2 - b^2)^(7/2) + (b^7*exp(2*x))/(- a^2 - b^2)^(7/2) + (a*b^6)
/(- a^2 - b^2)^(7/2) + (3*a^3*b^4)/(- a^2 - b^2)^(7/2) + (3*a^5*b^2)/(- a^2
 - b^2)^(7/2) + (3*a^2*b^5*exp(2*x))/(- a^2 - b^2)^(7/2) + (3*a^4*b^3*exp(2
*x))/(- a^2 - b^2)^(7/2) + (a^6*b*exp(2*x))/(- a^2 - b^2)^(7/2))/(2*(- a^2
- b^2)^(1/2)) - x/(2*b) + (4*b^3*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^
4 + 16*a^2*b^2) + (4*a^2*b*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4 + 16
*a^2*b^2)
```

3.1034 $\int \frac{\cosh^2(x)}{a+b\sinh(2x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)} dx = -\frac{\text{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(a+b\sinh(2x))}{4b}$$

```
[Out] 1/4*ln(a+b*sinh(2*x))/b-1/2*arctanh((b-a*tanh(x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {995, 648, 632, 212, 642, 12, 266}

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)} dx = -\frac{\text{arctanh}\left(\frac{b-a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log\left(-a\tanh^2(x)+a+2b\tanh(x)\right)}{4b} + \frac{\log(\cosh(x))}{2b}$$

```
[In] Int[Cosh[x]^2/(a + b*Sinh[2*x]),x]
```

```
[Out] -1/2*ArcTanh[(b - a*Tanh[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2] + Log[Cosh[x]
]/(2*b) + Log[a + 2*b*Tanh[x] - a*Tanh[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 995

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x^2)(a+2bx-ax^2)}\,dx, x, \tanh(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{2bx}{1-x^2}\,dx, x, \tanh(x)\right)}{4b^2} - \frac{\text{Subst}\left(\int \frac{-4b^2+2abx}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh(x)\right) \\
&\quad + \frac{\text{Subst}\left(\int \frac{2b-2ax}{a+2bx-ax^2}\,dx, x, \tanh(x)\right)}{4b} + \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, \tanh(x)\right)}{2b} \\
&= \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b} \\
&\quad - \text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh(x)\right) \\
&= -\frac{\text{arctanh}\left(\frac{2b-2a\tanh(x)}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}} + \frac{\log(\cosh(x))}{2b} + \frac{\log\left(a+2b\tanh(x)-a\tanh^2(x)\right)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$
\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{1}{4}\left(\frac{2\arctan\left(\frac{b-a\tanh(x)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{\log(a+b\sinh(2x))}{b}\right)
$$

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[2*x]),x]
[Out] ((2*ArcTan[(b - a*Tanh[x])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[a + b*
Sinh[2*x]]/b)/4
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\ln(1+\tanh(x))}{4b}+\frac{\frac{\ln\left(a\tanh(x)^2-2b\tanh(x)-a\right)}{2}+\frac{b\ \text{arctanh}\left(\frac{2a\tanh(x)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{2b}-\frac{\ln(\tanh(x)-1)}{4b}$
risch	$\frac{x}{2b}-\frac{x\,a^2b}{a^2b^2+b^4}-\frac{x\,b^3}{a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{-ab+\sqrt{a^2b^2+b^4}}{b^2}\right)a^2}{4(a^2+b^2)b}+\frac{b\ln\left(\mathrm{e}^{2x}-\frac{-ab+\sqrt{a^2b^2+b^4}}{b^2}\right)}{4a^2+4b^2}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{-ab+\sqrt{a^2b^2+b^4}}{b^2}\right)\sqrt{a^2b^2}}{4(a^2+b^2)b}$

```
[In] int(cosh(x)^2/(a+b*sinh(2*x)),x,method=_RETURNVERBOSE)
[Out] -1/4/b*ln(1+tanh(x))+1/2/b*(1/2*ln(a*tanh(x)^2-2*b*tanh(x)-a)+b/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*a*tanh(x)-2*b)/(a^2+b^2)^(1/2)))-1/4/b*ln(tanh(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.83

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2ab\cosh(x)^2+2\left(3b^2\cosh(x)^2+ab\right)\sinh(x)^2+2a^2+b^2+4\left(b^2\cosh(x)^3+ab\,c\right.}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2a\cosh(x)^2+2\left(3b\cosh(x)^2+a\right)\sinh(x)^2}\right.}{}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a^2 +
b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) - 2*(b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + a)*sqrt(a^2 + b^2))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4
*(b*cosh(x)^3 + a*cosh(x))*sinh(x) - b)) - 2*(a^2 + b^2)*x + (a^2 + b^2)*lo
g(2*(2*b*cosh(x)*sinh(x) + a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^2*b + b^3)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \int \frac{\cosh^2{(x)}}{a+b\sinh{(2x)}}\,dx$$

```
[In] integrate(cosh(x)**2/(a+b*sinh(2*x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*sinh(2*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\log\left(\frac{be^{(-2x)}-a-\sqrt{a^2+b^2}}{be^{(-2x)}-a+\sqrt{a^2+b^2}}\right)}{4\sqrt{a^2+b^2}} - \frac{x}{2b} + \frac{\log\left(be^{(4x)}+2ae^{(2x)}-b\right)}{4b}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="maxima")
```

```
[Out] 1/4*log((b*e^(-2*x) - a - sqrt(a^2 + b^2))/(b*e^(-2*x) - a + sqrt(a^2 + b^2
)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(b*e^(4*x) + 2*a*e^(2*x) - b)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\log\left(\frac{\left|2\,be^{(2\,x)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(2\,x)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,\sqrt{a^2+b^2}} - \frac{x}{2\,b} + \frac{\log\left(\left|be^{(4\,x)}+2\,ae^{(2\,x)}-b\right|\right)}{4\,b}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(2*x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(2*b*e^(2*x) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(2*x) + 2*a +
2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2) - 1/2*x/b + 1/4*log(abs(b*e^(4*x) + 2*a
*e^(2*x) - b))/b
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.63

$$\int \frac{\cosh^2(x)}{a+b\sinh(2x)}\,dx = \frac{\mathrm{atan}\left(\frac{a}{\sqrt{-a^2-b^2}}+\frac{b\,\mathrm{e}^{2\,x}}{\sqrt{-a^2-b^2}}\right)}{2\,\sqrt{-a^2-b^2}} - \frac{x}{2\,b} + \frac{4\,b^3\,\ln\left(2\,a\,\mathrm{e}^{2\,x}-b+b\,\mathrm{e}^{4\,x}\right)}{16\,a^2\,b^2+16\,b^4} + \frac{4\,a^2\,b\,\ln\left(2\,a\,\mathrm{e}^{2\,x}-b+b\,\mathrm{e}^{4\,x}\right)}{16\,a^2\,b^2+16\,b^4}$$

```
[In] int(cosh(x)^2/(a + b*sinh(2*x)),x)
```

```
[Out] atan(a/(- a^2 - b^2)^(1/2) + (b*exp(2*x))/(- a^2 - b^2)^(1/2))/(2*(- a^2 -
b^2)^(1/2)) - x/(2*b) + (4*b^3*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4
+ 16*a^2*b^2) + (4*a^2*b*log(2*a*exp(2*x) - b + b*exp(4*x)))/(16*b^4 + 16*a
^2*b^2)
```

3.1035 $\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x}{2b} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}b}$$

```
[Out] 1/2*x/b-1/2*arctanh((a-b)^(1/2)*tanh(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/(a-b)^(1
/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1144, 214}

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x}{2b} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

```
[In] Int[Sinh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*Sqrt[
a - b]*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{x^2}{-a-b+2ax^2+(-a+b)x^4}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}\left(-1+\frac{a}{b}\right)\text{Subst}\left(\int \frac{1}{a-b+(-a+b)x^2}\,dx, x, \tanh(x)\right) - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tanh(x)\right)}{2b}\\ &= \frac{x}{2b} - \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}\,b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x+\frac{(a+b)\arctan\left(\frac{(a-b)\tanh(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[2*x]),x]
[Out] (x + ((a + b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{(a+b)\,\text{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}}-\frac{\ln(\tanh(x)-1)}{4b}+\frac{\ln(1+\tanh(x))}{4b}$	61
risch	$\frac{x}{2b}+\frac{\sqrt{(a+b)(a-b)}\,\ln\left(e^{2x}+\frac{a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a-b)b}-\frac{\sqrt{(a+b)(a-b)}\,\ln\left(e^{2x}-\frac{-a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a-b)b}$	103

```
[In] int(sinh(x)^2/(a+b*cosh(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b)/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*(a-b))^(1/2))
-1/4/b*ln(tanh(x)-1)+1/4/b*ln(1+tanh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 5.83

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \left[\frac{\sqrt{\frac{a+b}{a-b}}\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2ab\cosh(x)^2+2\left(3b^2\cosh(x)^2+ab\right)\sinh(x)^2+2a^2-b^2+4\left(b^2\cosh(x)^3+ab\cosh\right.}{b\cosh(x)^4+4b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2a\cosh(x)^2+2\left(3b\cosh(x)^2+a\right)}\right.}{4b}\right. \\ \left. -\frac{\sqrt{-\frac{a+b}{a-b}}\arctan\left(\frac{\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2+a\right)\sqrt{-\frac{a+b}{a-b}}}{a+b}\right)-x}{2b}\right]$$

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((a + b)/(a - b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a
^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b - b^2)*cosh(x)
^2 + 2*(a*b - b^2)*cosh(x)*sinh(x) + (a*b - b^2)*sinh(x)^2 + a^2 - a*b)*sqr
t((a + b)/(a - b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*cosh(x))
*sinh(x) + b)) + 2*x)/b, -1/2*(sqrt(-(a + b)/(a - b))*arctan((b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a + b)/(a - b))/(a + b)) - x
)/b]
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \int \frac{\sinh^2{(x)}}{a+b\cosh{(2x)}}\,dx$$

```
[In] integrate(sinh(x)**2/(a+b*cosh(2*x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*cosh(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = -\frac{(a+b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}b} + \frac{x}{2b}$$

```
[In] integrate(sinh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*(a + b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
+ 1/2*x/b
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.94

$$\int \frac{\sinh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x}{2b} - \frac{\ln\left(a\,b+2\,a^2\,e^{2\,x}-b^2\,e^{2\,x}+b\,\sqrt{a+b}\,\sqrt{a-b}+2\,a\,e^{2\,x}\,\sqrt{a+b}\,\sqrt{a-b}\right)\,\sqrt{a+b}}{4\,b\,\sqrt{a-b}} + \frac{\ln\left(b^2\,e^{2\,x}-2\,a^2\,e^{2\,x}-a\,b+b\,\sqrt{a+b}\,\sqrt{a-b}+2\,a\,e^{2\,x}\,\sqrt{a+b}\,\sqrt{a-b}\right)\,\sqrt{a+b}}{4\,b\,\sqrt{a-b}}$$

```
[In] int(sinh(x)^2/(a + b*cosh(2*x)),x)
```

```
[Out] x/(2*b) - (log(a*b + 2*a^2*exp(2*x) - b^2*exp(2*x) + b*(a + b)^(1/2)*(a - b
)^(1/2) + 2*a*exp(2*x)*(a + b)^(1/2)*(a - b)^(1/2))*(a + b)^(1/2))/(4*b*(a
- b)^(1/2)) + (log(b^2*exp(2*x) - 2*a^2*exp(2*x) - a*b + b*(a + b)^(1/2)*(a
 - b)^(1/2) + 2*a*exp(2*x)*(a + b)^(1/2)*(a - b)^(1/2))*(a + b)^(1/2))/(4*b
*(a - b)^(1/2))
```

3.1036 $\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx=\frac{x}{2b}-\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[Out] 1/2*x/b-1/2*arctanh((a-b)^(1/2)*tanh(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(1
/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1107, 214}

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx=\frac{x}{2b}-\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[In] Int[Cosh[x]^2/(a + b*Cosh[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[x])/Sqrt[a + b]])/(2*b*Sqr
t[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{a+b-2ax^2+(a-b)x^4}\,dx, x, \tanh(x)\right)$$
$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tanh(x)\right)}{2b} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a+b+(a-b)x^2}\,dx, x, \tanh(x)\right)}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x+\frac{(a-b)\arctan\left(\frac{(a-b)\tanh(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[2*x]),x]
[Out] (x + ((a - b)*ArcTan[((a - b)*Tanh[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{(a-b)\,\text{arctanh}\left(\frac{(a-b)\tanh(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}}-\frac{\ln(\tanh(x)-1)}{4b}+\frac{\ln(1+\tanh(x))}{4b}$	63
risch	$\frac{x}{2b}+\frac{\sqrt{(a+b)(a-b)}\,\ln\left(e^{2x}+\frac{a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a+b)b}-\frac{\sqrt{(a+b)(a-b)}\,\ln\left(e^{2x}-\frac{-a+\sqrt{(a+b)(a-b)}}{b}\right)}{4(a+b)b}$	99

```
[In] int(cosh(x)^2/(a+b*cosh(2*x)),x,method=_RETURNVERBOSE)
[Out] -1/2*(a-b)/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(x)/((a+b)*(a-b))^(1/2))
-1/4/b*ln(tanh(x)-1)+1/4/b*ln(1+tanh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 5.71

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \left[\frac{\sqrt{\frac{a-b}{a+b}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,ab\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+ab\right)\sinh(x)^2+2\,a^2-b^2+4\left(b^2\cosh(x)^3+ab\cosh\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,a\cosh(x)^2+2\left(3\,b\cosh(x)^2+a\right)}\right.}{4\,b}\right.$$

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((a - b)/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 +
b^2*sinh(x)^4 + 2*a*b*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + a*b)*sinh(x)^2 + 2*a
^2 - b^2 + 4*(b^2*cosh(x)^3 + a*b*cosh(x))*sinh(x) + 2*((a*b + b^2)*cosh(x)
^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + a^2 + a*b)*sqr
t((a - b)/(a + b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*
a*cosh(x)^2 + 2*(3*b*cosh(x)^2 + a)*sinh(x)^2 + 4*(b*cosh(x)^3 + a*cosh(x))
*sinh(x) + b)) + 2*x)/b, 1/2*(sqrt(-(a - b)/(a + b))*arctan(-(b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-(a - b)/(a + b))/(a - b)) + x
)/b]
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx$$

```
[In] integrate(cosh(x)**2/(a+b*cosh(2*x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*cosh(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = -\frac{(a-b)\arctan\left(\frac{be^{(2x)}+a}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}b}+\frac{x}{2b}$$

```
[In] integrate(cosh(x)^2/(a+b*cosh(2*x)),x, algorithm="giac")
[Out] -1/2*(a - b)*arctan((b*e^(2*x) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b)
+ 1/2*x/b
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

$$\int \frac{\cosh^2(x)}{a+b\cosh(2x)}\,dx = \frac{x}{2b}-\frac{\ln\left(\frac{e^{2x}(a-b)}{b^2}+\frac{\sqrt{a-b}\,(b+a\,e^{2x})}{b^2\sqrt{a+b}}\right)\sqrt{a-b}}{4b\sqrt{a+b}}+\frac{\ln\left(\frac{e^{2x}(a-b)}{b^2}-\frac{\sqrt{a-b}\,(b+a\,e^{2x})}{b^2\sqrt{a+b}}\right)\sqrt{a-b}}{4b\sqrt{a+b}}$$

```
[In] int(cosh(x)^2/(a + b*cosh(2*x)),x)
[Out] x/(2*b) - (log((exp(2*x)*(a - b))/b^2 + ((a - b)^(1/2)*(b + a*exp(2*x)))/(b
^2*(a + b)^(1/2)))*(a - b)^(1/2))/(4*b*(a + b)^(1/2)) + (log((exp(2*x)*(a -
 b))/b^2 - ((a - b)^(1/2)*(b + a*exp(2*x)))/(b^2*(a + b)^(1/2)))*(a - b)^(1
/2))/(4*b*(a + b)^(1/2))
```

3.1037 $\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] arctan((a*sinh(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3284, 65, 209}

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
```

```
[Out] ArcTan[Sqrt[a*Sinh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{ax}(1+x)}\,dx, x, \sinh^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sinh^2(c+dx)}\right)}{ad}\\ &= \frac{\arctan\left(\frac{\sqrt{a\sinh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{\arctan(\sinh(c+dx))\sinh(c+dx)}{d\sqrt{a\sinh^2(c+dx)}}$$

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a*Sinh[c + d*x]^2],x]
[Out] (ArcTan[Sinh[c + d*x]]*Sinh[c + d*x])/(d*Sqrt[a*Sinh[c + d*x]^2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(dx+c)}{\cosh(dx+c)^2\sqrt{a\sinh(dx+c)^2}},\sinh(dx+c)\right)}{d}$	39
risch	$\frac{i\ln\left(e^{dx}+ie^{-c}\right)\left(e^{2dx+2c}-1\right)e^{-dx-c}}{d\sqrt{\left(e^{2dx+2c}-1\right)^2 a\,e^{-2dx-2c}}}-\frac{i\ln\left(e^{dx}-ie^{-c}\right)\left(e^{2dx+2c}-1\right)e^{-dx-c}}{d\sqrt{\left(e^{2dx+2c}-1\right)^2 a\,e^{-2dx-2c}}}$	132

```
[In] int(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(sinh(d*x+c)/cosh(d*x+c)^2/(a*sinh(d*x+c)^2)^(1/2),sinh(d*x+c))
/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 11.17

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx$$

$$=\left[-\frac{\sqrt{-a}\log\left(-\frac{a\cosh(dx+c)^2+2\sqrt{ae^{(4dx+4c)}-2ae^{(2dx+2c)}+a}\left(\cosh(dx+c)e^{(dx+c)}+e^{(dx+c)}\sinh(dx+c)\right)\sqrt{-a}e^{(-dx-c)}-\left(ae^{(2dx+2c)}-a\right.}{\left(e^{(2dx+2c)}-1\right)\sinh(dx+c)^2-\cosh(dx+c)^2+\left(\cosh(dx+c)^2+1\right)e^{(2dx+2c)}}\right.}{ad}\right.$$

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log(-(a*cosh(d*x + c)^2 + 2*sqrt(a*e^(4*d*x + 4*c) - 2*a*e^(2*d*
x + 2*c) + a)*(cosh(d*x + c)*e^(d*x + c) + e^(d*x + c)*sinh(d*x + c))*sqrt(
-a)*e^(-d*x - c) - (a*e^(2*d*x + 2*c) - a)*sinh(d*x + c)^2 - (a*cosh(d*x +
c)^2 - a)*e^(2*d*x + 2*c) - 2*(a*cosh(d*x + c)*e^(2*d*x + 2*c) - a*cosh(d*x
 + c))*sinh(d*x + c) - a)/((e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 - cosh(d*x
 + c)^2 + (cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) + 2*(cosh(d*x + c)*e^(2*d*x
 + 2*c) - cosh(d*x + c))*sinh(d*x + c) - 1))/(a*d), 2*sqrt(a*e^(4*d*x + 4*c
) - 2*a*e^(2*d*x + 2*c) + a)*arctan(cosh(d*x + c) + sinh(d*x + c))/(a*d*e^(
2*d*x + 2*c) - a*d)]
```

Sympy [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \int \frac{\tanh{(c+dx)}}{\sqrt{a\sinh^2{(c+dx)}}}\,dx$$

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)**2)**(1/2),x)
[Out] Integral(tanh(c + d*x)/sqrt(a*sinh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{2\arctan\left(e^{(-dx-c)}\right)}{\sqrt{a}d}$$

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(e^(-d*x - c))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \frac{2\arctan\left(e^{(dx+c)}\right)}{\sqrt{a}d\mathrm{sgn}\left(e^{(3\,dx+3\,c)}-e^{(dx+c)}\right)}$$

```
[In] integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] 2*arctan(e^(d*x + c))/(sqrt(a)*d*sgn(e^(3*d*x + 3*c) - e^(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{\sqrt{a\sinh^2(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)}{\sqrt{a\sinh{(c+d\,x)}^2}}\,dx$$

```
[In] int(tanh(c + d*x)/(a*sinh(c + d*x)^2)^(1/2),x)
[Out] int(tanh(c + d*x)/(a*sinh(c + d*x)^2)^(1/2), x)
```

3.1038 $\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -arctanh((a*cosh(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3284, 65, 212}

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(c+dx)}\right)}{ad} \\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$
\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}
$$

```
[In] Integrate[Coth[c + d*x]/Sqrt[a*Cosh[c + d*x]^2],x]
[Out] -(ArcTanh[Sqrt[a*Cosh[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\cosh(dx+c)\,\mathrm{arctanh}(\cosh(dx+c))}{\sqrt{a\cosh(dx+c)^2}\,d}$	31
risch	$\frac{\ln(\mathrm{e}^{dx}-\mathrm{e}^{-c})\,(\mathrm{e}^{2dx+2c}+1)\,\mathrm{e}^{-dx-c}}{d\sqrt{(\mathrm{e}^{2dx+2c}+1)^2\,a\,\mathrm{e}^{-2dx-2c}}}-\frac{\ln(\mathrm{e}^{dx}+\mathrm{e}^{-c})\,(\mathrm{e}^{2dx+2c}+1)\,\mathrm{e}^{-dx-c}}{d\sqrt{(\mathrm{e}^{2dx+2c}+1)^2\,a\,\mathrm{e}^{-2dx-2c}}}$	125

```
[In] int(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a*cosh(d*x+c)^2)^(1/2)*cosh(d*x+c)*arctanh(cosh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.61

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$$

$$= \left[\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+a}\log\left(\frac{\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{ade^{(2\,dx+2\,c)}+ad},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4\,dx+4\,c)}+2\,ae^{(2\,d}}}{a\cosh(dx+c)e^{(2\,dx+2\,c)}+a\cosh(dx+c)}\right.}{ad}\right.$$

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] [sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*log((cosh(d*x + c) + sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/(a*d*e^(2*d*x + 2*c) +
 a*d), 2*sqrt(-a)*arctan(sqrt(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + a)*
sqrt(-a)/(a*cosh(d*x + c)*e^(2*d*x + 2*c) + a*cosh(d*x + c) + (a*e^(2*d*x +
 2*c) + a)*sinh(d*x + c)))/(a*d)]
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = \int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)**2)**(1/2),x)
[Out] Integral(coth(c + d*x)/sqrt(a*cosh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = -\frac{\log\left(e^{(-dx-c)}+1\right)}{\sqrt{a}d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{\sqrt{a}d}$$

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(e^(-d*x - c) + 1)/(sqrt(a)*d) + log(e^(-d*x - c) - 1)/(sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(coth(d*x+c)/(a*cosh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\sqrt{a\cosh^2(c+dx)}}\,dx = \int \frac{\coth(c+d\,x)}{\sqrt{a\cosh\left(c+d\,x\right)^2}}\,dx$$

```
[In] int(coth(c + d*x)/(a*cosh(c + d*x)^2)^(1/2),x)
```

```
[Out] int(coth(c + d*x)/(a*cosh(c + d*x)^2)^(1/2), x)
```

3.1039 $\int x \cosh(2x)\mathbf{sech}(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int x\cosh(2x)\text{sech}(x)\,dx = -2x\arctan(e^x) - 2\cosh(x) + i\,\text{PolyLog}(2,-ie^x) - i\,\text{PolyLog}(2,ie^x) + 2x\sinh(x)$$

```
[Out] -2*x*arctan(exp(x))-2*cosh(x)+I*polylog(2,-I*exp(x))-I*polylog(2,I*exp(x))+
2*x*sinh(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5581, 3377, 2718, 5557, 4265, 2317, 2438}

$$\int x\cosh(2x)\text{sech}(x)\,dx = -2x\arctan(e^x) + i\,\text{PolyLog}(2,-ie^x) - i\,\text{PolyLog}(2,ie^x) + 2x\sinh(x) - 2\cosh(x)$$

```
[In] Int[x*Cosh[2*x]*Sech[x],x]
```

```
[Out] -2*x*ArcTan[E^x] - 2*Cosh[x] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2, I*E^x]
 + 2*x*Sinh[x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5581

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (x\cosh(x) + x\sinh(x)\tanh(x))\,dx\\ &= \int x\cosh(x)\,dx + \int x\sinh(x)\tanh(x)\,dx\end{aligned}$$

$$= x\sinh(x) + \int x\cosh(x)\,dx - \int x\text{sech}(x)\,dx - \int \sinh(x)\,dx$$

$$= -2x\arctan(e^x) - \cosh(x) + 2x\sinh(x) + i\int \log(1-ie^x)\,dx - i\int \log(1+ie^x)\,dx - \int \sinh(x)\,dx$$

$$= -2x\arctan(e^x) - 2\cosh(x) + 2x\sinh(x) + i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) - i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right)$$

$$= -2x\arctan(e^x) - 2\cosh(x) + i\,\text{PolyLog}(2,-ie^x) - i\,\text{PolyLog}(2,ie^x) + 2x\sinh(x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int x\cosh(2x)\text{sech}(x)\,dx = -2\cosh(x) - i(x(\log(1-ie^x) - \log(1+ie^x)) - \text{PolyLog}(2,-ie^x) + \text{PolyLog}(2,ie^x)) + 2x\sinh(x)$$

```
[In] Integrate[x*Cosh[2*x]*Sech[x],x]
```

```
[Out] -2*Cosh[x] - I*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I)*E^x]
+ PolyLog[2, I*E^x]) + 2*x*Sinh[x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

method	result
risch	$2\left(-\frac{1}{2}+\frac{x}{2}\right)\mathrm{e}^{x}+2\left(-\frac{1}{2}-\frac{x}{2}\right)\mathrm{e}^{-x}+ix\ln\left(1+i\mathrm{e}^{x}\right)-ix\ln\left(1-i\mathrm{e}^{x}\right)+i\,\text{dilog}\left(1+i\mathrm{e}^{x}\right)-i\,\text{dilog}\left(1-\right.$

```
[In] int(x*cosh(2*x)*sech(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/2+1/2*x)*exp(x)+2*(-1/2-1/2*x)/exp(x)+I*x*ln(1+I*exp(x))-I*x*ln(1-I*e
xp(x))+I*dilog(1+I*exp(x))-I*dilog(1-I*exp(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.88

$$\int x\cosh(2x)\text{sech}(x)\,dx$$
$$= \frac{(x-1)\cosh(x)^2 + 2(x-1)\cosh(x)\sinh(x) + (x-1)\sinh(x)^2 + (-i\cosh(x) - i\sinh(x))\text{Li}_2(i\cosh(x)$$

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="fricas")
```

```
[Out] ((x - 1)*cosh(x)^2 + 2*(x - 1)*cosh(x)*sinh(x) + (x - 1)*sinh(x)^2 + (-I*co
sh(x) - I*sinh(x))*dilog(I*cosh(x) + I*sinh(x)) + (I*cosh(x) + I*sinh(x))*d
ilog(-I*cosh(x) - I*sinh(x)) + (I*x*cosh(x) + I*x*sinh(x))*log(I*cosh(x) +
I*sinh(x) + 1) + (-I*x*cosh(x) - I*x*sinh(x))*log(-I*cosh(x) - I*sinh(x) +
1) - x - 1)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int x\cosh(2x)\text{sech}(x)\,dx = \int x\cosh(2x)\,\text{sech}(x)\,dx$$

```
[In] integrate(x*cosh(2*x)*sech(x),x)
```

```
[Out] Integral(x*cosh(2*x)*sech(x), x)
```

Maxima [F]

$$\int x\cosh(2x)\text{sech}(x)\,dx = \int x\cosh(2x)\,\text{sech}(x)\,dx$$

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + (x - 1)*e^x - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x\cosh(2x)\mathrm{sech}(x)\,dx = \int x\cosh\left(2\,x\right)\mathrm{sech}\left(x\right)\,dx$$

```
[In] integrate(x*cosh(2*x)*sech(x),x, algorithm="giac")
[Out] integrate(x*cosh(2*x)*sech(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(2x)\mathrm{sech}(x)\,dx = \int \frac{x\cosh(2\,x)}{\cosh\left(x\right)}\,dx$$

```
[In] int((x*cosh(2*x))/cosh(x),x)
[Out] int((x*cosh(2*x))/cosh(x), x)
```

3.1040 $\int x \cosh(2x)\mathbf{sech}^2(x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 12

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = x^2+\log(\cosh(x))-x\tanh(x)$$

```
[Out] x^2+ln(cosh(x))-x*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5581, 3801, 3556, 30}

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = x^2-x\tanh(x)+\log(\cosh(x))$$

```
[In] Int[x*Cosh[2*x]*Sech[x]^2,x]
```

```
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 5581

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(x + x\tanh^2(x)\right)\,dx\\ &= \frac{x^2}{2} + \int x\tanh^2(x)\,dx\\ &= \frac{x^2}{2} - x\tanh(x) + \int x\,dx + \int \tanh(x)\,dx\\ &= x^2 + \log(\cosh(x)) - x\tanh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = x^2 + \log(\cosh(x)) - x\tanh(x)$$

```
[In] Integrate[x*Cosh[2*x]*Sech[x]^2,x]
[Out] x^2 + Log[Cosh[x]] - x*Tanh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 1.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

method	result	size
risch	$x^2 - 2x + \frac{2x}{1+e^{2x}} + \ln\left(1+e^{2x}\right)$	26

```
[In] int(x*cosh(2*x)*sech(x)^2,x,method=_RETURNVERBOSE)
[Out] x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 7.58

$$\int x\cosh(2x)\text{sech}^2(x)\,dx$$
$$= \frac{(x^2-2\,x)\cosh\left(x\right)^2+2\,(x^2-2\,x)\cosh\left(x\right)\sinh\left(x\right)+(x^2-2\,x)\sinh\left(x\right)^2+x^2+\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sin\right.}{\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2+1}$$

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="fricas")
[Out] ((x^2 - 2*x)*cosh(x)^2 + 2*(x^2 - 2*x)*cosh(x)*sinh(x) + (x^2 - 2*x)*sinh(x
)^2 + x^2 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(
cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = \int x\cosh\left(2x\right)\text{sech}^2\left(x\right)dx$$

```
[In] integrate(x*cosh(2*x)*sech(x)**2,x)
[Out] Integral(x*cosh(2*x)*sech(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = \frac{x^2+(x^2-2\,x)e^{(2\,x)}}{e^{(2\,x)}+1}+\log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="maxima")
[Out] (x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.92

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = \frac{x^2e^{(2x)} + x^2 - 2\,xe^{(2x)} + e^{(2x)}\log\left(e^{(2x)} + 1\right) + \log\left(e^{(2x)} + 1\right)}{e^{(2x)} + 1}$$

```
[In] integrate(x*cosh(2*x)*sech(x)^2,x, algorithm="giac")
[Out] (x^2*e^(2*x) + x^2 - 2*x*e^(2*x) + e^(2*x)*log(e^(2*x) + 1) + log(e^(2*x) +
 1))/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int x\cosh(2x)\text{sech}^2(x)\,dx = \ln\left(\mathrm{e}^{2\,x} + 1\right) - 2\,x + \frac{2\,x}{\mathrm{e}^{2\,x} + 1} + x^2$$

```
[In] int((x*cosh(2*x))/cosh(x)^2,x)
[Out] log(exp(2*x) + 1) - 2*x + (2*x)/(exp(2*x) + 1) + x^2
```

3.1041 $\int x \cosh(2x)\mathbf{sech}^3(x)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int x \cosh(2x)\text{sech}^3(x)\, dx = 3x \arctan(e^x) - \frac{3}{2} i \,\text{PolyLog}(2, -ie^x) + \frac{3}{2} i \,\text{PolyLog}(2, ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2} x\text{sech}(x)\tanh(x)$$

```
[Out] 3*x*arctan(exp(x))-3/2*I*polylog(2,-I*exp(x))+3/2*I*polylog(2,I*exp(x))-1/2
*sech(x)-1/2*x*sech(x)*tanh(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5581, 4265, 2317, 2438, 5563, 4270}

$$\int x \cosh(2x)\text{sech}^3(x)\, dx = 3x \arctan(e^x) - \frac{3}{2} i \,\text{PolyLog}(2, -ie^x) + \frac{3}{2} i \,\text{PolyLog}(2, ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2} x \tanh(x)\text{sech}(x)$$

```
[In] Int[x*Cosh[2*x]*Sech[x]^3,x]
```

```
[Out] 3*x*ArcTan[E^x] - ((3*I)/2)*PolyLog[2, (-I)*E^x] + ((3*I)/2)*PolyLog[2, I*E
^x] - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5563

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5581

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sinh, Cosh}, F] && MemberQ[{Sech, Csch}, G] && IGtQ[p, 0] && IGtQ[q, 0]
&& EqQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(x\text{sech}(x) + x\text{sech}(x)\tanh^2(x)\right)\,dx\\ &= \int x\text{sech}(x)\,dx + \int x\text{sech}(x)\tanh^2(x)\,dx\end{aligned}$$

$$= 2x\arctan(e^x) - i\int \log(1-ie^x)\,dx + i\int \log(1+ie^x)\,dx + \int x\text{sech}(x)\,dx - \int x\text{sech}^3(x)\,dx$$

$$= 4x\arctan(e^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x) - i\int \log(1-ie^x)\,dx + i\int \log(1+ie^x)\,dx \\ - i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) + i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right) - \frac{1}{2}\int x\text{sech}(x)\,dx$$

$$= 3x\arctan(e^x) - i\,\text{PolyLog}(2, -ie^x) + i\,\text{PolyLog}(2, ie^x) - \frac{\text{sech}(x)}{2} \\ - \frac{1}{2}x\text{sech}(x)\tanh(x) + \frac{1}{2}i\int \log(1-ie^x)\,dx - \frac{1}{2}i\int \log(1+ie^x)\,dx \\ - i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) + i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right)$$

$$= 3x\arctan(e^x) - 2i\,\text{PolyLog}(2, -ie^x) + 2i\,\text{PolyLog}(2, ie^x) - \frac{\text{sech}(x)}{2} \\ - \frac{1}{2}x\text{sech}(x)\tanh(x) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) \\ - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right)$$

$$= 3x\arctan(e^x) - \frac{3}{2}i\,\text{PolyLog}(2, -ie^x) + \frac{3}{2}i\,\text{PolyLog}(2, ie^x) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \frac{3}{2}i(x(\log(1-ie^x) - \log(1+ie^x)) - \text{PolyLog}(2, -ie^x) \\ + \text{PolyLog}(2, ie^x)) - \frac{\text{sech}(x)}{2} - \frac{1}{2}x\text{sech}(x)\tanh(x)$$

```
[In] Integrate[x*Cosh[2*x]*Sech[x]^3,x]
```

```
[Out] ((3*I)/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I)*E^x] + Pol
yLog[2, I*E^x]) - Sech[x]/2 - (x*Sech[x]*Tanh[x])/2
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

method	result	size
risch	$-\frac{e^{x}\left(x\,e^{2x}+e^{2x}-x+1\right)}{\left(1+e^{2x}\right)^{2}}-\frac{3ix\ln\left(1+ie^{x}\right)}{2}+\frac{3ix\ln\left(1-ie^{x}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1+ie^{x}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1-ie^{x}\right)}{2}$	75

```
[In] int(x*cosh(2*x)*sech(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)*(x*exp(x)^2+exp(x)^2-x+1)/(exp(x)^2+1)^2-3/2*I*x*ln(1+I*exp(x))+3/2
*I*x*ln(1-I*exp(x))-3/2*I*dilog(1+I*exp(x))+3/2*I*dilog(1-I*exp(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 7.70

$$\int x\cosh(2x)\mathrm{sech}^3(x)\,dx =$$

$$-\frac{2\,(x+1)\cosh{(x)}^{3}+6\,(x+1)\cosh{(x)}\sinh{(x)}^{2}+2\,(x+1)\sinh{(x)}^{3}-2\,(x-1)\cosh{(x)}+3\,\left(-i\cosh\right.}{}$$

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*cosh(x)^3 + 6*(x + 1)*cosh(x)*sinh(x)^2 + 2*(x + 1)*sinh(x)
^3 - 2*(x - 1)*cosh(x) + 3*(-I*cosh(x)^4 - 4*I*cosh(x)*sinh(x)^3 - I*sinh(x
)^4 + 2*(-3*I*cosh(x)^2 - I)*sinh(x)^2 - 2*I*cosh(x)^2 + 4*(-I*cosh(x)^3 -
I*cosh(x))*sinh(x) - I)*dilog(I*cosh(x) + I*sinh(x)) + 3*(I*cosh(x)^4 + 4*I
*cosh(x)*sinh(x)^3 + I*sinh(x)^4 + 2*(3*I*cosh(x)^2 + I)*sinh(x)^2 + 2*I*co
sh(x)^2 + 4*(I*cosh(x)^3 + I*cosh(x))*sinh(x) + I)*dilog(-I*cosh(x) - I*sin
h(x)) + 3*(I*x*cosh(x)^4 + 4*I*x*cosh(x)*sinh(x)^3 + I*x*sinh(x)^4 + 2*I*x*
cosh(x)^2 + 2*(3*I*x*cosh(x)^2 + I*x)*sinh(x)^2 + 4*(I*x*cosh(x)^3 + I*x*co
sh(x))*sinh(x) + I*x)*log(I*cosh(x) + I*sinh(x) + 1) + 3*(-I*x*cosh(x)^4 -
4*I*x*cosh(x)*sinh(x)^3 - I*x*sinh(x)^4 - 2*I*x*cosh(x)^2 + 2*(-3*I*x*cosh(
x)^2 - I*x)*sinh(x)^2 + 4*(-I*x*cosh(x)^3 - I*x*cosh(x))*sinh(x) - I*x)*log
(-I*cosh(x) - I*sinh(x) + 1) + 2*(3*(x + 1)*cosh(x)^2 - x + 1)*sinh(x))/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int x\cosh(2x)\,\text{sech}^3(x)\,dx$$

```
[In] integrate(x*cosh(2*x)*sech(x)**3,x)
[Out] Integral(x*cosh(2*x)*sech(x)**3, x)
```

Maxima [F]

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int x\cosh(2x)\,\text{sech}(x)^3\,dx$$

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="maxima")
[Out] -((x + 1)*e^(3*x) - (x - 1)*e^x)/(e^(4*x) + 2*e^(2*x) + 1) + 12*integrate(1
/4*x*e^x/(e^(2*x) + 1), x)
```

Giac [F]

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int x\cosh(2x)\,\text{sech}(x)^3\,dx$$

```
[In] integrate(x*cosh(2*x)*sech(x)^3,x, algorithm="giac")
[Out] integrate(x*cosh(2*x)*sech(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cosh(2x)\text{sech}^3(x)\,dx = \int \frac{x\cosh(2x)}{\cosh(x)^3}\,dx$$

```
[In] int((x*cosh(2*x))/cosh(x)^3,x)
[Out] int((x*cosh(2*x))/cosh(x)^3, x)
```

3.1042 $\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx=\frac{2x}{\sqrt{\operatorname{csch}(x)}}-\frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[Out] -4*sech(x)/csch(x)^(3/2)+2*x/csch(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6874, 5553, 3856, 2719, 2706}

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx=\frac{2x}{\sqrt{\operatorname{csch}(x)}}-\frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[In] Int[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
[Out] (2*x)/Sqrt[Csch[x]] - (4*Sech[x])/Csch[x]^(3/2)
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 5553

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)
^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^(p - 1)/(b*n*(p
 - 1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csch[a + b*x^n]
^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] &&
NeQ[p, 1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x\cosh(x)\sqrt{\operatorname{csch}(x)} - \frac{4\operatorname{sech}^2(x)}{\sqrt{\operatorname{csch}(x)}}\right) dx \\
&= -\left(4\int \frac{\operatorname{sech}^2(x)}{\sqrt{\operatorname{csch}(x)}}\, dx\right) + \int x\cosh(x)\sqrt{\operatorname{csch}(x)}\, dx \\
&= \frac{2x}{\sqrt{\operatorname{csch}(x)}} - \frac{4\operatorname{sech}(x)}{\operatorname{csch}^{\frac{3}{2}}(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x) - 4\operatorname{sech}(x)\tanh(x))\, dx = \frac{2(x\operatorname{csch}(x) - 2\operatorname{sech}(x))}{\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[In] Integrate[Sqrt[Csch[x]]*(x*Cosh[x] - 4*Sech[x]*Tanh[x]),x]
[Out] (2*(x*Csch[x] - 2*Sech[x]))/Csch[x]^(3/2)
```

Maple [F]

$$\int \sqrt{\operatorname{csch}\left(x\right)}\,\left(x\cosh\left(x\right)-4\,\operatorname{sech}\left(x\right)\tanh\left(x\right)\right)dx$$

```
[In] int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
[Out] int(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="fricas"
)
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \text{Timed out}$$

```
[In] integrate(csch(x)**(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x)-4\operatorname{sech}(x)\tanh(x))\,dx = \int \left(x\cosh\left(x\right)-4\,\operatorname{sech}\left(x\right)\tanh\left(x\right)\right)\sqrt{\operatorname{csch}\left(x\right)}\,dx$$

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="maxima"
)
[Out] integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x) - 4\operatorname{sech}(x)\tanh(x))\,dx$$
$$= \int (x\cosh(x) - 4\operatorname{sech}(x)\tanh(x))\sqrt{\operatorname{csch}(x)}\,dx$$

```
[In] integrate(csch(x)^(1/2)*(x*cosh(x)-4*sech(x)*tanh(x)),x, algorithm="giac")
[Out] integrate((x*cosh(x) - 4*sech(x)*tanh(x))*sqrt(csch(x)), x)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \sqrt{\operatorname{csch}(x)}(x\cosh(x) - 4\operatorname{sech}(x)\tanh(x))\,dx$$
$$= \frac{e^{-x}\sqrt{-\frac{1}{\frac{e^{-x}}{2}-\frac{e^{x}}{2}}}\,\left(e^{2x}-1\right)\,\left(x-2\,e^{2x}+x\,e^{2x}+2\right)}{e^{2x}+1}$$

```
[In] int(-(1/sinh(x))^(1/2)*((4*tanh(x))/cosh(x) - x*cosh(x)),x)
[Out] (exp(-x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/2)*(exp(2*x) - 1)*(x - 2*exp(2*x) +
 x*exp(2*x) + 2))/(exp(2*x) + 1)
```

3.1043 $\int \sinh(x)(\cosh(x)+\sinh(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 22

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=-\frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{2}$$

```
[Out] -1/2*x+1/2*cosh(x)*sinh(x)+1/2*sinh(x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3168, 2644, 30, 2715, 8}

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=-\frac{x}{2}+\frac{\sinh^2(x)}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[In] Int[Sinh[x]*(Cosh[x] + Sinh[x]),x]
```

```
[Out] -1/2*x + (Cosh[x]*Sinh[x])/2 + Sinh[x]^2/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3168

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \left(i\cosh(x)\sinh(x)+i\sinh^2(x)\right)\,dx\right)\\
&= \int \cosh(x)\sinh(x)\,dx+\int \sinh^2(x)\,dx\\
&= \frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)\\
&= -\frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\sinh^2(x)}{2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx=-\frac{x}{2}+\frac{\cosh^2(x)}{2}+\frac{1}{4}\sinh(2x)$$

```
[In] Integrate[Sinh[x]*(Cosh[x] + Sinh[x]),x]
[Out] -1/2*x + Cosh[x]^2/2 + Sinh[2*x]/4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

method	result	size
risch	$-\frac{x}{2}+\frac{e^{2x}}{4}$	11
default	$\frac{\cosh(x)^2}{2}+\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}$	17
parts	$\frac{\cosh(x)^2}{2}+\frac{\cosh(x)\sinh(x)}{2}-\frac{x}{2}$	17
meijerg	$-\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(2x)}{\sqrt{\pi}}\right)}{4}+\frac{i\sqrt{\pi}\left(\frac{2ix}{\sqrt{\pi}}-\frac{i\sinh(2x)}{\sqrt{\pi}}\right)}{4}$	44

```
[In] int(sinh(x)*(cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/4*exp(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx = -\frac{(2x-1)\cosh(x)-(2x+1)\sinh(x)}{4(\cosh(x)-\sinh(x))}$$

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="fricas")
[Out] -1/4*((2*x - 1)*cosh(x) - (2*x + 1)*sinh(x))/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \sinh(x)(\cosh(x)+\sinh(x))\,dx = \frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}+\frac{\cosh^2(x)}{2}$$

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x)
[Out] x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2 + cosh(x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sinh(x)(\cosh(x) + \sinh(x))\,dx = \frac{1}{2}\cosh(x)^2 - \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] 1/2*cosh(x)^2 - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \sinh(x)(\cosh(x) + \sinh(x))\,dx = -\frac{1}{2}x + \frac{1}{4}e^{(2x)}$$

```
[In] integrate(sinh(x)*(cosh(x)+sinh(x)),x, algorithm="giac")
[Out] -1/2*x + 1/4*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \sinh(x)(\cosh(x) + \sinh(x))\,dx = \frac{e^{2x}}{4} - \frac{x}{2}$$

```
[In] int(sinh(x)*(cosh(x) + sinh(x)),x)
[Out] exp(2*x)/4 - x/2
```

3.1044 $\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx=\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^2}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}$$

```
[Out] 1/4*ln(1-tanh(1/2*x))+3/4*ln(1+tanh(1/2*x))+1/2/(1-tanh(1/2*x))-1/2/(1+tanh
(1/2*x))^2+1/(1+tanh(1/2*x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4482, 12, 908}

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx=\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

```
[In] Int[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
```

```
[Out] Log[1 - Tanh[x/2]]/4 + (3*Log[1 + Tanh[x/2]])/4 + 1/(2*(1 - Tanh[x/2])) - 1
/(2*(1 + Tanh[x/2])^2) + (1 + Tanh[x/2])^(-1)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx\\
&= 2\text{Subst}\left(\int \frac{(1+x^2)^2}{2(1-x)^2(1+x)^3}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\
&= \text{Subst}\left(\int \frac{(1+x^2)^2}{(1-x)^2(1+x)^3}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}+\frac{1}{4(-1+x)}+\frac{1}{(1+x)^3}-\frac{1}{(1+x)^2}+\frac{3}{4(1+x)}\right)dx, x, \tanh\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\log\left(1-\tanh\left(\frac{x}{2}\right)\right)+\frac{3}{4}\log\left(1+\tanh\left(\frac{x}{2}\right)\right)\\
&\quad+\frac{1}{2\left(1-\tanh\left(\frac{x}{2}\right)\right)}-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^2}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx=\frac{x}{4}+\frac{\cosh(x)}{2}-\frac{1}{8}\cosh(2x)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sinh(2x)$$

```
[In] Integrate[(1 + Sinh[x]^2)/(1 + Cosh[x] + Sinh[x]),x]
[Out] x/4 + Cosh[x]/2 - Cosh[2*x]/8 - Log[Cosh[x/2]] + Sinh[2*x]/8
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{3x}{4}+\frac{\mathrm{e}^{x}}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2x}}{8}-\ln\left(1+\mathrm{e}^{x}\right)$	28
default	$-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}+\frac{3\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4}$	48

```
[In] int((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x+1/4*exp(x)+1/4*exp(-x)-1/8*exp(-2*x)-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{6\,x\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)^{3}+6\,\left(x+\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+2\,\sinh\left(x\right)^{3}-8\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\right.}{8\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\right.}$$

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
- 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1)
+ 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(51) = 102.

Time = 0.66 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.52

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = -\frac{x\tanh^3\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{x\tanh^2\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{x\tanh\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{x}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh^3\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh^2\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{4\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} + \frac{2\tanh^2\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{6\tanh\left(\frac{x}{2}\right)}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4} - \frac{4}{4\tanh^3\left(\frac{x}{2}\right)+4\tanh^2\left(\frac{x}{2}\right)-4\tanh\left(\frac{x}{2}\right)-4}$$

```
[In] integrate((1+sinh(x)**2)/(1+cosh(x)+sinh(x)),x)
```

```
[Out] -x*tanh(x/2)**3/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - x*tan
h(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x*tanh(x/2)
/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + x/(4*tanh(x/2)**3 +
4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(x/2)**3/(4*ta
nh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 4*log(tanh(x/2) + 1)*tanh(
x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*log(tanh(x/
2) + 1)*tanh(x/2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4*l
og(tanh(x/2) + 1)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) + 2*t
anh(x/2)**2/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 6*tanh(x/
2)/(4*tanh(x/2)**3 + 4*tanh(x/2)**2 - 4*tanh(x/2) - 4) - 4/(4*tanh(x/2)**3
+ 4*tanh(x/2)**2 - 4*tanh(x/2) - 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.42

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = -\frac{1}{4}\,x+\frac{1}{4}\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}+\frac{1}{4}\,e^{x}-\log\left(e^{(-x)}+1\right)$$

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="maxima")
[Out] -1/4*x + 1/4*e^(-x) - 1/8*e^(-2*x) + 1/4*e^x - log(e^(-x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{1}{8}\,(2\,e^{x}-1)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^{x}-\log\left(e^{x}+1\right)$$

```
[In] integrate((1+sinh(x)^2)/(1+cosh(x)+sinh(x)),x, algorithm="giac")
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{1+\sinh^2(x)}{1+\cosh(x)+\sinh(x)}\,dx = \frac{3\,x}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2\,x}}{8}-\ln\left(\mathrm{e}^{x}+1\right)+\frac{\mathrm{e}^{x}}{4}$$

```
[In] int((sinh(x)^2 + 1)/(cosh(x) + sinh(x) + 1),x)
[Out] (3*x)/4 + exp(-x)/4 - exp(-2*x)/8 - log(exp(x) + 1) + exp(x)/4
```

3.1045 $\int x^5 \cosh^7(a+bx^3) \sinh(a+bx^3)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int x^5 \cosh^7(a+bx^3) \sinh(a+bx^3)\, dx = -\frac{35x^3}{3072b} + \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{35\cosh(a+bx^3)\sinh(a+bx^3)}{3072b^2} - \frac{35\cosh^3(a+bx^3)\sinh(a+bx^3)}{4608b^2} - \frac{7\cosh^5(a+bx^3)\sinh(a+bx^3)}{1152b^2} - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2}$$

```
[Out] -35/3072*x^3/b+1/24*x^3*cosh(b*x^3+a)^8/b-35/3072*cosh(b*x^3+a)*sinh(b*x^3+
a)/b^2-35/4608*cosh(b*x^3+a)^3*sinh(b*x^3+a)/b^2-7/1152*cosh(b*x^3+a)^5*sin
h(b*x^3+a)/b^2-1/192*cosh(b*x^3+a)^7*sinh(b*x^3+a)/b^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

$= \{5481, 5429, 2715, 8\}$

$$\int x^5 \cosh^7(a+bx^3)\sinh(a+bx^3)\,dx = -\frac{\sinh(a+bx^3)\cosh^7(a+bx^3)}{192b^2} - \frac{7\sinh(a+bx^3)\cosh^5(a+bx^3)}{1152b^2} - \frac{35\sinh(a+bx^3)\cosh^3(a+bx^3)}{4608b^2} - \frac{35\sinh(a+bx^3)\cosh(a+bx^3)}{3072b^2} + \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{35x^3}{3072b}$$

```
[In] Int[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

```
[Out] (-35*x^3)/(3072*b) + (x^3*Cosh[a + b*x^3]^8)/(24*b) - (35*Cosh[a + b*x^3]*S
inh[a + b*x^3])/(3072*b^2) - (35*Cosh[a + b*x^3]^3*Sinh[a + b*x^3])/(4608*b
^2) - (7*Cosh[a + b*x^3]^5*Sinh[a + b*x^3])/(1152*b^2) - (Cosh[a + b*x^3]^7
*Sinh[a + b*x^3])/(192*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5481

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)
^(n_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Cosh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{\int x^2\cosh^8(a+bx^3)\,dx}{8b}\\
&= \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{\text{Subst}\left(\int \cosh^8(a+bx)\,dx, x, x^3\right)}{24b}\\
&= \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2} - \frac{7\text{Subst}\left(\int \cosh^6(a+bx)\,dx, x, x^3\right)}{192b}\\
&= \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{7\cosh^5(a+bx^3)\sinh(a+bx^3)}{1152b^2}\\
&\quad - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2} - \frac{35\text{Subst}\left(\int \cosh^4(a+bx)\,dx, x, x^3\right)}{1152b}\\
&= \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{35\cosh^3(a+bx^3)\sinh(a+bx^3)}{4608b^2}\\
&\quad - \frac{7\cosh^5(a+bx^3)\sinh(a+bx^3)}{1152b^2} - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2}\\
&\quad - \frac{35\text{Subst}\left(\int \cosh^2(a+bx)\,dx, x, x^3\right)}{1536b}\\
&= \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{35\cosh(a+bx^3)\sinh(a+bx^3)}{3072b^2} - \frac{35\cosh^3(a+bx^3)\sinh(a+bx^3)}{4608b^2}\\
&\quad - \frac{7\cosh^5(a+bx^3)\sinh(a+bx^3)}{1152b^2} - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2} - \frac{35\text{Subst}\left(\int 1\,dx, x, x^3\right)}{3072b}\\
&= -\frac{35x^3}{3072b} + \frac{x^3\cosh^8(a+bx^3)}{24b} - \frac{35\cosh(a+bx^3)\sinh(a+bx^3)}{3072b^2}\\
&\quad - \frac{35\cosh^3(a+bx^3)\sinh(a+bx^3)}{4608b^2}\\
&\quad - \frac{7\cosh^5(a+bx^3)\sinh(a+bx^3)}{1152b^2} - \frac{\cosh^7(a+bx^3)\sinh(a+bx^3)}{192b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int x^5\cosh^7(a+bx^3)\sinh(a+bx^3)\,dx$$
$$= \frac{1344bx^3\cosh(2(a+bx^3)) + 672bx^3\cosh(4(a+bx^3)) + 192bx^3\cosh(6(a+bx^3)) + 24bx^3\cosh(8(a+bx^3))}{73728b^2}$$

```
[In] Integrate[x^5*Cosh[a + b*x^3]^7*Sinh[a + b*x^3],x]
```

```
[Out] (1344*b*x^3*Cosh[2*(a + b*x^3)] + 672*b*x^3*Cosh[4*(a + b*x^3)] + 192*b*x^3
*Cosh[6*(a + b*x^3)] + 24*b*x^3*Cosh[8*(a + b*x^3)] - 672*Sinh[2*(a + b*x^3
)] - 168*Sinh[4*(a + b*x^3)] - 32*Sinh[6*(a + b*x^3)] - 3*Sinh[8*(a + b*x^3
)])/(73728*b^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.50

$$\frac{(8b\,x^3-1)\,\mathrm{e}^{8b\,x^3+8a}}{49152b^2}+\frac{(6b\,x^3-1)\,\mathrm{e}^{6b\,x^3+6a}}{4608b^2}+\frac{7(4b\,x^3-1)\,\mathrm{e}^{4b\,x^3+4a}}{6144b^2}+\frac{7(2b\,x^3-1)\,\mathrm{e}^{2b\,x^3+2a}}{1536b^2}+\frac{7(2b\,x^3+1)\,\mathrm{e}^{-2b\,x^3-}}{1536b^2}$$

```
[In] int(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x)
```

```
[Out] 1/49152*(8*b*x^3-1)/b^2*exp(8*b*x^3+8*a)+1/4608*(6*b*x^3-1)/b^2*exp(6*b*x^3
+6*a)+7/6144*(4*b*x^3-1)/b^2*exp(4*b*x^3+4*a)+7/1536*(2*b*x^3-1)/b^2*exp(2*
b*x^3+2*a)+7/1536*(2*b*x^3+1)/b^2*exp(-2*b*x^3-2*a)+7/6144*(4*b*x^3+1)/b^2*
exp(-4*b*x^3-4*a)+1/4608*(6*b*x^3+1)/b^2*exp(-6*b*x^3-6*a)+1/49152*(8*b*x^3
+1)/b^2*exp(-8*b*x^3-8*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(117) = 234.

Time = 0.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.07

$$\int x^5\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx = \frac{3\,bx^3\cosh\left(bx^3+a\right)^8+3\,bx^3\sinh\left(bx^3+a\right)^8+24\,bx^3\cosh\left(bx^3+a\right)^6+84\,bx^3\cosh\left(bx^3+a\right)^4-3\,\cosh\left(bx\right.}{}$$

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/9216*(3*b*x^3*cosh(b*x^3 + a)^8 + 3*b*x^3*sinh(b*x^3 + a)^8 + 24*b*x^3*co
sh(b*x^3 + a)^6 + 84*b*x^3*cosh(b*x^3 + a)^4 - 3*cosh(b*x^3 + a)*sinh(b*x^3
 + a)^7 + 12*(7*b*x^3*cosh(b*x^3 + a)^2 + 2*b*x^3)*sinh(b*x^3 + a)^6 + 168*
b*x^3*cosh(b*x^3 + a)^2 - 3*(7*cosh(b*x^3 + a)^3 + 8*cosh(b*x^3 + a))*sinh(
b*x^3 + a)^5 + 6*(35*b*x^3*cosh(b*x^3 + a)^4 + 60*b*x^3*cosh(b*x^3 + a)^2 +
 14*b*x^3)*sinh(b*x^3 + a)^4 - (21*cosh(b*x^3 + a)^5 + 80*cosh(b*x^3 + a)^3
 + 84*cosh(b*x^3 + a))*sinh(b*x^3 + a)^3 + 12*(7*b*x^3*cosh(b*x^3 + a)^6 +
30*b*x^3*cosh(b*x^3 + a)^4 + 42*b*x^3*cosh(b*x^3 + a)^2 + 14*b*x^3)*sinh(b*
x^3 + a)^2 - 3*(cosh(b*x^3 + a)^7 + 8*cosh(b*x^3 + a)^5 + 28*cosh(b*x^3 + a
)^3 + 56*cosh(b*x^3 + a))*sinh(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.87

$$\int x^5\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx = \begin{cases} -\frac{35x^3\sinh^8\left(a+bx^3\right)}{3072b}+\frac{35x^3\sinh^6\left(a+bx^3\right)\cosh^2\left(a+bx^3\right)}{768b}-\frac{35x^3\sinh^4\left(a+bx^3\right)\cosh^4\left(a+bx^3\right)}{512b}+\frac{35x^3\sinh^2\left(a+bx^3\right)\cosh^6\left(a+bx^3\right)}{768b}+ \\ \frac{x^6\sinh\left(a\right)\cosh^7\left(a\right)}{6} \end{cases}$$

```
[In] integrate(x**5*cosh(b*x**3+a)**7*sinh(b*x**3+a),x)
```

```
[Out] Piecewise((-35*x**3*sinh(a + b*x**3)**8/(3072*b) + 35*x**3*sinh(a + b*x**3)
**6*cosh(a + b*x**3)**2/(768*b) - 35*x**3*sinh(a + b*x**3)**4*cosh(a + b*x*
*3)**4/(512*b) + 35*x**3*sinh(a + b*x**3)**2*cosh(a + b*x**3)**6/(768*b) +
31*x**3*cosh(a + b*x**3)**8/(1024*b) + 35*sinh(a + b*x**3)**7*cosh(a + b*x*
*3)/(3072*b**2) - 385*sinh(a + b*x**3)**5*cosh(a + b*x**3)**3/(9216*b**2) +
 511*sinh(a + b*x**3)**3*cosh(a + b*x**3)**5/(9216*b**2) - 31*sinh(a + b*x*
*3)*cosh(a + b*x**3)**7/(1024*b**2), Ne(b, 0)), (x**6*sinh(a)*cosh(a)**7/6,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\begin{aligned}\int x^5\cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx &= \frac{\left(8\,bx^3e^{(8\,a)}-e^{(8\,a)}\right)e^{\left(8\,bx^3\right)}}{49152\,b^2} \\ &+ \frac{\left(6\,bx^3e^{(6\,a)}-e^{(6\,a)}\right)e^{\left(6\,bx^3\right)}}{4608\,b^2} \\ &+ \frac{7\left(4\,bx^3e^{(4\,a)}-e^{(4\,a)}\right)e^{\left(4\,bx^3\right)}}{6144\,b^2} \\ &+ \frac{7\left(2\,bx^3e^{(2\,a)}-e^{(2\,a)}\right)e^{\left(2\,bx^3\right)}}{1536\,b^2} \\ &+ \frac{7\,(2\,bx^3+1)e^{\left(-2\,bx^3-2\,a\right)}}{1536\,b^2} \\ &+ \frac{7\,(4\,bx^3+1)e^{\left(-4\,bx^3-4\,a\right)}}{6144\,b^2} \\ &+ \frac{(6\,bx^3+1)e^{\left(-6\,bx^3-6\,a\right)}}{4608\,b^2}+\frac{(8\,bx^3+1)e^{\left(-8\,bx^3-8\,a\right)}}{49152\,b^2}\end{aligned}$$

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/49152*(8*b*x^3*e^(8*a) - e^(8*a))*e^(8*b*x^3)/b^2 + 1/4608*(6*b*x^3*e^(6*
a) - e^(6*a))*e^(6*b*x^3)/b^2 + 7/6144*(4*b*x^3*e^(4*a) - e^(4*a))*e^(4*b*x
^3)/b^2 + 7/1536*(2*b*x^3*e^(2*a) - e^(2*a))*e^(2*b*x^3)/b^2 + 7/1536*(2*b*
x^3 + 1)*e^(-2*b*x^3 - 2*a)/b^2 + 7/6144*(4*b*x^3 + 1)*e^(-4*b*x^3 - 4*a)/b
^2 + 1/4608*(6*b*x^3 + 1)*e^(-6*b*x^3 - 6*a)/b^2 + 1/49152*(8*b*x^3 + 1)*e^
(-8*b*x^3 - 8*a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(117) = 234$.

Time = 0.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.99

$$\int x^5 \cosh^7\left(a+bx^3\right)\sinh\left(a+bx^3\right)\,dx = -\frac{a\left(e^{(2\,bx^3+2\,a)}+e^{(-2\,bx^3-2\,a)}\right)^4+8\,a\left(e^{(2\,bx^3+2\,a)}+e^{(-2\,bx^3-2\,a)}\right)^3+24\,a\left(e^{(2\,bx^3+2\,a)}+e^{(-2\,bx^3-2\,a)}\right)^2+32}{6144\,b^2} + \frac{24\,(bx^3+a)e^{(8\,bx^3+8\,a)}+192\,(bx^3+a)e^{(6\,bx^3+6\,a)}+672\,(bx^3+a)e^{(4\,bx^3+4\,a)}+1344\,(bx^3+a)e^{(2\,bx^3+2\,a)}+}{}$$

```
[In] integrate(x^5*cosh(b*x^3+a)^7*sinh(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/6144*(a*(e^(2*b*x^3 + 2*a) + e^(-2*b*x^3 - 2*a))^4 + 8*a*(e^(2*b*x^3 + 2
*a) + e^(-2*b*x^3 - 2*a))^3 + 24*a*(e^(2*b*x^3 + 2*a) + e^(-2*b*x^3 - 2*a))
^2 + 32*a*(e^(2*b*x^3 + 2*a) + e^(-2*b*x^3 - 2*a)))/b^2 + 1/147456*(24*(b*x
^3 + a)*e^(8*b*x^3 + 8*a) + 192*(b*x^3 + a)*e^(6*b*x^3 + 6*a) + 672*(b*x^3
+ a)*e^(4*b*x^3 + 4*a) + 1344*(b*x^3 + a)*e^(2*b*x^3 + 2*a) + 1344*(b*x^3 +
 a)*e^(-2*b*x^3 - 2*a) + 672*(b*x^3 + a)*e^(-4*b*x^3 - 4*a) + 192*(b*x^3 +
a)*e^(-6*b*x^3 - 6*a) + 24*(b*x^3 + a)*e^(-8*b*x^3 - 8*a) - 3*e^(8*b*x^3 +
8*a) - 32*e^(6*b*x^3 + 6*a) - 168*e^(4*b*x^3 + 4*a) - 672*e^(2*b*x^3 + 2*a)
 + 672*e^(-2*b*x^3 - 2*a) + 168*e^(-4*b*x^3 - 4*a) + 32*e^(-6*b*x^3 - 6*a)
+ 3*e^(-8*b*x^3 - 8*a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\int x^5 \cosh^7\left(a+bx^3\right) \sinh\left(a+bx^3\right)\,dx = \mathrm{e}^{-2\,b\,x^3-2\,a}\left(\frac{7}{1536\,b^2}+\frac{7\,x^3}{768\,b}\right) - \mathrm{e}^{2\,b\,x^3+2\,a}\left(\frac{7}{1536\,b^2}-\frac{7\,x^3}{768\,b}\right) + \mathrm{e}^{-6\,b\,x^3-6\,a}\left(\frac{1}{4608\,b^2}+\frac{x^3}{768\,b}\right) - \mathrm{e}^{6\,b\,x^3+6\,a}\left(\frac{1}{4608\,b^2}-\frac{x^3}{768\,b}\right) + \mathrm{e}^{-4\,b\,x^3-4\,a}\left(\frac{7}{6144\,b^2}+\frac{7\,x^3}{1536\,b}\right) - \mathrm{e}^{4\,b\,x^3+4\,a}\left(\frac{7}{6144\,b^2}-\frac{7\,x^3}{1536\,b}\right) + \mathrm{e}^{-8\,b\,x^3-8\,a}\left(\frac{1}{49152\,b^2}+\frac{x^3}{6144\,b}\right) - \mathrm{e}^{8\,b\,x^3+8\,a}\left(\frac{1}{49152\,b^2}-\frac{x^3}{6144\,b}\right)$$

```
[In] int(x^5*cosh(a + b*x^3)^7*sinh(a + b*x^3),x)
```

```
[Out] exp(- 2*a - 2*b*x^3)*(7/(1536*b^2) + (7*x^3)/(768*b)) - exp(2*a + 2*b*x^3)*
(7/(1536*b^2) - (7*x^3)/(768*b)) + exp(- 6*a - 6*b*x^3)*(1/(4608*b^2) + x^3
/(768*b)) - exp(6*a + 6*b*x^3)*(1/(4608*b^2) - x^3/(768*b)) + exp(- 4*a - 4
*b*x^3)*(7/(6144*b^2) + (7*x^3)/(1536*b)) - exp(4*a + 4*b*x^3)*(7/(6144*b^2
) - (7*x^3)/(1536*b)) + exp(- 8*a - 8*b*x^3)*(1/(49152*b^2) + x^3/(6144*b))
 - exp(8*a + 8*b*x^3)*(1/(49152*b^2) - x^3/(6144*b))
```

3.1046 $\int \frac{\cosh^2(x)}{1+e^x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{\cosh^2(x)}{1+e^x} \, dx = -\frac{1}{8}e^{-2x} + \frac{e^{-x}}{4} + \frac{e^x}{4} + \frac{3x}{4} - \log\left(1+e^x\right)$$

```
[Out] -1/8/exp(2*x)+1/4/exp(x)+1/4*exp(x)+3/4*x-ln(1+exp(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 908}

$$\int \frac{\cosh^2(x)}{1+e^x} \, dx = \frac{3x}{4} - \frac{e^{-2x}}{8} + \frac{e^{-x}}{4} + \frac{e^x}{4} - \log\left(e^x+1\right)$$

```
[In] Int[Cosh[x]^2/(1 + E^x),x]
```

```
[Out] -1/8*1/E^(2*x) + 1/(4*E^x) + E^x/4 + (3*x)/4 - Log[1 + E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x^2)^2}{4x^3(1+x)}\,dx, x, e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3(1+x)}\,dx, x, e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{1}{x^3}-\frac{1}{x^2}+\frac{3}{x}-\frac{4}{1+x}\right)\,dx, x, e^x\right)\\
&= -\frac{1}{8}e^{-2x}+\frac{e^{-x}}{4}+\frac{e^x}{4}+\frac{3x}{4}-\log\left(1+e^x\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{1}{4}\left(-\frac{1}{2}e^{-2x}+e^{-x}+e^x+3x-4\log\left(1+e^x\right)\right)$$

```
[In] Integrate[Cosh[x]^2/(1 + E^x),x]
[Out] (-1/2*1/E^(2*x) + E^(-x) + E^x + 3*x - 4*Log[1 + E^x])/4
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{3x}{4}+\frac{\mathrm{e}^{x}}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2x}}{8}-\ln\left(1+\mathrm{e}^{x}\right)$	28
default	$-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{1+\tanh\left(\frac{x}{2}\right)}+\frac{3\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{4}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4}$	48

```
[In] int(cosh(x)^2/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] 3/4*x+1/4*exp(x)+1/4*exp(-x)-1/8*exp(-2*x)-ln(1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx$$
$$= \frac{6\,x\cosh{(x)}^2 + 2\,\cosh{(x)}^3 + 6\,(x+\cosh{(x)})\sinh{(x)}^2 + 2\,\sinh{(x)}^3 - 8\,\left(\cosh{(x)}^2 + 2\,\cosh{(x)}\sinh{(x)} + \right.}{8\,\left(\cosh{(x)}^2 + 2\,\cosh{(x)}\right.}$$

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="fricas")
[Out] 1/8*(6*x*cosh(x)^2 + 2*cosh(x)^3 + 6*(x + cosh(x))*sinh(x)^2 + 2*sinh(x)^3
- 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*log(cosh(x) + sinh(x) + 1)
+ 2*(6*x*cosh(x) + 3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x) - 1)/(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \int \frac{\cosh^2{(x)}}{e^x+1}\,dx$$

```
[In] integrate(cosh(x)**2/(1+exp(x)),x)
[Out] Integral(cosh(x)**2/(exp(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{1}{8}\,(2\,e^x-1)e^{(-2\,x)} + \frac{3}{4}\,x + \frac{1}{4}\,e^x - \log{(e^x+1)}$$

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="maxima")
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{1}{8}\,(2\,e^x-1)e^{(-2\,x)}+\frac{3}{4}\,x+\frac{1}{4}\,e^x-\log\left(e^x+1\right)$$

```
[In] integrate(cosh(x)^2/(1+exp(x)),x, algorithm="giac")
[Out] 1/8*(2*e^x - 1)*e^(-2*x) + 3/4*x + 1/4*e^x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(x)}{1+e^x}\,dx = \frac{3\,x}{4}+\frac{\mathrm{e}^{-x}}{4}-\frac{\mathrm{e}^{-2\,x}}{8}-\ln\left(\mathrm{e}^x+1\right)+\frac{\mathrm{e}^x}{4}$$

```
[In] int(cosh(x)^2/(exp(x) + 1),x)
[Out] (3*x)/4 + exp(-x)/4 - exp(-2*x)/8 - log(exp(x) + 1) + exp(x)/4
```

3.1047 $\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx=-\frac{4}{5}(1+\operatorname{sech}(x))^{5/2}+\frac{2}{7}(1+\operatorname{sech}(x))^{7/2}$$

```
[Out] -4/5*(1+sech(x))^(5/2)+2/7*(1+sech(x))^(7/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4458, 1584, 1483, 641, 45}

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx=\frac{2}{7}(\operatorname{sech}(x)+1)^{7/2}-\frac{4}{5}(\operatorname{sech}(x)+1)^{5/2}$$

```
[In] Int[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
```

```
[Out] (-4*(1 + Sech[x])^(5/2))/5 + (2*(1 + Sech[x])^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1584

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4458

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}(1-x^2)}{x^4}\,dx, x, \cosh(x)\right)\\
&= -\text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \cosh(x)\right)\\
&= \text{Subst}\left(\int \sqrt{1+x}\left(-1+x^2\right)\,dx, x, \text{sech}(x)\right)\\
&= \text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \text{sech}(x)\right)\\
&= \text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, \text{sech}(x)\right)\\
&= -\frac{4}{5}(1+\text{sech}(x))^{5/2}+\frac{2}{7}(1+\text{sech}(x))^{7/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = -\frac{8}{35}\cosh^4\left(\frac{x}{2}\right)(-5+9\cosh(x))\operatorname{sech}^3(x)\sqrt{1+\operatorname{sech}(x)}$$

```
[In] Integrate[Sech[x]*Sqrt[1 + Sech[x]]*Tanh[x]^3,x]
[Out] (-8*Cosh[x/2]^4*(-5 + 9*Cosh[x])*Sech[x]^3*Sqrt[1 + Sech[x]])/35
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$-\frac{4(1+\operatorname{sech}(x))^{\frac{5}{2}}}{5}+\frac{2(1+\operatorname{sech}(x))^{\frac{7}{2}}}{7}$	18
default	$-\frac{4(1+\operatorname{sech}(x))^{\frac{5}{2}}}{5}+\frac{2(1+\operatorname{sech}(x))^{\frac{7}{2}}}{7}$	18

```
[In] int(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x,method=_RETURNVERBOSE)
[Out] -4/5*(1+sech(x))^(5/2)+2/7*(1+sech(x))^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 431, normalized size of antiderivative = 17.24

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx =$$
$$-\frac{2\left(9\cosh(x)^6+54\cosh(x)\sinh(x)^5+9\sinh(x)^6+27\left(5\cosh(x)^2+1\right)\sinh(x)^4+27\cosh(x)^4+3\right.}{}$$

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="fricas")
[Out] -2/35*(9*cosh(x)^6 + 54*cosh(x)*sinh(x)^5 + 9*sinh(x)^6 + 27*(5*cosh(x)^2 +
 1)*sinh(x)^4 + 27*cosh(x)^4 + 36*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 27*
(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 27*cosh(x)^2 + 54*(cosh(x)^5 +
2*cosh(x)^3 + cosh(x))*sinh(x) + (9*cosh(x)^7 + 7*(9*cosh(x) + 5)*sinh(x)^6
 + 9*sinh(x)^7 + 35*cosh(x)^6 + 7*(27*cosh(x)^2 + 30*cosh(x) + 7)*sinh(x)^5
 + 49*cosh(x)^5 + 35*(9*cosh(x)^3 + 15*cosh(x)^2 + 7*cosh(x) + 1)*sinh(x)^4
 + 35*cosh(x)^4 + 35*(9*cosh(x)^4 + 20*cosh(x)^3 + 14*cosh(x)^2 + 4*cosh(x)
```

```
 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 7*(27*cosh(x)^5 + 75*cosh(x)^4 + 70*cosh(x
)^3 + 30*cosh(x)^2 + 15*cosh(x) + 7)*sinh(x)^2 + 49*cosh(x)^2 + 7*(9*cosh(x
)^6 + 30*cosh(x)^5 + 35*cosh(x)^4 + 20*cosh(x)^3 + 15*cosh(x)^2 + 14*cosh(x
) + 5)*sinh(x) + 35*cosh(x) + 9)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 + 1) + 9)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^
2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*
(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*
cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = \int \sqrt{\operatorname{sech}(x)+1}\tanh^3(x)\operatorname{sech}(x)\,dx$$

```
[In] integrate(sech(x)*(1+sech(x))**(1/2)*tanh(x)**3,x)
[Out] Integral(sqrt(sech(x) + 1)*tanh(x)**3*sech(x), x)
```

Maxima [F]

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = \int \sqrt{\operatorname{sech}(x)+1}\operatorname{sech}(x)\tanh(x)^3\,dx$$

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="maxima")
[Out] integrate(sqrt(sech(x) + 1)*sech(x)*tanh(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = -\frac{2\left(((((((9e^x+35)e^x+49)e^x+35)e^x+35)e^x+49)e^x+35)e^x+9\right)}{35\left(e^{(2x)}+1\right)^{\frac{7}{2}}}$$

```
[In] integrate(sech(x)*(1+sech(x))^(1/2)*tanh(x)^3,x, algorithm="giac")
[Out] -2/35*(((((((9*e^x + 35)*e^x + 49)*e^x + 35)*e^x + 35)*e^x + 49)*e^x + 35)*
e^x + 9)/(e^(2*x) + 1)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.92

$$\int \operatorname{sech}(x)\sqrt{1+\operatorname{sech}(x)}\tanh^3(x)\,dx = \frac{\left(\frac{72\,e^x}{35}-\frac{24}{5}\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{(e^x+1)\,(e^{2x}+1)^2} - \frac{\left(\frac{16\,e^x}{7}-\frac{16}{7}\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{(e^x+1)\,(e^{2x}+1)^3} - \frac{\left(\frac{44\,e^x}{35}-4\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{(e^x+1)\,(e^{2x}+1)} - \frac{\left(\frac{18\,e^x}{35}+2\right)\sqrt{\frac{1}{\frac{e^{-x}}{2}+\frac{e^x}{2}}+1}}{e^x+1}$$

```
[In] int((tanh(x)^3*(1/cosh(x) + 1)^(1/2))/cosh(x),x)
```

```
[Out] (((72*exp(x))/35 - 24/5)*(1/(exp(-x)/2 + exp(x)/2) + 1)^(1/2))/((exp(x) + 1
)*(exp(2*x) + 1)^2) - (((16*exp(x))/7 - 16/7)*(1/(exp(-x)/2 + exp(x)/2) + 1
)^(1/2))/((exp(x) + 1)*(exp(2*x) + 1)^3) - (((44*exp(x))/35 - 4)*(1/(exp(-x
)/2 + exp(x)/2) + 1)^(1/2))/((exp(x) + 1)*(exp(2*x) + 1)) - (((18*exp(x))/3
5 + 2)*(1/(exp(-x)/2 + exp(x)/2) + 1)^(1/2))/(exp(x) + 1)
```

3.1048 $\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = -\frac{4}{3}(1+\operatorname{csch}(x))^{3/2} + \frac{4}{5}(1+\operatorname{csch}(x))^{5/2} - \frac{2}{7}(1+\operatorname{csch}(x))^{7/2}$$

```
[Out] -4/3*(1+csch(x))^(3/2)+4/5*(1+csch(x))^(5/2)-2/7*(1+csch(x))^(7/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4457, 1584, 1483, 711}

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = -\frac{2}{7}(\operatorname{csch}(x)+1)^{7/2} + \frac{4}{5}(\operatorname{csch}(x)+1)^{5/2} - \frac{4}{3}(\operatorname{csch}(x)+1)^{3/2}$$

```
[In] Int[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
```

```
[Out] (-4*(1 + Csch[x])^(3/2))/3 + (4*(1 + Csch[x])^(5/2))/5 - (2*(1 + Csch[x])^(
7/2))/7
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1584

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4457

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(
1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}(1+x^2)}{x^4}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \frac{\left(1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \sqrt{1+x}(1+x^2)\,dx, x, \text{csch}(x)\right)\\
&= -\text{Subst}\left(\int \left(2\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, \text{csch}(x)\right)\\
&= -\frac{4}{3}(1+\text{csch}(x))^{3/2}+\frac{4}{5}(1+\text{csch}(x))^{5/2}-\frac{2}{7}(1+\text{csch}(x))^{7/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \coth^3(x)\text{csch}(x)\sqrt{1+\text{csch}(x)}\,dx = -\frac{1}{210}\text{csch}^3(x)\sqrt{1+\text{csch}(x)}(-2+62\cosh(2x) - 117\sinh(x)+43\sinh(3x))$$

```
[In] Integrate[Coth[x]^3*Csch[x]*Sqrt[1 + Csch[x]],x]
[Out] -1/210*(Csch[x]^3*Sqrt[1 + Csch[x]]*(-2 + 62*Cosh[2*x] - 117*Sinh[x] + 43*S
inh[3*x]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4(1+\text{csch}(x))^{\frac{3}{2}}}{3}+\frac{4(1+\text{csch}(x))^{\frac{5}{2}}}{5}-\frac{2(1+\text{csch}(x))^{\frac{7}{2}}}{7}$	26
default	$-\frac{4(1+\text{csch}(x))^{\frac{3}{2}}}{3}+\frac{4(1+\text{csch}(x))^{\frac{5}{2}}}{5}-\frac{2(1+\text{csch}(x))^{\frac{7}{2}}}{7}$	26

```
[In] int(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -4/3*(1+csch(x))^(3/2)+4/5*(1+csch(x))^(5/2)-2/7*(1+csch(x))^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 7.32

$$\int \coth^3(x)\text{csch}(x)\sqrt{1+\text{csch}(x)}\,dx = -\frac{2\left(43\cosh(x)^6+2\,(129\cosh(x)+31)\sinh(x)^5+43\sinh(x)^6+62\cosh(x)^5+\left(645\cosh(x)^2+310\,\text{c}\right.}{105}$$

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="fricas")
[Out] -2/105*(43*cosh(x)^6 + 2*(129*cosh(x) + 31)*sinh(x)^5 + 43*sinh(x)^6 + 62*c
osh(x)^5 + (645*cosh(x)^2 + 310*cosh(x) - 117)*sinh(x)^4 - 117*cosh(x)^4 +
4*(215*cosh(x)^3 + 155*cosh(x)^2 - 117*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3
 + (645*cosh(x)^4 + 620*cosh(x)^3 - 702*cosh(x)^2 - 12*cosh(x) + 117)*sinh(
x)^2 + 117*cosh(x)^2 + 2*(129*cosh(x)^5 + 155*cosh(x)^4 - 234*cosh(x)^3 - 6
*cosh(x)^2 + 117*cosh(x) + 31)*sinh(x) + 62*cosh(x) - 43)*sqrt((sinh(x) + 1
```

```
)/sinh(x))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 -
1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*c
osh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh
(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [F]

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = \int \sqrt{\operatorname{csch}(x)+1}\coth^3(x)\operatorname{csch}(x)\,dx$$

```
[In] integrate(coth(x)**3*csch(x)*(1+csch(x))**(1/2),x)
[Out] Integral(sqrt(csch(x) + 1)*coth(x)**3*csch(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 10.51

$$\begin{aligned}
&\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx \\
&= \frac{124\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)} \\
&\quad - \frac{78\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-2x)}}{35\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)} \\
&\quad - \frac{8\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-3x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)} \\
&\quad + \frac{78\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-4x)}}{35\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)} \\
&\quad + \frac{124\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-5x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)} \\
&\quad - \frac{86\sqrt{-2e^{(-x)}+e^{(-2x)}-1}e^{(-6x)}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)} \\
&\quad + \frac{86\sqrt{-2e^{(-x)}+e^{(-2x)}-1}}{105\sqrt{e^{(-x)}+1}\sqrt{e^{(-x)}-1}(3e^{(-2x)}-3e^{(-4x)}+e^{(-6x)}-1)}
\end{aligned}$$

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="maxima")
[Out] 124/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x)
 - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) - 78/35*sqrt(-2*e^(-x) + e^
```

```
(-2*x) - 1)*e^(-2*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(
-4*x) + e^(-6*x) - 1)) - 8/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-3*x)/(sqr
t(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) +
78/35*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-4*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x)
 - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)) + 124/105*sqrt(-2*e^(-x) +
e^(-2*x) - 1)*e^(-5*x)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e
^(-4*x) + e^(-6*x) - 1)) - 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)*e^(-6*x)/(
sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
 + 86/105*sqrt(-2*e^(-x) + e^(-2*x) - 1)/(sqrt(e^(-x) + 1)*sqrt(e^(-x) - 1)
*(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1))
```

Giac [F]

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = \int \sqrt{\operatorname{csch}(x)+1}\coth(x)^3\operatorname{csch}(x)\,dx$$

```
[In] integrate(coth(x)^3*csch(x)*(1+csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(x) + 1)*coth(x)^3*csch(x), x)
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.59

$$\int \coth^3(x)\operatorname{csch}(x)\sqrt{1+\operatorname{csch}(x)}\,dx = -\frac{8\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}}}{35\,(e^{2x}-1)} - \frac{8\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}}}{35\,(e^{4x}-2\,e^{2x}+1)} - \frac{86\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}}}{105} - \frac{16\,e^x\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}}}{7\,(3\,e^{2x}-3\,e^{4x}+e^{6x}-1)} - \frac{16\,e^x\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}}}{7\,(e^{4x}-2\,e^{2x}+1)} - \frac{124\,e^x\sqrt{1-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}}}{105\,(e^{2x}-1)}$$

```
[In] int((coth(x)^3*(1/sinh(x) + 1)^(1/2))/sinh(x),x)
```

```
[Out] - (8*(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(35*(exp(2*x) - 1)) - (8*(1 - 1/
(exp(-x)/2 - exp(x)/2))^(1/2))/(35*(exp(4*x) - 2*exp(2*x) + 1)) - (86*(1 -
1/(exp(-x)/2 - exp(x)/2))^(1/2))/105 - (16*exp(x)*(1 - 1/(exp(-x)/2 - exp(x
)/2))^(1/2))/(7*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (16*exp(x)*(1 -
 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(7*(exp(4*x) - 2*exp(2*x) + 1)) - (124*ex
p(x)*(1 - 1/(exp(-x)/2 - exp(x)/2))^(1/2))/(105*(exp(2*x) - 1))
```

3.1049 $\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh^x(x)$$

```
[Out] cosh(x)^x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6874, 2633}

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh^x(x)$$

```
[In] Int[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
```

```
[Out] Cosh[x]^x
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\cosh^x(x)\log(\cosh(x)) + x\cosh^{-1+x}(x)\sinh(x)\right)\,dx\\ &= \int \cosh^x(x)\log(\cosh(x))\,dx + \int x\cosh^{-1+x}(x)\sinh(x)\,dx\\ &= \cosh^x(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh^x(x)$$

```
[In] Integrate[Cosh[x]^x*(Log[Cosh[x]] + x*Tanh[x]),x]
[Out] Cosh[x]^x
```

Maple [A] (verified)

Time = 12.68 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\cosh(x)^x$	5
default	$\cosh(x)^x$	5

```
[In] int(cosh(x)^x*(ln(cosh(x))+x*tanh(x)),x,method=_RETURNVERBOSE)
[Out] cosh(x)^x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh(x\log(\cosh(x))) + \sinh(x\log(\cosh(x)))$$

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="fricas")
[Out] cosh(x*log(cosh(x))) + sinh(x*log(cosh(x)))
```

Sympy [F]

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \int (x\tanh(x) + \log(\cosh(x)))\cosh^x(x)\,dx$$

```
[In] integrate(cosh(x)**x*(ln(cosh(x))+x*tanh(x)),x)
[Out] Integral((x*tanh(x) + log(cosh(x)))*cosh(x)**x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(4) = 8$.

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 5.25

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = e^{\left(-x^2 - x\log(2) + x\log\left(e^{(2x)}+1\right)\right)}$$

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="maxima")
[Out] e^(-x^2 - x*log(2) + x*log(e^(2*x) + 1))
```

Giac [F]

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \int (x\tanh(x) + \log(\cosh(x)))\cosh(x)^x\,dx$$

```
[In] integrate(cosh(x)^x*(log(cosh(x))+x*tanh(x)),x, algorithm="giac")
[Out] integrate((x*tanh(x) + log(cosh(x)))*cosh(x)^x, x)
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cosh^x(x)(\log(\cosh(x)) + x\tanh(x))\,dx = \cosh(x)^x$$

```
[In] int(cosh(x)^x*(log(cosh(x)) + x*tanh(x)),x)
[Out] cosh(x)^x
```

3.1050 $\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\frac{\left(e^{c+dx}\right)^nF^{a+bx}}{dn+b\log(F)}$$

```
[Out] exp(d*x+c)^n*F^(b*x+a)/(d*n+b*ln(F))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5767, 2319, 2325, 2225}

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\frac{F^{a+bx}\left(e^{c+dx}\right)^n}{b\log(F)+dn}$$

```
[In] Int[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
```

```
[Out] ((E^(c + d*x))^n*F^(a + b*x))/(d*n + b*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5767

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{c+dx}\right)^n F^{a+bx}\,dx \\
&= \left(e^{-n(c+dx)}\left(e^{c+dx}\right)^n\right)\int e^{n(c+dx)}F^{a+bx}\,dx \\
&= \left(e^{-n(c+dx)}\left(e^{c+dx}\right)^n\right)\int e^{cn+a\log(F)+x(dn+b\log(F))}\,dx \\
&= \frac{\left(e^{c+dx}\right)^n F^{a+bx}}{dn+b\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx = \frac{F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n}{dn+b\log(F)}$$

```
[In] Integrate[F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n,x]
[Out] (F^(a + b*x)*(Cosh[c + d*x] + Sinh[c + d*x])^n)/(d*n + b*Log[F])
```

Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{n(dx+c)}F^{bx+a}}{dn+b\ln(F)}$	27
gosper	$\frac{F^{bx+a}(\cosh(dx+c)+\sinh(dx+c))^n}{dn+b\ln(F)}$	34

```
[In] int(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*n+b*ln(F))*exp(n*(d*x+c))*F^(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx = \frac{(\cosh(dnx+cn)+\sinh(dnx+cn))\cosh((bx+a)\log(F))+(\cosh(dnx+cn)+\sinh(dnx+cn))\sinh((b}{dn+b\log(F)}$$

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((cosh(d*n*x + c*n) + sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*n
*x + c*n) + sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n + b*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx = \begin{cases} \frac{F^{a+bx}(\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)+dn} & \text{for } b \neq -\frac{dn}{\log(F)} \\ F^{a-\frac{dnx}{\log(F)}}x(\sinh(c+dx)+\cosh(c+dx))^n & \text{otherwise} \end{cases}$$

```
[In] integrate(F**(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((F**(a + b*x)*(sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) + d*n)
, Ne(b, -d*n/log(F))), (F**(a - d*n*x/log(F))*x*(sinh(c + d*x) + cosh(c + d
*x))**n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx = \frac{F^a e^{(dnx+bx\log(F)+cn)}}{dn+b\log(F)}$$

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] F^a*e^(d*n*x + b*x*log(F) + c*n)/(d*n + b*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 10.15

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx$$
$$=2\left(\frac{2\,(dn+b\log{(|F|)})\cos{\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(F)+\frac{1}{2}\,\pi a\right)}}{(\pi b\mathrm{sgn}\,(F)-\pi b)^2+4\,(dn+b\log{(|F|)})^2}-\frac{(\pi b\mathrm{sgn}(F)-\pi b)\sin{\left(-\frac{1}{2}\,\pi bx\right.}}{(\pi b\mathrm{sgn}\,(F)-\pi}\right.$$
$$+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\mathrm{sgn}(F)-\frac{1}{2}i\,\pi a\right)}}{i\,\pi b\mathrm{sgn}\,(F)-i\,\pi b+2\,dn+2\,b\log{(|F|)}}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\mathrm{sgn}(F)+\frac{1}{2}i\,\pi a\right)}}{-i\,\pi b\mathrm{sgn}\,(F)+i\,\pi b+2\,dn+2\,b\log{(|F|)}}\right)e^{(cn+(dn+b\log(|}$$

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)+sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] 2*(2*(d*n + b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*s
gn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2) - (p
i*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F) +
1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n + b*log(abs(F)))^2))*e^(c*n + (d
*n + b*log(abs(F)))*x + a*log(abs(F))) + I*(I*e^(1/2*I*pi*b*x*sgn(F) - 1/2*
I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b + 2*d*n
+ 2*b*log(abs(F))) - I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/2*I*pi*a*
sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b + 2*d*n + 2*b*log(abs(F))))*e
^(c*n + (d*n + b*log(abs(F)))*x + a*log(abs(F)))
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+bx}(\cosh(c+dx)+\sinh(c+dx))^n\,dx=\int F^{a+b\,x}\,{\left(\cosh(c+d\,x)+\sinh(c+d\,x)\right)}^n\,dx$$

```
[In] int(F^(a + b*x)*(cosh(c + d*x) + sinh(c + d*x))^n,x)
```

```
[Out] int(F^(a + b*x)*(cosh(c + d*x) + sinh(c + d*x))^n, x)
```

3.1051 $\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx=-\frac{\left(e^{-c-dx}\right)^n F^{a+bx}}{dn-b\log(F)}$$

```
[Out] -exp(-d*x-c)^n*F^(b*x+a)/(d*n-b*ln(F))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5767, 2319, 2325, 2225}

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx=-\frac{F^{a+bx}\left(e^{-c-dx}\right)^n}{dn-b\log(F)}$$

```
[In] Int[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
```

```
[Out] -(((E^(-c - d*x))^n*F^(a + b*x))/(d*n - b*Log[F]))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5767

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{-c-dx}\right)^n F^{a+bx}\,dx \\
&= \left(e^{-n(-c-dx)}\left(e^{-c-dx}\right)^n\right)\int e^{n(-c-dx)}F^{a+bx}\,dx \\
&= \left(e^{-n(-c-dx)}\left(e^{-c-dx}\right)^n\right)\int e^{-cn+a\log(F)-x(dn-b\log(F))}\,dx \\
&= -\frac{\left(e^{-c-dx}\right)^n F^{a+bx}}{dn-b\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = -\frac{F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n}{dn-b\log(F)}$$

```
[In] Integrate[F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n,x]
[Out] -((F^(a + b*x)*(Cosh[c + d*x] - Sinh[c + d*x])^n)/(d*n - b*Log[F]))
```

Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{F^{bx+a}(\cosh(dx+c)-\sinh(dx+c))^n}{b\ln(F)-dn}$	37

```
[In] int(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] 1/(b*ln(F)-d*n)*F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.38

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = -\frac{(\cosh(dnx+cn)-\sinh(dnx+cn))\cosh((bx+a)\log(F))+(\cosh(dnx+cn)-\sinh(dnx+cn))\sinh}{dn-b\log(F)}$$

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -((cosh(d*n*x + c*n) - sinh(d*n*x + c*n))*cosh((b*x + a)*log(F)) + (cosh(d*
n*x + c*n) - sinh(d*n*x + c*n))*sinh((b*x + a)*log(F)))/(d*n - b*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = \begin{cases} \frac{F^{a+bx}(-\sinh(c+dx)+\cosh(c+dx))^n}{b\log(F)-dn} & \text{for } b\neq\frac{dn}{\log(F)} \\ F^{a+\frac{dnx}{\log(F)}}x(-\sinh(c+dx)+\cosh(c+dx))^n & \text{otherwise} \end{cases}$$

```
[In] integrate(F**(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))**n,x)
```

```
[Out] Piecewise((F**(a + b*x)*(-sinh(c + d*x) + cosh(c + d*x))**n/(b*log(F) - d*n
), Ne(b, d*n/log(F))), (F**(a + d*n*x/log(F))*x*(-sinh(c + d*x) + cosh(c +
d*x))**n, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = -\frac{F^a e^{(-dnx+bx\log(F))}}{dne^{(cn)}-be^{(cn)}\log(F)}$$

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -F^a*e^(-d*n*x + b*x*log(F))/(d*n*e^(c*n) - b*e^(c*n)*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 8.81

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = -2\left(\frac{2\,(dn-b\log(|F|))\cos\left(-\frac{1}{2}\,\pi bx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\mathrm{sgn}(F)+\frac{1}{2}\,\pi a\right)}{(\pi b\mathrm{sgn}\,(F)-\pi b)^2+4\,(dn-b\log(|F|))^2}+\frac{(\pi b\mathrm{sgn}(F)-\pi b)\sin\left(-\frac{1}{2}\,\pi\right.}{(\pi b\mathrm{sgn}\,(F)-}\right. \\ +i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\mathrm{sgn}(F)-\frac{1}{2}i\,\pi a\right)}}{i\,\pi b\mathrm{sgn}\,(F)-i\,\pi b-2\,dn+2\,b\log(|F|)}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\mathrm{sgn}(F)+\frac{1}{2}i\,\pi a\right)}}{-i\,\pi b\mathrm{sgn}\,(F)+i\,\pi b-2\,dn+2\,b\log(|F|)}\right)e^{(-cn-(dn-b\log(}$$

```
[In] integrate(F^(b*x+a)*(cosh(d*x+c)-sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -2*(2*(d*n - b*log(abs(F)))*cos(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*
sgn(F) + 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2) + (
pi*b*sgn(F) - pi*b)*sin(-1/2*pi*b*x*sgn(F) + 1/2*pi*b*x - 1/2*pi*a*sgn(F) +
 1/2*pi*a)/((pi*b*sgn(F) - pi*b)^2 + 4*(d*n - b*log(abs(F)))^2))*e^(-c*n -
(d*n - b*log(abs(F)))*x + a*log(abs(F))) + I*(I*e^(1/2*I*pi*b*x*sgn(F) - 1/
2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(I*pi*b*sgn(F) - I*pi*b - 2*d*
n + 2*b*log(abs(F))) - I*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/2*I*pi*
a*sgn(F) + 1/2*I*pi*a)/(-I*pi*b*sgn(F) + I*pi*b - 2*d*n + 2*b*log(abs(F))))
*e^(-c*n - (d*n - b*log(abs(F)))*x + a*log(abs(F)))
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+bx}(\cosh(c+dx)-\sinh(c+dx))^n\,dx = \int F^{a+b\,x}\,(\cosh(c+d\,x)-\sinh(c+d\,x))^n\,dx$$

```
[In] int(F^(a + b*x)*(cosh(c + d*x) - sinh(c + d*x))^n,x)
```

```
[Out] int(F^(a + b*x)*(cosh(c + d*x) - sinh(c + d*x))^n, x)
```

3.1052 $\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 51

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} + \frac{\arctan\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tanh(b*x+a))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*tanh(b*x+
a))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1176, 631, 210}

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\arctan\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}b} - \frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

```
[In] Int[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b*x]^4)
,x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b)) + ArcTan[1 + Sqrt[2]*Tanh[
a + b*x]]/(Sqrt[2]*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}\\
&= -\frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}+\frac{\arctan\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx=\frac{\arctan\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

```
[In] Integrate[(Cosh[a + b*x]^4 - Sinh[a + b*x]^4)/(Cosh[a + b*x]^4 + Sinh[a + b
*x]^4),x]
[Out] ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

method	result
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4bx+4a}+2i\sqrt{2}\,\mathrm{e}^{2bx+2a}-1\right)}{4b}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4bx+4a}-2i\sqrt{2}\,\mathrm{e}^{2bx+2a}-1\right)}{4b}$
derivativedivides	$\frac{\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\right.\right.}{}}{b}$
default	$\frac{\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\right.\right.}{}}{b}$

```
[In] int((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/4*I*2^(1/2)/b*ln(exp(4*b*x+4*a)+2*I*2^(1/2)*exp(2*b*x+2*a)-1)-1/4*I*2^(1/
2)/b*ln(exp(4*b*x+4*a)-2*I*2^(1/2)*exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.76

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = -\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^{3}+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^{2}+\sqrt{2}\sinh(bx+a)^{3}+\left(3\sqrt{2}\cosh(bx+a)^{2}-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+}{4\left(\cosh(bx+a)^{3}-3\cosh(bx+a)^{2}\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^{2}-\sinh(bx+a)^{3}\right)}\right.}{2b}$$

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a
)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 -
7*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*co
sh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x +
a)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a)
)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \begin{cases} -x & \text{for } a=\frac{i\pi}{2} \wedge b=0 \\ \frac{x(-\sinh^4(a)+\cosh^4(a))}{\sinh^4(a)+\cosh^4(a)} & \text{for } b=0 \\ -x & \text{for } a=-bx+\frac{i\pi}{2} \\ \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sinh(a+bx)}{\cosh(a+bx)}-1\right)}{2b}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sinh(a+bx)}{\cosh(a+bx)}+1\right)}{2b} & \text{otherwise} \end{cases}$$

```
[In] integrate((cosh(b*x+a)**4-sinh(b*x+a)**4)/(cosh(b*x+a)**4+sinh(b*x+a)**4),x
)
```

```
[Out] Piecewise((-x, Eq(b, 0) & Eq(a, I*pi/2)), (x*(-sinh(a)**4 + cosh(a)**4)/(si
nh(a)**4 + cosh(a)**4), Eq(b, 0)), (-x, Eq(a, -b*x + I*pi/2)), (sqrt(2)*ata
n(sqrt(2)*sinh(a + b*x)/cosh(a + b*x) - 1)/(2*b) + sqrt(2)*atan(sqrt(2)*sin
h(a + b*x)/cosh(a + b*x) + 1)/(2*b), True))
```

Maxima [F]

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \int \frac{\cosh(bx+a)^4-\sinh(bx+a)^4}{\cosh(bx+a)^4+\sinh(bx+a)^4}\,dx$$

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="maxima")
```

```
[Out] 2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x - 4
*a) + e^(-8*b*x - 8*a) + 1), x) + 2*integrate(e^(6*b*x + 6*a)/(e^(8*b*x + 8
*a) + 6*e^(4*b*x + 4*a) + 1), x) + 2*integrate(e^(-6*b*x - 6*a)/(6*e^(-4*b*
x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx=\frac{\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(e^{(4\,bx+4\,a)}-1\right)e^{(-2\,bx-2\,a)}\right)}{2\,b}$$

```
[In] integrate((cosh(b*x+a)^4-sinh(b*x+a)^4)/(cosh(b*x+a)^4+sinh(b*x+a)^4),x, al
gorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/4*sqrt(2)*(e^(4*b*x + 4*a) - 1)*e^(-2*b*x - 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{\cosh^4(a+bx)-\sinh^4(a+bx)}{\cosh^4(a+bx)+\sinh^4(a+bx)}\,dx = \frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,e^{2\,a}\,e^{2\,b\,x}\,\sqrt{b^2}}{4\,b}\right)+\mathrm{atan}\left(\frac{\sqrt{b^2}\,\left(\frac{56\,\sqrt{2}\,e^{2\,a}\,e^{2\,b\,x}}{b}+\frac{8\,\sqrt{2}\,e^{6\,a}\,e^{6\,b\,x}}{b}\right)}{32}\right)\right)}{2\,\sqrt{b^2}}$$

```
[In] int((cosh(a + b*x)^4 - sinh(a + b*x)^4)/(cosh(a + b*x)^4 + sinh(a + b*x)^4)
,x)
[Out] (2^(1/2)*(atan((2^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(4*b)) + atan(((b^
2)^(1/2)*((56*2^(1/2)*exp(2*a)*exp(2*b*x))/b + (8*2^(1/2)*exp(6*a)*exp(6*b*
x))/b))/32)))/(2*(b^2)^(1/2))
```

3.1053 $\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 47

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = -\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}-\frac{1}{3b(1+\tanh(a+bx))}$$

```
[Out] -4/9*arctan(1/3*(1-2*tanh(b*x+a))*3^(1/2))/b*3^(1/2)-1/3/b/(1+tanh(b*x+a))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2099, 632, 210}

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = -\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}-\frac{1}{3b(\tanh(a+bx)+1)}$$

```
[In] Int[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b*x]^3)
,x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) - 1/(3*b*(1 + Tanh
[a + b*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x+x^2}{1+x+x^3+x^4}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)^2}+\frac{2}{3(1-x+x^2)}\right)\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= -\frac{1}{3b(1+\tanh(a+bx))}+\frac{2\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(a+bx)\right)}{3b}$$
$$= -\frac{1}{3b(1+\tanh(a+bx))}-\frac{4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(a+bx)\right)}{3b}$$
$$= -\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}-\frac{1}{3b(1+\tanh(a+bx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. $2(47) = 94$.

Time = 6.94 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx$$
$$= \frac{(-\cosh(a+bx)+\sinh(a+bx))\left(\left(3+8\sqrt{3}\arctan\left(\frac{\text{sech}(bx)(\cosh(2a+bx)-2\sinh(2a+bx))}{\sqrt{3}}\right)\right)\cosh(a+bx)+\left(-3\right.\right.}{18b}$$

```
[In] Integrate[(Cosh[a + b*x]^3 - Sinh[a + b*x]^3)/(Cosh[a + b*x]^3 + Sinh[a + b
*x]^3),x]
```

```
[Out] ((-Cosh[a + b*x] + Sinh[a + b*x])*((3 + 8*Sqrt[3]*ArcTan[(Sech[b*x]*(Cosh[2
*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Cosh[a + b*x] + (-3 + 8*Sqrt[3]*A
rcTan[(Sech[b*x]*(Cosh[2*a + b*x] - 2*Sinh[2*a + b*x]))/Sqrt[3]])*Sinh[a +
b*x]))/(18*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{e^{-2bx-2a}}{6b}+\frac{2i\sqrt{3}\,\ln\left(e^{2bx+2a}+i\sqrt{3}\right)}{9b}-\frac{2i\sqrt{3}\,\ln\left(e^{2bx+2a}-i\sqrt{3}\right)}{9b}$
derivativedivides	$\frac{\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2}{3\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\cdot\right.}}{b}$
default	$\frac{\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2}{3\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\cdot\right.}}{b}$

```
[In] int((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/6*exp(-2*b*x-2*a)/b+2/9*I*3^(1/2)/b*ln(exp(2*b*x+2*a)+I*3^(1/2))-2/9*I*3
^(1/2)/b*ln(exp(2*b*x+2*a)-I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.70

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx =$$
$$-\frac{8\left(\sqrt{3}\cosh\left(bx+a\right)^{2}+2\sqrt{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{3}\sinh\left(bx+a\right)^{2}\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}}{3\left(\cosh(bx+a)-\sin\right.}\right.}{18\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="fricas")
```

```
[Out] -1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh
(b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b*c
osh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.30

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \begin{cases} -x & \text{for } (a = \\ \frac{x(-\sinh^3(a)+\cosh^3(a))}{\sinh^3(a)+\cosh^3(a)} & \text{for } b = \\ \frac{4\sqrt{3}\sinh(a+bx)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(a+bx)}{3\cosh(a+bx)}-\frac{\sqrt{3}}{3}\right)}{9b\sinh(a+bx)+9b\cosh(a+bx)} + \frac{3\sinh(a+bx)}{9b\sinh(a+bx)+9b\cosh(a+bx)} + \frac{4\sqrt{3}\cosh(a+bx)\,\mathrm{atan}\left(\frac{2\sqrt{3}\sinh(a+bx)}{3\cosh(a+bx)}-\frac{\sqrt{3}}{3}\right)}{9b\sinh(a+bx)+9b\cosh(a+bx)} & \text{otherw} \end{cases}$$

```
[In] integrate((cosh(b*x+a)**3-sinh(b*x+a)**3)/(cosh(b*x+a)**3+sinh(b*x+a)**3),x
)
```

```
[Out] Piecewise((-x, (Eq(b, 0) | Eq(a, -b*x + I*pi/2)) & (Eq(a, I*pi/2) | Eq(a, -
b*x + I*pi/2))), (x*(-sinh(a)**3 + cosh(a)**3)/(sinh(a)**3 + cosh(a)**3), E
q(b, 0)), (4*sqrt(3)*sinh(a + b*x)*atan(2*sqrt(3)*sinh(a + b*x)/(3*cosh(a +
 b*x)) - sqrt(3)/3)/(9*b*sinh(a + b*x) + 9*b*cosh(a + b*x)) + 3*sinh(a + b*
x)/(9*b*sinh(a + b*x) + 9*b*cosh(a + b*x)) + 4*sqrt(3)*cosh(a + b*x)*atan(2
*sqrt(3)*sinh(a + b*x)/(3*cosh(a + b*x)) - sqrt(3)/3)/(9*b*sinh(a + b*x) +
9*b*cosh(a + b*x)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(40) = 80.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9\,b} - \frac{e^{(-2\,bx-2\,a)}}{6\,b}$$

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="maxima")
```

```
[Out] 4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*s
qrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3^(
1/4)*sqrt(2))))/b - 1/6*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2\,bx+2\,a)}\right)-3\,e^{(-2\,bx-2\,a)}}{18\,b}$$

```
[In] integrate((cosh(b*x+a)^3-sinh(b*x+a)^3)/(cosh(b*x+a)^3+sinh(b*x+a)^3),x, al
gorithm="giac")
```

```
[Out] 1/18*(8*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a)) - 3*e^(-2*b*x - 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\cosh^3(a+bx)-\sinh^3(a+bx)}{\cosh^3(a+bx)+\sinh^3(a+bx)}\,dx = \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{b^2}}{3\,b}\right)}{9\,\sqrt{b^2}}-\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{6\,b}$$

```
[In] int((cosh(a + b*x)^3 - sinh(a + b*x)^3)/(cosh(a + b*x)^3 + sinh(a + b*x)^3)
,x)
```

```
[Out] (4*3^(1/2)*atan((3^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(3*b)))/(9*(b^2)^
(1/2)) - exp(- 2*a - 2*b*x)/(6*b)
```

3.1054 $\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)} \, dx$

Optimal result

Integrand size = 39, antiderivative size = 11

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)} \, dx = \frac{\arctan(\tanh(a+bx))}{b}$$

```
[Out] arctan(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {4465, 209}

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)} \, dx = \frac{\arctan(\tanh(a+bx))}{b}$$

```
[In] Int[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b*x]^2)
,x]
```

```
[Out] ArcTan[Tanh[a + b*x]]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
```

```
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\arctan(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \frac{\arctan(\sinh(2a+2bx))}{2b}$$

```
[In] Integrate[(Cosh[a + b*x]^2 - Sinh[a + b*x]^2)/(Cosh[a + b*x]^2 + Sinh[a + b
*x]^2),x]
[Out] ArcTan[Sinh[2*a + 2*b*x]]/(2*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

method	result	size
parallelrisch	$-\frac{i(\ln(\tanh(bx+a)-i)-\ln(\tanh(bx+a)+i))}{2b}$	30
risch	$\frac{i\ln(\mathrm{e}^{2bx+2a}+i)}{2b}-\frac{i\ln(\mathrm{e}^{2bx+2a}-i)}{2b}$	40
derivativedivides	$\frac{-\frac{(-2+\sqrt{2})\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}-\frac{\sqrt{2}(2+\sqrt{2})\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}}{b}$	86
default	$\frac{-\frac{(-2+\sqrt{2})\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}-\frac{\sqrt{2}(2+\sqrt{2})\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}}{b}$	86

```
[In] int((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x,method=_R
ETURNVERBOSE)
[Out] -1/2*I*(ln(tanh(b*x+a)-I)-ln(tanh(b*x+a)+I))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(11) = 22$.

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.45

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = -\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="fricas")
```

```
[Out] -arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.82

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \begin{cases} -x & \text{for } a = \frac{i\pi}{2} \wedge b = 0 \\ \frac{x(-\sinh^2(a)+\cosh^2(a))}{\sinh^2(a)+\cosh^2(a)} & \text{for } b = 0 \\ -x & \text{for } a = -bx + \frac{i\pi}{2} \\ \frac{\operatorname{atan}\left(\frac{\sinh(a+bx)}{\cosh(a+bx)}\right)}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate((cosh(b*x+a)**2-sinh(b*x+a)**2)/(cosh(b*x+a)**2+sinh(b*x+a)**2),x
)
```

```
[Out] Piecewise((-x, Eq(b, 0) & Eq(a, I*pi/2)), (x*(-sinh(a)**2 + cosh(a)**2)/(si
nh(a)**2 + cosh(a)**2), Eq(b, 0)), (-x, Eq(a, -b*x + I*pi/2)), (atan(sinh(a
 + b*x)/cosh(a + b*x))/b, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.45

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-bx-a)}\right)\right)}{b}$$

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="maxima")
```

```
[Out] (arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*e^(-b*x - a))))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.00

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = -\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bx+a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bx+a)}\right)\right)}{b}$$

```
[In] integrate((cosh(b*x+a)^2-sinh(b*x+a)^2)/(cosh(b*x+a)^2+sinh(b*x+a)^2),x, al
gorithm="giac")
```

```
[Out] -(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*x + a))) - arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*e^(b*x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\cosh^2(a+bx)-\sinh^2(a+bx)}{\cosh^2(a+bx)+\sinh^2(a+bx)}\,dx = \frac{\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

```
[In] int((cosh(a + b*x)^2 - sinh(a + b*x)^2)/(cosh(a + b*x)^2 + sinh(a + b*x)^2)
,x)
```

```
[Out] atan((exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/b)/(b^2)^(1/2)
```

3.1055 $\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 22

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{1}{2b(\cosh(a+bx)+\sinh(a+bx))^2}$$

```
[Out] -1/2/b/(cosh(b*x+a)+sinh(b*x+a))^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {4470}

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{1}{2b(\sinh(a+bx)+\cosh(a+bx))^2}$$

```
[In] Int[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x]
```

```
[Out] -1/2*1/(b*(Cosh[a + b*x] + Sinh[a + b*x])^2)
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]
```

Rubi steps

$$\text{integral} = -\frac{1}{2b(\cosh(a+bx)+\sinh(a+bx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{\cosh(2a)\cosh(2bx)}{2b}+\frac{\cosh(2bx)\sinh(2a)}{2b}+\frac{\cosh(2a)\sinh(2bx)}{2b}-\frac{\sinh(2a)\sinh(2bx)}{2b}$$

```
[In] Integrate[(Cosh[a + b*x] - Sinh[a + b*x])/(Cosh[a + b*x] + Sinh[a + b*x]),x
]
```

```
[Out] -1/2*(Cosh[2*a]*Cosh[2*b*x])/b + (Cosh[2*b*x]*Sinh[2*a])/(2*b) + (Cosh[2*a]
*Sinh[2*b*x])/(2*b) - (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{{\mathrm e}^{-2 b x -2 a}}{2 b}$	15
parallelrisch	$\frac{\tanh(bx+a)}{b(1+\tanh(bx+a))}$	21
gosper	$\frac{\sinh(bx+a)-\cosh(bx+a)}{2b(\cosh(bx+a)+\sinh(bx+a))}$	36
derivativedivides	$\frac{\frac{2}{\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1}-\frac{2}{\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)^{2}}}{b}$	36
default	$\frac{\frac{2}{\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1}-\frac{2}{\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)^{2}}}{b}$	36

```
[In] int((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*exp(-2*b*x-2*a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{1}{2\left(b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2\right)}$$

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"fricas")
[Out] -1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)
^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = \begin{cases} \frac{\sinh\left(a+bx\right)}{b\sinh\left(a+bx\right)+b\cosh\left(a+bx\right)} & \text{for } b\neq 0 \\ \frac{x\left(-\sinh\left(a\right)+\cosh\left(a\right)\right)}{\sinh\left(a\right)+\cosh\left(a\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x)
[Out] Piecewise((sinh(a + b*x)/(b*sinh(a + b*x) + b*cosh(a + b*x)), Ne(b, 0)), (x
*(-sinh(a) + cosh(a))/(sinh(a) + cosh(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{e^{\left(-2\,bx-2\,a\right)}}{2\,b}$$

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"maxima")
[Out] -1/2*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

```
[In] integrate((cosh(b*x+a)-sinh(b*x+a))/(cosh(b*x+a)+sinh(b*x+a)),x, algorithm=
"giac")
```

```
[Out] -1/2*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\cosh(a+bx)-\sinh(a+bx)}{\cosh(a+bx)+\sinh(a+bx)}\,dx = -\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{2\,b}$$

```
[In] int((cosh(a + b*x) - sinh(a + b*x))/(cosh(a + b*x) + sinh(a + b*x)),x)
```

```
[Out] -exp(- 2*a - 2*b*x)/(2*b)
```

3.1056 $\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 14

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx=\frac{1}{b(1+\tanh(a+bx))}$$

```
[Out] 1/b/(1+tanh(b*x+a))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {32}

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx=\frac{1}{b(\tanh(a+bx)+1)}$$

```
[In] Int[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),x]
[Out] 1/(b*(1 + Tanh[a + b*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx,x,\tanh(a+bx)\right)}{b}\\&=\frac{1}{b(1+\tanh(a+bx))}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{\cosh(2a)\cosh(2bx)}{2b} - \frac{\cosh(2bx)\sinh(2a)}{2b} - \frac{\cosh(2a)\sinh(2bx)}{2b} + \frac{\sinh(2a)\sinh(2bx)}{2b}$$

```
[In] Integrate[(-Csch[a + b*x] + Sech[a + b*x])/(Csch[a + b*x] + Sech[a + b*x]),
x]
```

```
[Out] (Cosh[2*a]*Cosh[2*b*x])/(2*b) - (Cosh[2*b*x]*Sinh[2*a])/(2*b) - (Cosh[2*a]*
Sinh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{e^{-2bx-2a}}{2b}$	15
parallelrisch	$-\frac{\tanh(bx+a)}{b(1+\tanh(bx+a))}$	22
derivativedivides	$\frac{\frac{2}{\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)^{2}}-\frac{2}{\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1}}{b}$	36
default	$\frac{\frac{2}{\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)^{2}}-\frac{2}{\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1}}{b}$	36

```
[In] int((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2*exp(-2*b*x-2*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{1}{2\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="fricas")
```

```
[Out] 1/2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^
2)
```

Sympy [F]

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = -\int \frac{\operatorname{csch}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\right)dx$$

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x)
```

```
[Out] -Integral(csch(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x) - Integral(-sec
h(a + b*x)/(csch(a + b*x) + sech(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="maxima")
```

```
[Out] 1/2*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx = \frac{e^{(-2\,bx-2\,a)}}{2\,b}$$

```
[In] integrate((-csch(b*x+a)+sech(b*x+a))/(csch(b*x+a)+sech(b*x+a)),x, algorithm
="giac")
```

```
[Out] 1/2*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}{\operatorname{csch}(a+bx)+\operatorname{sech}(a+bx)}\,dx=\frac{e^{-2a-2bx}}{2b}$$

```
[In] int((1/cosh(a + b*x) - 1/sinh(a + b*x))/(1/cosh(a + b*x) + 1/sinh(a + b*x))
,x)
```

```
[Out] exp(- 2*a - 2*b*x)/(2*b)
```

3.1057 $\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 12

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx=-\frac{\arctan(\tanh(a+bx))}{b}$$

```
[Out] -arctan(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {210}

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx=-\frac{\arctan(\tanh(a+bx))}{b}$$

```
[In] Int[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a + b*x]^2
),x]
```

```
[Out] -(ArcTan[Tanh[a + b*x]]/b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\tanh(a+bx)\right)}{b}\\&=-\frac{\arctan(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx=-\frac{\arctan(\sinh(2a+2bx))}{2b}$$

[In] Integrate[(-Csch[a + b*x]^2 + Sech[a + b*x]^2)/(Csch[a + b*x]^2 + Sech[a + b*x]^2),x]

[Out] -1/2*ArcTan[Sinh[2*a + 2*b*x]]/b

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

method	result	size
parallelrisch	$\frac{i(\ln(\tanh(bx+a)-i)-\ln(\tanh(bx+a)+i))}{2b}$	30
risch	$\frac{i\ln(\mathrm{e}^{2bx+2a}-i)}{2b}-\frac{i\ln(\mathrm{e}^{2bx+2a}+i)}{2b}$	40
derivativedivides	$\frac{\frac{\sqrt{2}\,(2+\sqrt{2})\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{(-2+\sqrt{2})\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}}{b}$	84
default	$\frac{\frac{\sqrt{2}\,(2+\sqrt{2})\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{(-2+\sqrt{2})\sqrt{2}\arctan\left(\frac{2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}}{b}$	84

[In] int((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] 1/2*I*(ln(tanh(b*x+a)-I)-ln(tanh(b*x+a)+I))/b

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx=\frac{\arctan\left(-\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, algorithm="fricas")

[Out] arctan(-(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a) - sinh(b*x + a)))/b

Sympy [F]

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = -\int \frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\right)dx$$

```
[In] integrate((-csch(b*x+a)**2+sech(b*x+a)**2)/(csch(b*x+a)**2+sech(b*x+a)**2),
x)
```

```
[Out] -Integral(csch(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x) - Inte
gral(-sech(a + b*x)**2/(csch(a + b*x)**2 + sech(a + b*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.17

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = -\frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(-bx-a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(-bx-a)}\right)\right)}{b}$$

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="maxima")
```

```
[Out] -(arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(-b*x - a))) - arctan(-1/2*sqrt(2)*(sqr
t(2) - 2*e^(-b*x - a))))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx = \frac{\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bx+a)}\right)\right)-\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bx+a)}\right)\right)}{b}$$

```
[In] integrate((-csch(b*x+a)^2+sech(b*x+a)^2)/(csch(b*x+a)^2+sech(b*x+a)^2),x, a
lgorithm="giac")
```

```
[Out] (arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*x + a))) - arctan(-1/2*sqrt(2)*(sqrt(
2) - 2*e^(b*x + a))))/b
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{-\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}{\operatorname{csch}^2(a+bx)+\operatorname{sech}^2(a+bx)}\,dx=-\frac{\operatorname{atan}\left(\frac{e^{2a}\,e^{2bx}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

```
[In] int((1/cosh(a + b*x)^2 - 1/sinh(a + b*x)^2)/(1/cosh(a + b*x)^2 + 1/sinh(a +
 b*x)^2),x)
[Out] -atan((exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/b)/(b^2)^(1/2)
```

3.1058 $\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 47

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx=\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}+\frac{1}{3b(1+\tanh(a+bx))}$$

```
[Out] 4/9*arctan(1/3*(1-2*tanh(b*x+a))*3^(1/2))/b*3^(1/2)+1/3/b/(1+tanh(b*x+a))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2099, 632, 210}

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx=\frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b}+\frac{1}{3b(\tanh(a+bx)+1)}$$

```
[In] Int[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a + b*x]^3
),x]
```

```
[Out] (4*ArcTan[(1 - 2*Tanh[a + b*x])/Sqrt[3]])/(3*Sqrt[3]*b) + 1/(3*b*(1 + Tanh[
a + b*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1-x-x^2}{1+x+x^3+x^4}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)^2} - \frac{2}{3(1-x+x^2)}\right)\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{1}{3b(1+\tanh(a+bx))} - \frac{2\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(a+bx)\right)}{3b} \\
&= \frac{1}{3b(1+\tanh(a+bx))} + \frac{4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(a+bx)\right)}{3b} \\
&= \frac{4\arctan\left(\frac{1-2\tanh(a+bx)}{\sqrt{3}}\right)}{3\sqrt{3}b} + \frac{1}{3b(1+\tanh(a+bx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$
\begin{aligned}
&\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx \\
&= \frac{-8\sqrt{3}\arctan\left(\frac{-1+2\tanh(a+bx)}{\sqrt{3}}\right)+3\cosh(2(a+bx))-3\sinh(2(a+bx))}{18b}
\end{aligned}
$$

```
[In] Integrate[(-Csch[a + b*x]^3 + Sech[a + b*x]^3)/(Csch[a + b*x]^3 + Sech[a +
b*x]^3),x]
```

```
[Out] (-8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[a + b*x])/Sqrt[3]] + 3*Cosh[2*(a + b*x)] -
3*Sinh[2*(a + b*x)])/(18*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

method	result
risch	$\frac{\mathrm{e}^{-2bx-2a}}{6b}+\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{2bx+2a}-i\sqrt{3}\right)}{9b}-\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{2bx+2a}+i\sqrt{3}\right)}{9b}$
derivativedivides	$\frac{\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}+\frac{2}{3\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right.}}{b}$
default	$\frac{\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}-\frac{2i\sqrt{3}\,\ln\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+\left(-i\sqrt{3}-1\right)\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{9}+\frac{2}{3\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right.}}{b}$

```
[In] int((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/6*exp(-2*b*x-2*a)/b+2/9*I*3^(1/2)/b*ln(exp(2*b*x+2*a)-I*3^(1/2))-2/9*I*3^
(1/2)/b*ln(exp(2*b*x+2*a)+I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(40) = 80.

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.70

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = \frac{8\left(\sqrt{3}\cosh\left(bx+a\right)^{2}+2\sqrt{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{3}\sinh\left(bx+a\right)^{2}\right)\arctan\left(-\frac{\sqrt{3}\cosh(bx+a)+\sqrt{3}\sinh(b}{3\left(\cosh(bx+a)-\sinh(bx-\right.}\right.}{18\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="fricas")
```

```
[Out] 1/18*(8*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2)*arctan(-1/3*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(
b*x + a))/(cosh(b*x + a) - sinh(b*x + a))) + 3)/(b*cosh(b*x + a)^2 + 2*b*co
sh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = -\int \frac{\operatorname{csch}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\right)dx$$

```
[In] integrate((-csch(b*x+a)**3+sech(b*x+a)**3)/(csch(b*x+a)**3+sech(b*x+a)**3),
x)
[Out] -Integral(csch(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x) - Inte
gral(-sech(a + b*x)**3/(csch(a + b*x)**3 + sech(a + b*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = -\frac{4\left(\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-bx-a)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)\right)}{9\,b} + \frac{e^{(-2\,bx-2\,a)}}{6\,b}$$

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="maxima")
[Out] -4/9*(sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) + 3^(1/4)*
sqrt(2))) - sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-b*x - a) - 3^
(1/4)*sqrt(2))))/b + 1/6*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx = -\frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2\,bx+2\,a)}\right)-3\,e^{(-2\,bx-2\,a)}}{18\,b}$$

```
[In] integrate((-csch(b*x+a)^3+sech(b*x+a)^3)/(csch(b*x+a)^3+sech(b*x+a)^3),x, a
lgorithm="giac")
[Out] -1/18*(8*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*b*x + 2*a)) - 3*e^(-2*b*x - 2*a))/
b
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{-\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}{\operatorname{csch}^3(a+bx)+\operatorname{sech}^3(a+bx)}\,dx=\frac{e^{-2\,a-2\,b\,x}}{6\,b}-\frac{4\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,e^{2\,a}\,e^{2\,b\,x}\,\sqrt{b^2}}{3\,b}\right)}{9\,\sqrt{b^2}}$$

```
[In] int((1/cosh(a + b*x)^3 - 1/sinh(a + b*x)^3)/(1/cosh(a + b*x)^3 + 1/sinh(a +
 b*x)^3),x)
```

```
[Out] exp(- 2*a - 2*b*x)/(6*b) - (4*3^(1/2)*atan((3^(1/2)*exp(2*a)*exp(2*b*x)*(b^
2)^(1/2))/(3*b)))/(9*(b^2)^(1/2))
```

3.1059 $\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 51

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = \frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*tanh(b*x+a))/b*2^(1/2)-1/2*arctan(1+2^(1/2)*tanh(b*x
+a))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1176, 631, 210}

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = \frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} - \frac{\arctan\left(\sqrt{2}\tanh(a+bx)+1\right)}{\sqrt{2}b}$$

```
[In] Int[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a + b*x]^4
),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Tanh[a + b*x]]/(Sqrt[2]*b) - ArcTan[1 + Sqrt[2]*Tanh[a +
 b*x]]/(Sqrt[2]*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1-x^2}{1+x^4}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \tanh(a+bx)\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}\\
&= \frac{\arctan\left(1-\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\sqrt{2}\tanh(a+bx)\right)}{\sqrt{2}b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$
\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sinh(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}
$$

```
[In] Integrate[(-Csch[a + b*x]^4 + Sech[a + b*x]^4)/(Csch[a + b*x]^4 + Sech[a +
b*x]^4),x]
```

```
[Out] -(ArcTan[Sinh[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

method	result
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4bx+4a}-2i\sqrt{2}\,\mathrm{e}^{2bx+2a}-1\right)}{4b}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4bx+4a}+2i\sqrt{2}\,\mathrm{e}^{2bx+2a}-1\right)}{4b}$
derivativedivides	$\frac{\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}+2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\right.\right.}{}}{b}$
default	$\frac{\frac{i\sqrt{2}\,\ln\left(2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}+2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)-2\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}+1\right)}{4}-\frac{i\sqrt{2}\,\ln\left(-2i\sqrt{2}\,\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}+\tanh\left(\right.\right.}{}}{b}$

```
[In] int((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/4*I*2^(1/2)/b*ln(exp(4*b*x+4*a)-2*I*2^(1/2)*exp(2*b*x+2*a)-1)-1/4*I*2^(1/
2)/b*ln(exp(4*b*x+4*a)+2*I*2^(1/2)*exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.76

$$\int\frac{-\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}{\text{csch}^4(a+bx)+\text{sech}^4(a+bx)}\,dx$$
$$=\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3+\left(3\sqrt{2}\cosh(bx+a)^2-7\sqrt{2}\right)\sinh(bx+a)+7\sqrt{2}\cosh(bx+a}{4\left(\cosh(bx+a)^3-3\cosh(bx+a)^2\sinh(bx+a)+3\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3\right)}\right.}{2b}$$

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)
*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3 + (3*sqrt(2)*cosh(b*x + a)^2 - 7
*sqrt(2))*sinh(b*x + a) + 7*sqrt(2)*cosh(b*x + a))/(cosh(b*x + a)^3 - 3*cos
h(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a
)^3)) + sqrt(2)*arctan(-1/4*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))
/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx - \int \left(-\frac{\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\right)dx$$

```
[In] integrate((-csch(b*x+a)**4+sech(b*x+a)**4)/(csch(b*x+a)**4+sech(b*x+a)**4),
x)
```

```
[Out] -Integral(csch(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x) - Inte
gral(-sech(a + b*x)**4/(csch(a + b*x)**4 + sech(a + b*x)**4), x)
```

Maxima [F]

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = \int -\frac{\operatorname{csch}(bx+a)^4-\operatorname{sech}(bx+a)^4}{\operatorname{csch}(bx+a)^4+\operatorname{sech}(bx+a)^4}\,dx$$

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="maxima")
```

```
[Out] -2*integrate((e^(-b*x - a) + e^(-5*b*x - 5*a))*e^(-b*x - a)/(6*e^(-4*b*x -
4*a) + e^(-8*b*x - 8*a) + 1), x) - 2*integrate((e^(-4*b*x - 4*a) + 1)*e^(-2
*b*x - 2*a)/(6*e^(-4*b*x - 4*a) + e^(-8*b*x - 8*a) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(e^{(4bx+4a)}-1\right)e^{(-2bx-2a)}\right)}{2b}$$

```
[In] integrate((-csch(b*x+a)^4+sech(b*x+a)^4)/(csch(b*x+a)^4+sech(b*x+a)^4),x, a
lgorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*sqrt(2)*(e^(4*b*x + 4*a) - 1)*e^(-2*b*x - 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{-\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}{\operatorname{csch}^4(a+bx)+\operatorname{sech}^4(a+bx)}\,dx = -\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,e^{2a}\,e^{2bx}\,\sqrt{b^2}}{4b}\right)+\operatorname{atan}\left(\frac{\sqrt{b^2}\,\left(\frac{56\,\sqrt{2}\,e^{2a}\,e^{2bx}}{b}+\frac{8\,\sqrt{2}\,e^{6a}\,e^{6bx}}{b}\right)}{32}\right)\right)}{2\,\sqrt{b^2}}$$

```
[In] int((1/cosh(a + b*x)^4 - 1/sinh(a + b*x)^4)/(1/cosh(a + b*x)^4 + 1/sinh(a +
 b*x)^4),x)
```

```
[Out] -(2^(1/2)*(atan((2^(1/2)*exp(2*a)*exp(2*b*x)*(b^2)^(1/2))/(4*b)) + atan(((b
^2)^(1/2)*((56*2^(1/2)*exp(2*a)*exp(2*b*x))/b + (8*2^(1/2)*exp(6*a)*exp(6*b
*x))/b))/32)))/(2*(b^2)^(1/2))
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).___name___ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```